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*"THE EFFECT OF COULOMB DAMPING ON SINGLE AND MULTIDEGREE
OF FREEDOM STRUCTURES"*

*Thesis Submitted for the
Degree of Doctor of Philosophy*

- at the -

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New Zealand*

- by -

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ABSTRACT

This study compares the effects of viscous and coulomb damping on the dynamic response of single and multidegree of freedom structures.

It was found that coulomb damping was less effective than viscous damping in reducing the steady-state resonant amplitude of all non-linear structures when the steady-state amplitude was greater than two. Also, there is no single, simple relationship relating the amount of viscous damping to coulomb damping for equal resonant steady-state response of non-linear structures.

For the earthquake excitation of the single-degree-of-freedom structure it was found that coulomb damping was less effective than viscous damping in reducing the velocity and displacement spectral response values for short period structures whereas for longer period structures coulomb damping was much more effective than viscous damping. Both forms of damping had a paradoxical effect on the acceleration response of non-linear structures in that an increase in damping generally causes an increase in the acceleration response.

The closed solution of a multidegree of freedom structure with viscous and coulomb damping subjected to a sinusoidal forcing function is derived and used to obtain approximate values of viscous and coulomb damping from small amplitude vibration tests. It is clear from the results obtained that if coulomb damping is present in small amplitude vibrations of a structure then the concept of equivalent viscous damping will result in the structure being overdamped when subjected to an earthquake ground motion.

CONTENTS

	<u>Page</u>
ACKNOWLEDGEMENTS	i
ABSTRACT	ii
CHAPTER 1 INTRODUCTION	1
CHAPTER 2 DYNAMIC RESPONSE OF SINGLE-DEGREE-OF-FREEDOM OSCILLATORS SUBJECTED TO SINUSOIDAL EXCITATION	9
2.1 Introduction	9
2.2 Skeleton Curves	9
2.3 Hysteresis Loops	17
2.4 Properties of Hysteretic Behaviour	17
2.5 Properties of the Ramberg-Osgood Hysteresis Loops	20
2.6 Solution of the Basic Ramberg-Osgood Equation	21
2.7 Steady-State Response by Energy Method	22
2.7.1 Energy Dissipated per Cycle	22
2.7.2 Energy Input Per Cycle	29
2.7.3 Amplitude of Non-Linear Steady-State Response	30
2.7.4 Accuracy of the Energy Method in the Determination of the Resonant Steady-State Amplitude	38
2.8 Steady-State Oscillations by the Method of Slowly Varying Parameters	39
2.8.1 Introduction	39
2.8.2 Equations of the Response Curves	39
2.8.3 Evaluation of $S(x_0)$ and $C(x_0)$ for a Bilinear-Hysteretic Force Deflection Relationship	44
2.8.4 Evaluation of $S(x_0)$ and $C(x_0)$ for a Ramberg-Osgood Force- Deflection Relationship	47
2.9 Frequency Response Curves	52
2.9.1 Verification of the Response Curves	61
2.10 Infinite Response at Resonance	68
CHAPTER 3 RESPONSE OF SINGLE-DEGREE-OF-FREEDOM OSCILLATORS SUBJECTED TO EARTHQUAKE MOTION	73
3.1 Introduction	73
3.2 A Class of Non-Linear Structures	73
3.3 Application of Hysteretic Formulation to the Ramberg-Osgood Structure	79

	<u>Page</u>
3.4 Solution of the Single-Degree-of-Freedom Equation of Motion by the Exact Method	84
3.5 Computation of Spectra	99
3.6 The Computer Programs	100
3.7 Errors in Spectra Calculations	108
3.8 Accuracy of the Computer Programs RMBH1 and ROS03	112
3.9 The Energy Equation of a Non-Linear Structure Subjected to an Earthquake	120
3.10 Response of Various Non-Linear Structures to Earthquake Excitation	123
3.11 Presentation and Discussion of Results	124
CHAPTER 4 THEORY OF STRUCTURAL TESTING	155
4.1 Undamped System	163
4.2 Viscously Damped System	165
4.3 Viscously and Coulomb Damped System	167
4.4 Forced Vibration of Pure Normal Modes Containing Viscous Damping Only	170
4.5 Forced Excitation of Pure Natural Modes, by Synchronized Vibration Exciters Acting on Only One Mass of a System with Viscous Damping	172
4.6 Steady-State Amplitude of a Single-Degree-of-Freedom System with Coulomb Damping that Lags the Velocity of the System	173
4.7 Forced Excitation with Synchronized Vibration Exciters Acting on Only One Mass of a System with Viscous and Coulomb Damping	178
4.8 Determination of the Stiffness and Viscous Damping Matrix from Experimentally Determined Modal Properties	183
4.9 Finding the Natural Frequencies and Mode Shapes from known System Properties	191
4.10 Determination of Viscous Damping from Experimental Results	194
4.11 Determination of Viscous and Coulomb Damping from Experimental Results	201
CHAPTER 5 EXPERIMENTAL AND TEST RESULTS	204
5.1 Introduction	204
5.2 Solution of the Equation of Motion of an Elastic Multidegree of Freedom Structure	204
5.3 Nielson's Experimental Test Results	210

	<u>Page</u>
5.4 Determination of the Stiffness Matrix From the Experimental Results	217
5.5 Natural Frequencies and Mode Shapes Determined from the Model of the Structure	220
5.6 Determination of the Viscous Damping Matrix	223
5.7 Computer Programs	226
5.8 Determination of the Accuracy of the Computer Programs EMD01 and EMS01	230
5.9 Tests of the Closed Solution of a Structure with Coulomb Damping Subjected to a Sinusoidal Forcing Function	232
5.10 Determination of Approximate Values of Viscous and Coulomb Damping From Small Amplitude Vibration Tests	237
5.11 Response of a Multidegree of Freedom Structure with Viscous and Coulomb Damping Subjected to an Earthquake	241
CHAPTER 6 SUMMARY AND CONCLUSIONS	243
APPENDIX 1	
A.1.1 IBM 1130 Computer System	247
A.1.2 Program RMBH1	247
A.1.3 Program ROS03	263
A.1.4 Program EMD01	274
A.1.5 Program EMS01	286
A.1.6 Program PHLAG	296
A.1.7 Program COUPL	296
A.1.8 Program MODAL	296
A.1.9 Program FOPY	307
A.1.10 Programs SVPAR and SVPRO	307