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CONTROL OF HIGH VOLTAGE DC AND FLEXIBLE AC TRANSMISSION SYSTEMS

by

DRAGAN JOVCIC

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical and Electronic Engineering

The University of Auckland
Auckland, New Zealand

December 1999

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ABSTRACT

Analytical modeling of HVDC systems has been a difficult task without a to-date reported model convenient for serious analysis of practically reported HVDC stability problems. In order to cover the frequency range \( f<100\text{Hz} \), and to cater for different model requirements, three different HVDC-HVAC models are developed in this Thesis: Detailed linear-continuous model, simplified linear continuous model and linear discrete model.

Detailed HVDC-HVAC system model is intended for small signal analysis of HVDC-HVAC interactions and resulting stability problems. It demonstrates good response matching against PSCAD/EMTDC simulation, where the CIGRE HVDC Benchmark model is used as the test system. All model variables (states) and parameters have physical meaning, and the model consists of modules, which reflect actual physical subsystems.

Simplified HVDC-HVAC system dynamic model is developed as a fourth order dynamic model, which is less accurate but more convenient for the analysis, than the detailed model. The model proves to be reliable for controller design for mitigation of composite resonance and for the study of non-linear effects in HVDC systems. The developed linear discrete model is primarily intended for the system analysis at frequencies close to \( 100\text{Hz} \) on DC side of HVDC system.

A new approach in modeling of TCR/TCSC, based on the same principles for HVDC modeling, is presented in this Thesis. The model development is far less difficult than the similar models presented in literature. PSCAD/EMTDC simulation confirms the model validity.

The simplified, linear-continuous model is used for the analysis of dominant non-linear effects in HVDC systems. The analysis of non-linear mode transformation between constant beta and constant gamma operation, shows that limit-cycle oscillations are not expected to develop, for normal operating conditions. The analysis of converter firing angle modulation shows that converter behaves as a non-linear element only for some unlikely operating conditions.

In this Thesis, it is attempted to counteract the composite resonance phenomenon by modifying the resonant condition on DC system impedance profile. This is accomplished by designing a supplementary HVDC controller that acts on HVDC firing angle on both line ends. PSCAD/EMTDC simulation results show that significant reduction in DC side first harmonic component (in some cases to \( 1/4 \) of the original value) is possible with the newly designed controller.

Chapter 5 studies \( 100\text{Hz} \) oscillations on DC side of HVDC system. A methodology for designing a new controller to counteract negative affects of these oscillations is presented. Linear simulation of the detailed controller design confirms noticeable reduction in second harmonic on DC side.

The eigenvalue decomposition and singular value decomposition is used for small-signal analysis of HVDC-HVAC interactions. The analysis of sensitivity of the dominant system eigenvalues with respect to the AC system parameters, shows the frequency range for the possible oscillatory instabilities at rectifier and at inverter side. The rectifier side of the system is most likely to experience instability at higher frequencies, whereas at inverter side the instabilities can be expected at lower frequencies. Further analysis shows that reduction in the AC system strength
will predominantly affect the eigenvalues at lower frequencies, where the SCR reduction at inverter side much more affects the system stability. The analysis of interactions between AC and DC systems through the influence of inherent feedback loops gives recommendations for the possible control of interaction variables with the aim of system stability improvement.

The root locus technique is used for the stability analysis of HVDC control loops, where all conventional and some alternative control methods suggested in the literature are investigated. It is found that DC feedback, at rectifier side, significantly improves the system stability at lower frequencies, however, at frequencies close to the first harmonic this feedback control degrades the system stability, actually accelerating the development of composite resonance. At inverter side, most of the feedback loops improve the system stability in a certain frequency range, whereas at other frequencies they noticeably deteriorate stability. Among all reported inverter control methods, reactive current feedback is found to be the best option.

The last Chapter develops a new controller for HVDC system operation with very weak inverter AC systems. The selection of feedback signal for this controller, is based on the analysis of positioning of zeros for candidate feedback signals. It is found that AC current angle is the best inverter feedback signal. This feedback signal can move the unstable complex eigenvalues left, into the stable region, without significantly affecting remaining eigenvalues. For the additional improvement in the system performance, a second order filter, designed using $H_\infty$ control theory, is placed into the feedback loop. The main design objective is the system robustness with respect to the AC system parameters changes. The controller designed in this Thesis, tolerates very wide changes in system strength, $1.7<\text{SCR}<3.5$, with the nominal operating point at $\text{SCR}=2.5$. This wide change of operating parameters, although very unlikely in practice, demonstrates large improvement in the system performance with the new controller. The similar controller especially designed for extremely low SCR, shows that HVDC system can satisfactory operate with $\text{SCR}=1.0$ at inverter side and heralds the possible controller use instead of synchronous condensers or SVC elements at inverter AC bus.
ACKNOWLEDGMENTS

I express honest thanks to my supervisor Dr. Nalin Pahalawaththa for all his help during this project. As a supervisor of this project, his professional assistance was invaluable. However the sincere, friendly relationship was great stimulus for work and far more honored advantage.

I am grateful to Dr. Mohamed J. Zavahir, for valuable help, unreserved support and supervision of my research.

Many thanks also to colleagues from the Power Systems Group, especially to Ragu Balanathan and Jahan Preirs for all their help.

Foundation for Research, Science and Technology NZ Ltd is kindly acknowledged for financial support of this project.
CONTENTS

INTRODUCTION 1

I. BACKGROUND 1
II. THESIS OBJECTIVES 3
III. THESIS OUTLINE 5
References 7

CHAPTER 1
ANALYTICAL MODELING OF HVDC-HVAC SYSTEMS 9

1.1 INTRODUCTION 9
1.1.1 Background 9
1.1.2 Object classification 11
1.2 LINEAR, CONTINUOUS SYSTEM MODEL 12
  1.2.1 AC System model 12
  1.2.2 Phase Locked Loop Model 16
  1.2.3 HVDC controller model 19
  1.2.4 DC system (converter stations and DC line) model 22
  1.2.5 Interaction equations 25
  1.2.6 General DC system model 27
  1.2.7 Polar coordinate to $dq$ coordinate transformation 30
  1.2.8 HVDC-HVAC system model 31
  1.2.9 Model verification 33
1.3 SIMPLIFIED, LINEAR, CONTINUOUS MODEL 36
  1.3.1 Modeling approach 36
  1.3.2 DC system model 36
  1.3.3 AC system model 37
  1.3.4 HVDC system model 38
1.4 LINEAR DISCRETE MODEL 40
  1.4.1 Discrete converter modeling 40
  1.4.2 Discrete HVDC system model 44
  1.4.3 Discrete converter model 46
1.5 COMPARISON OF LINEAR CONTINUOUS AND LINEAR DISCRETE HVDC MODELS 47
1.6 CONCLUSIONS 49
References 50

CHAPTER 2
MODELING OF TCR AND TCSC 51

2.1 INTRODUCTION 51
2.2 TCR/TCSC ANALYTICAL MODEL 52
  2.2.1 Modeling approach 52
  2.2.2 Controller model 52
  2.2.3 Main circuit model 54
  2.2.4 TCR model 56
  2.2.5 Model accuracy 58
CHAPTER 3
ANALYSIS OF NON-LINEAR EFFECTS IN HVDC SYSTEMS
3.1 INTRODUCTION
3.2 CONTROL MODE CHANGES IN HVDC SYSTEMS
3.3 ANALYSIS OF CONSTANT BETA-GAMMA MODE CHANGE
3.3.1 Describing Function Derivation
3.3.2 System Stability Analysis
3.3.3 Influence of Controller Parameters
3.4 NON-LINEAR PHENOMENA IN AC-DC CONVERTERS
3.4.1 Introduction
3.4.2 Converter firing angle modulation
3.4.3 Non-linear converter gain
3.5 CONCLUSIONS
References

CHAPTER 4
COMPOSITE RESONANCE ON HVDC SYSTEMS
4.1 INTRODUCTION
4.2 TEST SYSTEM ANALYSIS
4.3 CONTROLLER DESIGN
4.4 CONTROLLER IMPLEMENTATION
4.5 SIMULATION RESULTS
4.5.1 Small disturbances
4.5.2 Transient performance
4.5.3 Controller robustness
4.5.4 Controller testing for second harmonic injection
4.6 CORE SATURATION INSTABILITY
4.7 DISCUSSION AND PRACTICAL ISSUES IN CONTROLLER IMPLEMENTATION
4.8 CONCLUSIONS
References

CHAPTER 5
CONTROL OF SECOND HARMONIC OSCILLATIONS ON DC SIDE OF AN HVDC SYSTEM
5.1 INTRODUCTION
5.2 HARMONIC TRANSFER BETWEEN AC AND DC SYSTEMS - ANALYTICAL REPRESENTATION
5.3 INVERTER EXTINCTION ANGLE OSCILLATIONS
5.4 CONTROLLING THE ADVERSE EFFECTS OF 100Hz OSCILATIONS ON THE OPERATION OF HVDC SYSTEMS
5.4.1 Control Strategy
5.4.2 The main application areas of the proposed control method
5.5 CONTROLLER DESIGN
5.5.1 Design objectives and technique 100
5.5.2 Second Order Compensator (Compensator 1) design 101
5.5.3 First Order Compensator (Compensator 2) design 103
5.5.4 Controller design with respect to the system operation in different control modes 105
5.5.5 Controller performance evaluation 107
5.5.6 Influence of system parameters on controller performance 109
5.5.7 Discussion and implementation issues 109
5.6 CONCLUSIONS 111
References 112

CHAPTER 6
ANALYSIS OF HVDC-HVAC INTERACTIONS 113

6.1 INTRODUCTION 113

6.2.0 EIGENVALUE DECOMPOSITION BASED ANALYSIS 114
6.2.1 Eigenvalue location as a consequence of system coupling 114
6.2.2 Eigenvalue sensitivity and participation factor analysis 116

6.3 INFLUENCE OF AC SYSTEM SCR 118

6.4 ANALYSIS OF INHERENT FEEDBACK LOOPS BETWEEN THE SYSTEMS 120
6.4.1 Method of analysis 121
6.4.2 Analysis of Rectifier side interactions 122
6.4.3 Analysis of Inverter side interactions 123

6.5 ANALYSIS OF INPUT-OUTPUT DIRECTIONS 125
6.5.1 Introduction 125
6.5.2 System analysis 126

6.6 CONCLUSIONS 129
References 130

CHAPTER 7
STABILITY ANALYSIS OF HVDC CONTROL LOOPS 131

7.1 INTRODUCTION 131

7.2 ANALYSIS OF RECTIFIER CONTROL MODES 133
7.2.1 Root locus analysis 133
7.2.2 Eigenvalue sensitivity analysis 134
7.2.3 System analysis with weak AC system 135

7.3 ANALYSIS OF INVERTER CONTROL MODES 136
7.3.1 Analysis of conventional control loops 136
7.3.2 Alternative inverter control strategies 138
7.3.3 Weak AC system 139

7.4 CONCLUSIONS 141
References 142

CHAPTER 8
HVDC OPERATION WITH WEAK RECEIVING AC
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>CIGRE HVDC Benchmark model</td>
<td>13</td>
</tr>
<tr>
<td>1.2</td>
<td>Influence of AC system dynamics on converter firing angle. a) time domain b) small signal model.</td>
<td>17</td>
</tr>
<tr>
<td>1.3</td>
<td>D-Q-Z type phase locked loop</td>
<td>18</td>
</tr>
<tr>
<td>1.4</td>
<td>HVDC current controller model. a) constant current controller, b) constant firing angle operating mode, c) constant extinction angle operating mode.</td>
<td>21</td>
</tr>
<tr>
<td>1.5</td>
<td>DC system representation.</td>
<td>23</td>
</tr>
<tr>
<td>1.6</td>
<td>Relationship between phase angles of AC system variables at rectifier terminal.</td>
<td>26</td>
</tr>
<tr>
<td>1.7</td>
<td>Structure of the general DC system model.</td>
<td>29</td>
</tr>
<tr>
<td>1.8</td>
<td>Input-output coupling between AC-DC systems.</td>
<td>32</td>
</tr>
<tr>
<td>1.9a)</td>
<td>System response following current order step change. Test system is the CIGRE HVDC benchmark model, Rec. SCR=2.5, Inv. SCR=2.5. Inverter operating angle $\beta = 40^{\circ}$</td>
<td>34</td>
</tr>
<tr>
<td>1.9b)</td>
<td>System response following rectifier AC voltage disturbance step change. Test system is the CIGRE HVDC benchmark model, Rec. SCR=2.5, Inv. SCR=2.5. Inverter operating angle $\beta = 40^{\circ}$</td>
<td>35</td>
</tr>
<tr>
<td>1.10</td>
<td>Schematic diagram of simplified HVDC system model</td>
<td>37</td>
</tr>
<tr>
<td>1.11</td>
<td>AC system representation</td>
<td>38</td>
</tr>
<tr>
<td>1.12</td>
<td>Schematic diagram of the system model</td>
<td>39</td>
</tr>
<tr>
<td>1.13</td>
<td>Converter direct voltage waveform</td>
<td>41</td>
</tr>
<tr>
<td>1.14</td>
<td>Block diagram of a converter as discrete system</td>
<td>42</td>
</tr>
<tr>
<td>1.15</td>
<td>HVDC system as a discrete system</td>
<td>45</td>
</tr>
<tr>
<td>1.16</td>
<td>Simplified block-diagram of discrete model.</td>
<td>46</td>
</tr>
<tr>
<td>1.17</td>
<td>Discrete HVDC controller model.</td>
<td>47</td>
</tr>
<tr>
<td>1.18</td>
<td>Open loop frequency response for continuous and discrete models</td>
<td>48</td>
</tr>
<tr>
<td>1.19</td>
<td>System response after disturbance on rectifier AC voltage</td>
<td>48</td>
</tr>
<tr>
<td>2.1</td>
<td>Test system in use. a) TCR test model, b) TCSC test model.</td>
<td>52</td>
</tr>
<tr>
<td>2.2</td>
<td>System response following voltage-reference step change. voltage-magnitude (kV) response is shown.</td>
<td>59</td>
</tr>
<tr>
<td>3.1</td>
<td>Steady-state HVDC system characteristic.</td>
<td>64</td>
</tr>
<tr>
<td>3.2</td>
<td>HVDC system block diagram at operating point A</td>
<td>65</td>
</tr>
<tr>
<td>3.3</td>
<td>System equivalent block diagram</td>
<td>66</td>
</tr>
<tr>
<td>3.4</td>
<td>System stability analysis at operating point A.</td>
<td>67</td>
</tr>
<tr>
<td>3.5</td>
<td>Direct voltage after firing angle modulation.</td>
<td>70</td>
</tr>
<tr>
<td>3.6</td>
<td>Figure 3.6. Direct voltage after firing angle modulation. $\alpha_0 = 15^{\circ}$ $M = 10^{\circ}$ (Input signal is the same as in Figure 3.5).</td>
<td>71</td>
</tr>
<tr>
<td>4.1</td>
<td>Frequency characteristic of New Zealand HVDC system.</td>
<td>76</td>
</tr>
<tr>
<td>4.2</td>
<td>Frequency response between $E_i^d$ and $I_3$, case 3.</td>
<td>80</td>
</tr>
</tbody>
</table>
4.3 Rectifier current controller. 81
4.4 Inverter controller. 81
4.5 Case 3. System response following a step change in rectifier AC voltage (analytical model) 82
4.6 Case 2. System response following 3.4% step change in the rectifier AC voltage (taping factor decrease at primary side). 82
4.7 Figure 4.7. Case 3. System response following 3.4% step change in the rectifier AC voltage (taping factor decrease at primary side). a) Simulation responses. Comparison between original system and new controller. b) Original system responses. Comparison between simulation and analytical model. 83
4.8 Case 2. System response following a close 0.1sec single-phase fault on inverter AC system. 84
4.9 System response following a 3.4% step change in the rectifier AC voltage, (taping factor decrease at primary side).case of very weak AC system (Inv SCR=1.5). 84

5.1 Block diagram of the uncontrolled system. 96
5.2 Block diagram of controlled system. 97
5.3 Controller gains for NZ HVDC system. 99
5.4 HVDC system discrete model with two cascade compensators. 101
5.5 Root locus, original system. 102
5.6 r-domain Bode diagram for HVDC system. 103
5.7 Root locus, compensated system. 104
5.8 System response after disturbance step change. 105
5.9 Current order step response. 106
5.10 Root locus for constant gamma mode. 107
5.11 Rectifier direct current response following a 100Hz disturbance at rectifier DC side (Rectifier AC voltage, DC side equivalent disturbance) fault on the rectifier side AC voltage. 108
5.12 Inverter direct current response following a 100Hz disturbance at rectifier DC side (Rectifier AC voltage, DC side equivalent disturbance) fault on the rectifier side AC voltage. 108
5.13 Influence of system gain changes on controller performance. 109

6.1 System response following inverter side AC disturbance. 1.6% taping factor decrease at primary side. Inv. SCR=2.5, Rec. SCR=2.5 (Original system), Rec. SCR=1.45, Inv. SCR=2.5 (Reduced SCR). 120
6.2 Interactions between AC and DC systems. 120
6.3 External control of interaction variables. 122
6.4 Effect of various AC/DC system interactions on HVDC system performance, x-Location of original eigenvalues, o-system eigenvalues when the interaction variable is ideally controlled. 122
6.5 System response following rectifier side disturbance (1.6% taping factor decrease at primary side) with increased PLL gains at rectifier side. 124
6.6 System response following rectifier side disturbance (1.6% taping factor decrease at primary side) with increased PLL gains at inverter side. 124
6.7 HVDC system as a MIMO system with decentralized control structure. 125
6.8 Principal singular values for disturbance input. 126
7.1 Control loops at rectifier side.
7.2 Root locus with direct current and fast power feedback. + eigenvalues of the uncontrolled system, position of zeros.
7.3 System response following reference signal step change with DC feedback and with fast power feedback.
7.4 Root locus for the system with very weak receiving AC system. \( \text{Inv SCR}=1.5 \).
7.5 Inverter controller.
7.6 Root locus for direct current and direct voltage feedback at inverter side.
7.7 Inverter direct current response following step change in the current reference. The instability occurs with high-gain DC feedback at inverter side.
7.8 Root locus for alternative control strategies at inverter side. Only the low frequency dynamics are shown.
7.9 Influence of different control strategies on the system with low SCR.
7.10 Inverter direct current response following a disturbance at inverter side.

8.1 Inverter controller.
8.2 Singular value decomposition for direct current feedback at inverter side.
8.3 Root locus for inverter DC-current and DC-voltage feedback. “h” and “i” locus branches can have unstable eigenvalues.
8.4 Root locus for AC-current-angle feedback. + poles, position of zeros.
8.5 AC system perturbations represented as a multiplicative uncertainty.
8.6 Weighting function \( W_s(s) \) determined on the basis of relative uncertainty.
8.7 Model uncertainty in frequency domain. \( h_3 \) - maximum model uncertainty at half the sampling frequency.
8.8 Frequency response of sensitivity function and complementary sensitivity function.
8.9 Controller frequency response.
8.10 Controller implementation.
8.11 AC current angle measurement.
8.12 System response following current order step change. Inverter \( \text{SCR}=1.7@78\text{deg} \).
8.13 System response following current order step change. Inverter \( \text{SCR}=1.2@76\text{deg} \).
8.14 System response following a single phase fault(0.05sec) at the inverter side. Inverter \( \text{SCR}=1.3@76\text{deg} \).
8.15 System response following a single phase fault(0.05sec) at the inverter side. Inverter \( \text{SCR}=1.2@76\text{deg} \).
8.16 System response following current reference step change at 0.05s and 0.3s. Inverter \( \text{SCR}=1.0@76\text{deg} \). Original system is unstable.
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>LOW ORDER BESSEL COEFFICIENTS</td>
<td>70</td>
</tr>
<tr>
<td>4.1</td>
<td>AC SYSTEM STRENGTH FOR THE TEST CASES.</td>
<td>77</td>
</tr>
<tr>
<td>4.2</td>
<td>EIGENVALUES OF THE ORIGINAL SYSTEM</td>
<td>77</td>
</tr>
<tr>
<td>4.3</td>
<td>DESIRED POLE LOCATIONS</td>
<td>80</td>
</tr>
<tr>
<td>4.4</td>
<td>CALCULATED CONTROLLER PARAMETERS</td>
<td>80</td>
</tr>
<tr>
<td>4.5</td>
<td>A). HARMONIC CURRENT MAGNITUDE [A]. 1.6 KV, 3 PHASE POS. SEQ. HARMONIC INJECTION TO THE RECTIFIER AC SYSTEM. B). HARMONIC CURRENT MAGNITUDE [A]. 1.6 KV, 3 PHASE POS. SEQ. HARMONIC INJECTION TO THE INVERTER AC SYSTEM.</td>
<td>85</td>
</tr>
<tr>
<td>5.1</td>
<td>CONTROLLED PARAMETERS</td>
<td>104</td>
</tr>
<tr>
<td>6.1</td>
<td>DC SYSTEM EIGENVALUES</td>
<td>115</td>
</tr>
<tr>
<td>6.2</td>
<td>RECTIFIER AC SYSTEM EIGENVALUES</td>
<td>115</td>
</tr>
<tr>
<td>6.3</td>
<td>INVERTER AC SYSTEM EIGENVALUES</td>
<td>115</td>
</tr>
<tr>
<td>6.4</td>
<td>16 MOST IMPORTANT SYSTEM EIGENVALUES</td>
<td>117</td>
</tr>
<tr>
<td>6.5</td>
<td>SYSTEM EIGENVALUES FOR REDUCED SCR.</td>
<td>119</td>
</tr>
<tr>
<td>6.6</td>
<td>SINGULAR VALUE DECOMPOSITION FOR TWO FREQUENCIES BELOW 20rad/s.</td>
<td>127</td>
</tr>
<tr>
<td>6.7</td>
<td>SINGULAR VALUE DECOMPOSITION AT 440RAD/S.</td>
<td>127</td>
</tr>
<tr>
<td>7.1</td>
<td>SENSITIVITY OF DOMINANT COMPLEX EIGENVALUES</td>
<td>135</td>
</tr>
<tr>
<td>7.2</td>
<td>POSITION OF ZEROS FOR INVERTER CONTROL MODES</td>
<td>139</td>
</tr>
<tr>
<td>8.1</td>
<td>SYSTEM EIGENVALUES FOR REDUCED SCR AT THE INVERTER SIDE</td>
<td>145</td>
</tr>
<tr>
<td>8.2</td>
<td>PARTICIPATION FACTOR FOR DOMINANT COMPLEX EIGENVALUES IN THE CASE OF VERY WEAK RECEIVING AC SYSTEM SCR=1.0</td>
<td>145</td>
</tr>
<tr>
<td>8.3</td>
<td>SENSITIVITY OF EIGENVALUES WITH RESPECT TO THE CONTROLLER PARAMETERS.</td>
<td>147</td>
</tr>
<tr>
<td>8.4</td>
<td>RGA AROUND CROSS-OVER FREQUENCY</td>
<td>148</td>
</tr>
<tr>
<td>8.5</td>
<td>CONDITION NUMBER AROUND CROSS-OVER FREQUENCY.</td>
<td>149</td>
</tr>
<tr>
<td>8.6</td>
<td>POSITION OF ZEROS FOR CANDIDATE FEEDBACK SIGNALS.</td>
<td>151</td>
</tr>
<tr>
<td>8.7</td>
<td>CALCULATED CONTROLLER PARAMETERS</td>
<td>160</td>
</tr>
</tbody>
</table>
## GLOSARY

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Transformer ratio</td>
</tr>
<tr>
<td>(a)</td>
<td>Magnitude of input signal, transformer ratio, elements of matrix A</td>
</tr>
<tr>
<td>A</td>
<td>System matrix, area, describing function coefficients</td>
</tr>
<tr>
<td>B</td>
<td>Input matrix</td>
</tr>
<tr>
<td>C</td>
<td>Output matrix, capacitance</td>
</tr>
<tr>
<td>e</td>
<td>Controller error</td>
</tr>
<tr>
<td>(e_s)</td>
<td>AC source voltage</td>
</tr>
<tr>
<td>(e_{ac})</td>
<td>Converter AC voltage</td>
</tr>
<tr>
<td>(E_{th})</td>
<td>Equivalent Thevenin source</td>
</tr>
<tr>
<td>(E_{ac})</td>
<td>AC system voltage (DC side of converter transformers)</td>
</tr>
<tr>
<td>(E_{a,b,c})</td>
<td>a,b,c Phase AC voltage</td>
</tr>
<tr>
<td>(E^d)</td>
<td>Disturbance voltage</td>
</tr>
<tr>
<td>f</td>
<td>Frequency, function, number of AC systems, feedback controller gain</td>
</tr>
<tr>
<td>g</td>
<td>Transfer function</td>
</tr>
<tr>
<td>G</td>
<td>Transfer function</td>
</tr>
<tr>
<td>i</td>
<td>Instantaneous current, inverter</td>
</tr>
<tr>
<td>(i_{ac})</td>
<td>Converter AC current</td>
</tr>
<tr>
<td>(i,j)</td>
<td>(\sqrt{-1}), Imaginary number.</td>
</tr>
<tr>
<td>j</td>
<td>Generic index 1- rectifier, 2- inverter</td>
</tr>
<tr>
<td>I</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>(I_d)</td>
<td>Direct current, d-component of AC current</td>
</tr>
<tr>
<td>(I_m)</td>
<td>Current margin</td>
</tr>
<tr>
<td>(I_{ord})</td>
<td>Current order</td>
</tr>
<tr>
<td>(I_q)</td>
<td>q-component of AC current</td>
</tr>
<tr>
<td>(I_r)</td>
<td>Rectifier direct current</td>
</tr>
<tr>
<td>(I_i)</td>
<td>Inverter direct current</td>
</tr>
<tr>
<td>J</td>
<td>Bessel coefficients</td>
</tr>
<tr>
<td>k</td>
<td>Controller gain</td>
</tr>
<tr>
<td>K</td>
<td>Linearised (converter) gain, gain matrix, controller, controller gain</td>
</tr>
<tr>
<td>l</td>
<td>Instantaneous value of reactance</td>
</tr>
<tr>
<td>L</td>
<td>Reactance</td>
</tr>
<tr>
<td>m</td>
<td>Harmonic order</td>
</tr>
<tr>
<td>M</td>
<td>Magnitude of input signal</td>
</tr>
<tr>
<td>(n(*))</td>
<td>Non-linear element</td>
</tr>
<tr>
<td>(N(*))</td>
<td>Describing function</td>
</tr>
<tr>
<td>P</td>
<td>Park’s transformation matrix, power, correction matrix</td>
</tr>
<tr>
<td>R</td>
<td>Real numbers, correction matrix, resistance</td>
</tr>
<tr>
<td>(R_c)</td>
<td>Equivalent commutating resistance</td>
</tr>
<tr>
<td>s</td>
<td>La Place operator</td>
</tr>
<tr>
<td>S</td>
<td>Sampler, matrix, correction matrix, sensitivity function</td>
</tr>
</tbody>
</table>
\( t \)  
Time interval

\( T \)  
Time constant, matrix, complementary sensitivity function, transformation matrix

\( T_f \)  
Filter time constant

\( T_s \)  
Sampling time constant

\( T_u \)  
Time delay

\( v \)  
Instantaneous voltage, any AC variable

\( V_{dc,i} \)  
Rectifier, inverter direct voltage

\( V \)  
Voltage magnitude, Output singular vector

\( V_{ac} \)  
AC system voltage (AC side of converter transformers)

\( V_{dac} \)  
D-component of AC voltage

\( V_{qac} \)  
Q-component of AC voltage

\( u \)  
System input, commutation overlap

\( U \)  
Input singular vector

\( w \)  
Disturbance

\( W \)  
Transfer function, matrix

\( x \)  
State variable

\( x' \)  
Single phase state variable

\( X_c \)  
Equivalent commutation reactance

\( y \)  
System output

\( z \)  
Z-transformation operator, impedance

\( Z \)  
Z-transformation

\( \alpha \)  
Converter firing angle

\( \beta \)  
Converter angle \( (\beta = \pi - \alpha) \)

\( \gamma \)  
Converter extinction angle, condition number,

\( \sigma \)  
Singular value

\( \partial \)  
Partial derivative

\( \psi \)  
Current phase angle

\( \varphi \)  
Voltage phase angle

\( \theta \)  
PLL output angle

\( \phi \)  
Converter actual firing angle, power factor angle

\( \omega \)  
Angular frequency

\( \Delta \)  
Model uncertainty

\( \Sigma \)  
Matrix of singular values

**abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCCM</td>
<td>Combined and Coordinated Control Method</td>
</tr>
<tr>
<td>CRC</td>
<td>Constant Reactive Current control method</td>
</tr>
<tr>
<td>DSF</td>
<td>Damping Sensitivity Factor</td>
</tr>
<tr>
<td>HVDC</td>
<td>High Voltage Direct Current</td>
</tr>
<tr>
<td>HVAC</td>
<td>High Voltage Alternating Current</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-input multi-output system</td>
</tr>
<tr>
<td>NP</td>
<td>Nominal Performance</td>
</tr>
<tr>
<td>NS</td>
<td>Nominal Stability</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>RP</td>
<td>Robust Performance</td>
</tr>
<tr>
<td>SCR</td>
<td>Short Circuit Ratio</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
</tbody>
</table>
TCR    Thyristor Controlled Reactor
TCSC   Thyristor Controlled Series Capacitor
INTRODUCTION

I. BACKGROUND

Since the first introduction of HVDC power transfer technology, various analytical models for HVDC analysis and design have been developed [1-4]. These modeling approaches are mainly concerned with accurate modeling of AC-DC converters. The developed models can only be used for the very basic system stability analysis.

As the number of HVDC links worldwide has grown, more accurate analytical models have emerged [5-8]. Much more light has been shed on HVDC modeling principles in these models. Possibly, a single most comprehensive model developed to-date is reported in [7]. Still, this model does not give detailed representation of AC-DC interactions, it does not represent PLL dynamics, and AC systems are overly simplified.

With the fast expansion and development of digital based technologies in the last 20 years, a number of very accurate non-linear HVDC simulation software have emerged with the prominent one described in [9]. These software have been used as the main tool in industry for stability study of HVDC links. They are accurate, reliable and user friendly. The inherent shortcoming of these models is the lack of analytical representation of internal system properties (dynamics). They can be used only for a trial and error study. Extensive and repetitive simulation runs are necessary for any serious system analysis and, general, qualitative conclusions are seldom derived. Controller design is also usually based on the testing of successively incrementing controller gains.

A convenient and accurate analytical system model offers a tool for qualitative study of the system behavior without a need for simulation of the system responses. Analytical models can offer conclusions applicable to all systems with similar structure, regardless of the values for their actual parameters. They offer a platform for readily designing controllers that can significantly improve the system properties. In this Thesis, it will be firstly attempted to develop an accurate and convenient analytical model of an HVDC system.

An important step in HVDC modeling is the development of CIGRE HVDC benchmark test system [10]. Although it does not represent any particular HVDC system, this test system is designed to emulate typical HVDC configuration with system parameters (SCR and resonant conditions) calculated to make a difficult-to-control case. In [11] this test system is further modified to create very weak inverter AC system. These two test system will be primarily used in this Thesis.

Apart from modeling of HVDC systems, the analytical modeling approaches for the basic FACTS elements (TCR and TCSC) will also be studied in this Thesis. Because of the use of AC-DC converters, PLL circuits and two different frequency domains, the modeling of TCR/TCSC has many similar aspects as modeling of HVDC systems.

The most influential TCR/TCSC analytical models are presented in [12], [13] and [14]. Although their accuracy is demonstrated, these models are suitable for a particular purpose only, and for a simple AC transmission system. In all three models, special discretisation process is
Introduction

necessary, and the model development becomes difficult and tedious. This Thesis offers a generalized approach to development a TCR/TCSC model, based on independent modeling of various subsystems in a manner similar to that used in HVDC modeling.

The most important HVDC operating problems reported to-date can be classified into several categories: converter composite resonance including core saturation instability, second harmonic oscillations on DC side of HVDC system and problems related to weak AC systems connected HVDC systems. The frequency range for the above operating difficulties can be broadly classified as $5Hz < f < 100Hz$ as seen on DC side. All of these stability problems are essentially AC-DC interaction problems. The most important factor for poor dynamic analysis of HVDC operating difficulties is the lack of suitable, reliable analytical models. In practice, soon after the difficulties are experienced these are usually successfully simulated on digital simulators and the mechanism for development is explained, in general. However, in order to accurately predict the occurrence of the instability, to identify the parts of the system mostly responsible, and to design an effective controller for its elimination, it is necessary to have good knowledge of the system internal dynamic properties and the change of these properties as the operating conditions change and as the control logic is changed.

Core saturation instability has occurred on at least two HVDC systems [15], [16]. Pole 1 of New Zealand HVDC system is known to have resonant conditions that can cause instability [17]. This form of instability is manifested as a spontaneous growth of second harmonic oscillations on AC side, and first harmonic on DC side of the system [18]. The instability is accelerated by the additional generation of second harmonic as a consequence of transformer core saturation. It can take form of fast growing oscillations until some of the protection circuits trip the affected part of the system and cause an outage. In some cases, because of the non-linear system characteristics, the oscillations of certain magnitude can persist for a longer time introducing distortion in the system [16], and affecting power quality. Composite resonance is a phenomenon of complementary AC-DC resonance; a high impedance parallel resonance on the AC side coupled with a low impedance series resonance at an associated frequency at DC side [19]. This form of instability may accelerate development of core saturation instability.

References [20-23] offer further study of the above instabilities, present reported cases and address some control solutions. Theoretical research reported in [24] shows that the most important factors for the development of core saturation instability are the DC side impedance profile together with DC controls, and the AC side impedance characteristics. Similarly, reference [19] emphasizes the importance of system impedance profile and DC controller gains for the composite resonance phenomenon on HVDC systems. Therefore, development of these form of instability will be determined by the inherent system damping of a small signal excitation at particular frequencies. As an example, the resonant frequency of the DC system is determined by the DC line natural impedance profile in combination with smoothing reactors at both line ends. Despite the known causes of the instability, and analysis of the existing control loops, very little research has been performed to offer a controller design procedures for the system under the above conditions. In this Thesis, it will be attempted to modify the natural impedance profile of DC system by using different HVDC control strategies.

New Zealand HVDC system was experiencing $100Hz$ on DC side for a long time [25]. Although it has not been reported on many other systems, the mechanism is universal [26], and similar oscillations could occur on other systems. It is known that these $100Hz$ oscillations on DC side are a consequence of an unbalanced AC supply voltage [26]. The phenomenon is manifested as $100Hz$ oscillations on DC current and on extinction angle gamma. In the case of constant beta
and predictive type gamma controller, the oscillations on extinction angle will reduce the minimum gamma values, and the safe commutation margin may be endangered. The available HVDC references do not offer any study on the possible (control) countermeasures for this phenomenon. In this Thesis, it will be shown how the existing HVDC controllers can be used to reduce the 100Hz oscillations.

Except for the reference [27], there has not been a significant research on the possible HVDC control methods at rectifier side. The same direct current feedback has been used on most of the HVDC systems since their first practical implementation. At inverter side, however, the applied control method varies with different systems. The most often used control strategies are direct current feedback, direct voltage feedback and AC voltage feedback, whereas a number of alternative control methods has also been reported [27], [28]. The various inverter control strategies have not been compared in the references, from the dynamic point of view, and there are no clear recommendations for the inverter controller in the future HVDC systems. This Thesis uses the detailed analytical system model to offer a small-signal analysis of reported HVDC control strategies at both, rectifier and inverter side.

The inability of HVDC systems to operate with weak AC systems can be regarded as the most important factor in limiting the use of HVDC power transfer technology, both in number and in capacity. In large number of HVDC links, the rating of the link has been determined on the basis of minimum safe Short Circuit Ratio (SCR). Also, some HVDC projects have been abandoned, or locations for some terminals had to be altered because of very weak receiving AC systems. The recommendation has been traditionally accepted that for successful HVDC system operation, the strength of connected AC systems should be $\text{SCR}>2$. In references [27] and [28] the dynamic instabilities caused by the weak receiving AC systems are presented. The main operating problems with the HVDC systems connected to weak AC systems are manifested as the high-magnitude AC voltage oscillations and the difficulty in recovery from disturbances [29], [30]. These AC voltage fluctuations, even without actual instability, can be harmful for AC equipment and the quality of power supply. On the DC side, the probability of commutation failure will increase.

The traditional, most commonly used option for strengthening of AC systems is the use of Synchronous Condensers. Since this method suffers high losses and high operational cost, an alternative option, the possibility of the use of HVDC controls, specially designed for weak AC systems, will be investigated in this Thesis.

II. THESIS OBJECTIVES

The first objective of this thesis is to develop an analytical model suitable for the study of the most common HVDC operating difficulties described in Section 1.0 above. The model should have high fidelity in the frequency range $f<100Hz$. It is also important that the model is convenient for analysis of HVDC-HVAC interactions, since all the above instabilities arise from HVAC-HVDC coupling. A complex HVAC-HVDC-HVAC system consists of a several subsystems whose individual properties are usually known. The properties of the overall system are however difficult to predict. The model should follow this modular structure with the clear physical meaning for each subsystem, which will enable qualitative conclusions about their influence on the dynamics of the overall system. The main subsystems can be identified as AC systems on both HVDC ends, Phase Locked Loops and their controllers and DC system with its controls. The preferable model form is the linear continuous, represented in state-space domain.
This model representation is convenient for the system analysis and controller design using the linear systems control theory.

The same modeling principles and approaches apply to the development of FACTS models. The Thesis attempts to develop a small-signal model of TCR and TCSC in the form convenient for the system analysis and design, yet accurate in the frequency range of interest.

By varying the operating conditions in the above model(s), the reported instabilities could be simulated. Various sensitivity indices could also be used to predict the operating difficulties. The well-known engineering terms which describe a particular system (like SCR, impedance profile, etc.) should primarily be used in the analysis, in order to give practically useful indicators of possible stability problems. The model with the physically meaningful variables and parameters would enable firm connection between the conclusion from the theoretical, dynamic study and the parameters and operating conditions in the actual system. The influence of various controller parameters and operating modes on the development of the instabilities should similarly be studied. Sometimes, the operating problems can be avoided just by properly tuning the existing controllers, or by compromising other system properties, like speed of response or reference tracking.

The majority of the design stage countermeasures for the operating problems described in previous sections, are directed towards main circuit modifications. As an example, because of known stability problems HVDC systems are not coupled with weak AC systems [28],[30]. In this case, a connection point for HVDC system is changed or power transfer level is limited or the AC system is strengthen. Also a special main circuit modifications are employed (TCR, AC filters) if a condition for composite resonance exist [15]. A number of HVDC control system studies and novel designs, for each particular problem, have also been presented. A good example are the controller studies for composite resonance (core saturation instability) [25-29], and alternative inverter control strategies [28],[31]. Further in this line, the role of traditional HVDC controls and a possible use of novel control techniques in the analysis of stability problems, will be primarily studied in this Thesis. A control solution, although it requires very high level of the system knowledge, detailed dynamic models, mathematically complex design procedures and highly skilled personnel, very often has advantage over the main circuit modifications in terms of the overall project cost. It is known that HVDC converters have very high controllability. Nevertheless, the present day HVDC control methods are very similar to those applied at first HVDC schemes, with only a partial utilization of its capacity, which is especially true at inverter side. One of the primary objectives of this Thesis is to try to use HVDC controls to its full potential. It is investigated in this Thesis if it is possible to use the inverter controller for other control objectives (stability improvement and similar) and if there is a frequency range for further control action at rectifier side in order to counteract a particular operating difficulty.

Most of the traditional HVDC control theory is based on the static (V-I) operating curves, and therefore valid only at very low frequencies. This Thesis is primarily concerned with small signal stability analysis around the nominal operating point and in the frequency range below the second harmonic on DC side. This dynamic system analysis is far more general than the analysis based on the static diagrams. The conclusions obtained from the dynamic system analysis are, in some cases, opposite to the conclusions about a “more stable” or “less stable” conclusions, often derived from the static diagrams and static stability factors.

Far too often in the traditional approach, HVDC system has been viewed as an isolated system, bounded by the converter transformers. The influence of the connected AC systems is overly
simplified in this type of analysis. In this Thesis, HVDC system is considered as a dynamic subsystem inside a large HVAC-HVDC-HVAC system. The design of HVDC controllers is directed towards improvement in the stability of the overall system, and not just the HVDC subsystem. In this way, a converter can be viewed as a powerful controlling element inside a large, complex transmission system.

The design of a controller (or modification of the existing controller) is the most important step in the process of elimination of the instabilities by the use of control methods. The analysis of possible control methods requires a thorough analysis of controller objectives and the possible compromises, which need to be made in other parts of the system, or in other frequency domains. The controller design itself is based on careful selection of controller inputs, selection of controller structure and feedback signals, and the calculation of filter parameters. There are numerous possible controller designs for the same controller goals. This Thesis tends to develop the best control solution for the particular operating difficulty under the study. More importantly, by developing suitable system models, and thoroughly discussing the controller objectives, this thesis paves the way for the future design of better, optimal HVDC control solutions.

III. THESIS OUTLINE

Chapter 1 presents three different HVDC modeling approaches. Different models are developed to cater for the different modeling purposes.

Detailed HVDC-HVAC model includes detailed dynamic models of both AC systems, dynamic DC system model, dynamic models of both PLL circuits and detailed representation of AC-DC converter interactions. It is intended for analysis of HVAC-HVDC interactions in the frequency domain $f<100\text{Hz}$. The model is of $45^{th}$ order.

Simplified, linear-continuous model is much easier to develop and it is much more convenient for the system dynamic analysis. It is used for the analysis of AC-DC composite resonance and for the analysis of non-linear effects in HVDC (control) systems.

Linear-discrete HVDC system model is developed using the sampled data systems theory. It is used in the frequency domain close to second harmonic, $f\approx100\text{Hz}$.

Chapter 2 presents a basis for a novel approach in TCR/TCSC modeling. Using Park’s transformation to connect the control system model with the main circuit model develops the model. Since the model uses the same principles as the detailed HVDC system model, the model can be readily coupled with HVDC model, for the purpose of analysis of HVDC-HVAC-FACTS interactions.

Chapter 3 is concerned with the analysis of non-linear effects in HVDC systems. Two groups of non-linearities are analysed: non-linear effects as a consequence of HVDC control mode changes and non-linear signal transfer through AC-DC converters.

Chapter 4 analyses the composite resonance phenomenon on HVDC systems. A new controller is designed by using the state feedback theory, in an attempt to eliminate or modify the DC system natural resonance condition. Simulation results show that a noticeable reduction in DC side first harmonic is possible.
Chapter 5 studies the phenomenon of second harmonic oscillations on DC side of HVDC system. The causes of the oscillations and the negative effects on HVDC-HVAC system are thoroughly analysed. A new control method, based on DC feedback, is presented and an example of controller design is given. The possible controller use for the elimination of other negative effects is also studied.

Chapter 6 is focused on small-signal analysis of HVDC-HVAC interactions. The eigenvalue sensitivity analysis is used to study the sensitivity of the dominant eigenvalues with respect to the AC parameters. The study of influence of SCR changes on the system dynamic stability reveals completely different influence at rectifier side from that at inverter side. Analysis of inherent feedback loops between the AC and DC systems offers conclusions about interaction variables that should be controlled and those that are better to be left uncontrolled since they inherently improve the system stability.

Chapter 7 studies the dynamic stability of HVDC control loops, at both rectifier side and inverter side. The benefits and possible stability degradation are presented for all of the conventionally used control loops and for some of the alternative control methods, suggested in literature. The possible use of these control loops in the case of very weak AC systems is also studied in this Chapter.

Chapter 8 is focused on stability problems with very weak AC systems. A particular oscillatory mode mostly affected by SCR reduction is identified. The crucial part in the design of new controller is the selection of proper feedback signal, the signal that will move the affected mode without significantly affecting the remaining system eigenvalues. To maximize the controller performance, a simple second order \( H_{\infty} \) controller is placed in the feedback loop. The primary controller objective is improvement in the system robustness with respect to the AC system SCR changes.

At the very end of the Thesis, a list of the references closely related to the Thesis work is shown.
REFERENCES:


Introduction

1882 to 1888.


CHAPTER 1

ANALYTICAL MODELLING OF HVDC-HVAC SYSTEMS

1.1 INTRODUCTION

1.1.1 Background

Analytical modelling of HVDC systems has traditionally been regarded as a very difficult task, and its importance has been very often underestimated in the presence of a large number of available electro-magnetic simulation packages.

However, the approach based on the analytical modelling, for reaching generalised conclusions, or performing in-depth analysis, can not be matched by the trial and error type of studies carried out using simulation software.

The main sources of difficulties in modelling of HVDC systems can be summarised as:

- Frequency conversion through AC-DC converters.
- Discontinuous and non-linear nature of signal transfer through converters.
- Complexity of interaction equations between AC and DC variables.
- Modelling of Phase Locked Loops (PLL).
- Dynamic modelling of higher order AC systems.

The analytical model, sought in this thesis, is intended for analysis of practically observed issues associated with the operation of HVDC systems. Of special importance are:

- The composite resonance phenomenon.
- Problems related to weak AC systems connected to a DC system.
- Second harmonic oscillations on DC side of HVDC link (100Hz on DC side).

The characteristic frequency for the first dynamic instability falls into a wide frequency range, but of special importance is the case of composite resonance close to DC side first harmonic (50Hz). Problems related to weak AC systems connected to a DC system, are generally manifested in the lower frequency domain \( f<10\text{Hz} \) as seen on DC side. The last phenomenon occurs exactly at frequency of 100Hz as seen on the DC side. The base frequency is 50Hz.

References [1] and [2] (and a series of related references), offer the most rigorous and accurate HVDC modelling theory for the era of its publishing (early ‘70s). The importance of accurate discrete converter modelling, was highly emphasised in these references. However these were at the expense of neglecting a number of very important features in HVDC system operation, particularly related to the HVDC-HVAC interactions.

Reference [3], presents the most comprehensive HVDC model reported to-date. It presents a systematic approach in model building based on subsystem concepts, and using the state space representation. The use of \(dq0\) transformation for AC-DC model coupling, although not previously unknown in HVDC modelling [4], is also systematically and effectively used in the system analysis. This model however, suffer several shortcomings:
The model is of a linear discrete type, which is a great handicap when classical control theory, so well developed for linear continuous systems, needs to be employed. The frequency range for the model use is not specified. For lower frequency domain, discretisation is not necessary, as it is shown later in this Chapter.

The model does not include dynamics of the Phase Locked Loops. The dynamic behaviour, and the controller settings of PLL on both ends of HVDC link, can have significant impact on the system stability.

A more convenient and a general method for representation of HVDC-HVAC interactions would enable effective analysis of interactions between the subsystems.

It will be shown in this Thesis, that a linear continuous model suffices for the investigation of most of the observed operating difficulties, in general in the frequency range less than 100Hz on DC side. Rather than discretising the model, the modelling approach in this Thesis highly stresses the importance of AC-DC interactions, and accurate representation of all dynamic elements in the system.

The significance of the HVDC model derived in Reference [5], is the high level of response matching with the PSCAD/EMTDC simulation. However, since the model is developed using identification methods, the usefulness of the model in the system analysis and controller design is very limited. The states in the model are entirely artificial; they do not represent actual physical variables. The very fact that identification methods are employed in this reference for the system modelling, clearly emphasise the complexity of the system itself.

In contrast to the above model, the model presented in this Thesis uses the states most of which can be readily and practically obtained. Majority of the model parameters also have physical meaning. In addition, all AC-DC interaction equations are purposely rearranged clearly showing the physical relationship between the subsystem interconnection variables. The physical meaning of the model variables and parameters is dependent on the adequate representation of CIGRE benchmark model [6], which is used as the test system. The parts of the original CIGRE model that do not accurately represent the actual system, i.e. parts obtained using identification techniques, will not have physical meaning in the model, as discussed later in the Chapter.

