Multi-Objective Optimization for Maintenance Decision Making in Infrastructure Asset Management

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Abstract:

Maintenance decision making, selecting appropriate maintenance strategies for a road network, is an important and complex part of infrastructure asset management (IAM). Multi-objective optimization (MOO) can help in clarifying and simplifying a decision making problem with multiple objectives and trading off objectives by identifying efficient solutions. Therefore, MOO is a helpful tool in decision making process. The aim of this paper is to analyze the optimization problem in practical decision making process using MOO and identify efficient solutions in the context of maintenance decision making.

To accomplish this aim, this paper (1) introduces decision making in IAM and the previous applications of optimization; (2) discusses the mathematical formulation of optimization problems of decision making; (3) proposes an optimization method named dichotomic approach (DA) to solve the optimization problems of decision making and identify efficient solutions; (4) compares DA with Nondominated Sorting Genetic Algorithm II (NSGA II) using a practical maintenance decision making case; and (5) discusses other issues related with DA, such as controlling the numbers of identified solutions, the identification of non-supported solutions and decision making with three or more objectives.

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Using DA, optimization problems of decision making can be solved. Comparing with NSGA II, DA identifies more and better solutions, i.e. solutions are guaranteed to be efficient. The performance of DA algorithm in terms of computation time and implementation is also good.

Keywords:

Multi-objective optimization, maintenance decision making, infrastructure asset management, road network.

1 Introduction

Road, as an important infrastructure, is one of the socio-economic backbones of any society. Due to growing demands, decay of infrastructure and increasing financial pressure, the importance of the effective and efficient management of road is amplified. Infrastructure Asset Management (IAM), attempting to identify and implement the appropriate maintenance strategies for a road network, plays a role of increasing importance.

Maintenance decision making, as an essential part of IAM, selects maintenance strategies for a road network so that the goals of IAM are achieved (Maunsell Limited 2004). Network maintenance involves a significant amount of investment and has great impact on the public (NAMS 2011), but also faces many challenges due to the complexity of decision making process. For example, network owners have to consider life-cycle cost, risk and level of service aspects, when often these outcomes are in conflict with each other. Hence, multi-objective optimization (MOO) is applied to help in dealing with the issues at hand and trading off conflicting objectives.

Most of the previous applications of MOO to maintenance decision making in IAM are based on heuristic methods. As introduced in Section 2.3, heuristic methods may identify unreliable solutions of poor quality, small coverage and bad distribution especially when analyzing practical decision making problems or big road networks. In this paper, a deterministic MOO method named dichotomic approach (DA) is introduced to help with decision making when more than one objective is pursued. Then this method is tested with a practical decision making case and compared with Nondominated Sorting Genetic Algorithm II (NSGA II) which is one of the most effective multi-objective heuristic methods for decision making problems (Bai et al. 2012).

2 Background

2.1 Maintenance decision making in Infrastructure Asset Management

Maintenance decision making attempts to select appropriate maintenance strategies for a road network. A maintenance strategy indicates types of interventions that will be applied to a segment of road at particular points in time. Strategies, containing different interventions at different times, have different effects on outcomes such as maintenance cost, pavement condition, level of service, etc. When selecting different strategies for the segments of a road network, the result in terms of outcomes is also different. The selections of strategies (a solution) are evaluated based on objectives and constraints of the maintenance decision. If a selection of strategies satisfies all constraints and has better outcomes in terms of objectives, this strategy is preferred.

However, there are many challenges faced during the decision making process including:

- Many objectives need to be considered simultaneously (Jaffe 2011). The objectives, originating from the owners of infrastructure, customers and/or agencies, may be in conflict with financial realities and/or the status/age of the road infrastructure (Harmon 2003). They are not commensurable. Hence, these objective should be analysed equally and individually;
- The number of strategies may be large. If a road network has n segments with m interventions over t years, then the number of alternative strategies is $n^t \times m$ (Harvey

2012). For long-term decision making such as life-cycle maintenance decision making, there may be a large number of alternative strategies. Thus, it is difficult to analyze these strategies and select appropriate ones in reasonable time; and,

- Trading off maintenance strategies under multiple conflicting outcomes could be difficult. The proper trade-offs require adequate knowledge of the decision making problem at hand and its alternative options of decision. However, acquiring this knowledge and the options may be difficult.

MOO assists in overcoming the challenges mentioned above as being detailed in the subsequent sections.

2.2 Multi-objective optimization

MOO analyzes optimization problems of maintenance decision making with multiple objectives and a number of constraints in such a manner that the overall return on the investment, such as financial benefit and improved network condition, is maximized at the network level. More specifically, MOO attempts to solve the optimization problems raised in decision making process and identify efficient solutions.

Efficient solutions, also named Pareto solutions, are a group of feasible solutions that cannot be improved in one objective without worsening at least another objective (Hillier and Lieberman 2005). For maintenance decision making, a feasible solution indicates a selection of strategies for the entire road network that satisfies all constraints of the optimization problem of maintenance decision making; while an efficient solution is a set of feasible solution that achieves respective objectives in the best possible manner. Fig.1 is an example of a ten-year decision making with practical data which tries to optimize the network condition and minimize cost. It has 237 alternative strategies for ten segments. After analyzing all possible selections of strategies for the ten segments, all the feasible solutions are shown as blue rhombic points. Efficient solutions, Solution A (selecting strategies with indexes of 0, 88, 118, 119, 129, 136, 137, 145, 153 and 219), Solution B (selecting strategies with indexes of 0, 88, 118, 119, 126, 136, 137, 152, 154 and 219) and Solution C (selecting strategies with indexes of 0, 88, 97, 119, 127, 133, 137, 147, 154 and 222), are shown as red square points.

