Numerical Modelling of Wave Runup on Breakwaters

by

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Abstract

The design of rubble mound breakwaters is typically based on empirical formulae and physical modelling. One limitation of this approach is that different aspects of wave interaction with a breakwater, such as the elevation of the runup tip and armour stability, are treated separately. Therefore the development of a numerical model of wave runup on a rubble mound breakwater was the primary objective of the research described in this thesis.

Because of the range of slope conditions encountered with rubble mound breakwaters and revetments, two types of armour layer are considered. The first is impermeable and so only the flow within the external region is modelled. The flow is assumed to be governed by the unsteady one-dimensional shallow water wave equations and only regular waves are considered. It is shown how the use of the finite element method with a mesh of isoparametric elements that deforms and is fitted to the runup tip has a number of advantages over the traditional use of the finite difference method with a fixed grid.

Reasonably good results were obtained for the numerical modelling of wave runup on a riprap armoured 1:3 impermeable slope indicating that the numerical model may, in conjunction with a physical model, be of practical use in the design of revetments. Wave runup on smooth and Dolos armoured 1:1.5 impermeable slopes was modelled poorly. Therefore the model is more appropriate for wave runup on a revetment than a rubble mound breakwater.

The second type of armour layer is permeable and so the flow within the external region and armour layer is modelled simultaneously by coupling numerical models for the respective regions. It is concluded that this approach is unlikely to give acceptable results for the runup of regular waves on a steep, permeable armour layer unless it also accounts for the non-hydrostatic distribution of pressure within the external region.

An experiment is described in which continuous time histories of wave runup and dynamic pressure due to regular waves on smooth and Dolos armoured 1:1.5 slopes were measured. The results are used to discuss the assumption of hydrostatic pressure.

A method of assessing armour stability requirements which takes into consideration the effects of armour unit interaction is proposed. It is recommended that this is examined further.

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Contents

List of Abbreviations

ALE	Arbitrary lagrangian-eulerian
BEM	Boundary element method
DTI	Discrete time interval
FDM	Finite difference method
FEM	Finite element method
MAC	Marker and cell
MOC	Method of characteristics
PIANC	Permanent International Association of Navigation Congresses
STE	Space-time element
SUMMAC	Stanford University marker and cell
SWL	Still water level
USACE	United States Army, Corps of Engineers
VOF	Volume of fluid

List of Notation

With the exception of the dimensionless variables defined in Appendix E, and **U**, bold upper case symbols are matrices with | | indicating 'the determinant of'. Similarly, with the exception of the dimensionless variables defined in Appendix E, bold lower case symbols and bold greek symbols are vectors. Global vectors and matrices are subscripted, g and, unless otherwise annotated, vectors and matrices are defined at element level and therefore comprise local values. The subscripts indicate an association with a particular node whereas the superscripts are generally iteration or time step counters.

For simplicity, specific notation is not used to distinguish between experimentally measured and averaged values or between experimentally measured and numerically computed values. Instead the distinction is noted within the text.

A	Surface area of armour layer
A, B	Dimensionless coefficients (Eqn 2.20)
$\mathcal{A}$	Amplitude factor
A_d, B_d	Dimensionless coefficients (Eqn 2.21b)
A_u, B_u	Dimensionless coefficients (Eqn 2.21a)
a,b	Forchheimer constants
a_x, b_x, c_x, d_x	Dimensionless coefficients (Eqn 5.5a)
a_y, b_y, c_y, d_y	Dimensionless coefficients (Eqn 5.5b)
B	Waist width of Dolos armour unit
C	Overall length of Dolos armour unit
C_f	Friction factor
C_p	Pressure distribution correction factor
c	Wave celerity
e	Volumetric porosity
F	Fractional volume of fluid
f'	Friction factor
$f_{1,m}^k$, $f_{2,m}^k$	Unbalanced element forces for the continuity and momentum equations respectively (mth local node, kth iteration)

