
Purposeful questioning in mathematics: A guiding framework

Trish Holster

This paper considers the role of questioning in the teaching and learning process. Concerns raised in the literature indicate the need for a guiding framework to support teachers in using questioning strategies in a more flexible and purposeful manner. In response to this articulated need Fraivillig, Murphy, and Fuson (1999) constructed the Advancing Children's Thinking (ACT) framework. This framework is described with reference to literature on effective questioning, the role of discourse in classroom culture, and the impact of its use by teachers on children's opportunities to learn. Strengths of the ACT framework, including alignment to the New Zealand Numeracy Development Project are summarised. Possible limitations and how these might be addressed are briefly considered.

Introduction

Teaching for conceptual understanding through co-construction of meaning is promoted in the Numeracy Development Project (Higgins, 2003) and in a variety of Ministry of Education (MoE) documents designed to support teachers of mathematics and literacy (MoE, 1992; MoE, 1996; MoE, 2003). Co-construction of meaning using collaborative approaches assigns an importance to the role of classroom culture in providing opportunities to learn, and specifically to the development of intellectual autonomy of students, as these students are required to evaluate the worth of solution methods and justifications in collaborative discussion.

The role of argumentation skills and sociomathematical norms in teaching for conceptual understanding, and their development through classroom discourse, are widely discussed in the literature (e.g. Yackel, Wood & Cobb, 1993; Hershkowitz & Schwarz, 1999; Whitenack & Yackel, 2002; Yackel & Cobb, 1996). Yackel and Cobb (1996) describe sociomathematical norms as social norms specific to the mathematics classroom. Social norms are general classroom expectations such as cooperation in solving problems and persistence on personally challenging problems (Yackel, Cobb & Wood, 1991). While social norms might be seen to provide opportunity for collaborative discussion and

— problem-solving, sociomathematical norms “regulate mathematical argumentation and influence the learning opportunities” (Yackel & Cobb, 1996, p.461). Some social norms have a corresponding sociomathematical norm.

Corresponding to the social norm that students will explain the methods by which they reached their solutions is the sociomathematical norm of what is an acceptable explanation, and the social norm that students will offer different solution methods is supported by a corresponding sociomathematical norm of what is a mathematically different answer (Yackel & Cobb, 1996). Opportunities for student learning that arise from collaborative activity are identified by Yackel et al. (1991):

“first, the opportunity to use aspects of another’s solution activity as prompts in developing one’s own solution; second, the opportunity to reconceptualise a problem for the purpose of analysing an erroneous solution method; third, the opportunity to extend one’s own conceptual framework in an attempt to make sense of another’s solution activity for the purpose of reaching a consensus” (p. 406).

The central role of teacher-student interactions in teaching and learning is also highlighted by Alton-Lee (2003) who states that quality teaching must be responsive to student learning processes, must provide opportunity to learn, and must promote thoughtful student discourse.

Classroom Culture

Classroom culture can be seen as encompassing a particular set of social norms, a particular participation structure, and characteristic forms of discourse, which sustain both the social norms and the participation structure (Wood, 2002). A participation structure refers to the classroom features that influence participation by the students in the classroom – who participates and when and how they participate. Features of a classroom, including social norms, patterns of discourse, teaching approaches and artefacts may influence students’ participation in a variety of ways. They might: encourage students to buy into opportunities to participate; provide students with the tools (e.g. language) they need to participate more fully; give students the (decision-making) power to influence the direction of their learning, or require children to actively

— engage with concepts or thinking tools. Children may have the opportunity to initiate, negotiate or make decisions about modes of participation. The participation structure may influence the flexibility of the lesson direction, and the sense of ownership by children of knowledge generated through classroom discourse. This ownership may in turn affect the likelihood of children engaging with ideas and using the range of thinking tools offered in classrooms.

Wood (2002) identifies three enquiry classroom cultures (see Appendix 1) which may create different participation structures giving rise, in turn, to a differing quality of explanations and justifications required. Wood concluded that the quality of children's reasoning about mathematics was affected by the quality of the children's explanations and justifications that were required in class. In Wood's *strategy-report* culture students are expected to describe their alternative strategies for solving a problem and to listen to, and try, the solutions of others. In an *inquiry* culture students are expected to give reasons for the strategy they have chosen and to seek clarification about the solutions offered by peers. In an *argument* culture social norms require that children justify or defend their own solution methods and challenge the solution methods of others.

It is worth stressing that participation in these cultures may be problematic for children who have not have mastered the tools (e.g. language or norms such as what constitutes a different answer) to defend or challenge ideas through discussion. Children may also lack the self-confidence or self-image required as a learner in a collaborative learning environment (Klein, 2001).

