The 'haves' and the 'have nots': Who is better off?

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The Numeracy Development Project has been claimed as successful in raising student achievement. The extent of the success, however, varies with the decile rating of the school, and the degree to which the project itself is responsible for improved achievement can be questioned. This paper explores some of the theoretical background to aspects of the project and raises issues around the nature of the evidence being used to make such claims.

Introduction

In 2004, the decile 9 school at which I teach had not yet participated in the Numeracy Development Project (NDP), the overall aim of which is to develop teachers' knowledge of number concepts, students' strategies, and instructional practice in order to improve the achievement of students. One of the teachers in the senior syndicate (Years 5-6) had participated in the Years 4-6 focused Advanced Numeracy Project (ANP) – one of five projects within the NDP – in 2001 at a previous school. While analysing student data from a syndicate-wide Assessment Tools for Teaching and Learning mathematics assessment, the NDP-trained teacher commented that if her students did not achieve at least as well as those in the other classes, particularly in number work, then the efficacy of the NDP professional development programme would have to be questioned.

The Minister of Education, Trevor Mallard (Mallard, 2004) has said, however, that there is "evidence that students in the [Numeracy Development] project have been learning better than those not yet involved [in the project]". To date, however, there appears to have been no rigorous comparative study made between the 'haves' and the 'have nots' to provide independent evidence to substantiate this claim. The question is whether there is a significant difference in numeracy achievement between those students whose teacher had participated in the NDP professional development programme, and those students whose teachers had not – and whether it was NDP that was the cause of such a difference. This paper sets out to critically review the literature surrounding this issue and positions it, not only within the wider context of the teaching of numeracy, but also explores the literature regarding the constructivist learning theory upon which the NDP is largely based.

Traditional approaches to learning and teaching

Many theories of learning and teaching have been proposed in the last century. Until recently, behavioural psychology (the study of human actions by analysis of stimulus and response) has influenced education to such a degree that it had a virtual stranglehold on how teaching and learning resources were defined, and how teachers planned and implemented lessons. In fact, for much of the early part of the 20th century Skinner's behaviourist ideas dominated educational theories and research. Behaviourist theorists argued that the best way to effect learning was through the study of observable phenomena/behaviours.

In the 1950s, however, educationalists began to look beyond behaviourist stimuli and feedback to examine the mental states of learners and how, through cognitive processes, learners acquire knowledge. Like behaviourism, such cognitive theories assume that the role of mental activities is to map the real world (Jonassen, 1991). Even Piaget, whose theories mark the beginning of constructivist philosophies (wherein knowledge is constructed by the learner), assumed that mental constructions were representations of the real world to which the learner had to "accommodate" (Bruner, 1986).

Most traditional mathematics instruction and curricula are based on the *transmission* or *absorption* view of teaching and learning. In this view, students passively absorb sets of established facts, skills, and concepts, which are transmitted from the teacher to the students (Clements & Battista, 1990). When computation dominated the mathematics curriculum the prevailing psychological view of mathematics learning was behaviourist, and attention was focused on observable behaviours, not on mathematical thinking (Battista, 1994). In recent years, however, there has been a significant shift in planned instruction; from behaviourism to cognitivism, and now to constructivism (Cooper, 1993).

Paradigm shifts

Kuhn (1970) suggested that scientific knowledge is developed within an underlying framework or paradigm, which controls what questions are asked, how answers are pursued, what data are acceptable as evidence, and what are considered as acceptable answers. Although Kuhn was elaborating on science as a 'way of knowing', his idea appealed to academics in numerous disciplines, and the construct of paradigms has found its way into the literature of almost every academic discipline, including mathematics. It is popular today to speak of paradigm shifts, and certainly major conceptual changes do occur in virtually all fields of study over time. According to Applefield, Huber and Moallem (2001) "paradigm shifts bring new perspectives, new conceptualisations, and new ways of thinking about a topic" (p. 35).

Theories and ways of thinking about education in the late 20th century not only changed, but underwent a paradigm shift in how education and the nature of learning are viewed (Cooper, 1993; Jonassen, 1991). Although by no means an entirely new conceptualisation of the learner and the process of being a learner (the beginnings can be traced to John Dewey and other progressive educators), constructivist perspectives on learning have become increasingly influential in the past twenty years, and are seen as representing "a paradigm shift in the epistemology of knowledge and theory of learning" (Applefield et al., 2001, p. 36).

