



<http://researchspace.auckland.ac.nz>

ResearchSpace@Auckland

Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage.

<http://researchspace.auckland.ac.nz/feedback>

General copyright and disclaimer

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library Thesis Consent Form.

Numerical Modelling of Wave Runup on Breakwaters

by

Gavin Noel Palmer

A thesis submitted in partial fulfilment of the requirements for
the degree of Doctor of Philosophy

Supervised by Dr C.D. Christian

Department of Civil Engineering
University of Auckland
Auckland
New Zealand
February 1994

Abstract

The design of rubble mound breakwaters is typically based on empirical formulae and physical modelling. One limitation of this approach is that different aspects of wave interaction with a breakwater, such as the elevation of the runup tip and armour stability, are treated separately. Therefore the development of a numerical model of wave runup on a rubble mound breakwater was the primary objective of the research described in this thesis.

Because of the range of slope conditions encountered with rubble mound breakwaters and revetments, two types of armour layer are considered. The first is impermeable and so only the flow within the external region is modelled. The flow is assumed to be governed by the unsteady one-dimensional shallow water wave equations and only regular waves are considered. It is shown how the use of the finite element method with a mesh of isoparametric elements that deforms and is fitted to the runup tip has a number of advantages over the traditional use of the finite difference method with a fixed grid.

Reasonably good results were obtained for the numerical modelling of wave runup on a riprap armoured 1:3 impermeable slope indicating that the numerical model may, in conjunction with a physical model, be of practical use in the design of revetments. Wave runup on smooth and Dolos armoured 1:1.5 impermeable slopes was modelled poorly. Therefore the model is more appropriate for wave runup on a revetment than a rubble mound breakwater.

The second type of armour layer is permeable and so the flow within the external region and armour layer is modelled simultaneously by coupling numerical models for the respective regions. It is concluded that this approach is unlikely to give acceptable results for the runup of regular waves on a steep, permeable armour layer unless it also accounts for the non-hydrostatic distribution of pressure within the external region.

An experiment is described in which continuous time histories of wave runup and dynamic pressure due to regular waves on smooth and Dolos armoured 1:1.5 slopes were measured. The results are used to discuss the assumption of hydrostatic pressure.

A method of assessing armour stability requirements which takes into consideration the effects of armour unit interaction is proposed. It is recommended that this is examined further.

Acknowledgements

The financial assistance provided by a University Grants Committee Postgraduate Scholarship and by the Wilkins and Davies Engineering Research Scholarship is gratefully acknowledged. The author would also like to sincerely thank the following people and organisations:

Dr C.D. Christian, for the advice and support given during his supervision of this research.

Professor P.G. Lowe, Head of the Department of Civil Engineering, and Professor J.D. Fenton, Professor of Fluid Mechanics, for their advice and support.

Staff of the Engineering Library, particularly Ms J. Rodgers, for obtaining reference material.

Staff of the Computer Centre for assistance in computing.

Professor N. Kobayashi, Assistant Director of the Center for Applied Coastal Research at the University of Delaware, for clarifying some aspects of the research by Mr J.H. Greenwald and for supplying information on similar research performed at the University of Delaware.

Environment Waikato for making their computer equipment available and for assistance in the production of this thesis.

Mr H. Kebbell for assistance with the electronic equipment used in the experimental work.

Finally, my parents, sisters and Kelly for their support and understanding. Without them this research would not have been possible.