Cobb and Yackel (1996) and Yackel and Cobb (1996) describe how social norms and sociomathematical norms negotiated through the discourse of an inquiry oriented classroom create the participation structure which maintains that inquiry culture. Social norms (e.g. explaining and justifying solutions) directly affect the participation structure while sociomathematical norms (e.g. what is a mathematically different answer) might be seen to empower all students to participate more equally within that structure. However, Klein (2001) questions the reality of learners' intellectual autonomy which is

— taken for granted in some constructivist perspectives. Such an implied autonomy is in sharp contrast to the student passivity, learned helplessness, and compliant behaviour that she argues is apparent in many classrooms.

Questions as discourse tools

Discourse here is taken to mean the mechanisms through which members of the classroom exchange ideas. Here the focus will be on questions, which are a vital mode of interactive discourse in the classroom and subject to considerable analysis, some of which will be considered here. Kawanaka and Stigler (2000) argue that inconsistent results of previous research indicate that “asking more higher order questions does not simply improve student learning” (p. 255). In their own analysis of lessons collected as part of the video component of the Third International Mathematics and Science Study, Kawanaka and Stigler (2000) categorised questions as yes/no, name/state, and describe/explain questions. These categories were intended to “capture the cognitive demands of teachers’ questions” (p. 258) with the describe/explain questions considered to be of highest-order.

Transcript analysis in Kawanaka and Stigler’s (2000) study shows similarity in the structures of discourse across the three countries studied/sampled (Japan, U.S.A., Germany). Similarities were seen in how often teachers ask questions and the kinds of questions asked. But while a similar/comparable proportion of high-level questions were asked by teachers from each country sampled, Kawanaka and Stigler (2000) argue that “teachers across the three countries have different pedagogical goals” (p. 277). Japanese lesson transcripts showed teachers using high-level questions in divergent (open-ended) problem-solving where students solved non-routine problems using any method they chose. In contrast, German and U.S. lesson transcripts showed teachers using high-level questions in convergent problem-solving where students practised solving more routine problems using a common solution method promoted by the teacher. Kawanaka and Stigler found that while German and U.S. teachers appeared to predominantly use high-level questions to check student understanding, Japanese teachers used these questions both to check understanding and to elicit student ideas. In the German and U.S. lessons,

— knowledge was controlled by the teachers, and student responses were often evaluated by the teacher as right or wrong, in contrast with Japanese lessons where students evaluated and questioned peer solution methods. This parallels the work of Mason (2000) who suggests three pedagogical purposes for questions: focusing attention of students, testing (monitoring students' understanding), and enquiry.

While question type was similar in all three countries, the pedagogical purpose of questions differed, and this appears to have created differences in classroom culture, participation structure and opportunity for intellectual autonomy to develop. Kawanaka and Stigler's (2000) initial coding of question types emphasised characteristics of the question rather than broader pedagogical intent.

Bauersfeld (1994, cited in Mason, 2000) talks of a 'funnelling effect' to describe a situation where a teacher attempts to impose a mathematical view on students (i.e. focusing attention) through the use of initially indirect but increasingly specific and focused questions. Mason (2000) suggests/implies that this attempt to focus students' attention on mathematical features obvious to the teacher in fact discourages high-level thinking, as the children attempt to read the teacher's mind rather than think through the concepts themselves. Such forms of questioning covertly affect the participation structure of the classroom as the students' role changes from active enquiry to passive mind-reading.

The ACT framework (Fraivillig et al., 1999) is a framework for analysing and structuring discourse, and recognises the broader pedagogical purpose of classroom questions (see Appendix 2). It is based on identifying three purposes of discourse in an enquiry classroom/culture: eliciting thinking, supporting thinking, and extending thinking. This framework can be linked to the work of Wood (2000) whose three classroom cultures can be reframed as emphasising one or other of the three purposes of discourse over the other two. A strategy-report culture implies an emphasis on eliciting thinking, while an inquiry culture emphasises supporting the thinking of the listener/speaker. An argument culture as described by Wood clearly emphasises the purpose of extending thinking.

Adhami (2001) and Mason (2000) illustrate how teachers can effectively use strategic and tactical questions as two distinct tools to make the potential mathematics more visible to students. Strategic questions are a sequence of pre-planned questions that provoke successive steps in thinking. For example, the Number Framework and supporting numeracy booklets scaffold New Zealand teachers in their use of strategic questions. Tactical questions, in contrast, are responsive questions, allowing the teacher to respond to teachable moments, often using phrases offered by students to previous questions (paraphrased or not). These borrowed phrases usually allow students to retain a sense of ownership of the developing ideas and to negotiate meaning. The ACT framework essentially provides tactical rather than strategic questions with its focus on the questions' pedagogical purpose.

