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ON THE EARTHQUAKE GENERATED RESPONSE OF TORSIONALLY UNBALANCED BUILDINGS

by

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ABSTRACT

An investigation is made into the coupled lateral-torsional response of torsionally unbalanced "shear" buildings to horizontally directed earthquake excitation. Attention is confined to analytical models that have linearly elastic, viscously damped responses.

The investigation involves three separate analyses:

Firstly, in a preliminary study, the earthquake response of an asymmetric single storey building model is analyzed and general expressions are derived for the location of the centre of stiffness and the orientation of the principal axes.

Secondly, an analysis is made of the coupled lateral-torsional response of a partially symmetric single storey building model to a single component of earthquake excitation. A modal solution of the two equations of motion is developed and a general criterion is derived for the existence of full modal coupling.

By employing the design spectrum concept, together with conservative rules for the combination of modal maxima, analytical results in dimensionless form are evaluated for an equivalent static shear and an equivalent static torque. The combination expressions are then modified to include an allowance for modal coupling before the final results are computed and tabulated. The results substantiate previous findings which have pointed to a possible link between strong modal coupling and severely coupled lateral and torsional responses. In particular, they indicate that those nominally symmetric buildings which exhibit strong modal coupling are liable to respond more strongly in torsion than has hitherto been recognised by most building codes. This effect has not in the past been quantified in analytical terms.

Although the results have practical applications in design, the analysis concerns itself primarily with the determination of realistic estimates for the dimensionless response quantities and no attempt is made to derive design rules.

Finally, the partially symmetric single storey model is extended to a special class of partially symmetric multistorey "shear" buildings. The importance of this final analysis derives from the similarity between the results for the single storey model and those for the continuous multistorey model.

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