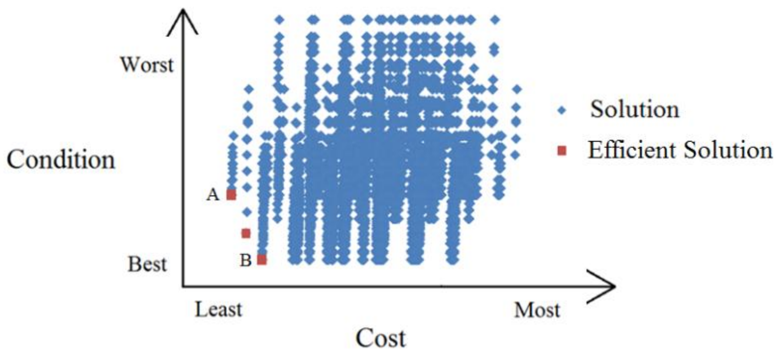

Fig. 1 Solution Example.

As a supporter of decision making, MOO with its efficient solutions can

(1) handle the conflicting objectives and different constraints. For example, in Fig. 1 two conflicting objectives (optimizing condition and minimizing cost) are analysed; and the solutions that achieve the objectives in best possible manner are identified. Several multi-objective decision making problems have been successfully solved using MOO as introduced in Section 2.3.

(2) largely simplify the decision making process. According to Fig. 1, practical decision making may have thousands of alternative feasible solutions even only ten segments are analysed. Because efficient solutions guarantee the best options of strategy selection for a decision making problem; decision makers only need to select one from the efficient solutions with the practical consideration such as policy or using methods such as Value Management (Lin and Shen 2007) without considering other possibilities. For example, in Fig. 1, a decision maker only needs to select from three

efficient solutions rather than thousands of feasible solutions. This largely reduces the workload of the decision maker. When more segments are analysed, much more feasible solutions exist, so the decision becomes more difficult without the help of MOO.

(3) show the achievable best outcomes of decision making. For example, in Fig. 1, solution A achieves the best outcome of one objective (least cost); and the solution B achieves the best outcome of the other objective (best condition). These two solutions indicate the range of achievable outcomes of objectives.

(4) Help with trade-offs. If identified efficient solutions are well spread, they also show the relationship of objectives. When sacrificing one objective, the return on another objective can be estimated by moving from one solution to another. Trade-offs balance objectives by adjusting the scarification and return. Hence, efficient solutions are also important for trade-offs.

Overall, MOO, identifying efficient solutions, helps in improving and understanding maintenance decision making and trading off.

2.3 Applied multi-objective optimization

MOO is increasingly applied to help with maintenance decision making in IAM (NAMS 2011). However, many researches (Hsieh and Liu 1997; Yang et al. 2003; Jian et al. 2009) only identify one preferred solution rather than a set of efficient solutions for their MOO problems in the decision making process. One preferred solution requires decision makers to have adequate knowledge about their problems, and cannot support decision making as mentioned in Section 2.2. Thus, it is less useful than a set of efficient solutions. Wu and Flintsch (2009) also discuss the importance of efficient solutions for the decision making process in IAM.

Some researchers solve their problems with multi-objective heuristic methods, including Genetic Algorithm (GA) (Hyari and El-Rayes 2006; Ge 2010; Sharma 2010), Particle Swarm Optimization (PSO) (Chen et al. 2006; Dashti et al. 2007) and others (Fang et al. 2005; Tee and Li 2011). However, heuristic methods have some weaknesses.

- Multi-objective heuristic methods try to identify solutions as close as possible to the efficient ones, but cannot guarantee the efficiency of the obtained solutions. Further, the gap between the identified and efficient solutions is unknown.
- The identified solutions may vary, even when the same optimization problem is solved more than once. The variance of the identified solutions may be big. Hence, the risk of identifying poor solution is high.
- Heuristic methods normally have parameters. Parameter calibration depends on the addressed problems, such as the number of strategies of a segment, the number of segments and the difficulty of constraints. It is difficult to do in a way that results in the consistently best possible performance of the algorithm.

Bai et al. (2012) also mention the difficulty of generating good solutions with heuristic methods. Hence, they add some efficient solutions named “Extreme Points” as initial solutions, which are obtained by lexicographically optimizing one objective each time. These efficient solutions improve the performance of the applied multi-objective heuristic method, but cannot overcome the weaknesses mentioned above.

Therefore, a deterministic method, solving MOO problems of maintenance decision making and being able to identify a set of efficient solutions, is necessary.

3 Formulation of the optimization problem in decision making

Before applying optimization, a decision making case should be mathematically formulated in a way that its goals and requirements are truly expressed using formulas. The optimization problem in decision making process is a combinatorial optimization problem. It can be easily formulated as integer programming problem (IP) that requires all decision variables to be integer and all the formulas representing objectives and constraints to be linear. Binary variables are used as decision variables $\boldsymbol{x} = (x_1, x_2 \cdots, x_n)$ representing the selection of strategies, as explained by Equation 1.

$$x_i = \begin{cases} 1 & \text{strategy } i \text{ is selected} \\ 0 & \text{strategy } i \text{ is not selected} \end{cases} \qquad \text{Equation 1}$$

For example, if a decision making case wants to efficiently keep a road network in an acceptable condition, its maintenance decision pursues the best financial investment considering the network condition and the related financial factors. This should be expressed by the formulation of its optimization problem. If this road network has m segments and $\mathcal{S}_j$ alternative strategies for segment j (n strategies in total), this decision making case can be formulated as Equations 2-6, where objectives are to obtain the maximizing present value (PV) benefit (Equation 2) and minimizing PV cost (Equation 3) with the acceptable pavement condition (Equation 4) and the annual budget of maintenance cost (Equation 5). Equation 6 ensures exactly one strategy is selected for each segment.

