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Off-Axis Stiffness Characterisation of Fibre Reinforced Plastics

by

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

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A new theoretical characterisation is developed for the off-axis stiffness of FRP materials. The theoretical model treats an off-axis unidirectional ply as an inhomogeneous material, and considers the effect of rigid body rotations of the fibres within the matrix material. Linear analytical, and nonlinear finite element solutions are developed for the model. The differences between the new model and the traditional homogenous orthotropic characterisation are functions of both the strain level, and the relative modulus ratio (E_f/E_m) of the constituent materials. For relative constituent moduli typical of most common FRP materials, there are significant differences between the new Rigid Body Motion (RBM) model and homogenous orthotropic characterisations at strains greater than 1%. In a 30° case with E_f/E_m = 100 and a strain level of 2%, the RBM theory predicts a longitudinal modulus 11% higher than the linear orthotropic theory. At small strain levels the RBM theory reduces to the homogenous orthotropic approximation.

A simple and reliable methodology is developed and verified for the experimental characterisation of off-axis tensile FRP specimens. The method applies a tensile load to a thin walled tubular specimen through a high strength, small diameter length of steel wire. The low torsional stiffness of the wire allows one end of the tube to rotate, thus preventing any torsional constraint. Analytical and experimental verifications both indicate that the required tensile load can be applied to tubular specimens without significant torsional constraint. The wire based testing method is used to measure the off-axis stiffness properties of carbon/epoxy tubular specimens at a range of fibre orientations.
Preface

For clarity of presentation, this thesis has been structured in three parts. Part A constitutes the main body of the thesis. The scope of the research is introduced, and a background given of related work and theories. The theoretical and experimental portions of this research are summarised, then their results presented and discussed.

Part B describes the details of the theoretical approaches used in the course of this work. Part C outlines and discusses the experimental portion of the work. Results which are relevant to the development of the various methods are presented within Parts B and C. The main results and conclusions of the work as a whole are presented in Part A.
Many people have assisted the development of this work. In particular, I gratefully acknowledge:

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Several organisations have also contributed to this work, and I gratefully acknowledge:

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<tr>
<td>$\theta$</td>
<td>fibre angle relative to load direction</td>
</tr>
<tr>
<td>$\phi$</td>
<td>fibre rotation relative to initial orientation</td>
</tr>
<tr>
<td>$V$</td>
<td>volume fraction</td>
</tr>
<tr>
<td>$w$</td>
<td>width fraction</td>
</tr>
<tr>
<td>$h$</td>
<td>height fraction</td>
</tr>
<tr>
<td>$d_f$</td>
<td>fibre diameter</td>
</tr>
<tr>
<td>$L_f$</td>
<td>fibre length</td>
</tr>
<tr>
<td>$P$</td>
<td>applied load</td>
</tr>
<tr>
<td>$F$</td>
<td>force</td>
</tr>
<tr>
<td>$M$</td>
<td>moment</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
</tr>
<tr>
<td>$W$</td>
<td>energy due to external work</td>
</tr>
<tr>
<td>$\mu$</td>
<td>specific strain energy</td>
</tr>
<tr>
<td>$SE$</td>
<td>strain energy</td>
</tr>
<tr>
<td>$U$</td>
<td>displacement</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>$v$</td>
<td>Poisson's ratio</td>
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<td>$G$</td>
<td>shear modulus</td>
</tr>
<tr>
<td>$S$</td>
<td>compliance</td>
</tr>
<tr>
<td>$f$</td>
<td>fibre</td>
</tr>
<tr>
<td>$m$</td>
<td>matrix</td>
</tr>
<tr>
<td>$c$</td>
<td>composite</td>
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Coordinate Axes:

The same coordinate axes, Figure 1, are used for the analytical and finite element modelling. The notation of the finite element software used defines the x and y axes as shown. The 1 and 2 axes are then defined by an anticlockwise rotation of angle θ. Note that the resulting axis systems differ from some others used in the literature.

![Coordinate Axes](image)

$x$ cartesian axis perpendicular to load direction
$y$ cartesian axis parallel to load direction
$z$ out of plane cartesian axis

1 cartesian axis transverse to fibre direction
2 cartesian axis parallel to fibre direction

Figure 1: Coordinate Axes

The terms longitudinal and transverse are used to define properties relative to a specimen's geometrical axes and the applied load. Thus the longitudinal modulus of a fibre denotes its modulus when loaded in its longitudinal direction (the 2 direction in Figure 1), while the longitudinal compliance of a tubular specimen is in the y direction.

Abbreviations:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>FRP</td>
<td>Fibre Reinforced Plastic</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon Fibre Reinforced Plastic</td>
</tr>
<tr>
<td>RBM</td>
<td>Rigid Body Motion</td>
</tr>
<tr>
<td>RVE</td>
<td>Representative Volume Element</td>
</tr>
<tr>
<td>ROM</td>
<td>Rule Of Mixtures</td>
</tr>
<tr>
<td>CCA</td>
<td>Composite Cylinder Assemblage</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
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Part A
During the last three decades, the use of advanced fibre reinforced polymer matrix composite materials in structural applications has increased significantly. Such materials are no longer used only in exotic applications, but have become accepted and valuable engineering materials.

Fibre composite materials have a range of potential advantages. Their high strength to weight and stiffness to weight ratios have led to their widespread use in weight critical applications such as transportation. The polymer matrices can provide good corrosion resistance in adverse environments. Other potential advantages include good fatigue properties, thermal stability, and thermal and acoustical insulation benefits. Integration of parts, a good surface finish and less waste can result in a reduced manufacturing cost compared to other materials.

The design and manufacturing processes for composite structures differ from those for structures using traditional materials. In the latter case, a structure is designed to suit a particular material with known mechanical properties. With composite materials, the material itself is designed and constructed at the same time as the structure. This control of the material provides great scope for tailoring the properties of the material to suit the application.

Perhaps the most significant difference between the behaviour of fibre reinforced plastic (FRP) materials and nominally isotropic materials such as most metals, is the directionality of the composite material’s properties. The stiffness and strength of unidirectional FRP’s often vary by more than an order of magnitude depending on the direction of the applied load. This directionality of properties complicates the analysis and testing of such composite materials, but provides valuable opportunities to the structural designer.

Not only can a structure be stiff and or strong only in the directions required for the load, but the directionality of the material can be exploited to tailor the structure to

---

1 In its general context, the term "composite material" covers a large range of material types. In this thesis, the designations "composite material", "fibre reinforced composite" and "fibre reinforced plastic (FRP)", are used interchangeably to mean materials which consist of fibres, (such as glass, Kevlar\textsuperscript{TM}, carbon, boron and similar), within polymer based resin matrices. The term "advanced" is taken to designate that the fibres are high modulus, \(\text{E}_f > 50 \text{ GPa}\), long in relation to their diameter, and unidirectionally oriented within a ply.
deform in a particular manner. The coupling that can exist between loads in one direction and deformations in another allows a composite structure to be designed to respond to a load in an active manner [1].

The coupling between modes of deformation complicates the testing of fibre composite laminates and structures. The behaviour of the material must be understood, and the test method designed appropriately, to ensure meaningful results are obtained [2].

To fully exploit the advantages of composite materials requires reliable, appropriate and accurate analytical theories. Theoretical models must accurately reflect the actual behaviour of the material, yet be simple enough to be of practical application. A similar compromise exists for experimental methods. Test methods must yield accurate and relevant results, at an acceptable level of complexity and cost.

This research is concerned with the stiffness of an off-axis, unidirectional ply of fibre reinforced polymer material. Such a ply, Figure 1.1a, has the fibres oriented at an angle (θ) to the applied load. The couplings inherent in such a material result in longitudinal, transverse, and shear deformations\(^2\) of the ply, as in Figure 1.1 b).

![Figure 1.1: Off-axis unidirectional ply](image)

\(^2\) The depiction of shear deformations depends on the reference axes chosen. Throughout this work shear deformations are portrayed as rotations of the left and right edges of a rectangular element. The element could equally be shown with rotations of all edges, representing the same shear deformation, but a with global rotation of the element.
The traditional elastic model of an off-axis ply uses a two stage process to calculate the deformation of the ply. Micromechanical models are used to calculate the on-axis properties of the ply based on the fibre and matrix properties and the proportions of each. The ply is then assumed to be a homogeneous orthotropic material, and classical tensor transformation theory used to calculate the off-axis behaviour of the ply. This approach assumes that the inhomogeneity of the ply does not affect its behaviour at the macromechanical level.

While a macroscopic averaging process is used to model the elastic behaviour of all materials, there are two significant differences in the case of fibre reinforced plastics; the relative stiffness of the constituent materials and the geometry of the inhomogeneity.

Most polymer resin based matrix materials have Young's moduli of less than 10 GPa. The modulus of typical fibres ranges from approximately 70 GPa for E-glass fibres through to more than 400 GPa for boron fibres. The modulus of advanced fibres is increasing as more highly oriented fibres are developed. Table 1.1 compares the longitudinal Young's modulus of various fibres to that of a typical thermosetting resin (\(E_m = 4\) GPa). The differences in moduli would be greater still for a low modulus resin.

<table>
<thead>
<tr>
<th>Fibre Type</th>
<th>(E_{L4}/E_m)</th>
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<tr>
<td>E-Glass</td>
<td>18</td>
</tr>
<tr>
<td>S-Glass</td>
<td>21</td>
</tr>
<tr>
<td>Kevlar™ 49</td>
<td>30</td>
</tr>
<tr>
<td>Carbon T300</td>
<td>58</td>
</tr>
<tr>
<td>Boron</td>
<td>103</td>
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The scale and geometry of the microstructure of FRP composites differ from other materials. The fibre diameters are much larger than typical metal grain structures, and unlike particulate composites, the fibre lengths can be of the same order as the dimensions of the component.

---

3 Most elastic theories for metals do not explicitly consider the deformations at the individual grain level, let alone at the molecular scale.
The analysis of the mechanical properties of composite materials appears to have originated with a paper by Einstein in 1906 [3], concerning the effective viscosity of a fluid containing a small amount of rigid spherical particles. Most subsequent work until the 1960's was concerned with particulate composites and polycrystalline aggregates. Once advanced fibre reinforced composites became available as engineering materials in the 1960's, development of theoretical models to describe their behaviour followed.

Many of the theoretical models were extensions of models previously developed for alloyed metals, and particulate composites with metal matrices. Unlike FRPs, both of the above classes of materials tend to be macroscopically isotropic, and do not usually have large differences in the stiffnesses of their constituents. In the case of particulate composites reinforced with spherical particles, the composite stiffness can only be increased to approximately 5 times that of the matrix and the composite strength may even be lower than that of the matrix [4].

The homogenous orthotropic characterisation of fibre reinforced composite materials materials was introduced at the early stages of theoretical modelling in the 1960's. Studies, such as those by Tsai [5] and Hahn and Erikson [6] found reasonable correlation with experimental data. However both of these studies only used very small strains (less than 500 microstrain in both cases) and had significant scatter in their experimental data.

The advent of higher stiffness fibres and lower stiffness polymeric resins has led to the development of theoretical models which can account for the observed non-linear behaviour of such materials. Many of these models (see section 2.2) only consider non-linear constitutive relations for the matrix materials. Some, such as those by Luo and Chou [7] and Kuo et al. [8], do acknowledge the geometrical non-linearity caused by the realignment of the fibres. These approaches only implicitly include the fibre rotation by using incremental homogeneous orthotropic based models.

The development of thermoplastic resin based composite materials has resulted in increased interest in the deformation mechanics of fibre reinforced composites. Not only do thermoplastic resins typically have relatively low cured moduli, but during forming of components substantial fibre displacement and rotations can occur [9].

A few experimental studies have observed behaviour thought to be caused by fibre rotation. Vangerko and Barker [10] observed changes in the Poisson's ratio of $\pm 45^\circ$ carbon/epoxy specimens. Cao et al. [11] observed fibre rotation of whiskers within a
SiC\textsubscript{W}/Al composite, and referred to observations of fibre rotation in Tungsten fibre reinforced epoxy composites.

The work described in this thesis is not an extension of existing theories, but an alternative solution method to the classical off-axis characterisation problem summarised by Figure 1.1. The basic premise of this work is that the relative stiffness of the constituents in a typical advanced fibre/polymer matrix composite is such that the fibres will tend to rotate within the resin matrix, towards the direction of the applied load. This rigid body motion of the fibres will cause deformations of the overall composite not recognised by the traditional homogenous orthotropic characterisation. By including such rigid body motions in a theoretical model a more accurate characterisation of material behaviour should be obtained.

The main aim of the research described in this thesis was to develop a theoretical model for the stiffness of an off-axis fibre reinforced composite material. The material was to be treated as an inhomogenous medium, and account to be taken of factors such as rigid body motion (rotation) of the fibres. The model was then to be used to study the significance of the rigid body effects on the behaviour of FRP composites with different constituent materials and at different levels of deformation.

To provide experimental verification of the theoretical modelling, an additional aim was to develop a simple and reliable methodology for the experimental characterisation of off-axis tensile specimens. The method was to be used to test a range of specimens with different fibre orientations.

The theoretical modelling work took two forms, analytical and numerical (finite element). Both methods were used to investigate two problems. The first concerned the rotation of rigid fibres within a compliant matrix. Solutions were developed using both analytical and finite element methods. The second problem investigated was the more general case of an off-axis unidirectional ply, as in Figure 1.1. Linear solutions were developed by both analytical and numerical methods, then the finite element model extended to the large deformation, geometrically non-linear case.

The geometrically non-linear finite element model was used to compare the rigid body motion model's predictions to those of linear and non-linear homogenous orthotropic characterisations, and to the data generated by the experimental work. Comparisons were made for a range of fibre orientations, material properties and strain levels.
Two sets of experimental work were performed. The first investigated dead weight loading of tubular off-axis specimens. A torsion measuring extensometer was developed and preliminary tests made of glass fibre/polyester resin tubes. The main testing programme developed and verified a method for testing off-axis tubular specimens in a standard tensile testing machine, then tested a comprehensive range of carbon fibre/epoxy resin specimens.

To assist the clarity of presentation, this thesis has been presented as three parts. Part A is the main body of the thesis, and as such contains the background, overviews of the theoretical and experimental work, the results and discussion, and the conclusions. Chapters A.3 and A.4 are summaries of the theoretical and experimental work, the details of each being presented as Parts B and C respectively.

Parts B and C provide full documentation of the theoretical approaches and the experimental work respectively. Results which are relevant to the development of the various methods are presented within Parts B and C. The main results and conclusions of the work as a whole are presented in Chapters 5 and 6 of Part A.

This structure requires a consistent numbering system to avoid confusion. Equations, figures and tables are numbered within each chapter, Figure 3.2 being the second figure of chapter 3. Equivalent numbering systems are used for the equations and tables. If only a numerical reference such as 3.2 is given to a chapter, equation, figure or table then the item referred to is in the same part of the thesis as the reference. On the rare occasion that a reference is made to items in another part of the thesis an alphanumeric code is used, Figure B.3.2 being the second figure of chapter three of part B.

Appendices are numbered sequentially throughout each part and are included at the end of each part. Appendix B.4 is the fourth appendix of part B. The literature references are numbered sequentially throughout the complete thesis. All of the literature references are listed at the end of part A.
A.2: Background

Stiffness Prediction and Measurement for Fibre Reinforced Plastics

An overview is given of the typical methods used to predict and measure the off-axis stiffness of fibre reinforced plastic materials. The various methods are discussed and compared in the context of this work.

2.1: Introduction

Fibre reinforced materials provide the opportunity to tailor the material properties, as well as the structure, to suit the particular application. To fully utilise this powerful capability requires the ability to accurately predict the behaviour of the final structure based on the constituent materials and the design of the material and structure. A wide range of theoretical methods have been developed for this purpose, virtually all based on the assumption that a fibre reinforced ply can be treated as homogeneous and orthotropic.

The traditional modelling process for a composite structure, Figure 2.1, consists of several stages, each building on the previous stage's predictions (or on equivalent experimental data).

Micromechanics

\[ \downarrow \]

Macromechanics

\[ \downarrow \]

Laminate Theory

\[ \downarrow \]

Structural Mechanics

Figure 2.1: Stages in traditional fibre composite modelling process

Micromechanics theories are used to predict the behaviour of an on-axis unidirectional ply based on fibre and matrix properties and the proportions of each. Macromechanics uses conventional orthotropic elasticity [12] to predict the off-axis
behaviour of the assumed homogeneous ply. Laminate theory assembles the oriented plys into a multiple layer laminate. Structural mechanics methods use the overall properties of the laminate to predict the behaviour of the structure.

This work is concerned with an alternative characterisation for the first two stages of this process. In view of this, Section 2.2 discusses the assumptions and solution methods used in traditional micromechanical and macromechanical models. Comparisons are made between the predictions of various theoretical models and experimental data.

In parallel with the theoretical requirements is the need to be able to carry out meaningful mechanical tests on such materials and structures. Tests can be used to verify theoretical predictions, to provide data for use in theoretical models, or to provide experimental data for situations which cannot be reliably modelled theoretically. In the case of mechanical stiffness measurements the specimen geometry, load application method and strain measurement techniques play important roles in the accuracy of the test results.

The field of mechanical testing of fibre reinforced plastics is wide, and a full critical review is well beyond the scope of this work. The discussions in Section 2.3 are restricted to the various methods used to measure the off-axis stiffness of fibre reinforced plastic materials. Specimen types and loading methods are compared and discussed, while strain measurement methods are compared and evaluated.

References are made to general texts and review papers, with detailed papers being referred to when relevant. The approach in this chapter is to discuss the concepts behind, and the significance of, different approaches rather than repeating detailed derivations available elsewhere.

This research is not a direct extension of existing theories, but attempts to propose an alternative solution method to the same basic micro/macromechanics problem. As a result of this, a significant proportion of the work referred to in this chapter dates from the early stages of theoretical modelling of fibre reinforced composites. In the case of micromechanics, most of the fundamental research took place in the 1960's and early 1970's. Most of the recent research has been refinements and extensions of the basic theories.
Chapter 2: Background

2.2: Off-Axis Stiffness Prediction

2.2.1: Introduction

The traditional model of a fibre reinforced plastic uses a two step process to predict the off-axis stiffness of a unidirectional ply. A micromechanical model is used to predict the on-axis properties of the ply. The ply is then treated as an effective continuum which is homogeneous, orthotropic, and usually linearly elastic. These assumptions allow conventional continuum mechanics transformation methods to be used to calculate the off-axis properties of the ply.

2.2.2: Micromechanics

In classical modelling of fibre composites, the term micromechanics is used to describe the process of calculating the effective on-axis properties of a lamina, based on the microstructural geometry and properties of the constituent fibres and matrix. In a more general sense, micromechanics denotes any model of composite behaviour which acknowledges the inhomogeneous nature of the material, or according to Jones [13]:

The study of composite material behaviour wherein the interaction of the constituent materials is examined in detail as part of the definition of the behaviour of the heterogeneous composite material.

In the case of mechanical stiffness properties, the classical micromechanics problem can be simply stated as:

Knowing the stiffnesses of the fibres and the matrix, and some details of the microstructural geometry of the composite, what are the effective on-axis stiffnesses of a unidirectional ply?

A wide range of micromechanical theories has been developed for fibre reinforced composites. The theories range from very simple models through to sophisticated elasticity solutions. Despite the differing levels of complexity, virtually all models are based on a common set of assumptions, which may be stated as follows:
• The matrix is homogeneous, linearly elastic and isotropic.

• The fibres are homogeneous, linearly elastic, isotropic (or transversely isotropic), perfectly aligned and regularly spaced.

• Fibre and matrix are free of voids, there is complete bonding and no transitional region between them.

• The lamina is macroscopically homogeneous and orthotropic, linearly elastic and initially stress free.

Clearly these assumptions over simplify the true situation. In particular, the fibres are not normally regularly spaced or perfectly aligned, an interfacial region does exist between them, and voids are normally present. Nonlinear elastic behaviour is often observed, both for the ply and for the matrix. Initial stresses are normally present as a result of the manufacturing process involved in the production of the composite. Various theories have attempted to include some of these factors. Several reviews, such as those by Hashin [4] and Tewary [14], have been published on micromechanical theories.

Fundamental to most of the micromechanical models is the concept of a representative volume element (RVE), sometimes termed a fundamental repeating cell. This is considered to be the smallest region over which the average stresses and strains of the composite can be treated as macroscopically uniform. This can be a single fibre with associated matrix, a group of fibres, or only a portion of a fibre and surrounding matrix if symmetry permits. Hashin [4] discusses the concept of a representative volume element in some detail.

The majority of the present micromechanical theories can be grouped under the headings of: mechanics of materials, bounding methods, exact elasticity solutions, self consistent models, statistical approaches, numerical modelling and semiempirical models. Each approach is briefly discussed below, followed by a comparison of the more commonly used theories. Further details and references can be found in the above review papers, or in texts [15, 16, 17, 18].
Mechanics of materials type approach

The most common mechanics of materials type micromechanics model is the well known Rule (or Law) of Mixtures, ROM. This approach assumes that the strains parallel to the fibre direction are equal in the fibre and the matrix, implying that sections parallel to the fibre direction remain plane. Displacement continuity and force equilibrium considerations are used to solve for the overall composite moduli.

Three different loading conditions are applied to a RVE consisting of a rectangular fibre with associated matrix blocks, Figure 2.2. Application of a uniform longitudinal displacement, $U_y$, yields the longitudinal Young's modulus and Poisson's ratio of the composite. Application of a uniform transverse stress, $\sigma_x$, yields the transverse moduli, and a uniform shear stress, $\sigma_{xy}$, results in the in-plane shear modulus of the composite.

\begin{align*}
E_y &= E_f V_f + E_m V_m \\
\frac{1}{E_x} &= \frac{V_f}{E_f} + \frac{V_m}{E_f} \\
\frac{1}{G_{xy}} &= \frac{V_f}{G_f} + \frac{V_m}{G_f}
\end{align*}

Figure 2.2: Rule of mixtures representative volume element

The two solution forms are also known as the parallel or Voigt, and series or Reuss, estimates respectively.
These somewhat trivial results are widely used as first approximations because of their simplicity. The longitudinal results differ only slightly from those obtained from more sophisticated models and generally correlate well with experimental results. The transverse and shear expressions predict moduli significantly lower than those measured experimentally or predicted by more sophisticated models [13].

The basic Reuss transverse solution violates the assumption of y-direction strain compatibility of the fibres and matrix. The solution assumes that each component is free to deform in the y-direction due to applied x-direction stress. However unless $\frac{v_f}{E_f} = \frac{v_m}{E_m}$ (which is very unlikely if $E_f \gg E_m$) then y-direction stresses will be induced in the two components. The accuracy of the transverse modulus prediction can be improved by invoking the assumption that $\varepsilon_{yf} = \varepsilon_{ym} = \varepsilon_{yc}$. In the case of transversely isotropic fibres this results in equation 2.4:

$$\frac{1}{E_x} = \frac{V_f}{E_f} + \frac{V_m}{E_m} + C$$

Where:

$$C = \frac{-V_f V_m (2 \frac{E_m}{E_f} - 2 V_m v_{xyf} + V_m \frac{E_f}{E_m})}{(V_f E_{yf} + V_m E_m)}$$

The transverse modulus predicted by this equation is higher than that given by Equation (2.2), but is still less than that measured experimentally or predicted by more sophisticated models.

**Bounding methods**

This approach uses the techniques of variational elasticity to calculate upper and lower bounds for the composite moduli. It was first applied to multiphase materials by Paul in 1960 [19].

The lower bound is derived from the principle of minimum complementary energy. Traction are specified on the surface of an arbitrary body and an admissible stress field assumed. The strain energy in the body due to the applied load must be less than that caused by the assumed stress field. Application of this principle to a uniaxial tensile specimen yields an expression for the lower bound of the composite moduli of the same form as the Reuss solution described in Equation (2.2) above.

The principle of minimum potential energy is used to calculate the upper bound. Surface displacements are specified and an admissible strain field assumed. The strain energy in the body due to the applied displacements must be less than the strain
energy due to the admissible strain field. Application of this theorem to a unidirectional tensile specimen yields an expression for the upper bound of the composite moduli of similar form as the Voigt expression. In the case of $V_m=V_f$ this bound becomes exactly the Voigt equation.

Paul’s work was developed in the context of alloyed metals with isotropic constituents. His work was extended to fibrous composites by Tsai [5] and by Hashin and Rosen [19], who narrowed the bounds by considering hexagonal and random arrays of fibres. These models consist of a series of sets of concentric cylinders arranged in the particular array layouts. A cylindrical fibre is surrounded by a cylinder of matrix with relative diameters in proportion to the volume fractions of each component. In the random array case the outside diameters of the sets of cylinders are allowed to diminish so that a region of the composite body can be filled with the cylinders.

This Composite Cylinder Assemblage (CCA) model was originally analysed by variational methods [20]. A much simpler direct method of solution was presented by Hashin in [21] and summarised in [22]. Although this model appears similar to the self consistent models described below, it differs in that the CCA model involves multiple fibres [22].

Hill [23] developed similar expressions to those of Hashin and Rosen without making any assumptions about the geometry of the composite. He demonstrated that these bounds were the best possible without taking account of the detailed geometry. An important conclusion of his work was that the Voigt expression is a lower bound for $E_L$ of the composite.

The variational approach has been widely used for estimating properties of a range of composite materials. References [24, 13, 4, 14] discuss its use in much more detail than that outlined here.

**Exact elasticity solutions**

These methods involve assuming a geometrical description for the composite material and then solving the resulting elasticity problem by an appropriate means. The resulting elastic fields are then averaged to obtain the overall elastic properties of the composite. Methods used to solve such problems include Saint Venant semi-inverse, complex variable mapping, series development and numerical techniques.
The specific geometry assumed for the composite is very important to the relevance and complexity of the solution. Some approaches use particular periodic arrays of fibres while others use symmetry considerations to reduce the problem to a representative volume element (sometimes called a fundamental repeating element).

Many of these solutions are very complicated and of limited practical use. Given the variability in the geometry of a typical real composite it is questionable whether an exact solution for a particular geometry is any more relevant than a more general, albeit approximate theory. A relatively simple elasticity solution, the self consistent model, is described below.

**Self consistent models**

These models take two forms, the first of which was applied to fibre reinforced composites by Hill [25]. This model, Figure 2.3 (a), consists of a single cylindrical fibre surrounded by an infinite homogeneous medium. The homogeneous medium has the properties of the overall composite and is subjected to uniform loading at infinity.

![Self consistent model geometries](image)

**Figure 2.3: Self consistent model geometries**

The second model, Kilchinskii [26, 27] and Hermans [28], consists of three phases, a fibre, a cylinder of matrix, and a surrounding homogeneous medium, again loaded at infinity. A variant on this model is that developed by Whitney and Riley [29] which consists of a single hollow fibre embedded in a concentric cylinder of matrix material.

Due to the lack of fibre-to-fibre interaction these models become less accurate as the concentration of fibres increases. Chow and Hermans [30] developed a model called
the 'reflection' theory which considers the effect that each fibre has on the stress field around other fibres.

**Numerical modelling**

A wide range of computer based numerical methods has been used to evaluate the on-axis elastic constants of a fibre reinforced composite, particularly for transverse and shear properties. Most of these methods have made use of some form of representative volume element, or unit cell, such as in Figure 2.4. Appropriate boundary conditions and loads were applied to the boundaries of the unit cell to represent the symmetry of the composite material.

![Figure 2.4: Typical unit cells](image)

A range of different numerical techniques has been used to model such unit cells, including:

- Complex variables [31]
- Airy stress functions as an infinite series [32]
- Finite difference methods [33, 34]
- Finite element methods [35]

The use of such numerical techniques allowed the effects of factors such as volume fraction, interface debonds, fibre distribution, fibre anisotropy and material nonlinearities to be investigated. Tewary [14] gives a comprehensive summary of the early (pre 1973) numerical modelling work.

More recently, the development of commercial finite element analysis computer software has led to a resurgence in micromechanical numerical modelling. Many of these approaches use three dimensional unit cells such as Figure 2.4 (c), and involve more complicated material characteristics than those used in the early analyses. Examples include those by Adams and Crane [36], who modelled a unit cell with a transversely isotropic fibre and an isotropic, elastic-plastic matrix material. The
matrix material properties were treated as temperature and moisture dependent. The non-linear shear stress/strain behaviour of the composite was modelled under different environmental conditions.

Caruso and Chamis [37] used superelement analysis methods to assemble nine, three dimensional unit cells into a multi-cell composite. The model was used to assess the accuracy of a set of simplified mechanics of materials type micromechanics equations. Equivalent boundary conditions as used for the ROM type micromechanics theory were applied to the multi-cell model.

Unit cells consisting of concentric cylinders were used by Zhang and Evans [38] to predict the mechanical properties of a unidirectional fibre reinforced composite with anisotropic constituents. An energy equivalence approach was used to compare the constituent strain energies calculated from the finite element model to the analytical strain energy of an assumed homogeneous body.

Nimmer et al. [39] used a unit cell as in Figure 2.4 (c) to investigate the behaviour of the interface of a silicon carbide fibre/titanium matrix composite. The stress and displacement fields at the interface were investigated under a transverse tensile load and different temperatures.

All of the numerical modelling described above has been concerned with the behaviour of a unidirectional composite subjected to on-axis loads. An area of micromechanical modelling that has received significant recent interest is that of woven fabrics. Finite element modelling of woven fabrics has been carried out by researchers such as Zhang and Harding [40] who used a strain energy equivalence method to compare finite element energy results for a woven representative volume element to the strain energy of a homogeneous, orthotropic material. Their Representative Volume Element had regions of warp tows, weft tows and matrix material. Uniform displacements were applied in principal material directions, and the finite element calculations of component strain energies compared to overall "average" strain energies and displacements to calculate the overall elastic properties.
Statistical approaches

A real fibre reinforced composite does not have perfectly aligned fibres arranged in a periodic array. Statistical approaches, such as those of Haddad [41] and Nomura [42] allow these and other variables to vary with position in the composite body. Such methods tend to result in very complicated field equations requiring sophisticated solution methods.

Semi-empirical models

A range of semi-empirical micromechanics models has also been developed for several reasons. In a design situation a relatively simple model with some experimental backing can be of more value than a complicated theoretical model of unknown reliability. A model with some experimental input may be able to partially account for the variability typical in a composite material, and may reduce the significance of the restrictive assumptions made in most theoretical models.

The earliest of the models of this type seems to be that of Tsai [5] who introduced a contiguity factor (C) representing the packing of the fibres. If all filaments are isolated C becomes zero, and if all are touching then C is unity. This factor contributes to $E_x$, $\nu_{yx}$ and $G$ of the composite. A filament misalignment factor (k) is used to modify the longitudinal modulus expression. These factors are back calculated from experimental data.

Halpin and Tsai [43] have demonstrated that Hills self consistent model [25] can be reduced to the approximate form:

$$E_y = E_t V_f + E_m V_m$$  \hfill (2.5)

$$\nu_{yx} = \nu_f V_f + \nu_m V_m$$  \hfill (2.6)

$$\frac{M}{M_m} = \frac{1 + \frac{\xi}{\eta} V_f}{1 - \eta V_f}$$  \hfill (2.7)

Where:

$$\eta = \frac{(M_f / M_m) - 1}{(M_f / M_m) + \xi}$$

And:  

$M$ = composite modulus $E_x$, or $G_{xy}$  

$M_f$ = Corresponding fibre modulus, $E_f$, or $G_f$  

$M_m$ = Corresponding fibre modulus, $E_m$, or $G_m$
The factor $\xi$ depends on the fibre shape, packing geometry and loading conditions. Its value can be found by comparison with exact elasticity solutions or by back calculation from experimental data. A series of expressions for $\xi$ to match various exact solutions has been developed [13].

Tsai and Hahn have [15] described a similar model using stress partitioning parameters. These are defined as:

$$\sigma_{xm} = \eta_x \sigma_{xf} \tag{2.8}$$
$$\sigma_{xym} = \eta_s \sigma_{xyf} \tag{2.9}$$

The parameters measure the relative magnitudes of the average matrix stresses as compared to the average fibre stresses. Using these factors in the simple ROM solution results in the following equations for the transverse and shear moduli.

$$\frac{1}{E_x} = \frac{1}{V_f + \eta_x V_m} \left( \frac{V_f}{E_f + \eta_x \frac{V_m}{E_m}} \right) \tag{2.10}$$
$$\frac{1}{G} = \frac{1}{V_f + \eta_s V_m} \left( \frac{V_f}{G_f + \eta_s \frac{V_m}{G_m}} \right) \tag{2.11}$$

The parameters can be determined experimentally or can be calculated from the expressions based on concentric cylinder elasticity models [15]. The same parameter is used to calculate and graph reduced matrix/fibre volume ratios by Tsai in [44]. Chamis [45] and Wu [46] have also developed semi-empirical models based on the mechanics of materials approaches with experimentally determined parameters.
Anisotropic fibres

The highly oriented microstructure of some types of fibres results in anisotropic mechanical properties. The fibre longitudinal stiffness is typically much greater than the transverse or shear stiffness. The earliest micromechanical modelling with anisotropic fibres seems to be that of Whitney [47], who treated the fibres as transversely isotropic and directly substituted the anisotropic fibre stiffness coefficients into existing isotropic micromechanics theories. Bloom and Adams [48] generalised an existing hexagonal array model to the case of transversely isotropic fibres, while Chen [49] performed a similar analysis for rectangular and square array cases.

Hashin [50] used analogies between isotropic and transversely isotropic elasticity equations to derive expressions and bounds for the five effective elastic moduli of a unidirectional fibre composite with transversely isotropic fibres and matrix. His modelling made use of the Composite Cylinder Assemblage [21] micromechanics model.

