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"ASPECTS OF SOIL-PILE INTERACTION UNDER STATIC LOADS"

A thesis submitted in partial fulfilment of the requirements for the degree of
Doctor of Philosophy

Department of Civil Engineering
University of Auckland
Auckland
New Zealand

by

PETER R. GOLDSMITH
November, 1979
A QUOTATION FROM MARK TWAIN

(Out of Context of Course!)

"THE RESEARCHES OF MANY COMMENTATORS HAVE
ALREADY THROWN MUCH DARKNESS ON THIS SUBJECT,
AND IT IS PROBABLE THAT IF THEY CONTINUE WE
SHALL SOON KNOW NOTHING AT ALL ABOUT IT".

* * * * *
Synopsis

This model study is concerned with attempting to identify some of the mechanics of pile-soil interaction under the influence of static loads, as a pre-requisite to defining the mechanics of the soil response to pile transmitted dynamic (i.e. seismic) lateral soil loads.

The emphasis has been directed at the mechanics of the response of the soil to loads transmitted through the pile, rather than the more usual approach of defining an analytical pile model and assuming a soil response. The work contained herein follows the incremental deformations occurring within a soil mass throughout the process of installation through to the ultimate lateral loading condition. To enable this to be done the comparatively recent developments in soil mechanics involving the application of the techniques of stereophotogrammetry and radiography, have been employed.

The study is mainly involved with short rigid model piles in dry dense sand.

A more general aim of the research project has been to attempt to draw the more research orientated and practical aspects of the statically loaded pile problem closer together. To this end, state-of-the-art reviews of both the axial and lateral loading situations have been conducted and an attempt made to relate them to the mechanics of soil response, as identified both in this research project and from full scale tests reported in the literature.

An attempt has been made to apply some of the more general observations resulting from the study, to the Type A prediction, (i.e. before the event), of the ground line displacements of a full scale pile under real site conditions when subjected to various loads.
Acknowledgements

The work presented in this thesis was carried out at the School of Engineering, University of Auckland under the supervision of Dr. John Hughes, who is sincerely thanked, particularly for his enthusiastic and intimate involvement in the early stages of the project, and his enthusiasm in the role of "Devil's Advocate" at the typing stage. Particular thanks must also be extended to Dr. Mack Pender for his ready ear during John Hughes' sabbatical leave at the time that this thesis was being prepared; and especially for reading and helpfully commenting on the appendices and some chapters. Similarly, the writer is also indebted to Dr. Ian Meiland for checking the mathematics presented herein.

Grateful thanks is also extended to Gail Riddell for her readiness to translate various publications from the French, often to find in the final analysis that they were of little value.

In addition, I would like to extend my sincere thanks to the following:-

Major Peter Hunt, RE (Ptd.), Senior Lecturer in Photogrammetry, Department of Surveying, University of Otago, for producing independent plots of soil deformations from stereo pairs of photographs.

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Mrs. Glynis Margetts for her care, thoroughness and efficiency in the typing of the text.

During the course of this research project the writer was the grateful recipient of:-
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In addition a National Roads Board grant for equipment was also received, for which the writer is equally grateful.

Finally, I must thank my Mother for her continued encouragement throughout the course of this work, and to whom this thesis is dedicated.
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Chapter 1 Introduction

1.1 PHILOSOPHY

1.1.1 General

Considerations pertaining to the mechanics of pile-soil interaction are probably the most difficult problems likely to be encountered by a foundation engineer. The concept of piled foundations have existed for probably as long as man has attempted to build. With this familiarity has come a degree of complacency. The development in the last 30 odd years of sophisticated looking formulae which have been persuasively conveyed to the fraternity of practising engineers has further enhanced this sense of complacency in what is a particularly complicated problem; such that many engineers now consider the design of piled foundations as a particularly simple chore, requiring nothing more than the application of generalised formulae using equally generalised soil properties.

With this categorisation of the problem into the slot of "simplicity" has developed a general lack of consideration of the mechanics of the problem. It is too easy for the designing engineer to use "textbook solutions" to fundamental problems without maintaining an awareness as to the limitations of, or the assumptions inherent in the various proposed design methods.

One of the aims of this research project has been to bridge the gap between the research engineer and the practising engineer, with particular regard to achieving perspective between the usually practiced design considerations of statically loaded piled foundations and the fundamental mechanics of the problem. It is believed that if the mechanics of a problem are appreciated, then rationalisations with regard to particular design problems can be made more reliably. In addition, if such mechanical considerations are appreciated, not only by the designing engineer, but also by the contractor, and in particular the individual responsible for the execution of the designers requirements, then an efficient solution to the particular foundation problem must follow.

This interaction between the geotechnical design engineer and the constructor is a fundamental facet of foundation engineering and has been expounded by a number of authorities in the field (for example Terzaghi, 1958; Casagrande, 1965; Peck, 1969). More recently Endo (1977) has also addressed himself to the problem thus:-

[ideally] "THE ENGINEER CONSIDERS GEOTECHNICAL CONDITIONS FROM THE RESULTS OF INVESTIGATIONS OF GROUND AND SOIL PROPERTIES OBTAINED FROM FIELD AND LABORATORY TESTS, AND DESIGNS THE OBJECT SATISFYING THE PERFORMANCE REQUIREMENTS BASED ON PAST RESEARCH ACHIEVEMENTS AND HIS OWN EXPERIENCES. THE CONTRACTOR ON HIS PART TAKES THIS DESIGN, THOROUGHLY GRASPS THE GROUND CONDITIONS GIVEN, AND UPON COLLECTING NECESSARY INFORMATION, EXECUTES THE WORK MAKING FULL USE OF HIS TECHNICAL CAPABILITIES AND EXPERIENCE TO ACTUALLY PRODUCE THE REQUIRED STRUCTURE ..............................................................
THERE WOULD BE NO PROBLEM IF THE ENTIRE SEQUENCE PROGRESSES IDEALLY, BUT IT IS OFTEN THE CASE THAT VARIOUS DISCREPANCIES ARISE DURING THE TRANSFER PROCESS FROM ENGINEER TO CONTRACTOR.  

It is in this latter comment that the mechanical understanding of the particular problem, held individually by the engineer and the contractor, is often at variance. Most significant advances in the art of civil engineering could be claimed to have their roots in inspiration. It is however, one of the fundamentals of nature's laws that all inspiration, in the final analysis, is based on commonsense. It is this "commonsense" understanding of the mechanics of a particular problem that effects the bridge between the designer and the constructor. Possibly therein lies one aspect of the "art of geotechnical engineering".  

Endo has further commented that:-


UNLESS DESIGN AND CONSTRUCTION ARE CARRIED OUT BASED ON A THOROUGH UNDERSTANDING BETWEEN ENGINEER AND CONTRACTOR REGARDING THE DEGREE OF EFFECT [OF CONSTRUCTION DISTURBANCE], THE RESULT WILL BE THAT THE REQUIRED PROPERTIES OF THE FOUNDATION WILL NOT ACTUALLY BE SECURED."

In short, if the mechanics of the problem are not equally understood by the designer and the constructor, then the performance of a foundation is likely to be reduced to a matter of chance, dependent upon the interaction of a number of events outside the recognition or control of the parties involved.

1.1.2 The Research Project

The research project was initially conceived to study the manner in which cohesionless soils responded to the application of dynamic lateral loads transferred to the soil through nailed foundations. It was a relatively rapid and short regression to the realisation that the preeminate of assessing the mechanics of dynamic pile-soil interaction was an understanding of the simple case of the anodication of a static lateral load. This similarly regressed to the realisation that the usual method of pile installation in cohesionless soils is by driving which in turn establishes a further set of conditions modifying the soil behaviour. In between the considerations of static and dynamic lateral loading are the intermediate and equally complex modes of loading of reeated and cyclic loads. These various modes of lateral loading are defined in Hughes, Goldsmith and Fendall (1974).

In addressing the problem of soil response to pile installation and by association, axial loading, it became clear that the mechanics of this aspect of pile-soil interaction was not only ill defined but subject to a wide variety of conflicting opinions that had been promulgated through the literature over a period of years.
Nevertheless, to reliably assess the problem of the dynamic response of soils to pile transmitted lateral loads required initially attempting to resolve or identify the manner in which the soil responded to the elemental disturbances leading up to the general conditions pertaining at the application of the first cycle of static lateral loading. If the mechanics of this aspect of the problem can be resolved, then the more complex repeated, cyclic and finally dynamic soil responses will follow.

Thus the understanding of the static soil response problem is the link to the mechanical understanding of the dynamic problem.

The research project as reported herein is consequently considered to be the first step along this complex path. The primary aim of the project has been to assess the relative mechanics of the soil response to the pile transmitted disturbing forces, through the complete process of pile installation to static lateral loading.

The dynamic project as initially conceived was aimed at enabling the response of full scale piles (or alternatively, large scale model piles) to be studied. In the event the project was restricted to the study of small scale model piles. Aspects relating to model pile studies are discussed later.

Considerations relating to the dynamic analysis of piled foundations are contained in Hughes, Goldsmith and Fendall (1978) and Goldsmith (1979).

1.2 STATIC LOAD CONSIDERATIONS

1.2.1 General

The fundamental factors affecting the mechanics of the soil response to static pile loading are:

(i) The effects of the pile installation process
(ii) the nature of the applied static loading, be it axial or lateral

Traditionally the two loading conditions are considered independent of each other and the effects of pile installation, if considered at all, are usually based on the designer's judgment. The reliable assessment of both the axial and lateral static load carrying capacity of piled foundations involves full scale loading tests; a requirement that can be sustained economically on only a very small number of projects.

Various analytical processes exist for assigning static load carrying capacity to piles when subjected to axial or lateral loads. All of these processes however, require an assessment of the appropriate soil properties. Often as not the soil properties used are obtained from either laboratory tested disturbed samples which at best will reflect the conditions existing prior to pile installation, or alternatively from semi-empirical correlations with subjective in-situ tests such as the SPT test.

Although they are considered to be independent, the mechanics of the soil response to pile installation axial loading, and lateral loading are directly related; the response in the lateral loading mode being dependent upon the response in the axial mode which in turn is governed by the effects of the chosen method of pile installation.
1.2.2 Aspects of Soil Response to Static Loads Transmitted Through Piles

(i) Pile Installation and Axial Loading

A number of questions of considerable importance to the designer can be noted which relate to the manner in which the soil responds to loads transmitted through the pile. Some of these follow:-

(a) How is the undisturbed soil affected by the process of pile installation and how does this mass deform?
(b) What is the mechanism by which the volume of a progressively installed displacement pile is accommodated within the soil mass?
(c) The installation of a displacement pile must cause considerable shear strain in the soil, consequently does the state of stress about the pile vary substantially during the process of installation by driving?
(d) If so, how does this affect the distribution of load between the pile tip and shaft?
(e) In addition, how is the distribution of load between the pile tip and shaft affected by variations in shaft surface roughness and pile tip configuration?

(ii) Lateral Loading

The answers to the foregoing questions ((a) to (d)) could reasonably be assumed to generate the conditions existing immediately prior to loading the pile in the lateral mode. Some of the questions which must be asked relating to this mode of loading are:-

(a) What mass of soil is involved in the response to lateral loads transmitted through the pile and how does this relate to the mass of soil already affected by the process of pile installation and axial loading?
(b) Does the mass of soil involved differ about the pile, not only in dimension, but also in the manner in which it deforms and thus responds to load?
(c) What is the effect of dilation of a cohesionless soil on the load carrying capacity of a pile?
(d) What influence does the stress state created by the process of pile installation have on the response of the soil to lateral loading?
(e) What are the mechanics of the "ultimate" condition pertaining around a laterally loaded pile, and as a consequence, what is the final mode of failure?

(iii) Additional Factors

Each of these ill defined aspects of the mechanics of the soil response to pile transmitted loads are likely to generate additional factors requiring consideration. Such factors are likely to be related to a particular construction process or set of site conditions which will also affect the conditions existing in the soil, for example:-

(a) What are the effects of adjacent piles and the conditions of pile head fixity?
(b) Are the actual site conditions reflected in the rationalisations made in the design process? How reliable is the soils data used in the design, and how rigorously was the investigation conducted?
Is the pile finally installed perfectly vertical?
What is the position of the pile tin relative to the pile ton, particularly in the case of long piles?

Does the method of pile installation finally adopted by the contractor relate to the assumptions, if any, made during design.

Various other considerations affect the real pile problem. In general the research project has attempted to give some insight into the factors listed under (i) and (ii), and in so doing the potential effects of the other variables listed under (iii) have been ignored.

Thus the research programme initiated by the considerations contained in this thesis is essentially a model study in dry sand aimed at gaining a better understanding of the mechanics of pile-soil interaction than presently exists. The study is particularly related to the single pile subjected to static loads.

Because the behaviour of the soil within a few pile diameters of the ground surface in the lateral loading mode is fundamental to the behaviour of the system as a whole, the study has been almost entirely restricted to short rigid timber piles of 20mm square section.

A fundamental qualitative approach has been adopted as being a necessary first step towards a quantitative understanding of the static and ultimately the dynamic response of the soil to the loads imposed by the pile in both the axial and lateral modes. To this end radiographic and stereophotogrammatic techniques recently employed in observing soil displacements in such applications as retaining wall and hopper studies, have been adapted to the statically loaded pile problem.

In summary then, the aim has been to develop, from a fundamental viewpoint, a fastidious and systematically contrived understanding of the soil response to pile transmitted static loads.

1.3.1 Introduction

In a panel discussion on model scaling laws and model testing of deep foundations in sand Vesic' (1964) made the following comments:-

"AN INCREASED USE OF MODELS SHOULD BE EXPECTED AS A PART OF THE MAJOR TASK OF CHECKING THE BASIC PREMISES AND CONSEQUENCES OF THEORIES OF SOIL MECHANICS. BOTH QUALITATIVE AND QUANTITATIVE DATA WILL BE INCREASINGLY OBTAINED BY MODEL TESTS WHICH SIMULATE, AS CLOSELY AS POSSIBLE, IDEAL THEORETICAL CONDITIONS.

....... IN REALITY WE ARE JUST BEGINNING TO REALIZE WHAT THE PROBLEMS OF THE THEORY OF DEEP FOUNDATIONS ARE. IN ORDER TO MAKE ANY PROGRESS, WE NEED TO CLARIFY AT LEAST WHAT ARE THE ACTUAL DEFORMATION PATTERNS AROUND DEEP FOUNDATIONS IN DIFFERENT SOIL CONDITIONS, AND WHAT ARE THE PRINCIPLE SCALE EFFECTS ON THOSE PATTERNS ......

ULTIMATELY FROM MODELS SIMULATING ACTUAL FIELD CONDITIONS, WE MAY EXPECT TO OBTAIN DATA ON DEFORMATION AND STRESS PATTERNS THAT MAY BE USEFUL FOR A SUBSEQUENT ANALYTICAL INVESTIGATION OF THE REAL PROBLEM, IN WHICH, OF COURSE, SCALE EFFECTS WOULD BE INVESTIGATED"
Clearly the concepts pronounced in the foregoing (Sections 1.1 and 1.2) attempt in part to contribute to an improvement of the dearth of certainty associated with the soil response to deep foundation (i.e., pile) loading.

1.3.2 The Categories of Soil Mechanics Model Testing

Due to the complex mechanical behaviour of real soils there are considerable difficulties in obtaining accurate solutions to the boundary value problems of soil mechanics. The majority of present day methods of solution demand upon mathematical idealisations of the soil stress-strain behaviour to either perfectly plastic or perfectly elastic. For the majority of problems both of these idealisations are unrealistic, and as a consequence, the predictions made by conventional methods of analysis are prone to error.

Consequently experimental investigation of the load-displacement relationships for the boundary value problems of soil mechanics is essential. Testing of full scale prototype structures, although very necessary, is expensive and time consuming; therefore the incentive and need to perform representative model tests is great. James (1971) has broadly classified soils mechanics model testing, dependent upon the primary objectives, into three distinct but inter-related categories I to III.

(1) Category I Model Tests

These tests are defined as being concerned only with predicting the behaviour of a specific prototype structure from that of the model. Any special ground conditions such as soil stratification or ground water must be suitably simulated in the model. Consequently the results obtained do not have general applicability. Figure 1.1(a) shows this category of model testing as related to a pad foundation problem.

In this type of model test the principles of similarity (i.e., "scale") must be satisfied before the results can be of practical use. Centrifugal model testing falls into this category, and allows the use of "prototype" soil at prototype stress levels and under approximately correct conditions of stress and strain paths (for example James, 1971).

(II) Category II Model Tests

This approach to modelling considers that the model is a small prototype structure itself and attempts to compare its behaviour with that predicted by some method of analysis. For this approach to be useful it is of utmost importance that the model conforms with the assumptions inherent in the method of analysis adopted. Typical requirements would be that the soil is in a uniform state and that the influence of the boundaries of the container may be ignored. An example of this sort of model study is that associated with the bearing capacity of a spread footing as indicated in figure 1.1(b).

From the results of such tests it should be possible to assess the accuracy of various methods of analysis and also to confirm that the soil constants established from fundamental testing apparatus are relevant to the stress conditions in the model and can be used to predict the performance of the model. The results obtained from such tests are not necessarily of
FIG. 1.1 MODEL TEST CATEGORIES
(After James 1971).
immediate use in the design of a complex full scale prototype, but are of great value in establishing
certain design principles. Experiments of both this type and those of Category I are inherently
restricted by the need to simulate real problems; i.e. to be related to the full scale practical
problem, the considerations of relative scale must be taken into account.

(iii) Category III: Model Tests

In this category of model test the full scale prototype need never exist. These tests
are designed specifically to reveal detailed stress and deformation information about a problem.
The prime objectives being to increase the understanding of both the soil behaviour and soil
structure interaction, such that new methods of analysis may be developed which in turn will
lead to better design rules for use in the future. An example of this sort of test is the
retaining wall problem indicated in figure 1.1(c). A considerable amount of work has been done
using this category of model test related to the simple shear response of soils, (for example,
Stroud, 1971); the deformations about retaining walls (for example James and Bransby, 1970;
Bransby and Milligan, 1975; and Milligan and Bransby, 1976); and the flow of granular soils
in bunkers or hoppers (for example Bransby and Blair-Fish, 1974(a) and (b)). The work of
Bransby and Blair-Fish is reviewed in Goldsmith (1976).

1.3.3 Comment

James (1971) in his concluding remarks observed:

"THERE IS A SERIOUS GAP IN OUR THEORETICAL AND EXPERIMENTAL KNOWLEDGE OF THE KINEMATIC
BEHAVIOUR IN SOILS PROBLEMS WHICH URGENTLY NEEDS TO BE FILLED ...... THERE ARE
PRECIOUS FEW SOLUTIONS TO SOIL MECHANICS PROBLEMS WHICH [EVEN ATTEMPT TO]
SATISFY KINEMATIC CONSIDERATIONS, STATIC STRESS FIELD REQUIREMENTS, AND THE REAL
STRESS-STRAIN LAWS OF SOIL. ONLY WHEN ALL THREE REQUIREMENTS ARE SATISFIED SIMULTANEOUSLY
WILL SATISFACTORY LOAD-DISPLACEMENT PREDICTIONS BE POSSIBLE. THUS A MORE DETAILED
THEORETICAL AND EXPERIMENTAL STUDY OF SOIL KINEMATICS, VIA CATEGORY III TYPE
EXPERIMENTATION, IS LIKELY TO PROVE A REWARDING AND VALUABLE EXERCISE"......

The work reported in this thesis is primarily Category III type experimentation, extending
in some areas to Category II considerations. As a consequence it should be emphasised that no
attempt has been made to relate the experimental work to full scale considerations, but rather
is aimed at attempting to identify the fundamental mechanics associated with the problem.
In this context the attempts to rigorously follow the soil response throughout pile
installation, axial loading and finally laterally loading would appear to be unique.

1.4 "SCALE EFFECTS"

Because of the variety of soil investigatory techniques presently available and the
potential for these various methods to influence the result so obtained in one way or another, virtually
no precise evidence defining pile-soil interaction or the mechanics of load transfer exist. For example,
all analytical methods yield the same results quite independent of how the pile might have been installed,
even though questionable empirical methods have been evolved to attempt to handle the problem.
The end result is that the foundation designer is in an extremely perplexing position in that he is faced with the task of attempting to provide economical designs on the basis of limited information and, in general, inadequate experience. To find that in the last 30 odd years the available analytical tools have been reduced to virtually an art form only serves to complicate the situation. When the ability to design piles for static axial loads is considered probably the simplest pile problem, the difficulties facing the designer when considering the lateral loading of piles, in both static and particularly dynamic modes, are considerably more complex.

Because the art of pile load assessment is so dependent upon empirical considerations; the variations in model testing techniques, methods of pile installation, soil deposition (both in the laboratory and geologically), the difference in size between both model tests and the full scale pile, as well as between in-situ testing devices and full scale piles, are all of extreme importance.

For the purpose of this thesis these various effects have been assigned the general designation of "scale" effects. Clearly some of them are not, but are rather reflections on our understanding of the mechanics of the problem. For the convenience of discussion these various effects are considered in two separate categories:

(a) Those affecting the determination of in-situ soils properties from conventional tests and their relationship to the axial loaded pile problem. (Chapter 2).
(b) Those affecting the interpretation of both model and full scale field tests. (Chapter 5).

Although these aspects are considered separately, they are however, clearly related. Because of the complexity of the pile problem the various factors discussed are unlikely to reflect all the phenomena likely to influence the interpretation of the load carrying capacity of piles. In all probability other equally significant factors may remain yet undiscovered. An attempt has been made to assess the relative influence of the various factors discussed; such an assessment in itself being an extremely complex undertaking. It is possible then that some influences will negate others; whilst on the other hand, some effects may be cumulative.

1.5 Full Scale Prototype Testing

As discussed in section 1.1, this project had its roots in an attempt to assess the response of cohesionless soils to dynamic loads transmitted through the pile-soil system. The primary source of dynamic excitation of immediate concern is that induced by seismic influences. Under these conditions the primary exciting force is introduced into the structure via the piled foundations. This excitation induces a "feedback" response from the structure.

From this point of view the writer conceived the idea of introducing a dynamic excitation into the foundation via the controlled use of explosives. With the assistance of 161 Force Engineers of the Corps of EMZE, N.Z. Army, a series of trials were conducted in cohesionless soils on the Army's seaside demolition range at "Hannaharoa. The results were extremely encouraging and indicated that the magnitudes of ground accelerations typically recorded in earthquakes could be induced in the soil mass using comparatively small amounts of explosives at some distances from the location of, but in close
proximity to, the recorder. Using millisecond delays it was also possible to develop a series of peak accelerations separated by short periods of delay as is typically recorded on an earthquake record. The magnitude of these peaks could also be controlled by altering the distance from the explosive source to the object, or alternatively reducing the size of the explosive force whilst maintaining a constant object distance.

An attempt was made (Hughes, Goldsmith and Fendall, 1979) to apply some of the observations accruing from the writer's research, as reported herein, in conjunction with work concurrently being conducted on the two pile situation by H.D.M. Fendall, to a full scale static lateral loaded pile test at the 'Estgate Freeway Site, Melbourne, Australia. These considerations are discussed in Part 3 of this thesis.

1.6 THESIS OUTLINE

The work presented in this thesis comprises three separate aspects.

These are:-

(i) Part 1: Pile Installation and Axial Loads
(ii) Part 2: Lateral Loading

1.6.1 Part 1: Pile Installation and Axial Loads

The aim of part 1 of Chapter 2 has been to emphasise the often overlooked complexity of the piling problem as compared to more conventional foundations. Virtually each pile type and method of construction has a unique set of associated effects on the soil mass into which it is installed. In general these effects are ignored in applying the various analytical techniques currently available.

One particular method of pile installation, i.e. by driving, is considered in this thesis; along with a review of the various empirical considerations that are presently suggested, as means of modifying the results obtained from analytical considerations, to accommodate such effects.

Forming the main body of this chapter is a review of the state-of-the-art relating to the assessment of the axial load carrying capacity of piles. The wide range of varying considerations currently assigned to the axially loaded pile problem are indicated. In general, because the mechanics of soil response about deep foundations are not well defined, many of these considerations are difficult to relate to the soil deformation patterns reported in this thesis. In reality, the situation presently prevailing, and it is likely to remain so for some years hence, is that where reliable estimates of the axial load carrying capacity of friction piles are required, (the distinction being drawn between piles end bearing on a rigid stratum), the designer has little choice but to resort to full scale pile loading tests.

Chapter 3 presents the results of an experimental study in which the displacement fields developed about model piles are quantified during installation. The two independent techniques of stereophotogrammetry and radiography are used to enable such displacement fields to be obtained on a vertical plane of symmetry through the model piles. The displacement fields obtained using both
techniques are in extremely good agreement for sands placed to the same initial density and indicate an independence of the undisturbed $K_o$ condition. Different soil behaviour is observed above and below the pile tin respectively.

In addition, the mass of soil affected by the installation of model piles by driving into dense dry sand is indicated, as is an assessment of the accommodation of the embedded pile volume.

Chapter 4 presents an assessment of the strains developed about model driven piles at various stages of pile installation, as obtained from an analysis of the displacements reported in Chapter 3. By deducing both the cumulative volumetric and shear strains it is possible to make an assessment of the likely stress conditions existing after pile installation as reflected in the mobilised soil friction angles.

In all model pile tests (and for that matter, all model tests) the question of the significance of "scale" effects is raised. Such considerations also pertain to the various in-situ penetration tests, which in themselves are in fact model pile tests. The various presently identifiable aspects of scale are discussed in the first part of Chapter 5.

In addition it is suggested that the lateral stress existing in the soil, a presently little considered factor, could dominate some of these currently recognised scale effects; at least in drawing correlations between laboratory model tests and full size foundations. Not to be overlooked in such correlations is the potential for residual stresses to be developed in both the model and full size piles which could cloud the observations made.

The main feature of Chapter 5 is an attempt to derive an analytical expression for the pile tip axial load carrying capacity of the model piles taking into account the deformational mechanics reported in the preceding chapters. The analysis as conceived allows the mobilised soil friction angle and dilation rate to be varied, and in addition gives an indication of the soil displacement profile about the pile tip. The results of the analysis are compared with a series of model pile axial loading tests in which the shaft and pile tip resistances are measured independently. The results are encouraging, especially when compared to the range of potential loads able to be predicted from the various bearing capacity theories reported in the literature.

Also in Chapter 5 is a qualitative assessment of the probable effects of shaft roughness and pile tin configuration.

An assessment is also made of the possible mechanics of load transfer between the pile shaft and the surrounding soil. By the same qualitative technique, the variation in pile tin configuration is shown to have very little influence, if any, on the deformation fields developed about a model pile throughout installation by driving.

Chapter 6 presents a preliminary analysis in which the displacement fields reported in Chapter 3 are converted into stress fields on the basis of the stress-strain characteristics of the soil as determined from triaxial tests. Reasonable agreement is found between these stress fields, the values of stress predicted by the analysis of Chapter 5, and with axial pile loads determined from model tests.
1.6.2 Part 2: Lateral Loading

A review of the state-of-the-art relating to the assessment of the lateral load carrying capacity of piles is contained in Chapter 7. The problems of axial and lateral static loading of piles are traditionally considered independent of each other. In addition the soil properties assumed to apply are those determined prior to pile installation. If an allowance is made for the effects of pile installation it is usually done so on the basis of the engineer's judgement. As is the present situation pertaining to the axially loaded pile problem; a reliable estimate of the lateral load carrying capacity of piles can similarly only be made from full scale pile loading tests.

A qualitative comparison between the displacement fields developed about laterally loaded short rigid piles (poles) and long flexible piles is made in the first part of Chapter 8. In the case of model poles in sand it is shown that the nature of the soil conditions (i.e. dry, saturated, loose or dense) has little effect on the depth to the point of rotation of the pile. Similar soil deformational characteristics are observed about both flexible and rigid piles, at least in the first few pile diameters (d) below the ground surface level.

Using the same radiographic technique employed in Chapter 3, the incremental soil displacement fields developed at various stages of lateral loading are presented in the latter part of Chapter 8. In so doing, four distinctly different zones of soil response are identified, although the primary soil response to lateral loading appears to come from the few d of soil in front of the pile. The point of rotation for a rigid pole is shown to rapidly stabilise at a particular depth. In addition the inter-relationship between the soil mass affected by pile installation by driving and that resisting laterally applied pile loads is identified.

From the displacement fields of Chapter 8, cumulative volumetric and shear strain fields are developed as presented in Chapter 9. This is done firstly by assuming the affected soil mass is initially strain free, and subsequently by incorporating the strains developed due to installation by driving. The nett result indicated is that the residual driving strains dominate those developed during lateral loading. The trends for continued dilation of the soil mass indicated during driving are continued throughout lateral loading, but with an added measure of complexity.

On the basis of the observations made, a failure mechanism for a rigid pile is suggested.

1.6.3 Part 3: Full Scale Pile Test

An attempt to relate some of the observations made in this model study to a full scale laterally loaded pile test is presented in Chapter 10. The application takes the form of a Type A prediction (i.e. before the event) of the likely deflections to be experienced by piles at various stages of loading at the Westgate Freeway site in South Melbourne. The initial soils data and final test results were made available by the Victoria Country Roads Board in association with their consulting engineers, Coffey and Partners. In general the predictions made were reasonably good. Some wrong assessments were however, made. These have been identified.
1.6.4 Part 4: Conclusions

Conclusions and suggestions for future work are contained in Chapter 11.

1.6.5 Part 5: Appendices

The various secondary studies supporting the main line project reported in Chapters 2 to 11 are contained in Appendices 1 to 12.
Part One

Pile Installation And Axial Loading

"... Methods of calculating bearing capacity from measured soil properties have assumed great importance. This logical step has been the goal of foundation engineers since Paton in 1895 applied Rankine's theory of conjugate stresses to obtain a solution for the bearing capacity of a pile. Before that goal is reached however much more must be known about the changes in the ground caused by the process of pile installation so that a realistic system of effective stresses can be introduced into that elusive bearing capacity formula."

WHITAKER, T. New Civil Engineer
August 1979
Beneath The Surface Supplement
Chapter 2  The Determination Of The Axial Load Carrying Capacity Of Piles

2.1 INTRODUCTION

Piling is both an art and a science (Tomlinson 1977). The art lies in selecting the most suitable type of pile and method of installation for the ground conditions and the nature of the loading. The foundation designer needs to understand how the performance of foundations is influenced by the properties of the supporting soil and, particularly in the case of piled foundations, how these properties are modified either permanently or for limited periods by the method of installation of the foundation.

Thus, science enables the engineer to predict the behaviour of the piles once they are in the ground and subjected to loading.

In this chapter the complexity of piled foundations are discussed, the presently available means of accounting for the process of pile installation by driving are considered, and the various analytical concepts and methods of assigning axial loads to piles in cohesionless materials are reviewed albeit not exhaustively. The existence of a scale effect in making correlations between model and full scale piles are also discussed.

2.2 FUNCTION OF PILES

Piles are relatively long and slender columnar elements of a foundation which have the function of transferring loads from the superstructure through weak compressible strata or through water, onto stiffer or more compact and less compressible soils, or onto rock. They may be required to carry a variety of loads, for example:-

(1) Axially compressive loads (their more commonly recognised function).

(2) Uplift loads, such as those associated with the support of tall structures subjected to overturning forces from winds, waves or even possibly earthquake.

(3) Pure lateral loads such as likely to be applied to marine structures from wave action or berthing ships.

(4) A combination of vertical and horizontal loads. This is probably the more usual load case and may be developed in a variety of situations:-

(i) where piles are used to support retaining walls

(ii) bridge piers and abutments

(iii) machinery foundations

(iv) where piles are subject to lateral forces induced by seismic activity.
The manner in which the pile-soil system responds to the load situation is further compounded by the nature of the loading - i.e. be it static or dynamic.

Dynamic forces may be generated by a number of phenomena, for example, seismic events, wave or wind action, or may even be machinery induced vibrations.

2.3 DETERMINATION OF SOIL RESPONSE TO LOADING

The complexity of the pile problem is highlighted by comparing a single axially loaded pile with a relatively "simple" foundation such as a spread footing subjected to a uniform applied pressure. The application of the principles of soil mechanics to the determination of the ultimate bearing capacity of a pile differs immensely from the application of these same principles to shallow spread footings.

2.3.1 Shallow Spread Foundation

In the case of the shallow spread footing the entire area of soil supporting the foundation is exposed and can be inspected and sampled to ensure that its bearing characteristics conform to those deduced from exploratory boreholes and soil tests. Provided that the correct constructional techniques are used, the disturbance to the soil is limited to a depth of only a few centimetres below the excavated founding level of the footing. Virtually the whole mass of soil stressed in response to the pressure (loading) applied to the footing (ρ in figure 2.1(a)) remains undisturbed and unaffected by the construction activities. The factor of safety against a general bearing capacity (shear) failure of the soil beneath the footing (and thus failure of the footing) under the design working load is able to be predicted from a knowledge of the physical characteristics of the undisturbed soil.

The degree of certainty involved depends only on the complexity of the soil stratification, the thoroughness of the soils investigation, and the sophistication of the analytical techniques used.

2.3.2 Piled Foundation

In contrast the conditions which govern the load carrying capacity of the piled foundation are quite different. Irrespective of whether the pile is installed by driving, jetting, vibration, jacking, screwing or drilling, the soil in contact with the pile face is completely disturbed by the method of installation. Significantly it is from this soil that the pile derives its support:-

(i) By skin friction in the axial loaded situation, and
(ii) For its resistance to lateral load.

In a similar manner the soil or rock beneath the toe of a pile may be compressed (or loosened) to an extent which may significantly affect its resistance to loads transmitted through the pile tip. The complex nature of conditions about a pile after installation are indicated in figure 2.1(b). To further compound the problem, changes may take place in the conditions at the pile-soil interface over a period ranging from days to months or even years.

These changes may be due to:-

(i) Dissipation of excess pore pressures set-up by pile installation in cohesive materials
(ii) The effects of pile-soil movement on friction characteristics
(iii) Long term degradation of the pile itself due to the influence of groundwater, chemical and electrochemical effects, soil bacteria, and in the case of timber piles attack by marine organisms.
FIG. 2.1 COMPARISON BETWEEN SPREAD FOOTINGS AND PILED FOUNDATIONS IN COHESIVE SOILS.
The problem assumes a further order of complexity when pile groups are considered. For example the effects of the process of installation on adjacent piles already installed may result in changes in both their load carrying and settlement characteristics.

In the present state of knowledge, the effects of the various methods of pile installation on load carrying and deformation characteristics cannot be calculated by the strict application of soil or rock mechanics theory. In general the best that can be done is to apply semi-empirical factors to the shear strength or compressibility characteristics of the undisturbed foundation material. (In this context, "undisturbed", is intended to mean, "prior to pile installation" and is not intended to reflect the degree of sample disturbance associated with any particular method of soils sampling or investigation).

Tomlinson (1977) has observed that:-

"ATTEMPTS TO STUDY THE BEHAVIOUR OF PILES IN THE LABORATORY HAVE NOT PRODUCED ANY WORTHWHILE DESIGN RULES BECAUSE OF THE IMPOSSIBILITY OF REPRODUCING INSTALLATION EFFECTS IN MODEL PILES. AT BEST THEY HAVE GIVEN AN INSIGHT INTO THE FUNDAMENTAL MATTERS OF THE TRANSFER OF LOAD FROM PILE TO SOIL, AND OF GROUP EFFECTS".

One of the aims of the model study reported in this thesis is to attempt to show the effects the method of installation has on driven piles in dense sands.

It is contended that whilst much effort has been assigned to refining analytical techniques, relatively little effort has been assigned to trying to define the physical nature of the manner in which the soil around a pile responds to loading.

A subsequent chapter in this thesis describes attempts to make a Tyne A prediction of the load-deflection response of a full scale pile subjected to a variety of lateral loads. The results clearly indicate that the relative degree of sophistication of the analytical technique adopted is secondary to

1. Understanding the physical nature of pile-soil interaction.
2. The interpretation of the soils data, and
3. The employment of reliable insitu soils testing techniques.

2.4 TYPES OF PILES

The British Standard Code of Practice for Foundations (CP2004:1972) defines three main types of piles in common use:

1. Displacement Piles
   This group comprises piles which are either solid throughout, or of a hollow section with one end closed. They are driven or jacked into the ground and thus displace at least their own embedded volume of soil.

2. Small Displacement Piles
   These are also driven into the ground but have a smaller cross-sectional area. This group includes rolled steel H or I sections and pipe or box sections driven with an open end such that the soil enters the hollow section.
   In general small displacement piles are used if ground displacement and disturbance must be kept to a minimum.
FIG. 2.2 CLASSIFICATION OF BEARING PILES (CIRIA).
(3) **Non-Displacement Piles**

These are generally formed by boring or excavation. The resultant void is then filled with concrete. Such piles are usually referred to as "bored and cast-in-place" piles. Permanent or temporary support is often given to the sides of the excavation. Permanent support is usually provided by steel or precast concrete casing whilst drilling mud (bentonite) is sometimes used to provide temporary support.

The CP2004 classification has been amended by Wiltman and Little (1977) to combine displacement and small displacement piles. This amended classification for bearing piles is shown in figure 2.2. A similar classification is given by Simons and Menzies (1976).

The advantages and disadvantages of various piling methods and systems are discussed by a number of authors, for example, Tomlinson (1977) and Simons and Menzies (1976).

### 2.5 METHODS OF ASSESSING THE LOAD CARRYING CAPACITY OF AXIALLY LOADED PILES

In general the methods of assessing the axial loads in piles are:-

(a) ultimate load method (i.e. bearing capacity theories)

(b) methods which enable the loads and displacements to be determined prior to the ultimate condition being reached (i.e. elastic theories).

The more usual methods are those associated with ultimate load determination; consequently in the following sections such methods are considered more fully than the alternative methods.

### 2.6 ULTIMATE LOAD CARRYING CAPACITY OF PILES

#### 2.6.1 Introduction

The methods of assessing the ultimate load carrying capacity of axial loaded piles in cohesionless soils are:-

(a) Analytical methods (i.e. bearing capacity theories)

(b) Empirical methods (i.e. Dutch Cone tests, Standard Penetration tests, pile driving formulae, Ménard pressuremeter)

(c) Full scale tests.

In recent years (as reported by Tomlinson, 1977) much attention has been given by research workers to methods of calculating the axial load carrying capacity of single piles based on the theoretical understanding of soil mechanics. They postulate that the skin friction on a pile shaft can be determined by a simple relationship between the coefficient of earth pressure 'at rest' \( K_o \), the effective overburden pressure, and the drained angle of shearing resistance of the soil \( \phi' \) and \( \phi \) respectively. However, they generally recognise that the coefficient of earth pressure must be modified by a factor which takes into account the method of pile installation. Similarly a number of researchers believe that the end-bearing resistance of a pile can be calculated by classical soil mechanics theory based on the undisturbed shearing resistance of the soil surrounding the pile tip.
FIG. 2.3 DIFFERENT ASSUMED FAILURE PATTERNS UNDER DEEP FOUNDATIONS (After Vesic, 1967).
An extremely significant comment made by Tomlinson is:-

"OBSERVATIONS MADE ON FULL-SCALE INSTRUMENTED PILES HAVE SO FAR ONLY SERVED TO REVEAL
THE EXTREME COMPLEXITIES OF THE PROBLEMS OF ASSIGNING AXIAL LOAD CARRYING CAPACITY,
AND HAVE SHOWN THAT THERE IS NO SIMPLE FUNDAMENTAL DESIGN METHOD".

The assessment of the static axial load carrying capacity of piles is based on the concept of
the separate evaluation of shaft friction and tip resistance thus:-

\[ Q_u = Q_t + Q_s - W_p \]  \[2.1\]

where \( Q_u \) = the ultimate load carrying capacity of the pile
\( Q_t = q_u A_{pt} \)
where \( q_u \) = the unit ultimate resistance of the pile tip
\( A_{pt} \) = the cross-sectional area of the pile
\( Q_s = f_s A_{ps} \)
where \( f_s \) = the unit ultimate resistance of the pile shaft
\( A_{ps} \) = the surface area of the pile shaft
\( W_p \) = the weight of the pile

The weight of the pile is generally small in comparison with the ultimate load carrying
capacity of the pile, and as a consequence, is usually ignored.

Situations however, do exist where the weight of the pile could be significant: for example marine
structures in deep water where a considerable length of the pile extends above the sea bed.

2.6.2 A Selective Review of Bearing Capacity Theories and Associated Observations of the Mechanics
of Pile-Soil Interaction

The following review is broadly divided into the assessment of pile tip and shaft load carrying
capacities.

(1) The Assessment of Pile Tip Capacities

The different failure patterns assumed to represent the ultimate resistances developed at the
pile tip are indicated in figure 2.3. Of these the proposals of De Neer (1945), Jaky (1948)
and Meyerhof (1951) are not kinematically possible (figure 2.3 (b)) in that inherent in the
assumption that the slip lines revert to the pile shaft is the requirement that the boundary
represented by the pile shaft deforms. All solutions assume that the ultimate unit pile
tip resistance is of the form:-

\[ q_u = q_o N^*_q \]  \[2.2\]

where \( q_o \) = the stress acting in the plane of the pile tip.

The wide range of values for the bearing capacity factor \( N^*_q \) proposed by various authors based
on small scale model tests as well as several well documented full scale tests on piles and
piers has been presented by Vesic' (1963, 1970). His diagram is reproduced in figure 2.4.
FIG. 2.4  BEARING CAPACITY FACTORS FOR CIRCULAR DEEP FOUNDATIONS.
(After Vesic, 1967).

FIG. 2.5  JAKY'S "PRESSURE SULB"
(After Jaky, 1948).
Jaky (1948) defined the point resistance of a pile as "essentially the bearing capacity of a shallow pier foundation at very great depth" and that the friction acting on the pile shaft was due directly to earth pressure. Jaky developed an expression from which the dimensions of the "pressure bulb" about a pile could be determined.

With reference to figure 2.5, these dimensions are:

\[
M = D \left( \tan(45 + \frac{\phi}{2}) e^{\frac{\pi}{2} \tan \phi} + \frac{1}{2} \cos \phi e^{\frac{3\pi}{4} + \frac{\phi}{2} \tan \phi} \right) \tag{2.3}
\]

\[
m = \frac{D}{m} \left( \frac{\cos \phi}{\cos(45 + \frac{\phi}{2})} e^{\frac{\pi}{4} + \frac{\phi}{2} \tan \phi} \right) \tag{2.4}
\]

\[
B = D \left( \frac{\cos \phi}{\cos(45 + \frac{\phi}{2})} e^{\frac{3\pi}{4} + \frac{\phi}{2} \tan \phi} \right) \tag{2.5}
\]

These dimensions are very sensitive to the choice of the soil friction angle (\(\phi\)).

For example, from the triaxial test results reported in Appendix 5, the critical state friction angle (\(\phi_{cs}\)) is 32\(^{\circ}\), however, the peak mobilised friction angle (\(\phi_{p}\)) is approximately 44\(^{\circ}\) (figure A6.14, Appendix 5).

Substituting these friction angle values into equations 2.3 to 2.5 yields the "pressure bulb" dimensions of table 2.1.

**TABLE 2.1**

**PRESSURE BULB DIMENSIONS FOR VARIOUS VALUES OF \(\phi\)**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>(\phi_{cs} = 32^{\circ})</th>
<th>(\phi_{p} = 44.4^{\circ})</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>14.550</td>
<td>54.70</td>
</tr>
<tr>
<td>m</td>
<td>1.70</td>
<td>2.90</td>
</tr>
<tr>
<td>B</td>
<td>10.10</td>
<td>28.20</td>
</tr>
</tbody>
</table>

Gisbourne (1970) in a similar analysis obtained "pressure bulb" dimensions for a friction angle, for the sand used in his experiments, of 40.5\(^{\circ}\). This friction angle was deduced from direct shear tests. The dimensions obtained by Gisbourne are shown in table 2.2.

Also shown in this table are the "pressure bulb" dimensions obtained from Jaky's formulae for the same value of \(\phi\).
TABLE 2.2
PRESSURE BULB DIMENSIONS FOR
\( \phi = 40.5^\circ \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>340</td>
<td>370</td>
</tr>
<tr>
<td>( m )</td>
<td>2.00</td>
<td>4.50</td>
</tr>
<tr>
<td>( B )</td>
<td>16.70</td>
<td>12.00</td>
</tr>
</tbody>
</table>

The "pressure bulb" dimensions indicated in Table 2.2 infer that to develop a complete bearing bulb, it would be necessary to install the pile to an embedment depth \( \left( \frac{D}{2} \right) \) of at least 13 for a friction angle of \( 32^\circ \), or over 50 for a friction angle of \( 44^\circ \). The potential range of values suggested by the theory is thus large.

The "pressure bulb" concept is, as are all the bearing capacity theories, dependent upon the assumption that the bulb perimeter is in fact a slip surface with the soil mass within the perimeter being in a state of plastic deformation.

Even though the results of Gisbourne (1970) (See table 2.2) are in agreement with Jaky's considerations, his work is believed to suffer from the same physical disparities as that of Jaky. Essentially these are that the inferred fields of plastic deformation bear no relationship to the observed limits of deformation subsequently reported by the writer. Implicit in this statement is the premise that some straining or deformation has to occur to mobilise stress states in excess of the ambient stress existing in the soil.

Gisbourne actually measured displacement fields about a uniformly inserted penetrometer (in fact a model pile), using a technique similar to the stereo-photogrammetric technique discussed in Appendix 1. However, even though he uses the displacements so obtained to calculate strains and thus "confirm" his theoretical analysis, he has not in fact published any of these displacement or strain fields. The writer finds this surprising, especially in view of the major contribution to the understanding of the mechanics of pile-soil interaction that such a contribution would have made at that time (1970).

From the writer's observations, as reported later in this thesis it would appear that the limits proposed by Jaky (1948) and Gisbourne (1970) are likely to be too large.

Jaky (1948), further suggests that the pile tip resistance in dense sand, remains constant up to an embedment depth \( \left( \frac{D}{2} \right) \) of 42, beyond which it increases linearly with depth. His considerations are represented diagrammatically by ABC in figure 2.6. This is quite the reverse to observations reported by a number of authors since 1948, for example Meyerhof (1951). Such works have shown in fact that the pile tip resistance tends to a limiting value at an embedment depth of about 10 as indicated by DEF in figure 2.6.

Meyerhof (1951) extended his bearing capacity analysis for shallow footings to the deep foundation situation (i.e. the pile). Meyerhof's concepts and failure modes were not dissimilar to those proposed by Jaky, however, his analytical considerations were more
exhaustive. As commented by Whittaker (1970), Meyerhofer's theory provides a working basis for explaining some of the known phenomena associated with pile behaviour. Meyerhofer's considerations can be generally described with reference to figure 2.7:

(a) Below the base of the pile is a central zone, (ABC in figure 2.7), which remains in an elastic state of equilibrium and acts as part of the foundation (See also the results of the installation of a square ended model pile in Chapter 5).

(b) On each side of zone ABC are two plastic zones (ACD and BCE). These zones are zones of radial shear.

(c) The slip lines so described revert to the pile shaft through zones of plane shear, as defined by ADF and BEG in figure 2.7.

As the base of a displacement pile of diameter D is forced into the sand under load, the progression of the failure zone is assumed to cause shearing within a cylinder of diameter 'a'. Surrounding this shear zone it is further assumed that a cylindrical zone of compaction of diameter 'b' is set-up. These zones are indicated in figure 2.8. Typical dimensions of these assumed zones are indicated in table 2.3.

**TABLE 2.3**

**TYPICAL DIMENSIONS OF ASSUMED SHEAR ZONES**

<table>
<thead>
<tr>
<th>SOIL STATE</th>
<th>'a' (SHEAR ZONE)</th>
<th>'b' (COMPACTION ZONE)</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose</td>
<td>40</td>
<td>6 to 80</td>
<td>Meyerhof (1959)</td>
</tr>
<tr>
<td>Dense</td>
<td>30</td>
<td>50</td>
<td>Kerisel (1961)</td>
</tr>
</tbody>
</table>

The dimensions for dense sand do not appear to agree with the observations of tables 3.1, 3.2 and 3.3, (Chapter 3).

Jaky's (1948) analysis was based on considerations of a weightless soil. Meyerhof (1951) took the soil mass into account and combined two separate failure "bulbs" to produce a final failure surface with dimensions which are difficult to define. Gisbourne (1970) states that Meyerhof's failure bulbs have approximately the same widths as proposed by Jaky, but with a lesser depth of penetration required for complete mobilisation of the "pressure bulb". On the other hand Meyerhof, as indicated by figure 2.8, has placed no fixed dimensions on the pressure bulb, but has instead appeared to arbitrarily fit the shape of the surface to semi-empirical observations (Meyerhof, 1959).

Meyerhof's bearing capacity theory is essentially an extension of Terzaghi's (1943) proposal, as indicated in figure 2.9.
FIG. 2.6 SUGGESTED VARIATIONS IN PILE TIP RESISTANCE IN DENSE SAND. (Diagrammatic).

FIG. 2.7 THE ZONES OF SHEAR AROUND THE BASE OF A PILE ACCORDING TO MEYERHOF.
Terzaghi's bearing capacity relationship is represented by the general expression:

\[ q_u = C N_c + q_o N_q + \gamma \frac{B}{2} N_\gamma \]

where \( q_u \) = the ultimate unit bearing capacity at the pile tip
\( C \) = cohesion
\( q_o \) = the normal pressure on the shear plane at the level of the base of the foundation
\( \gamma \) = the unit density of the soil (= \( \rho g \))
\( B \) = the foundation width, which for the pile situation is usually the pile "diameter" \( D \).
\( N_c, N_q \) and \( N_\gamma \) = the associated bearing capacity factors.

In cohesionless soils this general equation reduces to:

\[ q_u = q_o N_q + \gamma \frac{B}{2} N_\gamma \]

Terzaghi's original considerations assumed that the yield surface followed the same profile at depth as for a shallow footing, the additional load contribution with depth, in addition to the increasing value of \( q_o \), coming from shear on the surfaces of a right cylinder as indicated in figure 2.9.

The developments made by Jaky (1948) and Meyerhof (1951) were to assume that the yield surface reverted to the shaft at some distance above the pile tip. (See figure 2.5), however, as indicated earlier, this is not kinematically admissible.

The bearing capacity factors derived for the pile situation are semi-empirical for a number of reasons:

(a) The theory is based on considerations of rigid strip footings and subsequently modified in a semi-empirical manner to accommodate circular and both square and rectangular foundations.

(b) The compressibility of the soil is usually taken into account by an empirical alteration to the assessed shear strength of the soil. Meyerhof (1959), however, proposed further empirical modifications to the assessment of the bearing capacity parameters based on the results of penetrometer tests as discussed in a later section.

(c) The influence of the method of pile installation is assessed based on empirical evidence (See also (b) above).

(d) For any degree of reliability in the assessment of the load carrying capacity of driven piles in cohesionless soils, correlations should be drawn with cone penetration tests. Such correlations are discussed subsequently in section 2.6.7.
FIG. 2.8 THE ZONES OF SHEARING AND COMPACTION ASSUMED AROUND A DRIVEN PILE. (After Whittaker, 1970).

FIG. 2.9 FAILURE ZONES AS CONSIDERED BY TERZAGHI (1943) AND MEYERHOF (1951).
Since the basis of the "bearing capacity" theory is that the theoretical movements of the material in the plastic zones is parallel to the failure surfaces, Meyerhof considered that with increasing depth of pile penetration, the soil movement would tend to change from a general downward and outward movement to an upward one, which for a deep pile would be practically vertical. He believed that soil movements towards the shaft would in practice be unlikely.

The movements of the soil towards the shaft has in fact been observed by the writer at pile embedment depths \( \frac{D}{b} \) greater than 6. (See figures 3.50, Chapter 3).

The same criticisms made with respect to Jaky (1948) and Gisbourne (1970) can, however, be levelled at Meyerhof's considerations. Nevertheless such semi-empirical considerations produce bearing capacity values that do bear some relationship to those obtained from full scale pile tests, thus in the absence of more precise analytical methods, such methods provide a useful tool to the designer.

Meyerhof's (1951) test results from model driven piles in dense sand were found to generally exceed the theoretically determined capacities.

Rebazantzev and Yaroshenko proposed a further solution for determining the ultimate bearing capacity of piles in cohesionless soils. Their solution is similar to that proposed by Terzaghi (1943). (See figure 2.9) and is based upon the slip line analysis of Sokolovsky (1954). It would appear that they have attempted to model the shape of experimentally obtained "slip lines".

Their solution is of the form:-

\[ q_n = A_n q_B + \gamma n q_H \]  

which can be represented in a form similar to equation 2.7, thus:-

\[ q_u = \gamma q N_q + \gamma n N_H \]  

The author's correlations with experimental results appear good, however, they remain subject to the criticism that the nature of their testing procedure and the full state of stress initially existing in the soil are not defined (See Chapter 5).

Chronologically, Meyerhof (1959), as discussed in a later section, made further empirical allowances in his semi-empirical bearing capacity formulae for the effects of soil compaction by piles installed by driving in sand.

Of significance is that the plastically deforming zones, reported by Meyerhof, of up to 12D total diameter about the pile, and extending up to 5D beneath the pile tip, are surrounded by an elastic zone with limits of approximately 17D diameter about the pile and extending to approximately 12D beneath the pile tip.
Vesic' (1963) conducted tests on buried foundations in sands. However, whilst observing the considerable differences in load carrying capacity obtained between model piles in sand placed by showering and those in soil compacted by vibration, failed to comment. (See Chapter 5).

Bishop, Hill and Mott (1945) considered the problem of the expansion of a spherical cavity inside an infinite mass of an ideal solid, enabling the bearing capacity to be expressed in a form analogous to equation 2.6 excepting that the soil was considered to be weightless (i.e. the $y$-term was omitted).

For an infinite mass of soil, or at very great depths, the cavity expansion solutions indicate that the bearing capacity of a pile tip should be practically independent of its size and be proportional to the overburden pressure (similar conclusions can be drawn by the writer, Chapter 5).

Based on these conclusions and supported by some experimental evidence it has been generally accepted (for example Vesic, 1963), that the point bearing capacity of piled foundations in sands should be equal to the point resistance at depth of a cone penetrometer in applying the bearing capacity theories to piles in cohesionless soils. Vesic' commented:-

"SOME EXPERIENCES WERE NOT ENCOURAGING. FOR INSTANCE SCALE EFFECTS OF A NATURE OPPOSITE TO THOSE PREDICTED BY THE THEORIES HAVE BEEN REPORTED. OBSERVATIONS OF SHEAR PATTERNS IN SAND AROUND DEEP FOUNDATIONS [PRESUMABLY WITH BURIED PILES, AS WERE VESIC'S TESTS] SHOWED FAILURE SURFACES TO BE LOCALISED TO THE IMMEDIATE VICINITY OF THE FOUNDATION BASE. TO THE AUTHOR'S KNOWLEDGE [1963] NOT A SINGLE TEST EVER INDICATED FAILURE SURFACES REVERTING TO THE SHAFT".