A model with physically meaningful parameters is extremely important for the system analysis and study of various stability problems in the system. Simply, by changing the parameters of interest (simulating different operating conditions) and observing the changes in the system behaviour, the influence of the actual physical parameters can be investigated. The parameters that are varied need not have absolutely accurate physical meaning, so long as they can simulate particular operating condition in the actual system. A good example is the variation in AC system Short Circuit Ratio, which can be accommodated through a simplified harmonic-impedance AC system representation. The main advantage of having model with physically meaningful states/variables comes in the controller design stage. A very simple and robust feedback controllers are possible, without the need for complex observers, when the model states have their physical equivalent. It is very important that the measurable states, and physically close to the controller input, have physical meaning in the model, whereas remote states of very little use in the system analysis need not have accurate physical meaning.

In the study described in reference [7], a linear (continuous) HVDC-HVAC model is used for the analysis of second harmonic instability at Chateauguay HVDC link. However, the modelling methods have not been revealed.
The original idea to develop a general HVDC system model suitable for controller design at all frequencies, with a good accuracy yet very convenient, has been abandoned in this thesis. The system under consideration is excessively complex for such a task. Instead, a purpose-built model approach is used. To reflect particular modelling requirements, three different analytical HVDC models are developed:

1. **Linear, continuous, detailed model.** This model is intended for the analysis of HVDC-HVAC interactions and stability problems arising from these interactions. The model accurately represents dynamic properties of the overall system. This model probes into details of actual system, as much as it is possible by using linear-systems modelling approach. The main disadvantage of this model is its complexity.

2. **Linear, continuous simplified model.** Being of lower order, the model is convenient for basis system analysis and it is used for controller design for elimination of composite resonance phenomenon and for the study of non-linear HVDC effects. The model derivation is relatively straightforward, and in addition it is suitable for the application of basic controller design theories.

3. **Linear discrete system model.** This model is used for controller design in the frequency domain above 100Hz. The model gives accurate representation of the discrete nature of signal transfer through the converter. From the dynamic point of view, the model is relatively simple.

As a study system, the CIGRE HVDC Benchmark model [6], is used for the model development and in the analysis throughout this Thesis. This model is developed to represent a typical HVDC system, however with system configuration that may cause known operating difficulties.

**1.1.2 Object classification**

From the control system point of view, an HVDC system can be classified as:

- A non-linear system. The non-linear properties are manifested in the nature of the signal transfer through AC-DC converters. When a linearised system model is used it is important to clearly restrict the model use to the domain where the assumption of the linear behaviour is valid. In Chapter 3, the scope of validity of assumption of linear system is studied.
- A discrete system. The discrete system nature is a consequence of thyristor firings at discrete time intervals (once in a cycle for each valve). With the firm restriction for the model use in the lower frequency domain, and well below half the sampling frequency, it is possible to use the linear-continuous model for the representation of a discrete system. Chapter 1 studies the use of discrete versus continuous model.
- A multi-input multi-output system. These properties, which arise from the use of two control inputs, are studied in Chapter 6.
- A system with time-varying parameters. Some aspects of the variation of the model parameters, which are a consequence of operating conditions changes, are addressed in Chapter 8.
- A system with multiple time delays. Time delays are a consequence of commutation overlap, as it is discussed in development of discrete system model, later in this Chapter.
**1.2 LINEAR, CONTINUOUS SYSTEM MODEL**

**1.2.1 AC System model**

The AC system model developed here includes (Figure 1.1):

- The AC voltage source, which is assumed to be of constant magnitude, phase and frequency \( (e_{yj} = \text{const}) \) where \( j=1,2 \) for rectifier and inverter respectively. All phase angles are measured with respect to the source voltage of one selected phase.
- The equivalent AC system harmonic impedance characteristics. The adopted second order AC equivalent impedance will suffice for most studies. Inclusion of higher order AC system representation is a straightforward extension of the model. The independent phase system representation of the three phase AC system also enables modeling of the system phase asymmetry.
- AC system harmonic filters.
- Capacitors for reactive power compensation.

Similarly as in the original CIGRE mode, the effect of electro-mechanical dynamics of machines are not represented in this model.

The AC system from the CIGRE HVDC Benchmark model, Figure 1.1, includes all the above system features. For the frequency range of interest, any AC system can be reduced to an equivalent used with CIGRE model, also shown in Appendix A, Figure A.1. In this AC system model, the branch that represents equivalent AC system impedance (branch that connects to the AC source) is very simplified representation of the actual system, obtained using identification techniques. The states and parameters in this model segment, therefore, will not have physical meaning. As shown later in the Thesis, this model portion is used only for variation in the AC system strength. System Short Circuit changes can be sufficiently accurately simulated with the above model. Other model segments, like AC filter models, are sufficiently accurate. AC system model has equivalent structure for rectifier and inverter side.

Appendix A has the same structure as the Section 1.2. In parallel with the model development in this Section, Appendix A gives further explanations and details the model coefficients.

For any phase of the study system, the states are chosen to be the instantaneous values of currents in the inductors and voltages across the capacitors. As an example for phase “a”, the selected states are:

\[
\begin{bmatrix}
    i_{L2} \\
    i_{L1} \\
    v_{acj} \\
    i_{L3} \\
    v_{c3} \\
    v_{c2} \\
    i_{L4} \\
    v_{c4}
\end{bmatrix}_a
\]  \hspace{1cm} (1.1)

The phase “a” system model is written using dynamic electrical equations, and “s” domain representation as:

Dynamic electrical representation:

\[
L_2 \frac{dL2}{dt} = e_{sj} - v_{acj} - i_{L2}(R_3 + R_2) + i_{L1}R_2 \]  \hspace{1cm} (1.2)
Figure 1.1. CIGRE HVDC Benchmark model.
Analytical modelling of HVDC-HVAC systems

\[ L_1 \frac{d}{dt} i_{L1} = i_{L2} R_2 - i_{L1} (R_1 + R_2) \]  
(1.3)

\[ C_1 \frac{d}{dt} v_{ac1} = -i_{L2} + i_{L3} - \frac{1}{R_5} (v_{acj} - v_{c3}) - i_{L4} - \frac{1}{R_6} (v_{acj} - v_{c4}) \]  
(1.4)

\[ L_3 \frac{d}{dt} i_{L3} = v_{acj} - v_{c3} - v_{c2} - i_{L3} R_4 \]  
(1.5)

\[ C_3 \frac{d}{dt} v_{c3} = \frac{1}{R_5} (v_{acj} - v_{c3}) + i_{L3} \]  
(1.6)

\[ C_2 \frac{d}{dt} v_{c2} = i_{L3} \]  
(1.7)

\[ L_4 \frac{d}{dt} i_{L4} = v_{acj} - v_{c4} \]  
(1.8)

\[ C_4 \frac{d}{dt} v_{c4} = i_{L4} + \frac{1}{R_6} (v_{acj} - v_{c4}) \]  
(1.9)

s-domain equivalent:

\[ s x_1 L_2 = -x_1 - x_1 (R_3 + R_2) + x_2 R_2 + u_2 \]  
(1.2a)

\[ s x_2 L_1 = x_1 R_2 - x_2 (R_1 + R_2) \]  
(1.3a)

\[ s x_3 C_1 = -u_1 + x_1 - x_4 - \frac{1}{R_5} (x_3 - x_5) - x_7 - \frac{1}{R_6} (x_3 - x_8) \]  
(1.4a)

\[ s x_4 L_3 = x_3 - x_5 - x_6 - x_4 R_4 \]  
(1.5a)

\[ s x_5 C_3 = \frac{1}{R_5} (x_3 - x_5) + x_4 \]  
(1.6a)

\[ s x_6 C_2 = x_4 \]  
(1.7a)

\[ s x_7 L_4 = x_3 - x_8 \]  
(1.8a)

\[ s x_8 C_4 = x_7 + \frac{1}{R_6} (x_3 - x_8) \]  
(1.9a)

where index \( j = 1, 2 \) (shown for interaction variables) stands for rectifier and inverter respectively.

The above model takes as inputs instantaneous values of converter current and source voltage, whereas the output is AC bus voltage:

\[ u_1 = i_{acj} \quad u_2 = e_j \quad y = x_3 = v_{acj} \]  
(1.10)

The complete three phase model is expressed in the state space form, using Laplace operator, as:

\[ s x'_{ac} = A'_{ac} x'_{ac} + B'_{ac} u_{ac} \]  
(1.11)

\[ y'_{ac} = C'_{ac} x'_{ac} \]  
(1.12)
where:  
\[
\begin{align*}
\chi'_{ac} &= \begin{bmatrix} \chi'_{aca} \\ \chi'_{acb} \\ \chi'_{acc} \end{bmatrix}, \\
\mathbf{u}_{ac} &= \begin{bmatrix} i_{acj} \\ i_{acb} \\ i_{acc} \end{bmatrix}, \\
\mathbf{v}_{ac} &= \begin{bmatrix} v_{acj} \\ v_{acb} \\ v_{acc} \end{bmatrix} \\
\end{align*}
\]

\[ (1.13) \]

\[
\begin{align*}
A'_{ac} &= \begin{bmatrix} A'_{a} & 0 & 0 \\
0 & A'_{b} & 0 \\
0 & 0 & A'_{c} \end{bmatrix}, \\
B'_{ac} &= \begin{bmatrix} B'_{a} \\
B'_{b} \\
B'_{c} \end{bmatrix}, \\
C'_{ac} &= \begin{bmatrix} C_{a} & C_{b} & C_{c} \end{bmatrix}
\end{align*}
\]

\[ (1.14) \]

with the model matrices defined in Appendix A. In a general case \( A'_{a} \neq A'_{b} \neq A'_{c} \) and \( B'_{a} \neq B'_{b} \neq B'_{c} \), whereas for perfectly balanced AC system these matrices are the same. The model is therefore capable of taking into account the network imbalances.

The presented model is derived using instantaneous values for all AC quantities. In order to represent the AC system together with DC system in the same frequency frame, the effect of frequency conversion through AC-DC converter is accommodated using Park’s transformation [8]. All three-phase AC system variables are converted to \( dq \) variables through:

\[ \chi_{dq0} = P \chi'_{ac} \]

\[ (1.15) \]

where \( P \) represents \( abc \) to \( dq0 \) Park’s transformation. The \( abc-dq0 \) transformation procedure is given in[8], where the form convenient for the analytical modelling in this Thesis is shown in Appendix A.

The developed system model is linearised around the nominal operating point, and all states represented as variations of \( dq \) components of corresponding variables. The variables pertinent to the zero sequence component in (1.13) and (1.14) are neglected, since it is known that the zero sequence components do not produce any DC voltage. Also, the source voltage \( e_{sj} \) in (1.2), becomes nullified in \( dq \) coordinate frame, since it is assumed to be of constant magnitude and phase. The AC system model for rectifier or inverter AC system (\( j=1,2 \) respectively) is expressed in its final form as:

\[ s\xi_{aj} = A_{aj}\xi_{aj} + B_{acjdc}\mathbf{u}_{acjdc} \]

\[ (1.16) \]

\[ y_{acjdc} = C_{acjdc}\xi_{aj} \]

\[ (1.17) \]

where:

\[ \mathbf{u}_{acjdc} = \begin{bmatrix} I_{jd} \\ I_{jq} \end{bmatrix}, \quad \mathbf{v}_{acjdc} = \begin{bmatrix} V_{jd} \\ V_{jq} \end{bmatrix} \]

\[ (1.18) \]

and where the model states are \( dq \) components of the states derived in (1.2a-1.9a). The subscript notation denotes the input-output connection between the subsystems. As an example, \( B_{acjdc} \) is the input matrix of rectifier AC system for input signals coming from the DC system.

The above model takes \( dq \) components of corresponding current and voltage as inputs and outputs, respectively. The model inputs are obtained form the DC system model, whereas the model outputs are used as inputs for the DC system model. As shown later in Section 1.2.7 on DC sys
tem modelling, these input and output variables need to be transferred to its magnitude-angle components.

No external disturbances for the AC system are considered in (1.16). The commutating AC voltage is taken as the system outside disturbance, however it is included in the DC system model presented in Section 1.2.6. The AC-AC interactions are not considered in the above model. It is clear that they can be readily incorporated in (1.16).

The above presented detailed representation of AC systems, enables the analysis of influence of various AC system parameters including SCR variations, on the system stability.

In the following three Sections, the models for Phase Locked Loop (PLL), HVDC controller, and DC system are developed. These three models are combined to form a general DC system model, as presented in Section 1.2.6, which can be connected with the above AC system model.

**1.2.2 Phase Locked Loop Model**

As a common practice, Phase Locked Loops (PLL) have been neglected in the earlier HVDC modelling approaches. It has been erroneously assumed that its dynamics are too slow to be included into the system model. It will be shown in Chapter 6 that PLL dynamics have significant influence on the behaviour of the overall system. Also, PLL model must be included into the HVDC model since all PLL schemes have their own controller with two adjustable parameters/gains. The advantage of having two additional controller gains can be used to maximise the system stability, and possibly to counteract the stability problems by simply adjusting the available controllers.

The main role of PLL in HVDC systems is to give a reference signal for converter firing controller which is synchronised with the commutation voltage. If the PLL controller is tuned for fast tracking of the AC system dynamics, than the reference signal will be closely following the changes in the AC system voltage angle. In this case, the actual firing angle will not be deviating from the ordered firing angle. In the opposite case, with low PLL controller gains, the reference signal will have only slow dynamic changes with very loose tracking of the AC system dynamics. Normally, as applied to a typical HVDC system [12], PLL gains are tuned to low values.

Figure 1.2 shows the influence of the AC system dynamics on the actual firing angle. When the system gets perturbed, as shown by the dotted line in Figure 1.2 a), the actual position of the voltage crossings will subsequently change. If the firing angle order, sent from the controller is $\alpha$, than the actual firing angle seen by the converter $\phi$, becomes different, because of the change in the reference point. This result would correspond to the case of very low PLL gains, which is the case in most practical systems.

Figure 1.2 b) uses the small-signal block-diagram representation to depict the role of PLL in deriving the actual converter firing angle. When PLL is set for fast tracking of the system dynamic changes, the PLL output $\theta$ will closely follow the AC voltage angle changes $\phi$, and the actual firing angle will closely follow the firing angle ordered from the controller.

Phase locked loops used in HVDC converters, have traditionally evolved over several stages [9], and there is a wide range of PLL types currently in use in HVDC schemes worldwide. The
latest type of PLL (the \textit{D-Q-Z} type) as presented in [9],[10] is considered in this document. This PLL type, shown in Figure 1.3, is also used in [12]. The earlier PLL schemes are somewhat simpler and their description can be found in [10].

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Influence of AC system dynamics on the converter firing angle. a) time domain, b) small signal model.}
\end{figure}

The PLL model consists of the following units:

- Vector transformer model
- PLL controller model
- VCO model

The model for the vector transformer in Figure 1.3, is based on the analytical representation as shown below, where the details on the model derivation are shown in Appendix A:

The $\alpha$ and $\beta$ components of AC voltages for \textit{D-Q-Z} type PLL, are defined as:

\begin{align}
\alpha &= \frac{2}{3}v_a - \frac{1}{3}v_b - \frac{1}{3}v_c \\
\beta &= \frac{1}{\sqrt{3}}(v_b - v_c)
\end{align}

The two output, feedback signals from VCO are:

\begin{align}
\alpha &= V_m \sin \left(\omega t - \frac{\pi}{3} + \theta \right) \\
\beta &= V_m \cos \left(\omega t - \frac{\pi}{3} + \theta \right)
\end{align}

Assuming that the linearised equations for vector transformation (1.19),(1.20) are multiplied by the inherent PLL feedback loops (Figure 1.3) the error signal “$e$” can be derived.

\begin{equation}
e = v_\alpha \alpha + v_\beta \beta
\end{equation}
Analytical modelling of HVDC-HVAC systems

The error signal is a function of AC voltage angle $\phi_j$, and reference angle $\theta_j$ (PLL output). This error signal is used as a PLL controller input.

The PLL controller in Figure 1.3 is of PI type:

$$\Delta v_e = k_{cPLL}(k_{pPLL} + \frac{k_{dPLL}}{s})\Delta e$$  \hspace{1cm} (1.24)

The Voltage Controlled Oscillator in Figure 1.3 is represented by the following dynamic equation:

$$\theta = \frac{1}{s}(v_c + \omega_0)$$ \hspace{1cm} (1.25)

Considering the dynamics of Vector transformer, PLL controller VCO, the PLL model is derived in state space domain as (details on the model derivation are given in Appendix A):

$$sx_{PLL} = -k_{ipPLL}x_{2PLL} + k_{ipPLL}u_{PLLj}$$ \hspace{1cm} (1.26)

$$sx_{2PLL} = -k_{ipPLL}k_{pPLL}x_{2PLL} + k_{ipPLL}k_{pPLL}x_{1PLL} + k_{ipPLL}k_{pPLL}u_{PLLj}$$ \hspace{1cm} (1.27)

$$y_{PLLj} = x_{2PLL}$$ \hspace{1cm} (1.28)

where:

$$x_{PLL} = \begin{bmatrix} k_{ipPLL} \Delta \phi_j - \Delta \theta_j \\ \Delta \theta_j \end{bmatrix}$$ \hspace{1cm} (1.29)

with inputs and outputs to the PLL model:

$$u_{PLLj} = \Delta \phi_j, \quad y_{PLLj} = \Delta \theta_j$$ \hspace{1cm} (1.30)

where the angle index “$j=1,2$” denotes rectifier and inverter respectively.

![Figure 1.3. D-Q-Z type phase locked loop.](image)
Analytical modelling of HVDC-HVAC systems

The input to PLL model (AC system voltage phase angle) is obtained from the AC system model, whereas the output (AC voltage phase angle reference) is used as input to the DC system model.

### 1.2.3 HVDC controller model

The HVDC current controller scheme used in this Thesis is shown in Figure 1.4, and follows the HVDC control principles described in [8]. Figure 1.4 a) shows the current controller which would normally be employed at rectifier side. If the converter is inverter, three operating modes are considered, as presented in [8] and as shown in the Figure: Figure 1.4 a) shows constant current controller with two limiters that use the signals derived from constant beta \( \alpha_{\text{nom}} \) controller and constant extinction angle \( \gamma \). Figures 1.4 b) and 1.4 c) show constant beta \( \alpha_{\text{nom}} \) and constant gamma \( \alpha_{\text{max}} \) mode which are simplified representation of the actual controls where a number of non-linear elements are neglected. These operating modes assume that higher level controllers are neglected.

If the converter is rectifier, the current margin is \( I_m = 0 \), and the difference between current order \( I_{\text{ord}} \) and measured direct current \( I_d \) will be used to derive the firing angle signal as shown in Figure 1.4 a). The maximum limits \( \alpha_{\text{nom}} \) and \( \alpha_{\text{max}} \) are normally much higher than the firing angle calculated from the PI current controller, in the case of rectifier controller. Therefore only the control scheme shown in Figure 1.4 a) is normally active in the rectifier converter.

If the converter is operating as rectifier the firing angle order (controller output) can be expressed as (Figure 1.4a):

\[
\alpha = (k_p + k_i / s)(I_{\text{ord}} - I_d)
\]  

If the controller hits the maximum or minimum limit, the firing angle becomes:

\[
\alpha = \alpha_{\text{nom}} = \text{const} \quad \text{or} \quad \alpha = \alpha_{\text{min}} = \text{const}
\]  

Minimum limit is necessary to ensure sufficient forward biasing voltage on the valve electronics for a successful turn-on.

In the case of inverter controller, current margin will be imposed on the current order, and the PI controller will derive the firing angle larger than the two maximum limits, shown in Figure 1.4 a). During normal operation, \( \alpha_{\text{nom}} \) will be active, whereas during disturbances \( \alpha_{\text{max}} \) will require still smaller values for the firing angle.

If the converter is inverter there are three possible operating modes:

- **constant firing angle mode.** Since at inverter side ignition advance angle \( \beta \) is often used (\( \beta = 180 - \alpha \)) this mode is also referred as constant beta mode [8]. Firing angle is calculated as (Figure 1.4 b)).
Analytical modelling of HVDC-HVAC systems

\[ \alpha_{\text{nom}} = 180 - \cos^{-1} \left[ \cos \gamma_{\text{nom}} - \frac{6 X_{ci} I_{\text{ord}}}{E_{ac0} \pi} \right] = \text{const.} \]  

(1.33)

which is derived from the basic AC-DC interaction equations, shown in Appendix B. For a given current order and nominal AC voltage \( E_{ac0} \), this firing angle will be constant. Constant firing angle mode implies no control action at inverter side. Having constant firing angle, as it is shown in Appendix A, the perturbations in inverter direct voltage become:

\[ \Delta V_{di} = R_{ci} \Delta I_{di} \]  

(1.34)

The above equation gives the “positive slope” on the static HVDC operating diagram [8].

Some of the control schemes have proportional part of the current controller active in this control mode [11]. This would correspond to the case when integrator maximum limit is \( \alpha_{\text{max}} = \alpha_{\text{nom}} \), whereas the existing \( \alpha_{\text{nom}} \) limit, after the PI controller, is disabled (Figure 1.4 a)). The firing angle and inverter direct voltage for this control mode becomes:

\[ \alpha_{\text{nom}} = 180 - \cos^{-1} \left[ \cos \gamma_{\text{nom}} - \frac{6 X_{ci} I_{\text{ord}}}{E_{ac0} \pi} \right] + k_p \left( I_{\text{ord}} - I_d \right) \]  

(1.35)

\[ \Delta V_{di} = (R_{ci} + k_p K_{ac2}) \Delta I_{di} \]  

(1.36)

where \( K_{ac2} \) stands for the linearised converter gain, derived in Appendix A.

Throughout this thesis, constant firing angle operating mode will be used predominantly as inverter control strategy. Constant firing angle mode makes a common background for the comparison against different inverter control strategies, which use an additional feedback signal.

- **constant extinction angle mode** (Figure 1.4 c)). If a higher control levels are neglected, most HVDC control systems use constant extinction angle operating mode at inverter side [8],[13],[14],[15],[16]. This mode is also used with systems operating in beta constant (or other additional feedback loops), in order to keep safe commutation margin during disturbances [11]. In this mode, gamma is controlled to a reference value, which is selected as a minimum practically possible value in order to reduce reactive power, harmonics and equipment size [13]. This control mode is implemented in one of the following two control algorithms [13],[14],[15]:

1. Controller with direct compensation of disturbance [8], shown in (Figure 1.4 c)). Inverter firing angle is derived as:

\[ \alpha_{\text{max}} = 180 - \cos^{-1} \left[ \cos \gamma_{\text{min}} - \frac{6 X_{ci} I_d}{E_{ac0} \pi} \right] \]  

(1.37)

The above equation is the same as equation (1.33), except that in this case the outside “disturbances” \( (E_{ac0}, I_d) \), are measured, and the value of firing angle is corrected in order to
keep the safe commutation margin. The value $u_c$ in the Figure, is the calculated compensation for the outside disturbances, whereas $\cos\gamma_{\min}$ uses the calculated minimum gamma which is also a function of converter AC voltage.
\[ \alpha_{\text{nom}} = 180 - \cos^{-1} \left( \frac{\cos \gamma_{\text{nom}} - \frac{6X_{d}I_{\text{nom}}}{E_{\text{volt}}}}{\frac{3}{2}} \right) \]

\[ \beta_{\text{nom}} = 180 - \alpha_{\text{nom}} \]
2. Controller with gamma feedback [13]. This control method resembles the one used in [12]. The non-linear element that takes only the minimum gamma values over the last 6 firing instants as presented in [13] and used in [12] is neglected in this Thesis. For the detailed verification of the results obtained using this model, the above non linear element should be included. Inverter firing angle is in this case derived as:

$$\alpha_{\text{max}} = (k_{pr} + \frac{k_{\gamma}}{s})(\gamma_{\text{min}} - \gamma)$$  \hspace{1cm} (1.38)

This control algorithm (not shown in Figure 1.4), use the feedback control with PI controller \((k_{pr}, k_{\gamma})\) to keep gamma at the safe level during disturbances.

For both control schemes, as shown in Appendix A, the inverter direct voltage perturbations become:

$$\Delta V_{dl} = -R_{ci}\Delta I_{di}$$  \hspace{1cm} (1.39)

The above equation defines less stable “negative slope” on static HVDC diagram [8].

- **constant current mode.** This mode is active only during disturbances which can reduce direct current below \(I_{ord} - I_m\). Usually this happens in the case of severe AC voltage depressions at rectifier side. The inverter terminal in this mode takes over current control and the firing angle is derived as:

$$\alpha = (k_p + k_i / s)(I_{ord} - I_m - I_d)$$  \hspace{1cm} (1.40)

In some schemes, inverter is made operating in other “unconventional” modes. Constant voltage operating mode is one of the examples. Firing angle, in this case, is made proportional to the variations of inverter direct voltage. These alternative control modes at inverter side are studied in Chapter 7.

The above HVDC controller representation is very simplified, tailored to be used with a small signal linear continuous model. The conclusions derived in the later analysis, therefore may not apply to the actual non-linear controllers.

### 1.2.4 DC system (converter stations and DC line) model

This section describes the elementary DC system model. It does not consider any aspects of AC-DC coupling. This model is the basis for development of general DC system model.

The DC system model considered here is shown in Figure 1.5 (also described in Appendix A), and it comprises:

- DC line represented as a “T” section.
- Rectifier and inverter current controllers. HVDC controller model is developed in previous section.
- Smoothing reactors; lumped with line inductance.
- Impedance of converter transformers represented as an average reactance over one firing period, as described in Appendix A, in a manner similar to that described in [1].

23
Analytical modelling of HVDC-HVAC systems

Figure 1.5 DC system representation.

The electrical dynamic equations for the DC system model are shown below:

\[ L_r \frac{dI_r}{dt} = -R_r I_r + V_{dr} - V_{cs} \]  
\[ C_s \frac{dV_{cs}}{dt} = I_r - I_i \]  
\[ L_i \frac{dI_i}{dt} = R_i I_i + V_{cs} - V_{di} \]

where the subscript notation “r” for rectifier and “i” for inverter is used.

In the above equations (1.41) and (1.43) converter direct voltages are a non-linear function of direct voltage, AC voltage and firing angle. These original non-linear equations and a set of other basic AC-DC converter equations often used in this Thesis, are shown in Appendix B. The expressions for direct voltages in (1.41) and (1.43) are linearised as shown in appendix A, and expressed as:

\[ V_{dr} = K_{ac1} \phi_1 + K_{vac1} E_{ac1} + R_{c1} I_r \]  
\[ V_{di} = K_{ac2} \phi_2 + K_{vac2} E_{ac2} + R_{c2} I_i \]

Because of the assumptions normally used with linearisation, the following restrictions for their use will apply:

- The model is accurate only for small perturbations around the nominal operating point,
- The model is valid only for particular nominal operating point.

Appendix A gives all the linearised coefficients. The above equations (1.41-1.43) are represented in perturbations and together with (1.44-1.45) transformed to “s” domain.

The above DC system model, which includes HVDC current controller dynamics, is represented in “s” domain as:

\[ sL_r x_1 = -R_r x_1 - x_4 + K_{ac1} \phi_1 + K_{vac1} E_{ac1} \]  
\[ s x_2 = k_{\mu} I_{rf} - k_{\mu} I_{ord} \]  

24
Analytical modelling of HVDC-HVAC systems

\[ sL_{r1} x_3 = -R_{r1} x_3 + x_4 + K_{ac2} \phi_2 - K_{ac2} E_{ac2} \]  
\[ sC_s x_4 = x_1 - x_4 \] (1.48) (1.49)

where the model states are:

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
\end{bmatrix} = \begin{bmatrix}
  I_r \\
  k_{ir} / s(I_{ir} - I_{ord}) \\
  I_i \\
  V_{ce} \\
\end{bmatrix}
\] (1.50)

and where the model parameters are given in Appendix A. The states in the above model are deviations from the nominal value of pertinent variable. Note that \( R_{r1}, R_{i1} \) include the corresponding contribution from commutating overlap equivalent resistance, and \( L_{r1}, L_{i1} \) similarly include the contribution from averaged converter transformer reactance [1], as shown in Appendix A.

The state \( x_2 \) in equation (1.47) and (1.50) represents the output from the integral part of the rectifier PI current controller, as modelled in the previous section (eqn. 1.31). The inverter is assumed to be in constant beta mode.

The above model takes as inputs:

- \( \phi_1, \phi_2 \) - actual firing angle at rectifier, inverter converter,
- \( I_{ir} \) - rectifier direct current feedback signal (after the feedback filter),
- \( I_{ord} \) - current order as an external input,
- \( E_{ac1}, E_{ac2} \) - rectifier, inverter AC commutating voltages (\( E_{ac1} = a_{aci} V_{ac1} \) where \( a_{aci} \) stands for transformer ratio).

The model inputs are obtained from the AC system model and from the interaction equations, presented later in this Chapter.

The model outputs are \( I_r \) (rectifier direct current) and \( I_i \) (inverter direct current). These inputs and outputs are not suitable for direct coupling with the AC system models. For this reason, the developed DC system model is later expanded to incorporate various AC-DC interaction equations.

### 1.2.5 Interaction equations

This section addresses salient aspects of coupling between the subsystem models. The subsystem models developed earlier can not be directly connected because of different input-output variables. The modelling aim is to build an overall system model which consists of conveniently interconnected subsystems. The number of interaction variables should be reduced as much as possible. Similarly, interaction variables should be consistent throughout the model, and with a physical meaning.
The static, converter AC-DC interaction equations, which are detailed in Appendix B and [8], are shown below:

\[ I_{acj} = I_{dc} \frac{2\sqrt{6}}{\pi} a_{acj} \sqrt{2} \]  

(1.51)

\[ \cos \Phi_{acj} = \cos \phi_f - \frac{R_{acj} I_{dc} \pi}{6\sqrt{3} a_{acj} V_{acj}} \]  

(1.52)

The equation (1.51) will be referred here as a “dc-ac current equation”. The equation (1.52) gives power factor angle as a function of AC and DC system variables. This new variable, power factor angle (\( \Phi_{acj} \)), can be eliminated from interaction equations by knowing AC voltage angle and AC current angle.

Because of the complexity of interaction equations, particularly eqn. (1.52), the interaction variables (AC current and voltage angles in this case) can not be represented as a function of a variables coming from a single-subsystem. It is seen that the angle \( \Phi_{acj} \) in (1.52), depends upon DC side variables \( I_{dc}, \phi_f \) and AC side variable \( V_{acj} \). Direct use of this equation on the right side of differential equations (either AC or DC) would imply multiple loops between the subsystems.

To avoid complex loops between the subsystems, two of these equations (at each side) are artificially made dynamic equations with a small time constants. A new set of states is introduced in this way, which can easily be accessed in the system model.

The two following examples, illustrate the method of creating dynamic equations from the corresponding static interaction equations, at rectifier side.

With reference to the DC system model (eqn. 1.46-1.49), and Figure 1.2 b) on PLL modelling, the static equation for the actual rectifier firing angle is expressed as:

\[ \phi_1 = x_2 + k_{pr} I_{sf} - k_{pr} I_{ord} + \varphi_1 - \theta_1 \]  

(1.53)

where, the variables are defined as:

\( \phi_1 \)  

d actual rectifier firing angle.  

\( k_{pr} I_{sf} - k_{pr} I_{ord} \)  

proportional part of the firing angle order,  

\( (k_{pr} / s(I_{sf} - I_{ord}) + k_{pr} (I_{sf} - I_{ord}) = \alpha \)  

firing angle order from rectifier controller)  

\( \varphi_1 \)  

d rectifier AC voltage angle,  

\( \theta_1 \)  

d output angle from PLL,

As presented in Section 1.2.2, the term \((\varphi_1 - \theta_1)\) will approach zero, when the PLL action is sufficiently fast.

The equation (1.53) is transformed to a dynamic equation in the following way:

\[ sT_f \phi_1 = -\phi_1 + x_2 + k_{pr} I_{sf} - k_{pr} I_{ord} + \varphi_1 - \theta_1 \]  

(1.54)
with $T_f = 1/6000\,\text{sec}$. The selected value for time constant will enable equations (1.53) and (1.54) to have equivalent responses in the frequency domain of interest in this Thesis. The corresponding dynamic equation for actual inverter firing angle is derived in a similar manner.

The AC current angle, which is used as the DC system output, can be obtained from the AC voltage angle and power angle as shown in Figure 1.6. The AC current angle is therefore expressed as:

$$\psi_1 = \Phi_{ac1} - \phi_1$$  \hspace{1cm} (1.55)

where the angle between voltage and current vector is obtained by using equation (1.52), which is written here for the rectifier converter:

$$\cos \Phi_{ac1} = \cos \phi_1 - \frac{R \cdot I \cdot \pi}{6\sqrt{3}E_{ac1}}$$  \hspace{1cm} (1.56)

If the linearised coefficients from the above equation are defined as:

$$c_{2ac1} = \frac{\partial \Phi_{ac1}}{\partial \phi_1}$$  \hspace{1cm}  $$c_{3ac} = \frac{\partial \Phi_{ac1}}{\partial I_r}$$  \hspace{1cm}  $$c_{4ac1} = \frac{\partial \Phi_{ac1}}{\partial E_{ac1}}$$  \hspace{1cm} (1.57)

(Appendix A gives the values for the linearised coefficients), the dynamic equation for AC current angle is written:

$$sT_f \psi_1 = -\psi_1 - \phi_1 + c_{2ac1} \phi_1 + c_{3ac1} I_r + c_{4ac1} E_{ac1}$$  \hspace{1cm} (1.58)

The equation for inverter AC current angle is derived in a similar manner.

The equations (1.54) and (1.58) introduce two new states in the system model, which enable convenient access to the actual converter firing angle and interaction variable, AC current angle, in the system model, without much increase in the model complexity and much effect on the system dynamics.

![Figure 1.6. Relationship between phase angles of AC system variables at rectifier terminal.](image)
1.2.6 General DC system model

Interaction equations must be incorporated into either AC or DC system model. DC subsystem is chosen as more convenient, since in a multiterminal case there will still exist only one DC system and several AC systems. For this purpose, the DC system model is generalised and expanded to include the dynamics of PLL and interaction equations.

By using the interaction equations, the PLL model and the DC system model, the generalised DC system model can be obtained as shown by the equations below, and as schematically represented in Figure 1.7.

**DC system model equations:**

\[
\begin{align*}
sl_1x_1 &= -R_1x_1 - x_4 + K_{ac1}x_{10} + K_{ac1}E_{ac1} + K_{ac1}E_{ac1}^d \\
sl_2x_2 &= k_p x_4 - k_p I_{ord} \\
sl_3x_3 &= -R_3x_3 + x_4 + K_{ac2}x_{11} - K_{ac2}E_{ac2} + K_{ac2}E_{ac2}^d \\
scx_4 &= x_1 - x_4
\end{align*}
\]

**Rectifier PLL equations:**

\[
\begin{align*}
ssx_5 &= -k_{ppll}x_6 + k_{ppll}\phi_{ac1} \\
ssx_6 &= -k_{cpll}k_{ppll}x_6 + k_{cpll}x_2 + k_{cpll}k_{ppll}\phi_{ac1}
\end{align*}
\]

**Inverter PLL equations:**

\[
\begin{align*}
ssx_7 &= -k_{ppll}x_8 + k_{ppll}\phi_{ac2} \\
ssx_8 &= -k_{cpll}k_{ppll}x_8 + k_{cpll}x_7 + k_{cpll}k_{ppll}\phi_{ac2}
\end{align*}
\]

**Current transducer equation:**

\[ssT_acx_9 = -x_9 + x_1\]

**Actual firing angle (rectifier and inverter) dynamic equations:**

\[
\begin{align*}
ssT_f x_{10} &= -x_{10} + x_2 + k_p x_9 - k_p I_{ord} + \phi_{ac1} - x_6 \\
ssT_f x_{11} &= -x_{11} - x_8 + \phi_{ac2} + \beta
\end{align*}
\]

**AC current angle (rectifier and inverter) dynamic equations:**

\[
\begin{align*}
ssT_f x_{12} &= -x_{12} + \phi_{ac1} - c_{2ac1}x_{10} - c_{3ac1}x_1 + c_{4ac1}E_{ac1} + c_{4ac1}E_{ac1}^d \\
ssT_f x_{13} &= -x_{13} + \phi_{ac2} - c_{2ac2}x_{11} - c_{3ac2}x_3 + c_{4ac2}E_{ac2} + c_{4ac2}E_{ac2}^d
\end{align*}
\]

where the model states are defined as:
Analytical modelling of HVDC-HVAC systems

The generalised DC system model is structured in the following way:
• Equations (1.59)-(1.62) represent DC system model, equations (1.44-1.47) developed in Section 1.2.4.
• Equations (1.63)-(1.64) represent dynamics of rectifier side PLL, developed in Section 1.2.2.
• Equations (1.65)-(1.66) represent dynamics of inverter side PLL.
• Equation (1.67) is the dynamic equation of DC current transducer, modelled as a first order filter.
• Equations (1.68)-(1.71) are artificially introduced equations and they do not represent actual system dynamics. Equation (1.71) is obtained from the earlier derived equation (1.54). Equation (1.70) is obtained from equation (1.57). The corresponding equations for inverter side (1.69) and (1.71), are derived in a similar manner.

The above general DC system model can be expressed in the matrix form as:

\[ s\mathbf{x} = A_{dc} \mathbf{x} + B'_{dcac1} u_{ac1} + B'_{dcac2} u_{ac2} + B'_{dcinp} u_{inp} \]  (1.73)
\[ y'_{dcac1} = C'_{dcac1} \mathbf{x} \quad y'_{dcac2} = C'_{dcac2} \mathbf{x} \]  (1.74)

where the model inputs and outputs are defined as:

\[ u'_{ac1} = \begin{bmatrix} E_{ac1} \\ \phi_1 \end{bmatrix}, \quad u'_{ac2} = \begin{bmatrix} E_{ac2} \\ \phi_2 \end{bmatrix}, \quad u'_{inp} = \begin{bmatrix} I_{ord} \\ E_{ac1}^d \\ E_{ac2}^d \\ \beta \end{bmatrix} \]  (1.75)
\[ y'_{dcac1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad y'_{dcac2} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad y'_{dcac2} = \begin{bmatrix} I_r \\ I_i \\ \psi_1 \\ \psi_2 \end{bmatrix} \]  (1.76)
The model input and output matrices are shown in Appendix A. The external inputs to the above model are: current order $I_{ord}$, rectifier and inverter AC voltage disturbance $E_{ac1}^{d}, E_{ac2}^{d}$, and also $\beta$. The firing angle order at inverter side $\beta$, is used in Chapter 7 and 8 in order to study different inverter control strategies.

**Nomenclature:**

- $I_r, I_i$: rectifier, inverter DC current,
- $I_1, I_2$: rectifier, inverter AC current magnitude,
- $\psi_1, \psi_2$: rectifier, inverter AC current phase angle,
- $E_{ac1}, E_{ac2}$: rectifier, inverter AC voltage magnitude (secondary side),
- $\varphi_1, \varphi_2$: rectifier, inverter AC voltage phase angle,
- $\alpha, \beta$: rectifier, inverter firing angle order,
- $\theta_1, \theta_2$: rectifier, inverter PLL output angle,
- $\phi_1, \phi_2$: rectifier, inverter actual firing angle,
- $E_{ac1}^{d}, E_{ac2}^{d}$: rectifier, inverter AC voltage disturbance

*Figure 1.7 Structure of the general DC system model.*
Analytical modelling of HVDC-HVAC systems

The converter AC bus voltages are taken as the place to apply system disturbances \( E_{ac1}^d, E_{ac2}^d \), as shown in Figure 1.7. Because of the transformer tap changer operation and various switching actions (circuit breakers, disconnectors), the AC bus voltage is the most likely to experience direct outside disturbances.

1.2.7 Polar coordinate to \( dq \) coordinate transformation

This Section presents the final transformations of input and output DC model matrices, which will enable DC model connection with the AC system models.

The equations for transformation from magnitude-angle representation of AC voltages and currents to \( dq \) coordinate frame, for a general \( j \)-th AC system, are shown below:

\[
\begin{align*}
V_{jd} &= \sqrt{3}/2V_{acj} \sin \varphi_j \quad V_{jq} = \sqrt{3}/2V_{acj} \cos \varphi_j \\
I_{jd} &= \sqrt{3}/2I_{acj} \sin \psi_j \quad I_{jq} = \sqrt{3}/2I_{acj} \cos \psi_j
\end{align*}
\] (1.77)

For rectifier side, in matrix notation, for small perturbations, these equations become:

\[
\begin{bmatrix}
\Delta I_{1d} \\
\Delta I_{1q}
\end{bmatrix} = R_{dcac} \begin{bmatrix}
\Delta I_{ac1} \\
\Delta \psi_1
\end{bmatrix}
\] (1.79)

where:

\[
R_{dcac} = \sqrt{\frac{3}{2}} \begin{bmatrix}
\frac{\partial I_{1d}}{\partial I_{ac1}} & \frac{\partial I_{1d}}{\partial \psi_1} \\
\frac{\partial I_{1q}}{\partial I_{ac1}} & \frac{\partial I_{1q}}{\partial \psi_1}
\end{bmatrix}
\] (1.80)

and

\[
\begin{bmatrix}
\Delta V_{1d} \\
\Delta V_{1q}
\end{bmatrix} = P_{dcac} \begin{bmatrix}
\Delta V_{ac1} \\
\Delta \varphi_1
\end{bmatrix}
\] (1.81)

where:

\[
P_{dcac} = \sqrt{\frac{3}{2}} \begin{bmatrix}
\frac{\partial V_{1d}}{\partial V_{ac1}} & \frac{\partial V_{1d}}{\partial \varphi_1} \\
\frac{\partial V_{1q}}{\partial V_{ac1}} & \frac{\partial V_{1q}}{\partial \varphi_1}
\end{bmatrix}
\] (1.82)

The coefficients in the above matrices are shown in Appendix A.

The final DC system model for a two terminal system is obtained from (1.73-1.74) as:

\[
\begin{bmatrix}
x \\
y_{dcac1}
\end{bmatrix} = A_{dcac} \begin{bmatrix}
x \\
y_{dcac1}
\end{bmatrix} + B_{dcac1} u_{dcac1} + B_{dcac2} u_{dcac2} + B_{dcinp} u_{inp}
\] (1.83)

\[
y_{dcac2} = C_{dcac1} \begin{bmatrix}
x \\
y_{dcac1}
\end{bmatrix}
\] (1.84)

where the model inputs are:
Analytical modelling of HVDC-HVAC systems

\[ u_{\text{dcac}1} = \begin{bmatrix} V_{1d} \\ V_{1q} \end{bmatrix}, \quad u_{\text{dcac}2} = \begin{bmatrix} V_{2d} \\ V_{2q} \end{bmatrix} \]  

(1.85)

and the model outputs:

\[ y_{\text{dcac}1} = \begin{bmatrix} I_{1d} \\ I_{1q} \end{bmatrix}, \quad y_{\text{dcac}2} = \begin{bmatrix} I_{2d} \\ I_{2q} \end{bmatrix} \]  

(1.86)

and where the final DC system matrices for input-output (rectifier side) coupling with AC subsystem are obtained as:

\[ C_{\text{dcac}1} = R_{\text{dcac}1} S_{1}^{-1} C_{\text{dcac}1}^{\prime}, \quad B_{\text{dcac}1} = B_{\text{dcac}1}^{\prime} S_{1} P_{\text{dcac}1}^{-1} \]  

(1.87)

where:

\[ S_{1} = \begin{bmatrix} a_{\text{ac1}} & 0 \\ 0 & 1 \end{bmatrix}. \]  

(1.88)

with rectifier commutation transformer ratio denoted as \( a_{\text{ac1}} \). In a similar way, the above correction matrices can be included into the AC system model, to correct the AC system input output matrices. In this case we have:

\[ C_{\text{ac1dc}} = C_{\text{ac1dc}}^{\prime} S_{1} P_{\text{dcac1}}^{-1}, \quad B_{\text{ac1dc}} = B_{\text{ac1dc}}^{\prime} R_{\text{dcac1}} S_{1}^{-1} \]  

(1.89)

where, \( B_{\text{ac1dc}}^{\prime} \) and \( C_{\text{ac1dc}}^{\prime} \) stand for the earlier derived input and output matrices of AC system. In this case the interaction variables are magnitude and angle of AC voltage and current.

Figure 1.8 shows the schematic diagram of the AC-DC coupling through the use of correction matrices, where the correction matrices are included into the DC system model.

The DC model building as shown in Sections 1.2.4-1.2.7 considers a two terminal case. For a multi-terminal DC system, the original DC equations in Section 1.2.4 need to be changed. Further multi-terminal model building is straightforward expansion of the above development procedure.

1.2.8 HVDC-HVAC system model

The linearised, continuous, state-space model of the overall system can be obtained by combining those models for the subsystems: rectifier and inverter AC systems, and final DC system model, derived above.

The \( j \)-th AC system model is obtained in the form as in (1.16-1.17), whereas the final DC system model is shown in (1.83-1.84).
The final HVDC-HVAC system model is derived in matrix form as:

\[
\begin{align*}
    s\mathbf{x} &= A_s \mathbf{x} + B_s \mathbf{u}_{\text{inp}} \\
    \mathbf{y} &= C_s \mathbf{x}
\end{align*}
\]

(1.90) (1.91)

where matrix $A_s$ consists of the system matrices of subsystems, and interaction matrices between the subsystems. For a two terminal system, the system matrix is:

**Nomenclature:**

- $I_{jd}, I_{jq}$: $d,q$ components of AC current,
- $V_{jd}, V_{jq}$: $d,q$ components of Ac voltage,
- $P_{dcac}, R_{dcac}$: $d,q$ frame voltage, current input-output correction matrices
- $S_1, S_2$: transformer ratio correction matrices

*Figure 1.8 Input-output coupling between AC-DC systems.*
Analytical modelling of HVDC-HVAC systems

\[
A_s = \begin{bmatrix}
A_{dc} & B_{dc1} & C_{dc1} & B_{dc2} & C_{dc2}
B_{ac1d} & A_{ac1} & 0
B_{ac2d} & 0 & A_{ac2}
\end{bmatrix}
\]

(1.92)

and input, output matrices are:

\[
B_s = \begin{bmatrix}
B_{dcinp}
0
0
\end{bmatrix},
C_s = \begin{bmatrix}
C_{dcout} & 0 & 0
0 & C_{ac1out} & 0
0 & 0 & C_{ac2out}
\end{bmatrix}
\]

(1.93)

For a general multiterminal case with \( f \) AC systems, the model matrices are in the form:

\[
A_s = \begin{bmatrix}
A_{dc} & B_{dc1} & \cdots & B_{dcf} & C_{dcf}
B_{ac1d} & A_{ac1} & \cdots & B_{ac1f} & C_{ac1f}
\cdots & \cdots & \cdots & \cdots & \cdots
B_{acfd} & B_{acf1} & \cdots & B_{acf} & C_{acf}
\end{bmatrix}
\]

(1.94)

and input, output matrices are:

\[
B_s = \begin{bmatrix}
B_{dcinp}
0
0
\end{bmatrix},
C_s = \begin{bmatrix}
C_{dcout} & 0 & 0
0 & C_{ac1out} & 0
0 & 0 & C_{ac2out}
\end{bmatrix}
\]

(1.95)

where: \( A_{dc} \in \mathbb{R}^{13 \times 13}, A_{ac1}, A_{ac2}, \ldots \in \mathbb{R}^{16 \times 16} \), and for a two terminal case, dimensions of the final matrices are: \( A_s \in \mathbb{R}^{45 \times 45}, B_s \in \mathbb{R}^{45 \times 3} \).

