

Epipolar Geometry in Polycentric Panoramas

Fay Huang¹, Shou Kang Wei¹ and Reinhard Klette¹

Abstract

We introduce a new class of panoramic images, called polycentric panoramic images, which is a generalization from more specific classes such as single-center, multiple-center, or concentric panoramic images. This paper focuses on the derivation of an epipolar curve equation for polycentric panoramic images. The epipolar curve equation derived provides a unified approach for the epipolar geometry in any of the more specific classes of panoramic images.

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Abstract

We introduce a new class of panoramic images, called polycentric panoramic images, which is a generalization from more specific classes such as single-center, multiple-center, or concentric panoramic images. This paper focuses on the derivation of an epipolar curve equation for polycentric panoramic images. The epipolar curve equation derived provides a unified approach for the epipolar geometry in any of the more specific classes of panoramic images.

1 Introduction

This paper presents a unified approach for the epipolar geometry of various types of specific classes of panoramic images such as single-center, multiple-center, or concentric panoramic images. The focus is on the derivation of a unique epipolar curve equation that can be used in the intended range of panoramic image classes. The term *polycentric panoramic image* is used as a generalized characterization of different types of more specific panoramic images. The exact definition of a polycentric panoramic image is given in Section 2.

A commonly used camera setup for panoramic image acquisition is: a pinhole camera rotates where the optical center of the camera coincides with the rotation center, and images are captured consecutively at equidistant angles. Since all the pixel data in a panoramic image are collected through projections with respect to a single optical center, it is referred to as *single-center panoramic image*. The epipolar geometry of this kind of panoramic images has been studied in [MB95, KS97].

Another class of panoramic images receiving much attention recently is in use for applications of 3D scene visualizations and reconstructions [PBE99, PPBE00, SH99, SKS99, SS99]. These are referred to as *multiple-center panoramic images*. Such a panoramic image allows to associate disjoint regions in it (typically several image columns) with different optical centers. Unfortunately, only some initial work in studying epipolar geometry has been done so far for this class of panoramic images. For instance, H-Y. Shum and R. Szeliski [SS99] showed that epipolar geometry

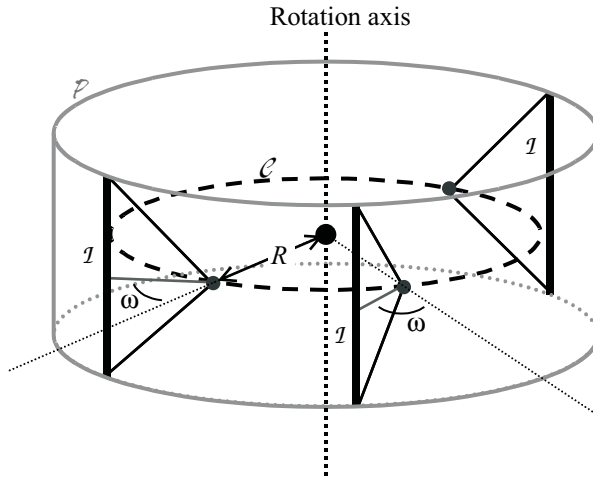


Figure 1: The image acquisition model of a polycentric panoramic image. To acquire a panoramic image \mathcal{P} a slit camera rotates with respect to a fixed 3D axis and captures one slit image \mathcal{I} in every constant angular interval. Each slit image contributes to one column of \mathcal{P} . See text for further details.

consists of horizontal lines if two concentric panoramic images are symmetric with respect to a normal vector of the rotation axis. Many other possible classes such as those symmetric with respect to a tangential vector, focal point etc; or those of non-symmetric situations have not been studied so far. Our paper derives a general epipolar curve equation for all kinds of panoramic images above-mentioned or equivalent.

The paper is organized as follows. The acquisition model and the mathematical model of polycentric panoramic images are given in Section 2. The derivation of the epipolar curve equation through various geometric transformations is elaborated in Section 3. In Section 4 the epipolar curves of some particular panoramic images are presented mathematically and graphically. Concluding remarks and comments on future work are in Section 5.

2 Image Acquisition Model

Polycentric panoramic images can be acquired by different imaging methods [WHK00]. One of the possible ways is using a slit camera. A slit camera is characterized geometrically by a single focal point and a 1D linear image slit ¹. Ideally, the focal point lies on the bisector of an image slit. The parameters of a slit camera used in this paper are the length of the image slit and the effective focal length of the camera.

