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A distributed-mass model with end-compliance effects for simulation of building pounding

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Abstract

Pounding damage in buildings has been observed after almost every urban earthquake. Pounding occurs when the out-of-phase vibrations of buildings causes them to close the at-rest separation and collide together. The relative motion can be caused by the differing dynamic properties of the structures, foundation flexibility effects or the spatial variation of ground motion. The principal objective of this doctoral research is to develop a numerical force model suitable for simulation of building pounding. The model has to be reasonably accurate in predicting both displacement and acceleration responses due to pounding.

The first part of the study consists of the experimental and numerical evaluation of numerical pounding models available in the literature. Shake table investigation of pounding between steel portal frames and impact tests between reinforced concrete slabs were carried out, along with numerical simulations. It is shown that the existing numerical models have many limitations which make them unsuitable for general purpose application in pounding simulation. Therefore, a damped Sears impact model is developed and verified in the second part.

The proposed model is based on the Sears impact model. It was observed that the original model cannot include the effects of (i) damping and attenuation of impact-induced stress waves, (ii) higher longitudinal modes of vibration of floors and (iii) the storey-stiffness of the buildings. The damped Sears model overcomes these limitations by analysing the stress propagation as superposition of viscously damped longitudinal modes of vibration of the floors. The storey-stiffness is incorporated by substituting the corresponding floor displacement of a lumped-mass model of the building in place of the translation mode of freely moving bars.

For validation of the model, the results from numerical simulation were compared with the experiment response from pounding between suspended RC slabs and between steel beams in three configurations: (i) impact between pendulums, (ii) impact between one-storey frames, and (iii) impact between a one-storey frame and a two-storey frame. Other existing models failed to predict both impact-induced acceleration and the transfer of momentum between colliding masses with any consistency. In contrast, the damped Sears model gave reasonable simulation of these quantities.
Dedications

To my Parents, Sisters and Wife

With Love and Gratitude
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Symbols and notations

\( A \) \hspace{1em} \text{Cross-sectional area}

\( c \) \hspace{1em} \text{Constant or instantaneous damping of an impact element}

\( C_i \) \hspace{1em} \text{Damping of } i^{\text{th}} \text{ structure}

\( c_v \) \hspace{1em} \text{One-dimensional wave propagation velocity of material}

\( c_{v,i} \) \hspace{1em} \text{One-dimensional wave propagation velocity of material of } i^{\text{th}} \text{ diaphragm}

\( E \) \hspace{1em} \text{Young's modulus of elasticity}

\( e \) \hspace{1em} \text{Coefficient of restitution}

\( \text{EBP} \) \hspace{1em} \text{End building pounding}

\( \text{ELC} \) \hspace{1em} \text{El Centro ground motion}

\( F \) \hspace{1em} \text{Instantaneous pounding or impact force}

\( \text{gap} \) \hspace{1em} \text{At-rest separation between colliding bodies}

\( \text{Hd} \) \hspace{1em} \text{Corrected Hertzdam impact element}

\( i \) \hspace{1em} \text{Subscript to denote the mass, frame or building being considered}

\( i, j \) \hspace{1em} \text{Subscript to denote } j^{\text{th}} \text{ floor of } i^{\text{th}} \text{ building or frame}

\( \text{JDS} \) \hspace{1em} \text{Japanese design spectrum}

\( k \) \hspace{1em} \text{Linear or nonlinear stiffness of an impact element}

\( K_i \) \hspace{1em} \text{Stiffness of } i^{\text{th}} \text{ structure}

\( \text{LHC} \) \hspace{1em} \text{Linear Hunt-Crossley impact element}

\( L_i \) \hspace{1em} \text{Length of longer diaphragm, floor or bar}

\( \ln( ) \) \hspace{1em} \text{Natural logarithm}

\( L_s \) \hspace{1em} \text{Length of shorter diaphragm, floor or bar}
LVe  Linear viscoleastic impact element

$m$  Mass

$m_e$  Equivalent mass of the system

$m_i$  Mass of $i^{th}$ system

$m_{i,j}$  Mass of $j^{th}$ floor of $i^{th}$ building

MHd  Modified Hertz damp impact element

MLVe  Modified linear viscoelastic impact element

NE  Normalized error

NIVe  Nonlinear viscoelastic impact element

NZDS  New Zealand design spectrum

RBP  Row building pounding

$R_i$  Cross-sectional radius of $i^{th}$ bar

$r_i$  Radius of the rounded end of $i^{th}$ bar

TBP  Two building pounding

$T_i$  Fundamental period of $i^{th}$ diaphragm, frame or a building

SRSS  Square root of sum of the squares

$u$  Instantaneous displacement

$\dot{u}$  Instantaneous velocity

$\ddot{u}$  Instantaneous acceleration

$\ddot{u}_g$  Instantaneous ground acceleration

$u_i$  Instantaneous displacement of $i^{th}$ mass being considered

$\dot{u}_i$  Instantaneous velocity of $i^{th}$ mass being considered

$u_{i,j}$  Instantaneous displacement of $j^{th}$ floor of $i^{th}$ frame or building
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<td>$\dot{u}_{i,j}$</td>
<td>Instantaneous displacement of $j^{th}$ floor of $i^{th}$ frame or building</td>
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<tr>
<td>$u_{\text{max}}$</td>
<td>Maximum displacement without pounding</td>
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<tr>
<td>$\delta$</td>
<td>Instantaneous relative displacement of two bodies</td>
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<tr>
<td>$\dot{\delta}$</td>
<td>Instantaneous relative velocity of two bodies</td>
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<tr>
<td>$\delta_0$</td>
<td>Relative impact velocity at the initiation of impact</td>
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<td>Instantaneous relative displacement of $j$th floors</td>
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<tr>
<td>$\dot{\delta}_j$</td>
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<td>$\zeta$</td>
<td>Displacement proportional damping factor</td>
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<tr>
<td>$\mu$</td>
<td>Poisson's ratio</td>
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<td>$\mu_{\text{max}}$</td>
<td>Maximum displacement of structure undergoing pounding</td>
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<td>Constant or instantaneous damping ratio of an impact element</td>
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<td>$\rho$</td>
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Chapter 2 is essentially the following journal article:

| Nature of contribution by PhD candidate |  
|----------------------------------------|---|
| Experimental design, conducting shake table tests, data analysis and manuscript writing |

| Extent of contribution by PhD candidate (%) | 85% |

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Chapter 3 is essentially the following journal article:

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Chapter 4 is essentially the following journal article:


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Chapter 5 is essentially the following unpublished article:

Nature of contribution by PhD candidate: Experimental design, conducting impact tests, data analysis and manuscript writing

Extent of contribution by PhD candidate (%): 80%

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Chapter 6 is essentially the following journal article which has been accepted for publication:


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Chapter 7 is essentially the following unpublished article:

Nature of contribution by PhD candidate: Experimental design, conducting impact tests, data analysis and manuscript writing

Extent of contribution by PhD candidate (%): 90%

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Chapter 8 is essentially the following unpublished article:

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Chapter 1
Introduction

Collision between adjacent structures or adjacent parts of a structure has often been observed in past earthquakes, e.g. Mexico City 1985 (Rosenblueth and Meli 1986), Loma Prieta 1989 (Kasai and Maison 1997), Bhuj 2001 (EERI 2001) and Christchurch 2011 (Chouw and Hao 2012; Cole et al. 2012). Such collisions, known as seismic pounding, occur because the at-rest separation is insufficient to accommodate the relative displacement of the colliding elements. The out-of-phase vibrations can be caused by different structural characteristics (e.g. Anagnostopoulos 1988; Maison and Kasai 1990), soil structure interaction (e.g. Chouw and Schmid 1995; Rahman et al. 2001; Chouw 2002) or the spatial variation of ground motion (e.g. Athanassiadou et al. 1994; Li et al. 2012; Bi and Hao 2013).

Many buildings before the introduction of mandatory separation in building codes, e.g. pre-1982 buildings in Taiwan (Jeng and Tzeng 2000) and pre-1976 buildings in New Zealand (Bothara et al. 2008), have been built with no gap between neighbouring structures. Some even share a common wall with adjoining structures. In such cases, pounding is inevitable as no two buildings have identical dynamic properties. Even in the case of buildings with separation distance compliant with requirements at the time of construction, subsequent revision of building codes might mean that they do not comply with current requirements. Similarly, buildings might be susceptible to pounding if the original calculations of required separation did not include foundation compliance (Chouw 2002).

Pounding structures exert repeated hammer-like blows on each other which may cause minor non-structural or severe structural damage (Figure 1.1) that may even lead to the complete collapse. Rosenblueth and Meli (1986) reported that “In over 40% of collapsed or seriously damaged buildings, there was pounding with adjacent structures. Sometimes pounding caused minor damage. In 15% of all cases it led to collapse”. This assessment was later revised to “In 15% of building with major damage or collapse (not only collapse),
evidence of pounding was found” (Anagnostopoulos and Karamaneas 2008). However, pounding was considered a significant factor in 20-30% of these cases.

Several urban seismic vulnerability surveys have identified pounding as one of the major hazards (e.g. Jeng and Tzeng 2000; Bothara et al. 2008). Bothara et al. (2008) considered pounding as a critical structural weakness in the seismic assessment of Wellington city in New Zealand and found the presence of a large number of susceptible buildings in the central business district. Jeng and Tzeng (2000) assessed the pounding vulnerability of Taipei City and found hundreds of mid-rise buildings susceptible to damage or even collapse from pounding. Jeng and Tzeng also identified five building configurations that are prone to pounding damage. Though pounding can occur between any two buildings with insufficient gap, more damage have been observed when a building is: (i) adjacent to a more massive building, (ii) adjacent to a building with fewer stories, (iii) subject to eccentric pounding, (iv) at the end of a row of buildings, and (v) subject to mid-column pounding. A building is more vulnerable if it possesses more of these weaknesses. Cole et al. (2010) considered an additional vulnerability: (vi) if the building is made of brittle
unreinforced masonry. One year later, the extensive pounding damages to masonry buildings (Figure 1.2) observed in Christchurch 2011 earthquake (Cole et al. 2012) proved the vulnerability of such buildings.

Figure 1.2 Pounding damage to unreinforced masonry building in 22 February 2011 Christchurch earthquake (Khatiwada and Chouw 2014)

The frequent occurrence of pounding damage has led to a substantial body of literature. Since the study of building pounding started in the 1980s, considerable progress has been made. Starting from the analysis of buildings idealized as single degree-of-freedom (SDoF) models (e.g. Wolf and Skrikerud 1980; Anagnostopoulos 1988; Davis 1992; Athanassiadou et al. 1994), the studies quickly evolved to multiple degrees-of-freedom (MDoF) models (e.g. Papadrakakis et al. 1991; Anagnostopoulos and Spiliopoulos 1992; Kasai et al. 1992). The influence of soil-structure interaction (e.g. Chouw and Schmid 1995; Rahman et al. 2001; Chouw 2002), spatial variation of
ground motion (e.g. Athanassiadou et al. 1994; Hao and Zhang 1999; Hao et al. 2000; Jankowski 2012) and impact surface characteristics (e.g. Guo et al. 2009; Jankowski 2010) have been considered. Simulations have also been carried out for three-dimensional relative motion between the building floors (e.g. Papadrakakis et al. 1996; Zhu et al. 2002; Mouzakis and Papadrakakis 2004; Jankowski 2008b). Results from several large and small scale experiments have been published (Leibovich et al. 1994; Filiatrault et al. 1995; Papadrakakis and Mouzakis 1995; Zhu et al. 2002; Chau et al. 2003; Jankowski 2008a; Guo et al. 2009; Jankowski 2010; Guo et al. 2012; Leibovich et al. 2012). However, despite these accomplishments, the pounding simulations have several persistent limitations.

The existing numerical procedures predict substantially different pounding forces with no consensus on which models are more accurate. The experimental studies have found several models to reasonably simulate the structural responses for the considered cases, e.g. Lagrange multiplier method (Papadrakakis and Mouzakis 1995), linear viscoelastic impact element (Zhu et al. 2002) and nonlinear viscoelastic element (Jankowski 2008a). However, substantially different results can be obtained when several of these models are applied to a single case (Jankowski 2005). A parameter, e.g. coefficient of restitution, shows a substantial effect in some studies (e.g. Anagnostopoulos and Spiliopoulos 1992; Muthukumar and DesRoches 2006), while insignificant effects in other studies (e.g. Anagnostopoulos 1988; Athanassiadou et al. 1994).

1.1 Motivation for the study

The practical application of building pounding simulation has been severely limited due to these uncertainties (Bothara et al. 2008; Cole et al. 2010). Since a reasonable prediction of pounding force is required to design retrofitting measures for susceptible buildings, currently the more common practice is to avoid pounding. Either the buildings are connected together so there can be no relative motion, or the individual buildings are strengthened to limit so the relative motion is less than the available separation. The first method requires a detailed modelling of the coupled buildings to avoid torsional response. As is common, if the adjacent buildings have different owners, there can be many legal and logistical issues in addition to the engineering ones. The second method, i.e. limiting the relative motion, can be done by either strengthening the building or adding viscous dampers. However, it only works if there is at least some
separation. In many cities around the world, e.g. Wellington (Bothara et al. 2008), Taiwan (Jeng and Tzeng 2000) and Thessaloniki (Anagnostopoulos 1988), often there is no separation.

In the PhD student’s home country Nepal and other countries in South Asia, the building on central streets are almost exclusively built without separation. In many cases, it is not possible to connecting these buildings together because the structural systems are completely different, e.g. a RC building in between two unreinforced masonry ones. The economic capacity of a large number of homeowners also does not permit the government to require that such buildings be demolished. Therefore, the original motivation of the study was to investigate the effects of row building pounding to develop a more applicable retrofit measure. Towards this motivation, the first part of the research focussed on identifying the most correct model.

However, as shown in the next section, it was found that the currently existing simulation methods are completely inadequate. Thus, the motivation for the latter half of the study was to derive a more accurate numerical force model and verify it experimentally.

1.2 Methodology

To investigate the effect of row building pounding, three types of steel frames were built. The frames were almost identical except for their stiffness. The mass of these frames could be increased by adding extra steel plates to the beam. In total, eight different mass-frame configurations were included in the tests. Two of the mass-frame configurations were selected as reference frames. These frames were subjected to pounding with all eight other configurations in three types of setups: (i) two building pounding (TBP), (ii) row building pounding (RBP), and (iii) end building pounding (EBP). In TBP, the reference frame was pounding with only one other frame. In RBP, the reference frame was in the middle of two other frames. EBP also had three frames but the reference frame the reference frame was on one end and the two frames were together. All three arrangements were subjected to pounding under five ground motions. The amplification of displacement due to pounding, i.e. the ratio of maximum displacements with and without pounding, was always higher for three frames than in TBP. However, the location of the frame (EBP or RBP) did not show much effect. This contradicts the post-seismic observations (Anagnostopoulos 1996) that the buildings on the end of a row always showed more damage than those in the middle.
A numerical analysis of TBP case was carried out to evaluate the existing numerical force models. The beams of all the frames have identical size and length. For such cases, the distributed-mass models (e.g. Malhotra 1998; Cole et al. 2011) predict no energy loss during impact. However, the test results clearly showed that significant energy was lost. Therefore, only lumped-mass models were considered. Responses simulated from five models were compared with experimental results. It was found that the existing numerical models cannot predict the pounding response of even very simple single-degree-of-freedom structures.

A review of literature showed that all viscoelastic contact elements had some problems. Either they predicted partially tensile contact force, or large discontinues were present in the predicted force or the derivations were made with some large approximations. Thus, a numerically exact viscoelastic force model without any such limitations was derived. A comparison with previous impact test results (van Mier et al. 1991) showed a much better performance than other viscoelastic models.

Further study showed that the uncertainty in input parameters, i.e. coefficient of restitution and contact element stiffness, cause substantial limitations in the performance of even the numerically exact model. A detailed numerical analysis was conducted to demonstrate these limitations. The numerical analysis also showed the problems with the existing distributed-mass models. Similarly, a detailed critical review of previously published experimental simulations showed that there were significant issues which limited the applicability of those results; either the instruments or sampling rates were not suitable, or the test setup had some problems.

A series of impact tests were carried out between two RC slabs and between two steel beams. A parametric investigation was carried out by varying the contact surface geometry, mass, impact velocity and support conditions. The results were significantly different from numerical predictions.

It was discovered that an impact model, proposed by Sears (1912), could explain several of the apparently contradictory results observed in the past experiments. The model incorporates the characteristics of both lumped-mass and distributed-mass models. However, it can only consider elastic stress propagation through freely moving bodies. Thus, the model was reformulated as damped Sears model which can model viscoelastic energy loss during stress propagation. The damped Sears model can also consider the
effect of column supports. This model was compared with the acceleration response from RC and steel impact tests.

Finally, a sample numerical simulation of multi-storey buildings has been carried out with the damped Sears model.

1.3 Outline

This doctoral thesis is a compilation of journal manuscripts which have been either published or submitted for publication at the time of writing. Each manuscript is presented as a chapter. Some information has been included in several of the manuscripts. Therefore, they have been slightly modified to maintain coherence in the thesis and avoid repetition. However, some repetition has been allowed where removing information seemed to adversely affect the flow and clarity of a chapter.

Chapter 2: Shake table simulation of row building pounding

The objective of the chapter is to compare the amplification of maximum deflection response because of TBP, EBP and RBP, measured as a ratio of the response with and without pounding. The frames were first allowed to vibrate alone under five different earthquakes. Then, they were subjected to pounding under the three specified setups and the amplification was calculated.


Chapter 3: Evaluation of numerical pounding force models

Five lumped mass models were selected for evaluation. Numerical simulations were conducted for TBP with all the models to calculate the amplification of maximum displacement. The numerical results are compared with those from the experiment.

Chapter 4: Numerically exact viscoelastic force model: derivation and validation

A numerically exact solution of Hunt-Crossley family of models was derived. The prediction of the proposed model, and three existing models, was compared with experimental force obtained by van Mier et al. (1991). A sample numerical simulation is presented for all five force models.


Chapter 5: Impact of RC slabs: influence of mass, velocity and contact surface geometry

The details of specimen preparation, test setup and instrumentations are presented. The peak acceleration and coefficient of restitution were obtained from impact between two RC slabs suspended as pendulums. The influence of the specified parameters on the response quantities is shown.


Chapter 6: Limitations in state-of-the-art and introduction to the Sears impact model

A critical review of previous published experimental and numerical simulations is presented. Examples of numerical simulations with both lumped-mass and distributed-mass models are provided to show that results from all existing models are unreliable. It is also shown that many experimental studies on pounding are affected by several issues which severely limit their applicability. Finally, the Sears impact model is presented to address some problems in the numerical formulae.


Chapter 7: Damped Sears impact model: derivation and validation

The two major weaknesses in the Sears impact model are removed by reformulating the model as damped Sears model. In this chapter, impact between two non-rigid diaphragms
Introduction

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is analysed as modal superposition of viscoelastic longitudinal vibration. The numerical predictions are compared with experimental acceleration response from impact between two RC slabs and between two steel beams. The effect of variation in mass of the slabs, and the diaphragm material, is shown.

Chapter 8: Damped Sears impact model: application to building pounding

A procedure is developed to apply Sears impact model to diaphragms supported by columns. The numerical model is validated by comparison with acceleration response obtained from impact of steel portal frames and impact of a portal frame with a two-storey frame. A sample numerical simulation of multi-storey RC buildings is presented. The effects of variation in effective wave propagation velocity and contact surface stiffness are shown.

Chapter 9: Conclusions and recommendations for future research

Included manuscript: Khatiwada S, Chouw N, Larkin T. A damped Sears impact model for structural pounding analysis: application to building pounding. Submitted to Earthquake Engineering and Structural Dynamics.
Chapter 2
Shake table simulation of row building pounding

The damage due to end building pounding had been identified as early as 1977 (Anagnostopoulos 1996). It is one of the most prevalent vulnerabilities as cities around the world are full of city blocks with rows of buildings in contact with each other, especially in CBDs (Anagnostopoulos 1988; Jeng and Tzeng 2000; Bothara et al. 2008; Cole et al. 2010). Thus, the subject has received considerable research attention. Anagnostopoulos (1988) conducted a numerical simulation on pounding of a row of buildings idealized as single degree of freedom systems, and concluded that the exterior structures in a row experience higher amplification of response than the interior structures. The response of interior structures were found to be amplified or reduced depending on whether their fundamental period was smaller or higher than the adjacent structures; stiffer structures typically receiving amplification and flexible structures undergoing reduction of response. The same stiffer structure in the middle of the row received less amplification than when they were placed externally. The study was one of the first to model energy loss during impact with a viscoelastic spring. Athanassiadou et al. (1994) carried out similar simulations including the effect of phase difference in ground motion due to the velocity of the seismic waves. They found that the stiffer structure, regardless of its position in a row, always suffered the most response amplification. The interaction between adjacent structures and their subsoil can also have a significant influence on the development of the relative displacement (Shakya et al. 2010).

Anagnostopoulos and Spiliopoulos (1992) observed in numerical simulation of three multi-storey buildings that sometimes end building pounding produced higher response amplification than for middle building, but mostly the amplifications were comparable. There were even some cases where the amplification for interior building was higher. Ohta et al. (2006) analysed pounding between two and three buildings using finite element program SAP2000 and observed that, only in some cases for the same building response amplification of the end building in three buildings configuration is higher than in two
pounding. The number of such cases was found to increase with a larger number of stories of participating structures.

From the observation in post-earthquake surveys and based on numerical studies, end building pounding is identified as more vulnerable than when the building is located in the middle of a row. Cole et al. (2010) included external building in a row as one of the six configurations susceptible to pounding damage. Bothara et al. (2008) also considered end building in a row more vulnerable than those within the row. In contrast, damage survey from Christchurch 2011 showed several cases where the buildings in the middle of the row were badly damaged, while buildings at the end of the same row survived (Chouw and Hao 2012; Cole et al. 2012).

Several experimental studies on pounding of two building have been performed in the past. Papadrakakis and Mouzakis (1995) subjected two storey concrete frame structures to floor to floor pounding and found that structures nearest to their resonance amplify the displacement of the adjacent structure. Filiatrault et al. (1995) conducted experiments on pounding of unequal height steel structures to validate the performance of FE analysis software to predict pounding response. Chau et al. (2003) studied the pounding between equal height steel structures. Rezavandi and Moghadam (2007) have experimentally evaluated the effectiveness of various mitigation measures in pounding of steel frames. The authors could not find any such experimental studies on three building pounding though it has often been numerically predicted as more hazardous than two building poundings. Similarly, no comparative experiments between two and three building pounding were found. To the authors’ best knowledge an experimental validation of the conclusions derived from past numerical studies on row building pounding has never been reported.

In this work a parametric shake table study of pounding between two and three steel portal frames was conducted. The frames were subjected to five ground motions. A frame was designated as the reference frame and its response amplification were investigated with three configurations: (i) two building pounding (TBP), (ii) row building pounding (RBP) and (iii) end building pounding (EBP). The reference frame was kept at the centre of two other identical frames for RBP while it was placed to the right of the identical frames for EBP. The top displacements of the frames were measured, and the amplification of the maximum displacement is employed as the measure of severity of pounding. The impact
forces have not been measured as the inclusion of any kind of force measuring device can alter the pounding force development and subsequently, response of the frames.

2.1 Experimental setup

Figure 2.1 Schematic drawing of the steel frame with 75 x 3 mm column
Figure 2.1 shows steel frames fabricated for this study. The frames had three different column sizes: 50 x 3 mm, 75 x 3 mm, and 100 x 3 mm. The stiffness of these three different types of frames are displayed in Table 2.1. The inside dimension between the beams in Figure 2.1 (Section A – A) varied according to the column size. The beams supported a 200 x 150 x 10 mm plate, which could be loaded with additional identical plates as shown by the dotted lines. Four mass setups were used for the test as shown in Table 2.2. The masses varied among different frames because the load plates were identical for all frames while the sizes of columns and horizontal bracings are different. The columns were connected to a separate base as shown in the expanded details on bottom left side of the figure, so that the details for all the columns in multi-storey frames would remain identical. There were two horizontal bracings between the beams. An accelerometer was attached to one of these bracings during the tests. A strain gauge is installed on each column just above the base joint. The strain gauges were calibrated against the displacement of each frame relative to its base.

### Table 2.1 Stiffness ID for different frames

<table>
<thead>
<tr>
<th>Stiffness ID</th>
<th>Column size (mm)</th>
<th>Lateral stiffness (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k1</td>
<td>50 x 3</td>
<td>5,926</td>
</tr>
<tr>
<td>k2</td>
<td>75 x 3</td>
<td>8,889</td>
</tr>
<tr>
<td>k3</td>
<td>100 x 3</td>
<td>11,852</td>
</tr>
</tbody>
</table>

### Table 2.2 Masses considered

<table>
<thead>
<tr>
<th>Mass ID</th>
<th>Additional load plates</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>k1</td>
</tr>
<tr>
<td>m0</td>
<td>0</td>
<td>8.04</td>
</tr>
<tr>
<td>m1</td>
<td>2</td>
<td>12.75</td>
</tr>
<tr>
<td>m2</td>
<td>4</td>
<td>17.46</td>
</tr>
<tr>
<td>m3</td>
<td>6</td>
<td>22.17</td>
</tr>
</tbody>
</table>

A 150 x 10 x 3 mm steel strip is employed as the contact interface as shown in the details of the right end of the beam. The strip is glued and welded to a 150 x 50 x 10 mm steel plate, which is bolted to an identical plate welded to the beams. The top left end of the frame had a similar detail but did not have the 3 mm middle strip. Thus, a plain surface contact in 150 x 10 mm area was assumed when the frames were placed end to end.
Table 2.3 shows the fundamental period of the mass-frame combinations considered in the experiments. Snap back tests were conducted to determine the fundamental period and the damping constant of the structures. The actual periods of the frames were found to be within ±2% of the theoretical values.

Table 2.3 Fundamental period T(s) for selected mass-frame combinations

<table>
<thead>
<tr>
<th>Mass</th>
<th>Frame Stiffness</th>
<th>k1m0</th>
<th>k2m0</th>
<th>k3m0</th>
</tr>
</thead>
<tbody>
<tr>
<td>m0</td>
<td>T = 0.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m1</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>m2</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>m3</td>
<td>T = 0.38</td>
<td>T = 0.32</td>
<td>T = 0.28</td>
<td></td>
</tr>
</tbody>
</table>

The experiment was conducted on a displacement-controlled, 10 kN shake table at the University of Auckland with a maximum displacement capacity of ±15 cm. Table 2.4 lists the frame pairings investigated and their fundamental period ratio. The medium stiffness frame (k2) was selected as the reference frame for identifying the effect of pounding against frames with a greater (k3) or a smaller stiffness (k1). The tests were performed for two different masses, i.e. m0 = 8.59 kg and m3 = 22.72 kg, on the reference frame. These two masses produce the greatest and the least possible natural frequency considered. The frames k2m0 and k2m3 are designated respectively reference frame 1 and reference frame 2.

The test setups are shown in Figure 2.2. For two building poundings, the reference frame was subject to pounding against a second frame, as shown in Table 2.4. Thus, the second frame either had a same stiffness and different mass (e.g. Cases 3-6) or same mass and different stiffness (e.g. Cases 1, 7) as the reference frames. The Cases 2, 8, 9 and 15 were added so that both the reference frames were subjected to pounding with the same set of frames in all cases.
Table 2.4 Period ratio of the frames considered

<table>
<thead>
<tr>
<th>Case</th>
<th>Configurations (from Table 3)</th>
<th>( T_2 ) (s)</th>
<th>( T_1/T_2 ) (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>k2m0-k1m0</td>
<td>0.23</td>
<td>0.82</td>
</tr>
<tr>
<td>2</td>
<td>k2m0-k1m3</td>
<td>0.38</td>
<td>0.49</td>
</tr>
<tr>
<td>3</td>
<td>k2m0-k2m0</td>
<td>0.19</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>k2m0-k2m1</td>
<td>0.24</td>
<td>0.79</td>
</tr>
<tr>
<td>5</td>
<td>k2m0-k2m2</td>
<td>0.28</td>
<td>0.68</td>
</tr>
<tr>
<td>6</td>
<td>k2m0-k2m3</td>
<td>0.31</td>
<td>0.60</td>
</tr>
<tr>
<td>7</td>
<td>k2m0-k3m0</td>
<td>0.16</td>
<td>1.15</td>
</tr>
<tr>
<td>8</td>
<td>k2m0-k3m3</td>
<td>0.27</td>
<td>0.70</td>
</tr>
<tr>
<td>9</td>
<td>k2m3-k1m0</td>
<td>0.23</td>
<td>1.36</td>
</tr>
<tr>
<td>10</td>
<td>k2m3-k1m3</td>
<td>0.38</td>
<td>0.82</td>
</tr>
<tr>
<td>11</td>
<td>k2m3-k2m0</td>
<td>0.19</td>
<td>1.66</td>
</tr>
<tr>
<td>12</td>
<td>k2m3-k2m1</td>
<td>0.24</td>
<td>1.32</td>
</tr>
<tr>
<td>13</td>
<td>k2m3-k2m2</td>
<td>0.28</td>
<td>1.13</td>
</tr>
<tr>
<td>14</td>
<td>k2m3-k2m3</td>
<td>0.31</td>
<td>1.00</td>
</tr>
<tr>
<td>15</td>
<td>k2m3-k3m0</td>
<td>0.16</td>
<td>1.92</td>
</tr>
<tr>
<td>16</td>
<td>k2m3-k3m3</td>
<td>0.27</td>
<td>1.15</td>
</tr>
</tbody>
</table>

\( T_1 \) of reference frame 1 (k2m0) is 0.2 s
\( T_1 \) of reference frame 2 (k2m3) is 0.32 s

The second and third frames were identical for each EBP and RBP configurations. A three letter prefix will be added to the case number to identify the type of configuration being employed, e.g. when a k2m0 frame is pounding with a k2m3 frame, it will be called TBP6; when a k2m0 frame is between two k2m3 frames, it will be called RBP6; and when a k2m0 frame is on a side of the two k2m3 frames, it will be called EBP6. Fig. 4 shows the pounding arrangement for Case TPB7.

