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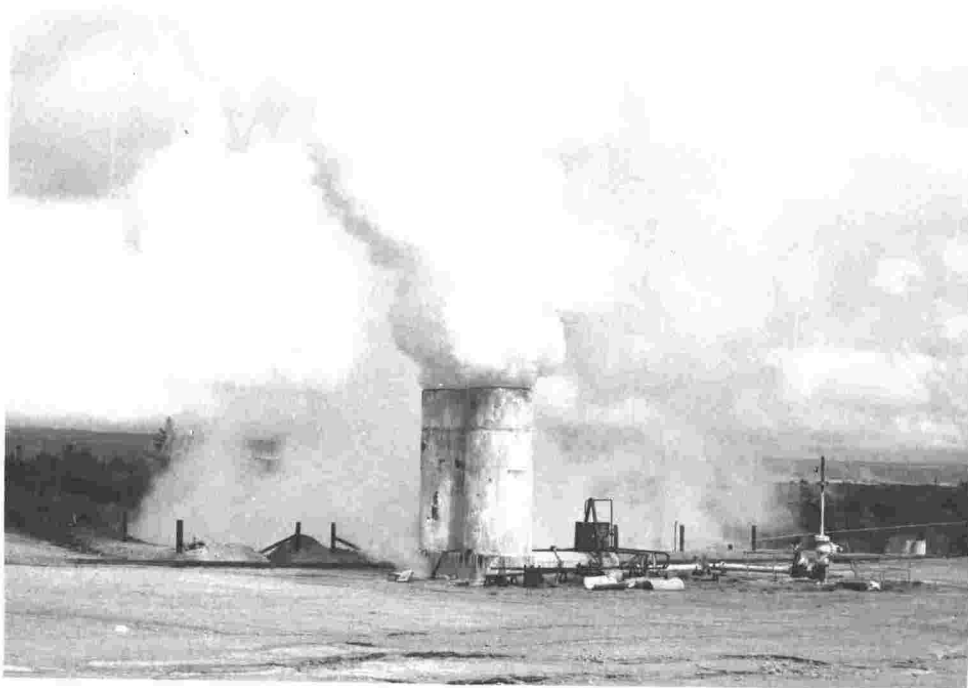
"EXPLOITATION OF GEOTHERMAL
RESERVOIRS"

A thesis submitted in partial fulfilment of
the requirements of the degree of Doctor of Philosophy
at the University of Auckland

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ABSTRACT

This work presents a numerical model for simulating the response of a geothermal reservoir to exploitation. The techniques developed are more efficient and in many ways superior to those of previous investigators. The model is capable of yielding a description of transient mass and heat flow in either a one- or two-dimensional reservoir defined by cartesian or cylindrical coordinates. The techniques enable simulation of a geothermal flow in all three thermodynamic states - compressed water, two-phase and superheated steam regions - and transitions between these states. The model is able to simulate a geothermal system where the presence of carbon dioxide as a second component influences exploitation response. Results are presented for a range of reservoir states. The effects of different physical parameters are considered. The usefulness of the model for looking at real systems is demonstrated by simulating the development of Wairakei and Broadlands geothermal areas in New Zealand.

ACKNOWLEDGEMENTS

I would especially like to thank Assoc. Prof. M.J. O'Sullivan for his supervision and advice throughout the course of study leading to the presentation of this thesis.

Also I am indebted to many helpful discussions with Dr G.A. Zyvoloski during his stay at the University of Auckland.

Thanks are due to my wife, Janet, for her understanding and encouragement during the later stages of this research.

