TWO-PHASE GAS-LIQUID FLOW -
WITH PARTICULAR EMPHASIS ON HOLDUP
MEASUREMENTS AND PREDICTIONS.

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ABSTRACT

The work described in this thesis is an analytical and experimental study of two-phase gas-liquid horizontal flow in a conduit with particular emphasis on holdup measurements and predictions. Holdup and pressure drop, their inter-relationships, and their flow pattern dependence were investigated. A simple method for flow pattern determination was presented so that the appropriate prediction method may be selected for a particular situation. The results were discussed by comparison with a wide range of experimental data and the relevant literature. Two simple devices for holdup measurements were developed in this work and their behaviours were also found to be flow pattern dependent. The results are as follows:

In the analytical study, the original Lockhart-Martinelli formulation was treated analytically for ideal stratified flow giving equations which agree with experimental pressure drop and holdup data and the more rigorously derived relationships of Johannessen and Taitel & Dukler. For ideal annular flow, the derived equations predicted pressure drop in large diameter pipes reasonably well giving results which are in agreement with the modified equation of Baker. Poor prediction was achieved for small diameter pipes. The holdup equations derived for annular flow were also in poor agreement with experimental data although a slight modification resulted in an equation that was not only suitable for holdup prediction, but also may be used to represent the original Lockhart-Martinelli holdup correlation over the entire operating range.

A correlation was presented for the frictional pressure drop in annular flow based on laboratory air-water data and geothermal steam-water data. The correlation was found to predict pressure loss values which agreed with data from various different sources. The correlation exhibited a point of inflexion which was believed to be due to the transition from a ripple wave type of interfacial disturbance to one of roll wave-droplet entrainment. An extensive literature survey showed that such a transition at high gas rate occurs at all flow orientations and is governed by a critical liquid rate given by a definite value of the Weber number defined in terms of the liquid phase.

The Butterworth form of holdup equation was justified by assuming ideal stratified and annular turbulent-turbulent and viscous-viscous flows. A full set of equations for stratified flow covering the cases of liquid-gas, turbulent-viscous and viscous-turbulent were also derived. It was found that the variation in the coefficients and exponential factors in the Butterworth equation was due to at least three factors: the flow pattern, the flow regimes of the phases, i.e., viscous or turbulent, and the range of the value of the ratio of the liquid holdup to the voidage. Furthermore, experimental data were found to behave according to whether the flow pattern was stratified, slug and plug or annular. Equations for determining these flow patterns were presented, based on the derived stratified flow equations, and were checked to be in agreement with the flow pattern maps of Mandhane et al and Taitel & Dukler, and the experimental flow pattern observations of this work.

Since the derivation from the original Lockhart-Martinelli formulation did not yield a completely satisfactory relationship for the holdup and pressure drop in annular flow, such a relationship was examined in terms of the film flow equations, Newton's law of viscosity and the Prandtl's mixing length. This was also compared with the analysis of Levy of annular-mist flow using the mixing length theory.

Throughout the analysis, the results were compared with various sources of laboratory air-water data and geothermal steam-water data, and the discrepancies, if any, were discussed.

The rise velocity of Taylor bubbles in conduits was also examined in terms of the film flow equations, the Newton's law of viscosity, the Prandtl's mixing length theory and the universal velocity distribution equations. The rise velocity of a Taylor bubble as derived by the Prandtl's mixing length theory has the same form as that derived by Dumitriscu and Davies & Taylor who used the classical potential flow theory. The analysis was extended to justify the Armand equation for holdup for slug and plug flows.

Thus, to summarise the analytical work presented in this work, given a set of input conditions, the flow pattern may be predicted as one of three: stratified, slug and plug, annular. From a knowledge of the flow pattern, appropriate methods of holdup and pressure drop prediction may be chosen. The inter-relationships between holdup and pressure drop for stratified and annular flow have also been shown.

In the experimental study, the application of two simple devices, developed in this work, one of which was subsequently patented, for holdup measurement was investigated. Both devices were found to be flow pattern dependent in their behaviour and require calibrations. During the study of these two devices, pressure drop, holdup and flow pattern data were also generated and were used for the comparison with the analytical part of this work.