Four selected time histories were applied to each configuration case, i.e. two artificial ground motions simulated based on the New Zealand design spectrum (NZDS 1 and NZDS 2) and two based on the Japanese design spectrum for hard soil condition (JDS 1 and JDS 2). The simulated time histories were scaled down so that the maximum ground displacement was ±10 cm. In addition, the most well-known earthquake, the 1940 El-Centro ground motion (ELC) is also considered. The El-Centro ground motion was not scaled. The scaled displacement time histories are shown together with the El-Centro excitation in Fig. 5. Since the ground motion directions can have a significant effect on the pounding response, the ground motions were applied twice: once from left to right (termed positive direction) and secondly from right to left (termed negative direction). The zero
separation was considered as past studies have found that the pounding response decreased as the gap size was increased. This is also a common configuration of buildings in CBDs of many big cities (Chouw and Hao 2012). The displacement response of the frames was measured by the strain gauges placed on the columns just below the beams. After the tests were finished, the calibration was checked again, and no significant difference was found between the initial and final calibration factors. The pounding interface also did not show any indentation or other permanent deformation after any of the tests.

Figure 2.2 Test setups for (a) Two building pounding (TBP), (b) Row building pounding (RBP), and (c) End building pounding (EBP)
Figure 2.3 Adjacent structure without seismic gap. (a) Frames on shake table, (b) pounding interface and (c) pounding elements without (top) and with middle strip (bottom)

Figure 2.4 Ground motions considered
2.2 Results and discussion

The maximum deflections of the reference frames are shown in Figure 2.5. For reference frame 1, the scaled NZDS loadings caused the maximum floor displacement while the scaled JDS loadings have the least floor displacement. The El-Centro ground excitation produced median deflection among the five time-histories in both directions. For reference frame 2, the El Centro ground motion caused the maximum floor displacement. Ideally, the deflection of the frame without pounding with the adjacent buildings should not be affected by the direction of the excitation but it can be seen that there is some slight effect likely due to inadvertent lack of symmetry in the model frames.

![Figure 2.5 Maximum deflection $u_{\text{max}}$ of the reference frame without pounding](image)

2.2.1 Two building pounding

For the study of pounding effects, a factor $\mu_{\text{max}}/u_{\text{max}}$ is used where $\mu_{\text{max}}$ and $u_{\text{max}}$ are the maximum deflection of the reference frame with and without pounding with adjacent frames, respectively. Thus $\mu_{\text{max}}/u_{\text{max}}$ is the amplification of maximum deflection due to pounding. Figure 2.6 shows $\mu_{\text{max}}/u_{\text{max}}$ of the reference frame 1. The results show that pounding not only amplified but also reduced the maximum deflection of the participating structures. Under the most demanding time history, NZDS 2, pounding reduced the maximum deflection in all configurations while for NZDS 1 the amplification was seen only for Case 2. For JDS and EIC time histories, the maximum $\mu_{\text{max}}/u_{\text{max}}$ occurred when the second frame was the most flexible (Case 2). It can be seen that, for a given frame pairing, the direction of ground motion can have a significant impact on $\mu_{\text{max}}/u_{\text{max}}$. For example,
\( \mu_{\text{max}}/u_{\text{max}} \) for Case 6 under JDS 1 increased almost 20% when the ground motion direction was reversed.

The experimental results for reference frame 1 agree with the previous numerical studies that \( \mu_{\text{max}}/u_{\text{max}} \) of reference frame 1 is highest when the other frame was most flexible (Case 2) and least when the other frame was most stiff (Case 7). When both frames have the same stiffness, amplification increased with increase in mass (Cases 3 to 6). For the second frame of similar mass the amplification decreased with stiffness (cases 2, 6 and 8). \( \mu_{\text{max}}/u_{\text{max}} \) was consistently less than one when \( T_1/T_2 \) was greater than 0.8 (Cases 1, 4 and 7). Some pounding was observed even when the second frame had the same mass and stiffness as the reference frame. It could be due to some slight difference in natural frequency even though every effort was made to keep the properties identical. Such pounding caused reduction in maximum displacement under all ground motions considered.

The results show that the displacement amplification due to pounding depends more on the fundamental periods of the two structures than on the mass. In Cases 6 and 8 the mass of the second frame was equal but \( T_2/T_1 \) in Case 6 was higher and so was \( \mu_{\text{max}}/u_{\text{max}} \). The displacement amplification was similar in Cases 5 and 8 which have second frames with different masses but almost equal fundamental periods.

Figure 2.7 shows that the reference frame 2, for Cases 9 to 16, underwent reduction in displacement in all cases of TBP. The reduction was more for larger difference in period.
For Case 10, where the second frame was more flexible, maximum displacement was reduced in both frames. Similar to the reference frame 1, some poundings were observed in Case 14 even though the two frames are nearly identical. The $\mu_{\text{max}}/u_{\text{max}}$ values also seemed to be affected by the $u_{\text{max}}$ of the frame. For instance, the deflection of reference frame 1 under El-Centro loading was similar to JDS earthquakes, and the $\mu_{\text{max}}/u_{\text{max}}$ values from the three ground motions were similar. While for reference frame 2, the $u_{\text{max}}$ and $\mu_{\text{max}}/u_{\text{max}}$ under ElC were similar to that produced by NZDS ground motions.

2.2.2 Row building pounding

The displacement amplification in RBP configurations is shown in Figure 2.8. The results are presented only for the positive ground motion. It was observed that the displacement amplification was always greater than TBP for $T_1/T_2 < 1$ (Cases 1, 2, 4, 5, 6 and 8). The maximum increase is in Case 2, the $\mu_{\text{max}}/u_{\text{max}}$ of reference frame 1 under increased from 1.51 to 2.03. Similarly, when the reference frame was flexible than the adjacent frames (for instance Cases 9 and 11), the maximum deflection of reference frame was even more reduced in RBP (Figure 2.8 (b)) than in TBP configurations (Figure 2.7).
2.2.3 End building pounding

Figure 2.9 shows the displacement amplification of both reference frames due to EBP. It was observed that, except in a few isolated cases, the amplifications were similar to RBP. Thus pounding of three frame in a row seems to be always more severe for a stiffer structure irrespective of its position in the row. Similarly, the most flexible structure always had reduced displacement.
Figure 2.10 Displacement time history under JDS loading for Case 2. (a) Reference frame 1 (bold frame in the top sketch), (b) 2nd frame (thin frame) and (c) 3rd frame (dashed frame).

The displacement time history of reference frame 1 pounding against the most flexible frame (Case 2), under JDS1 ground motion in the three different configurations is presented in Figure 2.10(a). The displacement of the reference frame was skewed to the positive in TBP and EBP but it was almost symmetric in RBP. The second frame, which was at the left end in all tests, had negative skew in all cases. The third frame also had comparable maximum deflection in both EBP and RBP. Even though the maximum deflections of the frame are similar in RBP and EBP, the frames attained higher peaks more often when they were placed at the end. The identical frames at the both ends in RBP had considerably different displacements (see the second row, middle result and last row, left result of Figure 2.10). The maximum displacement of flexible frame in Case 2 i.e. frame k1m3, was 38mm without pounding. It can be seen that the pounding reduced the
maximum deflection in all the cases, but the reduction was much more pronounced when it was placed at the centre.

The displacement response amplification of the stiffest frame under consideration, \( k_3m_0 \) under NZDS 2 and JDS1 ground motions are presented in Figure 2.11. The amplification is high when pounding against reference frame 2 and low against reference frame 1. The pounding response is very low for NZDS ground motions compared to JDS excitations.

It is apparent from the results that pounding of three buildings is intrinsically more hazardous to the stiffer structure than two building pounding. The location of the stiffer structure whether at the end, or in the middle of the adjacent two frames, did not appear to have any bearing on the hazard posed. In many cases, the reference frame 1 suffered more displacement amplification in RBP but there were several cases where EBP was more hazardous. Even when two stiffer frames were pounding with the more flexible reference frame, either one could have more amplification, dependent upon the ground motion, or even its direction (Figure 2.11). When frames of similar time period suffered pounding, the displacement response was reduced in almost all cases.

![Figure 2.11](image.png)

Figure 2.11 \( \mu_{\text{max}}/\mu_{\text{max}} \) of frame \( k_1m_0 \) in different pounding arrangements: (a) JDS2 and (b) NZDS2 ground motions.

In all cases the response amplification due to NZDS earthquakes is much smaller than that from JDS or ElC. Except for a few isolated cases, the NZDS excitation induced pounding caused a reduction in maximum displacement. When the displacement was amplified, the amplification factor was always lower than in JDS. Amplification under El Centro seemed...
to depend upon the non-pounding response of the frame. The reference frame 1 had similar maximum deflection under ElC and JDS ground motion, and the $\mu_{\text{max}}/u_{\text{max}}$ values were also similar (Figure 2.6), while for reference frame 2 both $u_{\text{max}}$ and $\mu_{\text{max}}/u_{\text{max}}$ under ElC ground motions are close to that from NZDS ground motions (Figure 2.7). This suggests that, for the frames under consideration, pounding tends to cause more amplification in the frames that have lower $u_{\text{max}}$, and less amplification when $u_{\text{max}}$ is higher. The behaviour may be related to the increased energy loss for higher velocity impact as observed in past studies (e.g. Jankowski 2010). Since no significant changes in calibration factor of strain gauges was found before and after the pounding experiments and no permanent deformation at the pounding location was observed, this behaviour cannot be attributed to plastic deformations.

### 2.3 Summary

A parametric shake table investigation of pounding between two buildings and three buildings in a row was conducted. Five different ground motions were applied to steel portal frames with three different stiffness and four different masses. Two of the frames were selected as reference frames, and each was subjected to pounding against eight other frames. Each reference frame was subjected to pounding with two identical frames on either side and with the two identical frames on one side. The eight symmetrical configurations were termed row building pounding and the eight asymmetrical arrangements were for end building pounding. The displacement amplification ratio due to pounding was calculated by dividing their absolute maximum deflection by no-pounding deflection under the same time history. In total 480 tests were performed.

The following conclusions can be drawn:

- Pounding between a row of buildings is always more hazardous to the stiffer building than pounding between two buildings.
- The location of the stiffer building did not seem to have an effect. Thus, contrary to the accepted state of the art based mainly on numerical investigations, a stiff building in the middle of a row is not any safer than that at the end of the row. The relative hazard depends only on the ratio of fundamental period with respect to the adjacent structures of the row.
- When buildings of similar fundamental periods, i.e. with period ratio of 0.8 to 1.2 underwent pounding, the maximum displacement of all the buildings is always reduced.

- For the same frame arrangement, if a time-history produced higher maximum displacement of the stiffer frames without pounding, the amplification due to pounding was lower and vice-versa. This could be related to the observations from impact mechanics that higher velocity of impact can cause more energy loss.

The pounding force was not measured during the test; thus no conclusion can be drawn on the possible damage at the contact locations. With this caveat, the results strongly suggest that the buildings located in the middle of a row should not be assumed safer than those at the end, even when the floor heights are same.
Chapter 3

Evaluation of numerical pounding force models

Seismic pounding is a form of impact where large impulsive forces act between the participating bodies resulting in a near-instantaneous change in momentum of the colliding bodies. The impact is called elastic if there is no loss of kinetic energy during the collision. If there is some energy loss, the impact is termed inelastic. Pounding is a complex phenomenon involving heterogeneous materials and different types of contact surfaces, supporting structure and foundation system. There may be several collisions within a single earthquake and the impact velocities for each event will be different. As the current state of the art in pounding research cannot account for these factors, the numerical simulations include many assumptions such as lumped mass models, a constant coefficient of restitution and insensitivity of pounding forces to the contact surface geometry.

The lumped-mass model assumes that the building frame or bridge deck acts as a diaphragm, and that the whole mass of the floor or deck contributes to the impact even though classical physics states that only a small part of the colliding mass participates in the contact (Goldsmith 2001). The model also assumes that the actual structural stiffness and the masses of the non-colliding floors above or below the pounding stories have no effect on the build-up of impulsive forces. During numerical simulations, displacement of the structures are computed considering all structural properties. However, when the displacements time history of the neighbouring buildings intersect the contact forces are calculated only from the colliding floor masses and relative velocity of impact.

The coefficient of restitution, defined as the ratio of the final relative velocity to the initial relative velocity of the colliding bodies, is employed as a measure of the elasticity of an impact. In an elastic impact, the coefficient of restitution equals one while it is zero for a completely plastic impact. Coefficient of restitution signifies the loss of kinetic energy due to several complex processes such as material damping, surface friction, surface yielding,
residual internal vibrations, generation of sound and heat during collision etc. These processes are not fully understood even for the relatively simple collisions of identical spheres at known velocities. In the absence of large scale experimental results, the numerical simulations have to assume values based on the experiments on collision of small masses. The value is also assumed as invariable throughout the simulations even though past experiments have shown it varies according to the mass and the initial relative velocity of the colliding bodies (Goldsmith 2001; Jankowski 2010).

The constraints and approximations have created a dichotomy in pounding research. Most pounding models are validated based on impact force and the numerical evaluation studies of these models also rely on structural acceleration, velocity and relative impact velocities. However, the researchers studying the effect of pounding on structures focus on amplification of structural displacement, bending moment, base shear and storey shear. Past studies that evaluated these models have mostly been based on single impact experiments which do not provide a measure of performance through a full ground motion. Shake table studies on pounding have usually evaluated only one model and performance have not been compared against other models.

To the authors’ knowledge, no past study has attempted a parametric comparison of the predictions of various pounding models with experimental deformation amplification. This study presents the results of a shake table investigation of floor to floor pounding between two steel portal frames and compares them with the results from elastic numerical analysis. A contact element model is recommended for use based on the comparison of predicted maximum displacement amplification with the experimental displacement amplification values. The contact elements are compared based on their displacement response because structural displacement has been found mostly insensitive to the contact element stiffness which cannot be determined with certainty. Structural drift is also the main kinematic parameter of interest to designers as all the internal forces in structures can be computed from displacement. The contact surface between the two frames was kept flat instead of the commonly employed hemispherical (van Mier et al. 1991; Chau et al. 2003) or cylindrical interfaces (Filiatrault et al. 1995).

### 3.1 Numerical models considered for evaluation

Contact element models have been employed frequently to predict pounding forces and structural responses. Such models define a constant “gap” between the buildings and if the
relative closing displacement between the two buildings is more than the “gap”, the contact force is activated. The most common form of contact model assumes the presence of an elastic spring with or without a viscous damper to model energy loss. These models have an advantage that they can be implemented in most existing numerical time history analysis software without significant programming modification. Other approaches like Lagrange-multiplier and Laplace-domain methods have also been employed in some studies (e.g. Papadrakakis et al. 1991; Chouw 2002) but their application is limited because they have not been implemented in commercial FE software and need special programming by the users.

This chapter compares the displacement amplification predicted by various viscoelastic contact-element models with the experimental results and select the best performing model. The linear elastic and nonlinear elastic contact elements have not been included in the comparison because they cannot simulate energy loss during impact. The numerical models considered for the evaluation are described below with their underlying assumptions and their intended performance priorities, such as modelling energy loss or better prediction of pounding force.

3.1.1 Linear viscoelastic model (LVe)

The linear viscoelastic model, also called the Kelvin model, has the same form as a Kelvin-Voigt material. A linear spring and a linear viscous damper are placed in parallel. The pounding force is given by:

\[ F = k \delta + c \dot{\delta} ; \quad \delta \leq 0 \]
\[ F = 0 ; \quad \delta > 0 \]

where \( F \) is the instantaneous pounding force

\( k \) is the impact element’s stiffness,

\( \delta = (u_2 + \text{gap} - u_1) \) is the instantaneous relative displacement, also called apparent deformation, of the masses; \( u_1 \) and \( u_2 \) are the displacements and gap is the at-rest separation of the two masses,

\( \dot{\delta} \) is the instantaneous relative velocity, and

\( c \) is the impact element’s constant damping given by,
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\[ c = 2 \xi \sqrt{\frac{k}{m_1 + m_2}} \]  
\[ 3.1(a) \]

where \( m_1 \) and \( m_2 \) are the colliding masses and \( \xi \) is the damping ratio given by

\[ \xi = \frac{\ln(e)}{\sqrt{\pi^2 + \ln(e)^2}} \]  
\[ 3.1(b) \]

and \( e \) is the coefficient of restitution described earlier.

The formulations were first derived by Anagnostopoulos (1988; 2004). The expressions for the damping coefficient, \( c \), and damping ratio, \( \xi \), were derived so that the energy loss is the same as that in the stereo-mechanical model for the same \( e \). The model, with respect to the displacement amplification of the structures, is found to be insensitive to the values of contact element stiffness. The impulsive acceleration calculated during impact varies according to the stiffness adopted, but this variation does not translate into variation in structural displacement. This model produces tensile force near the end of the contact which does not agree with experimental force time histories. Several objections to the model’s uniform damping throughout the contact have also been raised and various amendments to the value of \( \xi \) have been proposed.

3.1.2 *Modified linear viscoelastic model (MLVe)*

The modified linear viscoelastic element has an approach only dashpot. A similar model was initially proposed by Valles-Mattox and Reinhorn (1996) as impact Kelvin model. However, the impact Kelvin model has not been used in any numerical simulations perhaps due to its complex formulation. The proposing study also did not provide any numerical investigation to assess its performance. The MLVe model (Equation 3.2) was later proposed by Mahmoud (2008). The relationship between \( \xi \) and \( e \) in LVe model has been modified so that the total viscous damping can be incorporated within the approach period.

\[ F = k \delta + c \dot{\delta} \quad ; \quad \delta \leq 0; \quad \dot{\delta}(t) > 0 \]
\[ F = k \delta \quad ; \quad \delta \leq 0; \quad \dot{\delta}(t) < 0 \]
\[ F = 0 \quad ; \quad \delta > 0 \]  
\[ 3.2 \]

where all the parameters except \( c \) are as defined for the LVe model. \( c \) is also calculated using Equation 3.1(a), however, the damping ratio \( \xi \) is obtained from:
Mahmoud and Jankowski (2011) later proposed a second expression for the damping constant:

\[
\xi = \frac{1 - e^2}{e[2e - 2 + 2]} \quad 3.2(b)
\]

The two coefficients show only slight variation (Mahmoud and Jankowski 2011) and the first formulation (Equation 3.2(a)) gives a value nearer to that of the linear viscoelastic model. Thus, equation (2a) is used for the numerical modelling throughout this thesis.

Mahmoud and Jankowski (2011) acknowledged that both formulations of MLVe model performed inferior to the LVe in all cases. As intended, the modified model removed the tensile force from the \( F - \delta \) relationship of the linear viscoelastic model but performed worse when compared with experimental results.

### 3.1.3 Nonlinear viscoelastic model (NLVe)

The nonlinear viscoelastic model was proposed by Jankowski (2005) to simulate pounding force more precisely by removing the disadvantages in both LVE and nonlinear elastic models (Davis 1992; Chau and Wei 2001). A non-uniform viscous damping is added to the nonlinear elastic model to simulate energy loss during impact. The uniform damping and tensile contact force in LVe model is also avoided. Similar to the MLVE model, the viscous damper is active only when the masses are approaching each other.

\[
F = k \delta^{\frac{3}{2}} + c \dot{\delta} \quad \delta \leq 0; \quad \dot{\delta}(t) > 0
\]

\[
F = k \delta^{\frac{3}{2}} \quad \delta \leq 0; \quad \dot{\delta}(t) < 0
\]

\[
F = 0 \quad \delta > 0
\]

where \( k \) is a nonlinear stiffness and the instantaneous damping \( c \) is defined as:

\[
c = 2 \xi \sqrt{\delta \frac{m_1 m_2}{m_1 + m_2}} \quad 3.3(a)
\]

where \( \xi \) can be calculated from one of the following two relations (Jankowski 2006):
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\[
\xi = \frac{1-e^2}{\pi e} \quad \text{3.3(b)}
\]

\[
\xi = \frac{9\sqrt{5}}{2} \cdot \frac{1-e^2}{e[9\pi - 16] + 16} \quad \text{3.3(c)}
\]

and other parameters are same as defined for the previous models.

Jankowski (2005) analysed and compared several experimental results against both LVe and NIVe models. The NIVe model was found to be marginally better than the linear viscoelastic elastic model in predicting the pounding forces. It is surprising that the LVe model performed nearly on a par with the NIVe model. The former is optimized to model the energy lost in the system due to impact while the latter is optimized to match the pounding force time history.

The greatest strength of the NIVe model is its ability to simulate the force time history better than the other existing models if the stiffness and damping are known. However, it is handicapped by the inability to determine these parameters in advance. These parameters can be iteratively determined if the force time history is available but in the absence of such records its accuracy may not justify the extra complexity introduced in the calculations.

### 3.1.4 Hertzdamp model

The Hertzdamp model was introduced by Muthukumar and DesRoches (2006) to enable the nonlinear elastic model proposed by Hertz to model inelastic pounding. The model has been used previously in robotics and mechanical engineering applications. The numerical formulation of the model is as follows:

\[
\begin{align*}
F & = k \delta^{3/2} + c \dot{\delta}; \quad \delta \leq 0; \quad \dot{\delta}(t) > 0 \\
F & = 0; \quad \delta > 0
\end{align*} \quad \text{3.4}
\]

where \( k \) is a nonlinear stiffness as defined by Hertz (Goldsmith 2001) and the instantaneous damping \( c \) is defined as:

\[
c = \zeta \delta^{3/2} \quad \text{3.4(a)}
\]

where displacement proportional damping factor \( \zeta \) is given by:
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\[ \zeta = \frac{3}{4} \left[ \frac{k(1-e^2)}{\delta_0} \right] \quad 3.4(b) \]

where \( \dot{\delta}_o \) is the relative velocity at the initiation of contact and all other parameters are the same as in previous models.

The Hertzdamp and NIVe models both employ non-uniform damping. However, NIVe model assumes the damping during restitution is negligible while the Hertzdamp model assumes that the damping factor at any time is proportional to the apparent deformation \( \delta \).

\[ F(t) = k \delta(t) + c \dot{\delta}(t) \]

3.1.5 Modified Hertzdamp model (MHd)

Ye et al. (2009b) proposed a modification to Hertzdamp model because they found that the coefficient of restitution \( e \) calculated from the results of an impact simulation with Hertzdamp model was different from the initial value utilized in the computation. The error increased with decreasing value of the initially assumed \( e \). Ye et al. proposed to replace Equation 3.4(b) with Equation 3.5, which substantially reduced this error.

\[ \zeta = \frac{8}{5} \left[ \frac{k(1-e)}{e \delta_0} \right] \quad 3.5 \]

3.2 Experimental setup

The experimental part of this chapter is the two building pounding (TBP) presented in Chapter 2. The amplification of maximum displacement \( \mu_{\text{max}}/u_{\text{max}} \) obtained for reference frame 1 (Case 1 to 8 in Table 2.4) are the experimental results considered for evaluation of the numerical models. However, an additional impact experiment was conducted to find the impact element stiffness and coefficient of restitution.

The reference frame was struck with a pendulum striker of mass 1.7 kg. The contact area was kept 30 x 10 mm to maintain the same ratio of mass to contact area as in the frame pounding experiments. The suspended striker was pulled and allowed to impact the frame at 0.075 m/s speed. The test was repeated three times and accelerations of the frame and the striker were measured at a sampling rate of 4 kHz. The impact force time history was calculated from Newton’s second law. The post-impact velocities were calculated from the impact-momentum principle, i.e. the final momentum of the body is equal to its
momentum added to the area under the force time history. Because the impacting bodies
do not gain or lose mass from impact, the post-impact velocity was calculated as sum of
initial velocity and the area of the acceleration time history. The coefficient of restitution
was calculated from the velocities calculated from the post-impact response of the striker
pendulum and struck frame. The average values of coefficient of restitution and contact
elements stiffness were calculated and adopted for numerical simulations.

The coefficient of restitution was found to be 0.4. A value of 0.5 to 0.65 is usually
employed in numerical simulations, for instance: Anagnostopoulos (1988), Athanassiadou
(1994), Jankowski (2005), Mahmoud and Jankowski (2011). It is of note that Zhu et al.
(2002) also measured an experimental value of 0.4. However, Jankowski (2010) measured
values of 0.45 to 0.7 depending on impact velocity for steel spheres dropped on steel plates.
Jankowski’s value would be more than 0.7 for the impact velocity of 0.075 m/s adopted in
this test. The differences might be attributed to the difference in contact surface. Zhu et al.
(2002) and the current work had planar contact surfaces while Jankowski (2010) employed
spherical to plane contact interface.

The contact element models were used to simulate the same impacts, with a place holder
value for $k$. The value of stiffness $k$ for each impact element model is calculated by trial
and error such that the numerically simulated impulse and contact duration equal those
from the experimental record. Some previous studies (e.g. Jankowski 2010, Mahmoud and
Jankowski 2011) have identified these values by equating the maximum force and impulse.
However, some models showed considerably longer contact duration than the experimental
record in those comparisons. Therefore, this study adopted contact duration as a constraint
of optimization instead of the maximum contact force.

The experimental and simulated $F$-$t$ and $F$-$\delta$ curves are presented in Figure 3.1 and Figure
3.2 respectively. The impact stiffness calculated for three impact elements are shown in
Table 3.1. The MLVe and MHd models are derivatives of LVe and Hertzdamp models
respectively and they have been simulated with the $k$ obtained for the unmodified models.
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3.3 Numerical results and discussions

The frames employed in the shake table experiments and in the numerical simulations were subjected to the same time histories which were applied from both directions. The simulations were carried out for two coefficients of restitutions values, viz., 0.4 and 0.6. The value 0.4 was found from impact tests described in the previous section, while the value of 0.6 was used because the most commonly employed value in previous numerical studies is 0.6 to 0.65 (e.g. Ohta et al. 2006; Anagnostopoulos and Karamaneas 2008;
Jankowski 2008b; Shakya et al. 2008). Similarly, two $k$ values i.e. 200 kN/m from Table 3.1 and 100 times the maximum stiffness of participating structures were employed for the LVe and MLVe models. The second value was employed to study the effect on displacement amplification due to element stiffness variation. For the nonlinear impact elements, three $k$ values, viz. value from Table 3.1, a fifth of this value and five times this value were applied. Thus there were a total of 4 combinations each of $e$ and $k$ for linear models and 6 combinations each for the nonlinear models. The Equation (3.6) was solved for all these configurations with time-stepping integration at a time step of 0.1 ms.

$$
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_1 \\
    \ddot{u}_2
\end{bmatrix}
+ 
\begin{bmatrix}
    C_1 & 0 \\
    0 & C_2
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_1 \\
    \dot{u}_2
\end{bmatrix}
+ 
\begin{bmatrix}
    K_1 & 0 \\
    0 & K_2
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
+ 
\begin{bmatrix}
    F \\
    F
\end{bmatrix}
= 
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_g \\
    \ddot{u}_g
\end{bmatrix}
$$

where $\ddot{x}_i(t)$, $\ddot{x}_f(t)$, $u_i$, $m_i$, $C_i$ and $K_i$ are the horizontal acceleration, velocity, displacement, mass, equivalent viscous damping and stiffness, respectively, of the $i^{th}$ frame ($i = 1, 2$); $\ddot{u}_g$ is the ground acceleration and $F$ is the pounding force given by the particular numerical model while the frames are in contact. The stiffness and equivalent viscous damping were calculated from the period and damping ratio of frames measured by simple snap-back tests. The damping ratio varied according to the initial displacement, the tightness of the joints and the mass of the frame, so an average damping ratio of 0.016 was adopted for numerical analysis.