To acquire a polycentric panoramic image, denoted as \mathcal{P} , a slit camera rotates with respect to a fixed 3D axis (e.g. the rotation axis of a turntable) and captures one slit image, denoted as \mathcal{I} , in every constant angular interval γ . Each slit image contributes to one column of a polycentric

¹An image slit is defined by a line segment and the photon-sensing elements positioned on this line segment.

panoramic image. Unless the focal point is exactly incident with the rotation axis, e.g. [MW98], a polycentric panoramic image is usually associated with multiple focal points on a circle \mathcal{C} . The radius of the circle R , is the distance between the slit camera’s focal point and the rotation axis.

The orientation of the slit camera is defined by the angle ω between the normal vector of the circle \mathcal{C} at the associated focal point and the optical axis of the slit camera. The width $W_{\mathcal{P}}$ of a panoramic image, equivalently the number of slit images acquired for a panoramic image, is determined by the rotation angular interval γ , i.e. $W_{\mathcal{P}} = 2\pi/\gamma$. A set of panoramic images taken from the acquisition model characterized above with respect to the different rotation axes is called polycentric panoramas. Figure 1 depicts the acquisition model of the polycentric panoramic images and the symbols introduced.

3 Derivation of an Epipolar Curve Equation

This section considers the general case of polycentric panoramas. An epipolar curve equation is derived which holds for a large diversity of panoramic images.

3.1 Coordinate Systems

We define an 1D discrete image coordinate system for each slit image with the coordinate axis denoted by v . The unit of this coordinate system is defined in terms of an image pixel. We define another 1D real-number image-slit coordinate system with the coordinate axis denoted by y . The origins of the image and the image-slit coordinate systems are at the top and the center of the image slit respectively. Let v_c be the principle point ² in discrete image space. The conversion between these two coordinates v and y is $y = d(v - v_c)$, where d is the size of an image pixel in units of y .

We define a 2D discrete image coordinate system for each polycentric panoramic image. The coordinates are denoted as (u, v) , which is an image pixel at column u and row v . Each column itself is a slit image, thus the coordinate v here is identical to the coordinate v in the slit image coordinate system. We define another 2D real-number image-surface coordinate system for each polycentric panoramic image with the coordinates (x, y) . The origin of this coordinate system is defined at the center of the initial image slit. The conversion between these two coordinates (u, v) and (x, y) is $x = du$ and $y = d(v - v_c)$, where d is the size of an image pixel in units of y .

A 3D slit-camera coordinate system, shown in Fig. 2, is defined as follows. The origin coincides with the focal point of a slit camera, denoted as \mathbf{C} . The z-axis is perpendicular to the image slit and passes through the center of the image slit. The y-axis is parallel to the image slit towards the direction of the positive y value in the image-slit coordinate system. An image point \mathbf{p} on the image slit can be represented by the coordinates $(0, y, f)$, where f is the effective focal length of the slit camera. Another way of representing an image point \mathbf{p} is by an angular coordinate ϕ , which is the angle between the z-axis and the line passing through both the focal point and the image point. The conversion between the coordinates $(0, y, f)$ and ϕ is $\phi = \tan^{-1}(y/f)$.

²The center pixel of the slit image where the optical axis of the slit camera passes through the image.

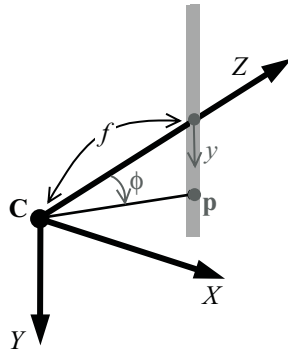


Figure 2: A 3D slit-camera coordinate system. An image point \mathbf{p} on the image slit can be represented by the coordinates $(0, y, f)$, where f is the effective focal length of the slit camera. See text for further details.

Each column of a polycentric panoramic image associates with a slit camera coordinate system. All the origins of the slit camera coordinate systems lie on the circle \mathcal{C} . A 3D turning-rig³ coordinate system is defined for each polycentric panoramic image. The origin, denoted as \mathbf{O} , coincides with the center of the circle \mathcal{C} . The z-axis passes through the center of the initial column of the panoramic image. The y-axis is parallel to all the slit images and towards the same direction as the y-axis of the slit camera coordinate system. We define an angle θ to be the angle between the z-axis of the turning-rig coordinate system and the segment $\overline{\mathbf{OC}}$. The orientation and the location of a slit camera coordinate system with respect to the turning-rig coordinate system can be described by a 3×3 rotation matrix \mathbf{R}_{oc} ,