$$\max \sum_{i=1}^{n} PVB_i * x_i \qquad \text{Equation 2}$$

$$\min \sum_{i=1}^{n} PVC_i * x_i \qquad \text{Equation 3}$$

$$\text{subject to} \quad \sum_{i=1}^{n} PPI_i * x_i \leq APPI \qquad \text{Equation 4}$$

$$\sum_{i=1}^{n} MC_{i,t} * x_i \leq Budget_t \;\; \forall t \qquad \text{Equation 5}$$

$$\sum_{i \in \mathcal{S}_j} x_i = 1 \qquad j = 1,2, \ldots m \qquad \text{Equation 6}$$

where, PVB_i PV benefit if strategy i is applied

PVC_i PV cost if strategy i is applied

PPI_i pavement performance index if strategy i is applied

$APPI$ acceptable pavement performance index

$MC_{i,t}$ maintenance cost in year t if strategy i is applied

$Budget_t$ annual budget of year t

The general formulation of an optimization problem for decision making is shown in Equations 7-9, where each of the K objectives can be formulated to be maximized or minimized. Constraints can be annual constraints and/or overall constraints.

$$\max/\min \quad f_k(\boldsymbol{x}) \qquad \text{for } k = 1,2,\cdots,K \qquad \text{Equation 7}$$

$$\text{s.t.} \quad g_l(\boldsymbol{x}) \leq \text{Limit}_l \quad \text{for } l = 1,2,\cdots,L \qquad \text{Equation 8}$$

$$\sum_{i \in \mathcal{S}_j} x_i = 1 \;\; j = 1,2, \ldots m \qquad \text{Equation 9}$$

where $f_k(\boldsymbol{x})$ objective function k

$g_l(\boldsymbol{x})$ constraint function l

Limit_l acceptable value of constraint l

K the number of objectives

L the number of constraints

4 Dichotomic approach (DA)

DA, proposed by Cohon (1978), Dial (1979) and Cohon et al. (1979), is a deterministic optimization method that transfers a MOO problem into single-objective optimization (SOO) counterparts in an iterative manner (Przybylski et al. 2010). In iterations, weights are calculated by pairs of consecutive identified solutions, which are used to weighted sum objectives and then establish new SOO counterparts. If a new feasible solution of a SOO counterpart is identified, this solution is inserted into the solution pool for further analysis in the next iteration. An iteration finishes when all pairs of consecutive identified solutions are analyzed. In this way, DA identifies efficient solutions for bi-objective optimization problems.

Fig. 2 contains a flow chart of the algorithm of DA; and Fig. 3 illustrates its steps for a problem where two objectives are maximized. The steps involved in the algorithm of DA are:

Step 1: Identification of endpoints $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$. The endpoints are the extreme points of the efficient frontier, which are identified by lexicographically optimizing each objective respectively. For example, in Fig. 3(b), endpoint $f(\boldsymbol{x}^1)$ corresponds to the objective vector of optimizing objective f_1 and endpoint $f(\boldsymbol{x}^2)$ to that of optimizing objective f_2.

Step 2: Calculation of weights. Weights (w_1 and w_2) are calculated based on two consecutive identified solutions ($\boldsymbol{x}^s$ and $\boldsymbol{x}^{s+1}$) using Equations 10 and 11, where s is the index of a solution. Initially, $s = N - 1$, where N is the number of identified solutions.

$$w_1 = f_2(\boldsymbol{x}^{s+1}) - f_2(\boldsymbol{x}^s) \qquad \text{Equation 10}$$

$$w_2 = f_1(\boldsymbol{x}^s) - f_1(\boldsymbol{x}^{s+1}) \qquad \text{Equation 11}$$

$$w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x}) \qquad \text{Equation 12}$$

Step 3: Establishment of a new optimization problem. In this step, new single-objective optimization counterparts are established by weighted summing the original objectives (Equation 12). The constraints remain unchanged. Fig. 3(c), (e) and (g) illustrate the new optimization problems. After using a pair of solutions, the index $s = s - 1$.

Step 4: Solving. This step solves SOO counterparts established in Step 3. Many standard algorithms and software tools can identify the optimal solution of SOO counterparts. In this paper, Gurobi is used as a SOO solver.

