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Digital Geometry: Introduction and Bibliography

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Abstract

Digital geometry deals with geometrical properties of "digital objects", which are usually taken to be sets of lattice points in the discrete space $\mathbf{Z}^n$. Such objects are often the result of applying a "digitization" process to objects in the Euclidean space $\mathbf{R}^n$. A central theme in digital geometry is how to characterize digital objects that could be the digitizations of "real" objects that have given geometric properties. The literature on digital geometry dates back to the late 1960's. The report includes a bibliography of more then 900 papers on the subject, organized by topic. It outlines the main lines of development of the field, and indicates areas in which interesting problems remain open.

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1 Introduction

Digital geometry [1]* is the study of geometrical properties of subsets of digital images. Such subsets, if the image has been properly segmented, often correspond to objects in the scene that gave rise to the image, and geometric properties are obviously important in describing these objects. If the objects are large relative to the resolution of the image, we may be able to regard the digital subsets as good approximations to the projections of the objects in the "real" image plane; if so, determining properties of the real object projections from the digital data can be regarded as a straightforward numerical process of approximation, and there is no need for a special "digital geometry" approach. In fact, however, objects are often small or thin, so that at least in some of their dimensions, they are comparable in size to the pixels (note that the pixels are typically only two or three orders of magnitude smaller than the entire image), and the approximations become very bad. It therefore becomes important to study how geometrical properties can be determined from the pixel subsets themselves.

The discrete nature of digital images causes difficulties when we attempt to define digital versions of standard geometric properties. We cannot simply represent the pixels as unit squares and the digital objects as unions of these squares, and define properties of the digital objects by using the corresponding conventional properties of the unions (on "digitization" schemes see Section 2); these properties may not adequately represent or approximate the properties of the real objects that gave rise to the digital objects. For example, suppose that the original real object is a disk, and that its digitization is the union of the unit squares (in a square tessellation of the plane) that the disk intersects. If the disk has radius r, this union of squares will be a polygon having on the order of r horizontal and vertical sides. Its area will be somewhat larger than that of the disk, and its perimeter will be significantly larger; for example, in Figure 1a, where $r = 5$, the perimeter of the polygon is 40, which is significantly larger than 10π. Note also that the digital "disk" is not convex, even though the disk was. As another example, suppose that the original real object is a circle (the boundary of the disk). Its digital version (Figure 1b) is a connected set, provided we regard corner-adjacent unit squares as being connected to one another; but if we do so, the complement of the digital circle is also a connected set, so that the digital circle violates the Jordan curve theorem.

In Sections 3–5 of this paper we discuss how to define various basic geometric properties of digital objects, including connectedness, convexity, area, diameter, perimeter, etc. We also consider three general questions about digital-geometric properties:

a) Property complexity: Measuring the properties' computational complexity (in particular, determining whether they can be computed using simple operations on the image, e.g. local operations).

b) Property preservation: Characterizing operations on the image that do not affect the values of the properties.

*These references are to sections of the bibliography.

(a) (b)

Figure 1: A disk of radius 5 on a grid of unit squares.

(a) Union of the squares intersected by the disk;
(b) union of the squares intersected by the circle.

c) Property recovery: Characterizing digital objects that could be the digitizations of real objects that have various conventional geometric properties.

In Section 6 we discuss the extension of digital geometry to three- (or higher-) dimensional digital images, e.g. obtained by segmenting 3D medical images. In Section 7 we briefly discuss other types of digital images, based on non-square tessellations (e.g., hexagonal) or on general types of discrete structures (e.g., graphs). Finally, in Section 8 we discuss how digital geometry can be extended to fuzzy subsets of digital images (e.g., to gray-level images in which the gray levels are regarded as representing membership values).

This paper also contains a bibliography of over 800 references on digital geometry. Sets of these references are cited in the appropriate sections of the paper. The references were collected by examining the annual bibliographies on image analysis and computer vision that the author has published since 1969. These bibliographies cover only a limited set of journals and conference proceedings; thus many relevant references have not been included.

2 Digitization [2]

A bounded subset S of the plane can be defined by its "characteristic function" χ_s—the function that has value 1 at points of S and 0 elsewhere. Since S is bounded, χ_s is zero outside some sufficiently large (say) square region R. We will assume here that we can use the same R for all the subsets S that we are interested in, and we choose R somewhat larger than necessary, so that a unit-width border of R never intersects S.

