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Numerical Modelling of Wave Runup on Breakwaters

by

Gavin Noel Palmer

A thesis submitted in partial fulfilment of the requirements for
the degree of Doctor of Philosophy

Supervised by Dr C.D. Christian

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Abstract

The design of rubble mound breakwaters is typically based on empirical formulae and physical modelling. One limitation of this approach is that different aspects of wave interaction with a breakwater, such as the elevation of the runup tip and armour stability, are treated separately. Therefore the development of a numerical model of wave runup on a rubble mound breakwater was the primary objective of the research described in this thesis.

Because of the range of slope conditions encountered with rubble mound breakwaters and revetments, two types of armour layer are considered. The first is impermeable and so only the flow within the external region is modelled. The flow is assumed to be governed by the unsteady one-dimensional shallow water wave equations and only regular waves are considered. It is shown how the use of the finite element method with a mesh of isoparametric elements that deforms and is fitted to the runup tip has a number of advantages over the traditional use of the finite difference method with a fixed grid.

Reasonably good results were obtained for the numerical modelling of wave runup on a riprap armoured 1:3 impermeable slope indicating that the numerical model may, in conjunction with a physical model, be of practical use in the design of revetments. Wave runup on smooth and Dolos armoured 1:1.5 impermeable slopes was modelled poorly. Therefore the model is more appropriate for wave runup on a revetment than a rubble mound breakwater.

The second type of armour layer is permeable and so the flow within the external region and armour layer is modelled simultaneously by coupling numerical models for the respective regions. It is concluded that this approach is unlikely to give acceptable results for the runup of regular waves on a steep, permeable armour layer unless it also accounts for the non-hydrostatic distribution of pressure within the external region.

An experiment is described in which continuous time histories of wave runup and dynamic pressure due to regular waves on smooth and Dolos armoured 1:1.5 slopes were measured. The results are used to discuss the assumption of hydrostatic pressure.

A method of assessing armour stability requirements which takes into consideration the effects of armour unit interaction is proposed. It is recommended that this is examined further.
Acknowledgements

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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ALE</td>
<td>Arbitrary lagrangian-eulerian</td>
</tr>
<tr>
<td>BEM</td>
<td>Boundary element method</td>
</tr>
<tr>
<td>DTI</td>
<td>Discrete time interval</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite difference method</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element method</td>
</tr>
<tr>
<td>MAC</td>
<td>Marker and cell</td>
</tr>
<tr>
<td>MOC</td>
<td>Method of characteristics</td>
</tr>
<tr>
<td>PIANC</td>
<td>Permanent International Association of Navigation Congresses</td>
</tr>
<tr>
<td>STE</td>
<td>Space-time element</td>
</tr>
<tr>
<td>SUMMAC</td>
<td>Stanford University marker and cell</td>
</tr>
<tr>
<td>SWL</td>
<td>Still water level</td>
</tr>
<tr>
<td>USACE</td>
<td>United States Army, Corps of Engineers</td>
</tr>
<tr>
<td>VOF</td>
<td>Volume of fluid</td>
</tr>
</tbody>
</table>
List of Notation

With the exception of the dimensionless variables defined in Appendix E, and \( \mathbf{U} \), bold upper case symbols are matrices with \( | \) indicating 'the determinant of'. Similarly, with the exception of the dimensionless variables defined in Appendix E, bold lower case symbols and bold greek symbols are vectors. Global vectors and matrices are subscripted, \( g \) and, unless otherwise annotated, vectors and matrices are defined at element level and therefore comprise local values. The subscripts indicate an association with a particular node whereas the superscripts are generally iteration or time step counters.

For simplicity, specific notation is not used to distinguish between experimentally measured and averaged values or between experimentally measured and numerically computed values. Instead the distinction is noted within the text.