The maximum deflections of the reference frame without pounding were calculated when subjected to various time histories. It was observed that the numerical and experimental deflection maxima in all the cases were very close. There was difference of just 2%-5% between the two values.

Figure 3.3 shows the comparison of experimental $\mu_{max}/u_{max}$ of the reference frame with those calculated by numerical models under positive JDS 2 time history. The results were similar for JDS 1 and El-Centro time histories. The numerical results are plotted only for the best performing contact element stiffness value while the results obtained for both the values of coefficient of restitution are shown for comparison. The contact elements were not very sensitive to change in stiffness of contact but there were some variations. The linear viscoelastic and modified linear viscoelastic elements agreed best with the
experimental results when $e = 0.4$, and $k = 100$ times maximum of stiffness of the participating structures. The nonlinear contact elements showed the best agreement with the shake table results when the experimentally measured contact element stiffness was used. The amplification was much greater when coefficient of restitution was 0.6 than when it was 0.4. The sensitivity of numerical models to the coefficient of restitution was not consistent. The variation of $e$ affected the amplification much more in some cases than in others.

The results from NZDS ground motions were significantly different from those observed for the JDS and El-Centro loadings. Figure 3.4 presents the experimental and numerical $\mu_{max}/u_{max}$ of the reference frame under positive NZDS 2 loading. The numerical results are for $e = 0.4$ and $k$ is the same as explained above. The experiment showed a reduction in maximum deflection in all pounding cases but numerical simulations did not reflect this. The trend was similar for all the contact element models for this time history applied through both directions. The displacement amplification was even greater when $e = 0.6$ for which the results are not shown here. Thus, for the frame configurations considered in this experiment, the commonly adopted value of coefficient of restitution predicted much more severe response than the experimentally determined value, which itself predicted higher displacement response than that from the experiments. The results due to NZDS 1, not presented here, were similar.

Figure 3.5 shows the predictive error in $\mu_{max}/u_{max}$ from different numerical models subjected to the JDS 2 loading. All the models overestimated the displacement amplification in Cases 1, 3, 5 and 8 but the amplification in Cases 4, 5, and 7 were predicted quite well by all except Hertzdamp model. The Hertzdamp model seldom underestimated the amplification but tended to overestimate severely in Cases 5, 6 and 8. Other models sometimes underestimated the amplification but the error was under 5%; the modified Hertzdamp model was as accurate as the linear viscoelastic model while the modified linear viscoelastic model generally had slightly higher error.

None of the models predicted consistently better than all others in all configurations; hence a simple criterion was applied to choose the best model. Figure 3.6 shows the plot of square root of sum of square (SRSS) of errors across all cases for all time histories. Four of the models perform very similarly but Hertzdamp model has almost twice the error magnitude.
Figure 3.3 Experimental and numerical $\mu_{\text{max}}/\mu_{\text{max}}$ of the reference frame under positive JDS 2 ground motion; (a) Linear Viscoelastic, (b) Modified Linear Viscoelastic, (c) Nonlinear Viscoelastic, (d) Hertzdamp, and (e) Modified Hertzdamp models.
From the discussion in Section 3.1, it can be seen that LVe is the simplest model and other models are significantly more complicated. However, the results show that the added complexity in newer models do not increase their accuracy. Thus linear viscoelastic contact element can be used in future numerical modelling of floor to floor pounding of two structures without any loss in accuracy. The model underestimates the severity of response amplification in some cases but the safety factors available in design of structures should be able to withstand a variation of 10%. Additionally, this model can already be found implemented as a linear viscoelastic link in most of the commercial FE software.
However, all the models significantly overestimate the displacement amplification in several cases for JDS ground motions and in almost every case for NZDS ground motions. If implemented in current form e.g. to develop measures for mitigating pounding effects, the models can substantially increase the cost. The tests also showed that the coefficient of restitution which is a main component of all the models is not well defined. The measured value was two thirds the value adopted in many numerical studies. When a value of 0.6, suggested by numerical studies, was implemented in analysis, displacement amplification was severely overestimated by all the models (Figure 3.3). Thus, significant refinement of the models is necessary before they can be recommended for use by design engineers.

![Figure 3.6 SRSS of errors in $\mu_{\text{max}}/\mu_{\text{max}}$ from various models.](image)

3.4 Summary

Numerical simulations were carried out for TBP configurations presented in Chapter 2. Five numerical pounding force models were selected and the simulation was carried out for each pair of frame under the five ground motions employed in the experiment. The amplification of maximum displacement due to pounding was calculated for the numerical models and compared with the experimental values. From the comparison of predicted and actual amplification, following conclusions can be drawn:

- The Hertzdamp model is the most inaccurate in these simulations. This was expected because the model was originally formulated for highly elastic impacts. However, other models also showed significant errors in predicting displacement amplification. Under JDS excitations the models showed reasonable agreement
with experiments. However, in several configurations they can overestimate the amplification by 20%. For more severe NZDS ground motions, the amplifications were significantly overestimated in almost all configurations. The linear viscoelastic element showed slightly better performance than modified linear viscoelastic, nonlinear viscoelastic and modified Hertzdamp models even though the latter models have much higher computation requirements. Thus, linear viscoelastic model is recommended for pounding simulations among the five models studied. However, significant refinement is necessary in all the models before they can be employed in designing retrofitting measures against pounding damage.

- None of the numerical models are sensitive to the change in contact element stiffness when predicting displacement amplifications. The models are sensitive to the change in coefficient of restitution, but no pattern could be identified. The numerical models were highly conservative for coefficient of restitution 0.6 which is the most commonly adopted value in past studies. Much better agreement with test results was observed when the experimentally measured value of 0.4 was adopted.

- The shake table results show agreement with the numerical models that the amplification of maximum displacement is more when the test frame collides with a more flexible adjacent frame and displacement is reduced when it collides with more rigid frames. But the numerical models mostly predict a higher amplification than observed in the experiments. Thus the current numerical models seem to overestimate the severity of displacement amplification due to seismic pounding.

- The displacement amplification due to pounding is higher if the difference in natural periods is higher. The mass difference between the two frames did not show as much effect as the difference in natural frequency.

- All numerical models were unable to simulate the reduced displacement for some of the time histories. A pattern was observed where the ground motions that produced higher displacement without pounding experienced less amplification due to pounding. Further experiments with many more ground motions will be required to ascertain if this is due to frequency content of the ground motions or a general behaviour.
Chapter 4

Numerically exact viscoelastic force model: derivation and validation

Pounding is an impact between two masses, e.g. building floors, walls, bridge decks or abutments, resulting into a transfer of momentum between the two masses. Most studies on building pounding idealize the participating structures as lumped-mass SDoF or MDoF systems. A number of methods have been proposed for including the pounding effects in time-history analysis of such structures, which can be broadly classified into two categories, viz. stereomechanical methods (impulse momentum principle) and impact element methods.

Stereomechanical methods interrupt the time-history analysis each time an impact is observed, update the velocities of the colliding structural masses according to the impulse momentum principles of classical physics and resume the time-history analysis. The loss of mechanical energy is incorporated by means of an experimentally determined or logically assumed value of coefficient of restitution, $e$, which is defined as the ratio of relative separation velocity to relative approach velocity of the colliding masses. These methods have a robust physical theory and historical research behind them; however, for structural engineers their main disadvantage is the inability to estimate the impact force due to the assumption of instantaneous change in momentum of colliding masses. Thus, these models can be used for global effects on structures but not for predicting local damage. Athanassiadou et al. (1994) utilized stereomechanics in parametric investigation of pounding between a row of buildings idealized as SDoF systems and subjected to spatially varying ground motion.

Impact element methods introduce a combination of gap and link elements between the colliding masses to simulate pounding (Figure 4.1). The pounding force will develop only if the gap closes, i.e. the relative displacement of the masses toward each other is more than the space between them and the elastic spring is compressed. The force is calculated
from the expression provided for the particular model being employed. If the contact is assumed elastic, the dashpot is omitted and the spring may be linear or nonlinear. If the models incorporate energy loss, the dashpot with equivalent viscous damping is added parallel to the spring. Experiments have found most impacts between large masses to be inelastic (Goldsmith 2001) and thus elastic models are seldom used for pounding analysis. The impact element method has often been used in literature because of the capability of the model to calculate pounding forces. Unfortunately, up to today the stiffness of the contact location cannot be predicted with certainty. Hence, several pounding models have been developed and the selection of the models in a study is often subjective.

Rigorous theoretical derivation exists only for elastic, nonlinear impact between spherical and cylindrical bodies and is known as the Hertz contact law (Goldsmith 2001) as shown in Equation 4.1.

\[ F = k \delta^{3/2} \]  

where at any time \( t \), \( F \) is the impact force, \( k \) is the stiffness of the nonlinear spring and \( \delta \) is the apparent penetration as described in Section 3.1.1.

Although attempts have been made to include viscous and plastic energy loss in the Hertz model theoretically (Pao 2009), such solutions are not yet appropriate for engineering applications. Instead, (Hunt and Crossley 1975) proposed a model (Equation 4.2) where the energy lost during impacts is accounted for by the damping that depends on the relative penetration and its rate of change.

\[ F = k \delta^{3/2} + \zeta \delta^{3/2} \dot{\delta} \]  

where \( \dot{\delta} \) is the rate of change of relative penetration and \( \zeta \) is the damping constant.
Because of its versatility, the model can be generalized as shown in Equation 4.3 and such models are known as Hunt-Crossley models (Zhang and Sharf 2004).

\[ F = k \delta^n + \zeta \dot{\delta}^n \dot{\delta} \]  \hspace{1cm} 4.3

Several approximate solutions for the damping constant \( \zeta \) in Equation (4.3) have been proposed for various mechanical engineering applications ranging from forces in ball-bearings to robotics, space docking and knee replacement simulations (Zhang and Sharf 2004). The solution for the damping constant in Equation 4.4, derived by (Lankarani and Nikravesh 1994), was first proposed for simulation of structural pounding by Muthukumar and DesRoches (2006) as ‘Hertzdamp model’.

\[ \zeta = \frac{3k(1-e^2)}{4\delta_0} \]  \hspace{1cm} 4.4

Ye et al. (2009) found that the approximations involved in the derivation of the damping constant introduced significant errors and the model was ineffective in maintaining the energy loss. For example, if \( e = 0.8 \) is used for calculation, the energy loss in the result is equivalent to \( e = 0.847 \). If \( e = 0.1 \) is assumed, the effective \( e \) is 0.665 which is an error of 565\%. A new formulation for \( \zeta \) was derived, as shown in Equation 4.5, and the performance was shown to be significantly improved. The authors acknowledged that there is still some error in the value of effective \( e \) when compared to the value assumed for the computations but the difference is very slight and much reduced from the previous formulation; effective \( e \) is 0.5769 at \( e = 0.6 \) and 0.0687 at \( e = 0.1 \) which is an error of 31\%. The authors recommended that the corrected relationship is suitable for use only for the impacts with \( e > 0.4 \) because the deviation is only 10\% at \( e = 0.4 \), and the relationship degrades rapidly below that.

\[ \zeta = \frac{8k(1-e)}{5e\delta_0} \]  \hspace{1cm} 4.5

Ye et al. (2009a) has presented a modified Kelvin impact model which is identical to linear form of Hunt-Crossley model (Equation 4.6). Because of the approximations involved in derivations, the damping \( \zeta \) is slightly different (Equation 4.7) from that of nonlinear form (Equation 4.5). The model shows slightly better accuracy than Equation 4.5 as at \( e = 0.3 \) the error in calculating the actual \( e \) value determined from the after impact velocities is 10\% while Equation 4.5 produces 15\% error.
This chapter presents an exact solution for the damping constant in Hertzdamp model, which removes these errors, and makes it suitable for all values of $e$. Since the relationship is derived for a generic $n$ in Equation 4.3, the relationship is applicable for linear as well as any nonlinear models. Henceforth, Hertzdamp model will refer to Equation 4.3 with $n = 3/2$ and a damping constant derived in Section 4.1. The linear form of Equation 4.3 is called linear Hunt-Crossley model hereafter. A comparison is presented in Section 4.1 between the post-impact errors in coefficient of restitution from the proposed solution and from Ye et al. (2009a, 2009b). The prediction of two forms of Hunt-Crossley models i.e. linear ($n = 1$) and Hertzdamp ($n = 3/2$) are compared with experimentally measured impact force between concrete bodies. The comparison also includes three other existing impact models and will show that the linear and Hertzdamp forms of Hunt-Crossley models are closer than other models in their prediction. Finally, a pounding simulation between two bridge segments is carried out using four pounding force models: Linear viscoelastic, nonlinear viscoelastic, linear Hunt-Crossley and Hertzdamp models. The resulting displacements and pounding forces are presented and discussed.

## 4.1 Derivation of exact expression for damping constant $\zeta$

During pounding, two bodies of mass $m_1$ and $m_2$ are moving in the same line with uniform velocities $v_1$ and $v_2$ respectively, as shown in Figure 4.1. If $v_1 > v_2$, a collision will occur. While in contact, the two bodies will first deform at the contact surface for some time-interval. When the maximum deformation is reached, the two bodies will start to regain their original shape. Finally, separation occurs, with velocity $v_2 > v_1$. During impact, the bodies impart equal impulsive forces on each other, producing velocity changes. Examples of typical $F$ - $t$ and $F$ - $\delta$ curves are provided in Figure 4.2. At the beginning of impact, $t_0$, $\delta = 0$, and $\dot{\delta} = \dot{\delta}_0$ while at the end, $t_f$, $\delta = 0$, and $\dot{\delta} = \dot{\delta}_f = - e \dot{\delta}_0$. There will be a time, $t_m$, when $\delta = \delta_m$ which is the maximum relative penetration. $\delta_m$ will not necessarily coincide with the occurrence of maximum force and the maximum force or maximum penetration do not necessarily occur halfway through the time of contact.
According to stereomechanics, the energy loss during impact, $\Delta E$, is provided by:

$$\Delta E = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (1 - e^{-\delta_0}) \dot{\delta}_0^2$$  \hspace{1cm} (4.8)

From Figure 2, it can be seen that the energy loss during contact can be expressed in terms of work done by the contact force as:

$$\Delta E = W_{\text{comp}} - W_{\text{rest}}$$  \hspace{1cm} (4.9)

where $W_{\text{comp}}$ is the work during the compression phase of impact, and $W_{\text{rest}}$ is the work during the restitution phase. Substituting $W = \int F \, d\delta$ in Equation 4.9,

$$\Delta E = \int_{\delta=0}^{\delta=\delta_m} (k \dot{\delta}^n + \zeta \ddot{\delta}^n \dot{\delta}) \, d\delta - \int_{\delta=0}^{\delta=\delta_f} (k \dot{\delta}^n + \zeta \ddot{\delta}^n \dot{\delta}) \, d\delta$$  \hspace{1cm} (4.10)

The total work done by the elastic component of the force is zero as shown by the nonlinear elastic curve in Figure 2(b) (dotted, monotonically increasing line). Thus Equation 4.10 reduces to:

$$\Delta E = \int_{\delta=0}^{\delta=\delta_m} (\zeta \ddot{\delta}^n \dot{\delta}) \, d\delta - \int_{\delta=0}^{\delta=\delta_f} (\zeta \ddot{\delta}^n \dot{\delta}) \, d\delta$$  \hspace{1cm} (4.11)

For the first expression on the RHS of Equation 4.11, integrating by parts,
The variables $\delta$ and $\dot{\delta}$ are coupled in Equation 4.12. One of the variables has to be expressed in terms of the other for the solution of Equation 4.11.

The impact Figure 1 can be represented as shown in Figure 4.3, by the transformation of the reference system. The Equation of motion for such a system will be:

$$m_e \ddot{\delta} + \zeta \delta^n \dot{\delta} + k \dot{\delta}^n = 0$$  \hspace{1cm} 4.13

where $m_e = \frac{m_1 m_2}{m_1 + m_2}$ is the equivalent mass of the system and $\ddot{\delta}$ is the rate of change of relative penetration velocity.

![Figure 4.3 Equivalent impact element model of contact](image)

The exact solution of this Equation, derived as shown in Appendix 4.A, is given by:

$$\frac{\zeta}{m_e \left( n + 1 \right)} \delta^{n+1} = k \ln \left| \frac{\dot{\delta} + k}{\zeta} \right| \dot{\delta} + C$$  \hspace{1cm} 4.14

where $C$ is the constant of integration.

At the start of contact, $t = t_0$, $\delta = 0$, and $\dot{\delta} = \dot{\delta}_0$. Thus,

$$C = \dot{\delta}_0 - \frac{k}{\zeta} \ln \left| \frac{\dot{\delta}_0 + k}{\zeta} \right|$$  \hspace{1cm} 4.15

At the end of contact, $t = t_f$, $\delta = 0$, and $\dot{\delta} = \dot{\delta}_f$. Substituting,
Numerically exact viscoelastic force model: derivation and validation

\[ C = \dot{f} \frac{k}{\zeta} \ln \frac{\dot{f}}{\dot{f} + \frac{k}{\zeta}} \]  \hspace{1cm} 4.16

Substituting the value of \(C\) from Equation 4.15 into Equation 4.14 for the compression phase of the impact,

\[ \frac{\zeta}{m_c} \frac{\delta^{n+1}}{n+1} = \dot{\delta}_0 - \frac{k}{\zeta} \ln \frac{\dot{\delta}_0 + \frac{k}{\zeta}}{-\ln \frac{\dot{\delta}_0 + \frac{k}{\zeta}}{\dot{\delta} + \frac{k}{\zeta}}} \]  \hspace{1cm} 4.17

Substituting the value of \(\frac{\delta^{n+1}}{n+1}\) in Equation 4.12 and simplifying, the first expression of the RHS of Equation 4.11 is found to be,

\[ \int_{\delta=0, \dot{\delta}=\dot{\delta}_0}^{\delta=\delta_m, \dot{\delta}=0} (\zeta \dot{\delta}^n \dot{\delta}) d\delta = m_c \left[ \frac{k^2}{\zeta^2} \left( \ln \left| \frac{\dot{\delta}_0 + \frac{k}{\zeta}}{\dot{\delta} + \frac{k}{\zeta}} \right| - \ln \left| \frac{k}{\zeta} \right| \right) + \frac{\dot{\delta}_0^2}{2} - \frac{\dot{\delta}_f^2}{2} \right] \]  \hspace{1cm} 4.18

Similarly obtaining the value of the second expression of the RHS of Equation 4.11 from Equation 4.16, substituting the two values into Equation 4.11 and simplifying,

\[ \Delta E = m_c \left[ \frac{k^2}{\zeta^2} \left( \ln \left| \frac{\dot{\delta}_0 + \frac{k}{\zeta}}{\dot{\delta} + \frac{k}{\zeta}} \right| - \ln \left| \frac{k}{\zeta} \right| \right) + \frac{\dot{\delta}_0^2}{2} - \frac{\dot{\delta}_f^2}{2} \right] \]  \hspace{1cm} 4.19

Equating the expressions for \(\Delta E\) in equations 4.8 and 4.19, and simplifying,

\[ (1 + e) = \frac{k}{\zeta \dot{\delta}_0} \ln \left| \frac{\frac{k}{\zeta \dot{\delta}_0} + 1}{\frac{k}{\zeta \dot{\delta}_0} - e} \right| \]  \hspace{1cm} 4.20

In practice, Equation 4.20 is an implicit equation since the objective is to solve for the quotient \(\frac{k}{\zeta \dot{\delta}_0}\) from which the value of \(\zeta\) can be obtained. Once this is achieved, force can be evaluated from Equation 4.3.

It is of note that Equation 4.20 can also be derived by equating the values of \(C\) from Equations 4.15 and 4.16. The lengthier derivation has been provided as it establishes the validation of viscoelastic energy loss. To the authors’ best knowledge, this is the first time
that a solution for the damping constant in Hunt Crossley model has been derived from both work-energy considerations and force considerations. Other approaches in the literature have utilized approximations and achieved a solution considering either force or energy approach, but not both.

Zhang and Sharf (2004) provides Equation 4.21 as the solution for Hunt-Crossley models, where $e = 1 - \alpha \dot{\delta}_0$. It was found that it yields the same values of $\zeta$ as Equation 4.20 although the argument of the logarithmic term in Equation 4.20 is based on absolute values while in Equation 4.21, the argument is based on algebraic values. Hence, in Equation 4.21 occurrence of negative values of argument destabilize the computations; while the argument is always positive in Equation 4.20. Note that Zhang and Sharf (2004) assumed a linear relationship between $e$ and $\dot{\delta}_0$ to arrive at the solution while the current derivation makes no such assumptions.

\[
k \ln \left[ \frac{\zeta \dot{\delta}_0 + k}{\zeta(1 - \alpha \dot{\delta}_0)\dot{\delta}_0} \right] - 2\zeta \dot{\delta}_0 + \alpha \dot{\delta}_0^2 = 0 \tag{4.21}
\]

To validate the proposed solution a numerical simulation was carried out. A steel sphere of mass 1 kg hitting on an identical sphere with velocity 1 m/s was performed. For different values of coefficient of restitution ranged from 0.01 to 1, the impact force and after impact velocities of the two bodies were calculated from Equation 4.3. The actual coefficient of restitution $e_{\text{post}}$ was calculated as the ratio of relative velocity after impact to the relative velocity before impact. The error in $e$ was computed as $\frac{\text{original } e - e_{\text{post}}}{\text{original } e}$. The simulation was performed for both the linear Hunt-Crossley and the Hertzdamp forms of Equation 4.3, with damping calculated from Equation 4.20. The simulations were repeated with Equation 4.5 for modified Hertzdamp model (Ye et al. 2009b) and Equation 4.7 for modified Kelvin impact model (Ye et al. 2009b), i.e. linear Hunt-Crossley model. The stiffness $k$ for $n = 3/2$ were calculated (Chau et al. 2003) to be 18 GN/m$^{3/2}$, while that for linear models was obtained to be 0.14 GN/m by linear regression of force against deformation in elastic Hertz contact. The percentage errors in post impact coefficient of restitutions are shown in Figure 4.4. The solution for $\zeta$ proposed in this work being numerically exact, had a maximum discrepancy of 0.07% in original $e$ and $e_{\text{post}}$, while those from Equations 4.5 and 4.7 have a large error, especially for lower values of $e$. 

Selection of existing viscoelastic force models for comparison

Comparison of the original Hertzdamp model against experimentally recorded impact force time histories was carried out by (Mahmoud et al. 2008) in which the model performed rather poorly. No such assessment has been carried out after the correction introduced by Ye et al. (2009b). Thus it was considered desirable to similarly evaluate the linear Hunt Crossley model (LHC) and the Hertzdamp (Hd) model using the value of the damping constant $\zeta$ from Equation 4.20. The performance of these models was compared against three other existing impact models, viz. Linear viscoelastic model (LVe, Equation 3.1), Modified linear viscoelastic model (MLVe, Equation 3.2) and Nonlinear viscoelastic model (NlVe, Equation 3.3). These models were described in Section 3.1 and the description has not been repeated here.

The LVe model has two discontinuities, at the start and the end of the contact. This causes a sudden jump in force from 0 to $c\dot{\delta}$ at the beginning and some tensile force at and near the end of contact. The modified linear viscoelastic model was introduced specifically to remove this tensile force from the linear model, as such negative force had not been seen in any experimental recordings of the pounding force, but it introduced an even bigger jump at the beginning of contact due to an increase in $c$. The impact element approach of pounding analysis rejects the assumption in stereomechanics that the contact is instantaneous and thus the initial jump seems unacceptable. The nonlinear viscoelastic model has no such jumps at the beginning or the end of the impact but the transition from deformation phase to restitution phase of contact is not smooth. No such discontinuities occur in the Hunt-Crossley models.
There are several models specialized only for particular cases such as the impact model presented by Polycarpou et al. (2013) for rubber shock absorbers which have not been included in the comparison with these more general models.

### 4.3 Assessment of pounding models using experimental data

van Mier et al. (1991) conducted a series of tests on the longitudinal impact of a freely swinging concrete mass against a concrete rod. The force time-history so recorded for the impact of a 570 kg mass with spherical contact surface on the plane face of a concrete rod at a speed of 0.5 m/s, was digitized for this study. The coefficient of restitution was calculated to be 0.19. In this time-history, it was found that the contact terminated when the relative penetration was still 1 mm. Since van Mier et al. had observed a flattening of the contact surface after impact, this permanent deformation seemed realistic. However, for simplicity, the numerical simulations for pounding force models were carried out so that the contact terminated when the relative penetration became zero.

The results achieved by van Mier et al. (1991) have previously been employed by Jankowski (2005), Mahmoud and Jankowski (2011) and Mahmoud et al. (2008) for evaluation of numerical force models. The coefficient of restitution was assumed to be 0.6 or 0.65 and the stiffness of the numerical models was estimated by trial and error so that the simulated maximum force equalled the recorded maximum force. A similar procedure was used for the current study. However, instead of the assumed valued, the coefficient of restitution was calculated directly from the measured impact force time history.

The normalized error (NE) in prediction, as defined in Equation 4.22 was calculated for each model. The values are presented in Table 4.1 with the damping ratio and normalized error of prediction. The simulated and experimental force time histories are presented in Figure 4.5.

\[
NE = \sqrt{\frac{\sum_{i=1}^{n} (F_m - F_i)^2}{\sum_{i=1}^{n} F_i^2}}
\]

where \(F_m\) is the force predicted by the model and \(F_i\) is the force recorded in the experiment.
Table 4.1 Stiffness, damping ratio and normalized error for various pounding models.

<table>
<thead>
<tr>
<th>Pounding Models</th>
<th>Stiffness ($k$)</th>
<th>Damping ($\xi$)</th>
<th>NE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVe</td>
<td>$7.83 \times 10^7$ N/m</td>
<td>0.47</td>
<td>61.44</td>
</tr>
<tr>
<td>MLVe</td>
<td>$6.77 \times 10^6$ N/m</td>
<td>1.64</td>
<td>70.36</td>
</tr>
<tr>
<td>NlVe</td>
<td>$1.81 \times 10^8$ N/m$^{3/2}$</td>
<td>2.84</td>
<td>63.55</td>
</tr>
<tr>
<td>Hd</td>
<td>$7.97 \times 10^8$ N/m$^{3/2}$</td>
<td>10.80</td>
<td>13.83</td>
</tr>
<tr>
<td>LHC</td>
<td>$3.07 \times 10^7$ N/m</td>
<td>10.80</td>
<td>30.77</td>
</tr>
</tbody>
</table>

Figure 4.5 Experimental and simulated impact forces.

It can be seen that the exact Hd model (Equation 4.20) has the least normalized error followed by LHC while all other models show at least twice as much error. The difference in results from those obtained in the previous studies (i.e. Jankowski 2005; Mahmoud and Jankowski 2011; Mahmoud et al. 2008) can be attributed to the large difference in the adopted values of coefficient of restitution.

The LHC model shows much better performance than the other two linear models because of the removal of the initial discontinuity. Researchers in bridge pounding have shown a marked preference towards using linear force models (e.g. Zhu et al. 2002; Raheem and Shehata 2009; Li et al. 2012), perhaps due to the apparent equivalence between axial stiffness of the girders and the stiffness of the contact element (Ye et al. 2009b; Malhotra 1998). The linear Hunt-Crossley model can maintain the equivalence, while removing the irrational discontinuities at the beginning and the end.
4.4 Numerical simulation of pounding structures

Muthukumar and DesRoches (2006) have presented the results of pounding simulation with the original HertzDam model and compared the results with other models, at coefficient of restitution 0.6 and 1.0, while no such comparison was found after the modification by Ye et al. (2009b). The performance of linear Hunt-Crossley model (Equation 4.7) was compared against linear viscoelastic model by Ye et al. (2009a). However, no comparison with nonlinear models was found in the literature. Thus an example of pounding simulation with these models is presented here and compared with the linear and nonlinear viscoelastic models.

The LHC model has been included in the comparison despite poorer performance than HertzDam in the experimental assessment, for three reasons: (i) The applicability of Hertz contact law to the contact between flat surfaces of prismatic elements is in doubt because the contact conditions do not satisfy any of the assumptions inherent in the derivation of the theory (Goldsmith 2001), (ii) the experimental data was measured for the contact of a spherical surface with a flat surface which is one of the conditions for which Hertz contact law has been derived, and by definition is superior to any linear model for such contact, and (iii) the apparent analogy between longitudinal stiffness of bridge girders and slabs and the stiffness of the contact element, due to which the stiffness, $k$, for the linear elements is simpler to identify than the stiffness for the nonlinear elements (Ye et al. 2009b; Malhotra 1998).

