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BRIDGE DECK ANALYSIS

being a thesis in two volumes

submitted for the

DEGREE OF DOCTOR OF PHILOSOPHY

at the

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THE UNIVERSITY OF AUCKLAND

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I.G. BUCKLE, B.E. (Civil) (Hons).

1964 - 1967

VOL. I.
SYNOPSIS

In this thesis the structural analysis of two basic types of bridge deck systems are discussed:

I. the multibeam bridge deck
II. the skewed anisotropic bridge deck.

The major difficulty in the analysis of I, the multibeam deck, arises from its lack of transverse bending stiffness; load distribution occurs by shear transference at interlocking shear keys. An analysis method, developed from transfer matrix theory is proposed and shown to be satisfactory for such a structure. Model studies on a quarter scale multibeam bridge deck are described together with field tests on the prototype decks - the southern motorway bridges crossing Slippery Creek. Agreement between theory, model studies and field tests is illustrated.

The satisfactory analysis of II, the skewed anisotropic deck, is complicated by its anistropic elastic properties and skewed geometry. An analysis procedure is introduced which is an extension of the finite element technique already established in other plate bending and plane stress problems. Using therefore the matrix displacement method and finite element discretization, the method has been programmed for solution by digital computer. Comparison of the computed displacements with those obtained by experiment on skewed isotropic and anisotropic steel plates is given. The finite element method is seen to be a powerful analytical tool, particularly because of its ability to handle elastic anisotropy and arbitrary geometric shapes.

This thesis is accordingly divided into two volumes as below:

Volume I: "Matrix Analysis and Structural Behaviour of Multibeam Bridge Decks."
Volume II: "Matrix Analysis of Skewed Anisotropic Bridge Decks."
ACKNOWLEDGEMENTS

The work that is described in this thesis was carried out at the School of Engineering under the joint supervision of Mr. R.A. Jones, Senior Lecturer, and Professor N.A. Mowbray, Head of the Department of Civil Engineering.

Mr. Jones directed the work through its initial stages and his enthusiastic support with the design, construction and testing of the quarter scale, multibeam model is gratefully acknowledged. His continuing advice and comments on later phases of the work were also appreciated.

On the registration of this project for a Ph.D degree, Professor Mowbray accepted sponsorship of the work and his helpful guidance and pertinent criticism, with particular reference to the finite element matrix work described in Volume II, is also acknowledged. For his encouragement and interest in all phases of the project, I owe my grateful thanks.

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These acknowledgements would not be complete without mention of the encouragement and help received from my family. My sincere thanks go to my mother for typing the draft manuscripts in a language entirely foreign to her, and for her continued patience and active interest in my work. (Similar thanks are expressed to Mrs. Shimmin, who typed the final script under the same
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VOLUME I.

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CHAPTER: ONE

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1.5 Summary, Recent Developments, Future Work and Appendices
1.0 Description

A multibeam bridge is constructed from a number of precast concrete beams, which are usually prestressed and laid along side one another to span longitudinally between piers. An alternative name for this type of deck is the term "pseudo-slab" since it is in effect an articulated slab but of non-uniform depth. (Figs 1.1 and 1.2)

Some measure of transverse reinforcement, prestressed or otherwise, is applied and in some cases a reinforced deck slab is poured over the beams and followed later with hot mix or tarsel. In other cases the hot mix is laid directly over the beams themselves.

Special edge beams may be placed to provide edge stiffening, a curb and support for the guardrails. The beams are usually seated through neoprene pads on to pier caps with some form of holding down reinforcement provided between the deck and the cap beams for the resistance of earthquake, wind, and other horizontal loads.

1.1 Importance

It will be seen that the multibeam deck is a comparatively simple form of bridge construction and as such possesses advantages over other bridge deck systems.

Simplicity, economy and reduced construction time have made this system popular, not only for short span bridges but also for walls, floors, roofs, and wharf deck systems. Recent years have shown an expansion in the precast, prestressed concrete industry and the importance of the multibeam deck system is becoming more evident. 1, 2, 3*.

A wide variety of size and shape of precast units are now available (fig 1.2) together with several types of shear keys and two systems of transverse reinforcement.

The superiority of the system is demonstrated by a recently completed two-span multibeam bridge deck on the Auckland-Hamilton Motorway which took just three weeks for construction from foundations to guardrails and showed a 20% reduction in total cost over that expected had a conventional deck type been used.

1.2 Structural Behaviour and Governing Parameters

Load distribution across the multibeam deck is effected by shear trans-

* Superscript numbers refer to the references cited in the bibliography.
FIG. 1.1 Perspective of a Multibeam Bridge Deck.

- guardrail
- transverse stressing cable at midspan
- shear keys
- 10 precast concrete beams
ference across the joint or interface between two adjacent beams; there is little or no transverse bending strength in these decks. Some decks are provided with shear keys while in others the reinforced deck slab ensures transference of the load between the beams. The transverse reinforcement may assist the load distribution but unless it is of relatively large proportions its effect is insignificant.

The parameters governing the structural behaviour of the multibeam deck are considered to be as follows:

(a) The ratio of transverse to longitudinal reinforcement.
(b) The size and shape of the precast beam.
(c) The ratio of the span to the width of the deck.
(d) The angle of skew of the deck.
(e) The size and shape of shear key.

It is unfortunate that despite its widespread adoption as a bridge deck and floor system, little has been recorded concerning the effect of these parameters on the structural behaviour of the multibeam deck. The development of a suitable analysis method appears to have suffered a similar fate with only slight progress being made since the introduction of the multibeam deck some years ago.

1.3 Existing Methods of Analysis

Analysis methods so far proposed by other workers 14, 19, 24, 27, 28, 30, 32, fall into two classes as will be considered in Chapter 2. These classes are distinguished by the particular theoretical model adopted for their respective methods. The orthotropic plate model is assumed in the first class and the articulated model in the second.

Methods based on the orthotropic theory are either inappropriate or complex and difficult to apply. Relaxation methods based on the articulated model, whilst simple in concept, are handicapped with tedious numerical calculations which are difficult to programme for solution by digital computer.

1.4 Purpose and Scope

It is the purpose of this thesis to introduce a new matrix analysis method, and by extensive experimental work examine the application of this theory to real multibeam decks and determine the significance of the governing parameters.
1.41 Proposed Method of Analysis

The proposed analysis method which is presented in Chapter 3, uses transfer matrix theory to determine the forces and deflections in the articulated model of the multibeam deck. Programmed for solution by an IBM 1620 Computer, this method once provided with the bridge dimensions, elastic properties and loading arrangements, will give the deflected profile, the load distribution and the shear force at any beam interface. Design charts are easily prepared; a ready analysis by hand calculation is also possible.

1.42 Experimental Work

To test the significance of the governing parameters and to check the reliability of the proposed analysis method a programme of experimental work was initiated. Both field and laboratory testing has been conducted and is described in Chapters 4 and 5 respectively.

1.421 Field Testing the Slippery Creek Bridges

With the co-operation of the New Zealand Government's Ministry of Works, the Southern Motorway Bridges crossing Slippery Creek, eighteen miles south of Auckland City, were load tested in May 1965. Five spans of the two bridges were investigated under different abnormal loads by observing changes in strain and deflection. All five parameters were examined during an eight week test programme. The nature of these tests and the procedure adopted for conducting them are described in Chapter 4; the results and conclusions drawn from this work are presented in Chapter 6.

1.422 Laboratory Testing the Model of the Slippery Creek Bridges

A quarter scale model of the Slippery Creek Bridges was built in the Structures Laboratory to reproduce the five spans tested at the site. Again tests were carried out to observe, under laboratory controlled conditions, the structural behaviour of the multibeam deck system. In addition more extensive tests were conducted on the model than were possible on the full scale structure; for example the determination of ultimate strength, the overload safety factors and the effect of transverse prestress. Construction of both the model and the permanent testing facilities necessary for these studies is described in Chapter 5. Correlation of model and full scale structures and conclusions from these tests are also presented and discussed in Chapter 6.
Summary, Recent Developments, Future Work and Appendices

Chapter 7 presents a Summary of the major results and conclusions that are presented in Chapter 6. Recent developments since the commencement of this thesis and possible future lines of research are also outlined in this Chapter. Twelve Appendices follow Chapter 7 listing Computer Programmes and additional experimental and theoretical work that has been necessary to the project as a whole.
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<td>(b) double-U</td>
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**FIG 1.2** Typical Units Available for Multibeam Structures
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2.5 Summary and Conclusion
2.0 Introduction

The historical development of load distribution theories, and therefore of bridge deck analysis, is extensive, well documented in Chapter 9 (Vol. II). This present Chapter is concerned with those theories that have been developed and applied to the multibeam bridge deck. These can be grouped into the two following categories:

A. Methods using an approximate solution to the classical differential equation governing the anisotropic bridge deck.
B. Methods using an exact solution to a physical approximation of the particular (multibeam) bridge deck.

It should be noted that all analysis methods so far proposed for bridge decks fall into one or other of these two classes but this volume is primarily concerned with the particular case of the multibeam bridge deck.

2.1 Analysis by the Orthotropic Plate Theory (A).

2.1.1 Historical Development

The fundamental equation for the flexure of thin isotropic plates was derived by Lagrange in 1811, but it was not until 1820 that Navier solved the equation for a rectangular plate with all edges simply supported. In 1829 Poisson published a paper on elasticity, and set forth the first acceptable derivation of this equation and a set of general boundary conditions. Plates of anisotropic material were first considered in 1879 by Boussinesq and the governing differential equation for the bending of thin anisotropic plates as set down by him is as follows:

\[ D_x \frac{\partial^2 \phi}{\partial x^2} + 2(D_x + 2D_{xy}) \frac{\partial^2 \phi}{\partial x \partial y} + D_y \frac{\partial^2 \phi}{\partial y^2} = q(x,y) \quad (3.1) \]

where \( D_x \) is the flexural stiffness of the plate in the \( x \) direction,
\( D_y \) is the flexural stiffness of the plate in the \( y \) direction,
\( D_1 \) is a constant incorporating the two Poisson ratios for the plate and the flexural stiffnesses \( D_x \) and \( D_y \),
and \( D_{xy} \) is the torsional stiffness of the plate.

It is usual to let \( H = D_1 + 2D_{xy} \).

The application of this theory to reinforced concrete slabs was not until 1914 when Huber published a series of papers on the subject. The principal
results are collected in his books 5, 6, which are concerned with the determination of the flexural and torsional rigidities $D_x$, $H$ and $D_y$ for concrete slabs. Later in 1946 and again in 1949 Guyon published his approach for isotropic slabs 7, 8, based on the anisotropic plate theory and developed a solution assuming a Levy series for deflection and a Fourier sine series for load. This approach was generalised by Massonet 9 in 1950 to include the effect of torsion and this work was further developed and extended in 1954 by Morice and Little 10, 11 and Rowe 12, 13 to provide a design procedure for the orthotropic bridge deck.

This design procedure, presented as a distribution coefficient method, is given in detail in Rowe's book "Concrete Bridge Design" 14 and apart from the discussion which follows in sec. 2.12, is again reviewed in Chapter 9.

At about the same time (1954) work was commenced at the Fritz Engineering Laboratory of the Lehigh University on the specific application of the orthotropic plate equation (2.1) to the multibeam bridge deck. This work was the basis of papers by Roesli 19, Walther 21, and Nasser 22 which proposed a design procedure for the lateral load distribution in multibeam bridges.

Further work on the application of the orthotropic plate theory to multibeam decks has been carried out by Cusens and Pama 24, in 1964. This paper presents an adaptation of the theory as presented by Guyon and Massonet and is therefore identical to that given by Rowe except for slight, but significant modifications made to the definitions of the governing parameters.

The above methods use approximations of one form or another to enable a solution to the orthotropic equation (eqn 2.1) to be obtained. The accuracy of the approximation and the suitability of the application form the basis of the following discussion.

2.12 First Method of Orthotropic Plates - Guyon, Massonet
(Method of Distribution Coefficients - Rowe, Morice, Little)

The basis of this method is the study of an equivalent elastic system obtained by replacing the stiffness of the longitudinal and transverse beams with a uniformly distributed system of the same overall stiffness. This equivalent system is a slab of uniform depth, with orthotropic elastic properties given by those of the original deck. It is convenient to combine the geometrical properties into one parameter which defines this relation between longitudinal and transverse properties. Denoted by $\theta$ it is defined by:

$$\theta = \frac{b}{2a} \sqrt[4]{\frac{1}{j}}$$

----------(2.2)
where $2b$ is the width of the equivalent slab

$2a$ is the actual span

$i$ is the longitudinal second moment of area per unit width

and $j$ is the transverse second moment of area per unit length.

In addition to the effect of $\theta$, the distribution coefficients are also sensitive to the degree of torsional stiffness exhibited by the bridge. An overall torsional parameter $\alpha$ is introduced and defined by

$$\alpha = \frac{G}{2E} \left( \frac{i_o}{j} + j_0 \right)$$

where $G$ is the torsional modulus

$E$ is the elastic of Young's modulus of the material of the bridge

$i_o$ is the torsional inertia per unit width

and $j_0$ is the torsional inertia per unit length.

In the case of a slab $\alpha$ reduces to unity and for a no-torsion grillage $\alpha$ is zero. Thus for any real bridge deck $\alpha$ will take on a value in the range: $0 \leq \alpha \leq 1$.

Using the Guyon-Massonet theory tabulated and graphed distribution coefficients have been prepared by Rowe using $\theta$ as the independent variable. Two sets of curves have been prepared for each of the two extreme values for $\alpha$: $\alpha = 0$ and $\alpha = 1$. For any intermediate value of $\alpha$ the appropriate distribution coefficients are given by the approximate interpolation formula suggested by Massonet:

$$K_\alpha = K_0 + (K_1 - K_0) \cdot \left(1 - \frac{\alpha}{\alpha_1} - \frac{\alpha}{\alpha_0}\right)$$

where $K_\alpha$ is the distribution coefficient for the particular required

$K_0$ is the distribution coefficient for $\alpha = 0$

and $K_1$ is the distribution coefficient for $\alpha = 1$.

Once extracted from the graphs the distribution coefficients give the profile shape, longitudinal moments and stresses relative to the mean deflection, mean longitudinal moment and stress at nine equally distributed standard positions across the deck width.

Transverse bending moments may also be determined using a different set of coefficients which are again plotted against $\theta$ as the independent variable for the two extreme values of $\alpha$. 
2.12 **Application to Multibeam Bridge Decks.**

It is unfortunate that the distribution coefficient method, although a powerful and widely adopted design tool has little application to the analysis of the multibeam bridge deck.

If an attempt is made to carry out such an analysis the evaluation of $\alpha$ and $\theta$ leads to difficulties since $j$ and $j_0$, measures of the transverse flexural and torsional stiffnesses, are assumed zero for the multibeam deck.

The shear keys provide load distribution by shear alone and neither bending moments nor twisting moments are distributed or resisted at the assumed hinge. However, in practice the transverse prestress provided may permit some development of bending and torsion moments of resistance by introducing compression and hence friction, between adjacent beams. This, however, is usually nominal and the effective values of $j$ and $j_0$ are zero.

2.122 **Application by Morice and Little**

Notwithstanding this difficulty Morice and Little load tested a small multibeam bridge in Hampshire and applied the distribution coefficient method to check its reliability.

The bridge selected for the test was a simple supported "pseudo-slab" of $15^\circ$ skew, containing 20 prestressed beams of 33ft 6in span transversely stressed together by cables lying parallel to the abutments. The bridge was designed to carry the Ministry of Transport distributed and knife edge loadings and a nominal transverse prestress of 70 lb/sq.in. was introduced to produce "composite action" of the individual beams.

The experimental distribution profiles which could be compared with theory were for those for which the transverse working load had not been exceeded. In that case, the structure was assumed to be "flexurally continuous" in each direction and the values of $\theta = \frac{h}{2a}$ and $\alpha = 1$ was assumed relevant as for a slab.

Neglecting the $15^\circ$ angle of skew, an initial analysis was made by Morice and Little for an equivalent right slab which gave $\theta = 0.4$. However, the actual distributions were markedly inferior to those calculated for $\theta = 0.4$, $\alpha = 1$ using the Massonet - Rowe analysis. In fact the distributions were ... "inferior to the Guyon ($\alpha = 0$) distribution which
represents the worst possible distribution for a given value of $\theta$. 15
A second analysis considered a shortened effective span giving $\theta = 0.5$.
The experimental behaviour now lay between the theoretical behaviour for $\alpha = 1$ and $\alpha = 0$. Yet no one value of $\alpha$ could be found which satisfied either all load positions or each point on a given transverse section for a given load position.

Eventually it was found that good agreement between theory and measurement existed for $\theta = 0.684$ and $\alpha = 0.52$ for the bridge. The value for $\alpha$ was found by a rearrangement of the interpolation expression given by equation (2.4):

$$\sqrt{\alpha} = \frac{K_\alpha - K_0}{K_1 - K_0}$$  \hspace{2cm} (2.4b)

where $K_\alpha$ was a set of distribution coefficients observed during the test on the bridge
and $K_0$ and $K_1$ the distribution coefficients as defined earlier.

Thus an equivalent value for $\alpha$ was determined empirically for the multi-beam deck under test, no reasonable explanation being found for such a value. It is, however, obvious that $\alpha$ would have a value less than unity if the total transverse prestress were insufficient to induce the same torsional, flexural, and shear properties found in the equivalent anisotropic slab. This condition ($\alpha < 1$) is more commonly found in practice where the transverse prestress is only nominal.

On the other hand the value of $\theta = 0.684$ is apparently explained by taking $2a$ in equation (2.3) as the length of span over which the transverse prestressing force was applied ignoring the effect of the $15^\circ$ angle of skew on the deck. This procedure can lead to difficulties when it is remembered that in the more recent types of multibeam decks only one single transverse stressing cable is used and $2a$ is then effectively zero.

The empirical approach used by Morice and Little to fit the distribution coefficient method to the observed behaviour has also been criticised by Hendry and Jaeger 15, 17, authors of the harmonic analysis method for grid frameworks and related structures. As they have pointed out any sort of assumption could be made on the behaviour of the bridge and by suitable selection of parameters results could be obtained apparently agreeing with those found by experiment in a particular case. In particular they emphasized ... that apparent agreement obtained by manipulation of parameters
in a given theory was meaningless in the absence of sound theoretical justification of the values selected. It could not be agreed by Hendry and Jaeger that the values of $\theta$ and $\varphi$ had been theoretically justified. Again the experimental observations did not reproduce the form of the transverse distribution profile both at mid span and at the quarter points. The distribution coefficient method insists on geometrically similar profiles at these points, since it means that one set of relative arithmetical coefficients may be used for defining the deflected shape of all transverse sections - the crux of the distribution coefficient method. This lack of similarity is illustrated in figures 180 and 182 of reference 15; a paper presented by Morice and Little to the Conference on the Correlation between Calculated and Observed Stresses and Displacements in Structures, held in London in 1955.

2.123 Application by Park. Still further evidence against the use of the distribution coefficient method in the analysis of multibeam bridges is provided in a thesis by Park from the University of Canterbury (1957).

Park's work involved the testing of five composite post-tensioned prestressed concrete slabs to determine their distributive properties and ultimate strength; the observed behaviour being compared with theory - the distribution coefficient method for elastic behaviour and Johansen's Yield Line theory for ultimate load behaviour. Since these slabs were in effect multibeam slabs Park's conclusions concerning the distribution coefficient method are of interest.

Park's experimental results showed that the theoretical estimation of transverse moments to be high (by 23%); Poisson's ratio and all terms of the load series being included in this estimation. In the case of longitudinal moments the theory was shown to underestimate the maximum value (by 20%) and thus in this case the error is serious. Possible discrepancies (as suggested by Park) in the analysis for longitudinal moments could be:

1. Over simplification in the initial assumptions.
2. Neglect of Poisson's ratio.
3. Consideration of only the first term of the load series.

Consideration has been given to these factors by the originators of the theory and a recommendation by Guyon of increasing the maximum theoretical bending moment by 10-15% was adopted by Rowe, Morice and Little. Rowe has since
shown that the inclusion of Poisson's ratio in the analysis makes little difference to his theoretical results - taking the ratio at 0.15 reduces the theoretical moment by approximately 2%.

It would appear, however, from Park's work that the theoretical maximum longitudinal moment should be increased not by 10% but by 20-30%. In doing so the distribution coefficient method would then arbitrarily estimate the maximum longitudinal moment and considerably over estimate the maximum transverse moment when applied to a multibeam deck.

Such a result is not difficult to reconcile when the physical nature of the multibeam deck is considered. Since there is no transverse bending medium the theoretical moments, derived assuming there is such a medium, will naturally over estimate the actual moment. Again, because of the lack of transverse stiffness a greater proportion of the applied load must be carried longitudinally and a theory which ignores this situation will under estimate the actual moment in this direction.

2.13 Second Method of Orthotropic Plates

(Lehigh Method - Roesli, Walther, Nasser)

The multibeam deck is again replaced by an orthotropic plate with elastic properties equivalent to those of the original deck. Taking equation (2.1) and dividing through by $D_x$ we have:

$$\frac{\partial^2 w}{\partial x^2} + 2\beta \frac{\partial^2 w}{\partial x \partial y} + \alpha \frac{\partial^2 w}{\partial y^2} = \frac{p(x,y)}{D_x}$$

(2.5)

where $\alpha = \frac{D_y}{D_x}$ and $\beta = \frac{H}{D_x}$.

Note that $\alpha$ has been redefined to conform with the notation used by Roesli, Walther and Nasser of the University of Lehigh, and should not be confused with the definition used by Rowe (equation (2.3)).

$\alpha$ and $\beta$ are therefore nondimensional parameters involving the ratio of longitudinal and transverse bending stiffnesses and a coefficient of torsional rigidity. A solution to equation (2.5) has been developed by the workers at Lehigh but no details are given either by Walther or Nasser and Roesli's paper is not readily available. The solution apparently involves and two further parameters relating the geometry of the whole deck to the geometry of a single beam.

The derivation of $\alpha$ is somewhat difficult and no theoretical approach has yielded satisfactory results since according to Walther "the lateral bending stiffness is by no means constant." It varies not only from point
to point in the bridge but is also dependent on the magnitude and location of the concentrated load - at least according to Walther - making it impossible to derive $\alpha$ theoretically. An experimental investigation was made by the team at Lehigh and empirical formulae were developed for an average value of $\alpha$ over the entire bridge, as a function of load and transverse prestress only. These are given as:

$$\alpha = 0.23 \sqrt{\frac{F}{F_c}}$$ for centre load \hspace{1cm} (2.6)

$$\alpha = 0.1 \sqrt[3]{\frac{F}{F_e}}$$ for edge load \hspace{1cm} (2.7)

where $F$ is the total transverse prestressing force.

and $P$ is the load applied at the centre or edge with the restriction that: $0 \leq F/P \leq 20$

The theoretical derivation for is also impossible and Roesli proposes:

$$\beta = 3k (1 - \alpha^{2/3}) + \alpha$$ \hspace{1cm} (2.8)

where $k$ is a constant of torsional rigidity of rectangular beams, dependent on their geometry.

Huber and Massonet have both given relationships which in effect link $\alpha$ with $\beta$, but according to Walther, "the assumptions made for these relationships proved to be incompatible with the theoretical considerations for orthotropic plates" - presumably when applied to multibeam bridge decks.

An interesting feature of this work is the consideration made for slip between adjacent beams. The effect of slip or incomplete interaction between the beams was determined experimentally and a modification is therefore made to the maximum moment coefficient during the design procedure. The modification takes the form of a percentage increase to this coefficient as read from a graph plotted with $F/P$ as the independent variable. It should also be noted here that similar work has been carried out by the University of Missouri Engineering Experimental Station during an experimental study on precast concrete bridge units. In this investigation to determine the effectiveness of load transfer between three such units observations were made on the relative slip between adjacent units and the results plotted against the load applied.

As with the distribution coefficient method of sec. 2.12, the theoretical derivation of the transverse stiffness and torsional properties of the
equivlent plate is extremely difficult and an estimation is made only after extensive testing. As already stated in sec. 2.12b, when the physical nature of the multibeam bridge is considered it is not surprising that such difficulties have been encountered. Both the Rowe - Morice - Little and the Lehigh University teams have resorted to the empirical determination of the governing parameters - a technique which will only prove a method for the particular conditions under which the tests were conducted.

2.14 Third Method of Orthotropic Plates

(Modified Method of Distribution Coefficients - Cusens, Pama)

Aware of the difficulties encountered by Morice, Little and others (sec. 2.12) Cusens has taken more care in the definition of the flexural and torsional parameters $\theta$ and $\phi$:

$$\theta = \frac{b}{2a} \sqrt{\frac{D_x}{D_y}}$$

$$\phi = \frac{T_x + T_y + 2D_1}{2D_x D_y}$$

Where $T_x = G_{ix}$

and $T_y = G_{iy}$, are the torsional stiffnesses with respect to the x and y axes. Cusens agrees that when the orthotropic plate equation (2.1) is applied to grillages the value of $D_1$ is negligible, but in the case of solid slabs $D_1$ is significant and this term should therefore be considered in the solution of multibeam decks. Comparison of equations (2.9) with Rowe's definitions of (2.2) and (2.3) will indicate Rowe's omission of $D_1$.

Again, in order to apply the orthotropic plate theory, the multibeam deck is assumed continuous in both longitudinal and transverse directions. Cusens notes that in the transverse direction the thickness of the slab actually varies from a maximum value equal to the depth of the individual beams to some value $\xi_d$ at the joint between beams. Provided that the ratio of beam width to overall deck width is small (Cusens suggests in the order of 10%) a conservative mean effective depth of the transverse section would be the value of $\xi_d$. It is suggested, therefore, that in the presence of shear keys for a bridge of negligible transverse prestress the effective depth should be taken to the base of the shear key. A similar suggestion has been put forward by Best of the Cement and Concrete Association who, after testing a prestressed concrete bridge which incorporated transverse mild steel shear...
connectors, assumed the concrete in the joints to be cracked to the level of the connector. The uncracked part of the concrete section is then used to calculate \( j \).

Cusens and Pana therefore propose the use of \( h \cdot \xi d \) in the calculation of the parameters \( \theta \) and \( \alpha \), where \( h \) is the thickness of the plate. Experimental tests were carried out at the SEATO Graduate School of Engineering in Bangkok and are described by the Authors. The proposed changes to the Distribution Coefficient method are contrasted against the observed experimental coefficients. Ultimate load tests are also described and analysed using the Yield Line theory. The value of the torsion parameter is shown to increase with the value of transverse prestress and also with the span/width ratio; its value as given by equation (2.9) is also indicated to be conservative.

The experimental tests also showed that the ratio \( i/j \) is independent of both load and transverse prestress which conflicts with the results of Walther's work \(^{21}\) (sec.(2.13)). In particular it appears that equation (2.3) is not generally valid and Walther's results should therefore be treated with care. Further, comparisons were made with the results the Lehigh University obtained on the Centerport Bridge \(^{20}\) and the Authors' theory shown to give closer agreement than that obtained by the Lehigh team. Refer sec. 3.51 and figures 3.8 and 3.9 of Chapter 3.

Cusens also compares his method with the tests performed by Morice and Little on the Hampshire Bridge (sec. (2.122)). Assuming \( h = 0.5d \) theoretical values of \( \theta = 0.65 \) and \( \alpha = 0.54 \) are obtained. These compare favourably with the empirically derived values of \( \theta = 0.68 \) and \( \alpha = 0.52 \).

It would appear, therefore, that a conservative, yet apparently satisfactory estimate of general structural behaviour may be obtained by assuming an effective depth \( (h \cdot \xi d) \) in a transverse direction and applying the Distribution Coefficient Method. However, it is seen that the Guyon-Massonet theory is invalid if one half the sum of longitudinal and transverse torsional stiffnesses exceeds the square root of the product of the corresponding flexural stiffnesses. In this case \( \alpha \) is greater than unity and the Distribution Coefficient Method, which is based upon this theory, is no longer applicable. This situation is particularly prevalent when the transverse prestress is small as in many practical cases or when the prestress is omitted completely (as seen in sec. 6.231 of Chapter 6 - analysis of model SA).
Aware of this fact Gusens and Pama have, since the publication of the above paper, developed an articulated Plate Theory in an attempt to handle this somewhat embarrassing situation. The method is reviewed in sec. 2.25.

2.2 Analysis by Articulated Plate Theory (B)

2.21 Preamble

In the following methods of analysis the multibeam deck is assumed to be an articulated plate composed of a number of separate beams interconnected by joints or assumed hinges. Only shear forces are assumed to be transferred through this joint which are thus free to rotate - the transverse flexural stiffness of the plate is zero. The articulated plate promises, therefore, to be a more realistic model of the multibeam deck.

2.22 First Method of Articulated Plates

(Illinois Method - Khachaturian et al)

The first of these analysis methods was proposed in 1960 by Duberg, Khachaturian and Fradinger 26 of the University of Illinois and later their work formed the basis of Bulletin 483 (authored by Pool, Arya, Robinson and Khachaturian) 27 of the Illinois Engineering Experiment Station (1965).

The method handles the analysis of single span, right, multibeam bridge decks having beams of solid or hollow section. It treats the beams as individual elements connected to one another by frictionless hinges. These hinges are assumed to act along the mid-depth of the shear key such that no relative movement except rotation is possible. Thus the transverse flexural strength of the bridge is assumed zero and load distribution occurs by transfer of longitudinal, vertical and transverse joint forces which are assumed to act at each hinge. The basic equations for finding these forces are developed from the compatibility conditions for displacement at each hinge.

As the beams are simply supported at their ends, Fourier series expansions for all forces and displacements are used to reduce the problem to the solution of sets of simultaneous linear algebraic equations. Unfortunately the Fourier series for the joint forces either do not converge or else converge too slowly for practical use - moments, however, are obtained by straightforward calculations. Based on a study of the asymptotic behaviour of the coefficient of the Fourier series a method of accelerating the convergence for the joint forces is adopted. This method is apparently "computationally practical" and allows the explicit determination of the most important characteristics of the joint forces.
Computed results for the joint force distributions, beam shears and beam moments are tabulated for four sets of a four-beam deck and one set of an eight-beam deck. No experimental justification of the analysis method is given in either reference and therefore there are no comparisons available between theoretical prediction of joint forces, shears and moments, and those values observed in practice. Such comparisons would be of interest since the method assumes the existence of three joint forces. It is considered that with low transverse prestress, as commonly found in practice, the horizontal and longitudinal joint forces may not develop. Only by the formation of substantial bond and friction forces between the concrete shear keys and adjacent beams will such forces arise. The resistance and transfer of these forces is therefore in doubt and an analysis based on the assumption of their existence may seriously underestimate the actual loads and stresses in the beams.

2.23 Second Method of Articulated Plates
(Relaxation Method - Norman, Nathan.)

In 1962, Norman of the Ministry of Works, New Zealand, published a method for the distribution of loads in precast floors systems which also assumed that shear and not bending moments could be transmitted laterally across the system (only vertical shear considered). The approach is made by considering the bending and torsional stiffnesses of a single beam whose mid-span section is likened to a rigid plate supported by a series of springs. Edge and centre stiffnesses for the beam are derived and a carryover factor, relating these stiffnesses, is defined. A distribution factor for each joint in the overall deck systems is also developed which is proportional to the relative edge stiffnesses of adjacent beams. The parameters thus established form, not surprisingly, the basis for a relaxation method of analysis which is exactly analogous to the Hardy Cross Moment Distribution Method well established in the structural analysis of frames.

It is unfortunate that Norman's definition for the carryover factor is incorrect, but this is only a technical error and the merit of the principle upon which the analysis is based (i.e. the step-by-step relaxation procedure) is not affected.

In the following year (1963) Nathan also presented a method based on exactly the same principle except that he bypassed the analogy of the elastic beam and equivalent spring system, working directly in the stiffnesses of the
beam itself. Nathan's presentation is slightly more extensive and a brief outline follows:

A single hinge or joint connection is assumed at midspan, vertical shears only being transmitted through the connection. In practice this joint runs the length of the beam but in the case of some roof T-beams it takes the form of steel connectors spaced at intervals along the beam. The analysis is, however, simplified by the assumption of only one joint and the computed force at this one joint may later be approximately distributed on the basis of a parabolic distribution.

Initially all beams are considered fixed in position at zero deflection under the action of the applied load and a number of holding forces (which are calculated from the applied loading). These holding forces or restraints are removed in turn and the deflection noted for that point together with the carryover or change in the two neighbouring restraints. The restraint is again replaced so that this joint is fixed in its new position; the next restraint is now removed. In its turn it will affect, by carryover, the restraints on the previous joint and the one following. The amount to be carried over from joint to joint is dependent on the relative distribution of the holding forces between beams and is therefore given by the distribution factor. The process is reiterated until the restraints have been reduced to zero or negligible values. At this stage the changes in deflection are summed and the deflected transverse profile of the deck obtained. The deflection that each beam would have if it alone withstood the applied load is calculated. The ratio of this value to that of the beam when forming part of the deflected structure is then a measure of the load distribution coefficient for that beam. Vertical shears between the beams may then be found and distributed along the length of key if necessary.

The method is thus seen to be practical from a computational point of view since the arithmetic is simple and can be carried out by hand. However, under certain conditions the calculation may become very tedious and inaccurate. This is particularly the case if the values of the flexural and torsional stiffnesses are markedly different from one another; for in such an instance the value of the carry over factor is close to unity and the convergence of the relaxation procedure is very slow requiring many cycles of calculation before a satisfactory result is obtained. See also sec. 6.232 where a carry over factor of 0.95 was obtained. Because the method of solution is a step-by-step relaxation technique it cannot be easily
programmed for solution by digital computer and the above situation is not readily avoided. Again, the preparation of design charts is also a difficult task.

The method is suitable for edge stiffened decks and decks which may not be simply supported. As with the Illinois method, no experimental justification is made either by Norman or by Nathan for their method but it will be inferred from experimental evidence given in Chapter 6 (sec. 6.232) that the relaxation method is applicable and sufficiently accurate for the analysis of the multibeam deck.

2.24 Third Method of Articulated Plates

(Differential Equation Method One - Spindel, Best)

The deck is assumed to have no transverse stiffness so that all distribution of load is by shear, and the differential equation

\[ F \frac{\partial^2 \omega}{\partial x^2} + G \frac{\partial^2 \omega}{\partial x \partial y} = p(x, y) \quad (2.10) \]

is assumed to be applicable. (F and G are the longitudinal bending and twisting stiffnesses per unit width). By expressing the applied loading in the form of a Fourier sine series in the x direction, Spindel has obtained a solution for \( \omega_n \), the \( n \)th term of the series representing the deflection. Values for longitudinal bending moment, torque and longitudinal and transverse shear are then obtained by suitable differentiation of the expression for deflection.

For deflection and longitudinal moment the first term of the series \( (\omega_1 \text{ and } M_1) \) is taken as being sufficiently close to the true sum. However, the expressions for the remaining quantities listed above are not found to be rapidly convergent and Spindel discusses this problem of convergence in order to give reasonably accurate answers.

Best has used the approach of Spindel and shows that the articulated plate theory gives better agreement than that obtained under a modified orthotropic plate theory such as that proposed by Cusens.

2.25 Fourth Method of Articulated Plates

(Differential Equation Method Two - Cusens, Pama, Ahmed)

Pama and Cusens (now of the University of St. Andrews) intend shortly (1966/1967) to publish two papers in which they have generalised the approach adopted by Spindel and considered the effect of longitudinal torsion and Poisson's ratio. The effect of edge stiffening beams can also be
considered by the method which is based on the differential equation for the
articulated plate:

$$D \frac{\partial^2 w}{\partial x^2} + (D_{xy} + D_{yx} + D_1) \frac{\partial^2 w}{\partial x \partial y} = p(x) \quad \text{(2.11)}$$

A solution to this equation is developed using a Levy series of the form

$$w = \sum_{n=1}^{\infty} Y_n \sin \alpha_n$$

where $\alpha_n = \frac{n\pi}{L}$, and $Y_n$ is a function of $y$ only; the load function being
derived by energy methods in conjunction with a Fourier series.

An edge stiffening parameter $\lambda$, relating the stiffness of the edge
beam to the torsional stiffnesses of the interior deck, and a flexure-torsion
parameter $\beta_n$, are defined. For a particular $\beta_n$, equations are given which
relate the maximum distribution coefficient $K_{max}$ to the load position for a
range of values of $\lambda$. An optimum value of $\lambda$ may therefore be determined
for the particular $\beta_n$ such that $K_{max}$ is a constant across the deck width.
Thus an optimum size of edge stiffening beam may be determined.

Comparisons are made between theory and tests conducted on the Langstone
bridge in England and the Centerport bridge in America (previously tested by
the Lehigh University 20). The Langstone bridge 16 was tested by Rowe and
had small fascia beams which were expected to have some edge stiffening effect.
Ojusens shows that for $\lambda = 0.124$ good agreement is obtained between the
observations and theory. The authors have also examined a bridge deck 32 for
which $\alpha = 1.26$, and is therefore outside the range of the Guyon-Massonet
theory. Close correlation was again obtained between experiment and the
theoretical results obtained using this articulated plate theory.

2.3 Analysis by Other Methods and Present Design Practice
2.3.1 Analysis by a Four Sided Grid - Gallia

A grid supported on four sides is regarded as the basic structure for a
multibeam bridge with stiffened exterior beams and diaphragms. The grid is
composed of longitudinal and transverse members represented by the precast
beams and "hidden" diaphragms.

The method was proposed by Gallia first in 1955 33 and again in 1958 34.
Stiffened exterior beams and diaphragms are not generally found in multibeam
decks since they require special construction details. However, the method
is of interest since the problem of transverse stiffness is avoided by building
into the deck transverse members (diaphragms); the analysis is then based on
Massonnet's method for grillages 6.
No evidence of experimental justification is made in either paper where only theoretical design curves relating the maximum moment in the grid members to the span are given.

2.32 Present Design Practice

Present day design practice is based largely on code recommendations of which at least two have been made regarding the distribution of live load or heavy concentrated loads in bridge decks. One was specifically made for the design of precast decks, (ACI 711-58) and the other for the design of ordinary slabs, but which has been used for many multibeam decks to date (AASHO : 1.3.2(c)).

2.321 ACI Standard 711-58

The former is the ACI standard for Precast Concrete Floors and Roof Units, section 412(b) of which states that extra or concentrated loads: ""may be considered to be uniformly distributed over not more than three identical units on either side thereof but never over a greater total width than 0.4 of the clear span distance ..." In section 6.222.2 of Chapter 6 this recommendation has been compared with observed behaviour and its suitability noted.