Contents

| | |
|--|------|
| Abstract | iii |
| Acknowledgements | v |
| Contents | vii |
| List of Abbreviations | xi |
| List of Notation | xiii |
| | |
| CHAPTER 1 INTRODUCTION | 1 |
| 1.1 Research Objectives | 1 |
| 1.2 Outline of Thesis | 3 |
| | |
| CHAPTER 2 WAVE INTERACTION WITH RUBBLE MOUND BREAKWATERS | 5 |
| 2.1 Introduction | 5 |
| 2.2 Rubble Mound Breakwaters | 5 |
| 2.3 Numerical Modelling of Wave Interaction With Breakwaters | 22 |
| 2.4 Numerical Modelling of Moving Boundary Problems | 43 |
| 2.4.1 Fixed Mesh Approach | 44 |
| 2.4.2 Deforming Mesh Approach | 48 |
| 2.5 Experimental Measurements of Data for Use in Numerical Modelling .. | 54 |
| 2.5.1 Maximum Runup and Maximum Rundown | 55 |
| 2.5.2 Methods of Experimentally Measuring the Motion of the Runup Tip | 70 |
| 2.6 Discussion | 73 |
| 2.7 Outline of Proposed Research | 75 |
| | |
| CHAPTER 3 EXPERIMENTAL STUDY OF WAVE RUNUP | 81 |
| 3.1 Introduction | 81 |
| 3.2 Equipment | 81 |
| 3.3 Procedure and Method of Analysis | 87 |
| 3.4 Results and Discussion | 89 |
| 3.4.1 Smooth 1:1.5 Slope | 91 |
| 3.4.2 Dolos Armoured 1:1.5 Slope | 98 |
| 3.5 Discussion | 110 |

| | | |
|-----------|--|-----|
| CHAPTER 4 | DESCRIPTION OF THE NUMERICAL MODEL OF WAVE RUNUP ON AN IMPERMEABLE ARMOUR LAYER | 111 |
| 4.1 | Introduction | 111 |
| 4.2 | Calculation of the Position of the Runup Tip | 111 |
| 4.3 | Mesh Rezoning | 114 |
| 4.4 | Solution of the Equations Governing Flow | 119 |
| 4.4.1 | Discrete Time Interval Method | 121 |
| 4.4.2 | Arbitrary Lagrangian-Eulerian Method | 128 |
| 4.4.3 | Space-Time Element Method | 134 |
| 4.5 | Additional Computations | 139 |
| 4.5.1 | Interpolation of Water Surface Elevation | 139 |
| 4.5.2 | Evaluation of Individual Terms of the Equations Governing Flow | 139 |
| 4.6 | Discussion | 140 |
| 4.6.1 | Discrete Time Interval Method | 140 |
| 4.6.2 | Comparison of the Arbitrary Lagrangian-Eulerian and Space Time Element Methods | 142 |
| 4.7 | Summary of Calculation Procedure | 143 |
| CHAPTER 5 | DESCRIPTION OF THE NUMERICAL MODEL OF THE FLOW WITHIN A PERMEABLE ARMOUR LAYER | 145 |
| 5.1 | Introduction | 145 |
| 5.2 | Calculation of the Elevation of the Internal Water Surface | 146 |
| 5.3 | Mesh Rezoning | 150 |
| 5.4 | Solution of the Equation Governing Flow | 152 |
| 5.5 | Additional Computations | 159 |
| 5.5.1 | Interpolation of Water Surface Elevation | 159 |
| 5.5.2 | Calculation of Fluid Velocities | 159 |
| 5.6 | Discussion | 160 |
| 5.7 | Summary of Calculation Procedure | 160 |
| CHAPTER 6 | DESCRIPTION OF THE NUMERICAL MODEL OF WAVE RUNUP ON A PERMEABLE ARMOUR LAYER | 161 |
| 6.1 | Introduction | 161 |
| 6.2 | Calculation of the Position of the Runup Tip and the Elevation of the Internal Water Surface | 162 |
| 6.3 | Mesh Rezoning | 164 |
| 6.4 | Solution of the Equations Governing Flow | 166 |
| 6.5 | Additional Computations | 169 |
| 6.6 | Summary of Calculation Procedure | 169 |

