

“Why did you say right angles?”: The place of questioning in mathematics

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Mathematics in the New Zealand Curriculum requires the classroom teacher to develop the learner’s critical thinking skills. Research indicates that critical thought is the final step in a four step process. The curriculum suggests that language development followed by conversation and dialogue development — steps one and two — will lead to critical thinking. The curriculum is silent on the third step, questioning, even though recent research suggests that all three are necessary to develop critical thinking. Critical thinking is one of the contributors to reflection. This paper discusses the four steps and promotes a model of the inter-relationship of the steps. From there, the paper proposes methods that the classroom teacher can use to improve both the teacher’s and the learner’s questioning skills in order to develop critical thinking.

Introduction

The impetus for this paper arose from an incident described by Dr Airini of Faculty Pasifika to the ‘Curriculum: Theory, Issues and Practice’ Master of Education [Auckland College of Education] class of Semester 1, 2002. A teacher was showing the class an overhead transparency of a trapezium and asking for a class member to talk about the angles. A boy stated that the angles were right angles. The teacher replied that his answer was wrong.

Dr Airini later spoke to the boy about the thinking behind his statement. He said that it looked like the table at home when you stood at one end of it. The table appeared to get narrower and yet he knew that the table’s corners were right angles. The picture on the overhead transparency was an optical illusion.

As a result of the teacher ending that discussion, that teaching point was lost forever. The teacher never learnt about the boy's thinking or his understanding of geometry or visual perspective. And, quite possibly, the boy decided to be less forthcoming in the future.

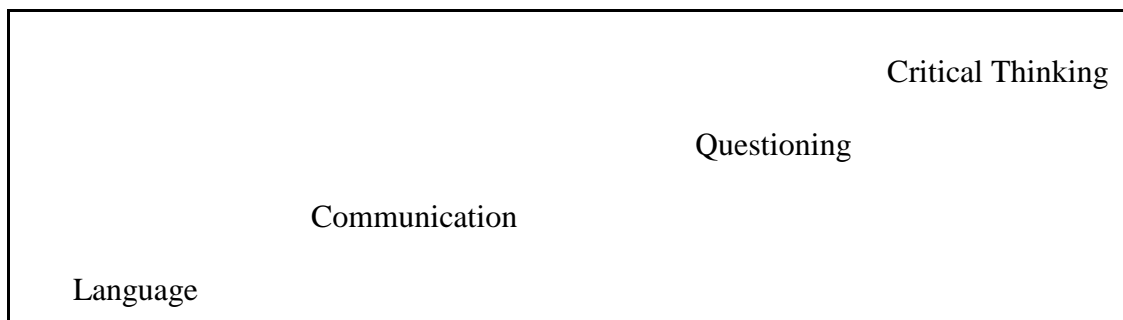
Why did the teacher end the discussion? Perhaps because the class teacher saw no reason to continue — the answer was wrong. It is unfortunate that the class teacher did not go on to question the boy. She could have elicited further information and understanding, and discussed the supposed error in a supportive manner.

All of this led me to wonder: "Why didn't the class teacher ask more questions?" and eventually to: "What is the place of questioning in mathematics?"

Questioning

Questioning, whether in mathematics or other curriculum areas, is one of the steps to critical thinking, which is part of the more complex reflective process. The first step of the learner's journey is the development of both the learner's own language and the understanding of the language of mathematics. From this comes the second step; the ability to communicate mathematically through dialogue and discourse. For the purpose of this paper, dialogue is discussion between two people; for example, the learner and the teacher or the learner and another learner, and discourse is discussion that takes place in the class as a whole or within small groups.

After communication comes the third step of questioning (recall, knowledge, thought provoking, and probing) and finally the fourth step of critical thinking which leads to reflection. These steps build on one another, each dependent on the former step and yet all are inter-related. This progression is based on the grounded theories developed from research undertaken by researchers such as Ellerton and Clements (1996), Fraivillig, Murphy and Fuson (1999), Kanen and Nisbet (1996), Malone (1996), Osuna-MacIsaac (1995), Reinhart (2000), Shaunessy (2000), Smith (2000), Sullivan and Clarke (1991) and Sullivan, Warren and White (2000). The four steps are presented below (Figure 1).