Constructivism

In contrast to both behaviourism and cognitivism, constructivism is not an objectivist theory in which reality is viewed as external to the learner (Cooper, 1993; Jonassen, 1991). Rather, constructivism presents a different view on how reality is perceived, and on the nature of knowledge: as being internal to the learner. In comparing constructivism to both behaviourism and cognitivism, Cooper (1993) states that:

> The constructivist … sees reality as determined by the experiences of the knower. The move from behaviorism through cognitivism to constructivism represents shifts in emphasis away from an external view to an internal view. Constructivists view reality as personally constructed, and state that personal experiences determine reality, not the other way round (p. 16).

The history of constructivism appears closely integrated with the evolution of educational psychology. Piaget (1896-1980) was perhaps the first in Western civilisation to explore learning and knowledge structures with a model that viewed children as the "builders of their intellectual structures" (Papert, 1980). At the same time, Vygotsky (1896-1934) was also exploring the nature of learning, developing 'dialectic theory', a social learning perspective that describes how children learn through interactions and dialogues with socialising agents (such as teachers, peers, and parents).

While both Piaget and Vygotsky are prominently mentioned in most texts on constructivist learning (for example, Duffy, Lowyck & Jonassen, 1993; Papert, 1980; Wilson, 1996), there are at least two further perspectives on constructivism to be considered: holistic and social constructivism. Holistic constructivism, wherein learners must begin with an understanding of the whole rather than its parts when constructing knowledge is an approach popular in, for example, the teaching of literacy (hence, the 'whole language' approach). Social constructivism differs from the Piagetian perspective and Vygotsky's description in that it defines learning as "socially shared cognition that is 'co-constructed' within a community of participants" (Bredo, 1994; John-Steiner & Mahm, 1996; Perkins, 1996, cited in Green & Gredler, 2002, p. 56).

The current emphasis on constructivism in education appears to have emerged, in part, in reaction to the 'overselling' of the computer as a metaphor for learning (Bredo, 1994) and, in addition, to the perceived 'transmission of knowledge' focus of information-processing theory (Marshall, 1996). Recently, modern theorists have begun to critically examine the implications of constructivist philosophy (Applefield et al., 2001; Green & Gredler, 2002; Ward, 2001). With only a few exceptions (for example, Brown & Campione, 1994; Cobb, Wood, Yackel, & Perlwitz, 1992; Palincsar & Brown, 1984) any systemic empirical research on mathematics constructivist classrooms has yet to be conducted. It remains to be seen whether the paradigm shift to constructivism in education will result in consistently improved teaching practice and better student outcomes.

Theory into Practice in the Mathematics Classroom

Brewer and Daane (2002) claim that teachers have historically based their philosophy of teaching on very little scientific evidence or knowledge of theory. They maintain that a strong foundation in the theory of effective teaching and learning, coupled with corresponding classroom instructional practices, can help promote a higher quality of mathematics education. Likewise, Battista (1999) maintains that one of the reasons that there has been very little progress in education, and in mathematics education in particular, is that teachers have failed to adhere to scientific methodology in their instructional practices. He suggests that, in order to achieve a quality mathematics programme, teachers must “make their practice consistent with scientific findings and principles” (p. 433). To do this teachers need to be able to make explicit their own theoretical understandings and how these inform their practice, that is, they need to be able to articulate the theory that drives their pedagogical decisions in mathematics (Brewer & Daane, 2002). This requires that they discuss these issues and according to Christiansen (1999) once teachers start thinking and talking about their own teaching and ideas about teaching there are no limits to the potential for development.

Steffe and Wiegel (1992) contend that mathematics education could be transformed by adopting constructivism as its philosophical basis. However, translating constructivist theory into practice in any classroom presents a challenge. Indeed, teachers have often distorted the original notion of constructivism simply because they want to be perceived as doing “the right thing” (Pirie & Kieren, 1992). A constructivist philosophy can provide teachers with a framework for teaching mathematics that encourages problem solving, reasoning, and communication (Simon & Schifter, 1993). Research studies have also shown that students in constructivist classrooms have a greater understanding of mathematics and experience more success in the mathematics classroom than those in traditional classrooms (Cobb et al., 1992). Battista (1994), commenting on the reform of mathematics education in the United States, observed, however, that many teachers have beliefs about mathematics that are incompatible with the constructivist philosophy underlying the reform movement and contends that “these beliefs play a critical role not only in what teachers teach but in *how* they teach it” (p. 462).