$\mathbf{f}^k$	Local vector of unbalanced forces
$\mathbf{f}_g^{\ k}$	Global vector of unbalanced forces
g	Gravitational acceleration
H	Wave height
H	Dimensionless wave height (Eqns E.4a and E.4b)
H_o	Unrefracted deepwater wave height
H_S	Significant wave height
h	Water depth
$\mathbf{h}$	Local vector of water depth
h_R	Water depth at runup tip
h_t	Still water depth
h_j^*	Interpolated previous time step water depth at the jth global node
$\mathbf{h}_g^{\ 0}$, $\mathbf{h}_g^{\ i}$, $\mathbf{h}_g^{\ \theta}$, $\mathbf{h}_g^{\ i+1}$	Global vectors of water depth at t=0, t^i, t^θ and t^{i+1} respectively
$\mathbf{h}^i$, $\mathbf{h}^\theta$, $\mathbf{h}^{i+1}$	Local vectors of water depth at $t=t^i$, t^θ and t^{i+1} respectively
$\mathbf{h}^*$	Local vector of interpolated previous time step values of water depth
i	Time step counter
J	Total number of global nodes
$\mathbf{J}$	Jacobian matrix
J_0, J_1	Bessel functions of the first kind
j	Global node counter
K	Hydraulic conductivity
K^e	Average hydraulic conductivity for element, e
K_D	Stability coefficient
k	Permeability: Iteration counter
k_Δ	Layer coefficient
L	Wavelength: Number of integration points for an element
L_o	Deepwater or offshore wavelength
l	Length of channel: Integration point counter
ℓ	Horizontal length of fluid element
ℓ_u	Slope-parallel length of the runup tip
ℓ_1, ℓ_2, ℓ_3	Total water depths at local nodes 1, 2 and 3 respectively
ℓ'	Minimum or maximum total water depth
ℓ^i, ℓ^θ, ℓ^{i+1}	Length of fluid element at $t=t^i$, t^θ and t^{i+1} respectively
M, N	Total number of nodes connected to an element
$\mathbf{M}$	Element mass matrix
m	Local node counter
m	Horizontal mass flowrate across the seaward edge of the armour layer
N_r	Total number of armour units
N_s	Stability number

n	Number of layers comprising primary armour layer: Local node counter
P	Average porosity of the primary cover layer
p	Pressure
p_D	Dynamic pressure
Q	Volumetric flowrate across the seaward edge of the armour layer
q	Bulk or macroscopic velocity
q_b	Volumetric flux
q'	Volumetric flow rate per unit horizontal length of the armour layer
$\bar{q}$	Average volumetric flow rate per unit horizontal length of the armour layer
R_d	Maximum rundown
R_u	Maximum runup
$R_{d,15}$	Maximum rundown at location where total water depth $(h+\eta)$ equals 15mm
$R_{u,15}$	Maximum runup at location where total water depth $(h+\eta)$ equals 15mm
r	Roughness and porosity correction factor
r	Vector of boundary and initial conditions
r_w	Waist ratio (=B/C)
r_x, r_y	Unit normal vectors in the x and y directions respectively
S	Element stiffness matrix
$\mathbf{S}^k$	Element tangent stiffness matrix
$\mathbf{S}_g^k$	Global tangent stiffness matrix
S_r	Specific gravity of an armour unit $(=\gamma_r/\gamma_w)$
T	Wave period
T	Dimensionless wave period (Eqns E.4a and E.4b)
T_S	Significant wave period
t	Time
t	Dimensionless time (Eqns E.4a and E.4b)
U	Depth-averaged horizontal velocity
U	Local vector of horizontal velocity
U	Dimensionless depth-averaged horizontal velocity (Eqns E.4a and E.4b)
U_e	Elemental velocity
U_p	Depth-averaged horizontal velocity within the armour layer
U_b	Depth-averaged horizontal velocity at seaward edge of armour layer
U_j^*	Interpolated previous time step horizontal velocity at the jth global node
U_R^i, U_R^θ, U_R^{i+1}, U_R^k	Horizontal velocity of the runup tip at $t=t^i$, t^θ, t^{i+1} and the kth iteration respectively
$\mathbf{U}_g^0$, $\mathbf{U}_g^i$, $\mathbf{U}_g^\theta$, $\mathbf{U}_g^{i+1}$	Global vectors of horizontal velocity at $t=0$, t^i, t^θ and t^{i+1} respectively
$\mathbf{U}^i$, $\mathbf{U}^\theta$, $\mathbf{U}^{i+1}$, $\mathbf{U}^k$	Local vectors of horizontal velocity at $t=t^i$, t^θ, t^{i+1} and the kth iteration respectively

