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# Controlling Inrush Currents in Inductively Coupled Power Systems

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# Controlling Inrush Currents in Inductively Coupled Power Systems

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*Abstract*— In an Inductively Coupled Power Transfer (ICPT) system, a multiplicity of (moving) loads take power from an elongated conductive loop (track) excited by a current in the range of 15-125A by magnetic induction at a VLF frequency of between 5-50kHz. In this application, the track performs the same function as a distribution line in a power system. However, frequency deviations cannot be tolerated in ICPT systems and therefore there are difficulties with inrush power surges as loads switch on. In severe cases, the inrush surge may compromise the security of the whole system.

This paper proposes a solution to this problem using an ICPT pickup controller with input shaping where the poles that can cause an inrush are not excited. The paper is supported by theoretical analysis and experimental measurements and is applicable across a wide range of ICPT sizes and applications. The solution reduces the inrush effects to 10% of an uncontrolled system.

*Index Terms*—inductively coupled power transfer (ICPT), input shaping, inrush current, power electronics, power system transients, rate limiting.

#### I. INTRODUCTION

An inductively coupled power transfer (ICPT) system is composed of a primary power supply and one or more secondary power pickups [1] as shown in Fig. 1. As opposed to a traditional power system in which the supply is an AC voltage source, the ICPT power supply may be regarded as an AC current source typically in the order of 15-125A at a frequency in the range of 5-50kHz. The current is typically fed through a track that may vary in length from 1-100m.

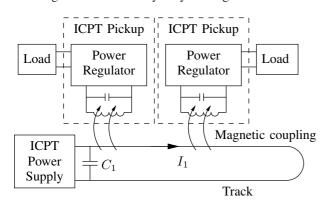


Fig. 1. A typical ICPT system.

Similar to a traditional power system, the ICPT power supply also requires compensation. Both series and parallel compensation are possible, with parallel compensation represented in Fig. 1 by the capacitor  $C_1$ . The energy stored within the compensating components is small and as such, when compared to traditional power systems, there is effectively no spinning reserve.

The authors are with the Department of Electrical and Computer Engineering, School of Engineering, The University of Auckland, Auckland, 1001, New Zealand. E-mail: j.boys@auckland.ac.nz Multiple ICPT pickups may be placed along the track and used to supply power to independent loads. The power is drawn from the track through magnetic coupling across an air gap and as such is dependent upon the amount of current in the track. The pickup appears as a loosely coupled transformer to the power supply [2].

Under steady state conditions, a simple ICPT pickup may be in one of two operational state that describe whether or not the pickup is drawing power from the power supply. An "on" state denotes a pickup which is drawing power from the power supply. Conversely, an "off" state denotes a pickup which has been decoupled from the power supply.

When a pickup switches on, there is a transient inrush of power to the pickup. As noted, the power supply has effectively no spinning reserve so that the inrush of power to the pickup transiently depletes the small amount of energy in the resonant elements of the track. This has the effect of transiently reducing the track current. This in turn adversely affects other pickups on the same track as they can no longer draw the same amount of power during the disturbance. It is therefore desirable to minimise the effects of the inrush of power to the pickup during startup.

This paper proposes a method of controlling the inrush power of the pickup at turn on. A DC equivalent model of the pickup [3] is utilised that describes the action of the limiting controller in conjunction with a shaping function to investigate and select an appropriate rate limit. The design is then verified experimentally.

## II. TRANSIENT BEHAVIOUR OF AN ICPT PICKUP

A standard parallel tuned ICPT pickup circuit is shown in Fig. 2. The circuit consists of an inductor  $L_t$  which is coupled to the track with a mutual coupling M. A current  $I_1$ flows through the primary track. The coupling of the inductor  $L_t$  to the primary track effectively acts as an AC voltage source referred to as  $V_{OC}$ . Capacitor  $C_t$  is used to tune the inductor. The inductor  $L_t$  and capacitor  $C_t$  together are typically referred to as a resonant tank. The AC waveform across the resonant tank is then rectified and regulated to provide a regulated DC output across a load  $R_L$ .

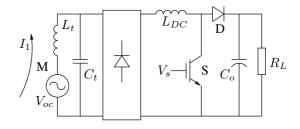


Fig. 2. The schematic for a parallel tuned ICPT pickup circuit.