2.322 AASHO Standard Specifications for Highway Bridges - 7th Edition

The latter is the recommendation due to Westergaard and adopted by AASHO in section 1.3.2(c) of their standard. The same recommendation has also been included by the Ministry of Works (N.Z.) in their Bridge Manual section 3.3.2(c) for the Distribution of Wheel Loads in Concrete Slabs.

For spans over twelve feet it is recommended that the width in feet over which a wheel load is to be distributed be given by

\[ E = \frac{10N + W}{4N} \]  

\[ (2.13) \]

where \( N \) is the number of lanes of traffic on the bridge and \( W \) is the width of roadway between curbs on the bridge.

Again further discussion on the value of this recommendation will be found in sec. 6.222.1 of Chapter 6.

2.323 Others

Roesli et al in his paper on the testing of the Centerport Bridge reports another recommendation that each beam be designed to carry 80% of the right or left wheel loads of an H20-S16 truck. Roesli's own experimental results do not support such a high percentage and nor does the experiment-
Index to Examples of Some of the Above Methods

In the following Chapters most of the above methods are again discussed with particular reference to the new method that is proposed in Chapter 3. Below is a short index indicating where in the text these methods may again be found.

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+ This method has been programmed in PDQ FORTRAN for use on an IBM 1620 machine; a listing and description of this programme is given in Appendix 2. Its use is described in the section indicated above (i.e. 6.231).

Summary and Conclusion

It would seem, therefore, that orthotropic plate theory is of limited application in the analysis of multibeam bridge decks. Only by the empirical determination of apparent transverse bending and torsional stiffnesses or in some cases by the estimation of an effective depth and subsequent application of the Distribution Coefficient Method can this theory be used. In any event a solution thus obtained should be interpreted with care because the above stiffnesses of a typical multibeam bridge arise by nominal transverse prestress forces and the doubtful ability of the shear keys to resist transverse moments.

In view of this uncertainty it is considered wiser to make the conservative assumption that the shear keys may only transfer shear forces. This assumption forms the basis of the articulated plate theory which is used in the development of the several analysis methods just reviewed. However, each suffers a particular disadvantage: for example the faulty basic assumption made by the Illinois method concerning the transmitted shear forces, and the heavy numerical work required by the other methods.

There is, therefore, a need for a new method which is founded on reasonable assumptions, and which is accurate and versatile, and above all easily calculated and programmed to give a solution by hand or digital computer.
Such a method is proposed in Chapter 3 which is based on the application of transfer matrices to the solution of the articulated plate model.
CHAPTER: THREE

TITLE: TRANSFER MATRIX ANALYSIS OF MULTIBEAM BRIDGE DECKS

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   3.21 Formulation of a Transfer Matrix Solution for a 10-beam Deck
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   3.25 Summary of General Matrix Equations for \( n \)-beam deck
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3.5 The Transfer Matrix Procedure Compared with Other Methods
3.51 The Methods of Roesli, Spindel, Cusens and Pama.
3.52 The Methods of Norman and Nathan and Rowe, Morice and Little.

3.6 Experimental Verification of Transfer Matrix Theory.
Notation

The symbols used in this chapter are defined where they first appear in the text. Below is a list of the more commonly occurring symbols including a brief definition and the page number where each is first introduced. They are divided into groups according to their nature, i.e. matrices, variables and Greek letters. Symbols used outside this chapter are also defined where they first appear but have not been listed below or elsewhere.

Matrices

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<td>2 x 2 square matrix product of all field and load matrices (35)</td>
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</tr>
<tr>
<td>B</td>
<td>2 x 1 column used in boundary vector determination (35)</td>
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<tr>
<td>C_F, C_P</td>
<td>2 x 1 columns load matrices for beam and joint loads resp. (38)</td>
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<tr>
<td>F_i</td>
<td>2 x 2 square field transfer matrix for the ith field (30)</td>
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<td>F_L, F_R</td>
<td>2 x 2 squares field transfer matrices for beams to left and right of the loaded beam resp. (38)</td>
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<tr>
<td>F_W</td>
<td>2 x 2 square field transfer matrix for the loaded beam (38)</td>
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</tr>
<tr>
<td>F_EL, F_ER</td>
<td>2 x 2 squares field transfer matrices for the left and right edge stiffening beams resp. (38)</td>
<td></td>
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<tr>
<td>I</td>
<td>2 x 2 square unit or identity matrix (30)</td>
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<tr>
<td>P_i</td>
<td>2 x 2 square point transfer matrix for the ith point (30)</td>
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<tr>
<td>P_W</td>
<td>2 x 2 square point transfer matrix for the loaded joint (38)</td>
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<tr>
<td>Z_i</td>
<td>2 x 1 column state vector at the ith point (30)</td>
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<td></td>
</tr>
</tbody>
</table>

Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>beam width (36)</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>beam depth (36)</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>elastic modulus of beam material (36)</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>shear or torsional modulus of beam material (36)</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>moment of inertia of a single beam (36)</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>torsional inertia of a single beam (36)</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>span (36)</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>total number of beams in the deck (40)</td>
<td></td>
</tr>
<tr>
<td>N_i</td>
<td>shear force at the ith joint (34)</td>
<td></td>
</tr>
</tbody>
</table>
Variables (Contd.)

\[ \begin{align*}
N_L, N_R & \quad \text{number of beams to the left and right of the load resp. (43)} \\
\tau & \quad \text{location of the loaded beam (40)} \\
W & \quad \text{load applied (36)} \\
x_i & \quad \text{deflection at the } i \text{th joint (34)} \\
X, X_E & \quad \text{flexural flexibility of inside and edge stiffening beams resp. (36, 38)} \\
Y, Y_E & \quad \text{torsional flexibility of inside and edge stiffening beams resp. (36, 38)} \\
\end{align*} \]

Greek Symbols

\[ \begin{align*}
\theta & \quad \text{rotation due to torsion; also angle of skew (36, 151)} \\
\Delta N & \quad \text{vertical deflection due to internal shears, } N_i \text{ (34)} \\
\Delta W & \quad \text{vertical deflection due to external loads, } W \text{ (39)} \\
\beta & \quad \text{torsion coefficient (36)} \\
\end{align*} \]
CHAPTER THREE. TRANSFER MATRIX ANALYSIS OF MULTIBEAM BRIDGE DECKS

3.0 Introduction

In this present chapter a new procedure is presented for the analysis of multibeam decks. The method uses transfer matrix theory to analyze the articulated plate model discussed in the previous chapter. The theory is applicable not only to bridge decks, but also to floor systems, wharves and roofs. It is suitable for a deck with simple or fixed support conditions, of any width or span, continuous or otherwise and for decks which may or may not be edge-stiffened. Skewed decks may be also considered. The analysis may be performed by hand although the method has been programmed for solution by digital computer.

3.1 Transfer Matrix Theory: Terminology, Definitions and Concept

The terminology and notation used in the following presentation is that of Pestel and Leckie. If the reader is unfamiliar with Transfer Matrix theory it is suggested that he examine Appendix 5 before proceeding further with this Chapter. In this Appendix the definitions and concept which follow are illustrated by application to a simple spring-mass system.

3.11 State Vectors

The state vector, $Z_i$, at a point $i$ of an elastic system is a column vector, the components of which are the displacements of the point $i$ and the corresponding internal forces.

3.12 Transfer Matrices

Two types of transfer matrices are defined:

3.121 Field Transfer Matrix, $F_i$

The field transfer matrix or field matrix, $F_i$, for the $i$ th field within an elastic system is a square matrix the components of which relate the state vectors that exist at those boundaries which define the limits of the field.

3.122 Point Transfer Matrix, $P_i$

The point transfer matrix or point matrix, $P_i$, at a point $i$ of an elastic system is a square matrix the components of which relate the state vectors which exist immediately to the left and to the right of that point.

Except when the $i$ th point is acted upon by an external force, $P_i$ will be equal to the identity matrix, $I$. 


3.13 Concept

Successive multiplication of state vectors with field and point transfer matrices allows the computation of all displacements and internal forces for elastic system to be made. Initial boundary conditions must be set and external loads known from which the initial state vector, \( Z_0 \), is calculated. Subsequent matrix multiplication gives the remainder of the state vectors.

The field transfer matrices are computed from elastic properties of the system and may therefore vary from field to field according to their distribution. Within a field, however, these properties are considered to be constant and discretization may therefore be necessary.

3.2 Transfer Matrix Theory: Application to Multibeam Decks

3.21 Formulation of a Transfer Matrix Solution for a 10-beam Deck

If the multibeam deck is assumed to act as an articulated plate in which the beams are interconnected only through assumed hinges, then the above transfer matrix theory may be successfully applied to the analysis of this an elastic (deck) system.

Consider a transverse section of a 10-beam multibeam deck (fig. 3.1). For convenience a 10-beam deck is discussed here but the theory is completely general and may be applied to a deck with any number of beams and of any width.

The general matrix equations for the solution of a deck with \( n \) beams is presented in sec. 3.25.

\[
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{bmatrix}
\]

Fig. 3.1

Let \( Z_0 \) be the state vector for the extreme left hand edge of the deck

\( Z_{10} \) be the state vector for the extreme right hand edge of the deck

\( F_1, F_2, F_3, \ldots F_{10} \) be the field transfer matrices for the system

\( P_1, P_2, P_3, \ldots P_{10} \) be the point transfer matrices for the system

\( W \) be the load applied to the deck (acting on beam 7).

The field transfer matrices of sec. 3.12 are in this application, beam transfer matrices. Again, the point matrices of the preceding section relate to the shear keys of the multibeam deck and except when the shear key
is itself loaded, this matrix will be unity. We may therefore write:

\[
\begin{align*}
Z_1^L &= F_1 Z_0^R \\
Z_1^R &= P_1 Z_1^L = P_1 F_1 Z_0 \\
Z_2^L &= F_2 Z_1^R = F_2 P_1 F_1 Z_0 \\
Z_2^R &= P_2 Z_2^L = P_2 F_2 P_1 F_1 Z_0 \\
&\quad \vdots \\
Z_6^R &= P_6 Z_6^L = P_6 F_6 P_5 P_4 F_4 P_3 F_2 P_2 P_1 F_1 Z_0 \\
Z_7^L &= F_7 Z_6^R + C_7 = F_7 F_6 P_5 P_4 F_4 P_3 F_2 P_2 P_1 F_1 Z_0 + C_7 \\
Z_7^R &= P_7 Z_7^L = P_7 F_7 F_6 P_5 P_4 F_4 P_3 F_2 P_2 P_1 F_1 Z_0 + P_7 C_7 \\
&\quad \vdots \\
\end{align*}
\]

and finally \( Z_{10}^L = F_{10} F_9 F_8 F_7 \cdots F_2 P_1 F_1 Z_0 + E_{10} F_{10} F_9 F_8 F_7 C_7 \)

\[
\text{-------------------------}(3.1)
\]

But since in general \( P_1 = P_2 = \cdots = P_{10} = I \), the unit matrix, and since all beams in a deck are usually the same size and stiffness, that is

\[
F_1 = F_2 = \cdots = F_{10} = F,
\]

equation 3.1 reduces to:

\[
Z_{10} = F^3 F_W F^6 Z_0 + F^3 C_F \text{-------------------------}(3.2)
\]

where the \( L \) and \( R \) superscripts are deleted, since the \( P_1 \)'s are unity and no distinction exists between \( Z_1^L \) and \( Z_1^R \):

and \( F_W \) is the field transfer matrix for the loaded beam \( (F_W = F_7, C_F = C_7) \)

Note: If edge stiffening beams are required in the deck, \( F_1 \) and \( F_{10} \) will not equal \( F \), but some other value \( F_E \), say.

In this instance, eqn 3.1 would be of the form:

\[
Z_{10} = F_E^3 F_W^2 F^5 F_E Z_0 + F_E^2 C_F \text{-------------------------}(3.3)
\]

Hence if \( Z_0 \), \( F \) and the field matrices are known, any intermediate state vector
Z_1 may be found by successive multiplication of the appropriate number of field matrices to Z_0, and the load matrix C_P. The progressive calculation terminates when Z_{10} is found. The derivation of Z_0, the starting point for the calculation, is explained in sec. 3.232.

3.22 Assumptions

It would be of value at this stage to list the assumptions made thus far and also those yet to be made in the derivation of the necessary transfer matrices and state vectors.

1. The beams are fixed-ended with respect to torsion.
2. The beams are simply supported with respect to longitudinal bending. In the field matrix derivations which follow (sec. 3.24) simple support conditions are assumed, but any known degree of fixity is permissible including beams continuous over intermediate supports.
3. The articulated plate model is assumed applicable. Therefore, only shear forces are transmitted across the assumed hinge between adjacent beams. No relative movement except rotation is possible at the joints; the transverse flexural strength is assumed zero. Due consideration of the shape and nature of the shear keys leads to the conclusion that only vertical shears need be considered.
4. A single such connection is assumed at midspan between adjacent beams, and a single concentrated force acts through this connection.

This is the major simplification made in the method but it can be shown (refer Appendix 6) that it is a reasonable one to make. In any event it will set a lower bound to the distribution of load out of the most heavily loaded beam and is therefore not only reasonable but also safe to make such an assumption.

5. Each individual beam is assumed to have infinite transverse flexural and torsional stiffness. This condition will be true if the width/ span ratio of a single beam is small. Some precast multibeam decks are composed of "planks" which are much wider than the usual beam and in such a case the transverse bending of the plank may have to be considered.

6. The applied loads are assumed to be concentrated loads and can take up any longitudinal position on the deck, but are
restricted in the transverse direction to either the centreline of the beam or a hinge line between adjacent beams.

7. The beams are assumed to behave elastically and superposition is assumed to hold with respect to torsional and flexural deflections. The normal theory for the bending of thin beams is also assumed.

8. Deflections and rotations under torsion are assumed small so that secondary effects such as bending in plan, axial changes in length, shear deformations, and cross section warping may be neglected.

These eight assumptions will be seen to be similar to those adopted by Norman and Nathan and to a certain extent by Khachaturian.

3.23 Derivation of State Vectors \( Z_1 \) and \( Z_0 \)

3.23.1 Determination of \( Z_1 \)

Since only vertical shears are considered at each shear key let their value be \( N_1 \). Again, let the vertical deflection of the key, which occurs under the action of the applied load \( W \), be given by \( x_1 \). Then \( Z_1 \), the state vector for the \( i \)th key will be the \( 2 \times 1 \) column matrix:

\[
Z_1 = \begin{bmatrix} x_1 \\ N_1 \end{bmatrix} = \begin{bmatrix} x \\ N \end{bmatrix}_i \]  

(3.4)

Fig. 3.2

3.23.2 Determination of \( Z_0 \)

\( Z_0 \) is determined by the boundary conditions at those edges parallel to the beams of the deck under consideration. Hence, it is possible to find \( Z_0 \) for both bridge decks and floor systems.

The equations 3.1, 3.2 or 3.3 can be reduced to:

\[
Z_{10} = A \cdot Z_0 + B \]  

(3.5)
where \( Z_{10}, Z_0 \) are 2 x 1 column state vectors, 
\( A \) is a 2 x 2 square matrix representing the matrix product
\[
F_E F_W^2 F_E^5 \text{ in equation 3.3,}
\]
and \( B \) is a 2 x 1 column matrix representing the matrix product
\[
F_E^2 C_F \text{ again in equation 3.3.}
\]
Expanding equation 3.5 we have:
\[
\begin{bmatrix}
  x
  \\
  N_{10}
\end{bmatrix}
= \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
  x
  \\
  N_0
\end{bmatrix}
+ \begin{bmatrix}
  B_{11}
  \\
  B_{21}
\end{bmatrix}
\]
(3.6)

3.232.1 \( Z_0 \) for Bridge Decks

Since \( N_0 \) and \( N_{10} \) represent the shear forces at the extreme left and right hand free edges of the bridge deck:

\[
\begin{bmatrix}
  N_0 \\
  N_{10}
\end{bmatrix} = \begin{bmatrix}
  0 \\
  0
\end{bmatrix};
\]
The unknown deflection in \( Z_0 \) (i.e. \( x_0 \)) is found from equation 3.6 which becomes:
\[
\begin{bmatrix}
  x
  \\
  0
\end{bmatrix}_0
= \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
  x
  \\
  0
\end{bmatrix}_0
+ \begin{bmatrix}
  B_{11}
  \\
  B_{21}
\end{bmatrix}
\]
(3.7)

which gives \( 0 = A_{21} x_0 + B_{21} \)

therefore \( x_0 = \frac{B_{21}}{A_{21}} \)

and hence \( Z_0 \) (for a bridge deck) = \[
\begin{bmatrix}
  -\frac{B_{21}}{A_{21}}
  \\
  0
\end{bmatrix}
\]
(3.8)

3.232.2 \( Z_0 \) for Floor Systems

In this case \( x_0 \) and \( x_{10} \), the deflections at the extreme left and right hand edges, must be zero because a floor system is supported at all four edges. The unknown shear forces \( N_0 \) and \( N_{10} \), are in fact the reactions at these supports. Equation 3.6 now becomes:
\[
\begin{bmatrix}
  0 \\
  N_{10}
\end{bmatrix}
= \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
  0
  \\
  N_0
\end{bmatrix}
+ \begin{bmatrix}
  B_{11}
  \\
  B_{21}
\end{bmatrix}
\]
(3.9)

which gives \( 0 = A_{12} N_0 + B_{11} \)

therefore \( N_0 = -\frac{B_{11}}{A_{12}} \)
and hence \( Z_0 \) (for a floor system) = \[
\begin{bmatrix}
0 \\
-B_{11} \\
A_{12}
\end{bmatrix}
\] \hspace{1cm} (3.10)

3.24 Derivation of Field Transfer Matrices

In this section expressions are derived for the field transfer matrices \( F, F_E \) and \( F_W \) (incl. \( C_F \)).

3.24.1 Definition of Beam Flexibilities \( X \), and \( Y \)

It is convenient at this point to define the bending and torsional flexibilities of a single beam.

3.24.1.1 Consider the \( i \)th beam loaded by a single point load, \( W \) (fig 3.3a)

Fig 3.3a

If the span is \( L \) and the flexural rigidity \( EI \)

then: \( \text{Maximum deflection} = \frac{1}{48} \frac{WL^3}{EI} \)

Therefore let \( X = \frac{1}{48} \frac{WL^3}{EI} \) \hspace{1cm} (3.11a)

3.24.1.2 Again consider the beam, but in torsion under the action of concentrated loads at midspan as in fig 3.3b.

Fig 3.3b

In this case, if \( \dot{\theta} \) be the rotation at midspan due to the action of \( N_i \) and \( N_{i+1} \) then:

\( \dot{\theta} = \frac{1}{4} \frac{La^2}{JG} (N_i + N_{i+1}) \)

Therefore let \( Y = \frac{1}{4} \frac{La^2}{JG} \) \hspace{1cm} (3.11b)

where \( 2a \) is the width of the beam and \( JG \) is the torsional rigidity of the section:

\[ \text{i.e. } JG = \beta \cdot 2a \cdot d^3 \cdot G \]

where \( \beta \) is a constant dependent on the geometry of the beam cross-section (refer Table 3.1 below)

\( d \) is the beam depth

and \( G \) is the modulus of torsional rigidity.

<table>
<thead>
<tr>
<th>TABLE 3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a/d</td>
</tr>
<tr>
<td>( \beta )</td>
</tr>
</tbody>
</table>

+ Note: the above expression for \( Y \) (eqn 3.11b) is a first approximation only.

An exact expression is derived in Appendix 7 where limits on the
use of equation 3.11b are given. In particular if the ratio

\[ \frac{L}{a \sqrt{\frac{12E}{3JG}}} \]

is less than or equal to 6, special care in the evaluation of

Y is needed: refer equations A7.4 and A7.5 of Appendix 7.

3.242 Derivation of \( F \) and \( F_E \)

Consider the \( i \)th beam of fig 3.2 isolated from the rest of the deck and under the action of the shear forces \( N_i \) and \( N_{i+1} \): if the load is to the right of this beam then it will behave as represented in fig 3.4a and if it is to the left, as in fig 3.4b.

Applied Load, \( W \)

\[ x_i = \Delta_N - a\theta \]
\[ x_{i+1} = \Delta_N + a\theta \]

Fig 3.4a

\[ x_i = \Delta_N + a\theta \]
\[ x_{i+1} = \Delta_N - a\theta \]

Fig 3.4b

The deflection of the beam may therefore be considered as the sum of two component deflections: one due to pure bending and the other to pure torsion. These are denoted as \( \Delta_N \) and \( a\theta \) respectively.
It is seen that:

\[
\Delta_N = (N_{i+1} - N_i) \times X
\]

and

\[
\varepsilon = (N_{i+1} + N_i) \times Y
\]

therefore

\[
x_i = \Delta - \varepsilon - N_i \cdot (X+Y) + N_{i+1} \cdot (X-Y)
\]

and

\[
x_{i+1} = \Delta + \varepsilon - N_i \cdot (X-Y) + N_{i+1} \cdot (X+Y)
\]

with little effort these equations are rearranged into the following matrix form:

\[
\left[ \begin{array}{c}
x \\
N_{i+1}
\end{array} \right] = \left[ \begin{array}{cc}
X+Y & LXY \\
X-Y & X-Y
\end{array} \right] \cdot \left[ \begin{array}{c}
x \\
N_i
\end{array} \right] - \left[ \begin{array}{c}
X+Y \\
X-Y
\end{array} \right] \cdot \left[ \begin{array}{c}
x \\
N_i
\end{array} \right]
\]

or

\[
Z_{i+1} = F_L Z_i
\]

Two field (beam) transfer matrices are therefore necessary for the deck according to the position of the field (beam) with respect to the load.

These are denoted by \( F_L \) and \( F_R \) for the beams to the left and right of the load respectively; they are defined by equations 3.12.

Hence equations 3.2 and 3.3 should read:

\[
Z_{10} = F_R^2 F_L^6 Z_0 + F_R^2 C_F
\]

and

\[
Z_{10} = F_R F_L^2 F_E^6 Z_0 + F_R F_C
\]

The field matrices are dependent only on \( X \) and \( Y \) which in turn depend only on the geometry and material of the deck (sec 3.241). Therefore, if a deck is to be edge stiffened this is reflected by the field matrices for the edge beams:

\[
F_{ER} \text{ or } F_{EL} = F \begin{vmatrix} X & X_E \\ Y & Y_E \end{vmatrix}
\]

3.243 Derivation of \( F_W \) and \( F_W \) (incl. \( C_F \) and \( C_P \))

3.243.1 \( F_W \) for a Loaded Beam

Consider the load applied to the centre line of the \( i \)th beam in a multi-beam deck as in fig 3.5.
Again these equations may be rewritten in matrix form:

\[
\begin{bmatrix}
  \mathbf{x}_i \\
  \mathbf{N}_{i+1}
\end{bmatrix} = \begin{bmatrix}
  \frac{X+Y}{X-Y} & \frac{LY}{X-Y} \\
  -\frac{1}{X-Y} & -\frac{X+Y}{X-Y}
\end{bmatrix} \begin{bmatrix}
  \mathbf{x}_i \\
  \mathbf{N}_i
\end{bmatrix} + \begin{bmatrix}
  -\frac{2XY}{X-Y} \\
  \frac{WX}{X-Y}
\end{bmatrix}
\]  \hspace{1cm} (3.15a)

and therefore, since

\[
\mathbf{Z}_{i+1} = \mathbf{F}_W \mathbf{Z}_i + \mathbf{C}_F
\]  \hspace{1cm} (3.15b)

then

\[
\mathbf{F}_W = \begin{bmatrix}
  \frac{X+Y}{X-Y} & \frac{LY}{X-Y} \\
  -\frac{1}{X-Y} & -\frac{X+Y}{X-Y}
\end{bmatrix}
\]  \hspace{1cm} (3.15c)

and

\[
\mathbf{C}_F = \begin{bmatrix}
  -\frac{2XY}{X-Y} \\
  \frac{WX}{X-Y}
\end{bmatrix}
\]  \hspace{1cm} (3.15d)
3.243.2 \( P_W \) for the Loaded Joint

Consider now the load applied to the \( i \)th joint in a multibeam deck as in fig 3.6

From the conditions for equilibrium at this joint:

\[
\begin{align*}
R_i &= x_i^R = x_i^L \\
N_i &= r_i^R = -N_i^L + W,
\end{align*}
\]

which in matrix form gives

\[
\begin{bmatrix}
x^R_i \\
N_i^R
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix}
x^L_i \\
N_i^L
\end{bmatrix} + \begin{bmatrix} 0 \\ W \end{bmatrix}
\]

but \( Z_i^R = P_W \cdot Z_i^L + C_P \)

hence \( P_W = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)

and \( C_P = \begin{bmatrix} 0 \\ W \end{bmatrix} \)

Fig 3.6

Note: For every other unloaded joint \( P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \), the unit matrix

and \( C_P = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)

3.25 Summary - General Matrix Equations for an \( n \)-beam Multibeam Deck

3.251 Case A - Load at the Centreline of the \( r \)th Beam

\[
A = F_{ER} F_{R}^{m-1} - F_{FW} F_{EL}^{2} \] (3.17a)
\[ B = F_{ER}^{n-1-r}r_C \]
\[ i = 0 \quad Z_0 = \begin{bmatrix} 0 \\
-\frac{B_{11}}{A_{12}} \end{bmatrix} \text{(floors)}; \quad \begin{bmatrix} \frac{B_{21}}{A_{21}} \\
0 \end{bmatrix} \text{(bridge decks)} \quad (3.18a) \]
\[ r \geq i \geq 1 \quad Z_i = F_L^{i-1}F_{EL}Z_0 \]
\[ n \geq i \geq r \quad Z_i = F_R^{i-r}F_L^{r-2}F_{EL}Z_0 + F_R^{i-r}r_C \quad \text{-- (3.19a)} \]
\[ i = n \quad Z_n = F_R^{n-1}F_W^{r-2}F_{EL}Z_0 + F_R^{n-1}r_C \]

3.252 Case B - Load at the Centreline of the \( r \) th Joint

\[ \text{Fig. 3.7b} \]

\[ A = F_{ER}^{n-1-r}r_P r_C \]
\[ B = F_{ER}^{n-1-r}r_C \]
\[ i = 0 \quad Z_0 = \begin{bmatrix} 0 \\
-\frac{B_{11}}{A_{12}} \end{bmatrix} \text{(floors)}; \quad \begin{bmatrix} \frac{B_{21}}{A_{21}} \\
0 \end{bmatrix} \text{(bridge decks)} \quad (3.18b) \]

\[ r \geq i \geq 1 \quad Z_i = F_L^{i-1}F_{EL}Z_0 \]
\[ \begin{align*}
n > i > r & \quad Z_i = F_i^{-r} P_i F_i^{-1} P_i^{-1} F_i Z_i + F_i^{-r} C_p \\
i = n & \quad Z_n = F_n^{-1} P_n F_n^{-1} P_n^{-1} F_n Z_n + F_n^{-1} C_p
\end{align*}\]

3.26 Catalogue of Matrices

\[
F_L = \begin{bmatrix}
\frac{X+Y}{X-Y} & \frac{X Y}{X-Y} \\
1 & \frac{X+Y}{X-Y}
\end{bmatrix}, \quad F_R = \begin{bmatrix}
\frac{X+Y}{X-Y} & -\frac{X Y}{X-Y} \\
-1 & \frac{X+Y}{X-Y}
\end{bmatrix}
\]

\[
F_W = \begin{bmatrix}
\frac{X+Y}{X-Y} & \frac{X Y}{X-Y} \\
1 & -\frac{X+Y}{X-Y}
\end{bmatrix}, \quad C_F = \begin{bmatrix}
-2 \frac{W X Y}{X-Y} \\
\frac{W V}{X-Y}
\end{bmatrix}
\]

\[
F_{EL} = F_L \quad X = X_E, \quad Y = Y_E
\]

\[
P_W = \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}, \quad C_P = \begin{bmatrix}
0 \\
W
\end{bmatrix}
\]

\[
X = \frac{1}{48} \frac{L^3}{EI}; \quad X_E = X, \text{ calculated for the edge stiffened beam (higher I)}.
\]

\[
Y = \frac{1}{4} \frac{L a^2}{J G}; \quad Y_E = Y, \text{ calculated for the edge stiffened beam (higher J)}.
\]

3.3 Analysis Procedure: Suitable for Hand Calculation

3.31 Basic Steps

The procedure has four basic steps:

1. From the dimensions and material properties of the deck find the flexibilities \( X \) and \( Y \) from equations 3.11; if edge stiffening is to be provided find \( X_E \) and \( Y_E \), also from equation 3.11.

2. Using these values for \( X \) and \( Y \) assemble the transfer matrices \( F_{ER}, F_{EL}, F_L, F_R, F_W \) and \( P_W \) (equations 3.20). Note the repetitive nature of the matrix elements which considerably simplifies the amount of work involved at this stage. Also find \( C_F \) and \( C_P \) using the applied load \( W \) and equations 3.20.

3. By successive post-multiplication of these field matrices find \( A \) and \( B \) (equations 3.17a and 3.17b). Hence obtain \( Z_0 \) from either equation 3.18a or 3.18b, depending on the position of
the applied load. The type of multibeam deck (bridge deck or floor system) also determines at this stage the value of $Z_0$.

4. Again, by using post-multiplication find the successive state vectors $Z_1$ to $Z_n$. A check on accuracy is provided by the condition, $Z_n = A.Z_0 + B$. These state vectors ($Z_i$) give the transverse deflection profile of the deck ($x_i$) together with the shear force transmitted through each shear key ($N_i$). From this information the load sustained by each individual beam is readily calculated and the load distribution amongst the beams of the deck is therefore found.

3.32 Notes and Aids to Calculation

3.321 Since matrix multiplication is involved, care must be taken to preserve the order of the multiplication. All matrices are post-multiplied but since the field matrices are only $2 \times 2$ squares, the amount of arithmetic involved is not large.

3.322 As already noted a check on accuracy is provided in the method; use should be made of the expression $Z_n = A.Z_0 + B$.

3.323 A recommended tabulation for finding the values of $X$ and $Y$ and the field matrices is presented in Table 3.2. A suggested scheme for the matrix multiplication of the field matrices and the state vectors is also given and presented in Table 3.3 (Specially drawn up for a ten-beam deck but the extension to a deck of any other width is obvious).

3.324 An example of the analysis of a ten beam deck by hand calculation is given in Appendix 8.

3.325 It is sometimes preferable to number the beams of a deck from right to left instead of left to right as in the above presentation. If this is done, an alteration in sign of the $(1,2)$ and $(2,1)$ elements of both the $F_L$ and $F_R$ field matrices is necessary.

The advantage of such a practice is a reduction in numerical work, since for example if $F_R$ is known, $F_L$ can be written down immediately, after a like change in sign.

Re-numbering is however only recommended if $N_R$ is greater than $N_L$.

See Appendix 8 for an example of its use.

3.4 Analysis by Digital Computer (IBM 1620)

Two computer programmes based on the above transfer matrix theory have been developed, which together analyse multibeam bridge decks and floor systems, for any known loads, elastic and geometric properties. Deflections, joint
CALCULATIONS FOR THE TRANSFER MATRIX ANALYSIS OF:

<table>
<thead>
<tr>
<th>Beam Position</th>
<th>Inside</th>
<th>Edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Width 2a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam Depth d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam Span L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastc Mod E</td>
<td></td>
<td>$\frac{2a}{d}$ (Table 3.1)</td>
</tr>
<tr>
<td>Shear Mod G</td>
<td></td>
<td>$\frac{1}{2a.d^3}$</td>
</tr>
<tr>
<td>Flex. Rig. EI</td>
<td></td>
<td>$I = 2a.d^3/12$</td>
</tr>
<tr>
<td>Tors. Rig. JG</td>
<td></td>
<td>$J = 3.2a.d^3$</td>
</tr>
<tr>
<td>$X, X_E = L^3/48EI$</td>
<td></td>
<td>$L^3$</td>
</tr>
<tr>
<td>$Y, Y_E = Ld^2/4JG$</td>
<td></td>
<td>$La^2$</td>
</tr>
<tr>
<td>$X+Y$</td>
<td></td>
<td>$F_L$</td>
</tr>
<tr>
<td>$X-Y$</td>
<td></td>
<td>$F_R$</td>
</tr>
<tr>
<td>$X'=Y$</td>
<td></td>
<td>$F_{EL}$</td>
</tr>
<tr>
<td>$X/Y$</td>
<td></td>
<td>$F_{ER}$</td>
</tr>
<tr>
<td>$2X'Y$</td>
<td></td>
<td>$F_{W}$</td>
</tr>
<tr>
<td>$X/Y$</td>
<td></td>
<td>$F_{W}$</td>
</tr>
<tr>
<td>$X$</td>
<td></td>
<td>$F_{W}$</td>
</tr>
<tr>
<td>$X'/Y$</td>
<td></td>
<td>$F_{W}$</td>
</tr>
</tbody>
</table>

REMARKS

$C_F$ & $C_P$

| TABLE 3.2 |
FIG 3.3 Matrix Scheme for the Analysis of the Multibeam Bridge Deck.

<table>
<thead>
<tr>
<th>LOAD</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>F_R</td>
<td></td>
<td></td>
<td>F_R</td>
<td></td>
</tr>
<tr>
<td>F_R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>F_R</td>
<td>F_R</td>
<td></td>
<td></td>
<td></td>
<td>F_R</td>
</tr>
<tr>
<td>F_R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>F_R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_W</td>
<td></td>
<td></td>
<td></td>
<td>F_W</td>
<td></td>
<td>F_W</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_L</td>
<td></td>
<td></td>
<td>F_L</td>
<td></td>
<td>F_L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>F_L</td>
</tr>
<tr>
<td>F_L</td>
<td></td>
<td>F_L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_E_L</td>
<td></td>
<td>E_E</td>
<td>F_E_L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10)</td>
<td></td>
<td></td>
<td>(6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. \[
\begin{align*}
Z_0 &= \begin{pmatrix}
\frac{B(2,1)}{A(2,1)} \\
0
\end{pmatrix} \\
N_0 &= 0
\end{align*}
\]

for bridge decks
shears, the load carried per beam and the load distribution expressed as a percentage of the total load, may be calculated and plotted (if required) using the 1620 Console Typewriter.

Due to the limited storage capacity of the 1620 (40K) two programmes were necessary for this task. These have been named OPUS TWO and OPUS THREE and are written in PDQ FORTRAN. The former calculates the deflections and shears from the data describing the load and deck concerned. The latter uses these results as data and computes the corresponding load distribution; the graph plotting subroutine is included in this programme.

Appendix 3 gives the details of these programmes including storage requirements, data specifications, flow charts, programme listing and a specimen set of results. See also Appendix 1 for a general description of the OPUS Programme Series.

3.5 The Transfer Matrix Procedure Compared with Other Methods

3.51 The Methods of Roesli, Spindel, Cusens and Pama

In order to compare the proposed matrix method with those of Roesli (sec 2.13), Spindel (sec 2.24) and Cusens and Pama (sec 2.25), an analysis was made of the Centerport Bridge details of which had been published by Roesli. The results obtained from the full scale testing of this Bridge have been used by the above authors from time to time to compare their own various methods.

Figs 3.8 and 3.9 repeat these results and thus compare the profiles predicted by these several methods. Shown in red is the transfer matrix prediction which is seen to compare most favourably with the experimental observations.

Some discrepancy does exist but more particularly for those profiles obtained under an outside edge loading (fig. 3.9). Other authors show similar divergence and Cusens suggests that "... the position of the load was not precisely as defined in the experimental report. The experimental distribution coefficients exhibit a characteristic to be expected of a load with slightly larger eccentricity." If allowance were made for the larger eccentricity then the transfer matrix theory would show even better agreement particularly in the vicinity of the load.

3.52 The Methods of Norman and Nathan and Rowe, Morice and Little

Comparison with the methods of Norman and Nathan (sec 2.23) and Rowe, Morice and Little (sec 2.12) are made in Chapter 6, sec 6.23, and illustrated in fgs 6.08a, 6.08b and 6.09.
FIGURE 3.8

CENTERPORT BRIDGE

Legend:
- Observed in Field Test
- Roesli et al
- Spindel
- Cusens and Pama
- Transfer Matrix Method

Description:
- Span... 32ft.
- Width... 27ft.
- Number of Units... 9
- Loaded on Beam 5 with 47,700
FIGURE 3.9  CENTERPORT BRIDGE

COMPARATIVE TRANSVERSE DISTRIBUTION COEFFICIENT PROFILES.

Description: Span...32ft; Width...27ft
Number of Units...... 9
Loaded on Beams 1 & 2 with a single axle load of 47,700 Lb.

Legend:
- Observed in field test
- Roesli et al.
- Spindel
- Cusens and Pama
- Transfer Matrix Theory.
Experimental Verification of Transfer Matrix Theory

Both field and laboratory studies have been conducted to further verify the proposed theory. Full scale tests have been performed on five spans of a multibeam bridge and similar tests conducted on a quarter scale model of this bridge.

Chapters 4 and 5 describe these tests and Chapter 6 gives the verification which is illustrated in figs 6.01, 6.02, 6.03 and 6.04 of Chapter 6.
# Chapter Four

## Title: Field Tests on the Multibeam Bridges at Slippery Creek

## Contents:

4.0 Introduction

4.1 Organisation, Purpose and Quarter Scale Model
    4.11 Organisation
    4.12 Purpose
    4.13 Model

4.2 Description of Slippery Creek Bridges
    4.21 General
    4.22 Description of Log-Beam Bridge
    4.23 Description of Hollow-Cored Bridge

4.3 Materials and Design of Slippery Creek Bridges
    4.31 Concrete
    4.32 Steel
    4.33 Design Stresses

4.4 Applied Loading and Special Test Vehicles
    4.41 General
    4.42 Lane Loader
    4.43 Point Loader

4.5 Instrumentation
    4.51 Deflection
    4.52 Strain

4.6 Test Programme and Procedure
    4.61 Lane and Point Loading Procedure
    4.62 Tests for the Effect of Transverse Reinforcement
    4.63 Tests for the Effect of Span/Width Ratio
    4.64 Tests for the Effect of Skew

4.7 Further Tests
    4.71 Additional Tests for Differential Slip
    4.72 Future Tests under Normal Traffic Conditions
4.0 Introduction

In 1964 the Ministry of Works of the New Zealand Government built by contract two multibeam bridges to span Slippery Creek, 18 miles south of Auckland City. The twin bridges lie on the Southern Motorway extension between Takanini and Drury. Each is three traffic lanes wide, one bridge being for northbound traffic into the City and the other for southbound traffic leaving the City. A set of the Contract Plans for these bridges has been included in Appendix 9.