$$\mathbf{R}_{oc} = \begin{bmatrix} \cos(\theta + \omega) & 0 & -\sin(\theta + \omega) \\ 0 & 1 & 0 \\ \sin(\theta + \omega) & 0 & \cos(\theta + \omega) \end{bmatrix},$$

where ω is the angle between the normal vector of the circle \mathcal{C} at \mathbf{C} and the optical axis of the slit camera, and a 3×1 translation vector

$$\mathbf{T}_{oc} = \begin{pmatrix} R \sin \theta \\ 0 \\ R \cos \theta \end{pmatrix},$$

where R is the radius of the circle \mathcal{C} . Figure 3 depicts the relationship between the slit camera coordinate systems and the turning-rig coordinate system. The conversion between the coordinate u and the angle θ is $\theta = (2\pi u)/W_{\mathcal{P}}$, where $W_{\mathcal{P}}$ denotes the width (in pixel) of the panoramic image.

A 3D world coordinate system is defined for the conversion between any pair of turning-rig coordinate systems for two polycentric panoramic images. The origin is denoted as \mathbf{W} . The relationship between the world coordinate system and a turning-rig coordinate system associated to a panoramic image can be described by a 3×3 rotation matrix \mathbf{R}_{wo} and a 3D translation vector \mathbf{T}_{wo} .

³For example, a turntable or a turning head on a tripod.

3D point on the ray $\ell_{o'}$. In other words, every (x', y') is possibly the corresponding point of the given image point (u, v) of \mathcal{P} . Let \mathcal{I}' denote the slit image contributed to the column x' of \mathcal{P}' and let \mathbf{C}' denote the associated slit camera's focal point. For each column x' , the corresponding y' value can be found by the following two steps. First calculate the intersection point, denoted as \mathbf{Q}' , of the ray $\ell_{o'}$ and the plane ϕ' passing through \mathbf{C}' and \mathcal{I}' . Second, project point \mathbf{Q}' to the slit image \mathcal{I}' to obtain the value of y' .

The associated angle θ' is $(2\pi x')/(W'_p)$, where W'_p is the width of the destination panoramic image. The position of the focal point \mathbf{C}' with respect to the turning-rig coordinate system of \mathcal{P}' can be described by $(R' \sin \theta', 0, R' \cos \theta')$, where R' is the radius of the circle \mathcal{C}' . A unit vector perpendicular to the plane ϕ' is $(-\cos(\theta' + \omega'), 0, \sin(\theta' + \omega'))$, where ω' is the angle between the normal vector of \mathcal{C}' at \mathbf{C}' and plane ϕ' . Therefore, the equation of plane ϕ' is

$$-\cos(\theta' + \omega')X + \sin(\theta' + \omega')Z = R' \sin \omega', \quad (2)$$

where the variables X and Z are with respect to the turning-rig coordinate system of the destination panoramic image \mathcal{P}' .

We substitute the x and z components of the projection ray $\ell_{o'}$ in Equ. 1 into the plane equation Equ. 2, and solve the value of λ . The intersection point \mathbf{Q}' can then be calculated from Equ. 1. We denote the obtained coordinates of \mathbf{Q}' as $(X_{o'}, Y_{o'}, Z_{o'})$. We have

$$\begin{bmatrix} X_{o'} \\ Y_{o'} \\ Z_{o'} \end{bmatrix} = \begin{bmatrix} X_{o'} \cos(\theta' + \omega') - Z_{o'} \sin(\theta' + \omega') + R' \sin \omega' \\ Y_{o'} \\ X_{o'} \sin(\theta' + \omega') + Z_{o'} \cos(\theta' + \omega') - R' \cos \omega' \end{bmatrix},$$

which transforms the point \mathbf{Q}' to the slit camera coordinate system associated to the slit image \mathcal{I}' and denote it as $(X_{c'}, Y_{c'}, Z_{c'})$.

A 3D point is allowed to project onto the slit image if and only if the x-component of the coordinates with respect to the slit camera coordinate system is equal to zero. Therefore, the projection of a 3D point $(0, Y_{c'}, Z_{c'})$ on the slit image \mathcal{I}' is

$$\begin{bmatrix} 0 \\ \frac{f' Y_{o'}}{X_{o'} \sin(\theta' + \omega') + Z_{o'} \cos(\theta' + \omega') - R' \cos \omega'} \\ f' \end{bmatrix},$$

where f' is the effective focal length of the slit camera acquiring \mathbf{p}' . Convert the projection of a 3D point in the slit image \mathcal{I}' back to the image-surface coordinate system of the panoramic image \mathcal{P}' . Given x' , the value of y' is

$$y' = \frac{f' Y_{o'}}{X_{o'} \sin(\frac{2\pi x'}{W'_p} + \omega') + Z_{o'} \cos(\frac{2\pi x'}{W'_p} + \omega') - R' \cos \omega'}.$$

To draw an epipolar curve in a discrete image, the coordinates (x', y') are converted to the discrete image coordinate system (u', v') by $u' = x'/d'$ and $v' = v_c + y'/d'$, where d' is the size of an image pixel in units of y' .

