The numeracy project: Foundations and development

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This paper will outline the factors leading to the development of the New Zealand Numeracy Project. It will posit that the Numeracy Project was a result of a society wanting to create a "knowledge society", which was seen as equating to a numerate society. It will demonstrate that the first model of the Numeracy Project was based on the "Count Me In Too" programme but that this model was extended, expanded, and changed in order to focus more on mathematical strategy development rather than simply teaching mathematical knowledge. It will outline the research base and teaching models underpinning the Numeracy Project and argue that the Numeracy Project is predicated on the premise that professional development of teachers will enrich student learning. Finally, the learning theories on which the Numeracy Project is based will be considered and the aspects of behaviourism and various forms of constructivism which are evident in the project will be outlined.

Introduction

The development of a numerate society has always been a goal of the New Zealand and international communities. New Zealand, in the latter stages of last century, responding to rapid and widespread technological change and the challenge of an economic climate of competitive and complex overseas markets, accepted the need to work towards a knowledge society. Within this context of social and political change mathematics was recognised to be of significant importance in its potential to equip students to participate and contribute to an evolving knowledge society (Education Review Office [ERO], 2000). Additionally, international comparison of student achievement brought about a ministerial review of New Zealand mathematics education (Higgins, 2001). Consequently, an increased focus on numerical literacy was enacted as part of a comprehensive numeracy strategic policy to raise achievement standards and this culminated in the development of the Numeracy Project (NP) (Irwin & Niederer, 2002).

Impetus for reform

The need for a reform of mathematical teaching and learning in New Zealand was supported by the Garden (1997) report of results of New Zealand Year 4-5 students in the 1994 “Third International Mathematics and Science Study” (TIMSS). This report signalled concerns about mathematical achievement standards of this age group based on their learning prior to the 1993 introduction of the new national curriculum “Mathematics in the New Zealand Curriculum” (MiNZC) (Ministry of Education [MoE], 1992). In order to investigate whether implementation of MiNZC in 1993 had improved results, a replication of TIMSS was administered to compare 1998 Year 5 students’ performance with those of their 1994 peers (Chamberlain, Chamberlain & Walker, 2001). Reform changes associated with MiNZC had not only included mathematical content but also change within the organisation and processes of classroom instructional practice. The second study results, however, indicated a “small non-significant increase in the mean mathematics achievement of students between 1994 and 1998” (Chamberlain et al., 2001, p. 63) and were a cause for ongoing concern.

Concern related specifically to mathematical achievement of primary aged students led to two investigative reports by ERO. In a comparative study of countries where students had significantly higher mathematical achievement levels, explanatory factors proposed for New Zealand students’ lower achievement results included differences in teaching practice and curriculum management (ERO, 2000). In their second report (ERO, 2002) it was proposed that in order to improve mathematics achievement levels in New Zealand some key issues need to be addressed, including pedagogical content knowledge, professional development, and teacher access to best practice teaching models.

The evolution of the Numeracy Project (NP)

Following the TIMSS report and concerns over the results of this report and subsequent studies, in 1997 the Ministry of Education put out a “Year Three Mathematics Contract”. All regions of New Zealand submitted plans, however, only two of these differed from the previous BSM style of teaching, which was loosely structured around a mixture of behaviourist objectives and radical constructivism. Both Auckland (Peter Hughes) and Waikato (Vince Wright) submitted plans based on numeracy concepts and Count Me In Too (CMIT) which aroused initial interest in the

CMIT programme (Higgins, 2001). A decision was made to implement the CMIT programme in 2000. In addition, a small exploratory study focusing on numeracy in Years 4-6 was undertaken. Initially the NP was influenced by CMIT, however subsequently CMIT was viewed as too focused on knowledge acquisition within numeracy. Therefore, other models were examined in order to incorporate the development of both knowledge and strategies which resulted in the development of the New Zealand NP. Explicit implementation goals of the NP focused on strengthening professional capability of teachers through provision of a Number Framework, an assessment tool, a teaching model which drew on the work of Pirie and Kieren (1989), and a radical reformation of teaching practice, particularly that of interaction patterns within the classroom environment (Thomas, Tagg & Ward, 2003). These goals complemented those of the previous two decades' broad-based reform movement in New Zealand and overseas (e.g., MoE, 1992). The reform agenda aimed to transform the ways teachers teach and the ways that students learn mathematics in order for students to know a "different" kind of mathematical practice to that experienced within a traditional pedagogy (Wood, 2002).