u, v	Horizontal and vertical components of velocity
u_A, v_A	Horizontal and vertical components respectively of vertical velocity at node A
u'	Horizontal velocity at the seaward edge of the armour layer
$\bar{u}$	Average horizontal velocity at the seaward edge of the armour layer
u_A^* v_A^*	Transformed horizontal and vertical components respectively of velocity at node A
u_C	Horizontal component of velocity at node C
W	Dry weight of an individual armour unit in the primary cover layer
W_l	Integration weight for the lth integration point
W_{50}	Median weight
w_r	Unit weight of an armour unit
X_j^{i+1}	Horizontal global coordinate of the jth global node at $t=t^{i+1}$
X_p	Horizontal global coordinate of pressure tapping
X_R^i, X_R^{i+1}, X_R^k	Horizontal global coordinates of the runup tip at $t=t^i$, t^{i+1} and the kth iteration respectively
X_A^{i+1}	Horizontal global coordinate of node A at $t=t^{i+1}$
X_C^{i+1}	Horizontal global coordinate of node C at $t=t^{i+1}$
x	Horizontal global coordinate
x	Dimensionless horizontal global coordinate (Eqns E.4a and E.4b)
x^*	Rotated horizontal global coordinate
x_g^0, x_g^i, x_g^θ, x_g^{i+1}	Global vectors of horizontal global coordinates at $t=0$, t^i, t^θ and t^{i+1} respectively
x^i, x^θ, x^{i+1}	Local vectors of the horizontal global coordinates of the nodes of an element at $t=t^i$, t^θ and t^{i+1} respectively
Y_A^{*i}, Y_A^{*i+1}	Transformed vertical global coordinates of node A at $t=t^i$, t^{i+1} respectively
Y_A^{i+1}	Vertical global coordinate of node A at $t=t^{i+1}$
Y_C^{i+1}	Vertical global coordinate of node C at $t=t^{i+1}$
y	Vertical global coordinate
y^*	Rotated vertical global coordinate
Z_p	Elevation of pressure tapping
Z_R^i, Z_R^{i+1}, Z_R^k	Elevation of the seaward slope at the location of the runup tip at $t=t^i$, t^{i+1} and the kth iteration respectively
α	Angle of seaward slope
α_c	Critical angle of seaward slope
Δt	Time step size
δ	Node position tolerance
δ_j	Potential distance moved by the jth global node
ϵ_{NR}	Newton-Raphson iteration convergence tolerance
ϵ_R	Runup tip elevation tolerance

ϵ_K	Hydraulic conductivity iteration tolerance
ϵ_w	Internal water surface elevation tolerance
η	Water surface elevation
z	Dimensionless elevation (Eqns E.4a and E.4b)
η_c	Elevation of crest of incident wave
η_H	Water surface elevation at location where total water depth equals h_H
η_{15}	Water surface elevation at location where total water depth equals 15mm
η_w	Water surface elevation within armour layer at location where total water depth equals h_w
η_p	Water surface elevation corresponding with X_p
η_0^{i+1}	water surface elevation at the seaward boundary at $t=t^{i+1}$
η_j^*	Interpolated previous time step water surface elevation at the jth global node
η_R^i, η_R^{i+1}, η_R^k	Elevation of the runup tip at $t=t^i$, t^{i+1} and the kth iteration respectively
η_g^0, η_g^i, η_g^θ, η_g^{i+1}	Global vectors of water surface elevation at $t=0$, t^i, t^θ and t^{i+1} respectively
η^i, η^θ, η^{i+1}, η^k	Local vectors of water surface elevation at $t=t^i$, t^θ, t^{i+1} and the kth iteration respectively
η^*	Local vector of interpolated previous time step values of water surface elevation
η	Local vector of water surface elevation
γ_r	Unit weight (saturated surface dry) of an armour unit
γ_w	Unit weight of water
θ	Time weighting parameter
λ	Dimensionless coefficient (Eqns E.4a and E.4b)
ξ	Surf similarity parameter (alternatively termed the Iribarren number, Ir): Local coordinate in spatial domain
ξ'	Local coordinate corresponding with water depth h'
ξ_H	Local coordinate corresponding with water depth h_H
ξ_b	Breaker surf parameter
ξ_o	Offshore surf parameter
ξ_j, σ_j	Local coordinates of the jth global node
ξ_l, σ_l	Local coordinates of the lth integration point
ξ_p	Local coordinate corresponding with X_p
κ	Element mass
π	Pi = 3.141..
ρ	Fluid density
τ	Friction coefficient
ϕ	Local vector of nodal values of η and U at $t = t^{i+1}$
ϕ	Piezometric head
$\Delta\phi_g^{i+1}$	Global vector of corrections to ϕ_g^k

ϕ_g^{k}, ϕ_g^{k+1}	Global vectors of nodal values of η and U at the kth and (k+1)th iterations respectively
ψ_n	Shape function for the nth local node of an element
Ω	Domain of integration
σ	Local coordinate in the time domain: Local coordinate in the vertical direction: Dimensionless variable (Eqns E.4a and E.4b)
ω	Weighting function
ω_m	Weighting function for the mth local node
c	Dimensionless coefficient (Eqns E.4a and E.4b)