4.4.1 Structural model

The structural arrangement employed by Malhotra (1998) for pounding of bridge decks at an expansion joint was analysed. The lumped-mass model is shown in Figure 4.6. Four pounding models were used, namely: nonlinear viscoelastic model, linear viscoelastic model, and linear and nonlinear Hunt-Crossley models. The time step of 0.1 ms was applied for the analysis, with four values of coefficient of restitution, i.e. 0.2, 0.4, 0.6 and 0.8. The impact model stiffness ($k$) for the girder impacts are provided by Muthukumar and DesRoches (2006): $2.8 \times 10^{10}$ N/m$^{3/2}$ for nonlinear models and $4.3 \times 10^9$ N/m for the linear models. The bridge decks were subjected to first 8 seconds of El Centro ground motion shown in Figure 4.7.
4.4.2 Results

The displacement response of the left girder due to pounding is shown in Figure 4.8. There is significant scatter in the results for the same configuration of structures and damping derived from the same coefficient of restitution. The scatter increased with time due to the cumulative effects. The results from the right bridge segment are similar. Figure 4.9 shows the acceleration response of the same deck for $e = 0.4$. The impact events did not show any significant scatter in time but there were significant differences in magnitude. It was observed that LHC model produced the strongest acceleration while LVe model produced the least even though the same impact stiffness was employed. Hd produced slightly lower accelerations than the NlVe. For $e = 0.8$, NlVe produced least acceleration and Hd produced the most. In contrast, NlVe produced highest acceleration for $e = 0.2$. 
Figure 4.8 Displacement of left girder, $e = 0.4$.

Figure 4.9 Acceleration time history of left girder, $e = 0.4$: (a) LHC, (b) LVe, (c) Hd, and (d) NIVe.
Figure 4.10 Calculated pounding forces at second and seventh pounding event with
(a) LHC, (b) LVe, (c) Hd, and (d) NIVe pounding models
The pounding forces due to second and seventh pounding events are shown in Figure 4.10. For all the impact models, the second pounding occurs at approximately 2.73 s. The relative impact velocities were 1.22, 1.21, 1.20 and 1.19 m/s for coefficient of restitutions 0.2, 0.4, 0.6 and 0.8 respectively. The onset of seventh pounding shows appreciable variance according to the coefficient of restitution. For LHC and Hd models, the impact velocities were 0.49, 0.60, 0.67 and 0.73 m/s for \( e = 0.2, 0.4, 0.6 \) and 0.8 respectively. For LVe model, the impact velocities were 0.53, 0.62, 0.69 and 0.73 m/s while for NIVe model, they were 0.49, 0.59, 0.67, 0.73 m/s respectively. A comparative study of the pounding force time histories during two impacts, for the four models and four values of \( e \) shows that the pounding force development in Hunt-Crossley models is less sensitive to an increase in damping, i.e. a decrease in \( e \) value, than the other two models. The shapes of the force time-history in the linear and nonlinear viscoelastic models are increasingly skewed with the decrease in \( e \). For \( e = 2 \), the peak force is achieved almost instantly for both these models. In the absence of suitable large scale impact experiments, it is difficult to decide which models are more accurate, but the authors suggest that the larger masses should also produce impact force time-histories similar to small impacts, and recommend the Hunt-Crossley models for pounding analysis.

4.5 Summary

Linear and nonlinear Hunt-Crossley models have been used in the past for simulating pounding forces. It was initially developed to accommodate viscous energy loss in elastic Hertz contact law in the analysis of small spherical masses and used extensively in mechanical engineering applications. Various approximate solutions for the deformation proportional damping have been developed. Muthukumar and DesRoches initially proposed the nonlinear Hunt-Crossley model (Hertzdamp model) for pounding application using one of these approximate solutions, derived by Lankarani and Nirkavesh. Ye et al. improved the Hertzdamp model so that it could be used for a wider range of coefficient of restitution. The linear Hunt-Crossley model (modified Kelvin impact model) was also proposed by Ye et al. However, all these models have difficulty in simulating the post-impact behaviour properly.

An exact solution for damping in the Hunt-Crossley models has been provided. While previous works used an approximation of the energy loss, the current work solved the
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differential equation of motion to obtain the exact damping. The solution presented here has been shown to be valid for all values of the coefficient of restitution.

The force prediction performance of the model was compared with other models, namely: Kelvin-Voight model, nonlinear viscoelastic model and modified linear elastic model, using a published impact force time-history from a past experimental study on collision of concrete bodies by van Mier et al. It was observed that the normalized error was least for the Hertzdamp model. The linear stiffness form of the Hunt-Crossley model also performed much better than other models which showed at least twice its normalized error. Thus, it is concluded that Hunt-Crossley models give much better agreement with measured impact force than other existing models employed in pounding analysis.

Additionally, a numerical pounding simulation of bridge decks was carried out with four models, and it was observed that despite the presence of some divergence in the displacement time-histories, the overall displacement response from all the models was similar. For the same simulation however, pounding force and acceleration time-histories show significant differences. The development of the pounding force with the time due to Hunt-Crossley models is similar to those recorded in previous small scale experiments but the shapes are surprisingly skewed in the linear viscoelastic and nonlinear viscoelastic models. For the same impact element stiffness but different coefficients of restitution, the nonlinear viscoelastic model has the most variation in peak pounding force while the Hertzdamp model has the least. The results from linear models lie in-between.

**Appendix 4.A**

The Equation of motion is,

\[ m\ddot{\delta} + \zeta \dot{\delta} + k \delta = 0 \]  

which has the same form as Lienard Equation,

\[ \ddot{x} + f(x) \dot{x} + g(x) = 0 \]  

Substituting \( y = \dot{x} \), Equation (ii) will convert to Abel’s Equation of the second form, i.e.

\[ yy' = f_0(x) y + f_1(x) \]  

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Substituting $z = \int x \, dx$ in Equation (iii), it will be converted to the canonical form,

$$w \, w' - w = R(z) \quad (iv)$$

where $R(z) = f_1(x) / f_0(x)$

The exact solution of Equation (iv), if $R(z)$ is a constant equal to $A$, is (Polyanin and Zaitsev 2003):

$$z = w - A \log_e |w + A| + C \quad (v)$$

where $C$ is the constant of integration.
Chapter 5

Impact of RC slabs: influence of mass, velocity and contact surface geometry

Floor to floor pounding between buildings principally occurs between reinforced concrete floors. However, there are very few experimental studies published on the subject. This chapter presents the results of a parametric investigation of impact between RC slabs. First a brief review of experimental literature on impact of reinforced concrete structures is presented, followed by the experimental setup and results.

Van Mier et al. (1991) subjected a 20 m long, stationary concrete beam to a longitudinal, horizontal impact by a square concrete bar. The stationary beam was 250 x 250 mm in cross-section and partially fixed at the other end by a buffer. A parametric study was conducted by varying the mass, velocity and contact surface geometry. The mass was varied by using two different strikers, i.e. 700 x 600 x 600 mm resulting in 570 kg mass and 650 x 400 x 400 mm resulting in 290 kg mass. Two velocities 0.5 m/s and 2.5 m/s were considered. Three striker surfaces i.e. hemispherical, conical and plane surface and two beam contact surfaces i.e. plane and corrugated were used. For each specimen, repeated impacts were conducted until the striker’s surface was extensively damaged. The force time history was obtained from the accelerometers attached to the mid-point of the beam. A peak value was observed in the force time history after which it remains constant until finally decaying. The rise time to the peak value and fall time from peak to zero was determined by the impact velocity, mass and contact surface geometry. Van Mier et al. calculated the stiffness of the hemispherical contact to be 44 kN/mm^{3/2}, which is significantly less than the theoretical value of 201.73 kN/mm^{3/2} obtained from the Hertz contact law (Muthukumar and DesRoches 2006).

An early work by Leibovich et al. (1994) involved conducting a shake table simulation of pounding between two RC slabs supported by steel frames. The coefficient of restitution was 0.35 for an impact velocity of 40 cm/s. It is not clear whether the measured coefficient of restitution was for concrete-concrete or concrete-timber-concrete pounding because a
timber plank was introduced at the contact interface to reduce the excessive out-of-plane movement.

Papadrakakis and Mouzakis (1995) conducted a shake table simulation of pounding between two storey RC frames. The slabs of the stiffer frame had a rectangular protrusion at the midpoint facing the second frame. The simulation was conducted without separation under a ramped sinusoidal motion. The excitation and fundamental frequencies of the flexible building were the same. Pounding increased the maximum displacement of the stiff frame by a factor of 2.5 approximately. In contrast the maximum displacement of the flexible frame was almost halved in comparison with that without pounding. The peak acceleration increased up to six-fold in both frames. The study compared the experimental results with that obtained from Lagrange multiplier method of pounding simulation. It was concluded that the numerical simulation produced good agreement with the experimental solutions. However, the results diverged substantially toward the end of the time window considered in both pounding and non-pounding cases.

Jankowski (2010) measured the coefficient of restitution when spheres, made of steel, concrete, timber and ceramic, were dropped on a plane surface of the same material. For each material, the impacts were repeated for seven different velocities for three different diameters. A cubic polynomial best represented the relationship between coefficient of restitution and impact velocity. The coefficient of restitution was independent of the mass of the ball. For the same series of tests, Jankowski (2007) found that the contact stiffness of the nonlinear viscoelastic force model (Section 3.1.3) was independent of velocity but varied with the falling mass. These experiments do not pay due regard to the influence of the undefined target mass. Jankowski (2010) also carried out a shake table simulation of pounding between two steel frames with the four types of contact interfaces i.e. concrete, steel, timber and ceramic. In comparison to experimental results, numerical simulations with impact velocity-dependent coefficient of restitution were slightly closer compared to those achieved using a constant coefficient of restitution. The constant coefficient of 0.65 is based on assumptions from previous studies e.g. Anagnostopoulos (1988).

Guo et al. (2009) carried out a shake table investigation on pounding reduction of base-isolated highway bridges. The bridge decks were loaded with RC masses but the deck and the impact interface were made of steel. Previous numerical studies, e.g. Zhu et al. (2002) and Muthukumar and DesRoches (2006), assumed the impact element stiffness of linear
viscoelastic model is equal to the axial stiffness of the decks. However, Guo et al. found that the impact stiffness is about 100 times smaller than the axial stiffness.

Leibovich et al. (2012) measured coefficient of restitution and impact acceleration for plane-ended circular concrete rods suspended as pendulums. Two bars of 1000 mm and bars of 1000 mm and 500 mm were considered. The coefficients of restitution calculated were 0.5 to 0.7 for the pounding between equal bars. In contrast to numerical predictions, e.g. Malhotra (1998) and Cole et al. (2011), the coefficient of restitution is higher when the longer bar was hit by the shorter bar. The recorded acceleration showed multiple peaks with subsequent oscillation around the zero value.

A shake table simulation of pounding between a base-isolated building against a moat-like wall was presented by Masroor and Mosqueda (2012b). The building was modelled by a steel frame. A concrete block was used as contact element with the surrounding wall. Two types of walls were used: (i) concrete with soil backfill and (ii) stiff steel plate without backfill. The wall substantially affected the characteristics of the impact force recorded by load cells placed between the concrete block and the mat foundation of the building. Numerical simulation by Masroor and Mosqueda (2012a) found that the contact force time history had two phases: (i) the impact phase where the contact force develops and (ii) the quasi-static passive-pressure phase between the wall and building. Thus, the impact force alone was not sufficient to separate the two structures.

The literature review above revealed an absence of experimental studies where both the pounding masses and the contact components are constructed of RC. In addition, no study on the effects of mass and contact surface geometry has been reported. This paper addresses the identified knowledge gap, i.e. the consequence of the mass of the pendulums, the relative velocities of impact and the geometry of the contact surface, for coefficient of restitution and peak impact acceleration. The experiment results are essential for validating numerical models for estimating the pounding force for structural seismic design.

5.1 Experimental setup

The experiment involved impact between two pendulums (Figure 5.1). The pendulum was made of single or multiple concrete slabs of size 550 x 500 x 120 mm. To avoid damage to the slabs removable contact elements were used. All the contact elements had a base of size 550 x 100 x 120 mm (see Figure 5.4). For all tests, the struck slab had a plane contact
surface, and the striker slab had spherical, cylindrical or rectangular surface. Table 5.1 lists the geometries of the contact elements.

The impact tests for each mass combination were performed at eight different velocities. The acceleration and displacement of the slabs were measured.

![Figure 5.1 Schematic drawing of the test setup](image)

5.1.1 **Preparation of specimens**

A design mix of cement : sand : 12.5 mm nominal aggregate of 1 : 1.5 : 3 respectively by volume was used. This is the nominal mix for M20 concrete recommended by IS:456 (Indian Bureau of Standards 2000). The M20 concrete is the most common floor-slab concrete in the PhD candidate’s home in South Asia, where buildings in almost all cities are built without seismic gap and are susceptible to pounding. Based on the candidate’s seismic assessment experience, this is also the most commonly used mix for old buildings in New Zealand, which are susceptible to pounding. The moulds for the slabs and attachment are shown in Figure 5.2. A steel reinforcement of 6 mm ϕ @ 120 mm c/c was used for each slab (Figure 5.3(a)). Six slabs and twenty contact elements were cast in ten different concrete batches. For each batch, three cylinder specimens were prepared and subjected to compression test after 28 days. The compressive strength of the cylinders was between 28 and 34 MPa.
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Figure 5.2 Moulds for (a) concrete slab and cylinder specimens and (b) cylindrical (left) and hemispherical (right) impact surfaces

Table 5.1 Contact elements

<table>
<thead>
<tr>
<th>Type</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>100 mm diameter</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>100 mm diameter</td>
</tr>
<tr>
<td></td>
<td>150 mm length</td>
</tr>
<tr>
<td>Plane</td>
<td>100 mm width</td>
</tr>
</tbody>
</table>

Figure 5.3 Casting of (a) RC slab and (b) contact elements and cylinder specimens
The pounding elements were cast with soft wooden dowels (Figure 5.4) of 12 mm diameter. These dowels were drilled out after curing was complete. A face of each slab was drilled and fitted with φ 12 mm anchor bolts to attach the pounding elements. Since the bolt heads would protrude and prevent contact for cylindrical and plane contact elements, a larger timber dowel was used near their face (see the bottom right contact element in Figure 5.4). This large dowel extended 20 mm in the concrete. The bolt heads fit within these larger holes and thus contact between striker and struck surfaces is achieved.

5.1.2 Test Setup and Instrumentation

The pendulums were constructed by suspending the concrete slabs from an overhead girder with four 6 mm φ steel cables (Figure 5.5). The cables were fitted with turnbuckles for levelling and alignment. The cables were connected to a 50 x 50 x 6 mm angle at their lower end. Two angles are placed on the top of the slabs, and two on the bottom. Two 10 mm φ bolts were used to hold the slabs and the angles together. The length of the pendulum is considered to be the vertical distance between the fastener of the cable with the angles and the fastener at the overhead girder. For single slabs, this length is 1.5 m. The period of any of the two pendulums matched that obtained from theoretical formula, 

\[ T = 2 \pi \sqrt{\frac{l}{g}} \]

A total of nine mass combinations (Table 5.2) were considered. Figure 5.5 illustrates two mass extreme mass combinations considered in the test.
Each pendulum had a ± 10 g accelerometer attached to the bottom face of the slab, under the centre of the impacting side. The accelerometers have a frequency range from 0.3 Hz to 10 kHz. The displacement of the slabs was measured by a laser sensor focused on the opposite (non-impacting) end of the slab. The sensors have a precision of 0.3 mm. They are not sensitive enough to measure the impact-induced relative displacement. The slab velocities are calculated from expressions for motion of a simple pendulum. An attempt was made to measure pounding force from strain gauges affixed to the reinforcement nets of the slab. However, the strain gauge results were below the noise threshold in the case of the impact velocities considered. A sampling rate of 10 kHz was adopted for the tests.
Table 5.2 Slab combination

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of slabs in pendulum</th>
<th>Striker mass (kg)</th>
<th>Struck mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1-m1</td>
<td>Single</td>
<td>109</td>
<td>108</td>
</tr>
<tr>
<td>m1-m2</td>
<td>Double</td>
<td>109</td>
<td>199</td>
</tr>
<tr>
<td>m1-m3</td>
<td>Triple</td>
<td>109</td>
<td>290</td>
</tr>
<tr>
<td>m2-m1</td>
<td>Single</td>
<td>200</td>
<td>108</td>
</tr>
<tr>
<td>m2-m2</td>
<td>Double</td>
<td>200</td>
<td>199</td>
</tr>
<tr>
<td>m2-m3</td>
<td>Triple</td>
<td>200</td>
<td>290</td>
</tr>
<tr>
<td>m3-m1</td>
<td>Single</td>
<td>291</td>
<td>108</td>
</tr>
<tr>
<td>m3-m2</td>
<td>Double</td>
<td>291</td>
<td>199</td>
</tr>
<tr>
<td>m3-m3</td>
<td>Triple</td>
<td>291</td>
<td>290</td>
</tr>
</tbody>
</table>

The impact was initiated by displacing the right mass (Figure 5.1) and allowing it to swing back and strike the left slab. Henceforth, the displaced slab is called striker slab and the second slab is called struck slab. Both masses move after the first impact and multiple impacts occur as they swing back and forth. The results were discarded if out-of-plane motion was observed after the first impact. However, the masses always showed out-of-plane motion after second or third impact. Since only the first impact was considered and processed in the results, the out-of-plane movement resulting from subsequent impacts has no relevance. Additionally, perfect contact could not be achieved for cylindrical and plane contact surfaces and thus, most of the tests were conducted only for the 100 mm φ hemispherical attachments. For comparison, some impacts were conducted for the case m1-m1 with the other pounding elements shown in Table 5.1, despite the presence of out-of-plane movement.

The initial displacement, \( d \) of the striker was between 1 and 7 cm. At the beginning of tests with each impact element, a few unrecorded impacts were conducted at \( d = 7 \) to 9 cm so as to remove asperities at the contact interface. This ensured repeatability of the test because otherwise the stiffness of the contact element may change at higher impact velocities due to damage accumulation. The hemispherical contact heads were slightly flattened (approx. 0.5 - 1 mm) while no effect was seen in cylindrical and plane contact surfaces. No other permanent deformations were observed at the end of the tests (Figure 5.6).

Before the tests, the striker was pulled back and allowed to impact with the adjacent slab several times, so that a small area at the contact location was flattened. This ensured that the contact location was sufficiently uniform to allow repeatability. The pounding location
for each slab was monitored and photographed after each test and no other damage was observed.

![Spherical pounding head at the end of the tests.](image)

Figure 5.6 The spherical pounding head at the end of the tests.

### 5.2 Results and discussion

The displacement and acceleration time-histories, when the striker slab was displaced 58 mm, are shown in Figure 5.7. The displacement shows two phases: (i) the period when the striker was displaced and stabilized at 58 mm displacement and (ii) the period when the striker was released and impacts and subsequent separations occurred. The velocity of the striker before the first impact was calculated to be 0.145 m/s. The immediate post-impact velocities of the striker and the struck slabs were 0.02 m/s and 0.126 m/s, respectively. The coefficient of restitution was 0.73. The largest acceleration magnitude of subsequent impacts decreased with each impact (Figure 5.7(b)). As anticipated the magnitude is due to decreasing impact velocity.

![Time history of displacement and acceleration](image)

Figure 5.7 Time history of (a) displacement of the slabs and (b) acceleration of the striker, initial displacement of the striker, $d=56$ mm
The accelerations show a surprisingly high negative component (approx. 1.85 g). As shown in Figure 5.8, the acceleration due to five successive impacts in the same test show minimal phase shift. The periodicity of the acceleration reflects a characteristic of impact-induced standing wave in the slabs.

![Figure 5.8 Impact accelerations for the first five impacts in Figure 5.7](image)

5.2.1 Effect of mass variation

![Figure 5.9 Effect on (a) coefficient of restitution and (b) peak impact acceleration, due to equal change in the mass of both pendulums](image)

Fig. 5.9 shows the effect of equal change in both masses. The coefficient of restitution decreased with the increasing mass. For the case with equal slabs e.g. case m1-m1, $e$ varies slightly with the impact velocity but there is no particular trend. For the impact of single slabs, $e$ increases with velocity. For the heaviest masses, i.e. case m3-m3, $e$ increased up to
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0.1 m/s impact velocity, remained almost constant between 0.1 to 0.15 m/s and thereafter decreased. For case m2-m2 no significant variations is observed. As anticipated, the peak acceleration magnitude is almost linearly proportional to the velocity in all three cases.

The effect of unequal variation in mass on coefficient of restitution is presented in Figure 5.10. The striker is kept constant and the mass of the struck pendulum is varied. The results are shown for (i) the lightest striker i.e. one slab and (ii) the heaviest striker i.e. three slabs. In both cases, $e$ decreased as the mass of the struck pendulum increased. For the lightest striker there is an increasing trend with higher velocity for all three mass combinations. For the heaviest striker an increase in impact velocity either had no effect or produced a smaller $e$. A comparison between case m1-m3 (Figure 5.10(a)) and case m3-m1 (Figure 5.10(b)) shows very different values for $e$. Thus, $e$ is affected not only by the total mass of the system but also by the ratio of the striker and the struck mass.

Similar to the case of equal masses, the peak acceleration in unequal masses shows almost linear relationship with impact velocity and appears independent of participating masses (Figure 5.11).

![Figure 5.10 Variation in coefficient of restitution for three different masses of struck slab when striker has (a) only one slab and (b) three slabs](image)

The impact response is significantly affected by the initial resistance of the struck mass. When a smaller mass is struck by larger mass (Figure 3.1, curve m3-m1), the coefficient of
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restitution decreases with velocity as observed in previous studies. However, when the struck mass is increased, a larger part of the pre-impact kinetic energy of the first mass is spent in overcoming the initial resistance to movement; for low velocities, this results in quite small coefficient of restitution. The coefficient increases for higher impact velocities, because the energy required to overcome the initial resistance is a smaller part of the pre-impact kinetic energy. The full curve i.e. increase, peak and decline, was observed for cases m3-m3 and m3-m2. The declining curve was not reached cases m1-m1, m1-m2 and m1-m3 etc.

Figure 5.11 Peak impact acceleration under various mass and velocity combinations for strikers with (a) a single slab and (b) three slabs

5.2.2 Effect of change in contact surface

Substantial out-of-plane motion was observed when the cylindrical and plane contact elements were used. The cylindrical and plane elements produce a line and area contact, respectively, while the spherical element produces a point contact. An instantaneous full contact cannot be achieved even with very careful preparation and execution; the contact develops over time and causes out-of-plane movement. The coefficient of restitution with cylindrical and plane contact elements was significantly smaller (Figure 5.12(a)) compared to that with spherical contact element. For low impact velocities the plane element produced only one impact. The pendulums then moved together. The coefficient of
restitution and peak impact acceleration (Figure 5.12(b)) increased with velocity all three surfaces.

The first acceleration pulse obtained with the three contact elements for the case m1-m1, at approximately 0.15 m/s impact velocity, is shown in Figure 5.13. The acceleration from spherical contact shows smooth lines while some small deviations are seen in cylindrical contacts. The acceleration obtained with plane contact surface has clearly visible discontinuities. Thus, the results establish that the details of the geometry of the contact surface play a significant role in the development of impact-induced response.
5.3 Summary

A series of impacts between reinforced concrete slabs as pendulums was conducted. A parametric investigation of coefficient of restitution and impact-induced peak acceleration was carried out by varying the velocities, pendulum masses and the contact surface geometry. A total of 95 impacts were performed.

The investigation reveals:

- The coefficient of restitution was influenced by the total mass of the striker and the struck pendulums, striker mass and the ratio of the masses. The coefficient decreased with increasing total mass.

- For similar impact velocity, the coefficient increased with a heavier striker mass; and decreased in the case of lighter striker impacting a heavier pendulum.

- With increasing impact velocity the coefficient of restitution did not show any general trend. For an impact of equal masses, the coefficient increased with velocity for the lightest mass considered. However, for heavier mass this is not the case. The coefficient also increased with velocity when a lighter mass struck a heavier mass, and decreased with velocity when the striker was heavier.

- The peak acceleration increased almost linearly with impact velocity and was almost insensitive to the striker, struck and total mass.

- The geometry of the impact element plays a significant role in the development of impact-induced peak accelerations.

The results of the experiments conducted in this study show for the first time that the coefficient of restitution, defined in conventional impact between two particle masses, cannot be applied in impact between components of structures. This is because the definition of the coefficient of restitution is based solely on velocities of participating masses without considering the distribution of masses, damping and stiffness along the structural members as well as structural boundary conditions.
Chapter 6

Limitations in state-of-the-art and introduction to the Sears impact model

The preceding chapters have shown significant problems with existing lumped-mass models for simulation of building pounding. This chapter discusses the more significant problems in the current state-of-the-art analysis of floor to floor pounding of buildings. The numerical simulations are discussed first, followed by a discussion of experimental studies. Finally, a short introduction is provided to the Sears impact model (Sears 1912) which was first proposed for building pounding simulation by Khatiwada et al. (2013d). It is shown that the model provides almost identical response to finite element analysis and it can address some of the limitations discussed in this study.

6.1 Limitations in numerical simulations

For numerical simulation of building pounding, a link connection along with a gap element is inserted between the two buildings at probable or predetermined pounding locations (Figure 6.1). While the buildings are not in contact, they can be analysed by any time-history integration method. When the gap between the floors is closed, an impact force model is employed to simulate the transfer of momentum. The models can be broadly classified into two types: (i) lumped-mass models and (ii) distributed-mass models.

6.1.1 Lumped-Mass Models

The lumped-mass models assume each floor mass to be concentrated at a point (Figure 6.1(a)) so that the effect of each collision is instantaneous over the whole floor. Some studies (e.g. Davis 1992; Maison and Kasai 1992; Filiatrault et al. 1995; Chau and Wei 2001) employ elastic models containing only a spring in series with the gap element. However, experiments have shown that there is always some loss of energy during impact (Goldsmith 2001). Thus, most current models include a viscous damper arranged parallel with the elastic spring (Figure 4.1). When the gap closes, pounding force is calculated with a numerical force model of the general form shown in 6.1, where $e$ is the coefficient of
restitution defined as the ratio of relative velocities after and before impact. For lumped-mass modelling, $e$ has to be defined at the start of the simulation as it is the factor determining the extent of viscoelastic loss of energy.

$$F_j = f_1(\delta_j) + f_2(e, \delta_j, \dot{\delta}_j)$$ \hspace{1cm} 6.1

where $F_j$ is the pounding force between the $j^{th}$ stories,

$$\delta_j = u_{1,j} - u_{2,j}$$ is the relative compression of the pounding stories,

$$\dot{\delta}_j = \dot{u}_{1,j} - \dot{u}_{2,j}$$ is the relative velocity of the pounding stories, and

$e$ is the coefficient of restitution.

![Figure 6.1 Model for analysis of building pounding. (a) Lumped-mass and (b) distributed-mass model](image)

Many pounding force models have been proposed using lumped-mass approximations. For the purpose of this study, five models (Table 6.1) are considered. It needs to be mentioned that there are many other force models (e.g. Valles-Mattox and Reinhorn 1996; Komodromos et al. 2007; Polycarpou et al. 2013), which will be referred in the following discussions where relevant.

The lumped-mass models are developed from stereomechanics (Goldsmith 2001), in which the contact is assumed to be instantaneous and energy loss is defined by $e$. In contrast to stereomechanics the lumped-mass model will produce a contact force with a finite duration. Though stereomechanics has been used in the past for building pounding simulation (Athanassiadou et al. 1994), it is rarely used nowadays because: (i) as no contact duration
is assumed, i.e. contact duration is zero seconds, the force time-history cannot be obtained, and (ii) the contact is resolved as a transfer of momentum, which requires a sudden change in velocity of the entire impacting floors. The local damage around the contact location cannot be considered in the absence of force. The sudden change in velocity necessitates recalculation of coefficients for subsequent time-history calculation which requires special coding in finite element software. The LVE model was formulated to facilitate an uninterrupted time-history analysis whose energy dissipation, for a certain value of $e$, was equal to that given by stereomechanics (Anagnostopoulos 2004). The other viscoelastic models were derived to predict pounding force in addition to the energy loss. They have much higher computational requirements. The limitations of lumped-mass models are discussed in the following subsections.