The system output matrices, although not explicitly defined in the model development, will depend upon the particular modelling purpose. As an example, if the DC currents are to be monitored the output matrix is:

\[
C_{dcout} = \begin{bmatrix}
1 & 0 & 1 & 0 & \cdots & 0 & \cdots & 0
\end{bmatrix}
\text{and } C_s \in \mathbb{R}^{2 \times 45}
\]

(1.96)

The modular structure of the model allows straightforward extension to a DC multiterminal configuration, where more than two AC systems are connected to the DC system. Since the model is built of subsystems, linked together via a common state space model, the model can be readily expanded to include new subsystem models such as: additional AC systems, AC voltage controlling elements and load dynamics.
As emphasised earlier, the model in this form is very convenient for the system dynamic analysis and controller design. The small signal stability analysis of the HVDC-HVAC system, can be carried out by performing the eigenvalue decomposition analysis of each subsystem, and of the overall system. Important conclusions about the influence of each subsystem on the overall system stability can be readily derived. Since the model uses physically meaningful AC-DC interaction variables, the role of each of these variables in the system stability improvement/degradation can be studied. For HVDC controller design, this model enables a systematic selection of DC system variables and/or AC system variables as suitable feedback signals.

1.2.9 Model verification

The developed model is converted to MATLAB software code and responses compared with non-linear digital simulation (PSCAD/EMTDC program [12],[17]). The test system used is the CIGRE HVDC Benchmark model [6],[12], having low SCR on both AC systems $Rec \ LS CR=2.5$ and $Inv \ LS CR=2.5$. The system data are shown in Appendix D. The controller parameters, including PLL controllers, are taken from [12], and also shown in Appendix D.

Figures 1.9 a) and b) compare the simulation responses and the model responses. Figure 1.9 a) uses current order reference step change (first input in (1.75)) as input, whereas Figure 1.9 b) uses rectifier AC voltage step change (second input in (1.75)) as input variable. For all of the system states, a good matching with the simulator responses, at lower frequencies, is observed. It is important that all model variables, including AC and DC variables, which can be easily physically identified, have satisfactory response fidelity.
Figure 1.9 a). System response following current order step change. Test system is the CIGRE HVDC Benchmark model, Rec. SCR=2.5, Inv. SCR=2.5. Inverter operating angle $\beta = 40\,\text{deg}$. 

![Graphs of Rectifier DC current, Rectifier AC voltage angle, Rectifier AC current angle, Inverter AC current angle over time](image)
Analytical modelling of HVDC-HVAC systems

Figure 1.9 b). System response following rectifier AC voltage disturbance step change. Test system is the CIGRE HVDC Benchmark model, Rec. SCR=2.5, Inv. SCR=2.5. Inverter operating angle $\beta = 40 \text{deg}$. 

Still, as can be seen from the above Figures, some mismatch is apparent for the dominant oscillatory mode. This mismatch is more pronounced in the disturbance response. The main reason for this difference is the neglected discrete nature of the actual system. The use of a non-discretised model will produce incorrect positioning of the system transmission zeros, which will in turn cause incorrect damping of the dominant oscillatory modes. Various neglected or linearised non-linear elements will also deteriorate the model fidelity. It is also seen that these differences are not significant. Further simulation studies with different system parameters and different controller gains also show good response matching. The simulation studies and system analysis presented in Chapter 6 and 7, demonstrate that the derived model is reliable for the system analysis and controller design at the frequencies around, and above the dominant oscillatory mode ($70\text{Hz}$) of the system.
It will also be shown later in Chapter 6, 7 and 8, that the root-locus analysis based on this model gives conclusions consistent with the results from the non-linear simulation and operating experience.

1.3 SIMPLIFIED, LINEAR, CONTINUOUS MODEL

1.3.1 Modelling approach

This model represents a compromise between a detailed HVDC-HVAC system modelling, as described in previous sections, and basic steady state HVDC modelling principles described in HVDC textbooks. To overcome the main shortcomings of the basic HVDC models, this model does not assume infinitely stiff AC voltages. It takes into account feedback influence through AC voltage magnitude and angle by using static AC system equations. In this way, the strength of the AC systems (or Short Circuit Ratio), as a very important indicator of the system behaviour, is represented in the model.

The model is however of much lower order than the detailed model developed in Section 1.2. This model does not require dynamic equations of AC systems and PLL systems. For practical, engineering purposes, the model has all advantages of a low order simple models.

1.3.2 DC system model

Figure 1.10 gives a schematic diagram of the system considered. The inverter is assumed to operate in constant $\alpha_{\text{max}}$ mode. The rectifier current controller is assumed to be proportional and integral (PI) type as presented in Section 1.2.3.

The DC system model is the same as the one developed in Section 1.2.4. The Laplace transformation of the linearised differential equations which describe the dynamics of the DC system are:

$$V_{dr} = L_r s I_r + R_r I_r + V_{cs}$$  \hspace{1cm} (1.97)
$$V_{cs} = L_i s I_i + R_i I_i + V_{di}$$  \hspace{1cm} (1.98)
$$V_{cs} = \frac{1}{C_s s} (I_r - I_i)$$  \hspace{1cm} (1.99)
$$\alpha = (k_{pr1} + k_{pr} / s)(I_{\text{ord}} - I_r)$$  \hspace{1cm} (1.100)

where $R_r$ and $R_i$ represent line and smoothing reactor resistance of the rectifier and inverter sides, as shown in Appendix A for the development of detailed model. $L_r$ and $L_i$ similarly denote the series reactance including line reactance and smoothing reactance. Transformer impedance is represented as an average reactance over one firing period as suggested in [1] and presented in Appendix A, and later added to $L_r$ and $L_i$. The symbol for small perturbations is omitted in these equations, for simplicity reasons. Assuming small perturbations around the nominal operating point, the converter direct voltages can be expressed (Appendix A) as:
Analytical modelling of HVDC-HVAC systems

\[ V_{dr} = K_{ac1} \alpha + K_{eac1} E_{ac1} + K_{eac1} E_{ac1}^d + R_c I_r \]  
\[ V_{di} = K_{ac2} \beta + K_{eac2} E_{ac2} + K_{eac2} E_{ac2}^d + R_c I_i \]  

Similarly to the model developed in Section 1.2, \( E_{ac1} \) and \( E_{ac2} \) represent the change in AC commutation voltages as a consequence of AC systems dynamics, whereas \( E_{ac1}^d \) and \( E_{ac2}^d \) represent the changes in AC voltages because of load changes and other disturbances. The linearised coefficients in the above equations are shown in Appendix A.

In equations (1.101-1.102), the controller firing angle order is used instead of the actual firing angle as used in detailed model in (1.44) and (1.46). Since the PLL dynamics are not considered in this model, and the AC system influence through the AC voltage angle is neglected, it is evident that \( \alpha = \phi_1 \), and \( \beta = \phi_2 \), in this model.

1.3.3 AC system model

This section develops a fundamental frequency AC system representation (static AC system model) which influences the DC system through static AC-DC interaction equations.

As far as the steady state operation of the rectifier is concerned, interaction between AC and DC systems can be described by the equations (1.51-1.52), also derived in Appendix B, which are slightly modified here:

\[ I_{acj} = I_{dc} \frac{2\sqrt{6}}{\pi} a_{acj} \sqrt{2} \]  
\[ \cos \Phi_{acj} = \cos \alpha_j - R_{acj} I_{dc} \pi \sqrt{3} E_{acj} \]  

where \( \Phi_{acj} \) denotes the angle between the AC current \( I_j \) and the AC voltage \( E_{acj} \), for rectifier and inverter (\( j=1,2 \) respectively). The equation (1.104) uses the controller firing angle instead of actual firing angle as used in (1.53). The rectifier AC system, as shown in Figure 1.10, can be represented by its equivalent circuit (and its phasor diagram) shown in Figure 1.11, where the following variables are used:

![Figure 1.10. Schematic diagram of the simplified system model.](image-url)
Analytical modelling of HVDC-HVAC systems

\[ E_{1ld} = |I_1||z_{1th}|, \quad E_{1ld} \] - voltage drop across the AC system equivalent \hspace{1cm} (1.105)

\[ z_{1th} = |Z_{1th}|\angle \xi = \frac{z_r z_{df}}{z_r + z_{df}} \] - Thevenin impedance \hspace{1cm} (1.106)

\[ E_{1th} = E_{rs} \frac{z_{df}}{z_{df} + z_r} \] and \[ E_{rs} = \text{const} \] , \[ E_{1th} \] - Thevenin voltage \hspace{1cm} (1.107)

\[ \tau = \xi - \Phi \] \hspace{0.5cm} with \hspace{0.5cm} \xi = \text{const} , \hspace{0.5cm} \tau \] - \[ E_{1ld} \] voltage phase angle \hspace{1cm} (1.108)

**Figure 1.11. AC system representation.**

The inverter AC system is modelled in a similar manner. Therefore, this model considers equivalent AC system representation at fundamental frequency, ie. Static AC system model. The unknown variable in Figure 1.11, which is used as input in DC system model, is the voltage \( E_{ac1} \).

The magnitude of this voltage can be obtained from the following equation, (Figure 1.11):

\[ |E_{1th}|^2 = |E_{ac1}|^2 + 2|E_{ac1}||E_{1ld}|\cos \tau + |E_{1ld}|^2 \] \hspace{1cm} (1.109)

or:

\[ |E_{1th}|^2 = |E_{ac1}|^2 + 2|E_{ac1}|I_1|z_{1th}|\cos \tau + |I_1|^2|z_{1th}|^2 \] \hspace{1cm} (1.110)

or when linearised:

\[ E_{ac1} = k_{ac1}I_1 + k_{\Phi ac1}\Phi_{ac1} \] \hspace{1cm} (1.111)

where the linearised coefficients are shown in Appendix C.

It can be seen that the change in the AC voltage \( E_{ac1} \) is affected by two parameters: the AC current magnitude \( |I_1| \) and the phase angle \( \Phi_{ac1} \) between AC current and voltage. These two variables can be expressed in terms of the DC side variables using equations (1.103) and (1.104) as it shown in Appendix C. Only the change of magnitude of the AC voltage \( E_{ac1} \) is considered in this model and not the change of the voltage phase angle.

### 1.3.2 HVDC System model

Using the equations (1.97-1.102), and substituting the expression for AC voltages (1.111), the system model is obtained as schematically shown in Figure 1.12.
Finally, the dynamic equations of the system model are written in the form:

\[
L_{rs} I_r = -(R_r + R_{ac1} + R_{ac2}) I_r - V_{cs} + (K_{ac1} + K_{ac2}) \alpha + K_{ac1} E_{ac1}^d
\]

\[
\alpha = \left( k_{pr} + k_p / s \right)(I_{ord} - I_r)
\]

\[
L_{is} I_i = -(R_i + R_{ac1} + R_{ac2}) I_i + V_{cs} - (K_{ac2} + K_{ac1}) \beta - K_{ac2} E_{ac2}^d
\]

\[
C_s V_{cs} = I_r - I_i
\]

Coefficients: \( R_{ac1}, K_{ac1}, R_{ac2}, K_{ac2} \), which are derived in Appendix C, represent the influence of the AC systems on the considered DC system. The change in strength of the AC system will directly affect the dynamic behaviour of the DC system by changing the controller gain (\( K_{ac1} \)) and changing the feedback coefficient (\( R_{ac1} \)) in the system dynamic equations. These coefficients will be zero for ideally strong AC systems (constant AC voltages). The model therefore represents the influence of short circuit ratio changes of the AC systems, however the dynamic properties of AC systems are not represented.

State variables, outside inputs, control inputs and outputs are chosen as:

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix} =
\begin{bmatrix}
    I_r \\
    \frac{1}{s}(I_{ord} - I_r) \\
    I_i \\
    V_{cs}
\end{bmatrix},
\begin{bmatrix}
    w_1 \\
    w_2
\end{bmatrix} =
\begin{bmatrix}
    I_{ord} \\
    E_{ac1}^d
\end{bmatrix},
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix} =
\begin{bmatrix}
    \alpha \\
    \beta
\end{bmatrix},
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} =
\begin{bmatrix}
    I_r \\
    I_i
\end{bmatrix}
\]

The system can now be represented in state-space domain as:

\[
sx = Ax + Bu + B_1 w \quad y = Cx
\]

\[\text{(1.116)}\]
where the model matrices are derived in Appendix C.

The main assumptions and simplifications, relative to the detailed model from Section 1.2, used in this model, can be summarised as:

- **Neglected AC system dynamics.** The model uses the static representation of AC systems. Because of this assumption, the model will not be reliable for the study of dynamic influence of very weak AC systems. The influence of various AC system parameters on the system behaviour is not truly represented. As an example, resonant conditions in AC systems (at lower frequencies) are not reflected in this model.
- **The AC-DC interactions are represented only through the AC voltage and AC current magnitude, whereas their phase angle changes are neglected.** In the case of severe phase angle changes in the system operation, this assumption will lead to erroneous results.
- **Neglected PLL dynamics.** The effect of PLL controller gains on the system dynamics can not be studied.

Despite the above observations, as it is shown in Chapter 4, the model is convenient yet fairly reliable for basic study of HVDC system dynamics such as analysis of AC-DC composite resonance. A new controller is designed in Chapter 4, based on this model, and the controller demonstrates improvement in the system responses.

### 1.4 LINEAR DISCRETE MODEL

#### 1.4.1 Discrete converter modelling

Since the firing of the thyristors occurs at discrete time intervals, a converter can be modelled as a discrete-linear system. The modelling approach adopted here follows the discrete converter modelling principles described in [1] and [2], with some simplifications and generalisations in the development of the overall HVDC system model.

Figure 1.13 shows the waveform of direct voltage for the period of approximately two thyristor firing cycles. The direct voltage curve $v$, which is obtained as the difference between phase voltages ($v_a - v_c, \ldots$), is controlled by changing the firing angle signal $\alpha$. The thyristor firing, beginning of thyristor conduction, happens only at discrete time intervals (shown as $t_1$ and $t_2$ in Figure 1.13). Between these firing intervals the control of direct voltage is not possible. Any change of firing angle between $t_1$ and $t_2$ will not have desired affect on the direct voltage waveform. In the frequency domain, this implies that input frequencies above 600Hz for 12-pulse system (300Hz for 6-pulse system) will be seen at the output as a lower frequency signal bands because of aliasing phenomenon.
For discrete modelling purpose, a converter is represented as a linear element plus an ideal sampler. The value which is sampled is the thyristor firing angle $\alpha$ and direct current $I_d$ [1], [2]. Converter firing angle and direct current have to be discretised, since they affect direct voltage only once per converter firing cycle. Direct current affects direct voltage through the commutation overlap angle. More detailed theory on discrete converter modelling can be found in [1],[2] and a number of related references, whereas the theory behind discrete systems in general, can be found in [18]. Figure 1.14 shows the block-diagram of a converter as a discrete system. The two samplers $S_1$ and $S_2$, have synchronised operation with the same sampling period.

The most simple discrete converter model can be derived if the two sampler and hold elements are used in conjunction with the linear continuous model developed in the previous section. This model would suffer similar shortcomings as the continuous model, since the average value of the converter direct voltage is used [1]. The model derived here is an impulse-discrete model, where the strength of each impulse corresponds to the perturbations of direct voltage around the nominal operating point.

Commutation overlap in the continuous model is also represented as an evenly distributed voltage reduction along the whole $\pi / 3$ period [1]. It should be emphasised that commutation overlap in the actual process happens only during the first $u$ degrees (Figure 1.13), and it does not affect voltage waveform in the remaining period till the beginning of the next cycle-period. Commutation overlap will also introduce a time delay, when direct voltage response to the changes in the firing angle is considered [1],[2].
Figure 1.14. Block-diagram of a converter as a discrete system.

Converter direct voltage can be represented as the ratio between the area under the direct voltage curve, as shown in Figure 1.13, and the sampling period.

\[
V_d = \frac{A}{T_s} \quad (1.118)
\]

\[
\Delta V_d = \frac{\Delta A}{T_s} \quad (1.119)
\]

\[
\Delta A(t) = -(\Delta A_1(t) + \Delta A_2(t - T_u)) \quad (1.120)
\]

where the time delay is defined as:

\[
T_u = u / \omega , \quad \omega = 2\pi f \quad f = 50Hz \quad (1.121)
\]

Time delay \( T_u \) (s) in (1.120), has to be introduced for the following reason: Any change of firing angle \( \alpha \), will impose two voltage area changes: \( \Delta A_1 \) at the same instant, and \( \Delta A_2 \) delayed by the length of commutation overlap \( u \) (Figure 1.13). Assuming infinitesimal changes of \( \alpha \), the area change \( \Delta A \) becomes:

\[
\Delta A_1 = \left( \frac{v_a + v_b}{2} - v_a \right) \Delta \alpha \quad (1.122)
\]

\[
\Delta A_1 = E_m \sin \left[ \frac{2\pi}{q} \sin \alpha \theta \right] \Delta \alpha \quad (1.123)
\]

where \( q \) represents the number of pulses, ie. For a six-pulse system \( q=6 \), and where \( E_m \) stands for the AC voltage magnitude.

The area \( \Delta A_2 \), which happens \( u \) degrees after the thyristor firing, is affected by the firing angle change and also by the commutation overlap change:

\[
\Delta A_2 = (v_b - \frac{v_a + v_b}{2}) \Delta (\alpha + u) \quad (1.124)
\]
Analytical modelling of HVDC-HVAC systems

\[ \Delta A_2 = E_m \sin \frac{2\pi}{q} \sin(\alpha^0 + u^0) \Delta(\alpha + u) \]  

(1.125)

If the known equation for AC-DC converter modelling, from Appendix B:

\[ I_d = \frac{\sqrt{3}E_m}{2\omega L_c} (\cos \alpha - \cos(\alpha + u)) \]  

(1.126)

is linearised, it becomes:

\[ \sin(\alpha^0 + u^0) \Delta(\alpha + u) = \frac{2\omega L_c}{\sqrt{3}E_m} \Delta I_d + \sin \alpha^0 \Delta \alpha \]  

(1.127)

The above equation is used to express the commutation overlap changes as a function of outside variables \( I_d \) and \( \alpha \). When (1.127) is substituted in (1.125), the area \( \Delta A_2 \) become:

\[ \Delta A_2 = E_m \sin \frac{2\pi}{q} \left[ \frac{2\omega L_c}{\sqrt{3}E_m} \Delta I_d + \sin \alpha^0 \Delta \alpha \right] \]  

(1.128)

Substituting equation (1.123), (1.128) and (1.120) into (1.119) gives (for 6-pulse operation):

\[ \Delta V_d(t) = -3 \frac{\sqrt{3}E_m}{2\pi} \Delta \alpha(t) - 3 \frac{\sqrt{3}E_m}{2\pi} \Delta \alpha(t - T_u) - \frac{3}{\pi} \omega L_c \Delta I_d(t - T_u) \]  

(1.129)

The last equation for \( T_u = 0 \) becomes the known equation for converter direct voltage as a function of thyristor firing angle [8], also used earlier in development of linear-continuous models (A.54). If Laplace transformation is applied to (1.129), we obtain:

\[ \Delta V_d(s) = -3 \frac{\sqrt{3}E_m}{2\pi} [1 + e^{-sT_u}] \Delta \alpha(s) - \frac{3}{\pi} \omega L_c e^{-sT_u} \Delta I_d(s) \]  

(1.130)

and finally for the impulse converter model [1]:

\[ \Delta V_d(s) = -3 \frac{\sqrt{3}E_m}{2\pi T_s} [1 + e^{-sT_s}] \Delta \alpha^*(s) - \frac{3}{\pi T_s} \omega L_c e^{-sT_s} \Delta I_d^*(s) \]  

(1.131)

where \( \alpha^* \) and \( I_d^* \) stand for discretised firing angle and direct current signals. Using equation (1.131), converter transfer functions from the Figure 1.14 can be expressed as:

\[ G_2(s) = \frac{\Delta V_d(s)}{\Delta \alpha^*(s)} = -3 \frac{\sqrt{3}E_m}{\pi T_s} [1 + e^{-sT_s}] \]  

(1.132)

\[ G_4(s) = \frac{\Delta V_d(s)}{\Delta I_d^*(s)} = - \frac{3}{\pi T_s} \omega L_c e^{-sT_s} \]  

(1.133)
Corresponding discrete transfer functions in $z$-domain can be derived once the input–output transfer function of the overall system has been derived in $s$-domain, as shown in the next section. Transfer functions $G_2(s), G_4(s)$ cannot be discretised prior to the development of the dynamic system model. It is known from $z$-transformation theory that the following rule must be obeyed:

$$Z\left\{W_1(s)W_2(s)\right\} \neq Z\left\{W_1(s)\right\}Z\left\{W_2(s)\right\}$$

(1.134)

The basics from $z$-transformation (and modified $z$-transformation) are shown in Appendix E. As a proof for the above equation (1.134), the equations E.1-E.3 from Appendix E, can be used.

Therefore, the converter transfer function cannot be discretised independently of the remaining part of the system. The overall $z$-transformation of HVDC system, depends upon the dynamics of each part of the system, i.e. controller dynamics and DC line dynamics must also be included. It is also important to emphasise that in this form of model derivation, the final transfer function also depends upon the particular position of the samplers between the parts of the system. The final transfer function for system with sampler placed after the controller output, as in Figure 1.14, differs from the transfer function when the sampler is placed before the controller, as it is normally the case in discrete control systems theory. This unusual location of sampler complicates the HVDC controller design, as it is shown later, in Chapter 5.

### 1.4.2 Discrete HVDC system model

The block diagram of the HVDC system as a discrete system is shown in Figure 1.15. The transfer functions $G_2(s)$ and $G_4(s)$, are a part of the converter model, from the previous section, whereas $G_1(s), G_3(s), G_5(s)$, stand for the rectifier controller, DC system and the current transducer transfer functions, respectively. The sampler $S_3$, which represent the discrete nature of the HVDC controller, will be omitted in the following analysis [1]. This sampler is assumed to operate at much faster rate than the sampler $S_1$. The sampler $S_3$ should be considered if the discrete controller model, as presented later in Section 1.4.3, is used.

Using the discrete-systems block-diagram algebra [18], the following equations are written:

$$I_r = G_3(s)G_2(s)\alpha^* + G_3(s)G_4(s)I_r^*$$

(1.135)

$$\alpha = G_1(s)(I_{\text{ord}} - I_r)$$

(1.136)

where the current transducer dynamics $G_5(s)$ are also neglected [1]. Equation (1.135) is discretised and $I_r$ can be replaced from (1.136):

$$\alpha^* = (G_1(s)I_{\text{ord}}(s))^* - (G_1(s)G_2(s)G_3(s))^*\alpha^* - (G_1(s)G_4(s)G_5(s))^*I_r^*$$

(1.137)
If (1.136) is discretised and \( \alpha^* \) eliminated from (1.135) and (1.136), and after the z-transformation is applied the following transfer function is derived:

\[
I_r(z) = \frac{I_{ord}G_1(z)G_2G_3(z)}{1 + G_1G_2G_3G_5(z) - G_3G_4(z) - G_1G_2G_3G_5(z)G_4(z) + G_1G_2G_3G_5G_6(z)G_2G_3(z)} (1.138)
\]

where the following notation is used:

\[
Z\{G_1(s)G_2(s)\} = G_1G_2(z) (1.139)
\]

It was shown in [1] that the sampler \( S_2 \) can be disregarded for most of the HVDC system modelling purposes. Signal which is sampled by the sampler \( S_2 \), has much less influence on the system behaviour than the signal sampled by the sampler \( S_1 \). The signal coming through sampler \( S_2 \) represents the voltage drop because of the commutation reactance \( X_c \). This voltage drop can be taken into account if the equivalent resistance \( R_c \) is added to the line resistance \( R_r \). The above assumption significantly simplifies the model from (1.138) with very little loss on accuracy [1].

Neglecting sampler \( S_2 \), transfer function (1.138) becomes:

\[
I_r(z) = \frac{G_2G_3(z)I_{ord}G_1(z)}{1 + G_1G_2G_3(z)} (1.140)
\]

Figure 1.16 shows the block diagram of the final discrete system model.
Analytical modelling of HVDC-HVAC systems

Figure 1.16. Simplified block-diagram of discrete model.

In a similar manner the transfer function with respect to the disturbance ($E_{acl}^d$ in Figure 1.16 and 1.15) can be derived:

$$I_r(z) = E_{acl}^d G_3(z) - \frac{G_2 G_3(z) K_{exc1} G_1 G_3 E_{acl}^d(z)}{1 + G_1 G_2 G_3(z)}$$

(1.141)

If the discrete controller model is used, equation (1.140) becomes:

$$I_r(z) = \frac{G_2 G_3(z) G_1(z) I_{ord}(z)}{1 + G_1 G_2 G_3(z)}$$

(1.142)

Therefore, the current order signal is now discretised prior to the use as an input to the system. This is much more common form of the discrete system model. In (1.140), each input signal has to be multiplied by the controller transfer function, prior to its discretisation.

It should be noted that, since the simplified linear continuous model is used for the remaining part of the system, the AC system dynamics, PLL dynamics and AC-DC interactions are disregarded in the above model derivation.

### 1.4.3 Discrete controller model

A great part of the HVDC control systems are digital, microprocessor based control systems. Sampling time of this control system is different for the different control levels and for the different control tasks at the same level. Constant current controller has the fastest operating cycle, with the sampling time equal to the firing frequency. Therefore, HVDC controller can be modelled as a discrete system. HVDC current controller as a discrete system, can be represented as a linear element preceded by a sample and hold element. However, because of the consistency with the developed discrete HVDC model, the controller sampling effect will be represented here with an ideal sampler and time delay element. Figure 1.17 shows the discrete HVDC controller.
1.5 COMPARISON OF LINEAR CONTINUOUS AND LINEAR DISCRETE HVDC MODELS

This section compares the dynamic characteristics of the discrete HVDC model, developed in the previous section, and corresponding linear continuous model. Continuous model is the simplified linear continuous model from Section 1.3, with infinitively strong AC systems. Except for the converter model, dynamic model of the remaining part of the system is identical in the two models in use. The purpose of this section is to highlight the frequency range where discrete HVDC modelling is important, and to emphasise benefits of discretisation. The above models are not expected to compare well with PSCAD simulation, because of their simple dynamic structure, presented assumptions and other simplifications used in their development. Further analysis in Chapter 4, shows that simplified continuous model can not give responses closely matching PSCAD/EMTDC.

Figure 1.18 shows the open loop frequency response for the two models, with the system parameters from Appendix D. The input is the controller input, with the DC current as output in the Figure.

The main differences between the models are in the frequency domain close to half the sampling frequency ($300\text{Hz} = 1884\text{rad/s}$), and at the frequencies around the zeros of the open loop transfer function which is represented by the first notch in Figure 1.18 (frequency around $40\text{Hz} = 251\text{rad/s}$). As shown in [1],[2],[15] discrete model will better represent HVDC system than linear continuous model because of discrete nature of AC-DC converter transformation process, as viewed from the control system.

It can be seen from the Figure that the two frequency responses differ mostly in the higher frequency domain. As the frequency approaches half the sampling frequency ($300\text{Hz} = 1884\text{rad/s}$) the discrete system gain becomes more attenuated and deviates from the corresponding continuous system. Since the gain for the continuous model is higher, there is a possibility of designing too conservative controller. It can be concluded that if the frequencies of interest are close to half the sampling frequency, discrete HVDC system model must be used. The most important shortcoming of the linear-continuous model is its inability to predict half the sampling instability (at $300\text{Hz}$) [18].

The difference is also evident around the zeros of the open loop transfer function, which is represented by the first notch in the Figure. This different position of the transfer function zeros will cause different parameters (residual values) in the numerator of transfer function, and conse
quentely the damping of inherent oscillatory modes will be inaccurate. It is concluded that continuous model should be used with confidence say below $30\text{Hz}=190\text{rad/s}$. In the lower frequency range, the continuous HVDC system model shows similar responses, and it has advantages over the discrete model because of its simplicity. It will be shown later however, that even around the dominant oscillatory mode, the controller designed using continuous model shows improvement in the responses. The benefits of discrete model are further discussed in Chapter 5.

Figure 1.18 Open loop frequency response for continuous and for discrete models.

Figure 1.19 compares disturbance step responses for the two models. As can be seen discrete model has more pronounced oscillatory mode.

Figure 1.19. System response after disturbance on rectifier AC voltage
1.6 CONCLUSIONS

Three different approaches to the analytical modelling of HVDC-HVAC systems are presented to meet different analysis purposes.

The detailed, continuous system model presents a novel approach in HVDC-HVAC modelling. The system model consists of three subsystems: Rectifier and inverter AC systems, and DC system. The linearised system dynamics are represented in a common frame of reference with respect to frequencies by transforming the AC system variables to its pseudo stationary $dq0$ components. The simulation responses confirm that all HVDC-HVAC interactions (including PLL dynamics) need to be represented in detail, whereas the model discretisation is not necessary if the model use is restricted to the frequency domain $f<100Hz$. Response matching with non-linear digital simulation is satisfactory in this frequency domain. As it was expected, some discrepancies in the responses are noticed at frequencies close to the dominant oscillatory mode. The advantage of the derived model is that it can be easily used for systematic analysis of AC-DC and control interactions as well as influence of various system parameters on the system performance. In model derivation, it was attempted to use as many as possible states with physical meaning. This resemblance with the physical system makes the model convenient for design of a new stabilising HVDC control loops. The model suffers from high complexity, which can be a handicap when a simple phenomena from HVDC operation need to be studied.

The simplified, linear continuous model is far easier to develop and it is more convenient for the controller design. It is derived as a fourth order continuous model with static representation of the AC systems and neglected PLL dynamics. The model will be used for controller design to counteract composite resonance phenomenon on HVDC systems. The main disadvantage of this model is the simplified AC-DC interactions and neglected AC system dynamics.

The developed discrete system model gives accurate representation of the discrete nature of the AC-DC conversion process. It should be used in the frequency domain where the discrete systems phenomena are prevailing ie close to half the sampling frequency. However it is well known that discrete systems models are usually more cumbersome for the system dynamic analysis and controller design. From the dynamic point of view, the model is very simple with the same shortcomings as the simplified linear continuous model.
REFERENCES:


CHAPTER 2

MODELLING OF TCR AND TCSC

2.1 INTRODUCTION

The Thyristor Controlled Reactor (TCR) and Thyristor Controlled Series Capacitor (TCSC) are rapidly becoming a part of modern power transmission system. The new thyristor based technology has enabled the speed of response and controllability characteristics of these elements to be far superior to any of the conventionally used voltage controlling (power system stabilising) elements [1].

The dynamics of TCR and TCSC and their controllers will have significant influence on the dynamic behavior of any HVDC-HVAC system, where they are incorporated. Mathematical representation of these elements is however very difficult. The main difficulties arise from the discrete nature of thyristor firing signals and the need for the system representation in two different coordinate frames (rotating and static frame). An accurate, convenient and general analytical model of TCR/TCSC has not been reported in the literature.

The most commonly used TCR/TCSC model is based on the fundamental-frequency-impedance system representation [2]. These models are accurate only in a very low frequency domain. They neither represent network dynamics nor they include various interactions between TCR and AC system.

A series of new discrete models for TCSC has been proposed recently [3],[4],[5]. Theoretically, these models can be used in the higher frequency domain. In [5], it is said that the model could be used in the frequency range $f < 100\,\text{Hz}$. However, because of the complex mathematical apparatus required for the model derivation, the practical value of these models is very limited. They are suitable for simple systems and only for the analysis purposes. The application of these models to a complex higher-order HVAC system would be very difficult. The controller design based on these models, also necessitates the use of discrete control theory.

The analytical model proposed in this Chapter uses the same modeling methodology presented with HVDC-HVAC modeling in Section 1.2. The TCR/TCSC model is developed as a stand-alone model which enables the model connection with any HVAC system of arbitrary complexity. The modeling approach is based on the Park’s transformation that enables convenient coupling between the three-phase main circuit side of TCR and the control circuit. One of the primary objectives in the model development is its compatibility with the earlier presented HVDC-HVAC model. An independent TCR/TCSC model enables building of a complex HVDC-HVAC-FACTS model with an arbitrary number and arbitrary location of FACTS elements. The primary aim is the development of model with physically meaningful parameters and variables. Similarly as in the development of HVDC model, all system dynamic elements and all converter interactions will be accurately represented.
2.2 TCR/TCSC ANALYTICAL MODEL

2.2.1 Modelling approach

The modeling approach adopted here uses two independently developed models: main circuit model (in the rotating coordinate frame) and general control circuit model (in the static coordinate frame). These models are joined together using Park’s transformation. Dynamic model of the main circuit represents all network dynamic elements including the TCR/TCSC inherent dynamics. Control circuit model comprises: voltage controller and detailed Phase Locked Loop (PLL) model. This modeling approach, being similar to the one used in Section 1.2, has all the advantages presented in building the detailed HVDC system model, and it will enable easy integration of FACTS models with the developed HVDC-HVAC system model. Figure 2.1 shows the schematic of the test systems in use, which is the slightly modified tutorial example from [6].

nomenclature:

\[ V \quad \text{AC voltage magnitude (controlled value)}, \]
\[ \phi \quad \text{AC voltage phase angle}, \]
\[ V_{\text{ref}} \quad \text{Voltage reference}, \]
\[ \theta \quad \text{PLL output angle}, \]
\[ \alpha \quad \text{Controller firing angle order}, \]
\[ \phi \quad \text{Actual firing angle}, \]

Figure 2.1. Test system in use. a) TCR Test model, b) TCSC Test model.

2.2.2 Controller model

This subsystem comprises: PLL model, TCR controller and interaction equations. The model can be mathematically written as:

PLL model:
Modelling of TCR and TCSC

\[
\begin{align*}
    x_1 &= -k_{iPLLc} x_2 + k_{iPLLc}\phi \\
    x_2 &= -k_{iPLLc} k_{pPLLc} x_2 + k_{iPLLc} x_1 + k_{iPLLc} k_{pPLLc}\phi \\
    x_3 &= k_{IC}(V_{ref} - x_4) \\
    sT_f x_4 &= -x_2 + x_3 - x_4 + k_{pc} V_{ref} - k_{pc} x_5 + \phi \\
    sT_{pc} x_5 &= -x_5 + V
\end{align*}
\]

TCR voltage controller

\[
\begin{align*}
    sT_f x_4 &= -x_2 + x_3 - x_4 + k_{pc} V_{ref} - k_{pc} x_5 + \phi
\end{align*}
\]

Voltage transducer dynamics:

\[
\begin{align*}
    sT_{pc} x_5 &= -x_5 + V
\end{align*}
\]

or in matrix notation:

\[
\begin{align*}
    s\bar{x} &= A_c \bar{x} + B_{cac} u_{cac} + B_{cimp} u_{imp} \\
    y_{cac} &= C_{cac} \bar{x}
\end{align*}
\]

where the model states are:

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
    x_5
\end{bmatrix} =
\begin{bmatrix}
    k_{iPLLc} \\
    -s \\
    \theta \\
    k_{IC} \\
    -s
\end{bmatrix}
\begin{bmatrix}
    (\phi - \theta) \\
    \theta \\
    k_{IC} (V_{ref} - x_5) \\
    \phi \\
    V_f
\end{bmatrix}
\]

where \( V_f \) denotes measured AC voltage, obtained after the feedback filter. To illustrate the above subscript notation, the following example is used: matrix \( B_{cac} \) is the controller input matrix for coupling with the AC system.

The inputs in the above model are:

\[
\begin{align*}
    u_{cac} &= \begin{bmatrix} V \\ \phi \end{bmatrix} \\
    u_{imp} &= \begin{bmatrix} V_{ref} \end{bmatrix}
\end{align*}
\]

\( V \) AC voltage magnitude
\( \phi \) AC voltage angle
\( V_{ref} \) Reference value for AC voltage

and the model output is:

\[
\begin{align*}
    y_{cac} &= x_4 = \phi
\end{align*}
\]

\( x_4 = \phi \) Actual firing angle.

The model matrices are derived in Appendix F.
The model is structured in the following way:
• Equations (2.1)-(2.2) represent PLL dynamic model, as presented in Section 1.2.2.
• Equations (2.3)-(2.4) represent the PI type TCR voltage controller, where (2.4) is artificially introduced, in the same way as presented in Section 1.2.6, and as shown below:

From Figure 1.2 b), the equation for the actual firing angle is written as:

$$\phi = \alpha - \theta + \varphi$$  \hspace{1cm} (2.11)

where the controller firing angle order is expressed as:

$$\alpha = k_{pc} (V_{ref} - V_f) + \frac{k_{tc}}{s} (V_{ref} - V_f)$$  \hspace{1cm} (2.12)

Equation (2.3) is derived using the integral part of the above controller equation. Equation (2.4) is obtained when equation (2.11) is artificially made dynamic equation as follows:

$$sT_f \phi = -\phi + \alpha - \theta + \varphi$$  \hspace{1cm} (2.13)

where $$T_f = 1/6000s$$.

• Equation (2.5) represents a first order voltage transducer (filter).

The PLL model in use is the same model presented earlier for HVDC systems, however the controller parameters are different, as shown in Appendix F. The TCR controller is assumed of PI type with only the basic voltage control loop. The higher level control loops (including the droop control), although readily incorporated, are not modeled at this stage.

### 2.2.3 Main Circuit Model

With reference to Figure 2.1, the main circuit dynamic equations, for one phase of TCR model, are written as:

$$sL_1i_{L1} = -R_1i_{L1} + R_2(-v - i_{L1}R_3)/(R_2 + R_3)$$  \hspace{1cm} (2.14)

$$sC_1v = i_{L1} + (-v - i_{L1}R_3)/(R_2 + R_3) - i_{acr} - i_{l2}$$  \hspace{1cm} (2.15)

$$sL_2i_{L2} = v - i_{L2}R_4$$  \hspace{1cm} (2.16)

$$si_{acr} = vB_{acr}$$  \hspace{1cm} (2.17)

where $$B_{acr} = 1/L_{acr}$$.

Since $$v$$ is a state variable and $$B_{acr}$$ is dependent upon firing angle signal, equation (2.17) is non-linear. However, assuming small deviations from the steady state AC operation, the non-linear product $$vB_{acr}$$ can be approximated with a linear summation as shown below.

By considering a small deviation from the steady state AC operating point:
Modelling of TCR and TCSC

\[ v^0 = V^0 \cos(\omega t + \varphi^0), \quad B^0 \]  \hspace{1cm} (2.18)

by the amounts \( \Delta v \) and \( \Delta B \) respectively, the new value for the AC voltage and the TCR susceptance can be considered as:

\[ v = (v^0 + \Delta v) = V \cos(\omega t + \varphi), \quad B_{tcr} = B^0 + \Delta B \]  \hspace{1cm} (2.19)

The equation (2.17) can be written as:

\[ s_i_{tcr} = (v^0 + \Delta v)(B^0 + \Delta B) \]  \hspace{1cm} (2.20)

where: \( v^0 = V^0 \cos(\omega t + \varphi^0) \), and \( V^0, \varphi^0, B^0 \) - constant.

Equation (2.20) is further rearranged as:

\[ s_i_{tcr} = v^0 B^0 + v^0 \Delta B + \Delta v B^0 + \Delta v \Delta B \]  \hspace{1cm} (2.21)

\[ s_i_{tcr} = v^0 B^0 + v^0 (B_{tcr} - B^0) + (v - v^0) B^0 + \Delta v \Delta B \]  \hspace{1cm} (2.22)

Assuming small variations around the nominal operating point, \( \Delta v \Delta B \approx 0 \), the above equation becomes:

\[ s_i_{tcr} = v B^0 + v^0 B_{tcr} - v^0 B^0 \]  \hspace{1cm} (2.23)

which is rewritten as:

\[ s_i_{tcr} = v B^0 + b_{tcr} V^0 - v^0 B^0 \]  \hspace{1cm} (2.24)

Note that in the above equation the right hand side is represented as a linear combination of state variables. The (rotating) variables used on the right side of equation (2.24) are:

\[ v = V \cos(\omega t + \varphi) \quad \text{and} \quad b_{tcr} = B_{tcr} \cos(\omega t + \varphi^0) \]  \hspace{1cm} (2.25)

In the above equations \( b_{tcr} \) is an artificial time varying variable, created by linearising dynamic equation for current through the thyristor controlled reactor (2.17). This variable has only one component as an actual variable, magnitude \( B_{tcr} \), which is a function of thyristor firing angle, whereas the phase angle \( \varphi^0 \) is a constant as seen in (2.25) and (2.20). The term \( v^0 B^0 \) in equation (2.24), is a rotating variable of constant magnitude and angle, which becomes nullified in static coordinate frame, and after the equation is linearised.

Using state-space variables the model is represented:

\[ sL_1 x_1 = -R_1 x_2 + R_2 (-x_2 - x_1 R_3) / (R_2 + R_3) \]  \hspace{1cm} (2.26)

\[ sC_1 x_2 = x_1 + (-x_2 - x_1 R_3) / (R_2 + R_3) \quad - x_4 - x_3 \]  \hspace{1cm} (2.27)

\[ sL_2 x_3 = x_1 - x_3 R_4 \]  \hspace{1cm} (2.28)
Modelling of TCR and TCSC

\[ sx_4 = B^0 x_2 + V^0 b_{tcr} - v^0 B^0 \]  \hspace{1cm} (2.29)

where the model matrices are shown in Appendix F.

The main circuit model (2.26)-(2.29) needs to be transferred to \(dq\) coordinate frame in order to be connected with controller model (2.1)-(2.5). The same transformation method described in Chapter 1 and Appendix A.1, can be used to this end. This modeling approach retains the same advantages of the modeling approach described for HVDC systems.

After Park’s transformation is applied, the main circuit model is represented as:

\[ s\dot{x} = A_{ac} x + B'_{acc} \dot{u}_{acc} \]  \hspace{1cm} (2.30)
\[ y'_{acc} = C'_{acc} x \]  \hspace{1cm} (2.31)

where the model takes as inputs \(dq\) components of \(h_{ac} :\)

\[ \dot{u}'_{acc} = \begin{bmatrix} b_{tcrd} \\ b_{tcrq} \end{bmatrix} \]  \hspace{1cm} (2.32)

and as outputs \(d-q\) components of \(v :\)

\[ y'_{acc} = \begin{bmatrix} v_d \\ v_q \end{bmatrix} \]  \hspace{1cm} (2.33)

2.2.4 TCR model

Prior to the coupling of the main circuit model and the controller model, it is necessary to correct the input and output matrices of the main circuit model, to enable matching of input and output variables.

The input model matrix is corrected as follows:

\[ B_{acc} = B'_{acc} S_{acc} T_{acc} \]  \hspace{1cm} (2.34)

where matrix \(S_{acc}\) is defined by the following equations:

\[ b_{tcrd} = B_{tcr} \sin \xi \hspace{1cm} b_{tcrq} = B_{tcr} \cos \xi \hspace{1cm} \begin{bmatrix} b_{tcrd} \\ b_{tcrq} \end{bmatrix} = S_{acc} \begin{bmatrix} B_{tcr} \\ \xi \end{bmatrix} \]  \hspace{1cm} (2.35)

\[ S_{acc} = \begin{bmatrix} \frac{\partial B_{tcrd}}{\partial B_{tcr}} & \frac{\partial B_{tcrd}}{\partial \xi} \\ \frac{\partial B_{tcrq}}{\partial B_{tcr}} & \frac{\partial B_{tcrq}}{\partial \xi} \\ \frac{\partial B_{tcrd}}{\partial B_{tcrq}} & \frac{\partial B_{tcrq}}{\partial \xi} \end{bmatrix} \]  \hspace{1cm} (2.36)
and where the coefficients of matrix $S_{acc}$ are shown in Appendix F.

The matrix $T_{acc}$ is obtained from:

$$\begin{bmatrix} B_{acc} \\ \xi \end{bmatrix} = T_{acc} \phi \quad T_{acc} = \begin{bmatrix} K_c \\ 0 \end{bmatrix}$$

(2.37)

with the linearised thyristor gain $K_c$ defined as

$$K_c = \frac{\partial B_{acc}}{\partial \sigma} \quad (\sigma \text{ - thyristor conduction angle})$$

(2.38)

and derived in Appendix F.

The final, output model matrix is represented as:

$$C_{acc} = C_{acc} Q_{acc}^{-1}$$

(2.39)

where:

$$v_d = V \sin \phi \quad v_q = V \cos \phi \quad \begin{bmatrix} v_d \\ v_q \end{bmatrix} = Q_{acc} \begin{bmatrix} V_t \\ \phi \end{bmatrix}$$

(2.40)

and where $Q_{acc}$ is obtained as:

$$Q_{acc} = \begin{bmatrix} \frac{\partial V_d}{\partial V} & \frac{\partial V_d}{\partial \phi} \\ \frac{\partial V_q}{\partial V} & \frac{\partial V_q}{\partial \phi} \end{bmatrix}$$

(2.41)

with the matrix coefficients given in Appendix F.

The final, main circuit model is written as:

$$sX = A_{acc} X + B_{acc} U_{acc}$$

(2.42)

$$Y_{acc} = C_{acc} X$$

(2.43)

The final TCR model consists of the two above derived models, controller model and main circuit model, and the correction matrices which enable coupling between the models. The model is represented as:

$$sX = A X + B U_{inp}$$

(2.44)

$$Y = C X$$

(2.45)

where:
Modelling of TCR and TCSC

\[
A = \begin{bmatrix}
A_c & B_{cac} & C_{acc} \\
B_{acc} & C_{acc} & A_{ac}
\end{bmatrix}
\]

(2.46)

\[
B = \begin{bmatrix}
B_{inp} \\
0_{8x1}
\end{bmatrix}
\]

(2.47)

and where the output matrix C is calculated for a particular output of interest.

The TCSC model can be derived in a similar manner, by using the circuit diagram from Figure 2.1 b).

2.2.5 Model accuracy

The main advantages of the above presented modeling approach can be summarized:

- Model building from the two physically meaningful subsystems.
- Convenient coupling between the main circuit model, PLL model and controller model.
- Physically meaningful model parameters and variables.
- The model represents a general AC system. The model can be readily expanded to AC system of any complexity.
- State-space, linear, continuous model form.

The only part in the model which has questionable accuracy is the equation (2.17), which gives TCR reactance as a function of thyristor firing angle. The actual object (process) which this equation represents, is highly non-linear and discrete. The equation is obtained from a Fourier series representation of the process, by neglecting all harmonics except the first one. Also, further linearisation of the equation, as shown by the gain \( K_{tc} \), rises the question of the gain accuracy at different values of the firing angle.

The above model, therefore, gives a platform for the building of a convenient, general, model of an AC system with FACTS element. More importantly, it presents the convenient method for coupling between the main circuit dynamic model, phase locked loops model and controller model. The focus for the further research should be on the improvement in the accuracy of equation (2.17).

If the identification methods (or some other modeling approaches) are focused on the accurate representation of the equation (2.17), and the linearised coefficient \( K_{tc} \), the model fidelity could be improved without any loss in the physical meaningfulness of the other model parameters.

2.3 SIMULATION RESULTS

Figure 2.2 shows the EMTDC/PSCAD verification of the developed model for TCR. As a perturbation, a step change in the system control input is considered. The test system is TCR model from Figure 2.1. The system data are given in Appendix F.
It is seen that the response matching is satisfactory for low frequency system studies. It is however evident that the model does not show satisfactory response matching in the higher frequency range. The main reason for model inaccuracy is the linearised equation for the TCR reactance.

Figure 2.2. System response following voltage-reference step change. Voltage-magnitude (kV) response is shown.
2.4 CONCLUSIONS

Analytical modeling of TCR/TCSC has been regarded as a very difficult task, and at present, in the available FACTS bibliography, there is no accurate and convenient model available.

This Chapter presents a convenient approach for building of TCR/TCSC model in the lower frequency range. The developed model comprises of: a detailed network dynamic model including the main circuit of TCR/TCSC, a controller model and a dynamic model of Phase Locked Loop. Park’s transformation is used for connection of rotating coordinate frames and static coordinate frames. The developed model can be readily included into the HVDC-HVAC system model, presented earlier in Chapter 1.