We can define a *digitization* $\hat{S}$ of S by subdividing R into (say) n by n unit squares, and letting $\hat{S}$ be the union of all the unit squares that intersect S. The characteristic function of $\hat{S}$ can be represented by an n-by-n array of 0's and 1's, where an element of the array is 1 iff the corresponding unit square intersects $\hat{S}$. Such an array is called a *digital image*; its elements are called *pixels*. The 1's of the array are called elements of $\hat{S}$, and the 0's are called elements of the complement $\bar{\hat{S}}$ of $\hat{S}$. If we regard the unit squares as closed, the digitization $\hat{P}$ of a single point P can consist of one, two, or four squares, depending on whether P is in the interior of a square, is on the common edge of two adjacent squares on the some row or column, or is the common vertex of a 2-by-2 block of squares. It is preferable to regard the squares as half open, so that each square contains (say) its north and east edges but not its south and west edges, and not its northwest and southwest vertices. When we do this, the squares constitute a partition of R (itself now regarded as a half-open square), and the digitization of a single point is always a single unit square.

3 Topology [3.a]

A pixel $\hat{P}$ in a digital image has two kinds of *neighbors*—four neighbors that are horizontally or vertically adjacent to $\hat{P}$ in the array, corresponding to the four unit squares that share edges with $\hat{P}$'s unit square, and four neighbors that are diagonally adjacent to $\hat{P}$. We will refer to the first four neighbors as the "4-neighbors" of $\hat{P}$ (or as being "4-adjacent" to $\hat{P}$), and to the set of all eight neighbors as $\hat{P}$'s "8-neighbors" ("8-adjacent" to $\hat{P}$). The union of $\hat{P}$ and its (4-,8-) neighbors is called its (4-,8-) *neighborhood*. A set $\hat{S}$ and a set $\hat{T}$ are called (4-,8-) adjacent if some pixel of $\hat{S}$ is (4-,8-) adjacent to some pixel of $\hat{T}$.

The (4-,8-) *border* of $\hat{S}$ is the set of pixels of $\hat{S}$ that have (4-,8-) neighbors in the complement $\bar{\hat{S}}$ of $\hat{S}$. The (4-,8-) *interior* of $\hat{S}$ consists of all the pixels of $\hat{S}$ that are not on its (4-,8-) border; hence all the (4-,8-) neighbors of a (4-,8-) interior pixel of $\hat{S}$ are in $\hat{S}$. A pixel of $\hat{S}$ is called (4-,8-) *isolated* if none of its (4-,8-) neighbors are in $\hat{S}$.

The border pixels of $\hat{S}$ can be identified by examining the entire image and finding pixels of $\hat{S}$ that have neighbors in $\bar{\hat{S}}$; but there are also algorithms that find border pixels by systematically "following" a border, starting from any one of its pixels. The number of steps needed to visit all the pixels of a border in this way is called the *perimeter* of the border. For other references on borders, and on the derivation of geometric properties of a region from its borders, see [3.b].

A sequence of pixels $\hat{P}_0 \cdots, \hat{P}_m$ is called a (4-,8-) *path* if $\hat{P}_i$ is a (4-,8-) neighbor of $\hat{P}_{i-1}$, $1 \leq i \leq m$. Two pixels $\hat{P}, \hat{Q}$ of $\hat{S}$ are called (4-,8-) *connected* in $\hat{S}$ if there exists a (4-,8-) path $\hat{P} = \hat{P}_0, \hat{P}_1, \cdots, \hat{P}_m = \hat{Q}$ such that all the $\hat{P}_i$'s are in $\hat{S}$. The maximal (4-,8-) connected subsets of $\hat{S}$ are called its (4-,8-) *components*. $\hat{S}$ is called (4-,8-) connected if it has only one (4-,8-) component.

The set $\hat{S}$ (4-,8-) *surrounds* the set $\hat{T}$ if any (4-,8-) path from a pixel of $\hat{T}$ to a pixel of the image border must intersect $\hat{S}$.

So far, the 4- and 8- definitions have played symmetrical roles; but in some respects, they are not interchangeable, and for some purposes it is desirable to use both of them, as we see from the following representative theorems (which will not be proved here):

Theorem 1 *$\hat{S}$ is the digitization of an arcwise connected set S if and only if $\hat{S}$ is 8-connected.*

Theorem 2 *If a 4-component of $\hat{S}$ and a 4-component of $\bar{\hat{S}}$ are 8-adjacent, they are also 4-adjacent.*

Theorem 3 *If a (4-,8-) component of $\hat{S}$ and an (8-,4-) component of $\bar{\hat{S}}$ are 8-adjacent, either the 4-component 8-surrounds the 8-component or the 8-component 4-surrounds the 4-component.*

Theorem 4 *Let C be any (4-,8-) component of $\hat{S}$. Then there is a unique (8-,4-) component of $\bar{\hat{S}}$ (4 or 8) adjacent to C that (4-,8-) surrounds C; it is called the* background *of C. All other (8-,4-) components of $\bar{\hat{S}}$, if any, that are (4- or 8-) adjacent to C are (8-,4-) surrounded by C; they are called* holes *in C. The (4-,8-) components of $\hat{S}$ and the (8-,4-) components of $\bar{\hat{S}}$, under the relation "is the background of", form a rooted tree in which the root is the (8-,4-) component of $\bar{\hat{S}}$ that contains the image border.*