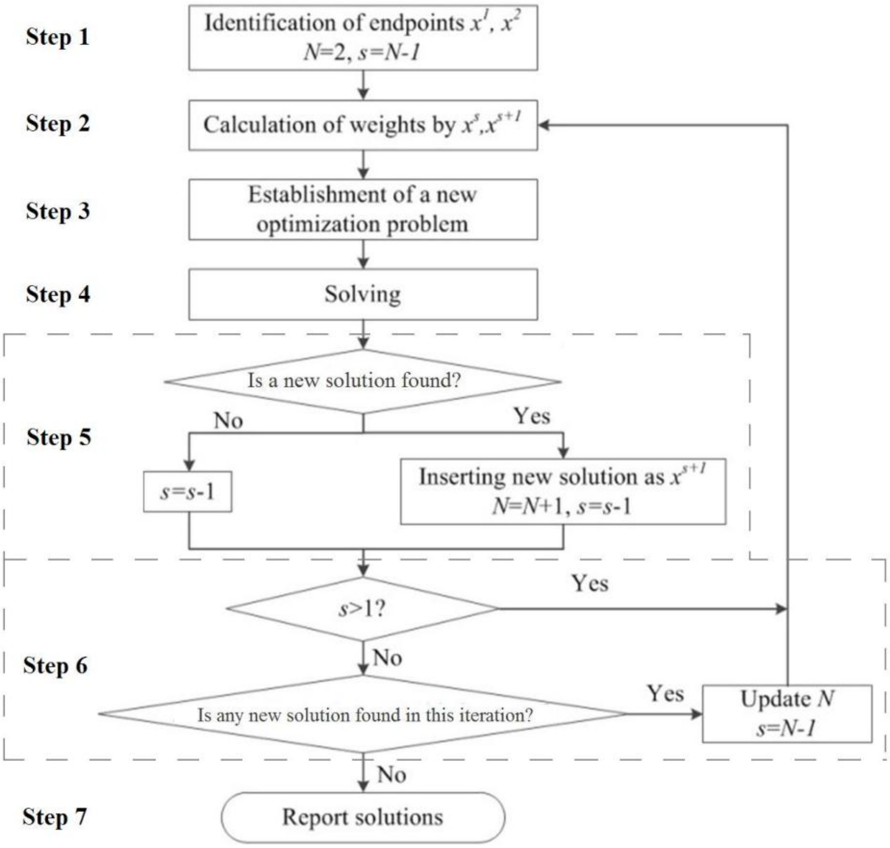

Fig. 2 Flowchart of Dichotomic Algorithm.

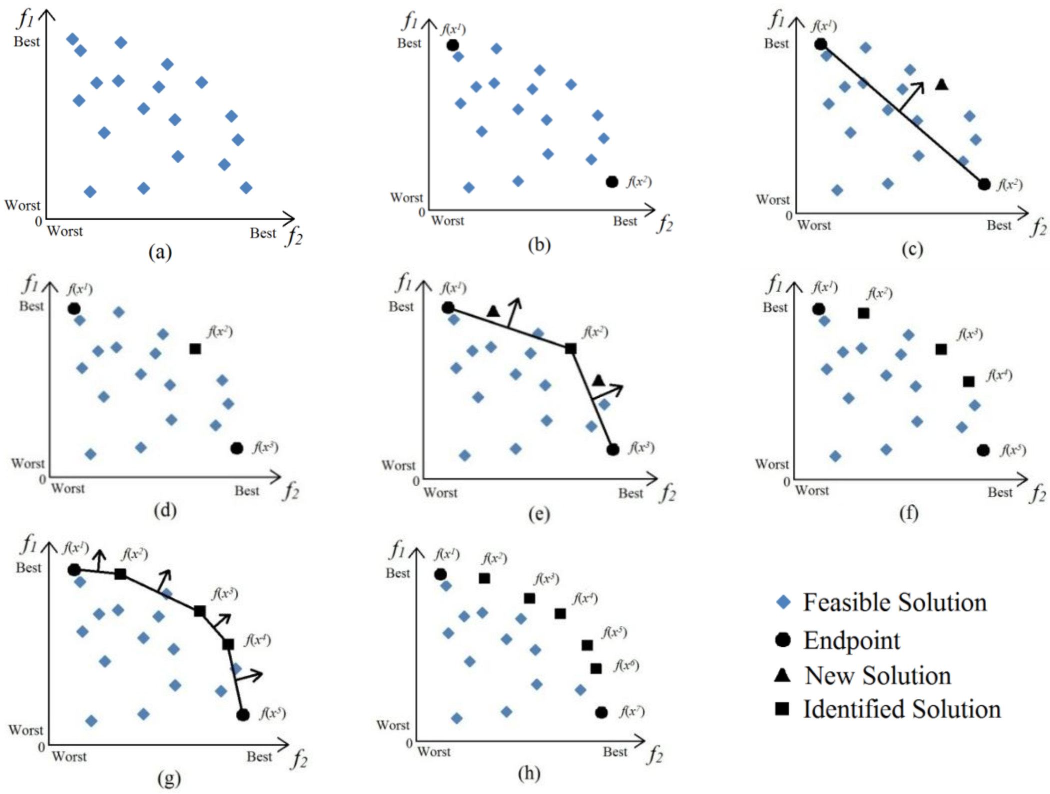

Fig. 3 Illustration of the Dichotomic Algorithm.

Step 5: If a new efficient solution is obtained from a SOO counterpart established by solutions $\boldsymbol{x}^s$ and $\boldsymbol{x}^{s+1}$, this solution is inserted into the solution pool with the index of $s+1$. The indexes of solutions located after the new solution (the index is greater than $s+1$) are updated. Fig. 3(d) and (f) are the examples of identifying new efficient solutions and adding them into the solution pool.

Step 6: Iteration. To identify more efficient solutions, all pairs of consecutive identified solutions should be used to calculate weights and establish SOO counterparts. When $s > 1$, a new pair of solutions x^s and x^{s+1} is sent to Step 2. When $s = 1$, all $(N-1)$ pairs of identified solutions have been analyzed. N is updated to the new number of efficient solutions; and s is updated to $(N-1)$. The iteration process repeats until no new solution is identified in the last iteration.

Step 7: Report solutions. All identified solutions are efficient solutions as shown in Fig. 3(h). Each solution represents an optional decision (a selection of strategies for the entire network) described by decision variables $\boldsymbol{x}$. These solutions consist of the selections of strategies described by $\boldsymbol{x}$. Outcomes, including objectives and constraints, can be estimated based on the selected strategies.

5 Tests

Computational tests are performed in this section to evaluate the performance of DA compared with NSGA II. NSGA II is one of the most advanced multi-objective heuristic methods (Deb et al. 2002), which is successfully applied to help with the trade-offs of a decision making case and achieves good result (Bai et al. 2012). According to the authors' experience, NSGA II is one of the most effective heuristic methods for addressing multi-objective optimization problems of decision making.