Table 6.1 Numerical force model

<table>
<thead>
<tr>
<th>Model</th>
<th>Proposed by</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear viscoelastic (LVE)</td>
<td>Anagnostopoulos (1988)</td>
<td>3.1, 3.1(a), 3.1(b)</td>
</tr>
<tr>
<td>Modified linear viscoelastic (MlVE)</td>
<td>Mahmoud and Jankowski (2011)</td>
<td>3.2, 3.1 (a), 3.2(b)</td>
</tr>
<tr>
<td>Nonlinear viscoelastic (NlVE)</td>
<td>Jankowski (2005)</td>
<td>3.3, 3.3(a), 3.3(b)</td>
</tr>
<tr>
<td>Hertzdamp (Hd)</td>
<td>Khatiwada et al. (2014a)</td>
<td>4.2, 4.20</td>
</tr>
<tr>
<td>Linear Hunt-Crossley (LHC)</td>
<td>Khatiwada et al. (2014)</td>
<td>4.6, 4.20</td>
</tr>
</tbody>
</table>

6.1.1.1 Uncertainties in Contact Stiffness

From Table 6.1, the contact stiffness $k$ is an integral part of all viscoelastic force models. Unfortunately, there is no accepted method of determining its value. The only analytical formula for $k$ was derived by Hertz (Goldsmith 2001) for contact of a curved surface against another curved surface or a plane surface. The NlVe and Hd models are derived by adding viscous damping to Hertz’s formula. However, the value of $k$ calculated from Hertz’s formula did not produce a reasonable agreement with experimental forces for contact of a spherical surface with a flat surface (Mahmoud et al. 2008). Additionally, the Hertz law has no formulation for contact of flat surfaces like building floors. Stiffness of linear elements has been related to the axial stiffness of slabs (Zhu et al. 2002; Ye et al. 2009a), however, impact experiments(e.g. Guo et al. 2009; Guo et al. 2012) have shown such values to be overestimated.
The effect of $k$ when a SDoF system with the fundamental period of 0.5 s was subjected to pounding with a SDoF with the period of 0.75 s under El-Centro ground motion is shown in Figure 6.2. $e$ is assumed to be 0.8. The amplification of maximum displacement is the ratio of maximum deflection of the building with and without pounding. The influence of the contact stiffness $k$ on the displacement time-history is presented in Figure 6.3. The corresponding effects on pounding force, relative penetration and relative velocity are shown in Figure 6.4. The duration of the most flexible contact is almost fifteen times that of the stiffest one. While the two softer configurations had only one impact in the time-window of 2.2 s to 2.52 s (solid and dotted lines in Figure 6.4(a)), the stiffer elements had two impacts in the same time window (dashed and dashed-dotted lines). The relative penetration and relative velocity, both, show significant and inconsistent dispersion at the end of the time window.

![Figure 6.2 Variation of the stiff structure's response with $k$](image1)

![Figure 6.3 Displacement time history for the SDoF of higher relative stiffness for four values of $k$.](image2)
Figure 6.4 Effect of contact stiffness on (a) pounding force, (b) relative deformation (δ) and (c) relative velocity.

Anagnostopoulos (1988) observed that the contact stiffness has considerable effect on pounding force and accelerations of the mass during contact, but negligible effect on the displacement response. However, this has led to the use of stiffness calculated from small-scale experiments (e.g. van Mier et al. 1991) to building pounding (Ohta et al. 2006; Jankowski 2008b). Anagnostopoulos’s conclusion was based on k located between points A and B, while the model in Jankowski (2008) lies well to the left of point B. Jankowski (2008a) found four times higher stiffness for concrete to concrete impact of 1.75 kg mass than for van Mier’s experiments on 570 kg mass (van Mier et al. 1991), which conclusively demonstrates that the stiffness obtained from small-scale experiments cannot be applied directly to building pounding.

The inability to specify contact stiffness has two major consequences: (i) the computed pounding force is not reliable so local damage at the pounding location cannot be estimated;
only the global response and the resulting possible damage to the building can be calculated, however, (ii) even this global response can be significantly over- or under-estimated depending on the value of $k$ (Figure 6.2). Since none of the existing models produce reliable results an extra computation effort cannot be justified. Thus, the simplest LVE model can be used for building pounding simulations.

6.1.1.2 Consequence of Existing Pounding Force Models
Several studies have attempted to identify the best performing lumped-mass models either by comparing forces obtained from single impact experiments (e.g. Jankowski 2005; Mahmoud and Jankowski 2011; Mahmoud et al. 2008) or displacements (e.g. Khatiwada et al. 2013a) and velocity (e.g. Jankowski 2005) response obtained from shake table simulations. In the comparisons of impact forces, the LVE model performed as well as other models. This is surprising because it records a tensile force near the end of contact, which cannot be observed in experiments. Additionally, the model was derived solely to equate the viscoelastic energy loss with that from the coefficient of restitution, without considering the value of pounding forces - while other models were derived specifically to predict pounding forces. The shake table evaluation of the numerical models by Khatiwada et al. (2013) was inconclusive, because all considered, lumped-mass models produced almost identical results which could be both very close to and very different from the experimental results.

![Figure 6.5 Contact forces simulated with various models (a) e = 0.8 and (b) e = 0.2](image)

Figure 6.5(a) shows the pounding force evaluated from the impact between two 100 kg masses with a relative impact velocity of 1 m/s and $e = 0.8$. The $k$ for the models was optimized so as to produce identical peak force. Figure 6.5(b) shows the impact forces for identical parameters when $e = 0.2$. Thus, the models differ significantly when predicting the impact force. However, when two SDoF systems of period 0.5 s and 0.75 s were
subjected to pounding under first 8 seconds of El-Centro ground motion, the displacements obtained using these five models are almost identical for each of $e = 0.8$ and $e = 0.2$ (Figure 6.6). Thus, displacement of pounding structures seems to be a function of $e$ only. Pounding models have no effect on displacement amplification which means that the least computationally demanding model \textit{i.e.} LVE can be used for all simulations. A computationally intensive model should be used only for pounding force prediction. Unfortunately, as discussed in section 2.1.1, any model can only be accurate if the correct value of $k$ is used – however this is currently very difficult to predict accurately.

![Figure 6.6 Displacement of the relatively stiffer SDoF system under el-Centro ground motion: (a) $e = 0.8$ and (b) $e = 0.2$](image)

6.1.1.3 Effect of support elements

The effect of supports of impacting structural members in development of the pounding force is not included in both lumped and distributed-mass models. In the lumped-mass models, the effect of columns is assumed to be included in the time-history integration. However it has not been experimentally verified. Additionally, most of the numerical studies assume a high $k$ value so that the system lies to the right of point A in Figure 6.2. Anagnostopoulos (1988) advised modifying $k$ for a high structural stiffness so that the $k$ is
always 20 times the collective stiffness of the considered storey. Unfortunately, the suggestion of using higher stiffness does not reflect the effect of the actual supporting system, e.g. floor with columns or floor with unreinforced masonry shear walls, on the pounding force development.

6.1.1.4 Model inaccuracies

Several studies employed models beyond the limitations resulting from the assumptions made in their respective derivations – an example is the attempts to calculate impact force from LVe model (e.g. Shakya and Wijeyewickrema 2009) even though the model produces improbable results. Similarly, attempts were made to simulate plane surface contacts in building pounding using the Hertz model originally derived only for spherical and cylindrical surfaces. Thus, there is no means to calculate stiffness in Hd or NlVe models except by back-calculations from the measured impact forces.

Valles-Mattox and Reinhor (1996) and Komodromos et al. (2007) modified the LVE model by introducing corrections in Equation 3.1 while keeping Equations 3.1(a)-(b) unchanged. These changes removed the tensile part of the impact force, but the models are computationally inconsistent, i.e. if post impact velocities are calculated by assuming \( e = 0.4 \) and effective \( e \) is checked by calculating the ratio of final relative velocity to initial relative velocity. The result is different from 0.4. Figure 6.7 shows these errors in simulation. Hertzdamp model by Muthukumar and DesRoches (2006) have similar inconsistency which is explained in detail by Ye et al. 2009a.

![Figure 6.7 Errors in e introduced by modifications to the LVe model](image)

Ohta et al. (2006) and Shakya and Wijeyewickrema (2009) simulated pounding in the finite element program SAP2000 by placing a viscoelastic link in series with a gap element (Figure 6.8). However, a gap element can only transmit positive force while a viscoelastic
link can generate negative as well as positive forces. The studies did not explore the ramifications of avoiding the transfer of tensile force with such arrangements.

Figure 6.8 Impact element adopted by Ohta et al. (2006) and Shakya and Wijeyewickrema (2009)

6.1.1.5 Coefficient of restitution

The Coefficient of restitution is a term used to define the elasticity of collision. Elastic impact \((e = 1)\) means that there is no loss of kinetic energy during impact. However, a completely inelastic impact \((e = 0)\) does not necessarily mean all kinetic energy is lost. Figure 6.9 shows the relationship between percentage loss of kinetic energy and coefficient of restitution. Thus, it does not seem rational to measure \(e\) for a ball falling on the ground because of gravitational acceleration and then to adopt the same value in the simulation of building pounding. This is because in the case of building pounding, similar masses are involved and no gravitational acceleration will be activated in the pounding direction. Furthermore, the tendency to assume all the energy is lost due to viscous damping, means that the effect of the contact surface is ignored even though van Mier et al. (1991) had demonstrated that damage development at contact locations can significantly affect the energy loss. Additionally, in conjunction with section 2.1.3, a building’s resistance against motion is affected not only by its mass, but also by its stiffness. Structures can store mechanical energy as strain energy and release it as kinetic energy later. Since the definition of \(e\) only includes velocity, it only forces a conservation of kinetic energy and not the total mechanical energy. Thus, \(e\) based models cannot be applied to floor to columns pounding, as columns are of smaller mass and the kinetic energy from the floors is stored as strain in the columns. Also, if the loss of energy is affected by the contact stiffness analogous to the effect of mass ratio (Figure 6.9), the current models cannot reflect such behaviour due to their reliance on \(e\).

Anagnostopoulos (1988) adopted \(e = 0.65\) for building pounding simulations. Perhaps because this value is from the most cited paper on building pounding, it has been disseminated as the most widely used and accepted value of \(e\). Thus, several studies (e.g.
Jankowski 2005; Mahmoud et al. 2008) have assumed $e = 0.65$ for analysing the experimentally measured impact forces, even though $e$ can actually be calculated from the impact force time-history by equating change in momentum of each mass to the area under the force time-history. If the initial velocities of the masses are known, the final velocities and $e$ can be calculated. However, $e = 0.6$ to 0.65 was used for evaluating a concrete to concrete impact, even though the force time-history suggests $e = 0.19$ (Khatriwada et al. 2014a). Similarly, Jankowski (2005) assumed $e = 0.65$ for a steel to steel impact force from Goland et al. (1955), however, the impulse-momentum principal provides $e = 1.5$, which is physically impossible. The experiment is discussed in more detail in section 3.

Figure 6.9 Effect of $m_2/m_1$ and $e$ on the loss of kinetic energy.

Thus, coefficient of restitution is inarguably the main limitation in pounding simulation with lumped -mass models. It forces the conservation of kinetic energy, i.e. the post impact velocity must produce the same $e$ irrespective of the strains in the structure. However, the value of such an important coefficient is also very uncertain. When it is combined with the uncertainty in $k$, the scatter in displacement results for the simulation defined in section 2.1.1 is shown in Figure 6.10. To the author’s knowledge, many studies have conducted separate parametric studies on $e$ and $k$. However, none of them have varied them both at the same time. Because of the computational demands, the ratio $k/\max(k_{s1}, k_{s2}) > 100$ has seldom been considered when varying $e$. From Figure 6.10, if the considered $k/\max(k_{s1}, k_{s2}) < 20$, $e$ has little effect on displacement amplification. However, for higher values of $k$ the effects of $e$ are quite pronounced. The results in Figure 6.10 are obtained from simulations assuming that the structures remain elastic during an earthquake. A deviation
might also be attributed to the structural yielding (Anagnostopoulos 1988) or spatial variation of ground motion (Athanassiadou et al. 1994) as shown in past studies.

![Figure 6.10 Combined effect of $e$ and $k$ on displacement amplification of stiffer SDoF system.](image)

6.1.2 Distributed-mass models

The distributed-mass models consider the building floors as flexible diaphragms (Figure 6.1(b)). The effect of pounding needs time to spread through the floors. These models are either directly derived from the wave propagation theory of impact, i.e. St. Venant’s principle (Goldsmith 2001) or validated against the theory. The St. Venant’s theory states that when two bars collide, stress waves are generated in both bars at the contact interface. These stresses propagate towards the far ends, changing the velocity of the sections through which they pass. Consequently, they get reflected back from these far ends. Separation occurs when the reflected wave in the shorter bar arrives at the contact interface. The contact duration and impact force for bars of identical material and cross-sectional area are shown in Figure 6.11. $L_s$ is the length of the shorter bar, $c$ is the propagation velocity of compressive waves in the material, $(v_1 - v_2)$ is the relative velocity of the bars at the time of impact, $\rho$ and $A$ are the density and cross-sectional area of the bars, respectively.

![Figure 6.11 Pounding force and contact duration $T_s$ from wave theory of impact.](image)
Malhotra (1998) analysed the pounding between two bridge segments with identical material and cross-sectional properties, and with 1000 modes of axial vibration. The study considered that the two colliding decks behave as if they are fused together as a single mass. The modes of vibrations are calculated for the fused bar. They remain fused as long as the axial strain at the contact location remains compressive. The decks are assumed to be separated when the strain becomes tensile. The axial force at $t = 1 \text{ ms}$ for a simulated impact between steel bars of 4 m and 6 m length and 0.1 m$^2$ cross-sectional area is shown in Figure 6.12. The relative velocity at the time of impact is 1 m/s. The pounding force is the axial force at the contact interface, i.e. 4 m from the left end of the fused bar. It can be seen that, the pounding force approaches that given by the wave theory of impact (Figure 6.11) when the number of modes are increased. However, the peak acceleration tends towards infinity (Figure 6.13), which does not seem realistic. Malhotra suggested calculating the coefficient of restitution for the pair of bridge decks under consideration from which their post-impact velocities can be obtained, and incrementally modifying the velocities of the bridge decks to achieve the calculated post-impact velocities over the contact duration.

![Figure 6.12 Axial force in the fused bar 1 ms after the initiation of impact.](image)

Watanabe and Kawashima (2004) showed that finite element modelling of the distributed masses can produce results similar to that from wave theory, provided the bridge decks are discretized into smaller segments and the contact stiffness is considered equal to the stiffness of these segments. The finite element analysis produces displacements and velocities similar to that from the wave theory of impact. However, the force and acceleration shows significant oscillations not seen in wave theory results (Figure 6.14). When the number of discrete segments is increased, the impact force is closer to that
predicted by wave theory. However, the oscillations are also more intense. Similar to the results presented by Malhotra (1998), the acceleration becomes progressively higher as the number of discrete segments is increased (Figure 6.15).

Figure 6.13 Acceleration in the 4 m bar 1 ms after the initiation of impact.

Figure 6.14 Pounding force when a 4 m steel bar strikes a 6 m steel bar with a relative velocity of 1 m/s and the number of discrete segments in the 4 m bar is equal to (a) 2, (b) 10, (c) 100 and (d) 200.

Cole et al. (2011) extended the work of Watanabe and Kawashima (2004) to building pounding simulations. Unlike Malhotra (1998), the model can simulate pounding between flexible diaphragms of different materials and unequal cross-sectional area. Similar to Malhotra (1998), the model cannot be directly applied to distributed masses with supporting structures. Therefore, an equivalent lumped-mass model was formulated which produces the same post-impact velocity and contact duration as the distributed model. The force is calculated from 6.2.
\[ F = \frac{(v_1 - v_2)}{\frac{T_1}{m_1} + \frac{T_2}{m_2}} \]

where \((v_1 - v_2)\) is the relative velocity at the initiation of impact,

\(T_1\) and \(T_2\) are the time required for the stress waves to travel twice through the shorter and longer diaphragm, respectively, and

\(m_1\) and \(m_2\) are the mass of the shorter and longer diaphragms, respectively

Figure 6.15 Acceleration of the centre of mass of the 4 m steel bar striking a 6 m steel bar at relative velocity of 1 m/s when the number of discrete segments in the 4m bar is equal to

(a) 2, (b) 10, (c) 100 and (d) 200.

The most significant weakness of the model is that it predicts a negative coefficient of restitution for certain configurations (Figure 6.16), which has no physical meaning. The second shortcoming is the absence of any proposal to apply it directly to structural pounding. Instead, the coefficient of restitution has to be calculated first, which is then used to calculate stiffness and damping of the equivalent lumped-mass model. Third, the theory assumes that the diaphragms separate when the stress wave has travelled twice through the shorter diaphragm, i.e. after time \(T_1\). However, in the sample analysis of one-story structures, the contact duration exceeded even \(T_2\). In contrast, the contact duration for two-story pounding was less than or equal to the \(T_1\). This suggests that the columns stiffness can significantly affect the collision time. Finally, like previous distributed-mass models, the predicted forces and acceleration show significant oscillations.
Limitations in state-of-the-art and introduction to the Sears impact model

Distributed-mass models are not affected by the uncertainties in $e$ as previously discussed. In contrast, they produce a value of $e$ which can be employed to evaluate their accuracy. The distributed-mass models assumed a full contact over a plane interface with no eccentricity. These conditions are almost impossible to duplicate in a laboratory environment, since any irregularity in contact surface or eccentricity in axis will mean that the compressive waves will not develop uniformly across the cross section as predicted. Thus, they cannot be directly evaluated against impact forces recorded from the experiments. However, inferences can be made based on contact duration, coefficient of restitution, and other responses. Unfortunately, the impact results from classical wave theory have mostly been disproved by past studies in physics (e.g. Tschudi 1921; Wagstaff 1924) because they produce a constant duration for impact of two given bars, while impact experiments have proven the duration to be based on velocity. Also, as discussed in Section 6.1.1, the applications can be limited due to the complexity of adding more factors like supports, material damping or cross-sectional properties. For instance, the model by Malhotra (1998) cannot incorporate unequal cross-sections while that by Cole et al. (2011) does not include the effects of collision damping.

6.2 Limitations in experimental simulations

The experimental studies on pounding are mostly limited to the elastic range to avoid permanent damage to the models. The models are placed on a shake table and subjected to harmonic or earthquake ground motions. The various measured responses of the structures can be compared with the numerical model under consideration. Some studies (e.g. Filiatrault et al. 1995; Papadrakakis and Mouzakis 1995; Zhu et al. 2002; Chau et al. 2003), have attempted to validate certain numerical models. Others (e.g. Khatiwada et al. 2013a),
tried to identify the best performing lumped-mass model. Guo et al. (2012) calculated the impact stiffness for different models from shake table pounding. Similarly, van Mier et al. (1991), Jankowski (2010) and Leibovich et al (2012) measured pounding forces and accelerations in single impacts. Jankowski (2010) and Leibovich et al (2012) also calculated the coefficient of restitution in the impacts. Some impact tests were conducted, in this PhD research, between concrete slabs as well as steel beams varying mass, velocity, contact surface and support conditions. The results indicate that the testing procedure demonstrated anomalies which led to the realization of several limitations in experimental simulation of pounding which cannot be observed in non-pounding cases. These limitations are presented in the following subsections. Possible occurrence of similar behaviour in the previously published experiments is also discussed. It has to be noted that these are neither the exhaustive list of all the experimental simulations, nor are the limitations proven to be definitely present. The presented material is indicative only, drawn from the student’s experiences, and there may quite possibly be other explanations for these observations.

### 6.2.1 Instrumentation

![Figure 6.17 Difference in impact accelerations recorded simultaneously by two instruments placed at the same location.](image)

Figure 6.17 shows the acceleration measured from the same impact of two 104 kg RC slabs by two different accelerometers placed at the same location. The low-pass accelerometer has bandwidth of 0 to 50 Hz while that of the broadband accelerometer ranges from 0.3 to 10 kHz. It can be seen that if only the low-pass accelerometer is used, the contact duration would be significantly overestimated while the contact force would be underestimated. The periodicity of acceleration after the first peak would also have not been detected.
It was discussed in section 6.1.1.5 that the experiments by Goland et al. (1955) produced a value of $e$ greater than one. The test involved dropping a steel sphere of 5/32 inch diameter on a steel hemisphere mounted upon a steel beam. The drop height was 2 inch. The impact forces were measured by a barium titanate sensor. A force time history was presented in the study. Three previous studies, viz. Jankowski (2005), Mahmoud and Jankowski (2011) and Mahmoud et al. (2008), have digitised the time-history to evaluate the predictions of various force models. The current authors found the area under the force time-history to be $6.55 \times 10^{-4}$ Ns. The mass and initial velocity of the sphere are 0.26 gram and 1 m/s, respectively. Thus, from the impulse momentum principle, the velocity at the end of the impact should be 1.566 m/s in the opposite direction. However, this is physically impossible as the kinetic energy after impact becomes larger than the kinetic energy before impact. Similar results have also been observed by the authors. Figure 6.18 presents the impact of an 8 kg steel beam with a stationary calibrated impact hammer. The acceleration of the beam was multiplied by its mass and compared with the force recorded by the impact hammer. A good agreement can be observed between the two results. However, when the impulse was applied to the beam’s momentum, the post impact velocity was found to be higher than the velocity at the start of the impact. The behaviour was consistent for several impact tests. It is improbable that two completely different instruments, i.e. an accelerometer and a calibrated impact hammer, produced identical errors. However, it is possible that the dynamic force measuring instruments calibrated for a certain setup cannot be effectively employed for all cases.

Figure 6.18 Force measured for a steel beam hitting a fixed impact hammer.

Figure 6.19 reproduces the impact force and relative impact velocity from Chau et al. (2003). All numerical models and impact experiments show that the impact force is larger for a higher velocity of contact. However, the two quantities in Figure 6.19 do not follow
this relationship. The sixth impact has considerable force but no contact velocity. The 29\textsuperscript{th} impact has one-tenth the velocity but is almost equal in force to the second impact. These results indicate the unsuitability of the impact force measuring instrument employed in the experiment.

![Figure 6.19 Relative impact velocity and pounding force of \( n \)\textsuperscript{th} impact when two steel towers were subjected to pounding under El-Centro ground motion (Chau et al. 2003)](image)

Filiatrault et al. (1995) employed load cells at the contact locations to measure pounding force during shake table simulations. Based on the results, these load cells considerably modified the contact properties. The experiment not only records negative contact forces but also records contact durations as long as 0.5 s. The contact duration might be lengthened by the load cells which acted as comparatively flexible impact elements (*k* = 12.8 kN/mm).

### 6.2.2 Sampling rate

In the course of this PhD, it has been observed that a minimum sampling rate was required to record the correct time-history. For the impact of a steel beam with the elastomeric impact head of the hammer, accelerations could be measured at 1 kHz. For the impact of 0.65 m long RC slabs, significant data was missed at 1 kHz. The best data was recorded at 10 kHz while any increase above that did not produce any change in acceleration time history. For the impact of 0.32 m long steel beams, a sampling rate of at least 30 kHz was required. Any increase beyond that, even up to 90 kHz did not produce any change in the time history. It was inferred that the sampling rate below a certain value could not record
the internal vibration of the masses. The displacement time-history of the masses, excluding the internal vibration, could be recorded with sufficient accuracy even at 1 kHz.

Papadrakakis and Mouzakis (1995) adopted a sampling rate of 1 kHz. The first mode of internal vibration of the slabs is about 1 kHz so the Nyquist rate is 2 kHz. Thus the impact accelerations cannot determine if there were any distributed-mass effects. Similarly, Chau et al. (2003) adopted a sampling rate of 4 kHz for a 1 m long steel slab, whose Nyquist rate is 5 kHz. The Nyquist rates would be higher for both experiments if higher modes of vibration occurred. Zhu et al. (2002) and van Mier et al. (1991) also had 1 kHz sampling rate. Leibovich et al. (2012) employed a 10 kHz sampling rate and observed oscillation of impact induced accelerations.

6.2.3 Apparently contradictory results

The experimental results found in the literature seem to be very disparate at first study. Papadrakakis and Mouzakis (1995) found Lagrange multiplier methods gave good prediction of the experimental results while Zhu et al. (2002) and Jankowski (2005, 2008a) have found impact elements based models to be accurate. Jankowski (2010) found the coefficient of restitution to be a cubic polynomial function of impact velocity, whose prediction does not match with experiments described in Refs. Zhu et al. (2002) and Leibovich et al. (1994). The values for $e$ in the latter two studies were much lower than that predicted by Jankowski’s relations for their respective impact velocities.

However, on further analysis, the observations actually do not contradict. From the discussions in section 3.2, Papadrakakis and Mouzakis (1995) and Zhu et al. (2002) cannot select or identify the best pounding force model because of the low sampling rates. It was also shown that pounding force models cannot be evaluated by comparing structural displacements responses. The two studies can only confirm that the value of $e$ adopted for numerical analysis was suitable. The same agreement can be obtained if any other model with a sufficiently high value of $k$ is employed. Section 2.1.5 showed that the impact force evaluations in Jankowski (2005) have been based on a faulty generalization of $e$. The results from Jankowski (2008a) that a Hertz contact based model can better predict pounding force in spheres does not conflict with any other studies, though it also may not be applicable to building pounding because of plane surface collision.
Numerical (e.g. Khatiwada et al. 2013d) and experimental (Goldsmith 2001 and the references therein) studies have shown that the coefficient of restitution depends on a multitude of factors like the nature of the contact surface, the materials, masses, lengths, length ratios and stiffness distributions of the impacting bodies and impact velocity. The studies on pounding have derived $e$ in various conditions where most of these factors are different. Thus, the different values of $e$ in these experiments are possible. However, the omission of other relevant parameters makes a generalization difficult, e.g. observation in Jankowski (2010) that $e$ is independent of the considered mass might be due to the accompanying increase in sphere diameter that results in increased contact stiffness.

### 6.3 Sears impact model

Khatiwada et al. (2013d) proposed the impact model, originally formulated by Sears (1912), for building pounding simulation. The Sears impact model includes both wave and lumped-mass behaviour. The pounding force is generated by the compression of the contact area with a stiffness that can be calculated from the Hertz contact law (Goldsmith 2001). The behaviour of the floor mass farther away from the contact location is governed by the material and cross-sectional properties, while the floor response during pounding is determined by the propagation of stress waves. The following theoretical discussions have been adapted from Sears (1912) and Goldsmith (2001).

![Figure 6.20 Impact of round-ended circular bars.](image)

For the collision of two circular bars (Figure 6.20) of densities $\rho_1$ and $\rho_2$, lengths $L_1$ and $L_2$, cross-sectional areas $A_1$ and $A_2$, compressive wave velocities $c_1$ and $c_2$, and rounded ends of radius $r_1$ and $r_2$, the contact force is calculated with the following assumptions:

- The impact force at the contact location can be modelled by using the Hertz contact law for spheres of the same radius as the ends of the impacting bar.
• The stress distribution in the bar is developed from a concentrated force acting normal to the surface of a semi-infinite solid.

• The deformation of the contact interface can be calculated as the deformation between the points $P_1$ located in bar 1 at a distance $d_1$ and point $P_2$ on the bar 2 at distance $d_2$ from the impact interface.

• The stress waves generated in the bars will reflect back and forth in the same bar, until the relative deformation between the two points is zero.

• The wave dispersion effects and material damping of the bars are assumed to be negligible.

According to the Hertz contact law, impact force at any time $t$ is given by,

$$F(t) = k \delta^{3/2} ; \quad \delta > 0$$

$$F(t) = 0 \quad ; \quad \delta > 0$$

where, $\delta$ is the relative displacement of points $P_1$ and $P_2$, and $k$ is the contact stiffness defined as,

$$k = \frac{4}{3\pi(h_1 + h_2)} \left[ \frac{r_1 r_1}{r_1 + r_2} \right]^{3/2}$$

where, material property $h_i = \frac{1 - \mu_i^2}{\pi E_i}$. $\mu_i$ and $E_i$ are the Poisson’s ratio and Young’s modulus of $i^{th}$ bar, respectively.

Sears (1912) defined the length of compressive zone $d_i$ as the sum of two values: (i) the distance $QP’$ (Figure 6.21) at which the stress due to $F(t)$ becomes uniform over the section, i.e. $\sigma_x = \frac{F(t)}{\pi R_i^2}$; and (ii) the displacement from $P’$ to $P_i$ relative to the far end of the $i^{th}$ bar. The two lengths are calculated by assuming that the distribution of stresses in the bars can be simulated as that in a semi-infinite solid (Figure 6.21). For the right bar (Goldsmith 2001):
\[ QP' = \frac{3}{2} R_2 \]

\[ P'P_2 = \frac{R_2}{2QP'} (1 + \mu)(3 - 2\mu) \]

Thus,

\[ d_i = R_i \left[ \frac{3}{2} + \frac{1}{\sqrt{6}} (1 + \mu)(3 - 2\mu) \right] \]

Figure 6.21 Right bar (un-hatched) considered as a part of semi-infinite solid (hatched parts outside the bar) at time \( d_2/c_2 < t < (d_2 + l_2)/c_2 \).