4 Epipolar Curve in Special Cases

We discuss the general equation in the context of a few special cases of panoramic images.

4.1 Epipolar Curve in Polycentric Panoramas

Consider two polycentric panoramic images, \mathcal{P} and \mathcal{P}' . The orientations and positions of their turning-rig coordinate systems with respect to the world coordinate system are: $\mathbf{R}_{wo} = \mathbf{R}_{wo'} = \mathbf{I}_{3 \times 3}$ and $\mathbf{T}_{wo} = (0, 0, 0)^T$ and $\mathbf{T}_{wo'} = (t_x, 0, t_z)^T$ respectively. Given is an image point (x, y) on \mathcal{P} , the equation of the epipolar curve on \mathcal{P}' is

$$y' = y \cdot \left(\frac{f'}{f} \right) \cdot \left(\frac{R' \sin \omega' - R \sin \left(\frac{2\pi x'}{W'_p} - \frac{2\pi x}{W_p} + \omega' \right) - t_x \cos \left(\frac{2\pi x'}{W'_p} + \omega' \right) + t_z \sin \left(\frac{2\pi x'}{W'_p} + \omega' \right)}{-R \sin \omega - R' \sin \left(\frac{2\pi x'}{W'_p} - \frac{2\pi x}{W_p} - \omega \right) - t_x \cos \left(\frac{2\pi x}{W_p} + \omega \right) + t_z \sin \left(\frac{2\pi x}{W_p} + \omega \right)} \right).$$

Figure 4 shows an example of a pair of polycentric panoramas in a synthetic scene. The top shows the source panoramic image with 30 test points in '*' labeled by numbers. The bottom shows the destination panoramic image with the corresponding epipolar curves. The turning-rig coordinate