LIST OF SYMBOLS

A	[-]	factor in Butterworth's holdup equation[61] of Eqn. (2.4.18) or (4.1.1).
A_L	$[L^2]$	Cross-section flow area for the liquid phase.
A_G	$[L^2]$	Cross-section flow area for the gas phase.
A_T	$[L^2]$	Total flow channel cross-section area, $= A_L + A_G$.
A'	[L]	function defined by Eqn. (5.4.4).
A_1	[-]	factor in Eqn. (4.3.5) with values given in Table (4.3.1).
A_2	[-]	factor in Eqn. (4.3.6) with values given in Table (4.3.2).
A_3	[-]	factor in Eqn. (4.3.10) with values given in Table (4.3.3).
A_4	[-]	factor in Eqn. (4.3.11) with values given in Table (4.3.4).
a	[-]	factor in Butterworth's equation[61], Eqn. (4.1.2).
a_A	[-]	factor used by Armand[15] in Eqn. (2.4.2).
a_B	$[T\ L^{-1}]$	factor used by Beattie[26] in Eqn. (2.5.71).
a_R	[L]	radial distance measured from pipe axis.
a_m	[-]	factor used by McManus in Eqn. (2.5.2).
a_s	[L]	radius of the gas-liquid interface at the tail-end of a rising Taylor bubble in a stagnant liquid enclosed in a vertical tube.
a_{TP}	[L]	radius of the gas-liquid interface at the tail-end of a rising Taylor bubble in a flowing two-phase mixture in a vertical tube.
a_1	[-]	factor in Eqn. (4.3.5) with values given in Table (4.3.1).
a_2	[-]	factor in Eqn. (4.3.6) with values given in Table (4.3.2).
a_3	[-]	factor in Eqn. (4.3.10) with values given in Table (4.3.3).
a_4	[-]	factor in Eqn. (4.3.11) with values given in Table (4.3.4).
B	[-]	factor in Eqn. (4.1.2) with values given in Table (4.2.2).
B_B	[-]	factor for the rise velocity equation of fine bubbles ofEqn. (2.5.89). Given by Levich[259] as 1.41 and Harmathy[181] as 1.53.
B^*	$[LT^{-1}]$	Initial function derived by Nguyen & Spedding[318]. Used in Eqn. (2.5.43) and (4.2.34).
B'	[L]	function defined by Eqn. (5.4.5).
b	[-]	factor in Eqn. (4.1.2) with values given in Table (4.2.2).
b_m	[-]	factor used by McManus in Eqn. (2.5.2).
C	[-]	Armand's factor, given in Eqn. (2.4.3) generally taken as equal to 1.2 for $\beta<0.9$. Equivalent to the reciprocal of K, the Bankoff parameter.
C_B	[-]	Factor used in Eqn. (5.1.1) by Dumitriscu and Davies & Taylor.
C_G, C_L	[-]	Factor in Blasius' equation, for gas, and liquid respectively. Equals 16.0 for turbulent flow and 0.046 for viscous flow.
C'	[-]	factor used by Chisholm in Eqn. (2.4.15).
C_1	[-]	factor used by Nicklin in Eqn. (2.5.38).

C_o'	[-]	factor used by Brown & Govier[55] in Eqn. (2.5.46).
C_o	[-]	Distribution parameter in Eqn. (2.5.42) used by Zuber & Findlay.
C_o^*	[-]	Distribution coefficient in Eqn. (2.5.43) and (4.2.34) derived by Nguyen & Spedding[318].
c	[-]	factor in Butterworth's equation, Eqn. (4.1.2).
c_1, c_2	[-]	Correlation factors used by Griffith & Wallis in Eqn. (2.5.35) for the velocity of bubbles.
c_m	[-]	factor used by McManus in Eqn. (2.5.2).
D	[L]	Pipe diameter.
D_i	[L]	Diameter of gas-liquid interface in annular flow.
$\overline{D}$	[L]	Hydraulic diameter, equals 4 times flow area divided by the wetted flow perimeter.
D*	[-]	Dimensionless number used by Wallis & Kuo and defined by Eqn. (2.7.10).
d_m	[-]	factor used by McManus in Eqn. (2.5.2).
$\left(\frac{dP}{dL}\right)$	$[M L^{-2} T^{-2}]$	Pressure gradient
E_D	[-]	fraction of liquid flow entrained as droplets.
F	[-]	Correlation function used by Levy in Eqn. (2.5.25).
Fr	[-]	Froude number, $Fr_L = U_{LS}^2/gD$; $Fr_{TP} = U_T^2/gD$; $Fr_f = \overline{U}_L/\sqrt{g\delta_L}$.
f	[-]	friction factor.
f_1	[-]	function defined by Eqn. (3.3.12).
f_2	[-]	function defined by Eqn. (3.3.13).
G	$[ML^{-2}T]$	mass flux.
g	$[LT^{-2}]$	acceleration due to gravity.
H	[-]	factor in Eqn. (4.1.3).
H_A	[-]	Corelation factor used by Armand in Eqn. (2.4.1).
h	[L]	height of liquid in the piezometer ring above the lowest point in the flow tube.
$\overline{h}$	[L]	mean wave height used in Eqn. (2.5.10) by Sekoguchi et al.
h_m	[L]	manometer deflection.
K	[-]	Bankoff's parameter in Eqn. (2.5.66). Equivalent to the reciprocal of Armand's factor C.
K'	[-]	Correlation factor defined by Gomezplata et al, given in Eqn. (2.5.55).
K"	[-]	Correlation factor defined by Yamazaki et al, given in Eqn. (2.5.82).
K_2, K_3	[-]	factors in Eqn. (2.5.41) used by Griffiths.
K_4	[-]	factor used by Hughmark given in Eqn. (2.5.44).
k_1	[-]	Corelation factor relating actual holdup to theoretical holdup, given in Eqn. (3.4.12).
k_f	$[L^{-1}]$	Wave number, used in Eqn. (3.7.3) equals $2\pi/\lambda_f$.
k_L	[-]	Correlation factor relating velocity gradient at the wall to the mean film velocity and film thickness when the film is in laminar flow.