The Education Review Office (2000) found that a significant number of teachers in New Zealand did not have sufficient content knowledge required for the quality teaching of mathematics. Holmes and Tozer (2004), drawing on international research confirm this in their paper on NDP stating that “a critical factor in the teaching of mathematics is teachers’ content knowledge and pedagogical content knowledge” (p. 60). They argue that improving teacher capability is fundamental to raising the achievement of children [in numeracy]. It is not enough, however, to focus on improving teachers’ content knowledge in the teaching of numeracy. If teachers are to participate in a numeracy development programme based on constructivism then they need to have a sound understanding of what constructivism means, to evaluate its promise, and to use it knowledgeably and effectively (Applefield et al., 2001; Battista, 1994).

Mathematics and Numeracy

According to Wheatley (1991), for a constructivist, mathematics is the activity of constructing patterns and relationships, which become part of a reality that is determined by the learner. While mathematics may be regarded as a subject in its own right, with a teaching progression and development, the term ‘numeracy’ brings with it connotations of real-life applications. Numeracy is the ability to understand and use numbers, especially the numbers encountered in everyday life. For this reason, it is often referred to as ‘number sense’ as in a person possessing ‘common sense about numbers’. Such a person would be considered numerate, that is, be able to master the basic skills and processes of “numbers, addition, subtraction, simple multiplication, simple division, simple weights and measures, money counting, and telling time”.(SIL International, n.d.). The New Zealand Ministry of Education (MoE) regards numeracy as part of everyday life, describing being numerate as having “the ability and inclination to use mathematics effectively – at home, at work, and in the community” (2005, p.49).

The use of student interviews and classroom interactions

A major tenet of the theory of constructivism is that understanding is personally constructed. Student interviews can provide teachers with a detailed, accurate, and complete picture of children’s mathematical understanding, helping them to understand how children construct knowledge, and giving them an opportunity to observe children’s attempts at solving problems in ways that make sense to the children. According to Buschman (2001), student interviews can change mathematical instructional practice in *some* classrooms, and influence instruction in *all* classrooms. Interviews provide teachers with an opportunity to determine students' prior and existing knowledge, with the information acquired being used to guide the teachers' planning for learning. Buschman's research found that interviews supported and enhanced instruction by making teachers more aware of what individual children knew and what tasks they could perform with this knowledge.

Cobb and Steffe (1983), cited in Moyer and Jones (2004), maintain that students' construction of knowledge is based on their experiences in those interactions in which the students determine *how* and *what* mathematical knowledge is constructed. Teachers' roles are critical in negotiating and establishing the quality of these classroom interactions. Recent changes to mathematical instructional practice mean that teachers must be able to make decisions in the midst of instruction (National Council of Teachers of Mathematics, 2000, cited in Sherin & van Es, 2003), rather than prior to, or following, instruction. Teachers are expected to listen closely to the ideas that students raise, and to the mathematics under discussion, and to then use that information to decide how to proceed.

Cognitively Guided Instruction (CGI), developed by Carpenter and Fennema (1992), is a research-based programme designed to assist teachers to identify students' standard and invented strategies in solving word problems. An essential part of the teacher's role in CGI is to know in advance which strategies are likely to be elicited in response to particular items, and to identify these responses as they move around the room observing students' work. This adaptive style of instruction requires, among other skills, being able to 'notice' critical features of classroom interactions (Sherin & van Es, 2003). Sherin and van Es lament, however, that for too long, teachers have been taught to 'do' rather than to 'notice'. Their call is for teachers to be trained to identify the strategies which students use and to 'notice' the interactions that occur – in other words, to build teacher capability in those aspects which are integral to quality mathematics teaching.

Effective Mathematics teaching

The MoE (2005) maintains that numeracy arises out of effective mathematics teaching. Other reports state that “effective teaching thrives in supportive school cultures, and in communities of professional practice” (MoE, 2004a, p. 16), describing these communities as being characterised by the analysis and open discussion of achievement information. One of the key findings in the Ministry’s analysis of the NDP data was that “students’ achievement in numeracy was enhanced by the participation of their teachers in one of the professional development projects” (MoE, 2004b, p. 32). It could be argued, however, that it is difficult to ascertain whether improvement in students’ achievement *was* purely as a result of their teachers’ participation in the NDP or, rather, was partly a result of their teachers already being part of supportive school cultures.