<table>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>( A )</td>
<td>Surface area of armour layer</td>
</tr>
<tr>
<td>( A, B )</td>
<td>Dimensionless coefficients (Eqn 2.20)</td>
</tr>
<tr>
<td>( A )</td>
<td>Amplitude factor</td>
</tr>
<tr>
<td>( A_0, B_0 )</td>
<td>Dimensionless coefficients (Eqn 2.21b)</td>
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<tr>
<td>( A_r, B_r )</td>
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<tr>
<td>( a, b )</td>
<td>Forchheimer constants</td>
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<tr>
<td>( a_r, b_r, c_r, d_r )</td>
<td>Dimensionless coefficients (Eqn 5.5a)</td>
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<tr>
<td>( a_r, b_r, c_r, d_r )</td>
<td>Dimensionless coefficients (Eqn 5.5b)</td>
</tr>
<tr>
<td>( B )</td>
<td>Waist width of Dolos armour unit</td>
</tr>
<tr>
<td>( C )</td>
<td>Overall length of Dolos armour unit</td>
</tr>
<tr>
<td>( C_f )</td>
<td>Friction factor</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Pressure distribution correction factor</td>
</tr>
<tr>
<td>( c )</td>
<td>Wave celerity</td>
</tr>
<tr>
<td>( e )</td>
<td>Volumetric porosity</td>
</tr>
<tr>
<td>( F )</td>
<td>Fractional volume of fluid</td>
</tr>
<tr>
<td>( f' )</td>
<td>Friction factor</td>
</tr>
<tr>
<td>( f_{i,m}^k, f_{s,m}^k )</td>
<td>Unbalanced element forces for the continuity and momentum equations respectively (mth local node, kth iteration)</td>
</tr>
</tbody>
</table>
Local vector of unbalanced forces
Global vector of unbalanced forces
Gravitational acceleration
Wave height
Dimensionless wave height (Eqns E.4a and E.4b)
Unrefracted deepwater wave height
Significant wave height
Water depth
Local vector of water depth
Water depth at runup tip
Still water depth
Interpolated previous time step water depth at the jth global node
Global vectors of water depth at t=0, t', t and t'+1 respectively
Local vectors of water depth at t=t', t' and t'+1 respectively
Local vector of interpolated previous time step values of water depth
Time step counter
Total number of global nodes
Jacobian matrix
Bessel functions of the first kind
Global node counter
Hydraulic conductivity
Average hydraulic conductivity for element, e
Stability coefficient
Permeability: Iteration counter
Layer coefficient
Wavelength: Number of integration points for an element
Deepwater or offshore wavelength
Length of channel: Integration point counter
Horizontal length of fluid element
Slope-parallel length of the runup tip
Total water depths at local nodes 1, 2 and 3 respectively
Minimum or maximum total water depth
Length of fluid element at t=t', t' and t'+1 respectively
Total number of nodes connected to an element
Element mass matrix
Local node counter
Horizontal mass flowrate across the seaward edge of the armour layer
Total number of armour units
Stability number
List of Notation