Research base for the NP

The NP's teaching model, drawing on the work of Pirie and Kieren (1989), emphasises using equipment followed by the process of imaging before learners are asked to abstract the situations and extract answers from their knowledge of number properties. The process of imaging is seen as a bridge to support concept construction from a concrete state to an abstract state. This differs from a traditional approach where students might be introduced to concepts through the use of concrete material but are then expected to extract meaning from the concrete as abstractions and to then apply these through procedural methods to solve problems (Irwin, 2003). Von Glasersfeld (1992) supports the notion of imaging leading to abstraction and contends that the teacher's role aids the process through generating situations that allow or suggest abstraction. However, this is extended within Pirie and Kieren's (1989) model of mathematical understanding which explicitly describes a view of understanding as a recursive rather than hierarchical process.

Pirie and Kieren (1989) argue that "mathematical understanding can be characterised as levelled but non-linear" (p. 8). In so doing they describe a process in which learners of mathematics may fold back or drop back to prior thinking. Thus, students may have a range of strategy levels upon which they rely within their repertoire. The NP supports this view of mathematical understanding as a recursive process and this is clearly apparent in the use of the diagnostic interview as an initial assessment tool. While students may display a range of strategies to solve the various problems, teachers are advised to group them according to the highest strategy level displayed (Hughes, 2002). In this sense the NP supports a view of children moving backwards and forwards through strategy levels rather than a hierarchical view whereby mathematical strategies are *mastered* before a new strategy is moved onto. This is illustrated by the statement that “students frequently revert to previous strategies when presented with unfamiliar problems” (MoE, 2002b, p. 1). In addition, the teaching model diagram uses double ended arrows to highlight the process of folding back and this is used in 75% of the NP teaching material (Hughes, 2002).

The NP is informed both by current research and the development of theoretical models to explain students’ acquisition of number concepts (Higgins 2003; Steffe & Cobb, 1988). It initially drew heavily on Wright’s development of the CMIT Pilot project and his “Mathematics Recovery” programme. The “Learning Framework for Number” was the basis of the Number Framework used in the Early Numeracy Project 2001 (Thomas, Tagg & Ward, 2003). However, this early Number Framework model was further adapted and the stages and strategy levels extended beyond counting. Hughes (2002) explains that the CMIT material was extended and modified because while it was developed for the first 3 years of school the New Zealand numeracy project was intended for the first 8 years of schooling, thus requiring extra stages to be added. The particular influence of the Mathematics Recovery programme is evident in the use of the concept of shielding or screening to force children to image rather than remain dependent on material (Hughes, 2002).

The Number Framework (MoE, 2002b) splits number into two sections: knowledge and strategy. The knowledge section is described as “key items of knowledge that students need to learn” while strategy is defined as “mental processes students use to estimate answers and solve operational problems with numbers” (MoE, 2002, p.1). The strategy stages are split into two broad areas – counting stages and part-whole stages. Learners move through stages (and at times may drop or fold back) within a hierarchical arrangement according to many theorists including the proponents of the NP (Piaget, 1978, Mulligan and Mitchelmore, 1997). Strategy levels are not equally distributed as the higher levels involve more demanding learning steps. Evaluation reports generated around the NP have supported the notion of uneven steps and demonstrated that student gains were greatest for those who started at the lower strategy levels (Irwin, 2003; Thomas, Tagg & Ward, 2003; Thomas & Ward, 2001).

The interdependence of strategies and knowledge are emphasised in the Number Framework (MOE, 2002) and it is necessary that students make progress in knowledge and strategy levels conjointly. This is illustrated by the fact that students at higher strategy levels also have, and need, more secure number knowledge (Thomas et al., 2003). However, the NP has a major focus on the teaching of mathematical strategies rather than the more traditional focus on the teaching of mathematical knowledge. Therefore most lessons are structured with a key emphasis on strategies and it is recommended that knowledge is taught during a ten-minute whole class warm-up at the beginning of lessons (MoE, 2002). In this way the NP illustrates a shift in teachers' emphasis on the teaching of knowledge to that of strategies as a critical factor (Higgins, 2001).