It is the writer's contention that the development of shear surfaces about the pile tip as reported by Vesic' are a direct consequence of the method of pile installation employed, (i.e. buried rectangular foundations).

It is felt that these shear patterns are in fact the bearing capacity failure surfaces normally developed in association with shallow strip foundations. In the buried pile situation, (where the soil is placed about the pile), apart from an increase in the surcharge pressure, the soil is not aware that it is supporting a "deep" foundation. Vesic's shear patterns could be further influenced by the fact that the model foundations used were rectangular and thus analogous to a strip footing.

The shear patterns observed are thus unrealistic in as much that in reality piles are not generally installed in the manner adopted by Vesic'.

As a result of his observations however, he proposed the failure pattern indicated in figure 2.10. Vesic, as a result of his studies, concluded that:-
(a) At embedment depths \(\left( \frac{d}{D} \right) > 15\), pile tip and shaft unit resistances remained constant with depth.

(b) The bearing capacity factor \(N_q\), the ratio of base resistance to vertical stress, is practically independent of foundation size and is a function of the relative density (and thus the angle of internal friction) of the soil.

(c) The fundamental fallacy of most conventional bearing capacity analyses for deep foundations in sands is in the assumption that the stress acting in the plane of the pile tip \(n\) in the bearing capacity equation is equal to the initial overburden stress at the level of the foundation base. As discussed subsequently, however, Meyerhof (1959) attempted to meet this criticism.

Vesic (1964) reported the results of a number of field and laboratory tests in sand involving 100 mm diameter driven steel piles. In discussing experimental findings reported to 1964, commented:

"MANY INVESTIGATIONS HAVE BEEN PERFORMED ON VERY SMALL MODEL PILES WITH PILE DIAMETERS NOT EXCEEDING 1 INCH (25 mm). QUALITATIVE DATA FROM SUCH INVESTIGATIONS CANNOT BE DIRECTLY RELATED TO THE PERFORMANCE OF ACTUAL SIZE PILES BECAUSE OF UNCLARIFIED SCALE EFFECTS. REPORTS OF AGREEMENT [BETWEEN] POINT RESISTANCE OF PILES AND PENETROMETERS, WHICH SUPPORT THE EXISTING [1964] THEORIES OF POINT ASSISTANCE ....... HAVE BEEN DISTURBED BY EVER INCREASING EVIDENCE OF UNEXPLAINED SCALE EFFECTS".

Vesic suggests that a further fundamental fallacy of pile bearing capacity computations may lie in the assumption that the stress conditions at failure around the pile are the same as those in an infinite soil mass prior to pile installation and loading. Such conditions are in fact only likely to apply in the case of bored piles.

Vesic (1964) reaffirmed that the linear increase in bearing capacity with depth only occurs up to embedment depths \(\left( \frac{d}{D} \right)\) of about 5; asymptoting finally to constant values at depths of about 30 for driven piles in dense sands. (cf. 20 for buried piles). He found that the ultimate point loads of driven piles were reached at smaller settlements than for buried piles. He also concluded that the point bearing capacities of driven piles are higher than those of bored piles only in as much as the sand density is increased by pile driving.

In 1964 Robinsky and Morrison (1964) reported on their study of the displacement and compaction of sand around model piles during installation by "pushing". Their results are discussed in detail in Chapters 3, 4 and 5.

The significant point demonstrated experimentally by the authors was that the soil conditions surrounding a driven displacement pile are not the same as those that existed before the commencement of pile installation when the soil properties were being studied to establish design criteria. The factors on which the pile design should be based, clearly depend upon the changed properties of the soil existing after the pile has been installed.
FIG. 2.10 "PRESSURE FIELD" PROPOSED BY VESIć (1963).
As discussed in a later section Meyerhof (1959) and Mordlund (1963) attempted to account for these changes in an empirical manner.

The generally conceived understanding of the soil deformations required by the ultimate bearing capacity theories is that shear surfaces are developed (Terzaghi, 1943; Jaky, 1948; Meyerhof, 1951; Beratzentzav et al., 1957 and 1961; Bisbourne, 1970).

Significantly Robinsky and Morrison failed to observe the development of such shear surfaces. In support of the author's observations, the writer also failed to identify the existence of shear failure planes, even though the piles were installed by driving in the writer's case.

Nurgumoğlu and Mitchell (1975) in studying the cone penetration test also postulated that above an undefined critical depth the penetration resistance was controlled by the failure of the soil in shear, however, their photographic results appear to indicate shear zones only when the device being penetrated was wedge shaped. (i.e. analogous to a strip footing). No shear surfaces were observed with circular penetrometers. In addition the authors reported no evidence of shear planes with wedge shaped penetrometers after only a few D penetration. These observations support the writer's earlier contentions with respect to Vesic's (1963) results.

Clearly then the problem of determining the axial load capacity of piles in sand has had to be considerably oversimplified due to the difficulty in assessing the effects of the various factors.

Robinsky and Morrison (1964) observed that

"THE DIRECT APPLICATION OF CONVENTIONAL EARTH PRESSURE THEORIES TO THE BEARING CAPACITY OF FRICTION PILES IN SANDS HAS PERHAPS ACCRUED WITHOUT DUE CONSIDERATION OF FACTORS SUCH AS THE LACK OF DEFINITE SHEAR PLANES, AND ALSO THE PRESENCE OF ARCHING".

This concept of "arching" along the pile shaft is discussed subsequently.

Broms (1966) summarised the methods of calculating the ultimate bearing capacity of axially loaded piles. A comment made by Broms which is considered by the writer to be of equal significance today (1979), was:

"'THERE DOES NOT EXIST TODAY (1966) A RELIABLE GENERAL METHOD TO DETERMINE, UNDER ALL CONDITIONS, THE BEARING CAPACITY OF PILES. GENERALLY CONSIDERABLE UNCERTAINTY EXISTS ABOUT THE ACTUAL BEARING CAPACITY OF A PILE WHEN LOAD TESTS HAVE NOT BEEN CARRIED OUT".

All of the semi-empirical methods of considering the axial load carrying capacities of axially loaded piles tacitly assume that both the shaft friction resistance and point resistance can be determined separately and are independent of each other. From consideration of the literature, Broms has reported that very small axial deformations are generally necessary to mobilise completely the shaft friction resistance. The magnitude of this deformation has, from tests on model piles ranging up to 180 mm diameter (7 inches), been
suggested by Vesic (1964) as being about 9 mm, and independent of both sand density and foundation dimensions.

In contrast, relatively large deformations are required to mobilise the maximum point resistance of piles driven into cohesionless soils and generally appear to increase in proportion to the shaft diameter. Thus Broms contends that the largest part of the applied axial pile load is carried by skin friction at low applied loads, while at high load levels, the largest part is carried by pile tip resistance. This assessment is at variance with the observations made by Robinsky, Sagar and Morrison (1964) who have indicated that the distribution of load between pile shaft and tip is very dependent upon the method of pile installation.

Broms suggests that in assessing the resistance of "relatively short" straight sided piles, it is generally sufficient to estimate only the value of the end bearing resistance as the shaft friction resistance for such piles is "often less than 30-40% of the ultimate load carrying capacity of the pile. This suggestion appears to the writer to be conservative.

With reference to the bearing capacity formula of equation 2.7, Broms suggests that, as the pile width (D) is usually very small compared to the embedded length of the pile (L), this equation can be further rationalised to:

\[ q_u = q_0 N_q = \gamma L N_q \]  \hspace{1cm} 2.10

Because field and laboratory tests appear to indicate that Meyerhofer's values for \( N_q \) actually over-estimate the pile tip load carrying capacity, Broms recommends that the bearing capacity values obtained by Berezantzev, Khristoforov and Golubkov (1961) in view of their apparent better agreement with load tests, be used. Tomlinson (1977), however, found that pile tip loads were still considerably over-estimated using these latter values.

In a lecture presented in 1967, Vesic (1967) made the observations that most ultimate load theories related to deep foundations assume that: -

(a) The foundation is placed into the ground without changing the initial stress distribution within the soil mass.
(b) Volume changes in the soil due to either shear or compression are negligible.
(c) The shear strength characteristics of a particular soil are defined by a single straight line Mohr envelope (See also Appendix 5).
(d) The shear strength at any point on this envelope is independent of strain.
(e) Elastic deformations are negligible compared to plastic deformations.

Possibly apart from point (e) these various assumptions are clearly questionable.

Vesic (1967) also comments, that as long as the major consequences of any theory agree with observations, it is reasonable to retain and use such observations unaltered.

From a critical appraisal of the various theories, he deduces that:
"...THERE IS SUFFICIENT EVIDENCE TO BELIEVE THAT THE EXPRESSIONS FOR
POINT AND SKIN RESISTANCE CAN YIELD RESULTS WHICH ARE IN OVERALL AGREEMENT
WITH EXPERIENCE IF \( q_o \) IS TAKEN EQUAL TO \( q_f \), THE EFFECTIVE VERTICAL STRESS AT
FAILURE AT THE LEVEL OF THE FOUNDATION BASE) AND IS THUS TAKEN ACCORDING TO
ITS TRUE MEANING AND NOT NECESSARILY EQUAL TO THE OVERBURDENED PRESSURE \( \sigma = \gamma L \)
AS RECOMMENDED BY BROMS (1966). (SEE EQUATION 2.10)…..[A] FALLACY IN THE
ANALYSES OF DEEP FOUNDATIONS IS THE INADMISSIBLE USE OF THE OVERBURDENED PRESSURE
\( \sigma \), INSTEAD OF \( q_f \).

However, given that this is a reasonable statement and that the various theories
can be found admissible, the problem remains in assessing the appropriate values of \( q_f \).
Today (1979), this still cannot be done.

The writer is thus inclined to reiterate the conclusions drawn by Vesic in 1967:

(1) In spite of significant progress made during recent years in understanding the
bearing capacity phenomenon around deep foundations in sand, there is still
not a satisfactory theory that could be recommended without reservation
to the practising engineer for the analysis of ultimate loads.

(2) Such a theory must include at least the following parameters:-
(a) foundation shape, relative depth and method of construction
(b) the shear strength of the sand
(c) the relative compressibility of the sand
(d) the volume change characteristics of the sand.

(3) The effect of scale must be fully recognised.

Given the state of the art then, this all means simply that only full-scale tests in
absolutely controlled conditions can yield conclusive data than can readily
be applied by the designer. As a consequence then, if a particular foundation design is
critical, for whatever reason, the designer should give serious consideration to
correlating his analysis with full scale tests. In so doing many of the imponderables
would be accommodated; i.e. variations in soil stratum, method of pile installation, pile-
soil interaction characteristics etc.

Whittaker (1970), in commenting that Terzaghi’s (1943) suggestion that a surcharge
effect exists due to frictional resistance on the pile shaft (i.e. \( q_o > q \)) has been
responsible for a number of theoretical solutions to this particular aspect of the pile
problem, observed that none of those put forward have come into practical use. The
general non-recognition of such theories is due mainly to a lack of adequate knowledge
of the nature of the stress field around a pile, and as a consequence, the reliance
on sometimes questionable assumptions. In addition, most theories, as has already been
indicated, do not take into account the volume changes in the soil resulting from pile
driving. Whittaker believes that if agreement is found between such a theory and the
results of a loading test, then either the effect of compaction is negligible, or the theory is in error by a compensatory amount.

Selby (1970) graphically illustrates the difficulties involved in attempting to assess axial load capacities of piles in cohesionless soils by means of the conventional methods (i.e. Meyerhof, 1951, Terzaghi, 1943, the Hiley pile driving formula), using soil properties determined during a conventional foundation investigation. The resulting wide range of calculated ultimate loads compared with actual measured loads are shown in Table 2.4 for various types of test piles driven into silty sands.

**TABLE 2.4**

PILE CAPACITIES DETERMINED BY VARIOUS METHODS

<table>
<thead>
<tr>
<th>PILE TYPE</th>
<th>LENGTH (m)</th>
<th>ULTIMATE AXIAL LOAD (tonnes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Hiley</td>
</tr>
<tr>
<td>Timber</td>
<td>14.2</td>
<td>157.5</td>
</tr>
<tr>
<td>Steel Tube</td>
<td>15.4</td>
<td>98.6</td>
</tr>
<tr>
<td>Steel Tube</td>
<td>22.4</td>
<td>139.3</td>
</tr>
<tr>
<td>Steel H</td>
<td>22.4</td>
<td>110.8</td>
</tr>
<tr>
<td>Steel H</td>
<td>15.4</td>
<td>89.5</td>
</tr>
</tbody>
</table>

In the case of the timber pile, the actual axial load carrying capacity was overestimated by a factor of 1.9 using the Hiley formula, 2.0 using Meyerhof, and 1.5 using Terzaghi (i.e. a range of 150 - 200%). In assessing the actual ultimate load, the criterion was similar to that suggested by Slack and Walker (1970) rather than as usually defined in the literature, (i.e. "the load beyond which the pile will begin to break into the ground" or "the point where the rate of change of load tends to infinity").

A significant and questionable feature of the present methods of ultimate axial pile load analysis is that, apart from the empirical provisions of Meyerhof (1959) and Nordlund (1963) as discussed subsequently, the methods of assigning axial loads to piles, putting aside the designer's skill, experience and intuition, are identical irrespective of how the pile is installed.
(2) The Assessment of Pile Shaft Capacities

As indicated by Vesic' (1964) pile shaft resistances have traditionally been considered analogous to the sliding of a rigid body in contact with the soil. Consequently the shaft resistance of an axially loaded pile is assumed to be proportional to the average soil overburden pressure but lying between the active and passive states as indicated by Meyerhof (1951).

The relationship for unit shaft resistance is usually expressed in the form:-

\[ f_s = K_s q_s \tan \delta \]

where \( f_s \) = the unit pile shaft resistance
\( K_s \) = the coefficient of lateral shaft pressure
\( q_s \) = the pressure acting in the soil in the vicinity of the pile
\( \tan \delta \) = the coefficient of friction between pile and soil.

The difficulty of assessing the ultimate axial load carrying capacity of a pile is thus compounded because, to enable the contribution made by the shaft to be assessed with any degree of confidence, both \( K_s \) and \( \tan \delta \) need to be determined experimentally and preferably from full scale pile tests.

Another factor that could affect the correlation between field and laboratory tests is the point on the load displacement curve where ultimate conditions are presumed to prevail. For example, the results reported by Vesic' (1964) are based on the definition that the ultimate loads are those at which the rate of displacement first reaches its maximum \( \left( \text{i.e. } \frac{d(\text{load})}{d(\text{strain})} \right) \)

This is similar to the technique proposed by Slack and Walker (1970) wherein the load-deflection results from a pile load test are plotted on logarithmic axes. The ultimate load is then defined as that associated with the change in the rate of displacement as discussed in section 2.6.11 and Appendix 5.

Of significance is the observation made by Vesic that the magnitude of shaft displacements necessary to mobilise the ultimate shaft resistances in the soil appeared to be independent of the pile dimensions and the initial density of the sand and were less than 0.35 inches (9 mm).

Vesic observed that both the bearing capacity factors and shaft resistances were the same for both driven and buried piles providing the driven piles were analysed using the mean soil density from those existing prior to and after pile driving. Vesic's results are indicated in figure 2.11.

Meyerhof's (1951) theoretical considerations showed that for axially loaded pile foundations, in cohesionless materials in particular, the contribution of shaft friction to the total ultimate load carrying capacity of the pile was important. He also showed that the earth pressure coefficient \( K_s \) acting on the shaft within the assumed failure zone (Jaky's "bulge of pressure") had an important influence on the theoretical bearing capacity factors derived for oiled foundations in cohesionless soils. Meyerhof placed \( K_s \) between the Rankine active and passive pressure coefficients, i.e.:-
FIG. 2.11 SHAFT RESISTANCES AND BEARING CAPACITY FACTORS FOR BURIED AND DrIVEN PILES IN SAND (After Vesić, 1964).

FIG. 2.12 AVERAGE UNIT SHAFT FRICTION ON DRIVEN PILES IN COHESIONLESS SOILS FOR $\frac{D}{L} > 20$. (After Tomlinson, 1977).
The further stated that "this coefficient depends mainly on the density, strength and deformation characteristics [of the soil], ... the method of [pile] installation ... and at present [1951] can only be obtained from the results of field tests." It is significant that for design purposes in 1979 this statement remains valid.

As discussed in a later section Nordlund (1963) attempted to provide for the effects of pile installation. Tomlinson (1977) discusses Nordlund's empirical method in some length and also presents worked examples for typical pile situations.

It is of interest that Tomlinson (1977) considers Nordlund's method to be invalid beyond a pile embedment depth (ϕ) of 20. Beyond this depth he instead presents his own correlations for shaft friction derived from the results of a number of published and unpublished pile loading tests. Tomlinson's recommendations are shown in figure 2.12.

Combining these recommended shaft friction values with the bearing capacity factors derived by Rereigmat, Khristoforov, and Golukov (1961), Tomlinson found that the base resistances and thus the pile ultimate load carrying capacities were over-estimated as indicated in figure 2.13.

Robinsky, Sagar and Morrison (1964) installed instrumented model piles by pushing, with a view to determining the lateral (i.e. shaft friction) and end bearing contributions to the ultimate load carrying capacity of the pile at various stages of installation. That the amount of load transferred to the pile wall varies in a complex manner throughout installation is indicated in figure 2.14. The beneficial effect of rough pile walls is indicated in figure 2.15. Indications of the effect of pile roughness are also presented in Chapter 5.

An interesting observation made by the authors was that the ultimate axial load carrying capacity of piles was found to vary less than 5% between those installed by pushing and those installed by driving. The distribution of load about the pile was however markedly affected. For example, they observed that pile installation by driving increased the proportion of load carried by the shaft by up to 20%, simultaneously reducing the pile tip contribution by the same amount. It is curious that contrary to the theory of Terzaghi and Peck (1948), wherein it was suggested that the capacity of axially loaded piles varied directly as the square of the depth, Robinsky, Sagar and Morrison, as indicated by figure 2.16, found in their model tests that the axial load capacity for a parallel sided pile varied directly with embedded pile length.

Meyerhof (1951) proposed values for the coefficient of shaft friction (κ_s) of 0.5 for loose sands and 1.0 for dense, thus inferring that the coefficient is independent of pile type and surface roughness. Because Meyerhofs recommended values result in calculated shaft resistances that are less than those obtained from test results, Brooms recommends values of κ_s.
FIG. 2.13 PREDICTED AGAINST OBSERVED AXIAL LOAD CAPACITY OF PILES DRIVEN INTO COHESIONLESS SOILS. (After Tomlinson, 1977).

FIG. 2.14 VARIATION IN SHAFT LOAD TRANSFER DURING DRIVING (After Robinsky, Sagar, and Morrison, 1964).
for concrete and timber piles as indicated in table 2.5.

<table>
<thead>
<tr>
<th>Pile Type</th>
<th>Soil Density</th>
<th>TABLE 2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loose</td>
<td>Dense</td>
</tr>
<tr>
<td>Concrete</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Timber</td>
<td>1.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

It is interesting to note, however, that Brinch-Hansen (1961) suggests values for \( k_s \) for piles in the range \( \frac{1}{10} \) of 0 to 20 of about 2 for loose soil (\( \theta \) about 20°) to about 100 for dense soils (\( \theta \) about 45°).

This considerable range of values serves only to reinforce the complexity of the pile-soil interaction problem and the fact that it is extremely difficult (if at all desirable) to generalise such effects.

Broms further reports that experimentally determined values of the pile-soil friction angle were found not to be influenced by the relative density of the surrounding soil. His recommended values are given in table 2.6, in which \( \theta' \) is the effective friction angle.

<table>
<thead>
<tr>
<th>Pile Type</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>3/4 ( \theta' )</td>
</tr>
<tr>
<td>Timber</td>
<td>2/3 ( \theta' )</td>
</tr>
</tbody>
</table>

As was the case in discussing Jaky’s (1948) theoretical considerations (Table 2.1), the writer again feels compelled to question exactly which friction angle is inferred in the values of table 2.6 (i.e. \( \theta_{cs} \) or \( \theta_0 \)).

Stroud (1971) has shown that the soil friction angle (\( \theta \)) can vary as much as 15° depending on soil density. However, as indicated in Chapter 4, the writer has shown that contours of cumulative shear strain tend to become parallel to the pile shaft with increased embedment depth (\( \frac{h}{D} \)). The magnitudes of the shear strains so achieved are associated with relatively constant values of the soil friction angle (i.e. they lie beyond point E on the stress-strain curve of figure A5.17, Appendix 5). Thus the apparent independence of the soil friction angle (\( \theta \)) from variations in density along the pile shaft as suggested by table 2.6,
FIG. 2.15 EFFECT OF SHAFT ROUGHNESS ON SHAFT LOAD TRANSFER.
(After Robinsky, Sagar and Morrison, 1964).

FIG. 2.16 VARIATION OF ULTIMATE AXIAL LOAD CAPACITY WITH EMBEDDED PILE VOLUME
(After Robinsky, Sagar and Morrison, 1964).
would appear reasonable.

According to Broms, however, his recommended values of $\phi$ are still likely to produce shaft resistances somewhat lower than actual values.

A number of authors, for example Hunter and Davisson (1969), Vesic' (1964 and 1970) and Robinsky and Morrison (1964), have shown that the distribution of pile-shaft friction loads indicate a reduction in mobilised friction towards the pile tip. Such trends are not accounted for by classical bearing capacity theories and thus identify a further inconsistency between theoretical considerations and recorded observations.

Vesic' (1970) reports that this low value in shaft friction immediately about a pile point in sand has been attributed to "arching" or stress-relief in a zone extending to perhaps 3D above the pile point. In addition to this reduction in friction adjacent to the pile point, Vesic' also reported a progressive increase in pile length (L) necessary to mobilise the full shaft friction, with increasing depth of pile embedment ($\frac{L}{D}$) as indicated in figure 2.17, in which the peak shaft friction can be seen to be mobilised at about 25% of the embedment length for an $\frac{L}{D}$ of 6.7. This proportion progressively increases until at an embedment depth of 33, the proportion is 86% of the embedment length.

Vesic' explains this phenomenon by a progressive loosening of the upper soil mass caused by continued pile penetration. As indicated by the volumetric strain results of Chapter 4 expansion (and thus loosening) of the soil mass does occur adjacent to the pile shaft. Whether this dilation is in fact responsible for Vesic' observed variation in load transfer is not apparent. Nevertheless, it would appear that the mechanics of load transfer along the pile shaft are considerably more complicated than presently envisaged. The parameters involved appear difficult to reliably assess in numerical terms.

A further feature identified by Hunter and Davisson (1969) which has tended to be overlooked in drawing correlations between model pile and full scale tests is that quite significant residual stresses (and thus "loads") are developed in piles installed by driving. Whilst these residual loads do not appear to affect the total measured load carrying capacity of a pile, they have been shown to affect the distribution of load between pile and shaft by as much as 70%.

Vesic' (1970) reported the development of coefficients of shaft friction ($K_s$) for full size piles in dense sand of the order of the active Rankine state of stress ($K_A$). It will be recalled that Meyerhof (1951) placed $K_s$ between $K_A$ and $K_p$ (equation 2.12). Vesic has also indicated that various authors have shown that a substantial portion of a pile load can be transmitted along the shaft. In the initial stages of loading practically the entire pile load has been observed to be taken by shaft friction, however, as the load on the pile increased towards ultimate, a greater proportion was observed to be carried by the pile tip. As ultimate load conditions were approached, the shaft load tended to remain constant while the tip load continued to increase.
FIG. 2.17 VARIATION OF SHAFT LOAD TRANSFER WITH INCREASING EMBEDMENT DEPTH IN SAND (After Vesić, 1970).
From the evidence then, it would appear that an adequately designed pile must always mobilise full friction along its shaft much earlier than it can mobilise its ultimate tip resistance. Because shaft resistance is frictional in nature it would then suggest that the rougher the pile the greater the zone of soil mobilised, (See also Chapter 5), thus the greater the initial axial load capacity of the pile.

Hanna and Tan (1973) conducted a series of model pile tests with piles ranging up to 38 mm diameter installed by burying in dense dry sands. This unrealistic method of pile installation was adopted by the authors to focus attention on the general mechanics of the pile problem. The authors comment that the mechanism of load development observed was not in agreement with any of the standard bearing capacity theories.

In the preceding review it has been well documented that the process of pile construction or installation causes major change in the stress-state and density of the soil mass in the vicinity of the pile and also at the pile-soil interface. A quantitative assessment of strain changes (and by inference, the stress changes) in the soil resulting from installation by driving is presented in Chapter 4. These changes are due to the obviously very complex and poorly understood loading to which the soil near the pile is subjected. Clearly then the soil is not stress and strain free after pile installation, however, as shown by Hanna and Tan, (and also as earlier reported by Hunter and Davisson, 1969) neither is the pile, even though it may be subject to no external load.

Their tests were directed towards obtaining a better understanding of the mechanism by which a pile develops resistance to applied load, arguing that until this mechanism can be reliably described, so real advances in pile test interpretation and behaviour prediction would be limited. Unfortunately the authors have considered the pile only rather than the soil, even though their general hypotheses was that the initial stress-state in the vicinity of the pile governed the shape and magnitude of the subsequent loading diagram. It is also the writer's view that the answer to the extremely complex problem of pile-soil interaction lies in understanding the deformation mechanisms in the soil.

During pile installation by driving the pile will be subjected to a large number of load - unload cycles, thus the position on the stress-strain curve from which the pile is loaded in a subsequent static load test is unknown. The importance of residual stresses locked into the pile due to the installation process has been well documented by Hanna and Tan as indicated in figure 2.18. Figure 2.18(a) shows typical shear stresses at the pile-soil interface with allowance made for the residual stresses in the pile. Figure 2.18(b) presents the same typical stress values but with the residual stresses in the pile neglected. Clearly the presence of residual stresses can have a significant effect on the apparent mechanism of pile-load-transfer. Hunter and Davisson (1969) made similar observations.

The experimental work reported by Hanna and Tan has shown that the distribution of load between end bearing and shaft resistance can vary widely depending on the allowance if any,
FIG. 2.18 THE SIGNIFICANCE OF RESIDUAL PILE STRESSES DUE TO THE INSTALLATION PROCESS (After Hanna and Tan, 1973).

FIG. 2.19 NATURE OF DEFORMATIONS REPORTED BY DEREZANTZEV AND YAROSHENKO (1957).
for the initial residual stress-state along the length of the pile. They have also
shown that classical soil mechanics assumptions which do not allow for the shear stresses on
the pile shaft, the change in soil state adjacent to the pile shaft, and the soil volume
changes associated with sand straining, are very unreal and completely misleading, not only
in the magnitudes of load so determined, but also in the mechanical interpretation of the
processes involved.

Tomlinson (1977) discusses some further aspects of the effects of pile installation
by driving on the development of pile shaft friction.

2.6.3 The Effects of Pile Installation by Driving

Piles installed by driving must necessarily displace a volume of soil, which, in an
incompressible soil, would equal that of the installed pile. In a dilatant soil the accommodation of
the embedded pile volume is complicated by volume changes occurring within the soil mass. In general
then, displacement piles will affect the properties of the soil mass such that they are no longer
representative of those existing prior to the commencement of driving.

Terzaghi (1943) recognised that the resistance of a pile to load depended not only on the
soil, but also on the manner in which the pile was installed. He stated:-

"OUR KNOWLEDGE OF THE INFLUENCE OF THE METHOD OF INSTALLING .... PILES ....
IS STILL RUDIMENTARY, AND THE PROSPECTS FOR EVALUATING THIS INFLUENCE BY
THEORY ARE VERY SLIGHT".

It is significant to note that in 1977, Tomlinson expounds much the same view, as noted
earlier.

Whilst some advances have been made since 1943, there are still no reliable quantitative
means of assessing the influence of pile driving.

Meyerhof (1951), in discussing his bearing capacity theory, observed that for driven
foundations in cohesionless soils the "actual" bearing capacities were less than estimated from his
theoretical considerations, and considered this due to the influence of material compressibility being
offset by "the increased density after installation".

Figure 2.19 shows observations made by Revezantzev and Yaroshenko (1957). They used
still and cine photography in conjunction with boxes with glass walls to investigate the deformations
of sands under foundations. The "core" shown in figure 2.19 is discussed later.

Clearly large deformations are occurring in the sand mass which must result in associated
changes in strain, and by inference stress, thus affecting the resistance to loading. The departure
of the deformation field indicated in figure 2.19 from coaxiality with the pile is probably due to
instability.

Chronologically Meyerhof (1959) made allowances in his semi-empirical bearing capacity
formula for the effects of compaction during installation of piles in sand by driving. Nordlund (1963)
also made allowances of a purely empirical nature.

These works are discussed in more detail in section 2.8.

Acknowledgement of the influence pile driving has on the properties of the soil mass was also made by Vésic (1963).

Vesic made some provision for density change by employing as the design density the mean of those existing before and after pile installation. He concluded that......

"THE POINT BEARING CAPACITY OF DRIVEN PILES ARE HIGHER THAN THOSE OF BORED PILES ONLY IN AS MUCH AS THE SAND DENSITY IS INCREASED BY PILE DRIVING".

Robinsky and Morrison (1964) employed radiography techniques to study the displacement and compaction of sand around model piles. They observed that.......

"SOIL PROPERTIES [AFTER PILES HAVE BEEN DRIVEN] ARE EXTREMELY DIFFICULT TO DETERMINE AND EVEN MORE DIFFICULT TO PREDICT PRIOR TO PILE DRIVING - THUS VALIDITY OF THE LATERAL EARTH PRESSURE APPROACH TO DESIGN IS CONTROVERSIAL".

It is of interest to note that Robinsky, Sagar and Morrison (1964) and Robinsky and Cragg (1973), observed that axial load capacity of piles varied directly with embedded volume; and thus by inference, the increased density about the pile due to the process of installation. Broms (1966) observed that the bearing capacity of piles driven into cohesionless soil depends primarily on the relative density of the soil, which he presumes is increased close to the pile during driving.

Vésic (1967) emphasises that one of the fundamental assumptions associated with ultimate pile load theories is that the foundation is placed into the ground without changing the pattern of the initial stress distribution in the soil mass. He later states, (1970), that for a proper understanding of the mechanics of load transfer it is necessary to consider the entire pile-soil system, as well as the construction procedures used to place the pile.

Whittaker (1970) observes that most pile ultimate load theories do not take into account compaction of a sand resulting from pile driving. Further, that if agreement is found between such theories and the results of loading tests, then either the effect of compaction is negligible, or the factors used in assessing load capacity are in error by a compensatory amount. Poulos and Mattes (1969), for example, base their analysis for the axially loaded pile on elastic theory with the simplifying assumptions that the soil is an homogeneous, isotropic, elastic material, whose properties are unaffected by the presence of the pile.

Full scale investigations by Hunter and Davisson (1969), using driven steel piles, indicated that "compaction" below the pile tip during driving caused a significant improvement in "soil strength", with an associated increase in ultimate pile tip capacity. They recommended that....

"FUTURE RESEARCHERS SHOULD CONSIDER THIS POSSIBILITY IN THEIR INVESTIGATIONS"
Figure 2.20 shows the concept expounded by Hanna and Tan (1973). They argue that because the installation of a pile into the ground causes deformations of the adjacent soil, and thus rotation of the planes of principal stress, as well as differential volume changes, an unloaded pile cannot be stress and strain free. Consequently the origins of the stress-strain relationships of pile-soil interface elements A and B in figure 2.20 will start at unknown isolated points. (such as suggested by points $A_0$ and $B_0$).

Whilst Hanna and Tan are concerned principally with the pile, the relative states of stress and strain suggested by figure 2.20 apply equally to the soil. The rotation of principal soil stresses is clearly demonstrated subsequently (Chapter 6).

Hanna and Tan contend that because the conditions in the pile change continuously with load, and as the origins of loading (represented by a particular point in stress-strain space) are also continuously changing, significantly different response must be expected from each of the elements. This concept is indicated in figure 2.21.

Consequently, under real conditions, a pile placed into the ground by a driving process will, prior to the application of any external loads, have probably already been conditioned to a number of load repetitions. It is consequently difficult to assess the stress-state existing in the soil prior to the start of a static load test.

Balaam, Poulos and Booker (1975) have developed a finite element analysis for an axially loaded pile. In recognition of the fact that the effect of installation of the pile is to create a zone of soil around the pile which has strength and deformation properties different from the main soil mass, they have analysed the pile and the soil as separated bodies. They also introduced a zone of disturbed soil about the pile.

Their theoretical results would appear to support the contention propounded by a number of authors, as discussed in this chapter, that the ultimate load of an axially loaded pile depends almost entirely on the strength characteristics of this disturbed zone.

Meyerhof (1976), in presenting the Terzaghi Lecture, made the observation that the behaviour of piles can "even now", (1976), only roughly be estimated from soil tests and semi-empirical methods of analysis based on the results of pile load tests, due to the complex interaction between the soil and the pile during and after installation.

Clearly then, the understanding of the influence of volume displacement due to pile driving, (particularly in sands), remains difficult to assess quantitatively. It is widely recognized however, and has been for some number of years, that the influence is real and could assume significant proportions.
FIG. 2.20 EFFECT OF INSTALLATION PROCESS ON PILE STRESS STATE. 
(After Hanna and Tan, 1973).

FIG. 2.21 THE EFFECT OF PILE LOADING HISTORY ON THE ORIGIN OF THE LOAD-DISPLACEMENT DIAGRAM 
(After Hanna and Tan, 1973).
Some Methods of Accounting for Volume Displacement Due to Pile Driving in Sand

1. Introduction

In the following section the methods of accounting for volume displacement about piles during the driving process, as conceived by Meyerhof (1959) and Nordlund (1963) are presented. The writer does not necessarily agree with the concepts as expressed.

2. Meyerhof (1959)

As indicated in the preceding section, the axial load capacity of a pile is usually estimated based on the assumption that the soil conditions about the pile are unaffected by the process of pile installation. It is clear, though, that when a pile is driven into cohesionless soils, the soil is subjected to changes in volume as a result of both:

(i) the effects of the mass of soil displaced by the intruding pile shaft, and

(ii) vibration associated with the pile installation process.

Both of these effects must result in permanent rearrangement, as well as prestressing, of the soil mass near the pile. Some crushing of the soil particles could also conceivably occur.

Meyerhof related both field and laboratory observations to conclude that static pressures (i.e. those associated with static axial loading) yields the smallest amount of "compaction". In comparison, intense vibration produced the greatest degree of "compaction". Impact pressures (i.e. those associated with pile driving) caused only a moderate amount of vibration and thus an intermediate degree of compaction.

An analogy was drawn between relative density - pressure relationships and void ratio - pressure curves obtained from static confined compression tests.

The following expression was thus derived to express the compaction occurring in the soil due to pile installation as a value of relative density ($D_R$):

$$D_R = D_2 - \left( \frac{D_2 - D_1}{1 + 2.3 (\frac{p}{p_C})^c} \right)$$

2.13

where $D_1$ and $D_2$ = initial and final densities respectively.

$p$ = the applied effective pressure

$p_C$ = a pressure constant.

$c$ = a compaction index obtained from the void ratio - pressure curves

Meyerhof in his study on loose sands concluded that to determine the degree of compaction about a pile during driving, it is necessary to calculate the magnitude of peak pressures at the base of the pile, and in the surrounding soil, ($p$ in equation 2.13), due to the applied energy of the hammer.

Figure 2.22 indicates Meyerhof's contention that the state-of-stress in the soil during the initial stages of compaction by driving, is similar to that of a deep circular footing, and is thus analogous to the states of stresses suggested by his bearing capacity theory.
FIG. 2.22 PLASTIC ZONES AND MAJOR PRINCIPAL STRESS AFTER DRIVING CIRCULAR PILE INTO COHESIONLESS SOIL.  
(After Meyerhof, 1959).

FIG. 2.23 COMPACTION OF SAND NEAR DRIVEN PILES.  
(After Meyerhof, 1959).
The shear strength of the soil is assumed to be fully mobilised and a plastic zone developed, as indicated in figure 2.22(a), while at a greater distance from the pile base, the soil is in an elastic state.

Meyerhof's consideration is that, in order to drive a pile, the stresses induced in the soil must be such that substantial permanent deformations are caused (i.e. a large proportion of the soil mass must reach a state of stress beyond peak values).

It follows then that the energy per blow of the pile driving hammer must be great enough to overcome the ultimate load carrying capacity of the soil, i.e.:-

Required minimum force for pile driving = (Pile base resistance) + (Pile skin friction).

= Dynamic resistance of the pile.

Meyerhof estimated the dynamic resistance using the Hiley formula:-

\[ R = \frac{n^* WH}{s+c/2} \]

where

\( n^* \) = the efficiency of the hammer blow.  
\( W \) = the weight of the hammer  
\( h \) = the free fall height of the hammer  
\( s \) = the penetration (i.e. set) of the pile base per hammer blow.  
\( c \) = the temporary elastic compression of the pile and soil

Meyerhof takes the major principal stress induced by the dynamic force as the pressure producing compaction of the soil. This stress was determined using plastic theory within the failure zone about the pile (determined from Meyerhof's bearing capacity considerations), whilst elastic theory, (Boussinesq-Mindlin equations) were used at a greater distance from the base.

Meyerhof suggests that the driving of a pile is more likely to be analogous to the expansion of the base of a caisson, in which case high stresses are induced about the base and are a maximum at the bottom of the shaft. The directions and values of the major principal stresses obtained by Meyerhof decrease roughly radially with distance from the bottom of the expanded base in both the plastic and elastic zones.

A limited number of cone penetration tests were conducted at various distances from single piles driven into sand. On the basis of this limited field data he contended that the increases in compaction indicated in figure 2.23 were substantiated.

(Petrasovits, 1973, made similar observations on the apparent increase in density about driven piles as a result of a series of penetrometer tests about driven model piles in sand).

Meyerhof thus concluded that, as a result of increased soil densities due to driving, the mobilised angle of internal soil friction (\( \phi_{moh} \)) about a driven pile increases in a roughly linear manner towards both the pile shaft and the base.
FIG. 2.24 VALUES OF $K$ CORRESPONDING TO MAXIMUM PASSIVE PRESSURE AS A FUNCTION OF $\phi$ FOR CASE OF HYPOTHETICAL HALL. (after Nordlund, 1963).

FIG. 2.25 TYPICAL DESIGN CURVE FOR EVALUATING $K_{e}$ (after Nordlund, 1963).
As a consequence of this study, bearing capacity factors, modified to accommodate the change in $\Delta_{mok}$ deduced from the foregoing considerations, were presented.

Thus, an attempt was made, dependent entirely upon the validity of the bearing capacity concept at the depths associated with pile foundations, to accommodate the effects of the installation of driven displacement piles in cohesionless soils.

(3) Nordlund (1963)

Nordlund argues that the greater the volume of pile per unit length, the greater is the compaction of soil in the immediate vicinity of the pile. Further, this increased compaction will have the effect of increasing the angle of friction, $\phi$, between the pile and the soil into which it is embedded.

In his considerations, Nordlund applied the variations to Meyerhofer's bearing capacity factor ($N_q$), as a function of $\phi$ as proposed by Rerezantzev, Khrisoforov and Golubkov (1961). He further assumes that for the case $\delta = \phi$, the coefficient of lateral earth pressure, $(K_g)$, increases to values approaching 20 as reported by Caquot and Kerisel (1948) (See Nordlund, 1963) and indicated in figure 2.24.

Nordlund expresses the view that the greater the volume of soil displaced by a pile, the greater the lateral movement of the soil, and thus the greater the passive wall pressure, $(K_g)$, that is developed. The volume displacement of the pile is taken into account empirically using charts such as those indicated in figures 2.25 and 2.26.

In figure 2.25 the angle $\omega$ represents the degree of outward taper of a pile wall from the vertical. In figure 2.26 curve 'a' is for pipe piles, 'b' for timber piles, 'c' for precast concrete piles, 'd' and 'e' for Raymond piles, and curve 'f' for H. piles.

Nordlund states that the location of curve 'b' has been determined from one load test, whilst that of curve 'c' is an estimate. He also states that the sole justification of the use of the curves in figure 2.25 is that their use yields reasonable results, as indicated in figure 2.27.

(4) Comment

Table 2.7 shows the ultimate load carrying capacity of a rough pile of volume 0.03 m$^3$/m (1 ft$^3$/ft) installed to an embedment depth $d$ of 10, determined using the methods of Nordlund and Meyerhofer. The pile diameter $D$ was taken as 0.3m (1 ft.), the unit density of soil as 1640 kg m$^{-3}$ and the coefficient of friction between the soil and the pile as equal to $\phi$ at 35°. In applying Meyerhofer's method to obtain the results of Table 2.7 the writer has assumed the increase in soil density to be 10% due to pile driving while the soil friction angle was assumed to increase from 35° to 40° at the base of the pile.
FIG. 2.26 RELATIONSHIP BETWEEN $V$ AND $\frac{c}{\phi}$ FOR VARIOUS TYPES OF PILES. (after Nordlund, 1963).

FIG. 2.27 COMPARISON OF COMPUTED AND OBSERVED REARING CAPACITY (after Nordlund, 1963).
TABLE 2.7
COMPARISON OF ULTIMATE AXIAL PILE LOADS
ACCOUNTING FOR EFFECT OF PILE INSTALLATION
USING THE METHODS OF MEYERHOF (1959) AND NORDLUND (1963)

<table>
<thead>
<tr>
<th>Author</th>
<th>Tip Load Expression</th>
<th>Calculated Value (kN)</th>
<th>Shaft Load Expression</th>
<th>Calculated Value (kN)</th>
<th>Total Load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyerhof (1959)</td>
<td>( Q_u = (\rho g)' \frac{D}{2} \cdot H_e' + K_{sf}' (\rho g)' D N_q' )</td>
<td>1454 for 10% increase in density</td>
<td>( Q_s = (C_p \cdot L) \cdot \frac{K_s}{2} (\rho g)' \cdot D \tan \delta' )</td>
<td>5.5 for 10% increase in density</td>
<td>1459</td>
</tr>
<tr>
<td>Nordlund (1963)</td>
<td>( Q_u = N_q A_b r_b' )</td>
<td>241</td>
<td>( Q_s = \frac{L^2}{L_0} K_0 q'_L \cdot \tan \delta )</td>
<td>( L^2 \cdot C_n \cdot dL )</td>
<td>117.6</td>
</tr>
</tbody>
</table>

Clearly there is considerable difference between the ultimate loads calculated by the two methods. It would appear that the presently available means of accounting for pile volume displacement during installation by driving and the subsequent modification to soil load carrying characteristics are at the best, inconclusive.

However, in the absence of other more soundly based theories empirical methods such as those proposed by Meyerhof and Nordlund must be accepted as viable, providing they can be demonstrated as capable of allowing ultimate loads to be assigned which are at least conservative, (figure 2.27).

2.6.5 A Reassessment of the Bearing Capacity Approach

(1) Introduction

Meyerhof, in presenting the Eleventh Terzaghi Lecture (Meyerhof, 1976) recognises some of the currently held limitations of the conventional bearing capacity theories, as discussed in the preceding sections, and introduces the concept of a critical depth. The realisation, by 1976, of the real complexity of the problem is probably exemplified by Meyerhof's own statement:
"ON ACCOUNT OF THE COMPLEX INTERACTION BETWEEN THE SOIL AND PILE DURING AND AFTER
CONSTRUCTION OF THE FOUNDATION, THE BEHAVIOUR OF SINGLE PILES .... UNDER [AXIAL]
LOAD CAN ONLY ROUGHLY BE ESTIMATED FROM SOIL TESTS AND SEMI-EMPIRICAL METHODS OF
ANALYSIS BASED ON THE RESULTS OF PILE LOAD TESTS".

However, Meyerhof still suggests that the result of installing a pile by driving into sand
is to compact the soil near the pile to a distance of a few pile diameters. (Density changes are
discussed by the writer in Chapter 4).

Based on simplifying assumptions a number of fairly recent approximate estimates have been
made of the deformation of sand near driven piles and the corresponding soil pressures on the
tip and shaft of a pile. One such example is Vesic (1972) in which he considers the expansion of a
spherical cavity and relates this to the pile problem. This work is discussed further in Chapter 5.

Such works however, indicate that the point resistance and average skin friction of a pile
would increase with greater depth of penetration. As has been seen in the preceding review, this
is only the case up to some critical depth beyond which the parameters asymptote to near constant
values. Since no satisfactory method of analysis of pile behaviour below the critical depth is
available, Meyerhof advocates the following empirical approach.

(2) Pile Tip Resistance

It has already been shown (equation 2.1) that the ultimate axial load carrying capacity
of a pile is divided into two separate and independently assessed components, which, neglecting
the weight of the pile, is given by:-

\[ \eta_u = \eta_t + \eta_s = \eta_{at} + \eta_{as} \]  \hspace{1cm} (2.15)

The recommended ultimate unit pile tip resistance is then given by:-

\[ \eta_{at} = q N_q \leq \eta_t \]  \hspace{1cm} (2.16)

where \( q \) = the effective overburden pressure at the pile tip

\( = \gamma' L \) (as suggested by Proctor, 1966).

\( N_q \) = the bearing capacity factor with respect to overburden pressure

\( \eta_t \) = the limiting ultimate unit pile tip resistance for \( \frac{L}{D} > \frac{L_c}{D} \)

where \( L_c \) = the critical depth of pile penetration

Meyerhof (1976) now considers that the bearing capacity factor \( (N_q) \) increases roughly linearly with
pile embedment length \( (\frac{L}{D}) \) and reaches its maximum value at an \( \frac{L}{D} \) of approximately 0.5 \( \frac{L_c}{D} \).

Beyond this penetration depth it is now considered by the author that conventional bearing
capacity theory no longer applies. Meyerhof has presented an approximate relationship between
the limiting pile tip resistance and the friction angle of the soil \( (\phi) \) thus:-

\[ \eta_{at} = 0.5 N_q \tan \phi \]  \hspace{1cm} (tons ft^{-2})  \hspace{1cm} (2.17(a))
Various other authors suggest different empirical relationships for this limiting pressure for example Chaplin (1977) and Biarez and Foray (1977).

Although the values of the bearing capacity and the ultimate limiting resistance depend mainly on the soil friction angle, they are also influenced, as has been seen in the preceding review, by a variety of factors including the compressibility of the soil and the method of pile installation. Meyerhof does not recommend any means to take these factors into account.

He emphasises that if the piles are driven into homogeneous soils to more than the critical depth, the unit point resistance cannot be estimated by conventional bearing capacity theories in terms of a bearing capacity factor ($N_q$). The corresponding ultimate unit point resistance ($a_u$) becomes practically independent of the overburden pressure at the pile tip (and thus of $N_q$) and depends on the value of the limiting pressure for piles where $L/D > 15$ to 20. Meyerhof supports this statement from the study of a number of pile load tests.

(3) Shaft Friction Resistance

The recommended average unit shaft friction in homogeneous sand is given by:

$$f_s = K_s \bar{a} \tan \delta \leq f_L$$

where $K_s$ = the average coefficient of earth pressure along the pile shaft.
$\bar{a}$ = the average effective overburden pressure along the pile shaft
$\delta$ = the angle of shaft friction
$f_L$ = the limiting value of the average unit shaft friction for $L/D \geq L_c/D$

Meyerhof reaffirms the observation made by a number of authors in the preceding review, that an estimate of the shaft friction, and particularly of the earth pressure coefficient, $K_s$, on the basis of the friction angle of the sand and the method of pile installation, is even more difficult than for the pile tip resistance. Reliable values of $K_s$ and $f_L$ can only be deduced from full scale load tests on piles at a particular site, nevertheless, rough estimates may be able to be made from the results of penetration tests.

With reference to the work of Vescic (1977) (see figure 2.17), Meyerhof considers that because of the recorded variation in shaft friction with pile embedment length, the corresponding local coefficient of earth pressure acting on the pile shaft ($K_{sz}$) decreases with depth (as the shaft friction angle increases) from a maximum near the top where it may approach the Rankine passive earth pressure coefficient, to a minimum near the pile tip where it may be less than $K_0$ (the coefficient of earth pressure at rest).

Meyerhof recommends that conventional shaft capacity theory in terms of $K_s$ not be used for piles longer than about 15 to 20m. This is because the corresponding value of the unit shaft friction ($f_s$) becomes practically independent of the average overburden pressure along the shaft, and is given by the limiting value ($f_L$).
2.6.6 Soil-Pile Parameters Required for use With Conventional Bearing Capacity Theories

The soil-pile parameters required for use with the conventional bearing capacity theories are:

(a) the unit weight of the soil (γ)

(b) the soil friction angle (φ) which may be obtained from conventional triaxial tests, unconfined compression tests, or from correlations with penetrometer tests. It is not clear, however, whether such values for the friction angle are appropriate. The preceding review has indicated that the fundamental assumption with the various bearing capacity theories is that the soil mass within the failure surface is shearing, in which case it would seem more appropriate to use the critical state friction angle values (φ_{cs}) (see Appendix 5). The use of a friction angle of magnitude less than the peak values obtained by the conventional tests already mentioned (φ_{cs} < φ_{o}) might go some way to reducing the general over-estimate of pile tip bearing capacities as produced by the present methods.

(c) the angle of shaft friction (δ) which may reliably be obtained from special shear box tests such as those described in Chapter 5.

(d) the coefficient of earth pressure (K_s), approximate values of which may be obtained from penetrometer tests or tabulated values. However, for any degree of confidence either large or full scale tests should be carried out.

2.6.7 Static and Dynamic Penetration Tests

(1) Definitions

(i) The Dynamic Penetration Test

The dynamic penetration test is also often called the "standard" or "spoon" penetration test (SPT). This test is used extensively in the USA and Canada. The test essentially comprises a 50 mm OD sampling tube of 35 mm ID which is driven into the ground at the bottom of a borehole under an energy of 350 ft·lb (475 mJ). The number of blows (N) per foot (0.3m) of penetration are recorded. The standard device comprises a 140 lb (63.5 kg) weight which is arranged to free fall from a height of 30 inches (0.762 m). Various authors have described the device, for example Terzaghi and Peck (1948).

Gibbs and Holz (1957) drew correlations between the SPT-N values and relative density (D_{R}) in sands, with allowance made for overburden pressures. The validity of this correlation has recently been thrown into serious doubt as a result of investigations stemming from the near failure of the Lower San Fernando Dam during the San Fernando Earthquake of February 1971 (Marcuson and Rienanousky, 1977).

Various other authors have attempted to draw similar correlations with relative density. A number of these are compared by Mitchell and Gardner (1975). The range of relative density values obtained are large, for example giving the same value of D_{R} for N values ranging from 40 to 60.
Because the SPT test tends to be very subjective, caution is recommended in its use (Fletcher, 1965). The influence of mechanical variables are discussed by McLean, Franklin and Dahlstrand (1975), Brown (1977) and Kovacs (1979).

(ii) The Static Penetration Test

Because of the potential for confusion by name between this test and the dynamic or SPT test, the static penetration test is usually called the "Dutch cone" or "cone" penetration (CPT) test. This test is used extensively throughout Europe.

The device is essentially a 60° cone of 36 mm base diameter which is pushed into the ground at a slow, but continuous rate. Variations on the basic device permit the shaft and point resistances to be obtained separately. The cone penetrometer is described by a number of authors, for example, Vermeiden (1948) and Haefeli and Bucher (1961). A number of factors affecting the cone penetrometer are discussed by a number of authors, and are summarised by Mitchell and Lunne (1978). A comprehensive discussion of the static penetration resistance of soils is given by Durgunoglu and Mitchell (1975).

(2) General

While static penetration methods appear to be generally preferred, they suffer from the disadvantage that in dense and very dense soils, a substantial reaction to jacking has to be provided. (Note that because the CPT test involves the continuous penetration of the cone, it is often referred to as a quasi-static test; i.e. QCPT).

CPT tests give a continuous record, but require additional boreholes for the identification of the soil.

SPT tests on the other hand generally furnish disturbed samples which enable index properties to be determined in the laboratory and are generally less costly than CPT tests. They are often difficult to carry out however, especially if high ground water conditions prevail.

All penetration tests become unreliable as the maximum particle size approaches the diameter of the penetrometer or sampling spoon.

The application of both the SPT and CPT tests to the determination of the in-situ shear strength of soils has been given by Schmertmann (1975).

(3) Application of SPT and CPT Test Results to the Determination of the Ultimate Axial Load Capacity of Piles

(1) Meyerhof

(a) General

Meyerhof (1956, 1976) has drawn and presented correlations between field tests using both devices, and both the point and shaft resistance of piles. The fundamental correlation drawn by Meyerhof was that:

\[ q = 4N \]
where $q_c$ = the static cone resistance (tons, ft$^{-2}$)

and $\mathcal{N}$ = the standard penetration resistance (blows per foot).

In metric terms, this correlation is:

$$q_c = 43MN$$  \hspace{1cm} (2.19b)

where $q_c$ is in kPa.

Meynerhof's correlations are summarised as follows:

(b) Pile Tip Resistance

When the pile tip is above the critical depth ($L_c$), Meyerhof draws the following approximate relationship between the ultimate unit point resistance for an axially loaded pile and the limiting static cone resistance thus:

$$q_u = \frac{q_c L_c}{D} \leq q_L$$  \hspace{1cm} (2.20)

where $L_c$ = the depth of embedment of the cone penetrometer, and $q_L = q_c$.

Meynerhof's correlation with the SPT test is:

$$q_u = \frac{4\mathcal{N} L_c}{100D} \leq 4\mathcal{N} \text{ (tons ft}^{-2}\text{)}$$  \hspace{1cm} (2.21a)

$$= 43\mathcal{N} \frac{L_c}{D} \leq 430N \text{ (kPa)}$$  \hspace{1cm} (2.21b)

where $\mathcal{N}$ = the average standard penetration resistance in blows per foot (blows per 0.3m).

(c) Shaft Friction Resistance

From field loading tests, Meyerhof (1956) initially proposed that the shaft friction acting on a driven pile in a homogenous soil deposit was related to the static shaft (skin) friction of a cone penetrometer thus:

$$f_s = 2 f_c = \frac{q_c}{20N} \text{ (tons ft}^{-2}\text{)}$$  \hspace{1cm} (2.22a)

$$= 0.54 q_c \text{ (kPa)}$$  \hspace{1cm} (2.22b)

where $f_c$ = the static cone shaft resistance.

However, by 1976, Meyerhof recommends the more conservative approach:

$$f_s = f_c = \frac{q_c}{40N} \text{ (tons ft}^{-2}\text{)}$$  \hspace{1cm} (2.23a)

$$= 0.27 q_c \text{ (kPa)}$$  \hspace{1cm} (2.23b)

In the case of the SPT test, he recommends that the limiting shaft friction ($f_L$) be given by:

$$f_L = \frac{N}{50} \text{ (tons ft}^{-2}\text{)}$$  \hspace{1cm} (2.24a)

$$= 2.15N \text{ (kPa)}$$  \hspace{1cm} (2.24b)
The expression represented by equation 2.24(a) was confirmed by Thorburn and Mac Vicar (1971).