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NOMENCLATURE

DIMENSIONAL VARIABLES

A	- recharge parameter [tL]
A	- area [L ²]
A _m	- total mass accumulation term [ML ⁻³]
A _c	- CO ₂ component accumulation term [ML ⁻³]
A _e	- energy accumulation term [ML ⁻¹ t ⁻²]
B*	- conductance of an interstice [L ²]
B	- average medium conductance [L ²]
c	- compressibility [L ⁻² t ²]
C	- specific heat [L ² t ⁻² T ⁻¹]
d	- reservoir depth [L]
d _v	- rate of vaporization [ML ⁻³ t ⁻¹]
D _m	- total mass transmissibility term [t]
D _c	- CO ₂ component mass transmissibility term [t]
D _e	- energy transmissibility term [L ² t ⁻¹]
D _{mg}	- total mass gravity term [ML ⁻³ t]
D _{cg}	- CO ₂ component gravity term [ML ⁻³ t]
D _{eg}	- energy gravity term [ML ⁻¹ t ⁻¹]
E	- specific internal energy [L ² t ⁻²]
f	- fugacity [ML ⁻¹ t ⁻²]
\bar{F}_m	- total mass flux term [ML ⁻² t ⁻¹]
\bar{F}_c	- CO ₂ component mass flux [ML ⁻² t ⁻¹]
\bar{F}_e	- energy flux term [Mt ⁻³]
F _m	- total mass residual [ML ⁻³ t ⁻¹]
F _e	- energy residual [mL ⁻¹ t ⁻³]
F _c	- carbon dioxide mass residual [ML ⁻³ t ⁻¹]

\vec{g}	- gravitational acceleration vector (g_x, g_y, g_z) [Lt^{-2}]
h	- specific enthalpy [L^2t^{-2}]
H	- rate of heat exchanged between rock and fluid [$ML^{-1}t^{-3}$]
H	- Henry's constant [$ML^{-1}t^{-2}$]
H_{soln}	- heat of solution [L^2t^{-2}]
\vec{J}	- conductive heat flux vector per unit cross-sectional area [ML^{-3}]
\vec{k}	- permeability tensor of the medium [L^2]
k	- isotropic permeability [L^2]
K_m	- thermal conductivity [$MLt^{-3}T^{-1}$]
l	- reservoir length [L]
m	- mass [M]
M	- molecular weight [M]
p	- pressure [$ML^{-1}t^{-2}$]
p^0	- vapour pressure of pure component [$ML^{-1}t^{-2}$]
\vec{P}	- viscous stress tensor [$ML^{-1}t^{-2}$]
P	- power rating of geothermal station [ML^2t^{-3}]
\vec{q}	- average flux velocity vector [Lt^{-1}]
Q_m	- total sink mass flow rate [$ML^{-3}t^{-1}$]
Q_e	- associated sink thermal energy flow rate [$ML^{-1}t^{-3}$]
Q_c	- sink mass flow rate of CO_2 component [$ML^{-3}t^{-1}$]
Q_{vap}	- rate of heat exchanged due to vaporization [$ML^{-1}t^{-3}$]
R	- recharge flow rate [Mt^{-1}]
R_o	- universal gas constant [$TL^{-2}t^2M^{-1}$]
S	- specific surface area [L^{-1}]
t	- time [t]
T	- temperature [T]
v	- molar volume [L^3M^{-1}]
V	- volume [L^3]
\vec{V}^*	- actual velocity vector in an interstice [Lt^{-1}]
\vec{V}	- average actual velocity vector [Lt^{-1}]
w	- reservoir width [L]
\vec{x}	- spatial coordinates (x,y,z) [L]
x,y	- areal coordinates [L]
z	- elevation coordinate (measured positive downwards) [L]

α	- solubility coefficient [$M^{-1}Lt^2$]
β	- formation compressibility [$M^{-1}Lt^2$]
Δr	- radial increment [L]
Δt	- timestep [t]
$\Delta x, \Delta y, \Delta z$	- spatial increments [L]
ϵ	- rate of energy dissipation [$ML^{-1}t^{-3}$]
λ	- thermal conductivity tensor [$MLt^{-3}T^{-1}$]
μ	- dynamic viscosity [$ML^{-1}t^{-1}$]
ρ	- density [ML^{-3}]
σ	- stress tensor [$ML^{-1}t^{-2}$]
∇	- vector differential operator [L^{-1}]

NON-DIMENSIONAL VARIABLES

a	- activity
B	- volume formation factor
C_0	- representative of mixture specific heat
d_0	- characteristic pore size
g	- magnitude of gravitational acceleration
k_0	- characteristic permeability
k_r	- relative permeability
L_0	- characteristic length of reservoir flow domain
n	- number of moles
N	- balance number
NX	- number of blocks in X coordinate direction
Q_m^0	- characteristic point sink mass flow rate (per unit volume)
Re^p	- Reynolds number for porous media
S	- saturation
S_e	- effective liquid saturation
S_{lr}	- irreducible liquid saturation
S_{vr}	- irreducible vapour saturation
\bar{T}	- average medium tortuosity tensor
T^0	- characteristic tortuosity of unconsolidated medium
T_0	- representative of temperature difference across reservoir domain
U_0	- representative of actual fluid velocity