k_T	[-]	Correlation factor relating velocity gradient at the wall to the mean film velocity and the film thickness when the film is in turbulent flow.
L_s	[L]	Slug length, used by Moissis & Griffith.
L	[L]	axial distance along flow channel.
l	[L]	mixing length.
l_v	[L]	length of bubble along minor axis.
M	[-]	function defined by Eqn. (4.3.3).
M_B	[-]	Ratio of two-phase to all liquid pressure drop across an orifice. Used by Baroczy and defined in Eqn. (6.2.6).
m_A	[-]	Correlation factor used by Armand in Eqn. (2.4.2).
m	[-]	exponent in Blasius' equation, equals 1.0 for viscous flow and 0.2 for turbulent flow.
m'	[-]	Power law exponent used by Bankoff in Eqn. (2.5.64).
N	[-]	function defined by Eqn. (4.3.9).
n	[-]	exponent in Blasius' equation, equals 1.0 for viscous flow and 0.2 for turbulent flow.
n'	[-]	Power law exponent used by Bankoff in Eqn. (2.5.65).
n_A	[-]	Correlation factor used by Armand in Eqn. (2.4.1).
n_L	[-]	factor used by Levy in Eqn. (2.5.25).
P	[-]	function defined in Eqn. (4.2.17); otherwise,
	$[ML^{-1}T^{-2}]$	Pressure.
p	[-]	factor in Butterworth's holdup equation of Eqn. (2.4.18) or (4.1.1).
Q	$[L^3T^{-1}]$	volumetric flow rate; Q_{Lf}, liquid flow rate in the film only, $= Q_L$ if $E_D=0$.
q	[-]	factor in Butterworth's holdup equation of Eqn. (2.4.18) or (4.1.1).
q_T	[-]	Correlation factor used by Turner & Wallis in Eqn. (3.4.11).
q_G, q_L	$[L^3T^{-1}]$	volumetric flow rate of gas and liquid respectively as collected by the subchannel sampler.
R	[L]	radius.
R'	[-]	density ratio function, used by Levy, given in Eqn. (2.5.25).
$\overline{R}$	[-]	Holdup, where $\overline{R}_L$ is the liquid holdup and $\overline{R}_G$ the voidage or gas holdup. $\overline{R}_S$ is the volume fraction occupied by spheres, Eqn. (2.5.60). $\overline{R}_{Lf}$ is the liquid holdup at the tail end of a large gas bubble in slug flow.
$\overline{R}_p$	[-]	pseudo holdup used in the correlation of μ_{TP} by Hagedorn & Brown for the homogeneous flow model.
$\overline{R}'$	[-]	fictitious holdup defined by Eqn. (6.5.1) and (6.5.2).
r	[-]	factor in Butterworth's holdup equation of Eqn. (2.4.18) or (4.1.1).
r'	[-]	factor used by Turner & Wallis in Eqn. (3.4.7).
r_G	[-]	local void fraction. r_{GC} is the pipe centre value.
Re	[-]	Reynolds number. Re_f, the film Reynolds number, $= \frac{4\delta_L\rho_L\overline{U}_L}{\mu_L}$. In the case of a circular tube, $Re_f \simeq Re_L$.
S	[L]	length of flow channel perimeter in contact with the flowing phase.
s	[-]	slip ratio, $= \overline{U}_G/\overline{U}_L$.
T, t		Turbulent flow.
U_s	$[LT^{-1}]$	slip velocity, used by Holmes & Russell[205] in Eqn. (2.5.78).
$\overline{U}$	$[LT^{-1}]$	True average velocity, e.g. $\overline{U}_G = Q_G/(A_T\overline{R}_G)$.