Since the developed model has all variables and parameters with physical meaning, the model is convenient for the system analysis and controller design.

The model, however, still uses the known very simplified representation of the relationship between the TCR reactance and thyristor firing angle.

The EMTDC/PSCAD simulation tests confirm the model fidelity. This model in the present form, can not offer high accuracy in the wider frequency range, because of the simplified, linearised equation for TCR reactance.
REFERENCES

Modelling of TCR and TCSC
CHAPTER 3

ANALYSIS OF NON-LINEAR EFFECTS IN HVDC CONTROL SYSTEMS

3.1 INTRODUCTION

This chapter offers a study of the most important non-linear effects associated with HVDC operation. Since linear system models are used throughout this document, this Chapter will demonstrate the usefulness of these models and the breadth of their application.

The most important non-linear effect is the change of control operating mode. The system operation in each control mode can be analysed by using linear systems theory. However these models cannot offer insight into the system behaviour during a transition between two modes. Some of these mode changes are a consequence of change of controller parameters, the others arise from the change in the system structure. The change in system structure occurs, as an example, when inverter takes over current control. This Chapter offers the basic system stability analysis when the transition occurs between constant beta and constant gamma mode.

It was emphasised earlier that a converter is a non-linear discrete system in its nature. Non-linear characteristics are reflected through variable system gains and through frequency transformation phenomenon. Under certain circumstances, as it is shown later in this chapter, gain can significantly change and frequency content of the output signal can deviate significantly from the input signal. These phenomena could cause instability problems if the system controller is based on a linear system model.

It is therefore important to determine the system conditions which will lead to these non-linear effects. The stability analysis of non-linear effects offers a complement to the basic steady state linear analysis, extending the scope of validity for analytical models.

The analytical studies of non-linear HVDC effects have not been well documented in HVDC bibliography. These effects are usually tested on digital simulator after the controller is designed.

In this Chapter, the simplified linear continuous model, developed in Section 1.3, is used for the studies. The test system data are given in Appendix D. It is important to consider that many non-linear effects are neglected in derivation of the model in use, and that conclusion may differ for actual non-linear controllers.

3.2 CONTROL MODE CHANGES IN HVDC SYSTEMS

Steady state characteristic of an HVDC system can be represented as in Figure 3.1. The most often, the system operating point is in the area around points 1, 2 or 3. Stability of these operating points has been practically proven and it can be analytically demonstrated using the linear systems theory. At points A, B and C, the system changes between two operating modes and consequently the system structure changes. At point A the inverter controller parameters/structure will change with rectifier controller maintaining current control. At point C, inverter takes over current control.
Analysis of non-linear effects in HVDC control systems

with the rectifier controller becoming essentially passive. Point \( B \) represents mode change to minimum alpha control at rectifier controller.

This section investigates the system stability when the system operates at point \( A \). Operating point \( A \) is the most significant of the three mode changing points. It represents the transition between the constant firing angle and the constant extinction angle operating modes. If the system normally operate in operating point \( I \), it goes through point \( A \) after inverter AC system disturbance or after increase in DC current. The disturbances which will cause this mode change are quite common in normal HVDC system operation. The CIGRE HVDC system, as implemented in [1] normally operates exactly at point \( A \).

3.3 ANALYSIS OF CONSTANT BETA-GAMMA MODE CHANGE

3.3.1 Describing Function Derivation

Figure 3.2 shows the system structure, when operating at point \( A \). Note that the variables shown \( (I_r, I_i, I_{ird}, \text{etc.}) \) are the deviations from the steady state operating point. The linear part of the system is very similar to the HVDC model developed in Section 1.3. The non-linear part, as represented by the block \( n_1(I_i) \), captures the inverter direct voltage changes as the system changes from constant beta to constant gamma mode.
With reference to Figure 1.10, and equations (1.97-1.99), the system dynamic equations for operating point $A$ are:

\begin{align*}
V_{dr} &= sL_1I_r + R_1I_r + V_{cs} \quad (3.1) \\
V_c &= sL_2I_i + R_2I_i + V_{di} \quad (3.2) \\
V_c &= \frac{1}{sC_s} (I_r - I_i) \quad (3.3)
\end{align*}

To derive the expression for non-linear element, the earlier developed equations for inverter direct voltage for constant beta operating mode (1.34 or 1.36) and for constant gamma operating mode (1.39) are used. Depending on the value of inverter direct current, the non-linear element is represented as (1.36 is used):

\begin{equation}
N_a(I_i) = \begin{cases} 
-R_c I_i & I_i > 0 \\
(R_{ci} + K_{ac2} k_{pi})I_i - I_m & I_m > I_i < 0 
\end{cases} \quad (3.4)
\end{equation}

where the inverter is assumed to operate with direct current feedback, as presented with derivation of (1.36). All model parameters in the above equations assume the same notation from Chapter 1 and Appendices A,B and C.

The Describing function method will be used to analyse the system stability. To this end the inverter current is expressed as [1]:

\begin{equation}
I_i = a \cos(\omega t) \quad (3.5)
\end{equation}

The non-linear element becomes (assuming $a < I_m$):
Analysis of non-linear effects in HVDC control systems

\[ n_1(a) = \begin{cases} 
- R_{cl} a \cos(\omega t) & 0 < \omega t < \pi \\
(R_{cl} + K_{ac2} k_{pi}) a \cos(\omega t) & \pi < \omega t < 2\pi
\end{cases} \quad (3.6) \]

When describing function method is applied the non-linear element is expressed as [2]:

\[ n(a) = A_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega t) + B_n \sin(n\omega t)) \quad (3.7) \]

with the coefficients calculated as:

\[ A_n = \frac{1}{\pi} \int_{0}^{2\pi} N(\omega t) \cos(n\omega t) d(\omega t) \quad B_n = \frac{1}{\pi} \int_{0}^{2\pi} N(\omega t) \sin(n\omega t) d(\omega t) \quad (3.8) \]

The describing function is defined as the ratio between output and input signals to the non-linear element:

\[ N(a) = \frac{n(a, \omega)}{a \cos(\omega t)} \quad (3.9) \]

Assuming the low-pass characteristics of the linear part of the system the describing function becomes:

\[ N(a) = \frac{A_1}{a} - \frac{B_1}{a} j \quad (3.10) \]

For the operating point A, the describing function can be derived:

\[ N_a(a) = \frac{K_{ac2} k_{pi}}{2} \quad \text{for} \quad a < I_m \quad (3.11) \]

The case when the amplitude of DC current oscillations is very large \( a > I_m \), is not considered, since this would rarely occur in practice.

3.3.2 System stability analysis

In order to perform system stability analysis, the system from Figure 3.2 is represented by its equivalent block diagram shown in Figure 3.3.

![Figure 3.3. System equivalent block diagram.](image)
Where \( G(j\omega) \) represents the transfer function of the remaining, linear part of the system, and where \( \text{dist.} \) is proportional to the Inverter AC voltage disturbance \( E_{ac}^d \), as in Figure 3.2.

The condition for existence of limit cycle is:

\[
G(j\omega) = -\frac{1}{N(a)}
\]  

(3.12)

Figure 3.4 shows the Nyquist diagram of the linear part of the system, and the describing function of the non-linear part. As can be seen from the Figure, the diagram \(-\frac{1}{N_i(a)}\) is outside the Nyquist diagram of the linear part and the system is stable. The stability is guaranteed for all values of direct current magnitude (\( a \) in (3.5)) since the resultant describing function from (3.11) does not depend upon input current magnitude. It is therefore concluded that for this mode change limit cycle does not exist.

3.3.3 Influence of Controller Parameters

It can be concluded from Figure 3.4 and equation (3.11), that the system becomes unstable if the inverter proportional gain is sufficiently increased (approximately 20 times for the test system).

Therefore if DC current feedback is used at inverter side and if the feedback gain is sufficiently large, the system can become unstable because of the mode changes to constant gamma mode.
3.4 NON-LINEAR PHENOMENA IN AC-DC CONVERTERS

3.4.1 Introduction

This section analyses the non-linear nature of the signal transfer through a converter. Two phenomena will be analysed: the input-output frequency transformation through converter, and non-linear converter gains.

3.4.2 Converter firing angle modulation

All HVDC converters use the firing angle as input, control signal, for controlling direct current (direct voltage). When converter is analysed in frequency domain using firing angle as input and direct voltage as output, it is assumed that the converter behaves as a linear element, i.e. without change of frequency between input and output. The models developed here also use this assumption. This is the basic requirement to apply frequency domain analysis techniques. In this section, the scope of validity of this assumption is investigated, and the converter behaviour beyond the limits for linear system is studied. This study is intended to confirm validity for the linear system models in use, and to offer basic conclusions about object controllability. The analysis is however simplified with many converter non-linearities neglected.

It is important to distinguish the frequency transformation phenomenon analysed here from the AC-DC frequency transformation studied in model development, in Chapter 1. AC-DC frequency transformation always complies to the known formula and it can be modelled by the means of Park’s transformation.

The equation for DC voltage [3], for a 12-pulse converter operation, (derivation is given in details later, in Section 5.1.2 eqn. (5.5)), can be expressed:

\[
V_{dc} = \sum_{m=1}^{\infty} \sum_{k=0}^{\infty} \frac{3V_m^+}{2} \left\{ A_{(12k-1)} \cos \left( (\omega_m + (12k - 1)\omega_0) t + (12k - 1)\alpha + \alpha_m^+ \right) + 
A_{(12k+1)} \cos \left( (\omega_m - (12k + 1)\omega_0) t - (12k + 1)\alpha + \alpha_m^+ \right) \right\}
\]

(3.13)

where:

- \(\alpha_0\) Firing angle,
- \(\alpha_m^+, V_m^+\) Positive sequence, phase angle and magnitude of harmonic \(m\),
- \(m\) Harmonic order.
- Value for coefficients \(A\) can be found in [3].

Formula (3.13) is derived for positive sequence components, whereas negative sequence components are neglected. Also commutation overlap effect is neglected.

If a converter is analysed in the frequency domain the input (firing angle) is assumed:

\[
\alpha = \alpha_0 + M \sin(\omega_0 t),
\]

(3.14)
The non-linear simulation of equation (3.13) and assuming firing angle input as in (3.14) is shown in Figure 3.5. Figure 3.5 a) shows that the system output (direct voltage) will have the same frequency as input signal (firing angle) when the nominal firing angle is large and modulation signal is 15°. In Figure 3.5 b) the conclusion is similar for nominal operating angle 15° and modulating signal of 10°. It is however noticed that the output becomes somewhat distorted as the nominal firing angle reduces. Therefore for all practical cases, the converter behaves like a linear element under the firing angle modulation.

The analysis that follows offers analytical explanation for the above findings. When the theory of Bessel function [4] is applied with assumption that the AC voltage is harmonic free \( V_m = 0 \ m=2,3,... \) and if harmonics of the order 12 and higher are neglected \( k=0 \), the direct voltage becomes:

\[
V_{dc} = V_{dc0} + \sum_{l=0}^{\infty} 3V_1 M \sin \alpha_0 J_{2l} (M) \cos((\omega_0 + 2l\omega_0)t) + \sum_{l=0}^{\infty} 3V_1 M \cos \alpha_0 J_{2l-1} (M) \cos((\omega_0 + (2l-1)\omega_0)t)
\]

\[ (3.15) \]

where \( J_l \) stands for Bessel coefficients [4].

It can be concluded that not only the input frequency component will be present at DC voltage but also all integer multiples of the input component (shown by coefficient \( l \)). The first sum in (3.15) gives the input frequency and all odd harmonics. The second sum gives the second harmonic and all even harmonics at the output. These harmonics are undesirable, since they can cause control problems in feedback control loops. It is therefore important to determine when these harmonics will have significant magnitude, relative to the input frequency component.

The magnitude of each harmonic will depend on the magnitude of the modulating signal \( M \) and upon nominal firing angle \( \alpha_0 \), as seen in (3.15). Table 3.1 shows that for small values of input signal magnitude \( M \), the zero Bessel coefficient is dominant [4] and other harmonics will be negligible. For values of \( M \) of the order of 0.2 radian, the first Bessel coefficient becomes \( J_1 (0.2) \approx 1/10 J_0 (0.2) \) and the second harmonic could have visible magnitudes. In this case, the system behaviour may somewhat deviate from the assumed linear behaviour. However, these values for modulation signal can not be expected in practice.
Analysis of non-linear effects in HVDC control systems

Figure 3.5. Direct voltage after firing angle modulation.

TABLE 3.1. LOW ORDER BESSEL COEFFICIENTS

<table>
<thead>
<tr>
<th>M(rad)</th>
<th>$J_0$</th>
<th>$J_1$</th>
<th>$J_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.2</td>
<td>0.99</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>0.4</td>
<td>0.96</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>0.6</td>
<td>0.91</td>
<td>0.29</td>
<td>0.04</td>
</tr>
<tr>
<td>0.8</td>
<td>0.85</td>
<td>0.37</td>
<td>0.08</td>
</tr>
<tr>
<td>1</td>
<td>0.77</td>
<td>0.44</td>
<td>0.11</td>
</tr>
<tr>
<td>1.2</td>
<td>0.67</td>
<td>0.5</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The magnitude of even harmonics depends also on $\cos \alpha_0$. It is also seen in (3.15), that the magnitude of oscillatory component with the same frequency as input frequency depends upon $\sin \alpha_0$. The worst case, in terms of generation of unwanted harmonics, occurs when the nominal firing angle is small (small $\sin \alpha_0$ and large $\cos \alpha_0$) and large modulation signal is applied ($M$...
large). It is however noted that, in practice, for large modulation signals it is necessary to have larger nominal operating angles. Figure 3.6 shows simulation of the equation (3.15), where the simulated operating conditions correspond to case b) in Figure 3.5. The slight mismatch with Figure 3.5 is a result of simplifications assumed with derivation of (3.15).

![Figure 3.6. Direct voltage after firing angle modulation. \( \alpha_0 = 15 \text{deg} \ M = 10 \text{deg} \) (Input signal is the same as in Figure 3.5).](image)

The above analysis confirms that converter will behave like a linear element, under input signal modulation, for most practical operating conditions.

### 3.4.3 Non-linear converter gain

Inspecting the second sum in equation (3.15) it is seen that the converter gain \( K_c \) can be expressed as:

\[
K_c = 3V_1^+ M \sin \alpha_0 J_{2l-1}(M)
\]  

(3.16)

The above expression is very simplified, and many non-linear converter characteristics are not considered.

The converter gain is therefore a non linear function of the following parameters: AC system voltage \( V_1^+ \), nominal firing angle \( \alpha_0 \) and magnitude of the input signal (firing angle) \( M \). As a consequence, the converter gain will change as these parameters change, and the system will have responses and behaviour different from the designed ones.

Usually, HVDC control systems have gain correction scheme for compensation of the firing angle change in (3.16). This control scheme is normally implemented as a gain scheduling adaptive control. In practical control systems, the remaining two factors are not compensated. Their change will directly affect the system gain. As an example, a temporary increase in AC voltage accompanied by large modulation signal can noticeably increase the system gain. Nevertheless it is known that these operating conditions are not likely to persist for a longer time in actual systems. Only if the control system is designed for large values of modulating signal, and if frequent voltage changes are expected, could the additional adaptive control loops be considered.
3.5 CONCLUSIONS

HVDC system can be regarded as a highly non-linear, discrete system. Although the linear system analysis can be applied for most operating conditions, it is still important to determine the conditions for non-linear system behaviour, and to analyse the system stability if the non-linear conditions prevail.

There are two main groups of non-linear effects in HVDC systems: Controller mode changes, and AC-DC converter non-linear effects.

The non-linear effects arising from the HVDC control system mode changes are studied using non-linear control systems theory based on the describing function method. It is concluded that the mode change from the constant beta to constant gamma mode on inverter side can lead to the unstable system operation only for very high controller gains. These conditions are not likely in the actual systems.

Analysis of the actual input-output converter behaviour under the firing angle modulation, shows that non-linear effects can not be expected for practical operating conditions. As the nominal operating angle reduces, and modulating signals magnitude increases, second harmonic content on the output will increase, however for all practical conditions second harmonic is negligible.

The open-loop gain of an AC-DC converter is a function of three system parameters: firing angle value, converter AC voltage and magnitude of the input signal. Since most HVDC control scheme use gain scheduling strategy to compensate for firing angle values only, the actual gain can still change as a function of operating conditions. However, the operating conditions which will significantly change the gain values are not likely in practice.

The use of the developed linear models is justified for the practical operating conditions.
REFERENCES

Analysis of non-linear effects in HVDC control systems
CHAPTER 4

COMPOSITE RESONANCE ON HVDC-HVAC SYSTEMS

4.1 INTRODUCTION

This chapter attempts to offer a practical solution for elimination of a composite resonance condition on AC-DC systems.

Composite resonance is a type of complementary resonance where a high impedance parallel resonance on AC side of the system is coupled with the low impedance series resonance at an associated frequency on the DC side [1]. Composite resonance can happen at any frequency, depending on the characteristics of AC and DC systems.

If viewed independently, both AC and DC systems can have its resonant frequency (dominant oscillatory mode), where the frequency and damping of this mode is determined by the system impedance profile. AC system resonant frequency is determined by the network topology together with the AC side converter filters and shunt compensating equipment. The resonant frequency of DC system is determined by the DC line natural impedance characteristics in combination with smoothing reactors at both line ends. The DC system impedance is also influenced by DC system controls.

Because of the interaction of system impedance through AC-DC converter, the resonant frequency resulting from AC-DC coupling will be different from the individual system resonant frequencies. This resulting resonance should be kept away from harmonic frequencies [1], and particularly from first harmonic on DC side.

It has been concluded in [2] that the system impedance characteristics at frequencies close to second harmonic (AC side) has been contributing factor in development the reported cases of core saturation instability. Core saturation instability, which is independently addressed further in this Chapter, has occurred on several practical systems [3-5]. New Zealand HVDC system is also known to have resonant conditions very close to the second harmonic [6], which can lead to this form of instability.

Based on the experimental results, reference [7] concludes that a large number of DC lines (cables and overhead transmission lines) have pronounced first resonance at a frequency between 45-75 Hz, i.e. close to the fundamental system frequency. The majority of DC systems are therefore naturally bound to cause operating problems at fundamental frequency. Figure 4.1 shows the measured frequency response of New Zealand HVDC link, where AC systems are assumed disconnected. The dominant resonant peak at frequency close to the first harmonic is clearly evident from the Figure.
References [1] offers the most comprehensive analysis of the AC-DC composite resonance phenomenon and its consequence. In this reference, a general case of composite resonance is discussed and it is analytically confirmed that AC-DC instabilities are a consequence of AC and DC system impedance. Further, the influence of the existing HVDC system controls on the resonance condition is thoroughly analysed. Reference [3] similarly uses Eigenvalue analysis to prove that AC-DC instabilities at lower harmonics are a consequence of the DC (and AC) system impedance profile. Reference [2] elaborates on the conditions for core saturation instability and it concludes that the development of core saturation instability is determined predominantly by the system impedance characteristics. The importance of system impedance and natural resonance conditions is therefore evident for both composite resonance and core saturation instability. This Chapter offers a practical solution for modifying the system impedance.

The influence of HVDC controls on the above instabilities is studied in [1] and also in [8-10]. These references, however, do not offer firm guidelines for the controller design under the resonant conditions on main circuit. The recommendations for the lower controller gain at particular frequency in [1] are not further developed to propose a complete design procedure. Particularly, it should be noted that any controller modifications, as a consequence of resonant conditions, will inevitably change the system behaviour at other frequencies, and alter the system performance like speed of response or disturbance rejection.

This Chapter offers a complete design procedure for the HVDC controller, including rectifier and inverter side controls, under the resonant conditions. Based on the above conclusions, the primary controller goal is to modify the DC system impedance in such a way to minimise or defer the conditions which would lead to the development of AC-DC composite resonance.
4.2 TEST SYSTEM ANALYSIS

The model in use is the simplified linear continuous model presented in Chapter 1 Section 1.2, with test system data given in Appendix D. Table 4.1 shows the Short Circuit Ratio (SCR) for the three cases studied. Case 3 is the system as implemented in [11]. Controller parameters are the same for the three test cases. Different cases are used to demonstrate generality and applicability of the analysis and design. The cases differ significantly, representing the extreme cases in HVDC practice.

Table 4.2 shows the eigenvalues of the linearised model for the three test cases. It can be seen that the dominant oscillatory mode for the first two cases (53Hz and 48Hz) is very close to the fundamental frequency (50Hz) with very light damping. For the third case damping is still smaller but oscillatory mode is at a somewhat lower frequency (36Hz). All three cases in Table 4.2 offer stable system, however it can be assumed that under the contingencies, the system may approach the instability boundary.

4.3 CONTROLLER DESIGN

Main design objectives for the new controller are:

- To move the oscillatory mode away from the fundamental frequency,
- To increase the damping of the dominant oscillatory mode,
- To design a decentralised (no communications required between controllers) and easy to implement controller.

The final model equations from Section 1.3 (equations 1.116,1.117) are rewritten here:

\[ s\mathbf{x} = A\mathbf{x} + B\mathbf{u} + B_w\mathbf{w} \quad y = C\mathbf{x} \]  

(4.1)

State variables, outside inputs, control inputs, and outputs are chosen as:
The rectifier firing angle as well as the inverter firing angle are considered as supplementary control inputs to the system. In most practical systems the inverter is in a minimum gamma mode, although a number of inverter feedback strategies exist, as discussed in Chapter 7, where [12] is one example. If only the rectifier terminal is used for current control, since at best only two states are available for feedback, very limited freedom is available for modifying the system dynamics. A more flexible controller can be achieved if the inverter terminal is also used as another controlled input. However, if the inverter is operated in minimum gamma mode (which would often be the case), as far as the DC current control is concerned, no contribution would be provided through the action of the inverter. In order to provide enough room for beta modulation, as is required from an inverter actively participating in DC current control, the inverter nominal operating beta angle has to be slightly increased above that calculated using the minimum gamma criterion.

A new controller is designed using state feedback and the pole-placement technique. Using the states as defined in (4.2), the state feedback would be difficult to implement. It could be difficult to identify a point on the DC line or cable which actually represents $V_{cs}$. Even if such a point is identified, transferring this information to either controller site would involve large time delays. To get more convenient state-space representation, the following new set of states, $\tilde{z}$, is defined:

$$\tilde{z} = \begin{bmatrix} \frac{\alpha}{\beta} \\ y \\ y \end{bmatrix}$$

(4.3)

The new states are therefore: $I_r I_i dI_{i1}/dt, dI_{i1}/dt$, which can be easily obtained at converter sites. The system dynamics can now be represented in terms of the new states as:

$$s\tilde{z} = TAT^{-1} \tilde{z} + (TAT^{-1} S + TB) u + S u$$

(4.4)

where: $T = \begin{bmatrix} C \\ CA \end{bmatrix}$, and $S = \begin{bmatrix} 0 \\ CB \end{bmatrix}$

(4.5)

The following control law is applied:

$$u = K \tilde{z}$$

(4.6)

In order to achieve an arbitrary pole placement for a fourth order system, feedback of all four states is necessary, [13]. The feedback matrix is therefore chosen as:

$$K = \begin{bmatrix} k_{pr1} & 0 & k_{dr} & 0 \\ 0 & k_{pl} & 0 & k_{dl} \end{bmatrix}$$

(4.7)
Such a feedback matrix ensures the decentralised operation of the controllers, thus avoiding the need for any form of communication between them. Zeros in the feedback matrix ensure that rectifier does not require inverter states and vice versa. The important aspects of controller implementation are shown in the next section.

The system is now represented as:

\[
sz = Wz, \quad (4.8)
\]
\[
W = [I - SK]^{-1}[TAT^{-1} + (TAT^{-1}S + TB)K] \quad (4.9)
\]

Using the pole-placement technique, given the desired poles locations, \( s_1^0, s_2^0, s_3^0, s_4^0 \), the unknown parameters \( k_{pr1}, k_{dr}, k_{pr}, k_{di} \) from (4.7) are determined directly, by equating with the corresponding polynomial coefficients in (4.10).

\[
\det(sI - W) = (s - s_1^0)(s - s_2^0)(s - s_3^0)(s - s_4^0) \quad (4.10)
\]

The “MATHEMATICA” software is used for calculation of coefficients in (4.10), in a symbolic form directly from the above equation. Once the left side of (4.10) is obtained in polynomial form, the corresponding coefficients are equated with the known coefficients on the right side. Appendix C shows the system matrix with the feedback coefficients in symbolic numbers.

The above presented design algorithm will always offer at least one real solution for controller gains, for any given location of the desired poles.

The unknown coefficients from (4.7) can not be determined directly using Ackerman’s formula [13], since the system has two inputs. Also, the generalised Ackerman’s formula for MIMO systems [14], can not be used since the matrix \( K \) in (4.7) must have four zero elements, at exact locations. The method of equating corresponding coefficients, although it requires complex calculus with symbolic numbers, remains the only option for the controller design.

The desired poles are chosen to meet the controller design objectives and acceptably limit the controller action on the inverter side. Placement of the desired poles far away from the original poles was avoided as the resulting gains can be large, resulting in large control signals (firing angles). The frequency of the dominant complex poles for the closed loop system is chosen to be around 30-40Hz, (to avoid possible subsynchronous interactions), whereas the damping ratio is chosen as \( \zeta \geq 0.7 \). Table 4.3 gives desired pole locations for the cases considered. A different set of desired poles for case 3 is selected to avoid large gain values. In this case, the real system poles are kept at the same position (Table 4.2), whereas the location of the dominant oscillatory mode is significantly improved.

The calculated gains are shown in Table 4.4. Note that the sign change for proportional gain means only a reduction of the original rectifier proportional gain (\( k_{pr} = 0.0315 \text{deg/}A \)) as shown in the controller implementation section. The frequency response of the system with the new and the original controllers (Figure 4.2), demonstrates elimination of the resonant condition with the new control strategy.
TABLE 4.3. DESIRED POLE LOCATIONS.

<table>
<thead>
<tr>
<th></th>
<th>$s_{1/2}$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>-170 ± 170j</td>
<td>-120</td>
<td>-110</td>
</tr>
<tr>
<td>case 2</td>
<td>-170 ± 170j</td>
<td>-120</td>
<td>-110</td>
</tr>
<tr>
<td>case 3</td>
<td>-300 ± 200j</td>
<td>-315</td>
<td>-140</td>
</tr>
</tbody>
</table>

TABLE 4.4. CALCULATED CONTROLLER PARAMETERS.

<table>
<thead>
<tr>
<th></th>
<th>$k_{pr1}$ [deg/A]</th>
<th>$k_{ap}$ [deg/s/A]</th>
<th>$k_{pr}$ [deg/A]</th>
<th>$k_{ai}$ [deg/s/A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>4.2215e-2</td>
<td>-3.4867e-5</td>
<td>7.3165e-3</td>
<td>1.3881e-4</td>
</tr>
<tr>
<td>case 2</td>
<td>3.0465e-2</td>
<td>-1.9468e-5</td>
<td>1.2232e-2</td>
<td>8.4854e-5</td>
</tr>
<tr>
<td>case 3</td>
<td>-2.7038e-2</td>
<td>-7.476e-5</td>
<td>1.34409e-2</td>
<td>5.0566e-6</td>
</tr>
</tbody>
</table>

Figure 4.2. Frequency response between $E_{ac1}^d$ and $I_1$, case 3.

4.4 CONTROLLER IMPLEMENTATION

The rectifier controller with the new supplementary damping controllers can be schematically represented as in Figure 4.3, which corresponds to Figure 1.4 a) in Chapter 1. The inverter controller is shown in Figure 4.4. The inverter supplementary controller will affect only the constant beta operating mode, as shown in Figure 1.4 b) in Chapter 1. Constant current and constant gamma mode controllers are not changed.
4.5 SIMULATION RESULTS

4.5.1 Small disturbances

Figure 4.5 shows the simulation responses for case 3, where the analytical model is used. The merit of the additional feedback control is easily observed from the Figure. This Figure shows the design stage simulation results, with the simplified linear model. As shown in Chapter 1, the model uses very simplified representation of AC system and AC-DC interactions, however it is convenient for the controller design.

EMTDC/PSCAD non-linear simulation is used for final controller testing. Test system is the original CIGRE HVDC Benchmark model as implemented in [11]. Core saturation is disabled in the tests. System performance was evaluated for a large range of step changes in the disturbance inputs and reference signal as given in equation (4.2). As a small disturbance on the AC side, a
Composite resonance on HVDC-HVAC systems

tap changer action on commutation transformers (3.4% voltage change) is simulated. Figures 4.6 and 4.7 show the disturbed system responses for cases 2 and 3. It is seen that the responses with additional control have better damping. Figure 4.7 b) also compares the responses between analytical model and PSCAD/EMTDC simulation. As shown in Section 1.3 on the model development, the model is very simplified representation of the actual system and it is not expected to demonstrate good response matching with digital simulation. The main simplifications are: neglected AC system dynamics neglected PLL controllers and simplified AC-DC interactions. Despite the simplified form, the model can be used for HVDC controller design. It is also noted that the model shows better response matching for cases with stronger AC systems (cases 1 and 2),

It should be noted however, that the controller is designed for one system, CIGRE Benchmark model, and the results are not tested for any other particular system.

Figure 4.5. Case 3. System response following a step change on rectifier AC voltage (analytical model)

Figure 4.6. Case 2. System response following a 3.4% step change in the rectifier AC voltage (taping factor decrease at primary side).
4.5.2 Transient performance

The new controller is designed for small disturbances around the nominal operating point. Controller behaviour for large disturbances can not be predicted during the design stage. In order to evaluate the system behaviour after large disturbances, the system response for single and three phase AC faults at the inverter side as well as rectifier side was simulated. Figure 4.8 shows the responses for one of the worst fault cases. The initial overshoot is reduced and the oscillations have better damping for most of the responses obtained. The temporary current reduction to half the reference value is a result of VDCOL action.

![Graph showing responses for one of the worst fault cases.](image)

**Figure 4.8.** Responses for one of the worst fault cases. The initial overshoot is reduced and the oscillations have better damping for most of the responses obtained. The temporary current reduction to half the reference value is a result of VDCOL action.

4.5.3 Controller robustness

Controller robustness is tested using the modified HVDC Benchmark model as presented in [17]. This model has a very weak inverter AC system (Inv. \(SCR=1.5\)). The same controller parameters from the case 3 are used for the tests. Figure 4.9 shows that the new control method tolerates changes in the AC system strength (and system parameters in general).

![Graph showing robustness test results.](image)

**Figure 4.9.** Controller robustness test results. The new control method tolerates changes in the AC system strength (and system parameters in general).
4.5.4 Controller testing for second harmonic injection

This section evaluates the controller under the external injection of second harmonic. The aim is to study the harmonic content throughout the system when second harmonic is injected into AC systems. The external second harmonic injection into the system is not uncommon, it can be caused by converters, switching elements or transformers.

To test the system for the conditions of additional second harmonic injection, the following scenario is applied: a small amplitude, positive sequence second harmonic (100Hz) voltage source is applied to the rectifier or the inverter side AC system. The magnitude of the fundamental component of DC current is then measured with and without the new supplementary controller. The second harmonic on both AC systems is also measured. Table 4.5 summarises these simulation results.
4.6 CORE SATURATION INSTABILITY

Core saturation instability is probably the most important form of reported instabilities on HVDC systems. At least two HVDC systems have experienced this form of instability, Chateauguay [3], and Kingsnorth [4-5] HVDC system. Pole 1 of New Zealand HVDC system is also found to be vulnerable to this form of instability [6].

The core saturation instability can occur as a consequence of composite resonance conditions between AC and DC systems at the frequency close to the second harmonic, as seen on the AC side [2]. In this case, a very small AC system excitation at the second harmonic can lead to a fast growing fundamental component on the DC side, which will in turn cause growing of second harmonic on the AC side.

The mechanism for instability is presented in [15] and further analysed in [16]. The mechanism can be summarised as: Assuming a small second harmonic disturbance on AC voltage, because of the frequency transformation through converters, the first harmonic will be present on DC side voltage. The magnitude of first harmonic will depend upon system parameters but also upon controller parameters. DC current will be contaminated with the first harmonic with the magnitude depending on the DC system impedance (including DC controls) at first harmonic. The fundamental component on the DC side of the system will cause a DC component and a second harmonic on the AC side. The induced DC component on the AC side will cause the converter transformers to saturate (even DC of less than 10% of magnetising current may cause saturation). Saturated transformers further generate multitude of harmonics, including positive sequence second harmonic. This additional second harmonic on AC current will cause second harmonic on AC voltage, amplified by the AC system impedance at second harmonic. The closed loop for instability is thus completed.

### TABLE 4.5 A. HARMONIC CURRENT MAGNITUDE [A]. 1.6 KV, 3 PHASE POS. SEQ. HARMONIC INJECTION TO THE RECTIFIER AC SYSTEM.

<table>
<thead>
<tr>
<th>case</th>
<th>system</th>
<th>100Hz on Rec. AC side</th>
<th>50Hz on Rec. DC side</th>
<th>50Hz on Inv. DC side</th>
<th>100Hz on Inv. AC side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>orig. system</td>
<td>18.6</td>
<td>23.4</td>
<td>25.6</td>
<td>25.2</td>
</tr>
<tr>
<td></td>
<td>supplem. contr</td>
<td>22.1</td>
<td>15.2</td>
<td>4.6</td>
<td>15.1</td>
</tr>
<tr>
<td>2</td>
<td>orig. system</td>
<td>17.7</td>
<td>26.6</td>
<td>15.2</td>
<td>28.8</td>
</tr>
<tr>
<td></td>
<td>supplem. contr</td>
<td>9.7</td>
<td>19.1</td>
<td>4.2</td>
<td>20.2</td>
</tr>
<tr>
<td>3</td>
<td>orig. system</td>
<td>24.9</td>
<td>32</td>
<td>18.1</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>supplem. contr</td>
<td>12.6</td>
<td>23.8</td>
<td>10.9</td>
<td>21</td>
</tr>
</tbody>
</table>

### TABLE 4.5 B. HARMONIC CURRENT MAGNITUDE [A]. 1.6 KV, 3 PHASE POS. SEQ. HARMONIC INJECTION TO THE INVERTER AC SYSTEM.

<table>
<thead>
<tr>
<th>case</th>
<th>system</th>
<th>100Hz on Rec. AC side</th>
<th>50Hz on Rec. DC side</th>
<th>50Hz on Inv. DC side</th>
<th>100Hz on Inv. AC side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>orig. system</td>
<td>23.9</td>
<td>41.5</td>
<td>36</td>
<td>40.7</td>
</tr>
<tr>
<td></td>
<td>supplem. contr</td>
<td>10.8</td>
<td>7.8</td>
<td>17</td>
<td>49</td>
</tr>
<tr>
<td>2</td>
<td>orig. system</td>
<td>19.1</td>
<td>34.2</td>
<td>35.3</td>
<td>40.9</td>
</tr>
<tr>
<td></td>
<td>supplem. contr</td>
<td>11.2</td>
<td>10.7</td>
<td>21.7</td>
<td>19.4</td>
</tr>
<tr>
<td>3</td>
<td>orig. system</td>
<td>21.8</td>
<td>15.1</td>
<td>40.6</td>
<td>21.3</td>
</tr>
<tr>
<td></td>
<td>supplem. contr</td>
<td>23.7</td>
<td>7.7</td>
<td>36.4</td>
<td>16.5</td>
</tr>
</tbody>
</table>
The development of the instability is influenced by the system parameters in the above feedback loop. It is clear that the HVDC control loops will have effect on the core saturation instability development. However, the full conditions for the development of core saturation instability are not simulated in this project, and the controller is not tested under this form of instability. Because of the highly non-linear nature of core saturation phenomenon, the benefit of the developed linear controller is difficult to estimate without a detailed simulation.

4.7 DISCUSSION AND PRACTICAL ISSUES IN CONTROLLER IMPLEMENTATION

The controller has been designed using the presented analytical system model and using the MATLAB software.

The simplified system model can not give results closely matching the responses obtained using a non-linear digital simulation in a wide frequency range, however it is convenient for the controller design. The discrepancies between the model and the system become more pronounced as the AC system SCR reduces, since the AC system dynamics and AC-DC interactions are neglected, as discussed in Chapter 1. Simulation results shown in Figures 4.6, 4.7 and 4.9, however, demonstrate that the controller can eliminate the dominant oscillatory mode without introducing any negative effects at other frequencies. The design procedure is still general enough to be applied to a wide range of different systems, as shown by the different test cases considered.

Table 4.5 confirms that the new control strategy significantly reduces the first harmonic magnitude on the DC side, in some cases to $1/4$ of its original value. This reduction is a consequence of improved system impedance at first harmonic as shown in Figure 4.2. The controller will evidently improve the system characteristics under the external injection of second harmonic.

It is seen in Table 4.5, that the AC second harmonic is also reduced. There are however, two cases in Table 4.5 which give certain amplitude increase in the AC side second harmonic. The model in use can not explain the harmonic content an AC side. It is presumed that this can be attributed to the increased generation of second harmonic (and multitude other harmonics) at the converter, which comes as a consequence of the increased firing angle modulation [16]. To fully evaluate the influence of the designed controller on AC side harmonics it would be necessary to use more accurate system representation, like models reported in [2],[16].

Despite the above positive influence on the dominant oscillatory mode, the controller performance under the conditions for core saturation instability can not be envisaged. This phenomenon is extremely complex involving various non-linear effects, and it can not be included in the linear controller design. Further controller exposure to the core saturation conditions would be required for confirmation of possible benefits.

The new control strategy implies operation in beta constant mode on the inverter side. This mode requires a larger nominal operating firing angle and greater reactive power consumption as consequence. The increased operating angle will also impose a somewhat increased characteristic harmonic generation on the inverter side. For the considered test case the normal operating angle (beta angle) was chosen to be $40\text{deg}$, which proved to give enough room for ordinary control action for ordinary disturbances. The nominal operating angle was around $39\text{deg}$ for the original CIGRE HVDC model.
The deficiencies mentioned above should be weighed against the much more stable operation offered by this control scheme. Some of the HVDC schemes (New Zealand HVDC link for example) already operate in the constant beta operating mode. For such systems the limitations mentioned above are not relevant.

The proposed control scheme relies on the two differentiators for DC current derivatives on both sides of the link. Differentiators amplify noise, and especially in the presence of a large amount of harmonic current they could be difficult to implement. The alternative method is to measure the direct voltages on both sides of the smoothing reactor. The current derivative is then obtained as:

\[
\frac{dI_{dc}}{dt} = \frac{1}{L}(V_{d2} - V_{d1})
\]

where \( L \) denotes the smoothing reactance whereas \( V_{d2}, V_{d1} \) are the corresponding direct voltages. This signal proved to give a better signal/noise ratio than the direct differentiation method.
4.8 CONCLUSIONS

The AC-DC composite resonance, occurs as a consequence of high impedance parallel resonance at AC side coupled with low impedance series resonance at associated frequency on the DC side.

The composite resonance at second harmonic can accelerate development of core saturation instability, which have been experienced on a number of practical HVDC systems.

The controller developed in this Chapter tends to modify the DC system impedance profile in order to remove conditions for AC-DC system composite resonance. The state feedback control theory is used to modify the system’s frequency response through the whole frequency range. All-states feedback enables an arbitrarily placement of the system poles and elimination of possible adverse effects between AC and DC systems in a wide frequency range. This design method enables simultaneous controller design with respect to low frequency design requirements and also composite resonance problems at somewhat higher frequencies. Non-linear PSCAD/EMTDC simulation confirms that the new control method improves the system transient responses and small signal step responses.

Simulation tests with second harmonic injection on AC system prove that the new controller will noticeably reduce the first harmonic component on DC side, in some cases to ¼ of the original values. The controller is not tested for core saturation instability. The tests with close AC system faults similarly demonstrate superior performance of the new controller.

In designing this new controller, special emphasis is given to the controller implementation feasibility. The controller can be implemented with measurement of states which are locally available at converter sites. The controller is based on four simple feedback loops without any need for complex higher order transfer functions in the feedback loop.

The robustness tests have confirmed that the controller also gives an improvement in the system responses if the AC system SCR is considerably changed (reduced).
REFERENCES

Composite resonance on HVDC-HVAC systems
CHAPTER 5

CONTROL OF 100Hz OSCILLATIONS ON AN HVDC SYSTEM

5.1 INTRODUCTION

100Hz oscillations (of gamma and DC current) on the DC side of the New Zealand HVDC link have been measured for a long time, and the phenomenon is well documented [1]. Although not many other HVDC links have reported similar operating problems, the mechanism for this instability, suggests that a large number of HVDC systems could be vulnerable, depending on the system design and operating conditions.

The main cause of 100Hz DC side current oscillations is the unbalanced AC supply voltage [2]. The mechanism for occurrence of 100Hz DC side oscillations is presented later in this Chapter. Modulation of the converter firing angle at 100Hz, which comes as the consequence of (improper) current controller action, depending on the control system in use, may also contribute to second harmonic DC oscillations.

The unbalance in the three phase AC voltage magnitudes (and phases) arises as a consequence of unbalanced load in the system, or as a result of AC system faults. The unbalanced load is a result of actual consumer load unbalance, or a result of long untransposed transmission lines.

Fundamental frequency component of three phase AC voltages can always be expressed in terms of positive, negative and zero sequences. An AC voltage unbalance will give rise to a negative sequence fundamental component. As a consequence of negative sequence fundamental component in an AC system, a second harmonic component will be present in voltage/current on DC side of the connected HVDC system. The mechanism for frequency transformation through AC-DC converters is presented later in this Chapter.

The adverse effects resulting from an AC system unbalance can be summarised as:

- Negative sequence fundamental voltage component on AC side of the system. Negative sequence AC currents will cause heating in electrical machines.
- Second harmonic oscillations on DC side current. Converter station equipment on DC side can withstand only small amount of harmonic currents and only in certain frequency range.
- Second harmonic oscillations on inverter extinction angle. The mechanism for the transfer of oscillations on gamma is presented later in this Chapter. 100Hz gamma oscillations may introduce lower gamma values (at oscillation minimums) and the safe commutation margin could be endangered [1]. This is dependent on the control system in use.

The response of the existing HVDC controller on 100Hz oscillations will be different for the different control modes, and different type of controller. The control modes presented in Section 1.2.3 are discussed below.

In the case of rectifier direct current controller, there is always a low pass feedback filter which will attenuate higher frequencies. Because of the limited controller gain and phase shift at 100Hz, the controller corrective action on 100Hz direct current disturbance will be very limited. It can be expected, however, that this controller will somewhat help reducing 100Hz oscillations on direct
current. If AC voltage unbalance occurs at the AC system at rectifier side of HVDC link, the
danger of commutation failure is not present, and only the adverse effects related to AC voltage
and DC current oscillations are applicable.

The system under study in this Chapter employs constant beta control at inverter terminal, in
order to create a more stable positive slope on static HVDC curve. The case with possible
additional current feedback, which usually have a low gain and a low pass filter, is also studied.
The controller is presented in Section 1.2.3, and the firing angle is derived as shown by the
equation (1.33) or (1.35). In this operating mode, extinction angle is not directly controlled.
Therefore, in a case of outside disturbance on extinction angle, the constant beta controller will
not respond to correct actual extinction angle. There will be some 100Hz action through direct
current feedback, if employed. In a case of large disturbances, the inverter will change to constant
extinction angle control in an attempt to keep gamma at a sufficiently large value. Therefore the
existing constant beta controller will have little influence on 100Hz component on DC side (DC
current or extinction angle), as it is discussed later in the Chapter.

It is also assumed that constant gamma controller is of predictive type with direct
compensation of direct current and AC voltage disturbance, as in Figure 1.4 c). Since the speed of
these two control loops (Direct current and AC voltage) is limited, and because of phase shift that
they introduce at 100Hz, they are not able to efficiently “protect” gamma from 100Hz outside
disturbance. As it is shown in [1], a 5deg gamma oscillations have been measured on New
Zealand inverter gamma, and despite mode change to gamma control, these oscillations persist.
These 100Hz gamma oscillations can cause very low minimum gamma angles and therefore the
safe commutation margin can be endangered. If the oscillations grow, extinction angle may fall
below the safe commutation margin, and commutation failure will occur. Consequently the
operation of the whole HVDC link will suffer. Neither constant beta nor constant gamma
controllers in such inverter controller will be able to adequately respond to this disturbance. The
effect of higher order HVDC controls, which could possibly correct nominal gamma values, is not
considered. One possible solution is to manually increase gamma reference [1], with the
inevitable drawback of increased reactive power consumption, larger harmonic generation, and
increased equipment stresses.

In the case of gamma feedback type constant extinction angle controller, assuming also usually
employed non-linear elements that keep minimum gamma values over the last six firings, the
response to this disturbance is somewhat better. Since the actual gamma minimum will be
effectively measured, the minimum commutation margin will be safeguarded. The drawback is, as
above, in the form of gamma average increase and consequently harmonic generation and reactive
power consumption is increased. This type of controller is not further studied in this Chapter.

It will be attempted in this Chapter to offer a controller design procedure to counteract some of
the above negative affects.
5.2 HARMONIC TRANSFER BETWEEN AC AND DC SYSTEMS - ANALYTICAL REPRESENTATION

This section presents the basis on the theoretical modelling for the harmonic transfer through an AC-DC converter. The presented model is the very similar to the model developed in [3].

Interaction between AC and DC systems through 3-phase converters in the harmonic domain can be described using modulation theory for converters [3],[4]. Using this theory, the converter is treated as a modulator and input-output relationships can be expressed in general form as:

\[ V_{dc} = v_a S_{va} + v_b S_{vb} + v_c S_{vc} \]  \hspace{1cm} (5.1)
\[ i_a = I_{dc} S_{ia}, \quad i_b = I_{dc} S_{ib}, \quad i_c = I_{dc} S_{ic} \]  \hspace{1cm} (5.2)

where \( S_{va}, S_{vb}, S_{vc} \) and \( S_{ia}, S_{ib}, S_{ic} \) are the switching functions for voltage modulation and for current modulation respectively. Assuming equidistant firing pulse HVDC control system and assuming zero commutation overlap, the current and voltage modulation functions become the same. Using Fourier analysis they can be expressed as [3],[4].

\[ S_a = \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \alpha) \]
\[ S_b = \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \frac{2\pi}{3} - \alpha) \]  \hspace{1cm} (5.3)
\[ S_c = \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \frac{2\pi}{3} - \alpha) \]

where:

\( \omega_0 \) - switching frequency,
\( \alpha \) - converter firing angle.

The firing angle \( \alpha \) is added in this analysis to generalise the model from [3], and make it applicable to the converters with firing angle delay control.

If the AC voltages are assumed as a fundamental component with a series of harmonics, they are expressed:

\[ v_a = \sum_{s=1}^{1} \sum_{m=1}^{\infty} V_{sm} \cos(\omega_m t + \alpha_{sm}) \]
\[ v_b = \sum_{s=1}^{1} \sum_{m=1}^{\infty} V_{sm} \cos(\omega_m t - \frac{2s\pi}{3} + \alpha_{sm}) \]  \hspace{1cm} (5.4)
\[ v_c = \sum_{s=1}^{1} \sum_{m=1}^{\infty} V_{sm} \cos(\omega_m t + \frac{2s\pi}{3} + \alpha_{sm}) \]

where:

\( s=1 \) - positive sequence component,
Control of 100Hz oscillations on an HVDC system

$s=-1$ - negative sequence component,  
$s=0$ - zero sequence component,

and:

- $V_m$ magnitude of $m$-th harmonic.
- $\alpha_m$ phase angle of $m$-th harmonic.
- $\omega_m = m\omega_0$ frequency of $m$-th harmonic (supply frequency).