The ACT Framework

The ACT framework is a theoretically based but practical framework for interpreting and responding to classroom interactions (Fraivillig et al., 1999). It is a description of types of discourse tools that helps teachers choose appropriate tools to advance students' thinking in mathematics. The ACT framework describes three distinct categories of discourse tools teachers use to facilitate students' conceptual development while acknowledging that some discourse approaches may serve more than one purpose. Fraivillig et al (1999) found that teachers who were judged as less effective predominantly used supporting approaches, with little eliciting of children's own thinking. The authors emphasised the value of extending thinking approaches in developing higher order thinking.

Eliciting thinking encourages the obtaining of many solution methods, elaboration of solution methods, collaborative problem solving, and use of students' explanations for lesson content (see Appendix 2). These provide a platform for an argumentation culture. Teacher access to students' current understandings through the varied responses of students is necessary for effective scaffolding. The teacher may then use this knowledge as the basis for the lesson, using students' responses as teachable moments. Eliciting many responses creates a greater opportunity for promoting cognitive conflict - in

—elaborating on their own solution methods children often recognise, and must wrestle with, the discrepancies in their explanations. Incorporating these explanations into ongoing discussion supports student ownership of negotiated meanings, and promotes intellectual autonomy as students rely on argumentation rather than teacher authority as the basis for judging the worth of their ideas (Fraivillig et al., 1999).

Effective decisions by the teacher about which solution methods will be discussed should ensure teachable moments occur, and maintain momentum and direction of the lesson. However, a tension appears to exist between a culture of full intellectual autonomy where students are free to decide which solution methods are discussed and a classroom where the teacher makes many of these decisions. Managing this tension is a task of the teacher, and would seem to be influenced by the sophistication of students' discourse skills. Deciding who needs an opportunity to speak publicly, and supporting their explanations, may ensure less articulate or less confident students are not intimidated by a classroom culture in which they are not yet fully equipped to participate. Promoting collaborative problem-solving (such as the opportunity to extend one's own conceptual framework or to reconceptualise a problem while making sense of another's solution or analysing an erroneous solution method) provides opportunities to learn not available in non-collaborative classrooms (Yackel et al., 1991).

Supporting thinking refers to types of discourse that support children when describing their own thinking and when they are listening to the ideas of others. These include reminding students of conceptually similar problem situations, supporting children to explain their solution method, recording symbolic representation of methods, and asking students to explain a peer's method (see Appendix 2). Such approaches to discourse serve to both support students' understanding and model discourse skills.

Extending thinking approaches encourage mathematical reflection by students, and prompt students to go beyond initial solution methods to use more varied or more efficient solution methods (see Appendix 2) (Fraivillig et al., 1999).

The ACT framework and teaching numeracy

The Numeracy Development Project (NDP) materials (e.g. MoE, 2005) offer teachers a source of common solution methods and learning progressions in children's concepts about number that reduce the cognitive burden on teachers, allowing them to pose tasks that are problematic to children at an identified developmental stage and to respond more flexibly to children's discourse with the purpose of making more salient the potential mathematics of those tasks. The ACT framework appears to be well aligned to the NDP support of teachers in its more focused and disciplined use of tactical questions within discussion of number strategies.

The ACT framework appears to offer teachers a broad range of discourse tools suitable for a range of pedagogical purposes, including development of metacognitive skills. It simplifies discourse decisions for teachers by grouping all pedagogical tools into three broad categories without attempting to reduce the complex act of teaching to a simplistic series of steps. It may also serve as a reflective tool for teachers and encourage teachers to use tactical questions more consciously. The framework structure may help teachers to integrate contributions from research rendering these insights manageable and meaningful to classroom teachers. Indeed the ACT framework appears to be a potentially powerful tool for change due to its flexibility, range of discourse tools, simplicity of structure, and alignment with current innovations and perspectives in New Zealand mathematics education.