It should not be overlooked that Meyerhof (1976) expresses caution in the use of these correlative values for both point and shaft resistances, and particularly with respect to penetrometer estimates of shaft friction, should be checked by representative pile load tests.

Recommendations for varying soil strata and pile groups are given in Meyerhof (1956, 1976).

(iii) Broms

Broms (1966) proposes values for the CPT test somewhat more conservative than those suggested by Meyerhof.

(a) Pile Tip Resistance

Broms recommends that the ultimate unit point resistance for an axially loaded pile be taken as the cone penetration resistance when the resistance is less than 100 ton ft$^{-2}$ (10.7 MPa) and that the limiting value ($q_u$) be taken as 100 ton ft$^{-2}$.

$$ q_u = q_c \leq q_z = 100 \text{ ton ft}^{-2} $$  \hspace{1cm} 2.25(a)

$$ = 10.7 \text{ MPa} $$  \hspace{1cm} 2.25(b)

Broms further recommends that the ultimate value ($q_u$) should be taken as the average CPT value ($\bar{q}_c$) measured over the length extending from 3.75D above the pile point to 10D below it.

Kerisel (1961) has shown that the ultimate bearing capacity of a pile can be lower than the point resistance measured by the cone penetrometer when the diameter of the pile is large. Broms therefore recommends that the results of cone test should be reduced when the pile diameter is larger than 0.5m.

Broms does not indicate the magnitude of the reduction, however this recommendation is clearly made in consideration of scale effects, which are discussed in Chapter 5 (see also (iii) below).

With regard to SPT tests, Broms, in view of the scatter of field results reported by Meyerhof (1956), recommends that the recorded lower limit be used, i.e.:

$$ q_u = 2.5 \text{N} \quad (\text{ton} \text{ ft}^{-2}) $$  \hspace{1cm} 2.26(a)

$$ = 268 \text{N} \quad (\text{kPa}) $$  \hspace{1cm} 2.26(b)

(b) Shaft Friction Resistance

Broms makes no recommendations for shaft resistance, however, Mohan, Jain, and Kumar (1963), have indicated that shaft friction resistances can be considerably larger than the empirical values suggested by Meyerhof. Vesić (1970) has drawn an empirical correlation with model and full scale load tests as a function of relative density ($\rho_r$) which can thus be used in conjunction with an SPT test:
(iii) Tomlinson

(a) Pile Tip Resistance

Tomlinson (1977) reports that extensive experience with the cone penetrometer in Holland has shown that in general:

\[ q_u = q_c \]  

It is usual, according to Tomlinson, to adopt Van der Veens' (1957) method in which the average cone resistance is taken over a shaft length of 3D above the pile tip and 1D below. (As noted in (ii) above, Broms recommends 3.75 and 1D respectively). In Holland the Van der Veen method has been modified thus:

\[ \bar{q}_c = \frac{1}{2} \left( \frac{1}{2} (q'_c + q''_c) + q''''_c \right) \]

where \( \bar{q}_c \) = the modified value of unit cone resistance

\( q'_c \) = the average cone resistance beneath the pile tip for the length: \( 0.7D < L < 4D \)

\( q''_c \) = the minimum cone resistance below the pile tip over the same range

\( q''''_c \) = the average envelope of minimum cone resistances above the pile tip over a height of 6 to 80. Values of \( q''''_c \) in excess of the minimum value selected for \( q''_c \) are neglected.

The relationship represented by equation 2.28 as established for Dutch soil conditions is not necessarily applicable to cohesionless soils outside of Holland, and possibly adjacent countries, due to vastly different geological histories and thus such varying factors as preconsolidation, mineralogy etc. (Meyerhof, 1976; Vesic', 1970). In support of this observation, Tomlinson (1977) records correlative reduction factors on measured cone resistances \( q_c \) ranging from 0.5 in Norway to 0.75 in Russia, however, he suggests from reported evidence that equation 2.28 appears to empirically accommodate the effects of pile installation in cohesionless soils. Nevertheless it is advocated that field trials be conducted to correlate the CPT test with pile loading tests, for a particular geographic location, to establish the appropriate relationship between the two.

Tomlinson also suggests that cone results be reduced, as recommended by Broms (1966), for pile diameters greater than 0.5m, (i.e. that a "scale" effect exists). The writers interpretation of Tomlinsons suggestion is (see also sub-section (4)):

\[ q_u = \frac{L_p^*}{L_p} q_c \]

where \( L_p^* \) = the actual pile embedment into the bearing stratum

\( L_p \) = the required depth of embedment to develop the same point resistance as the cone penetrometer.
where \( D_p \) = pile diameter
\( D_c \) = cone diameter
\( L_c \) = the depth of cone embedment into the bearing stratum.

(b) Shaft Friction Resistance

It is usual in Holland (Tomlinson 1977) to determine the unit shaft friction \( f_s \) from the relationship:

\[
f_s = \frac{\bar{q}_c}{250} \quad \text{(tons ft}^{-2}) \quad 2.31(a)
\]

\[
f_s = 0.4 \bar{q}_c \quad \text{(kPa)} \quad 2.31(b)
\]

(c) Recommended Procedure

Tomlinson's recommended procedure when using the static cone penetrometer is as follows:

(1) Inspect the cone resistance results and choose a provisional pile founding depth \( L_p \) that will permit the maximum shaft resistance to be mobilised.

(2) For this value of \( L_p \) obtain values of \( q_c \) for a distance of 30 above and 10 below the pile tin.

(3) Calculate \( q_u \) from equation 2.28, or in the absence of any correlation:

\[
q_u = 0.5 \bar{q}_c \quad 2.32
\]

(4) Either increase the pile founding depth to match the embedment depth of the penetrometer, or reduce \( q_c \) below that obtained in (3).

(5) Calculate \( f_s \) over the full pile embedment length and apply an appropriate factor of safety (Tomlinson suggests 2.5. See also section 2.6.12).

(6) Check the working load against the permissible working stress on the pile shaft.

(iv) Comment

In applying any of these empirical relationships, particularly those pertaining to the SPT test, designers should be reminded that the results of such a test can be highly subjective as discussed by Fletcher (1965), and Brown (1977), and as indicated by the wide range of correlative values expressed in figure 7.22 (Chapter 7).
The various correlations listed in the foregoing section are summarised in Table 2.8

**Table 2.8**

**Summary of Correlations of Unit Shaft and Tip Load Carrying Capacities SPT and CPT Tests**

<table>
<thead>
<tr>
<th>Author</th>
<th>Unit Tip Load ( (q_u) ) (kPa)</th>
<th>Unit Shaft Load ( (f_s) ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPT</td>
<td>CPT</td>
</tr>
<tr>
<td>Meyerhof</td>
<td>( \frac{q_c L_c}{100} )</td>
<td>( 1.08 )</td>
</tr>
<tr>
<td>(1956, 1976)</td>
<td>( \leq 430 )</td>
<td>( \leq f_x = 2.15 )</td>
</tr>
<tr>
<td>Broms</td>
<td>( 268 )</td>
<td>( q_c \leq q_x = 1070 )</td>
</tr>
<tr>
<td>(1966)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vesic</td>
<td></td>
<td>( 2.68 ) ( (10) )</td>
</tr>
<tr>
<td>(1970)</td>
<td></td>
<td>( D_R ) ( = ) relative density</td>
</tr>
<tr>
<td>Tomlinson</td>
<td>( q_c^* = q_c ) adjusted</td>
<td>( 0.4 q_c )</td>
</tr>
<tr>
<td>(1977)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Most of the authors referred to above provide correlations with cohesive soils, however, a useful, but not entirely current guide is also provided by Hadfield (1969).

Parry (1977) proposes further correlations between the SPT tests and the bearing capacity of sands. So far however, he has only considered shallow rectangular foundations in making the correlation. Mitchell and Lunne (1978) reported the comparisons of a number of correlations with the CPT test as advocated by various authors and draw some useful conclusions.

4. "Scale" Effects

1. The Application of CPT Results to the Ultimate Bearing Capacity of Piles

De Beer (1963) discussed at some length his justification of the scale adjustment generally applied throughout Holland and Belgium. In so doing he defined this scale effect, for a given depth, as "the variation of penetration resistance per unit area with transversal dimensions". Such observations were identified in the preceding sections with reference to Broms (1966) and Kerisel (1961). This scale effect is essentially then, the effect due to the difference in diameter between the penetrometer and the pile. The interpretation of this scale effect is dependent upon the acceptance of the nature of the shear zones given by bearing capacity considerations such as Meyerhof (1951).

Considering a homogeneous soil, the embedment depth \( (L_c) \) of a small diameter probe such as a cone penetrometer (35mm diameter) is likely to result in the shear zones so developed being entirely within the bed as indicated in figure 2.28(a). For the situation where a pile of
diameter say 10 times greater than that of the cone \((D_p = 10 D_c)\) is installed to the same depth as the penetrometer, \((L_c = L_p)\), the shear zones, assuming the validity of the bearing capacity theories at depth, will be as shown in figure 2.28(h). To enable the full development of the failure surface so that the general conditions pertaining about the pile tip are the same as those about the tip of the probe, the pile would have to penetrate to a considerably greater depth as indicated in figure 2.28(c).

It follows then that if the same bearing capacity law is applied to both the pile and the penetrometer, their penetrations into the soil mass must be governed by their scale ratios. Given a weightless soil, the required similitude would be given directly by a ratio of diameter, i.e.

\[
L_p = L_c \left(\frac{D_p}{D_c}\right) \quad 2.33
\]

This, however, is a simplification of the problem, as both overburden pressures and the degree of confinement afforded by the soil increase with the increased depth of penetration. Thus the problem is not one of simple considerations of scale. The method presented by De Reer (1963) is an attempt to take these considerations into account. His method, as summarised below, is related to the entry of the penetrometer into a bearing stratum at some depth. The considerations can also be applied to non-stratified beds.

In carrying out a CPT test, it is usual to observe penetration resistances every 200 mm. As the penetrometer enters the bed the observed resistance may increase over a number of observations. This increase is approximately linear and is taken to indicate the increase in resistance as the assumed shear zone becomes progressively enclosed within the bed. This linear increase is indicated by \(AB\) in the idealised curve of figure 2.29. Once the cone has penetrated beyond point \(B\) the observed penetration resistances are approximately the same, showing a much slower rate of increase (i.e. \(B\) to \(C\) in figure 2.29). This more gradual uniform increase is taken to represent the effects of increased overburden pressure.

Point \(B\) then represents the condition of full enclosure of the shear zones about the penetrometer tip. Thus the vertical distance \(AB\) (figure 2.29) is the development length necessary to develop the full ultimate tip resistance.

For a pile ten times the diameter of the cone, the corresponding depth to ensure full development of the shear zone about the pile tip would be ten times the development length of the pile (i.e. \(AC' = 10AB\) from equation 2.33). The assumption then is that the increase in resistance recognised by the pile tip would approximately follow the line \(AC\) in figure 2.29. The effect of overburden is recognised in the increase in resistance from \(B\) to \(C\).

Thus providing the pile could withstand the driving forces, and providing the stratum extended a sufficient depth below point \(C\) for the shear zone to continue to be within the stratum (4 to 5 \(D_p\)), then the pile would need to be driven to the level of point \(C\) to develop the maximum resistance for the minimum penetration afforded by the stratum.
FIG. 2.28 DEVELOPMENT OF SHEAR ZONES RELATIVE TO FOUNDATION SIZE.

FIG. 2.29 DE BEER'S METHOD OF DETERMINING THE DEPTH TO WHICH A PILE SHOULD PENETRATE TO CORRECT FOR THE SCALE EFFECT.
In practice the results of a cone penetrometer test are somewhat more complicated than suggested by the idealised curve of figure 2.29 (see figures 10.6 and 10.7, Chapter 10). and thus the interpretation of the results are similarly more difficult.

From a practical viewpoint Tomlinson (1977) suggests that this method can produce required penetration depths that are either uneconomic or impracticable. He reports that Vesic has shown that the ultimate base resistance can be obtained at a penetration of not more than 200 and suggests therefore that this be taken as a limiting penetration depth for the full development of the ultimate base resistance of a pile.

(ii) Other Considerations

Durgunoglu and Mitchell (1975) in analysing the static penetration resistance of soils have made a number of observations which must additionally affect the transposition of CPT results to the assessment of the axial load carrying capacity of piles. The significant points observed by the authors are that the resistance to cone penetration into soils depends on:

(a) Cone geometry and size
(b) Surface roughness of the cone
(c) Soil strength parameters
(d) Cone penetration depth
(e) The in-situ lateral soil pressure.

The effects of cone penetration depth have been discussed in (i) above. Because the test is an in-situ test the effects of the in-situ lateral soil pressure and soil strength parameters are automatically related between the penetrometer and the full size pile. However, the effects of (a) and (b) above could quite reasonably be manifested as "scale" effects that are not taken into account using De Reer's method. Even though the cone geometry and shape as well as the surface roughness are standardised, (in general), the applicability of the results so obtained relative to a pile of different tip configuration or surface frictional characteristics is open to question. Chapter 5 shows the influence of pile shaft friction. Even though the affected soil volume appears to vary little with differing surface roughnesses vastly differing effects are developed along the length of the shaft.

The penetrometer is essentially smooth, though notwithstanding the difference in surface roughness between the penetrometer and pile tips, some similarity must obviously exist, however, the relevance of the CPT test in the assessment of shaft frictional characteristics is questionable.

It could well be then that the apparent correlations as suggested by De Reer (1963) are in fact manifestations of an interaction of phenomena considerably more complex than the relatively simple geometric and overburden "scale effects".
2.6.8 Pile Driving Formulae

(1) General

Until comparatively recently (Tomlinson, 1977) all piles were installed by driving using a simple falling ram or drop hammer. A relationship was found to exist between the downward movement of a pile under a blow of given energy, and its ultimate resistance to static load. Thus, when all piles were driven by a drop hammer or falling ram, a considerable amount of experience was accumulated which enabled simple empirical formulae to be determined. Those formulae enabled the ultimate load carrying capacity of a driven pile to be calculated from the 'set' of the pile due to each hammer blow at the final stages of driving. However, as discussed by Tomlinson, because of the advances in modern piling equipment, pile driving formulae are now largely discredited as a means of predicting the resistance of piles to static loading.

Tomlinson expresses the opinion that there is no place for such "dynamic" formulae in the calculation of the ultimate static axial resistance of a pile. He also considers that the "possibility that such formulae could be used to calculate the deformation of a pile under working loads is quite inconceivable". Tomlinson prefers the "soils-mechanics" approach as outlined in the preceding sections. On the other hand however, Simons and Menzies (1976) comment that some piling formulae are convenient to use and give reasonable predictions of the ultimate bearing capacity of driven piles in cohesionless soils. They however, observe that the relationship between the dynamic and static resistance of a pile should be independent of time if the formula is to have any validity. As the load carrying capacity of clays are very much time dependent (See Chapter 10), clearly then, pile driving formulae should only be applied to cohesionless soils, if at all.

Flaate (1964) has investigated the validity of the three most commonly used pile driving formulae. In general it has been found that both the Hiley and Janbu formulae give "excellent correlations with load test results. (See Flaate, 1964, or Simons and Menzies, 1976). However, the variations found from application of the Engineering News formula are so great that it is generally not recommended as an economic design tool. It is generally held that pile driving formulae tend to lead to conservative designs.

(2) The Hiley Formula (Hiley, 1925)

This pile driving formula is probably the most widely used in this country (New Zealand). The static ultimate load carrying capacity of an axially loaded pile is given by:

$$Q_u = \frac{KHWH}{3.5}$$

where:
- $K$ = a hammer coefficient
- $W$ = the weight of the hammer
- $H$ = the drop of the hammer
- $n$ = the efficiency of the blow
- $s$ = the final pile set (penetration per blow)
- $c$ = the sum of the temporary elastic compression of the pile ($c_p$), the pile cap ($c_c$) and the ground ($c_g$).
Values of \( K, n, c_c, c_p \) and \( c_d \) can be readily obtained from either such nilino guides as the BSP Pocket Book (1969) or suitable foundation engineering texts such as Simons and Menzies (1976). Flaate (1964) recommends that a factor of safety of 2.7 be used with this pile driving formula.

(3) **The Janbu Formula** (Janbu, 1953)

\[
\eta_u = \frac{1}{K_u} \frac{n WH}{s}
\]

where

\[
K_u = c_d \left( 1 + \left(1 + \frac{\lambda e}{c_d} \right)^{1/2}\right)
\]

\[
c_d = 0.75 + 0.15 \frac{W}{p}
\]

\[
\lambda e = \frac{n W H L}{A_p E_p s^2}
\]

where \( W_p \) = the weight of the pile, 
L = the length of the pile, 
\( A_p \) = the cross-sectional of the pile, 
\( E_p \) = Modulus of elasticity for the pile material

A design aid for use with this pile driving formula is available in Simons and Menzies (1976). Flaate (1964) recommends that a factor of safety of 3.0 be used with this formula.

(4) **The Engineering News Formula**

This pile driving formula is widely used in the USA. The ultimate pile axial load capacity is given by:

\[
Q_u = \frac{W H}{s + c^*}
\]

where \( c^* = 1.0 \) for gravity hammers
\( = 0.1 \) for steam hammers
\( = 0.1 \frac{W_p}{E_p} \) for very heavy steel and concrete piles

Flaate (1964) recommends a factor of safety of 6.0 if this formula is used.

(5) **Other Pile Driving Formulae**

(1) **General**

Broms and Hellman (1971) have reported on the current methods of assessment of the point load carrying capacity of axially loaded piles as permitted by the Swedish Building Code.

The two methods applied are the Kreuger pile driving formula and the stress wave equation. The Kreuger formula is used when the applied loads are low and the penetration per blow during driving is relatively large (> 3 to 5 mm). The stress wave equation is used when the penetration is small (i.e. < 3 to 5 mm).
(ii) The Kreuger Formula (Kreuger, 1915)

\[ n_u = \frac{n \cdot W \cdot H}{s + 0.5} \frac{W + 0.25 W_p}{W + W_p} \]  

This equation is similar to the Hiley formula of equation 2.34. It is of interest to note that Broms and Hellman observe that both the Hiley and Janbu formulae appear to give the best agreement between ultimate loads calculated using pile driving formulae, and those measured for piles in cohesionless soils.

(iii) The Stress Wave Equation

\[ q_u = 2a^* - \left( \frac{s E_p W_p}{L} \cdot \frac{s*}{n.75 W} \right)^{1/2} \]  

where \( a^* \) = the maximum intensity of the stress wave.

The recommended factor of safety for use with this formula is 3.0.

The detailed derivation of this formula is given by Broms and Hellman (1971).

2.6.9 Ménard Pressuremeter

(1) General

Baguelin, Jézéquel and Shields (1978) present a novel method of analysing the ultimate load carrying capacity of axially loaded piles, using the Ménard pressuremeter. The basic characteristics of the Ménard pressuremeter are outlined in Appendix II. The results of field tests presented by the authors, while not indicating an at all satisfactory agreement with the empirical considerations, are probably no worse than the correlations on which are based the various empirical and semi-empirical methods of load assessment already discussed.

(2) Pile Tip Resistance

The authors propose that both the ultimate unit point resistance and the ultimate unit shaft resistance of the pile can be determined from the nett limit pressure \( n_u^{**} \) as obtained from a Ménard pressuremeter test. The nett limit pressure is defined by the authors as the difference between the limit pressure measured by the Ménard pressuremeter, \( n_L^{**} \) and the in-situ soil pressure \( n_o^* \). These reference pressures are indicated on figure A11.4. (Appendix II).

Thus

\[ n_u^{**} = n_L^{**} - n_o^* \]  

similarly

\[ q_u^{**} = q_u - q_0 \]  

The bearing capacity relationship suggested by the authors is:

\[ q_u = q_0 + k (n_L^{**} - n_o^*) \]  

which rearranges to:

\[ q_u^{**} = k n_L^{**} \]  

where \( q_0 \) = the unit overburden pressure at the pile tip

\( n_o^* \) = the in-situ pressure measured by the pressuremeter.
The authors present a number of tables giving values of $k$ for various soil types and conditions, and for both bored and driven piles. Values of $k$ recommended by the authors for driven piles in sand are indicated in figure 2.30. The adjustments in the value of $k$ are explained as being due in part to the fact that the pressuremeter is small compared to a pile and so reaches a critical depth much sooner than large foundations (i.e. $k$ is a scale factor).

Baquelin, Jézéquel and Shields, however, express difficulty in being able to understand why $k$ varies with soil type and condition. They consider that, as the pressuremeter test is an in-situ test, variations in soil type should be automatically accommodated. They suggest that the variation in $k$ is in part due to the fact that the pressuremeter only measures the passive pressure portion of the failure zone, as indicated in figure 2.31. It is the writer's understanding however, that, because of the assumed nature of the soil flow associated with the failure zones adopted for various bearing capacity theories, the whole of the soil mass, or at least the significant part of it, is in fact resisting the load in a passive manner. The writer would suggest that because the Ménard pressuremeter first requires the preparation of a borehole into which the pressuremeter is then inserted, that, by the time the borehole receives the pressuremeter, the borehole has relaxed significantly, thus modifying the state of stress in the soil and thus the manner in which the soil responds to loading.

An indication of the potential for borehole relaxation associated with the Ménard pressuremeter test is indicated by the results of a series of self-boring pressuremeter tests conducted by the writer (Hughes and Goldsmith, 1977) in heavily overconsolidated clays. (The characteristics of the self-boring pressuremeter are outlined in Appendix 11).

Figure 2.32(a) shows the pressure-expansion curve for an undisturbed test. Clearly the ultimate strength of the soil is reached at radial strains of less than 2% (this result is typical of the series from which this representative curve was extracted).

The results shown in figure 2.32(b) however, were obtained by first installing the instrument, removing it, and then subsequently reinserting the instrument into the prebored borehole after some elapsed time. The process was thus analogous to the technique required using the Ménard pressuremeter. The effects of borehole relaxation are striking. Clearly the ultimate strength of the soil has not been reached even at 5% radial strain; i.e. after some 250% increase in deformation.
FIG. 2.30 PRESSUREMETER DERIVED "BEARING CAPACITY" FACTOR FOR DRIVEN PILES IN SAND AND GRAVEL. (after Baguelin, Jézéquel and Shields, 1978).

FIG. 2.31 RELATIONSHIP OF LIMIT PRESSURE ($q^*$) TO BASE RESISTANCE OF AXIALLY LOADED PILE. (after Baguelin, Jézéquel and Shields, 1978).
2.6.10 Full Scale Load Tests on Piles

Clearly, from the preceding evidence, the only reliable method of assigning ultimate axial load carrying capacity is by the use of full scale loading tests. Unfortunately this is an extremely costly operation and is only likely to be justified on large projects. Because soil characteristics are unique and can vary significantly over relatively short distances, especially in New Zealand, the results of field load tests are unlikely to be able to be transferred between sites, even when these sites are in relatively close geographic proximity. The problem is further compounded by the fact that relatively minor variations in the method of pile installation can greatly modify the soil response.

The methods of conducting full scale pile tests have been outlined by a number of authors, for example, Chellis (1969) and Tomlinson (1977). The documented results of such tests are also well reported, for example, Selby (1970), Vesic (1970), Thorburn and MacVicar (1971), plus a large number of others extending over the last 30 odd years.

A useful summary of pile load test methods is given locally by Hadfield (1969). More comprehensive discussions are presented by Tomlinson (1977) and Woodward, Gardner and Greer (1972) for example.

2.6.11 Definition of Ultimate Load

As discussed earlier in this chapter, the ultimate resistance of an axially loaded pile is generally defined as the stage at which there is general shear failure of the soil beneath the pile tip. (i.e. point N in figure 2.34). Clearly this stage of the load-settlement process is only of academic importance to the designer.

From the designer's point of view, a piled foundation has failed in its engineering function when the pile settlement, (or relative settlement between piles in the case of pile groups), causes either intolerable distortion of the framework of the supported structure, and/or damage to claddings and finishes in the case of buildings. This stage may have been reached at a level of load such as that represented by point F in figure 2.34.
FIG. 2.32 PRESSURE-EXPANSION CURVES FROM SELF-DORING PRESSUREMETER TEST IN HEAVILY OVERCONSOLIDATED CLAY.

FIG. 2.33 PRESSUREMETER DERIVED SHAFT FRICTION VALUES.
(after Baguelin, Jézéquel and Shields, 1978).
Some authors, for example, Selby (1970), as discussed earlier, have defined the point represented by point E as that at which the rate of deformation \( \frac{d\text{load}}{d\text{settlement}} \) significantly changes. Slack and Walker (1970) define this point as that defining the change in slope of a log(load) versus log(displacement) plot.

The writer has related such considerations to the average stress-strain curve obtained from a series of triaxial tests associated with the experimental study reported herein, as discussed in Appendix 5. From Appendix 5 it can be seen that the point coinciding with the change in slope of the log-log plot represents a factor of safety on the ultimate strength of the soil of 1.33. This would then suggest that a factor of safety on deformation of 2 say, would then require a factor of safety on the ultimate strength, and thus by analogy, the ultimate axial load carrying capacity of the pile, of at least 2.66.

### 2.6.12 Allowable Loads on Piles

As discussed by Tomlinson (1977), a perfect design method for calculating allowable loads on piles would be one which predicted the load-deformation curve through all stages from initial loading to the point of ultimate failure. From such a predicted curve, the structural designer would then be able to distribute the load on to the piles to keep the deformation of the structure within tolerable limits. The foundation engineer would also then be able to satisfy himself that there was an adequate safety factor on the ultimate resistance to provide a safeguard against accidental overloading of the piles, and also allow for variations in the properties of the soil.

Given the present state-of-the-art, this cannot be done with any degree of confidence, unless of course full scale tests are employed.

As has been demonstrated there is still no reliable theoretical basis on which to assess the distribution of load between pile tip and shaft, and also accommodate the various influences of the method of pile installation.

In general then, the accepted approach to assessing the allowable axial loads on piles is to predict the ultimate resistance of the pile from a knowledge of the physical properties of the undisturbed soil. To this is applied an arbitrary factor of safety to obtain the allowable (or working) load. The value of the factor of safety so chosen depends on a number of considerations:

(a) The variability of the measured soil properties.
(b) The confidence, or otherwise, of the engineer in the empirical methods of predicting the ultimate pile resistance.
(c) The magnitude of pile movement considered acceptable at the working loads.

Tomlinson (1977) suggests from experience from a "very large number of loading tests taken to failure" of a variety of pile types in both clays and sands, that a factor of safety of 2.5 is likely to produce working load settlements of a maximum of 10mm. This view and recommendation is shared by Thorburn and Macnicar (1971).
FIG. 2.34 PILE LOAD SETTLEMENT CURVE FOR AXIAL LOADING TO COMRESSIVE FAILURE.
Both authors emphasise that such considerations should be used for preliminary design purposes only and should be confirmed by full scale testing. Notwithstanding the economics of a project, they recommend that the omission of pile testing should only be acceptable where the engineer has previous experience of pile behaviour in similar soil conditions.

For design purposes then, it would appear reasonable to suggest:

(i) Where pile test loading is intended to be carried out, or where the designer has considerable experience with a particular method of load assessment, method of pile installation, and in similar soil conditions, that a minimum factor of safety of 2.5 be used.

(ii) Where the recommended minimum factor of safety of 2.5 is used and load testing is omitted, that the structure be designed for potential settlements up to 10mm with some allowance being made for differential settlements based on both this potential settlement and any variation in observed soil conditions at a particular site.

(iii) Where the designer is lacking in adequate experience in all three of the factors listed in (i) above, and full scale tests are not likely to be carried out on the basis of economics or for some other reason, then the designer should provide for the criteria in (ii) above, and in addition use a minimum factor of safety of say 3 or greater, depending on both relative economics and the designer's confidence in rationally and reliably assessing the various factors involved.

It is believed that most design criteria will fall into this third category.

2.6.13 Conclusion

As can be seen from the foregoing review of the ultimate load carrying capacity of axially loaded piles, that in general, the only means available to the designer in assessing these ultimate loads are empirical relationships or semi-empirical formulae.

Since empirical formulae are, by definition, derived from observation and experiment, then the accuracy of the formulae is entirely dependent on the accuracy and scatter of the test data from which the formulae are derived.

Unfortunately, as can be seen from such figures as 2.4, 2.11, 2.12, and 2.13, which are typical of the nature of the results reported at some stage by almost all the authors quoted, the empirical data from which the currently available pile design correlations have been drawn can barely be classed as being well defined by close fitting curves.

Thus such design methods should be used with caution with the full utilisation by the designer of both experience and judgement.

Finally, it is re-emphasised that some authors, for example Thorburn and Mac Vicar (1971) recommend that a factor of safety of not less than 2.5 be used with such design methods, further, where possible these design methods should be used for preliminary design purposes only, with final design being based on full scale tests (Meyerhof, 1976).
2.7 ANALYTICAL DEVELOPMENTS TOWARDS THE ASSESSMENT OF THE LOAD-DISPLACEMENT CHARACTERISTICS OF AXIALLY LOADED PILES

2.7.1 Introduction

The preceding general review of the axially loaded pile problem, although being orientated towards consideration of the ultimate load carrying capacity of straight sided piles, does, however, highlight the generally extreme complexity of the problem of load transfer between the pile and soil. A number of the factors affecting the problem, for example the effects of pile installation, are virtually impossible to evaluate in numerical terms.

"Elastic" methods have been considered by a number of authors over a great period of years. For example, Morrison, as reported by Chellis (1951), proposed a method of assimilating load to axially loaded piles using the Boussinesq equations. Morrison's method was to assume that one quarter of the total pile load acted at each quarter element of the pile as indicated in figure 2.35(a). By using the Boussinesq equations to compute the distribution of pressure at the midpoint of each quarter element, the pressure bulbs indicated by figure 2.35(b) were obtained.

Vesic (1970), from a detailed study of the literature has broadly classified the approaches not depending on some semi-empirical ultimate bearing capacity theory into two broad methods of consideration:-

(1) The transfer function approach, and
(2) The elastic continuum approach.

These two fundamental approaches are indicated diagrammatically in figure 2.36.

2.7.2 The Transfer Function Approach

The transfer function approach (See figure 2.36(b)) divides the pile into a number of elements (n), each of which are considered as compressible short columns of length, 4L. The method of analysis is not dissimilar to the Modulus of Subgrade Reaction (Hinkler) method for analysis of laterally loaded piles, as discussed in Chapter 7: in that the method of load transfer to the soil, using the transfer function approach, is to replace the soil surrounding the pile with a set of non-linear springs supporting the pile at the mid-point of each element. These springs are entirely independent of adjacent springs. Each of the vertical compressible elements represented in figure 2.36(b) is acted upon by axial forces Q and shaft friction resistances, f.

Providing the distribution of axial force in the pile is known, then from simple statics the shaft resistances may be computed. Thus for element 'i', the shaft resistance \( f_i \) is given by:

\[
\frac{f_i}{p} = \frac{Q_i - Q_{i-1}}{\alpha L} = \frac{\alpha Q_i}{\rho_1 L}
\]

where \( p = \) the length around the pile perimeter.

Providing the modulus of elasticity and cross-sectional area of the pile shaft are known, \( E_p \) and \( A_p \) respectively, then the relative vertical displacements of the centroids of each of the elements can be computed from the simple Hookean relationship:-
where $\Delta L^* = \text{the change in length of } L \text{ in the general expression.}$

For the consideration of change in length of an elemental element, $\varepsilon$ may be replaced by:

$$\varepsilon_L = \frac{\Delta L}{L}$$

Thus the vertical displacements of the pile elements are given by:

$$\Delta z_i = \Delta z_{i+1} - \Delta z_i = \frac{0_i \Delta L}{E_p A_p}$$

Clearly then, a set of simultaneous equations can be derived which, providing the axial forces are known, and one boundary displacement known or assumed, permit the displacements to be determined along the pile. The forces $f_i$ acting on the pile are computed from the "transfer function" which may be an empirical or semi-empirical relationship of the form:

$$\Delta 0_i = 0_i - 0_i + 1 = \kappa (\Delta z_i)$$

This equation may be represented in the form of a diagonal matrix thus:

$$f_i = [\kappa](z_i)$$

The "transfer function" is intended to provide a unique relationship between the load transferred from an element and the displacement of that element. It contains also the hidden assumption that the displacements along any element are not affected by the loads transferred by other elements.

However, as with the Vinkler concept of a beam on a flexible foundation, the behaviour of neighbouring points do influence each other (Vesic, 1961), thus the concept of a unique transfer function is in obvious contradiction with reality. A number of transfer functions, as reported by Vesic (1970) and extended by the writer, are listed in Table 2.9.

### Table 2.9

**Typical Transfer Functions**

<table>
<thead>
<tr>
<th>Source</th>
<th>Transfer Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seed and Reese (1955)</td>
<td>Experimentally determined values $\kappa(x, y, z, \theta)$</td>
</tr>
<tr>
<td>Kezdi (1957)</td>
<td>Experimentally determined values $\kappa(x, y, z, \theta)$</td>
</tr>
<tr>
<td>Reese (1964)</td>
<td>Experimentally determined values $\kappa(x, y, z, \theta)$</td>
</tr>
<tr>
<td>Coyle and Reese (1966)</td>
<td>Experimentally determined values $\kappa(x, y, z, \theta)$</td>
</tr>
<tr>
<td>Reese, Hudson and Vijayvergiya (1969)</td>
<td>Experimentally determined values $\kappa(x, y, z, \theta)$</td>
</tr>
<tr>
<td>Parker and Reese (1970)</td>
<td>Experimentally determined values $\kappa(x, y, z, \theta)$</td>
</tr>
</tbody>
</table>

In general these transfer functions are based on full scale pile tests.
FIG. 2.35 MORRISON'S SOIL PRESSURE DISTRIBUTION ABOUT AXIALLY LOADED FRICTION PILE. (after Chellis, 1951).

FIG. 2.36 PILE-SOIL LOAD TRANSFER ANALYSIS. (after Vesic, 1970).
2.7.3 The Elastic Continuum Approach

The elastic continuum approach overcomes the inconsistency evidenced in the transfer function method in that the effects of transmitted shaft loads on pile elements both above and below the point being considered, are provided for. The concept fundamental to this method is that the soil is a homogeneous, elastic, isotropic continuum defined by the modulus of elasticity, \( E_s \), of the soil, and its Poisson's ratio \( \nu_s \) and \( \nu_s \) respectively. The conceptual arrangement is shown in figure 2.3K(a). It is generally assumed that \( E_s \) and \( \nu_s \) are unaffected by the presence of the pile.

In the analysis the soil and pile displacements are evaluated and equated at the element centres. The analysis further assumes that when the pile is loaded and thus displaces in the vertical direction, the elements of the pile remain rigid and thus incompressible. The element displacement are obtained in general by evaluating Mindlin's equations for the displacements due to a point load within a semi-infinite mass.

The fundamental assumption that the pile remains rigid requires that the displacements of all the pile elements are the same. The soil elements are then represented by a set of equations of the form:-

\[
(\Delta_s)_i = \frac{D}{E_s} \sum_{j=1}^{n} I_{pj} f_j + I_{po} q_p
\]

and a general equation for the pile displacement thus:-

\[
(\Delta_p) = \frac{D}{E_s} \sum_{j=1}^{n} I_{pj} f_j + I_{np} q_p
\]

In matrix form then:-

\[
\begin{bmatrix} \Delta_s' \\ \Delta_s'' \end{bmatrix} = C \begin{bmatrix} I^* \\ I^{**} \end{bmatrix} \{f\}
\]

\[
\begin{bmatrix} \Delta_p' \\ \Delta_p'' \end{bmatrix} = C \begin{bmatrix} I^* \\ I^{**} \end{bmatrix} \{q\}
\]

where \( C \) = the elastic constant = \( D/E_s \)

\( I^* \) and \( I^{**} \) = elastic influence factors.

Thus:  \( \Delta_s = \Delta_s' + \Delta_s'' \) = total elemental soil displacements

Similarly:  \( \Delta_p = \Delta_p' + \Delta_p'' \) = total elemental pile displacements

In a general representation then:-

\[
\begin{bmatrix} \Delta_s \\ \Delta_p \end{bmatrix} = C \begin{bmatrix} I^* \\ I^{**} \end{bmatrix} \{F_s\}
\]

where \( F_s \) = the combined soil resistance due to shaft and end friction.

Thus by requiring displacement compatibility at the pile-soil interface, \( \Delta_p = \Delta_s \), the forces acting on the soil can be determined. Slip between pile and the soil, \( \Delta_p \neq \Delta_s \), is able to be accommodated by limiting the friction term.

A number of methods are listed in table 2.10.
Even though the elastic continuum method has the fundamental advantage that the influence of adjacent points is considered it has the limitation of the assumption that the soil response to loading can be described by only two parameters \(E_s\) and \(u_s\), which may however, be varied with depth.

Poulos (1972) correlates various values of \(E_s\), back figured from reported test results, with various field testing and soil strength parameters.

Mitchell and Gardner (1975) provide correlations to enable the determination of \(E_s\) from a range of in-situ testing devices including, SPT and CPT tests and the Ménard pressuremeter.

The results obtained from such correlations are open to considerable interpretation as indicated by the large range of values suggested by various authors in Chapter 7.

### 2.7.4 Finite Element Analyses

The potential exists for finite element analyses to overcome most of the shortcomingsof the elastic continuum approach. For example Ellison (1969), as reported by Vesic (1970), developed a finite element method for the general analysis of an arbitrary pile in a soil mass with a non-linear stress-strain response. The analysis permitted the introduction of stress and displacement conditions imposed by the method of pile installation, and considers the fact that the pile presence in the soil affects the stress distribution.

Desai (1974) proposed a finite element method of analysis for the determination of the load-deformation behaviour of axially loaded piles in sands. His results are generally encouraging, however, the effects of pile installation still cannot be handled numerically, principly because they cannot be quantified. Desai's analysis sets the critical stresses in the soil and the pile at the same value.

A further limitation is that the dilational characteristics of cohesionless materials cannot readily be accommodated.
A finite element analysis for an axially loaded pile in which the pile and soil were analysed as separated bodies, with equilibrium and displacement compatibility being imposed at the pile soil interface, was presented by Ralaam, Poulos and Brooker (1975). The authors used Meyerhof's (1959) results to assess the effective increase in the elastic modulus of the soil \( E_s \) due to pile installation, as indicated in figure 2.37. (The results obtained by Meyerhof have been commented upon earlier).

Whilst the work of Ralaam et al is encouraging, the writer finds it surprising that their analysis leads them to conclude that the load settlement behaviour of a driven pile in sand is not significantly different from that which would be assessed using undisturbed soil modulus \( E_s \) values.

Randolph and Wroth (1978) have presented an approximate closed form solution to the pile load transfer problem and have presented a set of design aids which provide apparent good agreement with the various numerical methods of analysis discussed in the preceding three sections. This work represents a further significant analytical advance, but has yet to be substantiated as having any validity when related to the mechanics of the real problem.

2.8 CONCLUSION

In the course of some 30 years we have seen the concepts of bearing capacity theories first presented as a justifiable solution to deep foundation problems, (Terzaghi, 1943, Meyerhof, 1951), only to see their validity eroded away with the improved understanding that comes with time, until now (1979), a reliable assessment of the ultimate load carrying capacity of axially loaded piles of any significant depth in cohesionless materials, would appear to be entirely dependent upon full scale pile load tests (or possibly large scale model pile field tests). The problem is compounded when non-homogeneous and layered soils are taken into account. Attempts to rationalise the ultimate bearing capacity considerations for a variety of soil conditions are given by Meyerhof and Valsangkar (1977), and Meyerhof and Sastry (1978a and b).

Given the present state-of-the-art then the most reliable method of assigning axial loads to piles unquestionably involves full scale load tests. Even though serious doubt has been attached to the admissability of bearing capacity theories based on the concepts of classical soil mechanics and correlations with in-situ penetrometer tests, such methods still tend to be the most widely used methods of analyses.

Pure elastic methods of analysis are questionable with regard to their practical significance, at least as far as driven piles are concerned, however, it is felt that the potential exists for finite element methods of analysis to ultimately produce an analytical method which will satisfy all the presently uncoordinated aspects of the pile problem.

Pile driving formulae are fairly subjective and are tending to lose their validity with the developments in modern pile driving equipment.

The writer's understanding of the inter-relationship of the present analytical methods and understanding of the axial loaded pile problem is shown diagrammatically in figure 2.38. The broken lines in this figure indicate where cross correlation is presently employed or could ultimately be achieved.
FIG. 2.37 EFFECTS OF PILE INSTALLATION IN SAND AS SIMULATED BY BALAAM, POULOS AND ROOKER (1975).

FIG. 2.38 INTER-RELATIONSHIP OF THE VARIOUS METHODS OF AXIAL PILE LOAD ANALYSIS.
It is projected, as indicated in figure 2.38, that the finite element methods of analysis will ultimately provide the analytical link between the axial and laterally loaded pile problems, which at present tend to be treated as two separate and unrelated phenomena, finally providing a set of simplified design rules.

Such developments are unlikely to take place for some time in the future as the integration of the whole pile problem is dependent upon a thorough understanding of the mechanics of pile-soil interaction. As has been demonstrated by the review presented in this chapter, such an understanding has not yet evolved.

Much analytical work (as indicated by the references throughout this thesis), has been done on assigning techniques of varying degrees of sophistication to the load carrying capacity of the foundation system (i.e. pile-soil system).

In contrast the amount of research effort assigned to understanding the mechanics of the phenomena (i.e. pile-soil interaction) is recognised only by its scarcity.

Having come to terms with the complete static problem (i.e. axial and lateral loading), ideally the dynamic problem could then be investigated with some measure of confidence.

In practical terms however, research, because of the continuing needs of the designer, is multi-faceted, resulting in all aspects of a problem tending to be considered simultaneously. This method, whilst serving the designer's needs, results in a relatively uneconomic technological advance in that, because all aspects of a problem are involved, gross assumptions are often made, ultimately resulting in many researchers covering the same ground over a long period of time before significant advances are realised.
Chapter 3  Displacement Fields Developed About Model Piles During Installation By Driving

3.1 INTRODUCTION

Chapter 2 discussed the various methods of assessing the axial load carrying capacity of piles and presently available means of considering the effects of pile installation by driving. It is generally considered that the problem is extremely complex with a potentially large number of variable conditions.

The effects of pile-soil interaction can be broadly subdivided into the two fundamental soil types:-

(i) Cohesive soils, and
(ii) Cohesionless soils.

From an experimental viewpoint, granular materials are much easier to work with in the laboratory as problems associated with the preparation of cohesive soil samples, such as consolidation and repeatability are minimised. In the study reported in this and following chapters attempts have been made to quantify displacement and strain fields developed in the soil about driven model piles in dense sand throughout the complete process of pile installation by driving and subsequent lateral loading.

The literature abounds with the reports of model pile tests, with the piles being installed by a variety of methods. It would appear that most model tests involve buried foundations (i.e. the soil being placed around the prepositioned piles), or alternatively the pile is installed into the ground by pushing (i.e. jacking). Comparatively few model tests appear to have been conducted on piles installed by driving.

In contrast a considerable number of piles used in engineering works, especially in cohesionless materials, are installed by driving. For the model piles studied the process of pile installation by driving has been employed.

Various researchers have attempted to observe the mechanics of pile-soil deformation in response to the application of axial loads. For example Rerezantzev and Yaroshenko (1957) used still and ciné photography in conjunction with boxes with glass walls (see figure 2.19, Chapter 2).

Vesic (1963, 1967) used layers of sand impregnated with coloured cements. After loading the pile water was added which, after setting of the cement, allowed the hardened blocks to be cut through and the deformation patterns observed.

In this chapter soil displacements have been studied throughout the process of pile installation using the independent techniques of stereophotogrammetry and radiography, as discussed in Appendices 1 and 7.
The displacement fields obtained using the photogrammetric technique employ model half piles constrained to displace against a glass plate representing a neutral axis of the pile. The radiographic technique involves measuring the displacements of a plane of lead shot placed in the plane of a neutral axis of a "full" model pile. This technique was used in part to assess the influence of the glass plate on the displacement fields obtained using the model half piles. Both sets of displacement information were found to be in good agreement.

3.2 SIZE OF TEST CELL

Gisbourne (1970) correlated the test cell sizes of 18 researchers in an attempt to ascertain a reasonable size for a test cell for his model penetrometer tests. His correlations, including the ratios he chose himself, are reproduced in figure 3.1. The cross hatched band on this figure is the range of maximum observed effects reported by Robinsky and Morrison (1964) and Broms (1966) and is in close agreement with observations reported later by the writer (See table 3.1).

It should be noted that figure 3.1(a) in fact shows the depth of sand beneath the model pile when fully installed.

The significant dimensions of the test cells actually employed by the writer are also indicated on this figure. The cell used to obtain the preliminary displacement fields from stereo-photographs was undersize with respect to depth beneath the base of the model pile when fully installed. However, as discussed in Appendix 7, it was necessary to keep the size of the test cell to an absolute minimum to facilitate its use with the Radiography equipment available to the writer.

3.3 A QUALITATIVE ASSESSMENT OF THE DISPLACEMENT FIELDS DEVELOPED ABOUT MODEL PILES INSTALLED BY DRIVING

Figures 3.2(a) and (b) show respectively contours of vertical and horizontal displacements, and the associated vector field of displacement developed as a result of pile installation by driving into dense dry sand. These diagrams were obtained using the stereo-photogrammetric technique discussed in Appendix 1. They compare the undisturbed soil mass prior to pile installation with that existing after the pile has been fully driven to the depth shown.

Clearly the mass of soil which is influenced by the pile penetration is quite large and can be seen to extend to a number of pile diameters in radius about the pile and in depth beneath it.

Robinsky and Morrison (1964) employed radiographic techniques to study the displacement and compaction of sand around model piles.

In their model test series Robinsky and Morrison pushed rather than drove their piles into place. The initial sand densities studied ranged from loose to medium dense. The shape of the zone of influence obtained from their model studies is shown, for loose sand, in figure 3.3. The largest deformations appear to be occurring beneath the pile tip.

The tendency for the disturbed zone to reduce in diameter close to ground level has not been observed by the writer in comparisons with either loose or dense sands (for example figures 1.7 and 1.8) Appendix 1. It is difficult to conceive that this is a manifestation of the difference between
FIG. 3.1  SIZE OF TEST CELLS FROM 18 AUTHORS AS COLLATED BY GISBOURNE (1970).
FIG. 3.2 DISPLACEMENT AND VECTOR FIELDS ABOUT MODEL PILE INSTALLED BY DRIVING IN DENSE DRY SAND.

FIG. 3.3 ZONES OF INFLUENCE AND DEFORMATION IN LOOSE SAND
(After Robinsky and Morrison, 1964).
installation by driving and installation by pushinng. The writer would agree with the authors however, that the zone of influence beneath the pile tip appears to approximate the surface of a section of a sphere. (This consideration is discussed in more detail subsequently).

Robinsky and Morrison also comment that no shear failure planes were observed. A similar observation has been made by the writer, and is supported by the experimental data presented in this chapter.

Figure 3.4(a) is a "double exposure" photograph of a model half-pile driven into dense sand to a depth of embedment to pile diameter ratio \((\frac{h}{D})\) of about 6. Figure 3.4(b) is a vector displacement field obtained from the same stereo-pair that were used to obtain figure 3.4(a). The boundary to the displacement field has been defined as the "apparent limit of zero observable displacements". No shear zone as such is able to be identified, rather, the displacements simply gradually drop off until they cannot be measured with any degree of reliability. The position where this occurs has been defined as the limit to the displacement field. In Chapter 4 it will be shown from shear strain considerations, that within a little over one pile diameter of this apparent boundary the soil is at or near the "ultimate" state of stress.

It can clearly be seen in figure 3.4 that a zone of "drag down" exists about a pile. (The influence of pile roughness on this zone is discussed in Chapter 5). Robinsky and Morrison deduced that this drag down had a loosening effect resulting in a thin sleeve of loose sand about the pile. (The apparent state of density in this zone is discussed with respect to calculated volumetric strains in Chapter 4). They suggest that a cylinder of dense soil, originally contacted by the pile tip, encircles the loosened sand and prevents by arching the development of the full lateral earth pressure on the pile. Thus they contend that little load transfer can occur by friction from a pile to the surrounding soil mass.

Figure 3.5 shows contours of equal relative density, developed during pile installation, as obtained by Robinsky and Morrison in loose sand. These results indicate the apparently erratic development of high and low density areas. Although the writer reports an irregular development in volumetric strains in Chapter 4, the pattern is quite different from that suggested by Robinsky and Morrison. This could be due to the fact that both the initial soil density and method of pile installation employed by them differs from that of the writer.

Figure 3.6 shows the displaced shot images for a pile installed to an embedment depth of about 80 and was obtained by superimposing traces of lead shot locations from radiographs taken prior to pile installation and with the pile installed. Clearly the nature of the displacements are similar to those obtained from the photogrammetric technique shown in figure 3.2.

Roms (1966) suggested that the relative density of the soil increased close to the pile during driving in the areas indicated in figure 3.7.

This diagram infers that all the volume change that occurs about a pile is compaction, as indicated by the ground settlement line. Whilst this holds for driven piles in loose sand (figure 31.7), heave clearly occurs about piles driven into dense sands as indicated by figure 3.2(b), at least up to
FIG. 3.4 PHOTOGRAF AND VECTOR FIELD OF SOIL DISPLACEMENTS FOR INCREMENTAL PILE INSTALLATION BY DRIVING.

FIG. 3.5 CONTOURS OF EQUAL RELATIVE DENSITY (%) (After Robinsky and Morrison, 1964).
**Fig. 3.6** TOTAL DISPLACEMENT FIELDS OBTAINED BY SUPERIMPOSING RADIOGRAPHS.

**Fig. 3.7** COMPACTION OF COHESIONLESS SOILS DURING DRIVING OF PILES

*(After Broms, 1966)*.
embedment depths, \( \frac{1}{2} \), of about 6. Thus the surface profile for dense sands should be similar to the profile ABC superimposed by the writer on Figure 3.6.

The limits to the zones of soil modified by the driving process indicated by Broms in Figure 3.6 agree favourably with both those obtained by the writer, and those recorded by Robinsky and Morrison (1964), as indicated in Table 3.1.

### Table 3.1
**Limits of the Zone of Soil Modified by Pile Driving**
**in Dense Sand**

<table>
<thead>
<tr>
<th>RESEARCHER</th>
<th>ZONE DIAMETER</th>
<th>DEPTH PNEATH PILE TIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broms</td>
<td>7 to 120</td>
<td>3 to 50</td>
</tr>
<tr>
<td>Robinsky and Morrison</td>
<td>9 to 110</td>
<td>3 to 4.5D</td>
</tr>
<tr>
<td>(Radiographs)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goldsmith</td>
<td>8 to 120</td>
<td>3 to 50</td>
</tr>
<tr>
<td>(Photographs and Radiographs)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 3.1, D represents the pile diameter, as indicated in Figure 3.7.

Figure 3.2 shows the nett soil particle displacements obtained from displacements observed between the undisturbed state and that when the pile is fully installed.

As can be seen from Figure 3.4, which shows the displacements obtained for about a 10 increment of pile penetration, the soil particles follow incremental paths which are distinctly curved throughout the pile installation process (as supported by Berezantzev and Yaroshenko, 1957, Figure 2.10, Chapter 2). Total displacement paths for particular soil elements, comprising a number of incremental displacements, are indicated by the paths a to b and c to d in Figure 3.2(b).

Clearly the nett vector paths (e to e' in Figure 3.2(b)) do not give a true representation of the total soil deformations. It is thus contended that an accurate indication of the changing soil state can only be obtained by considering the relative positions of the soil particles at regular stages throughout the pile installation process. For this reason the subsequent study is made on piles installed at about 10 increments of pile penetration, thus enabling a more reliable assessment of the total soil deformations to be made (Section 3.5).

### 3.4 The Accommodation of the Volume of a Displacement Pile Installed into Sand by Driving

Figures 3.8(a) and (b) show contours of horizontal and vertical displacement and vector fields of displacement respectively for a model pile driven into loose sand using the stereophotogrammetric technique of Appendix 1. These figures were obtained by comparing a photograph of the undisturbed soil mass with that of the pile installed to the depth shown. The displacements are consequently nett displacements and don't necessarily reflect the actual trajectories followed by the soil particles. Of significance is the change in surface level. Clearly the effect of pile installation into the loose sand is to cause compression of the soil mass and indicates that this compression is in fact greater than
**FIG. 3.8** DISPLACEMENT AND VECTOR FIELDS ABOUT MODEL PILE INSTALLED BY DRIVING IN LOOSE DRY SAND.

**FIG. 3.9** DISPLACEMENT FIELDS FOR $\frac{D}{d}$ OF 8.0 AND 9.5.
the installed volume of the pile.

The displacement fields of figures 3.9(a) and (b) are for model piles driven into dense sand. In contrast to figure 3.8, surface heave is clearly occurring. As was the case for figure 3.8, the fields of figure 3.9 also represent nett displacements, thus the change in surface level is the nett change induced in installing the piles to the depths shown. The pile in figure 3.9(a) is installed to an embedment depth $h$ of 8 whilst that of figure 3.9(b) is at an $h$ of 9.5.

A diagrammatic representation of the vertical displacement occurring within the sand mass is indicated in figure 3.10 where (+) signs indicate upward vertical movement whilst (-) signs indicate downward. In this figure a-a is a free surface, thus volume change due to the combined effects of pile penetration and expansion or compaction of the soil mass is manifested in an upward movement of the soil surface.

Commencing at the plane h-b and extending through the plane c-c, the vertical displacements are downwards, but are not necessarily indicative of compaction. This is due to the complex interaction between both the horizontal displacements and the out of plane (circumferential displacements), which all contribute to the nett change in volume of a particular soil element.

Figures 3.11 and 3.12 show the measured changes in vertical displacement for the displacement fields of figures 3.9(a) and (b) respectively. In these figures the distance out from the pile centreline (horizontal axis) has been normalised with respect to the radius of the pile, and thus expressed as the ratio: $(r/R)$. The displacements on an intermediate plane, (designated curve 2), between those on plane a-a and h-b, (Curves 1 and 3 respectively), have been indicated on figure 3.11.

Clearly the geometric form of the vertical displacements are similar right throughout the displacement field, with the nett vertical displacements manifested at the surface level being indicated by curve 1.

Curve 4 in figure 3.11 indicates the vertical displacements occurring about the pile tip. In figure 3.12 only the curves equivalent to 1 and 4 in figure 3.11 are shown.