5.1 Studied Case

In this section, a road network of a city is analyzed. This city wants to maximize the return of investment on road maintenance. Hence, a bi-objective optimization problem is established to maximize the maintenance benefit and minimize the maintenance cost under annual budget and condition requirement.

This road network is divided into 699 segments. For every segment, ten-year maintenance strategies are generated by dTIMS CT 8 (Deighton Associates Limited 2008). Each strategy indicates types of treatments that are designed to be applied to a segment during ten years. In this case, the number of alternative strategies for a segment ranges from 8 to 161 based on the available treatments of this segment. Outcomes of a strategy are estimated based on its treatment using dTIMS CT 8. For example, Strategy 1 only applies a resurface treatment in Year 1 to Segment 1, which costs 60695 dollar in Year 1; while Strategy 64 applies a construction treatment in Year 8 to Segment 2, which costs 186630 dollar in Year 8. The yearly cost of strategies is recorded using a matrix, where $MC_{i,t}$ is the maintenance cost in Year t if Strategy i is applied. In this case, $MC_{1,1} = 60695$ and $MC_{64,8} = 186630$. Other outcomes can be constructed in the same way such as PV benefit PVB_i, PV cost PVC_i and pavement performance index PPI_i. Then the objectives and constraints can be formulated as Equations 2-6 in Section 3.

To analyze problems of different size, we consider the entire road network or only parts of it as shown in Columns (2) and (3) in Table 1. The applied NSGA II has a population of 300 and 200 parents. The stopping criterion is the maximum number of iterations (300 for Case 1, 700 for Case 2 and 1000 for Case 3).

Computational tests are conducted on a PC with Intel(R) Core™ i5 processor, 3.33 GHz CPU, 4.00 GB RAM. The computer program is written using Python 2.7.3. Gurobi 5.5.0 is used as the single-objective optimization solver. The computation time is measured as the CPU time.

After constructing all matrixes of outcomes and a list of decision variables, the optimization problem established for this decision making case (Equations 2-6) is solved by DA. The algorithm of DA introduced in Section 4 is based on maximizing both objectives. In this case, the objective of minimizing PV cost can be converted to Equation 11; while the other objective of maximizing PV benefit remains the same (Equation 10).

$$\max f_1 = \sum_{i=1}^{n} PVB_i * x_i \qquad \text{Equation 10}$$

$$\max f_2 = -\sum_{i=1}^{n} PVC_i * x_i \qquad \text{Equation 11}$$

According to the algorithm of DA, weights $\boldsymbol{w} = (w_1 + w_2)$ are iteratively calculated; and the SOO counterparts of the original bi-objective optimization problem of this decision making case can be established based on the weight as shown by Equations 12 -15. The SOO counterparts are solved by Gurobi; and solutions are obtained.

$$\max w_1 f_1(x) + w_2 f_2(x) \qquad \text{Equation 12}$$

$$\text{subject to} \quad \sum_{i=1}^{n} PPI_i * x_i \leq APPI \qquad \text{Equation 13}$$

$$\sum_{i=1}^{n} MC_{i,t} * x_i \leq Budget_t \;\; \forall t \qquad \text{Equation 14}$$

$$\sum_{i \in \mathcal{S}_j} x_i = 1 \qquad j = 1,2, \ldots m \qquad \text{Equation 15}$$

5.2 Results

The solutions of the optimization problem of the studied decision making case are shown in Fig. 4. In this case, one objective (PV Benefit) is maximized and the other (PV Cost) is minimized. Hence, efficient solutions are located differently with Fig. 3 where both objectives are maximized. The computational result is shown in Table 1.