At the time of construction the bridges were both three spans in length but differed in the type of multibeam deck used for their separate superstructures. In March, 1966 heavy flooding caused severe scouring under the southern abutments of the bridges and both decks have now been lengthened from three to five spans. Plate 1 shows the bridge after the extensions had been completed and the bridge opened for traffic.

4.1 Organisation, Purpose and Quarter Scale Model

4.11 Organisation

Knowing that the information available concerning the behaviour of multibeam decks was inadequate, the Ministry of Works announced early in 1964, their intention to conduct load tests on the Slippery Creek bridges. Subsequent negotiation between the Ministry of Works and the School of Engineering of the University of Auckland, resulted in the author conducting these tests on behalf of the Ministry of Works. The tests were, however, financed by the Ministry of Works, who also made available senior staff, cadet personnel, plant, machinery, equipment and other facilities, necessary for a field project of this size.

The tests took place in April and May, 1965, and lasted for a period of eight weeks. At this time the bridges were each only three spans long and five of the possible six spans were investigated for vertical deflection and longitudinal strain as the bridges were loaded with the special test vehicles.

4.12 Purpose

In brief, the tests were conducted to determine the importance of the several design parameters, the accuracy of recently proposed design methods and the transverse distribution of load amongst the several beams of these, two of the most common types of multibeam bridge decks.
More specifically, there were six objectives in mind during the Slippery Creek Tests. These were:

1. To check the validity of design methods of both those in current use and those recently proposed (especially the transfer matrix method of Chapter Three).

2. To determine the effect on the behaviour of the decks of transverse reinforcement, both ordinary mild steel and prestressed reinforcement.

3. To determine the relative behaviour of hollow-cored units and log-beam units.

4. To determine the effect of variation in the span/width ratio on the behaviour of the hollow-cored units.

5. To determine the effect of fifteen degrees of skew on the behaviour of both decks.

6. To obtain influence lines for deflection and
   (a) compare with results observed under lane loading
   (b) compare with results of model testing.

4.13 Model

Concurrent with the preparation and execution of the field testing, a quarter scale model of the Slippery Creek Bridges was built in the Structures Laboratory. Testing of the model actually commenced before that in the field, and preliminary results from the model influenced to some extent, the tests made on the prototype (refer sec. 4.31). The construction and testing of the model is discussed in Chapter 5. The results, and their analysis, of both the field and model testing is presented jointly in Chapter 6.

4.2 Description of the Slippery Creek Bridges

4.21 General

Both bridges have been built in concrete using cast-in-situ piles, capped with ordinary reinforced concrete beams to act as piers. Each deck consists of precast, prestressed beams which lie side by side across the width of the deck and are reinforced transversely to provide the transverse shear connection. Again, both bridges exhibit the same $15^\circ$ of skew and each consisted (at the time of testing) of three, 37 foot spans (skewspan). The normal width of each deck is 36 feet with no provision for footpaths; the guard rails are bolted to the sides of the deck. Refer to the Contract Plans in Appendix 9.

Although apparently identical in appearance the decks of the twinbridges differ in the type of prestressed unit or beam used to compile the deck.
4.22 Description of Log-unit Bridge - Figs 4.1 and 4.2a

The eastern bridge (that carrying Southbound traffic) was constructed from the Ministry of Works designed "log" beam. Each pretensioned beam was 35'-11" long, 11'-10" wide, 14" deep and carried a 1" longitudinal nib along each lower outer edge. Thus when the beams were assembled together to form the decks a 2" gap occurred between the units. This gap was filled with concrete after the fourteen 1" diam. m.s. tie rods had been threaded through the deck. A further 4" of deck, reinforced at 8" centres with 3/" diam. rod, was then poured over the beams. Shear reinforcement between the beams and this extra deck slab was provided in the form of spiral coils (4" diam. m.s. rod) set into the beams when they were precast.

Eighteen of these log beams were used to obtain the full width for each of the three spans, but for the purposes of this load testing programme the degree and type of transverse reinforcement was varied as the tests proceeded - such variations are described in sec. 4.52.

4.23 Description of Hollow-Cored Bridge - Figs 4.1 and 4.2b.

The western bridge was constructed from the Certified Concrete designed "hollow-core" beam. Each beam was 36'-11" long, of two widths, 3'-6.5" and 3'-8.5", 16" deep and hollow. Three hexagonal cores run the length of the units except where diaphragms have been inserted at each end and at midspan. Provision for a shear key was made by casting a 4" x 1" recess into the side of the units. After the deck was assembled and the shear keys poured the single transverse cable at midspan was stressed. No deck slab was provided and the decks were ready for immediate use.

For the purpose of these load tests, each span of this bridge was a different width; the first was the full width comprising ten of the hollow-cored units, the second, half the width, comprising five units and the third was quarter the width comprising only three units. In the second and third spans the remainder of the beams to make up the full complement of ten, were in place during the tests but free from that portion of the deck under test.

The log and hollow-cored decks are summarised in Table 4.3 which follows in sec. 4.4.

4.3 Materials and Design of Slippery Creek Bridges

4.3.1 Concrete

The mix used for the manufacture of the precast beams for the Slippery Creek Bridges was as indicated in Table 4.1.
FIG 4.1 Slippery Creek Bridge

(a) Elevation along Skew

(b) Plan of twin bridges
FIG 4.2

(a) Typical Cross-sections of the Log Beam Deck

15 transverse 1”Ø m.s. bolts at 30” c.c.s.

(b) Typical Cross-sections of the Hollow-cored Beam Deck

shear key 7”x 2”

single cable at midspan

34/3⁄8 Ø strands

9”x 7” void

44 1/2”

16”

36’-6”

10 beams
TABLE 4.1
CONCRETE MIX DESIGN FOR SLIPPERY CREEK BRIDGES

<table>
<thead>
<tr>
<th>Description</th>
<th>Sieve Analysis</th>
<th>Fineness Modulus (F.M.)</th>
<th>Weight per batch (Lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2&quot; aggregate</td>
<td>passing 1/2&quot; and retained 1/2&quot;;</td>
<td>6.6</td>
<td>2550</td>
</tr>
<tr>
<td>1/2&quot; aggregate</td>
<td>70% retained on 3/4&quot;</td>
<td>3.2</td>
<td>960</td>
</tr>
<tr>
<td>beach sand</td>
<td>passing No. 50</td>
<td>1.36</td>
<td>370</td>
</tr>
<tr>
<td>cement</td>
<td>Wilsonite Rapid Hardening</td>
<td></td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>Total Weight per Batch:</td>
<td></td>
<td>4580</td>
</tr>
</tbody>
</table>

Note: 1. The amount of water added was governed by apparent workability.
2. The aggregate/cement ratio was 5.55.
3. The units were steam cured for 8 hours; release of prestress at 12 hours (concrete strength not less than 5000 lbs/sq.in.).
4. The cement, aggregate and water, block tests, mix proportions, admixtures, measurement of materials and the handling and placing of the concrete was required to conform to the Ministry of Works Specification for Precast Pretensioned Bridge Units.
5. Laboratory tests at the School of Engineering for the crushing strength and the elastic modulus for this concrete gave the following results:
   - Crushing Strength: 11,300 lbs/sq.in.
   - Elastic Modulus: 5.70 x 10^6 lbs/sq.in.

Refer also sec 5.612.

4.32 Steel

Wire strand of 3/8" nominal diameter was used to pretension both the log and hollow-cored units. The various properties of this high tensile wire are summarised in Table 4.2 below:

TABLE 4.2
HIGH TENSILE STEEL PROPERTIES FOR SLIPPERY CREEK BEAMS

<table>
<thead>
<tr>
<th>Ultimate Strength of Wire</th>
<th>100 - 110</th>
<th>tons/sq.in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Diameter of Wire</td>
<td>0.128</td>
<td>ins.</td>
</tr>
<tr>
<td>Nominal Diameter of Strand</td>
<td>0.375</td>
<td>ins.</td>
</tr>
<tr>
<td>No. of Wires to the Strand</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>Min. Breaking Load</td>
<td>21,000</td>
<td>lbs/strand.</td>
</tr>
<tr>
<td>Min. Proof Load at 0.1% Elongation</td>
<td>18.250</td>
<td>lbs/strand.</td>
</tr>
<tr>
<td>No. of strands per log beam</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td>No. of strands per hollow-cored beam</td>
<td>3/4</td>
<td>-</td>
</tr>
<tr>
<td>Max. Camber allowed (36 ft. span)</td>
<td>13/16ths</td>
<td></td>
</tr>
</tbody>
</table>

-
Details of nonprestressed reinforcement can be found in the Contract Drawings for the Slippery Creek Bridges which are found in Appendix 9.

### 4.33 Design Stresses, Loads and Dimensions for Log and Hollow-Cored Decks

In the following Table (4.3) a comparative listing of design stresses, design loads, design dimensions, transverse tie arrangements and live load distribution is presented for both the log and hollow cored multibeam decks.

#### TABLE 4.3

**DESIGN STRESSES, LOADS, DIMENSIONS, TRANSVERSE TIE AND LIVE LOAD DISTRIBUTION**

<table>
<thead>
<tr>
<th></th>
<th>Log</th>
<th>H/C</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive Stress at transfer</td>
<td>2500</td>
<td>2700</td>
<td>Lbs/sq.in.</td>
</tr>
<tr>
<td>Tensile Stress at transfer</td>
<td>400</td>
<td>200</td>
<td>Lbs/sq.in.</td>
</tr>
<tr>
<td>Compressive Stress under Design Load</td>
<td>2000</td>
<td>2200</td>
<td>Lbs/sq.in.</td>
</tr>
<tr>
<td>Tensile Stress under Design Load</td>
<td>200</td>
<td>200</td>
<td>Lbs/sq.in.</td>
</tr>
<tr>
<td>Initial Load per 3&quot; Strand</td>
<td>14.7</td>
<td>14.7</td>
<td>Kips</td>
</tr>
<tr>
<td>Final Load (assuming 20% losses) per Strand</td>
<td>11.8</td>
<td>11.8</td>
<td>Kips</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Log</th>
<th>H/C</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Dead Load</td>
<td>150-170</td>
<td>150-170</td>
<td>Lbs/ft³</td>
</tr>
<tr>
<td>Live Load</td>
<td>H20S16</td>
<td>H20S16</td>
<td>Lbs/ft³</td>
</tr>
<tr>
<td></td>
<td>T16</td>
<td>T16</td>
<td></td>
</tr>
<tr>
<td>Additional Surfacing</td>
<td>-</td>
<td>16</td>
<td>Lbs/ft²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Log</th>
<th>H/C</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Width (overall)</td>
<td>11/10&quot;</td>
<td>31/6&quot;</td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>31/6&quot;</td>
<td></td>
</tr>
<tr>
<td>Unit Depth (overall)</td>
<td>18&quot;+</td>
<td>16&quot;</td>
<td></td>
</tr>
<tr>
<td>Tolerances:</td>
<td>Width</td>
<td>1/2&quot;</td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>1/2&quot;</td>
<td>1/2&quot;</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>1/2&quot;</td>
<td>1/2&quot;</td>
<td></td>
</tr>
<tr>
<td>Number of Units required per Span</td>
<td>18</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total number required for Bridge</td>
<td>54</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Cross-sectional Area of Deck</td>
<td>216+</td>
<td>144</td>
<td>in²/ft.width.</td>
</tr>
<tr>
<td>Moment of Inertia of Deck</td>
<td>5832+</td>
<td>3829</td>
<td>in⁴/ft.width.</td>
</tr>
<tr>
<td>Section Modulus of Deck</td>
<td>648+</td>
<td>478</td>
<td>in⁶/ft.width.</td>
</tr>
</tbody>
</table>

**DESIGN TRANSVERSE TIE**

Either: 14, 1" m.s. diam. transverse bolts together with 4" heavily reinforced deck
OR: 14, 0.276" H.T. single tendons stressed to 30,000 lbs final load.
No extra deck slab but shear keys cast.

Single four wire 0.276" diam tendon stressed to 30,000 lbs final load.
4.4 Applied Loading and Special Test Vehicles

4.41 General

It was originally planned to use a single eight ton load in the form of a concrete pipe to be lifted into the various loading positions by mobile crane. As a result of model tests made at the School of Engineering, it was decided to increase the load in order to obtain more significant deflections and strains (sec 5.63). Accordingly, two abnormal loading vehicles were assembled and transported to the site.

4.42 Lane Loader

For lane loading a Falcon, 25 ton Compactor was used (Plate 3A). The boxes were filled with sand and a further two 4 ton concrete blocks were added to give a total all-up weight of 67,600 lbs (30.2 tons). The inner pair of wheels were removed to provide a single axle loading through tyres 6'6" apart. The tow-bar was lengthened to allow a tractor to position the Compactor without resting on the span itself. See also Fig 4.36.

4.43 Point Loader

For point loading a forty foot long trailer loaded with five 4 ton concrete blocks was used (Plate 2). The trailer was supported on a single axle and carried two DC 3 aircraft tyres spaced at 2'-6" centres. The load was thus slightly unstable and outriggers were provided to help stabilise this 64,000 lbs (28.6 ton) load. However, the advantage of this vehicle lay in the fact that the load was, to all intents and purposes, a "point" load. Such a load was invaluable when analysing the results of the tests, in the application of design theories and in the derivation of influence lines, since the superposition effects of other loads need not be considered. At the commencement of testing with this trailer the load was jacked off the
POINT LOADING TEST VEHICLE
28.6 tons all-up weight

LANE LOADING TEST VEHICLE
(25 ton Falcon Compactor)
30.2 tons all-up weight
ground with a 100 ton Tangye hydraulic jack to obtain a true point load. A comparison between deflections observed in this position and later in the "wheels down" position showed no appreciable difference and jacking was discontinued. See also Fig 4.3a.

### 4.5 Instrumentation

The behaviour of the decks under load was observed by measuring changes in deflection and longitudinal strain.

Refer to Appendix 10 for the dial and strain gauge recording sheets used during the tests.

#### 4.51 Deflection

Thirty dial gauges of various travel (\(\frac{1}{2}\)" to 2") and manufacture (Mercer and Batty) were mounted under each span in turn, where two separate systems of scaffolding had been erected. One set supported the dial gauges and the other carried the walkways giving access for the six engineering cadets who read the gauges. The dial gauges were attached through aluminium blocks to Dexion fastened to the scaffolding (Plate 3B).

Ten gauges were mounted under each quarter, half and three-quarter line, one gauge at the centre of every second beam except where two were provided on each side of the centre line of the deck (Fig 4.4).

#### 4.52 Strain

Forty-four electrical resistance foil strain gauges, of 4" gauge length and Saunders-Roe manufacture, were mounted longitudinally under the five spans tested. The gauges were glued to the under surface of the decks which had been suitably prepared by wire brushing and the application of degreasing and neutralising agents (trichloroethylene and zinc chloride respectively); the cement used was Araldite, Casting Resin D.

Details of these gauges have been included in Table 5.7 of Chapter 5.

Strain gauges were provided only at midspan and were primarily installed to check on the possible overloading of the outside beams under the abnormal loading - two gauges were cemented to each outside beam for this reason. On spans A and D the gauges were, however, distributed across the whole width in a pattern similar to that of the dial gauges. This enabled a transverse profile of longitudinal strain for both the log and hollow-cored decks to be obtained (Fig 4.4).

Cables leading from the gauges to a mobile control and recording centre
Fig 4.4a Typical Sections of the Slippery Creek Bridges showing the transverse distribution of both dial & strain gauges.

- dial gauges  * strain gauges

Hollow Deck
spans D, E, F

spans A, C
Log Deck

Fig 4.4b Plan View of the Bridges showing the Overall Distribution of both dial and strain gauges; legend as above.

(hollow-cored beams)

NORTH

SOUTH

(log beams)
Fig. 4.5 Transverse Section through the Slippery Creek Bridges during Field Testing showing a typical arrangement of strain gauges and relative positions of load testing vehicle and the recording centre, and a schematic representation of the scaffolding used under the bridges for the observation of the dial gauges.
allowed the strains to be observed remote from the span under test (Plate 4A and Fig 4.5). Automatic electronic switching apparatus and a Phillips Null Point Strain Gauge Bridge were used for the observations. Both the dial and strain gauges were read for zero load at each occasion the load was off the span. This occurred at the conclusion of every third set of readings whilst the loader was being changed from lane to lane or row to row. A running check on the zero shift was thus maintained.

It was found that a compensating gauge cemented to an 8" x 4" block of the bridge concrete was subject to greater variation of temperature than the decks themselves. Large zero shifts and inaccurate observations resulted. This was corrected by using a gauge on a span not under/ as the compensator.

Originally it was planned that strains should be recorded automatically by pen recorder and a Direct Reading Bridge. This, however, proved impractical because of the large drift which was experienced with this Strain Bridge. This was due to the fact that the very small changes in strain, which occurred even under these abnormal loads, necessitated the use of the highest sensitivity range in the Bridge circuits, which in turn amplified the already considerable variation in line voltage available on the site. Such variations rendered the drift impossible to control without the installation of more elaborate electronic apparatus.

4.6 Test Programme and Procedure

The programme adopted for testing the bridges was such as to include the necessary tests to satisfy the objectives of sec. 4.12.

Appendix 10 includes prints of Tables issued in a report written for the Ministry of Works indicating a proposed testing timetable and a suggested loading procedure which was subsequently adopted.

4.61 Lane and Point Loading Procedure

Lane loading was accomplished by placing the Compactor in each of four lanes for each full width span. Readings of deflection and strain were taken when this vehicle was positioned at the three-quarter (3Q), half (H) and quarter (Q) points.

Similarly, when point loading, the forty foot trailer was placed in nine rows across the width of the deck for spans A and C and in ten rows for span D, five for E and three for F. The nine rows for spans A and C correspond to the nine standard load positions proposed in the Rowe-Morice-Little bridge deck design method (refer sec. 2.12). The ten, five and three rows correspond to
a point load on each beam of the hollow-cored decks. Again, whilst in each row the load was positioned in turn on the three-quarter, half and quarter points and readings made of deflection and strain.

Some difficulty was experienced when loading over the outside edge beams and the guardrails had to be removed in order to allow the trailer and compactor to be manoeuvred into these positions.

4.62 Tests for the Effect of Transverse Reinforcement

Variation in transverse reinforcement was achieved by testing and retesting the spans of the eastern bridge (log-beam bridge: spans designated A, B and C as in fig. 4.1b) with gradually increasing amounts of reinforcement. Span A was transversely reinforced with 1" diam. m.s. bolts threaded at each end and tightened prior to grouting. Span B was not tested and Span C was transversely reinforced with 0.276" diam. prestressing tendons stressed to 9 Kips. and grouted. Initial tests began on span A with only one bolt in position at midspan, and on span C with no transverse reinforcement at all, except for that provided in the 4" deck slab (3" diam. rod at 8" c.c.s.)

The tests for transverse reinforcement then proceeded as below:

**Span A**

Series A1: single bolt at midspan of bridge deck.
- A2: two bolts added at quarter points: total now 3 bolts at 3Q, H & Q.
- A3: four bolts added: total now 7 bolts at 5'-0" centres.
- A4: eight bolts added: total now 15 bolts at 2'-6" centres;
  i.e. uniformly spaced along the span.

**Span C**

Series C0: no transverse prestress provided.
- C1: two 0.276" diam. prestressing tendons each 1'-3" from midspan.
- C2: two tendons added at quarter points: total now 4 at 3Q, H & Q.
- C3: four tendons added: total now 8 tendons at 5'-0" centres.
- C4: six tendons added: total now 14 tendons at 2'-6" centres;
  i.e. uniformly spaced along the span.

4.63 Tests for the Effect of Span/Width Ratio

Comparative tests on spans D, E and F (span/width ratios of 1.0, 2.0 and 3.33) were conducted to test for the effect of this ratio on deflection, strain and load distribution.

4.64 Tests for the Effect of Skew

These were performed by taking sets of readings of deflection and strain after positioning the Compactor first with its wheels parallel to the sides
of the deck and then with the wheels at right angles to the abutments; the difference in readings, if any, is a measure of skew.

4.7 Further Tests

4.71 Additional Tests for Differential Slip

As discussed in later sections (Chapter 6) slip was observed between the beams of span E during the above tests. It was subsequently decided to conduct further tests of those extra spans added to the bridge after the flood damage of March 1966 (sec 4.0). These were carried out in October 1966 using a twin axle, self propelled compactor with a maximum axle load of 31,000 lb. A device, which was essentially a $1/10,000$th inch dial gauge clamped to a heavy base plate, was used for observing the differential movement between beams under various load positions. These tests took only half a day to run through and required four men to operate the compactor and record the results.

4.72 Future Tests under Normal Traffic Conditions

At the completion of these tests the site was cleared of scaffold, the guardrails replaced, the hot mix surfacing laid and the bridges opened for traffic. The strain gauges glued to the soffit of decks were sealed off against the weather with Plasti-Bond Fibreglass Cement. Only the mini-plug socket at the end of the short cable soldered to the gauge was left exposed. If future tests for dynamic characteristics of these bridges are contemplated these same gauges could be used. More sophisticated electronic strain gauge apparatus with good sensitivity would be necessary, but the information from such tests would be valuable (refer sec 7.22).
CHAPTER: FIVE

TITLE: MODEL TESTS ON THE MULTIBEAM BRIDGES AT SLIPPERY CREEK

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CHAPTER FIVE  MODEL TESTS ON THE MULTIBEAM BRIDGES AT SLIPPERY CREEK

5.0 Introduction

When it was announced early in 1964 that the Ministry of Works intended to load test the Slippery Creek Bridges it was decided to construct a quarter scale model of the Slippery Creek Bridge decks, for load testing in the Structures Laboratory at the School (Plate 1).

The basic reason for the construction and testing of the model was to determine the load distribution behaviour of multibeam decks. However, the investigation expanded to include tests to establish the validity of the proposed transfer matrix theory as well as to determine the effect of transverse prestress, span/width ratio, 15 degrees of skew, and the type of precast unit used in the deck. In short, the testing programme for the model embraced at least five of the six objectives listed in sec. 4.12 for the testing of the full scale structure. The tests, thus duplicated on the model decks gave the opportunity to confirm the behaviour observed at the site and also the necessary circumstances to check model and prototype correlation. An important byproduct of this work was therefore concerned with model studies on concrete structures.

Tests on the model were conducted under better conditions than were possible on the full scale structure since they were independent of the weather, flood and tide conditions and motorway construction. In addition, the range of tests performed on the model was much greater than that permissible at the site. A study of transverse prestress, an examination of the shape of shear key and ultimate tests to failure have been possible during the model studies. There were, therefore, several advantages to be gained in carrying out both model and field studies.

The theory used for the design of the model, its construction and testing programme, are described below. Three model spans were investigated (SA, SB and SHC), and the various tests performed in separate stages to correspond to all five spans tested at the site. The presentation and discussion of the results of both the model and field test programmes is given jointly in Chapter 6.

5.1 Design of Model

5.11 Dimensional Analysis

The following presentation gives the basic concepts for model design based on a dimensional analysis of the statics of model and full scale structures.
The reproduction in a model of a particular state of deformation in a full scale structure requires that at every point the corresponding strains must be equal.

That is: \[ \frac{e_s}{e_m} = 1 \] \hspace{1cm} (5.1)

where \( e \) is the strain and \( s, m \) are the suffixes representing the structure and model respectively.

If, as seemed reasonable in these studies, time-dependent phenomena are ignored, and the two free quantities, length and stress, are fixed, then the choice of two scales

\[ \lambda = \frac{l_s}{l_m} = \frac{e_s}{e_m} \] \hspace{1cm} (5.2)

\[ K = \frac{\sigma_s}{\sigma_m} = \frac{p_s}{p_m} \] \hspace{1cm} (5.3)

determines every other quantity to be considered. In these equations

\( l \) represents length
\( \delta \) represents deflection
\( \sigma \) represents normal stress
\( p \) represents pressure

and \( s, m \) the suffixes as before (eqn 5.1).

Hence the scales for concentrated forces and loads (\( F \) and \( P \)) and volume (or density) forces (\( \rho \)) are given by:

\[ \alpha = \frac{F_s}{F_m} = \frac{P_s}{P_m} = K \lambda \] \hspace{1cm} (5.4)

\[ \gamma = \frac{\rho_s}{\rho_m} = \frac{\sigma_s}{\sigma_m} = K \lambda \] \hspace{1cm} (5.5)

The proof of these relationships for \( \alpha \) and \( \gamma \) are easily demonstrated using dimensional analysis techniques.

It should be noted that the choice of \( K \) in eqn (5.3) determines the elastic and ultimate strength characteristics of the model material because:

\[ K = \frac{\sigma_s}{\sigma_m} = \frac{E_s E}{E_m E_m} = \frac{E}{E_m} = \frac{R_s}{R_m} \] \hspace{1cm} (5.6)

where \( R \) is the ultimate strength

and \( E \) is the elastic modulus of the material.

Thus deformation characteristics under non-linear or inelastic conditions such yielding or settlement are fixed provided they are independent of time. On the other hand, such phenomena such as viscous flow, creep, relaxation were excluded from these model studies.
5.12 Modelling Criteria

In practice a distinction is made between two kinds of models: Direct and Indirect Models. Direct models can be further subdivided into three types:

I Models in which \( \lambda > 1 \); \( K = 1 \) \hspace{1cm} (5.7a)

II Models in which \( \lambda > 1 \); \( K > 1 \) \hspace{1cm} (5.7b)

III Models in which \( \lambda > 1 \); \( K > 1 \) and which \hspace{1cm} (5.7c)

are tested only in their elastic range; i.e. are distinct from type II in that their behaviour does not reproduce that of the full scale structure once their elastic load range has been exceeded.

Models belonging to types II and III are usually constructed from materials of low elastic modulus in order to either reduce the intensity of the required loading or to magnify the small deformations involved thus facilitating their measurement. However, the choice of model material is an important factor particularly for type II models; for example, the successful use of concrete mixes incorporating pumice aggregates has been reported. Type III models permit a much wider range of materials; for example, celluloid, phenolic-ethylene resins, rubber, cork, paraffin wax etc. Such models are widely used in the preliminary design stages of the more complex structure.

5.13 Choice of Model Type and Dimensions

The Slippery Creek Bridge Model was designed as a direct model, type I. It was therefore built to a geometrical scale of one quarter and used concrete for its base material.

Thus:

\[ K = 1; \quad \lambda = 4; \]

\[ \alpha = 16; \quad \gamma = \frac{1}{2} \] \hspace{1cm} (5.8a)

hence

\[ \alpha = 16; \quad \gamma = \frac{1}{2} \] \hspace{1cm} (5.8b)

Each span of the model was therefore 9'-3" along the skew span 9'-0" right width, either 4" or \( 4\frac{1}{2} " \) deep, and exhibited the same 15 degrees of skew. Since two spans were built at a time the overall dimensions of the model measured about 20 ft by 10 ft and was seated about 3 ft above the floor (Plate 5).

Not only were the decks scaled but also the pier caps and pier columns: three 6" diameter boiler tubes supported the 9" x 9" pier caps on which the deck was seated through a bright steel bearing pad. A foundation beam was provided for each pier to maintain stability and lift the deck to a convenient height.
5.14 Special Problems associated with Type I Models of Concrete Structures.

5.14.1 Model Mix Design Criteria

The choice of a suitable mix for the concrete of the model was made with care. The mix had to satisfy the following two requirements:

1. The elastic modulus of the model concrete must equal that of the full scale structure for $K$ to be unity (eqn 5.8a). The simplest way to ensure this was to use the same mix in both structures, but difficulties with aggregate size arose as described in (2) below.

2. Good workability and bond strength must be maintained to ensure adequate transfer of prestress from tendons to concrete in these, pretensioned, prestressed units. Aggregate size was thus important and the full scale aggregate had to be reduced before use in the model. The actual amount of reduction was dictated by the spacing between the stressing tendons and cover requirements in the model units. In general, therefore, the aggregate was also scaled down by $\lambda = 4$ and hence a maximum aggregate size of $\frac{4}{4}''$ was used; the guarantee of equal elastic moduli being lost.

A compromise between (1) and (2) had been indicated and the mix design used was an attempt to satisfy both criteria - see sec 5.311 and also 5.612 for a determination of $K$.

5.14.2 Scaling the Transmission Length

Since the beams were all pretensioned units the transfer of prestress from tendon to concrete was effected by bond between concrete and tendon. The length at each end of each beam for the build up of stress in the concrete is of great importance and is called the transmission length. This length is of the order of 55 wire diameters$^{38}$ and thus for a 0.200 inch diameter tendon it was about 11 inches. It determines the effective span of the member over which the full working prestress may be developed and it is therefore desirable to keep the ratio of this transmission length to the overall span as small as possible. In the Slippery Creek bridge deck members this ratio is of the order of $2\frac{1}{2}$.

Although linear dimensions and aggregate size may be reduced it is not possible to scale down the transmission length without a considerable reduction
in wire diameter. Without such a reduction being made the ratio of transmission length to overall span for the model would have been as large as 10%. However, the use of a reduced wire diameter to give a shorter transmission length had three disadvantages:

1. From test results published\(^{38}\) the probable correct choice of wire would be 0.064" diameter. About 50 of these wires would have been required to carry the necessary prestressing force which would have made the placing of the concrete difficult and insertion of transverse ducts impossible within the confined section of these scaled units.

2. Special stressing jacks and anchorages would be required to stress the system.

3. This high tensile wire was unobtainable in New Zealand.

It was therefore decided to use the 0.200" diameter wire since this was readily available as was the associated PSC One Wire Stressing System. To overcome this problem of transmission length mechanical anchorages were buried at each end of each wire inside the beam itself. These served to reduce the transmission length down to the overall length of each anchorage (a barrel and wedge: \(1\frac{3}{8}'' \times 1/16''\)).

5.143 Tests for Ultimate Load Characteristics

5.143.1 Use of Oversized Stressing Tendons

The use of 0.200" diameter stressing tendons as discussed in sec 5.142 violated the scale factor relationship for steel cross-sectional areas. However, since the model was a prestressed structure the amount of prestressing steel provided would not affect the flexural properties of the deck. Thus tests within the elastic range will be comparable with similar tests on the full scale structure provided. Of course, the total prestress force had been scaled according to eqns 5.4 and 5.8b. On the other hand the model will not faithfully predict the behaviour of the full scale structure outside these conditions, that is under ultimate load conditions.

5.143.2 Dead Load Equivalence

Consideration of eqn (5.5) will indicate that for \(k = 1\) and \(a = 4\) the scale of densities for full scale and model structures should be in the ratio of 1 : 4 respectively. Obviously when the model is constructed from the same material as the full scale structure such a scale factor relationship is also violated.
The density of the model should be 4 times that of the structure and the dead load of the model should therefore be increased by the addition of a uniformly applied load. For tests within the elastic range such applied "dead" load is not necessary, but for tests under conditions of ultimate load it plays an important part in the mode of failure and affects the safety factors against collapse of the model. One series of these tests on the model was therefore conducted with six tons (162, 83 lb. ingots) of lead uniformly distributed over the surface (Plate 12) but for practical reasons alone the lead was not used for similar tests on other model spans. The calculation for this applied dead load (P) was made by equating

\[ \rho_m = 4 \rho_s \]

where

\[ \rho_m = 150 + \frac{P}{\text{volume}} \text{ Lbs/ft}^3 \]

i.e.

\[ 150 + \frac{P}{29.5} = 4.150 \]

therefore

\[ P = 13,280 \text{ Lbs, say 6 tons.} \]

assuming the density of concrete at 150 lb/ft^3 for both model and full scale structures.

5.144 Casting Model Cores

As noted previously and illustrated in fig. 5.3b one set of model units were hollow. Three hexagonal hollow cores were to be cast into each unit and allowance made for a diaphragm at each end and one at midspan. For the full scale structure, the hollow cores were cast using a collapsible form which could be withdrawn and the end diaphragms cast subsequently.

For the model units the manufacture and use of withdrawable formwork was not warranted and in its place lengths of polystyrene, cut to the shape and size of the scaled core, were buried in the units. These cores could later have been dissolved out with petrol but this was not considered necessary since they contributed neither weight nor strength to the beam itself. See Plate 7A.

5.145 Maintaining Cover Requirements

A further problem arose when boxing up and placing the reinforcement cage (sec 5.32) in the pier caps of the model. Whereas a clear cover of 3" in a full scale structure makes due allowance for normal tolerances in formwork and bending steel, should this cover be reduced to \( \frac{3}{4} \)" as in the model much greater care and closer tolerances are needed. If such attention is not given then it is possible as was found here that the cover will be non-existent or at most very small; again, a small aggregate size is essential.
5.2 Construction of Model Testing Facility

Before construction and testing of the model could itself take place, it was necessary to design and build requisite testing equipment and facilities within the Structures Laboratory. A test floor, loading frame and hydraulic remote controlled loading system were designed, built and assembled together to form the model facility as described below.

5.21 Construction of Test Floor

A special heavy duty test floor was laid in the laboratory which provided holding down sockets at convenient intervals throughout the area. The floor, measuring 20 ft long and 12 ft wide, was set 18" into the ground beneath the existing laboratory floor. Heavily reinforced with railway tees, the floor took 13 yds of concrete and 3 tons of steel giving a total dead load of approximately 27 tons.

Thirty, 1½" diameter sockets were set into the floor and welded to the steel tees. Each socket was capable of transmitting 10 tons to the floor, it being considered that a maximum, full scale structure load of 160 tons would be adequate to cause failure. The layout of the socket was such as to accommodate not only any position of the loading frame associated with these model tests but also for future tests on other similar projects.

5.22 Design and Assembly of Loading Frame

A loading frame was designed and built in the School's Workshop to span across the model and carry an hydraulic jack for the application of load to any desired point on the model decks.

The frame was supported on two triangular legs and four 6" diam. castors. It could be bolted to the test floor by locating any one of three possible sockets under each frame leg. (fig 5.1a, b, c). The frame was designed to carry a ten ton load on each leg and a twenty ton point load at mid span of the two 10" x 3½" channel head beams. The legs were each pinned to the channel beams so that any angle of skew may be taken up; one pin was fixed and the other sliding so that the distance separating the legs could also be adjusted.

5.23 Hydraulic Load System

For the application of point loads to the model deck an hydraulic jack rated at a capacity of ten tons was also designed and built. This jack was operated from a remote console fitted with a small pumping unit run off the
230 volt mains supply. Supplied with return and needle valves, the direction of motion of the loading ram could be reversed allowing the loading and unloading cycles to be carried out smoothly and quickly. The jack itself was suspended from the loading frame by a skate board and a 2" diameter spherical steel ball sandwiched between two adjustable plates. This adjustment allowed the line of action of the jack to be made vertical and the ball prevented the application of any torque to the deck. For the same reason a second steel ball was provided between the jack and the deck; this also ensured "point" contact which was made through a small square of pine used to locate the ball over the desired point on the deck. (Plate 10A).

The load actually applied to the deck was measured by the deflection of a proving ring suspended between the piston of the jack and the second steel ball. This ring was calibrated up to 5 tons (1 division = 14.18 lbs; maximum load 800 divisions) and was used for all the elastic tests performed on the model. The jack itself weighed about 180 lbs, was about 16" in overall height and carried a 6" diameter piston with 4.5" maximum travel.

5.3 Construction of the Model

5.3.1 Materials - Concrete and Steel

5.3.1.1 Concrete

Bearing in mind the considerations of sec. 5.131 a suitable mix was "designed" by adopting a trial and error approach. Three trial batches were made up and tested for workability and strength. The same 1/2" aggregate and beach sand as used on the full scale structure was used in these trials (refer sec 4.31). Six 8" x 4" blocks were made from each trial and a final mix chosen from the results of crushing tests on these blocks. Table 5.1 below presents details of the mix used for all model units.

<table>
<thead>
<tr>
<th>Description</th>
<th>Sieve Analysis</th>
<th>Weight per Batch (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4&quot; aggregate</td>
<td>passing 1/4&quot;:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90.9% passing No. 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>56.2% passing No. 8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>41.4% passing No. 16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30.9% passing No. 30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>21.8% passing No. 50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13.4% passing No. 100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.2% passing No. 200</td>
<td>105.0</td>
</tr>
<tr>
<td>beach sand</td>
<td>passing No. 50</td>
<td></td>
</tr>
<tr>
<td>cement</td>
<td>Guardian</td>
<td></td>
</tr>
<tr>
<td>calcium chloride</td>
<td>2% by weight of cement</td>
<td></td>
</tr>
</tbody>
</table>

continued over.
Fig. 5.1 Model Testing Facility

Elevation on A-A

Elevation on B-B
<table>
<thead>
<tr>
<th>Description</th>
<th>Sieve Analysis</th>
<th>Weight per Batch (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>water/cement ratio 0.45</td>
<td>13.5</td>
</tr>
<tr>
<td>Total Weight per Batch</td>
<td></td>
<td>164.1</td>
</tr>
</tbody>
</table>

Note:
1. A water/cement ratio of 0.45 was maintained throughout by the determination of the moisture content of both aggregates before each mix was prepared and the subsequent calculation of the additional water that needed to be added to bring the total amount per batch up to 13.5 lbs. Both aggregates were stored in open bins outside the laboratory and thus subject to a variation in moisture content that was governed by the local rainfall.  
2. Following the practice adopted in the manufacture of the Slippery Creek units, Wilsonite Rapid Hardening Cement was used, at least in the initial stages. However, because of industrial shortages of this brand of cement it became necessary to change to Guardian Ordinary Cement which, together with the addition of calcium chloride, gave the mix a sufficiently high early strength. This was necessary to allow the release of the units at 48 hours in order that a turn-around for the casting beds based on a two day cycle could be maintained. In fact the strength was sufficient at 24 hours to allow one set of 18 log beams to be poured on a one day cycle. The casting procedure is explained in sec. 5.33.  
3. The quantities in Table 5.1 are per batch; two batches were sufficient to make one log beam and six 8" x 4" blocks with spillage; three batches were necessary for each hollow cored unit with six 8" x 4" blocks and spillage. Quantities used for individual parts of the model are summarised in Table 5.2.  
4. No steam curing facilities were available in the laboratory, but the units were kept wet under hessian for at least seven days after pouring and then allowed to dry. The 8" x 4" control blocks were similarly treated.