Total volume changes were obtained by numerical integration of the change between the initial and final surface profiles, the results of which are compared with the embedded pile volumes in figure 3.13. The line OA in figure 3.13 represents the locus where the embedded pile volume $V_p$ is equal to the volume change measured from the soil surface profile $V_s$. Above line OA, $V_p > V_s$; below OA, $V_s > V_p$.

Hence if the compaction within the soil mass is greater than the volume of soil displaced by the pile, the appropriate point will lie above OA. If soil expansion is significant, the reverse applies.

It would thus appear, notwithstanding the seeming crudity of the comparison, that the greater proportion of the volume change in medium dense dry sand necessary to account for the volume of soil
FIG. 3.10 DIAGRAMMATIC REPRESENTATION OF SOIL VOLUME CHANGES.

FIG. 3.11 VERTICAL DISPLACEMENT PROFILES FOR R OF 8.0.
FIG. 3.12 VERTICAL DISPLACEMENT PROFILES FOR $L/D$ OF 9.5.

FIG. 3.13 COMPARISON BETWEEN EMBEDDED PILE VOLUME AND CALCULATED SOIL VOLUME CHANGE
FIG. 3.14 SIMPLIFIED CUMULATIVE VOLUMETRIC STRAIN FIELD AT 5D PENETRATION BY DRIVING INTO DENSE DRY SAND.
displaced by the pile, is provided by the upward change in the soil surface profile (i.e. is manifested in surface heave) at least for the installed pile depths considered.

Figure 3.14 shows a simplification of cumulative volumetric strains developed in driving the model pile to a depth of 50 as subsequently discussed in Chapter 4. A complex arrangement is developed where some zones are compacting whilst others are expanding.

The results reported in this thesis only consider pile penetrations up to about 100 (i.e. \( \frac{h}{D} = 10 \)), beyond this depth and at increasingly greater depths it would seem probable that the effects of volume changes within the soil mass would become increasingly significant.

This would then infer that at great depths the compaction within the soil is likely to achieve such proportions that it is able to account for the additional pile volume. This view is supported by observations made by Gishbourne (1971) from model tests with "piles" pushed into dense sand. Gishbourne found that after about 100 pile penetrations there was little change in the heaved surface profile.

It would then appear reasonable to expect this compaction zone to extend inwards with depth. A tendency for the zone of compaction to increase with depth is indicated from the strain analysis discussed in Chapter 4.

3.5 INCREMEHTAL DISPLACEMENT FIELDS DEVELOPED AROUN MDISPLACEMENT PILES INSTALLED BY DRIVING

3.5.1 Displacement Fields Obtained Using Stereo-Photogrammetry

In section 3.4 figures 3.9(a) and (b) were compared. Both diagrams showed total displacement fields obtained about driven model half-piles by comparing stereo-pairs of photographs consisting of a plate of the undisturbed soil mass (i.e. prior to pile installation), and the plate obtained after the model pile had been installed to the depths shown. The method for obtaining both displacement fields was that described in Appendix 1.

Clearly the displacement fields so obtained follow the same general pattern.

The difference between figures 3.9(a) and (b) is that the contours of figure 3.9(a) were obtained manually by the writer, using a Wild ST4 portable mirror stereoscope and parallax bar with micrometer, after having had considerable exposure to the technique and the nature of the displacement fields likely to be developed. In contrast the displacement fields of figure 3.9(b) were obtained by an operator with no knowledge of what to expect. This second diagram was plotted by Major Peter Hunt (Retd) of the Department of Surveying, University of Otago, on their Galileo Stereoscopicmometer SM63. Only the displacement fields obtained from the stereonlotter are subsequently reported.

The displacement fields obtained about model timber half-piles of 19.5mm diameter, driven into dense sand placed using the showering technique described in Appendix 4, are presented in figures 3.15 to 3.21 for embayment depths \( \frac{h}{D} \) ranging from 1.33 to 8.82.

Figure 3.22 shows the pile driven to an \( \frac{h}{D} \) of 10.22. Clearly the vertical displacement contours beneath the pile tin are being influenced by the rigid boundary presented by the base of the test cell, as
FIG. 3.15 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 1.33.

FIG. 3.16 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 2.46.

FIG. 3.17 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 3.74.
FIG. 3.18 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 4.92.

$\frac{L}{D} = 4.92$

FIG. 3.19 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 6.05.

$\frac{L}{D} = 6.05$
FIG. 3.20 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 7.39.

FIG. 3.21 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 8.82.
FIG. 3.22 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 10.22.

FIG. 3.23 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{D}$ OF 1.33
was discussed earlier with reference to figure 3.1. The average increment of pile penetration for figures 3.15 through 3.22 is 1.28D. The widths of the zone of disturbance about the pile and depth beneath the pile tip throughout the installation process, expressed in terms of pile diameter (D), are shown in table 3.2.

<table>
<thead>
<tr>
<th>L/D</th>
<th>WIDTH FROM PILE CENTRELINE (m)</th>
<th>DEPTH BENEATH PILE TIP (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.33</td>
<td>4.24</td>
<td>1.28</td>
</tr>
<tr>
<td>2.46</td>
<td>5.9</td>
<td>4.36</td>
</tr>
<tr>
<td>3.74</td>
<td>5.04</td>
<td>4.47</td>
</tr>
<tr>
<td>4.92</td>
<td>4.15</td>
<td>4.4</td>
</tr>
<tr>
<td>6.05</td>
<td>4.0</td>
<td>2.6</td>
</tr>
<tr>
<td>7.39</td>
<td>4.15</td>
<td>3.0</td>
</tr>
<tr>
<td>8.82</td>
<td>4.26</td>
<td>2.7</td>
</tr>
<tr>
<td>10.22</td>
<td>3.6</td>
<td>-</td>
</tr>
</tbody>
</table>

The average zone dimensions, from Table 3.2, are:

(i) Average width from pile centreline = 4.42D
    (Total width of field = 8.84D)
(ii) Average depth beneath pile tip = 3.26D

These average values of the displacement field dimension of total width 8.84D and depth below the pile tip of 3.26D are in agreement with the values quoted in table 3.1.

Fields of vector direction and contours of equal displacement are shown in figures 3.23(a) to 3.29(a) for embedment depths (h) of 1.33 to 8.82. These diagrams are the resolution of the horizontal and vertical contours of displacement for the approximate embedment depths of figures 3.15 to 3.21.

The vector directions shown in these diagrams indicate the resultant directions in which the soil is deforming, whilst the contours of equal displacement represent lines joining points defining elements of soil which have experienced vectoral displacements of magnitude defined by the contour.

The associated vector fields of displacement are presented in figures 3.23(b) to 3.29(b). As a qualitative check on the validity of these displacement fields, the stereo pairs of photographic plates used to determine the displacement fields were superimposed to produce "double exposure" photomontages at various stages of pile installation. These qualitative vector fields along with the experimentally determined vector fields for pile embedment depths (h) of 1.33, 2.46 and 6.05 are presented in figures 3.30(a) and (b), 3.31(a) and (b), and 3.32(a) and (b) respectively. The agreement would seem to indicate the absence of gross errors in the measured fields.

At all stages throughout the driving process there appears to be a "drag-down" zone adjacent to the pile wall of width varying from about 2.5D at the surface level. Apart from a shallow zone close to the surface, this drag-down zone appears however to be generally less than 10D in width.
FIG. 3.24 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{H}$ OF 2.46.

FIG. 3.25 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{H}$ OF 3.74.
FIG. 3.26 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{R}$ OF 4.92.

FIG. 3.27 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{R}$ OF 6.05.
FIG. 3.28 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR \( \frac{1}{5} \) OF 7.39.

FIG. 3.29 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR \( \frac{1}{5} \) OF 8.82.
FIG. 3.30 QUALITATIVE AND EXPERIMENTAL VECTOR DISPLACEMENT FIELDS FOR $\frac{h}{b}$ OF 1.33.

FIG. 3.31 QUALITATIVE AND EXPERIMENTAL VECTOR DISPLACEMENT FIELDS FOR $\frac{h}{b}$ OF 2.46.
FIG. 3.32 QUALITATIVE AND EXPERIMENTAL VECTOR DISPLACEMENT FIELDS FOR $L/D$ OF 6.05.
This zone appears to extend the full embedded depth of the pile to the pile position from which driving is recommenced at each increment of installation. Unfortunately this zone, in being so close to the pile wall, is in an area where displacements were difficult to measure, particularly using the stereo-photogrammetric technique.

Probably the most significant feature of the displacement fields presented in this section is the nature of the contours of equal vectorial displacement shown in figures 3.23(a) to 3.29(a). Clearly from a pile embedment depth \( R \) of approximately 2 right through to the end of pile installation, there is a remarkable uniformity in the shape defined by these contours. With comparatively minor variation the incremental 1D pile penetration develops contours of equal displacement which suggest the expansion of a spherical cavity. This observation is supported by the qualitative displacement fields of figures 3.30, 3.31 and 3.32. This concept is expanded in Chapter 5.

3.5.2 Displacement Fields Obtained Using Radiography

In the preceding section the displacement fields developed about a model half pile driven into dense sand, obtained using the stereo-photogrammetric technique of Appendix 1, were presented. Certain characteristic features of the displacement fields were identified and discussed. In this section a more rigorous method of obtaining the displacement fields is employed in an attempt to confirm the existence of these features. The radiographic technique used to obtain the displacement fields presented in this section is described in Appendices 7 and 8.

The size of the test cell used for these experiments is indicated in figure 3.1.

The displacement fields obtained about model timber piles of 10.5mm diameter driven into dense dry sand are presented in figures 3.33(a) to 3.41(a) for embedment depths ranging from 1.26 to approximately 9. In view of the symmetrical nature of the displacement fields indicated by those obtained using the photogrammetric techniques (figures 3.15 to 3.21) only the displacements developed about the left hand side of the pile centralines have been measured.

The continued axial symmetry of these displacement fields is indicated in figures 3.33(b) to 3.41(b). These figures show the symmetrical nature of the complete displacement fields and were obtained by superimposing traces of the lead shot images from each pair of radiographs. The approximate increment of pile penetration for figures 3.33 through 3.41 is 1m. During the installation process the piles were set to be installed in 1D increments, however, the actual incremental penetration achieved was not able to be measured because the poor contrast of the radiograph rendered it virtually impossible to define the ground surface level with any degree of reliability.

The widths of the zone of disturbance about the pile and depth beneath the pile tip throughout the installation process, expressed in terms of pile diameter (D), are shown in table 3.3.
DISPLACEMENTS:
(m.m) Horizontal — 1.0 —
Vertical — — 2.0 —

(c) Displacement Field
(b) Lead Shot Trace

FIG. 3.33 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 1.26.

FIG. 3.34 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 2.
Approximate surface level

Pile position prior to driving

\( \frac{L}{D} = 3 \)

(a) Displacement Field

(b) Lead Shot Trace

**FIG. 3.35** DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 3.

DISPLACEMENTS:

(mm) Horizontal --- 1.0 ---

Vertical --- 2.0 ---

Approximate surface level

Pile position prior to driving

\( \frac{L}{D} = 4 \)

(a) Displacement Field

(b) Lead Shot Trace

**FIG. 3.36** DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 4.
Approximate surface level

Pile position prior to driving

\[ \frac{L}{D} = 5 \]

3.4 D

(a) Displacement Field

(b) Lead Shot Trace

FIG. 3.37 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 5.

Approximate surface level

Pile position prior to driving

\[ \frac{L}{D} = 6 \]

3.23 D

5.3 D

(a) Displacement Field

(b) Lead Shot Trace

FIG. 3.38 DISPLACEMENT FIELD FOR \( \frac{L}{D} \) OF 6.
Approximate surface level

(a) Displacement Field

Pile position prior to driving

D = 19.5 mm

(b) Lead Shot Trace

DISPLACEMENTS:
(mm) Horizontal — 1.0 —
Vertical — 2.0 —

Approximate surface level

(a) Displacement Field

Pile position prior to driving

D = 19.5 mm

(b) Lead Shot Trace

FIG. 3.39 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 7.

FIG. 3.40 DISPLACEMENT FIELD FOR $\frac{L}{D}$ OF 8.
APPROMIMATE SURFACE LEVEL

Pile position prior to driving

(a) Displacement Field

(b) Lead Shot Trace

FIG. 3.41 DISPLACEMENT FIELD FOR \( \frac{b}{D} \) OF 9.
Approximate surface level

FIG. 3.42 LEAD SHOT TRACE FOR $\frac{L}{D}$ OF 10.
TABLE 3.3
DIMENSIONS OF DISPLACEMENT FIELDS
(from Radiographic results)

<table>
<thead>
<tr>
<th>L</th>
<th>WIDTH FROM PILE CENTRELINE (D)</th>
<th>DEPTH BENEATH PILE TIP (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26</td>
<td>4.8</td>
<td>1.7</td>
</tr>
<tr>
<td>2</td>
<td>7.09</td>
<td>3.04</td>
</tr>
<tr>
<td>3</td>
<td>4.78</td>
<td>3.13</td>
</tr>
<tr>
<td>4</td>
<td>4.89</td>
<td>2.98</td>
</tr>
<tr>
<td>5</td>
<td>5.17</td>
<td>3.4</td>
</tr>
<tr>
<td>6</td>
<td>5.3</td>
<td>3.23</td>
</tr>
<tr>
<td>7</td>
<td>5.28</td>
<td>2.55</td>
</tr>
<tr>
<td>8</td>
<td>4.79</td>
<td>3.15</td>
</tr>
<tr>
<td>9</td>
<td>5.55</td>
<td>3.4</td>
</tr>
</tbody>
</table>

The average zone dimensions from Table 3.3, are:

1. Average width from pile centreline = 5.29D
   (Total width of field = 10.58D)

2. Average depth beneath pile tip = 2.95D

These average values of the displacement field dimensions agree well both with those obtained by other researchers as reported in table 3.1 and those obtained using the stereo-rhotogrammetric technique presented in table 3.2 (viz. 4.42m, 8.84D and 3.26D respectively). Considering that the limit of zero displacement tends to be subjective and somewhat difficult to determine, the correlation is considered reasonable.

Figure 3.42 presents the displacement field for an embedment depth \( \left( \frac{h}{A} \right) \) of about 10, obtained in the same manner as figures 3.33(b) to 3.41(b). Unfortunately, as discussed in Appendix 7, the final radiograph making up this pair (FPD10) was accidentally destroyed in processing. Before the loss was realised the X-ray source had been shifted to check the "in-plane" characteristics of the displacement fields, thus it could not be reproduced. Figure 3.42, however, was able to be obtained by superimposing the base radiograph for the lateral loading series (FPD00) with the last of the driving series (FPD09). Because the projection of the reference points was no longer coincident, the radiographs were brought into coincidence using the undisturbed shot images on the boundaries. The radiograph FPLL00 is identical to the destroyed radiograph FPD10 in all aspects except in the location of the reference points.

Notwithstanding this, the characteristics displayed in the displacement fields of figures 3.33 to 3.41 are clearly being perpetuated in figure 3.42 (i.e. at an \( \frac{h}{A} \) of 10).

The fields of vector direction and contours of equal displacement derived from the displacement fields of figures 3.33 to 3.41 are shown in figures 3.42(a) to 3.51(a). The associated vector fields of displacement are presented in figures 3.43(b) to 3.51(b).
Fig. 3.43 Vector directions and contours, and vector field of displacement for $\frac{L}{D}$ of 1.26.

Fig. 3.44 Vector directions and contours, and vector field of displacement for $\frac{L}{D}$ of 2.
FIG. 3.45 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{D}$ OF 3.

Pile scale:
(mm) 10 20 30
Vector scale:
(mm) 5 10 15

(a) Vector Directions And Contours  
(b) Vector Displacement Field

FIG. 3.46 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{D}$ OF 4.
**FIG. 3.47** VECTOR DIRECTIONS AND CONTOURS AND VECTOR FIELD OF DISPLACEMENT FOR $L/D$ OF 5.

**FIG. 3.48** VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $L/D$ OF 6.
FIG. 3.49 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{D}$ OF 7.

FIG. 3.50 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{D}$ OF 8.
FIG. 3.51 VECTOR DIRECTIONS AND CONTOURS, AND VECTOR FIELD OF DISPLACEMENT FOR $\frac{L}{D}$ OF 9.
3.5.3 Radial Displacements and Drag-Down Zones as Obtained From Stereo-photogrammetric and Radiographic Results

(1) Radial Displacements

The "spherical" nature of the displacement contours observed in figures 3.23 to 3.29 are confirmed by the displacement contours of figures 3.43 to 3.51.

The displacement profiles measured at the base of the pile shaft for the displacement fields obtained from the stereo-photographs (i.e. figures 3.15 to 3.21) are shown in figures 3.52(a) to (g). These displacement profiles are consolidated onto one diagram in figure 3.53. Clearly there is very little variation in the displacement profile right throughout the pile installation process up to an embedment depth \( \frac{h}{3} \) of about 10.

The reason for the regularity in these displacement profiles, and for that matter the limiting diameter to the displacement fields (Table 3.1) is not able to be explained by the writer.

The corresponding displacement profiles measured at the base of the pile shaft for the displacement fields obtained from the radiographs, (i.e. figures 3.33 to 3.41), are consolidated in figure 3.54.

Even though the spread of the radial displacement profile values appear greater for those obtained from radiography (compare figures 3.53 and 3.54), the average profiles are in extremely close agreement as indicated in figure 3.55.

Clearly the fundamental characteristics of the displacement fields obtained using both stereo-photogrammetric and radiographic techniques are confirmed. The other significant feature obtained from the correlation of figure 3.54, is that the friction between the sand particles and the glass plate, as occurs in using the stereo-photogrammetric technique, appears to have a very minor effect on the recorded displacements.

It can thus be reasonably concluded that the possible modification of displacements obtained using stereo-photogrammetry, whilst being recognised as existing, can justifiably be ignored. A similar conclusion was reached by Arthur and Roscoe (1965) in association with work with model retaining walls.

(2) Drag-Down Zones

The apparent "drag-down" zone adjacent to the pile wall identified in figures 3.23 to 3.29 using the stereo-photogrammetric technique, is also able to be identified in the radiographic results presented in figures 3.43 to 3.51. As was discussed in section 3.5.1 the zone identified using the photogrammetric process was in an area close to the model pile wall where displacements were difficult to measure. Notwithstanding the difficulty of determining the contour of "zero" displacement, (given the accuracy limits of the displacement measuring technique, Appendix B), it is felt that the nature of the "drag-down" zone obtained using radiography is likely to be more precise than that discussed in section 3.5.1.
FIG. 3.52 RADIAL DISPLACEMENT PROFILES AT BASE OF PILE SHAFT.
FIG. 3.53 VARIATION OF RADIAL DISPLACEMENT WITH DEPTH OF PENETRATION (Photographs).

FIG. 3.54 VARIATION OF RADIAL DISPLACEMENT WITH DEPTH OF PENETRATION (Radiographs).
FIG. 3.55 AVERAGE RADIAL DISPLACEMENT PROFILES FROM PHOTOGRAPHS AND RADIOGRAPHS.
As a comparison the zone characteristics obtained from both techniques (stereo-photogrammetric and radiographic) are listed in table 3.4

### TABLE 3.4

<table>
<thead>
<tr>
<th>TABLE 3.4</th>
<th>&quot;DRAG-DOWN&quot; ZONE CHARACTERISTICS OBTAINED USING PHOTOGRAHMmetric AND RADIographic TECHNIQUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/D</td>
<td>STEREO-PHOTOGRAMMETRIC ZONE WIDTH (D) APPROXIMATE DEPTH (D)</td>
</tr>
<tr>
<td>1.37</td>
<td>Not evident 1.26</td>
</tr>
<tr>
<td>2.46</td>
<td>0.26 0.37</td>
</tr>
<tr>
<td>3.74</td>
<td>1.22 1.48</td>
</tr>
<tr>
<td>4.92</td>
<td>0.47 2.89</td>
</tr>
<tr>
<td>6.05</td>
<td>0.79 3.96</td>
</tr>
<tr>
<td>7.39</td>
<td>0.63 5.1</td>
</tr>
<tr>
<td>8.82</td>
<td>0.45 6.4</td>
</tr>
<tr>
<td>L/R</td>
<td>RADIographic AVERAGE ZONE WIDTH (D) APPROXIMATE DEPTH (D)</td>
</tr>
<tr>
<td>1.26</td>
<td>Not evident 1.26</td>
</tr>
<tr>
<td>2</td>
<td>Not evident 2</td>
</tr>
<tr>
<td>3</td>
<td>0.38 3</td>
</tr>
<tr>
<td>4</td>
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<td>1 4</td>
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<td>6</td>
<td>1 5</td>
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<td>7</td>
<td>0.7 6</td>
</tr>
<tr>
<td>8</td>
<td>0.4 7</td>
</tr>
<tr>
<td>9</td>
<td>0.7 2.8</td>
</tr>
</tbody>
</table>

From table 3.4, the significant observation is that the width of the "drag-down" zone clearly increases up to a pile embedment depth (L/D) of about 6. Beyond this depth the zone reduces in width, and as indicated by the radiographic results, appears to disappear altogether apart from about the first 3D of depth.

3.6 **SUMMARY OF OBSERVATIONS**

The following comments summarise the general observations relating to displacement fields developed about model piles during installation by driving in dense dry sand. These general observations, with particular reference to figures 3.43 to 3.51, can be described thus:-

(a) The development of the contours of equal displacement (figures 3.23 to 3.29 and 3.43 to 3.51) suggest the expansion of a spherical cavity, at least in the vicinity of the pile tip. This concept has formed the basis of an analysis of the pile tip load carrying capacity of axially loaded driven piles as discussed in Chapter 5.

(b) The deformation fields developed beneath the pile tip appear to remain relatively uniform throughout the pile installation process.

(c) Up to a pile embedment depth (L/D) of about 4 to 5, the soil flow adjacent to the pile shaft appears to be relatively uniform towards the ground surface.

(d) From pile embedment depths of about 6 onwards the development of the displacement fields becomes more irregular above the pile tip with a tendency for the soil to flow back towards the shaft.
FIG. 3.56 EFFECT OF PILE INTERFERENCE DURING DRIVING FOR VARIOUS PILE SPACINGS IN DENSE DRY SAND. (After Hughes, Goldsmith and Fendall, 1978a).
(e) Up to a pile penetration depth of about 6 ft, the soil tends to be forced upwards and outwards from the pile tip. Beyond this depth the soil flow above the pile tip tends to revert towards the pile shaft.

A significant feature able to be identified from the experimental results reported in this chapter is that shear failure planes were not able to be seen about the deformable mass of soil during pile installation. As mentioned in Section 3.3, Robinsky and Morrison (1964) made similar observations.

It is interesting to compare the observations listed in (b) to (e) above with the results of work being done contemporaneously by P.D.N. Fendall, concerned with the two pile situation.

Figure 3.56 shows results obtained by Fendall defining the influence that the installation of a second pile has on a pile already in place.

Even though the test results embody a degree of scatter, a significant change is observed between pile spacings of 4 and 60 ft; the effect appearing to be most pronounced when the second pile is driven to a depth of about 60 ft. This is compatible with the zone of influence of radius of approximately 4 to 60 ft due to the installation by driving of a single pile, as reported in tables 3.1, 3.2 and 3.3.

Up to a 60 ft depth of penetration, the pile already in place tends to be pushed away from the pile being installed. Beyond this depth the tendency is for the direction of movement to be reversed. The nett result is that the pile already in place initially moves away from the pile being installed, finally moving back towards it.

This phenomena is possibly a manifestation of the observations summarised in this section that the soil flow is, up to about 60 ft penetration, everywhere upwards and outwards from the pile tip, hence the installed pile tends to be forced away from the pile being driven. Beyond 60 ft penetration the direction of soil flow above the pile tip changes, enabling the previously installed pile to move back towards its original position.

Some of the observations discussed in this section have been reported earlier in Hughes, Goldsmith and Fendall (1978).
Chapter 4 Strain Fields Developed About Model Piles During Installation By Driving Into Dense Sand

4.1 INTRODUCTION

In the foregoing chapters the effects of pile installation into dense sand have been discussed.

It has been shown that below the pile tip the displacement fields appear to be very regular and vary little with depth or change in lateral stress in the soil. In contrast however, the nature of the soil movements above the pile tip are dependent on the pile embedment depth, showing a change in flow patterns at an embedment depth \((b/\ell)\) of about 6. The embedded volume of the pile would appear to be accommodated by a combination of surface heave and internal soil volume changes up to an \(b/\ell\) of about 9 to 10, after which depth it would appear that sufficient volume changes are occurring in the soil to accommodate the additional increments of inserted pile volume.

The results of a series of triaxial tests in dense dry sands at confining pressures ranging from 25 to 300 kPa, (Appendix 5), have indicated that some limiting stress ratio \(\frac{\sigma_2}{\sigma_1}\) should be reached at the stage that no further volume changes occur in the soil mass (i.e. the critical state is reached.)

As indicated by figure 4.1 (also figure A5.15 of Appendix 5), the soil will shear at some peak stress ratio, \(\frac{\sigma_2}{\sigma_1}\), prior to the critical state being reached. This peak stress ratio has been shown to occur at relatively low levels of cumulative shear strain, \(\gamma\), of about 2.75\% irrespective of confining pressure, and to remain essentially constant up to cumulative shear strains in excess of 20\%. In addition, as the confining pressure has been increased so the shear strains at which the critical state is approached have also increased; the soil continuing to shear at about the peak stress ratio with a rapid reduction in stress ratio towards the critical state condition. Thus during the process of pile installation, providing dilation continues to occur (i.e. \(\nu\) does not tend to zero), and assuming there is no stress relief from other causes, the soil mass will generally be at this peak stress ratio at shear strains greater than 2.75\% (i.e. about 3\%).

From the observations of Chapter 3, it would appear that a constant volume condition is never reached in the soil about a driven displacement pile in cohesionless media, at least for the conditions considered. It would thus follow that a critical state condition is likewise never reached (except for possibly small local zones), thus the stress conditions developed in the soil are everywhere likely to be those associated with the peak values of the stress ratio \(\frac{\sigma_2}{\sigma_1}\) as indicated in Appendix 5 (figure 5.17), and figure 4.1, providing stress relief does not occur (See Chapter 5).
In this chapter the volumetric and shear strain conditions about model piles driven into dense sand are determined from consideration of the displacement fields obtained using the radioisotopic technique of Appendix 7 and as presented in Chapter 3. The method of calculating the strains from the displacement fields so obtained is discussed in Appendix 10.

The volumetric and shear strains enable the dilation characteristics of the soil to be examined at various stages of pile penetration. Separately, the shear strains give an indication of the stress ratios achieved in the soil and thus the mobilised friction angles; while the volumetric strains are a direct reflection of the changes in density that are occurring within the soil mass. Where expansion represents loosening.

4.2 FIELDs OF CUMULATIVE SHEAR STRAIN

The cumulative shear strain fields calculated from the observed displacements of Chapter 3 using the method of Appendix 10 are shown in figures 4.2 to 4.10 for 10 increments of displacement up to an embedment depth \( \frac{h}{d} \) of approximately 9. Also shown on these figures are the limits to the zero observable displacements as defined in figures 3.33 to 3.41 (Chapter 3). Clearly the growth of these contours of cumulative shear strain appears to be uniformly parallel to the shaft wall with depth. The magnitude of the cumulative shear strains increase with depth of pile penetration, reaching particularly high values by the time the pile has been installed to an embedment depth \( \frac{h}{d} \) of 9.

The significant feature identified in figures 4.2 to 4.10 is that almost all the soil affected by the process of pile installation rapidly reaches cumulative shear strains greater than that associated with the first attainment of the peak stress ratio (about 2.75%). Consequently, it would appear that, at least about the pile tip, the mobilised soil friction angle (\( \phi \)) is probably the peak angle (\( \phi_p \)).

Figures 4.11(a) and 4.12(a) show incremental shear strain fields for pile installation from \( \frac{h}{d} \) of 7 to 8 and 8 to 9 respectively. The concept that incremental pile penetrations and thus soil response to axial pile loads are analogous to the expansion of a spherical cavity, at least about the pile tip, would appear to be supported. For comparison, figures 4.11(h) and 4.12(h) show the vector directions and contours of equal displacement for the same embedment depths. These diagrams were presented earlier as figures 3.50(a) and 3.51(a) (Chapter 3).

4.3 FIELDs OF CUMULATIVE VOLUMETRIC STRAIN

Cumulative volumetric strain fields are shown in figures 4.13 to 4.21 for the same embedment depths as figures 4.2 to 4.10. The negative volumetric strains in figures 4.13(a) to 4.21(a) represent volume contraction (i.e. increased density) while the positive volume strains indicate that the soil mass is expanding (i.e. reduced density; i.e. looser).

Figures 4.13(h) to 4.21(h) have been included to make it easier for the reader to visualise the nature of the volume changes and thus the change in soil densities. The shaded areas in these figures are zones in which the soil is expanding (i.e. getting less dense).
FIG. 4.1 STRESS RATIO - SHEAR STRAIN RESULTS FROM TRIAXIAL TESTS.

FIG. 4.2 SHEAR STRAIN FIELD (%) FOR $\frac{D}{d}$ OF 1.26.

FIG. 4.3 CUMULATIVE SHEAR STRAIN FIELD (%) FOR $\frac{D}{d}$ OF 2.
FIG. 4.4  CUMULATIVE SHEAR STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 3.

FIG. 4.5  CUMULATIVE SHEAR STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 4.
FIG. 4.6 CUMULATIVE SHEAR STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 5.

FIG. 4.7 CUMULATIVE SHEAR STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 6.
FIG. 4.8  CUMULATIVE SHEAR STRAIN FIELD (%) FOR \( \frac{L}{D} \) OF 7.

FIG. 4.9  CUMULATIVE SHEAR STRAIN FIELD (%) FOR \( \frac{L}{D} \) OF 8.
FIG. 4.10 CUMULATIVE SHEAR STRAIN FIELD (%) FOR \( \frac{L}{D} \) OF 9.
FIG. 4.11 INCREMENTAL SHEAR STRAIN FIELD (%) FOR PILE INSTALLATION FROM $\frac{L}{D}$ OF 7 TO $\frac{L}{D}$ OF 8.

FIG. 4.12 INCREMENTAL SHEAR STRAIN FIELD (%) FOR PILE INSTALLATION FROM $\frac{L}{D}$ OF 8 TO $\frac{L}{D}$ OF 9.
FIG. 4.13 VOLUMETRIC STRAIN FIELD (%) FOR $\frac{1}{D}$ OF 1.25.

FIG. 4.14 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{1}{D}$ OF 2.

FIG. 4.15 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{1}{D}$ OF 3.
FIG. 4.16 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 4.

FIG. 4.17 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 5.
FIG. 4.18 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 6.

FIG. 4.19 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 7.
FIG. 4.20 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 8.

FIG. 4.21 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 9.
FIG. 4.22 DEVELOPMENT OF ZONE OF LOOSE SAND ABOUT DRIVEN MODEL PILES IN DENSE SAND.
At shallow depths (i.e. up to about 50 penetration) the greater mass of the soil influenced by the pile installation process is expanding with relatively small amounts of compaction occurring around the perimeter of this expanding zone. However, with increased penetration of the pile, the change in volumetric strain becomes more complex with some elements going through a series of reversals between successive increments of installation where alternately compaction and expansion are occurring. Of significance however, is the "sleeve" of expanding, i.e. loose sand, that is maintained about the pile shaft, progressively reducing in width until, after an embedment depth (h) of 6 to 7, it appears to remain relatively static at, on the average, about 10 wide. Clearly then the existence of a "sleeve of loose sand" suggested by various authors (for example Rhinsky and Morrison, 1964; Vesic, 1963) is confirmed.

The other significant observation that can be made is that in dense sands the large zones of compaction adjacent to the pile shaft and tip as suggested by various authors (for example Cooke, 1978, Meyerhof, 1959, 1976; Roms, 1966) in fact do not exist at least for the conditions considered in this study. Meyerhof's considerations as to the soil volume changes about driven piles in cohesionless soils are discussed in Chapter 3.

In general the supposed verification of the existence of a zone of compaction about the tip of a driven pile in medium dense to dense cohesionless soils has been based on cone penetrometer tests. (Kerisel, 1961, Petrasovits, 1973). Under these circumstances the penetration test is probably only measuring the increase in stress in the soil due to the installation of a pile rather than increases in density. These increases in stress will probably decrease above the pile tip as suggested by Bennett and Gisbourne (1971) and would thus agree well with the results of the penetration tests reported by Kerisel, Petrasovits and others. Figure 4.27 shows the nature of the variations in stress reported by Bennett and Gisbourne. The logical extension to this reasoning is that the various correlations between cone penetration resistance and relative density, for example Gibbs and Holtz (1959), are questionable as it could be argued that given a sand of high relative density but low K<sub>0</sub> conditions, the penetrometer would in fact suggest a loose sand. Rather than reflecting density it is suggested that the penetrometer in fact reflects the packing structure and thus the K<sub>0</sub> conditions existing in the soil.

Figure 4.22 correlates the outer limits to the zones of loosening(expansion) about driven model piles in dense sand. The expanding zones are obviously widest at ground level. With increasing depth the zone width reduces to about 10 wide adjacent to the shaft and above the pile tip. About the pile tip the loosening effect appears to extend to a radius of about 4.0D, irrespective of depth of pile penetration, and appears to extend 2.5 to 3.0D below the pile tip.

Figures 4.23 and 4.24 show incremental volumetric strain fields for pile installation from an L of 7 to 8 and 8 to 9 respectively. From these incremental fields it would appear that the primary soil expansion is occurring about the pile tip. These loose zones initially set up about the tip are subsequently compacted by the passage of the shaft. It would however appear from figure 4.22 that this subsequent compaction is insufficient to completely recompact this loosened material at least over the penetration depths studied.
FIG. 4.23 INCREMENTAL VOLUMETRIC STRAIN FIELD (%) FOR PILE INSTALLATION FROM $\frac{L}{D}$ OF 7 TO $\frac{L}{D}$ OF 8.

FIG. 4.24 INCREMENTAL VOLUMETRIC STRAIN FIELD (%) FOR PILE INSTALLATION FROM $\frac{L}{D}$ OF 8 TO $\frac{L}{D}$ OF 9.
4.4 **GENERAL COMMENTS ON THE COMBINED VOLUMETRIC - SHEAR STRAIN EFFECT**

The significant feature indicated by figures 4.2 to 4.24 is that the large zone of soil beneath the pile tip is, irrespective of the embedment depth of the pile, subject to high compressive stresses, large shear strains, and significant expansional volumetric strains. These are then the general conditions suggested earlier and also discussed in Appendix 5 necessary for the soil to reach the peak stress ratio as indicated by figure 4.1. It can thus be concluded that about the pile tip the soil is in general at the peak stress ratio, \((\frac{\sigma}{\sigma_n})\), though in reality it is more probable that some soil elements will be at peak whilst others will be less than peak, but probably greater than critical state.

As discussed earlier, up to about a cumulative shear strain of 20% the soil is generally at the peak friction angle, (as determined from triaxial tests). Thus about the pile tip, as indicated in figure 4.25 it is considered that the mobilised friction angle is probably the peak value, however, at higher shear strains it is not clear what friction angle applies, but probably lies between the peak and critical state conditions. In terms of the mobilised soil friction angle, the conditions existing about the pile tip can then be expressed as:

\[
\phi_{cs} \leq \phi_{mob} \leq \phi_n
\]  

4.1

In the case of the model studies it has thus been assumed in the subsequent analysis (Chapter 5), that the appropriate mobilised friction angle is \(\phi_p\), which, from figure 4.1, is 44.4°.

As mentioned earlier, beyond a pile embedment depth \((\frac{L}{D})\) of 4 to 5, the width of the expanding zone reduces to about 10 from the pile shaft. Thus above this depth the same conditions in general exist in this zone as those about the pile tip; i.e. significant expansion and large shear strains. However, when the pile has penetrated beyond an \((\frac{L}{D})\) of 4 to 5, a large proportion of the soil mass about the pile shaft is in fact compacting, and in some positions by quite large amounts (i.e. c to c' on figures 4.20 and 4.21 respectively), even though still being subjected to quite large shear strains. This therefore suggests that the characteristics of the soil response have changed significantly as indicated in figures 4.26(a) and (b). For example: a typical soil element initially located beneath the pile tip (i.e. point A in figure 4.22) will follow the path OA across the state surface indicated pictorially in \(e, s, \varepsilon\) space in figure 4.26(a). The corresponding path followed by this soil element in \(z' - z'\) space is indicated in figure 4.26(b).

At point A then, the soil element has experienced an increase in stress and shear strain accompanied by a significant loosening due to the shearing deformation induced in the soil beneath the pile tip.

If the sand is continuously sheared, the state path followed by such a soil element would have been that indicated by OAC in figures 4.26(a) and (b); i.e. the critical state condition is reached with increasing shear strain, stress, and volume expansion; at which point the soil element is shearing at constant stress and in a state of constant volume.
FIG. 4.25 PRORABLE VARIATION IN MOBILISED SOIL FRICTION ANGLE ABOUT THE PILE TIP.
Loose sand

State surface from restructuring due to mechanical disturbance

(a) Stress Path in e, s, εν Space
(adapted from Stroud, 1971)

(b) Corresponding Path in εν - εν Space

FIG. 4.26 STRESS RELIEF DUE TO VOLUME CHANGE.
FIG. 4.27 VARIATION WITH PILE PENETRATION OF PRESSURE AT A POINT (After Bennett and Gisbourne, 1971).

FIG. 4.28 VARIATION IN AVERAGE DILATION RATE WITH PILE EMBEDMENT (ALL DATA).
If stress relief occurred after the passage of the pile tip and was accompanied by continued volume expansion and increasing shear strains, then the conditions representing the soil element would probably change from A to A' on the state surface.

Figure 4.26(b) indicates the changes in $\sigma - \varepsilon$ space experienced by the soil element with passage of the pile tip. As indicated by points c and c' in figures, 4.20 and 4.21, the soil element initially expands, then contracts. This condition is represented by the change from point A to point B in figure 4.26(h). In moving from A to B the soil element must pass through a constant volume stage represented by point A* in both figures 4.26(a) and (b).

The occurrence of volume compaction in conjunction with increased shear strains is a physical impossibility in terms of the stress-strain characteristics of the soil as defined by the state surface in figure 4.26(a); unless of course some other phenomena occurs. It is thus suggested that for the condition of volume contraction and increased shear strains to occur, it is necessary to postulate a further state surface which intersects the surface containing the path OAC.

With the existence of this intersecting surface the soil element would then follow the path OAA*M. At point B the soil element has experienced volume contraction, increased shear strains, and probably a considerable reduction in stress.

It is suggested that this postulated intersecting state surface is induced by the pile installation process itself. In following the path OA, the soil element, which is still located beneath the pile tip, has expanded to a loose state in which the packing structure is less stable than that existing at lower shear strains (point O for example). As the soil element passes out of the immediate confinement represented by the pile tip, it enters the zone where the soil flow is generally upward parallel to the shaft (see Chapter 3) and also comes under the influence of the transient disturbances due to pile driving (i.e. vibrations). This mechanical disturbance is sufficient to cause a restructuring of the soil particles, which, it is suggested, will follow the path AA*M.

During lateral loading further shearing occurs during which the soil element has been shown experimentally to deform along the path R to B' in figure 4.26(h). In figure 4.26(a) the state path must therefore be of a form similar to the corresponding RR' path. A reduction in stress after the passage of the pile tip, such as that postulated by the path AA*M has in fact been reported by Bennett and Gisbourne (1971) as indicated in figure 4.27.

4.5 THE VARIATION IN THE RATE OF DILATION WITH THE DEPTH OF PILE INSTALLATION

4.5.1 General

As discussed in Appendix 5 the rate of dilation ($\psi$) is given by:

$$\psi = \sin^{-1} \left( -\frac{\theta}{\psi} \right)$$

Thus, from the data used to construct the contour diagrams of figures 4.2 to 4.24, it is possible to obtain an indication of the variation in the rate of dilation throughout the pile installation process.

Appendix 10 briefly discussed the computer program developed for the strain analyses and indicated that average dilation rates were obtained using a least squares regression analysis.
An analysis of all the calculated strain data, (i.e. every constant strain triangle experiencing any deformation), yields the variation in dilation rate indicated in figure 4.28. In this diagram, the dilation rate has been expressed both as the ratio \( \frac{\dot{V}}{\dot{V}_0} \), and as the angle of dilation (\( \psi \))

\[ \frac{\dot{V}}{\dot{V}_0} = \sin \psi \]

These dilation rates are in fact the average values representing the deformation occurring throughout the soil mass, and are in terms of octahedral strains as indicated in Appendix 10.

Clearly there is a well defined trend for the average dilation rate so expressed to tend towards a constant value after the pile has reached an embedment depth \( \frac{h}{D} \) of 9 or 10. This observation thus supports the earlier speculation that because it is a fundamental characteristic of dense sands to expand on shearing, and because at even considerable depths such expansion is likely to occur, at least for driven displacement piles in sand, then the conditions developed within the soil mass about the pile are unlikely to reach a state of constant volume (i.e. critical state) at least at cumulative shear strains of less than 20\%; however, closer to the pile tip part or all of the soil may have reached this condition. It is, however, difficult to reliably measure the displacements in this highly deformed zone about the pile tip (see also figure 4.25). It would appear that the average dilation rate in fact asymptotes to a constant value at embedment depths \( \frac{h}{D} \) of about 10. From figure 4.28, this would appear to be at a dilation rate of about 10\(^{0} \) for the model tests.

At shallower embedment depths, for example, close to the ground surface, the recorded average dilation rates show a definite trend towards the maximum limiting value of 0.5 as indicated in Appendix 5. This tendency is clearly a reflection of the lack of confinement existing at shallow depths. The trend for the dilation rate to reduce with depth probably reflects the high stresses, and thus apparent confinement, being generated in the soil during the pile installation process; i.e. the soil is being prestressed.

As discussed in Chapter 3 the radiograph recording the displacements at a pile embedment depth \( \frac{h}{D} \) of 10 was accidentally destroyed. However, the nature of the displacement fields appeared to maintain the same uniformity (at least below the pile tip) as indicated for all increments up to an \( \frac{h}{D} \) of 9. (See figure 3.42, Chapter 3). On the basis of this similarity in displacement profiles the probable strain fields developed at the pile embedment depth of 10 have been reconstructed by applying the same displacement field developed at the embedment depth of 9 to the greater depth at an \( \frac{h}{D} \) of 10. The strain fields so reconstructed are shown in figures 4.29 and 4.30(a) and (b) for the cumulative shear and volumetric strains respectively. These reconstructed strain fields obviously bear good agreement with those obtained for the preceding pile embedment depths, as can be seen by comparing figure 4.29 and 4.30 with figures 4.19 and 4.21 respectively. This correlation is further substantiated by a comparison of the average dilation rate determined from these reconstructed strain fields, as indicated by point A on figure 4.28. The average dilation rate so determined fits the trends of figure 4.28 remarkably well.

On the basis of these correlations, these reconstructed strain fields have been taken as representing the soil state at the completion of pile installation by driving and thus the conditions existing immediately prior to the lateral loading of the pile (the aspects of lateral loading are
FIG. 4.29 CUMULATIVE SHEAR STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 10. (RECONSTRUCTED).

FIG. 4.30 CUMULATIVE VOLUMETRIC STRAIN FIELD (%) FOR $\frac{L}{D}$ OF 10. (RECONSTRUCTED.)
discussed in subsequent chapters).

The displacement fields reported in Chapter 3 indicate substantially different characteristics above to those developed below the pile tip. Similar differences are indicated by the cumulative volumetric strain fields of figures 4.13 to 4.21 and 4.30. Above the pile tip the soil is essentially straining in a vertical direction, whilst about the pile tip radial straining is occurring. Consequently the soil dilational characteristics have been separated to match these two zones of differing soil response, and thus by inference, the nature of the pile response to axial loading. In the following subsections the division is made between the soil mass above the pile tip, (i.e. that likely to contribute to pile shaft loads), and that below the tip (i.e. that which will directly resist the pile tip loads).

The approximations made in the subdivision of the triangular elements formed from the lead shot "nodal" points, are indicated in figure 4.31. Because the actual boundary was not quite as neat as that suggested by the rationalisation of figure 4.31, the triangles not fitting the general trend along with those where significant connection was occurring, were eliminated from the following analyses. The group of triangles that were eliminated from a particular analysis, because they in fact represent conditions on the other side of the rationalised boundary, comprised less than 20 points out of the total of 288 triangles considered. In figure 4.31, all triangles lying above the "tip level" indicated, were included in the "shaft" analysis. The inverted pyramid in figure 4.31 represents the actual pile tip level.

4.5.2 Soil Dilation Characteristics About the Pile Shaft

From an analysis of the data above the rationalised boundaries indicated in figure 4.31 the average dilational characteristics indicated in table 4.1 were obtained.

<table>
<thead>
<tr>
<th>( \frac{L}{T} )</th>
<th>Slope of Regression Line ( \frac{\sigma}{\tau} )</th>
<th>Dilation Rate ( \frac{\sigma}{\tau} )</th>
<th>Intercept (strain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.272</td>
<td>15.78</td>
<td>0.038</td>
</tr>
<tr>
<td>3</td>
<td>0.431</td>
<td>25.52</td>
<td>0.012</td>
</tr>
<tr>
<td>4</td>
<td>0.385</td>
<td>22.61</td>
<td>0.021</td>
</tr>
<tr>
<td>5</td>
<td>0.393</td>
<td>17.65</td>
<td>0.012</td>
</tr>
<tr>
<td>6</td>
<td>0.339</td>
<td>19.24</td>
<td>0.026</td>
</tr>
<tr>
<td>7</td>
<td>0.316</td>
<td>18.44</td>
<td>0.026</td>
</tr>
<tr>
<td>8</td>
<td>0.287</td>
<td>16.69</td>
<td>0.024</td>
</tr>
<tr>
<td>9</td>
<td>0.253</td>
<td>14.63</td>
<td>0.024</td>
</tr>
</tbody>
</table>
FIG. 4.31 ANALYTICAL SEPARATION OF SOIL MASS ABOVE AND BELOW PILE TIP FOR EACH INCREMENT OF PILE INSTALLATION.
At the pile embedment depth \( \left( \frac{L}{D} \right) \) of 1.26, there is insufficient shaft embedded in the soil to have any significant influence. For this reason no values have been recorded at this depth in Table 4.1. Apart from the results for an \( \frac{L}{D} \) of 2 the results show a general reduction with depth similar to that indicated in figure 4.27. The anomaly at an \( \frac{L}{D} \) of 2 is considered due to the fact that there are insufficient points to give a good representation (i.e. only 10, see figure 4.31).

The cumulative volumetric - shear strain \((\gamma - \dot{\gamma})\) plots from which the values of Table 4.1 were obtained are presented in figures 4.32 to 4.39. The regression lines of Table 4.1 are also indicated. Figure 4.40 presents the average dilation rates about the pile shaft in the same manner as figure 4.28, for the values of Table 4.1 and figures 4.32 to 4.39. These values show almost a linear decrease in the dilation rate \((\nu)\) about the pile shaft with depth.

### 4.5.3 Average Dilational Characteristics Developed Below The Pile Tip

The average dilational characteristics indicated in Table 4.2 were obtained from an analysis of the data below the rationalised boundaries indicated in figure 4.31. Again anomalous results are indicated at an \( \frac{L}{D} \) of 2. Apart from the possibility that the results are simply in error, no other explanation can be offered for the departure from the apparent trend set by the remainder of the results. The cumulative volumetric - shear strain \((\gamma - \dot{\gamma})\) plots from which the values of Table 4.2 were obtained are presented in figures 4.41 to 4.49 with the appropriate regression lines also shown. With increasing depth of pile penetration the \( \nu - \dot{\gamma} \) results from below the pile tip appear to show less spread than the corresponding values above the pile tip, as indicated by a comparison between figures 4.32 to 4.39 and figures 4.41 to 4.49.

### Table 4.2

**Average Dilational Characteristics Existing Below The Pile Tip**

<table>
<thead>
<tr>
<th>( \frac{L}{D} )</th>
<th>Slope of Regression Line ((\frac{\gamma}{\dot{\gamma}}))</th>
<th>Dilation Rate ((\nu))</th>
<th>Intercept (Strain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>0.450</td>
<td>26.73</td>
<td>0.008</td>
</tr>
<tr>
<td>2</td>
<td>0.274</td>
<td>15.89</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.406</td>
<td>23.97</td>
<td>-0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.339</td>
<td>19.91</td>
<td>-0.006</td>
</tr>
<tr>
<td>5</td>
<td>0.317</td>
<td>18.48</td>
<td>-0.005</td>
</tr>
<tr>
<td>6</td>
<td>0.292</td>
<td>16.98</td>
<td>-0.003</td>
</tr>
<tr>
<td>7</td>
<td>0.192</td>
<td>11.06</td>
<td>-0.006</td>
</tr>
<tr>
<td>8</td>
<td>0.190</td>
<td>10.95</td>
<td>-0.004</td>
</tr>
<tr>
<td>9</td>
<td>0.187</td>
<td>10.78</td>
<td>-0.004</td>
</tr>
</tbody>
</table>
FIG. 4.32 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{1}{3}$ OF 2.
(Data above Pile Tip).

FIG. 4.33 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{1}{3}$ OF 3.
(Data above Pile Tip).
FIG. 4.34 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{L}{D}$ OF 4.
(Data above Pile Tip).

FIG. 4.35 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{L}{D}$ OF 5.
(Data above Pile Tip).
Fig. 4.36 Cumulative Volumetric - Shear Strain Plot for $\frac{L}{D}$ of 6.
(Data above Pile Tip).

Fig. 4.37 Cumulative Volumetric - Shear Strain Plot for $\frac{L}{D}$ of 7.
(Data above Pile Tip).
FIG. 4.38 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR \( \frac{1}{4} \) OF 8.
(Data above Pile Tip).

FIG. 4.39 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR \( \frac{1}{4} \) OF 9.
(Data above Pile Tip).
FIG. 4.40 VARIATION IN AVERAGE DILATION RATE WITH PILE EMBEDMENT (Data above Pile Tip).
FIG. 4.41  VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{1}{D}$ OF 1.26.  
(Data below Pile Tip).

FIG. 4.42  CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{1}{D}$ OF 2.  
(Data below Pile Tip)
FIG. 4.43 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{L}{D}$ OF 3.
(Data below Pile Tin).

FIG. 4.44 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{L}{D}$ OF 4.
(Data below Pile Tin).
FIG. 4.45 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{L}{h}$ OF 5.
(Data below Pile Tip).

FIG. 4.46 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR $\frac{L}{h}$ OF 6.
(Data below Pile Tip).
FIG. 4.47 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR \( \frac{1}{2} \) OF 7.
(Data below Pile Tip).

FIG. 4.48 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR \( \frac{1}{2} \) OF 8.
(Data below Pile Tip).
FIG. 4.49 CUMULATIVE VOLUMETRIC - SHEAR STRAIN PLOT FOR \( \frac{l}{H} \) OF \( \theta \).
(Data below Pile Tin).

FIG. 4.50 VARIATION IN AVERAGE DILATION RATE WITH PILE EMBEDMENT
(Data below Pile Tin).
Figure 4.50 presents the average dilation rates below the pile tip, in the same manner as figures 4.28 and 4.40, for the values of tables 4.2 and figures 4.41 to 4.45.

Apart from the anomalous value at an \( \frac{L}{D} \) of 2, similar trends to figures 4.28 and 4.40 are indicated; i.e. close to the surface the values tend towards the limiting value of \( \frac{\dot{V}}{V} \) of 0.5 at the surface. At depth the values appear to be asymptotically towards a constant value, of about 0.17 (i.e. 10\(^0\)). Below the pile tip the state of stress would appear to be purely compressional thus the conditions could be claimed to be roughly analogous to the conditions prevailing in a triaxial test.

Figure 4.51 shows an approximate correlation with the triaxial data of Figure 45.6 (Appendix 5). This approximate correlation has been made on the basis that the average confining stresses at the pile tip are represented by approximately one third of the peak stress acting on the pile tip. This peak stress was that backfigured from the model pile tests reported in Chapter 5. Beyond a "confining" pressure of 300 kPa, the correlation is not good. This is probably because the triaxial data of Appendix 5 had to be extrapolated beyond 300 kPa and is thus unreliable. This only affects two points in Figure 4.51, viz., these for \( \frac{L}{D} \)'s of 8 and 9 respectively. The remaining seven points however, show a very encouraging agreement with the triaxial results. The correlation represented by OA in Figure 4.51 is in fact less than 1.2:1, indicating that, on the basis of average "confining" pressures and dilation rates, the values obtained about the piles from experimental considerations underestimate the corresponding triaxial values by less than 20%.

4.5.4 Discussion

In the foregoing it has been shown that although the cumulative shear strains about the pile develop in a regular manner, the cumulative volumetric strains are, in contrast, much more irregular. The development of areas where large compactional volume strains are occurring can result in stress relief about the pile shaft. In general however, the average dilation rate about the pile tip was approx. 2 to 5\(^0\) lower than the average values about the pile shaft. This comparison is expressed as the ratio \( \frac{\dot{V}_{\text{shaft}}}{\dot{V}_{\text{tip}}} \) in Figure 4.52. At depth the ratio appears to indicate a limiting value of about 0.7.

It is thus obvious that the assumption of a constant average rate of dilation (\( \dot{V} \)) applies only at a particular depth. Throughout the soil mass the dilation rate is varying as a direct function of the degree of confinement able to be established. It follows then that for shallow or surface footings the dilation rate likely to apply is the limiting value of \( \frac{\dot{V}}{V} \) of 0.5. However, the wider the footing the higher the degree of "apparent confinement" and thus the lower the dilation rate that could be anticipated. Nevertheless, as long as the soil mass is able to dilate to a free surface in close proximity to the founding level of the foundation, high dilation rates could be expected.

The large negative volumetric strains (i.e. zones of volume compaction) developed in the zone above the pile tip serve to explain the observation made in Chapter 3 that beyond an embedment depth \( \frac{L}{D} \) of 0 to 10, the additional installed pile volumes are accommodated by compaction of the soil mass. Consequently, beyond this depth the ground surface profile remains essentially static (i.e. no further surface heave).
FIG. 4.51 APPROXIMATE CORRELATION BETWEEN DILATION RATES OBTAINED FROM TRIAXIAL RESULTS AND THOSE OBSERVED ABOUT THE PILE TIP.

FIG. 4.52 RATIO OF SHAFT TO TIP DILATION RATES.
It is interesting to note, in view of the foregoing experimental results, that Fassett (1970) suggested that the effect of the installation of a displacement pile into sands by driving is for the material adjacent to the pile to undergo a predominantly lateral displacement. Extrapolating the results of plane strain work to the radially symmetric pile problem, he suggested that a zone of material adjacent to the pile will be strained to the critical state as indicated in figure 4.53(h) and that the coefficient of lateral earth pressure will be as indicated in figure 4.53(a). Clearly his suggestion that the critical state is reached is questionable.