In a political sense the Government's support of the Numeracy Project may be seen as recognising that better learning is to occur, the nature of quality teaching needs to be clearly defined and efforts made to produce and foster those who do it. The NP acts as a professional development programme for teachers advocating best practice in numeracy teaching. As a model, and a programme of professional development, it indicates a view of teachers as professionals rather than technicians advocating development rather than simply monitoring teachers' practice. This is in contrast to the curriculum document, which Biddulph, Taylor, Hawera and Bailey (2002) describe as a document of contradictions that can be seen to exist within a broader curriculum of social control. The NP is situated within the MOE's Literacy and Numeracy strategy. Key themes of this strategy are identified by Thomas et al., (2003) as raising expectations, improving professional capability, and involving the community in this process. The NP acknowledges professional development as a key to integrating theory and practice for quality outcomes in mathematics education. Through improving the professional capability of teachers, students' performance in numeracy is also improved. This is supported by research which indicates that these professional development programmes have improved outcomes for all children (Thomas et al., 2003).

The NP recognises the constructive nature of learning for teachers and students alike. It promotes the notion of good practice by providing teachers with research material to read, reflect on, and model, while also having teachers involved in mathematical research in collaboration with expert others or as experts of the classroom context. Teachers who have undergone professional development in the Numeracy Project report an increase in mathematics content knowledge and pedagogical knowledge (Thomas & Ward, 2002). Additionally, they report greater understanding of number and how they might teach it effectively (Higgins, 2001). The Number Framework provides teachers with knowledge of how children acquire number concepts, therefore supplying teachers with an increased understanding of how to assist children’s progress; the associated diagnostic tool is an effective means of accessing children’s levels of thinking in number (Thomas et al., 2003).

Learning theories and the NP

A number of learning theories have influenced the NP and are implicit within the Number Framework and associated teaching material. The first of the learning theories which may be seen within the NP is behaviourism. This theory influenced the teaching of mathematics through the idea that school mathematics could be carefully organised into a precise sequence of small steps in such a way that a clear learning path would be evident to the learner and result in sound outcomes. In this way, it was presented within a *behaviouristic* approach that all students could learn if instructional tasks were arranged in appropriate and carefully adjusted learning sequences (Neyland, 1995). Irwin and Irwin (2000) describe behaviourist learning theory as recognising that what we do depends on the circumstances both before we act and the consequences of our actions. Aspects of behaviourism may be clearly seen in the NP which advocates various activities for children at different levels on the framework. At lower stages and in terms of knowledge acquisition behaviourism is apparent in the NP. For example, activities such as teaching children to count out loud in multiples may be viewed as behaviourist type activities. The students say the required number out loud in chorus and are positively reinforced to say the right sequence through listening to their peers or encouragement by the teacher (or are negatively reinforced for saying the incorrect sequence). Behaviourism is recognised as effective in terms of the achievement of lower order skills, and therefore its use is valid in this type of activity (Neyland, 1995).

The predominant learning theory evident throughout all aspects of the NP, however, is that of constructivism. Wood (2002) argues that "currently there is widespread acceptance of the view that learning is an active constructive process" (p. 61). There are divergent theories within the realm of constructivism and the apparent impact of some of these theories on the NP will be outlined below. Followers of constructivist theory contend that "human learners have the capacity to invent or construct general theories about their experience" (Hughes, Desforges, Mitchell & Carre, 2000, p. 12). Therefore teaching involves understanding the learners' intellectual development, identifying their existing schemas and prior knowledge, arranging experiences to challenge their schemas (Hughes et al; 2000). A central belief across all theories of constructivism is that new knowledge is built on previous learning (Kanuka & Anderson, 1999). Therefore the influence of constructivist theory is apparent in the use of the diagnostic interview in the Numeracy Project. Through the use of this tool teachers are able to evaluate the level of the students against the Number Framework and are also given guidance as to the next steps to challenge the learners' schemas using the numeracy booklets and materials. These provide a range of activities from which teachers can then choose as most appropriate.