| | | |
|------------|--|-----|
| CHAPTER 7 | NUMERICAL MODELLING OF WAVE RUNUP ON AN IMPERMEABLE ARMOUR LAYER | 171 |
| 7.1 | Introduction | 171 |
| 7.2 | Smooth 1:15.4 Slope | 173 |
| 7.3 | Smooth 1:4 Slope | 192 |
| 7.3.1 | Progressive Wave | 194 |
| 7.3.2 | Bore | 208 |
| 7.4 | Riprap Armoured 1:3 Slope | 213 |
| 7.5 | Smooth 1:1.5 Slope | 239 |
| 7.6 | Dolos Armoured 1:1.5 Slope | 247 |
| 7.7 | Discussion and Summary | 254 |
| CHAPTER 8 | DISCUSSION | 257 |
| 8.1 | Number of Spatial Dimensions | 257 |
| 8.2 | Spatial Discretisation | 257 |
| 8.3 | Solution of Equations Governing Flow | 258 |
| 8.4 | Numerical Modelling of Wave Runup | 259 |
| 8.5 | Assumption of Hydrostatic Pressure | 259 |
| 8.6 | Accounting for Slope Permeability | 260 |
| 8.7 | Assessment of Armour Stability Requirements | 261 |
| CHAPTER 9 | CONCLUSIONS | 263 |
| REFERENCES | | 267 |
| APPENDIX A | PROCEDURE FOR THE CALCULATION OF AN AVERAGED TIME HISTORY | 283 |
| APPENDIX B | RESULTS OF THE EXPERIMENTAL STUDY OF WAVE RUNUP | 287 |
| B.1 | Introduction | 287 |
| B.2 | Smooth 1:1.5 Slope | 287 |
| B.3 | Dolos Armoured 1:1.5 Slope | 309 |

| | | |
|------------|---|-----|
| APPENDIX C | DETAILS OF THE NUMERICAL MODEL OF WAVE RUNUP ON AN IMPERMEABLE ARMOUR LAYER | 335 |
| C.1 | Introduction | 335 |
| C.2 | Calculation of the Position of the Runup Tip | 335 |
| C.2.1 | Specification of Elevation | 336 |
| C.2.2 | Specification of Horizontal Velocity | 336 |
| C.3 | Mesh Rezoning | 337 |
| C.4 | Solution of the Equations Governing Flow | 340 |
| C.4.1 | Discrete Time Interval Method | 340 |
| C.4.2 | Arbitrary Lagrangian-Eulerian Method | 345 |
| C.4.3 | Space-Time Element Method | 350 |
| C.5 | Additional Computations | 354 |
| C.5.1 | Interpolation of Water Surface Elevation | 354 |
| C.5.2 | Evaluation of Individual Terms of the Equations Governing Flow | 357 |
| APPENDIX D | DETAILS OF THE NUMERICAL MODEL OF THE FLOW WITHIN A PERMEABLE ARMOUR LAYER | 359 |
| D.1 | Introduction | 359 |
| D.2 | Calculation of the Elevation of the Internal Water Surface | 359 |
| D.3 | Mesh Rezoning | 362 |
| D.4 | Solution of the Equation Governing Flow | 362 |
| D.5 | Additional Computations | 362 |
| D.5.1 | Interpolation of Water Surface Elevation | 362 |
| D.5.2 | Calculation of Fluid Velocities | 363 |
| APPENDIX E | ANALYTICAL SOLUTIONS OF THE UNSTEADY ONE-DIMENSIONAL SHALLOW WATER WAVE EQUATIONS | 365 |
| E.1 | Introduction | 365 |
| E.2 | Standing Wave in a Closed Channel | 365 |
| E.3 | Wave Runup on a Smooth Impermeable Slope | 366 |
| APPENDIX F | NUMERICAL MODELLING OF A STANDING WAVE IN A CLOSED CHANNEL | 369 |
| APPENDIX G | NUMERICAL MODELLING OF WAVE RUNUP ON AN IMPERMEABLE ARMOUR LAYER | 379 |
| G.1 | Introduction | 379 |
| G.2 | Smooth 1:1.5 Slope | 379 |
| G.3 | Dolos Armoured 1:1.5 Slope | 382 |

List of Abbreviations

| | |
|--------|--|
| ALE | Arbitrary lagrangian-eulerian |
| BEM | Boundary element method |
| DTI | Discrete time interval |
| FDM | Finite difference method |
| FEM | Finite element method |
| MAC | Marker and cell |
| MOC | Method of characteristics |
| PIANC | Permanent International Association of Navigation Congresses |
| STE | Space-time element |
| SUMMAC | Stanford University marker and cell |
| SWL | Still water level |
| USACE | United States Army, Corps of Engineers |
| VOF | Volume of fluid |

List of Notation

With the exception of the dimensionless variables defined in Appendix E, and U , bold upper case symbols are matrices with $| |$ indicating 'the determinant of'. Similarly, with the exception of the dimensionless variables defined in Appendix E, bold lower case symbols and bold greek symbols are vectors. Global vectors and matrices are subscripted, g and, unless otherwise annotated, vectors and matrices are defined at element level and therefore comprise local values. The subscripts indicate an association with a particular node whereas the superscripts are generally iteration or time step counters.