Isotropic fibres can be characterised by measurement of the axial modulus and shear modulus. The characterisation of transversely isotropic fibres requires the measurement of five independent stiffness coefficients, a virtually impossible task by mechanical tests. Dean and Turner [51] used an elastic wave velocity approach to measure the stiffness of two types of carbon fibres. Their work was extended by Kriz and Stinchcomb [52] who curve fitted Hashin's [50] equations to ultrasonic results in order to calculate fibre properties. The wave propagation in composites with anisotropic fibres has also been investigated by Datta, Ledbetter and Kriz [53].

Material Properties

Any micromechanical theory is only as accurate as the fibre and matrix properties used as its input. Epoxy resins are basically isotropic and the stiffnesses of various formulations are well documented [54]. Reliable mechanical tests can easily be carried out on resin samples [55]. However, the size and geometry of typical fibres (diameters of the order of 10 μm) renders mechanical tests difficult and in some cases virtually impossible to perform. Longitudinal stiffness is the only fibre property which can be readily measured, and even in this case alternative methods give different results [56, 57]. The fibre axial shear modulus can be measured by a torsion pendulum but the results are not considered to be very reliable [50]. The fibre
transverse stiffness, Poisson's ratios and transverse shear modulus are all extremely difficult to measure.

Isotropic fibres such as glass can be characterised by measuring the longitudinal stiffness and estimating the Poisson's ratio. However, some fibres such as carbon are highly anisotropic, typically being treated as transversely isotropic with 5 required stiffness coefficients [54, 52, 50]. Due to the difficulty of direct measurement of fibre properties the usual approach taken (in the case of isotropic fibres also) is to measure on-axis ply properties and then use a micromechanical model to back calculate the fibre properties.

Different micromechanical models predict different composite properties based on the same constituent data. The converse is also true: they will calculate different constituent properties based on the same composite data. It is thus dangerous to use quoted constituent properties if their source is unknown. In many cases the data is only an inferred property based on a particular model, and can yield misleading results if used in a different theory.
2.2.3: Macromechanics

Classical theory

The classical macromechanics model of fibre reinforced composites treats the composite ply as a plane stress, linear elastic, homogenous, orthotropic material. The standard tensor transformation equations of anisotropic elasticity [12] are then used to calculate the properties of the ply at any orientation. The classical macromechanics theory is well documented in the wide range of composite material texts [13, 15, 16, 17]. Only a very brief description and discussion will be given here.

The ply on-axis elastic behaviour is described by four independent coefficients, which can be expressed as stiffnesses, compliances, or engineering constants. Equation 2.12 presents the on-axis strain/stress relationship in terms of compliances.

\[
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{pmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix}
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{pmatrix}
\]

(2.12)

Tensor transformations of the on-axis compliances result in the off-axis strain/stress relationship of equation 2.13.

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix} =
\begin{bmatrix}
\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\
\bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\
\bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66}
\end{bmatrix}
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}
\]

(2.13)

Where:

\[
\begin{align*}
\bar{S}_{11} &= S_{11} \cos^4 \theta + (2S_{12} + S_{66}) \sin^2 \theta \cos^2 \theta + S_{22} \sin^4 \theta \\
\bar{S}_{12} &= S_{12} (\sin^4 \theta + \cos^4 \theta) + (S_{11} + S_{22} - S_{66}) \sin^2 \theta \cos^2 \theta \\
\bar{S}_{22} &= S_{11} \sin^4 \theta + (2S_{12} + S_{66}) \sin^2 \theta \cos^2 \theta + S_{22} \cos^4 \theta \\
\bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66}) \sin \theta \cos^3 \theta - (2S_{22} - 2S_{12} - S_{66}) \sin^3 \theta \cos \theta \\
\bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66}) \sin^3 \theta \cos \theta - (2S_{22} - 2S_{12} - S_{66}) \sin \theta \cos^3 \theta \\
\bar{S}_{66} &= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66}) \sin^2 \theta \cos^2 \theta + S_{66} (\sin^4 \theta + \cos^4 \theta)
\end{align*}
\]

Nonlinear models

Several forms of non-linearities are observed in the stress/strain behaviour of unidirectional fibre reinforced composites. Both polymer and metal matrix composites usually exhibit non-linear shear behaviour. Many metal matrix
composites, and some polymer matrix composites have non-linear transverse strain characteristics. Some unidirectional composites also demonstrate a small degree of fibre direction non-linearity. Both micromechanics and macromechanics approaches have been used to characterise the nonlinear behaviour of fibre reinforced composites.

As the shear non-linearity is the most common, it was the first to have significant attention paid to it. Tsai and Hahn [58], added a 4th order constant to the ply on-axis shear constitutive relation to get improved matching of non-linear experimental observations. Sun and Chen [59] developed a single parameter plasticity model to perform a similar function of matching experimental data. Both of these macromechanical models treated the composite as homogeneous and orthotropic.

The non-linear high temperature behaviour of composites with resin matrices was modelled by Sung Kyu and Springer [60] who developed an incremental model of the ply behaviour. In an analogous manner to [58] and [59] the required parameters were obtained by mechanical tests.

The most common micromechanical approach has been to model the matrix material as either nonlinear elastic or elastic-plastic and use one of the traditional micromechanical models to solve for the composite properties. Nemat-Nasser and Iwakuma [61] treated the matrix as elastic-plastic and used Hill's self consistent method to estimate the overall instantaneous elastic-plastic moduli. Aboudi [62] formulated a plasticity based micromechanical model which used a doubly periodic array of square fibres. A composite cylinder based theory with linear elastic fibres and a non-linear elastic matrix was developed by Frost [63].

The large strain behaviour of composites, particularly those with very flexible matrices, has been described by iterative models. These models, such as those by Luo and Chou [7] and Abu-Farsakh [64, 65], typically increase the strain incrementally and recalculate the material non-linear coefficients and fibre orientation at each step.

The effect of fibre waviness on the longitudinal properties of a unidirectional composite has been evaluated by several researchers, including Mittelman and Roman [66], Luo and Chou [7] and Kuo et al. [8]. A typical approach is that of [7], where the fibre is treated as sinusoidally shaped, with each misaligned section of the fibre treated as an off-axis lamina.
2.3: Off-Axis Stiffness Measurement

2.3.1: Introduction

The typically anisotropic nature of fibre reinforced materials significantly complicates the mechanical testing of such materials. The coupling between different modes of deformation can greatly influence the results of tests. The inhomogeneity of the material makes the choice of test specimen dimensions and shape crucial to the reliability of a testing program. End effects from load introduction can be much more significant than in the case of isotropic materials [57].

In the case of an unconstrained off-axis tensile specimen, an applied tensile load will result in longitudinal, transverse, and shear deformations. Figure 2.5. portrays the deformed shapes of two types of off-axis specimens, the rectangular coupon and the thin walled tube.

The tension/shear coupling causes inplane rotation of the ends of the rectangular specimen. In the case of the tube, the ends remain parallel, but rotate about the longitudinal axis of the tube. Unlike the tubular specimen, the rectangular coupon has a finite width which can cause edge effects.

Both specimen types have been used for off-axis testing. The tensile coupon configuration is the most common, mainly because it is economical to manufacture and can be tested in a conventional testing machine. However the loading method for the coupons must be carefully designed if reliable results are to be obtained.

The tubular specimen is more expensive to manufacture than the tensile coupon and requires specialised fixtures for testing. However it can provide a more uniform stress field than the rectangular specimen, with no edge effects and potentially less end effects. The tubular specimen also provides the capability for combined loading. With
appropriate test fixtures the tube can be subjected to a tensile load, a torque and an internal pressure.

**2.3.2: Coupon specimens**

The rectangular off-axis tensile coupon test has been the subject of much research. The coupon specimen is economical to manufacture, and can be tested in a conventional tensile testing machine. It is easier to instrument than the tubular configuration if strain measurements are required. The main disadvantage of the coupon specimen is the difficulty of obtaining a uniform state of stress within the specimen. Conventional testing machine clamping grips do not allow shear or transverse strain of the ends of the specimen, thus causing a nonuniform stress state. The nonuniform stress state influences measured stiffness properties, and induces premature failures when strength testing.

Pagano and Halpin [2] discussed the issue of end constraint. They used a simple analytical model to demonstrate that the end constraint caused in-plane bending of the specimens, and hence nonuniform longitudinal stresses. A nylon-reinforced rubber was used for qualitative experimental comparisons.

A rotating clamp arrangement was developed by Wu and Thomas [67], which allowed the ends of the specimen to rotate in plane. A series of 15° specimens with length to width ratios of 5, 4 and 2.5 were tested with the rotating clamps. At the low stress levels used, the stress field was relatively uniform.

Finite element modelling was used by Rizzo [68] to study the effects of rigid clamping, with and without end rotation. Typical material properties for glass, graphite and boron fibre reinforced composites were used for the analyses. Rizzo concluded that the prevention of grip rotation was more significant than the clamping. For length to width ratios of greater than 10 the end conditions did not significantly influence the stresses at the centre of the specimen.

Richards et al. [69] used a finite element model of a 45° tensile coupon specimen to verify Pagano and Halpin's analytical model. For a specimen length to width ratio of 12 the maximum predicted error was approximately 5%. Neither Rizzo's [68] or Richards' [69] analyses investigated the transfer of load into the specimen. In both cases the boundary conditions were applied directly to the specimens.
Nemeth et al. [70] also verified Pagano and Halpin's analytical model by comparing finite element models of coupon specimens to experimental moiré interferometry results. They investigated specimen aspect ratio and recommended the use of aspect ratios of 15 or larger for stiffness measurement.

End tab designs have been studied by several researchers. Cole and Pipes [71] used highly tapered end tabs with the fibre orientation of the end tab the same as that of the specimen. Sun and Berreth [72] evaluated an end tab fabricated from woven glass fibre cloth and a silicon rubber matrix. The tab allowed shear deformation, and reduced the stress concentrations at the grips. They suggested that such a specimen and tab design could be suitable for shear strength testing.

Cron et al. [73] used finite element modelling and experimental testing to demonstrate that control of the end tab clamping and selective location of the clamp rotation point could virtually eliminate nonuniform stresses. Sandhu [74] carried out a comprehensive study of the design of off-axis coupon specimens. Of specific interest was the optimisation of the specimen design for the study of biaxial stress states, and for determining the shear stress-strain response of unidirectional laminates. Specimen geometry, fibre orientation and end tab design were optimised for specific types of tests.

**2.3.3: Tubular specimens**

Thin walled tubular specimens have been used to obtain a range of mechanical properties. In addition to off-axis properties, a tube is also acknowledged as the optimum specimen configuration for combined loading [47] and in-plane shear characterisation [75]. The main disadvantages of tubular specimens are their cost, and the need for specialised testing equipment.

An early use of thin walled composite tubes to determine moduli of fibre reinforced composite materials is that of Feldman et al. [76]. They applied internal pressure, axial tension and compression and torsion (independently) to 11 filament wound tubes. Strain gauges and dial gauges were used to measure strains and deformations. Only strains of up to 500 microstrain were applied.

Pagano and Halpin [77] carried out an analytical and experimental study of the end constraints on anisotropic tubes. This work paralleled their study of the rectangular off-axis specimen [2]. They concluded that provided that the tube was thin walled, and free to rotate, a uniform stress field could be obtained. They were not able to
explicitly calculate the thickness required for a particular accuracy. Their experimental work used weights to apply a load to a wire attached to the ends of the nylon reinforced rubber specimens. They also carried out an investigation of tension buckling of clamped end tubular specimens.

Whitney and Halpin [78] extended the above work to the case of combined loading. They demonstrated that a tube clamped at one end, while free to rotate at the other end could be used to completely characterise the mechanical properties of a fibre reinforced composite material. A freely rotating clamp with thrust bearings was used to apply the tensile load to similar specimens as those in [77].

Other work which paralleled the studies of the coupon specimen includes that of Rizzo and Vicario [79]. They carried out a finite element analysis of tubular specimens to investigate thickness to diameter ratio, length to diameter ratio and helical angle effects. They concluded that for modulus determination the thickness should be less than 0.05 of the internal diameter, and gave an expression for calculating the required length. A finite element study was also made of specimen gripping [79, 80]. A grip design with internal and external gripping provided more uniform stresses than a reinforced design with external clamping only.

A similar analytical parametric study was made by Pagano and Whitney [81]. They combined a modified plane strain elasticity solution with shell theory and studied the effects of tube geometry. They concluded that while the required thickness was very dependent on the degree of anisotropy, a conservative estimate of specimen length was given by \( L = 4 \times \text{Radius} + \text{gauge length} \).

Whitney, Pagano and Pipes [82] gave an overall view of the (1971) state of the art in testing of composite tubular specimens. They discussed specimen dimensions, end attachments and specimen fabrication. A testing method was outlined which used measured strains near the end attachments to actively control the displacements of the end fixtures. The end fixture displacements were mechanically forced to match the displacements of an unrestrained, loaded specimen.

Other tubular testing of the same era includes that of Bert and Guess [83] who performed uniaxial, biaxial and shear loading tests on filament wound carbon/carbon composite tubes and rings. A pressurised bladder was used to apply the internal pressure. Their results demonstrated a significant lack of symmetry in the measured stiffness matrix of the material. Wu [84] carried out tension-internal pressure and tension-torsion tests on carbon epoxy specimens. To reduce the constraint of the end
fittings an externally bonded end fixture was used for the tension-internal pressure testing, and an internally bonded plug for the tension-torsion tests. Servo-hydraulic based testing machines were used to apply the tensile and torsional loads.

Toombes et al. [85] used finite element modelling to optimise the end conditions for a biaxially loaded graphite epoxy tube. A combination of glass fibres, a low modulus epoxy resin and an aluminium ring were used to minimise the stress concentrations at the tube ends. A loading system with a rubber pressure bladder and external steel end caps was used in a displacement controlled, servo-hydraulic testing machine. Failure strains were higher than those obtained using flat coupon specimens, implying that a more uniform state of stress was achieved.

Tubular specimens have also been used for flexural testing. Goldman and dos Reis [86] used a finite difference scheme to solve the general boundary value for an edge loaded cylindrical shell. They presented results for a cantilevered composite cylinder under two cases, an applied bending moment, and an applied shear force.

A comprehensive study of the elastic properties of graphite epoxy tubes was carried out by Hahn and Erikson [6]. Three specimens at 0, 15, 30, 45, 60, 90 degrees were tested under the following four loading cases.

- axial loading, tension and compression
- torsional Loading, positive and negative
- positive axial and torsional combined loading
- negative axial and torsional combined loading

End fittings were bonded internally and externally and an MTS closed loop testing system was used to apply the loads. The measured compliances at the different angles were used to calculate the orthotropic transformation invariants. The averaged invariants were then used to predict the on-axis and off-axis compliances. Good agreement was found between the predicted and measured compliances, within the bounds of the (significant) experimental scatter.

This procedure used to predict the compliances has the effect of using the measured data to predict the same compliances. Thus the good agreement between the predicted and measured compliances is not surprising.

Derstine et al. [87] studied the nonlinear constitutive behaviour of graphite/epoxy tubes of a [15/0/±10/0/-15]_s configuration. They added a nonlinear constitutive
model to an existing orthotropic elasticity solution and used a numerical solution scheme. Flat coupon specimens, and a few unidirectional tubes were used to characterise the material properties, then one tube type was tested in torsion and torsion dominated biaxial loading. A friction type end fitting with external and internal gripping was used to apply the loads. The grip system was adequate for torsion testing but caused premature failures in tensile tests. A biaxial servo-hydraulic testing machine was used for the tube testing.

The nonlinear theoretical model gave more accurate predictions than the linear model. Resin rich ply interfaces were treated as individual resin layers, improving the accuracy of the predictions. The inclusion of residual stresses in the model reduced the correlation with the experimental data.

2.3.4: Strain Measurement

Strain measurement methods used on fibre reinforced composites include electrical resistance strain gauges, mechanical contact extensometers, and optical methods such as photoelasticity and moiré fringe techniques. References [88, 57] discuss these and other less common methods. The most common technique used is electrical resistance strain gauging.

The anisotropic and inhomogeneous nature of fibre reinforced composites can significantly affect the accuracy of strain measurements. In the case of electrical resistance strain gauges the main areas requiring consideration are gauge selection, gauge bonding, transverse sensitivity, misalignment, temperature compensation and local reinforcement effects.

Polymer matrix composite materials are generally poor thermal conductors, hence the gauge size and grid resistance need to be maximised to reduce local heating effects of the gauge and specimen [89, 90]. The size of the gauge needs to be large enough to be unaffected by local inhomogeneity of the material. Typically 350 ohm gauges of at least 3 mm gauge length are used.
Surface preparation can damage the specimen if care is not taken. Ideally lead wires should be presoldered before bonding to prevent thermal damage to the specimen. Correct adhesives and cleaning agents should be used [90].

Electrical resistance strain gauges have a small sensitivity to strains transverse to the gauge direction. The anisotropic stiffness of fibre reinforced composites can result in this transverse sensitivity causing significant errors in the indicated strain [90, 91, 92]. Corrections for the transverse sensitivity can be made [91] provided that the full 2 dimensional strain field is known. In the case of an unknown strain field this requires the use of rosette gauges. When the direction of the principal strains are known, as in a unidirectional tensile coupon test, two single gauges are adequate [91].

The highly directional stiffness of fibre reinforced composites makes the alignment of strain gauges more critical than with isotropic materials [92].

The thermal expansion coefficients of fibre reinforced composites vary markedly with direction. This renders the traditional concept of matching gauge coefficients meaningless. A dummy gauge can be used provided that the gauge is identically oriented on an exact replica of the test specimen [88].

The transverse stiffness of unidirectional fibre reinforced composites is typically very low (similar to that of the resin matrix). It is therefore possible for the gauge and adhesive to reinforce the area local to the gauge and affect the measured strain. This is very difficult to quantify, however correction methods are described in references (88, 93).
A.3: The Rigid Body Motion Theory

The basis of the Rigid Body Motion (RBM) theoretical model is presented and discussed. A summary is given of the theoretical methods used to solve the model. The full theoretical details are contained within Part B of this thesis.

3.1: Introduction

The traditional modelling process for the off-axis stiffness of a fibre reinforced plastic (FRP) composite consists of two distinct stages. Micromechanical models (see Section 2.2.2) are used to predict the on-axis properties of the unidirectional ply. The material is then treated as homogenous and orthotropic, and the tensorial transformation theories of classical linear elasticity (see Section 2.2.3) are used to calculate the ply's stiffness at any other angle. This approach assumes that the inhomogeneity of the ply does not affect its behaviour at the macromechanical level.

As discussed in Section A.2.2.3, several forms of nonlinearities are observed in the stress/strain behaviour of unidirectional fibre reinforced composites. FRP composites usually exhibit nonlinear shear behaviour and some FRP composites have non-linear transverse strain characteristics. Some unidirectional composites also demonstrate a small degree of nonlinearity in the fibre direction. Both micromechanics and macromechanics approaches have been used to characterise the nonlinear behaviour of fibre reinforced composites.

This theoretical model is based on the premise that the inhomogeneity of an off-axis FRP composite should not be ignored. Because of the relative modulus of the constituent materials in a typical advanced fibre/polymer matrix composite, Table 1.1, the fibres will tend to rotate within the resin matrix, towards the direction of the applied load. This rigid body motion of the fibres will cause deformations of the overall composite not recognised by the traditional homogenous orthotropic characterisation. By including such rigid body motions in a theoretical model, a more accurate characterisation of material behaviour can be obtained.

Theoretical modelling of the actual deformation of an off-axis FRP composite is very complicated, since the fibres are only approximately circular, are not uniformly distributed within the matrix, and are not necessarily perfectly aligned. This
modelling, as do all theoretical characterisations, makes a variety of approximations and simplifications. These are discussed in Section 3.2, the most significant approximation being the assumed geometry of the material. This is taken to be the same as that of the "Rule Of Mixtures" (ROM) on-axis micromechanics theory. The fibre reinforced ply is assumed to consist of an array of square fibres interspersed with rectangular matrix elements. Clearly the rectangular shape of the fibres does not accurately represent the true case of approximately round fibres completely surrounded by matrix material. This approximation is acceptable for this work, as this analysis is primarily intended to be a investigation of the general effects of rigid body fibre rotation, rather than an exact solution. The RBM model differs from the traditional "Rule of Mixtures/Orthotropic" modelling approach in that the traditional assumptions of homogeneous orthotropy are not made, and the off-axis ply is treated as an inhomogeneous material.

The theoretical modelling process comprised several stages. The rotation of a rigid fibre within a compliant matrix material was modelled analytically and using finite element methods. Attempts were made to develop an analytical solution for the overall deformation of the Rigid Body Motion model by superimposing the rotation solution on the individual deformations of the fibre and the matrix. Difficulties in apportioning the load between the fibres and the matrix prevented an overall solution from being developed in this manner. The subsequent overall deformation modelling did not use the rotation solution, but treated the composite material as general off-axis ply. The linear deformation of an off-axis ply was modelled analytically and using finite element methods. The finite element solution was extended to the nonlinear, large deformation case, and a comprehensive study made of the behaviour of the Rigid Body Motion model.

3.2: The Rigid Body Motion model

The basic premise of this theoretical model is that stiff fibres in a unidirectional layer of fibre reinforced composite will rotate towards the direction of applied load. In addition to this rotation there will be deformations of the fibres and the matrix due to the load acting on each. The combination of these individual deformations and the rotation of the fibres results in the overall deformation of the composite.

The model chosen to represent a fibre reinforced ply consists of an array of fibres interspersed with rectangular matrix elements. The fibres are oriented at an arbitrary angle (θ) to the direction of the load, as in Figure 3.1.
The load applied to the composite is distributed between the fibre and the matrix, resulting in the following components of deformation:

- The fibres rotate (\( \phi \)) towards the direction of the applied load.
- The fibres deform.
- The matrix deforms, displacing the fibres relative to each other.

These deformation components result in overall longitudinal, transverse and shear deformation of the composite element, as in Figure 3.2. The fundamental difference to the orthotropic theory is the effect of the rigid body rotation of the fibres.

**Figure 3.1: Rigid Body Motion Model**

**Figure 3.2: Deformed rigid body motion model**

(Deformations greatly exaggerated)
In common with most other micromechanical theories (Section A2.2.2), the following assumptions are made:

- The fibres are linearly elastic and homogeneous.
- The matrix is linearly elastic and homogeneous.
- The composite is free of voids.
- There is complete bonding at the interface of the constituents and there is no transitional region between them.
- The ply is initially in a stress free state.
- The fibres are regularly spaced, straight, and are aligned.

In addition to these, and the geometrical assumptions in Figure 3.1, the Rigid Body Motion (RBM) theory assumes that the fibre is much stiffer than the matrix. The significance of this assumption is investigated using the finite element model of the rotation component. (Section B.3.2.4).

3.3: Solution methods

Analytical and finite element modelling methods were used to solve two load cases of the model outlined in Section 3.2. The first case studied the behaviour of the model when only the fibres were loaded. Analytical and finite element solutions were developed for the rotation of the fibres. There was very good agreement between the solutions. Attempts were made to develop an analytical solution for the overall deformation of the Rigid Body Motion model by superimposing the rotation solution on the individual deformations of the fibre and the matrix.

Solutions were then developed which treated the composite material as general off-axis ply, with the load applied to both the fibre and the matrix. The linear deformation of an off-axis ply was modelled analytically and using finite element methods. The finite element solution was extended to the nonlinear, large deformation case. Both solution methods treated the constituent materials as linear elastic materials in a state of plane stress.
3.3.1: Analytical

Fibre rotation

The fibre rotation solution considers the case shown in Figure 3.3. If the load is applied only to the fibres, each fibre will tend to rotate about its midpoint due to the load component transverse to its direction, $P_{f1}$. This rotation deforms the matrix between the fibres, generating stresses within the matrix and at the fibre/matrix interface. These stresses balance the applied load once the fibre has rotated ($\phi$) to its new equilibrium position. The fibre is assumed to be rigid, and hence the fibre axial load plays no part in the analysis of the fibre rotation.

Figure 3.3: Rigid body rotation of fibres

Using the geometrical assumptions of Figure 3.1, and considering the displacements of the fibres as they rotate, the strains in the matrix can be calculated in terms of the fibre rotation ($\phi$) and geometry. Two methods were used to solve the fibre rotation versus load relationship. The first method used the equilibrium of a single fibre as its basis, the second performed an energy balance between the applied loads and displacements and the resulting strain energy within the matrix. The two methods yielded virtually identical solutions.
Overall deformation

The geometry of the overall deformation model is identical to that of the fibre rotation model. An array of rectangular fibres and matrix blocks are oriented at an angle to the applied load, as in Figure 3.4. Unlike the rotation case the overall deformation model distributes the load between the fibres and the matrix.

![Overall deformation model](image)

**Figure 3.4: Overall deformation model**

The initial attempts to solve the overall deformation model used the rotation solution as a basis and superimposed the individual deformations of the fibre and the matrix. Difficulties in apportioning the load between the fibres and the matrix prevented a solution from being developed in this manner.

The second approach to the overall analytical model did not consider the fibre rotation as a separate component of deformation, but used equilibrium and compatibility considerations to solve for the constituent stresses and strains, and hence the overall deformation of the model. An incremental methodology was developed to include the fibre rotation effects in the overall deformation.
3.3.2: Finite element

Fibre rotation

Finite element (FE) modelling of the fibre rotation provided verification of the theoretical analysis and enabled parametric studies to be made of assumptions made in the analytical model. The finite element model used the same geometry as the analytical theory, i.e. square fibres interspersed with rectangular blocks of matrix. The required fibre angles and spacing were achieved by altering the mesh shape. The rotation component of the fibre load was applied to each end of the fibres as a nodal force. Fibre rotation was calculated from the displacements of fibre nodes at the centre of the fibres.

A range of mesh shapes were investigated, with a parallelogram shaped model consisting of one fibre and one matrix block, Figure 3.6, being used for most of the parametric studies. The multi point constraint type boundary conditions used for this initial modelling imposed a condition of no transverse strain on the model. This condition was acceptable for the parametric studies as the analytical model could be formulated to model this case, as well as the more realistic situation of no transverse stress.

Figure 3.6: Parallelogram shaped FE model

Parametric studies were made of the effects of fibre stiffness, fibre load, fibre rotation and fibre volume fraction. Very good agreement was found between the analytical and finite element solutions in all cases.
Overall deformation

Parallelogram shaped meshes were used for the initial finite element modelling of the RBM theory due to the ease of their mesh generation. The parallelogram geometry results in a simple mesh shape, but complicates the boundary conditions of the model. Significant effort was applied to investigating suitable boundary conditions for the parallelogram meshes and evaluating alternative model geometries such as tubes. Appropriate parallelogram model boundary conditions were not able to be developed for all required loading conditions. Tubular model geometries generated additional boundary condition difficulties, were complicated to mesh, and expensive to analyse.

A rectangular shaped off-axis model, Figure 3.7, is more complicated to mesh than a parallelogram shaped model, but has relatively simple boundary condition requirements. Similar models to that shown in Figure 3.7 were created to represent each of the fibre orientations (0°, 15°, 30°, 45°, 60°, 75°, 90°) tested in the experimental program.

![Rectangular shaped FE model](image)

**Figure 3.7: Rectangular shaped FE model**

The behaviour of these models was verified by investigations of:

- On-axis behaviour
- Load application methods
- Homogenous orthotropic models
- Mesh density effects
- Nonlinear convergence
- Comparisons with the analytical solution

The models were then used to study the behaviour of the RBM model at large strains, different angles, and for ranges of material properties. Corresponding analyses were also made with the models formulated with linear and nonlinear orthotropic materials.
3.4: Summary

A new theoretical model for the off-axis stiffness of FRP composite materials has been presented. The model treats an off-axis ply as an inhomogeneous material, and includes the rigid body rotation of the fibres towards the direction of the load.

Analytical and finite element modelling methods were used to solve two load cases of the Rigid Body Motion model. The first case studied the behaviour of the model when only the fibres were loaded. Analytical and finite element solutions were developed for the rotation of the fibres. There was very good agreement between the solutions.

Solutions were then developed which treated the composite material as a general off-axis ply, with the load applied to both the fibre and the matrix. The linear deformation of an off-axis ply was modelled analytically and using finite element methods. The finite element solution was extended to the nonlinear, large deformation case, then used to study the effects of material properties, fibre angle, and strain levels on the behaviour of the RBM model.

The differences between the RBM model and homogenous orthotropic characterisations are functions of both the strain level, and the relative modulus ratio ($E_f/E_m$) of the constituent materials. There are significant differences between the RBM model and homogenous orthotropic characterisations at strains greater than 2%. At small strain levels (0.2%), the RBM theory reduces to the homogenous orthotropic approximation. At moderate strain levels (2%), there are significant differences between the RBM model and homogenous orthotropic characterisations for relative constituent moduli typical of most common FRP materials with high modulus (>50 GPa) fibres.

Part B of this thesis describes the details of the theoretical modelling and presents some intermediate results and parametric studies. The main results of the theoretical modelling can be found in Chapter A5.
A.4: Off-axis Testing

The aims of the experimental work are presented and discussed. A summary is given of the experimental methods and the structure of the experimental programme. The full experimental details are contained within Part C of this thesis.

4.1: Introduction

The overall aim of the experimental portion of this research was to accurately measure the off-axis stiffness of a unidirectional FRP material, in order to allow comparisons to be made with the predictions of the Rigid Body Motion theoretical model. The limited experimental data which is available at a comprehensive range of fibre angles, such as that in [5, 6, 76], is only at very small strains (typically less than 500 microstrain), and shows significant scatter. This aim required the choice of an appropriate specimen type, the development of a suitable load application method, and the use of accurate strain measurement techniques.

Two testing programmes were carried out, the preliminary one investigating "dead weight" loading of glass fibre/polyester resin thin-walled tubes. The main testing programme developed and verified a method for testing off-axis tubular specimens in a standard tensile testing machine, then tested a comprehensive range of carbon fibre/epoxy resin specimens.

In the case of an unconstrained off-axis tensile specimen (see Section A.2.3.1), an applied tensile load will result in longitudinal, transverse, and shear deformations. The tension/shear coupling causes in-plane rotation of the ends of a rectangular specimen. In the case of a tube, the ends remain parallel, but rotate about the longitudinal axis of the tube. A tubular specimen is continuous in the transverse direction, and hence does not suffer from edge effects as do rectangular coupon specimens.

Both tubular and rectangular specimen types have been used for off-axis testing. The tensile coupon configuration is the most common, mainly because it is economical to manufacture and can be tested in a conventional testing machine. However the loading method for the coupons must be carefully designed if reliable results are to be obtained. The coupon geometry is easier to instrument than the tubular configuration if strain measurements are required. The main disadvantage of the coupon specimen is the difficulty of obtaining a uniform state of stress within the specimen.
Chapter 4: Off-axis Testing

Conventional testing machine clamping grips do not allow shear or transverse strain of the ends of the specimen, thus causing a nonuniform stress state. The nonuniform stress state influences measured stiffness properties, and can induce premature failures when testing strength.

The tubular specimen is more expensive to manufacture than the tensile coupon and requires specialised fixtures for testing. However it can provide a more uniform stress field than the rectangular specimen, with no edge effects and potentially less end effects. Thin walled tubular specimens have been used to obtain a range of mechanical properties. In addition to off-axis properties, a tubular specimen is also acknowledged as the optimum specimen configuration for combined loading [87] and in-plane shear characterisation [75]. The main disadvantages of tubular specimens are their cost, and the need for specialised testing equipment.

Taking account of the factors described above, and considering the literature detailed in Section A.2.3, thin walled tubular specimens were chosen as the most appropriate specimen type for this test program.

The basic requirement of a loading method for off-axis tubular specimens is that it should apply a controllable tensile load, while allowing one end of the tube to rotate freely [78]. A range of different methods have been used to achieve this. Pagano and Halpin [77] used weights to apply a load to a wire attached to the end of nylon reinforced rubber specimens. Their work was a demonstration of the end effects for tubular specimens, not an actual material characterisation study. Whitney and Halpin [78] used a freely rotating clamp with thrust bearings, but did not give any details of the system, or its behaviour under load. Whitney, Pagano and Pipes [82] outlined a testing method which used measured strains near the end attachments to actively control the displacements of the end fixtures. The end fixture displacements were mechanically controlled to match the displacements of an unrestrained, loaded specimen. Hahn and Erikson [6] used an MTS closed loop tension/torsion testing machine to control the axial and torsional loads.

Two loading methods were investigated in the course of this study. The first was an application of Pagano and Halpin's [77] dead weight method to an actual material characterisation situation. The second, and that used for the main testing programme, used a standard tensile testing machine and applied the load through a length of small diameter, high strength steel wire. The diameter and length of the wire were chosen to ensure that the torsional stiffness of the wire was negligible, thus allowing the end of the tube to rotate freely.
The dead weight tube testing used extensometers to measure the longitudinal and shear strains of the specimens. The measurement of the transverse (hoop) strains of such a tubular specimens by extensometers is very difficult. To provide a full characterisation of the strain field, the main testing programme used rosette resistance foil strain gauges, supplemented by extensometer measurement of the longitudinal and shear strains.

4.2: "Dead weight" testing

A testing program was carried out on a batch of glass fibre reinforced polyester tubes. Tubes of varying fibre angles were constructed. Each tube had nominally 90% of its fibres oriented at either 0°, 5°, 15°, 30°, 45° or 55° to the longitudinal axis of the tube, with the remaining 10% of the tube’s fibres being oriented at 90° to the 90° direction.

