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"ELASTIC ANALYSIS OF SOIL MEDIA"

A Thesis for the degree of
Doctor of Philosophy

submitted to the
University of Auckland

- by -

GRAHAM RAMSAY

MAY 1973

Thesis

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CORRIGENDA

A. Corrections:

The following corrections are to be made. The corrections are underlined below:

- Page 45, 5th line from bottom to read: 'counteracted by a base live load'
- Page 46, 4th paragraph, 2nd line to read: 'occurs for $N_S = 3 - 4 \dots$ '
- Fig. 3.2b: Delete dashed line and associated points
- Fig. 3.4c: Caption to read: ' $v = .30$. Smooth Base D/S $> .575$ '
- Fig. 3.5: Amend key to 'EQUIVALENT STRATUM ---'
- Page 57, 5th paragraph, 5th line to read: 'For D/S = .67 the resulting ...'
- Fig. 3.19: Curves are for rough base strata
- Fig. 3.21a: " " " " " "
- Fig. 3.22a: " " " " " "
- Page 64, 2nd para, 4th line to read: '.. and rough or smooth based strata ...'
- Page 86, Equation 5.26 to read: $\tau_{xy}/\alpha = 0$
- Page 93, Table 5.2 Heading 'z/h' to read: 'y/h'
- Page 98, 3rd para, 3rd line to read: 'range from 40 to 2 times ...'
- Page 98, Table 5.4 Heading 'z/h' to read: 'y/h'
- Page 106, Fig. 5.26b, $a/h = .167$. The curve intercepts the x/a axis at 1.8 not 1.6 as shown (i.e. the cusp shown near the axis is incorrect).
- Page 114, 4th para, 5th line to read: 'with $B/h = 10.0$, resulting in decreases of ...'
- Page 133, 5th para, 6th line '... grease were unsuccessful ...'
- Page 137, equation 7.13 should read: $\tau = \frac{\epsilon}{a + b\epsilon}$
- Page 141, Delete first line
- Page 144, 4th line to read 'a $1/a \propto \sigma_{oct}$ '
- Page 151, 4th para, 5th line to read: '... correspond to the major stress changes ...'
- Page 154, 4th para, 1st line to read: '... from methods 3 and 4 include ...'

B. Additional Explanatory Notes:

1. Page 93, 2nd para, 3rd line:

A trend of increasing base vertical stress with increasing Poissons Ratio similar to that shown by the author's results is evident in the results of Poulos (1967) (Figs. 30 - 33).

2. Page 108. Insert the following paragraph after the first paragraph:

'... of the strip.

For wide footings an almost constant contact stress would be expected in the central section of the footing. The unexpected slight decrease in contact stress away from the centreline shown for $a/h = 1.0$ in Fig. 5.26c arises from the fact that when the contributions from the three contact stress components are evaluated to give constant displacements at the three selected points, the parabolic component becomes negative. Better accuracy would result if more terms were used in the assumed contact stress polynomial distribution. A slight initial decrease in contact stress away from the centreline is also evident in results from the Elemental Strip method.'

3. Page 110. Insert the following paragraph after paragraph (v):

'... with decreasing a/h .

The initial reduction of contact stress away from the centreline shown in Fig. 5.28 is evident in the Quartic Contact Stress results but not in the Elemental Strip Method results. This arises from the approximations associated with assuming a Quartic Contact Stress distribution as noted for the vertical load case.'

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ABSTRACT

Elastic solutions can be used to predict stresses and displacements in loaded soil media provided the non-linear stress-strain behaviour of the soil is recognised and allowance made for it in calculating displacements.

This thesis examines the methods of elastic analysis suitable for examining problems in Soil Mechanics and the determination of suitable stress-strain parameters.

The use of the Finite Element method in Soil Mechanics is reviewed, and a computer program for examining linear elastic plane stress and plane strain problems is presented. The program has been used to examine a range of problems involving infinitely long excavations of rectangular cross-section in the surface of a stratum overlying a rigid base. The excavations were analysed for a uniform vertical loading in the base of the excavation and for lateral wall loads representing the effect of the release of the insitu stresses during excavation. Cases in which the lateral movement of the walls is completely restrained are also considered, and the effect of wall restraint on the vertical displacements due to the vertical base loading is shown to be small. Results obtained by the author suggest that care should be exercised in the use of the Finite Element method to analyse problems involving loaded rigid footings or the stresses behind rigid retaining walls, and that the solutions in these cases may be inaccurate.

The solutions available for surface loadings on layered elastic strata are reviewed and results obtained from a Fourier Series approach are presented for a range of problems involving surface loadings on a single rigid base elastic stratum. Uniform vertical and horizontal strip loadings and rigid strip footings with vertical, horizontal and moment loadings are considered. A uniform vertical strip load on the surface of a multilayer stratum overlying a half space or rigid base is also analysed. The computer programs and full mathematical derivations for the strip loading cases are presented, and the extension of the solutions to rectangular loadings is discussed and the basic mathematical derivations outlined. The application of the Fourier Series solutions to the calculation of consolidation settlements is discussed.

Work by other investigators suggests that a hyperbolic model may be suitable for approximating soil stress-strain curves and that fundamental stress-strain behaviour should be examined in tests in which the bulk stress remains constant during the application of the shear stress. Equipment designed to carry out slow drained triaxial tests in which the bulk stress is constant during the shearing phase is described, and the results of a short series of tests used to proof test the equipment are presented.