That the critical state is reached along the shaft as suggested by Fassett, has not been detected by the writer, however it is suspected that a zone of material in which the critical state is reached, of probable width much less than the diameter of the pile, could exist about the pile tip.

This suggestion is indicated in figure 4.54.
FIG. 4.53 ZONE OF MATERIAL ABOUT A DRIVEN PILE STRAINED TO THE CRITICAL STATE AS SUGGESTED BY BASSETT (1970).

FIG. 4.54 POSSIBLE ZONE OF SOIL AT THE CRITICAL STATE.
Chapter 5 Axial Loads On Model Piles Installed By Driving

5.1 INTRODUCTION

In this chapter the various factors affecting correlations between field and laboratory load tests are discussed.

Based on the experimental evidence of the foregoing chapters a theory is developed in an attempt to meet the specification for an axial pile load theory suggested by Vesic (1967).

In addition a qualitative assessment is made of the affects of pile shaft roughness and tip configuration.

5.2 CORRELATIONS BETWEEN FIELD AND LABORATORY LOAD TESTS

5.2.1 Introduction

Much of the work reported in the literature, and made as recommendations to the designer, is based generally on empirical correlations between theoretical considerations and model tests. In some cases further additional correlations are made between model studies and full scale tests. Invariably the authors claim "good agreement in the results".

The writer has made observations which question the validity of such correlations between model and field studies, and in particular correlations drawn by independent authors between model studies carried out by a variety of unrelated researchers. Notwithstanding the effects of the various methods of pile installation, which immediately limit any correlations that can be made to studies involving piles installed in exactly the same manner, the method of sample preparation in the laboratory and the ambient stress states existing in the field can significantly affect the validity of any correlations.

Various other researchers, for example Hanna and Tan (1973) and Hunter and Davisson (1969), have questioned the results of both model and full scale pile tests, with particular respect to the sharing of load between pile shaft and tip, as a result of residual stresses developed during pile installation.

These considerations, along with others relating to the "scale effect" on bearing capacity factors, are discussed in the subsequent sections.

5.2.2 The Effects of In-Situ Lateral Soil Pressures

Most natural deposits of cohesionless materials are the result of fluvial deposition, or alternatively, sedimentation in a fluvial, marine, or lacustrine environment. The soil structure so resulting is probably most closely akin to that achieved in the laboratory by pluviation (showering) in air. Some degree of lateral particle movement occurring in this process, as would be likely to occur
under natural conditions of deposition (See figure A6.11, Appendix F). Laboratory pluviated samples are thus likely to bear a closer structural relationship to naturally occurring sand deposits than would laboratory samples produced by vibration.

The naturally occurring sand deposit, after initial deposition to a normally consolidated state, will with time be subjected to gradual increases in overburden pressure. This gradual increase in overburden pressure will probably cause the sand to experience some structural rearrangement at the particulate level, finally reaching the condition of equilibrium at which it is found in the natural or undisturbed state. The properties of the deposit are likely to be further influenced by the geologic history to which it may have been subjected. The geological effects may involve:-

(i) An increase in density of initially loose sands if the effective overburden pressure increases significantly,

(ii) An increase in the coefficient of earth pressure at rest \((K_0)\) if the soil is over-consolidated due to deposition and subsequent erosion, or due to seismically induced cyclic strains (Mori, Seed and Chan, 1978).

(iii) The inducement of a more stable structure due to the seismic history of the geographic location; i.e. further compaction due to vibration.

(iv) Increased stability due to other factors, for example cementation due to the minerology of the sand or overlying stratum through which waters are able to percolate.

The final properties of a naturally occurring sand deposit are thus likely to be dependent on at least the following five factors:-

(a) The density of the soil
(b) The structure at placement or deposition
(c) Cementation, if any
(d) Seismic history
(e) Overconsolidation, affecting \(K_0\).

Traditionally, naturally occurring deposits of cohesionless materials are investigated from the geotechnical point of view in terms of relative density obtained from some form of penetration test. (Factors related to these penetration tests, and interpretation of the relative density expression, are discussed in Chapter 2 and Appendix F respectively).

Often this restrictive form of investigation is due to the difficulty in obtaining undisturbed samples for laboratory analysis, consequently it is generally not possible to obtain an indication of any of the factors affecting the final properties of a naturally occurring sand deposit, least of all the magnitude of the horizontal stresses existing in the soil. Thus, in general, the influences of geological and geophysical phenomena such as the manner in which the sand has been deposited or whether it has been subjected to past seismic vibration, are not considered.

Recent investigations into the mechanics of the liquefaction of sands, (for example, Mullis et al, 1977, and Mori, et al, 1978), have indicated that these factors can have a significant effect on the
FIG. 5.1 EFFECT OF LABORATORY SAND PLACEMENT TECHNIQUES ON STRENGTH

FIG. 5.2 LATERAL LOAD-DISPLACEMENT CURVES FOR MODEL PILES IN VIBRATED AND SHOWERED DENSE DRY SAND
(After Hughes, Goldsmith and Fendall, 1978a).
capacity of the soil to resist load. Figure 5.1 shows the results of a detailed investigation reported by Mulilus et al (1977).

Clearly the weakest samples were those formed by pluviation of the soil through air, while the strongest were those formed by vibrating the soil in a moist condition. The naturally occurring circumstances analogous to these laboratory conditions are volcanic ash showers, and rearrangement and redeposition of sands beneath the ground water surface, or in lacustrine situations, by seismic activity.

Comparing the cyclic shear stresses necessary to cause liquefaction at 10 cycles, the differences in strength was about 20% (i.e. A to B in figure 5.1). It should be noted that in figure 5.1, the cyclic shear stress has been normalised in terms of the initial effective confining pressure.

Load-displacement curves obtained from laterally loaded model pile tests are presented in figure 5.2. Curve A in figure 5.2 was obtained in sand placed in thin layers and compacted by vibration. Curve B was obtained using the same model pile installed to the same embedment depth and using the same method of installation, however, in this case the sand was placed by showering. The same initial density (1670 kg m\(^{-3}\)) was achieved with both methods of commination.

Clearly the effect of particle rearrangement on denosition (structure) and the associated influence on the lateral stress developed in the soil can be significant. The difference in the ultimate load carrying capacity between curves A and B in figure 5.2 is about 20% of curve A, the changes in stiffness represented by the secant modulus at a particular displacement are however somewhat larger. It has been found, depending on the vibration technique used, the intensity of vibration, and the thickness of the placed sand layers, that this difference in ultimate load carrying capacity can be increased to as much as about 50% of curve A. (See figure 8.21, Chapter 8). In this latter case the sand was placed to an initial density of 1640 kg m\(^{-3}\).

Clearly then, because of the significant influence of the magnitude of the initial lateral stress, the abovementioned correlations, be they with model or full scale tests, are doubtful at the best, unless of course the initial conditions are expressed in terms of both density and lateral soil stress (\(K_0\)).

For the results obtained by showering as discussed in this thesis, the initial value of \(K_0\) was 0.4 (See Appendix 9). That achieved by vibration would appear to be somewhat higher. From the earlier considerations of the factors affecting natural denosition, this value could reasonably be taken as being indicative of the conditions existing in a naturally occurring normally consolidated deposit.

An observation of considerable significance is that while the manner in which a soil sample is placed, and thus the initial lateral stress, can have a significant effect on its ultimate load carrying capacity, the displacement fields developed about model piles at various stages of installation or lateral loading are virtually unaffected. This is evidenced by figure 5.3, (previously presented as figure 3.55, Chapter 3, but presented again here for the convenience of the reader), in which the average radial displacements at the level of the pile tip throughout pile installation by driving are compared using stereophotogrammetric and radiographic techniques.
FIG. 5.3  AVERAGE RADIAL DISPLACEMENT PROFILES FROM PHOTOGRAPHS AND RADIOGRAPHS.
The curve obtained from the photographs was in sand placed by vibration. That obtained from the radiographs was in sand placed using the showering technique described in Appendix 6. The range of displacement profiles comprising the average curve obtained from the radiographs was however, greater than that obtained from the photographs, as can be seen by comparing figures 3.53 and 3.54. Though no reason was given earlier for this range of values, it is now suggested that because the lateral stress is lower in the case of showered samples placed to the same density, the measure of confinement likely to affect particle reorientation will be reduced by a corresponding amount. Conceivably this could result in a somewhat erratic development in displacements due to the rotational and other affects associated with the relative displacements of irregularly shaned sand grains.

It would thus appear that the influence of the initial lateral stresses existing in a mass of soil (both naturally and in the laboratory) could form the basis of a separate line of research necessary to clarify the mechanics of the pile problem.

It is of interest to note that Durgunoglu and Mitchell (1975) in their cone penetration test studies also noted the significant effect of the increased lateral soil stress due to the compaction of sands by vibration, recording a "much greater penetration resistance than in normally consolidated deposits at the same void ratio".

These observations, while being made with direct reference to the pile problem, must be directly applicable to any foundation problem associated with which empirical or theoretical observations are justified on the basis of correlations between model and full scale tests, or between tests reported by individual authors. With the development of in-situ exploratory devices aimed at measuring the undisturbed stress state in the soil, for example the self-boring pressuremeter (SBP) (Hughes, Hrloth and Hinkle, 1977), a better measure of the lateral stresses existing in an undisturbed soil mass may be able to be obtained.

The features of the self-boring pressuremeter are discussed in Appendix 11.

5.2.3 The Existence of Residual Stresses

As discussed in Chapter 2, the existence of residual stresses has tended to be overlooked in drawing correlations between model pile and full scale tests. Whilst they do not affect the total load measured in a pile test, they have been shown to affect the distribution of load between the pile and shaft by as much as 70%. (Hunter and Davisson, 1969; Hanna and Tan, 1973). Hanna and Tan, as indicated in figure 2.18 (Chapter 2) have graphically shown the effect these residual stresses have in distribution of load along a pile shaft.

Thus where comparisons are made between the total axial load carrying capacity of model and full scale driven piles, the influence of residual pile loads is likely to be insignificant. However, in the case where the full scale pile is fully instrumented, comparisons on the basis of load sharing between the pile and the tin can be grossly in error unless these residual stresses are taken into account.
5.2.4 Scale Effects on the Bearing Capacity Factor \( N_y \)

Steenfelt (1977) attempts to show how the scale effect represented by the grain size : foundation size ratio can significantly affect the bearing capacity factor \( N_y \). He chose to use a series of stacked rods so that the "material" could then be considered to be purely frictional with a straight Mohr-Coulomb failure envelope. However, it has been shown in the literature (for example Rowe, 1962) that even rods will rearrange to adopt a new packing structure, and thus may undergo compaction and expansion with associated changes in lateral stresses.

The author's results whilst related to shallow rectangular footings and exhibiting a considerable scatter, do however indicate the possibility of a scale effect reflecting the apparent large size of sand grains as seen by the scaled down foundation.

De Beer (1965) identified a similar effect which he called the "scale effect on the progressive rupture" phenomenon, the general conclusion being that the bearing capacity factor, \( N_y \), decreases with increasing footing width, the practical effect of which is that for high densities, care should be taken when extrapolating values of the bearing capacity factor, \( N_y \), from small model tests to full scale foundations.

Yamaguchi, Kimura and Fujii (1977) on the basis of centrifuge tests confirmed De Beer's observations. They concluded that the generally held concept of most bearing capacity theories of constant shearing strain throughout the yielding mass is incorrect. In the case of a narrow footing they show that the mobilised shear strains are uniform along the slip surface and have low values coinciding with peak friction angles; however, with wide footings, the shear strains are much larger and are non-uniform along the slip plane with friction angles less than peak (i.e. approaching or at a residual or critical state value.) An indication as to the reason for the variation of \( N_y \) with foundation width has thus been given.

The significance of this effect with regard to oiled foundations has not been investigated. In view of the significance of the \( N_y \) term compared to the \( N_y \) term in the bearing capacity equation (Equation 2.7, Chapter 2), and Broms (1966), suggestion that the \( N_y \) term has a minor effect in comparison to the embedded length of the pile, this aspect of "scale" may be able to be ignored altogether.

5.3 DISCUSSION

Robinsky, Sagar and Morrison (1964) expressed doubt as to the usefulness of model pile tests, primarily on the basis of the difficulty associated with relating the results of model tests to full scale tests. Various other authors have commented on the effects of scale, for example Vesic (1967). His observations were far ranging generally concluding that "as the scale of deep foundations vary so do all the significant parameters influencing bearing capacity". He quotes such factors as the effects of dilation (i.e. the curvature in Mohr's Envelones at low stress levels), the shear strength of sands, the relative compressibility of the sand mass, and volume change within the sand mass, as significantly influencing the apparent scale effects. Vesic's final observation that "only full-scale tests in absolutely controlled conditions can yield conclusive data", cannot be disagreed with.
However, there is a place for model tests which could become increasingly important once the apparent effects of "scale" are resolved.

With regard to correlations between in-situ soils investigation techniques and the end bearing capacity of piles, certain aspects have been questioned herein.

In the case of correlations between model and full scale pile tests the generally considered scale effects can be summarised as:

(i) The effects of in-situ lateral pressures
(ii) The effect of residual loads as a result of pile installation by driving
(iii) The effect of the relative sizes of the soil particles to foundation width
(iv) The effects of pile installation.

These are the "scale" effects able to be presently identified with any degree of confidence.

Of these, putting aside, the effects of pile installation, it is the writer's consideration that the effect of in-situ lateral pressures has probably the greatest influence on the obviously wide variations in correlative results reported in the literature.

It is suggested that every effort be made in future works to draw correlations between model and full scale tests, or between individual research efforts, on the basis of:

(a) the method of pile installation,
(b) the initial density of the soil,
(c) the initial lateral stress state in the soil (K₀).

Whilst the foregoing comments have been generally related to the axially loaded pile problems, they apply equally to all aspects of pile loading, and all aspects of model and full scale foundation testing.

5.4 FACTORS AFFECTING THE ASSESSMENT OF AXIAL LOADS ON PILES

The fundamental difficulties in relating theoretical considerations to both model and full scale pile tests have been outlined, albeit briefly, in the foregoing sections. The suggestion was made that the single greatest correlative factor, the effect of in-situ lateral stress, and the packing structure of cohesionless soils, has in the past not been taken into consideration. Various authors have observed the effect of higher loads registered in sands compacted by vibration, but have, to the writer's knowledge, not questioned the mechanics of the observation. (For example Vesic, 1963; Durgunoglu and Mitchell, 1975).

Consequently it is suggested that many of the correlations used as a basis for the various semi-empirical techniques discussed in Chapter 2 have probably been fortunate accidents. It is not unlikely that various unidentified factors have interacted favourably to produce the desired results.

Vesic (1967), as discussed in Chapter 2, suggested that because of the complexity of the pile problem, there was not one theory in existence that could be unreservedly recommended to the practising foundation engineer for the assessment of ultimate axial pile loads. He suggested that any such theory must include at least the following parameters:
(i) Foundation shape, relative depth and method of construction.
(ii) The shear strength of the sand.
(iii) The relative compressibility and volume change characteristics of the sand.
(iv) An accommodation for the generally unclarified scale effects.

In addition to these, the writer would add one further parameter:-
(v) The displacement fields developed about the pile tip. (Indirectly, these are a reflection of (i) and (iii) above).

Clearly, as indicated in Chapter 2, the conventional bearing capacity theories fail to accommodate these basic requirements, apart from by the application of empirical adjustment factors. Neither can one of the other currently available design tools, for example, elastic analyses, pile driving formulae etc., be considered to hold much rationality when considered against these requirements.

Romh (1966) reported that very small axial deformations are generally necessary to mobilise completely the skin friction resistance along a pile shaft. Vesic (1964) has indicated that this deformation is of the order of about 9 mm and is independent of shaft diameter and lateral pressures.

In contrast relatively large deformations are required to mobilise the maximum point resistance of piles driven into cohesionless soils. As indicated by various authors (for example, Vesic, 1967), the magnitude of the required displacement appears to increase as the pile diameter increases.

On the basis of these observations, it would seem reasonable to conclude that the pile tip effect, due to the application of an axial load, is analogous to the incremental displacement of a pile. Chapter 3 has shown that the effect of incremental pile installation is to generate contours of equal displacement which are near circular about the pile tip on the mid-plane of the pile. The radial displacement profile so developed has also been shown to vary little, irrespective of the embedment depth of the pile. The projection of these circular contours through 3600 results in the generation of approximately spherical surfaces of equal displacement.

The obvious conclusion, is that the incremental pile installation, and thus by the earlier analogy, the application of axial load to the pile, is analogous to the expansion of a spherical cavity. An analytical representation of this concept is attempted in the following sections and verification sought from a series of model tests.

Various other authors have also considered the pile situation as being analogous to a cavity expansion problem, for example Bishop, Hill and Mott (1945); Baligh (1976); Butterfield and Ramjee (1970), and Vesic, (1972).

These various works are discussed briefly in the following.
5.5 VARIOUS CAVITY EXPANSION CONSIDERATIONS AS APPLIED TO THE AXIALLY LOADED PILE PROBLEM

5.5.1 Introduction

In recent years a considerable amount of work has been done on cavity expansion problems, especially with the development of such in-situ measuring devices as the pressuremeter. (Discussed in Appendix II). An indication of the volume of this work is given, for example, by Hughes (1973). The following brief discussion is thus intended only to give an indication of the variety of approaches.

5.5.2 Pile Installation Modelled as the Expansion of a Cylindrical Cavity

(1) Butterfield and Banerjee (1970)

Butterfield and Banerjee (1970) present an analysis in which the effect of pile installation is modelled as the expansion of a cylindrical cavity. Their idealised system is shown in figure 5.4. With this model the pile is taken to be rigid and long and any localised end effects adjacent to the ground surface and pile tip are ignored. It is assumed that conditions about the pile are rotationally symmetric and that the horizontal layers in the system do not change in thickness. The deformation of the ground produced by driving a pile of a particular radius is treated as analogous to the expansion of a cylindrical cavity from zero radius to a finite radius equal to that of the pile.

This assumption will result in the creation of a cylindrical "plastic" zone about the pile inside an outer elastic zone as indicated in figure 5.4. This assumption will also result in contours of equal displacement essentially parallel to the shaft. Such displacement fields would bear no resemblance to those actually observed as reported in Chapter 3.

Clearly then, the assumption that the expansion of a cylindrical cavity adequately models the effects of pile installation by driving in sand, is inappropriate. A preliminary analysis carried out by the writer in which such an assumption was made yielded results which bore no relationship to those actually obtained from the model tests reported later in this chapter.

Of significance however, was the adoption by the authors of large displacement theory considerations. In general most cavity expansion analyses start from the assumption that the cavity has a finite initial radius \( r_0 \) as indicated in figure 5.5(a). The expansion strains are usually expressed in terms of the undeformed radius of the cavity, the assumption being that the incremental displacements are small with respect to the undeformed radius. (i.e. small displacement theory).

\[
\varepsilon = \frac{u}{r_0}
\]

where \( \varepsilon \) = the expansion (circumferential) strain
\( u \) = the incremental expansion of the cavity wall
\( r_0 \) = the initial radius of the cavity.

This assumption however, presents a problem when the cavity is being expanded from zero radius. This problem is overcome by adopting the considerations of large displacement theory. Essentially the effect of large displacement theory is to assume that the strains are more correctly expressed in
FIG. 5.4 THE IDEALISED SYSTEM OF RUTTERFIELD AND RANERJEE (1970).

FIG. 5.5 STRAIN DEFINITIONS.
terms of the current radius of the expanding cavity (i.e. \( r_0 + u \)). Thus the associated strain expression becomes:

\[
e = \frac{u}{u + r_0} \neq a \quad \text{when} \quad r_0 = 0
\]

When \( r_0 \gg u \) then equation 5.2 \# equation 5.1.

(ii) Massarsch and Brons (1977)

Massarsch and Brons (1977) assumed that the change in stress conditions in the soil during the driving of a pile could be modelled as the expansion of a cylindrical cavity, with the soil along the shaft being mainly displaced radially away from the pile. Essentially their work employs the solution of Vesic (1972) for a cylindrical cavity. (This solution is similar to that discussed subsequently for Vesic's solution of the spherical problem, the difference being due to the different equation of equilibrium as applied to the cylindrical cavity analysis i.e.

\[
\frac{d\sigma_r}{dr} + m \left( \frac{\sigma_r - \sigma_0}{r} \right) = 0
\]

where \( m = 1 \) for the cylindrical problem

\( m = 2 \) for the spherical problem).

The authors extend the solution to accommodate the development of pore water pressures.

The inapplicability of the cylindrical cavity expansion solution to the axially loaded driven pile problem has already been discussed. In the absence of other experimental evidence, it would seem reasonable to assume that displacement fields with characteristics similar to those observed about driven piles in sand, are likely to be developed about driven piles in clay, or at least as a result of axial loading.

5.5.3 Pile Installation Modelled as the Expansion of a Spherical Cavity.

(i) Bishop, Hill, and Mott (1945)

The work of Bishop, Hill and Mott (1945) was in fact in conjunction with the indentation of ductile materials (metals) by cylindrical punches. Their fundamental assumptions were that the load on the punch would depend upon:-

(a) The shape and size of the tool.
(b) The coefficient of friction between the indenting tool and the material.
(c) The yield point and strain hardening properties of the material.

The similitude between these assumptions and the pile problem is obvious, i.e.:

(a) The shape and size of the pile.
(b) The coefficient of friction between the pile and the soil.
(c) The ultimate strength characteristics of the soil.

The authors' work was based on the existence of an initial hollow sphere. In addition the assumption was made that the material was non dilatant and frictionless (i.e. there were no volume changes and \( \sigma \) was equal to zero).
The yield stress of the material was the limiting boundary condition at the elastic/plastic interface. Contours of hardness as obtained by the authors, (figure 5.6) representing conditions of plastic flow, are clearly analogous to some of the considerations applied to the pile problem, for example the displacement fields of Robinsky and Morrison (1964) and the soil pressure fields of Chellis (1951) (figure 3.3 Chapter 3 and figure 2.35 Chapter 2, respectively).

(ii) Vesic (1972)

Virtually all the analytical work prior to about the beginning of the 1960's, as reported by Vesic, assumed the soil to behave as a rigid-plastic incompressible solid in a plastic region surrounding the cavity, and as a linearly deformable solid beyond that region, as indicated in figure 5.7. In general the effects of volume change within the plastic zone are not considered analytically. Vesic presents a semi-empirical correlation to enable the effect of volume change within the yielding mass of the soil to be considered. His tables are presented for the expansion of both cylindrical and spherical cavities, but, however, assume an initial cavity of known radius \( R_i \) in figure 5.7. The author's analytical concept is discussed briefly as follows:

The spherical cavity of initial radius \( R_i \) is expanded by a uniformly distributed pressure \( p \), acting on the inside of the cavity wall. With expansion a spherical zone about the cavity passes into a state of plastic equilibrium. This plastic zone will continue to expand until the cavity pressure reaches an ultimate value \( p_u \) at which indefinite expansion of the hole will occur.

At the instant the ultimate pressure \( p_u \) is reached, the cavity will have a radius \( R_u \), and the plastic zone around the cavity will extend to a radius \( R_p \). Beyond that radius the rest of the mass remains in a state of elastic equilibrium.

The author assumes that the soil in the plastic zone behaves as a compressible plastic solid defined by Mohr-Coulomb shear strength parameters \( (c, \phi) \), as well as an average volumetric strain, \( \Delta \). These parameters are determined from the known state of stress within the plastic zone \( (c, \phi) \) as well as from the stress-volume change characteristics of the soil \( \Delta \).

The technique proposed by the author for determining volumetric strains is semi-empirical in that it is based on correlations drawn with "typical" laboratory tests on the same sand at the same density. (The difficulties of obtaining in-situ soil densities have already been discussed in Chapter 2 (Section 2.6.7)).

An initial average volumetric strain is assumed and the solution iterated for the determined stress-state in the plastic zone, until successive values of \( \Delta \) are in close agreement.

Beyond the plastic zone, the soil is assumed to behave as a linearly deformable, isotropic solid defined by a modulus of elasticity \( E_s \) and a Poisson's ratio \( \nu_s \). It is further assumed that prior to the expansion of the cavity, the entire soil mass has an isotropic effective stress, \( q \), and that the body forces within the plastic zone are negligible when compared with existing and newly applied stresses.
Thus a spherically symmetric problem of a weightless medium is considered. With the foregoing considerations, the equation of equilibrium is:

\[
\frac{d\sigma_r}{dr} + \frac{2(\sigma_r - \sigma_\theta)}{r} = 0
\]

5.4

where \( \sigma_r \) = the radial stress acting on a typical soil element
\( \sigma_\theta \) = the circumferential stress acting on the same element (See figure 5.7).

\( r \) = the radius to the soil element being considered.

\( \sigma_r \) and \( \sigma_\theta \) are the major and minor principal stresses respectively. From the Mohr's state of stress shown in figure 5.8 for a cohesive material, it can be shown that the failure condition, defined as being reached when the stress circle is tangential to the "failure" surface is given by:

\[
(\sigma_r - \sigma_\theta) = (\sigma_r + \sigma_\theta) \sin \theta + 2c \cos \theta
\]

5.5

where \( c \) = the cohesive strength of a soil.

Substituting equation 5.5 into equation 5.4 and solving for \( \sigma_r \) yields, given the boundary condition that \( \sigma_r = p_u \) when \( r = R_u \):

\[
\sigma_r = (p_u + c \cot \theta)(\frac{r}{R_u}) - c \cot \theta
\]

5.6(a)

For a cohesionless soil (i.e. a pure sand) the 'c' term can be dropped, reducing the expression to:

\[
\sigma_r = p_u (\frac{r}{R_u})
\]

5.6(b)

To determine the ultimate pressure \( p_u \) and the instantaneous radius of the plastic zone \( R_p \) Vesci adopts a relationship stating that "THE CHANGE OF VOLUME OF THE CAVITY IS EQUAL TO THE CHANGE IN VOLUME OF THE ELASTIC ZONE PLUS THE CHANGE IN VOLUME OF THE PLASTIC ZONE".

Denoting the radial displacement of the limit of the plastic zone as \( u_p \), this relationship can be expressed thus: (See also figure 5.7).

\[
p_u^3 - p_1^3 = p_p^3 - (p_D - u_p)^3 + (p_D^3 - p_u^3)\Delta
\]

5.7

The radial displacement at the elastic/plastic interface is obtained from Lamé's solutions (Timoshenko and Goodier, 1970 or Salmon, 1957), as:

\[
u_p = (\frac{1 + \nu_p}{2E_p}) p_p (\sigma_p - \alpha)
\]

5.8

where \( \sigma_p = \alpha \) when \( r = p_p \)

Rearranging and combining these various expressions yields the ratio of the radius of the plastic zone to that of the cavity thus:

\[
\frac{p_p}{p_u} = 3 \sqrt{\frac{r}{1 + \nu_p \Delta}}
\]

5.9

where \( I_p \) = a rigidity index, which is essentially the ratio of the shear modulus of the soil (\( G_p \)) to its initial shear strength.
FIG. 5.6 HARDNESS CONTOURS AFTER PUNCHING INTO ANNEALED COPPER
(After Bishop, Hill and Mott, 1945).

FIG. 5.7 EXPANSION OF A CAVITY.
(After Vesić, 1972)

FIG. 5.8 MOHR'S CIRCLE OF STRESS FOR A C-4 MATERIAL.
Vesic tabulates values of \( I_r \) for various soil friction angles. With this ratio known \( (p_d/p_u) \), then the ultimate pressure on the cavity may be obtained from:

\[
\eta_y = c F_c + \alpha F_q
\]

which for a cohesionless soil reduces to:

\[
\eta_u = q F_q
\]

where \( F_q \) = a dimensionless spherical cavity expansion factor

\[
i.e. \quad F_q = \frac{3(1 + \sin \theta)}{3 - \sin \theta} \left( \frac{I_r}{1 + I_r^2} \right)^{\frac{d \sin \theta}{3(1 + \sin \theta)}}
\]

This expression (equation 5.10(b)) is equivalent to the bearing capacity relationship of equation 2.10, Chapter 2.

\[
i.e. \quad (p_u = ) q_u = q N_q
\]

When related to the pile problem however this solution remains semi-empirical for the following reasons:

(a) No attempt is made to match displacements developed due to the expansion of the cavity.
(b) Because these displacements are not considered, the effects of pile installation cannot be taken into account.
(c) The expansion of the cavity is considered to commence from a finite radius.
(d) The manner in which the soil volume changes are taken into account is limited in that only contracting soils are considered. Soils which expand in volume are treated as if they were incompressible.
(e) The solution requires that a value be assumed for Poisson’s ratio \( (\nu_g) \).

In general, the fundamental requirements set by the author himself in 1967 (Section 5.4), are not met. When expressed in terms of a frictionless material of constant volume, Vesic’s solution is similar to that obtained by Bishop, Hill and Mott (1945).

(iii) Baligh (1976)

Baligh extends Vesic’s solution to consider the case where the Mohr-Coulomb “failure” envelope in sands is curved, such as those obtained by the writer and reported in figure A5.10, Appendix 5.

Vesic’s analysis is based on high pressure triaxial tests which indicate that at the pressures developed under deep foundations (i.e. about 3000 kPa) only compactive volume changes occur, even in dense granular materials. His associated model tests were on 100 mm (4 inch) diameter piles.

However as indicated in Chapter 3, it would appear that as long as the soil can flow during driving towards a free surface or that the postulated intersecting state surface exists, volume expansion is probably able to continue to occur, even at high stresses. Consequently the triaxial compression test is unlikely to represent conditions existing about a driven pile, except possibly at considerable depths, i.e.

where \( \frac{L}{D} >> 10 \).

Baligh presents his results as a modification on Vesic’s (1972) work thus:—
where \( \varepsilon \) = a modification factor depending on the curvature of the Mohr-Coulomb "failure" envelope.

\[ \rho_y = \text{the expression given by equation 5.10(h).} \]

Tabulated values of \( \varepsilon \) are presented by the author.

This solution is semi-empirical for the same reasons listed with regard to Vesic (1972).

### 5.6 A FURTHER ATTEMPT TO MODEL THE AXIALLY LOADED PILE SITUATION AS A CAVITY EXPANSION PROBLEM

#### 5.6.1 Introduction

The analytical solution to the spherical expansion problem presented in this section is developed specifically in an attempt to provide a rational method of assessing the load carrying capacity of an axially loaded pile. In so doing all of the fundamental requirements suggested by Vesic' (1967) (See section 5.4), are accommodated either directly or indirectly.

Hughes, Wroth and Hindle (1977) presented a method for determining the angle of internal friction (\( \phi \)) and the angle of dilation (\( \psi \)) from pressuremeter tests in sands. This work then enables the in-situ values of \( \phi \) and \( \psi \) to be obtained, thus automatically accommodating the effects of stress history and other factors affecting the engineering properties of naturally occurring sand deposits.

Gibson and Anderson (1961) interpreted data from pressuremeter tests in sands assuming that the sand behaved perfectly elastically until failure was reached, beyond which the sand continued to fail at a constant stress ratio and at constant volume. The assumption of constant stress ratio has been well verified, (see figure A5.14, Appendix 5), however, it is in the further assumption that the sand deforms at constant volume that the major flaw in their analysis lies (see figure A3.1, Appendix 3). Vesic' (1972), as previously discussed, proposed a solution to the cylindrical cavity expansion problem (i.e. the pressuremeter problem) which requires that a semi-empirical assessment be made of the volume changes occurring within the sand mass. Al-Awkat, as reported by Schmertmann (1975) introduced the effect of volume change by using an empirical correlation between volumetric strain and field values of relative density (\( \rho_R \)). (The applicability of relative density criteria are discussed in Chapter 2 and Appendix 6).

Clearly then, prior to the presentation of the work of Hughes, Wroth and Hindle (1977), the only means of obtaining a reliable value for the friction angle of sands from in-situ tests was by applying empirical considerations to the results of in-situ pressuremeter tests to take into account the effects of their known dilatant behaviour. Their solution to the problem is based on the results of a large number of plane strain shear tests on Leighton Buzzard sand (Stroud, 1971) and interpreted in terms of the theory of stress dilatancy (Rowe, 1962, 1971).

The spherical analysis subsequently reported is in fact an extension of the work of Hughes, Wroth and Hindle (1977). Their concepts were adopted because the writer considered it eminently sensible to base the analysis on parameters that could be reliably obtained from in-situ tests: thus providing a means of overcoming some of the factors affecting the credibility of many of the methods of assigning axial loads to piles as discussed in Chapter 2.
From the experimental results of Chapter 3 and from the earlier discussion in this chapter, a simplification of the writer's concept of the manner in which the soil resists the axial loads transmitted by the pile is shown in Figure 5.0.

Basically the concept is that the pile replaces the upper hemisphere of a sphere of soil undergoing prestressing as a result of the expansion of a spherical cavity created by the penetration of the pile tip during installation.

5.6.2 The Analysis

(1) An Expression for the Radius of an Expanding Sphere in a Dilatant Material

As indicated in Appendix 5, the ratio of volumetric to shear strains and thus the dilation rate, \( \nu \), (where \( \nu = \sin^{-1} \left( \frac{3}{5} \right) \), See Appendix 5), remains sensibly constant over a substantial range of strain for constant values of the peak stress ratio \( \left( \frac{3}{5} \right) \). James and Pransby (1970) analysed the strain fields developed behind a model retaining wall as these were failed in passive modes by rotation separately about the top and toe. They found that although the magnitudes of the strains varied significantly throughout the field at any one stage of the test, the ratios of volumetric to shear strain remain reasonably constant, although at high levels of strain they tended to become more irregular as indicated in Figure 5.10. Similar observations have been made by the writer with regard to the strain fields developed during the installation of model piles by driving into sand, as discussed in Chapter 4.

Appendix 3 discusses Rowe's stress dilatancy theory which enables the stress ratio and dilation rate to be expressed as:

\[
P = KD
\]

where \( \phi = \frac{\sigma_1}{\sigma_3} \)

\[
K = \tan^2 \left( 45 + \frac{\phi}{2} \right) = \frac{1 + \sin \phi}{1 - \sin \phi}
\]

\[
D = 1 - \frac{\phi}{\sigma_1}
\]

From the typical Mohr's circle of stress for a sand indicated in Figure 5.11, the ratio of the maximum shear stress to the mean normal stress \( \left( \frac{5}{3} \right) \) is given by:

\[
\frac{t}{5} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \sin \phi
\]

Rearrangement gives:

\[
\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \phi_{\text{Moh}}}{1 - \sin \phi_{\text{Moh}}} = p
\]

In the spherical case the actual strain conditions are the same as those assumed to apply in the triaxial test: these are that the intermediate and minor principal stresses and thus strains are of equal value (i.e. \( \varepsilon_1 = \varepsilon_2 < \varepsilon_1 \)).

Thus the dilation rate \( \left( \frac{2}{3} \right) \) is the same as that derived for the triaxial tests in Appendix 5. i.e.
FIG. 5.9 THE SPHERICAL MODEL.

FIG. 5.10 MEAN VOLUMETRIC STRAIN-SHEAR STRAIN CURVE FOR TRIANGULAR SOIL ELEMENTS IN PASSIVE EARTH PRESSURE PROBLEM
(After James and Bransby, 1970)
\[
\frac{\varphi}{\psi} = \frac{2(1 - D)}{2 + D} = -\sin \psi
\]

This rearranges to:
\[
\phi = \frac{2(1 + \sin \psi)}{(2 - \sin \psi)}
\]

Hence equation 5.13 can be rewritten as:
\[
\frac{1 + \sin \theta_{mob}}{1 - \sin \theta_{mob}} = \left[ \frac{1 + \sin \phi}{1 - \sin \phi} \right] \left[ \frac{2(1 + \sin \psi)}{2 - \sin \psi} \right]
\]

It is assumed that \( \theta_{mob} \) and \( \psi \) remain essentially constant at a particular depth once the peak stress ratio is reached (See figures A5.5 and A5.14, Appendix 5).

When a sphere expands, because of the condition of diametric symmetry, all displacements are in a radial direction only. As indicated in figure 5.12 the strain (and thus displacement) conditions are of the same magnitude at any particular radius on the sphere. Because of this symmetry the principal stresses acting on any soil element are \( \sigma_r \) and \( \sigma_\theta \). Spherical coordinates are used and the radial displacement of any point is denoted by 'u'. Using "large strain" theory and taking compressive strains as positive, the current radial and circumferential strains (\( \varepsilon_r \) and \( \varepsilon_\theta \) respectively) are given by:

\[
\varepsilon_r = \frac{d u}{d u + d r} = \frac{\frac{d u}{d r}}{1 + \frac{d u}{d r}}
\]

\[
\varepsilon_\theta = \frac{u}{u + r}
\]

The volumetric strain for a typical soil element is given by:

\[
\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + (\varepsilon_1 \varepsilon_2 \varepsilon_3) - (\varepsilon_1 \varepsilon_2) - (\varepsilon_2 \varepsilon_3) - (\varepsilon_1 \varepsilon_3)
\]

For the spherical problem two of the principal strains are equal, i.e.:

\[
\varepsilon_1 = \varepsilon_r
\]

\[
\varepsilon_2 = \varepsilon_3 = \varepsilon_\theta
\]

Thus

\[
\varepsilon_v = \varepsilon_r + 2\varepsilon_\theta + (\varepsilon_r \varepsilon_\theta^2) - 2(\varepsilon_r \varepsilon_\theta) - \varepsilon_\theta^2
\]

Assuming that the products of the strains are small compared to the sum of the strains, then:

\[
\varepsilon_v = \varepsilon_r + 2 \varepsilon_\theta = \phi_{oct}
\]

This assumption is able to be justified for small strains (i.e. at some distance from the pile tip), but may be less valid at large strains (i.e. close to the pile tip).

Also:

\[
\phi_{oct} = \frac{1}{\sqrt{3}} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2}
\]

\[
= \frac{\varepsilon_r - \varepsilon_\theta}{\sqrt{3}}
\]

In equation 5.16, the dilatation rate is expressed as:

\[
\frac{\varphi}{\psi} = -\sin \psi
\]
FIG. 5.11 MÖHR'S CIRCLE OF STRESS FOR A * MATERIAL.

(a) Expanding Sphere

(b) Strain Conditions For All Elements

FIG. 5.12 SYMMETRY OF EXPANDING SPHERE.
\[
\varepsilon_1 + \varepsilon_3 \\
\varepsilon_1 - \varepsilon_3
\]
for conditions of plane strain 5.21(a)

This expression is also considered to hold in the triaxial compression test (see Appendix 5).

Assuming spherical expansion about a pile tip it is considered that the dilation angle \( \psi \) is more likely to be best represented by the spherical state of strain, i.e. those existing on the octahedral plane. Thus equation 5.21(a) is rewritten as:-

\[
\psi_{\text{oct}} = - \psi_{\text{oct}} \sin \psi
\]
5.21(b)

That this expression remains approximately constant for a particular depth of pile penetration is indicated in Chapter 4.

Substituting for equations 5.19(c) and 5.21(b):-

\[
\frac{\varepsilon_r + 2 \varepsilon_\theta}{\varepsilon_r} = - \left( \frac{1}{\sqrt[3]{3}} \right) \left( \frac{\varepsilon_r - \varepsilon_\theta}{\varepsilon_r} \right) \sin \psi
\]
5.21(c)

Letting \( A = \frac{1}{\sqrt[3]{3}} \sin \psi \), equation 5.21(c) rearranges to:-

\[
\frac{\varepsilon_r}{\varepsilon_\theta} = \frac{A - 2 \varepsilon_\theta}{A + \varepsilon_\theta}, \text{ or}
\]

\[
\frac{\varepsilon_r}{\varepsilon_\theta} = -\eta^*
\]
5.22(a)

where

\[
\eta^* = \frac{2 - \frac{1}{\sqrt[3]{3}} \sin \psi}{1 + \frac{1}{\sqrt[3]{3}} \sin \psi}
\]

Substituting for values of \( \varepsilon_r \) and \( \varepsilon_\theta \) as equation 5.18, equation 5.22 can be expressed in terms of the instantaneous displacement and position of the soil element thus:-

\[
\left( \frac{du}{dr} \right) \left( \frac{u + r}{u} \right) = -\eta^*
\]
5.22(h)

Rearranging this equation yields:-

\[
\frac{du}{dr} = -\eta^* \left( \frac{u + r}{u} \right) \left( 1 + \frac{du}{dr} \right)
\]
5.23(a)

which in turn rearranges to:-

\[
\frac{du}{dr} = -\eta^* \frac{u}{u + r} \left( 1 + \frac{du}{dr} \right)
\]
5.23(h)

and thus:

\[
\frac{du}{dr} = -\eta^* \frac{\frac{u}{r}}{1 + \frac{u}{r}(1 + \eta^*)}
\]
5.23(c)

Letting \( \frac{u}{r} = m \), \( \varepsilon_\theta \) may be expressed as a first order derivative thus:-

\[
\frac{du}{dr} = m + r \frac{dm}{dr}
\]
5.24
Equating equations 5.23(c) and 5.24:-
\[ \frac{du}{dr} = m + r \frac{dm}{dr} = \frac{-n^m}{1 + m (1 + n^p)} \]

Rearranging this yields:-
\[ r \frac{dm}{dr} = - \left\{ \frac{(1 + n^p) m (1 + m)}{m (1 + n^p) + 1} \right\} \]

Let \( R = 1 + n^p \), and rearrange equation 5.26, simultaneously gathering "like" terms:-
\[ \frac{dr}{r} = - \left\{ \frac{Dm + 1}{nm (1 + m)} \right\} dm \]

which can be reduced to the first order differential equation of:-
\[ \frac{dr}{r} = - \left\{ \frac{1 + m + \frac{1}{m}}{1 + m} \frac{1}{1 + m} \right\} dm \]

Integrating this and substituting back for \( B \) and \( m \) yields:-
\[ z_n (r) = - \left\{ \frac{n^p}{(1 + m + \frac{1}{m})} z_n \left( \frac{r}{r} + \frac{1}{1 + m} \right) + z_n \left( \frac{r}{r} + \frac{1}{1 + m} \right) \right\} \]

(ii) The Boundary Conditions

As discussed with respect to figure 5.7, as the plastic zone expands outwards it carries with it an interface or "frontier" where compatibility is maintained between the elastic and elastically deforming zones. Defining the radius of the pile as 'a' and the radius to the elastic-plastic boundary as '\( r_p \)' (See figure 5.13), the boundary conditions become:-

(a) At the pile face: \( r = a \) when \( u = a \)

(b) At the elastic-plastic boundary:
\[ r = r_p \] when \( u = u_p \)

Applying this latter boundary condition first and making the approximation that because the displacement on this outer boundary will be very small compared to the radius to the boundary, (i.e. \( u_p \approx r_p \)), then the current radius at the boundary associated with the displacement \( u_p \) is approximately equal to the radius of the boundary, (i.e. \( r_p \approx u_p + r_p \)); then an expression for this radius may be obtained from equation 5.28 thus:-
\[ r_n = r \left\{ \left( \frac{r + u}{r_u} \right) \left( \frac{r}{r_u} \right) \left( \frac{u}{r_u} \right) \right\} \]

Applying the other boundary condition then,
\[ \text{i.e. when } r = a; \text{ } u = a:-\]
\[ r_n = a \left\{ \frac{n^p}{(2^{1/p})^p} \right\} \]

From Lamé's solution for the displacement at the elastic-plastic boundary (equation 5.8):
\[ \text{i.e. } \]
\[ u_p = \left( \frac{1 + u_p}{2} \right) r_n \left( \sigma_n - \sigma_p \right) \]

a value for \( r_n \) in terms of the radius of the pile and the soil parameters, accounting for volume change, is able to be determined.
FIG. 5.13 ANALYTICAL BOUNDARY CONDITIONS.
The shear modulus of a soil, \( G \), may be determined directly from either laboratory triaxial tests or in-situ pressuremeter tests, thus it is convenient to rewrite equation 5.2 in terms of the shear modulus where:

\[
G = \frac{E}{2(1 + \nu_s)}
\]

Obtaining a value for \( G \) directly from a test enables the elastic parameter to be determined without having to assess a value for Poisson's ratio.

Thus the displacement on the elastic-plastic boundary becomes:

\[
u_D = \frac{r_n}{2n} (\sigma - \sigma_A)
\]

where \( \sigma_D \) = the stress at which yield of the soil first occurs, i.e. the stress at which the stress path followed by the deforming soil first intersects the "yield" surface.

\( \sigma_A \) = the in-situ or ambient stress.

As can be seen from figure 5.13(a) this stress varies about the hemisphere depending on both depth and orientation. A conservative approximation would be that it is equal to the in-situ undisturbed stress at the level of the mid-plane of the hemisphere (i.e. \( \sigma_A \) in figure 5.13(a) (i.e. \( \sigma_0 \)).

Substituting for \( u_D \) into equation 5.30:

\[
r_n = \frac{1}{2(1 + n^*)} \left( \frac{\Delta n}{\sigma_n - \sigma_A} \right)^{1+n^*}
\]

(iii) The Yield Condition

The differential equation of equilibrium which must be satisfied for an expanding sphere is as equation 5.3, i.e.:

\[
\frac{d}{dr} (\sigma_r) + 2(\sigma_r - \sigma_0) = 0
\]

It is now assumed that the sand mass is at the failure stress ratio when \( \frac{t}{s} \) is constant (See figure A5.15, Appendix 5). From equations 5.14 and 5.15, a constant value of \( \frac{t}{s} \) yields a constant value of \( \frac{\sigma_r}{\sigma_0} \) (i.e. \( \sigma_1 \) of:

\[
\frac{\sigma_r}{\sigma_0} = \frac{1 + \sin \phi'}{1 - \sin \phi'}
\]

The right hand side of equation 5.34 is usually referred to as \( 
\]

\[
\sigma_n = N \sigma_r
\]

Substituting for \( \sigma_0 \) in equation 5.3:

\[
\frac{d}{dr} (\sigma_r) + 2(\sigma_r - \sigma_0) = 0
\]
Rearranging:

\[ \frac{h}{\sigma_r} + 2(1 - N) \frac{dr}{r} = 0 \]  

Integrating and applying the boundary condition that:

\[ r = r_p \text{ when } \sigma_r = \sigma_p \]

\[ \ln \left( \frac{\sigma_r}{\sigma_p} \right) = 2(1 - N) \left\{ \ln \left( \frac{r_p}{r} \right) \right\} \]

Thus:

\[ \sigma_r = \sigma_p \left( \frac{r_p}{r} \right)^{2(1-N)} \]

From equation 5.20:

\[ \frac{r_p}{r} = (r + u)^{\frac{n^*}{1 + n^*}} \left( \frac{u}{r} \right) \left( \frac{r_p}{u_p} \right) \]

Substituting this into equation 5.38(b), then:

\[ \sigma_r = \sigma_p \left( \frac{r + u}{r} \right)^{\frac{n^*}{1 + n^*}} \left( \frac{u}{r} \right) \left( \frac{r_p}{u_p} \right) \]

Substituting for \( u_p \) from equation 5.32, then:

\[ \sigma_r = \sigma_p \left( \frac{r + u}{r} \right)^{\frac{n^*}{1 + n^*}} \left( \frac{u}{r} \right) \left( \frac{\Delta A}{\sigma_p - \sigma_A} \right)^{\frac{1}{1 + n^*}} \]

Equation 5.40(b) thus provides an expression for the radial stress acting on the pile tip for the deformation due to the pile tip in terms of all the known parameters of the soil. Thus at the pile tip, where \( u = r = a \):

\[ \sigma_{ra} = \sigma_p \left( \frac{2}{1 + n^*} \right) \left( \frac{\Delta A}{\sigma_p - \sigma_A} \right)^{\frac{1}{1 + n^*}} \]

where

\[ N = \frac{1 - \sin \phi_i}{1 + \sin \phi_i} \]

\[ n^* = \frac{2 - \sqrt{2} \sin \psi}{\sqrt{3} \sin \psi} \]

With reference to Powe's stress dilatancy equation (5.17), it can be shown that this can be rewritten:

\[ N_{\text{mob}} = N_{\text{f}} f_n \left( \frac{1}{n^*} \right) \]

thus indicating that Powe's criteria have been met.

It remains to ascertain the parameters to employ in equation 5.41 and to test it against the test loading of a model pile. In summary, these parameters are:

(a) the soil friction angle (\( \phi \))
(b) the ambient stress acting in the soil (\( \sigma_A \))
(c) the "yield" stress of the soil (\( \sigma_p \))
(d) the soil shear modulus (\( G \))
(e) the rate of dilatation (\( \nu \))
(a) The Soil Friction Angle (φ)

Appendices 5 and 9 show that the following parameters apply:

(i) $\phi_C = 32^\circ$
(ii) $\phi_{(mob)} = 44.4^\circ$
(iii) $K_0 = 0.4$

The peak and critical state friction angles are best discussed in terms of an elastic stress path in principal stress space as indicated in figure 5.14. In figure 5.14, the line OA'G is the "space diagonal" (or isotropic line). It is along this line that the major and minor principal stresses, (i.e. $\sigma_V$ and $\sigma_H$, or $\sigma_{r}$ and $\sigma_{g}$), are of equal value. Thus if the initial stress conditions acting in the soil were such that in-situ vertical and horizontal stresses were equal (i.e. $K_0 = 1$), then the elastic stress path followed by the soil would commence from the in-situ stress state represented by point A' and proceed with deformation or loading along the stress path A'AR for the case of an expanding cylinder ($\sigma_{r} = \sigma_{g}$) to an intercept with the yield surface at R. In fact, as the measured ratio of horizontal to vertical stress is $0.4$ ($K_0 = 0.4$, see Appendix 9), the stress path actually commences from the point marked A.

The stresses existing on the inside of the spherical cavity are given by:

$$\sigma_{\phi} = \frac{-\sigma_{r}}{2}$$  \hspace{1cm} (5.43)

Thus the stress path followed for spherical expansion is that indicated by AB'DE.

If by the time the stress path intercepts the critical state failure surface the soil is shearing at constant friction angle ($\phi_C = 32^\circ$) and at constant volume (i.e. $\nu = 0$), then the soil would follow the stress path AR'C along the failure surface, with unlimited deformation occurring for no change in the failure stress ratio.

In the series of model pile experiments reported herein it has been observed that most of the soil is likely to be at values of $\phi$ greater than the critical state. As indicated in Chapter 4 most of the soil mass continues to dilate hence constant volume (i.e. critical state) conditions are not reached except for possibly a thin zone about the tip. From Appendix 5 it would appear that most of the soil continues to shear at about the peak mobilised friction angle ($\phi_{(mob)}$) of about $44.4^\circ$ at a constant average dilatation rate. Thus the soil about the pile follows the stress path ADE in figure 5.14, where shearing occurs along the apparent failure surface for $\phi_p = 44.4^\circ$ defined by ADE. As constant volume conditions are approached the stress path will tend towards the critical state surface (ONC) finally intersecting it at point F when all volume change has ceased.

Because drained conditions apply (i.e. the soil mass is dry), the stress paths shown are effective stress paths.
(b) Values of the Ambient and Yield Soil Stresses ($\sigma_A$ and $\sigma_D$)

The ambient stress-state is defined by the stress acting at point A in Figure 5.15. The vertical and horizontal components of $\sigma_A$ are $\sigma_V$ and $0.4\sigma_V$ respectively. The yield stress $\sigma_D$ is obtained from triangle ABD when the stress path AD intersects the "neak yield surface" NO at N.

The flow factor $N_3$ is:

$$N_3 = \frac{1 + \sin \alpha}{1 - \sin \alpha} = \frac{\sigma_1}{\sigma_3} = \frac{(\sigma_V)_{0}}{(\sigma_D)_{0}} = \frac{1}{5.66} = 0.56$$

Thus:

$$\sigma_D = \sigma_V = \frac{(\sigma_D)_{0}}{(\sigma_D)_{0}}$$

where

$$(\sigma_D)_{0} = \frac{1}{5.66} = 0.177$$

Thus

$$\sigma_D = (\sigma_D)_{0} = (\sigma_D)_{0} \sigma_D = 0.177 \sigma_D$$

and

$$\sigma_H = \sigma_V + 2(\sigma_D)$$

which yields:

$$\sigma_D = 1.329 \sigma_V$$

(c) Values of the Shear Modulus $G$

The shear modulus $G$ is the slope of OA in Figure 5.16. Figure 5.16 is the idealised stress-strain curve of Figure 5.17 (Appendix 5). The stress conditions associated with point H are thus those applying to the "failure surface" given by:

$$\frac{1}{\sin \phi} = 0.55$$

Thus:

$$\phi = 33.37^\circ$$

and

$$N_6 = 3.44$$

The related stress space is indicated in Figure 5.17:

$$(\sigma_D)_{f} = 0.29$$

Thus:

$$(\sigma_1)_{f} = 1.139 \sigma_V$$

$$(\sigma_3)_{f} = 0.29 \sigma_V$$

The shear modulus $G$ is given by:

$$G = \frac{1}{\tan \phi}$$

$$\tau = \frac{(\sigma_1 - \sigma_3)_{f}}{2} = \frac{(1.139 - 0.33)\sigma_V}{2} = 0.405 \sigma_V$$
FIG. 5.14 ELASTIC STRESS PATH IN PRINCIPAL STRESS SPACE.

FIG. 5.15 STRESS AT FIRST "YIELD" ($\sigma_p$).

FIG. 5.16 IDEALISED STRESS-STRAIN CURVE FROM TRIAXIAL TESTS.
Thus from figure 5.16:

\[ \sigma_0 = \frac{0.495 \sigma_v}{0.70055} = 73.6 \sigma_v \]  

5.46(c)

(d) Values of the Dilation Rate (v)

The values in table 5.1 are collated from the experimentally determined volume change characteristics of figure 4.50 (Chapter 4). These values are in fact the average of those observed to have developed about the pile tin.

<table>
<thead>
<tr>
<th>L/M</th>
<th>v₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26</td>
<td>27.3</td>
</tr>
<tr>
<td>2</td>
<td>25.1</td>
</tr>
<tr>
<td>3</td>
<td>22.8</td>
</tr>
<tr>
<td>4</td>
<td>20.5</td>
</tr>
<tr>
<td>5</td>
<td>18.2</td>
</tr>
<tr>
<td>6</td>
<td>15.8</td>
</tr>
<tr>
<td>7</td>
<td>13.5</td>
</tr>
<tr>
<td>8</td>
<td>11.7</td>
</tr>
<tr>
<td>9</td>
<td>10.0</td>
</tr>
</tbody>
</table>

(e) Summary

In addition to table 5.1, the other parameters as derived are summarised in table 5.2 for the failure surface associated with the peak stress ratio yielding the peak friction angle of 44.4°.