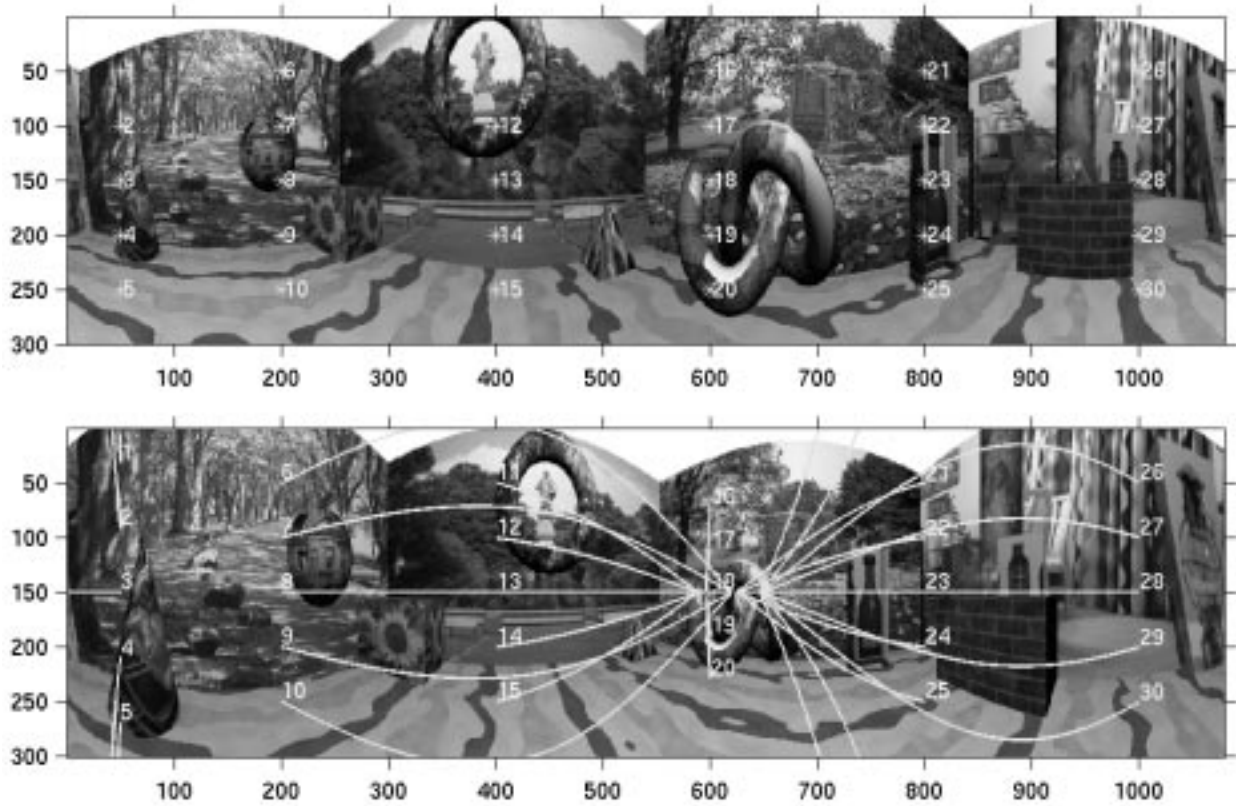


Figure 4: A pair of polycentric panoramas in a synthetic scene. The top shows the source panoramic image with 30 test points in '*' labeled by numbers. The bottom shows the destination panoramic image with the corresponding epipolar curves.

systems associated to the top panoramic image is set to the world coordinate system. The bottom panoramic image was acquired at the same level⁴ but 200 mm to the east and 100 mm to the north of the top one. The orientations of these two panoramic images are set to be identical. The effective focal lengths of the slit cameras used for acquiring these two panoramic images are both equal to 35.704 mm. The radiuses of the circles, where slit camera's focal points lie on, are both equal to 40 mm. The orientations of the slit cameras with respect to each rotation axes are both equal to 45°. Each slit camera takes 1080 slit images for one panoramic image. Both image pixel's width and height are equal to 1/6 mm.

4.2 Epipolar Curve in Concentric Panoramas

A set of polycentric panoramic images is called *concentric panoramic panoramas* [HP00] if the associated turning-rig coordinate systems are all coincident. Consider two concentric panoramic images, \mathcal{P} and \mathcal{P}' . Given an image point (x, y) on \mathcal{P} , the equation of the epipolar curve on \mathcal{P}' is

$$y' = y \cdot \left(\frac{f'}{f} \right) \cdot \left(\frac{R' \sin \omega' - R \sin \left(\frac{2\pi x'}{W_{\mathcal{P}'}} - \frac{2\pi x}{W_{\mathcal{P}}} + \omega' \right)}{-R \sin \omega - R' \sin \left(\frac{2\pi x'}{W_{\mathcal{P}'}} - \frac{2\pi x}{W_{\mathcal{P}}} - \omega \right)} \right). \quad (3)$$

Figure 5 shows an example of the epipolar curve in a pair of concentric panoramas. The effective focal lengths of slit cameras are both equal to 35.704 mm. The radiuses of the circles are both equal to 40 mm. The orientation of the slit camera with respect to the rotation axis of the top panoramic image is equal to 10° and of the bottom image is equal to 300°.

In particular $\omega' = (2\pi - \omega)$, the two concentric panoramic images are called *symmetric pair* [SS99, SKS99]. An important property about the symmetric panoramic image pair is that the epipolar curves become straight lines and coincide with image rows. The property can be shown from equation Equ. 3 by setting $f = f'$, $R = R'$, $W_{\mathcal{P}} = W_{\mathcal{P}'}$, and most critically $\omega' = (2\pi - \omega)$, we have

$$\begin{aligned} y' &= y \cdot 1 \cdot \left(\frac{R \sin(2\pi - \omega) - R \sin \left(\frac{2\pi x'}{W_{\mathcal{P}'}} - \frac{2\pi x}{W_{\mathcal{P}}} + 2\pi - \omega \right)}{-R \sin \omega - R \sin \left(\frac{2\pi x'}{W_{\mathcal{P}'}} - \frac{2\pi x}{W_{\mathcal{P}}} - \omega \right)} \right) \\ &= y \cdot \left(\frac{-\sin \omega - \sin \left(\frac{2\pi x'}{W_{\mathcal{P}'}} - \frac{2\pi x}{W_{\mathcal{P}}} - \omega \right)}{-\sin \omega - \sin \left(\frac{2\pi x'}{W_{\mathcal{P}'}} - \frac{2\pi x}{W_{\mathcal{P}}} - \omega \right)} \right) \\ &= y. \end{aligned}$$

The value of y' is equal to y . Figure 6 shows an example of the epipolar lines in a symmetric pair of concentric panoramas. The parameters are the same as the previous settings. Only the orientations of the slit cameras are different. One is equal to 10° and the other is equal to 300°. Note that all the epipolar curves become straight lines and coincide with image rows.