The contact force arises due to the deformation of the compression zones \( P_1Q \) and \( QP_2 \). Outside this zone, the force is transmitted by the propagation of compressive stress. Thus, the force requires a time \( \frac{x}{c_i} \) to reach point \( P \) at a distance of \( x \) from \( Q \) (Figure 6.20); at time \( t \), the effective force at \( P \) is equal to \( F' \left\{ \frac{t - x}{c_i} \right\} \), i.e. the force at point \( Q \) at time \( \left\{ \frac{t - x}{c_i} \right\} \).

If \( t < \frac{x}{c_i} \), the contact force has not arrived at the section and \( F' \left\{ \frac{t - x}{c_i} \right\} = 0 \). When \( t > \frac{x}{c_i} \), the difference of forces at the two faces of the element with a length \( dx \) is:

\[
\frac{\partial F}{\partial x} \, dx = \frac{1}{c_i} \frac{\partial}{\partial t} \left\{ \frac{t - x}{c_i} \right\} F' \, dx
\]
The motion of the section at \( x \) is given by

\[
\rho_1 A_i \ddot{u}_{i,x} = \frac{1}{c_i} \frac{\partial}{\partial t} F \left( \frac{x}{c_i} \right)
\]

6.8

where \( \ddot{u}_{i,x} \) is the acceleration of the section at \( t \).

Integrating twice with respect to time,

\[
\dot{u}_{i,x} = v_i t + \frac{1}{\rho_1 A_i c_i} \int_0^t \left[ F \left( \frac{x}{c_i} \right) + F \left( \frac{x - 2(L_i - x)}{c_i} \right) \right] dt
\]

where \( v_i \) is the velocity of the \( i^{th} \) bar at the initiation of contact.

Equation 6.9 is valid only until the stress reflected from the far end gets back to the section. Then two stress waves will act upon the section and the displacement \( u_{i,x} \) is given by:

\[
\dot{u}_{i,x} = v_i t + \frac{1}{\rho_1 A_i c_i} \int_0^t \left[ F \left( \frac{x}{c_i} \right) + F \left( \frac{x - 2(L_i - x)}{c_i} \right) \right] dt
\]

6.10

The stress wave will again get reflected from the impacting end, which will add another force term \( F \left( \frac{x - 2(L_i - x) + 2x}{c_i} \right) \) within the square brackets in Equation 6.10. Thus, a new term is added after each subsequent reflection until the bars separate.

Considering that the impact force acts towards the negative direction in the left bar, the displacements of points \( P_1 \) and \( P_2 \) are:

\[
u_{P_1} = u_{1,d_1} = v_1 t - \frac{1}{\rho_1 A_1 c_1} \int_0^t \left[ F \left( \frac{d_1}{c_1} \right) + F \left( \frac{d_1 + d_1 + 2l_1}{c_1} \right) + F \left( \frac{d_1 + 2l_1 + 2d_1}{c_1} \right) + \ldots \right] dt
\]

6.11

\[
u_{P_2} = u_{2,d_2} = v_2 t + \frac{1}{\rho_2 A_2 c_2} \int_0^t \left[ F \left( \frac{d_2}{c_2} \right) + F \left( \frac{d_2 + 2l_2}{c_2} \right) + F \left( \frac{d_2 + 2l_2 + 2d_2}{c_2} \right) + \ldots \right] dt
\]

6.11(a)
The deformation of contact interface $\delta$ is obtained by subtracting $u_{p_2}$ from $u_{p_1}$:

$$
\delta = (v_1 - v_2) t - \frac{1}{\rho_1 A_1 c_1} \int_0^t \left[ F \left( t - \frac{d_1}{c_1} \right) + F \left( t - \frac{d_1 + 2l_1}{c_1} \right) + F \left( t - \frac{d_1 + 2l_1 + 2d_1}{c_1} \right) + \ldots \right] dt
$$

$$
\left( 6.12 \right)
$$

$$
\delta = (v_1 - v_2) t - \frac{1}{\rho_2 A_2 c_2} \int_0^t \left[ F \left( t - \frac{d_2}{c_2} \right) + F \left( t - \frac{d_2 + 2l_2}{c_2} \right) + F \left( t - \frac{d_2 + 2l_2 + 2d_2}{c_2} \right) + \ldots \right] dt
$$

The value of $\delta$ at any time can be determined by any numerical integration procedure. Goldsmith (2001) provides a time-history integration approach given below:

$$
\delta_i = \delta_{i-1} + (v_1 - v_2) \Delta t - \frac{1}{\rho_1 A_1 c_1} \left[ F \left( t - \frac{d_1}{c_1} \right)_{i,m} + F \left( t - \frac{d_1 + 2l_1}{c_1} \right)_{i,m} + F \left( t - \frac{d_1 + 2l_1 + 2d_1}{c_1} \right)_{i,m} + \ldots \right] \Delta t
$$

$$
\left( 6.13 \right)
$$

where the time-step $\Delta t$ must be several times smaller than the smaller of the terms $d_i/c_i$ and $l_i/c_i$. The subscript $m$ denotes that the mean force acting during $\Delta t$ should be used in the computations.

Figure 6.22 shows an example of the development of pounding force and deformation through time. Initially, the deformation $\delta$ (left-most solid line) increases at a rate of $(v_1 - v_2)$ as if there is no resistance, and the force increases proportionately. When the wave passes through the point $P_i$ of the shorter bar, this causes some resistance against deformation and the rate of growth of the force is reduced (upper dashed line). The force follows a new path (dashed-dot-dot line) when the stress waves reach the point $P_i$ in the longer bar. The stress wave in the short bar gets reflected from the far end and when it reaches point $P_i$ again, the force follows the middle dashed line. Thus, each time the wave front passes through point $P_1$ or $P_2$, the force time-history diverges from the previous path. The rate of increment of $\delta$ decreases first, then becomes negative, and eventually separation occurs. In Figure 6.22, the separation occurs when the stress waves pass three times through the Point $P_i$ in the longer bar.
Limitations in state-of-the-art and introduction to the Sears impact model

Thus the theory imposes a finite rise time on the stress wave, and requires several reflections of the wave before separation of the bars. The first is due to the finite stiffness of the impact location which seems more congruous with the behaviour of structural elements, than the instant imposition of fully developed stress according to the wave theory.

The impact model was experimentally verified by Sears (1912). A parametric study was presented in Khatiwada et al. (2013d), that showed that the behaviour of the model is similar to lumped-mass models for short bars and similar to wave theory for longer bars. Bars of intermediate length show the presence of both properties. (Wagstaff 1924) reached the same conclusion from experiments on bars of various lengths.

The model is originally formulated for the contact of bars with curved ends because the analytical stiffness (Equation 6.4) is available only for such contacts, (from the Hertz contact law). As discussed earlier, building pounding occurs between two plane surfaces so Equations 6.3 and 6.4 cannot be applied as they are. However, the strength of the model is its versatility. For plane surfaces, the equations can be replaced by linear force deformation relations (Khatiwada et al. 2013d). The stiffness can be related with the floor slab’s axial stiffness by

\[ k = n \cdot \frac{EA}{L} \]

If the deformation zone is spread through the whole length of the slab, \( n \) equals 1. If the contact is stiffer, only a part of the slab is deformed, \( n \) is greater than 1, and the response of the rest of the slab is due to stress propagation. \( n \) can also be smaller than 1 if the contact stiffness is lower than the axial stiffness of the slab, as measured by Guo et al. (2012)
The performance of the Sears model was compared with results from the undamped, elastic impact of steel bars modelled in the finite element software, SAP2000. A bar of length 2.5 m struck a 1.5 m bar. Both bars are of 0.05 m in diameter. The length \( d \) is set at 0.1 m for the Sears model. The bars are modelled in SAP2000 as beams of adjacent portal frames. The effect of the columns is reduced by modelling them as very flexible (10 s period), and neglecting the gravity loads. The bars are also discretized into 0.1 m segments. The 2.5 m long bar was subjected to a short, high pulse load such that it hit the second bar. No external forces were active during the time of contact. Figure 6.23 shows a comparison between results from the two methods. For Figure 6.22(a), contact stiffness was equal to the axial stiffness of the whole shorter bar, while for Figure 6.22(b), the stiffness is calculated from one-fifteenth of the length of the bars. The force from softer contact is similar to that from lumped-mass models while the force from the stiffer contact is closer to that from the wave-theory of impact. Thus, it is apparent that the Sears model produces identical results as finite element models for the longitudinal impact of two free slabs. The slight differences in the two curves can perhaps be attributed to the effect of the columns.

The finite element discretization of bars had a significant influence. If the compressive stiffness of the segment is similar to the contact stiffness, the force shows oscillations similar to that observed by Cole et al. (2009). For a smooth force development, the segment stiffness must at least be twice the contact stiffness. Thus, the oscillations observed in force and acceleration from Watanabe and Kawashima (2004) seems to be
caused by the numerical instability induced by the assumption that the contact stiffness is equal to the stiffness of the adjacent segments of the colliding diaphragms. An analogous effect was observed in the Sears model when the time step $\Delta t$ was not small enough. For $k = n \left( \frac{EA}{L_i} \right)$, $\Delta t$ must be less than $L_i / (n c_i)$ i.e. the time required for the stress wave to travel through the $n^{\text{th}}$ part of the bar. Thus, the Sears model can be regarded as a closed form solution of undamped, elastic impact in finite element. The models by Watanabe and Kawashima (2004) and Cole et al. (2011) were also justified with finite element models. However, there is a significant difference. The models based on wave theory require an instantaneous achievement of the maximum force and the masses are discretized to achieve this. The models are considered progressively more accurate as the segments get smaller but this also produces more and more oscillations (Figure 6.14). For the Sears model, contact stiffness $k$ needs to be determined first. Then the masses have to be discretized such that the segments have stiffness of at least $2k$. The results do not show any significant changes for a finer discretization.

The Sears model can potentially address many of the limitations of the lumped-mass and distributed-mass models. However, more experimental work is needed to calibrate the parameters – such as the effective $k$ for contact between large masses. The model also does not include any collision damping in the current form, so it predicts an elastic collision for bars of equal length, which is not supported by the experiments (e.g. Leibovich et al. 2012). Thus, the attenuation of stress amplitudes needs to be assessed and incorporated in the model.

### 6.4 Summary

Building pounding has been considered over several decades. From the analysis of the pounding of two idealized SDoF systems, the current simulations can incorporate the effects of multiple degrees-of-freedom, spatial variation of ground motions, soil-structure interaction, as well as out-of-plane motions at the impact location. However, in design practice the validity of pounding analysis is limited. This is because up until now, none of the existing models have the capability to describe the pounding force development that has been observed experimentally.

This chapter discussed the various limitations in numerical and experimental simulations of structural pounding. The Coefficient of restitution has been identified as the other principle source of uncertainties in building pounding. The displacement response depends
almost exclusively on this coefficient. However, in many cases the coefficient is not reliable. All attempts to measure the coefficient have produced conflicting results.

Most of the experimental simulations are limited by the quality of instrumentation available, particularly in respect of insufficient sampling rate. The obtained results cannot identify the correct force model. Two cases are also presented to reveal that dynamic force measuring devices can provide incorrect results.

Although the distributed-mass models are independent of the coefficient of restitution, they still suffer from their own limitations, i.e. due to the requirement of instant full-surface contact, incapability to incorporate the effect of supporting structures and the generation of infinite acceleration.

The Sears impact model, which includes the effects of lumped-mass and distributed-mass models, is proposed in this work for simulations of building pounding. The model gives a closed-form solution for finite element analysis of undamped, elastic collision of freely moving bars.
Chapter 7

Damped Sears impact model: derivation and validation

The previous chapter introduced the Sears impact model (Sears 1912), which includes the effect of both contact interface and cross-sectional properties of colliding masses. In contrast, the lumped-mass models do not consider cross-sectional properties of the bars, while the existing distributed-mass models cannot include the effects of contact interface.

It was shown that the Sears model produces almost identical result as finite element modeling for elastic contact between undamped bars. However, the model has two drawbacks in its original form: (i) an inability to include damping of the propagating stress, and (ii) an inability to incorporate the effect of adjacent support elements.

This chapter proposes a damped Sears model which adds viscous damping of the stress waves to the original Sears impact model. Similar to the Sears model, the contact force is calculated from the apparent deformation of the colliding bodies with Hertz contact law. However, unlike the original Sears model, a modal analysis is for the rest of the structures. The performance of the model is validated with results obtained from experiments on impact between RC slabs presented in Chapter 5.

7.1 Numerical Derivation

The Sears model was derived for the collision of two bars with rounded ends (Figure 7.1(a)). When the bars come in contact, the zone between points P1 and P2 is compressed. Expressions for the length of deformation zones i.e. $d_1$ and $d_2$, were provided by Sears (1912) for the bars with rounded ends. However, Khatiwada et al. (2013d) observed that the contact responses were not sensitive to the distance between P1 and P2, perhaps because the contact stiffness is calculated only from the radii of the rounded ends. For simplicity, the effects of the non-uniform cross-section, i.e. rounded ends, on the stress propagation is also neglected. Therefore, a simplified idealization (Figure 7.1(b)) is adopted for the
development of the damped Sears model. The \(i\)th bar, where \(i\) equals 1 for the left bar and 2 for the right bar, has length \(l_i\), radius \(r_i\) and cross-sectional area \(A_i\). The rounded ends at the contact location are of radii \(r_i\). The materials properties i.e. Young’s modulus, density and Poisson’s ratio are \(E_i\), \(\rho_i\), and \(\mu_i\) respectively. The basic assumptions are:

- The contact stiffness is described by the Hertz contact law for spheres of the same radius as the ends of the bar.
- The contact causes compressive stresses at the contact interface which propagates to the far ends. At any section of the bar, the stress is uniformly distributed over the cross-section. Thus, the effect of initial concentration of the stress at the contact point is neglected.
- The contact-induced vibrational modes of the two bars are independent. They only affect each other in the generation of a contact force.
- The effects of wave dispersion and surface wave generation are neglected.
- The length of the rounded end is small compared to the length of the bars. The bars are assumed to be prismatic.

At any time \(t\), the displacement, velocity and acceleration of a section at a distance \(x\) from the left end of each bar is \(u_i(x, t)\), \(v_i(x, t)\) and \(a_i(x, t)\), respectively. At the beginning of
impact, i.e. \( t = 0 \), the bars are moving with uniform velocities \( v_{0,i} \). The displacements of the points \( P_1 \) and \( P_2 \) located on the contact ends of the bars, are denoted by \( u_{P1} \) and \( u_{P2} \) (see Figure 7.1).

According to the Hertz contact law, the impact force at any time \( t \) is given by,

\[
F = \begin{cases} 
  k \delta^{\frac{3}{2}} & ; \delta > 0 \\
  0 & ; \delta \leq 0
\end{cases}
\]  

where, \( \delta = (u_{P1} - u_{P2}) \) is the relative displacement of the contact surfaces.

\( k \) is the contact location stiffness defined as:

\[
k = \frac{4}{3 \pi \left( h_1 + h_2 \right)} \left[ \frac{r_1 r_2}{r_1^2 + r_2^2} \right]^{\frac{3}{2}}
\]  

where, the material property \( h_i \) is given by

\[
h_i = \frac{1 - \nu_i^2}{\pi E_i}
\]

The governing equation of motion for a homogenous, prismatic bar is (Graff 1991):

\[
E_i \frac{\partial^2 u_i(x,t)}{\partial x^2} - \rho_i \frac{\partial^2 u_i(x,t)}{\partial t^2} + \frac{f(x,t)}{A_i} = 0
\]

where \( f(x,t) \) is the total force at any section \( x \) at any time \( t \).

The motion of any point in the \( i^{th} \) bar can be expressed in terms of the first \( N \) modes of axial vibration as:

\[
u_i(x,t) = \sum_{n=0}^{N} q_{i,n}(t) U_{i,n}(x)
\]

where, \( U_{i,n}(x) \) is the \( n^{th} \) mode of the \( i^{th} \) bar, and

\( q_{i,n}(t) \) is the displacement of the same mode.
For the freely moving rods, the normal modes are given by:

\[ U_{i,n}(x) = \cos \left( \frac{n \pi x}{l_i} \right) \quad n = 0, 1, 2, 3, \ldots \]  

7.6

where, \( n = 0 \) is for the rigid body motion of the bar and \( n = 1, 2, 3 \ldots \) signifies the first and higher modes of axial vibration (Figure 7.2).

Substituting Equation 7.5 in Equation 7.4 and simplifying:

\[ \sum_{n=0}^{N} \left\{ c_i^2 q_{i,n}(t) U_{i,n}^\prime(x) - \ddot{q}_{i,n}(t) U_{i,n}(x) \right\} = - \frac{f(x,t)}{\rho_i A_i} \]  

7.7

where, \( U_{i,n}^\prime(x) = \frac{\partial^2 U_{i,n}(x)}{\partial x^2} \), \( \ddot{q}_{i,n}(t) = \frac{\partial^2 q_{i,n}(t)}{\partial t^2} \), and

\[ c_i = \sqrt{\frac{E_i}{\rho_i}} \]  

is the wave propagation velocity in the bar.

\[ U_{i,n}^\prime(x) = \left( \frac{n \pi}{l_i} \right)^2 U_{i,n}(x) \]  

and the natural frequency of the \( n \)th mode, \( \omega_n = \frac{n \pi c_i}{l_i} \). Thus, Equation 7.7 reduces to

\[ \sum_{n=0}^{N} \left\{ \ddot{q}_{i,n}(t) + \omega_{i,n}^2 q_{i,n}(t) \right\} U_{i,n}(x) =  \frac{f(x,t)}{\rho_i A_i} \]  

7.8

Multiplying both sides by \( U_{i,m}(x) \) and integrating from 0 to \( l_i \):
Damped Sears impact model: derivation and validation

\[
\sum_{n=0}^{N} \left\{ \ddot{q}_{i,n}(t) + \omega_{i,n}^2 q_{i,n}(t) \right\} \int_{0}^{l_i} U_{i,n}(x) U_{i,m}(x) \, dx = \frac{1}{\rho_i A_i} \int_{0}^{l_i} f(x,t) U_{i,m}(x) \, dx \quad 7.9
\]

Because of modal orthogonality, RHS of Equation 7.9 reduces to 0 for all \( m \neq n \). For \( m = n \),

\[
\left[ \ddot{q}_{i,n}(t) + \omega_{i,n}^2 q_{i,n}(t) \right] \int_{0}^{l_i} \cos^2 \left( \frac{n \pi x}{l_i} \right) \, dx = \frac{1}{\rho_i A_i} \int_{0}^{l_i} f(x,t) \cos \left( \frac{n \pi x}{l_i} \right) \, dx \quad 7.10
\]

Simplifying,

\[
\left[ \ddot{q}_{i,n}(t) + \omega_{i,n}^2 q_{i,n}(t) \right] = \frac{1}{\rho_i A_i} \int_{0}^{l_i} f(x,t) \cos \left( \frac{n \pi x}{l_i} \right) \, dx; \quad n=0
\]

\[
\left[ \ddot{q}_{i,n}(t) + \omega_{i,n}^2 q_{i,n}(t) \right] = -2 \frac{1}{\rho_i A_i} \int_{0}^{l_i} f(x,t) \cos \left( \frac{n \pi x}{l_i} \right) \, dx; \quad n \geq 1
\]

The force \( f(x,t) \) can be expressed as,

\[
f(x,t) = F \delta_x(x-l_i); \quad i = 1
\]

\[
f(x,t) = F \delta_x(x-0); \quad i = 2
\]

where \( \delta_x \) is the Dirac-delta function.

Substituting Equation 7.12 in Equation 7.11 and integrating,

\[
\ddot{q}_{1,0}(t) = -\frac{F}{\rho_1 A_1 l_1}
\]

\[
\ddot{q}_{2,0}(t) = \frac{F}{\rho_2 A_2 l_2}
\]

\[
\ddot{q}_{1,n}(t) + \omega_{1,n}^2 q_{1,n}(t) = -(-1)^n \frac{2F}{\rho_1 A_1 l_1}
\]

\[
\ddot{q}_{2,n}(t) + \omega_{2,n}^2 q_{2,n}(t) = \frac{2F}{\rho_2 A_2 l_2}
\]

Equation 7.13 in combination with Equations 7.1 and 7.5 can be used in conjunction with any time history integration methods to calculate total axial response. Equation 7.13 is derived from assumptions of elastic wave propagation. To incorporate viscoelasticity in the model, modal damping is added to the equation (Chopra 2001).
\[ \ddot{q}_{1,0}(t) = \frac{-F}{\rho_i A_i l_i} \]
\[ \ddot{q}_{2,0}(t) = \frac{F}{\rho_i A_i l_i} \]
\[ \ddot{q}_{1,n}(t) + 2\xi_1 \omega_{1,n} \dot{q}_{1,n}(t) + \omega_{1,n}^2 q_{1,n}(t) = -(-1)^n \frac{2F}{\rho_1 A_1 l_1} \]
\[ \ddot{q}_{2,n}(t) + 2\xi_2 \omega_{2,n} \dot{q}_{2,n}(t) + \omega_{2,n}^2 q_{2,n}(t) = \frac{2F}{\rho_2 A_2 l_2} \]  

where, \( \xi_i \) is the damping ratio of the \( i \)th bar.

It can be observed that the derivation is analogous to that given by Malhotra (1998), with certain significant differences. While Malhotra considered the two colliding bars to fuse together into a single bar for the entire contact duration, the proposed model analyzes the bars separately. Thus, the single bar in Malhotra undergoes free vibration with a mass weighted velocity at the contact instant, while the vibration in the proposed model is forced. The impact force is calculated from the deformation of the contact surfaces, while Malhotra calculated the impact force from strain at the boundary. Finally, Malhotra’s equation can only be used for contact between bars of identical materials and cross-sections because the equations become an order of magnitude more complex when the properties of bars are different. The proposed model is applicable for all combinations of materials and cross-sections.

Figure 7.3 shows the simulated impact force, velocities of points P1 and P2, and the acceleration of point P1 for damped and undamped collision of two steel bars with \( R_1 = R_2 = 0.05 \text{ m} \), \( r_1 = r_2 = 1 \text{ m} \) and \( l_1 = l_2 = 1 \text{ m} \). The first bar is moving with velocity 1 m/s while the second is at rest. The material properties for steel are \( E = 2 \times 10^{11} \text{ N/m}^2 \), \( \rho = 7850 \text{ kg/m}^3 \) and \( \mu = 0.3 \). The first hundred modes of internal vibration of each bar have been included in the analysis. The time history analysis was carried out at a time step of 0.1 μs. The damping had a substantial effect on the acceleration but there was a very small effect on the pounding force.

### 7.2 Experimental setup

The experimental setup is presented in Section 5.1. The results obtained for 100 mm φ spherical contact element is utilized in this chapter for verification of the Sears impact element.
Figure 7.3 Responses from damped and undamped impact between 1m long steel bars: (a) Pounding force, (b) acceleration and (c) velocity of first bar’s centre of mass and (d) velocity of second bar’s centre of mass.

7.3 Results and discussion

The displacement time history resulting from the impact of single RC slabs, with an impact velocity 0.14 m/s, is shown in Figure 7.4(a). The impacts due to the repeated contact of the two pendulums can be clearly observed. Figure 7.4(b) shows the accelerometer recordings for these impacts, indicated by the spikes.

The acceleration of the striker due to the first impact for a velocity of 0.14 m/s is shown in Figure 7.5. This experiment was originally planned to identify the best performing lumped-mass model. The first impact takes place when both the pendulums are vertical, so the weight of the pendulums is balanced by the tension in the cables. The only horizontal force is the impact force, which should be equal to the product of mass and acceleration of each slab. The acceleration time history in Figure 7.6 was unexpected. In the case of lumped mass model, the negative acceleration would suggest a horizontal force component resulting from gravity. However, the large negative acceleration activated, up to 1.8 g,
cannot be explained by the equilibrium mechanism of this lumped-mass model. This indicates that lumped-mass model cannot describe the observed behavior, which reflects the first shortcoming of this model. Therefore a distributed-mass model was required.

To reveal another shortcoming of the lumped-mass model, the force was calculated as product of mass and acceleration and only the first pulse of the acceleration of the striker was considered (See Figure 7.6). The measured accelerations were integrated to obtain the corresponding velocity and displacement time-histories. The force-relative displacement (δ) curve is shown in Figure 7.7. It was observed that the masses did not separate when force became zero at 4.6 ms. Instead, there is a small overlap of 0.12 mm (indicated by negative impact force in Figure 7.7). Both these shortcomings are contradictory to the basic assumption of the lumped-mass models, e.g. Equation 3.1, that impact forces develop only under compression. If a lumped-mass model is used, only one pulse-like compressive force will be obtained. However, Equation 3.1 cannot be used to reflect the observations in the experiment, i.e. the oscillating acceleration and consequently the time-history of force. The experimental observations has been confirmed in the work of Leibovich et al. (2012).
In the light of these shortcomings, the lumped-mass model is not pursued in this work. Instead, a damped Sears model is introduced. This model requires the longitudinal mode shapes of the slabs. For simplicity, the RC slabs in Figure 7.8(a) were idealized as 700 x 248 x 248 mm bar (Figure 7.8(b)), keeping the length, volume and mass equal in both cases. The second slab (without hemispherical protrusion) is also idealized similarly, which gives a 650 x 257 x 257 mm RC bar.
Young’s modulus for M20 concrete is 21 GPa and density is 2400 kg/m³. The stiffness of contact between the 0.05 m sphere and the flat surface was found to be 3.21 GN/m³/2 (7.2). The impact was simulated with the first 100 longitudinal vibration modes. The time step of simulation was 0.1 µs. The equations of motion (Equation 7.14) are solved by the interpolation of excitation method described in Chopra (2001).

![Figure 7.8 Simplification of experimental slab: (a) Actual RC slab and (b) idealized slab](image)

Figure 7.8 Simplification of experimental slab: (a) Actual RC slab and (b) idealized slab

![Figure 7.9 Acceleration of striker at 150 mm distance from contact location, for impact velocity 0.14 m/s](image)

Figure 7.9 Acceleration of striker at 150 mm distance from contact location, for impact velocity 0.14 m/s

The acceleration of the striker for impact velocity 0.14 m/s is presented in Figure 7.9. The numerical results are substantially different from the experimental accelerations in both magnitude and shape. Furthermore, even though a few small oscillations can be observed in acceleration, the time period is 0.45 ms, while that in the experimental result is 3.61 ms. The differences could be due to the effect of surface waves and lateral inertia (Graff 1991). In the theoretical derivations discussed in Section 2, the impact force was assumed to be applied uniformly at each section of the slabs. The effect of initial dispersion from the comparatively small contact area to the larger area (Figure 7.8(a)) is not considered in the idealized slab (Figure 7.8(b)). If the angle of dispersion is small, several internal reflections...
would be required before the stresses are uniformly distributed. In actual case, the area actively resisting compression force would be smaller than the total cross-sectional area of the idealized slab. Because of this decrease of area, the stiffness decreases which corresponds to a lower wave propagation velocity, \( c_i \). Therefore, the wave propagation velocity of the idealized slabs was gradually reduced to equate the numerical and experimental periodicity of acceleration. The closest agreement was observed when the effective wave propagation velocity, i.e. the velocity that results in coincidence between the acceleration periodicity of the physical and idealized slab, was one-eighth of the original velocity. It was also observed that with increased flexibility of the idealized slabs, the acceleration at different locations of the slabs varied substantially. The accelerations were calculated at 11 equidistant locations, where \( x = 700 \) mm is the contact surface, and \( x = 0 \) mm is the free end. Figure 7.10 shows the calculated acceleration at 4 locations in the bar, as well as the experimentally recorded acceleration at \( x = 150 \) mm. The peak acceleration at most of the sections is still 50% higher than the experimental results.

A comparison of numerical and experimental impulse is presented in Table 7.2. The impulse was calculated in both the experimental and numerical case from the change in velocity of the slabs following first impact. The following cases are considered:

(i) \( c = c_i \)  
(ii) \( c = c_i/8 \)  
(iii) \( c = c_i/8/(ns_i)^2 \)  
(iv) \( c = c_i/8/(ns_i)^2 \) and \( k/4 \)

where \( ns_i \) is the number of slabs.

As noted in the previous paragraph, there is a substantial effect of the wave velocity on the acceleration (see Figures 7.12 and 7.13). However, Table 7.2 shows that the two velocities produce very similar calculated impulse. The higher the impact velocity, the more reliable is the numerical calculation.
Damped Sears impact model: derivation and validation

(a)

Figure 7.10 Acceleration of striker slab at various locations for impact velocities (a) 0.14 m/s and (b) 0.095 m/s, where x is the distance from the impact location.