<table>
<thead>
<tr>
<th></th>
<th>44.4°</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>0.18</td>
</tr>
<tr>
<td>N</td>
<td>1.64</td>
</tr>
<tr>
<td>2(1-H)</td>
<td>1.329 σ_v</td>
</tr>
<tr>
<td>G</td>
<td>73.6 σ_v</td>
</tr>
<tr>
<td>σ_H</td>
<td>σ_v</td>
</tr>
<tr>
<td>σ_H</td>
<td>0.4 σ_v</td>
</tr>
</tbody>
</table>

5.6.3 Measured Axial Pile Loads

To enable the ultimate axial load capacity of the tin of a model pile of the same dimensions and surface roughness characteristics as the piles used for the experiments reported in Chapters 3 and 4 to be obtained, a matching model pile was constructed which permitted the shaft load carrying capacity to be separated from that of the tin, as indicated in figure 5.18.
FIG. 5.17 STRESS SPACE FOR ELASTIC SHEAR MODULUS

FIG. 5.18 MODEL PILE FOR INDEPENDENT MEASUREMENT OF SHAFT AND TIP LOADS.
The piles were installed using the same driving techniques as in Chapter 3, into sand placed to the same density with the same \( K_0 \) conditions (see Appendices 6 and 9). Load reaction and measurement was obtained using the triaxial equipment and load cell described in Appendix 9. The loads were applied using the triaxial apparatus in the manual mode. An interval of approximately 30 minutes was allowed between each load increment to allow the system to come to equilibrium. Complete load-displacement profiles were obtained at each embedment depth. Shaft and tin loads were measured at alternate embedment depths (i.e. tip loads were measured at L/D's of 1.26, 3, 5, 7, and 9, while shaft loads were measured at L/D's of 2, 4, 6 and 8). This arrangement was based on the observation made by Robin, Sagar and Morrison (1964), that only relatively small additional axial displacements were necessary to fully mobilise tip resistances after having already been taken to the ultimate value at a particular depth.

The special pile arrangement of figure 5.18 enables the pile tip to be kept coaxial with the shaft by means of a pair of frictionless teflon bearings. To prevent sand entering into the clearance gap provided between the pile shaft and tip, a 'shield' was fitted which itself was provided with a thin overlapping rubber membrane as a secondary safeguard. This arrangement proved to be very satisfactory.

The measured pile tip and shaft loads are indicated in figure 5.19. These results are the average of four tests. Clearly at shallow depths the maximum contribution to the axial load carrying capacity of the pile is provided by the pile tip. However, with increasing penetration the load carried by the pile tip tends to a constant value. Concurrently, as the embedment depth increases, so does the proportion of the shaft available to contribute load carrying capacity; thus at some embedment depth (L/D) the pile shaft and tip will carry equal loads. Beyond this depth the shaft load carrying capacity will be greater than that of the tip, and continue to increase with depth. Thus although the tip loading tends to become constant, the total load carrying capacity will continue to increase.

5.6.4 Radial Stress Profile

From Appendix 6, the dry density of the soil as placed was 1640 kg m\(^{-3}\). This yields an effective vertical stress with depth of:

\[
\sigma_v = (16.08)L \, kPa
\]

As indicated in figure 5.14 the horizontal stress existing in the soil prior to pile installation is \( K_0 \sigma_v \), thus the stress acting around the hemisphere will vary both with \( \sigma_v \) and radial direction. For simplicity in the following section the mean stress has been taken as \( \sigma_v \) at the level of the pile tip,

\[
\sigma_A = \sigma_v
\]

As indicated by figure 5.20, the foregoing theory is only likely to be appropriate when the pile embedment depth is sufficient to enclose the sphere defined by the radius \( r_D \).

Equation 5.33 defines the radius to the elastic-plastic boundary. The value of \( r_D \) is clearly dependent upon the soil dilation characteristics. If the soil were incompressible, for example, (i.e. \( v = 0 \)), the sphere defined by \( r_D \) would only just be beneath the soil surface at a pile embedment depth
FIG. 5.19 MEASURED MODEL PILE SHAFT AND TIP LOADS.

FIG. 5.20 INAPPROPRIATENESS OF THEORY AT SHALLOW DEPTHS.
As \( v \) increases, so also does \( r_n \), consequently the theory is probably inappropriate for the pile embedment depths considered in this project.

For the sake of a comparison however, the theory is compared with the results of the axially loaded model pile tests at embedment depths of 8 and 9.

Figure 5.21 shows the calculated pile tip loads, assuming \( \phi \) is equal to the peak triaxial value of 44.4°, for values of \( v \) of 90, 50 and the measured values of table 5.1.

Clearly the measured pile tip loads are considerably under-estimated; however if \( v \) is increased to 130° and \( \phi \) to 53° (i.e. increases of about 20% on the experimentally determined values), the measured loads are only under-estimated by about 20% (i.e. AB in Figure 5.21). The increase in \( v \), which is likely to be within the bounds of error associated with the assumption of small strain conditions, results however, in the surface of the hypothetical sphere projecting beyond the soil surface by about 2 to 30.

The increase in \( \phi \) is not inappropriate if the conditions about the pile tip are closer to plane strain rather than triaxial conditions.

Nevertheless it is obvious that the theory is inappropriate for the model pile depths considered and is thus more likely to apply at greater pile embedment depths.

The influence of the dilation of the soil mass is significant as indicated in Figure 5.21 wherein a dilation angle of approximately 10° accounts for almost a doubling in calculated tip loads: i.e. CD to EF.

It is of interest to compare the load prediction AB in Figure 5.21 with the ultimate tip capacities predicted by the various theoretical and empirical bearing capacity considerations reported in the literature.

The range of bearing capacity factors were collated by Vesic' (1963) and were presented earlier as Figure 2.4 (Chapter 2).

The pile tip axial load carrying capacity has been assessed for a peak friction angle of 44.4° (See Appendix 5) using the general relationship of equations 2.1 and 2.10 (Chapter 2), i.e.:

\[
\eta_t = \eta_u A_{pt} \quad (5.49(a))
\]

where

\[ \eta_u = a \eta_q \]

\( q_u \) = the effective over-burden pressure at the pile tip \( = q_g L = 16.08 \) kPa,

\( \eta_q \) = the bearing capacity factor

\( A_{pt} \) = the cross-sectional area of the pile tip.

Table 5.3 summarises bearing capacity factor for a \( \phi \) of 44.4° as reported by Vesic' (1963) (See figure 2.4).
FIG. 5.21 CALCULATED PILE TIP LOADS AT $\frac{L}{D}$ OF 8 AND 9.

FIG. 5.22 COMPARISON WITH BEARING CAPACITY THEORIES.
TABLE 5.3
RANGE OF BEARING CAPACITY FACTORS REPORTED IN THE LITERATURE
\( (\phi = 44.4^\circ) \) (after Vesc' 1963)

<table>
<thead>
<tr>
<th>Source</th>
<th>( n_n )</th>
<th>( n_u ) (kPa) at L/D of ( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Bound</td>
<td>Terzaghi (1943)</td>
<td>200</td>
</tr>
<tr>
<td>Medium Bound</td>
<td>Berezantzev (1961)</td>
<td>440</td>
</tr>
<tr>
<td>Upper Bound</td>
<td>De Beer (1945), Meyerhof (1953)*</td>
<td>3200</td>
</tr>
<tr>
<td>Writer</td>
<td>( \phi = 44.4^\circ ) ( \nu = \text{measured} )</td>
<td>575 (EF)</td>
</tr>
<tr>
<td></td>
<td>( \phi = 54^\circ ) ( \nu = 13^\circ )</td>
<td>990 (AP)</td>
</tr>
</tbody>
</table>

* for driven piles.

The various pile tip axial load carrying capacities so obtained are indicated, along with the writer's measured and predicted point loads, in figure 5.22. Clearly the results so obtained, given the nature of the analytical considerations, are encouraging.

5.6.5 Shaft Loads

Chapter 2 indicated that the uncertainty in assessing pile tip loads is relatively minor compared to that associated with the determination of pile shaft loads.

As a pile is driven into the ground the normal stress acting on the pile, at a fixed point in the soil \( (\sigma_n) \), will vary with depth of pile penetration. In addition, the normal stress at a fixed point on the pile will also vary with depth of installation.

As defined in equation 2.1, the pile shaft load \( (\eta_s) \) is given by:

\[ \eta_s = f_s A_{ps} \]  

\[ 5.50(a) \]

where \( A_{ps} \) = the pile shaft surface area

\( f_s \) = the average unit shaft friction.

It is usual to express \( f_s \) as an average value over the embedded length of the pile shaft; thus the normal stress acting on the pile is in fact an average value, \( \sigma_n \):

i.e.

\[ f_s = \sigma_n \tan \phi \leq f_L \]  

\[ 5.50(h) \]

where \( \phi \) = the angle of friction between the pile shaft and the soil

\( f_L \) = some limiting value at pile embedment depths somewhat greater than those considered in this study (See Chapter 2).
FIG. 5.23 PILE-SOIL FRICTION ANGLE FROM SHEAR BOX TESTS.

FIG. 5.24 DISPLACEMENT NECESSARY TO MOBILISE PEAK PILE-SOIL FRICTION ANGLE.
The average normal stress \( \bar{\sigma}_n \) is usually expressed as:

\[
\bar{\sigma}_n = K_s \, q \tag{5.50(c)}
\]

where \( q \) = some measure of the stress in the ground and may be variously expressed as the average overburden pressure: the average pressure in the soil due to pile installation, etc.,

\( K_s \) = a dimensionless number analogous to the earth pressure coefficient and relates \( q \) to \( \bar{\sigma}_n \).

Because \( \bar{\sigma}_n \) is usually assessed from the back analysis of load tests the value of \( K_s \) so determined will relate directly to the definition assigned to \( q \).

Various authors, for example Broms (1966) have indicated that the angle of friction (\( \delta \)) is constant. Broms suggests that for timber piles in sand, the following relationship hold:

\[
\delta = \frac{2}{3} \phi \tag{5.51}
\]

Figures 5.23 and 5.24 show the results of a series of shear box tests in which sand was showered into the test cell to an initial porosity of 38\% \( (K_o = 0.4) \) and the load and displacement necessary to mobilise the maximum friction angle (\( \delta \)) against a timber load platen for various values of confining stress were measured. The timber platen was of the same material as the model piles and was arranged to replace the normal shear box top platen; at the same time permitting sufficient displacement of the platen to occur to mobilise the peak timber-sand friction angle (\( \delta \)).

From figure 5.23, even though there is quite a large scatter, it would seem reasonable to assess \( \delta \) as being constant at 23.7\(^\circ\). Expressed in terms of the peak friction angle (\( \delta_p \)) of 44.4\(^\circ\) in the same manner as equation 5.51, yields:

\[
\delta = 0.53 \delta_p \tag{5.52}
\]

This value however, is not quoted as a general value, but rather relates specifically to the sand and pile material tested at a particular initial state of packing \( (n = 38\%) \).

Figure 5.24 shows that as the normal pressure acting against the shear box platen increases, so also does the displacement necessary to mobilise the peak friction value. For the conditions pertaining in the model pile test this displacement could be of the order of about 1 mm.

From equation 5.50 and the shaft load curve of figure 5.19, it is possible to assess the average unit shaft friction developed during the axial model pile load tests reported. The back analysed values are shown in figure 5.25. The point represented by the embedment depth \( (h_p) \) of 2 would appear to be anomalous, and is considered due to the difficulties associated with measuring such small loads. The pile embedment length axes in figures 5.25 and 5.26 are shown as the dual axes L and L°. L is the total embedment depth of the pile. L° indicates the embedded length of the shaft.

The tendency for the average unit shaft resistance \( (e_s) \) to increase initially at a much slower rate than beyond about 4 to 5 \( D \) penetration has been reported by other researchers, for example Vesic' (1967).
FIG. 5.25 BACK ANALYSED VALUES OF $\sigma_s$ AND $\sigma_n$ FROM MODEL PILE TESTS.

FIG. 5.26 COMPARISON WITH SHAFT LOAD CONSIDERATIONS.
Equation 5.50 can be rewritten as:

\[ \sigma_s = \beta_n \tan \phi \rho_n \]

\[ = 0.44 \beta_n \frac{\rho_n}{A_{ns}} \quad (\text{from equation 5.52:}) \]

(i.e. assuming \( \alpha \) is constant at the value of 23.70 obtained from the shear box tests)

\[ \phi_s = 2.27 f_s \quad (5.53(h)) \]

The values of \( \phi_s \) so obtained from the model pile test results are also indicated in figure 5.25.

Table 5.4 indicates the range of values suggested by various authors for \( K_s \) and \( \alpha \). Also indicated are the values of \( \phi_s \) which would be predicted for the model pile tests. In assessing values for \( K_s \) and \( \phi_s \), the peak friction angle of \( \phi = 44.40 \) has been taken.

Bennett and Gisbourne (1971) report the results of a test series in which a penetrometer was installed into a soil mass in which were located a number of load cells. The stress changes measured by these load cells indicated that the stress in the soil reaches a peak as the penetrometer (or model pile) tin passes the location of the load cell, then drops back to about 50% of the peak value measured at some distance out from the pile as indicated in figure 4.27 (Chapter 4). It would appear from their observations that this reduced stress, once the pile tip has passed a particular location, remains constant irrespective of the subsequent location of the pile tip at greater depths. This observation then tends to verify the observations made by various other researchers, for example Dobinson, Sagar, and Morrison (1964), who describe the phenomena as indicating "continuous changes in mobilised soil friction [along the pile shaft] as they are driven" and are explained as "the build-up and break-down of arching around areas of loosened sand".

The possibility of arching occurring adjacent to the pile shaft with associated stress relief due to the loosening effect would appear to be the only rational explanation presently available. Expansion (i.e. loosening) along the pile shaft has been reported by the writer in discussing the development of strains about driven piles in Chapter 4. However, as a result of these same strain analyses, the occurrence of any significant changes in the mobilised soil friction angles adjacent to the shaft wall would appear unlikely.

Thus in assessing the approach in table 5.4 as suggested by Vesic, \( \alpha \) has been assessed on the basis of average tip stresses back analysed from the model pile tests assuming:

(i) No stress relief
(ii) Stress relief of 50% of the developed tip stresses
(iii) Stress relief of 85% of the developed tip stresses
TABLE 5.4
RANGE OF VALUES OF $\bar{\sigma}_n$ SUGGESTED IN THE LITERATURE

<table>
<thead>
<tr>
<th>AUTHOR</th>
<th>$K_s$</th>
<th>$\bar{\sigma}$ (kPa)</th>
<th>$\bar{\sigma}_n$</th>
<th>$\bar{\sigma}_n$ from model tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roms (1966) (dense sand and timber piles)</td>
<td>4.0</td>
<td>the average effective overburden pressure along the shaft:</td>
<td>5.15</td>
<td></td>
</tr>
<tr>
<td>Meyerhof (1976)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meyerhof (1951)</td>
<td>1.0</td>
<td></td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>Meyerhof (1951)</td>
<td>$K_A = 0.18$ to $K_P = 5.66$</td>
<td></td>
<td>$0.23$ to $7.3$</td>
<td></td>
</tr>
<tr>
<td>Vescic' (1967, 1970) $K_s + K_A$</td>
<td>$K_A = 0.18$</td>
<td>the average stress along the shaft after pile installation</td>
<td></td>
<td>16.3 kPa</td>
</tr>
<tr>
<td></td>
<td>(1) No stress relief</td>
<td>(2) Assuming 50% stress relief</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3) Assuming 85% stress relief</td>
<td></td>
<td>54</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16.2</td>
<td></td>
</tr>
</tbody>
</table>

Applying the various approaches of table 5.4 to the model pile test results yields the comparisons of figure 5.26.

In this figure OA represents the upper bound of the various approaches employing average overburden pressures and is in fact that associated with a value of $K_s$ equal to the Rankine passive pressure coefficient. The values obtained using Vescic's consideration that $K_s$ approximates the Rankine active pressure coefficient but that the stress in the ground is that due to the process of pile installation is represented by the curve OB. This curve is in fact in terms of the average tip pressures for the particular pile embedment depths analysed from the model pile tests and assuming the stress relief adjacent to the shaft is 85%.

Clearly the proposal suggested by Vescic' more closely approximates the actual average conditions around the pile shaft, at least for the pile embedment depths considered. However, the difficulty remains in assessing the appropriate value of $\bar{\sigma}$, without recourse to full scale pile tests. Bassett (1979) on the basis of average effective overburden pressures suggested that $K_s$ could be as high as 10 to 20. The curve ON in figure 5.26, on the basis of average effective overburden pressures, represents a value of $K_s$ of about 14, but is probably only appropriate for the model pile tests and for the depth of pile penetration considered:-
From this section and section 5.6.4 the apparent load sharing between pile tin and shaft for the model pile installed to an embersment depth \( \frac{d}{D} \) of 0 is as indicated in figure 5.27. There is obviously a major change in magnitude of stress from the conditions existing at the tip to those existing along the shaft. The ratio of about 75 between pile tip stresses and average normal stresses acting on the pile shaft is in agreement with values indicated by Vesic \( \text{(1967)} \), which are in the range 30 to 150. Notwithstanding the large reduction in stress adjacent to the pile shaft, approximately 20\% of the total load carried by the model pile at an \( \frac{d}{D} \) of 0, is carried by the shaft.

### 5.6.6 Radial Displacement Profiles

From the foregoing analysis (Section 5.6.2) the equation relating radial displacement to the distance out from the pile face is equation 5.29, i.e.

\[
\begin{align*}
    r_p &= r \left( \frac{r + u}{r} \right) - \frac{1}{n+\gamma} \\
    &= r_p \left( \frac{r + u}{r} \right) - \frac{1}{n+\gamma}
\end{align*}
\]  

5.29

This equation can be rearranged to yield radial displacement profiles as follows:-

Substituting for \( u_p \) as equation 5.32 and rearranging yields:

\[
\begin{align*}
    r &= r_p \left( \frac{r + u}{r} \right) - \frac{1}{n+\gamma} \\
    &= r_p \left( \frac{r + u}{r} \right) - \frac{1}{n+\gamma}
\end{align*}
\]  

and thus:-

\[
\begin{align*}
    (r + u)^{n+\gamma} &= \frac{1}{r_p} \left( \frac{r + u}{r} \right)^{n+\gamma} \\
    &= \frac{1}{r_p} \left( \frac{r + u}{r} \right)^{n+\gamma}
\end{align*}
\]  

5.56

which finally reduces to:-

\[
\begin{align*}
    r &= \left\{ \frac{1}{r_p} \left( \frac{4u_n}{\sigma_p - \sigma_A} \right)^{n+\gamma} \right\}^{-1} - u \\
    &= \left( \frac{1}{r_p} \left( \frac{4u_n}{\sigma_p - \sigma_A} \right)^{n+\gamma} \right) - u
\end{align*}
\]  

5.57

Hence by setting a particular displacement \( (u) \), the radius associated with that displacement can be determined in terms of the radius to the elastic-plastic boundary \( (r_p) \) and the displacement at that boundary \( (u_p) \). For the values represented by \( A_n \) in figure 5.21 for pile embersment depths \( \frac{d}{D} \) of 8 and 0 respectively, the radial displacement profiles of figure 5.28 are obtained.

These radial displacement profiles are compared with the average measured profiles of figure 3.55 (Chapter 3) in figure 5.29. These calculated displacements are, when compared on the basis of radius, approximately twice those measured; as indicated in figure 5.30.

If the limit of zero observable displacements discussed in Chapter 3 is taken as indicating the elastic-plastic boundary, then the nature of the measured modelled soil response is as indicated in figure 5.31. Clearly the shape of revolution so defined is less than perfectly spherical and thus is
FIG. 5.27 APPARENT LOAD SHARING BETWEEN PILE TIP AND SHAFT AT PILE EMBEDMENT DEPTH \( \frac{L}{D} \) OF 9.

![Diagram of pile with load sharing](image)

FIG. 5.28 RADIAL DISPLACEMENT PROFILE OBTAINED FROM THE SPHERICAL ANALYSIS

![Graph of radial displacement vs. radius](image)

<table>
<thead>
<tr>
<th>( \frac{L}{D} )</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>o</td>
</tr>
<tr>
<td>9</td>
<td>□</td>
</tr>
</tbody>
</table>

Note: \( \phi \) constant at 13°
FIG. 5.29 CALCULATED AND MEASURED RADIAL DISPLACEMENTS.

FIG. 5.30 CORRELATION BETWEEN CALCULATED AND MEASURED RADIAL DISPLACEMENTS.
FIG. 5.31 COMPARISON BETWEEN OBSERVED AND MODELLED MASS OF DEFORMING SOIL ABOUT THE PILE TIP.
likely to result in an over-estimate of the radial displacement profiles

5.7 **COMMENT**

The analysis presented in section 5.6 has, in one sense, followed traditional concepts in that the point and shaft loads of axially loaded piles have been considered separately. On the other hand the manner in which the pile tip axial load carrying capacity has been assessed can be considered almost unique in that the analytical model is based on observed and verified phenomena. All the parameters used in the model are able to be obtained directly in the field from the application of recent developments in in-situ soil property measuring devices (i.e. the self-boring pressuremeter, Hughes, Wroth and Hindle, 1977).

No attempt has been made to provide physical, or analytical solutions to the mechanism of load transfer through the pile shaft. However, the complexity of the problem has been reaffirmed; along with an indication of the general unreliability revealed in the wide range of values suggested for the transfer function, \( K_s \).

The writer believes that the fundamental requirements for a satisfactory method of analysis, as described earlier in this chapter, have been considered. These fundamental requirements are:

(i) The foundation shame, relative depth and method of construction,
(ii) The shear strength of the sand,
(iii) The relative compressibility and volume change characteristics of the sand,
(iv) An accommodation for the generally unclarified scale effects,
(v) The nature of the displacements developed about the pile tip.

The foundation shame and method of construction are reflected in the nature of the displacements developed about the pile tip. Thus if these can be approximated, then the other criteria are, by association, also accommodated. The relative depth of the foundation and at least one of the scale effects are accommodated by considering the stress path followed by a typical soil element, commencing from an initial stress-state representative of the in-situ \( K_0 \) conditions. By attempting to follow a typical stress path the shear strength characteristics of the soil are taken into account in defining the "failure" surface. The compressibility and volume change characteristics of the soil are a fundamental feature of the analysis (i.e. the dilatation rate).

Whilst the volume change characteristics of the soil have been determined indirectly in this model study from a strain analysis of the displacement fields developed during pile installation, they can be determined directly in the field from a self-boring pressuremeter investigation using the technique of Hughes, Wroth and Hindle (1977).

A significant observation accruing from the analysis is that the value of \( v \) has a considerable effect on the results. This has been experimentally observed to reach a limiting value at an \( L_c \) of about 10.

Thus, although the analysis as presented does not produce a perfect solution, it has been shown to be, in its present form, no worse than the results obtained from the generally used theoretical and semi-empirical bearing capacity considerations at least for pile embedment depths \( L_c \) of 8 or 10.
In addition the form of the analysis enables the unrecoverable radial dislocations at the level of the pile tip to be approximated.

As a consequence, the writer considers that the analytical concept as presented holds considerable promise for refinement and development into a potentially useful design tool. It is likely that with further manipulation the model may be developed to better predict axial pile loads. However, before any such model can be unreservedly accepted it must first be able to reliably predict the results of model tests (both loads and soil displacement) and second should be tested against either full scale pile tests under well documented and controlled conditions, or alternatively against large scale model tests similarly controlled, to attempt to identify the potential existence of previously unrecognised scale factors.

The pressuremeter test in its present form is designed to fastidiously avoid disturbing the in-situ soil conditions. A suggested modification to this test to better model the effects of pile installation by driving, is discussed in Appendix II.

5.8 THE EFFECTS OF SHAFT ROUGHNESS AND PILE TIP CONFIGURATION

5.8.1 Introduction

In the foregoing study the general effects of the process of pile installation by driving have been considered for a pile of a particular surface roughness and tip configuration. In this section a qualitative assessment is made of the effects of varying these conditions. The "half-pile" arrangement as described in Appendix I has been employed, enabling stereo pairs of photographs to be obtained. These are then able to be superimposed yielding qualitative displacement fields.

For the purposes of the comparison, the pile shaft roughness was varied from perfectly smooth to perfectly rough. The pile tip effects were compared with those developed about a square ended pile.

5.8.2 The Effects of Pile Shaft Roughness

1) Introduction

In making the comparisons subsequently reported, the sand was placed to the same initial conditions as those existing for the radiographic studies (i.e. \( n = 38\% \), \( K_0 = 0.4 \), \( \rho = 1640 \text{ kg m}^{-3} \)). All model half piles were of the same basic dimension (i.e. \( D = 20 \text{ mm} \)). The perfectly smooth pile was constructed from brass, and subsequently polished to give the desired finish. The perfectly rough pile was in fact a timber pile coated with "Araldite" and surfaced with the finer fraction of the sand into which it was to be driven (i.e. the fraction less than \( D_{60} \), see Appendix 6).

The comparison was made between these perfectly rough and smooth piles and a "normal" pile, i.e. the timber model pile used in the experiments upon which the foregoing discussions and results have been based. All three piles had the same tip configuration and were driven to the same embedment depth (i.e. \( \frac{L}{D} = 6.5 \)).
(ii) The Results

The comparative results are shown in figures 5.32, 5.33, and 5.34 for the smooth, normal and rough piles respectively. Figures 5.32(a) to 5.34(a) show the "double-exposure" qualitative images obtained from the appropriate stereo pairs of photographs associated with driving the piles from an emplacement depth \( \left( \frac{h}{D} \right) \) of 5.5 to 6.5 (i.e. an incremental penetration of 1\( D \)). Figures 5.32(b) to 5.34(h) show the general trends of the displacement patterns indicated in the photographs. The general dimensions of the displacement fields so developed are indicated in table 5.5. The key to these dimensions is given in figure 5.35.

**TABLE 5.5**

VARIATION IN DISPLACEMENT FIELD DIMENSIONS WITH PILE ROUGHNESS

(Refer also figure 5.35)

<table>
<thead>
<tr>
<th>PILE TYPE</th>
<th>SMOOTH</th>
<th>NORMAL</th>
<th>ROUGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width at level of tip 'a'</td>
<td>3.70</td>
<td>4.250</td>
<td>5.250</td>
</tr>
<tr>
<td>Depth below pile 'b'</td>
<td>1.8</td>
<td>1.750</td>
<td>2.25</td>
</tr>
<tr>
<td>Depth below pile tin 'c'</td>
<td>2.8</td>
<td>2.75</td>
<td>3.25</td>
</tr>
</tbody>
</table>

These values are obviously in good agreement with the generally observed dimensions of table 3.1 (Chapter 3) which gave corresponding dimensions of:

\[ a = 4 \text{ to } 6D \ (3.7 \text{ to } 5.3D, \ \text{Table 5.5}) \]
\[ c = 3 \text{ to } 5D \ (2.8 \text{ to } 3.25D, \ \text{Table 5.5}) \]

Clearly then there is an approximate increase in the displacement field width of about 10\% between the perfectly smooth and perfectly rough conditions, with little difference apparent below the pile tin. The effect of pile roughness on the mass of soil involved in resisting axial loading is thus minor.

The significant feature indicated by figures 5.31 to 5.34 is the influence of the shaft roughness on the soil immediately adjacent to the shaft. In the case of the smooth pile there appears to be no interaction between the pile wall and the surrounding soil; i.e. the coefficient of friction between the pile and the soil is either zero or very small.

There is little evidence of "drag down" adjacent to the pile shaft apart from a very small mass of soil close to the ground surface.
FIG. 5.32 SAND DISPLACEMENTS ABOUT A SMOOTH PILE.

FIG. 5.33 SAND DISPLACEMENTS ABOUT A TIMBER PILE (i.e., a "Normal" pile).
FIG. 5.34 SAND DISPLACEMENTS ABOUT A ROUGH PILE.

FIG. 5.35 KEY TO TABLE 5.5.
In contrast the "drag down" effect adjacent to the shaft of the normal pile is more marked. This appears to initially be quite large close to the surface, but decreasing in width with increasing depth. Although it is not apparent from the photographs, it would be reasonable to assume that even at a considerable pile embedment depth the zone must extend for a width of at least a few sand grain diameters (i.e. <0.1" wide).

All these effects are accentuated when the roughness of the pile shaft is the same as that of the surrounding soil. The influence of the drag down zone at the ground surface is considerably more marked. In addition the width of this zone appears to be constant with depth to the vicinity of the pile tip. The width of this drag down zone is approximately 0.40.

Clearly then the roughness of the pile shaft can cause significantly different local characteristics in the displacement fields immediately adjacent to the shaft. In turn these variations must influence the stress conditions acting in the soil adjacent to the shaft, and thus the contribution of the load carrying capacity of the pile shaft to the total axial load carrying capacity of the pile; whilst however, not significantly affecting the nature of the displacements about the pile tip, and thus its ultimate axial load capacity.

An interpretation of the significance of these effects is discussed in the following subsection.

(iii) Discussion

Various authors have discussed the phenomena of pile shaft load transfer, as mentioned briefly in Chapters 2 and earlier in this chapter. Some further aspects are considered herein.

As has been previously discussed, a friction pile in cohesionless soil obtains support through the transfer of load through the pile wall to the surrounding soil, and from pile tip considerations. The load transferred by shaft friction has sometimes been neglected altogether on the assumption that it contributes only a small fraction of load to the total pile axial load capacity in cohesionless soils. When it is taken into consideration, the proportion of the load is usually estimated on the basis that it is a direct function of the vertical earth pressure. The inference from the discussion in Section 5.6.5, however, is that the lateral distribution of load through skin friction does not necessarily agree with conventional earth pressure theories.

As indicated by various authors, for example Robinsky and Morrison (1964), the consideration has long been held that the action of driving a friction pile into cohesionless soils will create a zone of compacted soil in the vicinity of the driven pile. Robinsky and Morrison were probably the first to recognise that in fact a zone of loose soil is generated close to the pile shaft.

This has been confirmed by the writer (See Chapter 4) and in addition has been shown to extend beneath and about the pile tip. This loose zone was suggested by Robinsky and Morrison as "caused by the drag down effect of the pile walls on the surrounding soil as the pile moves downward". However, as indicated in Chapter 4 and figures 5.32 to 5.34, the width of the "loose" zone, (i.e. the zone of expansion adjacent to the pile shaft), is likely to be significantly greater than the width of the drag-down zone actually developed.
Robinsky and Brorrison suggest that the overall effect is that of a cylinder of dense soil originally connected by the pile tip encircling an inner cylinder of loosened sand which prevents by "arching", the development of the full lateral earth pressure acting on the pile. In addition they suggest that the angle of internal friction (φ) of the loosened sand is considerably lower than that developed below the pile tip, and as a consequence of these combined effects, little load can be transferred by friction from a straight sided pile to the surrounding soil.

As indicated in Chapter 4 and Appendix 5 it is probable that such a reduction in the mobilised friction angle in fact does occur.

Various researchers, as reported for example by Broms (1960) and Vesic (1970) have identified that the shaft friction resistance reduces markedly in the vicinity of the pile tip. Vesic has attributed this to "arching or stress relief" in a zone extending to about 30 above the pile tip as earlier indicated in figure 2.17 (Chapter 2).

Bennett and Cisbourne (1971) indicated that stress relief occurred in the soil mass about the shaft after the passage of the pile tip; the level of stress at a particular point some distance out from the pile reducing to about 50% of that developed when the point was at the level of the pile tip (See also figure 4.27, Chapter 4).

(a) Smooth Shaft

Because the vertical displacements adjacent to the pile shaft are virtually parallel to the shaft (See figure 5.32), with no evidence of drag-down adjacent to the shaft wall because of friction, the radial stress acting at the pile wall can be represented as in figure 5.36. Defining the reduced radial stress as \( \sigma_n \), then the unit shaft friction \( (f_s^* \) acting at a particular depth will be:

\[
f_s^* = \sigma_n \tan \phi
\]

where \( \phi \) = the friction angle between the sand and the pile shaft

\[
\sigma_n \neq \sigma_{ra}
\]

where \( \sigma_{ra} \) = the stress at the pile tip at a particular depth due to pile installation

(b) "Normal" Shaft

As indicated in figure 5.33 the drag-down zone adjacent to the wall of a pile of intermediate shaft roughness appears to decrease significantly with depth and is likely to be much less than 0.10 wide. The radial stresses acting on the pile wall will then be further reduced from figure 5.36 as indicated in figure 5.37. The effect of the drag-down zone may be explained as follows:

In the left of the plane through \( A \) in figure 5.37 the soil tends to be moving upwards, due to the displacement occurring about the pile tip, whilst to the right of \( A \) the tendency is for the soil to move downwards under the influence of the pile shaft. The net effect is then analogous to the soil on the left of \( A \) remaining static and that to the right moving an amount equal to the sum of the relative movements of \( A \) to \( B \).
FIG. 5.36 POSSIBLE STRESSES ADJACENT TO A SMOOTH SHAFT.

FIG. 5.37 POSSIBLE STRESSES ADJACENT TO A TIMBER SHAFT.

FIG. 5.38 POSSIBLE STRESSES ADJACENT TO A ROUGH SHAFT.
This then represents the state of stress referred to by Terzaghi (1943), as "arching". In this situation the relative movement within the soil mass is opposed by a shearing resistance within the zone of contact between the "yielding" and the "stationary" masses, (i.e. a to b). Since the shearing resistance, (indicated in figure 5.37), tends to keep the yielding mass in its original position, it reduces the pressure on the "yielding" part of the support (c) and increases the pressure on the adjoining "stationary" part (d).

The "arching" effect, then, is this transfer of pressure from the yielding mass of soil onto the adjacent stationary parts. In these circumstances the soil is said to "arch" over the yielding part of the support.

The effect of arching is then to further reduce the state of stress such that \( \sigma_n \) is likely to be much less than \( \sigma_{ra} \). This further reduction in \( \sigma_n \) is, (as inferred from the arching theory) likely to be accompanied by an increase in either the vertical stress, \( \sigma_v \), or the circumferential stress \( \sigma_g \), or more likely, an increase in both.

(c) Rough Shaft

The drag-down zone adjacent to the wall of a Nile with a high degree of roughness has been shown to be approximately 0.3 to 0.40 wide (figure 5.34). The effect of this wider drag-down zone is to further reduce the radial stress acting on the Nile shaft, as indicated in figure 5.38, and thus to further increase either the vertical stress (\( \sigma_v \)), or the circumferential stress (\( \sigma_g \)), or both.

Thus where the drag-down zone exists the resultant stress acting on the Nile shaft is as shown in figure 5.39, and the unit shaft resistance, \( f_s^* \) is given by:

\[
f_s^* = \sigma_n^* \tan \delta^*
\]

being the effective vertical shearing resistance transferred to the Nile shaft wall. Equation 5.59 may be further expressed as:

\[
f_s^* = a \sigma_n \tan \delta^*
\]

where \( \sigma_n^* \) = the reduced radial pressure acting on the Nile shaft.
\( \sigma_n \) = the reduced radial pressure acting on the perimeter of the drag-down zone
\( a \) = the effect of arching
\( \delta \) = the effect of stress relief above the pile tin.

The coefficient \( a \) represents a stress reduction due to "arching" across the "drag-down" zone created by the relative roughness of the Nile wall. This drag-down zone is mobilised against the Nile face during installation, thus the significance of the effect is very dependent on the method of Nile installation and the surface friction characteristics of the Nile.
FIG. 5.39 STRESSES ACTING ON PILE SHAFT.
It is probable that as the pile surface roughness increases from the smooth case, so in fact does the pile-soil friction angle ($\phi$). Thus from equation 5.58 to 5.59, $\phi$ has been replaced by $\phi^*$, which in fact the pile-soil friction angle appropriate to the pile roughness under consideration.

From the results of the model pile tests (figure 5.10), it would appear that a greater pile penetration depth is necessary to limit the influence of the drag-down zone than is necessary to limit the tip pressures.

For a given depth then, temporarily ignoring the effect of the pile-soil friction angle ($\phi^*$), $\phi^*$ can be said to vary as:

$$\phi^*_{\text{rough}} < \phi^*_{\text{normal}} < \phi^*_{\text{smooth}}$$  \hspace{1cm} (5.60)

i.e. the rough surfaced pile is likely to have the least radial pressure acting against the shaft wall.

Whilst this may be the case, the value of $\phi^*$ has a significant influence on the actual amount of axial load transmitted to the soil as indicated by table 5.6.

<table>
<thead>
<tr>
<th>Type</th>
<th>Pile Material at the Interface</th>
<th>$\phi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth ($S$)</td>
<td>Brass</td>
<td>$\phi^<em>_S = \text{approx. zero; i.e. } \ll \phi^</em>_N &lt; \phi^*_D$</td>
</tr>
<tr>
<td>Normal ($N$)</td>
<td>Timber</td>
<td>$\phi^<em>_N \ll 0.5\phi$ (for the model pile and sand used; i.e. $\phi^</em>_D = 44.4^\circ$)</td>
</tr>
<tr>
<td>Rough ($R$)</td>
<td>Sand</td>
<td>$\phi^<em>_R \ll \phi^</em>_D$</td>
</tr>
</tbody>
</table>

Various values for $\phi^*$, as reported in the literature, have been indicated in section 5.6.5. In addition to these Lambe and Whitman (1969) suggest the following:

(a) for timber and concrete against soil;

$$\phi^* = \phi^*_{\text{CV}} \quad (\text{i.e. the critical state friction angle of the soil})$$

(b) for steel against soil;

$$\phi^* = \phi^*_{\text{iw}} \quad (\text{i.e. the interparticle friction angle of the soil } \neq 26^\circ \text{ to } 29^\circ \text{ for sand}).$$

As suggested then, the influence of the pile-soil friction angles indicated in table 5.6 in fact reverses the relativity indicated by equation 5.60, i.e.:

$$\phi^*_N \tan \phi^*_S < \phi^*_N \tan \phi^*_N < \phi^*_N \tan \phi^*_R$$  \hspace{1cm} (5.61)
(iv) Conclusion

The foregoing is an attempt to clarify the mechanics of the manner in which the pile shaft appears to contribute to the ultimate axial load carrying capacity of a pile, and also give an indication of the relative influence of the various parameters involved.

Clearly the fundamental factors that need to be identified to clarify the mechanics of pile shaft load transfer in sands alone, are:

(i) The change in stress in the soil mass, particularly adjacent to the shaft after the pile tin has passed a particular location.

This statement presupposes that the actual state of stress mobilised in the soil about the pile tin is able to be reliably determined by analytical means.

(ii) The mechanics of the further stress relief in the "drag-down" zone.

(iii) The variations in the width, and thus the influence of this drag-down zone for varying pile roughness and soil characteristics.

(iv) The nature of the relationship between the pile-soil friction angle (4) and soil characteristics, pile roughness and pile installation techniques.

Clearly these requirements present a daunting task. However, it is the writer's opinion that until the mechanics of pile-soil interaction accounting for these various factors are identified, then the influence of the factors cannot reliably be quantified. As long as such factors are unable to be quantified, so the designer cannot make universally agreeable rational assessments of not only the factors to be considered in the design process, but also the magnitudes to be assigned to them. As long as such conditions prevail, the only absolutely reliable design aid must remain full scale pile tests for each particular site.

5.8.3 THE EFFECT OF PILE TIP CONFIGURATION

(i) Introduction

As was the case for the comparisons made in section 5.8, the sand for the comparative observations reported in this section was placed to the same initial conditions as those existing for the radiographic studies. The model pile used was a "normal" pile, i.e. the timber model pile forming the basis of the work reported in this thesis; the difference being that the pile tin had been removed. The model pile was of the same basic dimension, i.e. \( d = 20 \text{ cm} \).

(ii) The Results

Figures 5.40(a) to 5.45(a) show the "double exposure" qualitative images obtained from the appropriate stereo pairs of photograms during driving a square ended pile at approximately 10 increments of pile penetration, to approximate embedment depths \((L/n)\) of 1, 2, 3, 4.5, 6, and 7 respectively. The general trends of the displacement patterns are indicated in figures 5.40(h) to 5.45(h) and can be seen to bear good agreement with the displacement patterns shown in figure 3.23(h) to 3.28(h), (Chapter 3), for the installation of an identical pile, except with a tanned tin, into sand of the same
FIG. 5.40 SAND DISPLACEMENTS ABOUT SQUARE ENDED PILE FOR $\frac{L}{D}$ OF 1.

FIG. 5.41 SAND DISPLACEMENTS ABOUT SQUARE ENDED PILE FOR $\frac{L}{D}$ OF 2.
FIG. 5.42 SAND DISPLACEMENTS ABOUT SQUARE ENDED PILE FOR $\frac{L}{D}$ OF 3.

FIG. 5.43 SAND DISPLACEMENTS ABOUT SQUARE ENDED PILE FOR $\frac{L}{D}$ OF 4.5.
FIG. 5.44 SAND DISPLACEMENTS ABOUT SQUARE ENDED PILE FOR $L/d$ OF 6.

FIG. 5.45 SAND DISPLACEMENTS ABOUT SQUARE ENDED PILE FOR $L/d$ OF 7.
initial properties, and to approximately the same embedment depths. The comparative depths of these displacement fields beneath the pile tin, measured relative to the point where the tin has the same cross-section as that of the shaft (i.e. from the base of the square ended pile) are shown in table 5.7.

Considering that the limits for the square ended pile have been based on visual displacements recorded on the photographs, this agreement is considered good: this indicating that the variation in pile tin configuration has little influence on the displacements developed about the pile tin during installation or on the mass of soil involved, at least within the bounds of normal pile tin shapes. As indicated by the photographs (figures 5.40(a) to 5.45(a)) and as shown diagramatically in figures 5.40(b) to 5.45(b), a wedge of sand of probable conical shape is carried down beneath the pile tin. This wedge carries out essentially the same function as the pile tin in that it causes the sand to be displaced by the penetration of the pile tin in a manner analogous to the expansion of a spherical cavity (see Chapter 3 and Section 5.6).

<table>
<thead>
<tr>
<th>L (approximate)</th>
<th>Tanered Pile Tin (Chapter 3)</th>
<th>Square Ended Pile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Photgraphs</td>
<td>Radiographs</td>
</tr>
<tr>
<td>1</td>
<td>2.3</td>
<td>2.7</td>
</tr>
<tr>
<td>2</td>
<td>5.4</td>
<td>4.0</td>
</tr>
<tr>
<td>3</td>
<td>5.5</td>
<td>4.1</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
<td>4.4</td>
</tr>
<tr>
<td>6</td>
<td>3.6</td>
<td>4.2</td>
</tr>
<tr>
<td>7</td>
<td>3.0</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Discussion

Most bearing capacity theories assume the development of an elastic zone beneath the pile tin, (or foundation width), as indicated by the triangle ABC in figure 5.46. In this zone the soil is assumed to remain in a permanent state of elastic equilibrium which moves with the pile or footing. This then implies a "dead" zone, or wedge of soil which is carried along with the foundation, and in the case of a driven displacement pile, to great depths. This wedge then acts as a natural pile tin as indicated by the qualitative deformation fields of figures 5.40 to 5.45.

During installation of the model half piles this "dead zone" was in fact seen to be developed and carried down beneath the pile tin.

Various other researches have observed this phenomena, for example Penezantzev and Yaroshenko (1957) as indicated in figure 2.19 (Chapter 2) and Rohinsky and Morrison (1944), as indicated in figure 5.47.

Penezantzev and Yaroshenko described this zone as a "compacted core" which "causes the displacement of soil along the slip surfaces" formed by the boundaries of the core, and moves as part of the
FIG. 5.46 ASSUMED ELASTIC ZONE BENEATH THE TIP OF A SQUARE ENDED PILE.

"Main compaction zone"
as defined byRobinsky andMorrison (1964)

FIG. 5.47 WEDGE OBSERVED BY ROBINSKY AND MORRISON (1964).
foundation. Porhinsky and Morrison however, did not identify this zone as having essentially rigid and moving with the foundation, even though the triangle ABC is clearly identified by the final shot positions in figure 5.47. Rather they have identified this zone as part of the "main connection zone" within which "vertical compression and two-directional horizontal expansion take place, accompanied by radial downward translation". This description has been shown by the results of Chapter 4, to be ill conceived.

The apex angle C of triangle ABC in figure 5.46 has been suggested from theoretical and experimental considerations to have a wide range of values as indicated in Table 5.8.

### TABLE 5.8
APEX ANGLE OF 2D WEDGE in FIG. 5.46

<table>
<thead>
<tr>
<th>Source</th>
<th>Angle (degrees)</th>
<th>( \theta = 30 )</th>
<th>( \theta = 40 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terzaghi (1943 (shallow footings))</td>
<td>180 - 2( \theta )</td>
<td>120</td>
<td>100</td>
</tr>
<tr>
<td>Jaky (1948)</td>
<td>90 - ( \theta )</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Meyerhof (1951)</td>
<td>90 - ( \theta )</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Berezantzev and Yaroshenko (1957)</td>
<td>(90 - 0.8( \theta )) to (90 - 0.1( \theta )) depending on depth and initial soil density</td>
<td>66 to 87</td>
<td>58 to 86</td>
</tr>
<tr>
<td>Vesic' (1963)</td>
<td>Approximately ( \theta )</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Robinsky and Morrison (1964) (from figure 5.47)</td>
<td>( \theta )</td>
<td>( \theta )</td>
<td>( \theta )</td>
</tr>
</tbody>
</table>

The writer's own observations would suggest an angle of approximately \( \theta = 40^0 \) or slightly less. Notwithstanding the wide range of potential "soil tin" configurations suggested in table 5.8 it would appear that the possible variation in pile tip configuration has an insignificant effect on the displacements developed in the soil. The significant feature confirmed in this section is that in the extreme cases of a tamed pile tin and a square ended shaft, the effect on the soil remains essentially that of the penetration of a rigid cone or wedge: thus reaffirming that the incremental penetration of a pile, and by analogy, the application of axial load, is itself analogous to the expansion of a spherical cavity from initially zero radius to the final radius represented by that of the pile. The analytical concept of section 5.6 is thus justified for all displacement piles installed in cohesionless soils by driving, irrespective of the tip configuration, providing it is not abnormally sharp (i.e. within the bounds of normal pile tin shales).
Chapter 6 Approximate Stress Fields Developed About Model Piles During Installation By Driving Into Dense Sand

6.1 INTRODUCTION

The logical extension to the work reported in this part of the thesis is the determination of the fields of stress developed in the soil mass at various stages of pile installation. If this could be done it would then be possible to complete the experimental/theoretical circle as follows:

(i) Apply a load to a model pile,
(ii) Measure the displacements mobilised in the soil in resisting the loads transmitted through the pile,
(iii) Determine the strain fields from the mobilised displacements,
(iv) From the calculated strain fields, determine the stress distribution throughout the soil mass,
(v) From the soil stresses acting at the pile-soil interface, reassess the pile load being resisted by the soil.

If the experimental and theoretical considerations are appropriate, then the applied loads in (i) above should be in reasonably close agreement with those assessed from the soil stress state in (v) above.

In this chapter an initial attempt is made to determine the mobilised soil stresses during pile installation by driving based on the observed displacement fields reported in Chapter 3.

6.2 CONSIDERATIONS AFFECTING THE DETERMINATION OF STRESSES

In attempting to convert the measured displacement fields into fields of mobilised soil stress the method proposed by Arthur, James and Roscoe (1964), as discussed in Appendix 12, has been employed. This technique is based on a rectilinear network of nodal positions. As indicated in Appendix 7 (figure A7.16), the network constructed for the assessment of strains reported in Chapter 4 was based on a series of constant strain triangles, and as a consequence, all the displacement data was stored on data cards in the appropriate triangular format. As an initial approximation the computer programme developed to analyse the nodal displacements to produce strain fields was modified to convert the triangular network of figure A7.16 into a rectilinear network as indicated in figure 6.1. In so doing the strains and directions of major principal strain appropriate to the rectilinear network were taken as the algebraic average of the values of each of the triangles within a particular rectangular element. The rectilinear network so constructed had the dimensions (1, J) of (21, 10); i.e. 210 elements. In columns J7 to J10 the approximation was made using elements (7, 2) and (7, 3) as examples, that the strain values associated with element (7, 2) were those represented by the constant strain triangle 22, while those associated with element (7, 3) were represented by the constant strain triangle 21. The coordinate positions of each rectangular element were similarly averaged.
FIG. 6.1 RECTANGULAR NETWORK FOR STRESS ANALYSIS
The assumptions fundamental to the analytical method of Arthur, James and Roscoe, are that the principal axes of stress and strain always coincide at any point within a soil mass prior to attaining the peak shear stress at that point, and in addition, that the stress-strain characteristics of the soil are able to be correlated to the model results on the basis of this coincidence of the principal axes of stress and strain. Endicott (1974) has indicated that it is reasonable to consider that for an elastic material the axes of principal stress and of total strain coincide, and that for a plastic material the axes of principal stress and of strain increment coincide. No such refinement has been attempted herein, instead the axes of major principal stress and cumulative strains are considered to be coincident throughout the process of soil deformation.

In determining the maximum value of the stress ratio \( \frac{\sigma_{x}}{\sigma_{y}} \) associated with the cumulative incremental shear strain for each position within the network, the idealised stress-strain curve of figure A5.17, (Appendix 5), was used. In determining the direction of the major principal strain (and thus of stress) (b) the total cumulative incremental element strains were used.

The stress ratios \( \frac{\sigma_{x}}{\sigma_{y}} \) acting on the soil boundaries were assessed on the basis of the initial \( K_{o} \) conditions (See Appendix 9). The assumed boundary conditions are indicated in figure 6.2. Close to the soil surface where the mobilised stresses will tend to zero it was assumed that the \( K_{o} \) conditions existing would approximate those developed at soil placement (i.e. \( K_{o} = 0.4 \)). Similar conditions were considered to apply at the walls of the test cell.

Thus, where:

\[
K_{o} = \frac{\sigma_{h}}{\sigma_{v}} = \frac{\sigma_{x}}{\sigma_{y}} = 0.4
\]  

Along the boundaries AB and AC in figure 6.2,

\[
\frac{\tau_{xy}}{\sigma_{x}} = AB,AC = \frac{\sigma_{y} - \sigma_{x}}{2} \frac{1}{\sigma_{x}} \left( \frac{\sigma_{v}}{\sigma_{x}} - 1 \right)
\]  

This rearranges to:

\[
\frac{\tau_{xy}}{\sigma_{x}} = AB,AC = \frac{1}{2} \left( 1 - \frac{K_{o}}{K_{o}} \right)
\]  

Substituting for \( K_{o} = 0.4 \),

\[
\frac{\tau_{xy}}{\sigma_{x}} = AB,AC = 0.75
\]  

Similarly,

\[
\frac{\tau_{xy}}{\sigma_{y}} = AD,AC = \frac{\sigma_{y} - \sigma_{x}}{2} \frac{1}{\sigma_{y}} \left( \frac{\sigma_{v}}{\sigma_{y}} \right)
\]

which rearranges to:

\[
\frac{\tau_{xy}}{\sigma_{y}} = AD,AC = \frac{1}{2} \left( 1 - K_{o} \right)
\]  

\[
= 0.3
\]
FIG. 6.2 ASSUMED BOUNDARY CONDITIONS
The initial stress acting on row 1 in Figure 6.1 was taken as 0.193 kPa, assuming that this network boundary was located approximately 12m below the ground surface, i.e.:

\[ \sigma_y(x,1) = \gamma y = (1640 \times 9.91 \times 10^{-3}) \times 0.012 = 0.193 \text{ kPa} \]  

In obtaining the results subsequently reported no attempt has been made to vary the direction of the line integration to follow the directions of increasing compressive stresses, or to iterate to assessed levels of stress. As a consequence, because the line integrations were conducted only in the y direction, significant anomalies have been created. In the preliminary results presented, such anomalies are indicated by broken lines.

A tacit assumption in this analysis is that the circumferential stress \( \sigma_y \) is the intermediate principal stress.

6.3 APPROXIMATE STRESS FIELDS FROM MEASURED DISPLACEMENTS

Applying the technique described in Appendix 12 in conjunction with the foregoing boundary conditions and within the limitations noted, the approximate directions of major principal strain and approximate contours of major principal stress are as indicated in figures 6.3(a) and (b) to 6.11 (a) and (b) respectively for pile embedment depths \( \left[ \frac{p}{2} \right] \) of approximately 1 to 9.

These preliminary approximate results are considered to be encouraging in that the general features identified at various stages of this study are reaffirmed. For example, the significant rotation of the major principal stress direction both in advance of the pile tip and after the pile tip has passed a particular point are clearly indicated. It should be noted that the directions of major principal strain presented are in good agreement with results previously presented by Gisbourne (1970) in conjunction with penetrometer studies at least above the pile tip. In addition, as suggested in Chapter 5 (Figure 5.39) wherein an attempt was made to assess the mechanics of soil-pile shaft load transfer, the direction of the major principal strain and thus stress acts, within a zone approximately 10 wide, in a direction commensurate with the transfer of load from the pile shaft to the surrounding soil mass (Figures 6.3(a) to 6.11(a)). Outside this narrow zone the direction of major principal strain and thus stress bends to approximately the horizontal at least above the pile tip, as could be expected, given the volume displacement occurring.

In addition an apparent large drop off in soil pressure is indicated adjacent to the pile shaft compared to that existing at the level of the pile tip. This would appear to be substantiated by the stress changes necessary to satisfy the distribution of load between pile shaft and tip as indicated in Figure 5.27 (Chapter 5).

It should be noted that the elements adjacent to or straddling the "apparent limit of zero observable displacements" indicated on figures 6.3 to 6.11 are likely to be subject to additional error where for example only one node of a rectangular element is experiencing displacement.

With increasing depth the stress fields indicated in Figures 6.3(h) to 6.11(h) are also compatible with the significant stress reductions suggested in Chapter 5 as likely to occur adjacent to the pile shaft.
FIG. 6.3 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 1.26.

FIG. 6.4 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 2.
FIG. 6.5  APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 3.

FIG. 6.6  APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 4.
FIG. 6.7 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 5.

FIG. 6.8 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 6.
FIG. 6.9 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR \( \frac{L}{D} \) OF 7.

FIG. 6.10 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR \( \frac{L}{D} \) OF 8.
FIG. 6.11 APPROXIMATE DIRECTIONS OF MAJOR PRINCIPAL STRAIN AND CONTOURS OF MAJOR PRINCIPAL STRESS FOR $\frac{L}{D}$ OF 9.
FIG. 6.12 VARIATION IN STRESS AT A POINT THROUGHOUT PILE INSTALLATION
Point A in figures 6.3(b) to 6.11(h) is an arbitrary point at which the variation in approximate stresses throughout pile installation can be assessed. The associated changes in mobilised soil stress levels are indicated in figure 6.12.

Also indicated on this diagram are the results obtained by Bennett and Gisbourne (1971) for model tests in which a penetrometer was continuously pushed into dense sand. Bennett and Gisbourne's results were obtained from an earth pressure cell located at a radius of approximately 1.3D from the shaft centreline and approximately 7.5D below the initial ground surface. The corresponding dimensions for point A in figures 6.3(b) to 6.11(h) are approximately 1 and 6D respectively. Bennett and Gisbourne's results were reported earlier in figure 4.27 (Chapter 4).