**Table 5.2**

**Quantities of Concrete in Model**
TABLE 5.2
QUANTITIES OF CONCRETE IN MODEL

<table>
<thead>
<tr>
<th>Description</th>
<th>Overall Dimensions</th>
<th>Volume</th>
<th>Weight</th>
<th>No. Read.</th>
<th>Totals</th>
<th>Cum. Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Floor</td>
<td>20'x12'x1½'</td>
<td>13.33yd</td>
<td>24 ton</td>
<td>1</td>
<td>13.33</td>
<td>13.33 c.yd</td>
</tr>
<tr>
<td>Foundation Beams</td>
<td>24&quot;x18&quot;x10'0&quot;</td>
<td>1.11</td>
<td>2.0</td>
<td>3</td>
<td>3.33</td>
<td>6.0</td>
</tr>
<tr>
<td>Cap Beams</td>
<td>9&quot;x9&quot;x9'6&quot;</td>
<td>0.20</td>
<td>0.36</td>
<td>3</td>
<td>0.60</td>
<td>1.08</td>
</tr>
<tr>
<td>Log Beams</td>
<td>5½&quot;x3½&quot;x9'0&quot;</td>
<td>0.0445</td>
<td>0.08</td>
<td>37</td>
<td>1.65</td>
<td>2.96</td>
</tr>
<tr>
<td>Hollow-core Beams</td>
<td>11⅓&quot;x4&quot;x9'0&quot;</td>
<td>0.0764</td>
<td>0.138</td>
<td>10</td>
<td>0.76</td>
<td>1.38</td>
</tr>
<tr>
<td>Deck Slab</td>
<td>9&quot;x9&quot;x1&quot;</td>
<td>0.25</td>
<td>0.45</td>
<td>2</td>
<td>0.50</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tests for crushing strength and elastic modulus of the 8" x 4" cylinders were carried out:

1. Immediately prior to the release of the beams from the casting bed; i.e. at either one to two days.
2. At the time of testing the model decks concerned, i.e. at six months, 1½ years and 2½ years.

A computer programme, written in PDQ FORTRAN, was developed for making a statistical analysis of the results obtained from these tests (refer sec 5.612; programme is listed in Appendix 4). A summary of the computed results for the model concrete is given in Table 5.3 and illustrated in Appendix 11.

TABLE 5.3
PROPERTIES OF CONCRETE IN MODEL

<table>
<thead>
<tr>
<th>Age</th>
<th>Crushing Strength</th>
<th>Elastic Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dec.</td>
</tr>
<tr>
<td>AT RELEASE:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 day</td>
<td>3710 +</td>
<td>13.66% +</td>
</tr>
<tr>
<td>2 days</td>
<td>5225</td>
<td>12.36%</td>
</tr>
<tr>
<td>7 days</td>
<td>10600</td>
<td>(5.5)</td>
</tr>
<tr>
<td>28 days</td>
<td>11400</td>
<td>(5.6)</td>
</tr>
<tr>
<td>AT TEST:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 mths</td>
<td>13210</td>
<td>7.97%</td>
</tr>
<tr>
<td>1½ year</td>
<td>13950</td>
<td>5.82</td>
</tr>
<tr>
<td>2½ year</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

+ values for crushing strength are expressed in lbs/sq.in.
standard deviation is given as a percentage of the mean.
++ see over.
figures in brackets are estimated from the crushing strength using Jensen's formula: refer sec. 5.612.

units for elastic modulus are also lbs/sq.in.

5.312 Steel

5.312.1 Prestressed

The prestressing steel used for all beams was 0.200" diameter, indented wire. The wire was supplied in a 5 ft diameter roll, approximately 3020 feet of it being used in the 47 model beams made. As discussed previously in secs 5.142 and 5.143.1 the single 0.200" diameter wire was chosen because the stressing facilities were already available in the laboratory.

The P.S.C. One Wire System uses a light weight jack which is compact and easily operated. The anchorage unit consists of a barrel and sleeve or wedge which locks on to the wire as it is drawn into the barrel by the tension in the wire.

For the transverse prestressing of the model deck SB. 0.276" diameter high tensile wire was threaded through each of the 14 ducts. Larger barrels and wedges were necessary but the same stressing system was used.

Tests on the 0.200" wire were conducted in the 100 ton Avery testing machine using an Amsler extensometer to record strain. Results of these tests are plotted in fig A11.3 of Appendix 11 (which also shows results for subsequent tests on the 0.276" wire) and summarised below:

\[ E_s \quad \text{(elastic modulus of H.T. steel)} \quad 28.4 \times 10^6 \text{ lbs/sq.in.} \\
U.T.S. \quad \text{(ultimate tensile strength)} \quad 109.6 \text{ tons/sq.in.} \]

5.312.2 Nonprestressed

Both the log and hollow cored units had a certain amount of nonprestressed reinforcement provided. This usually took the form of shear reinforcement, although some nonprestressed high tensile steel was provided in the hollow cored units to reinforce across the hexagonal cores for the resistance of local wheel load effects. Table 5.4 gives a summary of the reinforcement schedule for all steel used in the construction of the model.

| TABLE 5.4 |
| REINFORCEMENT SCHEDULE FOR MODEL |
TABLE 5.4
REINFORCEMENT SCHEDULE FOR MODEL

<table>
<thead>
<tr>
<th>Type</th>
<th>Place</th>
<th>Use</th>
<th>Quantity</th>
<th>Size</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESTRESSED</td>
<td>all beams</td>
<td>longitudinal prestress</td>
<td>3020 ft</td>
<td>0.200&quot;</td>
<td>6 wires/beam &amp; 8 wires/beam</td>
</tr>
<tr>
<td></td>
<td>deck SB</td>
<td>transverse prestress</td>
<td>170</td>
<td>0.276&quot;</td>
<td>7 1/2&quot; ccs.</td>
</tr>
<tr>
<td></td>
<td>deck SHC</td>
<td>transverse prestress</td>
<td>10</td>
<td>0.276&quot;</td>
<td>single wire at midspan</td>
</tr>
<tr>
<td>NONPRESTRESSED</td>
<td>log beams</td>
<td>stirrups</td>
<td>180</td>
<td>1/4&quot;</td>
<td>3&quot; ccs.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>spiral shear coils</td>
<td>625</td>
<td>0.064&quot;</td>
<td>2 coils/beam</td>
</tr>
<tr>
<td></td>
<td>hollow-core beams</td>
<td>core reinforcement</td>
<td>270</td>
<td>0.104&quot;</td>
<td>5&quot; &amp; 3/4&quot; ccs.</td>
</tr>
<tr>
<td></td>
<td>deck slabs</td>
<td>reinf of SA and SB</td>
<td>1075</td>
<td>0.200&quot;</td>
<td>2 1/2&quot; ccs.</td>
</tr>
<tr>
<td></td>
<td>cap beams</td>
<td>longitudinal reinf</td>
<td>120</td>
<td>5/8&quot;</td>
<td>4 bars/beam</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stirrups</td>
<td>345</td>
<td>1/4&quot;</td>
<td>3&quot; ccs.</td>
</tr>
</tbody>
</table>

5.32 Construction of Piers

The piers of the Slippery Creek Bridge were also reproduced to scale in the model. Three piers were constructed each using a 24" x 18" foundation beam, three 12" lengths of boiler tube to represent the three circular piles of the bridge and a 9" x 9" reinforced cap beam. The overall length of the foundation beams was 10'-0" and the cap beams 9'-5 3/8". To satisfy scale requirements for the support of the cap beams the boiler tube should have been 7 1/2" in outside diameter. This size was not readily available and 6" i.d. tube was used with a 7 1/2" diameter plate welded to one end. To obtain fixity between cap and pile the scaled equivalent reinforcement was welded to this plate and buried in the cap beam in a manner similar to that used at the site (ref. fig 5.2).

Whereas the clear cover to the reinforcing steel in the full scale structure was 3" this reduced to only 3/4" in the model. Extra care was therefore needed to ensure the boxing and reinforcing steel were assembled to a higher degree of accuracy than normally required for such work. Reduced aggregate sizes were again necessary to provide adequate compaction and cover within this limited space.
Four 9'-6" strips of 2" x ½" bright steel were cemented to the tops of the cap beams with Ciment Fondu to provide bearing pads for the multibeam decks. These strips were levelled and polished to provide a clean horizontal surface which was a scaled equivalent of the bearing area of the neoprene pads and pine used on the full scale structure. It may be argued that a knife edge support would have given better support conditions in that the actual span would have been known exactly. However, knife edges are never used in practice and it was felt that more profitable information on the actual behaviour of multibeam decks would be gained by using a bearing pad: for example, information on the effective span length and the effect of rigid seating on beam performance.

---

![Diagram of 24" x 18" beam](image_url)

Fig 5.2
Reinforcement of Piers.

5.33 Beam Manufacture

5.33.1 Stressing and Casting Bed

Both types of beam were cast in unit moulds specifically designed and built for this work. Refer to figures 5.3a, b, c, and also Plates 6A, 6B and 7A. In the unit mould method the wires are tensioned against the ends of the mould so that before the concrete is cast the whole prestressing force is carried by the mould which has to be designed as a strut to resist this compression. An inverted channel was used for the strut which carried the two side formers made of Oregon timber, strengthened with ½" bright steel strip and saturated in oil to prevent warping during use.

Each channel carried a fixed anchor plate and a fixed distribution plate (A and D in fig 5.3c) drilled in a pattern to allow the passage of the
0.200" tendons. A moveable anchor plate (A₂ in fig 5.3c) was provided at one end of each channel against which the One Wire Jack was used and the tendons anchored. Two 1/8" diameter bolts (B) separated these plates (i.e. D and A₂) and these allowed the gradual release of the prestress into the beam when necessary. The distribution plate (D) was provided as a safety measure to prevent the possible back lash of the tendons and anchor plate (A₂) should the bolts become dislodged (which is possible if the order of stressing be reversed).

Each bed was bolted to the floor to prevent it lifting at its ends under the action of the eccentric prestress forces; such forces tending to bow the bed and distort the beam section.

Any length of beam could be made up to a maximum equal to that of the side forms by simply adjusting the end blocks (C), though for these model units the lengths were 8\times11\frac{1}{4}" for the logs and 9\times0\frac{1}{2}" for the hollow cored units. The difference in length was due to the different end detail for the two beams - the log beams had square ends; the hollow cored units were skewed to follow the line of the piers (15 degrees).

5.332 Procedure

Table 5.5 gives a summary of the units made and also their release dates and age at test.

<table>
<thead>
<tr>
<th>Unit Mould</th>
<th>Beam Type</th>
<th>Section</th>
<th>Number Made</th>
<th>Used in</th>
<th>Age at release</th>
<th>Age at test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LOG</td>
<td>5\frac{1}{2}&quot; x 3\frac{1}{2}&quot; x 8\frac{1}{1} - 11\frac{3}{4}&quot;</td>
<td>18</td>
<td>Model SA</td>
<td>2 days</td>
<td>6mths</td>
</tr>
<tr>
<td></td>
<td></td>
<td>plus a 1\frac{1}{4}&quot; nib</td>
<td>18</td>
<td>Model SB</td>
<td>1 day</td>
<td>1\frac{1}{2} yrs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>along lower edges</td>
<td></td>
<td>Control</td>
<td>2 days</td>
<td>2\frac{1}{2} yrs</td>
</tr>
<tr>
<td>2</td>
<td>HOLLOW-CORED</td>
<td>11\frac{1}{8}&quot; x 4&quot; x 9\frac{1}{4}&quot;</td>
<td>10</td>
<td>Model SHC</td>
<td>2 days</td>
<td>2\frac{1}{2} yrs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>less 3 hexagonal cores and shear</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>keyways (2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There were ten basic steps in the procedure for casting these pretensioned prestressed beams:

1. The side forms were bolted in place, the moveable anchor plate adjusted and the bed oiled thoroughly.

2. The stressing tendons were threaded into the bed and the transmission anchorages included together with the end blocks and spacers - see Plate 6B. The bed was stressed beginning with the lowest wires to preserve stability of the moveable anchorage plate.
Fig. 5.3

Typical Details of Stressing Beds used for the manufacture of the model precast, prestressed concrete beams.
Each wire was initially strained to an average of $4.36 \times 10^{-5}$ as measured over the 10" gauge length of the Amsler extensometer clipped to the wire. Some difficulty was had in calculating the losses due to anchorage slip, but subsequent tests have confirmed the estimate made at the time of $125 \times 10^{-5}$. The net prestressing force per wire was therefore of the order of $2,800 \text{ lb} \pm 200 \text{ lb}$. (The desired force per wire to satisfy the scale factor relationship for prestress forces was $2,960 \text{ lbs}$.)

3. The polystyrene hexagonal cores were placed (hollow units only) followed by the stirrups and other nonprestressed reinforcement (Plates 6A and 7A).

4. The transverse rods which formed the ducts for later transverse stressing were oiled and placed.

5. The concrete was mixed, placed and vibrated into the bed; six 8" x 4" blocks for strength control were also made.

6. The laboratory was cleaned up and the beam covered with wet hessian.

7. The transverse rods were withdrawn within 12 hours and the side forms removed and cleaned.

8. The prestress was released into the beams when at least three control blocks exceeded a crushing strength of 3000 lbs/sq.in. With the inclusion of calcium chloride in the mix this occurred within 24 hours. Release was effected by screwing the moveable anchor plate towards its fixed counterpart.

9. The now relaxed tendons were cut and the beam lifted clear of the mould to be stored and covered with wet hessian for at least seven days.

10. The mould was cleaned and prepared for the next beam.

5.34 **Assembly of Multibeam Decks**

The beams as cast were, after individual testing (sec 5.51) assembled into three separate decks; designated SA, SB (for the two log beam decks) and SHC (for the single hollow cored deck). Plates 8A, 8B and 9A.

Variation in elastic properties across the width of each deck were eliminated as far as possible by staggering the separate beams according to their age. The oldest and youngest beams were thus placed side by side at one edge of the deck and the pattern continued as indicated in fig 5.1c.

Eleven series of tests were then performed on these decks under different conditions of depth, transverse prestress and width as detailed in Table 5.6.
### TABLE 5.6

<table>
<thead>
<tr>
<th>Series</th>
<th>Model</th>
<th>Unit Type</th>
<th>Depth</th>
<th>Width/Transverse Prestress</th>
<th>Tests Conducted elastic</th>
<th>Tests Conducted ultimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>SA</td>
<td>LOG</td>
<td>$4\frac{1}{2}''$; beams in place, 1'' slab poured, No keys.</td>
<td>full (1.0)</td>
<td>none</td>
<td>full set</td>
</tr>
<tr>
<td>II</td>
<td>SB</td>
<td>LOG</td>
<td>$3\frac{1}{2}''$; beams in place, no slab poured, but keys filled</td>
<td>full</td>
<td>ordinary reinforced</td>
<td>full set</td>
</tr>
<tr>
<td>III</td>
<td>SB</td>
<td>LOG</td>
<td>$3\frac{1}{2}''$; as above</td>
<td>full (1.0)</td>
<td>22% of long prestress</td>
<td>full set</td>
</tr>
<tr>
<td>IV</td>
<td>SB</td>
<td>LOG</td>
<td>$3\frac{1}{2}''$; as above</td>
<td>full (1.0)</td>
<td>44% of long prestress</td>
<td>full set</td>
</tr>
<tr>
<td>V</td>
<td>SB</td>
<td>LOG</td>
<td>$4\frac{1}{2}''$; beams in place and keys poured</td>
<td>full (1.0)</td>
<td>44% of long prestress</td>
<td>full set</td>
</tr>
<tr>
<td>VI</td>
<td>SHC</td>
<td>HOLLOW-CORED</td>
<td>4''; beams in place and keys poured.</td>
<td>0.3</td>
<td>single cable</td>
<td>full set</td>
</tr>
<tr>
<td>VII</td>
<td>SHC</td>
<td>HOLLOW-CORED</td>
<td>4''; as above</td>
<td>0.3</td>
<td>none</td>
<td>partial set</td>
</tr>
<tr>
<td>VIII</td>
<td>SHC</td>
<td>HOLLOW-CORED</td>
<td>4''; as above</td>
<td>0.5</td>
<td>single cable</td>
<td>full set</td>
</tr>
<tr>
<td>IX</td>
<td>SHC</td>
<td>HOLLOW-CORED</td>
<td>4''; as above</td>
<td>0.5</td>
<td>none</td>
<td>partial set</td>
</tr>
<tr>
<td>X</td>
<td>SHC</td>
<td>HOLLOW-CORED</td>
<td>4''; as above</td>
<td>1.0</td>
<td>single cable</td>
<td>full set</td>
</tr>
<tr>
<td>XI</td>
<td>SHC</td>
<td>HOLLOW-CORED</td>
<td>4''; as above</td>
<td>1.0</td>
<td>none</td>
<td>partial set</td>
</tr>
</tbody>
</table>

5.34.1 Deck SA

SERIES I Commencing with Deck SA, 18 of the log beams were laid side by side to span between two of the three piers. The key ways thus formed between two adjacent beams were stuffed with newspaper and the reinforcement laid (0.1040'' ms wire at 2'' ccs) for the 1'' thick deck slab. No transverse bolts or prestressing tendons were threaded through the ducts and this deck was tested right up to ultimate collapse after the deck slab had been poured and matured.
It is interesting to note at this point that all four corners of this deck lifted off the steel bearing pad after the deck slab had been poured. This was thought to be due to the shrinkage of the concrete in the slab as it set; the gap remaining between beam and bearing pad (about 1/16" max.) was packed with Ciment Fondu.

5.342 Deck SB

SERIES II The second span of log beams (SB) was tested in four stages. After the beams had been assembled side by side all fourteen transverse stressing tendons (0.276") were threaded into the deck. The key ways were then filled with concrete and the first series of tests on this deck carried out. The deck was therefore transversely reinforced but not prestressed and only 3\(\frac{1}{2}\)" deep.

SERIES III The fourteen transverse tendons were now stressed by first breaking the bond of the concrete at the keys and then jacking with two PSC One Wire Jacks from both ends of each tendon to eliminate as far as possible friction in the ducts. A force of 6400 lbs was jacked into each tendon so that after losses by anchorage slip the net force in each wire would be of the order of 6000 lbs and the ratio of transverse to longitudinal prestress would be 0.218 or 22%. The deck was now transversely prestressed but still only 3\(\frac{1}{2}\)" deep.

SERIES IV The stress in the transverse tendons was now increased to give an overall transverse prestress of 0.435 or 44\% of the longitudinal prestress in the beams. Again, by stressing from both ends to minimise friction losses, 12,800 lbs was jacked into each of the 14 tendons. Tests at this stage enabled the effect of a "high" percentage of transverse prestress to be observed; deck depth was still only 3\(\frac{1}{2}\)".

SERIES V The reinforcement was now laid for the extra 1" deck slab including the lifting of the flattened spiral shear coils cast into the top of each individual log beam. When this deck was poured the overall depth was increased to 4\(\frac{1}{2}\)" and transversely prestressed to 44\% of the longitudinal prestress. See Plates 8B, 10B and 11A.

5.343 Deck SHC

The third model span to be tested was the hollow cored multibeam deck. It was assembled on the set of piers used for SB above after both SA and SB had been removed by a 5 ton forklift hoist. This span was also tested in several (6) stages.

SERIES VI With only three beams laid together the two intermediate shear keys were poured and the single midspan transverse cable stressed...
SERIES VII Still with only three of the beams placed the stress in the transverse cable was released and some of the Series VI tests repeated. This provided information on the action of the shear keys and a special arrangement of dial gauges was set up to record this behaviour. (Refer fig 5.4).

SERIES VIII Two further beams were now added to the above three and the transverse cable again stressed to provide an initial 7500 lb force on the deck. The extra two shear keys were poured and tests conducted.

SERIES IX As in Series VII the transverse cable was now released and tests for the action of the shear keys conducted on this a five-beam deck.

SERIES X The final five beams were placed and the corresponding shear keys poured. The transverse tendon was again stressed to 7500 lbs initial force and tests conducted eventually up to ultimate collapse (after Series XI had been completed). See Plates 9A and 9B.

SERIES XI Again the transverse tendon was released and again the action of the shear key observed. The tendon was re-stressed prior to the ultimate load tests of Series X above.

5.4 Instrumentation

The instrumentation used for observing the behaviour of the model decks was either for measuring strain or deflection.

5.41 Strain

In general strain gauging was not a success, but nevertheless an attempt was made to record deck and pier strains as described below. Plate 12 indicates the general set-up for recording the changes in strain in the model SA during the ultimate load tests.

5.411 Deck Strains

Philips electrical resistance strain gauges were mounted on both the upper and lower surfaces of deck SA. Orientated to measure changes in longitudinal strain the 10 gauges were placed only on the midspan centre line and at 2"-3"
ccs. Five of the gauges were of 3" gauge length and the remainder of 1" gauge length (refer Table 5.7 for details of strain gauges and cements used). A check was therefore made to find the correct gauge length for use on concrete with a maximum aggregate size of \( \frac{1}{4}'' \). It was thus established,
though from somewhat small evidence, that both the 3" and 1" gauges were equally effective and that for reasons of economy and ease of application the 1" gauges would be used for future work. This conclusion falls in line with the generally accepted rule that the gauge length should not be less than four times the maximum aggregate size, to avoid the localised effects of the aggregate itself.

**TABLE 5.7**

**STRAIN GAUGE DETAILS**

<table>
<thead>
<tr>
<th>Gauge Type</th>
<th>PR 9217</th>
<th>PR 9810</th>
<th>4&quot; Foil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance, R</td>
<td>120 ± 0.5%</td>
<td>606 ± 0.5%</td>
<td>200 ± 0.25%</td>
</tr>
<tr>
<td>Gauge Factor, k</td>
<td>2.24 ± 1.5%</td>
<td>2.12 ± 1.5%</td>
<td>2.21 ± 1.5%</td>
</tr>
<tr>
<td>Temp. Coeff., αₜ</td>
<td>-(31 ± 2) × 10⁻⁶</td>
<td>-(35 ± 2) × 10⁻⁶</td>
<td>-10.10⁻⁶+++</td>
</tr>
<tr>
<td>Backing Material</td>
<td>paper</td>
<td>cresol</td>
<td>epoxy-ethylene</td>
</tr>
<tr>
<td>Cement</td>
<td>PR 9214/04</td>
<td>PR 9214/04</td>
<td>Araldite</td>
</tr>
<tr>
<td></td>
<td>PR 9214/05</td>
<td>PR 9214/05</td>
<td>Casting Resin D</td>
</tr>
<tr>
<td>Gauge Length</td>
<td>3&quot;</td>
<td>1&quot;</td>
<td>4&quot;</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>Philips</td>
<td>Philips</td>
<td>Saunders-Roe</td>
</tr>
<tr>
<td>Used On</td>
<td>MODEL SA &quot;E&quot; TRANSDUCER</td>
<td>MODEL SA</td>
<td>SLIPPERY CREEK</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BRIDGE DECKS</td>
</tr>
</tbody>
</table>

++ Units of αₜ are ohm/ohm °C

Unfortunately a complete set of strain observations was not made for all load conditions on this span. It was felt that for multibeam decks the observation of deflection alone would give adequate information on the load distribution characteristics and other properties of the decks. Observation of strain gave little return in terms of the information retrieved and the work and cost involved. The direction of strain for instance, is known beforehand to be longitudinal for these articulated plate decks (refer sec 2.2) and strain gauging would give no new information on this point. The use of dial gauges alone to investigate the behaviour of multibeam decks is supported by Cusens in his work²⁴ (sec 2.14). Strain gauging and strain observation was therefore discontinued for the other test series.

5.4.12 Pier Strains

As illustrated in fig 5.2 and discussed in sec 5.32 each pier cap was supported on three 6" i.d. boiler tubes. These tubes were each strain gauged with two, PR 9810 e.r.s.g. on opposite generators. The tubes were calibrated
Fig 5.4 DIAL AND STRAIN GAUGE DISTRIBUTION IN MODEL DECKS

Detail of dial gauge arrangement at midspan on the hollow-cored decks i.e. section A-A in fig 5.4b above.
for load before burial in the foundation beams so that the pier reactions could be observed whilst the model was under test.

However, the boiler tube was 4.08 sq.in. in cross-sectional area and with a stiffness of 122.4 lb/microstrain it was not sufficiently sensitive to record the actual changes in pier reaction. These reactions were of the order of 700 lb and could not, therefore, be observed for reasons of sensitivity and consequent accuracy. The sole strain gauge bridge available at the time could only read to within 10 microstrain. The recording of pier reactions was to be only a minor consideration in this study and its loss was not felt to be significant.

5.42 Deflection

5.421 Dial Gauges

Twelve 2" travel Mercer (0.001") dial gauges were indented together with the corresponding magnetic bases and clamps for use in these model studies. Two 3" x 1½" RSJ steel frames were made to stand under the model, to which the dial gauges were held by their magnetic clamps. Any position could be gauged by this system, but only the quarter (Q), three-quarter (3Q) and mid-span (H) deflections were observed. Refer figs. 5.6 a and b for the arrangement of the dial gauges under the model spans. A special arrangement was used for SHC in which gauges were placed on either side of each shear key to observe shear key behaviour. See Plates 8B, 11B and 12.

5.422 Precise Levelling

Whilst the dial gauges were adequate for load tests within the elastic range they were not satisfactory for ultimate tests where there was the danger the gauges should be damaged if the deck suddenly collapsed. In this case deflections were noted by levelling the deck with a staff and an Ertel INA automatic precise level.

5.5 Test Programme and Procedure

5.51 Tests on Individual Beams

5.511 Tests for Stiffness and Camber

All thirty-seven log beams were individually tested within their elastic range (Plate 7B). A point load was applied at midspan and deflections read at the quarter and midspan point. The load was applied in three equal increments up to 1000 lbs and then increased to just take out the camber in each beam. Deflections noted for each of these load increments gave sufficient information to obtain the flexural stiffness of the log beams and also the initial camber due to prestress and dead load. No similar tests were made on the hollow cored units.
The amount of initial camber in these units was found to be a function of at least two time variables: age at release of prestress into the beam and age at date of measurement of the camber (refer Appendix 11).

For example, the average camber in those beams released after two days curing was 0.415", whilst the average for those released at one day was 52\% greater, i.e. 0.633". Other variables affecting camber include mix properties, dead load and initial prestress.

5.5.12 Tests for Failure Load

As detailed in Table 5.5 one extra log beam was made and a load test to destruction was performed on it. Results are summarised below:

Cracking Load (first crack) : 520 lbs.
Collapse Load : 3250 lbs.

5.52 Test Programme for Model Decks

As can be seen from Table 5.6 of Sec 5.333 the Test Programme contained eleven series of tests designated I to XI. The programme, therefore, embraced both elastic and ultimate load tests on three separate model decks.

Load testing commenced in November 1964 and was finally completed in December 1966. Time out was taken during this two year period to organise and conduct the load tests on the full size bridge at Slippery Creek and also to develop and test the Finite Element Theory for Skewed Anisotropic Bridge Decks which forms the basis of the second volume to this Thesis.

5.5.21 Summarised Purpose

The eleven series of tests were conducted to establish:

1. the validity of the proposed transfer matrix theory, and the usefulness of other design methods;
2. the effect of transverse prestress;
3. the effect of the type of precast unit (solid or hollow core);
4. the effect of the span/width ratio;
5. the effect of the 15 degrees of skew;
6. the effect of the shape of shear key, and
7. the approximate ultimate load capacity of the decks.

Refer also to Table 5.6 of Sec 5.34 and also Table 5.8 below:

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Test for</th>
<th>Test Series</th>
<th>Results in Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Theoretical Verification of Transfer Matrix Theory</td>
<td>I, VI, VIII &amp; X</td>
<td>6.2</td>
</tr>
</tbody>
</table>
### Table: Test Purpose, Test for, Test Series, and Results

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Test for</th>
<th>Test Series</th>
<th>Results in Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Transverse Prestress</td>
<td>All series</td>
<td>6.3</td>
</tr>
<tr>
<td>3</td>
<td>Logs vs Hollow-core</td>
<td>I-V &amp; X</td>
<td>6.4</td>
</tr>
<tr>
<td>4</td>
<td>Span/Width Ratio</td>
<td>VI, VIII &amp; X</td>
<td>6.5</td>
</tr>
<tr>
<td>5</td>
<td>Fifteen (15) Degrees of Skew</td>
<td>All series</td>
<td>6.6</td>
</tr>
<tr>
<td>6</td>
<td>Shear Key Shape</td>
<td>VI - XI</td>
<td>6.7</td>
</tr>
<tr>
<td>7</td>
<td>Ultimate Behaviour</td>
<td>I, IV, V, &amp; X</td>
<td>6.8</td>
</tr>
</tbody>
</table>

5.53 Test Procedure for Model Decks

Single point loads were applied to the deck and in general deflections read for each load application. The scaled design wheel load was 1000 lb for the model but it was found that deflections under this load were negligible and that errors due to experimental technique would be significant in proportion to the actual deflection of the deck. For this reason the applied load was increased to 4000 lbs and a recommendation made to the Ministry of Works that the proposed load arrangements for the tests on the full scale bridge be also increased to 64,000 lb. (ref secs 4.13 and 4.31).

In general, therefore, for all tests except those under ultimate load, the load was applied in two stages: 2000 lb and 4000 lb. Hence with deflection readings at zero load and at each of these applied loads a check on possible experimental error was maintained - the deflection at 4000 lb should always have been twice that at 2000 lb. Constant zero checks eliminated error due to short term creep and dial gauge malfunction.

The loads were positioned over the centre of every beam at the quarter (Q), three-quarter (3Q) and midspan (M) transverse profile. Deflections were read under the deck at similar positions, refer sec 5.421 and figs 5.4 a and b.

During test to destruction it was necessary to remove the five ton proving ring (sec 5.23) and replace with a 25 ton Tangye Jack with pressure gauge. This increased the possible range of both applied load and deflection. Deflections were noted here for every 1000 lb increase in load until either collapse of the deck or the maximum capacity of the load system had been reached.

5.6 Determination of Actual Scale Factors

5.61 Determination of K

From equations (5.2) (5.6) (5.4) and (5.5) we have the scale factors
for the model

\[ K = \frac{E_s}{E_m} \]  \hspace{1cm} (5.6)

\[ \lambda = \frac{l_s}{l_m} \]  \hspace{1cm} (5.2)

and

\[ \alpha = K \lambda, \quad \gamma = K / \lambda \]  \hspace{1cm} (5.4), (5.5)

Thus \( \lambda = 4 \) as already noted in equation 5.8a and ideally \( K = 1 \).

It remains here to determine whether \( K \) was in fact unity, i.e. to determine \( K \) for the model in the "as built" condition.

This required the determination of the elastic moduli of the concrete used in the model and in the full scale bridge structure. Although a similar mix had been used in both structures it could not be guaranteed that these moduli would be equal (sec 5.131 (2)). Consequently, the three remaining \( 8'' \times 4'' \) control blocks from each beam of each deck were tested for \( E \) and \( f_c \) (elastic modulus and crushing strength) at an age similar to that at which the respective decks were tested.

5.611 E Transducer Mk II

A transducer was, therefore, developed to enable the determination of \( E \) to be made quickly and with reasonable accuracy on the 136, \( 8'' \times 4'' \) blocks that were available. This transducer (the Mark II model) is shown in Appendix 12, where all relevant dimensions and strain gauge bridge circuits are given. See Plate 17.

5.612 Results from the E Transducer

Once calibrated, the strain increment for a 20 ton increase in load on a \( 8'' \times 4'' \) block was noted in the transducer and this result fed into a computer programme for statistical analysis (refer Appendix 4 and also sec 5.311).

Such analysis by the 1620 gave the mean and standard deviation for the raw data (\( E \) and \( f_c \)) and also made a Chi-square test for Goodness of Fit between results observed and code recommendations for \( E \) given \( f_c \).

The following results were obtained:

for the model: \( E_m = 5.82 \times 10^6 \) (mean); 7.22\% (std. dev.)

for Slippery Creek: \( E_s = 5.70 \times 10^6 \) (mean); 3.78\% (std. dev.)

(The Goodness of Fit test indicated that Jensen's formula:

\[ E = \frac{6.0}{1.0 + 2000/f_c} \]  \hspace{1cm} (5.10)

gave the best prediction for \( E \) given \( f_c \), refer Appendix 4.)
There \( K = \frac{5.70}{5.82} = 0.98 \) \hspace{1cm} (5.11)

This is very close to the desired value of unity and because these values for \( E \) are at best, only first approximations, it was sufficiently accurate in this work to assume that \( K \) was in fact unity.

5.62 **Summarised Scale Factors**

From the above we have:

\[
K = 1, \quad \alpha = 4 \\
\alpha = 16, \quad \gamma = \frac{1}{4}
\] \hspace{1cm} (5.12)

hence the load scale factor was \( 1/16 \)

and the deflection scale factor was \( 1/4 \)
CHAPTER: SIX

TITLE: RESULTS OF MODEL AND FIELD TESTS ON THE SLIPPERY CREEK MULTIBEAM BRIDGES.

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       Collapse Mechanism.
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6.9 Miscellaneous Results.

6.91 Pier Reactions and Pier Deflections.

6.92 Experimental Check on the Stiffness of a Single, Full-Size,
     Hollow-cored Beam.

6.93 Check on Experimental Technique using Influence Lines.
6.1 Model Correlation with Full Scale Structure

This chapter is a joint presentation of the results of testing both the model and full scale structure and it is necessary to first demonstrate the validity of the model in order that prototype behaviour may be confidently predicted from model test results. These model-to-prototype forecasts were often made, either for confirmation of prototype behaviour or for information on parameters not investigated on the bridge itself.

Such validity was proved by using the model to predict deflection profiles for the full scale bridge under similar load and gauge positions. Comparisons were then made between the predicted and observed profiles and it was found that good correlation was achieved, after the necessary adjustments due to scaling (equations 5.12) had been made. Such correlation is illustrated in figs. 6.01, 6.02, 6.03 and 6.04 where predicted transverse deflection profiles from the model are compared with those observed at Slippery Creek for the spans and load positions as summarised in Table 6.1.

<table>
<thead>
<tr>
<th>DECK TYPE</th>
<th>SPAN</th>
<th>LOAD POSITION</th>
<th>MAXIMUM DEFLECTION (IN)</th>
<th>REFER ALSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOG</td>
<td>SA</td>
<td>Edge</td>
<td>0.222 0.230 3.5%</td>
<td>6.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Centre</td>
<td>0.118 0.125 5.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SHC VI</td>
<td>Edge</td>
<td>0.564 0.470 20.0%</td>
<td>6.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Centre</td>
<td>0.440 0.378 16.4%</td>
<td></td>
</tr>
<tr>
<td>HOLLOW-CORED</td>
<td>SHC VIII</td>
<td>Edge</td>
<td>0.382 0.340 12.4%</td>
<td>6.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Centre</td>
<td>0.263 0.262 0.4%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SHC X</td>
<td>Edge</td>
<td>0.356 0.355 0.1%</td>
<td>6.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Centre</td>
<td>0.160 0.158 1.1%</td>
<td></td>
</tr>
</tbody>
</table>

In spans E and F, discrepancies up to 20% will be noted indicating poorer model-prototype correlation than desired. This was accepted, since it was considered to be due to the poor seating of both the model and full scale decks (secs 6.442, 6.833) and the defective action of the shear keys (sec 6.7, also fig 6.03).
Disregarding these lesser width spans the maximum error was of the order of 6% and this was considered to be satisfactory. The use of the model to predict at least the elastic behaviour of the full scale, full width bridge decks had therefore been established.

6.2 Verification of the Transfer Matrix Analysis Method for Multibeam Bridge Decks

Using the PDQ FORTRAN programme OPUS TWO, developed for the analysis of multibeam bridge decks by the transfer matrix theory, both the log and hollow-cored multibeam decks were analysed for deflection and load distribution (refer Chapter 3, sec. 3.4; also Appendix 3 for details of OPUS TWO). These computed results were then compared with the corresponding results obtained from the physical testing of both the model and full scale structures.

6.2.1 Transverse Deflection Profiles

The same deflection profiles as used in sec. 6.1 for model correlation have again been chosen to illustrate the application and agreement attainable with the matrix theory. The theory was therefore checked against experimental results from four different decks:

1. an 18-beam, log multibeam deck: spans CO and SA
2. a 10-beam, hollow-cored multibeam deck: spans D and SHC
3. a 5-beam, hollow-cored multibeam deck: spans E and SHC
4. a 3-beam, hollow-cored multibeam deck: spans F and SHC

Each deck was considered in two load conditions: midspan edge and midspan centre positions.

Confirmation of the transfer matrix theory was also provided by the model results as plotted in figs. 6.01, 6.02, 6.03 and 6.04.

An examination of these profiles will show that for both full width spans of the log and hollow-cored decks excellent agreement has been obtained between observed, model predicted, and theoretical results. The maximum discrepancy between theory and observed behaviour is seen to be of the order of ±12%; that is, the theory has over predicted the maximum deflection of the deck by 12%. This difference does however, provide a safety margin and is due to the fact that the decks were not as "articulate" as assumed by the theory but still retained some transverse flexural strength due to either the reinforced deck slab or the slight transverse stressing.

The agreement is not so apparent for the decks of lesser width: spans E and F in figs. 6.03 and 6.04. Here other effects have become evident beside the simple mechanism assumed in the analysis. In span E (0.5 of the full width) differential slip between adjacent beams was observed as indicated
by the stepped transverse profile under the edge loading. This slip was
considered to be due to a lack of sufficient transverse prestress to
maintain the satisfactory action of the shear keys; such keys being able
to open and slide under the influence of abnormal vehicle loads. See
also secs. 6.32 and 6.7. Had this slip not occurred, better agreement
between theory and observed behaviour may well have been indicated. The
maximum difference recorded during a midspan centre loading condition was \(-4\%\).

Again in span F (0.3 of the full width), although little
differential slip occurred, apparent agreement is masked by the poor model
correlation (refer sec. 6.1). The maximum difference in profiles for the
centre loading condition was \(-2\%\) and for the edge loading condition was
either \(-2\%\) or \(-10\%\) depending on which set of observed profiles was assumed
correct. Such variation in observed behaviour was not typical of the
experimental results as a whole and was perhaps due to the poor seating of
this deck on the pier caps; this became evident only after completion of the
tests.

When the figures 6.01 to 6.04 are reviewed and the extreme
difference between theoretical and observed behaviour noted at 12\% it is
evident that the Transfer Matrix Analysis procedure has been satisfactorily
demonstrated and that its use for the analysis of multibeam decks has been
justified.

6.22 Theoretical Load Distribution Histograms

Following the verification of the transfer matrix theory as in the
previous section, the theoretical load distribution histograms were calculated
using OPUS TWO and a third PDQ FORTRAN programme OPUS THREE. OPUS THREE
contained the facility for plotting either the transverse deflection profile
or the load distribution histogram by the 1620 Console Typewriter. The actual
load carried per beam was computed from the difference in shear forces as
indicated by successive state vectors. Percentage distribution coefficients
were subsequently found.

6.221 Results

Figures 6.05, 6.06 and 6.07 show the load distribution histograms
for selected load positions on spans C0, D, E and F.