The Numeracy Development Project (NDP)

The NDP is described as “part of a key government initiative aimed at raising student achievement in mathematics by building teacher capability in mathematics teaching” (MoE, 2004b, p. 6). It has its roots in constructivism, through the use of a teaching model (MoE, 2004c), which guides the progression in the representation of mathematical ideas from a physical model to the abstract form. This strategy-teaching model is, in part, adapted from the Pirie and Kieren’s (1994) recursive theory of mathematical understanding. This theory is predicated on the assumptions that understanding is constructed by a learner, and that the learning environment plays a significant role in the content and processes of that learning. There are eight nested components to the theory, moving from the four central pre-verbal elements outward to those that can be articulated, such as *formalising, observing, structuring*, and *inventising*. Aspects of this theory together with principles of CGI, which focus on creating classrooms in which student inquiry and explanation of solution methods are encouraged (Carpenter & Fennema, 1992), form the basis of the projects. Alongside this sits Fravillig, Murphy, and Fuson’s (1999) pedagogical framework for advancing children’s thinking; the three components of this are: eliciting children’s solution methods, supporting children’s conceptual understanding, and extending children’s mathematical thinking.

Several key features characterise the numeracy projects, and reflect, in part, the theories described above:

1. The Number Framework – designed to aid understanding of the requirements of the Number strand of the mathematics curriculum document (MoE, 2004b). The framework makes a distinction between two inter-related components: *strategy*, and *knowledge*. *Strategy* describes the mental processes students use to solve problems with numbers, while *knowledge* describes key pieces of knowledge that students need to have in order to be able to build and use strategies effectively.
2. The diagnostic interview – designed to provide teachers with quality information about students' number knowledge and mental strategies so as to position the students on the Number Framework.
3. The professional development programme for teachers – a combination of workshops and in-school support.

Conclusion

Despite the paradigm shift from traditional teaching approaches to constructivism in mathematics, it remains to be seen whether students whose teachers have participated in the NDP show significant differences in achievement, when compared with students whose teachers have not. There are several variables to consider.

Firstly, according to Alton-Lee (2003) and Holmes and Tozer (2004), quality teaching is a key influence on student outcomes. It could be argued that, if all students are receiving quality teaching, regardless of their teachers' NDP participation (or non-participation), then it is entirely possible that there would be no difference in achievement. Secondly, the MoE's (2004c) review of the ANP found that "students at high decile schools seem to have benefited the most from the project" (p. 20). The school at which I teach is a decile 9 school – perhaps these students would 'benefit' from any form of mathematical instruction whether it is constructivist-based or otherwise?

The research into the NDP reveals that effective teachers "expect their students to succeed … [and] clearly define objective(s) for each session that help them focus the learning …" (MoE, 2004a, p. 16). These characteristics are also central to the Assessment for Learning (AfL) philosophy. Thus, it is conceivable that, in a school that uses the AfL approach, there could be a significant impact on the teaching of mathematics, and on student outcomes, regardless of participation in the NDP.

Fourthly, in any teaching situation, there are possibilities for numerous tensions and conflicts to exist. A particular tension is that between constructivist philosophy and that of the school ‘system’ or culture, if this is based on a different belief system. The way in which a teacher reacts to the tensions between these may have considerable influence on outcomes for the students. Moreover, given the challenges that teachers face when attempting to put theory into practice (Brewer & Daane, 2002; Pirie & Kieren, 1992), and given that the NDP requires a huge shift in teachers’ thinking (MoE, 2004b), it would take considerably longer than the period of the professional development contract for the theory to be embedded into practice. In addition, within even a small group of teachers there may be teachers who share the same practices, but hold different beliefs or, conversely, teachers who share the same beliefs, but different practices. Likewise, teachers’ own levels of content knowledge and pedagogical knowledge can vary from classroom to classroom within the same school. According to Applefield et al. (2001) the overriding goal of the constructivist in mathematics teaching is to “stimulate thinking in learners that results in meaningful learning, deeper understanding and transfer of learning to real world contexts” (p. 54). One could be forgiven for assuming that this has always been the goal of the conscientious, dedicated teacher, whether a constructivist, NDP-trained teacher or not.

Trevor Mallard’s claim (2004) of improved learning for those involved in the project needs to be further substantiated with a thorough investigation. As this statement was based solely on NDP reports and evaluations commissioned by the MoE, and not on any comparative or, indeed, independent study, it could be considered to be a somewhat dubious academic claim. Indeed, it may be misleading for the Minister of Education to make such a claim without more appropriate substantiation. Until such time as rigorous and thorough research is carried out, there will still be teachers waiting to be convinced that the ‘haves’ are, indeed, better off than the ‘have nots’.

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