To avoid the uncertainties associated with foil strain gauges on composite materials (see Section A.2.3.4), a specialised displacement measuring instrument was designed and constructed. This instrument clamps to the tube, and uses Linear Variable Differential Transformers (LVDTs) to measure longitudinal and shear displacements over a 100 mm gauge length. From these displacements the corresponding strains can be calculated. The gauge length of this instrument was later changed to 50 mm prior to the instrument being used for shear strain measurement during the carbon fibre tube testing programme.

The simplest loading method which provides a pure tensile load to a specimen while allowing it complete freedom to rotate is that of hanging a mass from the end of the specimen. This method was used for the testing of the glass/polyester tubes. The “dead weight” test method for these tubes is depicted in Figure 4.1. A platform loaded with lead masses was attached to the lower end of the tube. The load cell above the tube measured the tensile load. Because the platform was hanging freely, it could rotate to allow shear deformation of the tube.
Initial tests on these specimens [94] revealed several problems with the test method, the strain measurement instrumentation and the specimen type. In particular,

- The loading system was unacceptably time consuming and subject to inertial effects.
- The longitudinal strain measurement system was significantly affected by slight bending of the tubes.
- The bi-directional lamination sequence of the tubes was not compatible with the developmental stage of the analytical model.

4.3: Wire based test method

The problems described above were addressed by designing an improved loading method, by using more comprehensive strain measurement techniques, and by constructing more appropriate specimens. A range of unidirectional carbon fibre/epoxy specimens were designed and constructed. A method for loading these tubes in a tensile testing machine was developed and verified. Foil strain gauges, conventional longitudinal extensometers, and a specialised torsion extensometer were used to measure deformation of the tubes. All specimens underwent tensile testing, the 0 and 90° specimens also being subjected to torsion testing.

The loading method developed for the main testing programme allowed the use of a standard INSTRON mechanical tensile testing machine, which provided control over
the load and loading rate. The tensile load was applied to the tubes through a length of small diameter, high tensile steel wire. Due to its diameter (less than 2 mm) and length (500 mm), the torsional stiffness of the wire was very small, and hence had a virtually insignificant effect on the deformation of the tubes. The induced errors due to the wire were calculated analytically and investigated experimentally. For the geometry and materials used, the maximum predicted errors due to the wire torsional stiffness were in the region of 0.2%. The experimental verification process found no wire diameter effects on the measured strains for the diameter wires (less than 2 mm) used for the testing.

Figure 4.2 depicts the thin wire loading method. One end of the tubular specimen was attached through a universal joint to the testing machine's load cell. A second universal joint attached the lower end of the tube to one end of the steel wire. The other end of the steel wire was mounted to the testing machine's crosshead. Steel clamping jaws provided attachments to the wire.

Figure 4.2: Schematic of steel wire loading method

The wire based testing method is simple and effective. It allows conventional tensile testing machines to be used for testing off-axis tubular specimens, without the need for complicated and expensive fixtures.
4.4: Main experimental programme

Specimens were constructed with fibre orientations of 0°, 15°, 30°, 45°, 60°, 75° and 90°. Five specimens of each orientation were fabricated. The specimens consisted of six ply unidirectional carbon fibre/epoxy resin tubes, of nominal dimensions:

- Outside Diameter: 26.8 mm
- Inside Diameter: 25.0 mm
- Wall Thickness: 0.9 mm
- Length: 400 mm
- Fibre volume fraction: 0.54

The experimental programme had four stages:

- Confirming instrumentation reliability
- Developing and verifying the wire based testing method
- Tensile testing of the specimens
- Torsion testing of 0° and 90° specimens

The specimens' deformations were measured by foil resistance strain gauges and by extensometers. The tensile specimens were tested to longitudinal strains of approximately 2000 microstrain. In most cases each specimen was subjected to nine loading cycles, involving three groups of three loading cycles with the extensometers being reset between consecutive sets. The torsion testing of the 0° and 90° specimens, when combined with the 0° and 90° tensile data, provided a full characterisation of the material's on-axis tensile stiffness. The applied loads and resulting strains were recorded by a computer based data acquisition system.

The "raw data" from the tests was processed to yield stress/strain curves for each specimen. Corrections were made for the transverse sensitivity of the strain gauges. Regression was used to calculate the compliances from the stress/strain curves.
4.5: Summary

Two methodologies were evaluated for the tensile testing of off-axis FRP tubular specimens. The first method used lead masses to apply a "dead weight" to the end of the specimens, thus providing a tensile load while allowing the specimens freedom to rotate. The method was too time consuming to be suitable for testing a large number of specimens.

The second method used a conventional tensile testing machine. The load was applied to the tubular specimens through a small diameter, high strength steel wire. Due to its diameter and length, the torsional stiffness of the wire was very small, and hence had a virtually insignificant effect on the deformation of the tubes.

The wire based testing method was verified analytically and experimentally, then used to test a comprehensive range of off-axis carbon fibre/epoxy resin specimens. The deformations of the tubes were measured by extensometers and foil resistance strain gauges.

Part C of this thesis describes the experimental details, and discusses the validation of the test method. Appendix C.7 presents the stress/strain curves for each specimen. The overall results of the experimental programme are presented and discussed in Chapter A.5.
A.5: Results and Discussion

The results of the experimental and theoretical work are examined and discussed. Due to the quantity, and range of different types of results, it is not appropriate to include all of the results within this chapter. Where relevant, tables and graphs are used to summarise results. In other cases, a representative result is presented here, and the corresponding results are contained within an Appendix.

5.1: Introduction

This chapter consists of two sections, the first of which discusses the experimental portion of the research. The results of the test method validation process are detailed, then typical and summary results are presented for the deformation of the CFRP tubular specimens. The stress/strain characteristics and measured compliances of the tubes are examined. Comparisons are made between the strains measured by the strain gauges and by the mechanical extensometers.

The second section, 5.3, describes the results of the theoretical modelling. Comparisons are made between the analytical and finite element solution methods. The predicted behaviour of the Rigid Body Motion theory is compared to linear and nonlinear homogeneous orthotropic models for different fibre orientations, strain levels and material properties. The theoretical results are compared to the experimental data. Recommendations are made for future research directions.
5.2: Experimental Results

5.2.1: Introduction

The main aims of the experimental programme can be summarised as follows:

- develop and verify a simple and reliable methodology for the experimental characterisation of off-axis tensile fibre reinforced plastic (FRP) specimens

- measure the off-axis stiffness properties of one type of FRP material at normal strain levels for a range of fibre orientations

The experimental programme consisted of three stages. The first (discussed in Chapter C.2) investigated the dead weight testing of tubular specimens. A torsion measuring extensometer was developed, and preliminary tests carried out on glass/polyester tubular specimens. Difficulties with the loading system and the strain measurement system led to the second stage, the development of a wire based test method for tubular specimens. This test method was verified, then used to test a range of carbon fibre/epoxy resin tubular specimens.

In total, 30 tensile tests and six torsion tests were performed on the CFRP tubular specimens. Appendix C2 details each test. Most tensile tests comprised three sets of three loading cycles, resulting in 240 actual loading cases. Strains were measured by rosette resistance strain gauges and by extensometers, pairs of each being located on opposite sides of the specimens. The longitudinal extensometers were reset between sets of loading cycles. The raw data from sets of loading cycles was compared and checked for consistency, then a representative set of data from each test processed to yield the final results. The spreadsheet based data processing system applied calibration constants, corrected for the transverse sensitivity of the strain gauges and calculated strains and stresses relative to the tube geometrical axes. Least squares regression was used to calculate compliances from the numerical stress/strain data. Stress/strain graphs were plotted for each specimen. The results shown here are the actual experimental data, no smoothing or curve fitting has taken place.
5.2.1: Test method validation

A loading method for tubular off-axis tensile specimens is required to apply a tensile load to the specimen while allowing one end of the specimen to rotate freely. Analytical and experimental studies were carried out to investigate the torsional constraint applied to tubular specimens by the wire based loading method proposed in Section C.3.1.

The analytical study (Section C.6.1) predicted the strains in a homogenous orthotropic tube loaded through a high strength, small diameter wire. The strains were compared to those predicted for a tube which was completely free to rotate at one end. The material properties used were representative of those for the actual experimental specimens. As Table 5.1 shows, the predicted errors were very small.

<table>
<thead>
<tr>
<th>Fibre Angle</th>
<th>Wire Ø (mm)</th>
<th>Percentage Errors in Strains (Wire loading compared to unconstrained)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Longitudinal</td>
</tr>
<tr>
<td>15°</td>
<td>1.98</td>
<td>0.048</td>
</tr>
<tr>
<td>30°</td>
<td>1.60</td>
<td>0.023</td>
</tr>
<tr>
<td>45°</td>
<td>1.40</td>
<td>0.009</td>
</tr>
<tr>
<td>60°</td>
<td>1.40</td>
<td>0.004</td>
</tr>
<tr>
<td>75°</td>
<td>1.40</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Tubular specimens with fibre orientations of 15°, 30° and 45° were tested with a range of different diameter wires. Figure 5.1 shows the shear strain of the 15° specimen for three different wire diameters. As expected, the wire diameter does not seem to have any effect on the measured strains. The equivalent graphs for the other strain components and fibre orientations can be found in Section C.6.2 and Appendix C4. In all cases there was no apparent wire diameter influence on the measured strains.
Figure 5.1: Wire Diameter Effects on Shear Deformation of 15° Specimen
Test #6, Tube #10, 15°, SG

To further investigate the behaviour of the wire loading method, a 15° specimen was tested without a wire in the loading system. The tube was directly attached to the testing machine (through universal joints) at each end, and was thus restrained from large rotations. A small degree of rotational freedom existed due to free-play in the universal joints and the end fittings. The 15° specimen was chosen because the shear coupling, and hence anticipated errors, were greatest for this orientation. Figure 5.2 compares the shear strain for this case to the shear strain when a 1.98 mm wire was used.

Figure 5.2: Shear deformation of 15° specimen tested with and without wire
Test #28, Tube #10, 15°, SG
The amount of rotation required to allow shear deformation of the tube is relatively small. For the 400 mm specimen length used, a uniform 3000 microstrain shear would result in a relative rotation of the tube ends of only approximately 5°. The specimen length is also much greater than its diameter (approximately 15 times), which should allow a relatively uniform strain field to exist away from the tube ends. Despite both of these factors, Figure 5.2 demonstrates that the lack of wire in the loading system reduced the shear strain of the specimen by approximately 11%.

Corresponding graphs for the other strain components (Section C.6.2) show that the transverse strain was virtually unchanged, with only a 1% difference. The longitudinal strain was approximately 6% less than the wire loaded case. As the torsional constraint directly affects the rotation, and hence shear of the specimen, it is not surprising that the shear strain was the most influenced by the torsional constraint.

Even without the wire the tube was not fully constrained, and could still undergo some shear deformation. However the stiffness properties measured from such a specimen would be significantly in error, to the same extent as the errors in the strains. It is clearly not acceptable to rely on a long specimen length to achieve a uniform strain state in the centre region of a constrained tube.

The wire based testing method allows the specimen to rotate without significant constraint, and is simple and economical to use. The method could be further developed by investigations of different wire materials and lengths. A unidirectional FRP "wire" should provide a greater tensile strength/torsional stiffness ratio than a steel wire of similar dimensions. A stronger wire would allow the testing of specimens to higher strain levels. The wire based testing method was developed for stiffness measurement, and appears to perform very well in this role. It is less well suited to strength testing because of the significant difference in cross-sectional area, and hence strength, of the wire compared to typical specimens.

Given the effectiveness of the wire loading method it is possible that a shorter specimen length could be used than that used here (length/diameter = 15). The specimen length must be sufficient to ensure that the localised effects of the end fittings do not affect the tube's behaviour within the measurement gauge length.

Other experimental factors such as repeated loading effects, specimen loading rate and instrumentation stability are discussed in Section C.6.2. The results were acceptable in all cases.
5.2.3: Stress/Strain characteristics of CFRP tubes

Figure 5.3 shows the stress/strain response of a typical 15 degree specimen, in this case tube #10. All strain components were linear to the maximum strain levels applied. L, T and LT denote the longitudinal, transverse and shear components respectively, all relative to the geometric axes of the tube. There was good correlation between the extensometers (Ext) and strain gauge (sg) results.

![Stress/strain curves for 15° specimen](image)

**Figure 5.3: Stress/strain curves for 15° specimen**  
Test T28R10, Specimen 10

Stress/strain curves for each specimen can be found in Appendix C5. Most demonstrate similar correlation and linearity as Figure 5.3. The correlation between the strain measurement techniques is discussed in Section 5.2.5.

This work was intended to investigate the effects of fibre rotation at normal design loads, and hence the maximum strains applied to the specimens were between 2000 and 3000 microstrain. The strain levels were also restricted by the requirement to apply multiple loading cases to each specimen to check the consistency of the experimental results. To achieve this the specimens could not suffer any damage during each loading cycle. In the case of the 15° and 30° specimens the applied loads were also restricted by the tensile strength of the wires used in the loading system (Table 5.2, Section C.3.2 details the wire strengths).
5.2.4: Measured compliances of CFRP tubes

The conventional definitions of orthotropic lamina behaviour (Equation 2.13, Section A.2.2.3) were used to calculate apparent compliances for the tubular specimens. Least squares regression was used to calculate the gradients of the stress/strain curves. Longitudinal, \( \bar{S}_{22} \), and shear coupling, \( \bar{S}_{26} \), compliances were calculated from the strain gauge (sg) and extensometer (Ext) data\(^1\). The transverse strain, and hence the compliance, \( \bar{S}_{12} \), was only measured by strain gauges. Figures 5.4 a-c show the measured compliances plotted against fibre angle. Appendix C6 contains the numerical values.

---

\(^1\) The notation used for the compliances refers to Equation 2.13, Section A.2.2.3, and the axes defined on page ix.
Figure 5.4 c: Measured $\bar{S}_{26}$ compliance of CFRP tubular specimens

There is reasonable correlation at each fibre angle. The compliance variations with angle are clear. The apparent scatter in the results is strongly influenced by the absolute value of the compliances. This is particularly significant in the $S_{22}$ case where $S_{22}$ at 90° is much greater than that at 0°. To better portray the experimental variations, the compliances were normalised. The compliance data at each angle was averaged, then the individual compliances from each test divided by the average compliance for the particular angle and strain measurement method. Appendix C7 lists the normalised compliance values, Table 5.2 contains the averaged compliances, and Figures 5.5 a-c graph the normalised compliances.

Table 5.2: Averaged Compliances (TPa-1)

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>$\bar{S}_{22}$ sg</th>
<th>$\bar{S}_{22}$ ext</th>
<th>$\bar{S}_{12}$ sg</th>
<th>$\bar{S}_{26}$ sg</th>
<th>$\bar{S}_{26}$ ext</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8.860</td>
<td>9.114</td>
<td>-2.907</td>
<td>-0.575</td>
<td>-0.405</td>
</tr>
<tr>
<td>15</td>
<td>24.09</td>
<td>22.55</td>
<td>-9.667</td>
<td>-56.63</td>
<td>-56.41</td>
</tr>
<tr>
<td>30</td>
<td>57.48</td>
<td>59.06</td>
<td>-22.58</td>
<td>-84.87</td>
<td>-87.19</td>
</tr>
<tr>
<td>45</td>
<td>100.2</td>
<td>97.35</td>
<td>-31.30</td>
<td>-71.88</td>
<td>-69.86</td>
</tr>
<tr>
<td>60</td>
<td>147.4</td>
<td>135.2</td>
<td>-29.97</td>
<td>-48.22</td>
<td>-41.41</td>
</tr>
<tr>
<td>75</td>
<td>148.6</td>
<td>142.9</td>
<td>-14.89</td>
<td>-11.03</td>
<td>-16.16</td>
</tr>
<tr>
<td>90</td>
<td>137.6</td>
<td>162.9</td>
<td>-6.551</td>
<td>-0.344</td>
<td>-4.340</td>
</tr>
</tbody>
</table>
Figure 5.5 a: Normalised $S_{22}$ compliance of CFRP tubular specimens

Figure 5.5 b: Normalised $S_{12}$ compliance of CFRP tubular specimens (sg)

Figure 5.5 c: Normalised $S_{26}$ compliance of CFRP tubular specimens
The graphs of normalised compliances, Figures 5.3 a-c, portray the degree of experimental scatter more clearly than Figures 5.2 a-c. Figure 5.3a shows a similar degree of variation in $\tilde{S}_{22}$ for all fibre angles. This is not unexpected since all tubes were tested to similar longitudinal strain levels of approximately 2000 microstrain.

The scatter in the $\tilde{S}_{12}$ results, Figure 5.3b, is at a minimum at 45° and increases towards 0° and 90°. As the absolute value of $\tilde{S}_{12}$ is at a maximum at 45°, and a minimum at 0° and 90°, Figure 5.2b, it appears that the degree of scatter decreases as the absolute value of $\tilde{S}_{12}$ increases.

The shear coupling compliance $\tilde{S}_{26}$, Figure 5.3c, demonstrates equivalent behaviour to that of $\tilde{S}_{12}$. The variation in the normalised results increases as the fibre angle becomes greater and the magnitude of $\tilde{S}_{26}$ decreases, Figure 5.2c.

There is a common trend in the $\tilde{S}_{12}$ and $\tilde{S}_{26}$ compliances for the degree of variation to be approximately inversely proportional to the compliance being measured. Such a trend suggests that strain measurement errors may be the cause, rather than variations of specimen properties. An error of fixed magnitude will result in a greater percentage error for the cases with small absolute values of compliance. However in the case with the most variation, 75°, the normalised strain gauge and extensometer $\tilde{S}_{26}$ compliances are virtually equal for each specimen, (see Appendix C7), despite the actual compliances from each measurement method being substantially different (Appendix C6). This behaviour suggests that specimen variations and measurement factors both contribute to the scatter in the data.

Although all specimens were constructed from the same batch of materials, and by the same manufacturing method, it is still possible that significant variations exist between specimens. Errors when laminating the specimens may result in fibre angles differing from the nominal values. Slight differences in the cure cycle or the tension of the shrinkwrap tape could result in volume fraction and thickness variations between specimens, and within any particular specimen. Reasons for the experimental variations are discussed further in section 5.3.6.
5.2.5: Strain measurement

The longitudinal and shear strains were measured by extensometers and by electrical resistance strain gauges. Table 5.3 presents the percentage differences between the average compliances measured from the extensometers and from the strain gauges. Appendix C8 contains the comparisons for each specimen. The longitudinal extensometer, and both sets of the strain gauge results are the average of the strains measured on opposite sides of the specimens. Because of the layout and orientation of the strain gauge rosettes, the on-axis longitudinal strains are calculated from all three individual gauges of each rosette, while the shear strains are calculated from two gauges per rosette.

Table 5.3: Percentage differences between compliances from extensometers and strain gauges.

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Extensometer vs. Strain Gauge</th>
<th>% Difference in:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S_{22}</td>
<td>S_{26}</td>
</tr>
<tr>
<td>0</td>
<td>2.86</td>
<td>-29.6</td>
</tr>
<tr>
<td>15</td>
<td>-6.38</td>
<td>-0.38</td>
</tr>
<tr>
<td>30</td>
<td>2.74</td>
<td>2.73</td>
</tr>
<tr>
<td>45</td>
<td>-2.86</td>
<td>-2.81</td>
</tr>
<tr>
<td>60</td>
<td>-8.31</td>
<td>-14.1</td>
</tr>
<tr>
<td>75</td>
<td>-3.82</td>
<td>46.5</td>
</tr>
<tr>
<td>90</td>
<td>18.4</td>
<td>1163</td>
</tr>
</tbody>
</table>

In the case of S_{22} the differences are less than 10%, except for the 90° case which has 18% difference. Examination of the 90° actual and normalised compliance data (Appendices C6 and C7 respectively) shows poor correlation between the extensometer and strain gauge results. The specimen (#17) with the lowest S_{22} from the strain gauges (sg) has the largest S_{22} as measured by the extensometers (ext). The reverse is true for specimen 37 which has the largest S_{22} sg but the smallest S_{22} ext.

The differences in the average shear coupling compliances (S_{26}) measured by each strain measurement method are less than 3% for the 15°, 30° and 45° specimens. As the magnitude of S_{26} was very small at 0° and 90° (and should be zero), the large differences in these two cases are not significant. However the 60°, and particularly
the 75° specimens have large differences in the average compliances from each strain measurement method.

Examination of the differences for individual specimens, (Appendix C8) reveals that in 46% of the cases, the differences were less than 5%, 73% were less than 10% different, and in 83% of the cases there was less than 15% difference between the compliances measured by each strain measurement method. It is unusual that there was such good correlation between the torsional extensometer and strain gauges for fibre angles of 45° or less, yet very poor correlation at 60° and 75°. The reasons for the poor correlation between the strain measurement methods for the 60°, 75° and 90° specimens are discussed further in Section 5.3.6.
5.3: Theoretical Results

5.3.1: Introduction

Analytical and finite element (FE) methods were used to analyse the Rigid Body Motion (RBM) model described in Chapter 2. The details of the solution methods are described in Part B of this thesis. The analytical modelling (Chapter B.2) yielded a linear solution to the RBM problem, while the finite element model provided a nonlinear, large deformation solution.

This section begins by comparing the analytical solution to the linear FE solution. The nonlinear FE solution is then used to predict the behaviour of the RBM model for a range of situations. The cases presented are:

- Stress/strain behaviour for longitudinal strains of 0.2%, 2% and 10%
- Small changes in constituent properties
- Large changes in constituent properties
- Fibre angles ranging from 0° to 90° in 15° increments, at strain levels of 0.2%, 2% and 10%

The RBM model is compared to linear orthotropic and nonlinear orthotropic theoretical solutions, and to the results of the experimental testing.
5.3.2: Solution methods

Table 5.3 compares the results of the analytical RBM model (Chapter B.2) to the results of linear FE models. The linear FE models were identical to those used for the nonlinear (incremental) modelling, apart from the analysis control parameters in the input file.

Table 5.3: Overall deformation: analytical predictions compared to finite element results

<table>
<thead>
<tr>
<th>Case</th>
<th>% Differences in Overall Strain.</th>
<th>Analytical vs. Finite Element</th>
<th>( \varepsilon_x )</th>
<th>( \varepsilon_y )</th>
<th>( \varepsilon_{xy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_p/E_m ) (Angle = 30°)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>-0.19</td>
<td>-0.31</td>
<td>0.39</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td>-0.26</td>
<td>-0.23</td>
<td>0.12</td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
<td>-0.37</td>
<td>-0.31</td>
<td>-0.30</td>
</tr>
<tr>
<td>Angle (( E_p/E_m = 100 ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>no strain</td>
</tr>
<tr>
<td>15°</td>
<td></td>
<td></td>
<td>-0.36</td>
<td>-0.20</td>
<td>0.08</td>
</tr>
<tr>
<td>30°</td>
<td></td>
<td></td>
<td>-0.26</td>
<td>-0.23</td>
<td>0.12</td>
</tr>
<tr>
<td>45°</td>
<td></td>
<td></td>
<td>-0.33</td>
<td>-0.24</td>
<td>1.35</td>
</tr>
<tr>
<td>60°</td>
<td></td>
<td></td>
<td>-0.13</td>
<td>-0.04</td>
<td>2.25</td>
</tr>
<tr>
<td>75°</td>
<td></td>
<td></td>
<td>0.12</td>
<td>-0.08</td>
<td>2.02</td>
</tr>
<tr>
<td>90°</td>
<td></td>
<td></td>
<td>0.10</td>
<td>-0.09</td>
<td>no strain</td>
</tr>
</tbody>
</table>

There is very good correlation between the analytical and finite element solutions in all cases. The magnitude of the shear coupling, and hence the shear strain, is small for the 45°, 60° and 75° cases, which may contribute to the slight apparent error for these models.
5.3.3: Stress/strain characteristics

The finite element results were processed (see Section B.3.3.6) to create stress/strain graphs for each finite element model. The results of one model have been presented here as an example. The stress/strain graphs for the other cases are in Appendix B10. The models at other off-axis orientations showed similar characteristics to the one shown here. As expected, the 0° and 90° RBM models gave virtually identical results to the equivalent homogenous orthotropic cases.

Figure 5.6 presents the stress/strain curves from a 30° nonlinear finite element model. This is a Rigid Body Motion model at 30°, with materials matching those in the experimental specimens (material case 4, Table B.3.9). Longitudinal, transverse and shear strains are plotted against the applied longitudinal stress. These three graphs are for the large strain case, with the maximum longitudinal strain equal to 10%. The maximum strain level of 10% was arbitrarily chosen to highlight the effects of large strain levels on the behaviour of the various models shown. The transverse and shear strains are plotted to the same stress level as in the longitudinal strain graph.

The three sets of data plotted on each graph are denoted Rigid Body Motion, Orthotropic, and Linear Orthotropic. The first two data sets are the results from the geometrically non-linear finite element analyses of the inhomogeneous and homogenous cases respectively. The finite element solution for the non-linear orthotropic case (labelled Orthotropic on the graphs) allows the material properties to follow the deformed shape, thereby increasing the model's stiffness at large strains compared to the linear case. The Linear Orthotropic data is the linear analytical (which is the same as the linear FE) solution for the same on-axis material properties as the other two cases.

To further demonstrate the effect of strain level on the three theories shown, stress/strain graphs are presented for the same case, but with maximum longitudinal strains of 2%, Figure 5.7, and 0.2%, Figure 5.8. The 2% strain level represents a more realistic maximum strain level for a composite with a normal elongation matrix, while the 0.2% case is for comparison with the experimental results.
Figure 5.6 a: Longitudinal strain of 30 degree case 4, to 10% longitudinal strain

Figure 5.6 b: Transverse strain of 30 degree case 4, to 10% longitudinal strain

Figure 5.6 c: Shear strain of 30 degree case 4, to 10% longitudinal strain
Figure 5.7 a: Longitudinal strain of 30 degree case 4, to 2% longitudinal strain

Figure 5.7 b: Transverse strain of 30 degree case 4, to 2% longitudinal strain

Figure 5.7 c: Shear strain of 30 degree case 4, to 2% longitudinal strain
Chapter 5: Results and Discussion

Figure 5.8 a: Longitudinal strain of 30 degree case 4, to 0.2% longitudinal strain

Figure 5.8 b: Transverse strain of 30 degree case 4, to 0.2% longitudinal strain

Figure 5.8 c: Shear strain of 30 degree case 4, to 0.2% longitudinal strain
All three solutions diverge as the strain level increases. The RBM model is stiffer in the longitudinal and shear directions, but has greater transverse strains than the two orthotropic cases. At 10% longitudinal strain, Figure 5.6 a, the stress level for the RBM case is 9% greater than for the nonlinear orthotropic, and 53% greater than the linear orthotropic case. Table 5.4 quantifies the differences in longitudinal stress between the two nonlinear models at three strain levels.

Table 5.4: Percentage differences between RBM and nonlinear orthotropic off-axis theories at 30°

<table>
<thead>
<tr>
<th>Longitudinal strain</th>
<th>% difference in longitudinal stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>9</td>
</tr>
<tr>
<td>2%</td>
<td>2</td>
</tr>
<tr>
<td>0.2%</td>
<td>0</td>
</tr>
</tbody>
</table>

The higher longitudinal stiffness of the RBM theory is believed to be caused by the change in fibre orientation, and by the resulting deformations of the matrix which change the on-axis properties of the ply. While the nonlinear orthotropic solution does implicitly change the fibre orientation of the ply, it assumes that the on-axis properties of the ply remain the same.

The shear strain curve demonstrates similar behaviour to that of the longitudinal strain. Both nonlinear theories are stiffer than the linear characterisation, the stress for 10% shear strain being 18% higher for the RBM model than for the nonlinear orthotropic case.

At this fibre orientation (30°) the transverse strain of the RBM theory is less than that of the two homogeneous orthotropic models. As for the longitudinal and shear strains, the nonlinear orthotropic case is between the linear orthotropic and RBM results.

The differences between the theories depend upon the fibre angle. The RBM model is consistently stiffer than the others in the longitudinal direction. In the transverse and shear directions the sign of the difference depends on the fibre angle. The behaviour of the three theories at different fibre angles is discussed in Section 5.3.5. The stress/strain curves for the other angles can be found in Appendix B10.
As expected, the magnitude of the differences between the theories is very dependent on the strain level. There are significant differences between the models at large strains, Figure 5.6, but virtually no differences at small strains, Figure 5.8.

Graph 5.6a shows that at large strains the geometrical nonlinearity studied in this work has the effect of increasing the longitudinal stiffness of the composite material. Experimental observations of large strain off-axis behaviour, such as those of Luo and Chou [7], show reduced stiffnesses at large strains. This implies that the observed nonlinearities at large strains are a combination of the geometrical nonlinearities and the nonlinear material properties of the matrix, with the matrix material property nonlinearities (which reduce the stiffness) being greater than the geometrical effects. It is thus possible that at intermediate strain levels the geometrical and material property nonlinearities compensate for each other, thereby maintaining linearity longer than would otherwise be the case.

Figure 5.9 shows the fibre rotation as a function of the longitudinal strain. The fibre rotation is almost linearly proportional to the strain. At a strain level of 10% the fibre rotation is quite substantial, resulting in a final fibre orientation of approximately 23°.

![Figure 5.9: Fibre rotation of 30° RBM model](image)

It is clear from Figure 5.9 that the fibre rotations are significant at even relatively low strains. In such cases, caution should be taken in applying the traditional linear orthotropic characterisation which does not include this behaviour. At small fibre angles, a one or two degree change in fibre orientation can noticeably change the compliance (and hence stiffness) of the material.
5.3.4: Effect of constituent materials

Fibre and matrix properties which were back-calculated from the on-axis experimental results were used for most of the theoretical studies described in this chapter (Case 4, Table 5.5). Section B.3.3.3 describes how these properties were chosen. Two other groups of material properties were used as case studies to highlight aspects of the RBM model's behaviour. Table 5.5 lists the material property cases used for the FE modelling.

**Table 5.5: Material properties used in FE modelling**

<table>
<thead>
<tr>
<th>Material Case Description</th>
<th>Fibre</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_L$ GPa</td>
<td>$E_T$ GPa</td>
</tr>
<tr>
<td>1 Homogeneous # 1-3</td>
<td>112</td>
<td>6.67</td>
</tr>
<tr>
<td>2 Exp. Carbon/Epoxy H</td>
<td>203</td>
<td>8.2</td>
</tr>
<tr>
<td>3 Exp. Carbon/Epoxy L</td>
<td>203</td>
<td>7.4</td>
</tr>
<tr>
<td>4 Exp. Carbon/Epoxy M</td>
<td>203</td>
<td>7.9</td>
</tr>
<tr>
<td>5 Ef/Em=10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6 Homogeneous #5</td>
<td>5.86</td>
<td>2.08</td>
</tr>
<tr>
<td>7 Ef/Em=100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>8 Homogeneous #9</td>
<td>54.5</td>
<td>2.36</td>
</tr>
<tr>
<td>9 Ef/Em=1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>10 Homogeneous #9</td>
<td>540.5</td>
<td>2.39</td>
</tr>
</tbody>
</table>

The first study investigated the effect of small changes in the stiffness of the matrix. As discussed in Section B.3.3.3, the back calculated fibre shear modulus was very dependent on the matrix modulus. For the same on-axis shear stiffness, a 10% change in the matrix modulus resulted in a 400% change in the fibre shear modulus.

Cases 2, 3 and 4 represented three alternative sets of constituent properties back-calculated from the same on-axis properties (the experimental values). Case 1 represented the homogeneous orthotropic situation. Small changes in the matrix modulus resulted in large changes in fibre transverse and shear properties. The fibre was treated as transversely isotropic and the matrix as isotropic.

The 30° model was run with case 1, 2, 3 and 4 material properties. Figure 5.10 presents the resulting strains. The nonlinear, homogeneous orthotropic case is presented as a comparison, representing the situation of $E_m = E_f$. 

Figure 5.10 a: Constituent modulus effects on longitudinal strain

Figure 5.10 b: Constituent modulus effects on transverse strain

Figure 5.10 c: Constituent modulus effects on shear strain
The results in Figure 5.10 follow the expected trends. An increase in the fibre shear modulus results in a reduction of the longitudinal and shear strains, and an increase in the transverse strain (at a particular stress level). However the differences are small, despite the 400% increase in the fibre shear modulus from 20 GPa to 80 GPa.

This implies that if the fibre and matrix stiffnesses are substantially different (even in the Case 2 the fibre shear modulus is approximately 10 times that of the matrix), then the solution is not sensitive to the exact value of each. In a practical situation, provided that the constituent properties yield the correct on-axis properties then any small errors in the values for each should not significantly affect the accuracy of the off-axis results. This finding is particularly valuable because of the potential for errors when back-calculating constituent properties from on-axis results (see section 3.3.3).

The second study (Cases 5, 7 and 9) investigated the effects of large changes in relative fibre to matrix modulus (Ef/Em). The property cases did not represent any particular materials, although the (albeit unusual) combination of boron fibres and a low modulus resin could result in an Ef/Em of 1000. In these 30° models the fibre and matrix were treated as isotropic.

Because of the large range in material properties it was not possible to have only one homogenous orthotropic comparison. Cases 6, 8 and 10 represent the homogenous orthotropic cases with the same on-axis properties as cases 5, 7 and 9 respectively. Since cases 5, 7 and 9 have very different properties it is not appropriate to compare them in the same manner as Figure 5.10. Instead, the stress/strain graphs for each case have been processed to yield Figures 5.11 and 5.12.