Examination of these figures will show that the maximum load
carried in any one log beam was 17.65\% of the total applied load. The
corresponding figure for the hollow-cored deck was 26.33%. Therefore the
load carried per foot width of deck was 8.84% for the log beam deck and
7.16% for the hollow-cored deck.

The load was therefore more efficiently distributed amongst the
several beams of the hollow-cored deck than in the log deck. As a result it
would seem more prudent to use several wide units in a multibeam deck rather
than a large number of narrow ones for the same depth. However, practical
considerations dictate the most suitable width as discussed in a later section
(6.43) which concerns the optimization of the beam shape. Even from a
theoretical view point restrictions on width are desirable to limit the transverse bending and torsional stresses.

6.222 Comparison with the Codes

6.222.1 AASHTO Standard Specifications for Highway Bridges - 7th Edition 1957

The AASHTO recommendation for live load distribution in concrete slabs,
Case C, spans over 12 feet, is based on the Westergaard Method - see also
sec. 2.322.

The width of slab over which a wheel load is distributed is given by

$$E = \frac{10N + W}{4N}$$

where N is the number of lanes of traffic on the bridge
and W is the width of roadway between curbs on the bridge.

This recommendation (also adopted in the Ministry of Works Bridge Manual)
was used by the manufacturers of the hollow-cored units in their design for
spans D, E and F.

For the Slippery Creek bridges: \( N = 3, \ W = 36\)ft and therefore

$$E = \frac{30 + 36}{12} = 5\frac{1}{2}\)ft.$$

Thus 100% of the applied load is considered to be distributed over a width
of 5\(\frac{1}{2}\)ft. Figs 6.05 and 6.06 indicate that for the log and hollow-cored
decks only 43.66% and 36.79% of the applied load is in fact retained within
this area, the balance \(16\)% is effectively distributed throughout the
remainder of the deck width.

Alternatively this AASHTO Specification requires that the load
carried per foot width shall be 100/5.5 or 18.2% of the applied load. It is
thus evident that better load distributions (8.84% and 7.16% per foot width)
have been obtained for these multibeam decks than is in fact recommended by AASHO for an isotropic slab of the same dimensions. The fault lies with the AASHO formula, which apparently contains a safety margin of at least 100%.

6.222.2 ACI Standard (ACI 711-58)

Section 412 of this Standard for Precast Floor and Roof Units suggests that where heavy concentrated loads are to be expected, these may be considered as uniformly distributed over three units but not exceeding a total width of 0.4 of the clear span distance — see also sec 2.321.

The governing width here, was the width of three units for both decks. It was therefore implied that 100% of the applied load was to be withstood by those three units immediately next to or under the wheel load. Again figures 6.05 and 6.06 indicate that 46.99% and 62.33% of the applied load was sustained by these units. Again there is overestimation by the code although it is not so apparent if the distribution per foot width is considered. This ACI Specification would give 9.1% per foot width uniformly distributed over the three units (11ft); the corresponding value obtained here for the same units reduced from 7.16% (max) to 4.10% over the same width.

6.222.3 Others

A further recommendation reported by Roesli et al in his paper on the testing of the Centerport Bridge suggests that each beam or unit is to be designed as though carrying 80% of the right or left wheel loads of an H20 - S16 truck. The maximum predicted by figs 6.05 and 6.06 is 17.65% and 26.33% depending on the width and type of the beam. The conservative nature of the above recommendation is evident; a conclusion also reached by Roesli who reports maximum load-per-beam percentages in the range, 20 to 30%.

6.223 Verification of the Load Distribution Histogram by Model Test

During an ultimate load test on the Model Deck SA (series I, Table 5.6), it was observed that the first crack appeared at a load of 5,600 lbs (see sec. 6.811). Now a point load of 1,100 lbs on a single, 4½ inch deep beam would just crack the section. It may therefore be implied that the percentage of load carried by the cracked beam in the model deck was 1,100/5,600 or 19.7% of the applied load. This beam was in fact the outside edge beam of the deck for which the transfer matrix method predicted a load distribution percentage of 19.05%. Favourable agreement had thus been obtained.

6.23 Comparison of the Transfer Matrix Analysis with Other Methods
6.231 Comparison with the Distribution Coefficient Method of Rowe, Morice
and Little

The method of analysis due to Rowe, Morice and Little was described
in sec. 2.12 of Chapter 2 and is known as the Method of Distribution Coefficients
or the Guyon-Massonnet Method.

Figures 6.08a and 6.08b are presented to illustrate the application
of this method to the multibeam log deck. It was felt that, if such a method
was applicable, it would be shown to its best advantage by comparison with
the behaviour of the log beam decks, since of the two types considered, this
deck was the closer to the orthotropic plate assumed by the method.

In fig. 6.08a the distribution coefficient profiles for two separate
load positions are compared. Since the deck exhibited 15 degrees of skew
there was a certain amount of difficulty in determining the mean deflection
of the deck, which was necessary to find the distribution coefficients.
This difficulty, already discussed in sec. 2.122 and noted by Morice and
Little, is reflected in the two curves for observed behaviour that are
plotted for each load position. One curve assumes the skew span and right
width for computing the mean deflection and the other assumes the right span
and skew width in this calculation. Other combinations are possible but
these two present upper and lower bounds respectively to the mean deflection
and the true observed distribution coefficient profile is thus bounded by
these lines.

The theoretical profiles were calculated assuming a uniform slab
\((i = j)\) of the dimensions equal to those of the model deck SB. \* The value
of \(\theta\) was taken as 0.53 (skew span and right width adopted) and the
coefficient values for \(K_0\) and \(K_1\) read from the graphs published by Rowe.

Another PDQ FORTRAN programme was written, OPUS ONE, to accept
the matrices \(K_0\) and \(K_1\) and calculate \(K_\alpha\) (9 x 9 square arrays). Since the
value of the parameter \(\alpha\) was unknown, the programme was written so as to determine

\* If the model deck SA had been analysed by this method a uniform depth of
slab could not have been assumed and the value for \(\alpha\) would have been
as high as 13.7.
Kα for any desired value for this parameter. In particular this value could be incremented in any sized step (usually 0.1) from 0.0 up to 1.0 or alternatively any one value may be inserted if desired (see Appendix 2 for programme description and listing).

Figure 6.08a shows a selection of theoretical profiles for the following values of α: 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0, as computed by OPUS ONE. It can be seen that no one particular value of α will satisfy the observed behaviour, neither for the edge nor for the centre load positions. Values of α less than 0.7 indicate that uplift should occur on the edge remote from the load. This was never observed for it cannot occur in an articulated deck.

The distribution coefficient method is thus shown to be unsatisfactory for the analysis of this multibeam deck, for the reason that it was not possible to theoretically or experimentally predict the value of necessary for the analysis to be completed.

Figure 6.08b presents the distribution coefficient profiles for the model deck SB when under the influence of 44% transverse prestress. These profiles, when superimposed over figure 6.08a, indicate a better agreement between the distribution coefficient method and the observed behaviour. This is not unexpected since the transverse prestress promotes isotropic slab action (α = 1) for which this method of analysis was developed.

Comparison of figures 6.08a and 6.01 will indicate the superiority of the transfer matrix analysis method as proposed in this thesis over the distribution coefficient method of Rowe, Morice and Little.

6.232 Comparison with the Relaxation Method of Norman and Nathan

The Relaxation Method was also described in Chapter 2 (Sec 2.23) and is used here to predict the transverse deflection profile of the hollow-cored multibeam deck (span D) under edge loading. Figure 6.09 presents the results of this analysis along with that predicted by the transfer matrix method and that actually observed during the Slippery Creek Bridge tests.

Table 6.2 lists the necessary data for the relaxation process. It will be noted that the ratio of the bending stiffness to the torsional stiffness is large and therefore the carry-over factor is close to unity (-0.95). The convergence of the relaxation procedure was therefore very slow
and tedious to perform by hand. Thirty cycles of relaxation were required before the out of balance in holding forces was reduced to less than \( \frac{3}{8} \) of the applied load.

The results, however, show good agreement with those observed and compares favourably with the solution given by the transfer matrix method. This latter correlation was to be expected since identical assumptions were made in the initial stages of these two very different methods of analysis. It is considered, however, that the matrix method is superior, particularly where computer facilities, which need not be extensive, are available. This approach is independent of a convergence rate and gives an exact solution in four seconds on an IBM 1620 machine. Hand calculation for a similar problem takes just on one hour (see Appendix 8) whereas the corresponding time for the relaxation method is about 2\( \frac{1}{2} \) hours.

**TABLE 6.2**

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE FOR HOLLOW-CORED BEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( 1.484 \times 10^{14} )</td>
</tr>
<tr>
<td>( S )</td>
<td>( 54.91 \times 10^{14} )</td>
</tr>
<tr>
<td>( P = R + S )</td>
<td>( 56.394 \times 10^{14} )</td>
</tr>
<tr>
<td>( Q = R - S )</td>
<td>( -53.426 \times 10^{14} )</td>
</tr>
<tr>
<td>Distribution Factors</td>
<td>0.500 (all beams identical)</td>
</tr>
<tr>
<td>Deflection Factors</td>
<td>( \frac{1}{P} )</td>
</tr>
<tr>
<td>( \frac{Q}{P} )</td>
<td>( 0.01773 \times 10^{-4} )</td>
</tr>
<tr>
<td>Carry Over Factor</td>
<td>(-0.95)</td>
</tr>
</tbody>
</table>

6.3 The Effect of Transverse Prestress

The effect of both ordinary and prestressed transverse reinforcement was studied in tests on both the full sized bridge (sec 4.52) and the model (sec 5.34). The results are illustrated in figs 6.10, 6.11, 6.12 and 6.13 for the log beam deck and fig 6.14 for the hollow-cored deck.

6.31 Effect on the Behaviour of the Log Beam Deck

It can be seen at once that the effect of ordinary reinforcement (from zero up to fifteen, 1" deam. ms. bolts threaded transversely through
the deck) is very slight. Both figs. 6.10 and 6.11 indicate only small variations in the transverse deflection profiles over this range of reinforcement. The transverse distribution of load has been virtually unaffected by the insertion of the bolts into the deck (span A).

A similar conclusion can be reached concerning the effect of small amounts of transverse prestress. The maximum applied at Slippery Creek was only 2% of the longitudinal prestress in the beams and as can be seen only a small improvement in the distribution of load has been obtained. In fact the distribution when threaded with 15 bolts is almost identical to that transversely stressed to 2%; the maximum deflection being only 8.5% less in Row 5 and 14.2% less in Row 1 than when there are no bolts in the deck at all.

It can thus be inferred that neither the bolts nor the 2% of prestress are sufficient to develop any appreciable degree of transverse flexural rigidity and that these decks still behave as articulated plates. It is worth noting, however, that the transverse distribution of load did improve, albeit slight, as the amount of reinforcement was increased.

Also plotted figs 6.10 and 6.11 is the model prediction for the case where the transverse prestress is equal to 44% of the longitudinal prestress. These figures are based on the results of test series V (model SB).

Marked improvement in the transverse distribution of load was indicated particularly under a central load. This improvement takes the form of a 43.4% reduction in the maximum deflection of the deck. It is immediately obvious that the deck, under the influence of larger amounts of transverse prestress, to the extent of 44% of the longitudinal, acted as though a fully isotropic slab. In such instances the classical theory for the analysis of such slabs would be applicable. Refer Chapter 2 and also sec. 6.231, figs 6.08a and 6.08b.

It will be seen further, in figs 6.12 and 6.13, that the behaviour of the model deck was not significantly altered as the transverse prestress was increased from 22% to 44% of the longitudinal prestress. In these figures comparative behaviour is shown for the model under central and edge point loads when this deck (SB) was only 3\(\frac{1}{2}\) in deep.
It can therefore be summarised that:

1. 2% of transverse prestress was not sufficient to cause adequate friction and compression between adjacent units to prevent them separating at their lower nibs and the development of hinge action.

2. 22% of transverse prestress was sufficient to develop such friction and compressive stress as to promote isotropic slab action. The actual dividing line within this range (2 - 22%) between articulate and isotropic behaviour, is not clear and is difficult to predict theoretically.

3. The effect of mild steel bolts as transverse reinforcement was negligible and certainly not sufficient to warrant the expense or labour of installation.

6.32 Effect on the Behaviour of the Hollow-cored Beam Deck

It has already been seen in fig 6.03 and discussed in sec 6.21 that the effect of transverse prestress on the behaviour on the hollow-cored decks was to maintain satisfactory action of the shear keys. It could not be proved during the full scale tests that such was the cause of the differential slip noticed in the deflection profiles of fig 6.03. To give more information on this problem the model studies were extended to include a series of tests with no transverse prestress in the deck whatsoever.

A typical set of results under these conditions is given in fig 6.14. For comparison both fully stressed and non-stressed profiles are plotted, although there were not a sufficient number of dial gauges to obtain full profiles for the nonstressed decks. Nevertheless adequate information was obtained to record the same stepped profiles and indicate that slip past the keys was occurring. The amount of slip shown in fig 6.14 for the model deck was much greater than that observed on the full scale deck. The maximum observed in the field, where it is not known how much prestress was lost out of the central cable, was 14.8% of the theoretical deflection for that joint. However, the maximum slip observed in the model occurred during a central load and was measured at 73.2% of the observed deflection at the same joint after the full prestress had been applied. The largest slip (27.4%) observed when the deck was loaded on the left hand edge beam occurred at the same joint as above.

The question of shear key action and the mechanism of slip is further discussed in sec 6.7, but it is stated here that the transverse
The maximum changes in both deflection and strain were observed during an midspan edge, point loading test and one both seen to be about 34.7% greater in the hollow-cored decks than in the log beam decks. The increase in the "minimum" deflection observed can also be seen to be 29.4% of the log deck deflection and occurred during a central lane loading test.
These results are to be expected since the hollow-cored deck was about 34% lighter (weight per foot width) and 34% less rigid less rigid (flexural stiffness per foot width).

Figure 6.15 presents comparative deflection and strain profiles observed during the full scale tests. Figure 6.16 compares the theoretical load distribution curves for the two decks. In order that this comparison be valid the histograms of figs 6.05 and 6.06 are redrawn as smooth curves representing the load carried per foot width. Both the areas under these curves equal the total applied load.

6.42 Comparative Load Distributing Properties and Beam Optimization

The hollow-cored deck has already been seen (sec 6.22 and fig 6.16) to distribute the load more efficiently than the log beam deck. The appropriate figures in terms of maximum load carried per foot width of deck are 8.84% for the log deck and 7.16% for the hollow-cored deck.

This improvement in load distribution leads to the idea of optimization, the purpose of which is to determine the most efficient beam section. This study requires first, the isolation of that parameter which relates beam properties to load distribution and secondly, the examination of this parameter and the nature of its relationship to the load distribution characteristics of the deck. It has, however, been outside the scope of this project to develop this concept much further, but the following is suggested as a guide for future work.

It appears from the analysis given in Chapter 3 that the load carried per unit width is a function of

$$\gamma = \frac{1}{2a(1-Y)}$$

where X and Y are the flexural and torsional flexibilities of a single beam and 2a is its width.

X and Y have been defined in sec 3.24 and are seen to be functions of the dimensions of the beam, its span and its material properties. Therefore, for a given material, such as concrete, it should be possible to find a set of dimensions which will minimise \(\gamma\) for a given span, where \(\gamma\) is defined as:

$$\gamma = \frac{1}{2a(1-Y)}$$

For the log and hollow-cored beams examined above the value of \(\gamma\) was \(2.02 \times 10^3\) and \(1.39 \times 10^3\) respectively which corresponds to the maximum load distributions of 8.84% and 7.16% per unit width. Thus
the lower value of \(\gamma\) for the hollow-cored beams does, in this instance
at least, correctly predict a better load distribution characteristic for this deck.

Several practical considerations should, however, be borne in mind when proposing optimization. These include:

1. the total weight, which is of importance when transporting and placing the beams,
2. the natural period of vibration, which affects the dynamic characteristics of the deck under the impact of heavy traffic and
3. the standard of the factory control on dimensional tolerances and concrete quality. See also sec 6.44.2.

6.43 Model Tests on a Modified Log Beam Deck

At the conclusion of the first series of ultimate load tests on the Model SA it was felt that before proceeding with a second model of the same dimensions it would be profitable to check the behaviour of a log beam deck without the in-situ reinforced deck slab; i.e. at a depth of only 3\(\frac{1}{2}\)". Transverse shear connection was provided by 14, non-stressed, H.T. 0.276" diam. tendons threaded through the deck. The joints between the beams were filled with a sand-cement grout and tested through an identical range of loads as for the previous Model SA - except for the ultimate load tests. The results of these tests have already been presented in figs 6.12 and 6.13, where a satisfactory performance is indicated. The deflection profiles of this, a reduced log beam deck were now of similar shape and magnitude to those of the model, hollow-cored deck as plotted in fig 6.14.

A partial ultimate load test indicated cracking loads of 4,250 lbs and 7,500 lbs for the first and second cracks in the outside beam which was directly under the load. (Sec 6.821) At the time of these tests, the deck had been stressed to 44% of the longitudinal prestress but was still only 3\(\frac{1}{2}\)" deep. The overload safety factor to the first crack was therefore 4.25 and to the second crack, 7.5; the design load was taken as 1,000 lbs, being 1/16th of a 16 Kip wheel load of the AASH0,H20 S16 T16 truck. Both cracks closed on the release of the load and the deck continued to perform elastically in subsequent tests.

This simplified log beam deck was therefore shown to be at least as satisfactory as both its parent deck and the hollow-cored deck. Again practical details must be remembered and the difficulty of applying 22% transverse prestress or even just threading 14 transverse tie rods is
considered to be a handicap to the system — particularly in isolated areas
where stressing techniques may be unfamiliar or on sites with difficult
access to the sides of the bridge.

6.44 Advantages and Disadvantages of the Hollow-cored Deck

6.441 Advantages

The hollow-cored deck is simpler to erect, contains less materials
and requires less labour than the corresponding log beam deck. It is
therefore considerably cheaper and at present day (1966/1967) costs it is
quoted at 25/- a square foot in place — exclusive of the foundation and
substructure but inclusive of stressing, grouting and transportation. This
compares favourably with the 35/- square foot normally tendered for a deck
of equivalent log beams and represents a reduction of about 20% in the total
cost of any one structure. For a bridge the size of that at Slippery Creek
a saving of about £6,500 would have been made if the hollow-cored beams had
been used throughout.

From the structural point of view of resisting and distributing
live loads without overstressing the several members, there appears to be
no reason to prevent the use of hollow-cored beams in future structures.
As will be seen in sec 6.334 the safety factors against collapse for these
decks, whilst less than those for the log beam decks, are reasonable and
adequate.

6.442 Disadvantages

Being hollow, and therefore of a lighter weight, the beams have
a lower natural period of vibration and poorer characteristics under the
impact of heavy fast moving traffic; this behaviour may impair the structural
advantages of the system.

Reference should also be made to the possibility of individual
beams twisting in a cork screw manner along their length due to poor quality
control during manufacture. This may result in the subsequent deterioration
of bond and consequent non-uniform camber. Such behaviour is difficult to
control even under laboratory controlled conditions and particularly so when
the beam width is great. When seated through neoprene pads on to pier caps
the twisted beam is able to rock about one edge and under abnormal loading
the adjacent shear keys may be damaged.
The problem of differential slip which may occur between adjacent beams of this hollow-cored deck has previously been discussed in sec 6.32. Sufficient transverse prestress should be provided to prevent the keys sliding, for if this is not checked, repeated loading under heavy traffic may well cause deterioration of the keys and eventual total failure. Refer also sec 6.7 for the mechanics of shear key action.

6.5 The Effect of Span/Width Ratio

The span/width ratio was studied both in the field and on the model but only for its effect on the behaviour of the hollow-cored decks.

As expected and seen in figs 6.02, 6.03 and 6.04, the maximum deflection increases with an increase in span/width ratio for the same load and load position. Table 6.4 below summarises the range of values observed, together with the maximum theoretical load carried per beam in each case.

<table>
<thead>
<tr>
<th>SPAN</th>
<th>SPAN/WIDTH RATIO</th>
<th>MAX. DEF. OBSERVED</th>
<th>INCREASE IN DEF. OR LOAD</th>
<th>MAX. LOAD CARRIED PER BEAM</th>
<th>LOAD PER BEAM UNIFORM DISTRIB.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1.0</td>
<td>0.310</td>
<td>0.00</td>
<td>0.00</td>
<td>26.33%</td>
</tr>
<tr>
<td>E</td>
<td>2.0</td>
<td>0.325</td>
<td>4.84%</td>
<td>9.58%</td>
<td>28.85%</td>
</tr>
<tr>
<td>F</td>
<td>3.33</td>
<td>0.408</td>
<td>31.60%</td>
<td>41.40%</td>
<td>37.23%</td>
</tr>
</tbody>
</table>

A sharp increase in the value of the maximum sustained load and deflection is observed as the span/width ratio exceeds 2.0. In fact the load distribution in span F is almost perfect, that is, the load is almost shared equally amongst the beams comprising the deck. This is depicted in figs 6.07a and 6.17 but for convenience the relevant figures are listed below:

For an outside edge beam loading the distribution was: 37.23%, 33.04% and 29.73%.

Again for an inside beam loading the distribution was: 33.04%, 33.92% and 33.04%.

Both sets were computed using the transfer matrix analysis and OPUS THREE.

It is therefore concluded that this deck (Span F) could well be designed as a simple beam. However, for Span E such a procedure would lead to an overestimation of 8.85% in the maximum load carried per beam. This represents 44.2% of the uniform applied load and this practice would not be acceptable under these conditions.
It may therefore be generally concluded, though on somewhat small evidence, that for span/width ratios greater than or equal to 3.3 the deck may be designed as a simple beam with a uniform distribution of load.

6.6 The Effect of 15 Degrees of Skew

As described in sec 4.54 of Chapter 4, tests were carried out on the full scale bridge to determine the effect of 15 degrees of skew on the behaviour of the deck.

It was found that no appreciable difference in deflection readings could be observed for the two different load conditions - the wheels of the Compactor at right angles to the abutments and the wheels parallel to the sides of the deck. It was therefore thought that the angle of skew was so small that its effect could not be seen in these comparative deflection profiles and the tests were discontinued.

It was not, therefore, until final processing of the deflection profiles that the effect shown in fig 6.18 was discovered. In this graph the transverse profiles are plotted for the quarter (Q) and threequarter (3Q) gauge lines for Span D under both central and edge point loads of 64,000 lbs. Similar profiles were found in Spans A and C and to a lesser extent in E and F.

If the bridge was right, that is, of zero angle of skew, then these profiles would be coincident for a particular load position. The discrepancy between the profiles is due entirely to the skewed geometry of the deck and can be shown to be given by the following equation:

\[
\frac{w_Q - w_{3Q}}{w_Q} = 1 - \left(\frac{d^2 + c^2 - 2dcs\sin\theta}{d^2 + c^2 + 2dcs\sin\theta}\right)^\frac{n}{2}
\]  (6.2)

where \(w_Q\), \(w_{3Q}\) represent the deflections at the corresponding quarter and threequarter points respectively,

- \(c\) is the quarter span length,
- \(d\) is the distance from the load to the point under consideration, measured parallel to an abutment,
- \(n\) is arbitrary constant - probable value is 2,
- \(\theta\) is the angle of skew of the deck.

See also fig 6.18 for a sketch of a skewed deck and its geometry.

When \(d = 0\) equation 6.2 reduced to:

\[
\frac{w_Q - w_{3Q}}{w_Q} = 1 - (1)^\frac{n}{2} = 0
\]  (6.3)
that is, there should be no difference in deflection between the quarter and threequarter points of the loaded beam. Examination of fig 6.18 will not only show this to be true but also the fact that as \( d \) changes sign so the profiles intersect (at \( d = 0 \)) and \( w_Q - w_{\frac{3}{4}Q} \) also changes sign.

The maximum observed value of \( \frac{w_Q - w_{\frac{3}{4}Q}}{w_Q} \) was about 36\( \frac{1}{2} \%) \) and occurred in the beam furthest from the load, that is at maximum \( d \). The maximum error, therefore, in predicting deflection for a 15 degree skew bridge deck assuming it was a right deck - which is common design practice for decks under 20 degrees of skew - would be \( \pm 18\% \).

Although the equation (6.2) is neither proved nor rigorously defended, further data, such as that point in the deck where the maximum error in the estimation will arise, may be found by the partial differentiation of equation (6.2) with respect to the independent variable \( d \). Also, the effect of the span (4c) and greater angles of skew (e) may be determined.

It may, however, be generally concluded that the effect of skew on the behaviour of any bridge deck is not important for angles less than say 15 degrees or for large span/width ratios. In either case the deck tends to span parallel to the free edges and a sufficiently accurate analysis may be made by assuming the deck to be right. However, for angles of skew in excess of 15 degrees and/or for small span/width ratios such an approximation would not be valid since the deck now tends to span in a direction perpendicular to the abutments, that is across the shortest span. The angle of skew now assumes the greatest importance and no approximation based on solutions to a right deck will give design stresses of sufficient accuracy.

6.7 The Effect of Shear Key Shape

The effect of the shape of the shear key between adjacent hollow-cored units on the behaviour of the deck is now discussed.

Figs 6.03 and 6.14 illustrate the differential slip that may be observed in such a deck and an explanation of this mechanism is made using fig 6.19.
Consider the sectional elevation of figure 6.19 made through two adjacent beams and shear key ABCDEFGHK. It is postulated that, should there be little or no transverse force in the cable RST then, beam 2 is able to slide down the inclined face HG, thus separating from the key and its neighbouring beam at the face KHGFES. Both horizontal and vertical movements are therefore proposed, the vertical movement being the differential slip observed in figs 6.03 and 6.14. On the release of the load the stored strain energy in the deflected beam is sufficient to return the beam up the face GH and the differential slip is therefore recovered.

This idealization, whilst simple in concept, satisfies the observed facts; but for confirmation it was decided to measure the predicted horizontal movement (if any) during the corresponding model tests. Using a 1/10,000th dial gauge, such movement was indeed recorded; with a maximum of 0.0062" being observed at an applied load of 4000 lbs. This is just half of the maximum vertical movement observed during the same test. No such lateral expansion was observed after the deck had been stressed to 7500 lbs. This final result again confirmed that the slip was due to a lack of sufficient transverse prestress.

A further disadvantage of this type of key is its ability to shear off the corners of adjacent beams. This is particularly liable to occur if the beams are poorly seated as in the model and also at Slippery Creek. Cracking in the
upper surface surrounding these keys was observed and is described in sec 6.833. Attention to seating details at pier caps is thus indicated.

Changes in the shear key shape, such as lowering the inclination of the face HG (fig 6.19) until it was horizontal, would prevent the above-mentioned slip and ensure a positive interlocking key. It would, however, be difficult to achieve in practice since the adequate compaction of the concrete under such a surface during the pouring of the keys could not be assured. It would be far easier to ensure sufficient transverse prestress and use the original, wedge-shaped design.

Further tests were conducted at a later date (sec 4.71) on the extensions to the Slippery Creek Bridges to investigate slip in a different shaped key. This key is sketched in fig 6.20 where it is seen to be V-shaped and therefore much more dependent on sufficient transverse prestress for satisfactory performance.

![Diagram of Alternative Shear Key](attachment:fig6.20)

The surfaces AC and BC of the respective beams are formed against chequered steel plate to provide an interlocking surface between beam and key. Contact along AC and BC is maintained by the transverse prestress in the single midspan cable RST.

No appreciable slip was observed during these field tests; the maximum recorded under a 31,000 lb axle load being 0.0034". At first sight these keys seem as serviceable as the wedge-shaped keys discussed above - even more so, since the V-shaped keys are easier to pour. However, should the transverse prestress be lost from the cable RST, as for example in span E at Slippery Creek, then the deterioration in behaviour of this deck would be far greater, since the V-shaped key in itself offers no resistance to applied vertical loads. On the other hand, the wedge-shaped key continues to
operate under such conditions, for even though the keys may slip, the decks will still resist and satisfactorily distribute the live load, for a limited period at least, and failure would not be sudden but gradual providing time for repairs.

6.8 Ultimate Load Behaviour

Tests for ultimate load characteristics were carried out on all model decks though, for reasons already explained in sec 5.143 concerning model similitude, the results of these tests were qualitative rather than quantitative.

6.8.1 Ultimate Load Tests on Model SA

In sec 5.143 the calculation for maintaining dead load equivalence was given and accordingly 6 tons of lead was uniformly distributed over the surface of the model (SA) before the commencement of the ultimate tests (see plate 12).

6.8.11 Edge Loading

Since the midspan, edge beam loading was the most severe condition, the first test in this series was a point load applied in such a position. The load was increased in 1000 lb. increments until at a load of 8000 lb. slow yielding was observed in the deflection of the beam directly under the load. The dial gauges were removed and the behaviour under further increases in load observed by levelling the deck.

Fig 6.21 illustrates the observed load deflection behaviour for beams 1, 5, 9, 14 and 18 in the deck. Beam 18 is the loaded beam and the load-deflection curve for this beam shows that a change in flexural rigidity occurred at 5600 lbs. Such a singular change indicated the formation of a single crack in this beam and further increases in load developed this one crack instead of forming several others. Such a relationship between the various slopes of the load-deflection curves for a prestressed concrete beam and its crack pattern has also been noted by Jones; see also fig 6.22. The formation of a single crack is generally considered to be indicative of a bond failure and such was inferred here for this beam. It first appeared at a load of 5600 lb. which, as shown earlier in sec 6.223, confirmed the theoretical prediction of 5700 lb. (distribution coefficient of 19.05%).

The maximum load applied was 9000 lb. by which time a horizontal shear failure had occurred between the edge beam and the 1" deck slab, together with a localised compression failure in the slab itself. Further attempts to increase the load separated this beam further from the remainder of the deck.
The load continued to drop away until it was only supported by the beam itself. Large deflections in this beam were obtained, but even so, a load of approximately 2000 lbs. was sustained, until at a central deflection of 9" the beam dropped off one pier and collapsed to the floor. The "cable" action shown by this beam in resisting the load was assisted by the excessively large amount of steel present in the beam, which after bond failure and subsequent loss of prestress acted as an ordinary, over-reinforced beam.

6.812 Central Loading

At the loss of this outside beam the model deck SA was now only 17 beams wide but still structurally sound. It was therefore decided to carry out an ultimate test for a central load at midspan on beam 9.

Figure 6.22 illustrates the observed behaviour by showing the load deflection curves for beams 5, 9, 13 and 17. Once again these curves indicate crack development in beams 9, 5 and 13. At 23,000 lb. three cracks were evident in beam 9 but these closed on the release of the load which represented the maximum that could be applied by the hydraulic system. Consequently, a full test to failure could not be obtained. The load thus far applied (23,000 lb) had inflicted no structural damage to the deck except for the cracks observed above, the effect of which was to lower the flexural rigidity of beams concerned.

6.813 Summary and Safety Factor Against Collapse.

An applied load equal to 23 times the design load (taken as 1/16th of a 16 Kip wheel load of the AASHO, H20 S16 T16 truck - fig 6.26) did not permanently damage this deck when positioned centrally.

However, 9 times the design load applied in an outside lane caused a shear failure between deck slab and the outside log beam with serious damage evident.

It should be remembered that this model had no transverse reinforcement at all, except that provided in the 1" deck slab, and under such conditions it is quite remarkable that its minimum safety factor against collapse was approximately 9.0.

6.82 Ultimate Load Tests on Model SB

6.821 At 3\(\frac{1}{2}\)" Depth

A preliminary test, to find the initial cracking load only, was conducted on the model deck SB whilst 3\(\frac{1}{2}\)" deep but transversely prestressed to 44% of the longitudinal prestress.
The load was applied to an outside edge beam and incremented in 500 lb-steps: the load-deflection curves are plotted in fig 6.23. The maximum load was limited here to 8,000 lbs to prevent permanent damage to the deck which might have affected further elastic tests yet to be conducted.

The initial cracking load was found to be approximately 4,250 lbs whilst a second crack could have occurred at 7,000 lbs. Figure 6.23 indicates the not-very-distinct changes in slope which were observed during the tests and the above loads are therefore stated with some hesitancy.

6.822 At $\frac{1}{2}$" Depth

Further ultimate tests were made on this model after the 1" deck slab had been poured and the series of elastic tests had been completed. The load-deflection curve for the loaded edge beam is also plotted in fig 6.23, and indicates the formation of the first crack at 9,500 lbs. Again some hesitancy is expressed regarding these results because of the very linear nature of the load-deflection curve.

The load was increased in 500 lb increments up to a maximum dictated by the size of the proving ring (sec 5.23) of 11,500 lbs. Even at this load no sign of distress was evident in the deck indicating an improved behaviour over that of the model 3A, which, at a load of 9,000 lbs, failed in horizontal shear. Obviously the presence of transverse prestress was responsible for this improvement and it was felt that little more information could be gained by continuing these tests up to the maximum allowed by the system of 23,000 lbs. These tests were thus discontinued, a minimum safety factor against collapse of 11.5 having been indicated.

6.823 Summary and Safety Factor Against Collapse

Since the dead load of this deck was not equivalenced, the behaviour given above is just a qualitative description and the cracking loads represent only approximate orders of magnitude.

At a depth of $\frac{3}{2}$", 7 times the design load had not caused any serious damage whilst at a depth of $\frac{7}{2}$", 11 1/2 times this load had likewise inflicted no damage to the deck.

6.83 Ultimate Load Tests on Model SHC

The behaviour of the hollow-cored decks under ultimate load was determined after the model had reached its full width of ten units. The single point load tests used in the earlier models was replaced by a four point load, representing
the scale equivalent of the four heaviest wheel loads of the H20 316 T16 truck-trailer combination.

The scaled truck, in the form of a steel grillage, was positioned in an outside lane such that the line of action of the centre of gravity of the axle loads passed through the midspan centre line of the model deck: refer fig 6.26. The results of this test are given in figs 6.24, 6.25, 6.26 and 6.27 and discussed below under three headings:

1. Load deflection behaviour up to 80\% of ultimate load,
2. Collapse mechanism at ultimate load,
3. Shear key damage.

6.831 The Load-deflection Behaviour up to 80\% of Ultimate Load

This is shown in fig 6.24 and was observed using dial gauges at the midspan transverse section. The curves show the load-deflection behaviour for beams 1, 2, 3, 4, 6 and 8. Beams 1, 2, and 3 actually carried the model truck (fig 6.26) and thus show the greatest deflection for the given load. Changes in slope of these lines again indicate the formation of cracks. The first cracks appeared in beams 1 and 2 at a load of 6,000 lbs (total load); these were followed by cracks in beam 3 at 9,000 lbs and beam 4 at 16,025 lbs.

When a load of 20,150 lbs had been reached several shear keys exploded near the pier supports (sec 6.833) and this was reflected in the load-deflection behaviour by a marked drop in flexural rigidity in all beams as seen in fig 6.24. At a load of 23,150 lbs the damage to the shear keys was well established and accompanied by the opening of large tension cracks under the deck. Slow creep was observed in the deflection and the dial gauges were removed at this point for fear of sudden collapse and possible damage to the gauges. This load represented 80\% of the ultimate load and it was observed that should the load be released at any time thus far, full recovery of deflection was obtained.

6.832 The Load-deflection Behaviour at Ultimate Load i.e. Collapse Mechanism

Once the load of 23,150 lbs was reached and the dial gauges removed, the deflection of beam 1 was recorded for further increases in load by a 3 foot ruler. Observations made using the ruler are plotted in fig 6.25. Also observed during this test were the changes in transverse prestress force as the deck collapsed. The stressing jack was attached to the central cable and
Fig. 6.26  TENSION CRACK PATTERN in soffit of the hollow-cored model deck at collapse.

Shear Key Failures are not shown in this Plan View

Position of pads represent scaled H2O S16 T16 truck loading

H2O S16 T16 Design Vehicle (full size)

8K  14'  32K  14'  32K  14'  16K  12'  16K

clearance &/or lane width
changes in cable tension observed on the pressure gauge of the jack (Plate 14B).

In fig 6.25, OA is the load-deflection line for beam 1 already seen in fig 6.24. From A to B the load has increased to 28,550 lbs with a corresponding increase in deflection of $1\frac{1}{2}''$. During this period the main tension cracks developed, some compression-failure zones became evident and the shear keys continued to disintegrate.

As the load of 28,550 lbs was reached (point B) a 6'' block of concrete forming part of the lower soffit across one of the hollow cores of beam 3, dropped out of the slab. With the corresponding release in strain energy the load immediately fell off to 24,650 lbs and the deflection of beam 1 increased a further $1''$. This is depicted by the dotted line BC in fig 6.25.

At this stage the deck, particularly in the area of beams 1, 2 and 3 was badly cracked in tension with both well developed (1/16th inch wide) and fine hair-line cracks. Failure in localised compression zones had increased but there was little further damage to the shear keys.

Again the load was increased, from C to D in the figure, but at D (25,250 lbs), when another $1\frac{1}{2}''$ deflection had been observed, there was a sudden collapse of the deck caused by a compression failure in beams 1, 2 and 3 directly under one axle of the model truck (fig 6.26). The load had again dropped away during this collapse and was now only 14,875 lbs; another $1''$ of deflection being recorded. The dotted line DE represents this final collapse in fig 6.25. On release of the load the recovery indicated by EF was obtained.

The changes in transverse cable tension were observed as below. In fig 6.25, from O to A to B: 7,500 lbs force as specified for elastic tests,

from B to C: an increase to 8,700 lbs,

from C to D: a further increase to 9,800 lbs,

from D to E to F: a reduction back to 8,200 lbs.

The maximum increase in cable tension was therefore 31% of the normal working force and was caused by the distorted geometry of the deck under the abnormal load. Such a possible increase is of importance in the design of transverse tension in the midspan cable. Sufficient overload allowance must be made to prevent the failure of this cable and the associated complete collapse of the deck. Such an allowance appears to be in the order of 30 - 35%.

The maximum load sustained by the deck was therefore 28,550 lbs. This was the total applied load; the maximum individual wheel load was 7,140 lbs.
End Elevation on Beams illustrating irregular seating on pier cap
much enlarged scale
The maximum deflection observed was of the order of \( \frac{1}{8} \)" and the permanent set at release of load, after collapse, was \( \frac{1}{3} \)".

6.833 Shear Key Damage

As already mentioned in the above section, 6.831, the first permanent damage to be sustained by the model deck was the failure of those keys in the vicinity of the load. This is depicted in both figs 6.26 and 6.27, where it can be seen that a length of the key-ways between beams 1 and 2, 3 and 4, and 4 and 5, has failed by shearing off the upper corners of the respective beams. Refer particularly to figs 6.27a and 6.27c, and also Plates 15A, 15B and 16A.