Notwithstanding the difference in method of pile installation and approximate nature of the stress fields in figures 6.3 to 11, the agreement in the general trends indicated in figure 6.12 is considered promising.

From the spherical analysis developed in Chapter 5 the equilibrium equation 5.38(b) permits the radial stress profile to be determined throughout the plastically deforming mass in terms of the radius of the plastic boundary \( r_p \), which is itself able to be determined from equation 5.33, in terms of the dilatation characteristics of the soil.

Considering embedment depths \( \frac{L}{D} \) of 8 and 9 only and adopting the values of \( \phi \) of 54° and a v of 13°, as previously employed in Chapter 5, the analytically determined radial stress profiles at the level of the pile tip are obtained as indicated in figures 6.13 and 6.14. Also shown in these figures are the approximate radial stress profiles obtained at the same level from figures 6.10(b) and 6.11(b).

Clearly the agreement is encouraging.

6.4 COMMENT

The results indicated in figures 6.3 to 11, whilst being approximate in nature only and additionally being recognised as subject to possibly large anomalies, do nevertheless suggest that the experimental - analytical circle discussed in section 6.1 should in fact be able to be completed. The method suggested by Arthur, James and Poscoe (1964) would appear to be encouraging as related to the pile problem.
FIG. 6.13 RADIAL STRESS PROFILES FOR $\frac{L}{D}$ OF 8.

FIG. 6.14 RADIAL STRESS PROFILES FOR $\frac{L}{D}$ OF 9.
Summary Of Part One

1. **INTRODUCTION**

   The preceding chapters (i.e. Chapters 2 to 6) have considered the various aspects of the installation of model piles into dense sand by driving and the assessment of their ultimate axial load carrying capacity. The following summarises the significant observations made and points raised in each of these chapters.

2. **SUMMARY OF SIGNIFICANT POINTS OF CHAPTERS 2 to 6**

   **2.1 CHAPTER 2 - The Axial Load Carrying Capacity of Driven Piles in Sand**

   In this chapter the complexity of the pile problem is identified by comparing it with the simple analysis (in comparison) of a shallow spread footing. A further order of complexity is assigned when individual piles within groups which are installed within close proximity to each other are considered. In the case of displacement piles in cohesionless media, the method of pile installation can have a significant effect on the soil mass.

   In contrast to the relatively sophisticated analytical research work that has been carried out on the axially loaded pile problem in recent years, very little effort has been made to understand the fundamental mechanics of how the soil responds to the presence of a pile and as a consequence, resists the loads transmitted by the pile.

   It is fundamental to any problem that if the mechanics are understood then the analytical modelling can be more realistic, and hence there is likely to be quite good agreement with the initial real problem and the final analytical reconstruction of the problem.

   Displacement piles, by definition, displace a volume of soil equal to the embedded volume of the pile. Various semi-empirical approaches that have been suggested in the literature to account for these volume changes are discussed.

   This chapter also reviews the various methods of assessing the axial load carrying capacity of piles. Briefly these are based on the consideration that the contributions from the pile tip and shaft can be considered to act independent of each other such that one is not influenced by the other. The various methods currently available are:

   (i) Bearing capacity theories based on empirical and semi-empirical observations.
   (ii) Semi-empirical correlations between full size piles and penetration tests.
   (iii) Pile driving formulae based on empirical correlations.
   (iv) The Méndard pressurerometer which relates a form of in-situ testing to a wide range of field results, and is thus semi-empirical.
(v) "Elastic" methods of analysis using either a "winkler" or "continuum" approach.
(vi) Finite element analyses.
(vii) Full scale load tests.

Of the various analytical methods listed above, the bearing capacity theories are probably used most extensively in conjunction with penetration tests. The finite element methods of analysis are still very much in their infancy as a design aid. Not one of the analytical methods listed are able to model the influence of pile installation on the soil, or the volume change characteristics of the soil. In all cases the mechanics of the problem have not been able to be matched, simply because they have not been adequately identified. Virtually all analytical methods recommend that to reliably assess the pile shaft load carrying capacity, full scale load tests should be conducted.

Thus the situation remains where not one analytical technique exists which could unreservedly be recommended to the practising foundation engineer. For this reason, where reliable estimates of the axial load carrying capacity of piles which rely on interaction with the surrounding soil are required, (note that piles end bearing onto a rigid stratum present a somewhat different problem), the designer has no choice but to resort to full scale pile loading tests.

2.2 CHAPTER 3 - Displacement Fields Developed About Model Piles During Installation by Driving

It is shown that the mass of soil affected by the installation of a single pile into dense dry sand is quite substantial, having the general dimensions of 8 to 120 diameter concentric on the pile, and extending 3 to 5\(^\text{7}\) beneath the pile tip.

The effect of pile driving into dense dry sand is to cause surface heave which would appear to account for the volume of the embedded pile up to an installation depth of about 9 to 100. Beyond this pile embedment depth the surface heave is virtually non-existent, thus suggesting that the additional displaced soil volumes are accommodated by volume changes within the soil mass.

Measured displacement fields about model piles installed by driving into dense sand placed to the same initial density but to different initial values of \(K_0\), were virtually identical. The nature of the contours of equal vectorial displacement so obtained indicated that the effect of incremental pile installation is analogous to the expansion of a spherical cavity of initially zero radius.

The displacement fields showed distinctly different characteristics above the pile tip to those observed below the tip. Above a pile embedment depth \((z)\) of about 5 to 6, the general direction of the displacements above the pile tip is roughly parallel to the pile shaft. At greater pile embedment depths the general direction tends to revert towards the pile shaft.

A relatively narrow "drag-down" zone was observed to be developed adjacent to the pile shaft for the pile surface roughness used.

No definite shear planes were observed to develop about the deforming mass of soil during pile installation.
2.3 CHAPTER 4 - Strain Fields Developed About Model Piles During Installation by Driving

From an analysis of the displacements developed about model piles during installation by driving, fields of both volumetric and shear strain were assessed.

The significant feature identified in this chapter is that, based on the known characteristics of dense sand, where it has generally been held that the sand is compacting due to the installation of the pile, (i.e. getting denser), it is in fact expanding (i.e. getting less dense). The changes in volumetric strain about driven model piles have different characteristics above the pile tip to those below, as was observed to be the situation with respect to displacement.

As the pile penetrates beyond 4 to 5D large negative volumetric strains are developed around the shaft (i.e. compaction), outside an inner zone where the soil continues to expand. Expansion also occurs about the pile tip and is bounded by a narrow zone of compaction. The development of large negative volumetric strains with depth probably account for the additional inserted pile volume at depth (i.e. beyond about an $\frac{L}{D}$ of 9 where little surface heave is observed).

These soil volume changes are also likely to be associated with stress relief of the soil mass.

Of significance is that the dilation rate was observed to decrease with depth to a limiting value at an $\frac{L}{D}$ of about 9; the same depth where the pile tip capacity tends also to a limiting value.

2.4 CHAPTER 5 - Axial Loads on Model Piles Installed by Driving

The first part of this chapter indicates the difficulties that exist in attempting to relate model pile tests, for example penetrometer results, to full scale piled foundations. The problems are not only geometric, i.e. the ratio of grain size to foundation dimension, but have been shown to depend also on the in-situ stress-state in the soil (i.e. $K_o$).

The effect of $K_o$ conditions has been shown to be so significant that all correlations between test results reported in the literature are considered to be invalid unless they have been made on the basis of:

(i) Soil type and characteristics
(ii) Method of pile installation
(iii) Initial density
(iv) In-situ stress conditions (i.e. initial $K_o$)

It would appear that the $K_o$ effect could be far more significant than any of the other presently identified scale effects.

The main body of this chapter attempts to model the effects of axial load as the expansion of a spherical cavity, using the premise that the pile tip capacity of axially loaded driven piles is directly due to the prestressing of the soil during pile installation.

The form of the analysis is such that both the axial load carrying capacity of the pile and the displacements occurring about the pile tip are able to be modelled. Reasonable agreement is achieved with model tests.
An attempt to follow the stress path of a typical soil element located adjacent to the pile tip has been fundamental to the analysis.

In the final part of this chapter a qualitative study has indicated that, although the degree of shaft roughness does not significantly alter the mass of soil affected by the process of pile installation by driving, it does cause some quite noticeable and potentially important changes adjacent to the pile shaft.

With increasing pile shaft roughness a "drag-down zone" of increasing width is observed, reaching a maximum width of about 0.5D for the condition of optimum roughness (i.e. where the pile shaft has about the same roughness as the sand mass).

This drag-down zone occurs within the zone already loosened by dilation, thus all the conditions required for further stress relief by arching are identified as being present.

The soil-pile friction angle is shown to have the dominating effect resulting in the conclusion that a rough surfaced pile is likely to contribute considerably more axial load carrying capacity than a pile of similar dimension but with a smoother surface.

Consideration of the pile tip configuration shows that a cone of sand is carried along under the tip of a square ended shaft, thus causing virtually identical displacement fields to be developed as about a shaft with a tapered tip.

2.5 CHAPTER 6 - Approximate Stress Fields Developed About Model Piles During Installation By Driving

This preliminary analysis indicates the possibility of obtaining sensible stress fields about the model piles during driving on the basis of the measured soil displacements. The approximate stress fields so determined are in good agreement with those obtained from the spherical analysis which in turn are in reasonable agreement with the pile tip stresses back-analysed from model pile tests.

Considerable reductions in stress immediately adjacent to the pile shaft are indicated.

3. COMMENT

From the foregoing considerations it has been shown that the conditions existing in the soil mass around a driven pile are extremely complex, bearing little relationship to those that existed prior to pile installation. However, the conditions so developed are those that exist in the soil prior to the pile being subject to loading. Thus in the case of a pile which is likely to be subjected to lateral loads, the situation is infinitely more complex. In this mode of loading the major response from the soil is initially likely to come from the soil mass adjacent to the shaft. The soil in this zone is likely to be in a variety of different complex states of stress as a result of the process of pile installation, the mechanics of which cannot be confidently quantified, but are probably dependent on a variety of factors including:

(i) The distance of the soil element relative to the pile shaft.
(ii) The surface roughness characteristics of the pile shaft.
(iii) The stress relief that has occurred due to straining and other yet unidentified phenomena.
(iv) The likelihood of arching for some distance out from the pile shaft.
(v) The change from the initially mobilised soil parameters due to stress relief.

It is not unlikely that additional unidentified effects exist, nevertheless, it is in the light of this yet unclarified and complex situation that the design foundation engineer has to assess the likely response of a pile to lateral loading.

The situation is quite simply that where the mechanics of the axial loaded pile problem are ill-defined, no satisfactory attempt appears yet to have been made to identify those relating to the manner in which the soil responds to lateral loading.
Part Two

Lateral Loading
Chapter 7  The Determination Of The Lateral Load Carrying Capacity Of Piles

7.1  INTRODUCTION

In part 1 it has been shown that no analytical technique exists whereby the complete stress and deformation states acting within the soil mass, at the completion of pile installation or after axial loading, are able to be determined. However, it has been demonstrated that the soil mass is effectively prestressed to quite substantial levels on that which existed prior to pile installation and axial loading.

It is in the light of this complex situation that "rational" methods of analysis have been developed to enable a foundation designer to assess the likely response of a soil-pile system to lateral loads.

Traditionally the two problems are considered independent of each other; the justification being that the magnitude of applied axial load has little influence on the laterally loaded pile-soil system. Even if this can be verified beyond a shadow of doubt, the effect of pile installation remains conveniently ignored.

It has been the aim of this research project to attempt to bring some of the aspects affecting statically loaded pile foundations into perspective, one to the other, on the basis of the observed mechanics of pile-soil interaction.

7.2  GENERAL

The design of laterally loaded piles is generally governed by two conditions:

1. the deflections at working loads should not exceed the values which can be tolerated by the superstructure;
2. the ultimate strength should be sufficiently high to prevent collapse of the supported structure even under the most adverse combination of factors.

(The same two conditions apply equally to the design of axially loaded piles).

The design approach to foundation resistance to lateral loads has, until relatively recently, been comparatively simple. The solution has usually been found in the use of inclined (raked) piles. The concept inherent in their use is essentially the development of axial loads in the piles accompanied by an increase in structural rigidity to the foundation.

Where this technique has been employed to resist the lateral forces generated by dynamic loading (for example earthquake loads), their performance has often been far from satisfactory, as indicated by Margason (1975). Because they are very stiff in the lateral direction compared with the rest of the foundation, the raked piles are often called upon to resist very high lateral load
components. These high lateral loads damage either the pile or the pile cap, as shown in figure 7.1.

In the situation where it is necessary to consider the lateral resistance of a single pile, as opposed to an integrated foundation system, it is presently only possible to assess the failure load with any degree of confidence. Even in this area the problem seems to be far from resolved within acceptable measures of certainty, especially if one uses the number of papers published on the subject as an indicator.

The problem becomes considerably more complex when the pre-failure characteristics of the laterally loaded pile foundation are considered, and particularly so when the load-deflection characteristics of the pile-soil system are investigated. These characteristics are dependent on whether the method of lateral loading is static or dynamic. The dynamic forces may be generated by a number of phenomena, for example, seismic events, wave or wind action, or may even be machinery induced vibrations.

In this chapter a brief review of the analytical concepts applied to the laterally loaded pile problem is presented.

7.3 PILES SUBJECT TO LATERAL LOADING - A REVIEW

7.3.1 Introduction

As has been indicated, the behaviour of the pile-soil system is dependent on the rate of loading. The following review (which is not intended to be exhaustive) considers only static behaviour. Considerations pertaining to the dynamic behaviour of laterally loaded piles are discussed in Hughes, Goldsmith and Endall (1978 a and b) and Goldsmith (1979).

Traditional approaches to the lateral loading problem have invariably ignored any effect the pile installation process or the existence of axial loads on the pile may have in modifying the stress-state within the soil mass. Rather, soil response is generally taken to be dependent upon those parameters that may be obtained from laboratory or in-situ investigation of the "undisturbed" site materials, no allowance being made for the modifications known to occur.

It would seem reasonable to expect that the soil strength available to resist the laterally applied pile forces must be dependent upon the prior stress history of the soil. It is of interest however that Feda (1972) as reported by Broms (1972) found for a laterally loaded free-headed pile, that a vertical load did not have any appreciable effects on the measured lateral deflections. Their work indicates that a vertical load will probably only affect the behaviour of a laterally loaded pile when the axial load in the pile exceeds approximately 10% of the buckling load. Similarly Singh and Prakash (1971) found that an average vertical load in each pile of a model pile group in sand of the order of 4 to 5% of the buckling load of a single pile, does not, when subjected to repeated loading, appear to materially affect the overall behaviour.

In these tests the applied axial loads, while being a small proportion of the pile buckling load, were 1.7 to 2.0 times the applied lateral load. In most situations the design axial load is likely to be significantly less than the proportion of the pile buckling load considered by Singh and Prakash. It would thus appear that the existence of axial load on a pile subject to lateral loading is not a
FIG. 7.1  INTERACTION OF RAKED PILES AND CAPS DURING AN EARTHQUAKE
(After Margason, 1975).

FIG. 7.2  SCHEMATIC GENERAL AND SIMPLIFIED REPRESENTATION OF A PILE GROUP
(After Mair, Gray and Donovan, 1969)
significant factor; providing the magnitude of the axial load is small relative to the buckling load of the pile.

7.3.2 Pile Interaction

In contrast to the single pile situation, multiple pile analysis is considerably more complex, particularly if the group is large and if it has a cap in contact with the soil. In the case where the pile cap is not in contact with the soil, the general problem of a group subjected to lateral loads can be analysed assuming that each pile in the group behaves as an individual pile unaffected by other piles in the group. One such approach is to use the Elastic or Winkler methods of analysis, as discussed subsequently, to determine the components of the individual pile stiffnesses, (vertical, horizontal and rotational), resisting the movement of the pile cap.

Using these stiffnesses the displacements, (translation and rotation), of the pile cap when subjected to the annulled external loads can be calculated. The displacements thus imposed on the top of the individual piles by the pile cap are then used to compute the forces and displacements in each of the piles.

A number of methods for solving this particular problem exists, for example, Hrennikoff (1949), Saul (1968) and Aschenbrenner (1967).

An alternative approach involves considering the individual piles in the group as "equivalent cantilevers". The "equivalent cantilevers" are considered to be free bars, the effect of the soil being taken into account in the determination of the "equivalent length" (Donovan 1959, Francis 1964, Hair, Gray and Donovan, 1969). The cantilever length is chosen so that when subjected to the lateral loads and moments of the real pile, the equivalent cantilever will model either the deflections sustained by the real pile, or the moments, i.e. a separate equivalent cantilever is required for each mode. This process is shown schematically in figure 7.2.

Both of the foregoing procedures assume that all the piles in the group behave in exactly the same manner. Piles within a group however do interact with each other. This interaction results in an overall reduction in the load carrying capacity of each pile and can be of considerable significance, especially if the piles are located close together.

A number of researchers have carried out model tests which give an indication of the magnitude of this interaction. For example, Prakash (1962) and Navisson and Salley (1977) conducted tests on model pile groups in sand. They found that interaction existed between the piles up to a pile spacing of 3 pile diameters in the direction of the load and 3 pile diameters normal to the load; as indicated in figure 7.3. The interaction becomes particularly marked at lesser pile spacings as shown in figure 7.4. In figure 7.4 the measure of pile interaction is expressed as a proportion of the magnitude of the coefficient of horizontal subgrade reaction \( k_h \) of a single pile not influenced by adjacent piles.

In comparison Donovan (1959) observed after tests on model pile groups in cohesive soils, that the lateral resistance of piles was not affected when the pile spacing was greater than 4\( \sqrt{d} \). When the pile spacing was less than 2\( \sqrt{d} \) the soil enclosed between the piles was observed to behave as a unit.
FIG. 7.3 MINIMUM PILE SPACINGS IN SAND SUGGESTED BY PRakash (1962), AND DAVISSON AND SALLEY (1970).

FIG. 7.4 EFFECT OF PILE SPACING ON THE COEFFICIENT OF HORIZONTAL SUBGRADE REACTION \( (k_h) \) IN COHESIONLESS MATERIAL.
Under these circumstances it was concluded that the ultimate lateral resistance of the pile group was equal to the lateral resistance of an equivalent foundation with the same outside dimensions as the pile group.

Qualitative evidence of pile interaction has been obtained in many tests on full size pile groups. For example, Feagin (1937) concluded from full size tests that the lateral resistance of a pile group in sand increased at low load levels in proportion to the number of piles. As failure was approached, the resistance per pile became less as the number of piles increased, as indicated schematically in figure 7.5.

Similar findings have been reported by other authors, for example Mannilu et al (1977). However, little real data exists because of the difficulties and expense associated with testing full size pile groups.

For design purposes it is suggested, in the absence of more specific data, that allowances be made for pile interaction along the lines suggested by Prakash et al as indicated in figure 7.4. However, it must be emphasised that these suggested reductions in load carrying capacity are largely empirical and are not based on an understanding of the pile-soil interaction mechanism involved in causing the reduction in soil stiffness.

Recent analytical studies have computed pile interaction within groups using elastic theory; notably Poulos (1971 b) and Oteo (1972). In these studies the soil is assumed to be an ideal, elastic, isotropic material, consequently having equal strength in tension as in compression. Because of this the loads in the piles are computed to be symmetrical about the centre line of the group (AA in figure 7.6). The internal piles thus take the least load while the corner piles take the greater proportion of load.

Comparisons with available test data, as made by Broma (1972), indicate that elastic theory underestimates the interference between piles when the pile spacing is small. Conversely, overestimates of interference were observed at large pile spacings. However, notwithstanding these aberrations, the minimum pile spacings indicated in figure 7.3 are to some degree substantiated.

Pigeon and Toan (1978) present a computer program for the elasto-plastic analysis of pile groups. Essentially the program is a Winkler type static analysis which provides for degradation of the soil and allows, by iteration, the development of local plastic failure mechanisms in the individual piles. Iteration can be continued until an overall plastic collapse mechanism is formed by the pile group as a whole. This program would appear to differ from others to the extent that while many existing programs handle post-elastic behaviour of the soil, (e.g. Reese, O'Neill and Smith, 1970), they do not appear to accommodate post-elastic behaviour of the pile.

7.4 STATIC LOADING
7.4.1 General

Broadly the methods of considering static lateral loading of piles can be divided into three categories:
(a) Limiting (or ultimate) loads
(b) Elastic continuum
(c) Elasto-plastic discontinuum (the Winkler hypothesis).
FIG. 7.5  DIAGRAMMATIC REPRESENTATION OF THE QUALITATIVE OBSERVATIONS OF FEAGIN (1937) ON PILE INTERACTION

FIG. 7.6  HORIZONTAL LOAD DISTRIBUTION IN FIXED HEAD PILE GROUP
(after Poulos, 1971b)
The limit analyses concept permits only the ultimate load capacity of a pile-soil system to be determined with any degree of certainty. This contrasts directly with the elastic and elasto-plastic method. However, although both the method based on elastic theory and that on the elasto-plastic method (sometimes referred to as the "modulus of subgrade reaction" or the "Minkler" method) have been well described in the literature, understanding of the concepts relating to each is often confused.

7.4.2 Limiting Lateral Loads

Engineers have historically concerned themselves with conditions that would apply at failure, thus in contrast to the load-deflection characteristics, limiting load analyses have probably received the greatest amount of attention. Probably the most significant contribution prior to the late 1940's was that of Blum (1932); since then numerous authors have considered the same problem (for example, Broms 1964 a and b, Brinch Hansen 1961).

In limit load analyses the soil is essentially considered to fail in two distinct modes, as indicated in figure 7.7, (see also figure 8.14, Chapter 8).

1. The upper part of the pile where the response is influenced by the free ground surface.
   The presence of the free surface permits the soil adjacent to the pile to yield and move upwards in a wedge-like manner rather analogous to the passive failure condition behind a retaining wall.

2. The lower part of the pile where the soil yields locally around the pile in a horizontal direction.

Various authors have proposed limiting reaction pressures which can be mobilised in the soil to resist the lateral loads transmitted through the pile. The actual conditions and the general idealisation for cohesionless soils are shown in figure 7.8.

In cohesive soils it is generally indicated that the surface wedge action extends to a depth of about 3 pile diameters from the surface (e.g. Broms 1964 a). Figure 7.9 summarises the proposals of Reese, Ménard, Broms and Matlock for limiting soil reactions in cohesive soils.

In figure 7.9, \( \frac{d_c}{D} \) represents the critical depth at which the limiting pressure is assumed to apply, normalised in terms of pile diameters; and is the depth at which the soil tends to yield locally around the pile. The limiting pressure has been expressed in terms of the undrained cohesive strength of the soil \( (C_u) \).

In cohesionless soils the maximum soil resistance that can be mobilised against a pile is generally considered to increase with depth. Figure 7.10 shows the limiting soil reaction proposed by Broms (1964 b), Minkin (1950) as quoted by Davisson and Prakash (1963) and Brinch Hansen (1961).

If the shape of the soil pressure distribution along a pile is assumed, then the actual values of soil pressure associated with a particular applied load can be backfigured from static considerations alone.
FIG. 7.7 FAILURE MODES CONSIDERED TO APPLY UNDER LIMIT LOAD CONSIDERATIONS

(a) Actual Conditions. (b) Idealised Conditions.

FIG. 7.8 ACTUAL AND IDEALISED LIMIT PRESSURES FOR COHESIONLESS SOILS
FIG. 7.9  SOME PROPOSED LIMITING SOIL REACTIONS IN COHESIVE SOILS

FIG. 7.10  SOME PROPOSED LIMITING SOIL REACTIONS IN COHESIONLESS SOILS
FIG. 7.11 SOIL PRESSURE DISTRIBUTION FOR A POLE IN COHESIONLESS SOIL
(After Broms, 1964b).

FIG. 7.12 SIMPLIFIED ANALYTICAL MODEL
Broms (1964 b), for example, gives the soil pressure distribution against a short free headed pile in sand as indicated in figure 7.11. By applying this technique to model tests in dense vibrated sand, Fendall (see Hughes, Goldsmith and Fendall, 1978 a) has obtained soil reaction values in excess of 30 times the Rankine Passive pressure. It is not known why these values differ so markedly from those obtained by other authors.

From figures 7.9 and 7.10 there clearly appears to be a considerable difference of opinion. It would appear however, that the maximum moment developed in a pile for a given horizontal load is not particularly sensitive to the variations in soil reaction indicated in figures 7.9 and 7.10.

For design purposes, as discussed subsequently, it would seem reasonable to consider the pile fixed at an embedment depth varying between 3 to 100, from deflection considerations, and up to 20 from moment considerations; subsequently treating the problem as a simple cantilever as indicated in figure 7.12. If the moment criteria alone are adopted, the deflections (a) obtained using the same assumed depth to fixity could however be grossly in error. To fully describe the pile response using this technique it is necessary to choose the depth to the assumed point of fixity appropriately to the mode of behaviour being considered.

It should thus be obvious that indications of the order of magnitude of the ultimate load can be relatively easily determined, however, an assessment of the pre-ultimate loads and associated displacements are extremely difficult to make within an acceptable degree of certainty. In view of this complexity it is tending to become common practice, especially in Europe and the U.S.A., to resort to full scale testing rather than to rely on relatively inconclusive theory (for example Parker and Reese 1970).

A further degree of complexity is added to the pile problem when more than one pile is considered (i.e. pile groups), or the pile does not have a free head end condition (i.e. a pile cap or foundation beam is present which increase the degree of pile ton fixity).

As has been indicated for the single pile, the detailed behaviour and soil failure mechanism around a laterally loaded pile or group, is similarly not conclusively understood. Further, far less study has been made of the pile group problem than of the single pile situation.

7.4.3 Elastic Analysis

(1) The Analytical Technique

In this technique the pile is assumed to be a thin rectangular vertical strip of width \( D \), length \( L \) and constant flexibility \( E_D I_0 \) (the subscript \( D \) denotes pile). In analysing the analysis to a circular pile, the width \( D \) is taken as the diameter of the pile.

The idealisation of the actual system is shown in figure 7.13. The pile is divided into elements each of which is acted on by a uniform horizontal stress, \( n \), which is assumed to be constant across the width of the pile.

The soil is assumed to be an ideal homogenous, isotropic semi-infinite elastic material having a Young's modulus, \( E_s \), and Poisson's ratio, \( \nu_s \) (the subscript \( s \) denotes soil).

It is further assumed that \( E_s \) and \( \nu_s \) are unaffected by the presence of the pile.
FIG. 7.13 ELASTIC THEORY IDEALISATION OF PILE-SOIL SYSTEM

FIG. 7.14 ASSUMED RESPONSE OF THE PILE-SOIL SYSTEM TO A LATERAL LOAD
In the analysis, the soil and pile displacements are evaluated and equated at the element centres. The analysis further assumes that when the pile deflects, the elements of the pile remain rigid and move horizontally; thus the soil response at each element is analogous to the effect of applying a local horizontal load to the soil. The resistance of these elements in moving through the soil is accordingly able to be determined by the evaluation of Mindlin's equations over the surface of the elements. Mindlin's equations enable the horizontal displacement of a point within a semi-infinite mass due to a horizontal annulled load within the mass, to be evaluated. This assumed soil response is shown in figure 7.14.

The soil reaction to the pile is (from figure 7.14) given by:

\[(\text{Soil element } 1) \quad p_1 = I_{11} \Delta_{11} + I_{12} \Delta_{12} + I_{13} \Delta_{13} \ldots \ldots \cdot I_{1,n} \Delta_{1,n} \quad 7.1(a)\]

\[(\text{Soil element } 2) \quad p_2 = I_{21} \Delta_{21} + I_{22} \Delta_{22} + \ldots \ldots \cdot I_{2,n} \Delta_{2,n} \quad 7.1(b)\]

which can be written in matrix form thus:

\[
[p] = [I^*] [\Delta]_s \quad 7.1(c)
\]

where \(p\) = the soil reaction matrix

\(\Delta_s\) = the soil displacement matrix

\(I^*\) = the elastic influence matrix

where \(I^*\) = some function of \(E_s\) and \(u_s\).

Clearly, the soil reaction to the applied loads, such as \(p_1\) on element 1, are not independent of the other elements, but are a function of all the displacements \(\Delta_1\) to \(\Delta_n\).

To evaluate the influence matrix, \([I^*]\), the soil must be considered to be isotropic with a constant value of Young's modulus, \(E_s\). It will be recalled that this assumption is fundamental to the analysis. Such a situation is clearly ideal as most soils (including both homogeneous and heterogeneous soils) show an increase in \(E_s\) with depth as indicated in figure 7.15(a).

Poulos (1973) has suggested that it is possible to modify the influence matrix in an approximate way to accommodate this variation in \(E_s\). Any variation in \(E_s\) can only result in an approximate solution as the Mindlin equation (the basis of the technique) is strictly only applicable to a homogeneous soil. The method proposed for varying \(E_s\) assumes that the displacement of a point in a non-homogeneous mass can be obtained from the Mindlin solution by using the elastic modulus at that point.

As indicated in figure 7.16, the variation for a non-uniform \(E_s\) is to modify the value of the modulus used throughout the soil depending on the term of the matrix being considered. For example, when considering elements, the influence factor \(I_{22}\) would be evaluated assuming all the soil had a modulus \(E_2\). In contrast, \(I_{23}\) would be evaluated assuming all the soil had a modulus somewhere between \(E_2\) and \(E_3\). Similarly for influence factor \(I_{2,n}\). It is considered that the average value of \(E_s\) pertaining between the various elements being considered is probably appropriate, i.e.
FIG. 7.15 VARIATION IN SOIL STRENGTH WITH DEPTH

FIG. 7.16 ASSUMED RESPONSE FOR NON-HOMOGENEOUS SOIL MASS
Poulos' technique also allows for soil yield by assuming idealised stress deflection curves for each soil element. An iterative technique is employed until the commuted shear stresses in the soil do not exceed the yield values.

The response of the pile to the applied external loads and associated soil reactions on each of the elements is given, in a similar manner to equation 7.1, by:

(Load line) \[ H = K_{HH} + K_{H1} + K_{H2} \ldots + K_{H, n} \]
(Soil element 1) \[ 0 = K_{11} + K_{12} \ldots + K_{1, n} \]
(Soil element 2) \[ 0 = K_{21} + K_{22} \ldots + K_{2, n} \]

which can be expressed in matrix form thus:

\[ (F) = |K^*| \cdot (\Delta)_P \]

where \( F \) = the applied load matrix
\( \Delta_P \) = the pile displacement matrix
\( K^* \) = the pile stiffness matrix.

Displacement compatibility requires that \( \Delta_P = \Delta_s \), thus equations 7.1 and 7.3 can be solved to give the displacements and reactions on the pile.

A particular method of analysis is that of Poulos (1971). In this method the pile is divided into \( n + 1 \) elements, all elements being of equal length, \( \delta \), except those at the top and tip of the pile which are of length \( \delta/2 \). The method of analysis is shown diagrammatically in figure 7.17. Pile rotations are obtained in a similar manner, enabling the load line deflection \( \Delta_H \) to be evaluated as indicated in figure 7.14. Poulos further suggests that as pile-soil separation generally leads to groundline displacements approximately twice those given by the purely elastic solutions; that where pile-soil separation is considered possible, the effect he allowed for by reducing Young's Modulus by a factor of 2 in the area concerned. Poulos recommends that caution be exercised in arbitrarily applying this factor as overburden pressures influencing the top soil layer must also be taken into account.

Matthewson (1969) also analysed the laterally loaded pile in a uniform elastic half space.

The principle differences between the solutions of Matthewson and Poulos are:

(a) Poulos uses a finite difference form of the basic beam equation

\[ \frac{\Delta V}{dz} = \frac{w}{EI} \text{ to describe pile action (where } w = \text{load).} \]

Matthewson has used the integral equation directly.
FIG. 7.17 SCHEMATIC REPRESENTATION OF Poulos' METHOD OF ANALYSIS
(h) Poulos uses an integration of the Mindlin equation over a rectangle, given by Douglas and Davis (1964), to evaluate soil displacements. In comparison Matthewson uses a numerical integration technique which permits any desired degree of "accuracy".

The two solutions are compared in figure 7.18 using the tests reported by Kerisel and Adam (1967) and Gieser (1953). Matthewson (1969) and Poulos (1968) have independently addressed their techniques to this data by applying the same load to the same pile and selecting soil properties so that theoretical and measured top displacements were equal.

Clearly for practical purposes there is little difference between the two solutions, especially in view of the general uncertainty as to the process of pile-soil interaction.

The foregoing solutions derived from elastic theory, while not modelling real soils with any degree of accuracy, do have certain advantages. Primarily that the method of analysis takes into account the influence of movement of adjacent points. This interaction between elements cannot be readily handled by other systems of analysis. In these other systems allowance is usually made by resorting to full scale tests, as discussed earlier, or employing empirical modifications. For example, Yoshida and Yoshinaka (1972) propose a semi-empirical method for estimating the modulus of subgrade reaction used in the Winkler method of analysis, based on the average value of $E_s$ for the soil over the effective depth of the pile.

(ii) Determination of the Soil Modulus ($E_s$)

The quantity of primary importance in applying the foregoing elastic analysis is the Young's Modulus of the soil ($E_s$). Ideally, values of $E_s$ for undrained conditions and drained conditions should be determined so that predictions of both immediate and total final movements and rotations may be made. Theoretically, the drained and undrained values of $E_s$ may be determined from laboratory triaxial tests in which the stress path of a typical element in the ground is followed and the strains measured. The philosophy associated with stress path analyses and examples of the annihilation of the technique have been presented by a number of authors, for example Lambe (1967).

An introduction to the stress path interpretation of triaxial data has been given by Hughes and Goldsmith (1977). In reality such a stress path may be extremely complex and may depend largely on the method of installation of the pile. However, as both the initial and final stress states in the soil (i.e. after loading of the pile) are unable to be readily estimated, such laboratory procedures cannot presently be employed with any degree of confidence. Horizontal in-situ plate loading tests are suggested as a possible means of obtaining $E_s$ for bored piles; however, Poulos (1971b) suggests that the best method of obtaining $E_s$ at the present time is to carry out a full scale field loading test and to back figure $E_s$ from the measured deflections.

$E_s$ may be determined from a horizontal plate loading test by using equations 7.4 through 7.5.

$$k_n = \frac{p}{y} = \frac{E_s}{E_n (1 - v_s^2)} \sqrt{\frac{f}{E_n}} \tag{7.4}$$
FIG. 7.18 COMPARISONS OF THEORETICAL SOLUTIONS WITH MEASURED DISPLACEMENT PROFILES (After Matthewson, 1969).
where $k_h$ = coefficient of horizontal subgrade reaction

$p$ = average contact pressure under the plate

$y$ = plate penetration into the soil

$M_A$ = a shape factor which (after Slack and Walker 1970)

has the value: (i) circular plate, $M_A = 0.96$

(ii) square plate, $M_A = 0.95$

$\nu_s$ = Poisson's ratio

$A_p$ = plate area.

Thus for the undrained case where $\nu_s = 0.5$, equation 7.4 rearranges to:

(a) 300 mm x 300 mm square plate

\[
E_s = \frac{214p}{y}
\]

(b) Circular Plate

\[
E_s = \frac{1.28p\cdot r}{y}
\]

where $E_s$ = Young's Modulus of the soil (kPa)

$p$ = plate contact pressure (kPa)

$r$ = radius of circular plate (mm)

$y$ = plate penetration (mm).

All values of $E_s$ reported for cohesive soils in this section are for undrained conditions.

In the absence of any other data, Poulos makes the following recommendations backfigured from the results of full scale loading tests in cohesive and cohesionless soils reported by Brons (1964 a and b respectively), and assuming $E_s$ remains constant with depth. For cohesive soils the values of $E_s$ were found to lie within the range:

\[
E_s = 15 \text{ to } 95 \text{ } C_u \quad \text{i.e.} \quad \frac{E_s}{C_u} = 15 \text{ to } 95
\]

where $C_u$ = the undrained shear strength of the clay.

The lower values were associated with very soft clays and the higher values with stiff clays.

The average value for all the cases considered was:

\[
E_s = 40 \text{ } C_u \quad \text{i.e.} \quad \frac{E_s}{C_u} = 40
\]

These values appear to be particularly low when compared to the range of values reported by Simons (1974) for a number of structures on normally and slightly overconsolidated clays; and Butler (1974) for structures on heavily overconsolidated clays. The values of Simons and Butler are reproduced in Table 7.1 and were generally determined in association with settlement observations.

In comparison Skempton (1951) presented data showing that for most clays a representative range was:-
It is considered that the low values of $\frac{E_s}{C_u}$ reported by Poulos are a direct consequence of the nature of lateral pile loading. Most of the resistance to pile lateral loading is derived from the mass of soil within a few pile diameters of the ground surface where the degree of confinement is low. In comparison a relatively higher degree of confinement exists under vertically loaded foundations.

### TABLE 7.1
VALUES OF $\frac{E_s}{C_u}$ REPORTED BY SIMMONS (1974) AND BUTLER (1974)

<table>
<thead>
<tr>
<th>Site</th>
<th>$E_s/C_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normally and Slightly Overconsolidated Clays</td>
<td></td>
</tr>
<tr>
<td>Test embankment, Kings Lynn</td>
<td>40</td>
</tr>
<tr>
<td>Oil tanks, Arabian Gulf</td>
<td>50-70</td>
</tr>
<tr>
<td>Skaho Office Building, Oslo</td>
<td>150</td>
</tr>
<tr>
<td>Turnhallen (Heavy) Drammen</td>
<td>190</td>
</tr>
<tr>
<td>Tank, Shellhaven</td>
<td>220</td>
</tr>
<tr>
<td>Northeast Test Embankment, Boston</td>
<td>240</td>
</tr>
<tr>
<td>Preload test, Lagunillas</td>
<td>250</td>
</tr>
<tr>
<td>Preload test, Amuay</td>
<td>250</td>
</tr>
<tr>
<td>Loading test, Ska Erdeby</td>
<td>340</td>
</tr>
<tr>
<td>Storage tanks, South Portland</td>
<td>470</td>
</tr>
<tr>
<td>Satellite antenna tower, Fucino plains</td>
<td>450</td>
</tr>
<tr>
<td>Loading test, Fornebu</td>
<td>500</td>
</tr>
<tr>
<td>Loading test, Asrum</td>
<td>1000</td>
</tr>
<tr>
<td>Okerhørtan, Oslo</td>
<td>1500</td>
</tr>
<tr>
<td>Loading test, Mastemyr</td>
<td>3000</td>
</tr>
<tr>
<td>Heavily Overconsolidated Clays</td>
<td></td>
</tr>
<tr>
<td>Hendon</td>
<td>690</td>
</tr>
<tr>
<td>Moorfields</td>
<td>830</td>
</tr>
<tr>
<td>Hyde Park Cavalry Barracks</td>
<td>310</td>
</tr>
<tr>
<td></td>
<td>410</td>
</tr>
<tr>
<td></td>
<td>480</td>
</tr>
</tbody>
</table>

Average values of $E_s$ for cohesionless soils, again backfigured by Poulos from results reported by Broms, are given in Table 7.2.
Again the effects of different methods of pile installation have not been able to be assessed. Furthermore, the use of a constant value of $E_s$ with depth in sands is highly questionable (see figure 7.15(a)).

7.4.4 The Modulus of Subgrade Reaction Method

(1) The Technique

This method assumes that the reaction from the soil can be represented by a series of independent springs as indicated in figure 7.19. In comparison to the elastic method, this technique is far easier to handle as the influence matrix (equation 7.1(c) for the elastic case) only has terms on the diagonal, i.e. from figure 7.19:

$$
p_1 = k_s \Delta_1 0 0\quad 7.10(a)$$
$$p_2 = 0 k_s \Delta_2 0\quad 7.10(b)$$
$$p_n = 0 0 0 ... k_s \Delta_n ... \ \text{etc.} \quad 7.10(c)$$

which may be written in matrix form thus:

$$\begin{pmatrix}
p_1 \\
p_2 \\
p_n \\
\end{pmatrix} = \begin{pmatrix}
k_s & 0 & 0 \\
0 & k_s & 0 \\
0 & 0 & k_s \\
\end{pmatrix}\begin{pmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_n \\
\end{pmatrix} \quad 7.10(d)$$

In equation 7.10(d), $k_s$ is the equivalent spring stiffness of the soil. The influence matrix $[k_s]$ is equivalent to the influence matrix $[I^*]$ developed in the elastic method. This equivalent spring stiffness ($k_s$) is related to the coefficient of horizontal subgrade reaction ($k_h$) thus:

$$k_s = k_h \cdot D \cdot \delta L \quad 7.11$$

where $(D \cdot \delta L)$ is the frontal area of the pile element as presented to the soil.

Terzaghi (1955) has shown that for cohesionless soils the coefficient of subgrade reaction can be expected to increase linearly with depth and to decrease linearly with increasing width or diameter of the laterally loaded pile according to equation 7.12:-

$$k_h = n_h \frac{L}{h} \quad 7.12$$
FIG. 7.19 MODULUS OF SUBGRADE REACTION THEORY IDEALISATION OF PILE-SOIL SYSTEM
The modulus of horizontal subgrade reaction $k_h$ is usually expressed as:

$$k_h = k_h \cdot D$$  \hspace{1cm} (7.13)

These relationships are summarised in Table 7.3.

**TABLE 7.3**

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Unit</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$</td>
<td>Spring stiffness</td>
<td>$F$</td>
<td>$k_h \cdot \delta L$</td>
</tr>
<tr>
<td>$k_h$</td>
<td>Coefficient of horizontal subgrade reaction</td>
<td>$F \cdot L^3$</td>
<td>$D = \frac{k_h}{\gamma}$</td>
</tr>
<tr>
<td>$n_h$</td>
<td>Coefficient of horizontal subgrade reaction at a depth of unity for a pile with a width of unity</td>
<td>$F \cdot L^3$</td>
<td>$k_h \cdot \frac{D}{L}$</td>
</tr>
<tr>
<td>$k_h$</td>
<td>Modulus of subgrade reaction</td>
<td>$F \cdot L^2$</td>
<td>$k_h \cdot D = n_h \cdot L$</td>
</tr>
</tbody>
</table>

where $F = \text{force}$, $\delta L = \text{incremental pile length}$,$L = \text{length}$, $p = \text{pressure}$, $\gamma = \text{displacement}

Terzaghi (1955) and Rowe (1956) have recommended values of $n_h$ for sands as indicated in Table 7.4.

**TABLE 7.4**

| RECOMMENDED VALUES FOR $n_h$ IN SANDS |
|---|---|
| (kN m$^{-3}$) |

<table>
<thead>
<tr>
<th>Author</th>
<th>Loose Dry</th>
<th>Loose Submerged</th>
<th>Medium Dry</th>
<th>Medium Submerged</th>
<th>Dense Dry</th>
<th>Dense Submerged</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terzaghi (1955)</td>
<td>2500</td>
<td>1400</td>
<td>7500</td>
<td>5100</td>
<td>20 000</td>
<td>12 000</td>
</tr>
<tr>
<td>Rowe (1956)</td>
<td>2200</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>18 000</td>
<td>-</td>
</tr>
</tbody>
</table>

Davisson and Gill (1963) have reported values of $n_h$ for very loose submerged sands ranging from as low as 410 kN m$^{-3}$ under repeated loading, to over 27 000 kN m$^{-3}$ for very dense dry sands under static loading.
The load-deflection results from a model test presented in Hughes, Goldsmith and Fendall (1978a) is shown in figure 7.20. If a soil pressure distribution such as that shown in figure 7.11 is assumed, \( n_h \) can be backfigured from the slope of the load-deflection relationship using equation 7.14 as proposed by Broms (1964 b) for a short rigid pile:

\[
y_g = \frac{19H(1 + 1.33 \frac{e}{L})}{L^2 n_h}
\]

where 
- \( e \) = the level of load application above ground level
- \( L \) = embedded length of pile
- \( H \) = lateral load
- \( y_g \) = lateral displacement of the pile at the ground line.

From the nature of figure 7.20, it is obvious that because the load-deflection relationship is far from linear, any number of slopes are able to be chosen. The values of \( n_h \) corresponding to the slopes A, B and C of figure 7.20 are shown in Table 7.5.

<table>
<thead>
<tr>
<th>Slope</th>
<th>( n_h ) (kN m(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100 000</td>
</tr>
<tr>
<td>B</td>
<td>32 400</td>
</tr>
<tr>
<td>C</td>
<td>10 700</td>
</tr>
</tbody>
</table>

The value of \( n_h \) for dense sand recommended by Terzaghi, from Table 7.4, is 20 000 kN m\(^{-3}\). Slack and Walker (1970) present a technique whereby the load-deflection results from a pile load test are plotted on logarithmic axes, such as has been done in figure 7.21 for the model test results of figure 7.20. (See also Chapter 2 and Appendix 5). The point where the slopes of the straight lines so defined change, point S on figure 7.21, is a distinctive point referred to by Slack and Walker as a "yield" point - the point where the rate of deflection increases.

In the absence of any other definitive means of assessing a "yield" criteria for laterally loaded piles, it is felt that this technique provides a rational means of defining a "design ultimate load". For working loads an appropriate factor of safety could then be applied to this "design ultimate load". A similar suggestion was made with respect to axially loaded piles in Chapter 2.

Point S in figure 7.21 represents 75% of the maximum load able to be sustained by the model pile tested and is defined by the point associated with slope D on the load-deflection curve of figure 7.20. The slope of the line passing through this point yields a value of \( n_h \) of 15300 kN m\(^{-3}\) which is in closer agreement with the value recommended by Terzaghi for dense dry sand, than the values obtained from lines A, B and C as recorded in Table 7.5.
If the coefficient of subgrade reaction is constant with depth, then Timoshenko (1930) has shown that the shear stresses, bending moments, displacements and soil reactions are all functions of the parameter, \( k_0 \) (with units of length), generally referred to as the relative stiffness factor:

\[
 k_0 = \frac{4E \alpha I_n}{k_h D^3}
\]

where \( E \) is the pile flexibility.

The analytical approach described in this section permits both non-linear soils and soils with varying moduli to be used. However, as mentioned in the preceding section, the technique does not allow for the interaction of one point on another. Thus the soil spring stiffness, \( k_s \), associated with a particular element is assumed to be independent of displacements at other points along the pile.

In reality, however, just as with the Winkler concept of a beam on a flexible foundation, the behaviour of neighbouring points do influence each other. This has been indicated by Vesic (1961). This interaction is naturally more pronounced near the top of the pile where the relative displacements are largest. Because only empirical methods are available in assessing this interaction, judgement and experience are necessary in making a rational assessment of the appropriate coefficients of horizontal subgrade reaction to apply.

Both Ménard (1962) and Matlock (1970) suggest that the coefficient of horizontal subgrade reaction should increase linearly from the surface to a critical depth in a manner similar to that indicated for the proposed limiting soil pressures of figure 7.9. This was found to apply irrespective of soil type.

Baguelin, Frank and Said (1977) show analytically that the soil stiffness depends not only on the characteristics of the soil but also on the relative stiffnesses of the pile and soil, the pile length to width ratio, the existence of pile top fixity, and loading conditions. They have derived a technique for taking these factors into account in assessing the appropriate coefficient of subgrade reaction to use. Their somewhat complex analysis is not discussed in this review.

(ii) Methods for Determining the Coefficient of Horizontal Subgrade Reaction

Several methods exist for the determination of the coefficient of horizontal subgrade reaction. These may be broadly classified into field, laboratory and full scale test methods.

(a) Field Measurements

The historical method for determining the coefficient of subgrade reaction is that of Terzaghi (1955). Terzaghi assumed that the coefficient was not always constant but depended upon the magnitude of the load. This was subsequently confirmed by McCalland and Focht (1958). The original Terzaghi approach was to measure the stiffness of the soil when subjected to loading from a 300 mm x 300 mm plate. Thus the coefficient of horizontal subgrade reaction is equal to the pressure necessary to deflect the plate divided by the deflection of the plate (refer also Table 7.3):-
FIG. 7.20 LOAD-DEFLECTION RESULTS FROM A MODEL PILE TEST IN DENSE DRY SAND

FIG. 7.21 LOG-LOG PLOT OF LOAD-DEFLECTION RESULTS OF FIGURE 7.20.
\[ k_h = \frac{D}{y} \]  
\[ \text{where } p = \text{the pressure applied to the soil through the plate} \]
\[ = \frac{\text{load on plate}}{\text{plate area}} \]
\[ y = \text{plate penetration into the soil} \]

Variations on the original Terzaghi technique and empirical correlations between various methods of determining the soil modulus \( E_s \) are reported respectively by Slack and Walker (1971) and Yoshida and Yoshinaka (1972).

Yoshida and Yoshinaka have developed the following relationship for the coefficient of subgrade reaction in terms of the Ménard pressurerometer modulus, \( E_{sp} \) (the pressurerometer is discussed in Appendix 11). This process would appear to be iterative as it requires an initial estimate of the ground line deflection of the pile:

\[ k_h = 30 \times 10^3 \beta^* E_{sp} \left( \frac{1}{D_0} \right) \left( \frac{D}{D_0} \right)^{(0.75)(m-1)} y_g \text{ (cm)} \]  
\[ (\text{kN m}^{-3}) \]  
\[ \text{where } E_{sp} = \text{the average value of Young's Modulus for the soil determined from a Ménard pressurerometer test over the effective depth of the pile (approximately from the ground surface to the depth of the maximum bending moment of the pile) (kPa).} \]
\[ D_0 = \text{diameter of a standard 30 cm plate (cm)} \]
\[ D = \text{pile width or diameter (cm)} \]
\[ \beta^* = \text{constant with a value between } \frac{1}{1.0} \text{ and } \frac{1}{1.6}, (0.769 \text{ and } 0.625) \]
\[ y_g = \text{lateral deflection of the pile at the ground line (cm)} \]
\[ m = \text{coefficient which has the values of Table 7.6.} \]

**TABLE 7.6**

**EXPERIMENTAL VALUES OF COEFFICIENT (m)**

(after Yoshida and Yoshinaka 1972)

<table>
<thead>
<tr>
<th>Pile Type</th>
<th>Soil Conditions</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driven</td>
<td>(Elastic) Diluvial sediments or clay and claley soil Sand and Sandy Soil (Plastic)</td>
<td>0 to 0.3</td>
</tr>
<tr>
<td>Cast in Place</td>
<td>Generally where ( y_g &gt; 20 \text{ mm} )</td>
<td>0.4 to 0.7</td>
</tr>
</tbody>
</table>

They have also obtained the following approximate correlations for the Young's Modulus of the soil \( E_s \):-
where $E_{sp} = E_s$ obtained from a Ménard pressuremeter test
$E_{sc} = E_s$ from uniaxial or triaxial compression tests on undisturbed samples
$E_{s30} = E_s$ obtained from a 30 cm diameter plate loading test

$N =$ the number of blows from a standard penetration test.

It is significant to note that, as was found to be the case with recorded $E_s$ values, there is a similar large divergence of opinion as to the appropriate correlations to draw between the Standard Penetration Test blow count ($N$) and Young's Modulus for soil ($E_s$).

Table 7.7 presents additional correlations summarised by Sutherland (1974) made by D'Appolonia et al (1970) and Parry (1971).

### Table 7.7

<table>
<thead>
<tr>
<th>Author</th>
<th>Correlation</th>
<th>Source of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yoshida and Yoshinaka</td>
<td>$E_s = 700$ N</td>
<td>Full scale lateral loaded pile test</td>
</tr>
<tr>
<td>Parry (1971)</td>
<td>$E_s = 5000 N$</td>
<td>Settlement observations based on 24 published case studies</td>
</tr>
<tr>
<td>D'Appolonia et al (1970)</td>
<td>Preloaded Sand: $E_s = 54,000 + 1350$ N</td>
<td>Settlement observations from full scale structures and correlations with Dutch Cone penetration tests</td>
</tr>
<tr>
<td></td>
<td>Normally Loaded Sand: $E_s = 21,600 + 1060$ N</td>
<td></td>
</tr>
</tbody>
</table>

The values of Table 7.7 are further presented in figure 7.22.

Ménard (1962) related the modulus of horizontal subgrade reaction to the pressure expansion curve obtained from a pressuremeter test, thus:

$$K_h = 1.5 \text{ to } 2 E^*_{sp} \quad (= k_h D)$$

where $E^*_{sp} =$ the soil modulus obtained from a pressuremeter test, as defined by Ménard

i.e. $E^*_{sp} = \frac{\partial p^*}{\partial \psi}$ = the shear modulus of the soil

and $E_{sp} = (1 + v_s) \frac{\partial p^*}{\partial r}$ (Gibson and Anderson 1961)

= $2(1 + v_s) \frac{\partial p^*}{\partial r}$ in the undrained situation

= $2(1 + v_s) E_{sp}^*$
where \( \nu_s \) = Poisson's ratio for the soil
\( dp^* \) = the pressure required to expand the membrane of the pressuremeter against the borehole wall
\( dv^* \) = the volumetric strain associated with \( dp^* \).

For the undrained situation where \( \nu_s = 0.5 \), the in-situ Young's Modulus would then be given by 3.0 times the tangent of the pressure-expansion curve i.e. \( \rho_T \) in figure 7.23.

Ménard (1969) in an unpublished special report of the Centre d’Etudes Geotechniques, Paris, as reported by Baguelin and Jezequel (1972) developed curves similar to the p-y curves discussed subsequently, using his pressuremeter modulus \( E_{sp}^* \). This further correlation by Ménard yielded a relationship for the modulus of horizontal subgrade reaction \( K_h \) approximately 1.5 times greater than the earlier Ménard proposal of equation 7.19(a) i.e.:

\[
K_h = 2 \text{ to } 3 \frac{E_{sp}^*}{E_{sp}}
\]

Ménard’s construction is shown in figure 7.24. In figure 7.24, \( p_l^* \) is the limiting pressure while \( p_f^* \), as defined by Ménard, is the pressure associated with the onset of creep. (See also Appendix II). These values (\( p_f^* \) and \( p_l^* \)) are indicated on figure 7.23.

In determining the curve of figure 7.24, Ménard assumed that there was zero pressure on the face of the pile immediately prior to lateral loading. He further assumed that the soil response was represented by a constant stiffness up to an equivalent pile strain as that associated with the onset of creep \( (p_f^*) \) (point 'A' in figure 7.24). Beyond point 'A' i.e. unto the limiting pressure \( p_l^* \) (A to B in figure 7.24), the soil stiffness was half that of OA.