Figure 5.11 shows the percentage difference between the longitudinal modulus calculated from the RBM results, and that calculated from the respective nonlinear homogenous orthotropic cases. The percentage differences are plotted as a function of the relative constituent modulus, Ef/Em. Data is plotted for strain levels of 0.2%, 2% and 10%. Figure 5.12 has the same form as 5.11, but compares the RBM theory to the linear homogenous orthotropic cases.

Figures 5.11 and 5.12 demonstrate that the differences between the RBM and the orthotropic theory are functions of Ef/Em and strain level. The graphs quantify the errors implicit in using either the linear or nonlinear orthotropic approximations at this fibre orientation. Clearly at very high strain levels there are substantial differences, even if the fibre is only ten times the stiffness of the matrix. At the other extreme, for a strain level of 0.2%, even if Ef/Em = 1000 there are only very small differences (less than 2%) between the RBM and either of the orthotropic theories.
It is clear that the differences in longitudinal modulus increase rapidly initially, until \( \frac{E_I}{E_m} \) is approximately 100, and then increase more slowly above that level. Most conventional FRP composites have an \( \frac{E_I}{E_m} \) of between 20 to 100 (see Table 1.1, Chapter 1), and are hence likely to be affected by rigid body motions of the fibres. Current developments of even higher modulus fibres are resulting in composites with even more extreme relative constituent moduli.

In a case with \( \frac{E_I}{E_m} = 100 \) and a strain level of 2%, the RBM theory predicts a longitudinal modulus 3% higher than the nonlinear orthotropic, and 11% higher than the linear orthotropic theory. Such a situation could arise from Boron fibres with a normal stiffness matrix, or carbon fibres with a moderately low stiffness matrix. As the linear orthotropic theory would typically be used for a problem of this type, this suggests that such an approach could lead to significant errors.
5.3.5: Directional behaviour

Stress/strain graphs were created in the same format as Figures 5.6, 5.7 and 5.8 for the finite element models at each fibre orientation. While these graphs show the individual characteristics at each angle (see Appendix B10), they do not effectively illustrate the characteristics of the RBM theory with changing fibre orientation.

Figures 5.13, 5.14 and 5.15 were generated from the stress/strain graphs at each angle. They portray the normalised compliances\(^2\) at each fibre angle analysed. The data for the longitudinal (\(S_{22}\)) compliance graphs, Figures 5.13 a, 5.14 a and 5.15 a, was created by dividing the respective strain level (10%, 2% and 0.2%) by the stress level for each model at this strain. The data was then normalised with respect to the linear orthotropic longitudinal compliance at 0°.

The transverse coupling compliance (\(S_{12}\)) data was calculated from the transverse strain at the same stress as the longitudinal case. The data was normalised with respect to the linear orthotropic transverse coupling compliance at 0°.

In the same manner, the shear coupling compliance data (\(S_{26}\)) was calculated from the shear strain at the same stress as the longitudinal case. Since the shear coupling compliances are zero at 0°, the data was normalised with respect to the maximum linear orthotropic shear coupling compliance (at 30°).

The graphs clearly demonstrate that the differences between the three theories are very dependent on the strain level. Figure 5.13 shows the large differences at high strain levels, while Figure 5.14 indicates that there are few differences between the theories at small strains.

Even at large strains, Figure 5.13, the RBM theory agrees with the nonlinear orthotropic case at 0° and 90°, but not at the intermediate off-axis angles where the fibre rotation becomes significant. The RBM characterisation is consistently stiffer in the longitudinal direction, but becomes less stiff in the transverse and shear directions at large fibre angles. Note that in Figure 5.13 a, the linear orthotropic case is different to the nonlinear cases at 0°, although the scale of the graph does not make this apparent.

\(^2\) Use of the term "compliance" implies linear constitutive relations for the material. Clearly this is not the case for the nonlinear theories. Strictly the graphs show normalised stresses and strains, which for linear deformation are the same as the compliances. The compliance notation is used because it is more meaningful to the reader, and allows comparison with the experimental results, which are "nominal compliances" themselves if the stress/strain curves are not completely linear.
Chapter 5: Results and Discussion

Figure 5.13 a: Normalised $\bar{S}_{22}$ compliance at 10% longitudinal strain

Figure 5.13 b: Normalised $\bar{S}_{12}$ compliance at 10% longitudinal strain

Figure 5.13 c: Normalised $\bar{S}_{26}$ compliance at 10% longitudinal strain
Figure 5.14 a: Normalised $\bar{S}_{22}$ compliance at 2% longitudinal strain

Figure 5.14 b: Normalised $\bar{S}_{12}$ compliance at 2% longitudinal strain

Figure 5.14 c: Normalised $\bar{S}_{26}$ compliance at 2% longitudinal strain
Figure 5.15 a: Normalised $S_{22}$ compliance at 0.2% longitudinal strain

Figure 5.15 b: Normalised $S_{12}$ compliance at 0.2% longitudinal strain

Figure 5.15 c: Normalised $S_{26}$ compliance at 0.2% longitudinal strain
The most significant differences between the RBM model and the orthotropic theories appear in the shear compliance graphs. At 10% strain, Figure 5.13 c, there are large differences between the approaches, and even at 0.2% strain, Figure 5.15 c, the differences in the shear compliances are still apparent.

The graphs demonstrate that the rotation of the fibres results in more than just a realignment of the material axes, implying that the fibre rotation also results in a change in effective on-axis material properties. Although in the case of the longitudinal compliance, Figures 5.13-5.15a, the resulting compliance at any nominal fibre orientation is similar to that of a material with a reduced fibre angle, this is not the case for the transverse and shear coupling compliances.

Figure 5.15 shows that at the levels of strain used for the experimental specimens (approximately 0.2%) the differences between the models are very small. For these materials, at this strain level, the rigid body motion effects do not seem to be significant.
5.3.6: Comparisons with experimental results

Figures 5.16, 5.17 and 5.18 compare the experimental results of Figure 5.4 to the theoretical compliances as shown in Figure 5.15. The data from Figure 5.15 has been processed to yield the actual compliances.

Figure 5.16: $S_{22}$ Compliances, theoretical versus experimental

Figure 5.16: $S_{12}$ Compliances (-ve), theoretical versus experimental
Figure 5.18: $S_{26}$ Compliances (-ve), theoretical versus experimental

As discussed in Section 5.3.5 of this chapter, there are only very small differences between the theoretical models at this strain level used for the tubes testing (0.2% longitudinal strain). As the above three graphs show, the differences between the theoretical models are virtually insignificant compared to the variations in the experimental data. However comparisons of the theoretical predictions to the experimental data do provide useful information about the behaviour of the specimens and the experimental methodology.

The graph of the longitudinal compliance, Figure 5.16, shows good correlation between the experimental results and the three theoretical predictions. The only significant deviations are for two of the strain gauge values at 60°. The significant spread of the experimental data at 90° is also evident. Although the scatter at 90° is significant, it appears that the average value of the 90° compliances is not greatly in error, as is evidenced by the generally good agreement at 60° and 75°. The theoretical predictions at these two angles would be the most affected by errors in the 90° compliance.

The transverse coupling compliance ($S_{12}$), Figure 5.17, shows the greatest discrepancy between the experimental results and the theoretical predictions. There is reasonable agreement at 15° and 30°, but poor correlation at 45°, 60° and 75°. While
two sets of strain gauge data for 60° are clearly of questionable accuracy, as is apparent in Figures 5.16 and 5.18, the 45° and 75° data is in good agreement for the $\bar{S}_{22}$ and $\bar{S}_{26}$ compliances.

Figure 5.17 also shows noticeable differences between the measured $\bar{S}_{12}$ compliance at 0° and at 90°. Without alternative strain measurements it is difficult to conclude if this is actually the case, indicating asymmetry in the compliance matrix, or if it is a function of the strain gauge behaviour. The $\bar{S}_{12}$ compliance is calculated from the applied stress, and the measured transverse strain. Because of the layout of the strain gauge rosettes, the transverse strain is measured by one gauge on each side of the tube. These gauges are oriented at 90° to the large longitudinal strain, and are hence significantly affected by transverse sensitivity effects. While corrections are made for the transverse sensitivity effects, based on the strains indicated by the other gauges in the rosette, it is possible that the corrections do not fully account for the transverse sensitivity of the gauge. This could also be a factor in the poor $\bar{S}_{12}$ correlation for the other large fibre angle cases.

The graph of the shear coupling compliance ($\bar{S}_{26}$), Figure 5.18, demonstrates very good correlation, except for the two 60° strain gauge cases previously mentioned, and two sets of 30° data. The strain gauge and extensometer shear values for the 30° case are consistent for each sample, suggesting that it is not a strain measurement problem. Examination of the numerical values, Appendix C6, reveals that specimen #7 has consistently larger compliances than specimen #27, while most of the compliances of specimen #6 are less than those of specimen #27. While it is possible that the differing shear results for the 30 degree case are caused by variations in the fibre orientation, volume fraction, thickness, or matrix properties of the specimens, the longitudinal and transverse results for the 30° specimens demonstrate reasonably good correlation with the theoretical results. If the experimental results for the 30° case are representative of the true properties, then all three theoretical models underestimate the $\bar{S}_{26}$ compliance at this orientation.
5.4: Summary and Recommendations

This section summarises the results presented within this chapter. The overall implications of the results are discussed, and recommendations made for future extensions of this work.

The aims of the work described in this thesis can be summarised as:

- develop a theoretical model for the stiffness of an inhomogeneous, off-axis, fibre reinforced plastic (FRP) composite material.
- use the new model to study the significance of rigid body motion effects on the deformation of FRP composites with different fibre orientations, constituent materials, and at different strain levels.
- develop and verify a simple and reliable methodology for the experimental characterisation of off-axis tensile FRP specimens.
- measure the off-axis stiffness properties of one type of FRP material at normal strain levels for a range of fibre orientations, and compare these to the predictions of the new theoretical model.

Each of these aims has been successfully addressed in the course of this work. A new theoretical model was developed, and its behaviour has been presented for a range of different situations. At large strains, or for high fibre/matrix modulus ratios, the model's predictions differ significantly from either linear or nonlinear homogeneous orthotropic solutions. At small strains there are only very slight differences between the three theoretical solutions.

The RBM model predicts a higher longitudinal stiffness than the linear or nonlinear homogeneous orthotropic solutions for all fibre angles and material cases analysed. The relative magnitude of the transverse and shear coupling depends upon the fibre orientation.

The higher longitudinal stiffness, and different transverse and shear coupling, of the RBM theory are believed to be caused by the change in fibre orientation, and by the resulting deformations of the matrix which change the on-axis properties of the ply. While the nonlinear orthotropic solution does implicitly change the fibre orientation of the ply, it assumes that the on-axis properties of the ply remain the same.
The results presented demonstrate the effect of fibre rotation on the elastic behaviour of FRP composites with linearly elastic fibre and matrix materials. It is evident from the results that the fibre rotation has a significant effect on the overall behaviour at relatively large (greater than 1%) strain levels. To realistically model large strain behaviour it will be necessary to include the effects of nonlinear constituent material behaviour, particularly for the matrix material.

The wire based loading method developed and used for the main testing program appears to be a useful and reliable method. The analytical and experimental verifications both indicate that the required tensile load can be applied to tubular specimens without significant torsional constraint. The loading method is simple and economical to use, and allows the use of a conventional tensile testing machine. A possible further development for the test method would be to use a FRP based "wire" to apply the load, rather than a steel wire. Although attaching end fittings to such a wire could be more difficult than for steel wires, a FRP "wire" is likely to have a greater tensile strength/torsional stiffness ratio than a steel wire of similar dimensions.

The specimens tested demonstrated linear stress/strain characteristics to the maximum load levels applied. There was reasonable consistency between tests of similar samples, apart from a few cases, mainly at large fibre angles. Since the changes in property values with angle are generally less significant at large fibre angles, it appears that the observed variations may be partially due to strain measurement difficulties.

Although great care was taken with the strain measurement methods used, there were still significant variations between the strain gauge and extensometer results for some cases. At best, the differences were less than 1%, and at worst approximately 50%. It appears that while good agreement is possible, it can not be relied upon. The difference between the compliances measured by strain gauges, and those measured by extensometers, was less than 5% different in only 46% of the cases. In situations where properties are required to be measured to a high level of accuracy, the use of more than one form of strain measurement is strongly recommended.

Most of the strain measurement literature described in Section 2.3.4 discusses strain measurement on FRP materials from a theoretical standpoint, rather than based on experimental results. A detailed experimental and theoretical study of strain measurement on FRP materials would have merit.
The experimental work demonstrated some of the difficulties associated with mechanical characterisation of FRP composites. The combination of strain measurement problems, and variations between specimens, make the accurate determination of a representative property difficult to achieve. The variations in properties which can occur, both between test specimens and within a structure, render the concept of a representative property value less meaningful than for a more homogeneous material. The results presented here clearly portray the need to present (in some form) the degree of variation associated with any quoted data for FRP properties. The presentation of averaged or smoothed experimental data is likely to inadequately describe the behaviour of the material.

The factors described above complicate the experimental validation of any theoretical models. For the specimens and loads used in this case, the experimental variations were much greater than the differences in the theoretical models. While the measured longitudinal and shear coupling compliances correlated well with the theoretical models, the transverse coupling compliances demonstrated less agreement.

The theoretical modelling demonstrated that at small strain levels, and for moderate fibre/matrix stiffness ratios, there are only very slight differences between the new RBM model and the traditional homogenous orthotropic characterisation. The experimental specimens were chosen to be representative of materials in common use, and were tested to relatively low strain levels, both to represent typical design load levels and to allow multiple load cycles without damage. Unfortunately this means that the experimental work was not able to provide conclusive validation of the theoretical modelling.

Such validation is clearly desirable, and is recommended as a topic for further study. Specimens should be constructed with a low modulus, high elongation resin, to allow their large strain behaviour to be studied. At large strains, and for such materials, the nonlinear stress/strain behaviour of the resin is significant [7], and would need to be included into the theoretical model. Such a material characterisation could be readily incorporated in the finite element solution. Solution of the nonlinear analytical model would be of interest, but would need to include a nonlinear resin characterisation to be of practical value.
A new theoretical characterisation has been developed for the off-axis stiffness of fibre reinforced plastic materials. This characterisation was achieved by means of a theoretical model which treats a unidirectional ply as an inhomogeneous material. Linear analytical, and nonlinear finite element solutions were developed for the model.

The differences between the new Rigid Body Motion (RBM) model and the traditional homogenous orthotropic characterisations are functions of both the strain level, and the relative modulus ratio \( \frac{E_l}{E_m} \) of the constituent materials. In a 30° case with \( \frac{E_l}{E_m} = 100 \) and a strain level of 2%, the RBM theory predicts a longitudinal modulus 11% higher than the linear orthotropic theory.

For relative constituent moduli typical of most common FRP materials, there are significant differences between the RBM model and homogenous orthotropic characterisations at strains greater than 1%. At small strain levels the RBM theory reduces to the homogenous orthotropic approximation.

The higher longitudinal stiffness, and different transverse and shear coupling, of the RBM theory are believed to be caused by the change in fibre orientation, and by the resulting deformations of the matrix which change the on-axis properties of the ply.

A simple and reliable experimental methodology has been developed and verified for the study of off-axis properties of FRP composites. The method applies a tensile load to a thin walled tubular specimen through a high strength, small diameter length of steel wire. The low torsional stiffness of the wire allows one end of the tube to rotate, thus preventing any torsional constraint. Analytical and experimental verifications both indicate that the required tensile load can be applied to tubular specimens without significant torsional constraint.

The wire based testing method was used to measure the off-axis stiffness properties of carbon/epoxy tubular specimens at a range of fibre orientations. The specimens demonstrated linear stress/strain characteristics to the maximum load levels applied. There was reasonable consistency between tests of similar samples, apart from a few cases, mainly at large fibre angles.
It appears that the observed variations in measured properties may be due to strain measurement difficulties and variations in material properties of the specimens. Although great care was taken with the strain measurement methods used, there were significant variations between the strain gauge and extensometer results for some cases. In situations where properties are required to be measured to a high level of accuracy, the use of more than one form of strain measurement is strongly recommended.

For the specimens and loads used in the testing, the experimental variations were greater than the differences between the theoretical models. While the measured longitudinal and shear coupling compliances correlated well with the theoretical models, the transverse coupling compliances demonstrated less agreement.

The theoretical results demonstrate the effect of fibre rotation on the elastic behaviour of FRP composites with linearly elastic fibre and matrix materials. It is evident from the results that for materials such as those studied, with relative constituent moduli ($E_f/E_m$) of approximately 50:1, the effects of the rigid body fibre rotation start to become significant at relatively large (greater than 1%) strain levels.
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Part B: Theoretical Approaches

Analytical and finite element methods were used to solve the theoretical model outlined in Chapter A3. This section details the theoretical model and describes the analytical and finite element solution methods.
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B.1: Introduction

The basic premise of this theoretical model is that stiff fibres in a unidirectional layer of fibre reinforced composite will rotate towards the direction of applied load. In addition to this rotation there will be deformations of the fibres and the matrix due to the load acting on each. The combination of these individual deformations and the rotation of the fibres results in the overall deformation of the composite.

The model chosen to represent a fibre reinforced ply consists of an array of square fibres interspersed with rectangular matrix elements. The fibres are oriented at an arbitrary angle (θ) to the direction of the load, as in Figure 1.1.

![Figure 1.1: Rigid body motion model](image)

The load applied to the composite is distributed between the fibre and the matrix, resulting in the following components of deformation:

- The fibres rotate (θ) towards the direction of the applied load.
- The fibres deform.
- The matrix deforms, displacing the fibres relative to each other.

These deformation components result in overall longitudinal, transverse and shear deformation of the composite element, as in Figure 1.2. The fundamental difference to the orthotropic theory is the effect of the rigid body rotation of the fibres.
In common with most other micromechanical theories (Section A2.2.2) the following assumptions are made:

- The fibres are linearly elastic and homogeneous.
- The matrix is linearly elastic and homogeneous.
- The composite is free of voids.
- There is complete bonding at the interface of the constituents and there is no transitional region between them.
- The ply is initially in a stress free state.
- The fibres are regularly spaced, straight, and are aligned.

In addition to these, and the geometrical assumptions in Figure 1.1, the Rigid Body Motion (RBM) theory assumes that the fibre is much stiffer than the matrix. The significance of this assumption is investigated using the finite element model of the rotation component (Section B3.2.4).

The assumed rectangular shape of the fibres does not accurately represent the true case of approximately round fibres completely surrounded by matrix material. This analysis is intended to be a demonstration of the effect of rigid body fibre rotation, not an exact solution. The RBM model is based on the same material and geometrical assumptions as the traditional "Rule of Mixtures/Orthotropic" modelling approach (Section A2.2). The RBM model differs in that the traditional assumptions of homogeneous orthotropy are not made. The deformation of an off-axis ply is solved as an inhomogeneous material.
Chapter B.1: Introduction

The theoretical modelling process comprised several stages. The rotation of a rigid fibre within a compliant matrix material was modelled analytically and using finite element methods. Unsuccessful attempts were made to develop an analytical solution for the overall deformation of the Rigid Body Motion model by superimposing the rotation solution on the individual deformations of the fibre and the matrix. The subsequent overall deformation modelling did not use the rotation solution, but treated the composite material as general off-axis ply. The linear deformation of an off-axis ply was modelled analytically and using finite element methods. The finite element solution was extended to the nonlinear, large deformation case, and a comprehensive study made of the behaviour of the Rigid Body Motion model.

The theoretical modelling is described in the following two chapters. The descriptions of the various stages are grouped into analytical modelling and finite element modelling chapters. A few intermediate results and parametric studies are presented in the following sections. The main results of the theoretical modelling can be found in Chapter A5.
B.2: Analytical Modelling

2.1: Introduction

This chapter describes the formulation of the fibre rotation and overall deformation analytical models. In both cases an outline of the model is given, followed by an overview of the solution derivation.

In the case of traditional micromechanics models, the mechanics of materials approach provides simple estimates of properties and is often used for design purposes (Section A2.2). More refined elasticity solutions are used for more accurate estimates and for comparison to the mechanics of materials model.

Using a similar philosophy, the analytical modelling of the Rigid Body Motion theory is approached from a mechanics of materials viewpoint rather than pure elasticity. More sophisticated analyses can build from this base.

The analytical modelling was carried out in three phases. A analytical solution was developed for the rotation of rigid fibres within a compliant matrix. Attempts were then made to develop a solution for the overall deformation by combining the rotation solution with expressions for the individual deformations of the fibre and the matrix. Difficulties were encountered with the distribution of the applied load between the fibres and the matrix. A linear analytical solution was then developed for the general off-axis case. This model provided the basis of a large deflection non-linear solution which was solved by finite element modelling.
2.2: Fibre Rotation

2.2.1: Model description

Consider a single unidirectional ply of fibre reinforced material, oriented at an arbitrary angle ($\theta$) to the applied load (Figure 2.1). Any representative element such as those shown can be taken from this ply (Figure 2.2). In addition to the assumptions detailed in Chapter B.1, the analysis of the rotation component treats the fibre as completely rigid and the load as only being applied to the fibres.

![Figure 2.1: General unidirectional ply](image)

![Figure 2.2: Typical element](image)
As shown in Figure 2.3, each fibre will tend to rotate about its midpoint due to the load component transverse to its direction, $P_{f1}$. This rotation deforms the matrix between the fibres, generating stresses within the matrix and at the fibre/matrix interface. These stresses balance the applied load once the fibre has rotated ($\phi$) to its new equilibrium position. Due to the assumption of fibre rigidity the fibre axial load plays no part in the analysis of fibre rotation. It is however included in the overall deformation analysis.

![Figure 2.3: Rigid body rotation of fibres](image)

Using the previous geometry assumptions, and considering the displacements of the fibres as they rotate, the strains in the matrix can be calculated in terms of the fibre rotation ($\phi$) and geometry (Appendix B1). They are:

\[
\begin{align*}
\varepsilon_1 &= \frac{df(1 - \cos \phi) + A_0 \sin \theta \sin \phi}{A_0 \cos \theta - df} \\
\varepsilon_2 &= 0 \\
\varepsilon_{12} &= \frac{A_0 \sin \theta (1 - \cos \phi) - df \sin \phi}{A_0 \cos \theta - df} - \phi
\end{align*}
\]

With:

\[
A_0 = \frac{df}{V_f \cos \theta}
\]

With the usual (Section A2.2.2) micromechanics assumption of a ply being in a state of plane stress, and treating the matrix as isotropic, the stresses generated by these strains are given by:
\[ \sigma_1 = \frac{E_m}{(1 - \nu_m^2)} \left( \frac{d_f(1 - \cos\phi) + A_0 \sin\theta \sin\phi}{d_m} \right) \]  
(2.4)

\[ \sigma_2 = \frac{E_m \nu_m^2}{(1 - \nu_m^2)} \frac{d_f(1 - \cos\phi) + A_0 \sin\theta \sin\phi}{d_m} \]  
(2.5)

\[ \sigma_{12} = \frac{E_m}{2(1 + \nu_m)} \left( \frac{A_0 \sin\theta (1 - \cos\phi) - d_f \sin\phi - \phi d_m}{d_m} \right) \]  
(2.6)

Equations 2.4, 2.5 and 2.6 describe the stresses within the matrix, and hence acting at the fibre/matrix interface, for a given fibre rotation. It is assumed that the representative element, Figure 2.2, is sufficiently removed from free boundaries that the deformation and hence the matrix stresses are uniform along the fibre length.

### 2.2.2: Solution Methods

The fibre load versus rotation relationship can be solved using a variety of techniques. The two analytical approaches used in this work utilised equilibrium and energy conservation considerations. The two methods are described below.

**Fibre Equilibrium**

If the equilibrium of the fibre alone is considered, a free body diagram can be constructed as in Figure 2.4.
For moment equilibrium about the fibre centre, the applied moment due to the rotation component of the fibre load must equal the resisting moment due to the stresses at the fibre/matrix interface. Hence:

\[-P_{f1} + df^2 (\sigma_{m1} \tan (\theta) + \sigma_{m12}) = 0 \quad (2.7)\]

Substituting equations 2.4 and 2.6 into 2.7 results in a quadratic expression for the fibre rotation, \( \phi \):

\[A\phi^2 + B\phi + C = 0 \quad (2.8)\]

where:

\[A = d_f^2 (d_f \tan \theta - 0.5 A_o (1 - V_m) \sin \theta)\]

\[B = d_f^2 A_o (2 \sin \theta \tan \theta + (1 - V_m) \cos \theta)\]

\[C = \frac{-2 P_{f1} (1 - V_m^2) d_m}{E_m}\]

Due to the relative magnitudes of \( A, B, C \) and \( \phi \) the second order term is relatively insignificant. Parametric studies demonstrated that for small deformations the percentage error between the linear solution of equation 2.9 and the quadratic solution of equation 2.8 is typically less than 0.5 %. Using the linear solution, the fibre rotation (\( \phi \)) is given by:

\[\phi = \frac{-2 P_{f1} (1 - V_m^2) d_m}{E_m d_f^2 A_o (2 \sin \theta \tan \theta + (1 - V_m) \cos \theta)} \quad (2.9)\]

**Energy Conservation**

A typical element, Figure 2.2, can be further simplified to a fundamental representative cell, consisting of a single fibre surrounded by two half matrix blocks, as in Figure 2.5. For the \( \sigma_{m1} \) and \( \sigma_{m12} \) matrix stresses within this cell to be uniform, the \( \sigma_{my} \) and \( \sigma_{mxy} \) stresses must exist at the top and bottom matrix boundaries.
Figure 2.5: Fundamental representative cell

Since the fibre is assumed rigid, it can store no strain energy, and hence the work done by the forces at the top and bottom element boundaries must be equal to the strain energy within the deformed matrix. The work done results from the displacements of the external forces $P_{f1}, P_{my}$ and $P_{mxy}$. The total work done by the loads at each end of the fibre is given by:

$$W_f = 2 \int P_{f1} \, dl \tag{2.10}$$

and hence for small rotations:

$$W_f = L_f \int P_{f1} \, d\phi \tag{2.11}$$

Similarly the work done by the matrix forces is given by:

$$W_m = L_f \int dmdf(\sigma_{mxy} + \sigma_{mytan(\theta)}) \, d\phi \tag{2.12}$$

In the case of plane stress isotropy, the specific strain energy in the matrix is given by:

$$\mu_m = \frac{1}{2E_m} \left( \sigma_{xm}^2 + \sigma_{ym}^2 - 2 \, v_m \, \sigma_{xm} \sigma_{ym} + 2 \, (1 + v_m) \, \sigma_{xym}^2 \right) \tag{2.13}$$

For the specified geometry, the total strain energy stored in the matrix is then:

$$SE_m = \frac{d_l d_m L_c}{2E_m} \left( \sigma_{xm}^2 + \sigma_{ym}^2 - 2 \, v_m \, \sigma_{xm} \sigma_{ym} + 2 \, (1 + v_m) \, \sigma_{xym}^2 \right) \tag{2.14}$$
Substituting the stresses from equations 2.4, 2.5 & 2.6 into equation 2.14 results in the following expression for the strain energy:

$$SE_m = \frac{d f \, d_m \, L_f}{2 \, E_m} \left( E \Phi^4 + F \Phi^3 + G \Phi^2 \right) \quad (2.15)$$

where:

$$C = \frac{E_m}{2 \left(1 - V_m^2 \right) d_m}$$

$$D = \frac{A_o \sin \theta \, E_m}{4 \left(1 + V_m \right) d_m}$$

$$E = C^2 \, d_f^2 \left(1 - V_m^2 \right) + 2 \, D^2 \left(1 + V_m \right)$$

$$F = 4 \, C^2 \, d_f \left(1 - V_m^2 \right) A_o \sin \theta - \frac{8 D^2 \left(1 + V_m \right) \tan \theta}{\tan \theta}$$

$$G = 4 \, C^2 \, A_o^2 \left(1 - V_m^2 \right) \sin^2 \theta + \frac{8 D^2 \left(1 + V_m \right)}{\tan^2 \theta}$$

Combining equations 2.11, 2.12 and 2.15 gives:

$$\frac{2 \, E_m}{d_f \, d_m} \int \left( P_{fl} + d_m d_f (\sigma_{mxy} + \sigma_{my} \tan(\theta)) \right) d \phi = (E \Phi^4 + F \Phi^3 + G \Phi^2) \quad (3.16)$$

Substituting for $\sigma_{mxy}$ and $\sigma_{my}$ in terms of equations 2.4, 2.5 and 2.6, differentiating and simplifying results in a cubic expression for $\Phi$:

$$2 \, E \Phi^3 + E \Phi^2 + N \Phi - \frac{E_m \, P_{fl}}{d_f \, d_m} = 0 \quad (2.17)$$

where:

$$E = C^2 \, d_f^2 \left(1 - V_m^2 \right) + 2 \, D^2 \left(1 + V_m \right)$$

$$I = d_f \, V_m - d_m \left(1 - V_m \right) \sin^2 \theta$$

$$J = 2 \, A_o \sin \theta$$

$$K = (1 - V_m) \sin \theta \left(0.5 \, A_o \cos 2\theta - d_f \cos \theta \right)$$

$$L = - A_o \cos \theta \left(1 - V_m \right)$$

$$M = 3 \, F \frac{E_m^2 \left(K - I \tan \theta \right)}{\left(1 - V_m^2 \right) d_m}$$
\[ N = 2G - \frac{E_m^2 (L - J \tan \theta)}{(1 - v_m^2) \, d_m} \]

Due to the relative magnitudes of \( E, M, N \) and \( \phi \), the higher order terms are relatively insignificant. Using the linear solution, the fibre rotation (\( \phi \)) is given by:

\[ \phi = \frac{E_m \, P_{fl}}{N \, df \, d_m} \]  

(2.18)

A comparison of the equilibrium and energy solutions demonstrated that the linear analytical solutions to each method (Eqns. 2.9 and 2.18) predict identical behaviour. The results of the rotation model are compared to those of the finite element rotation model in Section 3.2.4.
2.3: Overall Deformation

2.3.1: Model Description

The geometry of the overall deformation model is identical to that of the fibre rotation model. An array of rectangular fibres and matrix blocks are oriented at an angle to the applied load, as in Figure 2.6. Unlike the rotation case the overall deformation model distributes the load between the fibres and the matrix.

![Figure 2.6: Overall deformation model](image)

The initial attempts to solve the overall deformation model used the rotation solution as a basis and superimposed the individual deformations of the fibre and the matrix. Difficulties in apportioning the load between the fibres and the matrix prevented a solution from being developed in this manner.

The second approach to the overall analytical model did not consider the fibre rotation as a separate component of deformation, but used equilibrium and compatibility considerations to solve for the deformation of the model. An incremental methodology is proposed to include the fibre rotation effects in the overall deformation.
2.3.2: Solution Method

Consider an off-axis element such as Figure 2.6. The applied force, $P_y$, can be treated as an average force per unit area, or nominal composite stress, $\sigma_{yc}$. A full description of the deformation of the element requires the calculation of the complete strain state of the fibre and the matrix. As for the fibre rotation solution it is assumed that the deformation of the element is uniform, with the effect that the fibre and matrix strains and stresses are constant within each constituent.