Using formula (5.1), (5.3) and (5.4) the DC side voltage, for 12-pulse converter, can be expressed as:

-for positive sequence components ($s=1$):

$$V_{dc} = \sum_{m=1}^{\infty} \sum_{k=0}^{\infty} \frac{3V_m^+}{2} \left\{ A_{(2k-1)} \cos\left[ (\omega_m + (12k-1)\omega_0) t + (12k-1)\alpha + \alpha_m^+ \right] + 
+ A_{(2k+1)} \cos\left[ (\omega_m - (12k+1)\omega_0) t - (12k+1)\alpha + \alpha_m^- \right] \right\}$$

(5.5)

-for negative sequence components ($s=-1$):

$$V_{dc} = \sum_{m=1}^{\infty} \sum_{k=0}^{\infty} \frac{3V_m^-}{2} \left\{ A_{(2k+1)} \cos\left[ (\omega_m + (12k+1)\omega_0) t + (12k+1)\alpha + \alpha_m^- \right] + 
+ A_{(2k-1)} \cos\left[ (\omega_m - (12k-1)\omega_0) t - (12k-1)\alpha + \alpha_m^+ \right] \right\}$$

(5.6)

whereas the zero sequence component can not be transferred to the DC side [3].

Assuming the AC supply voltages with only negative sequence fundamental frequency unbalance, ie $V_m^+ = 0, V_m^- = 0$ for $\forall m \geq 2$, DC side voltage can be expressed as:

$$V_{dc} = \sum_{k=0}^{\infty} \frac{3V_1^+}{2} \left\{ A_{(2k+1)} \cos\left[ (\omega_1 + (12k+1)\omega_0) t + (12k+1)\alpha + \alpha_1^+ \right] + 
+ A_{(2k-1)} \cos\left[ (\omega_1 - (12k-1)\omega_0) t - (12k-1)\alpha + \alpha_1^- \right] \right\}$$

(5.7)

Assuming also the supply frequency is equal to the converter switching frequency ($\omega_1 = \omega_0$), and $\alpha_1^\pm = 0$, the main oscillatory component on the DC side will be induced for $k = 0$, and DC voltage becomes:

$$V_{dc} = \frac{3V_1^+}{2} (A_{(1)} + A_{(-1)}) \cos \alpha + \frac{3V_1^-}{2} (A_{(1)} + A_{(-1)}) \cos (2\omega_0) t + \alpha$$

(5.8)

The first component in the above equation gives the known formula for DC voltage with commutation overlap neglected (A.54), (B.3). The second component shows that negative
sequence (fundamental frequency) unbalance on AC side induces the second harmonic component on DC side, with magnitude proportional to the magnitude of the original negative sequence component $V_1^-$. Harmonic content in DC side current will depend upon DC side impedance, as described by the equation:

$$I_{dc} = \frac{V_{dr}}{Z_{dce}}$$

(5.9)

where $Z_{dce}$ denotes the equivalent DC side impedance, which also includes remote converter, remote AC system and current controller.

In the case when commutation overlap is not neglected the above formula (5.8) will have different coefficients $A$ as shown in [3].

5.3 INVERTER EXTINCTION ANGLE OSCILLATIONS

The basic formula that gives the relationship between inverter extinction angle and DC current is [5] (Appendix B):

$$\cos \beta = \cos \gamma - \frac{2\omega_l L_c I_d}{\sqrt{3}E_m}$$

(5.10)

where:

- $\gamma$ extinction angle,
- $\beta$ inverter firing angle ($\beta=180-\alpha$),
- $E_m$ AC voltage magnitude (secondary side),
- $I_d$ DC current,
- $L_c$ transformer impedance.

It can be seen that inverter extinction angle (gamma) depends upon three variables: Inverter firing angle ($\beta$), DC current $I_d$ and AC voltage magnitude $E_m$. Therefore, the oscillations on the extinction angle will be induced by the disturbances (oscillatory component) on the above three variables. Similarly, using the formula (5.10), it is seen that gamma oscillations can be controlled by controlling the firing angle at particular frequency.
5.4 CONTROLLING THE ADVERSE EFFECTS OF 100Hz OSCILLATIONS ON DC SIDE OF HVDC SYSTEM

5.4.1 Control Strategy

The proposed control strategy is based on the DC current feedback. This control strategy requires modification of only the existing HVDC current controller.

Figure 5.1 shows the transfer function representation of the dynamics of a typical HVDC system. The DC current is considered as the system output, whereas AC system voltage and inverter firing angle are considered as disturbance and control inputs to the system, respectively. The AC voltage, DC current and angle gamma are assumed to be contaminated with 100Hz component.

\[
\Delta E_m \quad g_E(j\omega) \quad \Delta I_d \quad + \\
\Delta \beta \quad g_{\beta}(j\omega) \quad +
\]

*Figure 5.1. Block diagram of the uncontrolled system.*

In terms of input-output transfer functions, the system output is expressed as:

\[
\Delta I_d = g_E \Delta E_m + g_{\beta} \Delta \beta.
\]  
(5.11)

\(\Delta E_m\) and \(\Delta \beta\) are the system inputs, and the transfer functions:

\[
g_E = g_E(j\omega) = \frac{\Delta I_d}{\Delta E_m} \quad \text{and} \quad g_{\beta} = g_{\beta}(j\omega) = \frac{\Delta I_d}{\Delta \beta}
\]  
(5.12)

are assumed known for a given system. They are found experimentally using a simulator model, or similarly they can be calculated using an analytical model of the system.

By considering the detailed HVDC-HVAC system model, developed in Section 1.2, the above transfer functions are derived from the state-space system model (1.90-1.91), for the considered inputs and outputs, i.e:

\[
g_E(s) = \frac{I_d(s)}{E_m(s)} \quad \text{and} \quad g_{\beta}(s) = \frac{I_d(s)}{\beta(s)}
\]  
(5.13)

Considering Figure 1.7 and equation (1.73), where the firing angle \(\beta\) is shown as an additional control input, the feedback control strategy for the new controller is assumed to be:

\[
\Delta \beta = g_c(j\omega) \Delta I_d
\]  
(5.14)
In the case of New Zealand HVDC system, the firing angle depends upon direct current through the current control feedback loop for $\alpha_{\text{nom}}$ operating mode (constant beta mode) as shown in Figure 1.4 and equation (1.35). It is therefore, only necessary to modify the existing controller characteristics at the frequency of interest for this system. At steady state and lower frequencies, controller performances remains unchanged. For other systems, constant beta operating mode has to be introduced with the additional modulation signal supplied from the new controller.

The system with the new controller is shown in Figure 5.2. Since the intention of the controller is to reduce the 100Hz oscillations on DC side, the value of the controller transfer function at 100Hz is of particular importance. Hereafter, the magnitude of the feedback function at 100Hz will be defined as $k$. The feedback gain $k$, is in general, a complex number.

$$
\Delta E_m 
\begin{array}{c}
g_E(j\omega) \\
\Delta \beta \\
g_\beta(j\omega) \\
g_c(j\omega) 
\end{array} 
\Delta I_d
$$

Figure 5.2. Block diagram of the controlled system.

If the equation (5.10) is linearised around the nominal operating point, it becomes:

$$
c_\gamma \Delta \gamma = c_\beta \Delta \beta + c_i \Delta I_d + c_E \Delta E_m
$$

where the corresponding coefficients can be readily derived.

The feedback coefficient (gain) can be calculated by considering the controller goals. If the controller objective is to annul gamma oscillations, the feedback gain can be calculated from (5.11) and (5.15). If $\Delta E_m$ is eliminated from equations (5.11) and (5.15), and if $\Delta \gamma = 0$ is assumed, the following expression is derived:

$$
0 = c_\beta \Delta \beta + c_i \Delta I_d + c_E \frac{1}{g_E} (\Delta I_d - g_\beta \Delta \beta)
$$

If the equation (5.14) is replaced in (5.16), the feedback gain $k$ can be obtained as:

$$
k_\gamma = \frac{\Delta \beta}{\Delta I_d} = \frac{c_i + c_E \frac{1}{g_E}}{c_\beta - c_E \frac{g_\beta}{g_E}}
$$
This controller gain would completely eliminate gamma oscillations.

If all the AC-DC interactions have been properly considered in deriving the transfer function $g_E(j\omega)$, it can be assumed that the inverse of this transfer function also exist, i.e.

$$\Delta E_m = \frac{1}{g_E(j\omega)} \Delta I_d$$  \hspace{1cm} (5.18)

In this case, if the controller is intended for elimination of AC voltage unbalance, the feedback coefficient can be calculated from (5.11), by putting $\Delta E_m = 0$ as:

$$k_{Em} = \frac{1}{g_\beta}$$  \hspace{1cm} (5.19)

Since the controller is using DC current feedback, the attenuation in DC current oscillations will be achieved with negative feedback loop. The condition to be imposed on the controller gain for cancellation of DC current oscillations is given by:

$$k_{sd} < 0. \hspace{1cm} (5.20)$$

The actual attenuation of DC current oscillations will be proportional to the gain magnitude.

Any other control strategy can be readily derived from the above formulae. If the controller objective is known, with respect to all three variables ($E_m, I_d$ and $\gamma$), the controller gain can be calculated by combining the equations (5.17),(5.19) and (5.20). The equations for controller gains are in the complex domain, and as a result the controller gain is obtained in terms of magnitude and phase at frequency of 100Hz.

As an example for the above control strategy, the controller design procedure assuming the New Zealand HVDC system configuration is presented. More details on this controller design can be found in [6]. In the case of New Zealand HVDC system the controller has been designed for inverter side of the system. The controller objective was to minimise gamma oscillations with no negative effect on the AC system unbalance. Figure 5.3 shows the controller gains for this system. The gains are obtained experimentally (trial and error) using the FFT components in PSCAD/EMTDC.

The controller gain $k_{c3}$ is the gain which meets design requirements. The gain $k_{c3}$ is orthogonal with $k_E$, and no negative affect on AC voltage unbalance will be introduced. At the same time, this controller gain creates an angle with the gain $k_\gamma$ which is less then 90deg, and therefore gamma oscillations will be attenuated. It is however evident that this gain will actually increase oscillations on DC current.

Although theoretically feasible, the gain for cancellation of AC system unbalance ($k_{Em}$) has magnitude value which can not be practically implemented, since it would imply large firing angle modulation, and controller switching to the gamma minimum mode. In the case of New Zealand HVDC system, there is a 2deg margin between constant beta and constant gamma mode.
The gain $k_p$ in the Figure stands for the existing current controller gain. This existing current controller is primarily intended for system stability enhancement at lower frequencies. It is seen that this gain gives very little contribution towards attenuation of gamma oscillations. The controller gain $k_{c5}$ introduces the same AC voltage unbalance as the original controller, however with much more attenuation in gamma oscillations.

**nomenclature:**

- $k_\gamma$: controller gain for elimination of gamma oscillations,
- $k_{Em}$: controller gain for elimination of AC voltage unbalance,
- $k_{Idc}$: controller gain for attenuation of DC current oscillations,
- $k_p$: existing controller gain,
- $k_{c5}, k_{c3}$: new controller gains.

**Figure 5.3. Controller gains for New Zealand HVDC system.**

### 5.4.2 The main application areas of the proposed control method

The proposed controller can be beneficial in the following three circumstances:

1) An unbalanced AC system connected to the inverter side of HVDC link (approximately 1.6% negative sequence component was causing significant problems on New Zealand HVDC system): The controller will in these circumstances reduce steady-state gamma oscillations and the controller will improve the commutation margin during AC system disturbances. The best control strategy is the control of inverter gamma oscillations. However, the value of the controller gain should be a trade off between the reduction of gamma oscillations and increase in AC system unbalance. The final placement of the
controller gain in the complex plane will depend on restrictions placed on the AC system unbalance and on DC current oscillations.

2) A balanced AC system connected to the inverter side of HVDC link: In this case typically no steady-state gamma oscillations will prevail. However with the properly designed controller, the commutation margin can be improved in the case of AC system disturbances (single phase fault). The value to be controlled is inverter gamma, but different control strategy is suggested. The controller gain should be calculated to give maximum attenuation of gamma oscillations, i.e. the gain \( k_\gamma \) from eq. (5.17) should be used.

3) An unbalanced AC system connected to the rectifier side of HVDC link: The controller, under these circumstances can be used for control of either AC system unbalance or DC current second harmonic oscillations. Extinction angle oscillations and commutation failure are not of importance at rectifier side of the system. The value of the controller gain will depend on the desired emphasis placed on controlling AC system unbalance or DC current oscillations.

5.5 CONTROLLER DESIGN

5.5.1 Design Objectives and Technique

This section presents an example of controller design procedure. The earlier presented discrete model of the system will be used in the design. The primary controller objective is attenuation of second harmonic oscillations on DC current. For any other controller goal, the design is similar. The controller is designed for rectifier side of HVDC link, where the test system in use is the CIGRE Benchmark model [7].

The controller has structure of a low pass filter implemented in several stages, and placed in series with the existing controller (series compensation). Cascade compensation techniques for linear systems is used in an attempt to modify frequency response of the HVDC system. With respect to the system stability requirements and design objectives, the new controller must meet the following criteria:

- To extend system bandwidth beyond the second harmonic, with the controller gain as defined in (5.20).
- At frequencies far beyond 100Hz the controller should not contribute towards development of harmonic instabilities. Of special importance is that the controller ensures satisfactory gain (and phase) margin at 300Hz, in order to avoid aliasing, and half the sampling frequency instability.
- System stability in other operating modes should not be deteriorated.
- Implementation of the controller should be technically feasible.

To perform controller design in frequency domain bilinear transformation is used [8], given by:

\[
z = \frac{r - 1}{r + 1},
\]

which results in the frequency transformation given as:
\[ \omega_r = \tan\left(\frac{\omega T_s}{2}\right) \]  

(5.22)

where:

- \( \omega_r \) frequency in r-domain,
- \( \omega \) frequency in z (and s)-domain
- \( T_s \) sampling interval.

This transformation enables more accurate system representation at frequencies close to half the sampling frequency [8]. Half the sampling frequency in z-domain becomes infinity frequency in r-domain.

The newly designed compensator consists of two parts connected in series, as shown in Figure 5.4. Second order compensator (Compensator 1) is introduced to extend system bandwidth and to cancel the unwanted poles in the plant (pole-zero cancellation design). First order compensator (Compensator 2), will introduce desired gain margin at the higher frequencies. These two compensators can give basic shape to the frequency response curve, whereas final values for the system bandwidth can be adjusted by changing controller gain \( K_a \).
Control of 100Hz oscillations on an HVDC system

Figure 5.4. HVDC system discrete-model with two cascade compensators.

5.5.2 Second Order Compensator (Compensator 1) design

Figure 5.5 show root locus for the original system. Desired system roots can be found with respect to the design objectives and desired frequency response.

It can be noted from Figure 5.5, that the HVDC system has a pair of lightly dampened complex poles at a frequency close to the fundamental frequency. These poles indicate a natural oscillatory mode manifested as a consequence of the resonant DC line impedance combined with the smoothing reactances. In order to cancel the pair of plant complex poles, this compensator must be of the second order. Zeros of the compensator 1 are placed to cancel unwanted plant poles (pole-zero cancellation design method). These zeros can be determined directly in s-domain.
In order to extend the bandwidth of the system, poles for the compensator 1 must be added at a frequency beyond the existing crossover frequency and close to the value of the new crossover frequency. Frequency and damping ratio uniquely determine the location of each pole in z-plane. Very high damping ratio is chosen, in order to avoid new oscillatory modes in the system response. Therefore, the poles for compensator 1 should be located in area A in Figure 5.5. The exact location of these poles is not crucial for the design, since the final controller adjustment is done at a later stage, by changing the open loop gain.

One of the methods to determine the position of poles more accurately is by using the graphical techniques. Using the rules for mapping from s-plane into z-plane, for each particular frequency in s-domain corresponding frequency point on unit circle in z-domain can be found [8]. System gain at each frequency \( \omega \) can be calculated as, (Figure 5.5):

\[
A(\omega) = \sum_{i=1}^{4} d_i(\omega) - \sum_{i=1}^{4} l_i(\omega)
\]

(5.23)

If the gain is known at four frequencies of interest, location of poles can be obtained deterministically from the formula (5.23). The location of each pole is uniquely determined by its distance from two points on the circle. Since the location of zeros is known, four system gains are needed for calculation of parameters of compensator 1. Once the poles are determined in z-domain, they can be found in s-domain using inverse z-transformation.

Referring to the Figure 5.4, it can be seen that the sampler is placed between the compensators and the plant. This system structure implies two separate z-transformations: one for the controller and the other for the plant. This is demonstrated in the numerator of the final transfer function for the discrete model (1.140), in Section 1.4.2. Because of these separate transformations, ideal cancellation can not be achieved and a small fundamental oscillatory component will exist in the system response.
5.5.3 First Order Compensator (Compensator 2) design

Compensator 2 is chosen as a phase-lag in order to increase gain margin at half the sampling frequency. Phase-lag compensator will initially shrink bandwidth of the system because of the reduced gain at higher frequencies, however since gain margin is now increased there is much more room for increase of controller gain $K_c$. Increased controller gain will in turn extend the system bandwidth. Regarding noise amplification problem, phase-lag compensator also has an advantage over phase-lead compensator. The pole and the zero for the compensator 2 can be determined from Bode diagram in r-domain (Figure 5.6) using rules for design of linear continuous systems in frequency domain. Frequencies $\omega_d$ and $\omega_n$ correspond to the compensator pole and zero location, respectively (also area $B$ and $C$ in Figure 5.5). Since there is a unique transformation for poles between $z$ and $s$ domain, determined pole can be directly transferred to $z$ domain and $s$ domain.

![Figure 5.6. r-domain Bode diagram for HVDC system](image)

There is no unique translation between zeros in $s$-plane and zeros in $z$-plane. Zeros in $z$-plane also depend upon other zeros and location of poles in $s$-plane. This issue is more pronounced since modified $z$-transformation is used in parallel with $z$-transformation. When the determined zero is transferred to the $s$-domain it gives system which can not be physically implemented as a cascade compensator. To overcome this difficulty, an emulation design approach [9] is used instead. By knowing the location of compensator pole ($s_d$) in $s$-domain, the zero location is determined as:

$$s_n = as_d, \quad a < 1$$

(5.24)
Initial value for $a$ is chosen when the compensator transfer function is transferred from $r$ to $s$-domain independently of the remaining system, i.e., as:

$$
\frac{r + \omega_{nr}}{r + \omega_{dr}} > \frac{z + \omega_{nz}}{z + \omega_{dz}} > \frac{s + \omega_{ns}}{s + \omega_{ds}}
$$

(5.25)

Step responses and frequency response for the discrete system are then observed, and the initial value for $a$ is modified. As the value for $a$ is changed, the system behaves in a similar manner to a continuous system, and after several adjustments, the responses become satisfactory. Table 5.1 shows calculated controller parameters (corresponding to Figure 5.4). Frequency response for the compensated system is shown in Figure 5.6.

<table>
<thead>
<tr>
<th>$K_a$</th>
<th>$s_{n1}$</th>
<th>$s_{n2}$</th>
<th>$s_{d1}$</th>
<th>$s_{d2}$</th>
<th>$s_n$</th>
<th>$s_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.23</td>
<td>130690</td>
<td>90.2</td>
<td>900000</td>
<td>1690</td>
<td>12048</td>
<td>2857</td>
</tr>
</tbody>
</table>

Inspecting the calculated controller parameters for compensator 2, it is seen that the zero in $s$-domain $s_n$ is placed at frequency of 1900Hz. When transferred to $z$-domain, the compensator zero is located at 290Hz. Similar relationship stands for the compensator pole. This result, once again, confirms that an HVDC system cannot be modeled as a continuous system when frequencies of interest are close to half the sampling frequency. By using a linear continuous model, this zero would be at 1900Hz. This conclusion complements the results from comparison between continuous and discrete model in Section 1.5.

Figure 5.7. Root locus, compensated system

Referring to Figure 5.5, it is clear that if the same design procedure is followed and if more than two compensators are introduced, still better frequency response can be obtained. However, since the two compensators meet the design objectives, excessively complex controller structure is avoided. Figure 5.7 shows the compensated system root locus. Figures 5.8 and 5.9 show the disturbance step response and the current order step response respectively.
5.5.4 Controller Design With Respect To The System Operation In Different Control Modes

It is assumed that the system normally operates in constant beta mode, and the above presented controller design is carried out for this operating mode. However, during disturbances, system can switch to the constant (minimum) gamma operating mode or inverter in constant current mode. In the case of this mode change the system structure will change and the transfer function representing DC system in Figure 5.4 will change. In the case of change from constant beta to constant gamma mode, the expression for inverter direct voltage will change as per equations (1.34) and (1.39) and consequently system response, as seen from the rectifier controller, will change. Therefore the controller designed for beta mode might get detuned in gamma mode. Although the controller is designed for rectifier side, inverter mode changes must be taken into account to verify that the controller is robust against these different system structures. The main design objective for these two modes is the stable system operation, since the system is not expected to operate in these modes for prolonged period of time.

![Figure 5.8. System response after disturbance step change.](image)
The current controller on rectifier side remains the same regardless on the actual operating mode at inverter, and since robustness criteria are not considered in the design stage, the controller must eventually be tested for both, constant beta and constant gamma modes. HVDC system in constant gamma mode has different poles and they will not be cancelled with the same designed compensator. Some compensator parameters which give good results in the normal operating mode can even make the system conditionally stable in constant gamma mode.

Therefore, the zeros of the Compensator $I$ must be compromised to ensure stable system operation throughout all modes. Since the structural difference for constant gamma mode is not significant, only a small modification of compensator zeros is required. Figure 5.10 shows root locus for constant gamma mode with the same compensator parameters given in Table 5.1. Detail "A" shows that the whole root locus is inside the unit circle and the system is stable for all values of the controller gain $K_c$.

If the system moves to constant current operating mode at inverter side, rectifier controller hits minimum limit and designed controller becomes ineffective.

If the inverter is expected to take over current control for a longer period, then the inverter current controller should be modified in a similar way as the rectifier controller. System structure for inverter in constant current operating mode is similar to that in constant beta mode. Operating point is however different and some parameters in system model are different.
5.5.5 Controller Performance Evaluation

The improvement in the system step response is evident from the Figures 5.8 and 5.9. Settling time and overshoot are reduced and speed of response is increased. These improvements come from increased controller gain and from the extended system bandwidth.

To test the system for harmonic disturbances, a 100Hz oscillations are applied to the direct current (noise added to direct current in Figure 5.4) and to the rectifier (inverter) AC voltages $E_{ac1}^d, E_{ac2}^d$ in Figure 5.4. These harmonic disturbances can be assumed to be representing voltage phase unbalance in the actual AC system. The developed model does not employ Park’s transformation, and it can not accommodate frequency transformation through the converter. Consequently, 100Hz oscillations are simulated directly on DC side.

Figure 5.11 shows the rectifier current response, following a 100Hz disturbance on rectifier AC voltage (DC side equivalent), for the same controller parameters from Table 5.1. It can be observed that amplitude of oscillations is reduced to less than 30% of its original value. These responses can change with different controller gain values. Figure 5.12 shows the inverter direct current response after the same disturbance from the Figure 5.11. The effect of the new controller on inverter current oscillations is similar to the effect on rectifier direct current oscillations. If disturbance originate on rectifier side of HVDC system, rectifier controller can attenuate resulting oscillations on both ends of the HVDC system.

If the DC current magnitude (for original system), is compared in Figure 5.11 and in Figure 5.12, it is evident that the HVDC system naturally exhibits significant damping of 100Hz oscillations. Because of the HVDC line low pass characteristics, oscillations on inverter DC current are 4-5 times less then the rectifier DC current oscillations. This observation directly implies that the inverter current controller can not be used for damping of oscillations originating on rectifier side of the system. If the second harmonic originate on inverter side, for the same
reason the rectifier controller modifications will not be effective in damping the resulting oscillations. However, as far as the controller design procedure is concerned, inverter side design would follow the same methodology as presented above.

Figure 5.11. Rectifier direct current response following a 100Hz disturbance on rectifier DC side, (Rectifier AC voltage, DC side equivalent disturbance).

Figure 5.12. Inverter direct current response following a 100Hz disturbance on rectifier DC side, (Rectifier AC voltage, DC side equivalent disturbance).

5.5.6 Influence Of System Parameters On Controller Performance
The controller performance is tested for the system gain changes and compared with the original system. It is presented earlier in Chapter 3, that the converter gain is a complex non-linear function of several parameters and that each new controller must be tested against system gain variations. Figure 5.13 shows the DC current response after 30% gain reduction on rectifier side. It is evident that the original controller is much more sensitive to the gain variations. These conclusions are however obtained for CIGRE model and may not be valid for other systems.

![Figure 5.13. Influence of system gain changes on controller performance.](image)

5.5.7 Discussion and implementation issues

Referring to Figure 5.6, frequency response for original system shows that cross-over frequency is very low and that the original system is not able to reduce $100Hz$ oscillations on DC current. Only the increase in controller gain, for the same controller structure, is not possible since the oscillatory mode close to $50Hz$ would be more pronounced and gain margin would be reduced.

System with the designed new controller has cross-over frequency for open loop transfer function between third and fourth harmonic, and all oscillations below this frequency will be attenuated with new controller. However, the controller is not tested for third harmonic.

Gain margin for original system (infinity frequency in Figure 5.6), is relatively low indicating that the system could experience instability at $300Hz$. As a consequence of a change in operating conditions, the system gain could increase and stability at higher frequencies can be jeopardised. Since the gain margin is increased with the new controller, the system will be more robust to the open-loop gain changes.
Elimination of system natural oscillatory mode is done in this Chapter using pole-zero cancellation. It is well known that this method lacks robustness. Since the real system parameters are never exactly known, the perfect cancellation of unwanted poles is never achieved. The problem becomes still pronounced when the system is expected to operate between two different modes. For the HVDC systems with lightly damped complex eigenvalues below the second harmonic frequency, it is important to eliminate this oscillatory mode (possibly using different control strategies) before the presented controller design is carried out. Once the system frequency response is made well damped below the second harmonic, the task of bandwidth extension becomes less difficult. Also, for some other control strategies, it may be better option that the supplementary controller does not interfere with the existing controller at lower frequencies.

Practical implementation of designed controller, since it operates at higher frequencies, will depend upon the speed and the operating cycle time of the Converter Firing Control (CFC) computers of the particular HVDC system.
5.6 CONCLUSIONS

By using the known formulae for harmonic transfer through AC-DC converters it can be shown that the main reason for 100Hz oscillations on DC side of HVDC system is the voltage unbalance on the AC system. The main negative effects caused by the AC voltage unbalance arise from high magnitude second harmonic DC current oscillations and gamma oscillations.

If a direct current feedback is used at frequencies close to 100Hz, than some of the negative effects can be eliminated. By changing the controller complex gain (phase and magnitude), the new controller can eliminate AC voltage unbalance or direct current oscillations or gamma oscillations. The three controller gains for the three above extreme cases, should be calculated as a reference gains. Any other control strategy can be easily derived between the three extreme cases, if the controller primary aim is clearly defined.

The discrete HVDC system model should be used for controller design around 100Hz. The controller design is based on the root-locus technique from the discrete systems theory. The proposed modification consists of two cascade compensators with the transfer functions that can be readily and practically implemented. It is demonstrated in this Chapter, how the 100Hz oscillations on DC current can be reduced by applying modifications to the existing HVDC current controller.

The simulation results, on linear system model, show that the new controller can reduce direct current oscillations to less than $1/3$ of its original magnitude, on both ends of DC line. The original oscillations are however 3-4 times lower at the opposite end of HVDC line, relative to the source of oscillations.

The controller design is presented for elimination of direct current oscillations and for rectifier side only. A similar design should be followed for any other controller goal and for controller location at inverter side.
REFERENCES:


CHAPTER 6

SMALL SIGNAL ANALYSIS OF HVDC-HVAC INTERACTIONS

6.1 INTRODUCTION

The analysis presented in this Chapter offers a qualitative insight into small signal HVDC-HVAC interaction mechanism in the frequency domain \( f \leq 100\text{Hz} \), with emphasis on specific phenomena which have caused some of the experienced HVDC operating problems. Of major importance are the two groups of reported HVDC-HVAC interaction problems: composite resonance instabilities close to the second harmonic (including core saturation instability) and, problems related to the low Short Circuit Ratio (SCR) AC systems connected to DC systems.

The above presented operating problems have deserved a noticeable attention in the available HVDC bibliography. The composite resonance is best described in [1], whereas core saturation instability in the frequency domain is analysed in [2-4]. In these references, the implications of the phenomenon are well elaborated and the importance of AC-DC interactions are confirmed. These studies however do not analyse each particular interaction variable arising from coupling between AC-DC systems, and their implication on the phenomenon. In a similar manner, a recent paper [5] presents an excellent algorithm for analysis of the core saturation instability. This algorithm, based on the experimentally calculated AC-DC interaction matrices, can accurately predict whether or not the instability will occur for a particular system and at a particular frequency. However, algorithms of this type, can not give qualitative small signal conclusions about the mechanism and nature of the instability, as dependent on the subsystem properties. In a view of designing HVDC controllers under the conditions for instability, it is important to know the small signal system behaviour, and small signal properties of the system variables. Similarly, by knowing the properties of each independent system, it is important to be able to predict the possible issues when the systems are coupled together. It is shown that the controller designed in Chapter 4 can noticeably defer conditions for composite resonance. However, since a simplified HVDC model is used, the mechanism for composite resonance is not well studied in Chapter 4.

The problems associated with low SCR AC systems are also well presented in literature [6],[7]. Similar to the second harmonic instability, an in-depth analytical explanation of the problem, and particular aspect of AC-DC coupling, can not be found in the available literature.

The complexity of HVDC-HVAC interactions and well known difficulties in modelling of HVDC-HVAC coupling, are the main reason for the non-availability of a wide analytical background to these problems. In this Chapter, the earlier developed detailed, HVDC-HVAC model will be used.

The important points which need to be addressed in a study of HVDC-HVAC interactions can be summarised as:

- By knowing the independent behaviour of AC and DC systems, is it possible to foresee the behaviour and possible stability problems when the systems are coupled together?
- What is the nature and the frequency range for the expected problems at a particular interconnection point? What side of the system (rectifier or inverter) is more likely to encounter operating difficulties?
Since the SCR is a very important indicator of AC system characteristics, the analysis of eigenvalue movement as a consequence of SCR changes, would give invaluable practical information about expected stability problems.

The variables which characterise HVDC-HVAC interactions and which participate in inherent feedback loops between the systems, should be identified. Each one of these feedback loops and variables will naturally contribute either towards improvement or deterioration of the overall system stability. The frequency range where a particular variable mostly affects the system stability should also be determined. This analysis helps in determining the interaction variables which need to be controlled, and at the same time the interaction variables which are better to be left uncontrolled since they inherently improve the system stability.

Since there are two possible sources of disturbances on two ends of HVDC link, rectifier AC voltage and inverter AC voltage, it is also important to determine the worst case input-output directions which can cause stability problems. By knowing a particular combination of the two disturbances which will cause the worst stability problems, it is possible to study the control method that guarantees satisfactory operation for all operating conditions. Multi-input multi-output (MIMO) analysis of HVDC system compliments the basic dynamic analysis, to reveal the inherent dynamic properties that come forward when two different disturbances sources act simultaneously.

This chapter attempts to answer the above posed questions. The main method of the analysis is the eigenvalue decomposition and singular value decomposition. The model in use is the detailed linear continuous model, developed in Section 1.2, with the CIGRE HVDC Benchmark test system.

6.2 EIGENVALUE DECOMPOSITION BASED ANALYSIS

6.2.1 Eigenvalue placement as a consequence of system coupling

This section studies the eigenvalues of each subsystem independently, in order to reveal their influence on the overall system eigenvalues, when the systems are connected. By comparing the eigenvalue location of the overall system with the eigenvalues of subsystems, the negative side of AC-DC interactions can be studied.

Table 6.1 shows the dominant eigenvalues of the DC system, considered independently of the AC systems. The interactions between the systems are disabled in this analysis. From the DC system point of view, this would correspond to the case when AC voltages on both line ends are infinitely strong.

There are two pairs of complex eigenvalues (pair 2 and 3), which arise as a consequence of DC line dynamics. Eigenvalue pair 2 is a consequence of the DC system natural resonant condition close to first harmonic. Eigenvalue 1 represents the feedback filter time constant. Eigenvalue 4 comes from the artificially introduced dynamic equations for AC-DC interactions (there are four identical eigenvalues which are not shown). Eigenvalues 5-8 represent PLL dynamics at both ends.
Table 6.1 shows the eigenvalues of rectifier AC system. The system has these eigenvalues when DC system is not connected. The complex pair 4 is dominant. These eigenvalues represent the AC system resonant condition close to second harmonic, which is transferred to frequency close to first harmonic after the $dq$ transformation. Eigenvalues 5 (frequency is exactly $2\pi f$) are created from the AC system real poles.

Table 6.2 shows the eigenvalues of inverter AC system. The complex pair 5 is dominant with frequency close to second harmonic on AC side. Eigenvalues 6 are created from the AC system real poles.

Table 6.3 shows the 16 dominant eigenvalues of the test system (HVAC-HVDC-HVAC system). The frequency of the eigenvalues is shown as seen at the DC side, i.e., after $dq0$ transformation is applied. The eigenvalues are ranked on the basis of magnitude of their real part. If the eigenvalues of independent subsystems (AC and DC systems) are analysed and compared with the eigenvalues of the overall system, it can be found that the complex pair 1 is a consequence of the
rectifier side AC-DC coupling. These eigenvalues have been significantly moved towards the imaginary axis, compared to the original eigenvalues (pair 4 in table 6.2). Eigenvalues 4 originate from inverter AC system (pair 5 in Table 6.3), and their movement towards imaginary axis is slower than the rectifier AC system eigenvalues. Eigenvalues 1 and 4 reflect negative AC-DC interactions which can cause composite resonance (close to first harmonic on DC side) at rectifier and at inverter side respectively. These eigenvalues have been destabilised by AC-DC connection. It is evident that at rectifier side the resonant condition is more pronounced.

The eigenvalues 2 and 3 have been transformed from the rectifier/inverter AC system real eigenvalues (eig. 5 in Table 6.2 and eig 6 in Table 6.3). They do not move by the coupling with DC system, and for most of the considered system outputs, they appear with “pined” zeros (at exact location as poles), and thus have no importance in analysing the system behaviour. Hence, the real AC system poles do not seem to have any significance for AC-DC interactions. It is however noted that these poles might have influence on the development of core saturation instability.

The complex pair 5 is created from the DC system low frequency dynamics (eig. 3 in Table 6.1) with somewhat improved damping than in the case of independent DC system. The eigenvalues 6-11 are coming from the PLL dynamics and other DC line real eigenvalues, and they do not undergo significant movement by joining of AC and DC systems. These real eigenvalues can become low frequency complex pairs under different operating conditions, as presented later in this Chapter.

Although CIGRE Benchmark model is made to represent universal HVDC systems, the above conclusions have not been verified for any practical system.

### 6.2.2 Eigenvalue sensitivity and participation factor analysis

This section analyses the sensitivity of dominant system eigenvalues, with respect to the system parameters. The aim of this analysis is to determine which subsystem, when perturbed, can cause movement of the dominant eigenvalues. In this way, it is possible to determine the system parameters change, or change in load pattern or contingencies, which can destabilise particular eigenvalue. The frequency range of possible interaction problems as a consequence of change of operating conditions, can also be readily obtained from this analysis.

The column five in Table 6.4 shows the results from the eigenvalue sensitivity analysis [8]. The partial derivative \( \partial \lambda_k / \partial a_y \), which indicates the sensitivity of the eigenvalue to the changes in the system parameters, is calculated in this analysis. The considered parameters \([a_y]\) are the elements of the system matrix \( A \) (eqn. 1.94) in the state space system model. The four elements to which the considered eigenvalue is mostly sensitive are marked as “rec” or “inv” depending on whether they belong to rectifier AC system or to inverter AC system. Only the sensitivity to the AC system parameters are considered, since it is known that AC system parameters change considerably with the change of operating conditions (system loading). Parameters of the DC systems, on the other hand, do not change during operation.
Small signal analysis of HVDC-HVAC interactions

It is evident from these results that eigenvalues at higher frequency are more sensitive to rectifier AC system parameter changes. Even the eigenvalues originating from inverter AC system (eig. 4) are very sensitive to rectifier AC system parameters.

The eigenvalues at lower frequency are far more sensitive to inverter AC system parameters, as shown by the sensitivity analysis of eigenvalues 6-7 and 10-11(eigenvalues 8-9 are more sensitive to rectifier parameters). These eigenvalues (6-11) will give aperiodic system responses, for the present system configuration, however under different system parameters they can become pairs of low-frequency complex eigenvalues. This is well illustrated with reference to Figure 6.4 d) later in the Chapter, and Figure 7.8 a) in Chapter 7. In these Figures, a pair of real eigenvalues lying on the real axis (marked with crosses), becomes a complex low-frequency pair after a change in system parameters.

The column six shows the results from participation factor calculation. Participation factor indicates the degree of participation of each of the states in the considered eigenvalue. The indices of four states which participate mostly in the considered eigenvalue are shown in this column, where the states are grouped in the following manner, eqn (1.94), Section 1.2.8:

\[
\begin{align*}
  x_1 - x_{13} & \quad \text{DC system} \\
  x_{14} - x_{29} & \quad \text{Rectifier AC system} \\
  x_{30} - x_{45} & \quad \text{Inverter AC system}
\end{align*}
\]

**TABLE 6.4. 16 MOST IMPORTANT SYSTEM EIGENVALUES.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Damping ratio</th>
<th>Frequency [Hz]</th>
<th>Eigenvalue sensitivity</th>
<th>Participation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-25.25 ± 444.38i</td>
<td>0.0567</td>
<td>70.76</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>2</td>
<td>-43.78 ± 314.15i</td>
<td>0.1381</td>
<td>50</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>3</td>
<td>-43.79 ± 314.15i</td>
<td>0.1380</td>
<td>50</td>
<td>Inv,Inv,Inv,Inv</td>
</tr>
<tr>
<td>4</td>
<td>-76.44 ± 392.62i</td>
<td>0.1912</td>
<td>62.51</td>
<td>Rec,Rec,Inv,Rec</td>
</tr>
<tr>
<td>5</td>
<td>-124.18 ± 206.37i</td>
<td>0.5156</td>
<td>32.87</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>6</td>
<td>-110.08</td>
<td>Inv,Inv,Inv,Rec</td>
<td>38,23,30,31</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-53.17</td>
<td>Inv,Inv,Inv,Inv</td>
<td>31,23,38,39</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-29.19</td>
<td>Rec,Rec,Rec,Inv</td>
<td>31,30,39,15</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-12.06</td>
<td>Rec,Rec,Rec,Inv</td>
<td>15,30,31,45</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-8.52</td>
<td>Inv,Inv,Inv,Rec</td>
<td>30,15,31,45</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-7.03</td>
<td>Inv,Inv,Inv,Inv</td>
<td>30,31,45,42</td>
<td></td>
</tr>
</tbody>
</table>

The participation factor results are in agreement with eigenvalue sensitivity analysis for most of the eigenvalues. This implies that if a particular eigenvalue gets destabilised by perturbation, as an example, in rectifier AC system parameters, it can be observed and possibly stabilised by the use of rectifier AC system variables.

It can be concluded from the above analysis, that operating problems at rectifier side of HVDC link can be expected at higher frequencies, in the form of oscillatory (harmonic) instability. At inverter side, the instabilities can occur at much lower frequencies. Note, the frequency ranges for HVDC system analysis are defined in this Thesis as follows: lower frequency \( f<20\text{Hz} \), mid frequency \( 20\text{Hz}<f<50\text{Hz} \), and higher frequency \( f>50\text{Hz} \), as referred to the DC side.
Therefore, the rectifier side AC system should be designed not to have resonant peaks at frequencies around the second harmonic, since this resonant peak can cause instabilities when AC and DC systems are connected. The same resonant peaks at inverter AC system are not likely to cause instabilities. An explanation for these conclusions lies in the HVDC control structure. It is known that DC current controller structure in an HVDC system is asymmetrical, with rectifier controller regulating current and inverter controller being passive. The fast acting, high gain, controller at rectifier side will tend to excite “faster” eigenvalues from the connected AC system. Inverter AC system is connected to the passive end of HVDC link and it will tend to change the system time constants at lower frequencies, essentially contributing only to the “inertia” of the system. The influence of control loops on the system stability is further analysed in Chapter 7.

The majority of the above conclusions are a consequence of the adopted HVDC control structure, as further discussed in Chapter 7, and they will also apply for a more general HVDC system. In the case of CIGRE HVDC model they are very pronounced, since this model is purposely developed to demonstrate known operating difficulties.

6.3 INFLUENCE OF AC SYSTEM SCR

The AC system SCR have traditionally been considered as the best indicator of possible AC-DC interaction problems. It is known that an HVDC system connected to a low SCR AC systems \((SCR=2.0-3.0)\) may cause some operating problems [8]. AC systems with very low SCR \((SCR<2.0\) under normal operating conditions) are usually avoided. In this Thesis, the operating problems with weak AC systems are studied from the dynamic point of view.

This section uses the relative movement of the system eigenvalues to analyse the dynamic system behaviour under a reduced SCR. The SCR of each of the AC systems is reduced in small steps (keeping power factor constant) and the positions of the system eigenvalues are observed. Table 6. 5 shows the dominant eigenvalues, for rectifier side SCR reduced to \(SCR=1.7\), and for inverter side SCR reduced to the same value. More detailed analysis for the weak inverter AC systems is shown in Chapter 8.

It is evident from the Table, that the reduced SCR affects predominantly the eigenvalues at lower frequencies. This relates to both, rectifier and inverter side reduced SCR, however with some differences. Reduced SCR at inverter side causes rapid movement of newly created complex eigenvalues (eig. 7-8) towards the imaginary axis. The system becomes unstable for SCR close to 1.2. At \(SCR=1.9\), the system responses are so distorted that a simulation of start up procedure is very difficult. It is also seen that higher frequency eigenvalues slightly dislocate by the SCR reduction. This implies that composite resonance is somewhat affected by SCR changes.

In the case of reduced SCR at rectifier side, the eigenvalues 6-7 (with much better damping) move towards imaginary axis with slower rate. At \(SCR=1.3\), the system operates with very little changes in the responses. The system is also still stable at \(SCR=0.7\). As can be seen from Table 6. 5, the damping of dominant oscillatory mode is even improved with reduced SCR at rectifier side. Figure 6.1 shows PSCAD/EMTDC simulation responses with \(SCR=1.45\) at rectifier side, where inverter AC voltage disturbance is considered. The system response is not significantly deteriorated. It will be shown in Chapter 8, that the system is on the margin of stability when the inverter SCR is reduced to such low values.
It is therefore evident that the system is very sensitive to reduced SCR at inverter side, and the instability can be expected at lower frequency. At the same time, quite significant reduction of SCR at rectifier side is not expected to induce problems. This is an unfortunate fact, since it is known that the inverter side usually has lower SCR than the rectifier side. Power transfer direction is usually from stronger towards weaker AC systems.

As pointed earlier, the unsymmetrical HVDC control logic will cause different system behaviour at the two ends. High-gain DC feedback at rectifier side, as shown in the next chapter, will significantly improve stability of AC-DC connection point at rectifier side. At present, at inverter side, there is no proven HVDC control logic for system stability improvement.

The above conclusion also goes in favour of current control at inverter side. Since there is no practical reason for current control at rectifier side, except for reactive power consumption (and DC line faults), current control at inverter terminal could be considered as a normal operating mode when the inverter AC system is weak.

The analysis above also confirms that HVAC systems have large influence on the dynamic behaviour of the connected HVDC system, as it is emphasised in model development in Section 1.2.

The eigenvalues at higher frequencies did not move significantly under the perturbations applied, as shown by the dominant complex pairs (1 and 4) in Table 6.5, and also demonstrated in Figure 6.1. These conclusions, however, may be specific for the method of analysis applied and for the test system in use. It is noted that if SCR is reduced with accompanied changes in power factor and if different test system is used, there might be a possibility for further deterioration of composite resonance conditions.

The main conclusions on different affect of SCR changes on rectifier or inverter are a consequence of HVDC control structure, as further analysed in Chapter 7. These conclusions will apply to a more general case, without restrictions to CIGRE model.

<table>
<thead>
<tr>
<th>Table 6.5. System Eigenvalues for Reduced SCR.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original System</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
</tbody>
</table>
6.4 ANALYSIS OF INHERENT FEEDBACK LOOPS BETWEEN THE SYSTEMS

6.4.1 Method of analysis

This section examines the influence of inherent feedback loops between AC and DC systems. As it can be readily identified in Figure 6.2 (Figure 1.8), there are four AC-DC interaction variables: AC current magnitude, AC current angle, AC voltage magnitude and AC voltage angle. These interaction variables correspond to the case when correction matrices are used with the AC system input-output matrices, as shown in eqn. (1.89).

Figure 6.2 Interactions between AC and DC systems.

The interaction variables AC voltage magnitude and AC voltage angle are of special importance in this analysis, for these variables are either controlled, or they can be controlled by some conventional means. Converter bus voltage magnitude is controlled by using some of the voltage controlling elements (like SVC, STATCON or UPFC), whereas the AC voltage angle is “con-
trolled” by converter PLL. More precisely, PLL does not control AC voltage angle, but it can shield the DC system from AC voltage angle disturbances.

This section attempts to determine the specific feedback loops that negatively influence the system stability. Once they are determined it is also meaningful to know how “bad” they are, and at what frequency range their influence is predominant. This analysis is of importance in the case of a complex systems with higher number of controlling elements with adjustable parameters.

Considering the interaction of AC and DC systems through any physical variable (interaction variable), its influence on the system performance will be reduced (or it could be completely eliminated) if the variable is controlled by some external means such that it is kept as close to its steady-state value as possible. The degree of system interaction through this variable can be evaluated by comparing the performance of such a controlled system against the natural, uncontrolled system.

Let the system state equations are described in its usual form as:

$$\dot{x} = Ax + Bu, \quad y = Cx \quad (6.1)$$

The system equations from the analytical model in use, as shown in Chapter 1, are formulated such that the previously described interaction variables are represented as state variables.

To simulate external control of these state (interaction) variables the following method is used: The considered state variable $x_i$ is replaced by a new variable $x_{new}$ in (6.1), where the new state variable is obtained as shown in Figure 6.3.

The controller in Figure 6.3 is intended to control the new variable at its reference value, i.e. at $x_{new} = 0$. If the controller gain $k_c$ has very low values, then the new variable will have dynamics similar to that of the variable $x_i$. In the case of $k_c \to 0$ the two variables will be the same ($x_i = x_{new}$) and this system is referred here as the original system.

If the controller gain is increased, the new variable becomes faster controlled at its steady-state value and the influence of the original variable becomes attenuated. In the extreme case when $k_c \to \infty$, the new variable becomes suppressed at its nominal value ($x_{new} = 0$). In this case, the case of ideal control, the considered variable will not have influence on the dynamics of the rest of the system. The case of ideal control is equivalent to deleting the corresponding row and column from eqn. (6.1), i.e. $x_i = 0, \quad x_{new} = 0$.

By continuously varying the gain $k_c$, the influence of each of the interaction variable on the system stability can be examined.

Figure 6.4 shows the root locus when the gain $k_c$ is varied for each interaction variable independently. The general conclusions from this analysis with respect to inherent AC-DC interaction loops, are summarised in following sections.
6.4.2 Analysis of Rectifier side Interactions

From Figure 6.4 a), it can be observed that rectifier side AC voltage magnitude has a significant influence on the system stability, as can be seen by large possible dislocation of the original eigenvalues. By controlling this variable a considerable change in the system behaviour is possible. The location of ends of the root locus branches shows that a tight control of this variable, especially at mid-lower frequencies, would have deteriorating affect on the system stability. The system behaviour in the higher frequency range is of less importance, since it is known that with practically available voltage controlling elements no effective control action is imposed on AC voltage at such high frequencies.