Figure 1 The Four Steps of the Journey

Like the treads on a set of steps, if one of the treads is missing the steps are more difficult to climb. A missing tread makes using the steps difficult and the user may fall through the missing tread gap, and never make it to the final step of critical thinking.

In the following section these four steps are discussed in more depth.

Language: Own and Mathematical

Language is the basis of all that happens in classrooms. From learners' first days in educational settings, the place of language is paramount to further personal discovery, to discoveries of the surrounding world, and to exposures to and discovery of formal education. One of the core components of formal education is mathematics.

A pattern for the learner's discovery of school mathematics is set out in *Mathematics in the New Zealand Curriculum (MiNZC)* (Ministry of Education, 1992). This document directs teachers to assist learners to develop their own mathematical language; learners should "use their own words" (p.11) and then their own mathematical language.

This development of language is supported by Ellerton and Clements (1996). They state that the learner needs to use "natural language" (p.191) in order to comprehend mathematical terminology. This is especially important if the learner is from a non-

English speaking background (NESB). However, for the NESB learner the development of natural mathematical language can take up to three years as the learner moves from the Basic Interpersonal Communication Skills (BIC) stage to the Cognitive Academic Language Proficiency stage (CALP) (Osuna-MacIsaac, 1995, pp.49-50). At the same time the learner continues developing his/her own language — the mother tongue.

The BIC stage is the language of social interaction, e.g. “May I go toilet?” “Teacher, he hit me!” This form of language is the beginning of social English language and is learnt by NESB students when they first enter a New Zealand classroom. The CALP stage is the academic or school language. This uses precise terminology and allows the learner to express abstract thoughts, reasoning and concepts. The learner arriving in the classroom with English as their first language quickly moves to a stage where they can use and understand academic language. That learner has a definite advantage over the NESB learner, who may struggle to come to grips with BIC, let alone more complex academic language.

The development of the learner’s mathematical language at the CALP stage requires the teacher to encourage mathematical language, supported by the learner’s own language (Kanes & Nisbet, 1996). This can be facilitated by discussing the mathematical task being worked on (Reinhart, 2000) and encouraging learners to explain their reasoning (Sullivan et al., 2000). In summary, the teacher needs to talk and question about tasks and activities in order to develop a learner’s mathematical language.

As the learner’s mathematical language develops it becomes easier for the learner to communicate mathematical thinking. Teachers need to encourage dialogue and discussion (Fraivillig et al., 1999) so that learners improve their communication skills. These skills encompass an increased vocabulary range, and more fluent contextual structures.

Communication: Dialogue and Discourse

Communication is promoted as one of the eight essential skills in the *New Zealand Curriculum Framework* (Ministry of Education, 1993). *MinZC* (Ministry of Education, 1992) promotes the concept that communication in Mathematics needs to be “powerful, concise and unambiguous” (p.7), which implies a shared understanding. For this to happen, the teacher must engage the learner in meaningful dialogue about mathematics, and needs to model correct mathematical language whilst accepting the learner’s own language.

Malone and Ireland (1996) suggest that an important role of the teacher is the development of dialogue. An important part of dialogue is listening. Listening to the conversation permits “...better interpretation of the student’s assumptions and reasonings” (Smith, 2000, p.19). Smith identifies this as *hermeneutics* — the art of interpretation.

Thus the teacher should listen to what the learner has to say, question the learner, and engage in dialogue so that the learner has the opportunity to explain his/her thinking and understanding (Reinhart, 2000). Sullivan et al. (2000) share the belief that dialogue provides the teacher with information about what the learner is thinking and understanding, while allowing the learner to explain reasoning. Such dialogue between the teacher and learner increases the learner’s mathematical language and the learner’s skill in dialogue and discourse. In time this should lead to greater understanding of mathematics.