- w - relative permeability weighting parameter
- x - mole fraction
- z - compressibility factor

- γ - activity coefficient
- δ - difference operator
- λ_o - characteristic thermal conductivity
- μ_o - characteristic fluid viscosity
- ξ_α - mass fraction of component in phase α
- ρ_o - characteristic fluid density
- ϕ - porosity
- ϕ_o - characteristic porosity
- ω - over-relaxation parameter

SUBSCRIPTS AND SUPERSCRIPTS

- av - average
- B - block
- c - carbon dioxide component
- cg - carbon dioxide in gas phase
- cl - carbon dioxide in liquid phase
- cp - capillary pressure
- C - Corey form
- e - energy
- e - exterior radius
- f - flowing
- f - fluid
- k - k^{th} iteration
- l - liquid
- L - linear form
- m - mixture
- m - total mass
- n - known time level
- n+1 - updated time level

- r - rock
- r - characteristic of recharge
- r - reduced variable
- r - coordinate r
- s - steam
- sat - saturation
- t - time
- v - vapour
- w - water
- ws - H_2O component
- x - coordinate x
- y - coordinate y
- z - coordinate z
- 1 - component 1
- 2 - component 2
- α - phase α

ERRATA

- p6 last two paragraphs change "subsidence" to "subsidence"
- p10 second paragraph change "charge" to "change"
- p10 last paragraph change "considreable" to "considerable"
- p21 equation (2.2.6) change " $\nabla \cdot p_f$ " to " ∇p_f "
- p24 second paragraph change "Dagon" to "Dagan"
- p35 second paragraph change "(2.4.13) or (2.4.14)" to "(2.4.14) or (2.4.15)"
- p38 third paragraph change "(2.5.1) and (2.5.2)" to "(2.6.1) and (2.6.2)"
- p42 fourth paragraph change "replacing p" to "replacing p_c "
- p78 section [B] change " ρ_s " to " ρ_s "
- p78 section [B] change " ρ_{cv} " to " ρ_{cv} "
- p81 equation (3.4.18) change "m" to "M"
- p91 equation (3.5.3) change " S_ℓ " to " S_e "
- p91 equation (3.5.4) change " S_ℓ " to " S_e "
- p92 equations (3.5.5) change " S_ℓ^4 " to " S_e^4 "
- p99 second paragraph change "descretising" to "discretising"
- p102 bottom paragraph change "following timestep" to "second half of timestep"
- p105 third paragraph change "equations (4.2.11)" to "equations (4.2.12)"
- p118 equations (4.4.3) change " $D_{mg_{nz} g_z}$ " to " $D_{mg_{nz+\frac{1}{2}} g_z}$ "
- p118 equations (4.4.3) change " $D_{eg_{nz} g_z}$ " to " $D_{eg_{nz+\frac{1}{2}} g_z}$ "
- p119 top line change " $D_{m_{nz}}$ " to " $D_{m_{nz+\frac{1}{2}}}$ "
- p119 second line change " $D_{m_{nz}} = \frac{1}{2} [D_{m_{nz}}(h(T^*, p^*), p^*) + D_{m_{nz}}(h_{nz}, p_{nz})]$ "
to " $D_{m_{nz+\frac{1}{2}}} = \frac{1}{2} [D_m(h(T^*, p^*), p^*) + D_m(h_{nz}, p_{nz})]$ "