⁴The y-component of their associated rotation centers' coordinates with respect to the world coordinate system are equal.

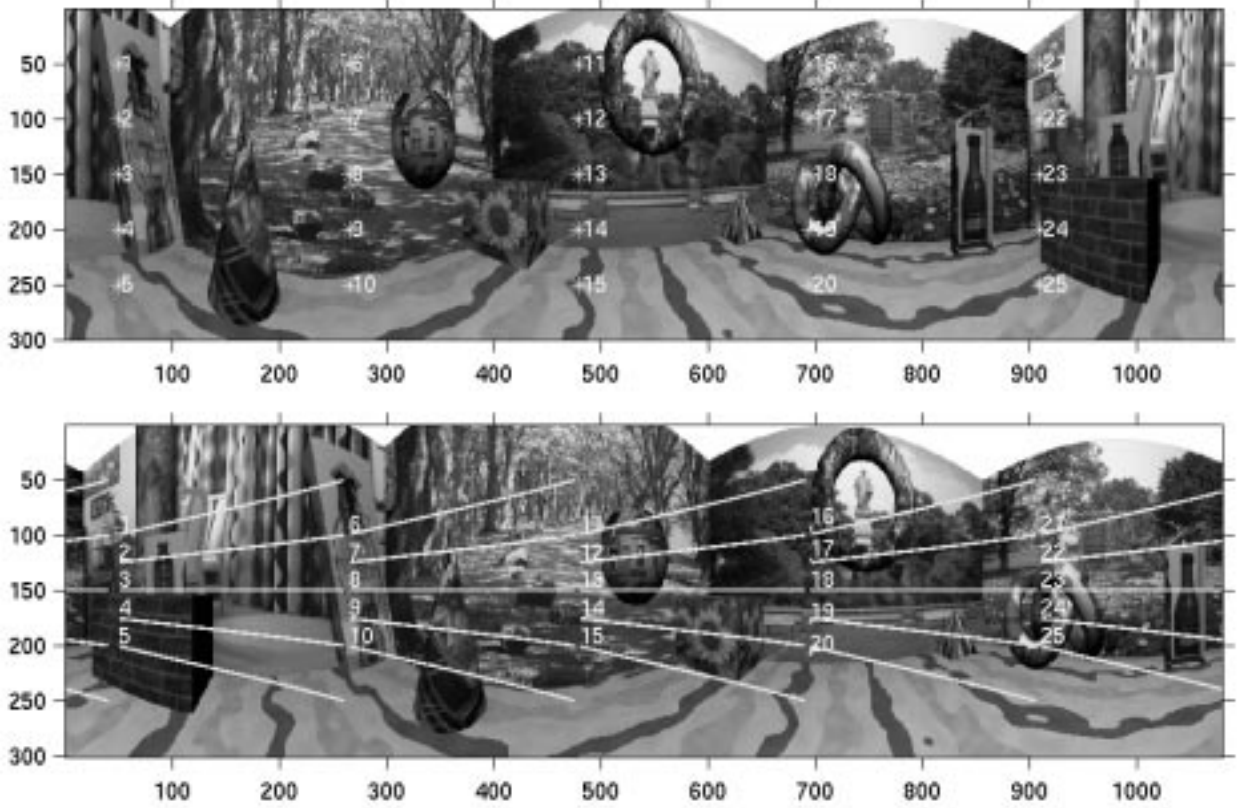


Figure 5: An example of a pair of concentric panoramas in a synthetic scene. The top image shows 30 test points labeled by numbers and the bottom shows the corresponding epipolar curves.