![Figure 6.4. Effect of various AC-DC system interactions on HVDC system performance, x - Location of original eigenvalues, o - system eigenvalues when the interaction variable is ideally controlled.](image)

For the case of weaker control of AC voltage magnitude, the system stability can be improved in the whole frequency range. It is therefore difficult to strictly label the influence of this variable...
as positive or negative. As a conclusion, the system stability can be improved by controlling the AC voltage magnitude, but careful tuning of controller gains is necessary.

Figure 6.4 b) demonstrates that control of AC voltage angle would improve the system stability in the mid and higher frequency range. Some negative influence is however introduced at lower frequencies. Since the AC-DC interactions at rectifier side at higher frequencies are of more importance, this loop can be regarded as negative, and it should be externally controlled. It is important that the damping of dominant oscillatory mode can also be improved by the AC voltage angle control.

The control of AC voltage angle can be accommodated indirectly, by changing the gains of PLL controller. Actual values for PLL gains are shown in appendix D. By tuning the PLL controllers, the DC system can be insensitive to the AC voltage angle changes. This can be explained with the use of Figure 1.2. b). If the PLL output angle is closely following the AC voltage angle, than the actual firing angle will follow the firing angle order, and the AC voltage angle changes will not be seen from the DC system.

Therefore, the inherent negative influence of this loop can be readily eliminated by increasing rectifier side PLL gains. Simulation results, shown in Figure 6.5, confirm this conclusion. The responses from this Figure show that a 20 times increase in rectifier PLL gains (gains \( k_{pPLL} \) and \( k_{iPLL} \) in Figure 1.3), can noticeably improve the system stability. The applied input in this Figure is the rectifier AC voltage disturbance step change. Consequently, if there is a resonant condition between AC and DC systems at rectifier side, an increase in rectifier side PLL gains could be considered as a possible countermeasure.

### 6.4.3 Analysis of Inverter side Interactions

Figure 6.4 c) shows that an ideal control of the AC voltage magnitude at inverter side would deteriorate the system stability in the whole frequency range. Conversely, with relatively low controller gains, the system stability is improved in the mid-lower frequency range. AC voltage magnitude can be recommended but with careful gain selection.

The potential control of AC voltage angle, as can be seen from Figure 6.4 d), would substantially degrade the system stability in the whole frequency range. The most pronounced negative influence is at lower frequencies. It is evident that the AC voltage angle at inverter side represents inherent negative feedback loop for the considered system, indicating that if this variable is externally controlled the system behaviour becomes worsen.

Therefore the PLL gains at inverter side should be tuned to low values. Figure 6.6 shows the simulated system response with inverter side PLL gains increased ten times. As can be seen, a new low-frequency oscillatory mode (\( \approx 15\text{Hz} \)) is looming and the stability is noticeably deteriorated.

The PLL gains at inverter side should be kept at lower values especially in the case of low SCR AC system, for it was shown in Section 6.3 that the reduced SCR predominantly affects the eigenvalues at lower frequencies. Traditionally, it has been a common practice to tune both, rectifier and inverter PLL gains, at very low values in order to maintain the system stability [9]. However, it is shown above, that the rectifier side PLL gains should be tuned to far larger values. De-
pending on the actual system and operating conditions, these conclusions may have more or less actual affect.

A typical controller gains, as shown in Appendix D, indicate that PLL gains are far lower than constant current controller gains (more than 30 times). Normally, the constant current controller is the main control loop in the system, controlling the power transfer at reference value. This controller is responsible for dynamic stability, disturbance rejection and system performance. The speed of this controller is also dictated by the outside requirements like speed of fault recovery or speed of tracking higher modulation signals or commutation failure considerations. It is evident that constant current controller must be sufficiently fast and it should remain the main control loop. However, following the above conclusions that PLL gains on both rectifier and inverter side have pronounced influence on the system stability, these controllers can now be used to assist constant current controller in meeting some of the above control goals. Because of a lack of their importance for the outside system control, PLL control loops can not be the main control loops. PLL controllers can not be considered to take over some of the tasks of constant current controller, but only to help the main controller, particularly in cases of operating difficulties.

Nevertheless, it is important to regard PLL controllers as any other control circuit in the system, and to use them to the full potential with respect to the system performance criteria. In the case of any stability problems, the settings of the available controllers should be firstly examined. Despite the above conclusions, the available HVDC bibliography does not give any recommendations on the PLL controller settings.

The analysis presented above, also provides an insight for the possible design of improved PLL controllers. By a selective control of AC voltage angle throughout the considered frequency range, a purposely designed PLL controller with cascade compensator(s) could further improve the system stability.
These conclusions also confirm that the dynamics of PLLs used with HVDC converters, have significant influence on the system stability. Their dynamic models must be included into the HVDC system model.

6.5 ANALYSIS OF INPUT-OUTPUT DIRECTIONS

6.5.1 Introduction

In this section, HVDC system is considered as a multi-input multi-output (MIMO) system and singular value decomposition [10] is used to determine the system input-output properties and the most important input-output directions. The objective of this analysis is to determine the system characteristics when more than one input and more than one output are considered.

A very brief background on singular value decomposition is given in Appendix G. More detailed description can be found in [10].

HVDC system becomes a MIMO system when at least two signals are considered as inputs, say two disturbance inputs or two control inputs, and at least two output signals. Figure 6.7 shows one of the configurations where HVDC system becomes MIMO system. The two disturbance inputs are bus voltage on rectifier and inverter side as defined in Figure 1.7 and equations (1.59-1.61), whereas two control inputs are rectifier and inverter firing angle. Inverter firing angle is considered as an input as defined in Figure 1.7 and equation (1.69). The rectifier side control input is added to the firing angle order as in equation (1.53). In this case, the system either operates without the existing rectifier current controller, or the system can operate with the existing controller and the new control signal, derived from the new controller in parallel with existing one.

The two control outputs can be any variables selected for control.

![Figure 6.7. HVDC system as a MIMO system with decentralised control structure.](image)

It is well known that because of the internal interactions between different inputs and outputs, a MIMO system can have entirely different behaviour from the corresponding SISO system [10],[11]. In a MIMO system, the magnitude of the output signal depends not only on the magnitude of disturbances, but also on the relative phase displacement between the disturbance signals.
Small signal analysis of HVDC-HVAC interactions

(disturbance directions). This section attempts to determine the input directions that will cause the worst case effect on the system outputs.

The system inputs and outputs have been scaled for this analysis, according to the recommendations in [10]. Normalisation of signals will enable direct comparison among different inputs and outputs. Without normalisation, it would be difficult to compare output signals which are measured, for example in thousands of volts and signals measured in degrees or radians.

6.5.2 System Analysis

Figure 6.8 shows the largest and smallest singular values, where the disturbances are considered as inputs. The two disturbances are rectifier and inverter bus voltage whereas the outputs are the two direct currents.

It can be seen that disturbance rejection is good in any direction, for the frequencies below 20rad/s. Since the gain values are very low, the disturbances will not have large effect on the considered outputs. It also can be seen that in this frequency range, the condition number is large, indicating that the system responses will largely depend on the disturbance directions. This is demonstrated by the large difference between largest and smallest singular values.

Table 6.6 shows singular value decomposition and condition number for two frequencies below 20rad/s.

![Figure 6.8. Principal singular values for disturbance input.](image)

126
Small signal analysis of HVDC-HVAC interactions

TABLE 6.6 SINGULAR VALUE DECOMPOSITION FOR TWO FREQUENCIES BELOW 20 rad/s.

<table>
<thead>
<tr>
<th>Frequency $\omega = 0.6\text{ rad/s}$</th>
<th>$\gamma(G) = 8.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output singular vectors</td>
<td>Singular</td>
</tr>
<tr>
<td>0.0035+0.6058i</td>
<td>-0.0018-0.7955i</td>
</tr>
<tr>
<td>0.0034+0.7955i</td>
<td>0.0004+0.6058i</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency $\omega = 12\text{ rad/s}$</th>
<th>$\gamma(G) = 8.46$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output singular vectors</td>
<td>Singular</td>
</tr>
<tr>
<td>0.1474+0.5761i</td>
<td>-0.0662-0.8013i</td>
</tr>
<tr>
<td>0.1566+0.7886i</td>
<td>0.0004+0.6058i</td>
</tr>
</tbody>
</table>

Considering as an example, the frequency of $12\text{ rad/s}$ in Table 6.6, the worst possible input direction is manifested by the input singular vector: $U = [0.668432 \quad -0.743772]^T$, which would correspond to the largest singular vector. This input vector suggest that the disturbance oscillations on rectifier and inverter AC voltage should be displaced by approximately $180\text{ deg}$, to create the worst possible response on DC current, i.e. the largest system gain. Therefore, if the two disturbances are in opposite phase and with the magnitude ratio $0.668/0.744$, the disturbance will have the maximum possible magnification. The phase direction of disturbances is readily explained from the practical point of view by knowing that, if voltages on DC line ends move in opposite directions, the voltage difference (and consequently DC current) will be mostly affected. The output direction for this case is $V = [0.1474 + 0.5761i \quad 0.1566 + 0.7886i]^T$. Therefore the inverter DC current will be more affected, as seen by the magnitude of the second element in the output vector. The existence of current controller on only one side of the link can be blamed for this output direction.

The condition number for this frequency, is regarded as large, according to recommendations in [10], and the system can be classified as difficult to control. Difficulties will be mostly pronounced in the case of decentralised controller. Because of the large system dependency on input directions, each controller needs information about both disturbances in order to derive proper action. A “completely” diagonal controller can not be expected to give best results in all disturbance directions. However, as it will be presented in Chapter 8, a decentralised controller is the only option for HVDC control structure.

For frequencies above $20\text{ rad/s}$ the disturbances will have much larger effect on the system. The worst case corresponds to the pronounced peak around $440\text{ rad/s}$, (Figure 6.8). This impedance peak represents the composite resonance case as discussed in Chapter 4. The exact values for SVD at $440\text{ rad/s}$ are shown in Table 6.7.

TABLE 6.7. SINGULAR VALUE DECOMPOSITION AT $440\text{ rad/s}$.

<table>
<thead>
<tr>
<th>Frequency $\omega = 440\text{ rad/s}$</th>
<th>$\gamma(G) = 4.91$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output singular vector</td>
<td>Singular</td>
</tr>
<tr>
<td>0.9015-0.0203i</td>
<td>0.1717+0.3968i</td>
</tr>
<tr>
<td>-0.3533-0.2492i</td>
<td>-0.2044+0.8783i</td>
</tr>
</tbody>
</table>

The results from Table 6.7, show that if a disturbance happens to be in the worst possible direction, the DC current (on rectifier side in this case) will be magnified $7.93$ times relative to the adopted scaling. In practical terms this means that, the voltage disturbance of $3.5\%$ will cause rectifier DC current to oscillate with magnitude of more then $25\%$ of rated value. This resonant frequency is at a safe distance from the DC side first harmonic ($314\text{ rad/sec}$). However, it is possible
that under different system configurations, or possibly with different controller settings, or different power transfer levels, the resonant peak might approach first harmonic. If this happens, a second harmonic disturbance of several percent on AC system (which is quite common) could cause very large magnitude DC side oscillations. The smallest singular value (σ(440rad/s) = 1.6164) is still very large, indicating that, at this frequency, the disturbances will always be significantly magnified. The above results once again confirm the system vulnerability to composite resonance close to second harmonic (AC side). The frequency range of around 440rad/s is therefore the area where the system performance improvement is necessary. The controller presented in Chapter 4 could be one of the practical solutions.

The above analysis demonstrates the importance of directions in HVDC system disturbances. Considering the singular values in Table 6.7, it is seen that the same magnitude of disturbance can be magnified by the gain in range 1.616 - 7.936 depending on the direction of disturbance on the other end of the link. If the system is considered as a SISO, the gain will be fixed value from the above range and the actual danger of composite resonance may be underestimated.
6.6 CONCLUSIONS

A small signal analysis of HVDC-HVAC systems is presented in this chapter. The eigenvalue sensitivity and participation factor analysis are used for the identification of frequency range for possible AC-DC interaction problems. The variations in rectifier side AC system parameters can cause composite resonance at frequencies around first harmonic. The coupling of DC systems with AC systems having low frequency resonance, at rectifier side, should be avoided since interactions between these systems can cause stability problems. The variations in inverter AC system parameters are likely to cause instability at a much lower frequency.

The analysis of influence of SCR changes reveals that the system is very sensitive to SCR reduction at inverter side. The possible instabilities can be expected at lower frequencies. At the same time, an excellent robustness to SCR reduction at rectifier side is noticed. A high gain direct current controller at rectifier side contributes significantly towards stability improvement at rectifier side.

The examination of inherent feedback loops between the subsystems shows that AC voltage angle changes at rectifier side can deteriorate the system stability if left uncontrolled. Consequently, the rectifier side PLL gains should be tuned to higher values. On the other hand, AC voltage magnitude should not be tightly controlled at rectifier side. Some stability improvement is possible with AC voltage control, if the controller parameters are carefully tuned.

At inverter side, the control of AC voltage angle deteriorates the system stability. This requires the corresponding PLL gains to be kept at lower values. Some stability improvement is possible by external control of AC voltage magnitude. Ideal control of AC voltage magnitude would deteriorate the system stability.

The singular value decomposition based analysis shows that for some applications it may be important to consider HVDC system as a MIMO system. At frequencies close to the first harmonic, the condition number is around five, which indicates large influence of disturbance directions on the system behaviour.
REFERENCES:

[4] A.R. Wood Arrillaga, Composite resonance; a circuit approach to the waveform distortion
ties of converter transformer core saturation instability” 6’th In Conf. On AC and DC
frequency domain analysis” IEE proc Gen, Trans & Dist, vol 143(1) Jan 1996 pp 75-81.
Instability at an HVDC Converter” IEEE Trans. on PD, vol. 11, no 4, October 1996, pp
Synchronous Compensators at an HVDC Inverter Bus in a Very Weak AC System” IEEE
ty Analysis of HVDC Systems” IEEE Trans. on Power Delivery, Vol. 10, no 4, October
ual
[10] S. Skogestad, I. Postlethwaite “Multivariable Feedback Control” John Wiley and sons,
1996.
CHAPTER 7

STABILITY ANALYSIS OF HVDC CONTROL LOOPS

7.1 INTRODUCTION

This Chapter analyses the dynamic influence of the most often used HVDC control loops, as well as some of the alternative control strategies suggested in the literature. The primary aim is to study the role which the existing HVDC control loops play in the development of the reported operating difficulties. It will be studied whether the conventional control logic improves the system stability, or in contrary whether it accelerates the development of (harmonic) instabilities. It is also important to demonstrate the actual dynamic benefit of some of the HVDC feedback loops. A few of the unconventional HVDC control algorithms discussed in the literature but not yet commonly used in practice, will be dynamically analysed and their benefits compared with those of the traditionally used control strategies. The small signal stability of the system in the frequency domain \( f < 100\text{Hz} \) (on DC side) is of the principal interest in this analysis. The fastest control loops are investigated with the master control level assumed inactive.

The rectifier side of an HVDC link, in the conventional control logic, is responsible for maintaining the power transfer at the required level. To maintain this task, direct current feedback with dominant integral action is usually used [1]. The current control at rectifier side will ensure a “stable” intersection point with the voltage controlling inverter on the static V-I curve. The actual dynamic effect of the DC feedback loop, on both, DC system and the connected AC system, in the whole frequency range, has not been well analysed in the available HVDC literature. It will be shown in this Chapter, that direct current feedback at rectifier side plays an important role in the development of AC-DC composite resonance phenomenon. In the case of low SCR AC system connected to the HVDC system, the direct current feedback also significantly changes the stability margins of the overall HVAC-HVDC-HVAC dynamic system.

Reference [2] suggests the use of fast (DC) power feedback at rectifier side. Most of the arguments for this control loop are based on the static V-I HVDC characteristic, whereas the small-signal stability analysis is not offered in the reference. The direct current and fast power feedback loops will be compared in this Chapter, from the dynamic point of view, by using the eigenvalue decomposition analysis.

The inverter side of an HVDC link is often in the constant extinction angle control mode [1]. This control mode has been chosen to enable minimum reactive power consumption at inverter side, and to keep the safe commutation margin. It is not intended for actual system stability improvement. Since inverter controller is not directly involved in the steady-state HVDC control in the nominal operating mode, there is more freedom for selection of inverter control signal and control strategy.

The constant firing angle operating mode (constant beta) at inverter side has been labeled as a “more stable” mode since it offers a “stabilising” positive slope on static V-I operating curve[1] [3]. If the inverter firing angle is used as an additional control input, there are further feasible feedback control strategies and a wide possibility for the system stability improvement. Reference [4] elaborates that direct current feedback or direct voltage feedback are sometimes used at inverter side for the system stability improvement. These feedback loops are introduced on the
Stability analysis of HVDC control loops

basis of a more stable, positive slope on the inverter static curve. A strong recommendations or critiques for these feedback loops, depending on the system parameters and operating conditions can not be found in the references. It will be shown in this Chapter that both direct current and direct voltage feedback are vulnerable to oscillatory instabilities in a certain frequency range, depending on the system characteristics.

The most influential alternative control strategies at inverter side are reported in [2] and [5]. Reference [2] proposes, the use of fast power feedback at inverter side. This control method (Combined and Coordinated Control method - CCCM control) consists of the direct voltage and direct current feedback, similarly as to the rectifier side, except that the positive voltage feedback is used at inverter side. The CCCM control method is simplified in this Thesis. The original control method as presented in [2] consists of rectifier and inverter feedback loops acting simultaneously. In this Thesis, the rectifier side control loops are used with the assumption of “classical” inverter control, and inverter control loops with usual current control at rectifier side.

Reference [5] suggests the use of reactive current feedback (CRC control method) as a result of study of $\Delta Q/\Delta V$ ratio. The alternative control techniques mentioned above, will be dynamically analysed in this Chapter and compared with conventional control logic.

This Chapter compares the dynamic effects of the practically used and suggested inverter/rectifier control strategies. In Chapter 8, the inverter controller objectives and aims will be discussed and new control method will be developed.

The small signal analysis of the system control loops, as offered in this Chapter, is important from several aspects:

- It indicates the possible dynamic instabilities.
- It presents the actual dynamic benefit of the considered feedback loop.
- It offers insight into the system disturbance rejection and speed of response.
- The eigenvalue sensitivity analysis indicates the system tolerance to the parameters changes.

The results and conclusions from this analysis are expected to assist in understanding the negative side of HVDC-HVAC interactions and can help in selecting a proper control strategy when the connected AC-DC systems are naturally bound to cause operating problems. The special emphasis will be placed on the operating problems caused by the second harmonic resonance and low SCR AC systems.

The test system used for the analysis is the CIGRE HVDC Benchmark system (Rec SCR=2.5, Inv SCR=2.5) from Appendix D and [6],[7]. The test system for the reduced SCR at inverter side is the modified CIGRE HVDC model (Rec SCR=2.5, Inv SCR=1.5) as proposed in [8].

The rectifier side control loops will be compared against uncontrolled system, i.e. the system with constant firing angles at both ends and no effective feedback control action. The inverter side control strategies will be compared against the system with constant firing angle at inverter side, and the usual direct current control at rectifier side.
7.2 ANALYSIS OF RECTIFIER CONTROL MODES

7.2.1 Root locus analysis

This section analyses the direct current feedback and the fast power feedback, with the rectifier controller structure as shown in Figure 7.1. The fast power feedback control strategy is taken from [2], and it consists of negative voltage feedback and negative current feedback.

As far as the small-signal analysis around the nominal operating point is concerned, the CCCM control method [2], consists of:
- negative voltage and negative current feedback at rectifier side and
- positive voltage and negative current feedback at inverter side.

In this Chapter, the rectifier side and the inverter side of CCCM control are analysed separately. Rectifier side is analysed with constant beta control at inverter side, and inverter side is similarly analyzed with the conventional HVDC control at rectifier side.

Figure 7.2 shows the root locus with direct current and fast power feedback, where the gain “k” from Figure 7.1 is varied. Figure 7.2 a) demonstrates that direct current feedback can substantially improve the system stability in the lower and mid frequency range. In the higher frequency range the DC current feedback negatively affects stability of the overall HVDC-HVAC system.

It is evident that the dominant oscillatory mode moves right, as the gain increases, and it becomes unstable for higher gain values (shown by branch “d”). If the dominant oscillatory mode is close to the first harmonic (on DC side), the conditions for core saturation instability may exist. Therefore, the DC current feedback loop will actually accelerate the development of AC-DC composite resonance and possibly also core saturation instability. This negative influence of controller gains on AC-DC composite resonance is in agreement with similar conclusions from [9].

The AC-DC composite resonance (second harmonic instability) has been traditionally viewed as a result of unfavorable resonant conditions between AC and DC systems [10], where the main circuit conditions are viewed as the main cause for the instability. The above analysis together with the results from [9] shows that the direct current feedback could have contributed towards development of core saturation instability in the some of the practically experienced cases.

![Figure 7.1. Control loops at rectifier side.](image)
Figure 7.2. Root locus with direct current and fast power feedback. + eigenvalues of the uncontrolled system, o - position of zeros.

Figure 7.2 b) shows that at lower frequencies, fast power feedback can not improve the system stability as it is possible with direct current feedback. The position of eigenvalues “C” can be improved only for large feedback gains. However for large feedback gains, another pair of complex eigenvalues will be close to the zeros marked “D”. In the mid (lower) frequency range, as shown by branch “e”, fast power feedback has advantage over direct current feedback. At higher frequencies, by considering the length of root locus branch “d”, fast power feedback appears to be a far less favorable control strategy. The fast power feedback has right-half-plane zeros further right, which makes the corresponding locus branch much longer. The eigenvalues which lie on the longer root locus branch will be more sensitive to the controller parameters changes, and to the system parameters in general. The eigenvalue analysis with respect to the system parameters is shown in the next section.

7.2.2 Eigenvalue sensitivity analysis

If the feedback gain is kept sufficiently low, the fast power feedback and direct current feedback can have similar position of dominant eigenvalues and consequently similar responses. As an example, the feedback gains can be tuned in such way that the dominant eigenvalues, the ones lying on the branch “d”, have the same location. However, since the eigenvalues for fast power feedback lie on the longer locus branch, these will be more displaced in the case of system parameters changes. This effect can be analysed by using the eigenvalues sensitivity methods.

Table 7. 1 shows the sensitivity of dominant complex eigenvalues with respect to the rectifier AC system parameters \( \frac{\partial \lambda}{\partial a} \), with the similar analysis method from Section 6.2.2. The dominant system eigenvalues have similar location for the two control strategies. The three largest sensitivity elements are shown in the Table. It is seen that the dominant complex eigenvalues are much more sensitive to the system parameters when fast power feedback is used. This implies that for the same change in the system parameters (change in loading etc.) the system performance will be much more changed if fast power feedback is used. More importantly, it can be expected that fast power feedback will be more vulnerable to negative AC-DC interactions at second harmonic.
Figure 7.3 shows the PSCAD/EMTDC simulation of possible instabilities with direct current and fast power feedback. As predicted by the analytical model, it is seen that both control strategies develop oscillatory instability with increased gains. Fast power feedback has somewhat higher frequency of dominant oscillations \( f_{fp} \approx 75\text{Hz} \) than DC feedback \( f_{dc} \approx 65\text{Hz} \). This conclusion from PSCAD/EMTDC simulation is also in agreement with root locus analysis from Figure 7.2 (DC current \( f_{dc} \approx 70\text{Hz} \) fast power \( f_{fp} \approx 79\text{Hz} \)). It is evident that there is some mismatch in the dominant oscillatory mode frequency, similarly as noticed in Chapter 1.

The above findings can serve as a vague confirmation of the fidelity of the analytical model in the frequency range around the dominant oscillatory mode.

### TABLE 7.1 SENSITIVITY OF DOMINANT COMPLEX EIGENVALUES.

<table>
<thead>
<tr>
<th></th>
<th>Direct current feedback</th>
<th>Fast power feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalues</td>
<td>(-25.25\pm444.38i)</td>
<td>(-28.61\pm483.08i)</td>
</tr>
<tr>
<td>1.</td>
<td>10.641</td>
<td>21.496</td>
</tr>
<tr>
<td>2.</td>
<td>7.802</td>
<td>18.846</td>
</tr>
<tr>
<td>3.</td>
<td>6.018</td>
<td>17.808</td>
</tr>
</tbody>
</table>

![Figure 7.3. System responses following reference signal step change with DC feedback and with fast power feedback.](image)

7.2.3 System analysis with weak AC system

Figure 7.4 shows the eigenvalues and root locus when the HVDC system is operating with very weak AC system at inverter end. By comparing the location of original eigenvalues in Figures: 7.2 a) and 7.4, it can be seen that reduction of SCR mostly affects low-frequency system dynamics, represented by eigenvalues “C” and “E” (the exact positions of these eigenvalues are shown in the Figures). Other eigenvalues are not significantly displaced. It is also seen that DC feedback can significantly improve the low-frequency system stability (branch “g”), if the feedback gain is sufficiently high. Therefore, an increase in DC feedback gain could be considered as a possible countermeasure when the AC system SCR becomes reduced. The value of this feedback gain is however limited by positioning of dominant complex eigenvalues at higher frequencies (branch...
“d”). If there is no sharp resonant condition at higher frequencies, or if the action of DC feedback is corrected around the first harmonic (possibly by designing a more complex controller), the high gain DC feedback could substantially improve stability of the overall HVAC-HVDC-HVAC system even with very low SCR AC systems.

![Figure 7.4. Root locus for the system with very weak receiving AC system. Inv. SCR=1.5.](image)

7.3 ANALYSIS OF INVERTER CONTROL MODES

7.3.1 Analysis of conventional control loops

This section analyses and compares the inverter control strategies traditionally used in industry [1]. Figure 7.5 shows the inverter controller. The supplementary signal is direct current feedback or direct voltage feedback.

The system input for the inverter feedback signals is the “inverter firing angle modulation $\beta$” as shown in Figure 1.7. The feedback signals under the study, can be readily obtained using the states from the detailed linear system model.

Figure 7.6 shows the root locus with direct current and direct voltage feedback at inverter side. It is evident from Figure 7.6 a), that direct current feedback can have negative influence at lower frequencies. The root locus crosses the imaginary axis around $15\text{Hz}$, and because of the long root–locus branch “h”, the corresponding eigenvalues will be very sensitive to the system parameters. Figure 7.7 shows the PSCAD/EMTDC simulation of this form of instability. The sign for direct current feedback in use, is chosen to enable positive slope on the static $V-I$ curve. This positive slope is recommended from the static HVDC operating diagram. This sign makes a negative (stabilising) feedback loop for the overall HVDC system.
Stability analysis of HVDC control loops

**Figure 7.5. Inverter controller.**

If the sign is reversed, then the effect of direct current control loop is similar to that at rectifier side (Figure 7.2 a)). The stability at lower frequencies is improved with significant deterioration at higher frequencies.

From Figure 7.6 b) it is seen that very little or no benefit is introduced with direct voltage feedback. The instability is possible at frequencies around 100Hz (branch “t”). We conclude from the above analysis, that a careful tuning of feedback gains is necessary if direct current or direct voltage feedback are used at inverter side. The stability improvement is possible only in a certain frequency range, whereas the system stability and system robustness become seriously degraded at some other frequencies.

The poor dynamic benefit of these two control loops is a consequence of the rectifier controller action. Since there is already one high-gain feedback loop in the system, the introduction of the additional control loop requires careful analysis of interactions between the controllers. Additional feedback loop can have completely different effect on the system to that when the feedback loop is solely used. The multi-input multi-output properties of the system, including interactions between the control loops, are analysed in Chapter 8.

The second column of Table 7.2 shows the location of zeros for the constant extinction angle control ($\gamma = const.$). The exact position of closed loop eigenvalues will be between the original eigenvalues and the zeros, depending on the feedback gain. Usually, for gamma control, the
feedback gain is high and the closed loop eigenvalues are close to the location of zeros. It is seen that gamma constant mode deteriorates the system stability in the whole frequency range. This result is in agreement with conclusions about “less stable” negative slope on the static V-I curve. However, since all zeros for constant gamma mode are in the left-half plane, and not far from the original eigenvalues, the system stability is only marginally degraded, without danger of actual dynamic instability. The use of beta control instead of gamma control will only marginally improve the system stability. The conclusions for constant gamma mode may be somewhat different when non-linear control elements (like gamma minimum calculation over the previous six cycles) are included.

![Inverter direct current response following step change in the current reference. The instability occurs with high-gain DC feedback at inverter side](image)

**Figure 7.7. Inverter direct current response following step change in the current reference. The instability occurs with high-gain DC feedback at inverter side**

### 7.3.2 Alternative inverter control strategies

The reactive current feedback, as proposed in [5], and CCCM control method developed in [2], will be analysed in this section.

The reactive current used for feedback is the \( I_{2q} \) component as shown in Figure 1.8 in Section 1.2.7. Using the instantaneous values, the measured current is \( i_{acj} \) as shown in Figure A.1, where the measurement point is the AC side of converter transformers. The method of measurement of AC side variables is shown in more detail in Chapter 8.

The low-frequency root locus for reactive current feedback is shown in Figure 7.8 a), whereas the influence at higher frequencies can be seen in Table 7. 2. The eigenvalues located on the branch “m” will always have improved damping, with the eigenvalues on the branch “l” having better location if the feedback gain is kept at lower values. More importantly, there are no right-half plane zeros with this feedback control, as shown in the third column in Table 7. 2. The benefit of reactive current feedback becomes more pronounced when the AC system strength is reduced, as it is presented in the next section.

The CCCM control method at inverter side consists of positive voltage feedback and negative current feedback. As it is seen from the low-frequency root locus (Figure 7.8 b)), there is a possibility for instability around 6Hz (branch “k”).

Although the improvement is evident at higher frequencies (Table 7.2), the actual dynamic benefit of this control loop is not clear. If the sign is reversed (i.e. negative voltage feedback and positive current feedback) than the effect is similar to that of direct voltage feedback solely used (Figure 7.6 b)). The root locus analysis shows that this control strategy has similar dynamic effect as only direct voltage feedback at inverter side.
TABLE 7.2 POSITION OF ZEROS FOR INVERTER CONTROL MODES.

<table>
<thead>
<tr>
<th></th>
<th>Constant firing angle (β) mode</th>
<th>Constant extinction angle (γ) mode</th>
<th>Reactive current feedback</th>
<th>CCCM control method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-25.25+444.46i</td>
<td>-18.78+449.42i</td>
<td>-39.11+460.68i</td>
<td>-25.12+319.51i</td>
</tr>
<tr>
<td>2</td>
<td>-25.25-444.46i</td>
<td>-18.78-449.42i</td>
<td>-39.11-460.68i</td>
<td>-25.12-319.51i</td>
</tr>
<tr>
<td>3</td>
<td>-76.46+392.64i</td>
<td>-62.88+378.59i</td>
<td>-36.88+410.84i</td>
<td>-166.60+272.31i</td>
</tr>
<tr>
<td>4</td>
<td>-76.46-392.64i</td>
<td>-62.88-378.59i</td>
<td>-36.88-410.84i</td>
<td>-166.60-272.31i</td>
</tr>
<tr>
<td>5</td>
<td>-110.12</td>
<td>-73.09+39.32i</td>
<td>-177.23</td>
<td>-70.30</td>
</tr>
<tr>
<td>6</td>
<td>-53.115</td>
<td>-73.09-39.32i</td>
<td>-59.44</td>
<td>+∞</td>
</tr>
<tr>
<td>7</td>
<td>-29.213</td>
<td>-26.42</td>
<td>-10.00</td>
<td>-9.22+2.07i</td>
</tr>
<tr>
<td>8</td>
<td>-12.069</td>
<td>-12.139</td>
<td>-10.00</td>
<td>-10</td>
</tr>
<tr>
<td>9</td>
<td>-8.526</td>
<td>-8.496</td>
<td>-9.10+2.18i</td>
<td>-10</td>
</tr>
<tr>
<td>10</td>
<td>-7.0369</td>
<td>-7.039</td>
<td>-9.10-2.18i</td>
<td>-9.22+2.07i</td>
</tr>
</tbody>
</table>

Figure 7.8. Root locus for alternative control strategies at inverter side. Only the low frequency dynamics are shown.

7.3.3 Weak AC system

Reduction of SCR at inverter AC system affects mostly the eigenvalues marked “F” in Figure 7.9, also located on the branch “g” in Figure 7.4. It is shown in Figure 7.9 how these eigenvalues change position with different inverter control strategies. The effect at higher frequencies is similar to that earlier presented in this chapter, since the eigenvalues at higher frequencies do not move significantly with SCR reduction.

As it can be seen, direct current feedback and CCCM control method can develop instability at lower frequencies, if used with low SCR AC systems. Also, the dominant eigenvalues are very sensitive to the system parameters with these control loops. As an example, it is necessary to change controller gains only by a factor of two to move the eigenvalues from point “M” to point “N. Point “M” would correspond to the best eigenvalue location with DC feedback at inverter side.
Although the direct voltage feedback is favorable at lower frequencies, it has strong negative influence at higher frequencies (similar to that in Figure 7.6 b)). The reactive current feedback, on the other hand, can eliminate the dominant oscillatory mode without serious stability deterioration at higher frequencies. The reactive current feedback can be identified as the best feedback signal for the systems with very low SCR. Figure 7.9 shows the simulation response with CRC method. The test system has very weak configuration at inverter side. It can be observed that the low frequency oscillatory mode is eliminated with the new feedback control.

It can be concluded, that the reactive current feedback would be the best inverter control strategy among all to-date reported control schemes. A more detailed analysis of inverter control strategies with respect to the overall control system objectives is performed in Chapter 8.

Figure 7.9. Influence of different control strategies on the system with low SCR.

Figure 7.10. Inverter direct current response following a disturbance at inverter side (taping factor decrease at primary side).
7.4 CONCLUSIONS

It has been shown that the use of direct current feedback control at rectifier side can significantly improve the system stability at lower frequencies, even with very weak AC systems at inverter side. However, at higher frequencies, this feedback loop has pronounced negative influence on damping of dominant oscillatory mode, and it may contribute development of composite resonance. The use of fast power feedback makes the system still more vulnerable to the composite resonance at higher frequencies. At lower frequencies, fast power feedback has some advantage over direct current feedback.

At inverter side, neither direct current feedback nor direct voltage feedback could be recommended from the root locus analysis. Both control strategies can develop oscillatory instability at some frequencies. The CCCM control method is also found to be vulnerable to dynamic instabilities at lower frequencies. The use of beta control instead of gamma control shows very little benefit.

The reactive current feedback is found to be the most favorable inverter control strategy. This control strategy also improves the system stability when the receiving AC system becomes very weak, without introducing any negative effects at higher frequencies.
REFERENCES:

CHAPTER 8

HVDC SYSTEM OPERATION WITH VERY WEAK RECEIVING AC SYSTEMS

8.1 INTRODUCTION

This chapter is focused on the stability problems caused by the weak inverter AC systems. The Short Circuit Ratio (SCR) of the AC system has been traditionally regarded as the most important indicator of the possible HVDC-HVAC operating problems [1]. Although modern inverter networks are becoming weaker, most of the existing HVDC systems, at present, operate with \( \text{SCR} \) greater than 2 at the inverter side. The systems with \( \text{SCR} \) less than 2 are known as very weak systems and they are avoided in practice.

References [2] and [3] describe the dynamic instabilities caused by the weak receiving AC systems. The main operating problems related to weak AC systems are the high-magnitude AC voltage oscillations and the difficulty in recovery from disturbances [4],[5]. AC voltage fluctuations are highly undesirable and can be harmful for AC equipment and the quality of power supply, whereas on the AC-DC converters they will increase the probability of commutation failure. High magnitude of the AC voltage oscillations is a consequence of high AC system impedance. If the AC system has high equivalent impedance, then small AC current perturbations will cause large voltage deviations.

The term “weak system” is used in this Thesis with the meaning “electrically weak”, i.e. the AC system with equivalent voltage source behind high equivalent impedance. A low inertia system weakness is not considered.

Traditionally, the use of synchronous condensers and/or SVCs have been the most commonly employed technique for the stability improvement of very weak AC systems. The disadvantages of the use of synchronous condensers, in terms of slow responses and high losses are well presented in [6]. Apart from being costly solutions, the use of SVCs has the disadvantage that it becomes less effective when the AC voltage level is reduced, i.e. during faults and large disturbances.

HVDC controls can also be used for the system stability improvement. Direct Current feedback and Direct Voltage feedback are sometimes used as stabilising signals at the inverter side [7]. Reference [6] proposes the use of AC voltage feedback for very weak AC systems. The recommendations for the above control strategies are based on the essentially static criteria (like static HVDC operating curve and Voltage Stability Factor \( VSF \)), whereas a thorough dynamic analysis is not offered in the references. It will be shown in this Chapter that AC voltage feedback can enhance the system stability only to a certain degree. This control strategy is vulnerable to the instabilities at higher frequencies, and consequently the feedback gain has to be suppressed to lower values.

In the previous Chapter, the HVDC system and HVDC control loops are analysed for the CIGRE HVDC Benchmark system with \( \text{SCR}=2.5 \) inverter side. This Chapter seeks to find the
best HVDC control strategy for operation with very weak AC systems. The fastest acting inverter control level (current control level) is used for the additional feedback control loop. Only the locally available variables at the inverter converter site, are used as candidates for the new feedback signal. The candidate signals are compared using the root-locus technique. In this way, their effect on the system dynamics and the possible dynamic instabilities, in the whole frequency range can be studied.

As far as the system dynamic stability and dynamic performance (responses) are concerned, there are two main difficulties with low SCR AC systems:

- The system responses become significantly altered as SCR is varied (large overshoots and poor damping),
- The system becomes dynamically unstable for very low SCR.

This Chapter develops a control method which gives the specified HVDC system performance for a very wide range of AC system parameters changes. Also, the designed controller enables stable HVDC system operation with very weak inverter AC system.

The potential development of a new technique for HVDC operation with very low SCR AC systems, would significantly extend HVDC application areas. The principal benefits involve:

- Possibility for further load increase at inverter AC system, without the necessity for AC system reinforcement.
- Additional increase of power transfer through existing HVDC links.
- Employment of HVDC links to connect remote and isolated AC systems with very low system strength (“dead” networks).
- Improved system stability would reduce the probability of commutation failure, and enable reliable power transfer.
- Possibility for replacement of synchronous condensers (SC) at inverter bus. Although their maintenance is known to be costly, SCs have been the most often used for system strengthening with weak inverter AC systems. Instead of employing SCs, the same system stability improvement could be achieved with a properly designed HVDC inverter controller.

8.2 SYSTEM ANALYSIS

8.2.1 Test system

This section offers dynamic analysis of the HVDC systems operating with weak receiving AC systems. The earlier presented detailed model of HVAC-HVDC-HVAC system is used in the analysis. The inverter AC system is however purposely modified to represent a system with very low strength. The changed AC system parameters are shown in Appendix H. The SCR is modified in this analysis by changing \( L_2 \) and \( R_3 \) in Figure 1.1, in such a way to keep the power angle constant.

8.2.2 Eigenvalue analysis

Table 8.1 shows the eigenvalue location when SCR is reduced at the inverter side, where the original system is the system studied in Chapter 6, Table 6.4. It is seen that the higher frequency eigenvalues are not much affected by the SCR reduction. Eigenvalues 9 and 10, however become
HVDC system operation with very weak receiving AC systems

significantly displaced. These eigenvalues are unstable for $SCR \approx 1.1$. The imaginary part of these eigenvalues shows the frequency of possible dynamic instabilities with low SCR AC systems. The instabilities can be expected in the lower frequency range $2Hz < f < 6Hz$. This is in good agreement with frequency for the practically reported instabilities, $5Hz$ in [2] and $6Hz$ in [3].

The controller design should be focused on moving these two eigenvalues to a stable position in the left half plane.

TABLE 8.1. SYSTEM EIGENVALUES FOR REDUCED SCR AT THE INVERTER SIDE

<table>
<thead>
<tr>
<th>Original System</th>
<th>Rec. SCR=2.5, Inv. SCR=1.7</th>
<th>Rec. SCR=2.5, Inv. SCR=1.2</th>
<th>Rec. SCR=2.5, Inv. SCR=1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-25.25+444.46i</td>
<td>-18.14+447.69i</td>
<td>-15.71 + 445.44i</td>
</tr>
<tr>
<td>2</td>
<td>-25.25-444.46i</td>
<td>-18.14-447.69i</td>
<td>-15.71 - 445.44i</td>
</tr>
<tr>
<td>3</td>
<td>-76.46+392.64i</td>
<td>-43.04+359.95i</td>
<td>-34.56 + 344.53i</td>
</tr>
<tr>
<td>4</td>
<td>-76.46-392.64i</td>
<td>-43.04-359.95i</td>
<td>-34.56 - 344.53i</td>
</tr>
<tr>
<td>7</td>
<td>-110.12</td>
<td>-153.230</td>
<td>-162.76</td>
</tr>
<tr>
<td>9</td>
<td>-53.116</td>
<td>-12.78+23.68i</td>
<td>-0.97 + 19.45i</td>
</tr>
<tr>
<td>10</td>
<td>-29.213</td>
<td>-12.78-23.68i</td>
<td>-0.97 - 19.45i</td>
</tr>
<tr>
<td>11</td>
<td>-12.069</td>
<td>-12.697</td>
<td>-13.34</td>
</tr>
<tr>
<td>12</td>
<td>-8.526</td>
<td>-8.207</td>
<td>-7.86</td>
</tr>
<tr>
<td>13</td>
<td>-7.0369</td>
<td>-6.566</td>
<td>-6.24</td>
</tr>
</tbody>
</table>

8.3 SELECTION OF CONTROL INPUT

This section investigates the use of rectifier or inverter side controller for stability improvement of the HVDC systems with weak inverter AC system. The aim is to study if the rectifier or the inverter control input should be used for the additional control signal. It should be firstly attempted to use the existing controller, to try to counteract the stability problem, and if the design is difficult a new control logic at the inverter end should be advised.

Table 8.2 shows the participation factor (four largest elements) for the mostly affected eigenvalues (eig. 9 and 10 in table 8.1). The locally available states at rectifier side and at inverter side are considered as feedback candidates in the analysis. Because of the physical distance between the converters, it is assumed that states at rectifier side are available for feedback at rectifier converter and inverter states at inverter converter.

TABLE 8.2. PARTICIPATION FACTOR FOR DOMINANT COMPLEX EIGENVALUES IN THE CASE OF VERY WEAK RECEIVING AC SYSTEM SCR=1.0

<table>
<thead>
<tr>
<th>States at rectifier side</th>
<th>States at inverter side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.0021</td>
</tr>
<tr>
<td>2.</td>
<td>0.0007</td>
</tr>
<tr>
<td>3.</td>
<td>0.0004</td>
</tr>
<tr>
<td>4.</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

It is evident from the table, that the inverter states “participate” much more in the considered eigenvalues. The participation factor is more than ten times larger for the inverter states. Therefore, it is necessary to use the inverter states and consequently the inverter input, for the dislocation of the eigenvalues responsible for the stability problems. Rectifier side control signal
HVDC system operation with very weak receiving AC systems

could also be used, but there is a danger that other eigenvalues with higher sensitivity could be affected. A very complex controller would be necessary in this case.

The new control signal will be provided to the inverter controller as “additional signal” as shown in Figure 8.1.

\[
\beta_{\text{cons}} + \gamma_{\text{meas}} \rightarrow \gamma_{\text{ref}} \rightarrow \beta_2 \rightarrow \beta_3 \rightarrow \beta_1 \rightarrow \alpha_{\text{inv}}
\]

\( \beta_1 \) beta constant mode
\( \beta_2 \) gamma constant mode
\( \beta_3 \) constant current mode

Figure 8.1. Inverter Controller.

8.4 SELECTION OF FEEDBACK SIGNAL

8.4.1 Method of analysis

Inverter side DC current and DC voltage, as well as all AC-DC interaction variables from Section 1.2.7, are considered as candidates for additional feedback signal. The magnitude-angle representation for three phase variables is used instead of \( d-q \) components. Magnitude–angle representation has more relevance to the actual physical variables. Corresponding \( d-q \) components of interaction variables can be obtained as a linear combination of magnitude-angle components, as presented in Section 1.2.7. Therefore for this reason \( V_d, V_q, I_d, I_q \) are not considered as feedback signals.

Two of the interaction variables are eliminated in the beginning: AC current magnitude, since it is directly proportional to the DC current, and AC voltage angle since it can be controlled by the existing PLL controller. Therefore the following signals are considered as candidates for the new inverter controller: DC current, DC voltage, AC current angle and AC voltage magnitude. All of these signals are easily accessible at converter site.

The following criteria are used to determine the most suitable feedback signal:

- Eigenvalue sensitivity with respect to the controller parameters [9]. This indicator shows how fast considered eigenvalue can be moved with the chosen feedback signal.
- Relative Gain Array (RGA) index and condition number for MIMO systems [9]. RGA indicates possible problems caused by interactions between the new inverter control loop and the existing control loop at rectifier side. Large RGA elements around cross-over frequency imply that the considered control loop will have significant impact on the output variable controlled at the other end of HVDC link. This negative interaction can, in the worst case, lead to instability. Condition number indicates how much the system behavior depends upon the direction of input signals. A large condition number is bad.
- Position of zeros for a particular output signal, where a theory on pole-zero calculation can be found in [9]. The open loop poles are the same for all considered outputs. Zeros are however different for each output. The position of zeros indicates maximum possible...
HVDC system operation with very weak receiving AC systems

movement of the considered eigenvalue, and it clearly indicates the direction of movement of the each of the important eigenvalues. This is the most important criterion in selecting the inverter controller feedback signal as it will be shown in this chapter.

### 8.4.2 Eigenvalue sensitivity with respect to controller parameters

The theory on how the eigenvalue sensitivity is used for selection of feedback signal is shown in Appendix G.1. More information on the theory can be found in [1] and [11].

The equation (I.7) from Appendix I.1, shows that the eigenvalue sensitivity can be calculated by using the eigenvectors of open loop system matrix. Table 8.3 shows the sensitivity of five pairs of open loop eigenvalues with respect to the candidate signals for the case of very weak receiving AC system, Inv. \( SCR=1.0 \). The AC voltage magnitude as a feedback signal, shows the largest sensitivity for the unstable pair of eigenvalues. This signal also gives movement of remaining eigenvalues in the correct direction (except for the pair 4). The second best signal is the AC current angle.

<table>
<thead>
<tr>
<th>Original Eigenval. Inv. SCR=1.0, Rec. SCR=2.5</th>
<th>DC current</th>
<th>DC voltage</th>
<th>AC current angle</th>
<th>AC voltage magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (-94 \pm 839.33i)</td>
<td>-117.2079</td>
<td>39.6387</td>
<td>22.3057</td>
<td>-89.6372</td>
</tr>
<tr>
<td>2 (-16.07 \pm 442.59i)</td>
<td>3.4718</td>
<td>0.2339</td>
<td>-0.2080</td>
<td>-1.0809</td>
</tr>
<tr>
<td>3 (-38.68 \pm 333.25i)</td>
<td>9.6782</td>
<td>3.1752</td>
<td>-0.0390</td>
<td>-0.4376</td>
</tr>
<tr>
<td>4 (-124.26 \pm 169.44i)</td>
<td>7.8962</td>
<td>4.8038</td>
<td>0.7596</td>
<td>1.0477</td>
</tr>
<tr>
<td>5 (5.42 \pm 2.78i)</td>
<td>-4.9238</td>
<td>-2.0734</td>
<td>-12.9637</td>
<td>-14.8384</td>
</tr>
</tbody>
</table>

The main disadvantages of using eigenvalue sensitivity for selection of feedback signals are:

- The open loop eigenvalue sensitivity, as presented above, is accurate only at the particular location of eigenvalues, and only for zero value of feedback gain. As a consequence, this method can be used with confidence only if a very small relocation of eigenvalues is desired. For any noticeable movement of eigenvalues an iterative procedure is necessary, as presented below. Very often in practice, however, the eigenvalues have to be moved considerably.