Six relationships are required to solve for the six unknown strain components, $\varepsilon_{xf}$, $\varepsilon_{yf}$, $\varepsilon_{xyf}$, $\varepsilon_{xm}$, $\varepsilon_{ym}$ and $\varepsilon_{xym}$. Conditions which can be applied to the off-axis ply include:

**Global equilibrium of the element:**

\[
\begin{align*}
\Sigma F_y &= 0 \quad (2.19) \\
\Sigma F_x &= 0 \quad (2.20) \\
\Sigma M_z &= 0 \quad (2.21)
\end{align*}
\]

**Compatibility of the fibres and the matrix:**

\[
\begin{align*}
\varepsilon_{2f} &= \varepsilon_{2m} \quad (2.22) \\
\sigma_{1f} &= \sigma_{1m} \quad (2.23) \\
\sigma_{12f} &= \sigma_{12m} \quad (2.24)
\end{align*}
\]

**Conservation of energy**

Work done by the applied load = total strain energy of the fibres and the matrix

\[
\int P_y \, dy = SE_f + SE_m \quad (2.25)
\]

The constitutive equations for the fibres and matrix materials allow each of these conditions to be expressed in terms of the $xy$ fibre and matrix strains or stresses. The overall deformation of the representative off-axis element can then be calculated from the constituent strains and the geometry of the element. Each of the above conditions is detailed below.
Global equilibrium of the element:

Consider the fibres and matrix at the edges of the off-axis element, as in Figure 2.7:

![Diagram of off-axis element](image)

**Figure 2.7: Edges of an off-axis element**

The width and height fractions of the fibre and matrix are defined as:

\[
\begin{align*}
    w_f &= \frac{W_f}{W_f + W_m} \quad (2.26) \\
    h_f &= \frac{H_f}{H_f + H_m} \quad (2.27) \\
    w_m &= \frac{W_m}{W_f + W_m} \quad (2.28) \\
    h_m &= \frac{H_m}{H_f + H_m} \quad (2.29)
\end{align*}
\]

Individual width and height fractions are used to allow the description of a deformed element. For an undeformed element of unit thickness:

\[
\begin{align*}
    w_f &= h_f = V_f \quad (2.30) \\
    w_m &= h_m = (1 - V_f) \quad (2.31)
\end{align*}
\]

For equilibrium of the element in the y direction: \( \sum F_y = 0 \)

\[
\sigma_{yc} = w_f \sigma_{yf} + w_m \sigma_{ym}
\]  

(2.32)

Similarly in the x direction: \( \sum F_x = 0 \)

\[
\sigma_{xc} = h_f \sigma_{xf} + h_m \sigma_{xm}
\]  

(2.33)

For equilibrium about the global z axis: \( \sum M_z = 0 \)

\[
\sigma_{xyc} = \frac{(w_f + h_f)}{2} \sigma_{yf} + \frac{(w_m + h_m)}{2} \sigma_{ym}
\]  

(2.34)
Compatibility of the fibres and the matrix:

For compatibility, the strains in the direction of the fibres must be equal to those of the matrix material:

$$\varepsilon_{2f} = \varepsilon_{2m} \tag{2.22}$$

For the case of isotropic fibre and matrix materials, in a state of plane stress, this can be expressed in terms of stresses as:

$$\frac{\sigma_{2f} - V_f \sigma_{1f}}{E_f} = \frac{\sigma_{2m} - V_m \sigma_{1m}}{E_m} \tag{2.35}$$

Transforming the stresses to the xy coordinate system:

$$\sigma_{1f} = \sigma_{x_f} \cos^2 \theta + \sigma_{y_f} \sin^2 \theta + 2 \sigma_{xy_f} \sin \theta \cos \theta \tag{2.36}$$

$$\sigma_{2f} = \sigma_{x_f} \sin^2 \theta + \sigma_{y_f} \cos^2 \theta - 2 \sigma_{xy_f} \sin \theta \cos \theta \tag{2.37}$$

$$\sigma_{1m} = \sigma_{x_m} \cos^2 \theta + \sigma_{y_m} \sin^2 \theta + 2 \sigma_{xym} \sin \theta \cos \theta \tag{2.38}$$

$$\sigma_{2m} = \sigma_{x_m} \sin^2 \theta + \sigma_{y_m} \cos^2 \theta - 2 \sigma_{xym} \sin \theta \cos \theta \tag{2.39}$$

Substitution of equations 2.36-2.39 into 2.35 results in the following expression for the global xy stresses:

$$A \sigma_{x_f} + B \sigma_{y_f} + C \sigma_{xy_f} = D \sigma_{x_m} + E \sigma_{y_m} + F \sigma_{xym} \tag{2.40}$$

Where:

$$A = E_m (\sin^2 \theta - V_f \cos^2 \theta) \quad D = E_f (\sin^2 \theta - V_m \cos^2 \theta)$$

$$B = E_m (\cos^2 \theta - V_f \sin^2 \theta) \quad E = E_f (\cos^2 \theta - V_m \sin^2 \theta)$$

$$C = -2 E_m \sin \theta \cos \theta (1 + V_f) \quad F = -2 E_f \sin \theta \cos \theta (1 + V_m)$$

For stress compatibility at the fibre/matrix interface the transverse stress of the fibres and the matrix must be equal, i.e.

$$\sigma_{1f} = \sigma_{1m} \tag{2.23}$$

Or in terms of xy stresses:

$$\sigma_{x_f} + \tan^2 \theta \sigma_{y_f} + 2 \tan \theta \sigma_{xy_f} = \sigma_{x_m} + \tan^2 \theta \sigma_{y_m} + 2 \tan \theta \sigma_{xym} \tag{2.41}$$

Similarly for the shear stresses:

$$\sigma_{12f} = \sigma_{12m} \tag{2.24}$$

And hence:

$$- \sigma_{x_f} + \sigma_{y_f} + \frac{(\cos^2 \theta - \sin^2 \theta)}{\sin \theta \cos \theta} \sigma_{xy_f} = - \sigma_{x_m} + \sigma_{y_m} + \frac{(\cos^2 \theta - \sin^2 \theta)}{\sin \theta \cos \theta} \sigma_{xym} \tag{2.42}$$
Conservation of energy

Work done by the applied load = total strain energy of the fibres and the matrix

\[ \int P_y \ dy = SE_f + SE_m \]  \hspace{1cm} (2.25)

For linear deformation of a unit volume of material:

\[ \frac{\sigma_{yc} \varepsilon_{yc}}{2} = \mu_f V_f + \mu_m (1 - V_f) \]  \hspace{1cm} (2.43)

Where the strain energy densities of the fibres and the matrix are given by:

\[ \mu_f = \frac{1}{2E_f} (\sigma_{xf}^2 + \sigma_{yf}^2 - 2 V_f \sigma_{xf} \sigma_{yf} + 2 (1 + V_f) \sigma_{xyf}^2) \]  \hspace{1cm} (2.44)

\[ \mu_m = \frac{1}{2E_m} (\sigma_{xm}^2 + \sigma_{ym}^2 - 2 V_m \sigma_{xm} \sigma_{ym} + 2 (1 + V_m) \sigma_{xym}^2) \]  \hspace{1cm} (2.45)

Calculation of xy stresses and strains.

Substitution and simplification of equations 2.32, 2.33, 2.34, 2.40, 2.41 and 2.42 results in six expressions for the xy stresses of the fibres and the matrix (Equations 2.46 to 2.51). The energy balance of equation 2.43 provides a check of the calculated values.

\[ \sigma_{xyf} = \frac{(G - H)}{(1 - J)} \]  \hspace{1cm} (Coefficients in Appendix B2)  \hspace{1cm} (2.46)

\[ \sigma_{yf} = \sigma_{yc} - \frac{\sigma_{xyf}}{\tan \theta} \]  \hspace{1cm} (2.47)

\[ \sigma_{xf} = - \frac{h_m \tan \theta}{w_m} \sigma_{xyf} \]  \hspace{1cm} (2.48)

\[ \sigma_{xym} = - \sigma_{xyf} \frac{w_f}{w_m} \]  \hspace{1cm} (2.49)

\[ \sigma_{ym} = \frac{(\sigma_{yc} - \sigma_{yf})}{w_m} \]  \hspace{1cm} (2.50)

\[ \sigma_{xm} = - \sigma_{xf} \frac{h_f}{h_m} \]  \hspace{1cm} (2.51)
The material constitutive relations allow the fibre and matrix strains to be calculated from the stresses. The global composite strains can be calculated from the constituent strains and the composite geometry:

\[
\varepsilon_{xc} = \varepsilon_{xf} w_f + \varepsilon_{xm} w_m \tag{2.52}
\]

\[
\varepsilon_{yc} = \varepsilon_{yf} h_f + \varepsilon_{ym} h_m \tag{2.53}
\]

\[
\varepsilon_{xyc} = \varepsilon_{yf} \frac{(w_f + h_f)}{2} + \varepsilon_{xym} \frac{(w_m + h_m)}{2} \tag{2.54}
\]

The rotation of the fibres can be calculated from the global strains of the composite:

\[
\phi = \text{atan} \left( \cos\theta \left( \varepsilon_{yc} - \varepsilon_{xc} \right) + \cos\theta \varepsilon_{xyc} \right) \tag{2.55}
\]

The solution process was coded as a Pascal computer programme. The programme listing is included as Appendix B3. Table 3.15, Section 3.3.3 compares the analytical model to the linear finite element models.

During the analytical modelling described above, the fibre and matrix materials have been treated as isotropic materials in a state of plane stress. The assumption of isotropy is valid for most polymer matrix materials, and for glass fibres. However as discussed in Section A2.2.2, carbon fibres are usually treated as transversely isotropic. The finite element modelling used an orthotropic material definition for the fibres (Section B3.3.3) to achieve the required constitutive behaviour. The analytical model could equally be derived for the case of transversely isotropic fibres.

The assumption of plane stress for the inhomogeneous ply was used for compatibility with existing micromechanical and macromechanical theories. In typical thin laminated composites each ply is generally considered to be in a state of plane stress (Section A2.2.3). The traditional mechanics of materials micromechanics models treat the constituents as plane stress (Section A2.2.2). In reality, the constituent materials will not be in a state of plane stress. The transverse stiffness of the fibres will provide a constraint to the out of plane deformations of the matrix material. Hence the matrix material will be in a condition between plane stress and plane strain, resulting in a slight increase in effective stiffness compared to the plane stress case.
Application of the overall deformation model

The model described above provides a solution to the linear deformation of a general off-axis ply. In the model’s linear form it represents an off-axis micromechanical theory, combining macromechanics and micromechanics into one solution. Its on-axis and off-axis predictions do not significantly differ from those of the conventional orthotropic solution method (Section A2.2).

To include the effects of fibre orientation into this model requires the use of an incremental approach, similar to that of Luo and Chou [7]. In the case of such a non-linear, homogeneous orthotropic model the principal material direction, and hence implicitly the fibre orientation, is altered at each incremental step.

The incremental Rigid Body Motion model explicitly alters the fibre orientation. The deformation of the matrix material has the additional effect of changing the spacing of the fibres. The differences between the homogenous orthotropic model and the Rigid Body Motion model arise because of this change in the microstructural geometry.

Each increment of the homogenous orthotropic theory results in a ply which has identical on-axis (relative to fibre direction) properties, but a changed orientation. In the RBM model the change of microstructural geometry results in the deformed ply representing a new situation with an altered fibre direction and different effective on-axis properties. The subsequent increment uses the deformed element, Figure 2.8, to calculate its initial geometry.

![Initial Element vs. Deformed Initial Element](Figure 2.8: Deformed RBM element)
The consequent steps in the incremental Rigid Body Motion model are outlined in Figure 2.9. The load is applied incrementally, with the linear model being used to solve for the incremental strains at each step. The fibre direction and geometry of the composite are updated between each increment.

![Figure 2.9: Steps in the incremental Rigid Body Motion model](image)

Attempts were made to solve the incremental Rigid Body Motion model analytically. The approaches were not successful due to difficulties in updating the geometrical parameters and the total strains. Finite element modelling of the overall model was carried out to provide a solution to the incremental model and to further investigate the microstructural deformations.
B.3: Finite Element Modelling

This chapter details the finite element modelling of the Rigid Body Motion theory. Results relevant to the verification of the finite element models are presented in this chapter. The overall results from the finite element analyses are compared with the analytical and experimental results in Chapter 5 of Part A.

3.1: Introduction

Chapter B.2 presented analytical solutions for the fibre rotation and overall deformation models described in chapter B.1. Both of these analytical solutions were for linear, small strain cases. The finite element modelling described in this chapter had two main aims: to provide verification of the two linear analytical solutions, and to allow the overall deformation model to be solved for the non-linear large strain case.

The finite element (FE) modelling consisted of two main stages. The first involved modelling the fibre rotation component of deformation only, providing verification of the analytical rotation solution, and allowing investigation of the validity of the various assumptions made in the modelling process. During the second stage, FE models were created to represent representative cells of the overall Rigid Body Motion theory. Models were created with fibre orientations which matched the experimental specimens, thus providing comparisons with both analytical and experimental results.
3.2: Fibre Rotation

3.2.1: Introduction

Finite element (FE) modelling of the fibre rotation has provided verification of the theoretical analysis and enabled parametric studies to be made of assumptions made in the analytical model. This section concentrates on the FE modelling of the fibre rotation only, the modelling of the complete Rigid Body Motion theory being described in section 3.3.

The general purpose geomechanical finite element analysis package ABAQUS [95], running on an IBM 4341 computer, was used to model the rotation component of the Rigid Body Motion theory. Although ABAQUS is a versatile and powerful software package, in this case its use was limited by the lack of preprocessing capabilities in the particular installation.

3.2.2: Model Development

The finite element model used the same geometry as the analytical theory, i.e. square fibres interspersed with rectangular blocks of matrix. The required fibre angles and spacing were achieved by altering the mesh shape. The rotation component of the fibre load was applied to each end of the fibres as a nodal force. Fibre rotation was calculated from the displacements of fibre nodes at the centre of the fibres. A typical ABAQUS input file for the single fibre model can be found in Appendix B4. A total of 82 FE analysis runs were performed of the rotation behaviour.

The initial analyses had three fibres within a semi-infinite matrix block. Figure 3.1 shows a typical deformed shape. Parameters investigated included fibre length, material stiffnesses, element types and mesh density. It was found that a mesh as in Figure 3.1 of 8 node shell elements (CPS8R, CPSE8R) gave results within 1% of a mesh using 20 node 3D brick elements (C3D20R). Doubling the mesh density of shell elements significantly increased computation time and gave results less than 0.5% different to the mesh density shown in Figure 3.1.
To better represent the continuous nature of the analytical model, the finite element mesh was modified to give an array of seven fibres interspersed with matrix blocks as in Figure 3.2. Multi Point Constraint (MPC) boundary conditions were used to make the x and y direction displacements (Ux and Uy) of the left and right sides of the mesh equal, thereby allowing continuous deformation without edge effects. This mesh was used for the development of appropriate boundary conditions and some initial investigations into the effects of material stiffness and fibre angle.

The use of the MPC boundary condition enabled the finite element model to be reduced to a single fibre with a single matrix block, Figure 3.3. By joining the left and right sides of the mesh with a MPC the model was effectively identical to the seven fibre case. The single fibre model gave results within 0.5% of the seven fibre model with substantially reduced computation time.
The single fibre model was used as the basis for all of the parametric studies described in Section 3.2.4. The models were treated as linear to conform to the state of development of the theoretical solution. The subsequent finite element modelling described in Section 3.3 investigated the large displacement, nonlinear behaviour of the overall rigid body motion theory.

The MPC type boundary conditions used for this initial modelling imposed a condition of no transverse strain on the model. This condition was acceptable for the parametric studies as the analytical model could be formulated to model this case as well as the more realistic situation of no transverse stress. The overall deformation modelling (Section 3.3) developed models and boundary conditions to suit the transverse stress free case.

**3.2.4: Parametric Studies**

**Fibre Stiffness:**

A key assumption of the Rigid Body Motion theory is that the relative stiffnesses of the fibres and the matrix are such that the fibre can be treated as virtually rigid in shear and bending. The finite element analysis allowed the validity of this fibre rigidity assumption to be investigated. Figure 3.4 demonstrates the change in fibre rotation with variations in relative stiffness of the two components.

When fibre and matrix moduli are equal there is no rotation, both components undergoing shear only. As the fibre / matrix stiffness ratio \( \text{Ef/Em} \) increases, the rotation increases and the shear of the fibre decreases. With a typical glass / thermoset resin stiffness ratio of 20/1 there is approximately 5% difference between the actual...
rotation (finite element) and that for the case of completely rigid fibres (finite element and analytical). At a stiffness ratio of 100/1 (e.g. Boron / thermoset resin) there is virtually no difference to the completely rigid case. In the case of carbon fibres and a thermoset resin matrix (50/1) there is approximately 2% difference. The errors would be even smaller for the case of less stiff resins.

![Figure 3.4: Fibre rotation vs. relative stiffness of fibre and matrix](image)

Fibre Load:

The finite element model confirms the linearity of the fibre rotation vs. load. Figure 3.5 demonstrates the very good agreement between the analytical model and the finite element results.

![Figure 3.5: Fibre rotation vs. transverse load](image)
Fibre Angle:

The analytical model predicts rotations almost identical to the finite element results, as in Figure 3.6. It should be noted that in this comparison the fibre load is constant with angle, thus a rotation exists even at 0°. In the true case the rotation component of the fibre load will vary with angle, and be zero at 0°.

![Figure 3.6: Fibre rotation vs. fibre angle](image)

Fibre Volume Fraction:

The agreement between the methods is worse at low fibre volume fractions than at high, as in Figure 3.7. This may be due to the restricted length of the finite element model. At low volume fractions the stresses from the finite element analysis were not completely uniform in the central region of the model.

![Figure 3.7: Fibre rotation vs. fibre volume fraction](image)
3.3: Overall Deformation

3.3.1: Introduction

Parallelogram shaped meshes were used for initial finite element modelling of the RBM theory due to the ease of their mesh generation. The parallelogram shape results in a simple mesh shape, Figure 3.3, but complicates the boundary conditions of the model. Significant effort was applied to investigating suitable boundary conditions for the parallelogram meshes and evaluating alternative model geometries such as tubes. Appropriate parallelogram model boundary conditions were not able to be developed for all required loading conditions. Tubular model geometries generated additional boundary condition difficulties, were complicated to mesh and expensive to analyse.

The finite element analyses of the overall deformation were carried out with EMRC NISA software, running on 80386 and 80486 based personal computers. The powerful pre-processing capabilities of the NISA package enabled the required mesh shapes to be readily created.

A rectangular shaped off-axis model, Figure 3.8(g), is more complicated to mesh than a parallelogram shaped model, Figure 3.8 (a,b,c,d), but has relatively simple boundary condition requirements. Similar models to that shown in Figure 3.8(g), were created to represent each of the fibre orientations tested in the experimental program.
3.3.2: Model Development

A range of mesh shapes and boundary conditions were investigated during the development of the final finite element models. Figure 3.8 presents examples of the various mesh geometries.

Figure 3.8: Finite element mesh geometries
Planar Models

The single fibre parallelogram shaped planar model, Figure 3.8.a, had the same geometry as used in the ABAQUS rotation modelling (Section 3.2.3) with two parallel rows of elements. The two fibre planar model, with four rows of elements, Figure 3.8.b, was created to enable the overall deformation of the composite to be calculated from the relative displacements of two fibres.

To evaluate suitable boundary conditions, the "one fibre" and "two fibre" planar models were initially modelled as homogeneous isotropic. The same isotropic material was used for both the fibre and matrix elements. It was anticipated that appropriate boundary conditions could be developed to represent the symmetry inherent in the representative volume element. Difficulties in this process led to the development of the wide planar models, the tubular models, and finally the rectangular models. Table 3.1 compares the stress fields generated within homogeneous isotropic "one fibre", "two fibre", and tubular models by combined tensile and shear loads.

Table 3.1: Stress fields within isotropic planar and tubular models

<table>
<thead>
<tr>
<th>Filename</th>
<th>Type</th>
<th>CPDISP</th>
<th>(\sigma_{xx})</th>
<th>(\sigma_{yy})</th>
<th>(\sigma_{xy})</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO1UY</td>
<td>1 Fibre</td>
<td>UY</td>
<td>-54.0</td>
<td>54.1</td>
<td>93.75</td>
</tr>
<tr>
<td>ISO1AL</td>
<td>1 Fibre</td>
<td>UY&amp;UX</td>
<td>16.23</td>
<td>54.1</td>
<td>93.75</td>
</tr>
<tr>
<td>ISO2UY</td>
<td>2 Fibre</td>
<td>UY</td>
<td>-54</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>ISO2AL</td>
<td>2 Fibre</td>
<td>UY&amp;UX</td>
<td>-28</td>
<td>54</td>
<td>93.6</td>
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<tr>
<td>SMLISO</td>
<td>Tubular</td>
<td>n/a</td>
<td>0</td>
<td>54.17</td>
<td>91.55</td>
</tr>
</tbody>
</table>

It is clear that neither of the coupled displacement boundary conditions adequately represented the symmetry required. The tubular model gave approximately the correct stress field. The planar models gave different results depending on the coupled displacements used.

To investigate the origin of the \(\sigma_{xx}\) stress, selected 1 fibre models were run with only the shear load, and a range of coupled displacements applied. Table 3.2 summarises the results.
Table 3.2: Single fibre models with shear load only

<table>
<thead>
<tr>
<th>Filename</th>
<th>Type</th>
<th>CPDISP</th>
<th>$\sigma_{xx}$</th>
<th>$\sigma_{yy}$</th>
<th>$\sigma_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO1SH</td>
<td>1 Fibre</td>
<td>UY</td>
<td>-54.1</td>
<td>0</td>
<td>93.75</td>
</tr>
<tr>
<td>SHISRZ</td>
<td>1 Fibre</td>
<td>UY&amp;ROTZ</td>
<td>-54.1</td>
<td>0</td>
<td>93.75</td>
</tr>
<tr>
<td>SHIXY</td>
<td>1 Fibre</td>
<td>UY&amp;UX</td>
<td>0</td>
<td>0</td>
<td>93.75</td>
</tr>
<tr>
<td>SHIXYZ</td>
<td>1 Fibre</td>
<td>UY,UX&amp;RZ</td>
<td>0</td>
<td>0</td>
<td>93.75</td>
</tr>
</tbody>
</table>

Apparently UY alone, or UY and ROTZ, did not adequately represent the degree of symmetry required. The -54.1 MPa $\sigma_{xx}$ was caused by the inadequate shear transfer between the left and right boundaries. The inclusion of UX achieved the required coupling. However, coupling UX resulted in the 16.23 MPa stress of ISO1AL (Table 3.1), caused by creating a zero strain constraint in the x direction. The 16.23 MPa stress was caused by Poisson effects when a tensile load is applied. To confirm this, tensile loads were applied to each coupling case, Table 3.3. The analytical cases are for left and right sides free or fixed:

Table 3.3: Single fibre models with tensile load only

<table>
<thead>
<tr>
<th>Filename</th>
<th>Type</th>
<th>CPDISP</th>
<th>$\sigma_{xx}$</th>
<th>$\sigma_{yy}$</th>
<th>$\sigma_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>Free</td>
<td></td>
<td>0</td>
<td>54.14</td>
<td>0</td>
</tr>
<tr>
<td>Analytical</td>
<td>Fixed</td>
<td></td>
<td>16.24</td>
<td>54.14</td>
<td>0</td>
</tr>
<tr>
<td>TNIUY</td>
<td>1 Fibre</td>
<td>UY</td>
<td>0</td>
<td>54.14</td>
<td>0</td>
</tr>
<tr>
<td>TNIIXY</td>
<td>1 Fibre</td>
<td>UY&amp;UX</td>
<td>16.2</td>
<td>54.14</td>
<td>0</td>
</tr>
<tr>
<td>TNIIXYZ</td>
<td>1 Fibre</td>
<td>UY,UX&amp;RZ</td>
<td>16.2</td>
<td>54.14</td>
<td>0</td>
</tr>
</tbody>
</table>

The results were as expected: UY was adequate for a tensile load with free edges, while UY & UX (with or without ROTZ) represented the fixed edge case. However, none of the combinations represented free edges for a general loading case.

Local coordinate systems were created parallel to the left and right boundaries of the model, and used as the basis of the coupled displacements. The fully coupled models (UX, UY and UX, UY, RZ) gave the same results as those for the previous case of the global coordinate axes. The other models gave different, and incorrect results (Table 3.4.).
Table 3.4: CPDISP with local coordinate systems on the boundaries

<table>
<thead>
<tr>
<th>Filename</th>
<th>Type</th>
<th>CPDISP</th>
<th>$\sigma_{xx}$</th>
<th>$\sigma_{yy}$</th>
<th>$\sigma_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical Free</td>
<td></td>
<td>0</td>
<td>54.14</td>
<td>93.7</td>
</tr>
<tr>
<td>LCSUYR</td>
<td>1 Fibre UY</td>
<td></td>
<td>-126</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>LCSXYR</td>
<td>1 Fibre UY&amp;UX</td>
<td></td>
<td>16</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>LCSYZR</td>
<td>1 Fibre UY&amp;RZ</td>
<td></td>
<td>-126</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>LCSALR</td>
<td>1 Fibre UX,UX&amp;RZ</td>
<td></td>
<td>16</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>LCSUXR</td>
<td>1 Fibre UX</td>
<td></td>
<td>162</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>LCSXZR</td>
<td>1 Fibre UX&amp;RZ</td>
<td></td>
<td>162</td>
<td>54.1</td>
<td>93.7</td>
</tr>
</tbody>
</table>

Since the degree of coupling appeared to be very dependent on the degree of UX constraint, a series of models was created with Multi Point Constraint (MPC) equations linking the UX of the left and right edges. All of the models (Table 3.5) used a CPDISP constraint to link the UY displacements. The models differed in the type of MPC equation used to prescribe the UX displacements. The three types were:

#1: The X displacements of the lower half side nodes were defined to be of equal magnitude and opposite sign to those of the respective nodes on the upper side. There were no direct X constraints between sides.

#2: The X displacements of the lower left side nodes were defined to be equal to negative that of the corresponding upper left side nodes, plus the x displacement of the centre left side node. The right side MPC was the same as #1. This MPC allowed Poisson induced transverse deformation.

#3: The X displacements of the left side nodes were prescribed to be equal to those of the corresponding right side nodes plus the X displacement of the base left side node. This modelled only the upper half of the previous cases, hence the nodes at the base of the model were constrained in the Y direction and the loads are only applied to the top. ISCHLF constrained the UX of the base left side node to be zero, hence modelling zero transverse strain.
Table 3.5: Single Fibre models with MPC equations prescribing UX of edges

<table>
<thead>
<tr>
<th>Filename</th>
<th>Type</th>
<th>Load Type</th>
<th>MPC</th>
<th>$\sigma_{xx}$</th>
<th>$\sigma_{yy}$</th>
<th>$\sigma_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHIMPC</td>
<td>1 Fibre</td>
<td>Shear</td>
<td>#1</td>
<td>0</td>
<td>0</td>
<td>93.7</td>
</tr>
<tr>
<td>ISOMPC</td>
<td>1 Fibre</td>
<td>Tensile &amp; Shear</td>
<td>#1</td>
<td>16.2</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>ISOMP2</td>
<td>1 Fibre</td>
<td>Tensile &amp; Shear</td>
<td>#2</td>
<td>-51</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>ISTMP2</td>
<td>1 Fibre</td>
<td>Tensile</td>
<td>#2</td>
<td>0</td>
<td>54.1</td>
<td>0</td>
</tr>
<tr>
<td>ISSMP2</td>
<td>1 Fibre</td>
<td>Shear</td>
<td>#2</td>
<td>-52</td>
<td>0</td>
<td>93.7</td>
</tr>
<tr>
<td>ISOHLF</td>
<td>1 Fibre</td>
<td>Tensile &amp; Shear</td>
<td>#3</td>
<td>-51</td>
<td>54.1</td>
<td>93.7</td>
</tr>
<tr>
<td>ISCHLF</td>
<td>1 Fibre</td>
<td>Tensile &amp; Shear</td>
<td>#3</td>
<td>16.2</td>
<td>54.1</td>
<td>93.7</td>
</tr>
</tbody>
</table>

For case #1 the results were completely consistent with the previous models which have resulted in zero transverse strain. The results for MPC #2 were as expected for cases which allowed transverse strain but did not adequately couple the shear deformation. There was a slight difference (4%) to previous cases in the transverse stress generated by the shear load. The half models (case #3), behaved exactly as the full length models. If the left base node was constrained then the results were as for no transverse strain, if not then they matched those of case #2.

**Wider models with no coupling between edges**

Since the edge constraints were a critical factor in the behaviour of the models, a range of wider models were created, with the aim of reducing the influence of the edges. Models that were 5, 10 and 50 fibres wide were constructed, Figure 3.8(e). A Y direction coupled displacement (CPUY) was used between the left and right edges, but no X direction coupling (CPUX) was used. In all cases the shear force related transverse stress was still significant. All stress components were much more nonuniform than with the various UX constraints previously used. It appeared that the width of the models would have to become very large before the transverse stress would become insignificant.
Tubular Models

One reason for the choice of a tubular shape for the experimental specimens was to avoid the difficulties associated with the edge effects of off-axis specimens. Following the same approach, tubular finite element models were constructed to attempt to characterise the overall deformation of composite system, without the edge related difficulties encountered with the previous planar models. Appendix B5 summarises the behaviour of the tubular models, which gave good results for homogeneous material properties but had unacceptable deformed shapes with inhomogeneous materials. The models were also complicated to mesh, and time consuming to analyse.

Summary

One reason for the initial choice of parallelogram shaped models was to allow a simple mesh shape, and to make use of the symmetry inherent in the composite material. However the parallelogram shape resulted in complicated boundary condition requirements. Tubular shaped models, on the other hand, had difficulties setting up the boundary conditions and were time consuming to create and analyse. The various difficulties encountered led to the development of the rectangular models, which were more complicated to mesh than the parallelogram models, but had simpler boundary condition requirements.
3.3.3: Rectangular Models

Introduction

The off-axis rectangular models had more complicated mesh shapes than the parallelogram shaped models due to fibres crossing the boundaries at an angle, as in Figure 3.8(e). However, the boundary conditions were much simpler than for the parallelogram shaped models because there was no shear transfer required between the left and right edges of the model. When appropriately formulated, a rectangular model could be used to model two fibre orientations: i.e. a 30° model became a 60° model when the loads and boundary conditions were rotated by 90°.

Rectangular meshes were created to represent 0°, 15°, 30°, 45°, 60°, 75° and 90° fibre orientation cases. The on-axis models (0° and 90°) represented the traditional rule of mixtures on-axis micromechanics problem, and as such were used to verify the materials, geometry and solution methods used in the off-axis models.

The models were treated as large displacement, geometrically nonlinear problems to accurately simulate the finite strains and fibre rotation. An incremental, total Lagrangian formulation with energy, force and displacement tolerances was used. The solution process used a full Newton-Raphson iterative scheme, with the stiffness matrix updated at each iteration [96].

Mesh Details

As for the analytical case, the finite element models consisted of rectangular fibres interspersed with blocks of matrix material. The number of fibres varied between 5 to 15 depending on the fibre orientation. Each fibre angle required a different number of fibres to ensure compatibility between opposite edges. The models were meshed with a volume fraction of 0.54, as for the experimental specimens. The fibre width, and hence model thickness, was 7 μm for all cases. To improve numerical accuracy, units of mm were used thus minimising the numerical order of the various input parameters.

All models used quadratic, plane stress, isoparametric elements, (NISA NKTP 1,11, NORDR 2). Most elements were 8 node rectangular (NKTP 1), with 6 node triangular elements (NKTP 11) being used where required at mesh edges. Appendix B6 contains plots of the mesh shapes for each fibre orientation. This section presents typical mesh
examples and discusses the characteristics of the meshes. An example NISA input file for an off-axis rectangular model is included as Appendix B7.

**On-axis Models**

A finite element model, Figure 3.9, was constructed to represent a cell of composite material with the fibres parallel to the cell sides. This mesh shape could be used to model the 0° and 90° cases. These two cases, with the corresponding in-plane shear case, constituted the traditional on axis micromechanics problems. For the RBM theory these cases represented the two extremes of the solution. As no fibre rotation can take place at 0° or 90° the RBM theory should reduce to the ROM at these angles.

![Figure 3.9: 0° & 90° finite element mesh shape](image)

The mesh shown in Figure 3.9 consisted of 100 quadratic, plane stress elements (NISA NKTP 1, NORDR 2). A similar model with 200 elements was used to investigate mesh density effects. While X axis symmetry considerations would allow the model size to be reduced for these on axis models, identical boundary conditions as the off-axis models were used for consistency.

**Off-axis Models**

Figure 3.10 shows a typical off-axis mesh shape, in this case for the 30° and 60° models. The 11 fibres were oriented at 30° to the global Y direction axis. This mesh could be used to represent the 30° and 60° fibre orientation cases by a 90° rotation of the boundary conditions. The rectangle’s dimensions differed slightly in each
direction to provide compatibility between opposite edges. A requirement of the boundary conditions used (discussed in the following section) is that each free boundary fibre and matrix node corresponds to an equivalent node on the opposite side of the mesh.

![Figure 3.10: 30° and 60° mesh shape](image)

The mesh consisted of 170 quadratic plane stress elements. The majority of the elements (140) were rectangular (NKTP 1, NORDR 2), with 30 triangular elements (NKTP 11, NORDR 2) where necessary to achieve the correct boundary shape.

The other orientation meshes were of similar form and size to the above model. The number of fibres and the proportions of the models varied slightly in order to ensure edge compatibility of each model. Higher density meshes were constructed for the 0° and 30° cases for verification purposes. Table 3.6 details the various rectangular meshes. Appendix B6 contains mesh plots for each fibre orientation.

### Table 3.6: Rectangular Mesh Details

<table>
<thead>
<tr>
<th>Model</th>
<th>Elements</th>
<th>Nodes</th>
<th>DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/90°</td>
<td>100</td>
<td>341</td>
<td>627</td>
</tr>
<tr>
<td>0/90° High Density</td>
<td>200</td>
<td>1181</td>
<td>3945</td>
</tr>
<tr>
<td>15/75°</td>
<td>395</td>
<td>1226</td>
<td>2397</td>
</tr>
<tr>
<td>30/60°</td>
<td>120</td>
<td>389</td>
<td>735</td>
</tr>
<tr>
<td>30/60° High Density</td>
<td>482</td>
<td>1525</td>
<td>2991</td>
</tr>
<tr>
<td>45°</td>
<td>544</td>
<td>1633</td>
<td>3203</td>
</tr>
</tbody>
</table>
**Boundary Conditions**

Each of these finite element models was a representative volume element (RVE) of a larger continuum. The boundary conditions applied to the model should also represent those occurring when the RVE is within the continuum. To achieve this, the boundary conditions must ensure that equilibrium and compatibility of the RVE are maintained. The applied boundary conditions must be sufficient to achieve the required equilibrium and compatibility, but must not overly constrain the deformation of the model.

As in the experimental work (Part C of this thesis), for this theoretical programme the general continuum can be considered to be of thin walled tubular form, subjected to a uniform tensile load, as in Figure 3.11. Such a tube will extend in the direction of the load, contract in the hoop direction, and rotate due to shear. The significance of assuming such a geometry is that the deformed upper and lower surfaces of the tube, and hence the RVE, remain parallel to their original positions. In the case of a rectangular tensile specimen, the total shear remains the same but all sides of the specimen may rotate relative to their original positions.

![Tubular Geometry](image1)

![Deformed RVE](image2)

**Figure 3.11: Representative volume element within a tubular continuum**

Allowing this form of deformation within the FE models requires that corresponding Y direction displacements on the left and right sides of the model are equal. This was achieved by coupling the Y direction displacements of corresponding nodes on the left and right edges, as in Figure 3.12. The X direction displacements were
unrestrained to allow transverse and shear deformation of the model. The boundary requirements of the rectangular model were simpler than the parallelogram models (Section 3.3.2) because shear transfer was not required between the left and right sides for overall equilibrium of the rectangular RVE. Although the FE models were in overall equilibrium, one node was constrained to prevent rigid body movement of the complete model. A fibre centre node as near as possible to the overall centre of the model was constrained by $U_x = U_y = 0.0$. This node also provided the reference point for calculating deformations of the model.