The reason for this poor performance at the joints was traced to the inadequate seating of the piers for beams which had twisted during manufacture - refer sec 6.442. All the model hollow-cored beams were observed to show some measure of twist due, as already suggested, to bond deterioration and non-uniform camber. The beams would therefore have been able to rock at their supports if they had not been prevented by the shear keys which locked adjacent beams. Under ultimate loading there was not sufficient strength in the joint to prevent the corners shearing off, a mode of failure which progressed down the length of the beam until the whole beam cross-section was able to rest on the pier - see fig 6.27d. This type of failure was also observed in the hollow-cored spans of the Slippery Creek Bridge, photographs of which are included in Plate 4B.

Since elimination of the twist of these extra wide units is difficult in practice, the remedy for such a problem would be to assemble the beams on a wet mortar pad instead of the conventional neoprene bearing pad. The beams would then be fully supported at the piers and the tendency for failure to occur would thus be prevented.

6.834 Safety Factors Against Collapse

The indicated safety factor against collapse for these hollow-cored decks is therefore 7.14. However, as already explained in the introduction to this section (6.8) the dead loads of this model deck and its prototype were not equivalenced and therefore the above estimate is qualitative only.

The safety factor against exceeding the elastic limit is also given here at 5.79 but the same note of caution is made regarding its application.

Working values for these safety factors could be taken at 7 and 5 respectively.

6.84 Safety Factor Summary - Both Decks
In Table 6.5 below the safety factors are summarised for both log and hollow-cored decks.

TABLE 6.5

<table>
<thead>
<tr>
<th>DECK TYPE</th>
<th>MODEL</th>
<th>SAFETY FACTORS</th>
<th>LOAD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>WORKING LOAD</td>
<td>ULTIMATE LOAD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EDGE</td>
<td>CENTRE</td>
</tr>
<tr>
<td>LOG (No Transv. Reinforcement)</td>
<td>SA</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>LOG (44% T,PS-3_{1/2})</td>
<td>SB</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>LOG (44% T,PS-4_{1/2})</td>
<td>SB</td>
<td>9_{1/2}</td>
<td>-</td>
</tr>
<tr>
<td>HOLLOW-CORED</td>
<td>SHC</td>
<td>5</td>
<td>-</td>
</tr>
</tbody>
</table>

Working Load Safety Factor is defined as \( \frac{\text{Load to cause first tensile crack}}{\text{Design load}} \)

Ultimate Load Safety Factor is defined as \( \frac{\text{Load to cause permanent damage}}{\text{Design load}} \)

6.9 Miscellaneous Results

6.91 Pier Reactions and Pier Deflections

As described in sec 5.912 an attempt to observe pier strains on the model was made, but with little success. The model reactions had an average value of 700 lbs which, due to the high stiffness of the model piers (6" i.e. boiler tube), caused a change in strain of only 6 microstrain. The available recording equipment was not sufficiently sensitive to observe these changes with any degree of reliability and the observations were discontinued.

Pier deflections were checked with a dial gauge graduated to \( \frac{1}{10,000} \) of an inch. No deflection was observed in the cap beams even during the ultimate load tests. This was not an unexpected result since the cap beams were well proportioned, heavily reinforced and each monolithic with three pier columns. The assumption of a simple, rigid support, often made during the design of these decks, is therefore sound.

6.92 Experimental Check on the Stiffness of a Single, Full Size Hollow-cored Beam

Ten of the unused hollow-cored beams in spans E and F of the bridges at
Slippery Creek were individually tested for stiffness. The results of these tests have a mean value for X (the flexibility of a single beam - see sec 3.241) of 0.183 x 10^{-4} ins/lb. Since these tests were conducted only to check uniformity, little attention was paid to supreme accuracy - a 3 foot ruler and two, 4\(\frac{1}{2}\) ton concrete blocks were used for the tests. The agreement to within \(8\%\) was thus acceptable.

6.93 Check on Experimental Technique using Influence Lines

As will be noted from sec 4.32, lane load tests were also conducted on the Slippery Creek Bridge spans. These tests were much fewer in number than those carried out with the point loader, but the results obtained were useful in checking the data recorded and the experimental technique.

For example, using the deflection profiles obtained whilst point loading on span D, influence lines of deflection were drawn for the individual beams in the deck as in fig 6.28. Using these lines predicted deflections were made for those lane load tests performed on the same span. These were checked against experimental observations and confirmation obtained as in Table 6.6 below.

**TABLE 6.6**

SEE ALSO FIG 6.28

**INFLUENCE LINE PREDICTION OF BEHAVIOUR OF SPAN D**

<table>
<thead>
<tr>
<th>LOAD IN LANE</th>
<th>DEFLN OF BEAM NO.</th>
<th>DEFLN FROM L.H. WHEEL</th>
<th>DEFLN FROM R.H. WHEEL</th>
<th>SUM=(3)+(4)</th>
<th>PRED’D</th>
<th>OBSERVED</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.267</td>
<td>0.150</td>
<td>0.417</td>
<td>0.156</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.088</td>
<td>0.120</td>
<td>0.208</td>
<td>0.078</td>
<td>0.077</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.123</td>
<td>0.065</td>
<td>0.188</td>
<td>0.071</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.131</td>
<td>0.156</td>
<td>0.287</td>
<td>0.108</td>
<td>0.109</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.082</td>
<td>0.042</td>
<td>0.124</td>
<td>0.046</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.155</td>
<td>0.116</td>
<td>0.271</td>
<td>0.102</td>
<td>0.108</td>
</tr>
</tbody>
</table>
**INFLUENCE LINE PREDICTION OF BEHAVIOUR OF SPAN D**

<table>
<thead>
<tr>
<th>LOAD IN LANE (1)</th>
<th>DEFLN OF BEAM NO. (2)</th>
<th>DEFLN FROM L.H. WHEEL (3)</th>
<th>DEFLN FROM R.H. WHEEL (4)</th>
<th>SUM = (3)+(4) (5)</th>
<th>DEFLECTION</th>
<th>PRED’D (6)</th>
<th>OBSERVED (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
<td>0.036</td>
<td>0.024</td>
<td>0.060</td>
<td></td>
<td>0.022</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.101</td>
<td>0.069</td>
<td>0.170</td>
<td>0.064</td>
<td>0.069</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1. All deflections are expressed in inches.
2. Column (6) = 0.375 x Column (5)

The experimental technique, i.e. the method of observing deflections, load assessment and load positioning, was thus shown to be appropriate and accurate.
Figure 6.01  Multibeam / 18 Logs

Slippery Creek Bridge  *  Span Co

Transverse Deflection Profiles at Midspan

For 64,000 lb point load in Row 1 and Row 5

Legend:
- Model Prediction: 16.11.64
- Observed in Field Test: 6.5.65
- Computed by Transfer Matrix Theory: 20.10.65
SLIPPERY CREEK BRIDGE  *  SPAN D
TRANSVERSE DEFLECTION PROFILES AT MIDSPAN
for 64,000Lb point load on beam 1 and beam 5

LEGEND:

--- Model Prediction
- - - Observed in Field Test
- - - - Computed by Transfer Matrix Theory
FIGURE 6.03  HOLLOW-CORE / 5 BEAMS

SLIPPERY CREEK BRIDGE  *  SPAN E

TRANSVERSE DEFORMATION PROFILES AT MIDSPAN
for 64,000Lb point load on beam 1 and on beam 3

LEGEND:

- Model Prediction
- Observed in Field Test
- Computed by Transfer Matrix Theory

4 max. slip. = 0.043"  
i.e. 14.8% of theoretical defln.
SIIPPERY CREEK BRIDGE

TRANSVERSE DEFLECTION PROFILES AT MIDSPAN for 64,000Lbs on beams 1 and 2 (point load)

LEGEND:
- Model Prediction
- Observed in Field Test
- Computed by Transfer Matrix Theory
Computation using Transfer Matrix Theory;
Coefficient expressed as a percentage of total applied load (Rows 1 and 5)
FIGURE 6.05

HOLLOW-CORED/10 BEAM

THEORETICAL LOAD DISTRIBUTION HISTOGRAMS

FOR SLIPPERY CREEK BRIDGE * SPAN D

Computed using Transfer Matrix Theory;
Coefficient expressed as a percentage of total
applied load (Beams 1 and 5)
THEORETICAL LOAD DISTRIBUTION HISTOGRAMS
FOR SLIPPERY CREEK BRIDGE  *  SPANS E & F
Computed using Transfer Matrix Theory
Coefficient expressed as a percentage of total applied load (Beams 1 and 5)
Theoretical (Rowe-Morice-Little) Distribution Coefficient Profiles for various values of $\alpha$, as indicated.

FIGURE 6.08a

QUARTER SCALE MODEL SA
DISTRIBUTION COEFFICIENT PROFILES AT MIDSPAN based on Method of Distribution Coefficients as proposed by Rowe-Morice-Little;
COMPARSED WITH EXPERIMENTAL COEFFICIENTS observed on the multibeam model SA

Theoretical Profiles are given for the following values of $\alpha$: 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0
FIGURE 6.09  HOLLOW-CORED/10BMS

COMPARISON BETWEEN NATHAN'S RELAXATION METHOD OF ANALYSIS AND
(a) EXPERIMENTAL OBSERVATION ON SLIPPERY CREEK - span D: deflection profile
(b) TRANSFER MATRIX ANALYSIS OF SAME SPAN: deflection profile and load distribution histogram

LOAD: 64,000lb on beams 1 and 9

Note: Above relaxation profiles were plotted after 30 cycles at a carry-over of 0.95
SLIPPERY CREEK BRIDGE * SPANS CO, C4, A4
TRANSVERSE DEFLECTION PROFILES AT MIDSPAN
for 64,000Lb point load in Row 1.

TO DEMONSTRATE THE EFFECT OF ORDINARY AND
PRESTRESSED TRANSVERSE REINFORCEMENT.
including model prediction for 44% trans.
prestress.

LEGEND AS SHOWN.
SLIPPERY CREEK BRIDGE * SPANS CO,C4,A4
TRANSVERSE DEFLECTION PROFILES AT MIDSPAN
for 64,000lb point load in Row 5.

TO DEMONSTRATE THE EFFECT OF ORDINARY AND
PRESTRESSED REINFORCEMENT (TRANSVERSE).
including model prediction for 44% trans.
prestress.

LEGEND AS SHOWN.
44% transverse prestress, model depth: 4\(\frac{1}{2}\)".

14/0.276"Ø tendons, not stressed.
14/0.276"Ø tendons, stressed to 22% of the longitudinal prestress.
14/0.276"Ø tendons, stressed to 44% of the longitudinal prestress.

-all at a model depth of 3\(\frac{1}{2}\)".

**FIGURE 6.12**

**SLIPPERY CREEK MODEL**  
**SPAN SB : SERIES III IV and V.**

**TRANSVERSE DEFLECTION PROFILES AT MIDSPAN**  
for 4000Lb point load in Row 1.

**TO DEMONSTRATE THE EFFECT OF PRESTRESSED TRANSVERSE REINFORCEMENT AND DECK DEPTH**  
on model span SB at 0, 22 and 44% transverse prestress and at depths of 3\(\frac{1}{2}\) and 4\(\frac{1}{2}\) inches.
SLIPPERY CREEK MODEL * SPAN SB SERIES III, IV, & V.
TRANSVERSE DEFORMATION PROFILES AT MIDSPAN
for 4000Lb point load in Row 5.

TO DEMONSTRATE THE EFFECT OF PRESTRESSED
TRANSVERSE REINFORCEMENT AND DECK DEPTH
on model span SB at 0, 22 and 44% transverse prestress
and at depths of 3½ and 4½ inches.

LEGEND AS SHOWN.
SLIPPERY CREEK MODEL * SPAN SHC
TRANSVERSE DEFLECTION PROFILES AT MIDSPAN
for 4000Lb point load on beams 1 and 5.

TO DEMONSTRATE THE EFFECT OF TRANSVERSE
PRESTRESS on structural behaviour of
multibeam decks with shear keys.

Above profiles were observed when all
transverse prestress had been released
from the deck.
FIGURE 6.16  18 LOGS vs 10 HOLLOW-CORES

SLIPPERY CREEK BRIDGE  * SPANS A and D

RELATIVE BEHAVIOUR OF LOG AND HOLLOW DECKS
being Load Distributions per Unit Width.

LEGEND:

- LOG beam deck
- HOLLOW-CORED beam deck
SLIPPERY CREEK BRIDGE * SPANS D, E, and F
LOAD DISTRIBUTION HISTOGRAMS FOR VARIOUS SPAN/WIDTH RATIOS
indicating an approach to uniform load distribution with increase in this ratio

LEGEND AS SHOWN
FIGURE 6.18  HOLLOW-CORE / 10 BEAMS

SLIPPERY CREEK BRIDGE  *  SPAN D

TRANSVERSE DEFORMATION PROFILES AT QUARTER POINTS
to show the effect of 15° of skew.

LEGEND:
- - Profiles under central load
- - Profiles under edge load
FIGURE 6.21  MODEL SA LOG/18 BEAMS

LOAD - DEFLECTION BEHAVIOUR NEAR ULTIMATE LOAD

Loaded on beam 18 which shows a change in stiffness at 5,600 Lbs corresponding to the formation of a single midspan crack.
FIGURE 6.22 MODEL SA LOG/18 BEAMS

LOAD - DEFLECTION BEHAVIOUR NEAR ULTIMATE LOAD

Loaded on beam 9, which shows 3 changes in stiffness as numbered, corresponding to the formation of 3 cracks in this beam; also seen, are cracks in B17, B5 and B13.
Two tests at different depths of deck are shown; in each case different beams were loaded; again changes in stiffness indicate cracks.
FIGURE 6.24  10 BEAM, HOLLOW-CORED DECK

LOAD - DEFLECTION BEHAVIOUR NEAR ULTIMATE

showing sudden change in stiffness at 20,150lb

corresponding to the deterioration of the
shear keys due to inadequate seating details.
Figure 6.25: Load - Deflection Behaviour at Ultimate Load

For detail of section OA see Fig. 6.24, beam 1.
Total Weight: 48,000 Lbs

**Figure 6.28** Hollow-Core/10BMS

Influence lines for deflection for a unit load of 64,000 Lbs on beams 1 and 5.

Slippery Creek Bridge * Span D

-used for confirmation of experimental technique; see sec 6.93 & Tab 6.6
CHAPTER: SEVEN

TITLE: SUMMARY AND CONCLUSIONS, RECENT DEVELOPMENTS AND FUTURE WORK.

CONTENTS:

7.1 Summary and Conclusions.

7.11 Model - Prototype Correlation.
7.12 Verification of the Transfer Matrix Analysis Procedure.
7.13 The Effect of Transverse Prestress.
   7.131 Log Beam Decks.
   7.132 Hollow-cored Decks.
7.14 The Relative Behaviour of the Log and Hollow-cored Decks.
7.15 The Effect of Span/Width Ratio.
7.16 The Effect of 15 Degrees of Skew.
7.17 The Effect of Shear Key Shape.
7.18 The Behaviour of these Multibeam Decks at Ultimate Load.
   7.181 Log Beam Decks.
   7.182 Hollow-cored Decks.

7.2 Recent Developments and Suggestions for Future Work.

7.21 Recent Developments.
   7.211 Log Beam Multibeam Decks.
   7.212 Hollow-cored Multibeam Decks.
7.22 Suggested Future Work.
SUMMARY, CONCLUSIONS AND RECENT DEVELOPMENTS CONCERNING
MULTIBEAM BRIDGE DECKS

7.1 Summary and Conclusions

In the following sections the results of the model and field tests on the Slippery Creek Bridges as presented in the preceding Chapter are summarised.

7.1.1 Model - Prototype Correlation

Good correlation was achieved between the quarter scale, direct model (type I) and its full scale prototype. Agreement to within 6% was observed in the full width spans; but this deteriorated as the span/width ratio increased due to the greater influence on deck behaviour of poor seating and shear key action.

The model studies allowed the behaviour of the full scale structure to be predicted for conditions that were not possible to reproduce at the site. For instance an investigation into the effect of higher percentages of transverse prestress, an investigation of the shear key mechanism and the determination of the safety factors against collapse have been made.

The value of the model studies has been demonstrated by indicating the manner in which they may be usefully employed in the verification of new analysis procedures, the development of new and improved bridge deck types, the examination of particular design parameters and the estimation of safety factors against cracking or ultimate collapse.

7.1.2 Verification of the Transfer Matrix Analysis Procedure

The transfer matrix analysis procedure as developed and proposed in Chapter 3 of this thesis for the solution of multibeam bridge decks has been verified by the comparison of transverse deflection profiles with experimental observations made on both the model and full scale Slippery Creek Bridge. Figs 6.01, 6.02, 6.03 and 6.04.

Further verification was provided by comparison with the experimental work of Roesli et al on the Centerport Bridge in the United States. This bridge was also used to compare the theoretical methods of Roesli, Spindel, Cusens, Pama and Ahmed, which have been applied to this structure. Figs 3.8 and 3.9. Again the transfer matrix method has been compared with the
Distribution Coefficient method of Rowe, Morice and Little and the Relaxation method of Norman and Nathan. Figs 6.08a, 6.08b and 6.09.

In all cases the transfer matrix method shows agreement with the experimentally observed behaviour that is as good as, and often superior to, the theoretical predictions of the above authors. The assumptions upon which the theory has been founded are therefore reasonable and justified; the method has proved to be not only quick and versatile but also accurate and reliable.

7.13 The Effect of Transverse Prestress

7.13.1 Log Beam Decks

The inclusions of 14 or 15 transverse bolts into the log beam decks made no appreciable difference to the behaviour of these decks; nor did the application of a nominal 2% transverse prestress. The load distribution characteristics were therefore unaffected and two alternative design recommendations are concluded which suggests improvements in the log beam deck. These are:

- either (a) to use the same system of log beams and the 4" additional reinforced deck slab, but without any further transverse reinforcement in the form of either mild steel bolts or applied prestress;
- or (b) to use only the log beams without the 4" additional deck slab, but to reinforce the beams transversely with either a number of mild steel bolts or a nominal amount of applied prestress.

This latter recommendation is based on the results of testing the model SB whilst only 3½" deep but transversely reinforced with 14 rods.

The effect of large amounts of transverse prestress was also investigated on this model and it was found that a prestress, equal to 22% of the longitudinal prestress, a significant improvement in behaviour had been obtained. Further increases in prestress - up to 44% max. - made little advance over that observed at 22% and it is concluded that for a given load and load position, there is a certain level of transverse prestress that induces sufficient friction (for twisting moments) and compression (for transverse bending moments) between adjacent beams, for these multibeam decks to behave as isotropic slabs. This critical level of prestress lay somewhere
between 2% and 22% for the log decks considered in this work. Above this level an isotropic theory for slabs will be applicable, for example the Distribution Coefficient method (see fig 6.08b), but below the level, articulated plate theory will govern and the transfer matrix theory would be suitable.

7.132 Hollow-cored Decks

The effect of transverse prestress, provided by the single midspan cable of the hollow-cored deck, is to ensure the satisfactory action of the shear keys between adjacent beams. Evidence from the field tests suggested a series of tests on the model SHC which examined the mechanism of the shear keys and their reliance on transverse prestress for satisfactory performance. Lack of sufficient prestress allows the wedge-shaped keys to expand the deck laterally accompanied by differential slipping across the keys. This is evidenced by the stepped profiles seen in the transverse deflection curves of fig 6.03 and reproduced in the model as in fig 6.14.

7.14 The Relative Behaviour of the Log and Hollow-cored Decks

The hollow-cored deck was 35% less stiff than the log beam deck and for the same load the former deflected 35% more than the latter; similar increases in the maximum strain were also observed.

In general the hollow-cored deck distributed the applied load more efficiently than the log beam deck despite its comparative lack of transverse "interconnection" and lower longitudinal stiffness. The maximum loading in the former was 7.16% of the applied load per foot width of deck and in the latter 8.84% per foot width. It was therefore concluded that the wider the individual beam the more economic the deck, for the better will be the load distribution. However, practical considerations such as weight, dimensional tolerances, local wheel load effects and dynamic characteristics will set a limit to the maximum desirable width.

Present day estimates (1966/1967) indicate that the hollow-cored deck is approximately 10/- a square foot cheaper than the corresponding log deck (this figure is not inclusive of the substructure costs which are common to both). The use of hollow-cored units instead of log beams for a structure the size of the Slippery Creek Bridges would represent a saving of approximately £6,500 or 20% of the total cost. The extra cost of the log beam deck lies in the large amount of in-situ work required for the
placing the 4" additional deck slab.

The hollow-cored beams, however, show greater tendencies to twist during manufacture and storage which results in poor seating at the pier caps and are more dependent on transverse prestress for satisfactory performance. Their vibrational characteristics may also be a disadvantage when subjected to heavy traffic.

7.15 The Effect of Span/Width Ratio

As the span/width ratio increased - 1.0 through 2.0 to 3.3 - both the maximum recorded deflection and the maximum load carried by any one beam also increased, a result that was not entirely unexpected. Of more importance, however, was the approach to a uniform distribution of load amongst the several members of each deck as the ratio increased.

The conclusion based on this study is that for span/width ratios in excess of 3.0, a uniform distribution of load amongst the beams of the deck may be assumed.

7.16 The Effect of 15 Degrees of Skew

The transverse deflection profiles at the quarter and three quarter lines, when drawn for a load in any midspan position, were not found to be identical, as would be expected for a right deck. This discrepancy was due to the skewed geometry of the deck. At Slippery Creek the maximum difference between these profiles was observed at 36\(\frac{1}{2}\) of the quarter span deflection at the point considered.

The maximum error therefore in predicting deflection, strain or moment, in a 15 degree skewed deck assuming it were right - which is common design practice for bridges under 20 degrees of skew - would be approximately 18\%. Equation 6.2 of the preceding chapter proposes a formula for the prediction of the above error at any other angle of skew, span, or load position. Again, the effect of skew on deck behaviour has been noted to be dependent primarily on the span/width ratio; the greatest effect occurs for the least real value of this parameter.

7.17 The Effect of Shear Key Shape

The shear keys used between the hollow-cored units both in the field and in the model were wedge-shaped. Under the action of an applied vertical load, positioned anywhere on the deck, the keys tended to open the joints laterally and induce vertical slip under the mechanical advantage of the wedge.
Such movement could be prevented by the application of sufficient transverse prestress (refer sec. 7.132 above) or by a change of shape of key. This latter alternative is not desirable since the wedge allows good compaction of concrete in the key as it is poured; the provision of adequate transverse prestress is not considered to be too far outside the capabilities of the ordinary bridge contractor.

The above shear keys were also able to burst off one corner of an adjacent beam if this unit was not seated satisfactorily. Almost any shaped key will do this and the best remedy is to seat the beams in a wet mortar and thus prevent any rocking movement.

7.18 The Behaviour of these Multibeam Decks at Ultimate Load

7.181 Log Beam Decks

Under the action of a point edge load the model SA collapsed, first by a horizontal shear failure between the outside beam and the 1" additional deck slab, and then by the failure of this beam as it fell off one pier under conditions of extreme geometrical distortion. This deck had no transverse reinforcement except that provided in the 1" deck slab and its failure at approximately 9 times the designed load, indicates a remarkable reserve of strength and adequate distribution of load.

Tests to destruction were not possible on the model SB which had been transversely stressed to 44% of the longitudinal prestress. The ultimate load capacity of this deck exceeded the maximum rated capacity of the model test facility.

7.182 Hollow-cored Deck

For these tests the heaviest axles of a scaled H20 S16 T16, AASHO truck-trailer combination were positioned in an outside lane over the midspan centre line.

The first damage to be sustained by this deck occurred at 5 times the design load. It was the failure of shear keys between those beams actually supporting the load, but only in a zone near the piers. This was the effect of poor seating at these supports due to the individual beams having twisted during their period of curing and storage.

As the load was further increased both tension cracks and compression failure zones developed; the deck failed in compression directly under one axle at a load 7 times the design load. At this stage the deck had deflected
a maximum of $4\frac{1}{3}$" and on release, retained a permanent $1\frac{2}{3}$"; the beams directly under the load were badly damaged, as were their shear keys, but those beams remote from the load were not distressed in any noticeable fashion.

The tension in the midspan transverse cable increase by $31\%$ during the test indicating the importance of a sufficient overload allowance in the design transverse force to prevent catastrophic collapse of the deck should this cable fail.

7.2 Recent Developments and Suggestions for Future Work

7.21 Recent Developments

7.211 Log Beam Multibeam Decks

Based on the results of the field tests at Slippery Creek and the model tests in the Structures Laboratory, the Ministry of Works Design Head Office amended their Standard Specifications for log beam bridge decks and introduced a revised standard in October 1965. This standard for precast, pretensioned, hollow bridge units (MOW 11116) describes a 3 foot wide, precast, double-U-shaped unit, which when in place at the site, takes a 4" deck slab reinforced with $\frac{2}{3}$" beam rod at 8" centres, i.e. as for the original log beam decks. Only two central transverse tie rods are specified for the deck but it is felt that little saving in cost will be achieved because a large amount of in-situ work is still required.

7.212 Hollow-cored Multibeam Decks

These have been further refined to include a deeper shear key and circular cores rather than the hexagonal type used in the Slippery Creek and model structures. As a result of the above tests, this multibeam deck system is now acceptable for use in Government Contracts and the extensions made to the Slippery Creek Bridges in 1966 for flood protection used these hollow-cored units but with a V-shaped key.

7.22 Suggested Future Work

Several possible avenues of future research work are listed:

1. Optimise the shape of precast beam to give the best load distribution per unit width (see sec. 6.42).

2. Improve the assumption that has been made in the transfer matrix theory regarding the shear force distribution at the shear keys.

3. Investigate shear key action with a view to redesign and the prevention of differential movement.
4. Investigation ultimate load behaviour by the use of models with appropriate adjustments for dead load equivalence.
5. Investigate vibration characteristics and behaviour under dynamic loads of multibeam bridge decks.
6. Extend the computer programme OPUS TWO to include any load position and any number of loads.
7. Investigate multibeam floor systems.
8. Edge Stiffening.
APPENDICES

ONE  Digital Computer Facilities and the OPUS Programme Series
   A1.1  Facilities
   A1.2  The OPUS Programme Series

TWO  OPUS ONE: Determination of the Distribution Coefficient Matrix, K
   A2.0  Introduction and Description
   A2.1  Storage Required and Approximate Speed
   A2.2  Input - Output: Data required, Sense Switch Settings
   A2.3  Flow Chart
   A2.4  Programme Listing and Specimen Result

THREE OPUS TWO and OPUS THREE: Computer Programmes for the Transfer Matrix
   Analysis of Multibeam Structures
   A3.0  Introduction
   A3.1  Description
   A3.2  Storage Required and Approximate Speed
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   A4.4  Flow Chart
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   A5.2  Transfer Matrices
   A5.3  Use of Field and Point Transfer Matrices
   A5.4  Generalised Transfer Matrix Theory

SIX  An Approximate Theoretical Justification for Single Point Distribution
      of Load at the Shear Keys of a Multibeam Deck
   A6.1  Application to a Beam Supported Elastically Along Both Free Edges.
SEVEN Torsional Analysis of a Simply Supported Beam

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TEN Timetable and Load Arrangement used during the Field Testing of the Slippery Creek Bridges

ELEVEN Material Properties of the Model
A11.1 Variation of Camber, Crushing Strength, and Elastic Modulus of Model Concrete with time
A11.2 Steel Properties

TWELVE Transducer for the Determination of Elastic Modulus of Concrete Blocks
A12.1 Description
A12.2 Calibration
A12.3 Strain Gauge Circuit
A12.4 Use of the Transducer
A1.1 Facilities

The digital computer available for the computational work involved in this thesis was an IBM 1620 machine with a 1622 Card Read Punch and a 1623 Storage Unit. This contained a further 20,000 storage positions giving the system a total capacity of 40,000 locations.

An IBM 026 Card Punch, an 870 Off-Line Printer and a card sorter complete the facilities available. These are at present (1967) being extended to include the IBM 1130 Computing System.

A1.2 The OPUS Programme Series

In the course of this thesis project various programmes have been written and these have been grouped into seven different classes according to their function. Written in PDQ FORTRAN C2 with Fixed Format Subroutines, these seven formed the basis of what was called the OPUS (Objective Programmes for Understanding Slabs) Programme Series, and were numbered consecutively from one to seven as below.

| Identification | Function | |
|----------------|----------| |
| OPUS ONE       | Distribution Coefficient Analysis of Bridge Decks | |
| OPUS TWO       | Transfer Matrix Analysis of Multibeam Decks | |
| OPUS THREE     | Load Distribution in Multibeam Decks, using State Vectors from OPUS TWO; graph subroutine included. | |
| OPUS FOUR      | Graph Plotter for any specified variable; maximum capacity, 73 variables; includes sin 4θ as a demonstration programme. | |
| OPUS FIVE      | Statistical Analysis Programme for Concrete; includes a Goodness of Fit test for E and f_c. | |
| OPUS SIX       | Finite Element Analysis of Skewed Anistropic Bridge Decks; in five parts: 6/1, 6/2, 6/3, 6/4A and 6/4B. | |
| OPUS SEVEN     | Matrix Inversion Programmes. | |

Several of these programmes will be found in the following three appendices of this volume, but OPUS SIX and SEVEN are not included until appendices 13 and 14 of volume II.

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<td>TWO and THREE</td>
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<td>Appendix</td>
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<td>-------------</td>
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<tr>
<td>SIX and SEVEN</td>
<td>Refer Appendices 13 and 14 of Volume II.</td>
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APPENDIX TWO: OPUS ONE: DETERMINATION OF THE DISTRIBUTION COEFFICIENT MATRIX, K

A2.0 Introduction and Description

This programme was written in FORTRAN to calculate \( K_\alpha \) from \( K_0 \) and \( K_1 \) for any value of \( \alpha \) according to the expression

\[
K_\alpha = K_0 + (K_1 - K_0).
\]

\( K_0 \) and \( K_1 \) are matrices describing the theoretical distribution coefficients for a bridge deck with zero and full torsional stiffnesses respectively (\( \alpha = 0 \) and \( \alpha = 1 \)). \( K_\alpha \) is the matrix of coefficients for a bridge deck where \( \alpha \) is given by \( \frac{G}{2E} \left( \frac{i_0 + j_0}{ij} \right) \); see also sec 2.12 of Chapter 2.

The programme was thus based on the Method of Distribution Coefficients due to Guillon and Massonnet and later Rowe, Morice and Little - the method is described in the above-named section in some detail.

The computer was therefore used to find the distribution coefficients for the Slippery Creek Bridge, since it was not possible to find \( \alpha \) theoretically. Using a trial and fit approach, sets of \( K_\alpha \) were found to best suit the observed data.

A2.1 Storage Required and Approximate Speed

OPUS ONE occupies 15,330 storage locations exclusive of subroutines and takes approximately three seconds to calculate \( K_\alpha \), for one particular value of \( \alpha \). About sixty seconds are required to list the flexibility matrix \( (K_\alpha) \) and the load transfer matrix.

A2.2 Input - Output: Data Required, Sense Switch Settings

The sequence of data cards as required by the programme is as follows:

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<th>Format Specification</th>
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<td>1 - 3</td>
<td>No. of equivalent wheel loads</td>
<td>F3</td>
</tr>
<tr>
<td></td>
<td>4 - 6</td>
<td>Actual No. of wheels on deck</td>
<td>F3</td>
</tr>
<tr>
<td>2</td>
<td>1 - 2</td>
<td>Increment on ( \alpha ), if ( \alpha ) is not to be specified</td>
<td>F2</td>
</tr>
<tr>
<td>3</td>
<td>1 - 6</td>
<td>Equivalent Load Matrix</td>
<td>F6.3</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3+</td>
<td>1 - 6</td>
<td>( K_0 ), Coefficient Matrix Element</td>
<td>F6.3</td>
</tr>
<tr>
<td>Card</td>
<td>Column Numbers</td>
<td>Description</td>
<td>Format Specification</td>
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<td>----------------</td>
<td>-------------</td>
<td>----------------------</td>
</tr>
<tr>
<td></td>
<td>7 - 12</td>
<td>$K_{1}$, Coefficient Matrix Element</td>
<td>F6.3</td>
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<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

If $K_{1}$ is to be computed for a particular value of $\alpha$, then sense switch 1 must be ON, and the value entered by card:

| 3++ | 1 - 6 | $\alpha$ | F6.3 |

If sense switch 1 is OFF, the programme will enter a DO loop and calculate $K_{\alpha}$ for a range of values for $\alpha$ from 0.0 up to 1.0 according to the size of increment specified on the second data card above.

The programme lists $K_{\alpha}$ under the heading "Flexibility Matrix" together with a summation of the columns of $K_{\alpha}$ according to the actual magnitude and position of the applied loads; this last row is listed under the heading of "Load Transfer Matrix".

**A2.3 Flow Chart**

A simplified, schematic flow chart for the programme is given in Fig A2.1.

**A2.4 Programme Listing and Specimen Result**

A PDQ FORTRAN listing of OPUS ONE is included in this appendix to give the source statements and their addresses together with the symbol table locations.

A specimen set of results as calculated for the model, multibeam deck of Chapter 5 is included after the programme listing.
Fig. A2.1 Flow Chart of OPUS ONE
PROGRAMME TO DETERMINE LOAD TRANSFER MATRIX FOR BRIDGE DECKS

READ 7, N, M
READ 27, L
DIMENSION A(9,9), B(9,9), P(9,9), Y(9,9), C(9), D(9)
DO 4 I=1, N
DO 4 J=1, 9
4 READ 20, P(I, J)
DO 2 I=1, 9
JC=1
DO 2 J=JC, 9
2 READ 3, A(I, J), B(I, J)
DO 6 I=1, 9
DO 6 J=1, 9
IF (I-J) 6, 6, 5
5 A(I, J)=A(J, I)
B(I, J)=B(J, I)
CONTINUE
6 IF (SENSE SWITCH 1) 24, 23
DO 17 I=1, 11, L
ALPHA=1/I
ALPHA=(ALPHA-1.)/10.
GO TO 26
24 READ 25, ALPHA
BETA=SQRT(ALPHA)
DO 8 I=1, 9
DO 8 J=1, 9
8 X(I, J)=A(I, J)+(B(I, J)-A(I, J))*BETA
DO 10 I=1, N
DO 10 J=1, 9
10 Y(I, J)=Y(I, J)+P(I, K)*X(K, J)
DO 11 J=1, 9
C(J)=0.
DO 11 K=1, N
11 D(J)=C(J)+Y(K, J)
DO 12 J=1, 9
AM=M
12 D(J)=C(J)/AM
PRINT 13
PRINT 14, ALPHA
PRINT 15
DO 18 I=1, 9
18 X1=/(1, 1)
X2=X(1, 2)
X3=X(1, 3)
X4=X(1, 4)
X5=X(1, 5)
X6=X(1, 6)
X7=X(1, 7)
X8=X(1, 8)
X9 = X(1,9)

18 PRINT 16, X1, X2, X3, X4, X5, X6, X7, X8, X9
19 PRINT 19
20 PRINT 21, N, D(1), D(2), D(3), D(4), D(5), D(6), D(7), D(8), D(9)
22 IF (SENSE SWITCH 1) 22, 17
23 CONTINUE
24 PAUSE
25 GO TO 22
26 FORMAT (42H MR DISTRIBUTION COEFFICIENT DETERMINATION)
27 FORMAT (F6.3, F6.3)
28 FORMAT (F6.3)
29 FORMAT (13, 13)
33 FORMAT ((//3H N, 5X21H LOAD TRANSFER MATRIX)
34 FORMAT (F7.3, F7.3, F7.3, F7.3, F7.3, F7.3, F7.3, F7.3, F7.3)
35 FORMAT (19H FLEXIBILITY MATRIX)
36 FORMAT (E14.8)
37 FORMAT (///6H ALPHA)
38 FORMAT (F6.3)
39 FORMAT ((12)

C PROG Sw 1 ON TO READ IN A PARTICULAR VALUE OF ALPHA

END

L9999 SIN
L9959 EXP
L9919 SORT
L9879 DRH
L9839 SN
L9799 M
L9599 B
L5719 C
L5519 J
L5479 SN
L5439 SN
L5399 ALPHA
L5359 SN
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L5279 SN
L5239 SN
L5199 X3
L5159 X7
L5119 SN
L9989 SINF
L9949 EXPF
L9909 SORTF
L9869 DRHF
L9829 SN
L9789 SN
L9149 X
L5629 D
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L5469 SN
L5429 SN
L5389
L5349 SN
L5309
L5269 AM
L5229 SN
L5189 X4
L5149 X8
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### DISTRIBUTION COEFFICIENT DETERMINATION

#### PI-IA (0.000000E-50)

**Flexibility Matrix**

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**Load-Transfer Matrix**

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**Load-Transfer Matrix**

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<td>1.190</td>
<td>1.160</td>
<td>1.120</td>
<td>1.090</td>
</tr>
<tr>
<td>5.100</td>
<td>0.600</td>
<td>0.700</td>
<td>0.830</td>
<td>1.000</td>
<td>1.160</td>
<td>1.320</td>
<td>1.390</td>
<td>1.410</td>
</tr>
<tr>
<td>4.100</td>
<td>0.500</td>
<td>0.590</td>
<td>0.720</td>
<td>0.900</td>
<td>1.200</td>
<td>1.390</td>
<td>1.620</td>
<td>1.900</td>
</tr>
<tr>
<td>3.300</td>
<td>0.400</td>
<td>0.500</td>
<td>0.640</td>
<td>0.820</td>
<td>1.090</td>
<td>1.410</td>
<td>1.800</td>
<td>2.290</td>
</tr>
</tbody>
</table>

**Load-Transfer Matrix**

<table>
<thead>
<tr>
<th></th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
</tr>
</thead>
</table>
Appendix Three
OPUS Two and OPUS Three;
Computer Programmes for the Transfer Matrix Analysis of
Multibeam Structures

A3.0 Introduction

These programmes, written in PDQ FORTRAN, were developed specifically for the analysis of multibeam bridge and floor deck systems, using the transfer matrix method as proposed in Chapter 3. This Appendix presents details of the programmes and includes programme listings, storage requirements and data format.

A3.1 Description

The theoretical basis for the method has been given in Chapter 3 and the programmes have been organised to analyse either bridge decks or floor systems of any width and span for any load applied along the midspan central line. Deflection and shears are calculated for all joints and these are used to find the actual load carried per beam and a percentage load distribution coefficient.