There are two steady state modes of operation for the pickup [4]:

- 1) The pickup is off. In this state, the switch S is conducting. This presents an effective short circuit across the resonant tank and decouples the pickup from the power supply. No power is delivered from the power supply to the resonant tank or the load in this case.
- 2) The pickup is on. In this state, the switch S is nonconducting. Power is delivered from the track to the resonant tank and subsequently delivered to the load. The amount of power delivered to the load is proportional to the product of the current in the track and the resonant tank voltage.

A typical controller for a pickup will switch between these two modes of operation so that the average amount of power drawn from the track is equal to the amount of power delivered to the load. Switching may be performed at frequencies in the order of 1-30kHz (fast switching) or at low frequency 10-100Hz (slow switching). Each method of switching provides advantages and disadvantages.

This paper is concerned with the transient response as a result of the pickup switching on (switch turning off). During this transition, significant amounts of transient overshoot in the resonant tank voltage is typical in a slow switching ICPT pickup as shown in Fig. 3.

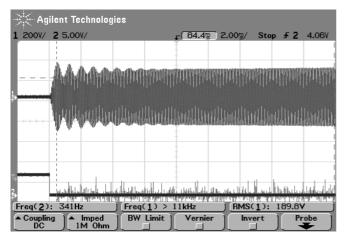


Fig. 3. The top trace is the resonant tank voltage across an ICPT pickup. The bottom trace is the drive to the switch. The resonant voltage is seen to overshoot when the switch is turned off.

When the switch is turned off, the pickup transiently draws more power than during steady state from the power supply due to the overshoot. If the energy stored in the resonant elements of the power supply is insufficient to supply the transient demands of the pickup, the track current will temporarily drop as shown in Fig. 4. In a multiple pickup situation, the collapse of the track current may cause a temporary loss of power to other pickups and consequently to their loads. This is an undesirable situation.

To prevent temporary loss of power as a result of transient overshoots, the ICPT power supply must be rated at the amount of power required transiently as opposed to the lower steady state power. Components on the pickup are also required to be rated for the transient overshoot. As such, the cost of an ICPT system is driven higher as a result of the transient overshoot.

Transient undershoots do not pose the same issue. Due to the removal of load the track current may be observed to increase as shown in Fig. 5. This transiently increases the

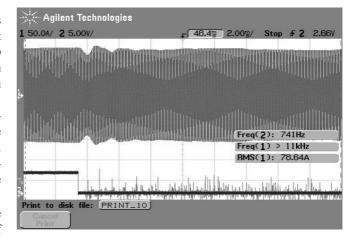


Fig. 4. The top trace is the track current as supplied by an ICPT power supply. The bottom trace is the drive to the switch. The track current is seen to temporarily collapse when the switch is turned off.

amount of available power to other pickups. In addition, the rectifier bridge across the resonant tank acts to clamp any undershoot.

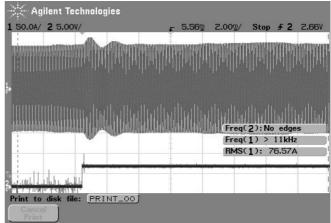


Fig. 5. The top trace is the track current as supplied by an ICPT power supply. The bottom trace is the drive to the switch. The track current is seen to temporarily increase when the switch is turned on.

It should be noted that near 100% voltage overshoots are to be expected in high power ICPT systems. The system shown in Figs. 3 and 4 was restricted to a 30% voltage overshoot as the track current collapsed by 15% at the same time.

#### III. ICPT PICKUP MODEL

The circuit of Fig. 2 is extremely difficult to analyse and contains four poles so simple expressions for damping factors and overshoot are not attainable. However, for control purposes, a DC equivalent circuit may be constructed to describe the envelope of the AC voltages within the ICPT pickup circuit [3]. The DC equivalent is also desirable as it eliminates the non-linear diodes and the resonant components [3]. The DC equivalent circuit is shown in Fig. 6. A further simplification used here is the replacement of the output capacitor  $C_o$  and load  $R_L$  with an equivalent DC supply (sink)  $V_{DC}$ . This approximation is made as the output voltage is nominally held constant on the (large) output capacitor by the controller. The equivalent circuit allows an expression describing the relationship between resonant tank voltage envelope  $V_r$  and switching duty cycle d to be determined using state space averaging techniques [3], [5], as shown in (1).