![Diagram](image)

**Figure 3.12: Left, right and centre boundary conditions**

The boundary conditions in Figure 3.12 were sufficient for homogeneous orthotropic materials. For the inhomogeneous case, additional constraints were required for compatibility of the top and bottom edges. These constraints minimised the localised deformation caused by the application of a uniform stress to materials of different stiffnesses. The constraints, Figure 3.13, took the form of coupling the Y direction displacements of the midpoint edge nodes of the fibre and matrix blocks. The nodes at the fibre/matrix boundary interface were not constrained. The effects of this constraint are discussed further in the model verification section which follows. For consistency, the boundary conditions of Figures 3.12 and 3.13 were applied to all models, including the homogeneous cases. For the 60°, 75° and 90° models the boundary conditions were rotated through 90° to correspond to the applied loads.
Chapter B.3: Finite Element Modelling

Figure 3.13: Top edge boundary conditions
(Similarly for bottom edge)

Loads

A uniform tensile stress (as a pressure on element faces) was applied to the appropriate edges of the models as in Figure 3.14. For the $0^\circ$ shear case, and during initial development of the off-axis models, the distributed loads were applied as nodal forces. As expected, when the nodal forces were distributed correctly to suit the quadratic elements, there was no difference in results compared to when the load was applied as an edge pressure. Nodal forces were required for the $0^\circ$ shear case, as an edge pressure could not be used to simulate the shear force. The magnitudes of the applied loads varied with fibre orientation, and hence model stiffness.

Figure 3.14: Load application
Material Properties

The material properties of the constituents play a crucial role in the accuracy and relevance of any micromechanical model, analytical or numerical. Epoxy resins are basically isotropic and the stiffnesses of various formulations are well documented [54]. Reliable mechanical tests can easily be carried out on resin samples [55]. However, the size and geometry of typical fibres (diameters of the order of 10 μm) render mechanical tests difficult and in some cases virtually impossible to perform. Longitudinal stiffness is the only fibre property which can be readily measured, and even in this case alternative methods give different results [56, 57]. The fibre axial shear modulus can be measured by a torsion pendulum but the results are not considered to be very reliable [50]. The fibre transverse stiffness, Poisson's ratios and transverse shear modulus are all extremely difficult to measure.

Isotropic fibres such as glass can be characterised by measuring the longitudinal stiffness and estimating the Poisson's ratio. However carbon fibres are highly anisotropic, typically being treated as transversely isotropic with five required stiffness coefficients [50, 52, 54]. Due to the difficulty of direct measurement of fibre properties a common approach taken (in the case of isotropic fibres also) is to measure on-axis composite ply properties and then use a micromechanical model to "back calculate" the fibre properties. The micromechanical models used include simple strength of materials theories [13], semiempirical models [44] and elasticity solutions of varying sophistication [47, 50, 51, 52, 53]. Clearly the fibre properties estimated by such a process depend on the micromechanical theory used. Hence there is a danger in using material properties which have been "back calculated" from one theory as the input to a different theory, particularly for the purposes of theoretical validation.

A computer program, MICROMECH, (Appendix B8), was written to use six conventional micromechanical theories:

- Rule of mixtures [15, 13, 17]
- Rule of mixtures with transverse Poisson's effects [15]
- Empirical stress partitioning [13]
- Theoretical stress partitioning [15]
- Chamis' simplified equations [97, 98]
- Composite Cylinder Assemblage model [50, 4]

The MICROMECH program was used to "back calculate" the fibre properties from the averaged 0°, 90° and shear tube test results. Table 3.7 lists the fibre properties
predicted by each theory. The empirical stress partitioning model was not used for this comparison because it performs the reverse process of matching known constituent properties to experimental data.

**Table 3.7: Carbon fibre properties back calculated from tubular test results**

<table>
<thead>
<tr>
<th>Micromechanics Model</th>
<th>Calculated Fibre Engineering Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_L$ (GPa)</td>
</tr>
<tr>
<td>ROM</td>
<td>203</td>
</tr>
<tr>
<td>ROM Poisson</td>
<td>203</td>
</tr>
<tr>
<td>Stress Partitioning</td>
<td>203</td>
</tr>
<tr>
<td>Chamis</td>
<td>203</td>
</tr>
<tr>
<td>CCA</td>
<td>203</td>
</tr>
</tbody>
</table>

For $E_m = 5$ GPa, $v_m = 0.35$, fibre volume fraction = 0.54,

$E_{LC} = 112$ GPa, $E_{TC} = 6.67$ GPa, $v_{LTC} = 0.31$, $G_{LTC} = 3.93$ GPa

* n/a denotes variable not applicable to particular model

As discussed in Section A 2.2.2, the ROM theories agreed well with the other models for $E_L$ and $v_{LT}$, however, they did not accurately model $E_T$ or $G_{LT}$. There are very large differences between the fibre shear moduli calculated by the ROM theories and those calculated by the other approaches. The three more sophisticated models provided similar estimates of $E_T$. Chamis' equations gave a significantly different value for $G_{LT}$ than the stress partitioning or CCA models.

The stress partitioning and CCA models use $v_{TT}$ and $E_T$ to predict $E_T$ of the composite, resulting in two variables to be extrapolated from one experimental result. Fortunately $E_T$ is only weakly dependent on $v_{TT}$ allowing an estimate of $v_{TT}$ to be made and then $E_T$ to be calculated from this and the experimental value for $E_{TC}$. A parametric study of the two theories revealed that variations in $v_{TT}$ from 0.20 to 0.40 caused percentage differences in predicted $E_{TC}$ of only 0.5% and 1% respectively for each of the theories. As the fibres are widely accepted to be transversely isotropic [50, 52, 54], it is unlikely that $v_{TT}$ will be outside of this range. An intermediate value of 0.30 was used for this work. A selection of quoted properties for carbon fibres is presented in Table 3.8. As in this study, most of these values were "back calculated" from lamina properties.
Table 3.8: Engineering constants for carbon fibres

<table>
<thead>
<tr>
<th>Source (Reference)</th>
<th>How Calculated</th>
<th>Fibre</th>
<th>$E_L$ GPa</th>
<th>$E_T$ GPa</th>
<th>$\nu_{LT}$</th>
<th>$G_{LT}$ GPa</th>
<th>$\nu_{TT}$</th>
<th>$G_{TT}$ GPa</th>
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</thead>
<tbody>
<tr>
<td>47</td>
<td>from experimental data</td>
<td>Thornel 25</td>
<td>165</td>
<td>14</td>
<td>0.30</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>From 47?</td>
<td>?</td>
<td>165</td>
<td>14</td>
<td>0.30</td>
<td>0.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>CCA from ultrasonic data</td>
<td>Modmor II</td>
<td>233</td>
<td>15.6</td>
<td>0.35</td>
<td>23.8</td>
<td>0.45</td>
<td>5.5</td>
</tr>
<tr>
<td>51</td>
<td>CCA from ultrasonic data</td>
<td>Modmor I</td>
<td>400</td>
<td>8.6</td>
<td>0.35</td>
<td>13.7</td>
<td>0.53</td>
<td>2.8</td>
</tr>
<tr>
<td>52</td>
<td>CCA from ultrasonic data</td>
<td>Modmor II</td>
<td>232</td>
<td>15</td>
<td>0.28</td>
<td>24</td>
<td>0.49</td>
<td>5.02</td>
</tr>
<tr>
<td>50</td>
<td>CCA from experimental data</td>
<td>?</td>
<td>345</td>
<td>9.66</td>
<td>0.20</td>
<td>2.07</td>
<td>0.30</td>
<td>3.72</td>
</tr>
<tr>
<td>15</td>
<td>?</td>
<td>T300</td>
<td>230</td>
<td>16.6</td>
<td>0.20</td>
<td>8.27</td>
<td>0.41</td>
<td>5.89</td>
</tr>
<tr>
<td>99</td>
<td>?</td>
<td>T300</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This work</td>
<td>Stress Partitioning</td>
<td>T300</td>
<td>203</td>
<td>7.7</td>
<td>0.28</td>
<td>9.2</td>
<td>0.30</td>
<td>2.96</td>
</tr>
<tr>
<td>This work</td>
<td>Chamis</td>
<td>T300</td>
<td>203</td>
<td>7.6</td>
<td>0.28</td>
<td>6.6</td>
<td>2.88</td>
<td></td>
</tr>
<tr>
<td>This work</td>
<td>CCA</td>
<td>T300</td>
<td>203</td>
<td>7.6</td>
<td>0.28</td>
<td>9.2</td>
<td>0.30</td>
<td>2.88</td>
</tr>
</tbody>
</table>

The higher longitudinal modulus fibres are expected to have reduced transverse and shear stiffnesses due to the more aligned nature of their microstructure, and this is confirmed in the above data. However there are substantial variations between the quoted values for the intermediate modulus fibres also. There are significant differences between the various transverse modulus values and the longitudinal shear modulus values. A wide range of transverse Poisson's ratio values are quoted, presumably due to the apparent insensitivity of the transverse modulus of the composite to this parameter.

The calculated longitudinal stiffness of the fibres in this work appears to be lower than for similar (T300) fibres in the other studies presented above. Given the accepted reliability of the fibre direction micromechanical models it seems likely that experimental factors have contributed to this. Small variations in fibre orientation will have a significant effect on the modulus of a specimen, as will variations in the fibre volume fraction. The largest variations in experimental results were for the zero degree specimens, with moduli ranging from 102 to 123 GPa. It is possible that the averaged experimental value used for the above micromechanical calculations represents the case of fibres slightly off-axis rather than the true zero degree situation. At a fibre volume fraction of 0.54, a composite modulus of 123 GPa equates to a longitudinal fibre modulus of 225 GPa, which is closer to the quoted values from references [15, 99]
Fibre properties were "back calculated" from the global on-axis (0°, 90° and 0°/90° shear) experimental results using the conventional micromechanical theories in the MICROMECH computer programme. Selected sets of these properties were used as initial values in the 0°, 90° and 0°/90° shear finite element models. The results of the on-axis modelling were used to choose the sets of constituent properties which accurately represented the on-axis behaviour of the Rigid Body Motion theory. These properties were used as the input for the off-axis models. Additional sets of material properties were chosen to demonstrate aspects of the RBM theory's behaviour not significant in the case of the experimental materials.

The finite element modelling of the on-axis cases confirmed that the ROM Poisson micromechanics model most accurately predicted the behaviour of the geometrical case considered in this modelling (Table 3.10). Thus, this model was used to calculate the required constituent and lamina properties listed in Table 3.9.

**Table 3.9: Material properties used in FE modelling**

<table>
<thead>
<tr>
<th>Material Case Description</th>
<th>Fibre</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_L$ GPa</td>
<td>$E_T$ GPa</td>
</tr>
<tr>
<td>1 Homogeneous #1-3*</td>
<td>112</td>
<td>6.67</td>
</tr>
<tr>
<td>2 Exp. Carbon/Epoxy H</td>
<td>203</td>
<td>8.2</td>
</tr>
<tr>
<td>3 Exp. Carbon/Epoxy L</td>
<td>203</td>
<td>7.4</td>
</tr>
<tr>
<td>4 Exp. Carbon/Epoxy M</td>
<td>203</td>
<td>7.9</td>
</tr>
<tr>
<td>5 Ef/Em=10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6 Homogeneous #5</td>
<td>5.86</td>
<td>2.08</td>
</tr>
<tr>
<td>7 Ef/Em=100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>8 Homogeneous #9</td>
<td>54.5</td>
<td>2.36</td>
</tr>
<tr>
<td>9 Ef/Em=1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>10 Homogeneous #9</td>
<td>540.5</td>
<td>2.39</td>
</tr>
</tbody>
</table>

* Numbers are explained in the following text

Cases 1 to 4 were based on the experimental on-axis (0° and 90°) properties. Case 1 represented the homogeneous orthotropic situation. Cases 2, 3 and 4 represented three alternative sets of constituent properties back-calculated from the same experimental
on-axis properties. Small changes in the matrix modulus resulted in large changes in fibre transverse and shear properties. The fibre was treated as transversely isotropic and the matrix as isotropic.

Cases 5, 7 and 9 were chosen to demonstrate the effects of significant changes in relative fibre to matrix modulus ($E_f/E_m$). They did not represent any particular materials, although the (albeit unusual) combination of boron fibres and a low modulus resin could result in an $E_f/E_m$ of 1000. Cases 6, 8 and 10 represented the respective homogenous lamina properties. In these models the fibre and matrix were treated as isotropic.
Model Verification

A series of studies were carried out to verify different aspects of the behaviour of the finite element models. Factors investigated were:

- Correlation with on-axis micromechanics theories
- Load application method
- Behaviour of homogeneous orthotropic models
- Mesh density effects
- Convergence of the nonlinear solution process
- Correlation with analytical RBM solution

Each of these investigations is described below.

**On-axis behaviour**

The 0° and 90° mesh was used to analyse the three load cases of the traditional on-axis micromechanics problem, 0° tensile, 90° tensile and on-axis shear. For the RBM theory these cases represent the two extremes of the solution. As no fibre rotation can take place at 0° or 90°, the RBM theory should reduce to the ROM at these angles.

Two sets of fibre and matrix properties estimated from the experimental results were used for the analyses. Both sets of properties (Case 2 and Case 4, Table 3.9) were estimated from the ROM Poisson micromechanics theory with a slightly different matrix modulus. Table 3.10 compares the overall global XY strains of the models to the analytical predictions for the experimental laminate stiffnesses. There was good agreement between the finite element models and the theoretical solution for all cases.

**Table 3.10: Accuracy of On-axis models**

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Material Case</th>
<th>Percentage Error in Strains</th>
<th>εx</th>
<th>εy</th>
<th>εxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0° Tensile</td>
<td>Case 2</td>
<td>+0.7</td>
<td>0.0</td>
<td>zero strain</td>
<td></td>
</tr>
<tr>
<td>0° Tensile</td>
<td>Case 4</td>
<td>+0.7</td>
<td>0.0</td>
<td>zero strain</td>
<td></td>
</tr>
<tr>
<td>90° Tensile</td>
<td>Case 2</td>
<td>+0.7</td>
<td>-1.1</td>
<td>zero strain</td>
<td></td>
</tr>
<tr>
<td>90° Tensile</td>
<td>Case 4</td>
<td>+0.7</td>
<td>-1.1</td>
<td>zero strain</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>Case 2</td>
<td>zero strain</td>
<td>zero strain</td>
<td>+0.2</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>Case 4</td>
<td>zero strain</td>
<td>zero strain</td>
<td>- 0.2</td>
<td></td>
</tr>
</tbody>
</table>

Note: x and y refer to the global cartesian coordinate system of the FE models, as in Figure 3.9.
Load Application

The application of a nominally uniform distributed load to an inhomogeneous medium can cause localised deformations which influence the overall behaviour of the model. Without prior knowledge of the precise load distribution, and not to influence the solution by assuming a particular distribution, it is necessary to apply the load in such a manner as to ensure an appropriate load distribution. Figure 3.14 portrays the deformed shape of a 0° model without constraints on the load application edges. Alternative methods of load application were evaluated on the 0° model, since the effects are most significant in this case, and because theoretical comparisons can be made. The effects of the method developed were checked for the off-axis situation.

Figure 3.14: 0° model without constraints on load application edges.

The load application problem is identical to that of the analytical model: the applied load has to be distributed correctly between the fibres and the matrix. The initial method evaluated an effective medium approach, analogous to that of [25]. Additional homogeneous material, Figure 3.15, was added to the ends of the model to distribute the applied loads into the fibre and matrix.
Figure 3.15: Effective medium load application method

The difficulty with this method is in choosing the properties of the homogeneous material. In particular, if the Poisson's contraction of the end material is not equal to that of the inhomogeneous region then transverse stresses are generated in the model. In a general off-axis case, the overall transverse behaviour of the inhomogeneous region is unknown (it is part of the required solution), hence the material properties of the homogeneous region are also unknown. An iterative trial and error approach could be adopted, however this method significantly complicates the solution process.

An alternative approach, and that utilised in this work, is to apply the load directly to the edge nodes of the inhomogeneous region, and constrain the nodes appropriately to maintain an edge shape compatible with the Representative Volume Element concept. As shown in Figure 3.14, if unconstrained, the more compliant matrix deforms significantly more than the fibres. This results in a displacement incompatibility between the top and bottom edges of the model. In the 0° case the increased longitudinal strain of the matrix results in a corresponding transverse strain of the matrix, and hence the composite.

The Rule Of Mixtures (ROM) on-axis solution imposes a uniform longitudinal strain on both the fibres and the matrix. A similar, although not quite as strict constraint, can be imposed by coupling the Y direction displacements of the centre nodes only of the fibres and matrix, as in Figure 3.16.
In the 0° case, the $U_y$ of the interface nodes will equal that of the centre nodes. In an off-axis inhomogeneous case this need not be so. The effects of this constraint were evaluated on the 0° model by imposing a fixed displacement to the loading edges, as in the traditional rule of mixtures micromechanics model, and comparing this with the theoretical solution and the constrained and unconstrained finite element cases. Table 3.11 confirms the excellent agreement between the coupled displacement FE model and the applied displacement FE and theoretical cases.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Overall Stiffness of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Longitudinal Modulus (GPa)</td>
</tr>
<tr>
<td>No Constraints</td>
<td>110</td>
</tr>
<tr>
<td>Coupled Displacements</td>
<td>112</td>
</tr>
<tr>
<td>Applied Displacement</td>
<td>112</td>
</tr>
<tr>
<td>Theoretical</td>
<td>112</td>
</tr>
</tbody>
</table>

The unconstrained and coupled displacement cases were compared for the 30° off-axis case. Adding the coupled displacements increased the global Y direction strains by 1% and increased the global shear strain by 0.1%. The transverse strain was virtually zero in this case. Coupled displacements were used for all of the finite element models.
Homogeneous models

At each fibre orientation a case was analysed with homogeneous orthotropic material properties. The identical input file to the inhomogeneous model was used, with only the material data changed. The material properties of the matrix and fibre elements were both altered to be those of the overall experimental laminate. These models provided verification of the loads, boundary conditions, material orientations and strain measurement calculations. Table 3.12 compares the results from these analyses to theoretical orthotropic results. Very good agreement existed in all cases.

Table 3.12: Accuracy of Homogeneous Orthotropic Models

<table>
<thead>
<tr>
<th>Fibre Angle (°)</th>
<th>Percentage Difference to Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_x$</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>0.00</td>
</tr>
<tr>
<td>30</td>
<td>0.00</td>
</tr>
<tr>
<td>45</td>
<td>0.00</td>
</tr>
<tr>
<td>60</td>
<td>0.06</td>
</tr>
<tr>
<td>75</td>
<td>0.00</td>
</tr>
<tr>
<td>90</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Mesh Density effects

As detailed in Table 3.10, the models used for the 0 and 90° analyses gave good agreement with theoretical results. For comparison, higher density 0 and 30° models were also created. The meshes, Appendix B6, had approximately double the number of elements as the standard density models. Table 3.13 compares the results of the high density models to those of the low density cases. There is very good agreement between the models.

Table 3.13: Results of High Density Models

<table>
<thead>
<tr>
<th>Fibre Angle (°)</th>
<th>Percentage Difference to Low Density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_x$</td>
</tr>
<tr>
<td>0</td>
<td>+0.00</td>
</tr>
<tr>
<td>30</td>
<td>+0.17</td>
</tr>
</tbody>
</table>
This theoretical programme is concerned with the overall deformation of the inhomogeneous material rather than the detailed microstructural behaviour of the constituents. Neither of the mesh densities used can realistically model the stress and strain gradients at the interface between the fibres and the matrix. In any case, the geometrical approximations made in the modelling process render such detailed analysis meaningless.

Nonlinear convergence

A series of comparisons were made to investigate the effects of the parameters controlling the nonlinear solution process. Table 3.14 compares the effects of the various parameters. The default parameter settings of 10 equal increments, force, displacement and energy tolerances = 0.001, are used as the base values for the comparison.

<table>
<thead>
<tr>
<th>Parameter Change</th>
<th>Percentage Change</th>
<th>Overall Strain</th>
<th>Relative Solution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium off: (no iterations)</td>
<td>+ 0.5</td>
<td>- 43</td>
<td></td>
</tr>
<tr>
<td>With Equilibrium off; 20 increments</td>
<td>- 0.5</td>
<td>+ 8.5</td>
<td></td>
</tr>
<tr>
<td>10 Auto increments</td>
<td>no change</td>
<td>- 28</td>
<td></td>
</tr>
<tr>
<td>Tolerances = 0.01</td>
<td>no change</td>
<td>+ 1</td>
<td></td>
</tr>
<tr>
<td>Tolerances = 0.0001</td>
<td>no change</td>
<td>+ 20</td>
<td></td>
</tr>
</tbody>
</table>

Even with no iterations the overall deformation of the model did not alter significantly. The insensitivity to the tolerance value confirmed the good convergence of the model. Although automatic incrementing reduced the solution time, 10 fixed increments were used in the modelling to create the required number of data points for the overall stress/strain curves. The convergence tolerances were set equal to 0.001.
Correlation with RBM analytical model

A Pascal computer programme (Appendix B3) was written to calculate the analytical RBM solution described in chapter B.2. The program calculated the fibre and matrix stresses and strains, and the overall effective composite strains. The input to the programme comprised fibre and matrix material properties, the fibre volume fraction and $\sigma_{yc}$, the applied y direction nominal stress. Table 3.15 compares the predictions of the analytical model to results of linear finite element analyses.

**Table 3.15: Overall deformation: analytical predictions compared to finite element results**

<table>
<thead>
<tr>
<th>Case</th>
<th>% Differences in Overall Strain.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical vs. Finite Element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_x$</td>
<td>$\varepsilon_y$</td>
</tr>
<tr>
<td>$Ef/Em$ (Angle = 30°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.19</td>
<td>-0.31</td>
</tr>
<tr>
<td>100</td>
<td>-0.26</td>
<td>-0.23</td>
</tr>
<tr>
<td>1000</td>
<td>-0.37</td>
<td>-0.31</td>
</tr>
<tr>
<td>Angle (Ef/Em = 100)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>15°</td>
<td>-0.36</td>
<td>-0.20</td>
</tr>
<tr>
<td>30°</td>
<td>-0.26</td>
<td>-0.23</td>
</tr>
<tr>
<td>45°</td>
<td>-0.33</td>
<td>-0.24</td>
</tr>
<tr>
<td>60°</td>
<td>-0.13</td>
<td>-0.04</td>
</tr>
<tr>
<td>75°</td>
<td>0.12</td>
<td>-0.08</td>
</tr>
<tr>
<td>90°</td>
<td>0.10</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

There is good correlation between the analytical and finite element solutions in all cases. The magnitude of the shear coupling, and hence the shear strain, is small for the 45°, 60° and 75° cases, which may contribute to the slight apparent error for these models. Any undue constraint from the finite element boundary conditions is likely to be more significant for these cases than for the other orientation models.
3.3.4: Analysis Structure

The finite element analysis of the overall rigid body motion model consisted of three stages:

1) Investigation and development of alternative model types.
2) Verification of the rectangular models.
3) Analysis of different material property and angle cases.

This section details the structure of the third stage, the first and second stages being described in sections 3.3.2 and 3.3.3 respectively. The development and verification stages contained significantly more analyses than the final results stage. A total of 110 analysis runs were made of the rectangular models.

The aim of stage 3 was to highlight the significant characteristics of the rigid body motion theory, and to compare its predictions to those of the traditional homogeneous orthotropic model. The analysis programme was also required to yield results which would enable comparisons to be made with the experimental data.

Models of each fibre orientation (0°, 15°, 30°, 50°, 60°, 75°, 90°) were analysed with the Case 4 constituent material properties of Table 3.9. These material properties were obtained by back-calculation from the on-axis experimental data, as described in section 3.3.3. Equivalent models with homogenous orthotropic materials (Case 1, Table 3.9) were analysed to provide the homogeneous orthotropic comparison.

The 30 degree model was analysed with two different groups of material properties. The first group of properties consisted of Cases 1 to 4 (Table 3.9), which represented alternative constituent material cases with the same on-axis properties as the experimental specimens.

The second group of materials, Cases 5 to 10, were sets of constituent properties with fibre to matrix stiffnesses of 10:1, 100:1 and 1000:1. Each of these inhomogeneous material cases had a corresponding homogenous orthotropic model. Section 3.3.3 discusses the characteristics of the material cases in more detail. Table 3.16 lists the stage 3 finite element analyses.
### Table 3.16: Stage 3 finite element analyses

<table>
<thead>
<tr>
<th>Run #</th>
<th>Fibre Angle (°)</th>
<th>Material Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>7</td>
<td>75</td>
<td>4</td>
<td>RBM, experimental properties</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>11</td>
<td>30</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>12</td>
<td>45</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>13</td>
<td>60</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>14</td>
<td>75</td>
<td>1</td>
<td>Homogenous, experimental properties</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>4</td>
<td>RBM shear, experimental properties</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>1</td>
<td>Homogenous shear, experimental properties</td>
</tr>
<tr>
<td>17</td>
<td>30</td>
<td>2</td>
<td>RBM, alternative experimental properties</td>
</tr>
<tr>
<td>18</td>
<td>30</td>
<td>3</td>
<td>RBM, alternative experimental properties</td>
</tr>
<tr>
<td>19</td>
<td>30</td>
<td>5</td>
<td>RBM, $E_f/E_m=10$</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>6</td>
<td>Homogenous, $E_f/E_m=10$</td>
</tr>
<tr>
<td>21</td>
<td>30</td>
<td>7</td>
<td>RBM, $E_f/E_m=100$</td>
</tr>
<tr>
<td>22</td>
<td>30</td>
<td>8</td>
<td>Homogenous, $E_f/E_m=100$</td>
</tr>
<tr>
<td>23</td>
<td>30</td>
<td>9</td>
<td>RBM, $E_f/E_m=1000$</td>
</tr>
<tr>
<td>24</td>
<td>30</td>
<td>10</td>
<td>Homogenous, $E_f/E_m=1000$</td>
</tr>
</tbody>
</table>
3.3.5: Typical Results

The results presented in the Results and Discussion chapter, Section A.5, are processed results, mainly in the form of stress/strain graphs and graphs of normalised deformation components. This section describes the results processing methods and presents examples of typical deformed shapes and stress/strain graphs. Appendix B9 contains a deformed shape for each orientation rectangular model. Stress/strain graphs for each model are contained in Appendix B10. Detailed discussion of the results is in Chapter A.5.

Results Processing

The main parameters of interest from the analyses were those describing the overall deformation of the representative volume element, i.e., the final shape of the finite element mesh. For comparison with the traditional homogeneous orthotropic model, and the experimental data, this was measured by the "effective overall strain", which uses the definitions of homogenous strain to describe the deformation of a non-homogenous body.

The effective overall strains were calculated from the relative nodal displacements of fibre centrelines. Finite strain theory was used to accurately characterise the large deformations. Several definitions of strain can be used to describe finite strains [100]. For this modelling the engineering definition of strain was used, as is usual for thermoplastic composite modelling [9] and sheet metal forming [101].

The required nodal displacements were output from the NISA/DISPLAY post-processor and imported into a customised spreadsheet template containing the strain analysis equations and the required stress/strain graphs. The resulting overall stress/strain data and graphs formed the basis of the further results processing.

To highlight the behaviour of the models at different strain levels, global longitudinal, transverse and shear stress/strain graphs were generated for longitudinal strains of 0.2%, 2%, and 10%. Nine stress/strain graphs were thus generated for each model described in Table 3.15. Section A.5 (Results and Discussion) discusses the generation of the stiffness/orientation and material property effect graphs.
Deformed Shape

Figure 3.17 shows the deformed shape of the 30° model from run 4, Table 3.15. Appendix B9 contains deformed plots for the other orientation models. Slight localised deformation is apparent near to the model edges, however the deformation is uniform throughout the central region of the model.

![Deformed 30° model](image)

Figure 3.17: Deformed 30° model

Stress/Strain curves

Figure 3.21 presents three stress/strain curves from run 4, Table 3.16. This is a Rigid Body Motion model at 30°, with case 4 materials (Table 3.9). Longitudinal, transverse and shear strains are plotted against the applied longitudinal stress. These three graphs are for the large strain case, with the maximum longitudinal strain equal to 10%. The transverse and shear strains are plotted to the same stress level as in the longitudinal strain graph. Appendix B10 contains stress/strain curves for each fibre orientation.

The three sets of data plotted on each graph are denoted Rigid Body Motion, Orthotropic, and Linear Orthotropic. The first two data sets are the results from the geometrically non-linear finite element analyses of the inhomogeneous and homogeneous cases respectively. The Linear Orthotropic data is the linear analytical solution for the same material properties as the other two cases.
Figure 3.21 a: Longitudinal strain of 30 degree case 4, to 10% longitudinal strain

Figure 3.21 b: Transverse strain of 30 degree case 4, to 10% longitudinal strain

Figure 3.21 c: Shear strain of 30 degree case 4, to 10% longitudinal strain
All three solutions diverge as the strain level increases. At 10% longitudinal strain, Figure 3.21 (a), the stress level for the RBM case is 9% greater than the nonlinear orthotropic, and 53% greater than the linear orthotropic case. The finite element solution for the non-linear orthotropic case allows the material properties to follow the deformed shape, thereby increasing the model's stiffness at large strains compared to the linear case. The transverse strain of the RBM model is less than the other two cases for a given stress level. The shear strain graph is of similar form to that of the longitudinal strain.

3.4: Summary

Two finite element based analysis studies were made of the Rigid Body Motion theory. The first study modelled the fibre rotation only, and investigated the effects of fibre stiffness, load, angle and volume fraction. Good agreement was found between the analytical solution and the finite element results.

The second study modelled the overall deformation of the RBM model. A series of different model geometries and boundary conditions were investigated. Rectangular off-axis models were created to match the fibre orientations of the experimental specimens. The models were verified and compared to the RBM analytical solution, then used to study the effects of material properties and strain levels on the behaviour of the RBM model. The results were compared to those of non-linear homogeneous orthotropic finite element models, and the conventional linear homogeneous orthotropic analytical solution.
Theoretical Appendices

B1: Fibre rotation strains
B2: Coefficients for Equation 2.46
B3: OFFAXIS computer programme
B4: ABAQUS input file
B5: Tubular NISA models
B6: Rectangular FE meshes
B7: NISA input file
B8: MICROMECH computer programme
B9: Deformed FE meshes
B10: Predicted stress/strain curves
Appendix B1: Fibre rotation strains

As the fibres rotate, the matrix between them deforms. If it is assumed that the strain in the matrix is uniform, then the strains can be calculated from the geometry of the deformation, as in Figure 1.

![Rotated fibre geometry](image)

**Figure 1: Rotated fibre geometry**

The 2 direction strain is defined to be zero by the assumption of fibre rigidity. The 1 direction, and 12 shear strains can be calculated from the displacements of points A and B, Figure 1.
Define:

\[ \beta_1 = \tan \left( \frac{d_f}{2} \right) \]
\[ \beta_2 = \tan \left( \frac{d_f}{2} \right) \]
\[ L_{1}' = \frac{d_f}{2 \sin \beta_1} \]
\[ L_{2}' = \frac{d_f}{2 \sin \beta_2} \]
\[ A_0 = \frac{d_f}{V_f \cos \theta} \]

\((V_f = \text{Fibre volume fraction})\)

Where:

- \(L_1 = \text{Length of DA}\)
- \(L_2 = \text{Length of CB}\)
- \(L_{1}' = \text{Length of DA}'\)
- \(L_{2}' = \text{Length of CB}'\)

Then:

\[ L_2 = L_1 - A_0 \sin \theta \]

**1-Direction strains:**

Consider the 1-direction displacements of point A and point B, as they move to A' and B' respectively, as the fibres rotate by \(\phi\).

\[ U_{1A} = 0.5 \, d_f - L_{1}' \sin(\beta_1 - \phi) \]  \hspace{1cm} (1)
\[ U_{1B} = -0.5 \, d_f + L_{2}' \sin(\beta_2 + \phi) \]  \hspace{1cm} (2)

Then for infinitesimal strains,

\[ \epsilon_1 = \frac{U_{1A} - U_{1B}}{A_0 \sin \theta - d_f} \]  \hspace{1cm} (3)

And hence:

\[ \epsilon_1 = \frac{d_f (1 - \cos \phi) + A_0 \sin \theta \sin \phi}{A_0 \cos \theta - d_f} \]  \hspace{1cm} (4)

**Similarly for the shear strains:**

\[ U_{2A} = -L_1 + L_{1}' \cos(\beta_1 - \phi) \]  \hspace{1cm} (5)
\[ U_{2B} = L_2 - L_{2}' \cos(\beta_2 + \phi) \]  \hspace{1cm} (6)

Then for infinitesimal strains,

\[ \epsilon_{12} = \frac{U_{2A} + U_{2B} + \phi}{A_0 \sin \theta - d_f} \]  \hspace{1cm} (7)

And hence:

\[ \epsilon_{12} = \frac{A_0 \sin \theta (1 - \cos \phi) - d_f \sin \phi}{A_0 \cos \theta - d_f} \]  \hspace{1cm} (8)
The preceding equations have been formulated with the assumption of no x-direction displacements of the fibre centres \((A_0 \text{ constant})\). This provides compatibility with the fibre rotation finite element modelling which imposed this condition through its boundary conditions.