- The calculation of eigenvalue sensitivity at any other point of root locus requires re-calculation of system eigenvectors. If the eigenvalues need to be moved significantly from its open loop values, then the value of eigenvalue sensitivity should be calculated for several intermediate points along the root locus in order to get accurate information about the speed and direction of eigenvalue movement. The procedure for calculation is as follows:

  
  new feedback gain => system matrix => system eigenvalues and eigenvectors => eigenvalue sensitivity and new feedback gain.

The process is therefore iterative until the desired values of eigenvalues are obtained. This process can be very time-consuming.
HVDC system operation with very weak receiving AC systems

- Large eigenvalue sensitivity, usually implies long root-locus branch. Eigenvalues that lie on a long branches of root-locus are very sensitive to the system parameters changes. This implies poor robustness.
- If the root locus changes direction from the open loop poles to the position of zeros, than this method gives erroneous results. Appendix I.1 shows an example where eigenvalue originally move left, and then change direction giving the final eigenvalues with deteriorated damping.

Despite the above observations, this method is sometimes used in power systems controller design [11],[12],[13]. Based on Damping Sensitivity Factor (DSF) factor, this method was solely used in designing decentralized FACTS controller in [11]. Possibly, one of the better options would be the positioning of corresponding zeros for candidate feedback signals, as presented later in Section 8.4.4.

8.4.3 Relative Gain Array index and Condition Number

The RGA index shows how much the considered feedback loop affects the remaining feedback loops in the system. If the RGA index gets large values, the considered feedback loop may have significant negative influence on the remaining control loops in the system. The theoretical background is shown in Appendix I.2. More detailed derivation can be found in [9].

Table 8.4 shows the calculated RGA elements for the considered feedback signals. The rectifier side feedback loop is assumed to be in conventional configuration with DC current control. It can be seen that DC current is not a good choice for feedback signal at inverter side. This feedback signal gives RGA index that significantly differs from unity, and as a consequence considered feedback signal will be much affected by the action of other feedback loops. The remaining signals give RGA elements of similar values, not far from unity. The RGA index is normally used only to eliminate bad control loops.

<table>
<thead>
<tr>
<th>TABLE 8.4. RGA AROUND CROSS-OVER FREQUENCY.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Frequency [rad/sec]</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>120</td>
</tr>
</tbody>
</table>

Condition number is often used along with Singular Value Decomposition [9], to indicate the range of possible values for the system gains. Basic definition of condition number is also given in Appendix G.

Table 8.5 shows the condition number values around the cross-over frequency, for the candidate feedback signals. A relatively large value of condition number in Table 8.2, indicates that the system can not be regarded as a single input single output system (SISO) or as a MIMO with closely coupled signals. MIMO systems with closely coupled signals give low values for condition number. It is known that systems with a large condition number (\(\gamma > 10\)) are ill-conditioned and difficult to control [9]. Large condition number implies that the controller needs
information from all output signals, in order to derive proper control action. It is also known that pairing of inputs and outputs which give large condition number should be avoided in the case of decentralized control. Therefore it is evident from Table 8.5 that direct current is the least desirable feedback signal at inverter side.

<table>
<thead>
<tr>
<th>Frequency [rad/sec]</th>
<th>DC current</th>
<th>DC voltage</th>
<th>AC current angle</th>
<th>AC voltage magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>237</td>
<td>7.42</td>
<td>7.42</td>
<td>11.33</td>
</tr>
<tr>
<td>30</td>
<td>8.70</td>
<td>2.29</td>
<td>3.03</td>
<td>3.76</td>
</tr>
<tr>
<td>130</td>
<td>2.45</td>
<td>3.59</td>
<td>2.99</td>
<td>3.83</td>
</tr>
</tbody>
</table>

Figure 8.2 shows the singular value decomposition (SVD) for the case of direct current feedback. The basic theory on SVD is given in Appendix G. The inputs in this Figure are the firing angle signals at rectifier and inverter side, with rectifier and inverter DC currents as outputs. The goal of this analysis is to determine the controllability of the outputs with particular input signals [9].

A large difference between the principal gains, (consequently a large condition number) implies that the system will have different behavior for different directions of input signals, as discussed in Section 6.5.2. As a consequence, although the system parameters stay the same, the system behaves differently for change in input signals. A very small value of the minimum principal gain also implies that the system will have very poor gain for a certain combination of inputs. If the system has low gain values, the controller action will not be reflected on output, implying that disturbance can be propagated freely. The particular direction of input signals which give maximum or minimum system gains can be determined by calculating input singular vectors, in the manner described in 6.5.2. Direct current feedback is evidently a bad choice for the inverter feedback signal.

![Figure 8.2. Singular value decomposition for direct current feedback at inverter side.](image-url)
HVDC system operation with very weak receiving AC systems

The RGA, condition number and SVD for other feedback signals do not give any firm conclusion for the feedback signal selection. Using these criteria, only inverter DC feedback can be eliminated.

8.4.4 Position of zeros for candidate signals

To assess the benefit of each feedback loop and the impact on the system stability, the most appropriate method is the analysis of the corresponding root locus. A root locus shows the movement of eigenvalues, from its original position to the position which corresponds to infinity value of the feedback gain. The main drawback of this method is the computational burden in the case of complex (higher order) systems. Moreover, if the analysis is needed for large number of feedback signals, the time required for analysis may be excessive. On the other hand, calculating the location of zeros for each feedback signal does not require great computational effort. Given a transfer function system representation, the calculation of zeros as the roots of the transfer function numerator, is straightforward.

The location of zeros shows the maximum achievable movement of corresponding eigenvalues. The root locus for each eigenvalue starts at the open-loop location of the eigenvalue and ends at the location of corresponding zero. Therefore, the location of zeros will indicate the general direction of eigenvalue movement and the length of the root locus branch. They indicate the location of eigenvalues for large values of feedback gains.

The location of zeros in the right-half complex plane is undesirable, for it indicates instability at higher values of feedback gain. The feedback signals with unstable zeros far into the right-half plane can be eliminated, without drawing/analysing the whole root locus. In general, right-half plane zeros are undesirable at any frequency. However, because of the low-pass controller characteristics, the unstable zeros at very high frequencies are not of concern.

Zeros located close to the original eigenvalues imply that the considered feedback signal can not influence the particular mode. The most favorable position of zeros is far in to the left-half plane indicating improvement into the system stability margin with the chosen feedback control.

Table 8.6 shows the original system eigenvalues (for the case of very weak receiving AC system Inv. $SCR=1.0$) and the zeros for each of the candidate feedback signals. Observing position of zeros, the following conclusions can be drawn:

**DC current feedback signal.** If the DC current is used as inverter side feedback signal, the system stability can be improved at lower frequencies, however at higher frequencies this feedback signal significantly deteriorates the system stability as shown in Figure 8.3 a). The pair of high-frequency zeros in the right half plane (end of branch “$h$” in Figure 8.3 a)), indicates that eigenvalues of the dominant oscillatory mode will become unstable for higher gain values. Therefore, the “positive slope” on the HVDC static characteristic correctly indicates stability improvement at lower frequencies, but it fails to indicate strong negative effect at frequencies around the dominant oscillatory mode. If the AC system is strong, and if the sign of feedback signal is reversed this signal can be used for stability improvement around the dominant oscillatory mode.

**DC voltage feedback signal.** In the case of very weak AC system no benefit of this feedback loop is evident from Table 8.6 and Figure 8.3 b). The pair unstable open loop poles originally move left, and then they move even further right with very high rate (branch “$i$” in Figure 8.3 b)).
HVDC system operation with very weak receiving AC systems

In the case of stronger AC system and if the opposite sign is used, some improvement in the system stability in the mid-high frequency range is possible.

**AC voltage feedback signal.** This feedback signal is intuitively suggested as the first candidate, since a very important AC system variable is directly controlled with this loop. As shown in introduction, the variations of AC voltage are of main concern with low SCR systems. As can be seen from Table 8.6, this signal can stabilise the system at lower frequencies, but there is a possibility for instability at frequency close to 500 rad/sec. The main concern is the location of zeros far away in the right half plane (zeros in first two rows), which implies large movement of eigenvalues even for small feedback gains. Also, since these eigenvalues are very far away in right half plane, very large portion of the root locus is in the right half plane. A careful design of feedback filter is necessary in this case. However, with a properly designed filter in the feedback loop and if the feedback gain is kept at sufficiently low values, this signal could still achieve some stability improvement in the case of low SCR AC systems. The possibility for the use of feedback filter is further discussed later in this section.

**AC current angle feedback signal** This signal shows the best properties for stability improvement of very weak AC system. The pair of unstable poles can be considerably moved into left half plane with very little negative influence on the remaining poles. Since there are no right-half plane zeros, an infinity gain margin is possible with this feedback signal.

### Table 8.6. Position of Zeros for Candidate Feedback Signals.

<table>
<thead>
<tr>
<th>Open loop poles</th>
<th>DC Current</th>
<th>DC Voltage</th>
<th>AC Current angle</th>
<th>AC Voltage magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Eigenvalues</td>
<td>SCR=1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-94.80+839.33i</td>
<td>-527.33+1085.53i</td>
<td>-59.92+544.43i</td>
<td>-64.49+792.79i</td>
</tr>
<tr>
<td>2</td>
<td>-94.80-839.33i</td>
<td>-527.33-1085.53i</td>
<td>-59.92-544.43i</td>
<td>-64.49-792.79i</td>
</tr>
<tr>
<td>3</td>
<td>-16.07+442.59i</td>
<td>-12.78+425.76i</td>
<td>-14.31+442.15i</td>
<td>-16.27+442.63i</td>
</tr>
<tr>
<td>4</td>
<td>-16.07-442.59i</td>
<td>-12.78-425.76i</td>
<td>-14.31-442.15i</td>
<td>-16.27-442.63i</td>
</tr>
<tr>
<td>5</td>
<td>-38.68+333.25i</td>
<td>65.04+310.47i</td>
<td>-23.23+330.72i</td>
<td>-38.71+334.192i</td>
</tr>
<tr>
<td>6</td>
<td>-38.68-333.25i</td>
<td>65.04-310.47i</td>
<td>-23.23-330.72i</td>
<td>-38.71-334.192i</td>
</tr>
<tr>
<td>7</td>
<td>-124.26+169.44i</td>
<td>-125.15+188.58i</td>
<td>-112.72+166.50i</td>
<td>-123.15+170.52i</td>
</tr>
<tr>
<td>8</td>
<td>-124.26-169.44i</td>
<td>-125.15-188.58i</td>
<td>-112.72-166.50i</td>
<td>-123.15-170.52i</td>
</tr>
<tr>
<td>9</td>
<td>5.42+2.78i</td>
<td>-10.00</td>
<td>86.67+11.88i</td>
<td>-10.00</td>
</tr>
<tr>
<td>10</td>
<td>5.42-2.78i</td>
<td>-10.00</td>
<td>86.67-11.88i</td>
<td>-10.00</td>
</tr>
<tr>
<td>11</td>
<td>-15.64</td>
<td>-14.29</td>
<td>-10.00</td>
<td>-918.59</td>
</tr>
<tr>
<td>12</td>
<td>-6.71+1.21i</td>
<td>0.00</td>
<td>-9.86+8.97i</td>
<td>-8.81+2.41i</td>
</tr>
<tr>
<td>13</td>
<td>-6.71-1.21i</td>
<td>-7.59</td>
<td>-9.86-8.97i</td>
<td>-8.81-2.41i</td>
</tr>
</tbody>
</table>
HVDC system operation with very weak receiving AC systems

Figure 8.3. Root locus for a) inverter DC-current and b) DC-voltage feedback. “h” and “i” locus branches can have unstable eigenvalues.

Figure 8.4 shows the whole root locus for the case of AC current angle feedback. It is seen that the unstable eigenvalues “D” are moved into the stable region, which has good damping and good speed of response. Also, the damping of the second dominant complex pair “E” is somewhat improved. Therefore the AC-current-angle is chosen as the feedback signal for the new inverter controller.

According to the first two criteria (Eigenvalue sensitivity and RGA index), also the AC current angle feedback signal shows similarly good properties. The best choice for inverter side feedback is AC current angle.

It is important to discuss the use of a feedback filter for some of the candidates that show stability degradation with simple feedback loop, since some of these variable (like AC voltage feedback) are of paramount importance for AC-DC interconnection point.

A first order feedback filter will introduce a ten-times gain reduction per frequency decade, at frequencies above cross over frequency. The cross over frequency for such filter, in the case of above system, would be expected at approximately $50\text{rad/s}$. This implies that we could expect ten times reduced gain around $500\text{rad/sec}$. Consequently, the observed stability degradation at higher frequencies, can be compensated with such a feedback filter. However, for both AC voltage feedback and DC current feedback a significant eigenvalue movement far into the right half plane is evident at higher frequencies. Even with a feedback filter, in order to maintain stability margin at higher frequencies (no oscillatory responses), the feedback gain would have to be very low. A low feedback gain would imply poor stability improvement at lower frequencies. A noticeable dislocation of eigenvalues 9-10 is required to achieve satisfactory system performance at lower frequencies.

Further, since the root locus branch at higher frequency is relatively long (because of the zeros far into the right half plane), there is a possibility for significant dislocation (movement) of the eigenvalues lying on this branch. This implies that the system would have poor robustness with respect to the changes of controller parameters and system parameters in general. System behavior would change significantly with the change in operating parameters. Therefore, for the above reasons, direct current or AC voltage feedback could introduce only limited stability improvement and with not easy design of feedback filter.
HVDC system operation with very weak receiving AC systems

8.5 INVERTER CONTROLLER DESIGN

8.5.1 Design objectives

The results from the previous section demonstrate that the AC-current-angle control with a simple feedback gain can significantly improve the system stability. However, this stability improvement is guaranteed only for the system operation around the nominal operating point. A controller design based only on the root-locus analysis, can not offer insight into the controller tolerance to variations in the system parameters.

In this section the development of a feedback controller that will be insensitive to large variations in SCR and impedance angle, at the inverter side is discussed. Based on the AC-current-angle feedback, a second order filter is introduced into the feedback loop to further improve the controller performance and the system robustness.

A design of feedback controller is always a compromise between the achievable performance and the ability of the controller to tolerate changes in the system parameters. With respect to the HVDC inverter controller design, this can be viewed as designing a controller which can accommodate large variations in the AC system parameters (SCR changes), against designing a controller which will offer satisfactory system responses at very low SCR levels.

In the first instance, the system responses at nominal SCR level do not need significant improvement, but the controller is needed to enhance the system robustness, for example for the variation in the system strength from strong to weak AC system.

In the second case the nominal system may be unstable or marginally stable, because of the very low SCR, and the controller is required to improve the system stability and responses around the nominal operating point. For this design, the controller robustness to large change in the system SCR around an already low SCR value, will be somewhat compromised.
As the confirmation of the ability of the proposed control scheme to accommodate the above two extreme cases, two controller designs are described in this Chapter.

Robust Controller, intended to render the system stability and speed of response for very wide range of operating conditions (SCR and impedance angle values).

Controller for very weak AC systems, intended to enable system operation with very low SCR and with moderate changes of the system parameters around its nominal value.

These two designs, presented in Sections 8.5.3 and 8.5.6, may have applications in two different systems depending on their characteristics. Firstly, the model uncertainty is discussed, since a special approach is adopted to cater for the special practical requirements of the AC system representation.

8.5.2 Model uncertainty

Robustness is required to be an important characteristic of inverter side HVDC controller. A control system is robust if it is insensitive to differences between the actual system and the model in use. These differences should be clearly mathematically defined in order to investigate the controller robustness.

Two groups of model uncertainty are considered [9]:

1) Parametric uncertainty, where the structure of the model is known but some of the parameters are uncertain. Parametric uncertainty is sometimes called structured uncertainty since it models uncertainty in a structured manner.

2) Neglected and unmodeled dynamic uncertainty, (unstructured uncertainty) where the model is in error because of unknown dynamics of the system or because of unknown nature of the modeled process.

The parameters of inverter side AC system are assumed uncertain since these parameters substantially change as the AC system operating conditions change. The resistance $R_3$ and inductance $L_2$ form the inverter AC system, Figure 1.1 Chapter I and Figure A.1 appendix A, are considered as uncertain and they are expressed as:

$$L_{2\text{min}} < L_2 < L_{2\text{max}}, \quad R_{3\text{min}} < R_3 < R_{3\text{max}} \quad \text{for } G_p(s) \in \Pi_1$$

where the following notation is adopted throughout this Chapter:

- $\Pi$ a set of possible perturbed plant models,
- $\Pi_1 \in \Pi$ a set of possible perturbed plant models because of structured uncertainty,
- $\Pi_2 \in \Pi$ a set of possible perturbed plant models because of unstructured uncertainty,
- $G(s) \in \Pi$ nominal plant model with no uncertainty,
- $G_p(s) \in \Pi$ particular perturbed plant models.

The limits for parameters changes in (8.1), can not be expressed directly, since they would be meaningless from the practical point of view. To get the physical meaning for the maximum possible deviations of parameters, the boundary values in equation (8.1) are obtained from...
maximum expected changes in SCR and power factor angle. For robust controller design, the following changes are considered:

\[ 1.7 < SCR < 3.7, \quad 70 < \delta < 80 \]  \hspace{1cm} (8.2)

with nominal operating point at:

\[ SCR = 2.5, \quad \delta = 76^\circ. \]  \hspace{1cm} (8.3)

These changes are assumed to be the largest possible deviations of operating conditions for the studied AC system. It is certain that practical systems are not likely to experience such large parameters variations.

The model inaccuracy around half the sampling frequency is regarded as an unstructured uncertainty. This model error comes from the use of continuous model for representation of discrete system. The model error will be assumed to be starting at 50Hz and reaching 100% uncertainty at half the sampling frequency (300Hz).

To represent the uncertainty in frequency domain, the plant model \( G_p \) is expressed in terms of lumped multiplicative uncertainty, as shown in Figure 8.5. Plant model can be represented as:

\[ G_p(s) = G(s)(1 + W_f(s)\Delta_f(s)) \quad G_p(s) \in \Pi \]  \hspace{1cm} (8.4)

![Figure 8.5. AC system perturbations represented as a multiplicative uncertainty.](image)

**Nomenclature:**
- \( \beta \) - Inverter firing angle order,
- \( \psi_2 \) - Inverter AC current angle.

where:

\[ W_f(s) \] - weighting function,

\[ |\Delta_f(j\omega)| \leq 1, \forall \omega \]

\( K(s) \) - controller.

The multiplicative weight \( W_f(s) \) is obtained as:

\[ |W_f(s)| \geq l_f(\omega) = l_1(\omega)l_2(\omega), \forall \omega \]  \hspace{1cm} (8.5)

where, for structured uncertainty:
HVDC system operation with very weak receiving AC systems

\[
l_1(\omega) = \max_{G_r \in \Omega_1} \left| \frac{G_p(j\omega) - G(j\omega)}{G(j\omega)} \right| \quad \forall \omega
\]

(8.6)

and for unstructured uncertainty:

\[
l_2(\omega) = \max_{G_r \in \Omega_2} \left| \frac{G_p(j\omega) - G(j\omega)}{G(j\omega)} \right| \quad \forall \omega
\]

(8.7)

Figure 8.6 shows the graphical method for derivation of weighting function for multiplicative uncertainty, where both \(l_1\) and \(l_2\) are shown. In this Figure, the values for system parameters from (8.1) are changed in small steps and all the value for transfer function form (8.6) are drawn in frequency domain. The multiplicative weight, shown as an envelope for family of transfer functions, is chosen as the second order function:

\[
W_1(s) = 19.46 \frac{(s+4.2)(s+300)}{(s+24)(s+6800)}
\]

(8.8)

where:

\[
l_1(\omega) = 19.46 \frac{(s+4.2)300}{(s+24)6800}, \quad l_2(\omega) = \frac{(s+300)6800}{(s+6800)300}
\]

This multiplicative weight will ensure that the worst case of uncertainty is covered.

Figure 8.6. Weighting function \(W_1(s)\) determined on the basis of relative model uncertainty.

Figure 8.7 shows the uncertainty boundaries with respect to the nominal plant transfer function. It is evident that structured uncertainty allows similar model deviations along the whole frequency domain. Unstructured uncertainty, however, extends the boundary for model deviations.
only around half the sampling frequency. The allowed model uncertainty at half the sampling frequency ($h_s$), exceeds 100%.

8.5.3 Robust Controller design

The controller robustness can be explicitly considered in the design stage only in the $H_\infty$ control theory. To this end, it is necessary to consider robust stability as special requirement in the $H_\infty$ controller synthesis.

The mixed sensitivity approach from $H_\infty$ theory is used for controller synthesis. The basic theory behind this controller design method is shown Appendix J. More detailed theory can be found in [9] and [10].

![Diagram of model uncertainty in frequency domain](image)

**Figure 8.7. Model uncertainty in frequency domain.** $h_s$ - maximum model uncertainty at half the sampling frequency.

Since the controller robustness is of primary importance, the complementary sensitivity function will be determined entirely on the basis of multiplicative uncertainty condition. The measurement noise rejection is of less importance, and reference tracking is irrelevant since the designed controller is stabilising controller without a reference signal.

The condition for control signal magnitude is made relatively lose to enable the best results with the first two requirements. The condition for robustness is considered as the most important controller requirement, and it is not compromised during the design stage. The bound for sensitivity function is originally made overly-demanding and it is later relaxed during controller design, to enable controller implementation as a low order filter.

Figure 8.8 a) shows graphs of the chosen weight $W_1(j\omega)$ and of the sensitivity function $(S(s))$ for the system with $H_\infty$ optimal controller and with second order controller, where the controller order reduction is presented in Section 5.4. Similarly, Figure 8.8 b) shows graphs of
HVDC system operation with very weak receiving AC systems

complimentary sensitivity function \((T(s))\). It can be seen that both: condition for robust stability and condition for nominal performance are satisfied. The controller meets the performance objectives specified by \(T(s)\) and \(S(s)\). It can be seen, that the condition for robust performance (RP) derived as:

\[
RP \iff \|W_1(s)S(s)\| + \|W_3(s)T(s)\| < 1, \forall \omega
\]  
(8.9)

is also met within a factor of at most two. The following notation is used in (8.9):

- \(S(s)\) sensitivity function,
- \(T(s)\) complementary sensitivity function,
- \(W_1(s)\) weight for sensitivity function,
- \(W_3(s)\) weight for complementary sensitivity function,

The equation (8.9) follows from the following inequity [9]:

\[
\|W_1(s)S(s)\| + \|W_3(s)T(s)\| \leq 2 \max\left\{\|V_1(s)S(s)\|, \|V_3(s)T(s)\|\right\} < 2
\]  
(8.10)

This condition means that the system will have guaranteed performance specified by \(W_1(j\omega)\), for whole range of considered parameters changes.

![Figure 8.8. Frequency response of sensitivity function and complementary sensitivity function.](image)

The \(H_e\) optimal control algorithm gives as the result controller of the order equal to the system order plus order of weighting functions. Consequently the order of the original controller is \(45+2+2+2=51\). The controller of this complexity is difficult to implement and the next step in controller design is the controller order reduction.

**8.5.4 Controller order reduction**
Balanced truncation and balanced residualisation techniques can be used for controller order reduction [9]. In balanced truncation, the states which are least controllable and observable are discarded. This is done by ranking the states with respect to their Hankel singular values. This algorithm preserves the system gain at infinity, whereas steady state gain may be changed. In balanced residualisation, the derivatives of the states corresponding to the small Hankel singular values are simply set to zero. The system steady state gain is preserved in this way. Balanced residualisation is therefore chosen as more suitable for the controller in design.

The frequency response of the original $H_{\infty}$ controller and of the fourth order controller obtained using balanced residualisation are shown in Figure 8.9. The controller order is then reduced to the second order (also shown in Figure 8.9) by matching the controller frequency response with the fourth order controller. This reduction of controller order implies some degradation in the controller performance, as seen in Figure 8.8.

![Figure 8.9. Controller frequency response.](image)

### 8.5.5 Controller implementation

Figure 8.10 shows the controller block diagram. The controller is implemented in two stages: washout filter (differentiator with a first order filter) and an $H_{\infty}$ controller.

The first stage, washout filter, ensures that the steady state value of inverter firing angle is unaffected. This element consists of two branches: the upper branch will pass only the steady state component, whereas the lower branch is all pass filter. In steady state, the output of the element will be zero. During transients, however, the output will follow the input values, with the lowest passing frequency dependent on the time constant in the upper branch. As far as digital simulation is concerned, a washout filter implemented in this form has practical advantages since direct differentiation (which could cause numerical instabilities) is avoided. The overall transfer function of the element can be derived as
HVDC system operation with very weak receiving AC systems

Figure 8.10. Controller implementation.

\[
1 - \frac{1}{(sT_1 + 1)(sT_1 + 1)} = \frac{s(sT_1 + 2T_1)}{(sT_1 + 1)(sT_1 + 1)} \quad (8.11)
\]

which consists of differentiator, first order filter and phase lead component. The phase lead component does not have large influence on the overall performance of the element.

The second stage is the \( H_{\infty} \) controller implemented in the form of second order transfer function. It is known that much better performance can be achieved if a higher order controller is used. However a simple second order controller is chosen to facilitate controller practical implementation. Calculated controller parameters are shown in Table 8.7.

The feedback signal can be obtained by measuring the instantaneous values for AC current and using the on-line Park’s transformation to get \( d-q \) components as shown in Figure 8.11. By using this method in the case where synchronous machines are employed, it is important that the angular frequency, which is used for Park’s transformation, is synchronized with AC system frequency. This synchronization is not used with the simulation performed. The same method of getting a feedback signal from the AC system variables, can also be used for the control strategies studied in Chapter 7.

8.5.6 Design of Controller for Very Weak AC Systems

Controller for very weak AC systems is designed for the nominal operating point at: \( SCR = 13 \), \( \delta = 76 \)deg, with the system parameters shown in Appendix H. In designing this controller, strong emphasis is placed on the system stability (performance) and only moderate parametric uncertainty (robustness) is considered. Following the procedure similar to robust controller design, the controller parameters can be calculated (Table 8.7).

| TABLE 8.7. CALCULATED CONTROLLER PARAMETERS. |
|-------------------|---|---|---|---|
| \( K \) | \( T_1 \) | \( T_2 \) | \( T_3 \) | \( T_4 \) |
| contr 1 | 25 | 0.0085 | 0.12 | 0.18 | 0.14 |
| contr 2 | 40 | 0.0085 | 0.8 | 7 | 0.9 |
8.6 SIMULATION RESULTS

PSCAD/EMTDC simulation software is used for the controller testing. Figure 8.12 shows the verification testing of the robust controller. In order to demonstrate the controller robustness, the system for these tests is chosen to be on the edge of the controller operating range (SCR=1.7@78deg). The improvement in terms of the speed of response and reduced overshoot, is evident from the Figure. The main advantage of this controller is that the system will have similar (specified) performance for very wide range of operating conditions. Further testing of this controller with very low SCR (SCR=1.2) have also demonstrated considerable improvement in the responses.

Figure 8.13 portrays performance of the controller for very weak AC systems. Significant improvement in the system response can be noticed. The original system in this Figure with SCR=1.2 is on the margin of stability and the responses are very poor. The system with the new controller is stabilised with noticeable reduction in overshoot and settling time. It is important that the AC voltage responses are also substantially improved, as can be seen from the Figure, although this variable is not directly controlled. Since there must be a trade off between performance and robustness, controller for very weak AC systems tolerates only moderate changes in the system parameters.
HVDC system operation with very weak receiving AC systems

Figure 8.12. System response following current order step change. Inverter SCR=1.7@78deg.

Figure 8.13. System response following current order step change. Inverter SCR=1.2@76deg.

Figure 8.14. System response following a single phase fault(0.05sec) at the inverter side. Inverter SCR=1.3@76deg.
HVDC system operation with very weak receiving AC systems

It is very difficult to analytically express the lowest limit for possible SCR by using the above methods, since the original system is dynamically unstable for $SCR<1.0$. The control theory employed, and the method for structured uncertainty, does not consider controller design for unstable systems.

It is emphasized that the test system in the above Figures does not have any AC “stabilising” elements (synchronous condensers, SVC, etc.). The responses presented are obtained by using only HVDC controls. This result heralds the possible use of the above control method instead of conventionally used SCs or SVC with weak inverter AC systems.

Figures 8.14 and 8.15 show the system recovery after a single phase fault at the inverter AC bus. It is evident that the AC voltage has very large fluctuations with long and difficult recovery, for the case of the original system. The superiority of the new controller is easily observed. Figure 8.16, where the test system has $SCR=1.0@76\text{deg}$, further demonstrates benefits of the new control logic. The original system in this configuration, is unstable. From the steady state point of view, it is seen in the Figure that AC voltage drops following drop in DC current which causes reduction in power infeed in the inverter AC system.
The “blips” which can be noticed in some of the figures above, when system approaches steady state, can be attributed to numerical instabilities in PSCAD/EMTDC, which is a consequence of highly non-linear nature of the process. They can not be explained using linear systems theory.
8.7 CONCLUSIONS

By using the analysis of eigenvalue locations, it has been shown that reduction in strength of the inverter AC system affects predominantly the low frequency system dynamics. The existing HVDC controller at rectifier side can not be used for stability improvement, since the affected eigenvalues are far more sensitive to the states at inverter side. It is necessary to design new controller at inverter side, using the inverter firing angle as a control input.

The new control algorithm tends to make the HVDC control scheme more symmetrical, with a high-gain low-frequency controller at the inverter side, similar to the direct current feedback that conventionally exists at the rectifier side. According to the position of zeros for candidate feedback signals, the best feedback signal at the inverter side is found to be the AC-current-angle. Since the system robustness and the robust performance is the main control objective, the $H_\infty$ control theory is used in the controller design.

EMTDC/PSCAD simulation testing of the new controller, shows that the system can meet the design objectives from $H_\infty$ control theory, for very wide range of the AC system SCR changes, where the considered operating range is $1.7<SCR<3.7$ and $70<\delta<80$ for the test system. The controller designed for very low SCR proves that the HVDC system can give satisfactory fast responses even with SCR as low as $SCR=1.0$. The simulation responses following a single phase fault, further demonstrate that a properly designed HVDC controller can give significant improvement in the disturbance recovery.

The new control method can be used as an alternative for the traditionally used expensive solutions to the low-SCR related problems such as employing synchronous condensers and/or SVC elements at the AC bus.
REFERENCES:


CONCLUSIONS

I. GENERAL CONCLUSIONS

Because of the complexity of HVDC-HVAC systems, and to cater for the different model purposes, three different HVDC models are suggested in this Thesis: Detailed HVAC-HVDC model, simplified linear continuous model and linear discrete model.

Detailed HVDC-HVAC system model demonstrates excellent response matching against PSCAD/EMTDC simulation for wide range of system parameters and various controller gains and operating modes. For an accurate analytical HVDC system model the following aspects are of importance:

- Detailed modeling of DC system dynamics (fourth order model is sufficient).
- Detailed modeling of PLL dynamics (second order model).
- Dynamic model of AC system and AC filters at both ends (sixteenth order model is used).
- Detailed representation of AC-DC interactions.
- Use of Park’s transformation for AC-DC coupling.
- Model discretisation is not necessary, if the model is used in the frequency range $f<100Hz$.

A very important property of the detailed model is that the most of the model variables (states) and parameters have physical meaning and that the model consists of modules, which reflect actual subsystems. These properties can simplify system analysis and controller design procedure.

For some modeling purposes, it is more convenient to have simplified HVDC dynamic models. A simple, fourth order dynamic system model which however incorporates the influence of AC systems through static, equivalent impedance AC system equations, is also developed in this Thesis. This model can be used for the study of non-linear effects in HVDC systems. It also proves reliable for controller design for mitigation of composite resonance phenomenon.

For the system analysis closer to half the sampling frequency, a properly discretised model should be employed. The discrete model developed here is of impulse type, employing an ideal sampler for discretisation of firing angle signal. It is used for controller design for elimination of $100Hz$ oscillations on DC side of HVDC system.

In modeling of TCR/TCSC elements, a similar approach to HVDC modeling, with the use of Park’s transformation can be employed. In this way, the dynamic of AC system, PLL and TCR controller are represented as a separate subsystems. The model can be readily connected to the existing HVAC-HVDC system model.

The analysis of non-linear mode transformations in HVDC systems shows that limit-cycle oscillations are not likely to develop, for normal operating conditions, for the mode change from constant beta to constant gamma mode. The analysis of converter firing angle modulation shows that only for very small nominal angles and large magnitude of input signals, which do not occur in practice, the converter direct voltage could have noticeable magnitude of second harmonic superimposed on the fundamental signal. This implies that converter behaves as a linear element for most practical operating conditions.
Conclusions

One of the methods to counteract the possible composite resonance on HVDC-HVAC systems is to modify the resonant condition on DC system impedance profile. This can be done by properly designing HVDC controller that acts on HVDC firing angle on both line ends. PSCAD/EMTDC simulation results show that significant reduction in DC side first harmonic component (in some cases to $1/4$ of the original value) is possible with the newly designed controller. The controller is also robust to the AC parameters changes and it shows improvement in the responses to small signal disturbances.

The eigenvalue decomposition and singular value decomposition analysis applied to the detailed HVDC-HVAC system model can offer small-signal analysis of HVDC-HVAC interactions. The analysis of sensitivity of the dominant system eigenvalues with respect to the AC system parameters shows the frequency domain for the possible oscillatory instabilities and the side of the system which is the most likely to experience the instability. The rectifier side of the system is expected to experience instability at higher frequencies ($50\text{Hz}$ and over), whereas at inverter side the instabilities can be expected at lower frequencies. The reduction in the AC system strength will predominately affect the eigenvalues at lower frequencies as can be shown by the relative movement of the dominant system eigenvalues. The SCR reduction at inverter side has much more effect on the system stability. If the interaction between AC and DC systems is analyzed through the influence of inherent feedback loops, it can be shown that AC voltage angle at rectifier side should be tightly controlled. At inverter side the system stability is improved if a loose control is applied at AC voltage angle. Examination of influence of AC voltage magnitude shows that control of this variable could improve the system stability at both ends, however careful tuning of controller gains is necessary.

The stability analysis of HVDC control loops reveals the influence of each control loop in the whole frequency domain. At rectifier side, direct current feedback significantly improves the system stability at lower frequencies, whereas at frequencies close to first harmonic it degrades the system stability. DC feedback actually accelerates development of AC-DC composite resonance close to first harmonic frequency. At the inverter side, most of the feedback loops will improve system stability at some frequencies, with noticeable stability deterioration at other frequencies. Reactive current feedback is found to be the best inverter control strategy among all reported inverter control methods. This feedback signal does not have any unstable zeros, and consequently very high value of feedback gain could be used.

By analyzing the position of zeros for candidate feedback signals, it can be concluded that AC current angle is the best inverter feedback signal for HVDC operation with very weak inverter AC systems. This feedback signal can move the unstable mode left, into the stable region, without significantly affecting remaining eigenvalues. To maximize the benefit of this feedback loop, a second order filter, designed using the $H_\infty$ control theory, is placed into the feedback loop. In the case of inverter controller, the main design requirement should be the system robustness with respect to the AC system parameters changes. The controller designed in this Thesis, tolerates very wide changes in system strength, $1.7<\text{SCR}<3.5$, with nominal operating point $\text{SCR}=2.5$. This wide change of operating parameters is very unlikely to happen in practice. The controller specially designed for still lower SCR shows that HVDC system can satisfactorily operate with inverter $\text{SCR}=1.0$ and without any Synchronous Condensers or SVC elements at inverter AC bus. The new control method does not require significant investment.
II. THESIS CONTRIBUTION

The contributions from this thesis can be summarized in several points, as follows:

- Analysis of HVDC modeling principles. Since there has been a great variety of HVDC modeling approaches, it is important to postulate the most convenient model structure, model form and to identify the parts and operating conditions of the system that must be modeled.

- Development of an accurate and detailed HVDC system model. In this Thesis, an analytical model of CIGRE HVDC Benchmark test system, which has good response matching with non-linear digital simulation, is developed. The subsystems in the model, most model variables and model parameters have physical meaning. This physical resemblance is of outstanding importance in the system analysis and controller design.

- Analysis of negative effects caused by the AC system voltage unbalance. The study presented here paves the way for the use of HVDC controls for elimination of 100Hz DC side oscillations and/or AC voltage unbalance.

- Analysis and elimination of composite resonance on DC systems. A relatively simple yet very effective HVDC controller, developed here, can significantly defer development of composite resonance at frequencies close to first harmonic.

- Stability analysis of (inverter side) HVDC control loops. In the available HVDC bibliography, there is no clear recommendation for the inverted control method. This thesis offers comprehensive dynamic analysis of most of the reported HVDC control loops at the inverter side.

- Development of HVDC control method for HVDC operation with very weak AC systems. This controller is the single most significant and most advanced contribution. The controller enables reliable HVDC system operation with inverter SCR much lower than traditionally thought possible. This control method should be considered a possible substitute for the use expensive and power consuming synchronous condensers at inverter station.

- A method for selection of feedback signal in a complex dynamic system with large number of candidate feedback signals. It is shown in the Thesis that the position of zeros for feedback candidates is the single most reliable, still very convenient, method for fast scanning through a substantial number of candidate feedback signals.

Three new HVDC control algorithms have been developed in this Thesis: Controller for elimination of composite resonance on HVDC-HVAC systems, controller for elimination of second harmonic oscillations on DC side of HVDC system and the inverter controller for HVDC operation with very weak AC systems. All three controllers are completely new concept in HVDC control theory. They have been theoretically presented in this Thesis, and tested on industry accepted non-linear simulator PSCAD/EMTDC.

The first two controllers are intended as a cure for a particular operating difficulty. Under the normal operating conditions their implementation may not be justifiable. The third controller (for HVDC operation with weak AC systems) is however developed as the best control option for inverter converter. Since this controller increases the system robustness to the AC system parameters changes and AC system contingencies, the controller can be used as a normal HVDC control method at inverter side. Likewise, for the future HVDC systems, the control method could be an alternative to the traditional more expensive methods of the system stability improvement like synchronous condensers or SVC elements.
Conclusions

III. FURTHER STUDIES

Concerning the modeling HVDC systems in the frequency range $f<100\text{Hz}$, very little could be improved in the developed detailed HVDC model. In the frequency range $f>100\text{Hz}$ however, only the basic modeling principles are presented. The developed discrete model is overly simplified, from the dynamic point of view. The next step would be to apply the presented discretisation principles on the detailed dynamic model, with the use of two samplers (at rectifier and inverter) and two different time delays. It is certain that the development of this model would be tedious. Because of the higher order dynamic model, the use of two samplers and difficult system structure, the model would not be convenient for the system analysis and controller design. Also, at present, there is no incentive from the industry for the HVDC study in this frequency domain. As it is shown in the thesis, most of the to-date reported stability problems fall in the frequency range $f<100\text{Hz}$, and they can be successfully analyzed using the models presented here. In the higher frequency domain, the harmonic domain study is usually sufficient.

In this Thesis a new modeling approach to TCR/TCSC modeling is presented. The initial tests confirm model validity. As presented in the Thesis, the obvious point for the model improvement is the equation for the TCR reactance as a function of thyristor firing angle. If this relationship could be more accurately determined, possibly using other modeling principles, the resulting system model would be very powerful tool for the dynamic analysis of FACTS elements. The further work could also be performed to incorporate/connect this model with the developed HVDC-HVAC system model, for the purpose of analysis of FACTS-HVDC interactions. The model could similarly be used for research on tuning of FACTS controllers, for possible coordination between them and selection of location for future elements, providing the frequency boundaries for the model fidelity are verified.

The controller designed for elimination of $100\text{Hz}$ oscillations on DC system has been successfully tested for elimination of $100\text{Hz}$ gamma oscillations on New Zealand HVDC system. However discrepancies between the analytical model responses and the simulator responses were observed. Despite these differences, the final values for the controller parameters can be obtained relatively easily by using the modern features on digital simulator, like FFT and similar. For more demanding analysis and more accurate controller design, a further improvement in the discrete system model is necessary.

The controller for HVDC operation with very weak AC systems, has been extensively tested on CIGRE HVDC benchmark model, with very weak inverter AC system configurations. The next step is to test controller as the possible substitute for Synchronous Condensers. Using the presented results, it can be presumed that the new controller, together with properly sized fixed capacitors for reactive power support, can be adequate replacement for Synchronous Condensers, in terms of dynamic stability and steady state (reactive) power balance. The best direction for research would be to use the data of an actual HVDC system for the testing. Since majority of installed HVDC systems use Synchronous Condensers at inverter bus, the potential application area of this controller is enormous. A proper cost-benefit analysis should accompany these studies.
APPENDIX A DETAILED LINEAR CONTINUOUS MODEL

A.1 AC system model

With reference to Figure A.1, the dynamic electrical equations that describe each phase of the AC system can be derived as:

\[
L_2 \frac{d i_{L2}}{d t} = e_j - v_{acj} - i_{L2} (R_3 + R_2) + i_{L1} R_2 \\
L_1 \frac{d i_{L1}}{d t} = i_{L2} R_2 - i_{L1} (R_1 + R_2) \\
C_1 \frac{d v_{ac1}}{d t} = -i_{acj} + i_{L2} - i_{L3} - \frac{1}{R_5} (v_{acj} - v_{c3}) - i_{L4} - \frac{1}{R_6} (v_{acj} - v_{c4}) \\
L_3 \frac{d i_{L3}}{d t} = v_{acj} - v_{c3} - v_{c2} - i_{L3} R_4 \\
C_3 \frac{d v_{c3}}{d t} = \frac{1}{R_5} (v_{acj} - v_{c3}) + i_{L3} \\
C_2 \frac{d v_{c2}}{d t} = i_{L3} \\
L_4 \frac{d i_{L4}}{d t} = v_{acj} - v_{c4} \\
C_4 \frac{d v_{c4}}{d t} = i_{L4} + \frac{1}{R_6} (v_{acj} - v_{c4})
\]  

These equations are applied for both rectifier and inverter AC systems, where index \( j = 1, 2 \), will be used for rectifier and inverter respectively. The states, inputs and outputs are chosen as:

\[
\begin{bmatrix}
i_{L2} \\
i_{L1} \\
v_{acj} \\
i_{L3} \\
v_{c3} \\
v_{c2} \\
v_{L4} \\
v_{c4}
\end{bmatrix}
= u_{acjdc}
\begin{bmatrix}
i_{acj} \\
e_{acj}
\end{bmatrix}
= y_{acjdc} \begin{bmatrix}
v_{acj}
\end{bmatrix}
\]

The states are therefore chosen to be currents through inductors and voltages across capacitors.
Appendix

![AC system representation](image)

**Figure A.1 AC system representation.**

The state-space model for phase A is:

\[
sx_a = A_x x_a + B_x u_a, \quad y_{acj} = C_a x_a
\]  

(A.10)

where:

\[
A_a = \begin{bmatrix}
-(R_2 + R_3) & \frac{R_2}{L_2} & -1 & 0 & 0 & 0 & 0 \\
\frac{L_2}{R_2} & -(R_1 + R_2) & 0 & 0 & 0 & 0 & 0 \\
\frac{L_1}{R_1} & \frac{L_1}{R_1} & \frac{1}{C_1} (\frac{1}{R_5} + \frac{1}{R_6}) & \frac{-1}{C_1} & \frac{1}{C_1 R_5} & 0 & \frac{-1}{C_1} \\
0 & 0 & \frac{-1}{L_3} & \frac{-1}{L_3} & \frac{-1}{L_3} & 0 & \frac{-1}{L_3} \\
0 & 0 & \frac{1}{C_3 R_5} & \frac{1}{C_3} & \frac{1}{C_3 R_5} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{C_2} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{L_4} & 0 & 0 & 0 & \frac{-1}{L_4} \\
0 & 0 & \frac{1}{C_4 R_6} & 0 & 0 & 0 & \frac{-1}{C_4 R_6}
\end{bmatrix}
\]  

A.11

\[
B_a = \begin{bmatrix} 0 & 0 & -1/C_1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad C_a = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}
\]  

(A.12)

This model is transferred to a three phase system model, as shown in Chapter 1, and latter to \(dq\) coordinate frame, in order to obtain the final AC system model.
AC system model transformation from \( abc \) to \( dq0 \) coordinate frame

In Section 1.2.1, the three phase system model is derived in the form:

\[
\dot{x}_{abc} = A x_{abc} + B u_{abc}
\]  

(A.13)

where the model matrices, states and inputs are defined in (1.2a-1.9a) and (1.10).

In order to transform this model to \( dq \) coordinate frame, Park’s transformation is used, which is defined as:

\[
\mathbf{v}_{dq0} = P \mathbf{v}_{abc}
\]  

(A.14)

where:

\[
P = \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\cos \omega_0 t & \cos(\omega_0 t - 2\pi/3) & \cos(\omega_0 t + 2\pi/3) \\
\sin \omega_0 t & \sin(\omega_0 t - 2\pi/3) & \sin(\omega_0 t + 2\pi/3)
\end{bmatrix}, \quad \mathbf{v}_d = \begin{bmatrix} v_d \\ v_a \\ v_b \end{bmatrix}, \quad \mathbf{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}
\]  

(A.15)

and where \( \mathbf{v} \) represents any electrical AC variable such as voltage or current.

To get a model suitable for application of Park’s transformation, several model transformations are necessary. The following model transformation is introduced to rearrange system states as:

\[
\begin{bmatrix}
x_{1a} \\
x_{1b} \\
x_{1c} \\
x_{2a} \\
x_{2b} \\
\vdots \\
x_{nc}
\end{bmatrix} = T_1 \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix},
\]  

(A.16)

and the model becomes:

\[
T_1 \dot{x}_{abc} = T_1 A T_1^{-1} T_1 x_{abc} + T_1 B T_1^{-1} T_1 u_{abc}, \quad T_1 \in \mathbb{R}^{3n \times 3n}
\]  

(A.17)

\[
x_{abc} = A_{11} x_{abc} + B_{11} u_{abc}
\]  

(A.18)

where transformation matrix \( T_1 \) and model matrices \( A_{11}, B_{11} \) can be readily derived.

A new transformation is applied to equation (A.18) as:
Appendix

\[ P_{tr}^* \Sigma_{abc1} = P_{tr} A_{tr} P_{tr}^{-1} P_{tr} \Sigma_{abc1} + P_{tr} B_{tr} P_{tr}^{-1} P_{tr} \Sigma_{abc1} \text{, where } P_{tr} = \begin{bmatrix} P & 0 & \ldots & 0 \\ 0 & P & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & P \end{bmatrix}, P_{tr} \in R^{3nx3n} (A.19) \]

where n is the order of the original single-phase AC system model.