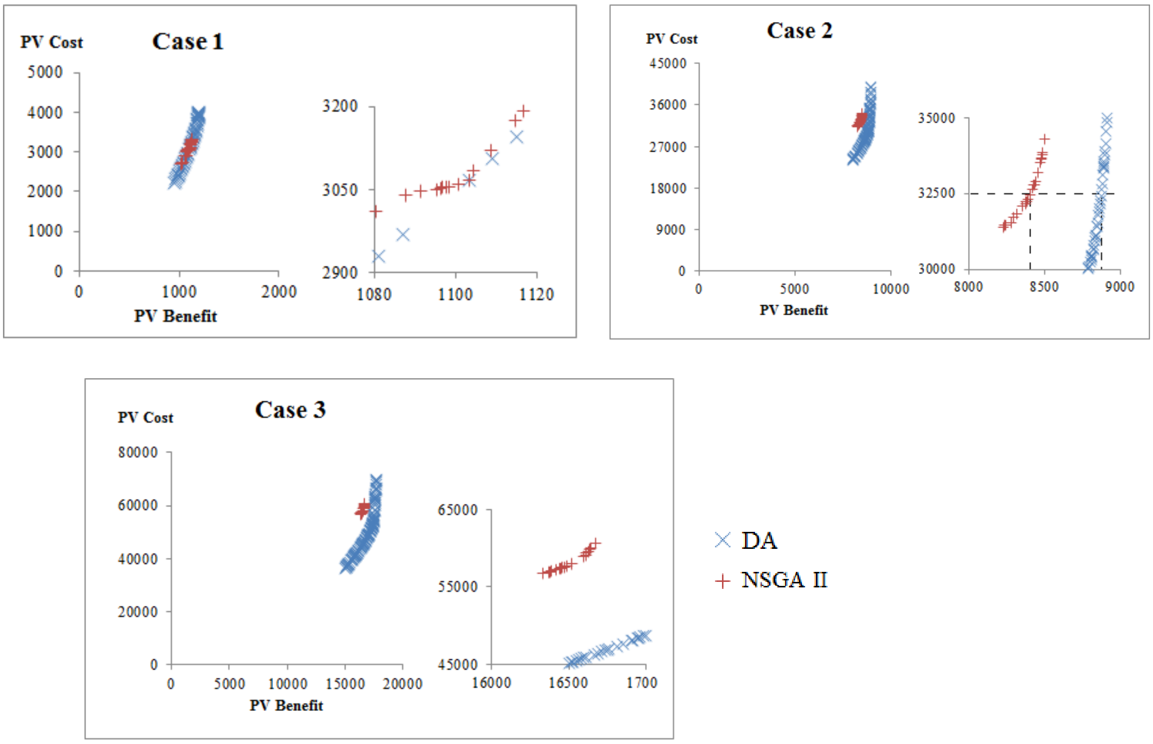

Fig. 4 Identified Solutions

Table 1 Summary of the Case Study

Case Index	Number of segments	Number of strategies	Number of identified solutions		Computation Time** (s)	
			DA	NSGA II	DA	NSGA II
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	50	1823	47	28 (7*)	19.88	38.67
2	400	11307	105	24 (0*)	206.35	273.55
3	699	20412	136	20 (0*)	475.86	547.85

* The number of efficient solutions.

** The computation time is the CPU time only for the optimization, excluding input and output data.

According to Table 1, DA can analyze bi-objective optimization problems in decision making process and identify efficient solutions in reasonable time. Comparing with NSGA II, DA identifies more and better solutions in less time in all cases. The superiority of DA is more obvious when more segments are analyzed. The detailed analysis of the results is discussed below.

Firstly, the solutions of DA are efficient solutions. According to Column (5) in Table 1, NSGA II identifies 7 efficient solutions when only 50 segments are analyzed. However, when 400 or more segments are analyzed, no efficient solution is identified, which means the solutions of DA achieve more PV benefit than the solutions of NSGA II when spending same PV cost. For example, in Case 2, when spending 32500 dollar (PV value) on the maintenance, the solution of DA generates 470 dollar more PV benefit than the solution of NSGA II. Furthermore, the distance between the two sets of solutions is obviously increasing with the growth of the segments, see Fig. 4. Hence, the solutions of NSGA II become worse with the growth of the segments.

Secondly, DA identifies more solutions than NSGA II. With the growth of the analyzed segments from 50 to 400 and 699, the problem becomes bigger, and DA identifies more solutions (47, 105 and 136 respectively). These solutions are all unique efficient solutions which are not worse than any other solutions. Even selecting different strategies, all these solutions satisfy the constraints of annual budget and condition requirement (acceptable average PPI). However, their objective vectors are different. The performance of NSGA II is worse as the identified solutions reduce from 28 to 24 and 20.

Thirdly, DA always finds identical solutions when run more than once for the same problem. Hence its solutions are more reliable and stable. However, the solutions of NSGA II may be different when solving the same problem more than once, due to its randomness.

Fourthly, the solutions of DA have good coverage. They cover the entire range of efficient solutions from the minimum-cost solution to maximum-benefit solution in this case; while the solutions of NSGA II only cover a small range of feasible solutions. The coverage becomes worse for bigger problems. In Fig. 4, the solutions of NSGA II cover around 50% of the coverage of the solutions of DA for Case 1 and only around 20% of DA for Case 3.

Fifthly, in all cases, DA identifies sufficiently enough efficient solutions (Column (4) in Table 1). As shown in Fig. 4, solutions identified by DA are enough to show a clear efficient frontier and the relationship of objectives.

Sixthly, the computation time of DA is shorter than that of NSGA II. According to Columns (6) and (7) in Table 1, as the optimization problem grows bigger, the computation time of DA also grows from 19.98 to 206.35 and 475.86 seconds, which are less than the computation time of NSGA II. NSGA II spends less time if fewer iterations are proceed. However, fewer iterations probably lead to worse solutions. Comparing with NSGA II, DA identifies better solutions in less time in all cases. The computation time of DA is also affected by the number of solutions. When too many solutions exist, the computation time may be long.

Finally, the application of DA is easy. No parameter needs to be calibrated by the decision maker. However, when applying NSGA II, the stopping criterion/criteria, population and parent size and mutation rate should be properly calibrated by the decision maker based on the addressed case. Therefore, the application of DA is easier and problem-independent.

Overall, DA can solve the optimization problems of the maintenance decision making and identify efficient solutions. Comparing with NSGA II, DA identifies better solutions in less computation time.

After identifying the efficient solutions, the decision maker only needs to analyze the outcomes of these efficient solutions. Taking Case 2 as an example, 105 efficient solutions are identified. All these solutions satisfy the annual budget and condition constraint; thus the annual budget and condition do not need to be considered. However, if the condition is a consideration of trade-offs, the average PPI of the entire network of an efficient solution can be calculated. Moreover, as efficient solutions, all the identified solutions achieve objectives in best possible manner. With the alternative strategies, this decision making case needs at least 24138 dollar PV cost to keep its network into the acceptable condition, and generates at most 8956 dollar PV benefit under annual budget. The relationship of PV benefit and PV cost is described by the trend of the efficient solution as shown in Fig. 4. The decision maker can estimate the scarification and return based on this relationship, and then determines the trade-off. After the trade-off is determined, an efficient solution that satisfies this trade-off could be selected or adjusted. Then the strategies selected by the solutions are implemented, such as the implementation method introduced by Chinowsky and Rojas (2003). However, no matter which efficient solution is selected, no other solution is better than the selected one measuring by objectives and constraints.