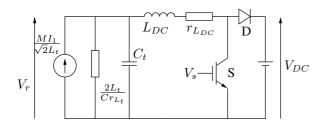


Fig. 6. A DC equivalent circuit used for analysis.

$$\frac{V_r(s)}{d(s)} = \frac{-\frac{V_{DC}}{L_{DC}C_t}}{s^2 + s\left(\frac{r_{L_t}}{2L_t} + \frac{r_{L_{DC}}}{L_{DC}}\right) + \frac{2L_t + r_{L_{DC}}r_{L_t}C_t}{2L_{DC}L_tC_t}} \quad (1)$$

From (1), it can be seen that both the winding resistance in the DC inductor,  $r_{L_{DC}}$ , and the winding resistance in the tuning inductor,  $r_{L_t}$ , is necessary in order to provide damping for the system. In a typical system however, both values are designed to be as close to zero as possible to minimise losses. As such, typical ICPT systems are underdamped and display oscillatory behaviour across the resonant voltage tank when excited by a change in the switch duty cycle.

The poles of the system can be seen to be have a relatively small real component due to the low value of the resistances. The frequency of oscillation is predominantly determined by  $L_{DC}$  and  $C_t$ , and is approximately  $\frac{1}{\sqrt{L_{DC}C_t}}$ .

# IV. CONTROLLERS

#### A. Traditional Controllers

Traditional controllers were investigated as a possible method to control the transient response. However, from (1), it can be seen that the system contains two complex poles with a relatively small real component. The poles are therefore close to the imaginary axis. The application of any reasonable gain into the feedback path would move the complex poles into the right half plane, thus resulting in system instability.

Due to the potential of system instability from the presence of the complex poles, it was decided that a classical controller would be inappropriate.

#### B. Rate Limiting Controller

Input shaping is a well known method of control that is often applied to control mechanical systems [6], [7]. Input shaping is commonly used to avoid exciting any resonances in a given mechanical system so that the transient responses may be as short as possible [8].

In the case of the ICPT pickup, a resonance is caused by the DC inductor  $L_{DC}$  and the tuning capacitor  $C_t$ . The resonance is excited by a step change in duty cycle or any other disturbance. Here when the pickup is turned on the switch is changed suddenly from a duty cycle of one to a duty cycle of zero. The idea is that this transition from one to zero should be more gradual and shaped to best effect. Although there are many possible input shaping functions [7], a slope (rate) limiting function was chosen. The rate limiter limits the maximum rate of change for the duty cycle, thus having the effect of changing a step change in duty cycle into a more gradual ramp. It has the advantage of being relatively simple to analyse and implement.

### C. System Analysis

The response of the system may be determined by performing a convolution between a ramped duty cycle function and the transfer function. The ramped duty cycle function d(t) is defined in Fig. 7.

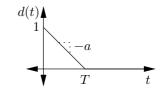


Fig. 7. A ramped duty cycle.

The ramp has a gradient of -a and reaches zero duty cycle after a time T. As such, aT = 1. For convenience, the system is treated at this stage as having a general transfer function h(t).

The ramp function d(t) may be expressed as shown in (2).

$$d(t) = 1 - a \cdot t \cdot u(t) + a \cdot t \cdot u(t - T) - u(t - T)$$
(2)

The result of the convolution is shown in (3).

$$v(t) = v(0) - a \int_{t-T}^{t} \int h(\tau) d\tau d\tau + \int h(\tau) d\tau \Big|_{\tau=0}, t \ge T$$
(3)

An examination of (3) reveals that after a time T, the time varying component of the transient is the difference between the same integral but where one is delayed by a time T. It is this time varying component that is being minimised.

If the function h(t) is periodic or nearly so with no DC component, then the integral of h(t) is also periodic. It follows therefore, that the double integral is also periodic. If the double integral of h(t) is assumed to have a periodic nature and if T is an integer multiple of the period, then the time varying expression in (3) should evaluate to zero. In this case, any transient overshoot is eliminated.

Should h(t) be an exponentially decaying sinusoid, the integral of h(t) is also predominantly an exponentially decaying sinusoid. Similarly, the double integral of h(t) is predominantly an exponentially decaying sinusoid. In this case, if T is an integer multiple n of the period, then the time varying expression in (3) should evaluate to the difference between the first period and the (n + 1)th period. In this case, the transient overshoot is reduced but not eliminated.

#### D. Application to the ICPT Pickup

In the case of an ICPT pickup, the transfer function describes the system as a lightly damped second order system. In traditional control solutions, it is normally desirable to increase the damping factor to reduce both the overshoot and the amount of time taken to reach steady state. However, in this case, a lightly damped system is an advantage that may be utilised to minimise the magnitude of the transient response.