4.3 Epipolar Curve in Single-center Panoramas

A set of polycentric panoramic images acquired with all the slit camera's focal points coincided at a single point is called *single-center panoramas*. Consider two single-center panoramic images, \mathcal{P} and \mathcal{P}' . The orientations and positions of their turning-rig coordinate systems with respect to the world coordinate system are: $\mathbf{R}_{w\mathcal{O}} = \mathbf{R}_{w\mathcal{O}'} = \mathbf{I}_{3 \times 3}$ and $\mathbf{T}_{w\mathcal{O}} = (0, 0, 0)^T$ and $\mathbf{T}_{w\mathcal{O}'} = (t_x, 0, t_z)^T$ respectively. Each associated circle \mathcal{C} becomes a single points and angle $\omega = 0$, we have $R = R' = 0$ and $\omega = \omega' = 0$. Given an image point (x, y) on \mathcal{P} , the equation of the epipolar curve in \mathcal{P}' is

$$\begin{aligned}
 y' &= y \cdot \left(\frac{f'}{f}\right) \cdot \left(\frac{-t_x \cos\left(\frac{2\pi x'}{W_{\mathcal{P}'}}\right) + t_z \sin\left(\frac{2\pi x'}{W_{\mathcal{P}'}}\right)}{-t_x \cos\left(\frac{2\pi x}{W_{\mathcal{P}}}\right) + t_z \sin\left(\frac{2\pi x}{W_{\mathcal{P}}}\right)}\right) \\
 &= y \cdot k \cdot \left(t_z \sin\left(\frac{2\pi x'}{W_{\mathcal{P}'}}\right) - t_x \cos\left(\frac{2\pi x'}{W_{\mathcal{P}'}}\right)\right),
 \end{aligned}$$

where

$$k = \left(\frac{f'}{f \left(t_z \sin\left(\frac{2\pi x}{W_{\mathcal{P}}}\right) - t_x \cos\left(\frac{2\pi x}{W_{\mathcal{P}}}\right)\right)}\right)$$

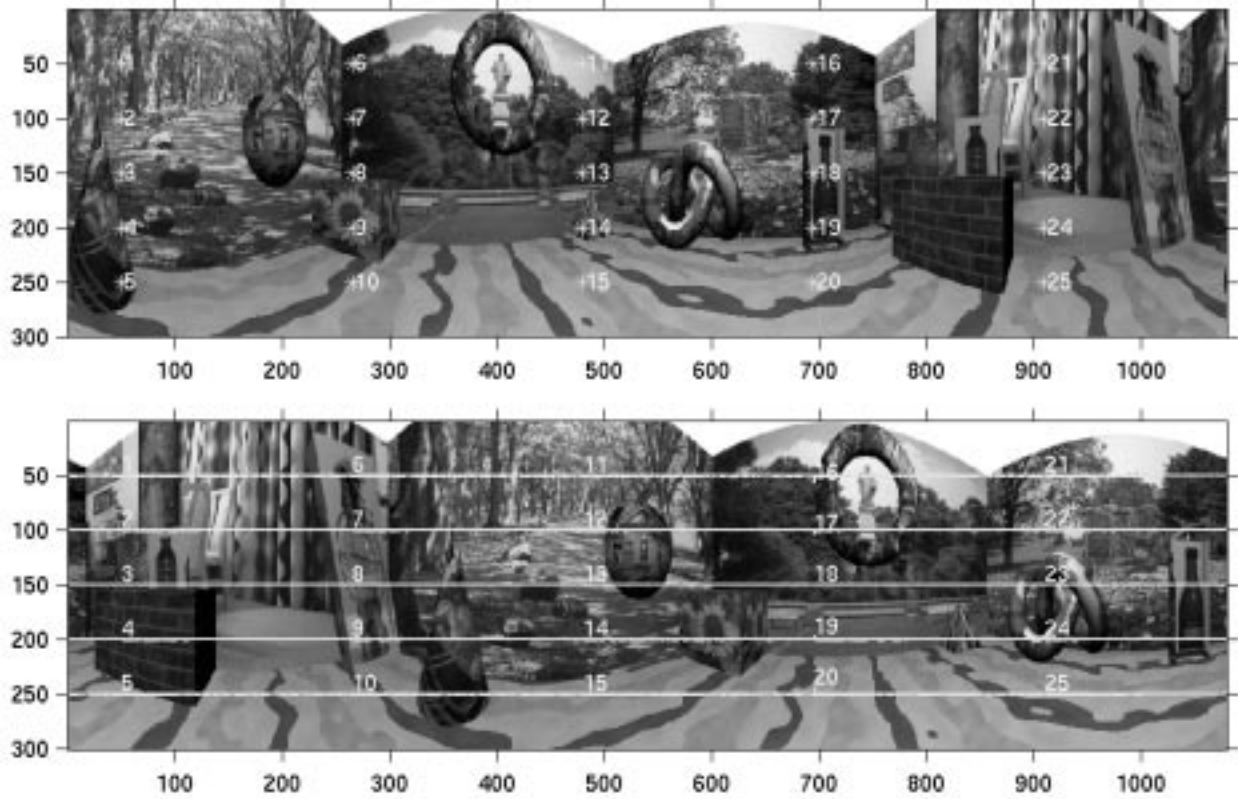


Figure 6: An example of the epipolar curves in a symmetric pair of concentric panoramas. All the epipolar curves become straight lines and coincide with image rows.

is a scalar. Figure 7 shows an example of the epipolar curves in a pair of single-center panoramas. The parameters of these two panoramic image acquisitions are identical to those of the polycentric panoramic pair except the orientations of the slit cameras are both equal to 0° and the radiuses of the circles are both equal to 0 mm.