6 Discussion

This section discusses some issues related with DA such as how to control the number of identified solutions, the fact that DA cannot obtain all efficient solutions, and the application of DA to problems with three or more objectives.

6.1 Controlling numbers of identified solutions

In practice, MOO problems of maintenance decision making may have too many efficient solutions. Some of the solutions may have similar outcomes. These similar solutions are not useful when analyzing trade-offs. However, obtaining and analyzing these solutions needs much time. For example, in Case 2 the distances between pairs of consecutive solutions (d) is shown in Fig. 5, where 75% of the identified solutions are close ($d \leq 200$).

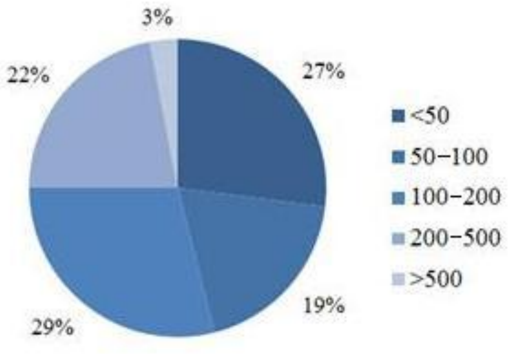

Fig. 5 Summary of the Distance *d* between Pairs of Consecutive Identified Solutions

In this paper, a filter, defined as the acceptable minimum distance, is added into the classic DA to control the number of identified solutions. If the distance between a new solution and its consecutive solution is greater than the filter size, the new solution is accepted and this new solution and its consecutive solution will be used for further analysis; otherwise, the new solution is inserted into the efficient solution pool but will not be used for further analysis.

The three cases in Section 5.1 are studied again with the filter, and the results are shown in Table 2. Comparing with the previous results (Column (3)), after determining the filter size (Column (2)), solutions that are too close (closer than the required filter size) are not identified; therefore the number of identified solutions (Column (4)) is largely reduced and the computation time (Column (5)) is also reduced by at least 79.69%. The quality and coverage of the identified solutions are still good. Fig. 6 compares the previous solutions and the solutions with filter for Case 2. According to this figure, the identified solutions are also efficient and cover the entire range of efficient solutions. Hence, the effectiveness of the algorithm is improved.

Table 2 Results of the Tests with Filter

Case Index	Filter	Number of efficient solutions		Computation time* (s)	Time reduced by (%)
		Previous	Identified		
(1)	(2)	(3)	(4)	(5)	(6)
1	50	47	18	4.06	79.69
2	200	105	27	23.19	88.76
3	2000	136	21	70.55	85.17

* Computation time is the CUP time only for the optimization.

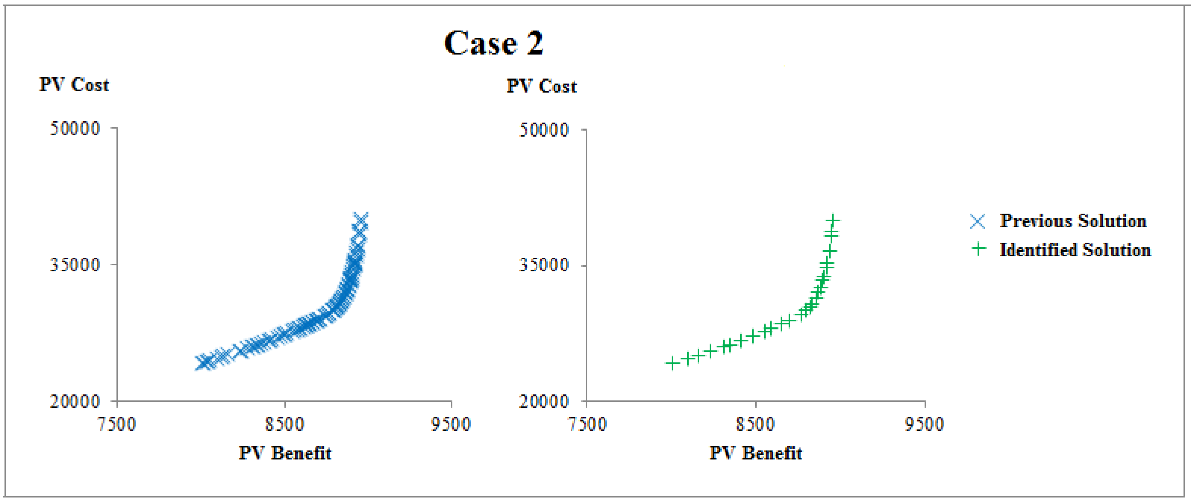

Fig. 6 Comparison of Previous and Identified Solutions for Case 2

Fig. 7 illustrates the number of efficient solutions identified with different filter sizes for Case 2. With the growth of the filter size, the number of identified solutions decreases from 105 to 2 (the minimum number is 2) while the constraints remain unchanged. A decision maker can control the number of identified solutions by assigning a proper filter size.