n  Number of layers comprising primary armour layer: Local node counter
P  Average porosity of the primary cover layer
p  Pressure
p_d  Dynamic pressure
Q  Volumetric flowrate across the seaward edge of the armour layer
q  Bulk or macroscopic velocity
q_b  Volumetric flux
q'  Volumetric flow rate per unit horizontal length of the armour layer
\bar{q}  Average volumetric flow rate per unit horizontal length of the armour layer
R_d  Maximum rundown
R_u  Maximum runup
R_d,15  Maximum rundown at location where total water depth \((h+\eta)\) equals 15mm
R_u,15  Maximum runup at location where total water depth \((h+\eta)\) equals 15mm
r  Roughness and porosity correction factor
\mathbf{r}  Vector of boundary and initial conditions
\rho_w  Waist ratio \((=B/C)\)
\mathbf{r}_x, \mathbf{r}_y  Unit normal vectors in the x and y directions respectively
S  Element stiffness matrix
S^e  Element tangent stiffness matrix
S^g  Global tangent stiffness matrix
\gamma_s  Specific gravity of an armour unit \((=\gamma/\gamma_w)\)
T  Wave period
T_d  Dimensionless wave period (Eqns E.4a and E.4b)
T_s  Significant wave period
t  Time
t  Dimensionless time (Eqns E.4a and E.4b)
U  Depth-averaged horizontal velocity
U^e  Local vector of horizontal velocity
\bar{U}  Dimensionless depth-averaged horizontal velocity (Eqns E.4a and E.4b)
U_k  Elemental velocity
U_p  Depth-averaged horizontal velocity within the armour layer
U_b  Depth-averaged horizontal velocity at seaward edge of armour layer
U_j^t  Interpolated previous time step horizontal velocity at the jth global node
U_{k,0}^{t}, U_{k}^{t}, U_{k}^{t+1}, U_{k}^{t+1}  Horizontal velocity of the runup tip at \(t=t^{'}, t', t'^{+1}\) and the kth iteration respectively
U_0^{t}, U_{r}^{t}, U_{r}^{t+1}, U_{r}^{t+1}  Global vectors of horizontal velocity at \(t=0, t', t'^{+1}\) respectively
U_i^{t}, U_{i}^{t}, U_{i}^{t+1}, U_{i}^{t+1}  Local vectors of horizontal velocity at \(t=t', t'^{+1}\) and the kth iteration respectively
\( u, v \) Horizontal and vertical components of velocity

\( u_A, v_A \) Horizontal and vertical components respectively of vertical velocity at node A

\( u' \) Horizontal velocity at the seaward edge of the armour layer

\( \bar{u} \) Average horizontal velocity at the seaward edge of the armour layer

\( u'_A, v'_A \) Transformed horizontal and vertical components respectively of velocity at node A

\( u_C \) Horizontal component of velocity at node C

\( W \) Dry weight of an individual armour unit in the primary cover layer

\( W_I \) Integration weight for the \( I \)th integration point

\( W_{so} \) Median weight

\( w_e \) Unit weight of an armour unit

\( X_i^{t+1} \) Horizontal global coordinate of the \( j \)th global node at \( t=t^{i+1} \)

\( X_p \) Horizontal global coordinate of pressure tapping

\( X_i^0, X_i^1, X_i^{i+1} \) Horizontal global coordinates of the runup tip at \( t=t^i, t^{i+1} \) and the \( k \)th iteration respectively

\( X_A^{i+1} \) Horizontal global coordinate of node A at \( t=t^{i+1} \)

\( X_C^{i+1} \) Horizontal global coordinate of node C at \( t=t^{i+1} \)

\( x \) Horizontal global coordinate

\( x^* \) Dimensionless horizontal global coordinate (Eqns E.4a and E.4b)

\( x_1, x_2, x_3, x_4 \) Global vectors of horizontal global coordinates at \( t=0, t^i, t^e \) and \( t^{i+1} \) respectively

\( x'_1, x'_2, x'_3, x'_4 \) Local vectors of the horizontal global coordinates of the nodes of an element at \( t=t^i, t^e \) and \( t^{i+1} \) respectively

\( Y_A^{i+1}, Y_A^{i+1} \) Transformed vertical global coordinates of node A at \( t=t^i, t^{i+1} \) respectively

\( Y_A^{i+1} \) Vertical global coordinate of node A at \( t=t^{i+1} \)

\( Y_C^{i+1} \) Vertical global coordinate of node C at \( t=t^{i+1} \)

\( y \) Vertical global coordinate

\( y' \) Rotated vertical global coordinate

\( Z_p \) Elevation of pressure tapping

\( Z_R^{i}, Z_R^{i+1}, Z_R^{k} \) Elevation of the seaward slope at the location of the runup tip at \( t=t^i, t^{i+1} \) and the \( k \)th iteration respectively