The light damping provides an exponentially decaying sinusoid where the difference in amplitude between successive sinusoids does not vary greatly. Here, this characteristic is used to an advantage by choosing the ramp duration T as an integral number of periods of the oscillation period, thus minimising the amount of ripple in the transient response.

This can be mathematically expressed by evaluating the time varying integral expression in (3). The transfer function h(t) is set as a standard damped sinusoid as shown in (4).

$$h(t) = e^{-\sigma t} \sin\left(\omega t\right) \tag{4}$$

The integral describing the time varying component of the transient response  $v_{tr}(t)$  is then evaluated as:

$$v_{tr}(t) = a \int_{t-T}^{t} \int h(\tau) d\tau d\tau$$

$$= a \int_{t-T}^{t} \int e^{-\sigma t} \sin(\omega t) d\tau d\tau$$

$$= \frac{a}{(\sigma^{2} + \omega^{2})^{2}} \{ (\sigma^{2} - \omega^{2}) e^{-\sigma t} \sin(\omega t) + 2\sigma \omega e^{-\sigma t} \cos(\omega t) - (\sigma^{2} - \omega^{2}) e^{-\sigma t} e^{\sigma T} \sin(\omega (t - T)) - 2\sigma \omega e^{-\sigma t} e^{\sigma T} \cos(\omega (t - T)) \}$$
(5)

In the ICPT pickup, the frequency of oscillation  $\omega$  is typically of the order of 1kHz. The damping factor  $\sigma$  is of the order of 0.05. The approximation can thus be made that  $(\sigma^2 \pm \omega^2) \cong \pm \omega^2$ . Given that  $\omega^2 \gg 2\sigma\omega$ , the cosine expressions can also be considered negligible. Thus the expression for  $v_{tr}(t)$  can be approximated by that shown in (6) (where *a* is replaced with its equivalent of  $\frac{1}{T}$ ).

$$v_{tr}(t) \cong \frac{e^{-\sigma t}}{T\omega^2} \left( e^{\sigma T} \sin\left(\omega \left(t - T\right)\right) - \sin\left(\omega t\right) \right), t \ge T$$
(6)

This expression describes the time varying transient voltage across the resonant tank for  $t \ge T$  as the difference between two sinusoids, one time delayed with respect to the other. If T is expressed as a multiple of the oscillation period  $(T = n \frac{2\pi}{\omega})$  where n is a real number (though ideally integer), then the substitution of T into (6) results in:

$$v_{tr}\left(t\right) \cong \frac{e^{-\sigma t}}{n2\pi\omega} \left(e^{\sigma n\frac{2\pi}{\omega}} \sin\left(\omega\left(t-n\frac{2\pi}{\omega}\right)\right) - \sin\left(\omega t\right)\right)$$
(7)

Equation (7) is true for  $t \ge n \frac{2\pi}{\omega}$ . It can be seen that the magnitude of the time varying transient component is inversely proportional to the value of n.

Worst Case Analysis: In the ideal case, the two sinusoids in (6) are in phase and thus  $v_{tr}(t)$  would evaluate to zero if the damping factor is ignored. It follows that the worst case amplitude is when the two sinusoids have opposite phase. This condition arises if T is selected to be half the oscillation period, that is  $n = \frac{1}{2}$ . The damping factor can be eliminated to maximise the difference between the two sinusoids, resulting in (8).

$$v_{tr}\left(t\right) \cong \frac{2\sin\left(\omega t\right)}{\pi\omega} \tag{8}$$

This can be compared with an equivalent expression for a step input (9).

$$v_{tr\_step}(t) \cong \frac{\cos\left(\omega t\right)}{\omega}$$
 (9)

It can be seen that in the worst case scenario, the amplitude of the transient response is reduced by a factor of  $\frac{\pi}{2}$  through the use of a rate limited input.

# E. Practical Results

The rate limited input controller described above was implemented in an ICPT pickup operating at a track frequency of 15kHz. The resonance due to the DC inductor  $L_{DC}$  and the tuning capacitor  $C_t$  has an oscillation period of approximately 800 $\mu$ s. Three experimental studies were undertaken as described below.

The first experiment is used to determine the response of the resonant tank and the effect on the track current in response to a full step change in duty cycle in a pickup controller without the rate limiter present.

The second and third experiments also investigates the response of the resonant tank and the effect of the track current in response to a change in switch duty cycle in the presence of a rate limiting controller. The second experiment is performed with the ramp delay T set to one oscillation period of  $800\mu$ s. The third is performed with T set at  $3200\mu$ s.