(b)

Table 7.1 Experimental and numerical impulse for impact between single slab pendulums

<table>
<thead>
<tr>
<th>Trial</th>
<th>Impact velocity (m/s)</th>
<th>Impulse (N s)</th>
<th>(Experimental - Numerical) / Experimental (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exp.</td>
<td>$c_1$</td>
</tr>
<tr>
<td>1</td>
<td>0.030</td>
<td>2.6</td>
<td>3.2</td>
</tr>
<tr>
<td>2</td>
<td>0.040</td>
<td>3.5</td>
<td>4.3</td>
</tr>
<tr>
<td>3</td>
<td>0.096</td>
<td>8.7</td>
<td>10.3</td>
</tr>
<tr>
<td>4</td>
<td>0.146</td>
<td>13.7</td>
<td>15.7</td>
</tr>
<tr>
<td>5</td>
<td>0.171</td>
<td>16.2</td>
<td>18.3</td>
</tr>
<tr>
<td>6</td>
<td>0.183</td>
<td>18.0</td>
<td>19.7</td>
</tr>
<tr>
<td>7</td>
<td>0.226</td>
<td>21.2</td>
<td>24.3</td>
</tr>
<tr>
<td>8</td>
<td>0.243</td>
<td>23.5</td>
<td>26.1</td>
</tr>
</tbody>
</table>

Notes: 1 calculated using $c_1$ as defined in Equation (7)
2 calculated using one-eighth of $c_1$

Figure 7.11 presents the accelerations when a second slab is placed on top of either the striker or struck slab. When the mass was increased twice, the wave velocity had to be reduced by the same factor. When the striker was loaded, the periodicity is almost twice that of the single slab. When the single slab struck the double slab, the periodicity of
Damped Sears impact model: derivation and validation

striker’s acceleration is same as that in single slab. However, the acceleration magnitudes are substantially different (see Figure 7.11 solid and dashed lines). The computations were repeated with one fourth the $k$ calculated from Equation 7.2 to obtain convergence. The numerical and experimental acceleration magnitudes become closer (Figure 7.12). However, at this moment, the only reason for this reduction in $k$ is that the compressive zone extends further from the contact interface for double slabs. The contact stiffness reduces because of the longer compressive zone caused by an increase in the activated contact force.

![Graph](image)

Figure 7.11 Acceleration of striker slab at various locations for impact velocity 0.085 m/s when a second slab was loaded (a) on the striker and (b) on the struck slab.

The numerical and experimental impulse for impact between a single slab and a double slab are presented in Table 7.2. The closest agreement in impulse is obtained for case (iii). In contrast, case (iv) produced the closest agreement in acceleration. The impulse results for the remaining six mass combinations (see Table 5.2) are presented in the Appendix (Tables 7.A1 – 7.A4) and show a similar trend. The impact occurs at the mid-height of single slabs and mid-height of the lowest slab in the case of multiple slabs. Consequently, in the latter case axial wave propagations alone cannot be expected.
Damped Sears impact model: derivation and validation

(a)

(b)

Figure 7.12 Acceleration of striker slab at various locations for $v_{0,1} - v_{0,2} = 0.085 \text{ m/s}$ and one fourth $k$, when a second slab was loaded (a) on the striker and (b) on the struck slab.

Table 7.2 Experimental and numerical impulse for impact between a single slab pendulum and a double slab pendulum

<table>
<thead>
<tr>
<th>Trial</th>
<th>Striker pendulum</th>
<th>Struck pendulum</th>
<th>Impact velocity (m/s)</th>
<th>Impulse (Ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Exp. $c_i$</td>
<td>$c_i/8$</td>
</tr>
<tr>
<td>1</td>
<td>Single</td>
<td>Double</td>
<td>0.042</td>
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Notes: 1, 2 see Table 2

3 calculated using $c_i$ modified according to the number of slabs in striker and struck pendulum

4 calculated with $c_i/8/(ns_i)^2$ and $k/4$ (Equation (2))
7.4 Summary

Two principal deficiencies were observed in the Sears impact model introduced in Chapter 6: (1) lack of damping mechanism and (2) lack of consideration of higher modes of axial vibration. Therefore a new damped Sears impact model was formulated. Similar to the Sears model, the proposed model is based on both lumped-mass and wave propagation models. It is assumed that the impact forces develop with the deformation at the contact location and stress waves spread to other parts of the structures.

In total 76 RC slab experiments were performed and used to validate the proposed model. The impact velocities considered were 0.02 m/s up to 0.24 m/s and the masses of the single, double and triple slabs having a ratio of 1 : 2 : 3 in various combinations.

The performance of the damped Sear model is significantly better than other existing models, i.e. lumped-mass model and distributed-mass model, because:

(i) In contrast to all lumped-mass models, impulse can be predicted without the need to specify a coefficient of restitution.

(ii) Unlike all lumped-mass and distributed-mass models, the damped Sears model can closely simulate the experimental impact accelerations, including the periodicity observed after the first pulse.

(iii) The momentum transferred between the structures can be reproduced more accurately.
### Appendix 7.A

Table 7.A1 Experimental and numerical impulse for impact between a single slab and a triple slab

<table>
<thead>
<tr>
<th>Trial</th>
<th>Striker pendulum</th>
<th>Struck pendulum</th>
<th>Impact velocity (m/s)</th>
<th>Impulse (N s)</th>
<th>(1c_i)</th>
<th>(2c_i/8)</th>
<th>(3c_i/(ns_i)^2)</th>
<th>(4k/4)</th>
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Notes: 1, 2, 3, 4 see Table 3

Table 7.A2 Experimental and numerical impulse for impact between two double slabs

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<th>(2c_i/8)</th>
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Notes: 1, 2, 3, 4 see Table 3
Table 7.5 Experimental and numerical impulse for impact between a double slab and a triple slab

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Notes: 1, 2, 3, 4 see Table 3

Table 7.6 Experimental and numerical impulse for impact between two triple slabs

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Notes: 1, 2, 3, 4 see Table 3
Chapter 8

Damped Sears impact model: application to building pounding

This study further extends the capabilities of the damped Sears model to describe building pounding by including the adjoining structural members. The governing equations for contact between curved surfaces are modified so that the influence of plane surfaces and the supporting structures can be simulated. This model is then validated with the experimental results from the impact between two steel framed structures. Finally, a practical example is presented for the simulation of pounding between two unequal height buildings.

8.1 Theoretical derivations

This section refers extensively to the equations presented in the previous chapter. The main purpose is to develop procedures to incorporate (i) contact between flat surfaces and (ii) effects of the elements supporting the colliding diaphragms.

8.1.1 Flat surface contact

The force-deformation relation in Equation 7.1 was derived by Hertz for contact between two curved surfaces or between a curved and a flat surface. The assumptions of derivation exclude the application of the relationship to contact between two flat surfaces. This is also evident from Equation 7.2 where the nonlinear stiffness $k$ becomes indeterminate if both $r_1$ and $r_2$ become infinite. To overcome this limitation, a modification is proposed as follows:

\[
\begin{align*}
F &= k_L \delta; & \delta > 0 \\
F &= 0; & \delta \leq 0
\end{align*}
\]

where, the linear stiffness $k_L$ is series combination of the contact location stiffness $k_{A,i}$ of the two bars, defined as
Damped Sears impact model: application to building pounding

\[ k_{A,i} = D \frac{EA_i}{l_i} \]  \hspace{1cm} 8.2

where, D is the fraction of the \( i \)th bar which takes part in development of the impact force.

If the whole length of the floor slab is assumed to deform during impact, \( D = 1 \). If the contact is assumed to be stiffer, \( D \) will be higher. The constant \( D \) cannot be determined from the current state of the art in building pounding studies. For a particular building and support system \( D \) can be estimated from corresponding physical experiments. However, this \( D \) value cannot be deemed applicable to the general building and support configurations.

8.1.2 Effect of the supporting columns

To the author’s knowledge, existing formulations of on distributed-mass models of pounding based on wave theory have not included the effects of the supporting columns directly. Malhotra (1998) included this effect by first calculating the time of contact and effective coefficient of restitution, \( e \), for the colliding bridge segments, then using this value to calculate the final velocities of the two decks, and finally changing the their velocities over the computed contact duration by incremental steps. The pounding force is calculated from the wave theory formulations. Cole et al. (2011) also first calculated the effective coefficient of restitution and contact duration for the colliding floors. An equivalent lumped mass model was then formulated, which is a linear viscoelastic element (Anagnostopoulos 2004) that produces the same contact duration and energy loss as the wave theory. Thus, the floor displacements and inter-storey drifts are calculated from the equivalent lumped-mass model while the pounding force is calculated from the original wave theory formulations. It was suggested in Khatiwada et al. (2013d) to calculate the pounding force on each floor from the Sears model for the initial velocity of each impact, and then to apply it to the building floors. However, subsequent study found that this force is insufficient to produce separation if the buildings had significant floor displacement at the time of contact. Therefore, a methodology has been developed to include these effects in pounding simulation with Sears impact model.

Two frames, as shown in Figure 8.1(a), are subjected to floor to floor impact against each other. The mass, stiffness and damping matrices are \( m_{ki}, k_{ki}, \) and \( c_{ki} \) respectively, where \( i = 1 \) for the left frame and \( i = 2 \) for the right frame. The left frame has \( J_1 \) floors and right...
frame has $J_2$ floors. If the $j^{th}$ floors of the frames collide, the impact force $F_j$ can be calculated from relative penetration $\delta_j$ from Equation 7.1 or 8.1 according to the contact surface geometry. The displacement $u_{i,j}$ of $j^{th}$ floor of $i^{th}$ frame, for a section at distance $x$ from the left end of the floor is:

$$u_{i,j}(x,t) = \sum_{n=0}^{N} q_{i,j,n}(t)U_{i,j,n}(x)$$  \hspace{1cm} 8.3

where, $U_{i,j,n}(x)$ is the $n^{th}$ mode of the $j^{th}$ floor of $i^{th}$ frame, and

$q_{i,j}(t)$ is the modal displacement.

![Figure 8.1 An example of (a) Impacting frames and (b) lumped mass model](image)

The modes of internal vibrations for each floor are obtained from Equation 7.6 as:

$$U_{i,j,n}(x) = \cos \left( \frac{n \pi x}{l_{i,j}} \right) \quad n = 0, 1, 2, 3, \ldots$$  \hspace{1cm} 8.4

However, the translation of the $j^{th}$ floor cannot be obtained from Equation 7.6 because the beams are not freely moving. Instead, the floor translation is governed by the overall motion of the frames. However, from the second assumption listed above, the $n = 0^{th}$ mode for each floor is same as the displacement of the $j^{th}$ floor in the equivalent lumped-mass model of the frames (Figure 8.1b). Hence the $0^{th}$ mode displacement of each floor can be obtained from the equation of motion for two coupled multi-degree of freedom structures, as follows:

$$\begin{bmatrix} \ddot{q}_{1,0} \\ \ddot{q}_{2,0} \end{bmatrix} \begin{bmatrix} m_{s,1} & 0 \\ 0 & m_{s,2} \end{bmatrix} + \begin{bmatrix} \dddot{q}_{1,0} \\ \dddot{q}_{2,0} \end{bmatrix} \begin{bmatrix} c_{s,1} & 0 \\ 0 & c_{s,2} \end{bmatrix} + \begin{bmatrix} q_{1,0} \\ q_{2,0} \end{bmatrix} \begin{bmatrix} k_{s,1} & 0 \\ 0 & k_{s,2} \end{bmatrix} = \begin{bmatrix} PF_1 \\ PF_2 \end{bmatrix}$$  \hspace{1cm} 8.5

where, $\dddot{q}_{i,0}$, $\dddot{q}_{i,0}$ and $q_{i,0}$ are the column matrix of floor acceleration, velocity and displacement, respectively, of lumped-mass systems, and

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Damped Sears impact model: application to building pounding

$PF_i$ is the pounding force on each building, where

\[
\begin{align*}
(PF_1)_j &= -F_j \\
(PF_2)_j &= F_j \\
(PF_1)_j &= (PF_2)_j = 0
\end{align*}
\]

if \( j \leq J_1 \) and \( j \leq J_2 \)

\[ (PF_1)_j = (PF_2)_j = 0 \quad \text{if } j > J_1 \text{ or } j > J_2 \]

Thus the first two sub-equations of Equation 7.14 are replaced by Equation 8.5 while the last two sub-parts are replaced by:

\[
\begin{align*}
\ddot{q}_{1,j,n}(t) + 2\zeta_{1,j} \omega_{1,j,n} \dot{q}_{1,j,n}(t) + \omega_{1,j,n}^2 q_{1,j,n}(t) &= -(-1)^n \frac{2F_j}{\rho_{1,i,j} l_{1,j}} \\
\ddot{q}_{2,j,n}(t) + 2\zeta_{2,j} \omega_{2,j,n} \dot{q}_{2,j,n}(t) + \omega_{2,j,n}^2 q_{2,j,n}(t) &= \frac{2F_j}{\rho_{2,i,j} l_{2,j}}
\end{align*}
\]

where, \( \zeta_{i,j} \) is the modal damping,

\[
\omega_{i,j,n} = \frac{n \pi c_i}{l_{i,j}}
\]

is the natural frequency of \( n^{th} \) mode for \( n = 1, 2, \ldots \), and

\( \rho_{i,j} \), \( l_{i,j} \) and \( A_{i,j} \) are the density, length and cross-sectional area, respectively, of the \( j^{th} \) floor of the \( i^{th} \) bar.

Each time-step in the simulation is composed of four parts: (i) The calculation of displacements due to translation (Equation 8.5); (ii) The modal analysis of internal vibration of each floor (Equation 8.7); (iii) The recombination of two results into displacement of various sections of each floor (Equation 8.4); and (iv) The computation of impact force from the apparent overlap (\( \delta \)) between adjacent floors. For seismic poundings, Equation (8.5) should be modified to,

\[
\begin{bmatrix}
\ddot{q}_{1,0} \\
\ddot{q}_{2,0}
\end{bmatrix}
\begin{bmatrix}
m_{s,1} & 0 \\
0 & m_{s,2}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_{1,0} \\
\dot{q}_{2,0}
\end{bmatrix}
+ \begin{bmatrix}
\ddot{q}_{1,0} \\
\ddot{q}_{2,0}
\end{bmatrix}
\begin{bmatrix}
c_{s,1} & 0 \\
0 & c_{s,2}
\end{bmatrix}
+ \begin{bmatrix}
\dddot{q}_{1,0} \\
\dddot{q}_{2,0}
\end{bmatrix}
\begin{bmatrix}
k_{s,1} & 0 \\
0 & k_{s,2}
\end{bmatrix}
+ \begin{bmatrix}
PF_1 \\
PF_2
\end{bmatrix}
\begin{bmatrix}
\dddot{u}_g \\
\dddot{v}_g
\end{bmatrix}
= \begin{bmatrix}
PF_1 \\
PF_2
\end{bmatrix}
\begin{bmatrix}
m_{c,1} \\
m_{c,2}
\end{bmatrix}
\]

where, \( \dddot{u}_g \) is the ground acceleration at time \( t \), and

\( m_{c,i} \) is the mass matrix with elements \( (m_{c,i})_{ij} = m_{ij} \)
8.2 Experimental setup

Figure 8.2 Steel beams for impact. All dimensions are in mm.

The test specimens consist of 300 x 50 x 50 mm steel beams (Figure 8.2). Two steel plates, 200 x 50 x 10 mm, were welded close to the either end of the beams (Figure 8.2 (top view) and Figure 8.3(a)). The mass of the beams could be increased by bolting additional 200 x 150 x 10 mm steel plates (e.g. four plates are added in Figure 8.3(b) and three plates in Figure 8.3(c)) to these plates. Finally, four 50 x 50 x 3 mm plates were welded to the bottom of the beam as shown in Figure 8.2 (side view). The frames (see Sub-section 8.2.2) are constructed by attaching 400 x 50 x 3 mm columns to these plates.

Figure 8.4 shows the three test setups adopted for the study, i.e. impact between (i) beams suspended as pendulums (Figure 8.4(a)), (ii) the beams of two single-storey frames (Figure 8.4(b), and (iii) between the beam of a single-storey frame and the first-storey beam of a two-storey frame (Figure 8.4(c)).

Similar to the RC impact tests Chapter 5, several detachable steel contact elements were prepared. The contact elements were of two types, i.e. plane (Figure 8.3(b)), and 30 mm hemisphere (Figure 8.3(c)). The elements were fixed to the beams with two M6
countersunk bolts. To ensure proper contact, one beam always had the plane contact element, while the other beam had the 30 mm hemispherical element. The beam assemblies with plane and hemispherical elements are 8.6 kg and 8.65 kg respectively.

A few impacts were conducted at 0.2-0.4 m/s velocity before the tests, to flatten the pounding head and ensure the repeatability of subsequent tests. Without this initial flattening, the pounding head can slowly develop permanent deformation with each impact which might cause variable contact conditions. Since the impact tests were conducted at less than 0.1 m/s velocities no further permanent deformation of the contact elements, besides this initial flattening of 0.3 mm, was observed at the end of the tests. The beams and columns also did not show any permanent deformation at the end of the tests.

Figure 8.3 Steel Beam employed for the test (a) unloaded beam, (b) loaded beam and (c) suspended beam with spherical impact head.

Figure 8.4 Test setups for impact between (a) pendulums, (b) single storey frames and (c) a single storey and a two-storey frames.
The impact accelerations of the beams were measured by two accelerometers of capacity ±10g and bandwidth 0.3 Hz to 10 kHz. For pendulum tests, the horizontal displacement of the beams was measured with laser sensors focused at the far end of each beam. The precision (0.3 mm) of the laser sensors is not sufficient to measure the relative deformation of the beams during contact, so they are used to obtain the maximum displacements of the beams before and after each impact. The immediate pre-impact and post-impact velocities of the pendulum masses are calculated from these peak displacement values. For the impact of beams supported by columns, strain gauges were attached to each column, see Figure 8.5(a), to measure inter-storey drifts of the frames. The velocities immediately prior to and post-impact were calculated from the free vibration of the frames. Although, a 10 kHz sampling rate was sufficient for the impact of RC slabs described in Chapter 5, the measurement of impact acceleration of the steel beams required 50 kHz sampling rate.

8.2.1 Impact between pendulums

The two steel beams were suspended from an overhead girder by four M12 eye-bolts (Figure 8.3(c)). Initially, the position of the beams was adjusted so that they had less than 1 mm gap but did not actually touch each other. The impact was induced by pulling one beam to some distance, and letting it swing free to strike the second beam. The initial displacements of the far ends of the beams were measured by laser sensors. The velocities at the beginning and end of each impact can be calculated from the equations of motion of
freely hanging pendulums. Impacts between pendulums were carried out at ten relative velocities.

### 8.2.2 Impact between two one-storey frames

The frames were constructed by bolting two steel columns (400 x 50 x 3 mm) to the 3 mm plates at the bottom of the beams (see right frame in Figure 8.5(a)). Two M6 bolts and two joint plates of size 100 x 50 x 3 mm were used to connect the column and the plate. Free vibration tests were conducted to obtain the natural period (0.13 s) and damping (0.5 %) of the frames. The stiffness of each storey was calculated from the mass and natural frequency of the frames.

For the impact tests, the frames were placed with the ends of the beams very close to each other, but not in actual contact. One of the frames was then displaced and released so that the beam of the first frame struck that of the second frame. The velocity of the striking beam at the time of impact was calculated from the initial floor displacement of the frame. Similarly, the after-impact velocity of each beam was calculated from the maximum floor displacements after the impact. Ten impact tests were carried out for this setup.

### 8.2.3 Impact between a one-storey frame and a two storey frame

A second storey was added to the struck frame in the previous section. For this purpose, a second type of beam (Figure 8.5(b)) was fabricated. The beam had two 50 x 50 x 3 mm plates welded to its top surface for connecting the second storey’s columns. It also had two less attachments on the bottom (see Figure 8.2 (side view) and Figure 8.5 (b)) to keep the mass equal in both types of beams. This beam was used as first floor in the two-storey frame (see the left frame in Figure 8.5(a)). The natural period and damping of the two-storey frame were identified by free vibration tests.

The frames were placed such that the beam of the one-storey frame was very close to but not in actual contact with the first floor beam of the two-storey frame (Figure 8.4(c) and Figure 8.5(a)). The impact was forced similar to that for the two one-storey frames. However, two different types of test were conducted for this setup: (i) the one-storey frame was displaced and allowed to strike the second storey frame, and (ii) the second storey frame was displaced and allowed to strike the one-storey frame. Each type of impact was repeated ten times.
8.3 Results and discussion

8.3.1 Impact between pendulums

The proposed model requires the longitudinal mode shapes of the beams. Therefore, the beams in Figure 8.2 were idealized as square bars of the same length, mass and volume. The beam with the 3 cm diameter hemispherical attachment was idealized as a 335 x 57.4 x 57.4 mm steel bar and the beam with plane attachment as 320 x 58.5 x 58.5 mm steel bar. The pendulum masses can be modelled as freely moving bars for the duration of contact. Equations 7.1 and 7.14 were used to simulate contact. The E, ρ and μ for mild steel are 200 GPa, 7850 kg/m³ and 0.3, respectively. From Equation 7.2, the contact stiffness k for a 3 cm steel hemisphere hitting a plane steel surface is 17.9 GN/m³/2. 5% damping was assumed for axial vibration of beams in all numerical simulations.

Figure 8.6 Impact induced accelerations. (a) Experimental (x = 300 mm) and (b) simulated acceleration of the striker beam at various locations for impact of pendulums at a velocity of 0.017 m/s.

Figure 8.6(a) shows the experimental acceleration of the striker at 35 mm from the contact interface, for an impact velocity of 0.017 m/s. The simulated accelerations were calculated for 11 equidistant sections incorporating the first 100 modes of axial vibrations of the beams. The results at four selected locations of the striker, at distance x from the non-impact end of the beam, are shown in Figure 8.6(b). Similar to the RC slab impact tests, it was found that the simulated accelerations were substantially different from the experimental record. The experimental acceleration has several peak values after the first
pulse with substantial negative components. However, the simulated acceleration has almost negligible oscillation magnitude. Similarly, the simulated periodicity of 0.145 ms is only a seventh of the experimental periodicity of 1.044 ms, averaged from the first nine cycles.

The periodicities matched (Table 8.1) when the wave velocity, $c_i$, of the beams was reduced to one-eighth that obtained from $c_i = \sqrt{\frac{E}{\rho}}$, the one dimensional wave velocity of a bar of constant cross section. This change is equivalent to reducing the effective cross-sectional area of the 0.05 x 0.05 m beam by a factor of 64 to maintain the same mass which reduces the effective axial stiffness. The reduced wave velocity may be attributed to the dispersion induced by the abrupt changes of cross-section in the bar and the finite geometry of the cross section (Graff 1991). The reduced velocity also produced different acceleration time history at different sections of the bar (Figure 8.7). However, the simulated acceleration time histories were still substantially different from the experimental result shown in Figure 8.6(a).

Table 8.1 Periodicity of acceleration time history for reduction in $c_i$.  

<table>
<thead>
<tr>
<th>Reduction factor</th>
<th>Periodicity (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.133</td>
</tr>
<tr>
<td>2</td>
<td>0.266</td>
</tr>
<tr>
<td>3</td>
<td>0.399</td>
</tr>
<tr>
<td>4</td>
<td>0.532</td>
</tr>
<tr>
<td>5</td>
<td>0.665</td>
</tr>
<tr>
<td>6</td>
<td>0.797</td>
</tr>
<tr>
<td>7</td>
<td>0.930</td>
</tr>
<tr>
<td>8</td>
<td>1.063 (cf. 1.044 above)</td>
</tr>
</tbody>
</table>

Figure 8.7 Simulated accelerations of the striker beam for 1/8 $c_i$ and $N = 100$. 

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By trial and error it was found that the results showed better agreement when only one mode of axial vibration \((N = 1)\) was used in simulations (Figure 8.8). The simulated acceleration time history at \(x = 301.5\) mm are in phase with experimental acceleration which was measured at \(x = 300\) mm. However, the peak simulated magnitude was only two-thirds of the experimental magnitude. Also, the period of first two acceleration cycles was only two-thirds of the latter cycles while the experimental acceleration has same periodicity throughout. From further trial and error, it was found that the acceleration magnitudes can be matched by considering a higher contact stiffness \(k\) or a lower stress wave velocity \(c_i\). Figure 8.9(a) compares the simulated and experimental accelerations when twice the contact stiffness was applied. The magnitudes became closer but the period of the first two cycles was reduced even more than that in Figure 8.8. Figure 8.9(b) presents the same comparison when the wave propagation velocity was reduced by half, i.e. \(c_i = \frac{1}{16} \sqrt{\frac{E}{\rho}}\). The periodicity and magnitudes of the first two cycles of simulated acceleration are in much closer agreement with the experimental results but the subsequent cycles had almost twice the period.

![Figure 8.8 Simulated accelerations of the striker beam for 1/8 \(c_i\) and \(N = 1\) in comparison with the experimental acceleration.](image)

The effect of the various modifications on the pounding force is shown in Figure 8.10. When \(c\) or \(k\) is reduced the peak pounding force decreases and contact duration increases. A comparison with Figure 8.8 and Figure 8.9 shows that the first two cycles of acceleration, discussed in the preceding paragraph, occur during contact while the rest of the oscillation is after separation. Thus, the case 1/16 \(c_i\), \(N = 1\) gives a better agreement with experimental accelerations during contact (see Figure 8.9(b)) while the case 1/8 \(c_i\), 2 \(k\), \(N = 1\) better simulates the oscillations after the beams separate (see Figure 8.9(a)). The results also contrast with RC slabs test where a good agreement between simulated and experimental accelerations was obtained when the first 100 modes of axial vibrations of the slabs were used in the calculations. The difference in behaviour could be because of the uniformity of
the steel in comparison to RC, which affects the propagation of the stress waves through them. The RC slabs also showed a considerably steeper reduction in magnitudes of oscillations after separation, while the reduction is more gradual for the steel beams (Figure 8.6(a)). Despite these differences, for both RC and steel impacts, the periodicity of acceleration after impact is same as that obtained when effective wave propagation velocity is one-eighth of the wave speed in the considered beam.

![Figure 8.9 Acceleration of striker beam at various locations for N = 1: (a) 1/8 c_i and 2 k and (b) 1/16 c_i and original k.](image)

![Figure 8.10 Simulated pounding force for simulations shown in Figures 7 to 10.](image)

8.3.2 Impact between one-storey frames

From free vibration tests, the period of one-storey frames (Figure 8.4(b)), with two columns and without additional plates, was found to be 0.13 s. The corresponding damping ratio was 0.5%. The horizontal displacement time-histories of the beams, when the right frame was pulled 1.95 mm, are shown in Figure 8.11. The initial gap between the frames was 1.25 mm. Thus the striker vibrates freely from the initial displacement of 1.95 to the right up to 1.25 mm to the left, where the first impact occurred. The striker swung back to
1.1 mm on the right while the struck beam displaced up to 2.42 mm to the left. The striker beam’s velocity just before the first impact was calculated to be 0.069 m/s. The striker’s acceleration time history (Figure 8.12) shows that there were five progressively softer impacts after which the frames vibrated freely. Considering, the one-storey frame cases, a detailed view of acceleration due to the first three impacts is shown in Figure 8.13. There is a distinctive similarity in the time history of all three impacts. The oscillation period of accelerations is 0.86 ms, as opposed to 1.04 ms observed in the pendulum’s impacts (Figure 8.6(a)). From Table 8.1, it can be seen that this equates to a reduction of the stress velocity $c_i$ by a factor of 6.5. However, a closer examination of Figure 8.13 reveals the influence of a second period of 4.04 ms which suggests a velocity reduction factor of 30.37.

![Figure 8.11 Displacement time history of one-storey frames pounding with each other, with 0.069 m/s relative velocity at first impact.](image1)

![Figure 8.12 Acceleration time history of the striker for the impact shown in Figure 8.11](image2)

![Figure 8.13 First 20 ms of acceleration time histories for the first three impacts in Figure 8.12.](image3)
The numerical simulation was first carried out for $1/6.5 \, c_i$. The simulated acceleration time history of the striker due to first impact is shown in Figure 8.14. The periodicity of acceleration matched but there was very large difference in the first two peak magnitudes. Figure 8.15 compares the accelerations simulated for $1/30.37 \, c_i$ against experimental results. The difference in magnitude was still quite large. A closer match was obtained when the first two axial modes of the beams were considered (Figure 8.16). However, the contact stiffness also had to be modified to reduce the magnitude discrepancy. From trial and error, the closest agreement in peak magnitudes was obtained only when the contact stiffness $k$ was reduced by a factor of 15 (Figure 8.17). This is similar to the effect of additional masses in the impact between RC slabs, and might be attributed to the same phenomenon, i.e. the compressive zone extended further from the contact interface, which caused a reduction in contact stiffness. Thus, the effect of the additional resistance provided by the columns is similar to that of the additional mass (dotted line and dashed and dotted line).