The fibre rotation model can also be derived for the case of no x-direction stress. In this case the 12 (shear) and 2-direction strains are the same. However the 1-direction strain is calculated from the condition that the x-direction stress is zero. By using the matrix constitutive relations, and tensorial stress and strain transformations, the 1-direction strain is given by Equation (9):

\[
\varepsilon_1 = \frac{\sin \theta \cos \theta (1 - \nu_m)}{(\cos^2 \theta + \sin^2 \theta \nu_m)} \left( \frac{A_0 \sin \theta (1 - \cos \phi) - d_f \sin \phi}{A_0 \cos \theta - d_f} - \phi \right)
\]
Appendix B2:
Coefficients for Equation 2.46

Equation (2.46) from Section A 2.3.2 reads:

\[ \sigma_{x,yf} = \frac{(G - H)}{(1 - J)} \]

The coefficients are given by:

\[ G = \frac{\sigma_{yc}}{w_m} (E - \tan^2 \theta (A h_m + D h_f)) \]
\[ H = \frac{\sigma_{yc}}{w_m} ((B w_m + E w_f) - \tan^2 \theta (A h_m + D h_f)) \]
\[ I = \frac{1}{w_m} ((C w_m + F w_f) - 2 \tan \theta (A h_m + D h_f)) \]
\[ J = \frac{1}{w_m \tan \theta} ((B w_m + E w_f) - \tan^2 \theta (A h_m + D h_f)) \]

Where:

\[ A = E_m (\sin^2 \theta - V_f \cos^2 \theta) \quad D = E_f (\sin^2 \theta - V_m \cos^2 \theta) \]
\[ B = E_m (\cos^2 \theta - V_f \sin^2 \theta) \quad E = E_f (\cos^2 \theta - V_m \sin^2 \theta) \]
\[ C = -2 E_m \sin \theta \cos \theta (1 + V_f) \quad F = -2 E_f \sin \theta \cos \theta (1 + V_m) \]
Appendix B3:
OFFAXIS Computer Programme

program offaxis;

{Calculates solution to linear RBM model}
{Mark Battley 1993}

var
  sxf, syf, sxym, sym, sxym, syc: double;
  exf, eyf, exym, eyym, exy, eyy, eyym, eyyt: double;
  A, B, C, D, E, F, G, H, I, J: double;
  Ef, vF, Em, vm, Volf, wF, hF, Wm, hm, theta, thetadeg, rotation: double;
  cs, sn, tn, cs2, sn2, tn2, sncs: double;
  LHS, RHS, error: double;
  i: integer;
  widthf, widthm, syapp, widthc, heightc: real;

{======================================================================}

function finished: boolean;
var
  ch: char;
begin
  writeln;
  write('Another set of data (Y/N)? '); repeat
    read(ch);
    until ch in ['y', 'Y', 'n', 'N'];
  finished := ((ch = 'n') or (ch = 'N'));
end;

{======================================================================}

procedure mfout;
{Outputs fibre and matrix strains and stresses}
begin
  writeln('exf= ', exf : 10 : 7);
  writeln('eyf= ', eyf : 10 : 7);
  writeln('exym= ', eyym : 10 : 7);
  writeln('eyym= ', eyym : 10 : 7);
  writeln('sxf= ', sxf : 10 : 2);
  writeln('syf= ', syf : 10 : 2);
  writeln('sxym= ', sxym : 10 : 2);
  writeln('sym= ', sym : 10 : 2);
end;
{===============================================================================}
procedure straincalc;
begin

  sn := sin(theta);
  cs := cos(theta);
  tn := sn / cs;

  snscs := sn * cs;
  sn2 := sn * sn;
  cs2 := cs * cs;
  tn2 := tn * tn;

  A := Em * (sn2 - vf * cs2);
  B := Em * (cs2 - vf * sn2);
  C := -2 * Em * snscs * (1 + vf);
  D := Ef * (sn2 - vm * cs2);
  E := Ef * (cs2 - vm * sn2);
  F := -2 * Ef * snscs * (1 + vm);

  G := syc * (E - tn2 * (A * hm + D * hf)) / wn;
  H := syc * ((B * wm + E * wf) - tn2 * (A * hm + D * hf)) / wn;
  I := ((C * wn + F * wf) - (A * hm + D * hf) * 2 * tn) / wn;
  J := ((B * wn + E * wf) - (A * hm + D * hf) * tn2) / (wm * tn);

  {calculate fibre stresses}
  sxyf := (G - H) / (I - J);
  syf := syc - sxyf / tn;
  sxf := -hm * tn * sxyf / wn;

  {calculate matrix stresses}
  sym := (syc - wf * syf) / wn;
  sxm := -sxf * hf / hm;
  sxym := -sxyf * wf / wn;

  {calculate fibre strains}
  exf := (sxf - vf * syf) / Ef;
  eyf := (syf - wf * sxf) / Ef;
  exyf := 2 * (1 + vf) * sxyf / Ef;

  {calculate matrix strains}
  exem := (sxm - vm * sym) / Em;
  eym := (sym - vm * sxm) / Em;
  exem := 2 * (1 + vm) * sxym / Em;

  {calculate composite strains}
  exc := exf * wf + exem * wm;
  eyc := eyf * hf + eym * hm;
  exyc := eyf * (wf + hf) / 2 + exem * (wm + hm) / 2;
  rotation := cs * (sn * (eyc - exc) + cs * exyc);

  writeln(exc * 100 : 10 : 4, eyc * 100 : 10 : 4, exyc * 100 : 10 : 4, rotation * 180 / pi : 7 : 3);
end;
{==============================================================}

procedure setinitial;
begin
writeln('Off-axis Fibre Reinforced Composite Analysis');
writeln('Rigid Body Motion Off-axis model');
writeln;
write('Fibre Stiffness (GPa) : '); readln(Ef);
Ef := Ef * 1000;
write('Fibre Poissons Ratio : '); readln(vf);
write('Matrix Stiffness (GPa): '); readln(Em);
Em := Em * 1000;
write('Matrix Poissons Ratio : '); readln(vm);
write('Fibre Volume Fraction : '); readln(Volf);
write('Fibre Angle (degrees) : '); readln(thetadeg);
theta := thetadeg * PI / 180;
write('Tensile Stress (MPa) : '); readln(sYc);
exc := 0;
eyc := 0;
exyc := 0;
widthf := volf;
widthm := 1 - volf;
widthc := 1;
heightc := 1;
wf := widthf / widthc;
hf := wf;
wm := widthm / widthc;
hm := wm;
writeln;
writeln(' exc eyc exyc rotation');
writeln;
end;

{==============================================================}

{main program}
begin
repeat
setinitial;
straincalc;
until finished;
end.
Appendix B4: ABAQUS input file

*HEADING
  RIGID BODY MOTION MODEL, FIBRE AT 30 DEGREES
*PREPRINT, HISTORY=NO, MODEL=NO
**
** FIBRE AT 30 DEGREES TO LOAD
**
** FIBRE DIAMETER = 0.0002 M
** 50 ELEMENTS LONG
** SINGLE FIBRE MODEL
** FIBRE CENTRE CONST
** PLANE STRESS ELEMENTS
** VERY HIGH STIFFNESS FIBRE
** SINGLE FIBRE MODEL, MPC TYPE 5 BOUNDARY BETWEEN LB AND RB
** NORMAL STIFFNESS MATRIX
** MARK BATTLEY
**
*NODE
  1,0.,0.
  101,0.005000,0.00866
  405,0.00046188,0.
  505,0.005462,0.00866
*NGEN, NSET=LB
  1,101,1
*NGEN, NSET=RB
  405,505,1
*NFILL
  LB,RB,4,101
*NSET, NSET=TOPLD
  202
*NSET, NSET=BOTLD
  102
*NSET, NSET=FIBCEN
  152
*NSET, NSET=OUTPUT 49,50,51,52,53,150,154,251,252,253,254,255
*ELEMENT, TYPE=CPS8R 1,1,203,205,3,102,204,104,2
*ELGEN
  1,50,2,1,2,202,50
*ELSET, ELSET=F1, GENERATE
  1,50,1
*ELSET, ELSET=M1, GENERATE
51,100,1
*ELSET, ELSET=FIBRES
  F1
*ELSET, ELSET=MATRIX
  M1
*ELSET, ELSET=ELOUT
  M1
*SOLID SECTION, ELSET=FIBRES, MATERIAL=HSTIFF 0.0002
*SOLID SECTION, ELSET=MATRIX, MATERIAL=EPOXY 0.0002
*MATERIAL, NAME=HSTIFF
*ELASTIC
  80.E19,0.3
*MATERIAL, NAME=EPOXY
*ELASTIC
  4.E9,0.3
*BOUNDARY
  FIBCEN,1,2
*MPC
  5,LB,RB
** *PLOT
** *DRAW, ELNUM
**
*STEP, LINEAR
*STATIC
*CLOAD
  TOPLD,1,-8.66
  TOPLD,2,5.0
  BOTLD,1,8.66
  BOTLD,2,-5.0
*EL PRINT, ELSET=ELOUT
*NODE PRINT, NSET=OUTPUT
** *PLOT
** *DISPLACED
** U
*ENDSTEP
Appendix B5: Tubular NISA models

A tubular finite element model was created (TUB11) with:
- 10 fibres, each with 20 elements
- 200 elements, 640 nodes total
- 20N FZ per fibre
- Tube mean diameter = 1.47021E-3m
- Tube thickness = fibre diameter = 2e-4m
- 1840 cpus to run

For verification purposes this model was tested with fibres and matrix both E=4GPa (RBMISO). Although mid-thickness results were correct (stresses 1%, strains 0.2%) there were variations in stresses (+/-1.5%) and strains (+/-3.4%) through the thickness. Models were created with half and quarter the original thickness, Table 1.

**Table 1: Tubular finite element models**

<table>
<thead>
<tr>
<th>Filename</th>
<th>Thickness mm</th>
<th>Diameter / Thickness</th>
<th>Variation in:</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Strain: through the thickness</td>
<td>Inside vs Outside</td>
</tr>
<tr>
<td>TBISO2</td>
<td>0.2</td>
<td>7.35</td>
<td>+/-3.4</td>
<td>27.2</td>
</tr>
<tr>
<td>TBTHIN</td>
<td>0.1</td>
<td>14.7</td>
<td>+/-1.1</td>
<td>13.6</td>
</tr>
<tr>
<td>VTHIN</td>
<td>0.05</td>
<td>29.4</td>
<td>+/-0.6</td>
<td>6.8</td>
</tr>
</tbody>
</table>

On the basis of these results a larger diameter model with 20 fibres was created (TB20F).
- 20 fibres, each with 20 elements
- 400 elements, 1280 nodes total
- 20N FZ per fibre
- Diameter = 2.94042E-3m
- Thickness = fibre diameter = 2e-4m
- 5200 cpus to run
In this case the midthickness strain was only 0.04% different to the analytical, and the strain varied by +/- 1.5% through the thickness.

The fibre properties were changed to 400GPa resulting in a very nonuniform deformation (TUBROT2). The fibre midpoint boundary conditions were changed to be restrained in both z and theta directions (previously only one node was held in the theta direction). This model (ROT) resulted in a barrel type deformed shape. Alternative boundary conditions appeared to be required to restrain the top and bottom edges of the tube.

Because of the time required to run the large diameter model (approx 1.5 hrs) it was decided to experiment with boundary conditions on the smaller diameter model then apply the final type to the large model.

RIGID links were applied in the radial direction between the ends and one centre point (SMLRIG): very nonuniform deformation. The ends of the tube were joined to each other by radial direction rigid links (SRIGL), again without success.

Models without rigid links were run with different fibre stiffnesses. (Still 20N axial load).

- **R400F: 400GPa fibres:** diameter remained almost constant
- **R4E13: 4E13 fibres:** diameter changed, stresses nonuniform
- **R4E15: 4E15 fibres:** diameter changed, stresses nonuniform

The applied forces were changed to be 10N per fibre at 90 degrees to the fibre, resulting in:

- **R411:** 4E11 GPa fibres: Remained virtually in plane, with uniform stresses and displacements.
- **R413:** 4E13 GPa fibres: The ends curved over substantially, the stresses were only uniform within the centre two elements.

To prevent diameter changes, all nodes were constrained in the radial (x) direction (ROTX). Coupled displacements were tried with normal and high stiffness fibres: CP411, CP413. Each end of every fibre was coupled to a single centre node. This resulted in:
CP411: Virtually no diameter change, fairly uniform deformation, SZZ very uniform
CP413: "Bulges" near ends of tube, SZZ stress only uniform in centre 2 elements.

An isotropic model with a tensile and shear load was run (SMLISO). Once *LDCASE was modified to give local stresses the results were very close to theoretical:

\[
\begin{align*}
\sigma_{\text{longitudinal}} : & \quad 0.2 \% \\
\sigma_{\text{Transverse}} : & \quad 2 \% \\
\sigma_{\text{shear}} : & \quad = 0 \text{ as expected.}
\end{align*}
\]

The material properties of this model were changed to create normal and high stiffness fibres. The deformation was as for R411, R413 but local stresses/strains were now available. For STR411 they were up to 20% less than analytical and for STR413 they were even lower. In particular \(\sigma_{\text{shear}}\) was nearly zero.
Appendix B6:

Rectangular FE meshes

0 degree low density

0 degree high density
30/60 degree low density mesh

High density 30 degree model
15/75 degree mesh

45 degree mesh
Appendix B7: NISA input file

ANALYSIS = NLSTATIC
NLTYPE=GEOM
REFC=TOTAL
MAXC=5000
LOAD=1
FILE = NLC4
SAVE=26,27
*TITLE
Nonlinear RBM, orthotropic materials (Case 4) at 30 degs
** CPDISP at top and bottom
** Low density 30 degree RBM model
** uniform pressure at top and bottom of 25 N/mm^2
*ELTYPE
  1, 1, 2
  2, 1, 11
*RCTABLE
  1, 8,1, 0
  7.0000002E-03, 7.0000002E-03, 7.0000002E-03, 7.0000002E-03,
  7.0000002E-03, 7.0000002E-03, 7.0000002E-03, 7.0000002E-03,
*READ,LD30.MSH
** Read in nodes and elements
*READ,MATCASE4.DTA
*MATDIR2
** Materials at 30 degrees
  1,0,0,0,0,120,0
  120,1,0,0,0,120,0
*READ,TOPCP.DTA
*SETS
** Elements
**   matrix:
  101,R,27,30,1,35,38,1
**   fibres:
  102,R,3,6,1,11,14,1,19,22,1
** Nodes
  201,R,80,84,1,123,127,1
  202,S,39
*TIMEAMP
333,1
1.0,1.0
*EVENT,ID=1
INCREMENTs=EQUAL,10
MAXITERATION=20
TIMEATEND=1.0
TOLERA=0.001,0.001,0.001
NEWT=FULL,1
EQUIL=ON,1
*NLOUT
1,2,-1,4,0,2,0
*PRINT
AVND,101,102
DISP,201,202
ELST,101,102
*POST
** Output unaveraged nodal strains and stresses for
postprocessing NDSTRN,0
NDSTRS,0
*SPDISP
** SPDISP SET = 1
   82,UX , 0.000000E+00,,,,,,,,, 0
   82,UY , 0.000000E+00,,,,,,,,, 0
*READ,PRE3ONL.DTA
** Read in uniform pressure data
*ENDDATA
Appendix B8: 
MICROMECCH Computer Programme

program MICROMECH;
{
Program to compare different micromechanics theories
Theories included:
- Basic Rule Of Mixtures
- ROM + transverse poisson's effects
- Stress Partitioning analytical
- Chamis' simplified equations
- Composite Cylinder Assemblage model
- Stress Partitioning empirical theory not active in this version
[For transversely isotropic fibres and isotropic matrix]
[Mark Battley, 24/2/93]
}

var
ELc, ETc, vLTc, GLTc, vTc, GTc, kTc: real;
ELf, ETf, vLTF, GLTF, vTF, GTf, kTf: real;
Em, Gm, vm, km: real;
Volf, Volm: real;
{kT = plane strain bulk modulus, GTT = transverse shear modulus}
ch: char;

{==========================================================--=================}
procedure ROM;
{Uses basic Rule of Mixtures}
begin
ELc := ELf * Volf + Em * Volm;
vLTc := vLTF * Volf + vm * Volm;
ETc := 1 / (Volf / ETf + Volm / Em);
GLTc := 1 / (Volf / GLTF + Volm / Gm);
Writeln;
writeln('Basic ROM',' ELc : 6 : 2, ETc : 6 : 2, vLTc : 6 : 3, GLTc : 6 : 2);
end;

{==========================================================--=================}
procedure ROMPoisson;
{Uses Rule of Mixtures with poisson's stiffening effect on transverse modulus included}
{ ref Tsai & Hahn pg 391 }
{ new Rule of Mixtures for EL and v}
begin
ELc := ELf * Volf + Em * Volm;
vLTc := vLTF * Volf + vm * Volm;
c := vLTF * vm * (vLTF * vLTF * Em / ELf + vm * vm * ELf / Em - 2 * vLTF * vm) / (vLTF * ELf + vm * Em);
ETc := 1 / (Volf / ETf + Volm / Em - c);
GLTc := 1 / (Volf / GLTF + Volm / Gm);
Writeln;
writeln(' ROM Poisson',' ELc : 6 : 2, ETc : 6 : 2, vLTc : 6 : 3, GLTc : 6 : 2);
end;

{==========================================================--=================}
procedure Stresspartitioningcalc;
{Uses Rule of Mixtures for EL and v}
(Uses Tsai/Hahn stress partitioning parameters for ET & G)
(Calculates stress partitioning parameters)
(Refer Tsai & Hahn pg 394-396)

\[
\begin{align*}
\text{var} & \quad nk, nG, nS, m: \text{real}; \\
\text{begin} & \quad ELc := ELf \times Volf + Em \times Volm; \\
& \quad vLTc := vLTf \times Volf + vm \times Volm; \\
& \quad nk := (1 + Gm / kTf) / (2 \times (1 - vm)); \\
& \quad nG := (3 - 4 \times vm + Gm / GTf) / (4 \times (1 - vm)); \\
& \quad nS := 0.5 \times (1 + Gm / GLTf); \\
& \quad kTc := \frac{volf + nk \times volm}{Volf + \frac{vm \times volm}{Em}}; \\
& \quad GTc := \frac{volf + nG \times volm}{Volf + \frac{vm \times volm}{Em}}; \\
& \quad m := 1 + 4 \times kTc \times vLTc \times \frac{vLTc}{ELc}; \\
& \quad ETc := \frac{4 \times kTc \times GTc}{(kTc + m \times GTc)}; \\
& \quad GLTc := \frac{volf + nS \times volm}{Volf + \frac{vm \times volm}{Em}}; \\
& \quad \text{writeln;} \\
& \quad \text{(writeln(' nS=', nS:4:2, ' nk=', nk:4:2, ' nG=', nG:4:2));)} \\
\end{align*}
\]

end;

{-------------------------------------------------------------------}

\textbf{procedure} Stresspartitioningexp;
(Uses Rule of Mixtures for EL and v)
(Uses Tsai/Hahn stress partitioning parameters for ET & G)
(Stress partitioning factors chosen to match experimental data)
(Refer Tsai & Hahn pg 394)

\[
\begin{align*}
\text{var} & \quad nT, nS: \text{real}; \\
\text{begin} & \quad nT := 0.7; \\
& \quad nS := 0.58; \\
& \quad ELc := ELf \times Volf + Em \times Volm; \\
& \quad vLTc := vLTf \times Volf + vm \times Volm; \\
& \quad ETc := (volf + nT \times volm) / (Volf / ETf + nT \times volm / Em); \\
& \quad GLTc := (volf + nS \times volm) / (Volf / GLTf + nS \times volm / Em); \\
& \quad \text{ writeln;} \\
& \quad \text{writeln('Stress Partition exp', ELc:6:2, ETc:6:2, vLTc:6:3, GLTc:6:2);} \\
& \quad \text{(writeln(' nS=', nS:4:2, ' nT=', nT:4:2));)} \\
\end{align*}
\]

end;

{-------------------------------------------------------------------}

\textbf{procedure} Chamis;
(Uses Rule of Mixtures for EL and v)
(Uses Chamis' simplified equations for ET & G)
(Refer ref #13)

\[
\begin{align*}
\text{begin} & \quad ELc := ELf \times Volf + Em \times Volm; \\
& \quad vLTc := vLTf \times Volf + vm \times Volm; \\
\end{align*}
\]
ETc := \frac{Em}{(1 - \sqrt{volf}) \times (1 - \frac{Em}{ETf})};
GLTc := \frac{Gm}{(1 - \sqrt{volf}) \times (1 - \frac{Gm}{GLTf})};
writeln;
writeln('Chamis ', ELc : 6 : 2, ETc : 6 : 2, vLTc : 6 : 3, GLTc : 6 : 2);
end;

{==================================================}

procedure CCA;
{Uses Hill's expressions for EL & vLT: refs #68, #73, #27}
{Uses Hashin's Composite Cylinder Assemblage model for ET & G}
{Refer to refs #68 pg 489, #73 pg 546, bk 14 pg 28}
var
  aa, bb, a, b1, b2, r, g, m: real;
begin
  kTc := \frac{(km \times (kTf + Gm) \times Volm + kTf \times (km + Gm) \times volf)}{((kTf + Gm) \times volm + (km + Gm) \times volf)};
  aa := \frac{(4 \times (vLTf - vm) \times (vLTf - vm) \times volm \times volf)}{(votm / kTf + Volf / km + 1 / Gm)};
  ELc := ELf \times Volf + \frac{Em \times Volm}{aa};
  bb := \frac{(vLTf - vm) \times (1 / km - 1 / kTf) \times volm \times volf}{(vom / kTf + Volf / km + 1 / Gm)};
  vLTc := vLTf \times Volf + \frac{vm \times Volm + aa}{bb};
  GLTc := \frac{Gm \times (Gm \times volm + GLTf \times (1 + volf))}{(Gm \times (1 + volf) + GLTf \times volm)};
  b1 := km / (km + 2 * gm);
  b2 := \frac{kTf}{(kTf + 2 * GTf)};
  g := GTf / Gm;
  a := \frac{(b1 - g \times b2)}{(1 + g \times b2)};
  r := \frac{(g + b1)}{(g - 1)};
  GTc := \frac{Gm \times (1 + ((1 + b1) \times \frac{volf}{(r - volf \times (1 + (3 \times b1 \times b1 \times volm \times volm)) / (a \times volf \times volf + 1))))}{(r - volf \times (1 + (3 \times b1 \times b1 \times volm \times volm)) / (a \times volf \times volf + 1))};
  m := 1 + \frac{4 \times kTc \times vLTc \times vLTc}{ELc};
  ETc := \frac{(4 \times kTc \times GTc)}{(kTc + m \times GTc)};
  writeln;
  writeln('CCA ', ELc : 6 : 2, ETc : 6 : 2, vLTc : 6 : 3, GLTc : 6 : 2, kTc : 6 : 2);
end;

{==================================================}

procedure input;
begin
  writeln('MICROMECHANICAL COMPOSITE ANALYSIS');
  writeln('Enter name of material: ');
  readln(matname);}
  writeln('Enter fibre engineering constants (transversely isotropic): ');
  writeln('Enter fibre engineering constants:
  (ET GLT GTT: )');
  readln(ETf, GLTf, GTf, vTf);
  writeln('Enter fibre engineering constants: (GPa): ');
  writeln('Enter fibre engineering constants: (GPa): ');
  writeln('Enter fibre engineering constants: (GPa): ');
  writeln('Enter fibre engineering constants: (GPa): ');
  writeln('Enter fibre engineering constants: (GPa): ');
  writeln('Enter fibre engineering constants: (GPa): ');
writeln;'Enter matrix engineering constants (isotropic): ''); writeln(' E v:'); readln(Em, vm); writeln;
write('Enter fibre volume fraction: '); readln(Volf);
writeln;
write('Engineering constants: EL ET vLT GLT k vTT GTT '); writeln;
Volm := 1 - Volf;
GTf := ETf / (2 * (1 + vTf));
Gm := Em / (2 * (1 + vm));
km := Gm / (1 - 2 * vm);
kTf := 1 / (4 / ETf - 1 / GTf - 4 * vLTf * vLTf / ELf);
writeln;
writeln;
write('Matrix ', Em:6:2, Em:6:2, vm:6:3, Gm:6:2, km:6:2);
end;
Appendix B9: Deformed FE meshes

The following plots show the deformed (bold) and undeformed (light) mesh shapes of the nonlinear RBM finite element models, with Case 4 material properties (Table 3.9, Section B 3.3.3). The deformations are shown at actual scale. The load directions for each model are as shown in Figure 3.14, Section B 3.3.3.

0° Model

![Diagram of deformed and undeformed mesh shapes for 0° Model.](image-url)
15° Model

Nonlinear RVE, Case 4 inhomogeneous orthotropic materials at 15 degs, 454

30° Model

Nonlinear RVE, orthotropic materials (Case 4) at 3 degs, 2500 MPa
45° Model

Displaced-Shape
- MX DEF = 1.12E-01
- NODE NO = 1569
- SCALE = 1.0
- (Actual Scaling)

60° Model

Displaced-Shape
- MX DEF = 4.73E-03
- NODE NO = 156
- SCALE = 1.0
- (Actual Scaling)

Nonlinear 60° Degree ROM model, Case 4 materials.
75° Model

DISPLAY III - GEOMETRY MODELING SYSTEM (92B) PRE/POST MODULE

UNIQUE RIV, Case 4 - inhomogeneous orthotropic materials at 75 degress, 118

90° Model

DISPLAY III - GEOMETRY MODELING SYSTEM (92B) PRE/POST MODULE

Nonlinear 90 Degree RIV model, 1000 MPa Transverse Stress, Case 4 material
Appendix B10: Predicted Stress/Strain Curves

This appendix presents the stress/strain curves predicted for each orientation of the experimental specimens. The 30° case is discussed in Section A 5.3.3.

Longitudinal, transverse and shear strains are plotted against the applied longitudinal stress. The graphs are for the large strain case, with the maximum longitudinal strain equal to 10%. The maximum strain level of 10% was chosen to highlight the effects of large strain levels on the behaviour of the various models shown. The transverse and shear strains are plotted to the same stress level as in the longitudinal strain graph.

The three sets of data plotted on each graph are denoted Rigid Body Motion, Orthotropic, and Linear Orthotropic. The first two data sets are the results from the geometrically non-linear finite element analyses of the inhomogeneous and homogeneous cases respectively. The finite element solution for the non-linear orthotropic case (labelled Orthotropic on the graphs) allows the material properties to follow the deformed shape, thereby increasing the model’s stiffness at large strains compared to the linear case. The Linear Orthotropic data is the linear analytical (which is the same as the linear FE) solution for the same on-axis material properties as the other two cases.
**0° Fibre Angle**

Figure 1: Longitudinal strain of 0 degree case 4, to 10% longitudinal strain

Figure 2: Transverse strain of 0 degree case 4, to 10% longitudinal strain

Figure 3: Shear strain of 0 degree case 4, to 10% longitudinal strain
15° Fibre Angle

Figure 4: Longitudinal strain of 15 degree case 4, to 10% longitudinal strain

Figure 5: Transverse strain of 15 degree case 4, to 10% longitudinal strain

Figure 6: Shear strain of 15 degree case 4, to 10% longitudinal strain
30° Fibre Angle

Figure 7: Longitudinal strain of 30 degree case 4, to 10% longitudinal strain

Figure 8: Transverse strain of 30 degree case 4, to 10% longitudinal strain

Figure 9: Shear strain of 30 degree case 4, to 10% longitudinal strain
45° Fibre Angle

![Graph showing longitudinal stress vs. longitudinal strain](image1)

**Figure 10:** Longitudinal strain of 45 degree case 4, to 10% longitudinal strain

![Graph showing transverse stress vs. transverse strain](image2)

**Figure 11:** Transverse strain of 45 degree case 4, to 10% longitudinal strain

![Graph showing shear stress vs. shear strain](image3)

**Figure 12:** Shear strain of 45 degree case 4, to 10% longitudinal strain
60° Fibre Angle

Figure 13: Longitudinal strain of 60 degree case 4, to 10% longitudinal strain

Figure 14: Transverse strain of 60 degree case 4, to 10% longitudinal strain

Figure 15: Shear strain of 60 degree case 4, to 10% longitudinal strain
75° Fibre Angle

Figure 16: Longitudinal strain of 75 degree case 4, to 10% longitudinal strain

Figure 17: Transverse strain of 75 degree case 4, to 10% longitudinal strain

Figure 18: Shear strain of 75 degree case 4, to 10% longitudinal strain
90° Fibre Angle

Figure 19: Longitudinal strain of 90 degree case 4, to 10% longitudinal strain

Figure 20: Transverse strain of 90 degree case 4, to 10% longitudinal strain

Figure 21: Shear strain of 90 degree case 4, to 10% longitudinal strain
A method was developed for testing of off-axis tubular specimens, then used to test a range of CFRP off-axis specimens. This section details the development and verification of the methodology, and details the experimental test programme.
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C.1: Introduction

1.1: Aim

The aim of this experimental programme was to provide reliable data on the measured off-axis stiffness of unidirectional fibre reinforced plastics. Two testing programmes were carried out, the preliminary one investigating dead weight loading of glass fibre/polyester resin tubes. The main testing programme developed and verified a method for testing off-axis tubular specimens in a standard tensile testing machine, then tested a comprehensive range of carbon fibre/epoxy resin specimens.

1.2: Specimen Choice

The typically anisotropic nature of fibre reinforced materials significantly complicates the mechanical testing of such materials. The coupling between different modes of deformation can greatly influence the results of tests. The inhomogeneity of the material makes the choice of test specimen dimensions and shape crucial to the reliability of a testing programme. End effects from load introduction can be much more significant than in the case of isotropic materials [57].

As discussed in section A.2.3.1, in the case of an unconstrained off-axis tensile specimen, an applied tensile load will result in longitudinal, transverse, and shear deformations. The tension/shear coupling causes inplane rotation of the ends of a rectangular specimen. In the case of a tube, the ends remain parallel, but rotate about the longitudinal axis of the tube. Unlike the tubular specimen, the rectangular coupon has a finite width which can cause edge effects.

Both specimen types have been used for off-axis testing. The tensile coupon configuration is the most common, mainly because it is economical to manufacture and can be tested in a conventional testing machine. However the loading method for the coupons must be carefully designed if reliable results are to be obtained. It is easier to instrument than the tubular configuration if strain measurements are required. The main disadvantage of the coupon specimen is the difficulty of obtaining a uniform state of stress within the specimen. Conventional testing machine clamping grips do not allow shear or transverse strain of the ends of the specimen, thus causing a nonuniform stress state. The nonuniform stress state influences measured stiffness properties, and can induce premature failures when strength testing.
The tubular specimen is more expensive to manufacture than the tensile coupon and requires specialised fixtures for testing. However it can provide a more uniform stress field than the rectangular specimen, with no edge effects and potentially less end effects. Thin walled tubular specimens have been used to obtain a range of mechanical properties. In addition to off-axis properties, a tube is also acknowledged as the optimum specimen configuration for combined loading [87] and in-plane shear characterisation [75]. The main disadvantages of tubular specimens are their cost, and the need for specialised testing equipment.

Taking account of the factors described above, and considering the literature detailed in Section A.2.3, thin walled tubular specimens were chosen as the most appropriate specimen type for this test programme.

1.3: Glass fibre tube testing

A preliminary testing programme was carried out on glass fibre reinforced polyester tubes [94]. These tubes were loaded by a dead weight system which ensured that there was no torsional constraint on the tubes' deformation. Longitudinal and shear strains were measured by a Linear Variable Differential Transformer (LVDT) based displacement instrument. Initial tests on these specimens revealed several problems with the test method, the strain measurement instrumentation and the specimen type. In particular,

- The loading system was unacceptably time consuming and subject to inertial effects.
- The longitudinal strain measurement system was significantly affected by slight bending of the tubes.
- The bi-directional lamination sequence of the tubes was not compatible with the developmental stage of the analytical model.

1.4: Carbon fibre tube testing

The problems described above were addressed by constructing more appropriate specimens, designing an improved loading method, and by using more comprehensive strain measurement techniques. A range of unidirectional carbon fibre/epoxy specimens were designed and constructed. A method for loading these tubes in a standard tensile testing machine was developed and verified. Foil strain gauges, conventional longitudinal extensometers, and a specialised torsion extensometer were used to measure deformation of the tubes. All specimens underwent tensile testing, the 0 and 90° specimens also being subjected to torsion testing.
A testing programme was carried out on a batch of glass fibre reinforced polyester tubes [94]. These tubes were sourced from Kilwell NZ Ltd, a fishing rod manufacturer based in Rotorua, New Zealand. The tubes were constructed from a glass fibre/polyester resin prepreg material. The tubes were constructed by rolling the prepreg material on to a steel mandrel at the required orientation, wrapping cellophane shrink-wrap tape tightly around the laminate and then curing the tube in a vertical oven. Tubes of varying fibre angles were constructed. Each tube had 90% of its fibres oriented at either 0°, 5°, 15°, 30°, 45° or 55° to the longitudinal axis of the tube, with the remaining 10% of the tube's fibres being oriented at 90° to the 90% direction. Burn off tests indicated a glass volume fraction of approximately 47%. Nominal dimensions of the tubes were:

- Outside Diameter: 28.6 mm
- Inside Diameter: 25.4 mm
- Wall Thickness: 1.6 mm
- Length: 400 mm

To avoid the uncertainties associated with foil strain gauges on composite materials (see Section A.2.3.4), a specialised displacement measuring instrument was designed and constructed. This instrument clamps to the tube, Figure 5.1, and uses Linear Variable Differential Transformers (LVDTs) to measure longitudinal and shear displacements over a 100 mm gauge length. From these displacements the corresponding strains can be calculated. The gauge length of this instrument was later changed to 50 mm prior to the instrument being used for shear strain measurement during the carbon fibre tube testing programme. Appendix C1 describes the design, development and calibration of this equipment.