Another constructivist approach which may be seen as relevant in regards to the NP is the formative approach. This approach as defined by Neyland (1995) as 'entirely learner-centred and aims to match learning opportunities in mathematics with learners' natural cognitive abilities' (p. 39). Formativist approaches may be seen as linked to the work of Piaget. A teacher using this approach aims to help learners develop mathematical concepts in tune with their development in thinking (Neyland, 1995). Therefore this approach supports teaching to the level the individual child is assessed as being at and catering to their individual needs. This is apparent in the Numeracy Project where children are individually interviewed and grouped according to their knowledge and strategy levels.

Links to constructivism in the NP may also be seen in the use of a model of children's learning. Steffe and Kieren (1994) identify models as important tools for constructivist researchers. They define models as coordinated schemes of actions and operations a researcher constructs from their experience of children's actions. By constructing a model of children's mathematical behaviour explanations of recurrent patterns as well as explanations of progress are made. Von Glasersfeld (1992) also argues that it is "necessary for a teacher to build up a model of the students' conceptual world" (p. 4). Constructivism is also seen to manifest in the classroom through problem challenges, small group work, and classroom discussions (Steffe & Kieren, 1994). All of these are aspects and learning tools used in the NP.

Constructivist theorists contend that mathematical knowledge is socially situated in an environment where communication is central and the language and symbols of mathematics are utilising the tools of society, and that these are mediated within the classroom community (Vgotsky, 1962 as cited in Kanuka & Anderson, 1999). Therefore for learning to take place there needs to be discussion. The process of children learning is seen to have a social nature "rich social interactions with others substantially contribute to children's opportunities for learning" (Wood, 2002, p. 61). Irwin (2003) writes that "constructivist classes are characterised by teachers listening to students and students listening to one another" (p. 50). The NP identifies discussion as an important tool both for assessment and learning with two types of classroom interaction seen as central to effective teaching; "asking students to explain their thinking and waiting for them to do so and the use of questioning and explanations of other students to help students progress in their thinking" (Thomas & Ward, 2002, p. iii). This aspect of the NP is also positively evaluated by teachers who have undergone the NP professional development, stating that they now have a "greater emphasis on questioning and students' explanations" (Higgins, 2001, p. iv) in their classroom programmes. Moreover, constructivist theorists argue that "learners will require a variety of learning experiences to advance to different kinds and levels of understanding" (Kanuka & Anderson, 1999, p. 62). Indeed, the Numeracy Project supplies multiple learning experiences that are available and recommended for each and every strategy level.

Radical constructivism promotes the importance of authentic learning activities and learning activities that develop learners' metacognitive skills (Kanuka & Anderson, 1999). Within the NP learners are encouraged to share their strategies and evaluate how they solved problems in their cooperative groups, thus modelling the practice of mathematicians. Situated cognition is another theory which has influenced the Numeracy Project, postulating that "knowledge cannot be separated from the context in which we learn it" (Hughes et al., 2000, p. 16). Through this lens learning is seen as a fundamentally social activity, defined by culture and the context of the learning setting, with knowledge situated in the context of learning. Knowledge can therefore be seen as the working practices of a culture of learning (Hughes et al., 2000). Situated cognition theory holds that students learn through authentic experience developing understandings in mathematics as they talk about mathematics and act as mathematicians do in the real world. Within a classroom setting this includes working in collaborative groups, analysing mathematical problems and approaches to understanding and solving problems. Using such approaches children are encouraged to listen to each others ideas and explain and prove their own as it is theorised that children's thinking and ideas can develop through listening to and challenging other children's ideas. This is seen in the Numeracy Project with its encouragement towards using collaborative groups to solve mathematics problems.

Summary

In conclusion, this paper has demonstrated that the Numeracy Project was, in part, the result of a nation striving towards a "knowledge society" and a numerate society. It has shown that research demonstrating a lack of performance of New Zealand primary age students in mathematics led to further research and the development of the NP. The paper has outlined the numerous influences on the construction of the New Zealand NP which include both teaching models and learning theories. It has also shown the effects of learning theory on the NP including both behaviourism and more importantly constructivism.

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