A3.2 Storage Required and Approximate Speed

OPUS Two occupies 19,439 storage locations exclusive of subroutines and takes four seconds to calculate deflections and shears in a ten-beam deck for a single load condition; a further 90 seconds is required to list these results.

OPUS THREE occupies 19,420 storage locations which is again exclusive of subroutines, and for ten-beam deck takes just 45 seconds to compute and list the load carried by each beam and the respective distribution coefficients (expressed as a percentage of the total applied load). If either the deflection profile or the load distribution curve is required these may be plotted by sense switch control on the 1620 Console Typewriter. The time taken to plot one graph including axes and axis values is about 3 minutes.

A3.3 Input and Output: Data Required, Operation and Sense Switch Settings

A3.3.1 OPUS Two

The sequence of data cards and operations for OPUS Two is as follows:

<table>
<thead>
<tr>
<th>Card</th>
<th>Column Numbers</th>
<th>Data Description</th>
<th>Format Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 10</td>
<td>Name of Structure</td>
<td>A5, A5</td>
</tr>
<tr>
<td></td>
<td>11 - 15</td>
<td>Beam Type</td>
<td>A5</td>
</tr>
<tr>
<td></td>
<td>16 - 20</td>
<td>Span Identification</td>
<td>A5</td>
</tr>
<tr>
<td></td>
<td>21 - 23</td>
<td>No. of Beams in the Deck</td>
<td>13</td>
</tr>
</tbody>
</table>
After reading this card, the page is headed and the programmes stop to ACCEPT the date through the Console Typewriter.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 - 10</td>
<td>Modulus of Elasticity, E</td>
<td>E10.4</td>
</tr>
<tr>
<td>11 - 20</td>
<td>Modulus of Rigidity, G</td>
<td>E10.4</td>
<td></td>
</tr>
<tr>
<td>21 - 26</td>
<td>Span Length</td>
<td>F6.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1 - 6</td>
<td>Beam Width (Right Width)</td>
<td>F6.3</td>
</tr>
<tr>
<td>7 - 12</td>
<td>Beam Depth</td>
<td>F6.3</td>
<td></td>
</tr>
<tr>
<td>13 - 15</td>
<td>Angle of Skew</td>
<td>F3.0</td>
<td></td>
</tr>
<tr>
<td>16 - 21</td>
<td>Breadth of Internal Core</td>
<td>F6.3</td>
<td></td>
</tr>
<tr>
<td>22 - 27</td>
<td>Depth of Internal Core</td>
<td>F6.3</td>
<td></td>
</tr>
<tr>
<td>28 - 29</td>
<td>Number of Internal Cores</td>
<td>F2.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1 - 6</td>
<td>Applied Load</td>
<td>F6.1</td>
</tr>
</tbody>
</table>

After reading this card, a data check is made and listed for programme debugging and identification.

The beam flexibility and torsion parameters, X and Y, are calculated and listed followed by the formation of the field transfer matrices.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5+</td>
<td>1 - 3</td>
<td>Number of beams to the left of the load</td>
<td>13</td>
</tr>
<tr>
<td>4 - 6</td>
<td>Number of beams to the right of the load</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

There are as many of these load-position cards as there are positions to be analysed, with a maximum equal to the number of beams in the deck.

The sequence of result cards from OPUS TWO, which are also the data cards for OPUS THREE, is as follows:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 14</td>
<td>Load Applied</td>
<td>E14.8</td>
</tr>
<tr>
<td>15 - 19</td>
<td>No. of Beams in the Deck</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>20 - 23</td>
<td>No. of Beams to the right of the load</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1 - 8</td>
<td>Deflection at joint 0</td>
<td>F8.5</td>
</tr>
<tr>
<td>3</td>
<td>1 - 10</td>
<td>Shear Force at joint 0</td>
<td>F10.2</td>
</tr>
<tr>
<td>4</td>
<td>1 - 8</td>
<td>Deflection at joint 1</td>
<td>F8.5</td>
</tr>
<tr>
<td>5</td>
<td>1 - 10</td>
<td>Shear Force at joint 1</td>
<td>F10.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The total number of these cards again depends on the number of beams in the deck and the number of load positions to be analysed.

The following use has been made of the sense switch settings in OPUS TWO

<table>
<thead>
<tr>
<th>Sense Switch Number</th>
<th>Use (OPUS TWO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>For the analysis of floor systems this switch is turned ON but for bridge decks it must be OFF.</td>
</tr>
<tr>
<td>2</td>
<td>On the control of this switch all deflections and shear forces are listed if ON, but only selected values are listed if turned OFF.</td>
</tr>
<tr>
<td>3</td>
<td>If the load lies above a beam centreline, this switch must be ON, but if applied to a joint then it must be OFF.</td>
</tr>
<tr>
<td>4</td>
<td>To trace all arithmetic instructions using the Floating Point Accumulator (FAC), sense switch 4 is turned ON.</td>
</tr>
</tbody>
</table>

A3.32 OPUS THREE

OPUS THREE uses the above output as data and lists deflections or load distribution as indicated in the sense switch settings below.

<table>
<thead>
<tr>
<th>Sense Switch Number</th>
<th>Use (OPUS THREE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>For the insertion of new axis values for the graph plotting subroutine, sense switch 1 is turned ON.</td>
</tr>
<tr>
<td>2</td>
<td>If the graphs are to be plotted automatically switch 2 is turned OFF.</td>
</tr>
<tr>
<td>3</td>
<td>If the deflection values are to be plotted and listed this switch is turned OFF, but if it is ON the load carried per beam and the percentage coefficients are listed and, if required, plotted.</td>
</tr>
<tr>
<td>4</td>
<td>The plotting of results is supressed if this switch is OFF and the programme automatically returns for new data. The trace feature is not included in this programme.</td>
</tr>
</tbody>
</table>

A3.4 Flow Charts

Schematic Flow Charts for the programmes are given in Table A3.1 and A3.2 respectively.
Programme Listings and Specimen Results

Programme Listings for both OPUS TWO and OPUS THREE are given here, including source statement addressed and symbol table.

The set of specimen results included in this Appendix were calculated for a ten-beam deck - span D of the Slippery Creek Bridges. Output listings are given for the deflections and shears from OPUS TWO and the load distribution and coefficients from OPUS THREE. A typical graph is also included.
START

Read: E, G, S

Read: RB, D, θ, BI, DI, PH

Read: P

Compute BMI

Compute X

ZA = B/D; find β

Compute BTJ

Compute Y

Compute $Z_{i+1} = F_i \cdot Z_i$

ON IF SS2 OFF

Print & Punch $Z_{i+1}$

Punch $Z_{i+1}$

Compute $Z_0 = \frac{A(1,1)}{B(1,2)}$ (floors)

Compute $Z_0 = \frac{A(2,1)}{B(2,1)}$ (bridges)

ON IF SS1 OFF

Compute $D = F_{NL}, W = F_{NR}, V = F_W$

A = DV, B = DC

Read: NL, NR

Punch: P, NBW, NR

IF SS3

Compute $F, C_F$ (beam P)

Compute $P, C_P$ (point P)

Print & Punch $Z_0$

Fig. A3.1 Flow Chart of OPUS TWO
Flow Chart of OPUS THREE

START

Read: P, NB, NR

Read: Z

Compute
DL(I), PDL(I)
SDL

ON IF SS3

Print: P, NB

ON IF SS4

Compute G(I)
Plot: DL(I) using SS1&2

Print: DL(I), PDL(I)
SDL

OFF

Print: P, NB

Compute G(I)
Plot: Defl'n using SS1&2

Print: Deflection

Fig. A3.2
PROGRAMME TO DETERMINE LOAD TRANSFER IN MULTIBEAM BRIDGE DECKS USING TRANSFER MATRICES.

DIMENSION T(9), R(9)
BEGIN TRACE
READ 201, H1, H2, H3, H4, NB
PRINT 202
FORMAT (A5, A5, A5, A5, 13)

PRINT 202
FORMAT (35HIGH TRANSFER MATRIX ANALYSIS OF THE)

PRINT 203
FORMAT (2X, A5, A5, 2X, 16HMULTIBEAM BRIDGE)

CONTROL 102
PRINT 204, H3, NB, H4
FORMAT (11HBEBAM TYPE*, A5, 12H NO IN DECK*, 13, 7H SPAN*, A5, 5H DATE)
ACCEPT 206, DATE

FORMAT (F8.2)
CONTROL 102
PRINT 207, E, G, S
FORMAT (E10.4, E10.4, F6.3)

READ 208, RB, DD, THETA, B1, DI, PH

READ 22, P
FORMAT (F6.1)

FORMAT (12HDATA CHECK*)

TYPE 205, E, G, S, RB, DD, THETA, B1, DI, PH, P

T(1) = 0.0
R(1) = 0.0
T(2) = 1.0
R(2) = 0.141
T(3) = 2.0
R(3) = 0.229
T(4) = 3.0
R(4) = 0.267
T(5) = 4.0
R(5) = 0.281
T(6) = 6.0
R(6) = 0.299
T(7) = 8.0
R(7) = 0.307
T(8) = 10.0
R(8) = 0.313
T(9) = 100.0
R(9) = 0.333

H = THETA*3, 1415927/180.
BB = RB/COS(H)
BMI = (DD**3*R*RB - D1**3*B1*PH)/12.
X = (S**3)/(48.*E*BMI)
ZA = BB/DD
-8290 100 DO 101 J=1,7
-8302  QQ=ZA-T(J)
-8362  RR=T(J+1)-ZA
-8422  IF (QQ) 100, 103, 102
-8473 102 IF (RR) 101, 104, 105
-8534 101 CONTINUE
-8570 105 TZ=T(J+1)-T(J)
-8654  TF=R(J+1)-R(J)
-8738  FA=R(J)+QQ*(TF/TZ)
-8822  GO TO 108
-8830 103 FA=R(J)
-8866  GO TO 108
-8874 104 FA=R(J+1)
-8910 108 CONTINUE
-8910  BTJ=DD**3*BB*FA*
-8970  Y=(BB*BB**S)/(16.*BTJ**G)
-9078  CONTROL 102
-9090  CONTROL 102
-9102  PRINT 209
-9114 209 FORMAT (8HBEAM FLEXIBILITY AND TORSION PARAMETERS X AND Y)
-9234 210 FORMAT (15H (SINGLE POINT))
-9300 214 PRINT 109
-9312 219 FORMAT (3X2HR6X1HB6X1HD 6X1HF7X3HBM16X3HB16X1HX13X1HY)
-9468  PRINT 110, RB, BB; DD, - FA, BM1, BTJ, X, Y
-9634  C
-9634  DIMENSION A(2, 2), B(2, 2), C(2, 1), D(2, 2), FL(2, 2) FR(2, 2) U(2, 2)
-9634  DIMENSION V(2, 2), W(2, 2), Q(2, 2), Z(2, 1). ZNR(2, 2) ZNL(2, 2) F(2, 2)
-9634  FR(1, 1)=(X+Y)/(X-Y)
-9718  FR(2, 1)=1,7(X-Y)
-9766  FR(1, 2)=(4.*X**Y)/(X-Y)
-9874  FR(2, 2)=FR(1, 1)
-9886  FL(1, 1)=FR(1, 1)
-9922  FL(1, 2)=-FR(1, 2)
-9953  FL(2, 2)=-FR(2, 2)
-9970  IF (SENSE SWITCH 3) 85, 86
-9990 86 F(1, 1)=1;
-9990  F(2, 1)=0.
-9990  F(1, 2)=0.
-9990  F(2, 2)=-1.
-9990  C(1, 1)=0.
-9990  C(2, 1)=P
-9990  GO TO 215
-9990 85 F(1, 1)=FR(1, 1)
-9990  F(2, 1)=-FR(2, 1)
-9990  F(1, 2)=FR(1, 2)
-9990  F(2, 2)=-FR(2, 2)
-9990  C(1, 1)=-2.*X**Y**P/(X-Y)
-9990  C(2, 1)=X**P/(X-Y)
-9990 215 CONTROL 102
-9990  CONTROL 102
-9990  PRINT 25
-9990 25 FORMAT (23HDEFLECTIONS AND SHEAR FORCES)
-9990 31 READ 46, NL, NR
J0546  NB=NL+1
J052  NBW=NR+NL+1
J0630  PUNCH 62,P,NB,W,NR
J0678  62  FORMAT (E14.8,14,14)
J0710  IF (SENSE SWITCH 3) 87,88
J0730  88  NL=NL+1
J0766  87  A(1,1)=1.
J0778  A(1,2)=0.
J0790  A(2,1)=0.
J0802  A(2,2)=1.
J0814  B(1,1)=1.
J0826  B(1,2)=0.
J0838  B(2,1)=0.
J0850  B(2,2)=1.
J0862  IF (NL-1) 32,33,33
J0930  33  DO 10 L=1,NL
J0942  IF (L-NL) 42,42,10
J1010  42  DO 9 I=1,2
J1022  DO 9 J=1,2
J1034  D(1,J)=0.
J1106  9  DO 9 K=1,2
J1118  9  D(I,J)=D(I,J)+FL(I,K)*A(K,J)
J1514  DO 49 I=1,2
J1526  DO 49 J=1,2
J1538  49  A(I,J)=D(I,J)
J1754  10  CONTINUE
J1790  GO TO 34
J1798  32  DO 44 I=1,2
J1810  DO 44 J=1,2
J1822  44  D(I,J)=A(I,J)
J2038  34  IF (NR-1) 35,36,36
J2106  36  DO 11 L=1,NR
J2118  IF (L-NR) 43,43,11
J2186  43  DO 12 I=1,2
J2198  DO 12 J=1,2
J2210  W(I,J)=0.
J2282  DO 12 K=1,2
J2294  12  W(I,J)=W(I,J)+FR(I,K)*B(K,J)
J2690  DO 50 I=1,2
J2702  DO 50 J=1,2
J2714  50  B(I,J)=W(I,J)
J2930  11  CONTINUE
J2966  GO TO 37
J2974  35  DO 45 I=1,2
J2986  DO 45 J=1,2
J2998  45  W(I,J)=B(I,J)
J3214  37  DO 13 I=1,2
J3226  DO 13 J=1,2
J3238  V(I,J)=0.
J3310  DO 13 K=1,2
J3322  13  V(I,J)=V(I,J)+F(I,K)*W(K,J)
J3718  DO 14 I=1,2
J3730  DO 14 J=1,2
J3742  U(I,J)=0.
J3814  DO 14 K=1,2
14  U(1, J) = U(1, J) + D(I, K) * V(K, J)
DO 15  I = 1, 2
    Q(1, 1) = 0,
DO 15  K = 1, 2
15  Q(1, 1) = Q(1, 1) + D(I, K) * C(K, 1)
IF (SENSE SWITCH 1) 54, 53
53  XNR = Q(2, 1) / U(2, 1)
ZNR(1, 1) = XNR
ZNR(2, 1) = 0.
GO TO 55
54  FNR = Q(1, 1) / U(1, 2)
ZNR(1, 1) = 0.
ZNR(2, 1) = FNR
55  PRINT 26, ZNR(1, 1)
PUNCH 23, ZNR(1, 1)
PRINT 27, ZNR(2, 1)
PUNCH 27, ZNR(2, 1)
IF (NR - 1) 93, 39, 39
39  DO 1  L = 1, NR
   IF (L - NR) 79, 79, 1
79  DO 4  I = 1, 2
   Z(1, 1) = 0.
4  Z(1, 1) = Z(1, 1) + FR(I, K) * ZNR(K, 1)
DO 51  I = 1, 2
51  ZNR(1, 1) = Z(1, 1)
PUNCH 23, ZNR(1, 1)
PUNCH 27, ZNR(2, 1)
IF (SENSE SWITCH 2) 81, 1
81  PRINT 26, ZNR(1, 1)
PRINT 27, ZNR(2, 1)
1 CONTINUE
93  IF (SENSE SWITCH 3) 38, 90
38  PRINT 28
PRINT 80, NB
GO TO 89
90  PRINT 91
PRINT 92, NL
89  DO 5  I = 1, 2
5  ZNL(1, 1) = 0.
DO 52  K = 1, 2
52  ZNL(1, 1) = ZNL(1, 1) + F(I, K) * ZNR(K, 1) + C(I, 1) / 2.
PRINT 26, ZNL(1, 1)
PUNCH 23, ZNL(1, 1)
PRINT 27, ZNL(2, 1)
PUNCH 27, ZNL(2, 1)
IF (NL - 1) 40, 41, 41
41  DO 6  L = 1, NL
6  IF (L - NL) 7, 7, 6
7  DO 8  I = 1, 2
8  Z(1, 1) = 0.
6  Z(1, 1) = Z(1, 1) + FL(I, K) * ZNL(K, 1)
J6574 DO 52  I=1,2
J6586 52 ZNL (1,1)=Z(1,1)
J6694 PUNCH 23, ZNL (1,1)
J6718 PUNCH 27, ZNL (2,1)
J6742 IF (SENSE SWITCH 2) 82,6
J6762 PRINT 26, ZNL (1,1)
J6786 PRINT 27, ZNL (2,1)
J6810 6 CONTINUE
J6846 40 CONTINUE
J6846 IF (SENSE SWITCH 2) 83,31
J6866 83 PAUSE
J6878 GO TO 31
J6886 46 FORMAT (13,13)
J6914 23 FORMAT (F8.5)
J6936 26 FORMAT (/F8.5)
J6964 27 FORMAT (F10.2)
J6986 28 FORMAT (/12H LOADED UNIT)
J7040 80 FORMAT (9H BEAM NO.,13)
J7088 91 FORMAT (/13H LOADED JOINT)
J7144 92 FORMAT (7H)

J7188 .C PROG SW 1 ON TO ANALYSE FLOOR SYSTEMS BUT OFF FOR BRIDGE DECKS
J7188 .C PROG SW 2 ON TO PRINT AND PUNCH OUT BUT OFF TO PUNCH OUT ONLY
J7188 .C PROG SW 3 ON FOR LOAD ON UNIT CL., BUT OFF FOR LOAD ON JOINT CL.
J7188 .C PROG SW 4 ON FOR TRACE BUT OFF FOR NO TRACE
J7188 END TRACE
J7188 END

L9999 SIN L9989 SINF L9979 COS L9969 COSF
L9995 EXP L9949 EXPF L9939 LOG L9929 LOGF
L9919 SORT L9909 SQRTF L9899 ABS L9889 ABSF
L9879 DRH L9869 DRHF L9859 ATAN L9849 ATANF
L9839 T 39759 L9749 R 39669 L9659 SN 0201 L9649 H1
L9633 H2 L9629 H3 L9619 H4 L9609 NB
L9599 SN 0202 L9589 SN 0203 L9579 SN 0204 L9569 SN 0206
L9559 DATE L9549 SN 0207 L9539 E L9529 G
L9519 S L9509 SN 0208 L9499 RB L9489 DD
L9479 THETA L9469 DI L9459 DI L9449 NH
L9439 SN 0022 L9429 P L9419 SN 0220 L9409 SN 0205
L9399 00000000000 L9389 51100000000 L9379 50141000000 L9369 51200000000
L9359 50229000000 L9349 51300000000 L9339 50267000000 L9329 51400000000
L9319 50281000000 L9309 51600000000 L9299 50299000000 L9289 51800000000
L9279 50307000000 L9269 52100000000 L9259 50313000000 L9249 53100000000
L9239 50333000000 L9229 H L9219 5131415927 L9209 0000
L9199 53180000000 L9189 BB L9179 BMI L9169 0000
L9159 001 L9149 002 L9139 52120000000 L9129 X
L9119 52480000000 L9109 003 L9099 ZA L9089 SN 0100
L9079 SN 0101 L9069 J L9059 QQ L9049 RR
L9039 SN 0103 L9029 SN 0102 L9019 SN 0104 L9009 SN 0105
L8999 TZ L8989 TF L8979 FA L8969 SN 0108
L8959 BTJ L8949 Y L8939 52160000000 L8929 SN 0209
L8919 SN 0210 L8909 SN 0212 L8899 SN 0214 L8889 SN 0109
L8879 SN 0110 L8869 A 38839 L8859 38799 L8789 C 38779
L8769 D 38739 L8729 FL 38699 L8689 FR 38659 L8649 U 38619
L8609 V 38579 L8569 W 38539 L8529 Q 38499 L8489 Z 38479
L8469 ZNR 38459 L8449 ZNL 38439 L8429 F 38399 L8389 SN 0085
| L8379 SN 0086 | L8369 SN 0215 | L8359 SN 0025 | L8349 SN 0031 | L8309 SN 0001 |
| L8339 SN 0046 | L8329 NL | L8319 NR | L8319 I | L8189 K |
| L8299 NDW | L8289 SN 0062 | L8279 SN 0087 | L8269 SN 0088 | L8229 L |
| L8259 SN 0032 | L8249 SN 0033 | L8239 SN 0010 | L8229 L | |
| L8219 SN 0042 | L8209 SN 0009 | L8199 I | L8149 SN 0035 | |
| L8179 SN 0049 | L8169 SN 0034 | L8159 SN 0044 | L8109 SN 0012 | |
| L8139 SN 0036 | L8129 SN 0011 | L8119 SN 0043 | L8069 SN 0013 | |
| L8099 SN 0050 | L8089 SN 0037 | L8079 SN 0045 | L8029 SN 0053 | |
| L8059 SN 0014 | L8049 SN 0015 | L8039 SN 0054 | L7989 SN 0026 | |
| L8019 XNR | L8009 SN 0055 | L7999 FNR | L7949 SN 0039 | |
| L7979 SN 0023 | L7969 SN 0027 | L7959 SN 0093 | L7909 SN 0051 | |
| L7939 SN 0001 | L7929 SN 0079 | L7919 SN 0004 | L7869 SN 0028 | |
| L7899 SN 0081 | L7889 SN 0038 | L7879 SN 0090 | L7829 SN 0092 | |
| L7859 SN 0080 | L7849 SN 0089 | L7839 SN 0091 | L7789 SN 0006 | |
| L7819 SN 0005 | L7809 SN 0040 | L7799 SN 0041 | L7789 SN 0006 | |
| L7779 SN 0007 | L7769 SN 0008 | L7759 SN 0052 | L7749 SN 0082 | |
C  OPUS THREE

C  LOAD DISTRIBUTION DETERMINATION AND GRAPH PLOTTER

C  DIMENSION CD(19), CF(19), DL(13), G(73), PDL(19)

C  PRINT 51

C  51 FORMAT ('/27HSET SENSE SWITCHES AS BELOW')

C  PRINT 54

C  54 FORMAT ('49HSW 2, TO INSERT NEW AXIS VALUES/AUTOMATIC PLOTTING')

C  PRINT 55

C  55 FORMAT ('47HSW 3, LOAD DISTRIBUTION CURVES/REFLECTION CURVES')

C  PRINT 61

C  61 FORMAT ('49HSW 4, TO SUPPRESS PLOTTING RESULT/GRAPHS PLOT')

C  READ 62, P, NBW, NR

C  62 FORMAT ('E14.3, I4, I4')

C  NBWO=NBW+1

C  DO 1 I=1, NBWO

C  1 FORMAT ('F8.5')

C  READ 2, CD(1)

C  DO 2 FORMAT ('F8.5')

C  READ 3, CF(1)

C  IF (NR-1) 30, 19, 19

C  3 FORMAT ('F10.2')

C  DO 19 L=1, NR

C  DL(L)=CF(I+1)-CF(I)

C  30 I=NR+1

C  DL(I)=P-CF(I+1)-CF(I)

C  1=I+1

C  IF (1-NBW)15, 15, 36

C  DO 17 L=1, NBW

C  DL(L)=CF(L)-CF(L+1)

C  17 CONTINUE

C  NB=NBW-NR

C  DO 35 L=1, NBW

C  PDL(I)=100.*DL(I)/P

C  SDL=DL(I)

C  DO 26 L=2, NBW

C  SDL=SDL(I)+SDL

C  DO 32 I=1, 73

C  32 G(I)=-1.

C  IF (SENSE SWITCH 3) 11, 13

C  PRINT 12

C  12 FORMAT ('29HTRANSVERSE REFLECTION PROFILE')

C  PRINT 13, P, NB

C  13 FORMAT ('3HFOR20X6HUNDER F10.2, 5H LOAD3H ON2H B13')

C  L=1

C  DO 22 L=1, NBWO

C  8713 G(L)=CD(I)

C  L=L+4

C  CONTINUE

C  IF (SENSE SWITCH 4) 43, 44

C  EXECUTE PROCEDURE 9979

C  PRINT 42

C  42 FORMAT ('26HDEFLECTION VALUES (INCHES)')


```
-9006  DO 74  I=1,NBWO
-9013   TYPE 41,CD(I)
-9066   41  FORMAT (F9.5)
-9088   IF(I-7) 74,71,73
-9156   73  IF(I-14) 74,71,72
-9224   72  IF(I-21) 74,71,74
-9292   71  CONTROL 102
-9304   74  CONTINUE
-9340   CONTROL 102
-9352   CONTROL 102
-9364   CONTROL 102
-9376   CONTROL 102
-9388   CONTROL 102
-9400   CONTROL 102
-9412   CONTROL 102
-9424   GO TO 10
-9432   11  PRINT 16
-9444   16  FORMAT (34HTRANSVERSE LOAD DISTRIBUTION CURVE)
-9536   PRINT 13, P, NB
-9572 L=3
-9594   DO 33 I=1,NBW
-9596   G(L)=DL(I)
-9663   L=L+4
-9704   33  CONTINUE
-9740   IF (SENSE SWITCH 4) 24,23
-9760   23  EXECUTE PROCEDURE 9979
-9772   24  PRINT 34
-9784   34  FORMAT (47HLOAD DISTRIBUTION AND DISTRIBUTION COEFFICIENTS)
-9902   CONTROL 102
-9914   DO 84 I=1,NBW
-9926   TYPE 21,DL(1),PDW(1)
J0010   21  FORMAT (F10.2,F9.4)
J0038   IF (I-3) 84,81,82
J0106   82  IF (I-6) 84,81,83
J0174   83  IF (I-9) 84,81,85
J0242   85  IF (I-12) 84,81,86
J0310   86  IF (I-15) 84,81,87
J0378   87  IF (I-18) 84,81,88
J0446   88  IF (I-21) 84,81,84
J0514   81  CONTROL 102
J0552   84  CONTINUE
J0562   28  PRINT 27,SDL
J0586   27  FORMAT (26HSUM OF DISTRIBUTED LOADS =F19.2)
J0668   CONTROL 102
J0680   CONTROL 102
J0692   CONTROL 102
J0704   GO TO 10
```
J07 12 begin procedure 9979
J07 40 control 102
J07 52 control 102
J07 64 control 102
J07 66 dimension jak(73)
J07 76 if(sense switch 2) 9914, 9917
J07 96 print 9916
J08 08 format ('1hi gb/jtb graph plotter subroutine, aug65')
J09 14 print 9980
J09 26 format('9hlow x val')
J09 68 accept 9981, xll
J09 92 format('f5.3')
J10 14 print 9982
J10 26 format('14hx div(1.3 ins)')
J10 73 accept 9981, xint
J11 02 print 9983
J11 14 format('9hlow y val')
J11 56 accept 9981, yll
J11 80 print 9984
J11 92 format('14hy div(1.0 ins)')
J12 44 accept 9981, yint
J12 63 print 9985
J12 80 format('32hset tabs at 20, 32, 44, 56, 68, 80, 92')
J13 63 print 9986
J13 80 format('15hset margin at 9')
J14 34 print 9987
J14 46 format('43hsw 1 on for margins and values press start')
J15 56 pause
J15 63 do 9991 = 1,73
J15 80 jak(1) = (g(1) - yll) * 6. / yint + 0.5
J17 12 continue
J17 48 control 102
J17 60 jakz=42
J17 72 execute procedure 9940
J17 84 do 9996 multy=1,6
J17 96 do 9995 multx=1,5
J18 08 klkz=1
J18 20 jakz=42 - 6*(multy - 1) - multx
J19 04 execute procedure 9940
J19 16 continue
J19 52 klkz=2
J19 64 if(sense switch 1) 9992, 9993
J19 84 yord=jakz/6
J20 32 yord=yord* yint+yll
J20 80 type 9997; yord
J21 04 format('f9.4')
J21 26 jakz=42 - 6*multy
J21 80 execute procedure 9940
J21 98 continue
J2234 DO 9998 MULTX=1.5
J2246 KLIKZ=1
J2258 JAKZ=6-MULTX
J2294 EXECUTE PROCEDURE 9940
J2306 9998 CONTINUE
J2342 9994 IF(SENSW SWITCH 1)9994, 9973
J2362 9994 TYPE 9997,YLL
J2393 9973 JAKZ=0
J2398 EXECUTE PROCEDURE 9960
J2410 9918 IF(SENSW SWITCH 1)9918, 9909
J2430 9918 XXX1=XLL+XINT
J2466 XXX2=XXX1+XINT
J2502 XXX3=XXX2+XINT
J2538 XXX4=XXX3+XINT
J2574 TYPE 9906, XLL
J2593 TYPE 9905, XXX1, XXX2, XXX3, XXX4
J2653 9906 FORMAT(F15.2)
J2680 9905 FORMAT(F18.2, F18.2, F18.2, F18.2)
J2718 9903 CONTINUE
J2728 9909 CONTROL 102
J2730 CONTROL 102
J2742 CONTROL 102
J2754 CONTROL 102
J2766 CONTROL 102
J2778 9909 IF(SENSW SWITCH 2)9939, 9913
J2793 9989 PRINT9907
J2810 9907 FORMAT(///43HIF NOT SATISFIED INSERT 490 AND PUSH RS)
J2946 PAUSE
J2953 9903 CONTINUE
J2958 END PROCEDURE 9979
J2966 BEGIN PROCEDURE 9940
J2994 CONTROL 108
J3006 DO 9920 I=1, 12
J3018 MARKS =0
J3030 DO 9921 IL=1, 12
J3042 NUIB = I+IL-1
J3090 IF (SENSE SWITCH 1) 9933, 9929
J3110 9933 IF (NUMB-1) 9929, 9928, 9929
J3178 9920 IF(JAK(NUMB)-JAKZ) 9930, 9922, 9930
J3270 9930 IF(KLIKZ-1) 9944, 9944, 9943
J3338 9944 TYPE 9942
J3350 9942 FORMAT (1HI)
J3376 MARKS=IL
J3396 GO TO 9921
J3403 9943 TYPE 9945
J3434 9945 FORMAT (1H-)
J3446 MARKS=IL
J3446 GO TO 9921
J3454 9929 IF(JAK(NUMB)-JAKZ) 9921, 9927, 9921
J3546 9927 MARK= IL-MARKS-1
J3594 IF(MARKE) 9922, 9922, 9926
J3650 9926 DO 9923 JOXX=1, MARKE
J3662 CONTROL 101
J3674 9923 CONTINUE
J3710 9922 EXECUTE PROCEDURE 9924
J3722 MARKS=IL
J3734 9921 CONTINUE
J3770 IF(MARKS-11) 9910, 9911, 9920
J3833 9910 CONTROL 103
J3850 GO TO 9920
J3853 9911 CONTROL 101
J3870 9920 CONTINUE
J3906 IF(JAK(73)-JAKZ) 9934, 9935, 9937
J3974 9934 IF(SENSE SWITCH 1) 9936, 9937
J3994 9936 IF(KLIKZ-1) 9946, 9946, 9947
J4062 9946 TYPE 9942
J4074 CONTROL 102
J4084 RETURN 9940
J4094 9947 TYPE 9945
J4106 CONTROL 102
J4113 RETURN 9940
J4126 9937 CONTROL 102
J4138 RETURN 9940
J4146 9935 EXECUTE PROCEDURE 9924
J4153 CONTROL 102
J4170 END PROCEDURE 9940
J4173 BEGIN PROCEDURE 9960
J4206 CONTROL 103
J4213 IF(SENSE SWITCH 1) 9969, 9971
J4233 9969 DO 9970 I=1, 55, 18
J4250 DO 9970 IL=1, 18
J4262 NUMB=I+IL-1
J4310 IF(JAK(NUMB)-JAKZ) 9950, 9951, 9950
J4402 9951 EXECUTE PROCEDURE 9924
J4414 GO TO 9970
J4422 9950 IF(NUMB-1) 9953, 9954, 9953
J4490 9954 TYPE 9956
J4502 9956 FORMAT (1H+)
J4523 GO TO 9970
J4536 9953 IF(NUMB-10) 9957, 9953, 9957
J4604 9953 TYPE 9955
J4616 9955 FORMAT (1HI)
J4642 GO TO 9970
J4650 9957 IF(NUMB-37) 9959, 9961, 9959
J4713 9961 TYPE 9955
J4730 GO TO 9970
J4733 9959 IF(NUMB-55) 9962, 9963, 9962
J4806 9963 TYPE 9955
J4833 9962 TYPE 9964
J4926 9962 FORMAT (1H-)
J4933 9964 CONTINUE
J4936 IF(JAK(73)-JAKZ) 9965, 9966, 9965
J5004 9966 EXECUTE PROCEDURE 9924
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### Beam Flexibility and Torque Parameters, X and Y (Single Point)

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<th>C</th>
<th>D</th>
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### Reflections and Shear Forces

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**Load Data**
SW 2: TO INSERT NEW AXIS VALUES/AUTOMATIC PLOTTING
SW 3: LOAD DISTRIBUTION CURVES/DEFLECTION CURVES
SW 4: TO SUPPRESS PLOTTING RESULTS/GRAPHS PLOTTED

TRANSVERSE LOAD DISTRIBUTION CURVE
FOR SLIPPERY CREEK "D" UNDER 64000.00 LOAD ON B 1

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<tr>
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<tr>
<td>1612.52 2.5195 1791.68 2.7995 2169.90 3.3904</td>
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<tr>
<td>2789.22 4.3581 3718.42 5.3100 5060.77 7.9074</td>
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<td>6965.33 10.3034 9643.89 15.0685 13393.99 20.9279</td>
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<td>16384.43 26.3348</td>
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SUM OF DISTRIBUTED LOADS = 64000.00

TRANSVERSE LOAD DISTRIBUTION CURVE
FOR UNDER 64000.00 LOAD ON B 2

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<td>3099.12 4.8423 4131.56 6.7555 5623.74 8.7360</td>
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<td>13393.89 20.9279</td>
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SUM OF DISTRIBUTED LOADS = 64000.00

TRANSVERSE LOAD DISTRIBUTION CURVE
FOR UNDER 64000.00 LOAD ON B 3

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SUM OF DISTRIBUTED LOADS = 63999.99

TRANSVERSE LOAD DISTRIBUTION CURVE
FOR UNDER 64000.00 LOAD ON B 4

<table>
<thead>
<tr>
<th>Load Distribution and Distribution Coefficients</th>
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</thead>
<tbody>
<tr>
<td>2789.21 4.3581 3099.11 4.8423 3753.34 5.3645</td>
</tr>
<tr>
<td>4824.57 7.5383 6431.84 10.0497 8753.73 13.6777</td>
</tr>
<tr>
<td>10270.51 16.0476 9373.04 14.6453 7739.28 12.0926</td>
</tr>
<tr>
<td>6965.37 10.3833</td>
</tr>
</tbody>
</table>

SUM OF DISTRIBUTED LOADS = 64000.00

TRANSVERSE LOAD DISTRIBUTION CURVE
FOR UNDER 64000.00 LOAD ON B 5

<table>
<thead>
<tr>
<th>Load Distribution and Distribution Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>3718.42 5.8100 4131.56 6.4555 5003.73 7.3133</td>
</tr>
<tr>
<td>6431.84 10.0497 8574.56 13.3977 9392.29 15.4567</td>
</tr>
<tr>
<td>8753.73 13.6777 6810.07 16.6407 5623.04 8.7360</td>
</tr>
<tr>
<td>5060.76 7.9074</td>
</tr>
</tbody>
</table>

SUM OF DISTRIBUTED LOADS = 64000.00
TRANSVERSE REFLECTION PROFILE
FOR SLIPPERY CREEK "D" UNDER 64000.00 LOAD ON B 1

1GB/JTB GRAPH PLOTTER SUBROUTINE... AUG65
LOW X VAL
0.085
X DIV(1.8 INS)
4.5RS
LOW Y VAL
0.285
Y DIV(1.0 INS)
-0.045
SET TABS AT 20, 32, 44, 56, 68, 80, 92
SET MARGIN AT 9
SW 1 ON FOR MARGINS AND VALUES, PRESS START
TRANSVERSE DEFLECTION PROFILE
FOR SLIPPERY CREEK "D" UNDER 64000.00 LOAD ON B 10

DEFLECTION VALUES (INCHES)
0.04547 0.04661 0.05009 0.05610 0.06492 0.07700 0.09295
0.11357 0.13990 0.13962 0.11274 0.09153 0.07491 0.06205
0.05232 0.04521 0.04037 0.03756 0.03664
APPENDIX FOUR  OPUS FIVE: STATISTICAL ANALYSIS PROGRAMME FOR CONCRETE

A4.0 Introduction

This programme, also written in PDQ FORTRAN, was used for the statistical analysis of the crushing strength and elastic modulus observed from tests on large numbers of 8" x 4" test cylinders. These tests have been previously discussed in secs 5.311 and 5.612.

A4.1 Description

The programme reads each block identification number, the crushing load observed for the 8" x 4" block and the strain increment in the block for a 20 ton change in load. The corresponding crushing strength and elastic modulus for each block is computed and listed. These are followed by the mean and standard deviation for both quantities; the standard deviation being expressed as a percentage of the mean.

If desired, (sense switch 1, ON) the estimated value for elastic modulus is computed and listed for each observed crushing strength, based on the following four formulae:

1. \( E = 1.8 + 0.46 \frac{f_c}{1000} \)  due to Hognestad
2. \( E = 1.8 + 0.50 \frac{f_c}{1000} \)  due to PCI
3. \( E = \frac{f_c}{1000} \)  due to ACI
4. \( E = \frac{6.0}{1 + \frac{2000}{f_c}} \)  due to Jensen

where \( E \) is the elastic modulus and \( f_c \) is the crushing strength; both have units of lbs/sq.in.

In addition a Chi-Square Goodness of Fit test is applied to find which of the above formulae best describes the observed data. Once the goodness of fit of one of the above formulae has been established for a particular sample of concrete then estimation of the elastic modulus at other known crushing strengths is possible. The goodness of fit test may be found in any text on statistical methods, e.g. Hoel\(^{41}\).

A4.2 Storage Required and Approximate Speed

The complete programme occupies 33,664 storage locations which does not include the subroutines. The relatively low speed of the typewriter in
listing the individual values makes the programme slow to run; the total
time is dependent on N, the number of blocks to be analysed.