The coupling between different modes of deformation in a typical off axis specimen makes the selection of specimen type and loading method crucial to the accuracy of the results. In the case of a tubular specimen the ends of the specimen remain parallel, the tension/shear coupling resulting in twisting about the specimen's longitudinal axis. For reliable tension test results it is necessary to apply an axial load to the specimen while allowing it complete freedom to twist.

The simplest loading method which provides a pure tensile load to a specimen while allowing it complete freedom to rotate is that of hanging a mass from the end of the
specimen. This method was used for the testing of the glass/polyester tubes. The “dead weight” test method for these tubes is depicted in Figure 5.1. A platform loaded with lead masses is attached to the lower end of the tube. The load cell above the tube measures the tensile load. Because the platform is hanging freely, it can rotate to allow shear deformation of the tube.

The end fittings were bonded within the tubes, as detailed for the carbon tube testing (Section 4.5). Clamping methods were not adequate for the applied loads.

Figure 5.1: Schematic of test method for glass/polyester tubes

The initial tests on these specimens revealed the following problems:

- The loading system was unacceptably time consuming and subject to inertial effects.
- The longitudinal strain measurement system was significantly affected by slight bending of the tubes.
The test procedure consisted of:

1. Instrumenting the specimen
2. Attaching the specimen ends to the crane and loading platform
3. Zeroing the instrumentation
4. Lowering the load platform to the floor and adding masses
5. Raising the load platform
6. Damping out torsional and transverse oscillations of the system
7. Reading instrumentation
8. Repeating steps 4 to 7 until maximum load reached.

The time consuming parts of this procedure were steps 4 and 6. In addition to the test method limitations, the bi-directional lamination sequence of the tubes was not compatible with the developmental stage of the analytical model. [94]

Although the dead weight loading method provided a tensile load to the specimen without any torsional constraint, the method was too time consuming to allow testing of a large number of specimens. An alternative method was developed, which allowed the use of a standard tensile testing machine. Experience gained in the preliminary tests led to the use of a much more comprehensive strain measurement system, using rosette strain gauges and extensometers on opposite sides of the specimen. A range of new unidirectional specimens were designed and constructed for the main test programme.

The remainder of Part C describes the development of the new test method, and details the experimental programme.
C.3: Wire Based Testing Method

3.1: General description

For the purposes of this research programme, the test method for the tubular specimens was required to:

- Apply a variable, controlled and measurable tensile load
- Allow the tube to rotate without any constraint
- Measure longitudinal, transverse (circumferential) and shear deformations

The choice of loading method was also influenced by practical constraints such as time, cost and equipment availability. While the dead weight loading method utilised for the glass/polyester tubes provided a theoretically pure tensile loading situation, for the reasons described in Section 5.2 it is not practically acceptable.

The loading method developed for the main testing programme allowed the use of a standard INSTRON mechanical tensile testing machine, which provided control over the load and loading rate. The tensile load was applied to the tubes through a length of small diameter, high tensile steel wire. Due to its diameter and length, the torsional stiffness of the wire was very small, and hence had a virtually insignificant effect on the deformation of the tubes. The induced errors due to the wire were calculated analytically and investigated experimentally. For the geometry and materials used, the maximum predicted errors due to the wire torsional stiffness were in the region of 0.2%. The test method validation is described in Section 5.6.

Figures 5.2, 5.3 and 5.4 depict the thin wire loading method. One end of the tubular specimen was attached through a universal joint to the testing machine’s load cell. A second universal joint attached the lower end of the tube to one end of the steel wire. The other end of the steel wire was mounted to the testing machine's crosshead. Steel clamping jaws provided attachments to the wire.
Figure 5.2: Schematic of steel wire loading method

Figure 5.3: Steel wire loading method
3.2: Wire Details

Different wire types and diameters were evaluated during development of the test method. The wire and its attachment method needed to have adequate tensile strength for the loads required by the tubes. The torsional stiffness of the wire had to be minimised to prevent torsional constraints or forces being applied to the tube. Since the length of the wire was restricted to a maximum of 500 mm by the dimensions of the testing machine, only the wire material, geometry and diameter could be varied.

The validation calculations (Section 5.6) for the wire based test method assumed the use of solid wire. However initial wire testing investigated the use of multi-strand wire for two reasons:
• The torsional stiffness would be less than a solid wire of similar diameter
• Loading eyelets could easily be formed at the ends of the wire

Strength tests were carried out on 2.5 and 3.0 mm diameter 1x19 stainless steel wires. This type of wire had 8 inner and 11 outer windings in opposite directions. It was anticipated that the reversed winding directions would prevent any torsional forces being generated by the loaded wire. Standard eyelets and brass crimps were used to create loading eyes at each end of the wires. Average failure loads for tensile loading of wire samples are given in Table 5.1.

Table 5.1: Stranded wire failure loads

<table>
<thead>
<tr>
<th>Wire Diameter (mm)</th>
<th>Failure Load</th>
<th>Average (kN)</th>
<th>Std Dev (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>3.6</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>5.5</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

A selection of 0° and 90° specimens were tested both with and without wires used in the loading system (Figures 5.5 and 5.6 respectively). As no tension/shear coupling should exist for these specimens, comparisons of experimental results indicated whether the wire had influenced the behaviour of the tubes.

Figure 5.5: Testing of zero degree specimen with stranded wire
It was clear that when carrying large tensile loads, such as in the 0° case, the stranded wires applied a significant torque to the tube. As the loads applied to the 90° tube were substantially less, and the shear stiffness of the tube was the same as that of the 0° specimen, the torque effects of the wire were less significant, although still unacceptable.

The stranded wire tests clearly demonstrated its unsuitability for the test programme. Strength tests were carried out on solid high tensile wires of various diameters, using alternative methods for applying the load to the wire (Table 5.2).

Table 5.2: Strength testing of solid wires

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Failure Load (kN)</th>
<th>UTS (MPa)</th>
<th>Loading Method</th>
<th>Failure Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.40</td>
<td>3.3</td>
<td>2144</td>
<td>Clamping</td>
<td>Fracture</td>
</tr>
<tr>
<td>1.60</td>
<td>4.2</td>
<td>2089</td>
<td>Clamping</td>
<td>Fracture</td>
</tr>
<tr>
<td>1.98</td>
<td>6.1</td>
<td>1980</td>
<td>Clamping</td>
<td>Fracture</td>
</tr>
<tr>
<td>2.38</td>
<td>6.3</td>
<td>n/a</td>
<td>Clamping</td>
<td>Slippage</td>
</tr>
<tr>
<td>2.38</td>
<td>5.6</td>
<td>n/a</td>
<td>Clamping &amp; Emery Paper</td>
<td>Slippage</td>
</tr>
<tr>
<td>2.38</td>
<td>2.0</td>
<td>450</td>
<td>Silver Soldered</td>
<td>Fracture</td>
</tr>
<tr>
<td>3.20</td>
<td>7.7</td>
<td>n/a</td>
<td>Clamping</td>
<td>Slippage</td>
</tr>
</tbody>
</table>

These test results demonstrated that the strength of the wires were greater than expected, and hence smaller diameter wires than initially anticipated could be used. Steel/steel
clamping provided adequate load transfer to the wires and was quick to attach (Figure 5.4). The minimum diameter wire required for adequate strength was used for each of the different orientation specimens (Table 5.3). Tests with different diameter wires demonstrated that the wire diameters used had no apparent effect on the deformation of the tubes (Section 5.6.1).

### Table 5.3: Wire diameters used for testing tubular specimens

<table>
<thead>
<tr>
<th>Fibre Orientation (°)</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire Diameter (mm)</td>
<td>None</td>
<td>1.98</td>
<td>1.60</td>
<td>1.40</td>
<td>1.40</td>
<td>1.40</td>
<td>None</td>
</tr>
</tbody>
</table>

### 3.3: Machine details

The main tensile testing programme was carried out in an INSTRON 1186 Tensile Testing Machine (Serial # H4240). A 200 kN load cell (Serial # UK130) was used for all tests.

Torsion testing of the 0° and 90° specimens was performed in a Tecquipment torsion testing machine, (Serial # B10371066).
C.4: Specimens

4.1: Type

The specimens consisted of unidirectional carbon fibre/epoxy resin tubes, of nominal dimensions:

- Outside Diameter: 26.8 mm
- Inside Diameter: 25.0 mm
- Wall Thickness: 0.9 mm
- Length: 400 mm

These dimensions result in a thickness to internal diameter ratio of 0.036, in keeping with Rizzo and Vicario's recommendations [79] that the ratio should be less than 0.05 for modulus determination. The specimen length is much greater than the minimum recommended by Pagano and Whitney [81] (4 x Radius + gauge length = 150 mm).

The specimens were constructed with fibre orientations of 0°, 15°, 30°, 45°, 60°, 75° and 90°. Five specimens of each orientation were fabricated, although in most cases three specimens were tested at each orientation.

4.2: Materials

The tubes were constructed from 300 mm wide Graphil 6 mil unidirectional carbon/epoxy prepreg. The material came from batch 254-2, with a nominal fibre weight of 147-148 gm/m² and a nominal 38% resin content by mass.

4.3: Construction method

The prepreg material was cut to the required dimensions for the particular orientation, taped into a rectangular panel and then manually rolled onto the ground steel male mandrels, resulting in a six ply unidirectional laminate. Strips of prepreg material were used to construct a boss to allow removal of the mandrel, then tensioned cellophane tape was wound around the exterior of the specimen. The specimens were cured hanging vertically, then the mandrels extracted with a hydraulic ram. The anisotropic thermal expansion characteristics of the material resulted in the force required for mandrel extraction varying significantly from low forces for the 90° specimens through to very high forces for the 0° specimens.
4.4: Fibre Volume Fraction

The fibre volume fractions of five specimens were measured by a computer based image analysis system. (Imageplus+ 5.76 on a Dapple computer). Cross-sectional samples from each specimen were mounted in epoxy resin and polished. A total of 228 cross-sections were viewed and analysed at 20x and 50x magnifications, resulting in an average measured volume fraction of 54%. Figures 5.7 and 5.8 show a typical cross-section at two magnification levels.

Figure 5.7: Cross-section of tube at 10x magnification

Figure 5.8: Cross-section of tube at 20x magnification
4.5: End fittings

Bonded aluminium plugs were used as end fittings for the tubes (Figure 5.9). The aluminium plugs were internally threaded to match standard INSTRON fittings. Due to thermal expansion anisotropy, the internal diameter of the tubes varied by approximately 0.1 mm between the 0° and 90° specimens. The plugs were machined to appropriate diameters for the different specimens, thus ensuring close fitting and hence coaxial end plugs.

![Diagram of end fittings for carbon/epoxy tubes]

Figure 5.9: End fittings for carbon/epoxy tubes

Proprietary high strength epoxy resin adhesive was used to bond the end fittings to the tubes. The aluminium plugs were wire brushed and solvent cleaned. The bond surface of the tubes was abraded with 80 grit emery paper and thoroughly solvent cleaned. The adhesive was cured at 80°C for 1 hour, followed by a minimum of 24 hours at room temperature. The elevated temperature cure and the surface preparation were crucial to obtaining an adequate bond between the epoxy adhesive and the cured epoxy of the tubes.
4.6: Individual Specimen Details

Table 5.4 lists the fibre angles, dimensions, strain gauge resistances and test numbers of the specimens tested.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Fibre Angle (°)</th>
<th>Test Number</th>
<th>Diameters</th>
<th>Strain Gauge Resistance (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Internal (mm)</td>
<td>External (mm)</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>T1, T26</td>
<td>24.96</td>
<td>26.80</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>T2, T3, T25</td>
<td>24.97</td>
<td>26.82</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>T27</td>
<td>24.96</td>
<td>26.85</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>T6, T8, T28</td>
<td>24.96</td>
<td>26.82</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>T9</td>
<td>24.95</td>
<td>26.78</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>T7</td>
<td>24.96</td>
<td>26.80</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>T11</td>
<td>24.98</td>
<td>26.76</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>T10</td>
<td>24.98</td>
<td>26.86</td>
</tr>
<tr>
<td>27</td>
<td>30</td>
<td>T12</td>
<td>24.99</td>
<td>26.86</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
<td>T13</td>
<td>24.99</td>
<td>26.86</td>
</tr>
<tr>
<td>21</td>
<td>45</td>
<td>T17</td>
<td>25.01</td>
<td>26.81</td>
</tr>
<tr>
<td>22</td>
<td>45</td>
<td>T21</td>
<td>25.00</td>
<td>26.89</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>T15</td>
<td>24.99</td>
<td>26.92</td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>T18</td>
<td>25.00</td>
<td>26.98</td>
</tr>
<tr>
<td>25</td>
<td>60</td>
<td>T20</td>
<td>25.02</td>
<td>26.85</td>
</tr>
<tr>
<td>13</td>
<td>75</td>
<td>T14</td>
<td>25.00</td>
<td>26.82</td>
</tr>
<tr>
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<td>T23, T30</td>
<td>25.03</td>
<td>27.02</td>
</tr>
</tbody>
</table>
C.5: Instrumentation

5.1: Introduction

The deformation of the tubes was measured by foil strain gauges and by displacement extensometers. The outputs from the extensometers, strain bridges and load cell were collected by an IBM PC based data acquisition system. The resulting data was processed and graphed in a standard spreadsheet computer software package. Figure 5.10 details the instrumentation system.

![Diagram of instrumentation system](image)

Figure 5.10: Instrumentation system layout

5.2: Strain gauges/gauging

Rosette strain gauges of various dimensions and resistances were used (Table 5.4). In all cases, pairs of rosette gauges were mounted on opposite sides of the specimen, allowing any minor tube bending effects to be measured and corrected for. Two types of gauges, Micro-Measurements EA-06-125RD-350 (Lot R-A38AD689) and Micro-Measurements EA-06-060RZ-120 (Lot R-A38AD533) were used. The 350Ω gauges were affixed with
Micro-Measurements AE 10 epoxy adhesive and cured for 2 Hours at 50°C. The 120Ω gauges were bonded with cyanoacrylate adhesive. Surface preparation of the specimens consisted of solvent cleaning, abrasion with 400 grit emery paper, dusting and then cleaning with alcohol followed by methyl ethyl ketone.

Micro-Measurements P3500 strain gauge bridge amplifiers were used to read the strain gauges. A 10 times analog amplifier was constructed to increase the P3500 signal output voltage to an adequate level for the data acquisition system.

The P3500 used a 2V supply voltage for the strain gauge bridge. This resulted in a 0.28 kW/m² power density for the 350 Ω gauges, well within the recommended range of 0.31-1.6 kW/m² for reinforced polymer materials [89]. Due to the lower resistance and smaller size of the 120 Ω gauges, this supply voltage resulted in an unacceptably high power density of 3.38 kW/m². External resistors were used to halve the supply voltage across the 120 Ω strain gauges, thereby reducing the power density to 0.84 kW/m². This modification to the gauge wiring, Figure 5.11, reduced the sensitivity of the gauges, but improved their temperature stability and hence accuracy.

Figure 5.11: Strain gauge wiring for 120 Ω gauges
5.3: Extensometers

Two conventional INSTRON clip gauge extensometers were used to measure the longitudinal strains of the tubes (Model G51-15, 50 mm gauge length). The extensometers were attached by rubber bands to each side of the tube, Figure 5.12. Light abrading of the tube surface was necessary for adequate extensometer location. The use of two extensometers allowed tube bending effects to be measured and accounted for. The extensometers were calibrated with a dial calibration rig before each test.

The LVDT based torsion measuring extensometer developed for the GRP tube testing was used to measure the shear strains of the tubes. The gauge length of the extensometer was reduced to 50 mm, allowing measurement of up to 3000 microstrain. This extensometer clamped to the tube at six points, Figure 5.12. Appendix C1 describes the design and calibration of this equipment.

Figure 5.12: Longitudinal and torsional extensometers
5.4: Data acquisition and processing

The 10 channels of data, Figure 5.10, were collected by an IBM PC based data acquisition system. This consisted of a 16 channel, 12 bit, +/-10 volt TECHMAR™ analogue to digital converter and data acquisition board. Although the board had 16 channels, this particular system had previously been used only for three channels and at frequencies of greater than approximately 30 Hz. Significant hardware and software modifications were required to achieve reliable use of the required ten channels at frequencies of approximately 1 Hz. Table 5.5 lists the channel numbers and data content.

<table>
<thead>
<tr>
<th>Channel #</th>
<th>Data</th>
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<tbody>
<tr>
<td>6</td>
<td>Strain Gauge #6</td>
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<tr>
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<td>Strain Gauge #5</td>
</tr>
<tr>
<td>8</td>
<td>Strain Gauge #4</td>
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<tr>
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<td>Strain Gauge #3</td>
</tr>
<tr>
<td>10</td>
<td>Strain Gauge #2</td>
</tr>
<tr>
<td>11</td>
<td>Strain Gauge #1</td>
</tr>
<tr>
<td>12</td>
<td>Torsion extensometer</td>
</tr>
<tr>
<td>13</td>
<td>Longitudinal ext. #2</td>
</tr>
<tr>
<td>14</td>
<td>Longitudinal ext. #1</td>
</tr>
<tr>
<td>15</td>
<td>Load</td>
</tr>
</tbody>
</table>

The ASCII data files from the data acquisition system were imported into a customised spreadsheet analysis template which allowed input of calibration constants, strain gauge sensitivities and tube dimensions. Spreadsheet formulae corrected for transverse sensitivity of the strain gauges, calculated on-axis strains and stresses, and used least squares regression to calculate compliances of the tubes. Spreadsheet macros were created to automate plotting of the required graphs of results.
C.6: Validation of Test Method

The errors induced by the wire based loading method were investigated analytically and experimentally. In the analytical investigation, comparisons were made between the specimen strains when loading by wire and the ideal specimen strains with a theoretically pure loading case. In the experimental study, specimens were tested with different diameter wires and the measured strains were compared.

The experimental validation of the test method also studied the behaviour of the experimental instrumentation. The effects of loading rate, load-hold-unload cycles and repeated loadings were investigated. The effects of removing and replacing the extensometers on the specimen were evaluated.

6.1: Analytical

The aim of the analytical modelling of the wire testing method was to calculate the errors in specimen strains caused by the torsional stiffness of the steel wire. The theoretical strains were calculated with the presence of a wire, and with a theoretically pure tensile load (no resistance to torsional deformation). Representative material properties were used to model the tubular specimens as homogeneous orthotropic thin shells. The layout and boundary conditions for each of the analytical cases are detailed in Figure 5.13.

![Analytical verification models](image)

Figure 5.13: Analytical verification models
**Case i: With wire**

Consider compatibility at point b: the twist of the specimen (R_{y_{sp}}) must be equal to that of the wire (R_{y_{w}}) at this point. i.e. \( R_{y_{sp}} = R_{y_{w}} \) \( (5.1) \)

For an isotropic, homogeneous wire:

\[ R_{y_{w}} = \frac{T_w L_w}{J_w G_w} \]  
\( (5.2) \)

For an orthotropic, homogeneous specimen:

\[ R_{y_{sp}} = \frac{2 \frac{\gamma_{xy}}{L_s} L_s}{d_s} \]  
\( (5.3) \)

Where the surface shear strain of the specimen,

\[ \gamma_{xy} = \bar{S}_{26} \sigma_y + \bar{S}_{66} \sigma_{xy} \]  
\( (5.4) \)

and the torque:

\[ T_{sp} = \frac{2 \sigma_{xy}}{d_{sp}} J_{sp} \]  
\( (5.5) \)

Substituting (5.2) and (5.3) into (5.1) gives:

\[ \gamma_{xy} = \frac{L_w d_{sp} T_w}{2 L_{sp} J_w G_w} \]  
\( (5.6) \)

Substituting (5.4) and (5.5) into (5.6), and with \( T_w = T_{sp} \), we find that

\[ \sigma_{xy} = \frac{\bar{S}_{26} \sigma_y}{\left( \frac{L_w J_{sp}}{L_{sp} J_w G_w} - \bar{S}_{66} \right)} \]  
\( (5.7) \)

Then for the specimen:

\[ \varepsilon_x = \bar{S}_{12} \sigma_y + \bar{S}_{16} \sigma_{xy} \]
\[ \varepsilon_y = \bar{S}_{22} \sigma_y + \bar{S}_{26} \sigma_{xy} \]  
\( (5.8) \)

\[ \varepsilon_{xy} = \bar{S}_{26} \sigma_y + \bar{S}_{66} \sigma_{xy} \]

with \( \sigma_{xy} \) given by equation 5.7, and \( \sigma_y = \frac{P}{A_{sp}} \) \( (5.9) \)

**Case ii: ideal loading**

If the tube is free to rotate then \( \sigma_{xy} = 0 \) and hence

\[ \varepsilon_x = \bar{S}_{12} \sigma_y \]
\[ \varepsilon_y = \bar{S}_{22} \sigma_y \]  
\( (5.10) \)

\[ \varepsilon_{xy} = \bar{S}_{26} \sigma_y \]
Results of analytical error estimation

The ideal case strains, the wire based strains and the percentage differences between the two situations were calculated for the full range of specimen angles and for a range of wire diameters. Appendix C3 contains the full table of results. Table 5.6 summarises the predicted errors for the wire diameters used for the actual testing. The required wire diameter was governed by the tensile loads applied to the specimens.

<table>
<thead>
<tr>
<th>Fibre Angle</th>
<th>Wire Ø (mm)</th>
<th>Percentage Errors in Strains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Longitudinal</td>
</tr>
<tr>
<td>15°</td>
<td>1.98</td>
<td>0.048</td>
</tr>
<tr>
<td>30°</td>
<td>1.60</td>
<td>0.023</td>
</tr>
<tr>
<td>45°</td>
<td>1.40</td>
<td>0.009</td>
</tr>
<tr>
<td>60°</td>
<td>1.40</td>
<td>0.004</td>
</tr>
<tr>
<td>75°</td>
<td>1.40</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The predicted errors are very small for all specimen configurations. Even in the 15° case (which has the largest loads of the off-axis specimens, and the most tension shear coupling) the maximum error is less than 0.2%.
6.2: Experimental

The experimental evaluation of the test method investigated the effects of the wire loading technique and studied the behaviour of the experimental instrumentation. The effects of repeated loadings, load-hold-unload cycles and loading rate were investigated. The effects of removing and replacing the extensometers on the specimen were evaluated.

Repeated Loading

Specimens were subjected to repeated loading cycles. The measured deformations were very consistent between cycles. Typical results are presented in Figures 5.14 and 5.15.

Figure 5.14: Repeated Loading; Longitudinal Deformation
Test #11, Tube #6, 30°, Ext. F

Figure 5.15: Repeated Loading; Shear Deformation
Test #11, Tube #6, 30°, Ext. SHR
Loading Rate

The crosshead speed of the INSTRON testing machine was varied from 1 to 10 mm/min for a specimen type which was normally tested at 5 mm/min, Figure 5.16. No loading rate effect was evident.

![Figure 5.16: Loading Rate Effects on Measured Deformation](image)

Test #8, Tube #10, 15", SG1 Data

Resetting of Extensometers

The gauge length setting of the longitudinal extensometers was crucial to their accuracy. To investigate the consistency of gauge length setting when fitting the extensometers, specimen tests were repeated with the extensometers being removed and replaced between tests. Figure 5.17 demonstrates the consistency of gauge length resetting.

![Figure 5.17: Resetting of Longitudinal Extensometer Gauge Length](image)

Test #13, Tube #20, 45", Ext F
Load-Hold-Unload Cycles

The stability of the instrumentation system was investigated by loading a specimen, holding the load at that level for a time interval and then unloading the specimen. Figure 5.18 shows the results. The measured voltages from all forms of instrumentation were stable and returned to their initial values.

![Diagram showing load-hold-unload cycles](image)

**Figure 5.18: Load-hold-unload behaviour of test system**

Test #10, Tube #7, 30°

Wire Diameter

The analytical modelling of the wire testing method predicted that the torsional stiffness of the wire should have an insignificant effect on the deformation of the tubes (Section 5.6.1). The maximum predicted errors in strains, 0.2%, were for the 15° specimen which had the largest loads and most tension-shear coupling of the off-axis specimens.

The effect of the wire loading method was investigated experimentally by testing 15°, 30° and 45° specimens with a range of different diameter wires. The measured strains for the 15° case are compared in Figures 5.19 to 5.21. The results for the 30° and 45° cases can be found in Appendix C4. The strain results given are from the strain gauges, the extensometer results demonstrated the same behaviour.
Figure 5.19: Wire Diameter Effects on Longitudinal Deformation of 15° Specimen
Test #6, Tube #10, 15°, SG

Figure 5.20: Wire Diameter Effects on Transverse Deformation of 15° Specimen
Test #6, Tube #10, 15°, SG
In all cases, the wire diameter did not appear to influence the deformation of the specimen. This confirmed the theoretical predictions of insignificant wire effects on the tube deformation. The wire diameter used for the actual test programme (Table 5.3) varied with the fibre orientation (and hence required load) of the specimen. The minimum diameter wire required for the applied load was used in each case. The diameter used was never greater than those tested above.

A 15° specimen was tested without using wire in the loading system. The tube was attached to the testing machine (through universal joints) at each end, and was thus restrained from large rotations. A small degree of rotational freedom existed due to free-play in the universal joints and the end fittings. Figures 5.22 to 5.24 compare the strains for this case to those for the tube tested with a 1.98 mm wire.
Figure 5.22: Longitudinal deformation of 15° specimen tested with and without wire
Test #28, Tube #10, 15°, SG

Figure 5.23: Transverse deformation of 15° specimen tested with and without wire
Test #28, Tube #10, 15°, SG
Graphs 5.28 and 5.30 demonstrate that the lack of wire in the loading system reduced the longitudinal and shear strains of the specimen. The transverse strain was virtually unchanged, with only a 1% difference. The most significant effect was in the case of the shear strain, which was reduced by approximately 11%. The longitudinal strain was approximately 6% less than the wire loaded case. As the torsional constraint directly affects the rotation, and hence shear of the specimen, it is not surprising that the shear strain suffered the greatest effect.

Even without the wire the tube was not fully constrained, and could still undergo shear deformations. However stiffness properties measured from such a specimen would be significantly in error.
7.1: Structure of programme

The experimental programme, Table 5.7, had four stages:

- Confirming instrumentation reliability
- Developing and verifying the wire based testing method
- Tensile testing of specimens
- Torsion testing of 0° and 90° specimens

7.2: Test Procedure

A standard procedure was followed for all tests. For the tensile tests the procedure consisted of:

**Preparation:**

- Allow instrumentation system to warm up and stabilise
- Attach torsion extensometer and measure calibration diameter
- Attach end fittings, mount specimen in testing machine
- Fit longitudinal extensometers to specimen
- Wire strain gauges to strain bridges
- Zero all instrumentation
- Load/unload specimen
- Reset instrumentation zeros
- Run calibration test (acquire zero and full scale values for instrumentation)

**Tensile testing:**

- Apply load to maximum value then unload, collecting data
- Repeat load cycle two times
- Reset gauge length of longitudinal extensometers
- Load cycle three times
- Reset gauge length of longitudinal extensometers
- Load cycle three times

**Torsion testing:**

- Fit specimen to torsion testing machine
- Set instrumentation zeros
- Apply load cycle
- Repeat load cycle three times
7.3: Test Details

Table 5.7 lists the tests carried out on the CFRP tubes. A "Standard Tensile" test consisted of nine loading cycles, involving three groups of three loading cycles with the extensometers being reset between consecutive sets. Appendix C2 details the loads and crosshead speeds for each test.

Table 5.7: List of carbon fibre tube tests

<table>
<thead>
<tr>
<th>Test #</th>
<th>Fibre Angle (°)</th>
<th>Specimen #</th>
<th>Wire Type</th>
<th>Test Description</th>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>None</td>
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</tr>
<tr>
<td>2</td>
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<td>Testing Instrumentation</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>15</td>
<td>3.0mm Stranded</td>
<td>Wire Evaluation</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>17</td>
<td>None, 2.5, 3.0mm Stranded</td>
<td>Wire Evaluation</td>
</tr>
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<td>90</td>
<td>17</td>
<td>1.98mm Solid</td>
<td>Wire Evaluation</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>10</td>
<td>1.40, 1.60, 1.98 mm Solid</td>
<td>Wire Evaluation</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>30</td>
<td>1.98 mm Solid</td>
<td>Standard Tensile</td>
</tr>
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<td>8</td>
<td>15</td>
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<td>1.98 mm Solid</td>
<td>Loading Rate</td>
</tr>
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<td>7</td>
<td>1.60, 1.98mm Solid</td>
<td>Wire Evaluation</td>
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<td>90</td>
<td>37</td>
<td>n/a</td>
<td>Torsion</td>
</tr>
</tbody>
</table>
C.8: Summary

Two methodologies were evaluated for the tensile testing of off-axis FRP tubular specimens. The first method used lead masses to apply a dead weight to the end of the specimens, thus providing a tensile load while allowing the specimens freedom to rotate. The method was too time consuming to be suitable for testing a large number of specimens.

The second method used a conventional tensile testing machine. The load was applied to the tubular specimens through a small diameter, high strength steel wire. Due to its diameter and length, the torsional stiffness of the wire was very small, and hence had a virtually insignificant effect on the deformation of the tubes.

The wire based testing method was verified analytically and experimentally, then used to test a comprehensive range of off-axis carbon fibre/epoxy resin specimens. The deformations of the tubes were measured by extensometers and foil resistance strain gauges.

Appendix C5 presents the stress/strain curves for each specimen, and Appendix C6 lists the compliances measured for each specimen. The overall results of the experimental programme are presented and discussed in Chapter A.5. of Part A.
Experimental Appendices

C1: LVDT based extensometer
C2: Test details
C3: Predicted wire loading errors
C4: Wire diameter effects
C5: Experimental stress/strain curves
C6: Measured compliances
C7: Normalised compliances
C8: Strain gauges vs. extensometers
Appendix C1: LVDT Based Extensometer

The Linear Variable Differential Transformer (LVDT) based extensometer constructed for this research was used in two configurations. The first (see Figure C 5.1), measured both longitudinal and shear deformations of tubular specimens and was used for the testing of the glass/polyester tubes. While the extensometer performed well for measuring shear deformations, a single measurement of the longitudinal displacements could not detect any slight bending of the specimens.

In its second configuration, Figure 1, the extensometer was used to measure the shear deformations of the carbon/epoxy tubes. In this case two conventional clip gauge extensometers were used to measure the longitudinal strains.

Figure 1: Extensometers

Each end of the torsional extensometer clamps to the tubular specimen at three points. A portion of aluminium tube is used as a guide when fitting the extensometer to a specimen to ensure that the ends are parallel and at a constant gauge length.
The LVDT measures the relative displacement of the two ends of the extensometer. The surface shear strain of the specimen is then calculated from the gauge length and the relative radii of the specimen and the LVDT, as in Figure 2:

![Diagram of LVDT](image)

\[ \gamma = \frac{U_s}{GL} \frac{R_1}{R_2} \]

Where:
- \( U_s \) = displacement at LVDT
- \( GL \) = gauge length (50 mm)
- \( R_2 \) = radius of LVDT
- \( R_1 \) = radius of specimen

Figure 2: Cross-section of extensometer

The LVDT is an Schaevitz Type 005 MS AC, with a nominal linear range of +/- 0.250 mm. A Daytronic 9130 conditioning amplifier was used for signal conditioning. The amplifier was adjusted to give 1 volt per 0.05 mm. Calibration checks demonstrated that the LVDT was linear to within 1% over a range of +/- 0.20 mm.

The torsion extensometer was evaluated by testing an aluminium tube and comparing the results to theoretical and strain gauge results. All three sets of data were within 3% for the normal operating range.
## Appendix C2: Test Details

<table>
<thead>
<tr>
<th>Test #</th>
<th>Fibre Angle</th>
<th>Specimen #</th>
<th>Number of Runs</th>
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<th>Maximum Load (kN)</th>
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# Appendix C3: Predicted Wire Loading Errors

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Loads are those required to achieve a maximum strain of 2000 microstrain.
Appendix C4:
Wire Diameter Effects

30 Degree Results:
Test #10, Tube #7, 30°, Strain gauge data

Figure 6.1: Wire Diameter Effects on Longitudinal Deformation of 30° Specimen

Figure 6.2: Wire Diameter Effects on Transverse Deformation of 30° Specimen
**45 Degree Results:**

Test #13, Tube #20, 45°, Strain gauge data
Figure 6.5: Wire Diameter Effects on Transverse Deformation of 45° Specimen

Figure 6.6: Wire Diameter Effects on Shear Deformation of 45° Specimen
Appendix C5: Experimental stress/strain curves

Section A 5.2.3 presents and discusses the characteristics of a stress/strain curve for one of the CFRP specimens. A total of 21 specimens were tested, each a number of times (see Appendix C2), resulting in a total of 240 loading cases. As it is impractical to present results for all of these situations, this appendix presents representative stress/strain curves for one specimen at each of the fibre orientations tested. Appendix C8 contains the compliances calculated from the stress/strain data for each specimen.

0°: Test 27, Run 2
15°: Test 27, Run 2

30°: Test 27, Run 2

45°: Test 27, Run 2
## Appendix C6: Measured Compliances

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### Appendix C8: Strain gauges vs. extensometers

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