Where does dialogue place the teacher? According to Fraivillig et al. (1999), it is necessary for the teacher to facilitate this discourse. They base this on some of Yackel’s writing (1995, as cited by Fraivillig et al., 1999), where it is suggested that the teacher’s role includes “...facilitating the discourse among the students while they engage in collaborative problem solving, and supporting children’s developing understanding of adequate mathematical explanations” (Fraivillig et al., 1999. p.149).

To be an effective facilitator a teacher may require professional development in the use of questioning. In particular a focus is needed on questions which encourage learners to think and explain what they have attempted in their mathematics work

(Kanes and Nisbet, 1996). This leads to the teacher using a range of questions such as the lower and higher orders of questions outlined in Bloom's Taxonomy (1956). Thus the teacher encourages the learner to recall information and knowledge (lower order), and also to consider mathematical thinking processes (higher order).

All of this dialogue is not only between the teacher and the learner, but also between the learner and classmates. The modelling by the teacher demonstrates a set of guidelines for the learner to follow in order to enhance meaningful conversation. The learner's confidence develops so that eventually each can "...discuss (his/her) mathematical ideas and explorations" (Gervasoni, 1999, p.236) with peers.

Research by Higgins (1992) provides evidence that discussions between learners during mathematics time, especially in junior classes during independent time as part of the *Beginning School Mathematics* programme, develops interaction and social knowledge skills. Although not stated in Higgin's work, the discussions between learners may also be reiterations of learners' mathematical ideas. It is on these interaction and social knowledge skills that the teacher is able to build the learner's dialogue and discourse skills. Using these dialogue and discourse skills the teacher is able to engage the learner in meaningful mathematical dialogue, which includes questioning.

Questioning: Recall and Thought Provoking

Questioning is a vital part in the process of learning and discovering mathematics. This needs to be supported by good language skills, both own and mathematical language, and sound communication skills (Hunkins, 1972). Notwithstanding this, *MinZC* (Ministry of Education, 1992) is silent on this point. Questioning, apart from being mentioned in Achievement Objectives — "...pose questions for mathematical exploration" (p.24) — is not part of the vocabulary of the official document, even though all the writing about developing mathematical language and dialogue and discourse are directing the learner and teacher towards the use of good questions.

Why is there this silence? Did the writers of the document not consider questioning important? This is unlikely. Did the writers assume that teachers would intrinsically

know how to question learners in order to enhance knowledge and understanding? Perhaps this assumption was ill-founded? Were there other factors that unduly influenced the curriculum writers? Whatever it was that caused the writers to leave out the vital questioning aspect of the journey from language development to critical thinking, and so render the curriculum less effective, is worth exploring.

Questioning should be an important aspect of the language, dialogue and discourse of the mathematics lesson. Not the questions that pose problems for investigation as outlined in some Suggested Learning Experiences in the curriculum document, but questions that are probing for information about learners' thinking and reasoning (Malone and Ireland, 1996). Such teacher questions need to encompass a range of question types such as those outlined in Bloom's Taxonomy (Bloom, 1956).

Questions worth asking are "...more than simply recall of information" (Nisbet, Putt, Taplin, 1996, p.179) and should focus on Bloom's higher order. The teacher's question should also be a result of a response given by the learner to an earlier question (Shaunessy, 2000) - questioning is a cyclic process where questions lead to responses which lead to further questions. The teacher needs to "...ask powerful questions" (Hunkins, 1972, p.9) because questioning plays such a vital role in learning.

In *Never say anything a kid can say!* Reinhart (2000) describes how she asks "...thought provoking questions that will lead to understanding" (p.481) rather than explains the answers to learners' problems. In this way, Reinhart improves his own questioning skills and, at the same time, models those skills for the learner. He suggests this improves learners' understanding of mathematics, because learners are expected to ask about problems they are experiencing in mathematics rather than adopting the 'I-can't-do-it approach' or the 'teacher-will-give-me-the-answer' syndrome.

In an attempt to overcome both syndromes, Reinhart (2000) developed a number of valuable questioning techniques. He provides a list which includes wait-time — allowing learners the time to think of their answers before accepting an answer.