U_{GS}, U_{LS}	$[L\,T^{-1}]$	superficial velocity of gas and liquid respectively, e.g. $U_{GS} = Q_{G}/A_{T}$. $U_T = U_{GS} + U_{LS}$. the superficial total velocity is also called the non-slip velocity.
U*	$[LT^{-1}]$	friction velocity, $= \sqrt{\tau/\rho}$ also defined in Eqn. (2.5.11) and (2.5.13).
U'	$[LT^{-1}]$	velocity of two-phase mixture at distance y from the channel wall. Used by Bankoff[19] in Eqn. (2.5.64).
U_{GD}	$[LT^{-1}]$	drift velocity relative to total flow, used by Zuber & Findlay in Eqn. (2.5.42).
U_B	$[LT^{-1}]$	Bubble velocity. U_{BTP} = bubble velocity in two phase flow. U_{BS} = bubble velocity when liquid is stangant. U_{BO} = velocity of rise of fine bubbles in stagnant liquids.
$\bar{U}_f$	$[LT^{-1}]$	Average velocity of equilibrium film associated with the rise of large bubbles, $\bar{U}_{fs}$ - associated with stagnant liquid, $\bar{U}_{fTP}$ - associated with flowing liquid.
U'_{LS}, U'_{GS}	$[LT^{-1}]$	Superficial velocity of liquid and gas respectively in a sampling probe or subchannel sampler.
U	[-]	function defined by Eqn. (4.2.19).
V, v		viscous flow.
We_L	[-]	Weber number, $\bar{U}_L \Big/ \left(\frac{\sigma}{\rho_L \delta_L}\right)^{1/2}$
w	$[MT^{-1}]$	mass flow rate.
x	[L]	in Chapter 5, distance below the tip of a rising Taylor bubble. Otherwise
	[-]	dryness fraction, $= W_G/(W_G + W_L)$.
y	[L]	Distance from pipe wall.
y+	[-]	distance parameter, $= yU^*/\nu_L$.
z_1, z_2	[-]	factors in Eqn. (2.3.7) used by Hagedorn & Brown.
z_3	[-]	factor in Eqn. (2.4.17) used by Chisholm.
z_4	[-]	factor in Eqn. (2.5.2) used by McManus.
z_5, z_6	[-]	Correlation factor in Eqn. (2.5.9) used by Shearer & Nedderman.

GREEK LETTERS

α_{LM}	[-]	function used by Lockhart-Martinelli, defined by Eqn. (3.2.3).
β	[-]	Volumetric ratio of input gas rate to total flow rate = $Q_G/(Q_G+Q_L)$.
β_{LM}	[-]	function used by Lockhart-Martinelli, defined by Eqn. (3.2.4).
δ	[L]	mean phase thickness.
δ_s, δ_{TP}	[L]	mean equilibrium liquid film thickness associated with the rise of a Taylor bubble in stagnant and flowing liquid respectively.
δ^+	[-]	Dimensionless film thickness = $\frac{\delta U^*}{\nu_L}$.
δ_r	[L]	relative film thickness used by Gill, Hewitt & Lacey[143], = δ_L - gas boundary layer thickness.
ε	[L]	characteristic roughness.
η	[-]	dimensionless film thickness used by Dukler[112], defined in Eqn. (2.5.11).
θ	[-]	angle between flow direction and the horizontal.
θ'	[-]	angle subtended at the axis of the flow tube by the chord formed by the gas-liquid interface during stratified flow.
λ	[-]	mixing length constant, = 0.4 for single phase flow near the wall region.
λ_f	[L]	wave length used in Eqn. (3.7.3).
μ	$[ML^{-1}T^{-1}]$	dynamic viscosity
ν	$[L^2T^{-1}]$	kinematic viscosity
ν_w	$[T^{-1}]$	Number of ripples or waves per unit time, used by Sekoguchi in Eqn. (2.5.10).
τ	$[ML^{-1}T^{-2}]$	shear stress.
ρ	$[ML^{-3}]$	density. Homogeneous density $\rho_H = \rho_G\beta + \rho_L(1-\beta)$. Two-phase mixture density $\rho_{TP} = \rho_G \bar{R}_G + \rho_L (1-\bar{R}_G)$.
σ	$[MT^{-2}]$	surface tension.
ϕ	[-]	Lockhart-Martinelli parameter. e.g. $\phi_G = \sqrt{\left.\frac{dP}{dL}\right\|_{TPF} \Big/ \left.\frac{dP}{dL}\right\|_G}$
X	[-]	Lockhart-Martinelli modulus defined in Eqn. (3.2.5).

SUBSCRIPTS

A	accelerational component.
c	centre of flow channel.
E	obtained by energy consideration.
F	friction component.
f	liquid film.
G	gas flowing alone.
GO	calculated by assuming flow was all gas but with the mixture mass flux.
H	hydrostatic component.
i	interface.
L	Liquid flowing alone.
LO	calculated by assuming flow was all liquid but with the mixture mass flux.
M	obtained by momentum consideration.
T	total or two-phase
TP	two-phase
w	wall.

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