For simplicity, specific notation is not used to distinguish between experimentally measured and averaged values or between experimentally measured and numerically computed values. Instead the distinction is noted within the text.

| | |
|------------------------|--|
| A | Surface area of armour layer |
| A, B | Dimensionless coefficients (Eqn 2.20) |
| A | Amplitude factor |
| A_d, B_d | Dimensionless coefficients (Eqn 2.21b) |
| A_u, B_u | Dimensionless coefficients (Eqn 2.21a) |
| a, b | Forchheimer constants |
| a_x, b_x, c_x, d_x | Dimensionless coefficients (Eqn 5.5a) |
| a_y, b_y, c_y, d_y | Dimensionless coefficients (Eqn 5.5b) |
| B | Waist width of Dolos armour unit |
| C | Overall length of Dolos armour unit |
| C_f | Friction factor |
| C_p | Pressure distribution correction factor |
| c | Wave celerity |
| e | Volumetric porosity |
| F | Fractional volume of fluid |
| f' | Friction factor |
| $f_{1,m}^k, f_{2,m}^k$ | Unbalanced element forces for the continuity and momentum equations respectively (mth local node, kth iteration) |

| | |
|---|--|
| \mathbf{f}^* | Local vector of unbalanced forces |
| \mathbf{f}_g^k | Global vector of unbalanced forces |
| g | Gravitational acceleration |
| H | Wave height |
| H | Dimensionless wave height (Eqns E.4a and E.4b) |
| H_o | Unrefracted deepwater wave height |
| H_s | Significant wave height |
| h | Water depth |
| \mathbf{h} | Local vector of water depth |
| h_R | Water depth at runup tip |
| h_t | Still water depth |
| h_j^* | Interpolated previous time step water depth at the j th global node |
| $\mathbf{h}_g^0, \mathbf{h}_g^i, \mathbf{h}_g^\theta, \mathbf{h}_g^{i+1}$ | Global vectors of water depth at $t=0, t^i, t^\theta$ and t^{i+1} respectively |
| $\mathbf{h}^i, \mathbf{h}^\theta, \mathbf{h}^{i+1}$ | Local vectors of water depth at $t=t^i, t^\theta$ and t^{i+1} respectively |
| \mathbf{h}^* | Local vector of interpolated previous time step values of water depth |
| i | Time step counter |
| J | Total number of global nodes |
| \mathbf{J} | Jacobian matrix |
| J_0, J_1 | Bessel functions of the first kind |
| j | Global node counter |
| K | Hydraulic conductivity |
| K^e | Average hydraulic conductivity for element, e |
| K_D | Stability coefficient |
| k | Permeability: Iteration counter |
| k_Δ | Layer coefficient |
| L | Wavelength: Number of integration points for an element |
| L_o | Deepwater or offshore wavelength |
| l | Length of channel: Integration point counter |
| ℓ | Horizontal length of fluid element |
| ℓ_u | Slope-parallel length of the runup tip |
| ℓ_1, ℓ_2, ℓ_3 | Total water depths at local nodes 1, 2 and 3 respectively |
| ℓ' | Minimum or maximum total water depth |
| $\ell^i, \ell^\theta, \ell^{i+1}$ | Length of fluid element at $t=t^i, t^\theta$ and t^{i+1} respectively |
| M, N | Total number of nodes connected to an element |
| \mathbf{M} | Element mass matrix |
| m | Local node counter |
| m | Horizontal mass flowrate across the seaward edge of the armour layer |
| N_r | Total number of armour units |
| N_s | Stability number |