\( \alpha \) Angle of seaward slope

\( \alpha_c \) Critical angle of seaward slope

\( \Delta t \) Time step size

\( \delta \) Node position tolerance

\( \delta_j \) Potential distance moved by the \( j \)th global node

\( \epsilon_{NR} \) Newton-Raphson iteration convergence tolerance

\( \epsilon_R \) Runup tip elevation tolerance
List of Notation

\( c_k \) Hydraulic conductivity iteration tolerance
\( c_w \) Internal water surface elevation tolerance
\( \eta \) Water surface elevation
\( z \) Dimensionless elevation (Eqns E.4a and E.4b)
\( \eta_e \) Elevation of crest of incident wave
\( \eta_H \) Water surface elevation at location where total water depth equals \( h_H \)
\( \eta_{15} \) Water surface elevation at location where total water depth equals 15mm
\( \eta_w \) Water surface elevation within armour layer at location where total water depth equals \( h_w \)
\( \eta_p \) Water surface elevation corresponding with \( X_p \)
\( \eta_{0\,i+1} \) Water surface elevation at the seaward boundary at \( t = t^{i+1} \)
\( \eta_{j} \) Interpolated previous time step water surface elevation at the \( j \)th global node
\( \eta_{R,j}, \eta_{R\,i+1}, \eta_{R}^{k} \) Elevation of the runup tip at \( t = t^i, t^{i+1} \) and the \( k \)th iteration respectively
\( \eta_{0\,i}, \eta_{0\,i+1}, \eta_{k}^{0} \) Global vectors of water surface elevation at \( t = 0, t^i, t^{i+1} \) respectively
\( \eta_{i}, \eta_{i}, \eta_{i+1}, \eta_{k} \) Local vectors of water surface elevation at \( t = t^i, t^i, t^{i+1} \) and the \( k \)th iteration respectively
\( \eta \) Local vector of interpolated previous time step values of water surface elevation
\( \eta \) Local vector of water surface elevation
\( \gamma_f \) Unit weight (saturated surface dry) of an armour unit
\( \gamma_w \) Unit weight of water
\( \theta \) Time weighting parameter
\( \lambda \) Dimensionless coefficient (Eqns E.4a and E.4b)
\( \xi \) Surf similarity parameter (alternatively termed the Iribarren number, \( Ir \)): Local coordinate in spatial domain
\( \xi \) Local coordinate corresponding with water depth \( h' \)
\( \xi_H \) Local coordinate corresponding with water depth \( h_H \)
\( \xi_b \) Breaker surf parameter
\( \xi_o \) Offshore surf parameter
\( \xi_j, \sigma_j \) Local coordinates of the \( j \)th global node
\( \xi_l, \sigma_l \) Local coordinates of the \( l \)th integration point
\( \xi_p \) Local coordinate corresponding with \( X_p \)
\( \kappa \) Element mass
\( \pi \) \( \pi = 3.141... \)
\( \rho \) Fluid density
\( \tau \) Friction coefficient
\( \phi \) Local vector of nodal values of \( \eta \) and \( U \) at \( t = t^{i+1} \)
\( \phi \) Piezometric head
\( \Delta \phi^{i+1} \) Global vector of corrections to \( \phi^k \)
\( \phi_k, \phi_{k+1} \)  
Global vectors of nodal values of \( \eta \) and \( U \) at the \( k \)th and \((k+1)\)th iterations respectively

\( \psi_n \)  
Shape function for the \( n \)th local node of an element

\( \Omega \)  
Domain of integration

\( \sigma \)  
Local coordinate in the time domain; Local coordinate in the vertical direction: Dimensionless variable (Eqns E.4a and E.4b)

\( \omega \)  
Weighting function

\( \omega_m \)  
Weighting function for the \( m \)th local node

\( c \)  
Dimensionless coefficient (Eqns E.4a and E.4b)