Reinhart's approach is best summed up in his words: "(M)y definition of a good teacher has since changed from 'one who explains things so well that students understand' to 'one who gets students to explain things so well that they can be understood' " (p.2, original emphasis).

Sullivan and Clarke (1991) researched the questioning techniques of teachers in Australian schools. The researchers believe that questions have to be a balance between recall questions and thought provoking questions, "...asking a range of types and levels of questions is likely to be important" (p.16).

All of this questioning is a vehicle for the teacher to elicit information and explanations from the learner. The teacher's questioning needs to support the learner as the teacher challenges the learner in a positive and supportive manner. These further questions result from the learner's earlier responses or as the teacher and learner discuss errors that they have made. Another part of the process, as outlined by Fraivillig et al. (1999), is questioning to extend the teacher's own knowledge of the learner's understanding, and extending the learner's own mathematical understanding.

Thinking: Critical and Reflective

Having been silent on questioning, *MinZC* (Ministry of Education, 1992) requires the teacher to develop the learner's "critical reflection" (p.11). How is this to be undertaken unless the learner is encouraged to question? Critical reflection flows from questioning of a divergent nature which "...foster(s) growth of higher level [order] thinking" (Shaunessy, 2000, p.15).

Reinhart's research (2000) supports this flow, from questioning to reflecting. He argues that asking good questions, those which cause the learner to think about what is happening in mathematics and reflect upon their thinking, is a necessary part of the mathematical process. He further suggests that the use of a process question (that is a question which requires reflection, and/or analysis, and/or explanation) is likely to lead to the learner having to think at much higher levels than that required for the product question's relatively straight-forward recall answer. Reinhart emphasizes the

importance of asking for explanations of the thinking which has led to answers. This could prove challenging for both learner and teacher.

Smith (2000) argues that the teacher use questions that stimulate learner thought processes. He proposes that this could result in the learner reflecting and consequently giving a reasoned answer. The evidence from some researchers points to a common theme: higher order questions require the learner to think about their mathematics and to explain what they have done to obtain an answer (Kanes & Nisbet, 1996; Sullivan & Clarke, 1991). Malone and Ireland (1996) suggest that the teacher needs to aim for “metacognitive development” (p.122) in which the learner constructs his/her own thoughts, and this then leads to reflection on thinking. Little in these arguments is new because Bloom (1956), promoting higher order thinking by appropriate questioning, outlined this in the Analysis, Synthesis and Evaluation sections of his taxonomy.

Where does all of this leave classroom teachers? Possibly confused and concerned. And, quite likely, wanting to improve their practice.

Where to Now?

For the learner the ultimate destination on this mathematical journey is to have “...the ability to communicate findings, to present an argument, and to exploit an intuitive approach to a problem.” (Ministry of Education, 1992, p.15). They also need to have developed the skills to think critically about mathematics. The learner’s journey traversed from the development of mathematical language, to ensuring sound mathematical communication skills, to questioning skills, and ultimately to “critical reflection” (p.11).

The learner’s mathematical journey should parallel similar journeys in other curriculum areas. And, as with other curriculum journeys, the ultimate destination is critical thinking, an integral part of reflection.

Research suggests that there is progression from one step to the next, and yet at the same time the previous step continues to develop (Fraivillig et al., 1999). Without the learner developing an adequate mathematical language then it is not likely that clear,

“concise and unambiguous” (Ministry of Education, 1992, p.7) communication will eventuate. Continuing the journey down a sound path of communication it becomes evident that questioning skills are needed in order to progress to a stage from which the learner can reflect critically. But surprisingly, as indicated, questioning is ignored in *MiNZC* (Ministry of Education, 1992).

What should the teacher do to develop the learner’s questioning skills and consequently lead to the learner becoming a critical thinker? The teacher could introduce the learner to the *5WH+2* plan of questioning. In this plan each letter represents a question starter that jogs the learner’s thoughts and can lead to further questions. The question starters are: What? Who? Why? When? Where? How? Plus what if; and so? Although developed for literature rather than mathematics, this plan gives the teacher a basis on which to build and develop questioning skills, as well as providing a model of questioning techniques.