| | |
|---------------------------------------|---|
| n | Number of layers comprising primary armour layer: Local node counter |
| P | Average porosity of the primary cover layer |
| p | Pressure |
| p_D | Dynamic pressure |
| Q | Volumetric flowrate across the seaward edge of the armour layer |
| q | Bulk or macroscopic velocity |
| q_b | Volumetric flux |
| q' | Volumetric flow rate per unit horizontal length of the armour layer |
| \bar{q} | Average volumetric flow rate per unit horizontal length of the armour layer |
| R_d | Maximum rundown |
| R_u | Maximum runup |
| $R_{d,15}$ | Maximum rundown at location where total water depth $(h + \eta)$ equals 15mm |
| $R_{u,15}$ | Maximum runup at location where total water depth $(h + \eta)$ equals 15mm |
| r | Roughness and porosity correction factor |
| \mathbf{r} | Vector of boundary and initial conditions |
| r_w | Waist ratio $(=B/C)$ |
| $\mathbf{r}_x, \mathbf{r}_y$ | Unit normal vectors in the x and y directions respectively |
| \mathbf{S} | Element stiffness matrix |
| \mathbf{S}^k | Element tangent stiffness matrix |
| \mathbf{S}_g^k | Global tangent stiffness matrix |
| S_r | Specific gravity of an armour unit $(=\gamma_r/\gamma_w)$ |
| T | Wave period |
| T | Dimensionless wave period (Eqns E.4a and E.4b) |
| T_s | Significant wave period |
| t | Time |
| t | Dimensionless time (Eqns E.4a and E.4b) |
| U | Depth-averaged horizontal velocity |
| \mathbf{U} | Local vector of horizontal velocity |
| U | Dimensionless depth-averaged horizontal velocity (Eqns E.4a and E.4b) |
| U_e | Elemental velocity |
| U_p | Depth-averaged horizontal velocity within the armour layer |
| U_b | Depth-averaged horizontal velocity at seaward edge of armour layer |
| U_j^* | Interpolated previous time step horizontal velocity at the jth global node |
| $U_R^i, U_R^\theta, U_R^{i+1}, U_R^k$ | Horizontal velocity of the runup tip at $t=t^i, t^\theta, t^{i+1}$ and the kth iteration respectively |
| $U_g^0, U_g^i, U_g^\theta, U_g^{i+1}$ | Global vectors of horizontal velocity at $t=0, t^i, t^\theta$ and t^{i+1} respectively |
| $U^j, U^\theta, U^{i+1}, U^k$ | Local vectors of horizontal velocity at $t=t^i, t^\theta, t^{i+1}$ and the kth iteration respectively |

| | |
|---------------------------------------|---|
| u, v | Horizontal and vertical components of velocity |
| u_A, v_A | Horizontal and vertical components respectively of vertical velocity at node A |
| u' | Horizontal velocity at the seaward edge of the armour layer |
| \bar{u} | Average horizontal velocity at the seaward edge of the armour layer |
| u_A^*, v_A^* | Transformed horizontal and vertical components respectively of velocity at node A |
| u_C | Horizontal component of velocity at node C |
| W | Dry weight of an individual armour unit in the primary cover layer |
| W_l | Integration weight for the l th integration point |
| W_{50} | Median weight |
| w_r | Unit weight of an armour unit |
| X_j^{i+1} | Horizontal global coordinate of the j th global node at $t=t^{i+1}$ |
| X_p | Horizontal global coordinate of pressure tapping |
| X_R^i, X_R^{i+1}, X_R^k | Horizontal global coordinates of the runup tip at $t=t^i, t^{i+1}$ and the k th iteration respectively |
| X_A^{i+1} | Horizontal global coordinate of node A at $t=t^{i+1}$ |
| X_C^{i+1} | Horizontal global coordinate of node C at $t=t^{i+1}$ |
| x | Horizontal global coordinate |
| \bar{x} | Dimensionless horizontal global coordinate (Eqns E.4a and E.4b) |
| x^* | Rotated horizontal global coordinate |
| $x_g^0, x_g^i, x_g^\theta, x_g^{i+1}$ | Global vectors of horizontal global coordinates at $t=0, t^i, t^\theta$ and t^{i+1} respectively |
| x^i, x^θ, x^{i+1} | Local vectors of the horizontal global coordinates of the nodes of an element at $t=t^i, t^\theta$ and t^{i+1} respectively |
| Y_A^i, Y_A^{i+1} | Transformed vertical global coordinates of node A at $t=t^i, t^{i+1}$ respectively |
| Y_A^{i+1} | Vertical global coordinate of node A at $t=t^{i+1}$ |
| Y_C^{i+1} | Vertical global coordinate of node C at $t=t^{i+1}$ |
| y | Vertical global coordinate |
| y^* | Rotated vertical global coordinate |
| Z_p | Elevation of pressure tapping |
| Z_R^i, Z_R^{i+1}, Z_R^k | Elevation of the seaward slope at the location of the runup tip at $t=t^i, t^{i+1}$ and the k th iteration respectively |
| α | Angle of seaward slope |
| α_c | Critical angle of seaward slope |
| Δt | Time step size |
| δ | Node position tolerance |
| δ_j | Potential distance moved by the j th global node |
| ϵ_{NR} | Newton-Raphson iteration convergence tolerance |
| ϵ_R | Runup tip elevation tolerance |