Baguelin, Jezequel and Shields (1978) have reported other correlations between \( k_h \) and the Ménard modulus. These yield values for \( k_h \) similar to those calculated using equations 7.19(a) and 7.19(c). Thus the value of \( k_h \) presently proposed by the Centre d’Etudes Ménard is:-

\[
\begin{align*}
(1) & \quad D > 0.6m \\
\frac{1}{k_h} &= \frac{2}{9} \frac{E_{sp}^*}{E_{sp}} \frac{D_0}{D} \left( \frac{D}{D_0} \times 2.65 \right)^a + \frac{a}{8} \frac{E_{sp}^*}{E_{sp}} \frac{D}{D_0} \\
\end{align*}
\]

where \( D_0 \) = reference diameter of 0.6m
\( a \) = a rheological factor depending on soil type and the ratio \( \rho = \frac{E_{sp}^*}{(p_f^* - p_0)} \)
= 1 for overconsolidated clays where \( \rho > 16 \)
= \( \frac{2}{3} \) for normally consolidated clays where \( 9 < \rho < 16 \)
= \( \frac{1}{2} \) for weathered and/or remoulded clay where \( 7 < \rho < 9 \).

Values of \( \rho \) for sands and silts are given in Baguelin et al (1973)

Equation 7.20(a) rearranges to:

\[
\begin{align*}
\rho = \frac{a}{8} \frac{E_{sp}^*}{E_{sp}} \frac{D}{D_0} \\
\end{align*}
\]

where \( [1.2 (4.429)^a] + (1.5) = 8 \)
FIG. 7.22 CORRELATIONS BETWEEN YOUNG'S MODULUS FOR SOIL ($E_s$), AND THE STANDARD PENETRATION TEST N VALUE

FIG. 7.23 TYPICAL STRESS-STRAIN CURVE FROM MÉNARD PRESSUREMETER TEST

FIG. 7.24 p-y CURVE OBTAINED FROM A MÉNARD PRESSUREMETER TEST (After Ménard, 1969)
(ii) $D < 0.6m$

$$\frac{1}{k_h} = \frac{D}{E_s} \frac{4(2.65)^2 + 3a}{18}$$

i.e.

$$k_h = \frac{18 E_s}{D[4(2.65)^2 + 3a]}$$

7.21(a)
7.21(b)

(b) (b) Laboratory Methods

(i) Constant Stiffness

On the basis of laboratory tests Terzaghi (1955) drew a relationship between the undrained cohesion of a soil and the modulus of horizontal subgrade reaction thus:

$$K_h = 67 C_u$$

7.22

McClelland and Focht (1956) from undrained triaxial tests drew the correlation that:

$$K_h = 11 E_{scu}$$

7.23

where $E_{scu} = E_s$ obtained from the undrained triaxial test.

Equations 7.22 and 7.23 thus yield the ratio:

$$\frac{E_s}{C_u} > 6$$

7.24

This value contrasts significantly with the values suggested by Poulos (1971 b) and those of Table 7.1.

(ii) Non-linear Stiffness

The concept of p-y curves and methods for their development were first introduced by Matlock (1970). Essentially these curves, as indicated in figure 7.25, represent the springs of figure 7.19 but allow for non-linear response from the soil. Thus the complete non-linear load deflection characteristics of the soil can be accommodated. The p-y curves, then, provide a varying coefficient of horizontal subgrade reaction.

Matlock developed his p-y curves by an empirical correlation with undrained triaxial tests on cohesive soils in the following manner.

The method described applies specifically to soft clays below the water surface. He suggested that the curve should have a cubic form and should pass through the origin, striking a plateau represented by the ultimate resistance of the soil, at some particular level of strain. In addition to the origin, the cubic is described by two points which are functions of $\varepsilon_{50}$, the strain corresponding to 50% of the maximum principal stress difference (i.e. 50% of the failure load of the undrained triaxial test) as indicated in figure 7.26(a).

In the absence of triaxial test data, typical values of $\varepsilon_{50}$ as given by Skempton (1951) are presented in Table 7.8.
FIG. 7.25 \( p-y \) CURVES AS DEVELOPED BY MATLOCK (1970)
### TABLE 7.8
TYPICAL VALUES OF $\varepsilon_{50}$ SUGGESTED BY SKEPTON (1951)

<table>
<thead>
<tr>
<th>Consistency of Clay</th>
<th>$\varepsilon_{50}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>0.020</td>
</tr>
<tr>
<td>Medium</td>
<td>0.010</td>
</tr>
<tr>
<td>Stiff</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The ultimate lateral soil resistance per unit length of shaft ($n_u$) is computed as a function of the critical depth for soil response ($L_c$) indicated in Figure 7.9.

$$L_c = \frac{6}{\gamma^2 + 0.5}$$  \hspace{1cm}  \text{(7.25)}

where $\gamma$ = the effective unit weight of the soil

$C_u$ = the undrained cohesive strength of the soil

Thus:

$$p_u = C_u N_p$$  \hspace{1cm}  \text{(7.26(a))}

where

$$N_p = 3 + 6 \frac{L}{L_c} \text{ for } 0 \leq L \leq L_c$$  \hspace{1cm}  \text{(7.26(b))}

and

$$N_p = 9 \text{ for } L \geq L_c$$

i.e. $p_u$ varies linearly from $9C_u$ at the critical depth to $3C_u$ at the surface as indicated in Figure 7.9.

The value of $p_u$ is obtained for each depth where a $p$-$y$ curve is required (i.e. at each designated spring location). Thus $p_u$ is computed from triaxial compression test data obtained from samples consolidated to the stress level equivalent to the undisturbed in situ stress associated with each spring location on the analytical model.

The relative displacement of the pile is then related to the undrained triaxial test at 50% of the ultimate strength by a factor of 5:

$$\left(\frac{y}{r}\right)_{50} = 5 \varepsilon_{50} \text{ at } \frac{n}{n_u} = 0.5$$  \hspace{1cm}  \text{(7.27)}

This defines point A on the $p$-$y$ curve of Figure 7.26(c).

The ratio $\frac{y}{r}$ is the equivalent pile strain defined in Figure 7.26(b).

The cubic so described is defined by the relationship:

$$\frac{n}{n_u} = 0.5 \left(\frac{y}{y_{50}}\right)^{1/3}$$  \hspace{1cm}  \text{(7.28)}

The final point on the $p$-$y$ curve construction, point B on Figure 7.26(c), is defined where:

$$p = p_u \text{ at } \left(\frac{y}{r}\right) = \left(\frac{y}{y_{50}}\right)_{50}$$  \hspace{1cm}  \text{(7.29(a))}
FIG. 7.26 MATLOCK'S P-y CURVE CONSTRUCTION FOR SOFT CLAYS BELOW THE WATER SURFACE
The resultant p-y curve is shown in figure 7.26(c).

Equation 7.27 is equivalent to the following relationship between the modulus of subgrade reaction at 50% ultimate load and the secant modulus at 50% of the ultimate strength from the triaxial test, for the case where \( L > L_c \):

\[
K_{50} = 1.8 E_{scu50}
\]

7.30

Similar techniques have been developed for stiff clays both above and below the water surface. For example procedures for developing p-y curves for stiff clays above the water surface are presented by Reese and Welch (1975) while Reese, Cox and Koop (1975) present procedures for stiff clays below the water surface.

Reese, Cox and Koop (1974) presented procedures for developing p-y curves for sand. In this technique the p-y relationship is defined by three straight lines and a curve as indicated in figure 7.27. The initial straight portion of the p-y curve represents "elastic" behaviour of the sand, while the horizontal portion represents the ultimate behaviour. The initial and final straight lines are joined by a curve and an intermediate straight line selected empirically to yield a shape consistent with p-y curves obtained experimentally. The slope of the initial portion of the p-y curve may be obtained from the values given in Tables 7.9 and 7.10. It is interesting to note that Reese et al obtained values of \( k_h \) that were 2.5 times the highest values for submerged sands reported by Terzaghi (1955). The parameters required for definition of the remaining parts of the curve are the pile width (or diameter) \( D \), \( 
\)

\[
\gamma
\]

The bulk density of the soil mass.

**TABLE 7.9**

**RECOMMENDED VALUES OF \( k_h \) FOR SUBMERGED SAND FOR INITIAL PART OF p-y CURVE CONSTRUCTION**

<table>
<thead>
<tr>
<th>Density</th>
<th>Loose</th>
<th>Medium</th>
<th>Dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>*( k_h (kN/m^3) )</td>
<td>5400</td>
<td>16300</td>
<td>34000</td>
</tr>
</tbody>
</table>

* \( k_h = p/y \)

** After Reese (1975)**

**TABLE 7.10**

**RECOMMENDED VALUES OF \( k_h \) FOR SANDS ABOVE THE WATER SURFACE FOR INITIAL PART OF p-y CURVE CONSTRUCTION**

<table>
<thead>
<tr>
<th>Density</th>
<th>Loose</th>
<th>Medium</th>
<th>Dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>*( k_h (kN/m^3) )</td>
<td>6300</td>
<td>24500</td>
<td>61000</td>
</tr>
</tbody>
</table>
FIG. 7.27 TYPICAL FAMILY OF n-y CURVES FOR SAND
(After Reese, Cox and Koon, 1974)

FIG. 7.28 SCHEMATIC REPRESENTATION OF THE DETERMINATION OF THE CRITICAL DEPTH \( z_c \) IN DEVELOPING n-y CURVES FOR SAND
The soil resistance is computed using the following equations:

1. **Ultimate Resistance Near Ground Surface**

   This equation was developed from an expression suggested by Reese (1962) for the ultimate passive force resisting the lateral movement of a rigid cylinder. The assumed passive wedge is shown in figure 8.3 (Chapter 8).

   \[ p_c^* = \frac{\gamma L K_a}{D} \tan (\beta - \alpha) \cos \alpha + \tan \beta (D + L \tan \beta \tan \alpha) \]

   \[ + K_a \frac{L \tan \beta (\tan \beta \sin \beta - \tan \alpha) - K_a D}{D} \]

   \[ 7.31 \]

   * denotes ultimate resistance near the ground surface.

2. **Ultimate Resistance Well Below Ground Surface**

   \[ p_c^{**} = K_a \gamma L (\tan \beta - 1) + K_o \gamma L \tan \alpha \tan \beta \]

   ** denotes ultimate resistance well below the ground surface.

   In equations 7.31 and 7.32

   \( \alpha = \beta/2 \)

   \( \beta = 45 + \alpha/2 \)

   \( K_a = \tan^2 (45 - \alpha/2) \)

   \( L = \) depth of wedge, and

   \( K_o \) is recommended by Reese et al as 0.4.

   This value of \( K_o \) is compatible with that obtained experimentally by the writer for sand deposited by pluviation in air (See Appendix 9).

   Values of \( p_c^* \) and \( p_c^{**} \) are calculated for a number of depths (\( z \)) respectively and the critical depth (\( z_c \)) as defined by the intersection of equations 7.31 and 7.32 and is located as indicated in figure 7.28. Above \( z_c \) equation 7.31 is subsequently used; below \( z_c \) use equation 7.32.

   The displacement, \( y \), (see figure 7.27) associated with the ultimate resistance of the soil is empirically established as

   \[ y_u = \frac{30}{80} \]

   \[ 7.33 \]

   That associated with the intersection of the curve and intermediate straight portion of the \( p-y \) curve is

   \[ y_m = \frac{D}{60} \]

   \[ 7.34 \]

   Reese et al found that the ultimate resistances calculated using equations 7.31 and 7.32 were in poor agreement with the values measured from two full size pile tests. The ultimate resistance values are consequently adjusted using an empirical factor derived from the observed values thus:

   \[ p_u = A p_c \]

   \[ 7.35 \]
\[ p_m = D p_c \]  

where \( p_c = n^*_c \) or \( n^*_c \) depending on the location of \( p-y \) curve being considered.

\( A \) or \( B \) are empirical adjustment factors for static loading given in figures 7.28 and 7.30.

The non-linear portion of the \( p-y \) curve is constructed by fitting equation 7.37 between points \( k \) and \( m \) shown on figure 7.27.

\[ p = cy^{1/n} \]  

Construction of the \( p-y \) curve, (figure 7.27), is as follows:-

(i) The initial straight line, (0-k in figure 7.27), is defined by:-

\[ p = k_h y \]  

where \( k_h \) is obtained from Tables 7.9 and 7.10 as appropriate. This equation applies up to point \( k \) where \( y_k \) is defined by equation 7.40.

(ii) The curve, (k-m in figure 7.27), is defined by:-

\[ p = cy^{1/n} \]  

where

\[ n = \frac{p_m}{by_m} \]  

\[ b = \frac{p_u - p_m}{y_u - y_m} \]  

\[ c = \frac{p_m}{y_m} \]  

Equations 7.39(a), 7.39(b), and 7.39(c).

\( p_u, p_m, y_u \) and \( y_m \) are defined by equations 7.33 through 7.36.

Curves (i) and (ii), defined by equations 7.38 and 7.37 intersect at point \( k \) where:-

\[ y_k = (\frac{c}{k_h})^{n-1} \]  

Equation 7.40.

(iii) The straight line (m-u in figure 7.27) is defined by:-

\[ p = b(y-y_m) + p_m \]  

Equation 7.41.

The above analytical sequence is repeated for each spring location.

It should be noted that each of the procedures referred to for developing \( p-y \) curves have been based on a limited number of experimental studies using full sized, instrumented piles.

(a) Full Scale Determination of \( p-y \) Curves

As indicated in the preceding section, a fair measure of uncertainty exists in developing \( p-y \) curves from laboratory data or based on empirical data derived from full scale pile tests in another part of the world. In view of this Matlock (1970) suggested a technique whereby full scale pile tests were conducted and back analysed to determine the appropriate \( p-y \) curves.
FIG. 7.29 VALUES OF THE NON-DIMENSIONAL COEFFICIENT, A, FOR NORMALISED PILE LENGTH
(After Reese, Cox and Koop, 1974).

FIG. 7.30 VALUES OF THE NON-DIMENSIONAL COEFFICIENT, B, FOR NORMALISED PILE LENGTH
(After Reese, Cox and Koop, 1974).
While the results so obtained would not be general, they would at least apply to similar piles within the immediate proximity of the test pile and subject to similar loading conditions and methods of pile installation. Specially instrumented piles are required to obtain the necessary data from a full scale test to enable accurate profiles of displacement and stress on the pile to be determined.

Full scale tests with any degree of instrumentation are extremely expensive, nevertheless resorting to such measures provides probably the only way to gain a reliable assessment of pile-soil response, at least until our analytical and mechanical understanding has developed further.

The method of back analysis is shown diagrammatically in figure 7.31. Such a technique was employed in assessing pile response on the Mangere Bridge site as reported by Priestley (1974 and 1977). Electrical resistance strain gauges were used to measure strains in the pile wall under both axial and lateral loading. The strains measured under lateral load were then used to calculate the distribution of moment along the pile. The coefficients of horizontal subgrade pressure so obtained are shown in figure 7.32.

Figure 7.33 shows the p-y curve associated with point A in figure 7.32. The value of $k_h$ represented by point A was obtained from the initial tangent to the p-y curve, i.e. 0-a in figure 7.33.

Determination of p-y curves from full scale pile tests overcomes the difficulties in determining the effects of pile-soil interaction and the other uncertainties such as scale effects associated with the other methods of determining the modulus of horizontal subgrade reaction. However, to have to resort to such measures clearly indicates that the mechanism of the behaviour of the pile-soil system when subject to lateral loads is not well understood.

7.4.5 The Equivalent Cantilever Method

Various authors have presented analytical solutions to the laterally loaded pile problem. One such solution is that of Kocsis (1968). In this solution, moments and deflections are expressed in terms of a depth to fixity. This depth is a function of both the pile and soil stiffnesses and is similar to the relative stiffness factor, $k_0$, obtained by Timoshenko (1934) (equation 7.15).

The depth to fixity is defined by Kocsis as the depth to the point on the pile where the lateral movement is zero irrespective of the magnitude of the load.

Kocsis's solution has been applied by Hughes, Goldsmith and Fendall (1978a) to determine equivalent cantilever lengths to enable simplified pile analysis for design purposes, as discussed earlier.

The method presented by Kocsis uses the modulus of subgrade reaction method, and assumes the soil remains elastic throughout the loading process (i.e. the force displacement relationship for each spring is linear). Further, $k_h$ is assumed to be constant with depth for clays and to vary linearly with depth for sands.
FIG. 7.31 DIAGRAMMATIC REPRESENTATION OF THE DETERMINATION OF p-y CURVES BY BACK ANALYSIS OF FULL SCALE PILE TESTS
FIG. 7.32 COEFFICIENT OF LATERAL SURGRAGE REACTION MEASURED AT MANGERE BRIDGE (N.Z.) (After Priestley, 1974).

FIG. 7.33 p-y CURVE ASSOCIATED WITH POINT A IN FIGURE 7.32.
The general arrangement proposed by Kocsis is indicated in figure 7.34. For the situation where the lateral load is applied above the ground line, the results obtained from the arrangements of figures 7.34(a) and (b) can, because of elasticity, be added algebraically as indicated in figure 7.34(c).

The equivalent cantilever is shown in figure 7.35. A separate equivalent cantilever length is chosen depending on whether deflection or moment is being modelled.

In figure 7.35:

\( L_{E M} \) = the equivalent cantilever length that will yield the same maximum moment as the actual loading arrangement.

\( L_{EA} \) = the equivalent cantilever length that will yield the same load line deflection as the actual loading arrangement.

The application of the technique is presented in detail in Hughes, Goldsmith and Fendall (1978a) and presupposes the pile is embedded a depth greater than the appropriate depths to fixity assessed in the analysis. Analytical expressions for these depths are given in Kocsis (1968) and Hughes, Goldsmith and Fendall (1978a).

To obtain a range of values, this technique has been applied to loose and dense sands and to clays ranging from stiff to very soft. Both timber and concrete piles have been considered, producing a range of pile stiffnesses from flexible to very stiff. The load line has been ranged from the ground line to an elevation of 60 above the ground line.

The results obtained are indicated in figures 7.36 and 7.37.

In these figures the line O-A is a pivot line. The diagram is entered at a particular load level, i.e. a-b in figure 7.36. The equivalent cantilever length is then obtained by reading down from point b, which, for the example, yields the range of equivalent cantilever lengths shown by c-d in figure 7.36.

The equivalent cantilever length for moment varies a relatively small amount for large variations in both soil and pile stiffness. Thus for a particular pile diameter and line of action of the load relative to the ground line, the probable range of values for the maximum moment can be readily determined with no knowledge of the stiffness characteristics of the pile or the soil.

In comparison, the equivalent cantilever length for moment for a given soil stiffness and load line varies less than 10% across the range flexible to very stiff piles. (See Hughes, Goldsmith and Fendall 1978a)

The equivalent cantilever length for load line displacement \( (L_{EA}) \), expressed relative to the ground line \( (L_{0} \) in figure 7.12) varied from 3 to 9.50 for the range of soil and pile soil stiffnesses and load line eccentricities considered. Similarly for moment, \( L_{M} \) varied from 0.2 to 20. These ranges have been stated earlier in section 7.4(2).

7.5 Conclusions

As indicated earlier, most existing methods of lateral load analysis of piles, apart from those employing full scale pile tests, or empirical data derived from full scale tests, are generally based on "undisturbed" soil parameters. The process of consolidating laboratory triaxial samples to in-situ
FIG. 7.34 NOMENCLATURE USED BY KOCSIS (1968).

FIG. 7.35 THE EQUIVALENT CANTILEVER.
stress levels representative of the appropriate depth may be reasonable for bored piles in that it would enable the stress path followed by the soil during pile loading to be simulated, providing of course any soil modification or stress relief associated with the boring process could be approximated. However, such a technique does little to model the structural changes in the soil mass due to the process of pile installation by driving.

The method of pile installation can cause significant changes in the soil conditions existing about the pile, particularly in the case of driven piles in granular materials. Various authors have recognised that these changes occur, for example, Rohinsky and Morrison (1964), Vesic (1970) and Meyerhof (1969). However, as indicated in Part 1 of this thesis, all such considerations have been related to the problem of the resistance of piles to axial load. The soil deformation process throughout the installation of the pile and the subsequent lateral loading appears to have received little previous attention.

Roscoe (1970) indicated that once the deformation process in the soil, is understood, under any loading situation, only then are we in a position to make rational assessments as to the mechanics of soil structure interaction.

In the preceding Chapters the writer has shown that the process of pile installation by driving can cause enormous changes in the strain and thus the stress state existing in the soil about a pile.

No analytical techniques currently exist which enable these varying states of stress to be reliably predicted, and thus allow the initial conditions existing prior to lateral loading of a pile to be determined; irrespective of how the pile may have been installed. Related to these changes in stress state are also complex changes in soil density ranging from local loosening to local compaction.

The writer has shown that the actual density changes about driven piles bear little relationship to those generally assumed in the literature. As a result the currently available methods of determining the initial soil conditions from in-situ tests such as the penetration test (a generally universal tool as far as cohesionless soils are concerned), are questionable. This in itself has been indicated by the wide range of SPT 'N' correlations indicated in figure 7.22.

The state of the art review presented in this chapter has indicated that a number of relatively sophisticated analytical techniques for the assessment of the lateral load carrying capacity of piles exist. None of these analytical techniques appear to recognise the complex changes likely to occur in the soil mass, not only due to pile installation, but also during the actual process of resisting the lateral loads transmitted to the soil from the pile.

Thus, irrespective of what method of analysis a designer might choose to adopt, he still remains in the difficult position of not only selecting the "right" soil parameters but also attempting to make some allowance for the changes likely to occur in whatever parameters he has adopted, as a result of the influence of pile installation on these soil parameters; if in fact he recognises that changes will occur.
FIG. 7.36 EQUIVALENT CANTILEVER LENGTHS FOR DEFLECTION $\left( \frac{L}{D} \right)$
(After Hughes, Goldsmith and Fendall, 1978a)

FIG. 7.37 EQUIVALENT CANTILEVER LENGTHS FOR MOMENT $\left( \frac{L}{D} \right)$
(After Hughes, Goldsmith and Fendall, 1978a).
In all reasonableness, the comments made in Chapter 2 with regard to the axially loaded pile problem apply equally to the laterally loaded situation. The statement made by Vesic (1967) can, with little alteration, be reiterated in the context of the laterally loaded pile problem in sands in particular:

(1) In spite of significant progress made in recent years there is still not a satisfactory theory that could be recommended without reservation to the practising engineer for the analysis of laterally loaded piles.

(2) Such a theory must include at least the following parameters:-
   (a) foundation shape, relative depth and method of construction,
   (b) the shear strength of the sand,
   (c) the relative compressibility and volume change characteristics of the sand, and in addition

(3) Such a theory should enable the displacements developed in the soil in resisting the applied pile loads to be modelled.

Given the present state of the art then, the most reliable method of assigning lateral loads to piles, as in the case of axial loaded piles, unquestionably involves full scale load tests. Unfortunately, the costs involved are prohibitive for most projects.

In the short term, at least until forms of analysis which account for all the fundamental aspects of the soils problem evolve, the solution to the problem must reside in attempting to determine in situ soils parameters which reliably represent those that exist after pile installation. No such methods, including the variety of pressuremeters recently developed, presently exist, at least for the driven situation.

Traditionally the problems of axial and lateral loading of piles are, considered independent of each other, as are the static and dynamic modes of loading. Clearly the mechanics of soil response affecting one are only a variation on, and are dependent upon, those affecting the other.

Much of the content of this chapter has been presented previously in Hughes, Goldsmith and Fendall (1978a and b).
Chapter 8 Displacement Fields Developed About Model Driven Piles During Lateral Loading

8.1 INTRODUCTION

The closure of Part 1 of this thesis was a summary of the significant aspects of the effects of pile installation producing the conditions existing in the soil prior to subjecting a pile to lateral loading. It has been shown in Part 1 that not only are these conditions extremely complex, but they bear little relationship to those existing in the soil prior to their modification by the process of pile installation. In Part 2 of this thesis, the continued deformations of the soil mass about the "full" model piles discussed in Chapters 3 and 4 are followed through lateral loading, and the associated volumetric and shear strains determined.

In this Chapter the displacement fields developed about model piles in dense dry sand are identified. The first part of this chapter presents a qualitative comparison between the total displacement fields developed about laterally loaded "poles" (i.e. short rigid shafts) and "piles" (i.e. long flexible shafts). This qualitative study was in fact the preliminary work leading up to the research project described in this thesis. From this preliminary work the radiographic technique, described in Appendix 7, was developed. The latter part of this chapter uses this radiographic technique to determine the incremental fields of soil displacement at various stages of lateral loading.

8.2 A QUALITATIVE COMPARISON OF THE DISPLACEMENT FIELDS DEVELOPED ABOUT LATERALLY LOADED MODEL PILES

8.2.1 General

With particular reference to the laterally loaded pile problem, various authors and researchers have indicated that a fundamental understanding of the mechanics of pile-soil interaction is lacking. For example Poulos (1968, 1971) has stated that it is not clear how soils perform and that there is often not good agreement between predicted and measured pile displacement profiles.

Prakash (1960), as commented on by Kondner and Green (1962), indicated that the majority of analytical solutions applied to the lateral stability of rigid piles, up to that time (1960), were based on a number of assumptions such as pressure distribution diagrams of various shapes and which expressed soil resistance, (in terms of Rankines states of pressure), as various functions of lateral displacement and depth below ground level.

Matlock and Reese (1960) discussed the convenience of repeated applications of elastic theory to account for the non-linear response of soils when subjected to lateral pile loads. They commented that the uncertainty inherent in estimating soil behaviour from conventional soil tests, is probably consistent with the uncertainties associated with, for example, the choice of a simple soil modulus-depth function.
Whilst it is possible that the effects of such uncertainties may be self compensating, quite the reverse is equally probable.

In 1956, McClelland and Focht recognised that the two distinct elements necessary to reliably predict pile behaviour under lateral loads are an accurate determination of soil stress-strain characteristics, and the subsequent mathematical analysis based on the parameters so obtained.

Roscoe (1970) similarly commented that the soil mechanician should be as equally concerned with soil response at working loads as he is at failure conditions. Only then can he make predictions of soil-structure interaction. Before this can he done a detailed study of the stress-strain behaviour of soils must be made.

It is well recognised (for example, Krause 1973, Broms 1964, 1965, Kondner et al 1962, Reese et al 1970) that the accurate determination of the characteristics of the soil within a few pile diameters of the ground surface is fundamental to the behaviour of a pile as a whole; be it a long flexible pile or a short rigid one. Below a certain depth pile performance is practically independent of soil characteristics. Nevertheless, the extent of soil mobilisation in providing the lateral resistance to a displacing pile, for various loads, and soil and pile characteristics, does not appear to have been reliably quantified.

Many authors have made a number of different assumptions on which predictions have been based.

Figure 8.1 shows Rowe's (1956) assumption indicating the boundary of a zone of large grain movements and inferred slip plane directions during friction angle mobilisation of a pile in sand or gravel. It is of interest to compare this speculated boundary, based on work done on continuous retaining walls, with the boundaries actually observed and presented herein.

Figure 8.2 shows the mode of failure suggested by Broms (1964). Here the soil in front of the pile is assumed to move in an upwards direction whilst that at the rear moves downward, filling the void left by the displacing pile. At relatively large depths the soil located in front of the pile is considered to move laterally to the back side of the pile instead of upwards.

Two similar modes of failure were assumed by Reese et al (1970, 1974) for a pile displacing laterally through soil. These were firstly, that failure planes developed near the surface that allowed the movement of a wedge of soil, and secondly, that for greater depths soil resistance is limited by the flow of sand around the pile as the pile is displaced laterally.

Figure 8.3 shows the wedge that is assumed to develop near the surface. Reese suggests that for the case of a rigid cylinder moving through the soil, the rupture surfaces are in fact curves.

The writer believes that the state-of-the-art is well served by analytical techniques. As is the case in the axially loaded pile situation, the development of these has probably outstripped the ability to confidently obtain appropriate soil parameters which can in turn be directly correlated between model interaction studies, full scale studies to overcome unidentified scale factors, and simple in-situ soils testing procedures.
FIG. 8.1 SLIP PLANE DIRECTIONS DURING \( a - \text{MORILISATION} \)
(after Rowe, 1956)

FIG. 8.2 PILE DEFLECTIONS AND SOIL MOVEMENT ASSUMED BY BROMS (1964)

FIG. 8.3 ASSUMED PASSIVE WEDGE FORMED NEAR THE SURFACE
(after Reese, et al, 1970)
On the basis of these observations and the lack of understanding of the mechanics affecting pile installation, it is felt that a better understanding of the mechanics of pile-soil interaction in the laterally loaded pile situation is likewise essential. Techniques whereby a study of the physical mechanics of interaction could be made, other than various methods for obtaining pile displacement profiles, do not appear to have previously been applied to this field.

In Appendix 1 displacements obtained by subjecting a model pile to a lateral load are compared with qualitative results obtained from model tests, in which the movements of sand are viewed in the plane that contains the line of action of the displacing force. In this chapter the technique discussed in Appendix 1 is used to compare the mechanisms associated with lateral loading of piles and flexible piles in both loose and dense dry sands and saturated dense sand.

Andrews and Butterfield (1973) have indicated that the smallest sand displacements able to be measured using this technique is about 0.005 mm.

The long term aim of the fundamental approach initiated in this research project is to gain a better understanding of the mechanics of soil-pile interaction, initially with regard to displacements and in the long term, stress mobilisation within the soil. It is hoped that ultimately a correlation between simple in situ soil tests and observed behaviour might be obtained.

As discussed in Appendix 1 (figure A1.6), the half pile concept clearly does not significantly modify the pile displacement geometry, from which it is concluded that the half pile technique gives a reliable qualitative representation of the mechanics of pile-sand interaction occurring on the plane of symmetry.

For the results subsequently reported, at least two tests were carried out for each of the conditions considered.

8.2.2 Qualitative Comparisons of Various Loading and Driving Conditions

(1) Lateral Loaded Pole Action in Loose and Dense Dry Sand

A short rigid model half pile (i.e. a pole) was displaced laterally in both loose and dense dry sand. Figures 8.4(a) and (b) show the vertical and horizontal displacement contours and displacement vector fields observed for loose and dense dry sands respectively.

Double exposure photographs giving a visual verification of these displacement and vector fields have previously been presented in Hughes and Goldsmith (1976 and 1977).

In both figures 8.4(a) and (b), zones of large movement are clearly activated in the lee of the pile. These zones of large displacements appear to extend, in all cases examined, to the vicinity of the point of rotation of the pole. The displacement contour fields for both density states, whilst exhibiting the general similarities expected from identical loading conditions, display decidedly different detail, as indicated by the soil deformations in zones 1, 2, 3 and 4.
Vector scale: 5 15 25 (mm)
Pile scale: 5 15 25 (mm)

**FIG. 8.4** DISPLACEMENT AND VECTOR FIELDS FOR LATERAL LOADED POLES IN LOOSE AND DENSE DRY SANDS

(a) Loose Dry Sand

(b) Dense Dry Sand
It is expected that the deformational modes will in general be compaction of the loose sands and expansion of the dense.

The vector fields presented in figures 8.4(a)(1) and 8.4(h)(1) show that for the same pile displacement, the resulting magnitude of sand grain displacement at some distance from the pile, (for example point a), is larger in the dense sand than the loose. This is probably due to compaction occurring in the loose sands.

The line b-h separates the predominant upward and downward movements in each case. It is obvious that in the dense sand the upward movement extends to a greater depth below the surface than in the loose sand.

The amount of material mobilised in each case, represented by the area contained within the limits of observable displacements, is about the same. However, the lateral pole displacement in the dense sand generates the wider overall zone of influence which, measured at the surface level, extends to some 5 to 6 pile diameters (D) in advance of the pole compared to approximately 4 to 5D in the loose sand. Surprisingly the zone of large displacements is greater in the lee of the pole in dense sand.

The bulk of the loose sand is likely to be tending to compact. The vertically upward movement in zone 1 is probably due to the occurrence of both volume expansion and a tendency for the sand to accumulate as it rides up in advance of the pole.

In zone 1 of the dense case the larger movements are probably associated with greater dilation occurring. The downward motion in zone 2 is probably directly due to the tendency for the sand (in both the loose and dense cases) to enter the "void" left by the pole tip.

Clearly the expected deformational modes are evident here.

An interesting feature is that in the dense case there is a relatively uniform rotational movement of the sand with the pole tip from zone 2 to zone 4; the vertical upward movement not quite extending to surface level. Less regularity in rotational movement is displayed in the loose sand, probably due to greater compaction in advance of the pole tip.

As indicated by figure 8.4(a)(2), the loose sands show a tendency, above the point of rotation, to displace uniformly in the horizontal direction of the pole displacement.

The zone of horizontal movement in the direction of pole displacement (zone 1) is greater in the loose sand, whilst in the lee of the tip (zone 2) the zone is greatest in the dense sand. The horizontal movements in the lee of the pole (zone 4) are much greater in the loose sand than in the dense.

There appears to be a greater tendency for the loose sand to move downward in the face of the advancing pole, as compared to the upward movement in the dense case.
FIG. 8.5 DISPLACEMENT AND VECTOR FIELDS FOR LATERAL LOADED POLES IN DRY AND SATURATED DENSE SANDS
A vertical downward movement extends in the face of the advancing pile tip (zone 3) and probably indicates that connection is occurring. The downward vertical movement adjacent to the zone of large displacements is clearly due to the inability of the loose sand to stand freely.

(2) **Pole Action in Dense Dry and Dense Saturated Sands**

Figures 8.5(a) and (h) show displacement vector fields and contours of horizontal and vertical displacement for laterally displaced poles in both dense dry and dense saturated sands respectively. Photographs of these displacement fields can also be seen in Hughes and Goldsmith (1976). In the dry test the sand is fully able to dilate irrespective of the rate of loading, whilst in the saturated test, which was a "fast" test, the amount of dilation able to occur is probably inhibited by the negative pore water pressures generated.

Unfortunately the model piles display different tip configurations, though both piles, apart from tip configuration, are of similar dimension and have been subject to the same displacement measured at surface level. Though the general character of behaviour should be unaffected, the different tips must influence the detailed soil response, especially in their immediate vicinity.

The saturated case exhibits more frictional resistance to the applied displacements due to the negative pore water pressures generated, and thus requires a larger force to displace it.

The vector fields for the dry and saturated cases are shown respectively in figures 8.5(a)(1) and 8.5(h)(1). As previously indicated, zones of large displacement are activated in the lee of the pile in each case and appear to extend to the vicinity of the point of rotation of the pole. The zone of large displacement is much greater in the lee of the pile in the dense dry sand (approximately 1.5D compared to less than 1D, measured at surface level), though outside this large displacement region (zone 4) more soil is mobilised in the saturated than in the dry case. The observable displacements at surface level extend approximately 4 to 5D beyond the zone of large displacements (zone 4) in the saturated sand, however no such leeward displacements are indicated for the dense dry case.

Although the pole in the dry case is embedded about one pile diameter deeper than that in the saturated case, due to different tip configurations, the displacement vectors, especially in the first 2 to 3D below surface level (zone 1), are considerably larger in the dry case, (for example, point a). This probably indicates the influence of more water in inhibiting dilation in a saturated soil when subjected to rapid loading.

Further differences in the limits of the observable displacements occur in zone 1. These are that the pole in dense dry sand mobilises the surface layers of the soil to a depth of approximately 2.5D and extending to 5 to 6D in advance of the pole. In contrast the surface layers mobilised in the saturated sand extend to some 3.3D in depth and only about 4D in advance of the pile.
Two significant features with regard to the vertical displacement contours are apparent. Firstly the boundary between upward and downward vertical movement between zones 1 and 2, joins the pile at a depth of approximately 2.5 m in the dry case, but when saturated, the boundary is about 2 m below the surface level. Secondlv, the upward vertical displacement of sand on the lee side of the pole (zone 4) extends to the surface only when the sand is saturated.

(3) Comparisons between a Short Rigid Pile; A Pile of Medium Flexibility and a Long Flexible Pile

The displacement vector fields and contours of horizontal and vertical displacement for a short rigid pile, a pile of medium flexibility and a long flexible pile subjected to a lateral displacement in dense dry sand are shown respectively in figures 9.6(a), (b) and (c). Photographs of these displacement fields have been presented in Hughes and Goldsmith (1976 and 1977).

The piles in each case have been subjected to about the same physical displacement at the ground surface level. The pile has been displaced approximately 0.3 m while both the medium flexible and the flexible piles have been displaced approximately 0.8 m.

The absolute width of the mobilised material above zone 4 extends about the same distance from the undisplaced pile centre line in each case. In both the flexible and medium flexible cases the observable displacements extend approximately 90. Another significant feature is that in all cases the zones of large displacements in the lee of the piles are almost identical. For example the width of the zone at surface level is approximately 4.0 times the physical displacement for the pile and 3.0 for both the medium flexible and flexible piles. In all cases the zones of large displacements extend to about the level of the point of rotation of the piles. This represents a depth of about 5.5 m for the pile and about 9.5 m for the flexible and medium flexible piles.

In drawing a comparison between the medium flexible and flexible piles, it is of interest to observe that their point of rotation is at about the same depth for the same imposed lateral surface displacement, even though the medium flexible pile is of the order 3 to 4 times stiffer than the flexible pile and founded at a depth of approximately 15 m compared to 17 m for the flexible pile.

Figures 8.6(a)(1), 8.6(b)(1) and 8.6(c)(1) show the displacement vector fields for the three cases being considered. It is clearly shown by the line describing the limit of observable displacement that the shapes of the velocity vector fields for the medium flexible pile and the pile are similar, although the pole tip configuration differs from that of the medium flexible and flexible piles. It is unlikely that the influence of the tips affect the overall trends in the displacement fields which clearly show the transitional change in the manner of soil mobilisation from the pile through to the flexible pile. Tip translation of the medium flexible pile is barely occurring but is still large enough to generate a significant zone of upward displacements. However, the displacement field in zone 1 of the medium flexible pile is clearly tending towards that displayed by the flexible pile rather than that of the pile.
FIG. 8.6 DISPLACEMENT AND VECTOR FIELDS FOR LATERAL LOADED POLE, MEDIUM FLEXIBLE AND FLEXIBLE PILES IN DENSE DRY SAND
It is obvious that in the vicinity of zone 3 for both the pile and medium flexible pile (i.e. about their tips) the sand flow is generally in an upwards direction. However, it is significant that below the apparent point of rotation on the flexible pile, the sand flow is dominantly horizontal. Such an assumption was made by Brooms (1964).

A significant feature, particularly in zone 1, is that the magnitudes of the displacements for pile action are greater than those for the medium flexible pile, which are in turn, greater than those for the flexible pile. It is felt that this could be a reflection of the different volumes of soil involved. However, the mechanism is not clear.

Both horizontal and vertical displacements are shown in figures 8.6(a),(2), 8.6(b)(2) and 8.6(c)(2) for the pile, medium flexible and flexible piles respectively. The horizontal displacement contours describe similar shapes in all three cases with a slight difference being observed in the region of zone 4 for the medium flexible pile.

The boundary between the upward and downward vertical displacement contours, between zones 1 and 2, is at about a depth of 8.5h for both the medium flexible and flexible piles as compared to 2.5h for the pile. In zones 2 and 4 there are noticeable differences in the development of the vertical displacement contours. Comparisons between the deformations obtained about the three pile types considered in this section are made later in the chapter in table 8.1.

The following ultimate physical compatibility conditions are thus likely to develop for the various pile configurations considered:-

(i) Flexible Pile

For pile action to develop the pile stiffness must obviously be vastly greater than the ultimate stiffness of the soil. Ultimately the critical state is failure of the soil when subjected to the loads necessary to cause large body rotations of the pile, rather than structural failure of the pile itself. (A failure mechanism for piles in cohesionless soils is suggested in Chapter 9).

(ii) Flexible Pile

If a pile was of very high flexibility in comparison to the soil, then under lateral loading the pile would fail by yielding about a point close to the soil surface; the total response to the loading being observed mainly in the pile yield mechanism rather than mobilising a significant amount of soil resistance. As the pile stiffness increases more soil resists in mobilising resistance to the passage of the pile through the soil. The limiting case for a pile of uniform flexibility and strength is when the curvature of the pile exceeds the ultimate limit of the pile material. Thus only sufficient soil strength is mobilised as is necessary to permit the development of the ultimate curvature of the pile.
FIG. 8.7 FORM OF ACTUAL FAILURE SURFACE OF POLE CONSTRAINED TO ROTATE ABOUT GROUND LEVEL

FIG. 8.8 FORM OF MOLILISED ZONE AT THE SURFACE LEVEL
(after Rowe, 1956)
From this qualitative study it would appear then that the necessary founding depth for a long flexible pile is intimately related to the additional length necessary to develop end "fixity". For the model pile considered, though the pile was not tested to failure, this zone of soil in which "fixity" is developed, extends, under these loading conditions, to a depth of about 70 below the apparent point of rotation.

(iii) Medium Flexible Pile

Obviously an intermediate stage between pole and flexible pile action is inferred. In effect the action could be broken into two components, consisting of flexible pile behaviour superimposed on a rigid body rotation. It appears that the flexibility of the pile is mobilised to the point where the ability of the soil to provide end "fixity" is exceeded, whence the pole action dominates. Thus, for small lateral loads or displacements, the pile will progressively mobilise more soil strength as the tolerable curvature is developed within the pile.

If the end "fixity" conditions are exceeded then the soil response undergoes a transition to that which would exist under rigid pole action. Providing the pile strength was not exceeded, one would then expect the ultimate failure mode to be that associated with rigid pole rotations.

8.2.3 Discussion

Roscoe and Schofield (1956) conducted model pole tests in a clean dry sand. Their pole, of circular cross-section, was constrained to rotate about the ground surface.

The three dimensional surface of the passive zone of failure obtained was of the form indicated in figure 8.7.

Rowe (1956) indicated a similar bound to the passive failure surface, as manifested at ground level, which is indicated by figure 8.8.

It appears from Roscoe and Schofield (1956) that the failure mode represented in figure 8.7 is in fact a passive pressure zone in the lee of the pole as indicated by the mode of failure shown in figure 8.9.

Neither Roscoe et al or Rowe appear to have indicated the limits of the active failure surfaces generated by the soil flowing to occupy the "void" left by the displacing pole, as has clearly been seen to occur in the qualitative study presented.

However, both analysts recognise that some contribution must be made from this flowing material in that active forces are mobilised as is indicated by Roscoe and Schofield's two dimensional theoretical failure surface, shown in figure 8.10, where any is considered to be an active zone and ecd a passive zone.

Czerniak (1957) suggested that a pole displaced laterally in sand may not recover to its original position after the load was removed, due to the sand filling the void initially left by the displacing pile.
FIG. 8.9 FAILURE MODE FOR POLE CONSTRAINED TO ROTATE ABOUT POINT 'O'

FIG. 8.10 ROSCOE AND SCHOFIELD'S THEORETICAL FAILURE SURFACE
(after Roscoe and Schofield, 1956)
Reese et al. (1970, 1974), consider the contribution of an active force from the lee of a pile displacing laterally in sand, as shown by $F_A$ in figure 8.3(h).

From the qualitative study presented, sand flow in the lee of a pile clearly occurs and extends, without exception, to about the depth of the point of rotation, (or apparent point of rotation for a flexible pile).

The magnitudes of these displacements are quite large, thus sufficient strain to develop the active characteristics of the soil is clearly occurring. Figure 8.11 shows the displacement field observed in plan about the full model pile used to define the displacement profile shown in figure A1.6 (Appendix 1) and was obtained using the techniques discussed in Appendix 1.

From the surface manifestations of the sand which has undergone observable displacements both in front and in the lee of the pile, the volume of material mobilised in the active zone is clearly minor compared to that inferred by the passive zone in front of the pile. It thus appears that the contribution made by such an active zone to the equilibrium of the pile could, as suggested by Pross (1964), be neglected.

The writer feels that simplified flow fields can be rationally envisaged that portray the sand flow that actually occurs about a pile in the plane of the displacing force.

That this sand flow is a complicating factor in determining soil response, is well recognised. Various authors (Spillers and Stoll 1964, Reese, Cox and Kooi 1974, Parker and Reese 1970) have either suggested or resorted to the simplified approach of a wedge geometrically defined by compatible slip surfaces on which Coulomb yield criteria are satisfied, as indicated in figure 8.3.

Table 8.1 presents a correlation of the observed soil response between the various tests. Each of the criteria considered in this table are indicated by dimension in figure 8.12.

From table 8.1 it can be seen that in all tests the piles were subjected to about the same lateral displacement (i.e. 7 to 9 mm). A significant feature is that, for the poles, virtually all the parameters correlated appear to increase from the dense saturated, through the loose dry, to the dense dry condition. The exceptions are that the depths to the boundary separating predominantly upward and downward movements in zone 1 are about the same, and that the extent of soil mobilisation in zones 4 and 3 decrease from the dense saturated through to the dense dry case.

On the basis of these correlations between the various qualitative tests, the flow fields of figures 8.13 and 8.14 are suggested.

Figure 8.13 shows simplified comparative flow fields for poles of the same stiffness subjected to about the same lateral displacement at the ground surface level in saturated dense, loose dry and dense dry sands.

Figure 8.14 shows a simplified flow field for a flexible pile in dense dry sand. This flow field is clearly similar to that indicated by Pross (1964), in that, below the apparent point of rotation, sand flow about the pile is predominantly in a horizontal direction to the back of the pile.
DISPLACEMENTS (mm)
Direction of load: \[\rightarrow\]
Normal to load: \[\rightarrow\]
Vector scale: \[\begin{array}{c} 5 \\ 15 \end{array}\]
Pile scale: \[\begin{array}{c} 20 \\ 60 \end{array}\]

(a) Horizontal Displacement Contours
(b) Vector Displacement Field

FIG. 8.11 PLAN VIEW OF THE DISPLACEMENT FIELD ABOUT A LATERALLY LOADED FLEXIBLE PILE

FIG. 8.12 DEFINITION OF THE DIMENSIONS REFERRED TO IN TABLE 8.1.
FIG. 8.13 SIMPLIFIED FLOW FIELDS FOR POLES

FIG. 8.14 SIMPLIFIED FLOW FIELD FOR A FLEXIBLE PILE
<table>
<thead>
<tr>
<th>Sand State</th>
<th>Pole</th>
<th>Medium Flexible Pile</th>
<th>Flexible Pile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dense* Saturated</td>
<td>Loose Dry</td>
<td>Dense Dry</td>
</tr>
<tr>
<td>Extent of soil mobilisation from front face of undisplaced pile measured at surface level (above zone 1)</td>
<td>mm</td>
<td>90</td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>pile diameter A</td>
<td>4.26</td>
<td>5.66</td>
</tr>
<tr>
<td>Width of zone of large displacements at surface level</td>
<td>mm</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>pile diameter</td>
<td>0.94</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>physical displacement</td>
<td>2.22</td>
<td>3.29</td>
</tr>
<tr>
<td>Depth of point of rotation beneath initial surface level</td>
<td>mm</td>
<td>110</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>pile diameter C</td>
<td>5.23</td>
<td>5.52</td>
</tr>
<tr>
<td>Horizontal pile displacement measured at initial surface level</td>
<td>mm</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>pile diameter D</td>
<td>0.42</td>
<td>0.34</td>
</tr>
<tr>
<td>Depth to boundary separating predominantly upward and downward movements in zone 1</td>
<td>mm</td>
<td>50</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>pile diameter E</td>
<td>2.39</td>
<td>2.21</td>
</tr>
<tr>
<td>Extent of soil mobilisation from back face of undisturbed pile (zones 3 and 4)</td>
<td>mm</td>
<td>96*</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>pile diameter F</td>
<td>4.58</td>
<td>3.10</td>
</tr>
</tbody>
</table>

**NOTE:**
1. Width of all poles in the plane of the glass plate = 20 mm
2. Width of all piles in the plane of the glass plate = 9.5 mm
3. All piles were subjected to about the same physical displacement at surface level
   * Indicates a fast test
   + Indicates measured at surface level

8.2.4 Conclusion

In this section, qualitative comparisons have been drawn between displacement fields obtained from pole action in loose and dense dry sands, and saturated dense sand; and between the action of a pole, a medium flexible, and a flexible pile, in dense dry sand.

In all the tests conducted the piles were subjected to about the same physical displacement at the ground surface. On the basis of this qualitative study simplified flow fields have been suggested for both poles and flexible piles and a comparison made between the volume of soil mobilised in each case. Lateral loading in most design applications for piles in sands and gravels are probably dynamic loads such as those induced quickly by such forces as earthquake, wind or water. Thus the results of the tests done
Approximate surface level

(a) Trace of Superimposed Radiographs.

FIG. 8.15  VECTOR FIELDS OF DISPLACEMENTS OBSERVED FROM SUCCESSIVE EXPOSURES OF RADIOPHGS
Approximate surface level

General direction of movement

Pile scale:

5 15 25 35

Vector scale:

5 10 15 20

\[ \frac{H}{H_u} \text{ approximately 80%} \]

(b) Apparent Displacement Vector Field.
in saturated sands in which pore water pressures can develop would be appropriate, although conditions could well exist where either the pore water pressures are not a significant factor, or the lateral loads are applied slowly whence the loose or dense dry results would be more applicable.

The material presented in this section has previously been published; Hughes and Goldsmith (1976) and, in part, Hughes and Goldsmith (1977).

In reaffirming the need to have a better understanding of the mechanics of pile-soil interaction, particularly as related to the laterally loaded pile problem, De Beer (1977 a) and in his "State of the Art" report at the Ninth International Conference on Soil Mechanics and Foundation Engineering (1977 b) commented favourably on the nature of this preliminary work, as reported by Hughes and Goldsmith (1977).

A significant feature observed from a comparison of the measured displacement fields in this section and the photographs of Hughes and Goldsmith (1976 and 1977), is, as was the case in the driven pile situation; that no definite shear failure planes have been observed in the deforming soil mass. This observation is in distinct contrast with the concepts of "failure planes" suggested by various authors as quoted in this chapter.

8.3 INCREMENTAL DISPLACEMENT FIELDS DEVELOPED ABOUT LATERALLY LOADED MODEL PILES

8.3.1 Introduction

The total displacement fields presented in section 8.2 are of the same nature as figure 3.1(b) (Chapter 3) in that they are not a true representation of the total deformation path that the soil particles have been subjected to. This is because the displacement fields were obtained from stereo pairs of photographs, taken at the end of pile installation and at the completion of lateral displacement respectively. As a consequence, any curvature occurring in the particle displacement path cannot be recorded. Accordingly it is necessary to apply the same technique adopted in Chapter 3 of viewing the soil particle trajectories at small increments of displacement.

Nevertheless, the total displacement fields obtained using radiography agree well with those obtained using stereo-photogrammetry; as can be seen by comparing figures 8.15 and 8.6. Figure 8.15 was obtained by superimposing traces of lead shot locations from radiographs taken prior to lateral loading and just before failure. The displacement field represents that developed in the soil at approximately 80% of the ultimate load.

The displacement fields presented in this section are thus incremental displacements observed between successive increments of lateral loading.

Because there is little difference between the incremental displacement fields obtained using the stereo-photogrammetric technique of Appendix 1 and those obtained using the radiographic technique of Appendix 7, only the latter displacement fields are presented. The general agreement between
the displacement fields obtained using both techniques was indicated in Chapter 3. Another reason for selecting these results was that they comprise the data subsequently used to calculate the strain fields about the laterally loaded model piles as discussed in Chapter 9. In addition they tend to be more detailed than the displacements obtained from the stereo-photographs in that they are constructed from displacements at specific points rather than a rationalisation of trends.

8.3.2 Displacement Fields Obtained Using Radiography

The incremental displacement fields obtained about model timber piles of 19.5 mm diameter driven into dense dry sand are presented in figure 8.16 to 8.20. Also shown on each of these figures is the apparent limit of the zone of soil modified by the process of pile installation. Clearly all the significant displacements, through the entire lateral loading process, and thus the resistance to lateral loading, occurs within the zone modified by driving.

Also indicated on figures 8.16 to 8.20 are the proportions of the ultimate load \( \frac{H}{H_u} \) at which the displacement field was obtained. The associated load displacement curve in terms of ground line displacements is shown in figure 8.21, on which the positions of the corresponding radiographs are also indicated.

Figure 8.22 shows this load displacement curve redefined in terms of logarithmic axes, as suggested by Slack and Walker (1970) and as previously discussed in Chapter 2 and Appendix 5. To recap, point A on figure 8.21 marks the point at which the rate of deformation significantly changes, (corresponding with A' in figure 8.21). It is suggested that for design purposes the load associated with point A should be defined as the "ultimate design load".

From figures 8.21 and 8.22 the displacement fields represented by the radiograph PLL05 lie beyond this point A.

Figures 8.23(a) to 8.27(a) show the fields of displacement vector direction and contours of equal vectorial displacement derived from the incremental displacement fields of figures 8.16 to 8.20. The associated vector fields of displacement are presented in figures 8.23(h) to 8.27(b).

Clearly the same general patterns as indicated in section 8.2 are established, even at very low loads; with the nature of the incremental displacement fields becoming more strongly defined as the lateral load on the pile increases. It should be noted that the displacement fields presented in Section 8.2 were obtained using the equipment described in Appendix 1, which was arranged to apply displacements to the head of the pile. The results presented in this section were obtained by applying loads at the head of the pile, (See Appendix 7), rather than displacements.

8.3.3 Discussion

An interesting feature of figure 8.23(h) is that under the initial application of load, the soil in front of the pile, (the soil mass primarily responsible for the resistance to the applied lateral pile loads), appears to initially compact as indicated by the downward direction of the soil displacement
**DISPLACEMENTS:**

(\text{mm})

- Horizontal: 1.0
- Vertical: -2.0

---

**FIG. 8.16** INCREMENTAL DISPLACEMENT FIELD FOR $\frac{H}{H_u}$ OF 0 TO 18%
Apparent limit of zone of soil modified by driving

Approximate surface level

Outline FPLL02

\[
L_R = 6.47 \text{ to } 7.34 \text{D}
\]

\[
\frac{H}{H_u} = 34\%
\]

FIG. 8.17 INCREMENTAL DISPLACEMENT FIELD FOR \( \Delta \left( \frac{H}{H_u} \right) \) OF 10 to 34%
DISPLACEMENTS:

$(m m)$

$D = 19.5 \text{mm}$

$D = 19.5 \text{mm}$

$L_R = 8.17 \text{ to } 8.30 D$

$\frac{L}{D} = 10$

$\frac{H}{H_u} = 50\%$

**FIG. 8.18** INCREMENTAL DISPLACEMENT FIELD FOR $\Delta \left( \frac{H}{H_u} \right)$ OF 34 to 50%
FIG. 8.19  INCREMENTAL DISPLACEMENT FIELD FOR $\frac{H}{H_u} \text{ of 50 to 65}%$
**Fig. 8.20** Incremental Displacement Field for $\Delta \left( \frac{H}{H_u} \right)$ of 65 to 80%
FIG. 8.21 LATERAL LOAD - GROUND LINE DISPLACEMENT CURVE FOR RADIOGRAPHS

FIG. 8.22 LOG LOAD vs LOG DISPLACEMENT