For the teacher wanting more information about questioning, Reinhart (2000) writes very fully about the techniques of questioning which he successfully employs. His techniques further his learners’ mathematical thinking. The main thrust of his questioning is to use “best possible questions” (p.479). That is, questions which cause learners to “reflect, analyse, and explain” (p.480) and which are generally open-ended.

Using Bloom’s Taxonomy (1956) of lower and higher order questions, the teacher can guide the learner towards higher order questions that encourage and challenge the learner to think critically about mathematics (Figure 2).

Figure 2 Bloom's Taxonomy related to mathematics

Bloom's Taxonomy (Bloom)	Skill (Bloom)	Mathematical use – example (example provided by M. Kilpatrick)
• Knowledge	• recall of information	<i>How many groups of 6 in 30?</i>
• Comprehension	• understanding	<i>Tell me how you could illustrate this multiplication fact — $3 \times 6 =$</i>
• Application	• use information	<i>Show how $2+2+2+2=8$ can be written as a multiplication fact.</i>
• Analysis	• seeing patterns	<i>Explain why 2×5 and 5×2 both give the same result but reach the result by different paths.</i>
• Synthesis	• use old ideas to create new ones	<i>Your calculator has developed a fault — the middle row of buttons has failed (4, 5, 6). How can you use your calculator to complete this equation? $5.739 \times 4 =$</i>
• Evaluation	• make choices based on reasoned argument	<i>You have to measure the perimeter of the school grounds. Decide how you approach this task, test your method, and report whether you would recommend your method or not and why.</i>

The six levels are divided equally between the lower-order — Knowledge, Comprehension, Application — and the higher-order — Analysis, Synthesis, Evaluation. Use of questions from both orders is beneficial to the learner's understanding as well as developing the learner's critical thinking.

Sullivan et al. (2000) introduce open-ended tasks which they see as a natural progression from good questioning. Their argument is that open-ended tasks promote thinking by the learner and consequently can provide greater information about what the learner knows, his/her thinking, and his/her ability to explain reasoning for answers. Open-ended tasks also provide the teacher with greater scope for learner

questioning. Open-ended tasks permit the learner to develop several answers, via several methods, all of which are feasible. This provides the teacher with other opportunities to question the learner in order to extend learner thinking and to encourage the learner to think critically about what they have done. This is opposed to closed-tasks which have only one answer and generally promote one method of arriving at this.

So what is the place of questioning in mathematics? From the evidence provided in this paper questioning is the penultimate step on the mathematical journey. Without the teacher's questioning, and consequent modeling for the learner, it is unlikely that the learner will develop questioning skills for themselves or develop "critical reflection" (Ministry of Education, 1992, p.11).

It is essential that teachers ask questions of learners about their mathematics thinking. To ignore the questioning aspect of the journey, as the curriculum document suggests by its silence on the matter, is detrimental to the learner's mathematical development beyond the first two steps of the journey. The ultimate destination is critical thinking. But this will not be achieved if the third step is ignored and a gap left in the pathway. The chance for the learner to bridge that gap is reduced when questioning is missing.

Back to an Alternative Beginning

So what would the boy have said if the teacher had asked "Why do you say right angles?" Quite possibly the boy would have explained about the table at home. That he knew that the corners were right angles even though they appeared otherwise.

And from this question and response, could the teacher have continued to elicit further information and understanding by questioning? In doing this, the teacher could have turned what was a negative learning situation into a supportive and positive learning environment. The discussion may have ranged over many aspects of geometry and the visual arts. Not only would the boy have had the opportunity to express his ideas and explain his mathematical thinking, but so would others. The teacher, and the boy's classmates, would have gained an insight into his knowledge and understanding. It would have been an enriching experience for all.

What does this mean for teachers? In the words of Rudyard Kipling, teachers, in Mathematics and other curriculum areas, need to be like the Elephant's child, "...full of 'satiabile curiosity...'" (Kipling, 1964, p.45). Teachers should never stop asking the "best possible questions" (Reinhart, 2000, p.479) of the learners in their classes.

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