| | |
|---|--|
| ϵ_K | Hydraulic conductivity iteration tolerance |
| ϵ_w | Internal water surface elevation tolerance |
| η | Water surface elevation |
| z | Dimensionless elevation (Eqns E.4a and E.4b) |
| η_c | Elevation of crest of incident wave |
| η_H | Water surface elevation at location where total water depth equals h_H |
| η_{15} | Water surface elevation at location where total water depth equals 15mm |
| η_w | Water surface elevation within armour layer at location where total water depth equals h_w |
| η_p | Water surface elevation corresponding with X_p |
| η_0^{i+1} | water surface elevation at the seaward boundary at $t=t^{i+1}$ |
| η_j^* | Interpolated previous time step water surface elevation at the j th global node |
| $\eta_R^i, \eta_R^{i+1}, \eta_R^k$ | Elevation of the runup tip at $t=t^i, t^{i+1}$ and the k th iteration respectively |
| $\eta_g^0, \eta_g^i, \eta_g^\theta, \eta_g^{i+1}$ | Global vectors of water surface elevation at $t=0, t^i, t^\theta$ and t^{i+1} respectively |
| $\eta^i, \eta^\theta, \eta^{i+1}, \eta^k$ | Local vectors of water surface elevation at $t=t^i, t^\theta, t^{i+1}$ and the k th iteration respectively |
| η^* | Local vector of interpolated previous time step values of water surface elevation |
| η | Local vector of water surface elevation |
| γ_r | Unit weight (saturated surface dry) of an armour unit |
| γ_w | Unit weight of water |
| θ | Time weighting parameter |
| λ | Dimensionless coefficient (Eqns E.4a and E.4b) |
| ξ | Surf similarity parameter (alternatively termed the Iribarren number, Ir): Local coordinate in spatial domain |
| ξ' | Local coordinate corresponding with water depth h' |
| ξ_H | Local coordinate corresponding with water depth h_H |
| ξ_b | Breaker surf parameter |
| ξ_o | Offshore surf parameter |
| ξ_j, σ_j | Local coordinates of the j th global node |
| ξ_l, σ_l | Local coordinates of the l th integration point |
| ξ_p | Local coordinate corresponding with X_p |
| κ | Element mass |
| π | $\pi = 3.141..$ |
| ρ | Fluid density |
| τ | Friction coefficient |
| ϕ | Local vector of nodal values of η and U at $t = t^{i+1}$ |
| ϕ | Piezometric head |
| $\Delta\phi_g^{i+1}$ | Global vector of corrections to ϕ_g^k |

| | |
|--------------------------|--|
| ϕ_g^k, ϕ_g^{k+1} | Global vectors of nodal values of η and U at the k th and $(k+1)$ th iterations respectively |
| ψ_n | Shape function for the n th local node of an element |
| Ω | Domain of integration |
| σ | Local coordinate in the time domain: Local coordinate in the vertical direction: Dimensionless variable (Eqns E.4a and E.4b) |
| ω | Weighting function |
| ω_m | Weighting function for the m th local node |
| c | Dimensionless coefficient (Eqns E.4a and E.4b) |