![Figure 8.14 Experimental and simulated acceleration of the striker because of the first impact ($v_i = 0.069 \, \text{m/s}$) in Figure 13 with $1/6.5 \, c_i$ and $N = 1$.](image1)

![Figure 8.15 Simulated and experimental accelerations for the frame impact at a velocity of 0.069 m/s for $1/30.37 \, c_i$ and $N = 1$.](image2)

The pounding forces using Equation Figure 7.1 and the apparent overlap, $\delta$, simulated for various cases considered in the previous paragraph, are presented in Figure 8.18. Similar to the impact between pendulums, reduction in $k$ decreased the magnitude and increased the
duration of pounding force. However, at high $k$ and low $c_i$ (1/30.37), the comparative flexibility of the beams resulted in two separate impacts.

Figure 8.16 Simulated and experimental accelerations for the frame impact at a velocity of 0.069 m/s for 1/30.37 $c_i$ and $N = 2$.

Figure 8.17 Simulated and experimental accelerations for the frame impact at a velocity of 0.069 m/s for 1/30.37 $c_i$, 1/15 $k$ and $N = 2$.

Figure 8.18 Simulated pounding force for impact of frames at velocity 0.069 m/s for variation of parameters shown in Figure 8.14 to Figure 8.17.

8.3.3 Impact between a one-storey and a two-storey frame

The one-storey frame in these tests is the striker frame in Sub-section 8.3.2 with a period of 0.13 s. From snapback tests, the period of the first two modes of the two-storey frame was found to be 0.21 and 0.08 seconds. First, the one-storey frame in Figure 8.4 was pulled 0.95 mm to the right and allowed to snap back and strike the first floor beam of the two-storey frame. Secondly, the two-storey frame was pulled to the left and allowed to strike the one-storey frame.
The displacement time histories of the frames when the one-storey frame was pulled 0.95 mm are presented in Figure 8.19. The initial gap between the frames was 0.72 mm. A total of five impacts can be observed in the acceleration time history of the striker beam (Figure 8.20). The relative velocity of the beams just before the first impact was calculated to be 0.029 m/s. The acceleration of the striker beam during the first impact is shown in Figure 8.21. The shape of the acceleration time history is very similar to that observed for impact between one-storey frames (Figure 8.13). The simulated accelerations were calculated for $1/30.37 c_i$ and $N = 2$ since these values give the closest approximation in the impact of one-storey frames. However, for the closest approximation, the simulations required the contact stiffness to be $1/10 k$ as opposed to the $1/15 k$ for impact between one-storey frames.

![Figure 8.19 Displacement time history of the pounding beams when the two-storey frame is struck by the one-storey frame and the relative velocity at first impact is 0.029 m/s.](image)

![Figure 8.20 Experimental acceleration of the beam of one-storey frame striking the first storey beam of two-storey frame shown in Figure 20.](image)

![Figure 8.21 Simulated and experimental acceleration of the first impact in Figure 20; for $1/30.37 c_i$, $1/10 k$, impact velocity = 0.029 m/s and $N = 2$.](image)
Figure 8.22 shows the horizontal displacement of the pounding beams, when the second storey beam of the two-storey frame was pulled 1.72 mm to the right and released. The initial displacement of the first storey beam was 0.87 mm. The acceleration time history of the struck beam (Figure 8.23) shows that there were a total of 6 impacts. While the successive impacts in the case of one-storey frames were progressively softer, the acceleration for the second impacts in Figure 8.20 and Figure 8.23 are higher than the first. The relative velocity for the first impact was calculated to be 0.028 m/s. Because of the almost equal velocities, the impact accelerations for the first impact in this case (Figure 8.24) is almost identical to that in Figure 8.21. The numerical simulations produced the closest results with the same parameters, i.e. $1/30.37 c_i$, $1/10 k$ and $N = 2$. 

![Displacement time history of the pounding beams when the two-storey frame struck the one-storey frame.](image1)

![Experimental acceleration of the beam of one-storey frame struck by the first storey beam of two-storey frame.](image2)

![Simulated and experimental acceleration of the first impact in Figure 20; for $1/30.37 c_i$, $1/10 k$, impact velocity = 0.028 m/s and $N = 2$.](image3)
8.3.4 Consequence of system flexibility

In comparison with impact between pendulums, the impact between frames is softer, e.g. the peak acceleration was 4.5 g for a pendulum impact with a velocity of 0.017 m/s (Figure 8.6(a)). In another case, for an impact velocity of 0.04 m/s the 10 g accelerometer was saturated. In contrast, an impact between frames produces a peak acceleration of just 8 g even though the velocity is 0.069 m/s (Figure 8.12). For impact between a one-storey frame and a two-storey frame, the peak acceleration was 6.2 g for a velocity of 0.029 m/s (Figure 8.20). This clearly demonstrates the effect of support conditions. Namely, with increasing fundamental frequency of the system the magnitude of the impact acceleration decreases and the duration increases.

To the author’s knowledge, the variation of $k$ according to the system has not been reported previously. However, a careful study of the experiment on pounding between two-storey, single bay RC building frames by Papadrakakis and Mouzakis (1995) shows clearly that the contact stiffness is not solely a function of contact conditions. From the gap and the acceleration time histories in Papadrakakis and Mouzakis (1995), the contact duration of pounding at the second storey floor slabs is almost half that of the first storey floor slabs, which means the effective contact stiffness between the second floors is higher than that between first floors. For the two-storey frames, the translational stiffness of the first storey is higher than that of the top floor. Therefore, it can be suggested that the effective contact stiffness is governed jointly by the contact surface details and the translational stiffness of the storey under consideration.

8.4 Practical example

The simplified buildings shown in Figure 8.25 are used as an example of the application of the proposed pounding model to illustrate its use in predicting pounding responses under earthquake loading. The buildings were subjected to floor to floor pounding under 8 seconds of El-Centro ground motion (Figure 8.26). The at-rest separation gap was set to zero at all floors. The building slabs are reinforced M20 concrete and $\rho$, $\mu$ and $E$ are 2500 kg/m$^3$, 0.15 and 21 GPa, respectively. The relevant characteristics of the building are provided in Table 8.2. The entire floor was assumed to take part in the contact force development, i.e. $D = 1$. From Equation 8.2, the contact stiffness was calculated to be $6.3 \times 10^8$ N/m. The effective stress propagation velocity was assumed to be $\frac{1}{8} \sqrt{\frac{E}{\rho}}$ as calculated
for the impact of RC slabs. The simulation was carried out with a time step of 10 μs. The first 100 modes were used as found necessary from the impact of RC slabs in Chapter 7. When the buildings are in contact, the pounding force was calculated from Equation 8.1. Equation 8.8 was solved by Newmark’s method and Equation 8.7 was solved by the interpolation of excitation method described by Chopra (2001).

![Figure 8.25 Adjacent buildings subjected to pounding](image1)

![Figure 8.26 El-Centro ground motion](image2)

Table 8.2 Building properties.

<table>
<thead>
<tr>
<th>Pounding Models</th>
<th>5-storey building</th>
<th>3-storey building</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor Cross-section (m)</td>
<td>8 x 0.12</td>
<td>8 x 0.12</td>
</tr>
<tr>
<td>Floor weight (tonnes)</td>
<td>48</td>
<td>29</td>
</tr>
<tr>
<td>Interstorey Stiffness (kN/m)</td>
<td>8.4 x 10⁴</td>
<td>6.7 x 10⁴</td>
</tr>
<tr>
<td>Longitudinal floor stiffness (GPa)</td>
<td>1.01</td>
<td>1.68</td>
</tr>
</tbody>
</table>

The displacement of the third floors of the buildings is shown in Figure 8.27. Pounding reduced the displacement for both buildings at this floor. Studies (e.g. Jeng and Tzeng 2000) have suggested that this type of pounding can be extremely hazardous to the taller
building since the fourth storey columns can be damaged because of the restricted sway at the third floor. However, the interstorey drift at the fourth floor was not significantly affected by the pounding (Figure 8.28). Similarly, the acceleration response of the 4th floor was not amplified by pounding (Figure 8.29(b)). However, the third floor acceleration was almost 8 g due to pounding (Figure 8.29(a)).

![Figure 8.27 Displacement of mid-point of the third floors of the buildings, (a) without pounding and (b) with pounding.](image)

![Figure 8.28 Interstorey drift at the 4th storey with and without pounding.](image)

The pounding force at the third floor is shown in Figure 8.30. The pounding force reaches 360 kN. The pounding force due to four different contacts beginning at 2.04 s, 2.56 s, 3.05 s and 5.29 s are shown Figure 8.30(b). The forces are significantly different from those predicted by either lumped-mass (e.g. Muthukumar and DesRoches 2006 and Jankowski 2005) or distributed-mass methods (e.g. Cole et al. 2011 and Malhotra 1998).
Figure 8.29 Accelerations of central section of (a) the 3rd and (b) the 4th floors of the 5-storey building.

Figure 8.30 (a) Pounding force at third floor level and (b) pounding force at four contacts beginning at the time instants shown in the legend.

Figure 8.31 The influence of effective wave propagation velocity on amplification of responses because of pounding: (a) 5-storey building and (b) 3-storey building.
A sensitivity analysis was carried out for $k$ and $c_i$ because of the uncertainties observed in Section 4. The effect of these two parameters on the response was calculated. The responses shown are floor displacements and inter-storey drift of the buildings. When $D$ was varied, $c_i = c/8$ was kept constant, and the effect of $c_i$ was studied for $D = 1$. From Figure 8.31, $c_i$ has a large influence on both these responses, especially for the three-storey structure. In contrast, the contact stiffness did not show appreciable effect as shown in Figure 8.32, where $k = D k_i(D = 1)$. However, the effect of $k$ on pounding force is quite large (Figure 8.33). These results indicate that these two parameters are critical for determination of possible damage.

![Figure 8.32](image1.png)  
**Figure 8.32** The influence of contact stiffness on amplification of responses because of pounding: (a) 5-storey building and (b) 3-storey building.

![Figure 8.33](image2.png)  
**Figure 8.33** The changes in pounding force due to: (a) contact stiffness and (b) effective wave propagation velocity.
8.5 Summary

This chapter extends the damped Sears model presented in the previous chapter to building pounding applications. The methodology is presented and validated with experiments on impact between steel beams. The experiments included: (i) impact between pendulums, (ii) impact between one-storey frames, and (iii) impact of a one-storey frame with a two-storey frame. With the appropriate selection of parameters, the model predicted impact accelerations in reasonable agreement with experimental results. Finally, a practical application of the model to building pounding is presented.

The following conclusions can be drawn from the work:

- The proposed model can predict the impact accelerations of the frames substantially better than the lumped-mass or distributed-mass models.
- The stress due to impact propagates at a considerably slower rate than predicted by material properties alone.
- The stress propagation velocity is less when the beams are supported by columns than when they were suspended as pendulums.
- The effective contact stiffness that achieved the closest agreement between experiment and theory was affected by the translational stiffness of the pounding beams. The stiffness factor $k$ was found to be 1/1, 1/10 and 1/15 for impact of pendulums, impact of a single storey frame against two-storey frame and impact of one-storey frames, respectively. This illustrates that effective stiffness controlling contact response, e.g. contact duration and acceleration, is a function of not only the contact interface details but also the structural details of the supporting system.
- From the practical example, the effective velocity of stress propagation has a large influence on floor displacements as well as pounding force. In contrast, for the case considered, the stiffness has substantial effect on the pounding force but not on the displacement.
Chapter 9

Recommended further development of damped Sears model for application to building design against pounding

The objective of this PhD research is to develop a numerical force model suitable for simulation of building pounding. A suitable model, i.e. damped Sears model has been derived in Chapter 7 and verified for the impact experiments between freely moving masses. As distributed-mass models of impact have traditionally suffered from an inability to account for effect of supporting elements, the model was further developed to incorporate supporting elements in Chapter 8. However, the model needs substantial further development before being implemented in real-life assessment and strengthening design of real-world buildings.

This chapter first identifies the limitations of the current model for application to real world building pounding and then outlines further research work required toward this development.

9.1 Limitation of the proposed model in its current form

An example simulation of building pounding with damped Sears model was presented in the previous chapter. Several simplifying assumptions were made in the simulation as follows:

- Both buildings have very regular and symmetrical configuration with only four corner columns in each building.
- Both buildings have same width and the impact is fully distributed over the whole area on the contact end.
- The floor slabs of both buildings are of same thickness and at exact same level, such that there is no offset between the edges of contact surface.
Conclusions and recommendations for future research

- The floor slabs are of uniform cross-section throughout their length, so that an unobstructed stress propagation path is available from end to end of each floor slab.
- The contact stiffness was calculated from the linear stiffness of a fraction of the length of the floor slab.
- Potential alteration of the contact location due to impact induced damage has not been considered.
- Potential effects of the non-linear behaviour of the columns have not been considered.

In addition, the small scale validation experiments presented in Chapters 7 and 8 have shown that the key parameters of the model, i.e. contact stiffness $k$ and effective stress propagation velocity $c$, have significant effect on the response of impacting masses. Furthermore, these parameters were affected by every change in the test configuration, e.g. change in mass, change in impact velocity, additional columns, additional storey etc.

Therefore, for the application of the model in its current form to real world buildings, following simplifications will be required:

- The building needs to be modelled as an MDoF system with lumped floor masses as shown in Figure 8.1(b) for the solution of Equation 8.8. The stiffness of each storey will need to be calculated as sum of the stiffness of all the columns and walls on that floor. Any effect of spatial distribution of columns and walls will be ignored.
- It has to be assumed that the contact location can withstand and transfer unlimited force, without undergoing permanent deformation. Similarly, the building walls and columns will also be assumed to remain elastic for the duration of the analysis.
- It has to be assumed that the impact is always over the full surface of the slabs at the contact interface. The effect of distribution of stress waves through the slab due to possible point or edge contact cannot be considered. Similarly, the supporting columns and walls will be assumed to act as point springs during impact, i.e. the stress waves will not propagate through the columns or walls.
- The floor slabs will need to be modelled as longitudinal bars (see floor slabs in Figure 8.1a) with effective cross-sectional area calculated as the quotient of volume divided by length. Any effect of floor slab openings, protrusions etc. will be ignored.
Conclusions and recommendations for future research

- Any offset in plan between the two buildings will have to be ignored as only the central, longitudinal impact between the floor slabs can be considered. This will ignore any torsional motion arising due to non-central impact.
- Finally, in the absence of relevant large scale test results on impact between distributed masses, the contact stiffness and effective stress velocity cannot be reliably estimated. As shown in the previous chapter, a sensitivity analysis will need to be conducted for these two parameters.

These are extreme simplifications and unlikely to be obtained in real world pounding scenarios, for example, the intensity of transferred force will be limited by the capacity of contact surface, beyond which permanent deformation will occur. Therefore, the model needs more development before it is ready for applications such as assessment of a building’s capacity to survive under pounding, or design of retrofitting scheme to ensure survival.

9.2 Recommendations for further development of the proposed model

The limitations outlined in the previous section can be divided into two categories, viz. (i) modelling limitations and (ii) knowledge limitations. The first category of the limitations can be rectified by modifying the model to include these behaviours in the current model by making analytical or numerical alterations. The second category will need laboratory tests, preferably large scale impact tests, to resolve because no relevant test results could be found in the literature.

The first three simplifications in the previous list can be primarily considered modelling limitations, while the last three items require more laboratory tests. Nevertheless, all items will need a measure of both modelling and experimental resolution, i.e. a modification to include the effect of edge or corner impact will need to be verified with experiments while the experimental results for torsional impact between buildings will then need to be incorporated into the numerical model. Following sub-sections present possible solutions to resolve these limitations.

9.2.1 Spatial distribution of lateral load resisting members

Post-earthquake surveys have shown that boundary columns and walls near the pounding location undergo more damage due to pounding than those farther from contact location. NZSEE (2006) suggests that the columns closest to the pounding location should be
strengthened if a building has insufficient separation distance with its neighbour. Therefore, the model should be expanded to consider the spatial distribution of walls and columns instead of modelling a building as a lumped-MDoF system. This can be accomplished by modifying Equation 8.8 so that the force, mass, stiffness and damping matrices represent a 3-D beam-column frame. While the M, K and C matrices can be modified relatively easily to incorporate the 3-D frame, the PF (pounding force) matrix will take considerable effort to be modified to accurately represent the effect of pounding-induced forces at each node.

9.2.2 Maximum pounding force transferred at the contact location

The small scale low velocity laboratory impacts conducted in this study did not produce any permanent damage in the contact surface area and the force transfer can be considered essentially elastic, whether linear (Eqn. 8.1) or nonlinear (Eqn 7.1). However, larger scale higher velocity impact tests (e.g. van Mier et al. 1991) have shown that permanent deformation (damage) of the contact surface due to large impact forces can impose an upper limit on the magnitude of impact force. Therefore, Equations 7.1 and 8.1 should have an upper limit of PF_{i,max}, which can be quite easily incorporated in the model as follows:

\[
F = k \delta^{3/2} \leq PF_{i,max} ; \quad \delta > 0
\]

\[
F = 0 \quad ; \quad \delta \leq 0
\]

However, the value of PF_{i,max} cannot be determined from the current state of the art. Large scale laboratory tests are recommended to estimate this value to the accuracy required for structural engineering purpose.

9.2.3 Contact between edge or corner of floor slabs

The theoretical derivation and numerical analysis as well as experimental tests in this study have assumed full-surface contact between floor slabs. However, this assumption may not be valid in majority of real-world buildings: the floor edges might not be perfectly parallel or the buildings might undergo torsional vibration due to unequal distribution of mass or stiffness. Most of the building design codes require an eccentricity of ±10% to be considered in analysis, which means that even if the buildings might have full-surface contact under one of the scenarios, the contact in all other situations will be between one edge, or even corner, or a floor and mid-face of the adjacent floor. The corner-to-face
contact differs from full-surface contact in two principal ways: (i) the maximum pounding force as discussed in Section 9.2.1 will be less for corner-to-face contact than for full-surface contact because the contact surface area is smaller and can be damaged at lesser force (for instance, compare cone to plane surface impact in van Mier’s tests with spherical to plane surface impact (van Mier et al 1991); and (ii) because of the large lateral dimensions of the floor slabs, the stress-waves will propagate transversely as well as longitudinally. The proposed model in the current form is based on longitudinal stress propagation and does not incorporate this transverse stress propagation, for which the model will need to be reformulated considering two- or three-dimensional stress propagation. However, considering the extent of other uncertainties present in the analysis of building pounding, the mathematical effort required for full theoretical derivation may not be worthwhile. As a first evaluation of possible effects of such corner-contacts, a series of laboratory tests is recommended to be carried out. If the tests show that the global effects of the corner-to-face pounding are less severe than those of full-surface pounding, the design engineers need to consider only the more severe full-surface contact and mathematical derivation for corner-to-face contact will not be required.

9.2.4 Longitudinal variation in cross-sectional area of floor slabs

Floor slabs often contain offsets and openings e.g. balconies, staircases and lift wells. The proposed model is based on longitudinal vibrations of prismatic bars. Therefore, the effects of such openings and offsets have not been considered. Because there can be numerous variations of size and location of such openings and offsets, it is recommended that laboratory tests be carried out to assess the effects. The model needs to be updated if significant effect is found.

9.2.5 Longitudinal variation in cross-sectional area of floor slabs

Often the plan dimensions of the adjacent buildings can be such that the lengths of the contact face are unequal (Figure 9.1). Therefore, while the assumption of longitudinal wave propagation might be true for one building (left building in Figure 9.1), the other building (right building in Figure 9.1) will have both transverse and longitudinal wave propagation.
Conclusions and recommendations for future research

Figure 9.1 Torsional pounding between adjacent floors

This case can be considered a combination of those in Sections 9.2.1 and 9.2.3 and 9.2.4. Therefore, it is recommended that any theoretical or experimental strategy be formulated after solving the previous cases.

9.2.6 Contact surface stiffness and effective wave propagation velocity

The effect of these two critical parameters was demonstrated in the previous two chapters. It was shown that the coefficient of restitution $e$ is reduced, i.e. viscoelastic loss of kinetic energy is greater, if the stiffness is increased or effective wave propagation velocity is reduced; high stress velocity or soft contact surface results in essentially elastic impact. Because these two values cannot be analytically determined with the current state of the art, a sensitivity analysis was carried out in the building pounding simulation in Chapter 8 and quite a substantial effect was observed on interstorey drift and pounding force (refer Figures 8.31 and 8.33). In the absence of large scale experimental verification of which of the scenarios is more likely, a design engineer will have to select the most severe scenarios for both cases. For a design engineer, this can be quite uneconomic when designing retrofitting measure for mitigation of pounding-induced damage in structures. Therefore, the values of these parameters need to be determined for adoption of the proposed model into structural design calculations.

It was observed in the Chapters 7 and 8 that these parameters can be affected by many parameters, e.g. floor mass, type of supports (suspended pendulum or supported by column) and number of storeys. Therefore, it is recommended that the values be determined from large scale parametric tests on impact between distributed concrete masses rather than complex theoretical derivations.
9.3 Summary

This chapter first identifies the limitations of the proposed model in its current form for application to real world building pounding simulations and designs, and then outlines further research work required toward this development.
Chapter 10

Conclusions

10.1 Conclusions

The principal objective of this doctoral research is to develop a numerical force model suitable for simulation of building pounding. The model has to be reasonably accurate in predicting both displacement and acceleration responses due to pounding. The first part of the study (Chapters 2-6) consists of the experimental and numerical evaluation of numerical pounding models available in the literature. It was discovered that these models have many limitations which make them unsuitable for general purpose application in pounding simulation. Therefore, a new force model is developed and verified in the second part (Chapters 7-8).

10.2 Evaluation of the existing force models

First, shake table experiments were carried out for pounding between two and three steel portal frames in a row. The amplification of maximum displacement of two reference frames due to pounding were calculated for three different configurations: (i) only with one other frame; (ii) with two identical frames, one on either side; and (iii) at one end of two identical adjacent frames. The two reference frames were subjected to pounding with eight different types of frames in the three configurations under five ground motion time histories. The time histories were applied once from left to right and then from right to left. A total of 480 tests were performed. Unlike the consistent observations in post-earthquake surveys that buildings located at the end of a row suffer more damage than those in the interior, however, the investigations showed that the displacement amplification was always the highest for the stiffest frame. As predicted by the previous numerical studies, it was observed that the three-frame arrangements always caused higher amplification than two-frame arrangements. Similarly, the experimental results validated the previous numerical observations that the response of the stiffer frame is amplified and the response of the more flexible frame is reduced. However, the experimental results did not agree with previous numerical simulations that a significantly heavier mass in a stiffer frame can amplify the response of the more flexible frame with a smaller mass. A pattern was
observed where the ground motions that produced higher displacement without pounding experienced less amplification due to pounding.

Second, an evaluation of five selected contact elements for lumped-mass modelling of pounding was carried out. These models were used to numerically simulate the 80 pounding tests in the two-frame configuration described in the previous paragraph. The amplification of maximum displacement of the reference frame obtained from the numerical simulations was compared with that obtained from the experimental results. All the numerical models except one produced essentially identical results. The Hertzdamp model predicted substantially higher amplification in all cases because the derivation is valid only for almost elastic contacts but the coefficient of restitution seems to be below 0.4 in these tests. The most commonly used value of coefficient of restitution, i.e. 0.6, overestimated the results in all the cases. With coefficient of restitution equal to 0.4, the numerical results were very inconsistent: the results for two of the time histories were quite good; however, the results for remaining three time histories were substantially worse. Overall, the tests did not verify any of the considered models as performing well.

Third, a new contact element for lumped-mass modelling was derived because a review of literature showed that all existing viscoelastic contact elements had some limitations. Either they predicted partially tensile contact force, or large discontinues were present in the predicted force or the derivations were made with some large approximations which severely limited the application potential. Therefore, a numerically exact viscoelastic force model without any such limitations was derived by solving the Hunt-Crossley family of equations which have been used in mechanical engineering since 1975. A comparison with previous impact test results showed a much better performance than the other viscoelastic contact elements.

Fourth, a series of impacts between reinforced concrete slabs as pendulums was conducted. A total of 95 impacts were performed for parametric investigation of coefficient of restitution and impact-induced peak acceleration was carried out by varying the velocities, pendulum masses and the contact surface geometry. The coefficient of restitution was found to be influenced by the total mass of the striker and the struck pendulums, striker mass and the ratio of the masses; the coefficient also decreased with increasing total mass. For similar impact velocity, the coefficient increased with a heavier striker mass; and decreased in the case of lighter striker impacting a heavier pendulum. With increasing
impact velocity the coefficient of restitution did not show any general trend. For an impact of equal masses, the coefficient increased with velocity for the lightest mass considered. However, for heavier mass this is not the case. The coefficient also increased with velocity when a lighter mass struck a heavier mass, and decreased with velocity when the striker was heavier. The peak acceleration increased almost linearly with impact velocity and was almost insensitive to the striker, struck and total mass. All these observations contradict the assumption and predictions of lumped-mass models of seismic pounding.

Finally, a detailed critical review of the literature was carried out. Various significant limitations were found in both numerical and experimental simulations on building pounding. The coefficient of restitution was identified as the principal source of uncertainties in lumped-mass models. The displacement response depends almost exclusively on this coefficient; however, in many cases the coefficient is not reliable and almost all attempts to measure the coefficient have produced conflicting results. Although the distributed-mass models are independent of the coefficient of restitution, they still suffer from their own limitations, i.e. due to the requirement of instant full-surface contact, incapability to incorporate the effect of supporting structures and the generation of infinite acceleration. Most of the experimental simulations are limited by the employed instruments and insufficiency of sampling rate. Some force measuring instruments produced erroneous results while some of them modified the property of the system and the results were not reliable. Due to low sampling rate in many experimental simulations, the obtained results could not identify the correct force model. The Sears impact model, which includes the characteristics of both lumped-mass and distributed-mass models, was proposed as a means to explain the experimental results. The model gives a closed-form solution for finite element analysis of undamped, elastic collision of freely moving bars. However, in the original form it cannot include the effects of (i) damping and attenuation of stress waves, (ii) higher longitudinal modes of vibration and (iii) the adjacent structural members supporting the colliding structural elements.

10.3 Damped Sears impact model: derivation and verification

The deficiencies in the Sears impact model have been corrected in Chapter 7 by adding higher modes of vibration and viscoelastic modal damping. The proposed damped Sears model continues the major advantage of the Sears impact model that it is based on both lumped-mass and wave propagation models. The impact forces develop with the
deformation at the contact location and the effects spread to other parts of the structures as compressive stress. A methodology is developed in Chapter 8 to include the effects of storey stiffness on the vibration response of the colliding floors so that the model can be used for simulation of multi-storey building pounding.

The damped Sears model was validated from the results of the impact of RC slabs presented in Chapter 5. In addition, impact between steel beams was conducted to validate the model for pounding of frames. Three configurations were considered: (i) impact between pendulums, (ii) impact between one-storey frames, and (iii) impact between a one-storey frame and a two-storey frame.

The validation efforts revealed that the experimental results could not be replicated with classical values of several parameters, e.g. one-dimensional stress propagation velocity of the materials and contact surface stiffness. These parameters had to be empirically modified for different setups. With appropriate selection of parameters, the performance of the damped Sears model was significantly better than other existing models, i.e. lumped-mass model and distributed-mass model. Following conclusions can be drawn:

- The proposed model can predict the impact accelerations of the suspended beams and slabs substantially better than the lumped-mass or distributed-mass models. Lumped-mass models are unable to simulate the periodicity observed after the first pulse while distributed-mass models always predict infinite instantaneous acceleration. The damped Sears model can closely simulate the experimental impact accelerations, including the periodicity observed after the first pulse.

- The model also excels in prediction of impulse, i.e. momentum transferred between the colliding masses. In contrast to all lumped-mass models, a coefficient of restitution is not necessary to predict impulse. Similarly, while the existing distributed-mass models cannot consider energy loss in impact between members of equal length because the coefficient of restitution in such cases is always predicted as 1. The damped Sears model has no such limitations.

- To the students’ best knowledge, the damped Sears model is the only non-FE model that can consider both the distributed-mass and the storey-stiffness effects in simulations. A reasonable prediction of experimental acceleration was obtained for the impact between single-storey, and between single- and multiple-storey frames.
Conclusions and recommendations for future research

- The effective stress propagation velocity was substantially lower than that predicted by the material properties alone. The effective velocity seemed to be significantly affected by both mass and system stiffness. The velocity became lower when additional masses were added or when the pendulum support was replaced by column supports.

- The stiffness of the contact surface also decreased with increasing mass and with increasing system stiffness. A higher contact stiffness was necessary for impact between pendulums than for impact between one- and two-storey frames, which in turn was larger than that needed for impact between two single-storey frames.

- From the practical example, the effective velocity of stress propagation has a large influence on floor displacements as well as pounding force. In contrast, for the case considered, the stiffness has substantial effect on the pounding force but not on the displacement.
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