Theorem 5 *The number of (4-,8-) components of $\hat{S}$, minus the number of (8-,4-) components of $\bar{\hat{S}}$, can be locally computed by counting the numbers of certain types of* 2*-by-*2 *blocks of pixels in the image (the details are omitted here; see* [3.c]*).*

The number of (4-,8-) components of $\hat{S}$ or of $\bar{\hat{S}}$ cannot be locally computed. To count them, the standard approach is to "label" them [3.d], i.e. to assign labels to the pixels in such a way that two pixels get the same label iff they belong to the same component, and then count the number of distinct labels that have been used. (Once the components are labeled, various geometric properties of the components can be computed; see Section 4.) Another approach is to "shrink" the components to isolated pixels, and count and delete these pixels as they are created [3.e]. The shrinking process uses simple local image operations; except when isolated pixels are deleted, it preserves all the connectedness-related properties of the image. (The related problem of "filling" regions, given their boundaries, will not be reviewed here.)

A (4-,8-) connected set C is called a (4-,8-) *curve* if every pixel of C has exactly two (4-,8-) neighbors in C. A connected subset of a curve is called an *arc*; if an arc A has more than one pixel, exactly two of its pixels have just one neighbor in A (they are called the *endpoints* of A), while the others have two such neighbors. On digital arcs and curves see [3.c].

A pixel is called (4-,8-) *simple* if it is (4-,8-) adjacent to only one (4-,8-) component of pixels of its type (0 or 1), and to only one (8-,4-) component of pixels of the opposite type, in its 8-neighborhood. Evidently, changing the type of a simple pixel does not change the (4-,8-) connectedness properties of the pixels of its type nor the (8-,4-) connectedness properties of the pixels of the opposite type [3.e].

4 Distance and size

The (4-,8-) *distance* between two pixels $\hat{P}$ and $\hat{Q}$ is the length of the shortest (4-,8-) path between them. If we associate integer Cartesian coordinates with $\hat{P}$ and $\hat{Q}$, the 4-distance turns out to be the familiar "city block" distance (the sum of the absolute differences of the x and y coordinates of $\hat{P}$ and $\hat{Q}$), while the 8-distance turns out to be the "chessboard" distance (the max of these absolute differences). If $\hat{P}$ and $\hat{Q}$ are elements of a set $\hat{S}$, the $\hat{S}$-*intrinsic* (4-,8-) distance between P and Q is the length of the shortest (4-,8-) path between them that lies in $\hat{S}$. The pixels at (4-,8-) distance 1 from $\hat{P}$ are just the (4-,8-) neighbors of $\hat{P}$. The (4-,8-) *disk* of radius r centered at $\hat{P}$ is a (diagonally oriented, upright) square of side length $(r\sqrt{2}, 2r+1)$.

If we label each pixel of $\hat{S}$ with its (4-,8-) distance to the nearest pixel of $\bar{\hat{S}}$, the resulting array of integer labels is called the (4-,8-) *distance transform* of $\hat{S}$. Distance transforms can be computed by repeatedly performing local operations. On digital distances and distance transforms see [4.a].

The 4-distance between two pixels is always at least as great as their Euclidean distance, which in turn is at least as great as their 8-distance. The Euclidean distance transform of an image cannot be exactly computed by repeatedly performing local operations, but good approximations to it can be computed [4.b].

The set of (nonstrict) local maxima of the (4-,8-) distance transform of $\hat{S}$ is called the (4-,8-) *medial axis* of $\hat{S}$. Evidently, $\hat{S}$ is the union of the (4-,8-) disks centered at the pixels of its medial axis and having radii equal to its (4-,8-) distance transform values at these pixels. This makes it possible to compute useful properties of $\hat{S}$ from its medial axis. On medial axes and their uses see [4.c].

The (4-,8-) (intrinsic) *diameter* of a set $\hat{S}$ is the greatest (4-,8-) (intrinsic) distance between any pair of pixels of $\hat{S}$. For other work on digital diameter, width, perimeter, etc. see [4.d]; for other work on paths, distances, etc. see [4.e].

5 Convexity and elongatedness

We recall that a (real) set S is called *convex* if, for any two points P, Q in S, the straight line segment $\bar{P}\bar{Q}$ is contained in S. It is not difficult to characterize digital sets $\hat{S}$ that could be the digitizations of convex sets [5.a], and digital (4-,8-) arcs A that could be the digitizations of straight line segments [5.b]. Other types of digital arcs are discussed in [5.c].