The ACT framework was designed to provide "instructional strategies [approaches] teachers could use once classroom social norms are established" (Fraivillig et al., p. 149). Indeed, it may be simplistic to assume that classroom social norms are established in a simple or stable form at all. Social and sociomathematical norms are continually renegotiated over time (Yackel & Cobb, 1991). As the cognitive and metacognitive skills of students become more sophisticated so perhaps do the social and sociomathematical norms which then support more sophisticated classroom discourse. Because these norms are developed through classroom discussion of mathematical ideas it seems that the ACT framework could be readily extended to include this evolutionary

— aspect of discourse. For example, development of sociomathematical norms such as ‘what is an adequate or different explanation’ might be incorporated into the ACT framework as ‘supporting describer’s thinking’. Social norms such as the expectation that all children will attempt to describe their solution would likewise fit under ‘orchestrates classroom discussions’, an aspect of the eliciting component. Inclusion or elaboration of discourse tools promoting social and sociomathematical norms might strengthen the framework and help maintain the credibility of collaborative learning as an option for teachers.

Conclusion

Relinquishing intellectual authority without relinquishing control of discourse is a key issue for teachers in an enquiry class culture. Maintaining control of classroom discourse, in which intellectually autonomous students do most of the talking, requires very sensitive control and coordination of discourse skills and pedagogical content knowledge. Vygotsky’s statement (1964, in Bruner 1993) that “the tutor serves the learner as a vicarious form of consciousness” appears to be at the heart of Bruner’s scaffolding concept. The ACT framework can provide a scaffolding function for teachers, serving as a vicarious form of consciousness as these teachers gain command of a greater range of discourse skills. Fraivillig et al (1999) state that “teachers require images of effective teaching to guide their professional development” (p. 168). The ACT framework offers images of classroom discourse tools that can be appropriated by teachers at any stage of professional development. While the range of discourse tools within the framework is extensive the framework itself is straightforward and its use may significantly enhance teaching and learning.

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Appendix 1

Features of Conventional and Reform Class Cultures**Conventional Class Culture**

Discussion Context	Mathematics Thinking	Explainers (student)	Listeners	
			Teacher	Students
Report correct answers	Recall answers & procedures	Tell answers Tell procedures	Evaluate Ask test questions	Pay attention

Reform Class Cultures

Discussion Context	Mathematics Thinking	Explainers (student)	Responsibility for Thinking ↓	Listeners	
				Teacher	Students
				Responsibility for Participation →	
Strategy Report	Compare Contrast	Tell different ways		Accept Elaborate	Solve same way
Inquiry	Reason to clarify or question	Clarify solutions Give reasons		Ask questions Provide answers	Way makes sense Ask questions
Argument	Reason to justify or challenge	Justify Defend solutions		Disagree Make challenges	Disagree Make challenges

From: Wood, T. (2002). What does it mean to teach mathematics differently? In B. Barton, K. Irwin, M. Pfannkuch, & M. J. Thomas (Eds.), Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia: *Mathematics Education in the South Pacific* (pp. 350 – 357). Sydney, Australia: Mathematics Education Research Group of Australasia.

Appendix 2
Framework for Advancing Children's Thinking in Mathematics

Eliciting	Supporting	Extending
<p>Facilitates responding Elicit many solution methods for one problem from the entire class.</p> <p>Waits for and listens to students' descriptions of solution methods.</p> <p>Encourages student elaboration of students' responses.</p> <p>Conveys accepting attitude toward students' errors and problem-solving efforts.</p> <p>Orchestrates classroom discussions</p> <p>Use students' explanations for lesson's content</p> <p>Monitors students' levels of engagement</p> <p>Decides which students need opportunities to speak publicly or which methods should be discussed.</p>	<p>Supporting describer's thinking Reminds students of conceptually similar problem situations.</p> <p>Provides background help for an individual student.</p> <p>Directs group help for an individual student</p> <p>Assigns individual students in clarifying their own solution method.</p> <p>Supporting listener's thinking Provides teacher-led instant replays</p> <p>Demonstrate teacher-selected solution method without endorsing the adoption of a particular method</p> <p>Support describer's & listener's thinking Records symbolic representation of solution methods on the chalkboard</p> <p>Asks a different student to explain a peer's method</p> <p>Supporting individual private help sessions Supports individuals in private help sessions.</p> <p>Encourage students to request help (only when required)</p>	<p>Maintains high standards and expectations for all students</p> <p>Asks all students to attempt to solve difficult problems & to try various solution methods</p> <p>Encourages mathematical reflection Encourage students to analyse, compare, and generalise mathematical concepts</p> <p>Encourage students to consider and discuss interrelationships among concepts</p> <p>Lists all solution methods on the chalkboard to promote reflection</p> <p>Goes beyond initial solution methods Pushes individual students to try alternative solution methods for one problem situation.</p> <p>Promotes use of more efficient solution methods for all students.</p> <p>Uses students responses, questions and problems as core lesson</p> <p>Cultivates a love of challenge</p>

From: Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education*. 30(2), 148 – 170