- p120 equations (4.4.7) change " $\left[Q_{m,nz}^{n+\frac{1}{2}} - D_{m,nz+\frac{1}{2}}^{n+\frac{1}{2}} \left(\frac{p^* - p_{nz}^n}{(\Delta z)^2} \right) \right]$ "
to " $\left[Q_{m,nz}^{n+\frac{1}{2}} + D_{m,nz+\frac{1}{2}}^{n+\frac{1}{2}} g_z - D_{m,nz+\frac{1}{2}}^{n+\frac{1}{2}} \left(\frac{p^* - p_{nz}^n}{(\Delta z)^2} \right) \right]$ "
- p120 equations (4.4.7) change " $\left[Q_{e,nz}^{n+\frac{1}{2}} - D_{e,nz+\frac{1}{2}}^{n+\frac{1}{2}} \left(\frac{p^* - p_{nz}^n}{(\Delta z)^2} \right) - K_m \left(\frac{T^* - T_{nz}^n}{(\Delta z)^2} \right) \right]$ "
to " $\left[Q_{e,nz}^{n+\frac{1}{2}} + D_{e,nz+\frac{1}{2}}^{n+\frac{1}{2}} g_z - D_{e,nz+\frac{1}{2}}^{n+\frac{1}{2}} \left(\frac{p^* - p_{nz}^n}{(\Delta z)^2} \right) - K_m \left(\frac{T^* - T_{nz}^n}{(\Delta z)^2} \right) \right]$ "
- p124 fifth paragraph change "Figure 4.7.1" to "Figure 4.7.1 and Figure 4.7.2"
- p127 equations (4.7.3) change " $\frac{\partial \rho_s}{\partial p}(p)$ " to " $\frac{\partial \rho_s}{\partial T}(p)$ "
- p127 equations (4.7.3) change " $\frac{\partial E_s}{\partial p}(p)$ " to " $\frac{\partial E_s}{\partial T}(p)$ "
- p150 equations (4.13.1) change " $VQ_\alpha^{n+\frac{1}{2}}$ " to " $-VQ_\alpha^{n+\frac{1}{2}}$ "
- p199 chapter heading change " $H_2O:CO_2$ " to " H_2O "
- p215 top line block 3 change "25.124" to "35.124"
- p216 replace second paragraph with the following:

"The reservoir profiles of pressure, enthalpy, saturation and temperature at various times of production are presented in Table 6.4.3. The pressure and temperature profiles in the reservoir have slightly increased after 360 days. This is unrealistic and is a consequence of using zero-flux boundary conditions at the top and bottom of the reservoir. This boundary condition fixes the pressure and temperature gradients but does not actually fix the pressure and temperature. More physically realistic results would be obtained by prescribing a constant pressure-temperature boundary condition at the top and a recharge boundary condition at the base of the reservoir (refer Chapter 8). However, the results presented in this section using the more simple zero-flux boundary condition formulation adequately demonstrate the gross features of the response to production."

- p235 second paragraph change "parameters" to "geometry"
- p253 replace first sentence second paragraph with the following:
"The reverse trend was established in the producing block for very high saturation ($S_\ell > 0.90$) reservoirs."
- p253 replace third paragraph by the following:
"Note that in this case discharge is almost totally from the liquid phase. The presence of the dissolved carbon dioxide decreases compressibility of the liquid and this spreads pressure drawdown over the outer blocks."
- p269 omitted coordinate label in Figure 8.2.3 should be:
"Field Pressure, Bars" with range 35 to 70
- p276 replace fourth paragraph with the following:
"Profiles used as initial conditions for the one-dimensional Wairakei model are listed in Table 8.4.3. It is evident that the fluid is close to boiling near the top of the Waiora aquifer [as described by Bolton (1970)]. Compressed water

under hydrostatic pressures is at deeper levels. The profile of Table 8.4.3. does not represent a true natural state of the Wairakei field. The numerical model did not yield a steady cross-sectional profile for natural Wairakei conditions. However, averaging over 600 timestep profiles generated by the numerical model (Table 8.4.3 lists one timestep profile) described an average natural mass discharge of 550 kg/s and an average total heat flow of 456 MW at the surface. The latter was composed of 424 MW of convective heat flow and 32 MW of conductive heat flow. These compare well with the observed measurements."

p277 top of Table 8.4.3 change "437kg/s" to "550kg/s"

p277 replace label for Table 8.4.3 with the following:

"Profile of the Wairakei geothermal field used as initial conditions for the One-dimensional model. Averaging 600 similar profiles yields average heat and mass discharge at the surface of 456 MW and 550 kg/s. The values shown at each grid block are pressure (p) in bars, mixture enthalpy (h) in kJ/kg, water saturation (S_w) and temperature (T) in °C."

p281 last paragraph change "natural" to "initial"

p298 first paragraph change "form (9.3.1)" to "form (9.3.2)"

p298 last paragraph change "(9.3.1)" to "(9.3.2)"

p322 fourth bottom line change "The choice" to "With the choice"