The *convex hull* $\langle S\rangle$ of a set S is the smallest convex set that contains S; the *concavities* of S can be defined as the connected components of the difference set $\langle S\rangle - S$. These concepts are also applicable to digital objects [5.d].

(Intrinsic) elongatedness is not a standard geometrical concept; it is not obvious how to formulate the fact that a coiled rope is an elongated object. However, images often contain objects that are (almost) everywhere elongated (blood vessels or drainage networks, alphanumeric characters, etc.), and it is desirable to be able to recognize this. The medial axis can be used, in principle, to detect elongatedness; a large piece of medial axis whose pixels have small distance values must correspond to an elongated object part. However, such axis pieces may not be easy to detect, since the axis is not necessarily connected and the piece is not necessarily arclike. There has been extensive work on "thinning" algorithms that are designed to reduce elongated object parts to centrally located arcs while preserving their connectedness [5.e.].

6 Three dimensions

Three-dimensional digital images, obtained by techniques such as computed tomography, are used extensively in analyzing three-dimensional objects such as the organs of the human body. A 3D digital image represents a bounded volume of space; the image is a three-dimensional array whose elements ("voxels") correspond to unit cubes which partition the volume. As in the two-dimensional case, we will assume here that the elements of the array take on only the values 0 and 1.

A unit cube has six "face neighbors" with which it shares a face; twelve "edge neighbors" with which it shares only an edge; and eight "vertex neighbors" with which it shares only a corner. As in the two-dimensional case, concepts of adjacency, connectedness, etc. can be defined using any of these types of neighbors; but in order to obtain simple algorithms involving these concepts, they must be used in the correct combinations.

The greater complexity of the neighborhood of a voxel makes it nontrivial to define such things as "simple" voxels, "thin" surfaces, "tunnels" (like the "hole" in a torus), and so on. Because of the growing importance of 3D images, the study of their geometry has given rise to considerable literature on 3D digital topological concepts [6.a] and on 3D borders (surfaces) [6.b]. Some work has also been done on 3D distance and size [6.c], convexity and straightness (or flatness) [6.d], and thinning [6.e].

7 Other grids [7]

In both 2D and 3D, we have assumed that the elements of the digital image correspond to the unit squares or cubes of a regular tessellation. There are advantages to using other tessellations; for example, in 2D, a pixel in a regular hexagonal tessellation has only one kind of neighbor (all neighbors are edge-adjacent), which significantly simplifies the study of topological properties. However, scanners are designed to sample an image at the points of a square grid; for this and other reasons, hexagonal (and triangular) grids are rarely used. Non-Cartesian 3D grids have also been proposed, but have not been used to any significant degree.

Generalizations of digital geometry can be formulated in which the "pixels" (or "voxels") are elements of an abstract discrete data structure, not directly related to 2D or 3D Euclidean space. The theoretical study of such structures is of interest because it provides unified treatments of large classes of models for digital images. However, it has had little impact on the development of practical algorithms.

8 Gray levels

Up to now we have assumed that our digital sets were "crisp": a pixel (or voxel) is either in the set or in its complement. A more general idea is to allow the pixels to have degrees of membership in the set; these can be defined, e.g., by real numbers in the interval [0,1]. A simple way to assign degrees of membership to the pixels in an image is to scale their gray levels to the range [0,1]; this amounts to treating lightness as membership, and may be appropriate if the image contains light objects on a dark background.

In ordinary geometry, fuzzy analogs of many standard geometrical properties can be defined. For example, a fuzzy set S is called *fuzzily connected* if, for any two points P, Q, there exists an arc with P, Q as endpoints such that all points on the arc have memberships in S at least as great as the lesser of the memberships of P or Q; and S is called *fuzzily convex* if, for any two points P, Q, all points on the line segment $\bar{P}\bar{Q}$ have memberships in S at least as great as the lesser of the memberships of P and Q. Such definitions can also be formulated for fuzzy subsets of digital images. Most of the work on fuzzy or gray-weighted digital geometry has involved medial axes and thinning [8a], but there also has been some work on connectedness, distance, and other topics [8b].

9 Concluding Remarks

Digital geometry is quite different from computational geometry; the latter deals with finite sets of geometrical objects such as points and lines, but these objects are assumed to exist in real Euclidean space, and computations of arbitrary precision are allowed. As indicated in Section 1, digital geometry also has little or nothing in common with the use of discrete grids in numerical approximation (e.g., in finite-element methods). On the other hand, some branches of discrete geometry, such as the geometry of lattice points (points with integer coordinates), are closer in spirit to digital geometry (the lattice points can be regarded as the centers of pixels); some of the methods and results of Minkowski's "geometry of numbers" are quite relevant to digital geometry. Continued study of discrete geometric concepts should lead, over the coming years, to a deeper understanding of the geometric properties of digital objects.

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