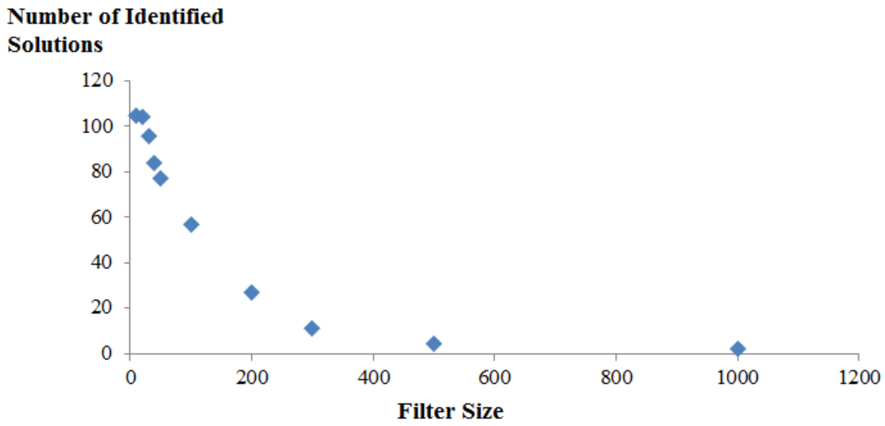

Fig. 7 Relationship of the Filter And the Number of Identified Solutions.

6.2 Non-supported solutions

DA cannot identify all efficient solutions. It only identifies supported solutions located on the boundary of the convex hull of feasible objective vectors such as solutions A, C and E in Fig. 8, but cannot find non-supported solutions located in the interior of the convex hull such as solutions B and D in Fig. 8 (Ehrgott 2005).

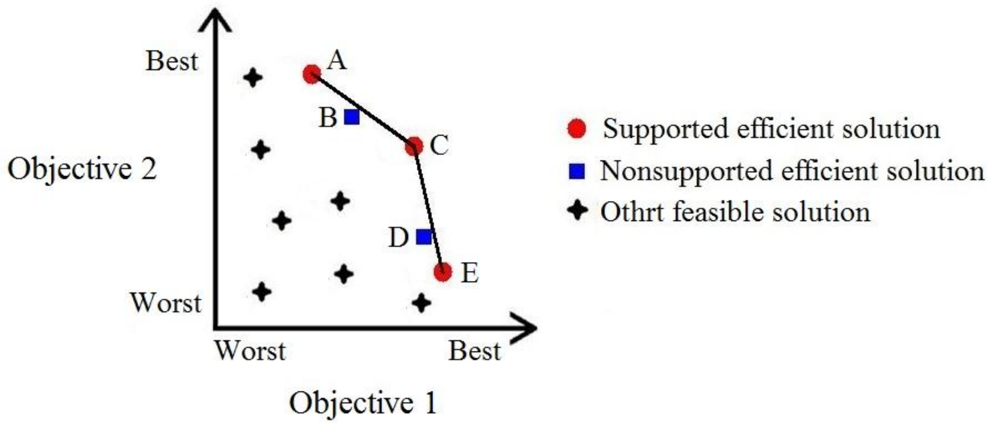

Fig. 8 Example of Supported And Nonsupported Solutions.

When supported solutions are not sufficient enough to show a clear relationship of objectives, an improved DA with extra constraints (Equations 13 and 14) can identify the non-supported solutions to fill the gap between the supported solutions in the efficient frontier. The idea of this method is similar with epsilon-constraint method (Changkong and Haimes 1983).

However, the computation time is longer than the classic DA. Heuristic methods can also be used to identify non-supported solutions (Chen et al. 2013).

$$\text{s.t.} \quad f_1(\boldsymbol{x}) \leq \max\big(f_1(\boldsymbol{x}^s), f_1(\boldsymbol{x}^{s+1})\big) - \varepsilon \qquad \text{Equation 13}$$

$$f_2(\boldsymbol{x}) \leq \max\big(f_2(\boldsymbol{x}^s), f_2(\boldsymbol{x}^{s+1})\big) - \varepsilon \qquad \text{Equation 14}$$

where ε is a small constant

6.3 Decision making with three or more objectives

The classic DA only solves bi-objective optimization problems. When three or more objectives are pursued, a different weighting method transferring a MOO problem into SOO counterparts should be used. For a K-objective decision making optimization problem, the weights can be calculated by the method introduced by Przybylski et al. (2010) in order to integrate objectives and identify new solutions. Once a new solution is identified, it is inserted into the solution pool to identify more solutions in the next iteration, which is similar with the classic DA. However, the effectiveness of DA when dealing with three or more objectives is not as good as the classic DA when dealing with two objectives.

7 Conclusion

This paper introduces the application of MOO to maintenance decision making and the importance of efficient solutions. Efficient solutions not only are the best options of the maintenance decision, but also simplify decision making process and show the relationship of objectives. Hence, they are necessary for decision making and trade-offs. To identify efficient solutions, a deterministic optimization method, DA, is proposed to support multi-objective maintenance decision making in IAM and tested with a practical case. According to the case study, DA shows great abilities when dealing with bi-objective optimization problems of maintenance decision making.

- DA guarantees to identify efficient solutions, all of which are supported. Comparing with NSGA II, DA identifies solutions with better quality, stability and coverage, especially when the problem is big.
- The computation time of DA is fast even for large problems. When many efficient solutions exist, the computation time may be long. Decision makers can control the number of identified solutions in order to improve the efficiency of DA.
- The application of DA is straight-forward. No parameter needs to be calibrated by the decision maker.
- This paper also discusses other issues, including controlling the number of identified solutions, the identification of non-supported solutions, and decision making problems with three or more objectives.

Despite being a great supporter for decision making, DA also has some weaknesses. Firstly, even the optimization for three- or more- objective problems is discussed in Section 6.3, the effectiveness of the algorithm is reducing when more objectives are analyzed. More researches are needed to effectively handle decision making problems with three or more objectives. Secondly, when a road network has many segments and alternative strategies; the computation time is a vital factor especially for large problems. How to identify efficient solution in less time is another critical research area. We present a solution to this by proposing a proper filter size to reduce the number of identified efficient solutions and computation time. Other methods that improve the efficiency of DA are also needed.

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