5 Conclusion

So far, only the epipolar curve equation itself is derived, no mathematical analysis has been done. Since there are many parameters involved in the equation, it is interesting to see how each of them affects the behavior of the epipolar curve. How to classify the epipolar curves based on different behavior of the curves? How many equivalent classes can be found? Given a set of uncalibrated panoramic images of one particular class, how many corresponding points are necessary to calibrate the desired parameters? In this paper, the panoramic image surface is chosen to be a perfect cylinder. However, there are other geometric forms such as an ellipse etc exist for use in some applications. It is interesting to derive a more general epipolar curve equation for those panoramic images.

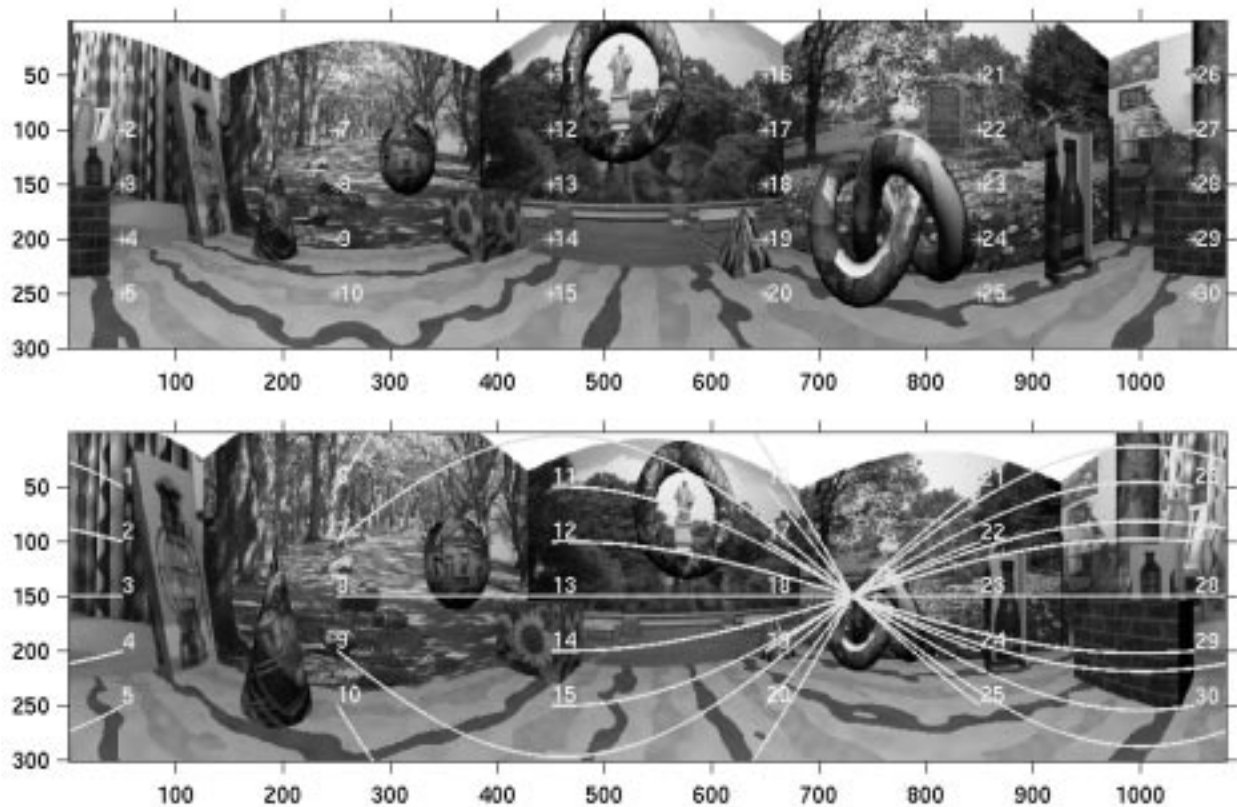


Figure 7: An example of the epipolar curves in a pair of single-center panoramas.

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