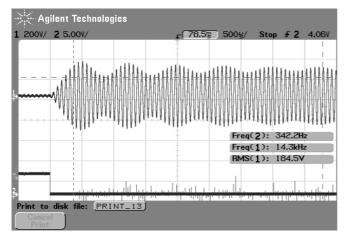
The rate limited ramp of d(t) is accomplished by switching the switch at a frequency of approximately 32kHz and 13kHz in the second and third experiment, respectively, with a varying duty cycle during the transient period.

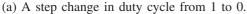
Figs. 8 and 9, (a), (b), and (c) show the measured response of the resonant tank and the track current from the first, second and third experiment, respectively. From Figs. 8 (b) and (c), it can be seen that with a rate limited change in duty cycle, the magnitude of the overshoot in the resonant tank voltages is greatly reduced. As expected, Figs. 9 (b) and (c) show that with a rate limited change in duty cycle, the impact upon the track current is lessened. In the case of the third experiment, the effect upon the track current was effectively negligible.

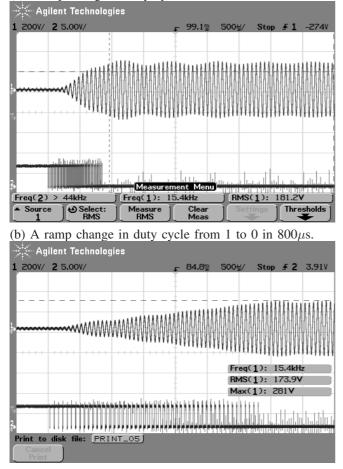
Practical Considerations: Ideally, the value of T should be equal to exactly one period of oscillation to minimise any transient response while allowing the steady state response to be reached in a minimum amount of time. However, in practice, this choice of T may not be feasible if the switching frequency used to vary the duty cycle is comparable to the oscillation frequency of the overshoot.

Should this be the case, then the approximation of a smoothly changing duty cycle no longer holds true as shown in Fig. 10. As such, the value of T should be chosen to be long enough such that the switching period is negligible when compared to T. Alternatively, the switching frequency may be increased if it is practically possible.

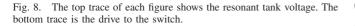
An additional advantage of selecting a larger value for T is that the magnitude of the transient voltage reduces. This

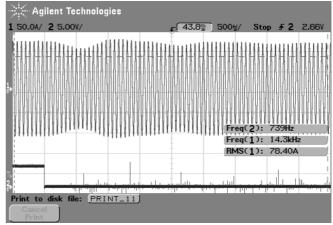




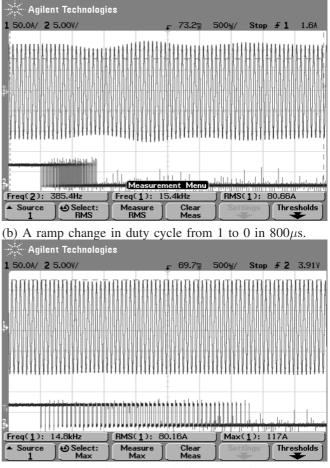


(c) A ramp change in duty cycle from 1 to 0 in  $3200\mu$ s.





(a) A step change in duty cycle from 1 to 0.



(c) A ramp change in duty cycle from 1 to 0 in  $3200\mu$ s.

Fig. 9. The top trace of each figure shows the primary track current. The bottom trace is the drive to the switch.

is because  $v_{tr}(t)$  is inversely proportional to T as shown in 6. Although T should ideally be an integer multiple of the oscillation period to minimise transient ripple, with a sufficiently large value of T, the amplitude of the transient voltage becomes negligibly small.

However, the value of T must be bounded. This is because T is now effectively the turn on and turn off period for the switch. The amount of time taken to reach steady state is therefore proportional to the value of T. As such, T should be relatively small compared to the slow switching period that is used to regulate the output voltage.

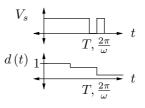


Fig. 10. Having too low a value of T compared to the switching period will invalidate the assumption that a smoothly changing duty cycle is present.

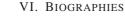
### V. CONCLUSION

In a traditional slow switched ICPT pickup, the voltage across the resonant tank circuit experiences transient overshoots as a result of a step change in the duty cycle of the power controlling switch. A controller that limits the rate of change in the duty cycle of this power switch can significantly reduce the amount of transient overshoot compared to the original circuit.

Within limits, the magnitude of the reduction is proportional to the delay of the rate limiting. Practical experiments have shown the effect of the transient overshoot upon an ICPT power supply to be reduced to near negligible amounts.

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