**A4.3 Input and Output: Data Required and Sense Switch Settings**

The sequence of data cards is as follows:

<table>
<thead>
<tr>
<th>Card</th>
<th>Column Numbers</th>
<th>Data Description</th>
<th>Format Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 3</td>
<td>Number of blocks to be considered (N)</td>
<td>.13</td>
</tr>
<tr>
<td></td>
<td>4 - 9</td>
<td>5% critical point for $\chi^2_{N-1}$</td>
<td>F6.3</td>
</tr>
<tr>
<td>2</td>
<td>1 - 3</td>
<td>Block number</td>
<td>F3.0</td>
</tr>
<tr>
<td></td>
<td>4 - 9</td>
<td>Crushing load</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>10 - 12</td>
<td>Strain increment</td>
<td>F3.0</td>
</tr>
</tbody>
</table>

The Output is listed as described in sec A4.1 above and illustrated in the specimen listing given in sec A4.5 below.

The following use has been made of the sense switch settings in OPUS FIVE.

<table>
<thead>
<tr>
<th>Sense Switch Number</th>
<th>Use (OPUS FIVE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If this switch is turned OFF the formulae predictions for $E$ given the observed $f_c$ are supressed.</td>
</tr>
<tr>
<td>2 and 3</td>
<td>Not used</td>
</tr>
<tr>
<td>4</td>
<td>To trace all arithmetic instructions this switch is turned ON.</td>
</tr>
</tbody>
</table>

**A4.4 Flow Chart**

A schematic flow chart for OPUS FIVE is given in Fig A4.1.

**A4.5 Programme Listing and Specimen Result**

The source statements for the programme are listed in this Appendix; such listing includes the statement addresses and symbol table locations.

A set of results for some of the concrete sampled from the model SA is included below. The number of observations (N) was 31 and $\chi^2_{N-1}$ was 43.7.

As indicated in sec 5.612 of Chapter 5, Jensen's formulae is shown to give the best prediction of elastic modulus, at least to a 5% level of significance.
Fig. A4.1 Flow Chart of OPUS FIVE
OPUS FIVE

STATISTICAL ANALYSIS OF CONCRETE

BEGIN TRACE

DIMENSION BNO(150), P(150), A1(150), XDF(150), X(150), FC(150), E(150)

DIMENSION XDF2(150), E1(150), E2(150), E3(150), E4(150)

DIMENSION G(150)

READ 1, N, X2N

FORMAT (13, F6.3)

SL = 0.9 * X2N

SU = 1.1 * X2N

DO 2 I = 1, N

READ 3, BNO(I), P(I), A1(I)

FORMAT (F3.0, F6.0, F3.0)

DO 4 I = 1, N

FC(I) = P(I) / 12.566368

DO 7 I = 1, N

E(I) = 2350.0 / A1(I)

BN = N

SFC = FC(1)

DO 8 I = 2, N

SFC = SFC + FC(I)

AFC = SFC / BN

SE = E(I)

DO 9 I = 2, N

SE = SE + E(I)

AE = SE / BN

XBAR = AFC

DO 10 I = 1, N

X(I) = FC(I)

EXECUTE PROCEDURE 20

SDVF = Z

XBAR = AE

DO 11 I = 1, N

X(I) = E(I)

EXECUTE PROCEDURE 20

SDVE = Z

DO 12 I = 1, N

E1(I) = 1.8 + 0.46 * FC(I) / 1000.0

E2(I) = 1.8 + 0.5 * FC(I) / 1000.0

E3(I) = FC(I) / 4000.0

E4(I) = 6.0 / (1.0 + 2000.0 / FC(I))

PRINT 94

FORMAT (36HCOMPVE STRENGTH ELASTIC MODULUS ANG)

PRINT 95

FORMAT (33HCODE RECOMMENDATIONS FOR CONCRETE)

CONTROL 102

CONTROL 102

TYPE 13

FORMAT (7HBLK NO., 3X, 2HFC, 4X 7HE (EXP))

IF (SENSE SWITH 1) 19 90
Goodness of Fit Test

**Type 18**

**FORMAT** (2X, 4HHGDN, 5X, 3HPCI, 5X, 3HACl, 5X. 3HJEN)

**DO** I = 1, N

**TYPE 15, BN0(I), FC(I), E(I)**

**IF** (SENSE SWITCH 1) 6, 14

**TYPE 5, E1(I), E2(I), E3(I), E4(I)**

**FORMAT** (F3.4, F3.4, F3.4, F3.4)

**CONTROL IO2**

**PRINT** 16, AFS, AE

**FORMAT** (*7HAVE VAL. F8.1, F8.4)

**PRINT** 17, SDVFC, SDVE

**FORMAT** (*7STD DEP. F3.4, F3.4)

**C GOODNESS OF FIT TEST**

**DO** I = 1, N

**X(I) = E1(I)**

**EXECUTE PROCEDURE 25**

**IF** (SL-S) 26, 27, 27

**IF** (SU-S) 26, 27, 29

**PRINT** 30

**FORMAT** (22. ACCEPT HYPOTHESIS THAT)

**PRINT** 35

**FORMAT** (33HE IS GIVEN BY HOGNESTAD S FORMULA)

**PRINT** 36, S

**FORMAT** (31. THAT 5 LEVEL OF SIGNIFICANCE Z = F3.3)

**GO TO** 31

**PRINT** 31, S

**FORMAT** (20. REJECT HOGNESTAD. Z = F3.3)

**GO TO** 34

**PRINT** 32, S

**FORMAT** (49. ACCEPTANCE OR REJECTION OF HOGNESTAD IS DOUBTFUL. Z = F3.3)

**CONTROL 102**

**DO** I = 1, N

**X(I) = E2(I)**

**EXECUTE PROCEDURE 25**

**IF** (SL-S) 38, 39, 39

**IF** (SU-S) 40, 40, 41

**PRINT** 30

**PRINT** 42

**FORMAT** (29HE IS GIVEN BY THE PCI FORMULA)

**PRINT** 36, S

**GO TO** 43

**PRINT** 44, S

**FORMAT** (46. REJECT PCI, Z = F3.3)

**GO TO** 43

**PRINT** 45, S

**FORMAT** (46. ACCEPTANCE OR REJECTION OF PCI IS DOUBTFUL Z = F3.3)
CONTROL 102

DO 46 I=1,N
X(I)=E3(I)
EXECUTE PROCEDURE 25
IF (SL-S) 47, 48, 49
IF (SU-S) 49, 49, 50
PRINT 30
PRINT 51
FORMAT (29HE IS GIVEN BY THE ACI FORMULA)
PRINT 36, S
GO TO 52
PRINT 53, S
FORMAT (4HREJECT ACI, z = F3, 3)
GO TO 52
PRINT 54, S
FORMAT (4HACCEPTANCE OR REJECTION OF ACI IS DOUBTFUL, z = F3, 3)
DO 55 I = 1, N
X(I)=E(I)
EXECUTE PROCEDURE 25
IF (SL - S) 56, 57, 58
IF (SU - S) 53, 53, 59
PRINT 30
PRINT 60
FORMAT (30HE IS GIVEN BY JENSEN S FORMULA)
GO TO 61
PRINT 62, S
FORMAT (17REJECT JENSEN, z = F3, 3)
GO TO 61
PRINT 63, S
FORMAT (4HACCEPTANCE OR REJECTION OF JENSEN IS DOUBTFUL, z = F3, 3)
CONTROL 102
GO TO 23
BEGIN PROCEDURE 20
DO 21 I=1,N
XDF(I)=X(I)-XBAR
XDF2(I)=XDF(I)*XDF(I)
SXDF2=XDF2(I)
DO 22 I=2,N
SXDF2=SXDF2+XDF2(I)
Z2=SXDF2/BN
Z=(SQRT(Z2)/XBAR)*100, 0
END PROCEDURE 20
BEGIN PROCEDURE 25
S=0.
DO 24 I=1,N
G(I)=(X(I)-E(I)1/(X(I)-E(I))2/X(I)
S=S+G(I)
END PROCEDURE 25
END TRACE
END
### COMPVE STRENGTH, ELASTIC MODULUS AND CODE RECOMMENDATIONS FOR CONCRETE

<table>
<thead>
<tr>
<th>BLK NO</th>
<th>FC (EXP)</th>
<th>E (EXP)</th>
<th>HOGD</th>
<th>PCI</th>
<th>ACI</th>
<th>JEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>13869:5</td>
<td>6.7335</td>
<td>8.1799</td>
<td>8.7347</td>
<td>13:9695</td>
<td>5:2433</td>
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<tr>
<td>81</td>
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<td>6.0411</td>
<td>8.3205</td>
<td>8.8375</td>
<td>14:175:1</td>
<td>5:2531</td>
</tr>
<tr>
<td>83</td>
<td>13800:3</td>
<td>5.1991</td>
<td>8.1481</td>
<td>8.7001</td>
<td>13:9003</td>
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<td>89</td>
<td>12320:4</td>
<td>5.4524</td>
<td>7.8739</td>
<td>7.6021</td>
<td>12:2042</td>
<td>5:2107</td>
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<td>93</td>
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<td>5.4524</td>
<td>7.6209</td>
<td>8.2353</td>
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<td>5:1930</td>
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<td>95</td>
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<td>6.0102</td>
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<td>104</td>
<td>11679:5</td>
<td>5.0193</td>
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<td>7.7465</td>
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<td>12067:9</td>
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<td>7.3512</td>
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<td>7.6011</td>
<td>11:7623</td>
<td>5:1230</td>
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<td>13425:5</td>
<td>5.4032</td>
<td>7.9757</td>
<td>8.5127</td>
<td>13:4255</td>
<td>5:2220</td>
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<td>14550:7</td>
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<td>8.1933</td>
<td>9.0753</td>
<td>14:5507</td>
<td>5:2749</td>
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<tr>
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<td>14550:7</td>
<td>5.8750</td>
<td>3.4933</td>
<td>3.9753</td>
<td>14:5507</td>
<td>5:2749</td>
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<td>7.6700</td>
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<td>12:7610</td>
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<td>5.2690</td>
<td>7.1786</td>
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<td>12:7610</td>
<td>5:1370</td>
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<td>195</td>
<td>12483:3</td>
<td>5.3167</td>
<td>7.5423</td>
<td>8.0416</td>
<td>12:4333</td>
<td>5:1714</td>
</tr>
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<td>201</td>
<td>11533:2</td>
<td>5.1086</td>
<td>7.0961</td>
<td>7.5566</td>
<td>11:5332</td>
<td>5:1119</td>
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<tr>
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<td>7.4786</td>
<td>7.9724</td>
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<tr>
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<td>7.6258</td>
<td>11.6517</td>
<td>5:1209</td>
</tr>
</tbody>
</table>

**AVE VAL** 13170.7 5.7927  
**STD DEV** 8.1296 7.4673

**ACCEPT HYPOTHESIS THAT**  
**E IS GIVEN BY HOGNESTAD'S FORMULA**  
**AT 5 LEVEL OF SIGNIFICANCE, Z = 17.393**

**ACCEPT HYPOTHESIS THAT**  
**E IS GIVEN BY THE PCI FORMULA**  
**AT 5 LEVEL OF SIGNIFICANCE, Z = 25.375**

**REJECT ACI, Z = 123.541**

**ACCEPT HYPOTHESIS THAT**  
**E IS GIVEN BY JENSEN'S FORMULA**  
**AT 5 LEVEL OF SIGNIFICANCE, Z = 3.014**
A5.1 State Vectors

In the simple case of a spring mass system Fig A5.1 the displacement of the point \( i \) is the linear displacement \( x_i \), and the corresponding internal force is the direct force \( N_i \) in the spring. For this case then the state vector, \( Z_i \), is given by:

\[
Z_i = \begin{bmatrix} X \\ N_i \end{bmatrix}
\]  

---

A5.2 Transfer Matrices

In Fig A5.1, the masses \( m_{i-1} \) and \( m_i \) are connected by a massless spring of stiffness \( k_i \). The state vector just to the right of mass \( m_i \) is denoted by \( Z^R_i \) and that to the left is denoted by \( Z^L_i \).

Fig A5.2a

A consideration of the equilibrium of the spring \( k_i \) (fig A5.2a) isolated from its neighbouring masses gives:
\[ N_i^L = N_i^R - 1 = k_i(x_i - x_i-1) \]  \hspace{1cm} \text{(A5.2)}

using the stiffness property of the spring.

This last equation can be rewritten
\[ x_i = x_i-1 + \frac{N_i^R}{k_i} \]  \hspace{1cm} \text{(A5.3a)}
\[ N_i^L = 0 + N_i^R_{i-1} \]  \hspace{1cm} \text{(A5.3b)}

which can be expressed in matrix form:
\[ \begin{bmatrix} x \\ N_i \end{bmatrix}_i = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_i-1 \\ N_i-1 \end{bmatrix}^R \]  \hspace{1cm} \text{(A5.4a)}
\[ \begin{bmatrix} x \\ N_i \end{bmatrix}_i = F_i \cdot \begin{bmatrix} x_i-1 \\ N_i-1 \end{bmatrix}^R \]  \hspace{1cm} \text{(A5.4b)}

Hence by means of the matrix \( F_i \) the state vectors \( \begin{bmatrix} x_i \\ N_i \end{bmatrix}^L \) and \( \begin{bmatrix} x_i \\ N_i \end{bmatrix}^{R}_{i-1} \) are related.

The matrix \( F_i \) is known as the field transfer matrix or field matrix.

The matrix relationship between the state vectors to the left and right of mass \( m_i \) is found by considering the forces acting on the mass (fig A5.2b). In addition to the spring forces \( N_i^L \) and \( N_i^R \) there is an external force \( W_i \) acting on \( m_i \).

From consideration of the equilibrium of \( m_i \):
\[ \begin{bmatrix} x_i^R \\ N_i^R \end{bmatrix} = \begin{bmatrix} x_i^L \\ N_i^L \end{bmatrix} \]  \hspace{1cm} \text{(A5.5a)}
\[ N_i^R = N_i^L - W_i \]  \hspace{1cm} \text{(A5.5b)}

Rewriting in matrix form:
\[ \begin{bmatrix} x \\ N_i \end{bmatrix}_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ N \end{bmatrix}_i^L + \begin{bmatrix} 0 \\ -W_i \end{bmatrix} \]  \hspace{1cm} \text{(A5.6a)}
\[ \begin{bmatrix} x \\ N_i \end{bmatrix}_i = P_i \cdot \begin{bmatrix} x_i \\ N_i \end{bmatrix}^L + C_i \]  \hspace{1cm} \text{(A5.6b)}

The matrix \( P_i \) is known as the point transfer matrix or point matrix.

A5.3 Use of Field and Point Transfer Matrices

The use of the field and point transfer matrices can be illustrated by again referring to fig A5.1.
Consider the mass \( m_{i+1} \):

\[
Z_{i+1}^R = P_{i+1} \cdot Z_{i+1}^L
\]

but

\[
Z_{i+1}^L = F_{i+1} \cdot Z_i^R
\]

but

\[
Z_i^R = P_i \cdot Z_i^L + C_i
\]

but

\[
Z_i^L = F_i \cdot Z_{i-1}^R
\]

but

\[
Z_{i-1}^R = P_{i-1} \cdot Z_{i-1}^L
\]

\[\text{using alternatively equations A5.4 & A5.6}\]

Therefore by substitution:

\[
Z_{i+1}^R = P_{i+1} F_{i+1} P_i F_i P_{i-1} Z_i^L + P_{i+1} F_{i+1} C_i \quad (A5.7)
\]

Thus the state vectors to the extreme left and right of the spring system are related through the successive matrix multiplication of the field and point matrices. It can be seen, therefore, that, if \( Z_{i+1}^L \) is known from the boundary conditions of the system, then \( Z_{i+1}^R \) may be determined; intermediate state vectors being given as the calculation proceeds.

In the example chosen (fig A5.1) if \( W_i \) was the inertia force acting on the mass \( m_i \), vibrating with circular frequency, \( \omega \), then

\[
W_i = m_i \omega^2 x_i \quad (A5.8)
\]

and equation A5.7, after absorbing \( W_i \) into the point matrix \( P_i \), would give the necessary expression for the natural period of vibration of the system.

Vibration analysis of elastic systems is the main field of application of transfer matrix theory although it is suitable in many other situations, particularly structural mechanics.

A5.4 Generalised Transfer Matrix Theory

The foregoing discussion can now be generalised and a consideration made of any system (fig A5.3), composed of discrete elements or fields which are interconnected at defined nodes or points.

![Diagram of node and joint](image)
Knowing the state vectors for the extreme left node usually determined by boundary conditions, it is possible to "transfer" across the system and determine successive, intermediate state vectors by the multiplication of alternate field and point matrices. Thus when the applied loading is known all displacements and internal forces acting at the nodal points may be computed for the system.

The size of the state vectors and transfer matrices need not necessarily be restricted to 2 x 2 columns and 2 x 2 squares as above. In more complex systems where each node may have six degrees of freedom and the associated six generalised forces, these vectors and matrices may then be of the order of 12 x 1 columns and 12 x 12 squares.
A6.1 Theoretical Solution to a Clamped Half-Plane

Consider a clamped half-plane of flexural rigidity, D, as in Fig A6.1, loaded by a single point load, P, at (a,0). From the classical theory of elasticity the deflection w at any point (x,y) in the half-plane is given by:

\[ w = \frac{P}{16\pi D} \left[ (r_1^2 \log r_1^2 - r_1^2) - (r_2^2 \log r_2^2 - r_2^2) \right] \quad (A6.1) \]

Subsequent differentiation and substitution gives the distribution of shear force \( S_x \) along the edge OY as:

\[ S_x = -\frac{2Pa^3}{\pi(y^2 + a^2)^2} \quad (A6.2) \]

This distribution has been plotted in Fig A6.2 where it can be seen that 45% of the load is carried over a length equal to 1.35a along the axis OY. Therefore, 90% of the load is supported symmetrically about the origin over a total length of 2.70a.

A6.2 Application to a Beam Supported Elastically Along Both Free Edges

There are several dissimilarities between the above hypothetical elastic system and the case of a loaded beam of finite length and width. However, the distribution of shear along the elastically supported free edges of the beam will be of the same form as that depicted in Fig A6.2, and it would be fair to assume that the same distribution is applicable. That is, 90% of the applied load may be assumed to be supported over a length of 2.7a of shear key about the midspan centre line. For beams with a span/width ratio of about 10, the length 2.7a represents 13\% of the span. It is therefore a reasonable assumption that this distribution of reaction can be replaced by a single concentrated load at midspan, as illustrated in Fig A6.3.
Figure A6.2
DISTRIBUTION OF SHEAR ALONG OY
ACCORDING TO:

\[ S_x = -\frac{2Pa^3}{(y^2 + a^2)^2} \]
Fig A6.3.

Equivalent Concentrated Load, $N_1$
APPENDIX SEVEN  TORSIONAL ANALYSIS OF A SIMPLY SUPPORTED BEAM

Consider a simply supported beam of span L, and subjected to a torque T, applied at midspan as in Fig A7.1a.

Fig A7.1a

Under the action of the torque T, the beam will twist through an angle θ, and thereby induce both torsional and flexural stresses in the beam. It is normal to neglect the flexural stiffness of the beam in an analysis of this type, but such an assumption is not sound for wide rectangular beams or I-shaped sections, as will be shown below.

A theoretical analysis gives the torque developed by the transverse shears, such as that acting on the element dy in Fig 7.1b, as

$$\frac{E I_y a^2 d^3 \theta}{12 d^3}$$

where $I_y$ is the moment of inertia about the OY axis and $E$ is the elastic modulus. This is the torque that is resisted by the flexural stiffness of the beam.

Further analysis, using the conventional approach, gives the torque resisted by the torsional stiffness of the section as

$$J G \frac{d \theta}{d x}$$

where J is the torsional moment of inertia

and $G$ is the modulus of rigidity.

The total torsion resisted by the beam section is therefore

$$-\frac{E I_y a^2 d^3 \theta}{12 d^3} + J G \frac{d \theta}{d x}.$$
This must equal the applied torque \( T \), and since this is constant over the length of the beam, \( \frac{dT}{dx} = 0 \).

Therefore:
\[
-\frac{EI}{y} \cdot \frac{\alpha^2 \theta}{12} + JG \cdot \frac{\alpha^2}{dx} \theta = 0
\]

This governing differential equation has a solution of the form:
\[
\theta = A \sinh \frac{x}{\alpha} + B \cosh \frac{x}{\alpha} + Cx + D
\]

where:
\[
\alpha = \sqrt{\frac{2 EI}{12 JG}}
\]

Substitution of the boundary conditions gives:
\[
\theta = \frac{\alpha T}{2JG} \left[ \sinh \frac{x}{\alpha} - \tanh \frac{L}{2\alpha} \cdot \cosh \frac{x}{\alpha} - \frac{L}{2\alpha} \right] \frac{\cosh \frac{L}{2\alpha}}{\cosh \frac{L}{2\alpha} - 1}
\]

Therefore the rotation at midspan \( x = 0 \) is
\[
\theta = \frac{T}{2JG} \cdot \frac{\frac{L}{2} \cosh \frac{L}{2\alpha} - \alpha \sinh \frac{L}{2\alpha}}{\cosh \frac{L}{2\alpha} - 1}
\]

Therefore, if \( T \) is a \( (N_i \pm N_{i+1}) \) as in equation 3.11b, then
\[
Y = \frac{2}{2JG} \cdot \frac{\frac{L}{2} \cosh \frac{L}{2\alpha} - \sinh \frac{L}{2\alpha}}{\cosh \frac{L}{2\alpha} - 1}
\]

Now if \( \frac{L}{2\alpha} \) is large (i.e. \( \geq 6 \)) then \( \cosh \frac{L}{2\alpha} \approx \sinh \frac{L}{2\alpha} \approx \) a large number, and
\[
Y = \frac{2}{2JG} \cdot \left( \frac{L}{2} - \alpha \right) \approx \frac{a}{4\mu JG} \cdot (L - 2\alpha)
\]

Now \( \frac{L}{2\alpha} = \sqrt{\frac{3EI}{4JG}} \) is frequently greater than 6 (for the beams considered in this work \( \frac{L}{2\alpha} \) lay in the range 8.4 to 18.6) and therefore the latter expression for \( Y \) (equation A7.5) is usually sufficient.

However, should \( L \) be much greater than \( 2\alpha \) then equation A7.5 reduces to:
\[
Y = \frac{L}{4\mu JG} \sinh \frac{La^2}{4\mu JG}
\]

the expression given in equation 3.11b. Therefore equation A7.6 gives a first approximation which, for beams considered in this work, was sufficiently accurate. However, for units of greater width, and therefore of a higher \( \alpha \),
the \( \frac{L}{2\alpha} \) ratio is smaller and some care is then required in the calculation of the parameter \( Y \). In particular if the ratio \( \frac{L}{\sqrt{\frac{1}{3} \frac{EL}{JG}} a} \) is less than or equal to 6, equation A7.4 must be used for \( Y \).
APPENDIX EIGHT  TYPICAL CALCULATIONS FROM THE TRANSFER MATRIX ANALYSIS
OF A MULTIBEAM BRIDGE DECK

The example chosen here to illustrate the transfer matrix method, is
that for a ten-beam bridge deck and in fact was taken from the calculations
made for the hollow-cored span of the Slippery Creek Bridges (Span D).

The tabulations which follow have been drawn up specifically for hand
calculation and show the ease with which the state vectors \((Z_0 \text{ to } Z_{10})\) are
obtained. The deflections and shear forces at all joints in the deck are
thus found for the applied load of 64,000 lb at midspan, on beam number 6.

Units are either inches, pounds, or lbs per sq. in., as the case may
be. Several notes on the calculation are also included.

Once the state vectors have been found the shears at two consecutive
joints may be subtracted to find the load carried by the intermediate beam.
<table>
<thead>
<tr>
<th>Beam Position</th>
<th>Inside</th>
<th>Edge</th>
<th>Inside</th>
<th>Edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Width $2a$</td>
<td>$44.5$</td>
<td>$2a/d$</td>
<td>$2.88$</td>
<td>$1.82 \times 10^5$</td>
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<tr>
<td>Beam Depth $d$</td>
<td>$16.0$</td>
<td>$I=2a.d^3/12$</td>
<td>$14,477$</td>
<td>$49,518$</td>
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<tr>
<td>Beam Span $L$</td>
<td>$408.0$</td>
<td>$J=\beta.2a.d^3$</td>
<td>$6.80 \times 10^6$</td>
<td>$2.16 \times 10^5$</td>
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<tr>
<td>Elastic Mod $E$</td>
<td>$5.8 \times 10^6$</td>
<td>$L^3$</td>
<td>$1.055$</td>
<td>$-1.872 \times 10^{-6}$</td>
</tr>
<tr>
<td>Shear Mod $G$</td>
<td>$2.4 \times 10^6$</td>
<td>$L^2a$</td>
<td>$-6.099 \times 10^4$</td>
<td>$1.055$</td>
</tr>
<tr>
<td>Flex. Rig. EL</td>
<td>$8.39 \times 10^{10}$</td>
<td>$F_L$</td>
<td>$1.055$</td>
<td>$1.872 \times 10^{-6}$</td>
</tr>
<tr>
<td>Tors. Rig. JG</td>
<td>$1.19 \times 10^{11}$</td>
<td>$F_R$</td>
<td>$6.099 \times 10^4$</td>
<td>$1.055$</td>
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<tr>
<td>$X, Y=1/48EI$</td>
<td>$1.685 \times 10^{-5}$</td>
<td>$F_{EL}$</td>
<td>$-5.991 \times 10^{-2}$</td>
<td>$0.0$</td>
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<tr>
<td>$Y, Y=1/48JG$</td>
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<td>$F_{ER}$</td>
<td>$65,780$</td>
<td>$64,000$</td>
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</tbody>
</table>

**Remarks:** In these calcs, the effect of skew and the hollowed section has been included in the determination of both $I$ and $J$. The signs have been reversed in $F_L$ and $F_R$ for easier arithmetic in the matrix multiplication over - see note 3, page 198.
TRANSFER MATRIX ANALYSIS OF THE SLIPPERY CREEK BRIDGE: SPAN 'D' - HOLLOW CORED, 10 BEAMS.

W=64,000Lbs.

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continued...
continued from page 196...

\[
\begin{array}{cc}
1.055 & -1.872 \times 10^{-6} \\
-6.099 \times 10^4 & 1.055 \\
1.055 & -1.872 \times 10^{-6} \\
-6.099 \times 10^4 & 1.055 \\
2.018 & -9.710 \times 10^{-6} \\
-3.163 \times 10^5 & 2.018 \\
\end{array}
\]

\[
\begin{array}{cc}
2.018 & -9.710 \times 10^{-6} \\
-3.163 \times 10^5 & 2.018 \\
3.729 & 19.903 \times 10^{-6} \\
-6.484 \times 10^5 & 3.729 \\
13.821 & 76.371 \times 10^{-6} \\
-24.880 \times 10^5 & 13.821 \\
\end{array}
\]

\[
\begin{array}{c}
0.0876 \\
5.060 \\
0.0830 \\
0.0 \\
-5.991 \times 10^{-2} \\
65.780 \\
-0.7596 \\
15.17 \times 10^4 \\
\end{array}
\]

(10) \quad x_0 = \frac{-15.17 \times 10^4}{-24.88 \times 10^5} \quad \text{for bridge decks (see fig 3.3, p45)}

\[
= 0.0610
\]
Remarks:

1. The sequence of operations is indicated by consecutive numbering from 1 to 22.

2. Three figure accuracy has been maintained as far as possible, a slide rule being used throughout.

3. Since $N_R$ is greater than $N_L$ the beams and joints have been numbered from the right and the calculation has proceeded from this side. This is contrary to the formulation in Chapter 3 and requires a reversal of sign in the field matrices $F_L$ and $F_R$. However, the change enables $F_L^4$ to be written down immediately from $F_R^4$ without further calculation. See also sec 3.325.

4. These results may be compared with the profiles plotted in fig 6.02 which were obtained by computation using the IBM 1620 Computer and OPUS TWO.
Appendix Nine  Contract Plans of the Slippery Creek Bridge

Four reduced photostats of the plans for the Slippery Creek Bridge are included. The set is not complete, for it does not contain the amendments showing the hollow-cored units used in the western bridge nor the revised, double-width log unit used in the eastern bridge. It does, however, give the dimensions, locality plan, details of substructure and guardrail layout for the bridges.

Since completion in 1964 flood damage to the southern abutment necessitated the extension in 1966 of each bridge to a total length of five spans. Testing, however, took place during the intervening year whilst the bridge was still the original length, as in the following photostats.
The following material describes the proposed testing timetable and also the load positions to be investigated. Also included are copies of the strain and deflection recording sheets used during the field tests.

The timetable suffered in many ways but its inclusion here may serve in the planning of future projects.
M.O.W LOG BEAMS
18 BEAMS/SPAN

SPAN A: Tests A1, A2, A3, A4
SPAN B: not to be tested
SPAN C: C1, C2, C3, C4

NORTH → Auckland

CERTIFIED CONCRETE HOLLOW-CORE BEAMS

SPAN D: 10 beams
SPAN E: 5 beams
SPAN F: 3 beams

SOUTH → Hamilton

FIG. PLAN OF DUAL BRIDGE OVER SLIPPERY CREEK

TABLE I TESTING SCHEDULE A – OVERALL

<table>
<thead>
<tr>
<th>DAY</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>A1</td>
<td>C1</td>
<td>D</td>
<td>A2</td>
<td>C2</td>
<td>A3</td>
<td></td>
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<tr>
<td>PREP</td>
<td>A1</td>
<td>A2</td>
<td>C2</td>
<td>A2</td>
<td>C2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TEST</td>
<td>A3</td>
<td>E</td>
<td>F</td>
<td>C3</td>
<td>A4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PREP</td>
<td>C3</td>
<td>A3</td>
<td>A4</td>
<td>C4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TEST</td>
<td>C4</td>
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<td></td>
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<td></td>
<td></td>
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<td>C4</td>
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</tbody>
</table>

WEEK 1

WEEK 2

WEEK 3

AUCKLAND – HAMILTON MOTORWAY BRIDGE
SLIPPERY CREEK LOADING TESTS

PW 412

MINISTRY OF WORKS P.W.O. FORMER

P.W.O. FORMER

P.W.O. FORMER

Chief Designing Engineer

P.W.D.
<table>
<thead>
<tr>
<th>DAY</th>
<th>8:00 - 9:30</th>
<th>9:30 - 12:00</th>
<th>12:00 - 12:30</th>
<th>12:30 - 3:30</th>
<th>3:30 - 4:30</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>SET UP DIAL</td>
<td>Span A1, one transv bolt at centre. Stage I, (27)*</td>
<td>LUNCH.</td>
<td>Span A1. Stages 2 &amp; 3, (15+6)</td>
<td>DISMANTLE</td>
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<tr>
<td>2</td>
<td>GAUGES AND</td>
<td>Span C1, one transv HT wire at centre Stage I, (27) contractor prepares span A2.</td>
<td></td>
<td>Span C1. Stages 2 &amp; 3, (15+6)</td>
<td>DIAL GAUGES</td>
</tr>
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<td>3</td>
<td>INSTALL STRAIN</td>
<td>Span D. Stage I, (30) contractor prepares span C2</td>
<td></td>
<td>Span D. Stages 2 &amp; 3, (15+6)</td>
<td>AND</td>
</tr>
<tr>
<td>4</td>
<td>GAUGE BRIDGES</td>
<td>Span A2, 3 transv bolts at quarter points Stage I, (27)</td>
<td></td>
<td>Span A2. Stages 2 &amp; 3, (15+6)</td>
<td>STRAIN GAUGE</td>
</tr>
<tr>
<td>5</td>
<td>AND SWITCHING</td>
<td>Span C2, 3 wires at quarter points Stage I, (27) contractor prepares span A3</td>
<td></td>
<td>Span C2. Stages 2 &amp; 3, (15+6)</td>
<td>BRIDGE GEAR.</td>
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<td>GEAR WITH</td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>PEN RECORDERS.</td>
<td></td>
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<td>9</td>
<td></td>
<td>Span E Stage I, (15) contractor prepares span A4.</td>
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<td>Span E. Stages 2 &amp; 3, (9+3)</td>
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<td>10</td>
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<td>Span C3, wires at 5'-0&quot; centres. Stage I, (27)</td>
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<td>Span C3. Stages 2 &amp; 3, (15+6)</td>
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<td>12</td>
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<td>Span F. Stages 1, 2 &amp; 3, (9+3+3)</td>
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</table>

*(27) numbers in brackets refer to no of load positions reqd. to complete testing for that stage.*
FIG. 2. NINE, SINGLE, POINT LOAD POSITIONS FOR SPANS A & C.

NOTE: Lanes 3, 4, 5 will only be tested if time permits.

FIG. 3. LANE POSITIONS FOR SPANS A, C & D.

FIG. 4. TEST FOR SKEW-1.

FIG. 5. TEST FOR SKEW-2.
**FIG. 6. TEN, SINGLE POINT LOAD POSITIONS FOR SPAN D.**

- 6'-6" Falcon compactor. (b)
- 2'-3" edge clearance for 40'-0" trailer. (a)
- Other point loads at centre of beams

**FIG. 7: (a) FIVE, SINGLE POINT LOAD POSITIONS**

- (b) THREE LANE LOAD POSITIONS FOR SPAN E.

- 6'-6" Falcon compactor (b)
- 2'-3" edge clearance for 40'-0" trailer (a)
- Other point load at centre of beam 2.

**FIG. 8: (a) THREE SINGLE POINT LOAD POSITIONS**

- (b) ONE LANE LOAD POSITION FOR SPAN F.
**DIAL GAUGE READINGS**

- **Q**: Quarter
- **H**: Half
- **3Q**: Three Quarter

**Sponsor**

- **A**
- **B**
- **C**
- **D**
- **E**
- **F**

**Traverse Tia Condition**

- **1**
- **2**
- **3**
- **4**
- **Test Stage**

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A11.1 Variation of Camber, Crushing Strength and Elastic Modulus of Model Concrete with Time.

Section 5.31 of Chapter 5 describes the materials used in the construction of the model. A certain amount of data was collected during its construction and testing, with regard to the properties of the concrete and their variation with time. This data is presented graphically in figs A11.1 and A11.2, where plots of crushing strength, elastic modulus and beam camber are shown.

The crushing strength and elastic modulus variations in fig A11.1 were plotted from Table 5.3 of Chapter 5. The camber variation was observed over a forty day period for those log beams released at two days from the stressing bed. The average camber during this time was 0.415" but considerable variation about the mean was observed. The general trend was for the camber to fall away with time as indicated in fig. A11.2.

A11.2 Steel Properties

Load-extension tests on the high tensile steel wire used in the model have been mentioned in sec 5.312.1 of Chapter 5. Tests on both the 0.200" and 0.276" diameter wire were conducted in the 100 ton Avery testing machine using an Amsler extensometer on a 10" gauge length to record extension. The results are plotted in fig A11.3 and give:

- an elastic modulus of \( 28.4 \times 10^6 \) lbs per sq in.
- and ultimate tensile strength of 109.6 tons per sq in.
**FIGURE A11.1  CONCRETE PROPERTIES**

**GRAPH OF VARIATION IN CRUSHING STRENGTH, \( f_c \), AND ELASTIC MODULUS, \( E \), WITH TIME.**

Units are Lbs/in\(^2\) for both \( f_c \) and \( E \)

For the Model therefore:
Average Crushing Strength (1½ yr): 13,170
Average Elastic Modulus (1½ yr): \( 5.79 \times 10^6 \)
40 day period during which observation were made.

FIGURE A 11.2 MODEL BEAM CAMBER

VARIATION OF MIDSPAN BEAM CAMBER OF MODEL LOG BEAMS WITH AGE

average camber: 0.415 inches
range of values observed: 0.325" (min) 0.476" (max)
FIGURE A11.3
LOAD-STRAIN CURVES FOR HIGH TENSILE STRESSING WIRE

E = 28.5 x 10^6; UTS = 109.6 t.s.i
- for 0.200"Ø wire
APPENDIX TWELVE  TRANSUDER FOR THE DETERMINATION OF ELASTIC MODULUS OF CONCRETE BLOCKS

A12.1 Description

This transducer was designed and built for the purpose of measuring the elastic modulus of 8" x 4" concrete test cylinders.

It was essentially a set of four steel arcs which linked together a pair of steel collars. These were clamped to the 8" x 4" block under test and strain in the block detected by electrical resistance strain gauges on the four steel arcs. Refer fig. A12.1 and Plate 17.

A12.2 Calibration

Calibration of the transducer was necessary and this was done by using a standard, calibrated, 4" steel tube, and running comparative tests with the transducer clamped to it.

Such tests gave a calibration factor of 2350 lbs per sq in., which meant that the elastic modulus of a block under test was given by \( \frac{2350}{A_l} \) lbs per sq.in., where \( A_l \) represented the average strain increment, for a 20 ton increase in load, as observed from the transducer when connected to a direct reading bridge as described in the following section.

A12.3 Strain Gauge Circuit

Eight, PR 9810 strain gauges were used on the four steel arcs and were connected in pairs to form the four arms of a fully active Wheatstone Bridge Circuit. The pairs were chosen from opposite diameters and wired in series to eliminate bending strains due to possible eccentric loading. The use of a fully active bridge gave maximum sensitivity and self compensation with regard to temperature. Details of the gauges have been given in Table 5.7 of Chapter 5; the system being particularly suitable for the Philips Direct Reading Bridge. See also the wiring diagram fig A12.2.

A12.4 Use of the Transducer

Once the gauge leads have been connected to the terminals of the strain gauge bridge, the transducer is clamped (finger tight) to the middle half of the block leaving a 2" clearance top and bottom. The block is then placed in the testing machine and the load cycled up to 25 tons and back to zero several times. This allows the transducer to settle on its contact pads and it also puts the block through its first half-dozen load cycles - the observed elastic
 modulus is higher during the initial load cycles but it falls away to a more or less constant value very quickly.

The strain increment for a 20 ton change in load is then recorded and another block chosen for test.

This increment is then punched on to a card together with the block identification number and its crushing strength and later fed into a computer programme (OPUS FIVE) for statistical analysis - see Appendix 4.

The transducer was found to be quick and reliable.

Fig. A12.1 — above —
'E' TRANSDUCER—DIMENSIONS

Bridge is four-arm & fully active
Above lettering corresponds to terminals of "Edac" (Philips) Direct Reading Strain Bridge
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Plate 17.  A general view of the transducer developed for measuring the elastic modulus of concrete blocks. In this view it is seen clamped to an 8" x 4" block under test in the 100 ton Avery testing machine. The direct reading bridge, used for the strain observation, is shown on the lower right, whilst the calibrating cylinder and its compensator are shown above the transducer itself.