If derivative of (A.14) is expressed as:

\[ \dot{\nu}_{dq} = P_{tr} \dot{\nu}_{abc} + P_{tr} \dot{\nu}_{abc} \] (A.20)

and using the known properties of Park’s transformation:

\[ P_{tr}^{-1} P_{tr}^{-1} = \begin{bmatrix} P P^{-1} & 0 & \ldots & 0 \\ 0 & P P^{-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & P P^{-1} \end{bmatrix} \]

and:

\[ P_{tr} Q P_{tr}^{-1} = Q, \text{ for } \forall Q \in R^{nxn}, \] (A.21)

where \( P_e = \omega_0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \) (A.22)

using (A.20) and (A.14), equation (A.13) can be transformed as:

\[ \dot{\Sigma}_{dq0} = P_{tr} P_{tr}^{-1} \Sigma_{dq0} + A_{tr} \Sigma_{dq0} + B_{tr} u_{dq0} \] (A.23)

\[ \dot{\Sigma}_{dq0} = (P_{tr} P_{tr}^{-1} + A_{tr}) \Sigma_{dq0} + B_{tr} u_{dq0} \] (A.24)

\[ \dot{\Sigma}_{dq0} = A_{tr} \dot{\Sigma}_{dq0} + B_{tr} u_{dq0} \] (A.25)

where:

\[ A_{tr} = P_{tr} P_{tr}^{-1} + A_{tr}, \quad B_{tr} = B_{tr} \] (A.26)

In a similar manner as in (A.16) the states are again rearranged, where the transformation matrix is defined as:
It is known that zero sequence components do not produce any DC voltage and therefore first \( n \) elements from the previous equations can be deleted. The final model becomes:

\[
\dot{x}_{dq} = A_{ac} x_{dq} + B_{ac} u_{dq}
\]

where:

\[
A_{ac} = T_2 A_{12} T_2^{-1}, \quad B_{ac} = T_2 B_{12} T_2^{-1}, \quad A_{ac} \in \mathbb{R}^{2m \times 2n}, \quad B_{ac} \in \mathbb{R}^{2n \times n}
\]

In general case, the dynamic model in \( s \) domain, for \( j-th \) AC system becomes:

\[
\dot{s}x = A_{acj} s x + B_{acjdc} u_{acjdc}
\]

\[
y_{acjdc} = C_{acjdc} x_{acj}
\]

where:

\[
\begin{bmatrix}
I_{jd} \\
I_{jq}
\end{bmatrix}, \quad \begin{bmatrix}
V_{jd} \\
V_{jq}
\end{bmatrix}
\]

The presented model for \( j-th \) AC system does not take into account interactions with other AC systems. The inputs and outputs relate to the DC system only. The inputs to the model are \( dq \) components of AC current. The outputs are either \( dq \) or magnitude-angle components of AC voltage, as shown in Section 1.2.7.

### A.2 PLL model

Mathematical model of the D-Q-Z type phase locked loop will be derived here considering each dynamic PLL part independently (Figure 1.3 in Chapter 1):

**vector transformer:**

The three phase voltages are expressed in its usual form as:

\[
v_a = V_m \cos(\omega t - \frac{\pi}{3} + \phi)
\]
The \( \alpha \) and \( \beta \) components of three phase voltages are defined as:

\[
v_a = \frac{2}{3} v_a - \frac{1}{3} v_b - \frac{1}{3} v_c \quad \text{(A.36)}
\]

\[
v_\beta = \frac{2}{3} v_c - \frac{1}{3} v_b + \frac{1}{3} v_a \quad \text{(A.37)}
\]

After substituting voltages from (A.33)-(A.35) the two components of vector transformer are:

\[
v_t = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cos (\omega t + \phi) + \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \sin (\omega t + \phi) = V_m \begin{bmatrix} \cos (\omega t - \frac{\pi}{3} + \phi) \\ \sin (\omega t - \frac{\pi}{3} + \phi) \end{bmatrix} \quad \text{(A.38)}
\]

\[
v_\beta = -\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \cos (\omega t + \phi) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \sin (\omega t + \phi) = V_m \begin{bmatrix} \cos (\omega t - \frac{\pi}{3} + \phi) \\ \sin (\omega t - \frac{\pi}{3} + \phi) \end{bmatrix} \quad \text{(A.39)}
\]

The two output, feedback signals from VCO are:

\[
v_{\sin} = V_m \sin \left( \omega t - \frac{\pi}{3} + \theta \right) \quad \text{(A.40)}
\]

\[
v_{\cos} = V_m \cos \left( \omega t - \frac{\pi}{3} \theta \right) \quad \text{(A.41)}
\]

The control error is defined as:

\[
e = v_\alpha v_{\sin} + v_\beta v_{\cos} \quad \text{(A.42)}
\]

after substituting in (A.38)-(A.41):

\[
e = V_m^2 \sin (\phi - \theta) \quad \text{(A.43)}
\]

and since at nominal operating point \( \phi^0 = \theta^0 \), the linearised control error becomes:

\[
\Delta e = V_m^2 (\Delta \phi - \Delta \theta) \quad \text{(A.44)}
\]

where superscript “0” stands for nominal values.

**PLL controller:**

The PLL controller is of PI type:

\[
\Delta v_c = k_{cPLL} (k_{PPLL} + \frac{k_{jPLL}}{s}) \Delta e \quad \text{(A.45)}
\]
Appendix

**VCO:**

The role of VCO is to generate a ramp function with the slope proportional to the control signal. Its dynamic equation can be written as:

\[ \theta = \frac{1}{s} (v_c + \omega_0) \]  
(A.46)

where \( \omega_0 \) denotes the fundamental frequency signal. Fundamental frequency component is necessary to keep the output angle constant at steady-state system operation.

When linearised (A.46) becomes:

\[ \Delta \theta = \frac{1}{s} \Delta v_c \]  
(A.47)

The final model for phase locked loop can be derived by closing the main PLL feedback loops:

\[ s x_1 = -k_e PLL x_2 + k_i PLL u \]
\[ s x_2 = -k_e PLL k_p PLL x_2 + k_i PLL x_1 + k_e PLL k_p PLL u \]  
(A.48)

\[ y = x_2 \]  
(A.49)

where:

\[ x = \begin{bmatrix} \frac{k_{PLL}}{s} (\phi - \theta) \\ \theta \end{bmatrix}, \quad u = \phi, \quad y = \theta \]  
(A.50)

**A.3 DC system model**

With reference to Figure A.2, the dynamic electrical equations for DC system can be expressed as:

\[ L_r \frac{dI_r}{dt} = -R_r I_r + V_{dr} - V_{cs} \]  
(A.51)

\[ C_s \frac{dV_{cs}}{dt} = I_r - I_i \]  
(A.52)

\[ L_i \frac{dI_i}{dt} = -R_i I_i + V_{cs} - V_{dl} \]  
(A.53)

The converter direct voltages in the above equations are obtained form the following equations (as shown in Appendix B):

\[ V_{dr} = 6E_m \sqrt{2} \cos \phi_r / (3 / \pi) x_{e1} I_r \]  
(A.54)
or when linearised:
\[
\Delta V_{\text{dr}} = \frac{\partial V_{\text{dr}}}{\partial E_{\text{ac1}}} \Delta E_{\text{ac1}} + \frac{\partial V_{\text{dr}}}{\partial \phi} \Delta \phi + \frac{\partial V_{\text{dr}}}{\partial I_r} \Delta I_r
\]  
(A.55)

where:
\[
K_{\text{ac1}} = \frac{\partial V_{\text{dr}}}{\partial \phi} = -6E_{\text{ac1}}^0 \sqrt{2} \sin \phi_1^0 / \pi
\]  
(A.56)
\[
K_{\text{eac1}} = \frac{\partial V_{\text{dr}}}{\partial E_{\text{ac1}}} = 6\sqrt{2} \cos \phi_1^0 / \pi
\]  
(A.57)
\[
R_{c1} = \frac{\partial V_{\text{dr}}}{\partial I_r} = 3X_{c1} / \pi
\]  
(A.58)

where superscript "0" denotes steady-state values.

The converter direct voltages are now expressed as:
\[
V_{\text{dr}} = K_{\text{ac1}} \phi_1 + K_{\text{eac1}} E_{\text{ac1}} + R \frac{\partial \phi}{\partial \phi} + R_{c1} I_r
\]  
(A.59)
\[
V_{\text{di}} = K_{\text{ac2}} \phi_2 + K_{\text{eac2}} E_{\text{ac2}} + R_{c2} I_i
\]  
(A.60)

and the resistances are defined:
\[
R_r \quad \text{rectifier side DC line resistance.} \\
R_{c1} \quad \text{commutating overlap equivalent resistance at rectifier side.}
\]

The equivalent resistance in (1.46) is therefore obtained as \( R_{c1} = R_r + R_{c1} \), with inverter resistance calculated in similar manner.

The equivalent reactance in (1.46) is obtained as \( L_{r1} = L_r + L_{c1} \)

where:
\[
L_r \quad \text{rectifier side DC line reactance and smoothing reactance.} \\
L_{c1} = L_{c1} \left( \frac{2\pi}{12} - u^0 \right) \frac{12}{2\pi} + \frac{1}{2} \frac{12}{2\pi} \frac{12}{2\pi} u^0 \text{ averaged converter transformer reactance, which is obtained under the assumption that while one diode is conducting transformer reactance is } L_{c1}, \text{ and during the commutation overlap transformer reactance is } \frac{1}{2} L_{c1}. \text{ The commutation overlap } u^0 \text{ can be calculated from the converter equations in Appendix B.}
\]

At rectifier and inverter side, the equations for controller firing angle order is obtained from (1.31) and (1.33) as:
\[
\alpha = (k_p + k_i / s)(I_{ord} - I_{if})
\]  
(A.61)
where $I_{1f}$ stands for measured rectifier DC current (after feedback filter). By using (A.49-A.53) and using the integral part of (A.61), the dynamic equations (1.46-1.49) are obtained.

\[
\beta = \text{const} 
\]  
(A.62)

From equation (A.60), for the inverter converter, assuming $\Delta E_{ac2} = 0$ the expressions for direct voltages (1.34) and (1.39) are derived:

- for constant beta mode ($\Delta \beta = 0$):
  \[
  \Delta V_{di} = R_{c2} \Delta I_i 
  \]  
(A.63)

- for constant gamma mode ($\Delta \gamma = 0$):
  \[
  \Delta V_{di} = -R_{c2} \Delta I_i 
  \]  
(A.64)

The above equation is obtained when $\Delta \gamma = 0$ is substituted in the converter equation from Appendix B:

\[
I_i = \frac{\sqrt{2} E_{ac2}}{2 \omega L_c} (\cos \gamma - \cos \beta) 
\]  
(A.65)

If now firing angle $\beta$ is eliminated from (A.60) and (A.65) we get:

\[
V_{di} = -\frac{3 E_{ac2}}{\pi} \sqrt{2} \frac{I_i 2 X_{c2}}{\sqrt{2} E_{ac2}} + R_{c2} I_i = -R_{c2} I_i 
\]  
(A.66)

### A.4 Interaction equations

The two equations that give relationship between AC and DC variables are shown below:
Appendix

\[ I_{acj} = I_{dc} \frac{2\sqrt{6}}{\pi} a_{acj} \sqrt{2} \]  \hspace{1cm} (A.67)

\[ \cos \Phi_{acj} = \cos \phi_j - \frac{R_{cj} I_{dc} \pi}{6\sqrt{3}a_{acj} V_{acj}} \]  \hspace{1cm} (A.68)

The linearised coefficients, defined for rectifier side as:

\[ c_{1ac1} = \frac{\partial A}{\partial r} \quad c_{2ac1} = \frac{\partial \Phi_{ac1}}{\partial r} \quad c_{3ac1} = \frac{\partial \Phi_{ac1}}{\partial \phi} \quad c_{4ac1} = \frac{\partial E_{ac1}}{\partial \phi} \]  \hspace{1cm} (A.69)

are obtained as:

\[ c_{1ac1} = 2\sqrt{6}a_{ac1} \sqrt{2} / \pi \]  \hspace{1cm} (A.70)

\[ c_{2ac1} = \sin \alpha^0 / \sin \Phi_{ac1}^0 \]  \hspace{1cm} (A.71)

\[ c_{3ac1} = \frac{R_{c1} \pi}{6\sqrt{3}E_{ac1} a_{ac1} \sin \Phi_{ac1}^0} \]  \hspace{1cm} (A.72)

\[ c_{4ac1} = \frac{R_{c1} I_{dc1}^0 \pi}{6\sqrt{3}(E_{ac1}^0 a_{ac1})^2 \sin \Phi_{ac1}^0} \]  \hspace{1cm} (A.73)

The equation (A.67), dc-ac current equation, gives only the magnitude value of converter AC current \( I_1 \), as a function of direct current \( I_{dc} \) (rectifier or inverter). The AC current angle is obtained from dynamic equation (1.58).

The above coefficients relevant to inverter side are obtained in similar manner.

A.5 General DC system model

The general DC system model developed in 1.2.6 is in the form:

\[ \begin{align*}
  s\dot{x} & = A_{dc} x + B_{d1ac1} y_{ac1} + B_{d2ac2} y_{ac2} + B_{dimp} u_{imp} \\
  y_{d1ac1} & = C_{d1ac1} x \\
  y_{d2ac2} & = C_{d2ac2} x
\end{align*} \]  \hspace{1cm} (A.74)

where the model inputs and outputs are defined as in (1.75-1.76).

The input and output matrices for the above model are:
The output matrix $C'_{\text{dacc1}}$ takes into account the “dc-ac current” equation through the linearised coefficient $c_{\text{lac1}}$. In this way the output from the model are AC current magnitude and AC current angle as shown in (A.76).

The above matrices need to be further corrected to accommodate for different coordinate frames used on AC and DC side.

**A.6 Polar coordinate to dq coordinate transformation**

Since AC system model is developed in “d-q” coordinate frame, whereas DC system model uses magnitude-angle representation of AC system variables, it is necessary to further modify the input and output matrices.

For the transformation equations as defined in (1.77-1.78) the transformation matrices are derived as:

$$R_{\text{dacc1}} = \frac{\sqrt{3}}{2} \begin{bmatrix} \sin \psi_{\text{lac1}}^0 & I_{\text{mac1}}^0 \cos \psi_{\text{ac1}}^0 \\ \cos \psi_{\text{ac1}}^0 & -I_{\text{mac1}}^0 \sin \psi_{\text{ac1}}^0 \end{bmatrix}$$

(A.78)

defined by (1.72) and:

$$R_{\text{dacc1}} = \frac{\sqrt{3}}{2} \begin{bmatrix} \frac{\partial A_{id}}{\partial \psi_1} & \frac{\partial A_{id}}{\partial \psi_1} \\ \frac{\partial A_{ac1}}{\partial \psi_1} & \frac{\partial A_{ac1}}{\partial \psi_1} \\ \frac{\partial A_{iq}}{\partial \psi_1} & \frac{\partial A_{iq}}{\partial \psi_1} \end{bmatrix}$$

(A.79)
Appendix

\[ P_{dcac} = \sqrt{3/2} \begin{bmatrix} \sin \varphi_{ac}^0 & E_{mac}^0 \cos \varphi_{ac}^0 \\ \cos \varphi_{ac}^0 & -E_{mac}^0 \sin \varphi_{ac}^0 \end{bmatrix} \] \hspace{1cm} (A.80)

defined by (1.82) and:

\[ P_{dcac} = \sqrt{3/2} \begin{bmatrix} \mathcal{N}_{ld} & \mathcal{N}_{ld} \\ \mathcal{N}_{ac} & \mathcal{N}_{ac} \\ \mathcal{N}_{lq} & \mathcal{N}_{lq} \\ \mathcal{N}_{ad} & \mathcal{N}_{ad} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{N}_{ld}}{\partial \varphi_{lq}} \\ \frac{\partial \mathcal{N}_{ld}}{\partial \varphi_{lq}} \\ \frac{\partial \mathcal{N}_{lq}}{\partial \varphi_{lq}} \\ \frac{\partial \mathcal{N}_{lq}}{\partial \varphi_{lq}} \end{bmatrix} \] \hspace{1cm} (A.81)

For inverter side, the matrices are derived in similar manner.
APPENDIX B BASIC EQUATIONS FROM HVDC THEORY

This Appendix shows a set of basic equations from AC-DC converters and HVDC theory, which are often used in this Thesis.

For an HVDC converter, the direct voltage can be expressed:

\[ V_{dc} = B \frac{3\sqrt{3}}{\pi} E_m \cos \alpha - B \frac{3}{\pi} X_c I_{dc} \]  

or

\[ V_{dc} = B \frac{3\sqrt{6}}{\pi} E_{LN} \cos \alpha - B \frac{3}{\pi} X_c I_{dc} \]  

or

\[ V_{dc} = B \frac{3\sqrt{2}}{\pi} E_{LL} \cos \alpha - B \frac{3}{\pi} X_c I_{dc} \]  

where:

- \( E_m \) maximum amplitude of AC voltage.
- \( E_{LN} \) line-neutral RMS value of AC voltage.
- \( E_{LL} \) line-line RMS value of AC voltage.
- \( X_c = \omega L_c \) equivalent transformer reactance at fundamental frequency.
- \( R_c = \frac{3}{\pi} X_c \) equivalent commutating resistance.
- \( I_{dc} \) direct current through converter.
- \( B \) number of bridges in converter. For six pulse system \( B=1 \), for 12-pulse system \( B=2 \).

The equation which shows relationship between firing angle and extinction angle is shown below:

\[ \frac{\sqrt{3}}{2} E_m (\cos \alpha - \cos \delta) = I_{dc} \omega L_c \]  

or for inverter side:

\[ \cos \beta = \cos \gamma - \frac{2 \omega L_c I_{dc}}{\sqrt{3} E_m} \]  

where:

- \( \alpha \) converter firing angle (\( \beta \) equivalent at inverter side)
- \( \delta \) converter extinction angle (\( \gamma \) equivalent at inverter side)

The equation which gives AC system power angle as a function of DC side variables is:

\[ \cos \Phi = \cos \alpha - \frac{R I_{dc}}{3\sqrt{3} \frac{\pi}{E_m}} \]  

183
Appendix

where:

\( \Phi \)  AC system power angle (angle between AC voltage and AC current)

The equation which shows relationship between AC and DC current is:

\[
I_{ac} = BI_{ac} \frac{\sqrt{6}}{\pi}
\]  (B.5)

where:

\( I_{ac} \)  RMS fundamental frequency current.
APPENDIX C LINEARISED COEFFICIENTS IN THE SIMPLIFIED LINEAR CONTINUOUS MODEL

Using the phasor diagram from Figure 1.11, the converter bus voltage can be obtained from the following equation:

$$ E_{ac}^2 = E_{ac}^2 + 2E_{ac}|I_1|z_{th}|\cos \tau + |I_1|^2|z_{th}|^2 $$  \hspace{1cm} (C.1)

When linearised around nominal operating point, the above equation becomes:

$$ -2E_{ac}^0 \Delta E_{ac} - 2E_{ac}^0 I_1^0 |z_{th}| \cos \tau^0 = 2E_{ac}^0 \Delta I_1 |z_{th}| \cos \tau^0 + 2E_{ac}^0 I_1^0 |z_{th}| (-\sin \tau^0) \Delta \tau + 2E_{ac}^0 \Delta I_1^0 |z_{th}|^2 $$  \hspace{1cm} (C.2)

and it can be further rearranged as:

$$ \Delta E_{ac} \left( 2E_{ac}^0 + 2I_1^0 |z_{th}| \cos \tau^0 \right) = -(2E_{ac}^0 |z_{th}| \cos \tau^0 + 2E_{ac}^0 I_1^0 |z_{th}|^2) \Delta I_1 + 2E_{ac}^0 I_1^0 |z_{th}| (-\sin \tau^0) \Delta \tau $$  \hspace{1cm} (C.3)

In equation (1.104):

$$ \Delta E_{ac} = k_{fac} \Delta I_1 + k_{\Phi ac} \Delta \Phi_{ac} $$  \hspace{1cm} (C.4)

the AC voltage is a direct function of phase angle $\Delta \Phi_{ac}$ since the equation (1.101) for small perturbations becomes:

$$ \Delta \tau = -\Delta \Phi $$  \hspace{1cm} (C.5)

Therefore the coefficients in (1.104) and (C.4) become:

$$ k_{fac} = \frac{-2E_{ac}^0 |z_{th}| \cos \tau^0 + 2E_{ac}^0 I_1^0 |z_{th}|^2}{2E_{ac}^0 + 2I_1^0 |z_{th}| \cos \tau^0} $$  \hspace{1cm} (C.6)

$$ k_{\Phi ac} = \frac{2E_{ac}^0 I_1^0 |z_{th}| \sin \tau^0}{2E_{ac}^0 + 2I_1^0 |z_{th}| \cos \tau^0} $$  \hspace{1cm} (C.7)

To get the phase angle $\Delta \Phi_{ac}$ as a function of DC side variables the equation (A.68) with linearised coefficients defined in (A.71-A.73) is used. The relationship between rectifier AC current and rectifier DC current is expressed using the equation (A.67) and linearised coefficient defined in (A.70).

Therefore, using (C.4) the AC bus voltage is expressed as:
Appendix

\[
\Delta E_{ac1} = \frac{k_{\phi ac1} c_{2 ac1}}{1 + k_{\phi ac1} c_{4 ac1}} \Delta I_r + \frac{k_{\phi ac1} c_{2 ac1}}{1 + k_{\phi ac1} c_{4 ac1}} \Delta \alpha
\]  

(C.8)

or:

\[
\Delta E_{ac1} = K_{ac1} \Delta I_r + K_{ac1} \Delta \alpha
\]  

(C.9)

Using the equations (1.97, 1.101, C.9) the equation (1.112) in the final model is obtained:

\[
L_{r1} s I_r = -(R_1 + R_{ac1}) I_r - V_m + (K_{ac1} + K_{ac1}) \alpha + K_{ac1} E^d_{ac1}
\]  

(C.10)

where the “AC” coefficients are obtained as:

\[
R_{ac1} = K_{ac1} K_{ac1} \quad \text{and} \quad K_{ac1} = K_{ac1} K_{ac1}
\]  

(C.11)

and as shown in Appendix A: \( R_{r1} = R_r + R_{ac1} \quad L_{r1} = L_r + L_{ac1} \)

(C.12)

The above coefficients for inverter side can be obtained in similar manner.

The above coefficients represent the influence of AC system on dynamics of DC system. Through these coefficients, the strength of the AC system will directly affect dynamic behavior of the DC system.

The final model is expressed in the form:

\[
sx = Ax + Bu + B_1 w \quad y = Cx
\]  

(C.13)

where the model matrices are:

\[
A = \begin{bmatrix}
- \frac{R_{r1} + R_{ac1} + (K_{ac1} + K_{ac1}) k_{pr}}{L_{r1}} & \frac{-(K_{ac1} + K_{ac1}) k_{pr}}{L_{r1}} & 1 & 0 \\
L_{r1} & 0 & -R_1 + R_{ac1} & 1 \\
0 & 0 & L_{r1} & 0 \\
0 & 0 & -1 & C_x \\
\end{bmatrix}
\]  

(C.14)

\[
B = \begin{bmatrix}
\frac{K_{ac1} + K_{ac1}}{L_{r1}} & 0 \\
0 & \frac{K_{ac2} + K_{ac2}}{L_{r1}} \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
\frac{(K_{ac1} + K_{ac1}) k_{pr}}{L_{r1}} & \frac{K_{ac1}}{L_{r1}} & 0 \\
L_{r1} & 0 & 0 \\
0 & 0 & -\frac{K_{ac2}}{L_{r1}} \\
0 & 0 & 0 \\
\end{bmatrix},
\]  

(C.15)
In the case of additional feedback loops for the controller designed in Chapter 4, matrix $A$ becomes:

$$
A = \begin{bmatrix}
\frac{R_1 + R_{sa} + (K_{sa} + K_{rad})k_{pr} + k_{pd}}{L_1 + (K_{sa} + K_{rad})k_{dl}} & \frac{(K_{sa} + K_{rad})k_{pr}}{L_1 + (K_{sa} + K_{rad})k_{vl}} & \frac{1}{L_1 + (K_{sa} + K_{rad})k_{dl}} & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{C_s} & 0 & \frac{-R_1 + R_{sa} + (K_{sa} + K_{rad})k_{pd}}{L_1 + (K_{sa} + K_{rad})k_{vl}} & \frac{1}{L_1 + (K_{sa} + K_{rad})k_{dl}} \\
\end{bmatrix}
$$

(C.17)

where the feedback gains $k_{pr}, k_{dr}, k_p, k_{di}$ are calculated as shown in Chapter 4.
APPENDIX D SYSTEM PARAMETERS FOR THE TEST CASES

D.1. CIGRE HVDC BENCHMARK MODEL
Rectifier SCR=10, Inverter SCR=2.5

TABLE D.1. TEST SYSTEM DATA

<table>
<thead>
<tr>
<th>DC system data</th>
<th>AC1 (rectifier) data:</th>
<th>AC2 (inverter) data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alphaacr=20.2 deg; Betaaci=40 deg; Lr=0.5968+0.086 H; Li=0.5968+0.0848 H; Rcr=28.4446-2.5; Rr=2.5; Ri=2.5; Cs=26e-6 F; aac1=213.4557/345; aac2=209.228/230; Vac1=342.05e3 V; (line-line RMS)</td>
<td>SCR=10@75deg R1=0.4156; L1=0.0206; R2=13.96; R3=0.4156; L2=0.0206; C1=3.342e-6 F; L3=0.1364 H; R4=29.76; C2=74.28e-6 F; R5=261.87; L4=0.0136 H; C4=6.685e-6 F; R6=83.32; Vac1=342.05e3 V; (line-line RMS)</td>
<td>SCR=2.5@75deg R1=0.7406; L1=0.0365 H; R2=24.81; R3=0.7406; L2=0.0365 H; C1=7.522e-6 F; L3=0.0606 H; R4=13.23; C2=167.2e-6 F; C3=15.04e-6 F; R5=116.38; L4=0.0061 H; C4=15.04e-6 F; R6=37.03; Vac2=215.05e3 V; (line-line RMS)</td>
</tr>
<tr>
<td>AC1 (rectifier) data:</td>
<td>AC2 (inverter) data</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>SCR=10@75deg R1=0.4156; L1=0.0206; R2=13.96; R3=0.4156; L2=0.0206; C1=3.342e-6 F; L3=0.1364 H; R4=29.76; C2=74.28e-6 F; R5=261.87; L4=0.0136 H; C4=6.685e-6 F; R6=83.32; Vac1=342.05e3 V; (line-line RMS)</td>
<td>SCR=2.5@75deg R1=0.7406; L1=0.0365 H; R2=24.81; R3=0.7406; L2=0.0365 H; C1=7.522e-6 F; L3=0.0606 H; R4=13.23; C2=167.2e-6 F; C3=15.04e-6 F; R5=116.38; L4=0.0061 H; C4=15.04e-6 F; R6=37.03; Vac2=215.05e3 V; (line-line RMS)</td>
<td></td>
</tr>
<tr>
<td>AC1 (rectifier) data:</td>
<td>AC2 (inverter) data</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>SCR=10@75deg R1=0.4156; L1=0.0206; R2=13.96; R3=0.4156; L2=0.0206; C1=3.342e-6 F; L3=0.1364 H; R4=29.76; C2=74.28e-6 F; R5=261.87; L4=0.0136 H; C4=6.685e-6 F; R6=83.32; Vac1=342.05e3 V; (line-line RMS)</td>
<td>SCR=2.5@75deg R1=0.7406; L1=0.0365 H; R2=24.81; R3=0.7406; L2=0.0365 H; C1=7.522e-6 F; L3=0.0606 H; R4=13.23; C2=167.2e-6 F; C3=15.04e-6 F; R5=116.38; L4=0.0061 H; C4=15.04e-6 F; R6=37.03; Vac2=215.05e3 V; (line-line RMS)</td>
<td></td>
</tr>
</tbody>
</table>

TABLE D.2. CONTROLLER DATA:

<table>
<thead>
<tr>
<th>Rectifier controller</th>
<th>Inverter controller:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p = 0.03148$ deg/ A</td>
<td>$k_p = 0.022$ deg/ A</td>
</tr>
<tr>
<td>$k_i = 4.77$ deg/ As</td>
<td>$k_i = 2.29$ deg/ As</td>
</tr>
</tbody>
</table>

D.2. CIGRE HVDC BENCHMARK MODEL
Rectifier SCR=2.5, Inverter SCR=2.5

AC1 data:
SCR=2.5@84deg
R1=0.0;
L1=0.151; 
R2=2160.633; 
R3=3.737; 
L2=0.0; 
C1=3.342e-6 F; 
L3=0.1364 H; 
R4=29.76; 
C2=74.28e-6 F; 
C3=6.685e-6 F; 
R5=261.87; 
L4=0.0136 H; 
C4=6.685e-6 F; 
R6=83.32; 
Vac1=352.6e3 V;  (line-line RMS)

All remaining data are the same as for the previous CIGRE HVDC model D.1.
**Appendix**

### D.3. Simplified linear continuous HVDC system model

With reference to Figure D.2, the general system data are shown below.

![Figure D.2 CIGRE HVDC system schematic diagram with AC system dynamics neglected](image)

\[
z_1 = 2.5 + 0.5968s + 27.1691\frac{s}{3\pi};
\]

\[
z_2 = \frac{1}{s(26 \times 10^{-6})};
\]

\[
z_3 = 2.5 + 0.5968s + 26.63\frac{s}{3\pi};
\]

\[ts = 1.6666667 \times 10^{-3} \text{s}; \quad (\text{used with discrete model})
\]

\[tu = 8.8471 \times 10^{-4} \text{s}; \quad (\text{used with discrete model})
\]

\[E_{ac10} = 212,837 \text{V};
\]

\[E_{ac20} = 203,770 \text{V};
\]

\[\beta_{nom} = 40 \text{ deg}
\]

Table D.3 shows the AC system parameters, as defined in Figure 1.11, at nominal operating point. Table D.4 shows the controller data.

**TABLE D.3. CALCULATED NOMINAL AC SYSTEM PARAMETERS:**

<table>
<thead>
<tr>
<th>parameter</th>
<th>rectifier</th>
<th>Inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td>[</td>
<td>\varphi_0</td>
<td>]</td>
</tr>
<tr>
<td>[\xi_0]</td>
<td>81.89\text{deg}</td>
<td>69.65\text{deg}</td>
</tr>
<tr>
<td>[\Phi_0]</td>
<td>32.42\text{deg}</td>
<td>30.86\text{deg}</td>
</tr>
<tr>
<td>[</td>
<td>I_i^0</td>
<td>]</td>
</tr>
</tbody>
</table>

**TABLE D.4. CONTROLLER DATA:**

<table>
<thead>
<tr>
<th>Rectifier controller</th>
<th>Inverter controller:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[k_{pr} = 0.0315 \text{deg/As}]</td>
<td>[k_{pi} = 0.022 \text{deg/As}]</td>
</tr>
<tr>
<td>[k_{\nu} = 1.77 \text{deg/As}]</td>
<td>[k_{ii} = 2.29 \text{deg/As}]</td>
</tr>
</tbody>
</table>
APPENDIX E  BASIS FROM Z-TRANSFORMATION AND MODIFIED Z-TRANSFORMATION THEORY

Assuming that the sampling interval is $T_s$ and the time delay is $T_u$ ($T_u < T_s$), the basic equations from $s$ to $z$ transformation are shown below.

\[ Z\left\{ \frac{1}{s+a} \right\} = \frac{z}{z - e^{-aT_s}} \quad (E.1) \]

\[ Z\left\{ \frac{1}{s} \right\} = \frac{z}{z - 1} \quad (E.2) \]

\[ Z\left\{ \frac{1}{s(s+a)} \right\} = \frac{z(1 - e^{-aT_s})}{(z-1)(z - e^{-aT_s})} \quad (E.3) \]

\[ Z\left\{ \frac{e^{-sT_u}}{s+a} \right\} = \frac{e^{-a(T_u-T_s)}}{z - e^{-aT_s}} \quad (E.4) \]

\[ Z\left\{ \frac{e^{-sT_u}}{s(s+a)} \right\} = \frac{1}{a(z-1)} - \frac{e^{a(T_u-T_s)}}{a(z - e^{-aT_s})} \quad (E.5) \]
F.1 Controller model

Following the model derivation from Section 2.1, the dynamic equations are written:

\[
\begin{align*}
sx_1 &= -k_{ipPLLc}x_2 + k_{ipPLLc}\varphi \\ 
sx_2 &= -k_{ipPLLc}k_{pPLLc}x_2 + k_{ipPLLc}x_1 + k_{ipPLLc}k_{pPLLc}\varphi \\
\frac{sx_3}{2} &= k_{ic}(V_{ref} - x_2) \\
\frac{sT_f x_4}{2} &= -x_2 + x_3 - x_4 + k_{ptc}V_{ref} - k_{ptc}V + \varphi \\
\frac{sT_f x_5}{2} &= -x_3 + V
\end{align*}
\]

or in matrix notation:

\[
\begin{align*}
sx &= A_c \bar{x} + B_{cac} \bar{u}_{cac} + B_{cinp} \bar{u}_{inp} \\
y &= C_{cac} \bar{x}
\end{align*}
\]

where model matrices are:

\[
A_c = \begin{bmatrix}
0 & -k_{ipPLLc} & 0 & 0 & 0 \\
k_{ipPLLc} & -k_{ipPLLc}k_{pPLLc} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -k_{ic} \\
0 & -1/T_f & 1/T_f & -1/T_f & 0 \\
0 & 0 & 0 & 0 & -1/T_{fe}
\end{bmatrix}
\]

\[
B_{cac} = \begin{bmatrix}
0 & k_{ipPLLc} \\
0 & k_{ipPLLc}k_{pPLLc} \\
0 & 0 \\
-k_{ptc}/T_f & 1/T_f \\
1/T_{fe} & 0
\end{bmatrix},
B_{cinp} = \begin{bmatrix}
0 \\
0 \\
k_{ic} \\
k_{ptc}/T_f \\
0
\end{bmatrix}
\]

\[
C_{cac} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]


**F.2 Main circuit model**

As shown in Section 2.2.3, using the state space variables, the main circuit model is represented as:

\[ sL_1x_1 = -R_1x_2 + R_2(-x_2 - x_1R_1) / (R_2 + R_3) \quad (F.11) \]

\[ sC_1x_2 = x_1 + (-x_2 - x_1R_1) / (R_2 + R_3) - x_4 - x_3 \quad (F.12) \]

\[ sL_2x_3 = x_1 - x_3R_4 \quad (F.13) \]

\[ sx_4 = B^0x_2 + V^0b_{te} \quad (F.14) \]

or in matrix notation:

\[ s\mathbf{x} = A_{ac}\mathbf{x} + B_{ac}\mathbf{u}_{ac} \quad (F.15) \]

\[ y_{acc} = C_{acc}\mathbf{x} \quad (F.16) \]

where the model variables are:

\[ \mathbf{x} = \begin{bmatrix} i_{L1} \\ v \\ i_{L2} \\ i_{te} \end{bmatrix}, \quad \mathbf{u}_{ac} = b_{te}, \quad y_{acc} = x_2 = v \quad (F.17) \]

and the model matrices are:

\[
A_{ac} = \begin{bmatrix}
-R_2R_3 / ((R_2 + R_3)L_1) & -R_1 / L_1 - R_2 / ((R_2 + R_3)L_1) & 0 & 0 \\
1 / C_1 - R_1 / ((R_2 + R_3)C_1) & -1 / ((R_2 + R_3)C_1) & -1 / C_1 & -1 / C_1 \\
1 / L_2 & 0 & -R_4 / L_2 & 0 \\
0 & B^0 & 0 & 0
\end{bmatrix} \quad (F.18)
\]

\[
B_{ac} = \begin{bmatrix}
0 \\
0 \\
0 \\
V^0
\end{bmatrix}, \quad C_{acc} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \quad (F.19)
\]

The above model is represented in three-phase coordinate frame and later transferred to \(dq\) coordinate frame in the same way as shown in Chapter 1 and Appendix A.1. The final model is written as:

\[ s\mathbf{x} = A_{ac}\mathbf{x} + B'_{ac}\mathbf{u}_{ac} \quad (F.20) \]

\[ y = C'_{acc}\mathbf{x} \quad (F.21) \]
Appendix

F.3 TCR model

The input matrix of the main circuit model is corrected as:

\[
B_{\text{acc}} = B'_{\text{acc}} S_{\text{acc}} T_{\text{acc}}
\]

where:

\[
\begin{bmatrix}
  b_{\text{tcrd}} \\
  b_{\text{treq}}
\end{bmatrix} = S_{\text{acc}} \begin{bmatrix}
  B_{\text{tcr}} \\
  \xi
\end{bmatrix}
\]

and where the matrix \( S_{\text{acc}} \) is derived from the following equations:

\[
b_{\text{tcrd}} = B_{\text{tcr}} \sin \xi \quad b_{\text{treq}} = B_{\text{tcr}} \cos \xi
\]

which are linearised and represented in matrix form. Matrix \( S_{\text{acc}} \) is obtained as:

\[
S_{\text{acc}} = \begin{bmatrix}
  \frac{\partial b_{\text{tcrd}}}{\partial B_{\text{tcr}}} & \frac{\partial b_{\text{tcrd}}}{\partial \xi} \\
  \frac{\partial b_{\text{treq}}}{\partial B_{\text{tcr}}} & \frac{\partial b_{\text{treq}}}{\partial \xi}
\end{bmatrix} = \begin{bmatrix}
  \sin \xi & B_{\text{tcr}}^0 \cos \xi^0 \\
  \cos \xi & -B_{\text{tcr}}^0 \sin \xi^0
\end{bmatrix}
\]

The matrix \( T_{\text{acc}} \) is obtained as:

\[
\begin{bmatrix}
  B_{\text{tcr}} \\
  \xi
\end{bmatrix} = T_{\text{acc}} \phi
\]

where \( T_{\text{acc}} = \begin{bmatrix} K_{\text{tc}} \\ 0 \end{bmatrix} \)

and where:

\[
B_{\text{tcr}} (\sigma) = \frac{1}{L_{\text{tcr}} (\sigma)} = \frac{\sigma - \sin \sigma}{\pi \sigma} \\ K_{\text{tc}} = \frac{\partial B_{\text{tcr}}}{\partial \sigma}
\]

The output model matrix is corrected as:

\[
C_{\text{acc}} = C'_{\text{acc}} Q_{\text{acc}}^{-1}
\]

where:

\[
\begin{bmatrix}
  v_d \\
  v_q
\end{bmatrix} = Q_{\text{acc}} \begin{bmatrix}
  V \\
  \phi
\end{bmatrix}
\]

and where the matrix \( Q_{\text{acc}} \) is derived from the following equations:

\[
v_d = V \sin \phi \quad v_q = V \cos \phi
\]

which are linearised and represented in matrix form, giving the matrix \( Q_{\text{acc}} \) as:
\[ Q_{\text{ac}} = \begin{bmatrix} \frac{\partial V_d}{\partial V} & \frac{\partial V_d}{\partial \phi} \\ \frac{\partial V_q}{\partial V} & \frac{\partial V_q}{\partial \phi} \end{bmatrix} = \begin{bmatrix} \sin \phi^0 & V^0 \cos \phi^0 \\ \cos \phi^0 & -V^0 \sin \phi^0 \end{bmatrix} \]  
(F.32)

F.4 Test system data

SVC test system data are shown below:

\[ k_{pPLLc} = 100 \]
\[ k_{ipPLLc} = 900 \]
\[ k_{ipPLLc} = 1 \]
\[ T_f = 1 / 6000 \]
\[ T_{fv} = 1 / 6000 \]
\[ k_{pfc} = 10 / 86 \]
\[ k_{pfc} = (1 / 0.002) / 86 \]

\[ R_1 = 3.58 \]
\[ L_1 = 0.009386 \]
\[ R_2 = 1000 \]
\[ R_3 = 0.1 \]
\[ C_1 = 1.473e-6 \quad (\text{Ptc r=167.0MVA}) \quad (C_{Lc} \text{ is included in } C_1) \]
\[ R_4 = 144 \]
\[ V_{\text{nom}} = 120kV \quad V_2 = 12.6kV \quad \text{L-L} \]
APPENDIX G  SINGULAR VALUE DECOMPOSITION THEORY

Any complex matrix \( A \) can be factorized into a singular value decomposition:

\[ A = U \Sigma V^H \]  \hspace{1cm} (G.1)

where unitary matrices \( U \) and \( V \) contain output and input singular vectors respectively. \( V^H \) is the complex conjugate transpose of \( V \). Square, diagonal matrix \( \Sigma \) contains singular values of matrix \( A \) in descending order.

\[ \Sigma = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\}, \quad \bar{\sigma} = \sigma_1 > \sigma_2 > \ldots > \sigma_n = \sigma \]  \hspace{1cm} (G.2)

where: \( \bar{\sigma} \) - largest singular value, \( \sigma \) - smallest singular value.

The singular values are the positive square roots of the eigenvalues of \( A^H A \), i.e.:

\[ \sigma_i(A) = \sqrt{\lambda_i(A^H A)} \]  \hspace{1cm} (G.3)

The singular values are sometimes called the principal values of principal gains, and the associated directions are called the principal directions.

If the singular values in (G.2) are dependent upon frequency, i.e. in (G.1) we consider \( A(j\omega) \), then the largest and smallest singular values are called the principal gains. The largest principal gain shows the maximum possible amplification of the input signal. The smallest principal gain shows the minimum possible amplification.

The column vectors of \( U \) denoted \( (u_i) \), represent the output direction of the plant. They are orthogonal and unit length (orthonormal), that is:

\[ \|u_i\|_2 = \sqrt{|u_{i1}|^2 + |u_{i2}|^2 + \ldots + |u_{in}|^2} = 1 \]  \hspace{1cm} (G.4)

where \( \|u_i\|_2 \) stands for Euclidean norm of vector \( u_i \).

Similarly, the column vectors of \( V \) denoted \( (v_i) \), are orthogonal and unit in length and represent input directions.

To determine a range of possible different input-output directions, the condition number is introduced:

\[ \gamma(A) = \frac{\sigma_1(A)}{\sigma_n(A)} = \frac{\bar{\sigma}(A)}{\sigma(A)} \]  \hspace{1cm} (G.5)
Therefore the condition number indicate the ratio between the largest and smallest singular values. If it is close to one, than the plant behaves similarly to a single input single output system. In the case of large condition number, the system behavior depends largely upon the directions of input signals.
APPENDIX H AC SYSTEM PARAMETERS FOR REDUCED SCR AT INVERTER SIDE

Table H.1 shows the values for the AC system parameters which are changed to get different strength of the considered system. The test system is CIGRE Benchmark model from Figure A.1. It is seen that the power factor is maintained constant in this analysis. The last column denotes the dynamic stability of the overall HVAC-HVDC system with the considered parameters.

<table>
<thead>
<tr>
<th>case</th>
<th>(R_1[\Omega])</th>
<th>(L_2[H])</th>
<th>dynamic stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCR=2.5@76deg</td>
<td>0.7406</td>
<td>0.0365</td>
<td>stable</td>
</tr>
<tr>
<td>SCR=1.7@76deg</td>
<td>3.0960</td>
<td>0.0661</td>
<td>stable</td>
</tr>
<tr>
<td>SCR=1.3@76deg</td>
<td>5.4123</td>
<td>0.0957</td>
<td>stable</td>
</tr>
<tr>
<td>SCR=1.2@76deg</td>
<td>6.2338</td>
<td>0.1061</td>
<td>stable</td>
</tr>
<tr>
<td>SCR=1.0@76deg</td>
<td>12.7976</td>
<td>0.1332</td>
<td>unstable</td>
</tr>
</tbody>
</table>
APPENDIX I  THEORY ON METHODS FOR SELECTION OF FEEDBACK SIGNAL

I.1 Eigenvalue sensitivity with respect to controller parameters

If the system is given in the state space form:

\[ s\mathbf{x} = A\mathbf{x} + b\mu \]  \hspace{1cm} (I.1)
\[ y = c^T x \]  \hspace{1cm} (I.2)

and let the control law be defined with:

\[ u = -fy \]  \hspace{1cm} (I.3)

the system matrix becomes:

\[ A' = A - BfC \]  \hspace{1cm} (I.4)

The sensitivity of eigenvalue with respect to the controller gain \( f \) can be calculated as:

\[ \frac{\partial \lambda_i}{\partial f} = \sum_k \sum_j \frac{\partial \lambda_i}{\partial a'_{kj}} \frac{\partial a'_{kj}}{\partial f} \]  \hspace{1cm} (I.5)

where \( a'_{kj} \) is \((k,j)\) element of matrix \( A' \), and the partial derivatives are expressed as:

\[ \frac{\partial \lambda_i}{\partial a'_{kj}} = \psi'_{ik} \phi'_{ji}, \text{ and } \frac{\partial a'_{kj}}{\partial f} = -b_k c_j. \]  \hspace{1cm} (I.6)

The sensitivity of eigenvalue in its open loop location is calculated as:

\[ \frac{\partial \lambda_i}{\partial f} = -\sum_k \sum_j \psi'_{ik} \phi'_{ji} b_k c_j \]  \hspace{1cm} (I.7)

Figure I.1 shows an example when this method can give erroneous results (The Figure is taken from Chapter 7, Figure 7.6).

We consider the movement of eigenvalues marked “S” in Figure I.1 b) by the use of DC voltage feedback. Eigenvalue sensitivity gives large values with correct sign for these eigenvalues, since they (originally) move left from the open loop positions. The conclusion would be that this feedback signal is a good choice for improvement in damping of eigenvalues “S”. However, it is clear from the Figure, that the root locus changes direction and this feedback signal is likely to reduce damping of eigenvalues “S” and cause stability problems.
Appendix

Therefore, if the root locus changes direction (especially if it changes direction close to the
original eigenvalues), the eigenvalue sensitivity method may lead to the erroneous results about
the selection of feedback signal.

![Diagram](image)

**Figure I.1 An example where Eigenvalue Sensitivity can give erroneous conclusions about
feedback signal selection.**

### I.2 Relative Gain Array as a measure of interactions in decentralised MIMO system

Assuming that a MIMO system is represented by a transfer function matrix:

\[ y = [G]u, \quad (I.8) \]

than the effect of \(i\) input on the \(j\) output can be expressed as:

\[ y_j = g_{yj}u_i \quad (I.9) \]

We can evaluate the effect \(\frac{\partial y_j}{\partial u_j}\), of a given input \(u_j\) on a given output \(y_i\), for two extreme cases:

Other loops open:

\[ \left( \frac{\partial y_j}{\partial u_j} \right)_{u_k=0, \forall k \neq j} = g_{yj} \quad (I.10) \]

Other loops closed:

\[ \left( \frac{\partial y_j}{\partial u_j} \right)_{y_k=0, \forall k \neq i} = \hat{g}_{ij} \quad (I.11) \]

where \(\hat{g}_{ij} = 1/[G^{-1}]_{ji}\)

The RGA matrix is defined as:
Therefore the RGA elements represent deviation in the gain value between output $y_j$ and input $u_j$, as a consequence of the action of other control loops.

Clearly the most desirable values for RGA are values close to unity. When $RGA=1$, the particular input has the same influence on the particular output regardless of the other control loops.
APPENDIX J  BASIS FROM $H_\infty$ CONTROL THEORY - MIXED SENSITIVITY APPROACH

The system block diagram, considered in $H_\infty$ synthesis, is shown in Figure J.1,

![System block diagram in $H_\infty$ synthesis.](image)

where $W_1(s), W_2(s), W_3(s)$ are weights for: sensitivity functions, control signal and complementary sensitivity function respectively. Assuming usual notation from $H_\infty$ control theory, $z$ denotes outputs to be optimised, $w$ stands for disturbances, and $v$ is the control error.

The $H_\infty$ optimal controller $K(s)$ is designed to satisfy the following condition:

$$\left\| F_1(P,K) \right\|_\infty = \max_{w(t) \neq 0} \frac{\left\| z(t) \right\|_2}{\left\| w(t) \right\|_2} < \gamma$$  \hspace{1cm} (J.1)

where:

$$z = F_1(P,K)w$$  \hspace{1cm} (J.2)

and where $\gamma$ should be reduced successively to get optimal solution. The matrices $P$ and $K$ are defined as:

$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} w \\ u \end{bmatrix}$$  \hspace{1cm} (J.3)

Therefore, in the mixed sensitivity approach, the values to be minimized are the system outputs, which are obtained when the sensitivity functions are multiplied by the corresponding weights.

The norm to be minimized in (J.1) can also be expressed as:

$$\left\| F_1(P,K) \right\|_\infty = \begin{bmatrix} W_1S \\ W_2KS \\ W_3T \end{bmatrix}$$  \hspace{1cm} (J.4)
with usual notation form robust control theory:

\[ S = (I + G(s)K(s))^{-1} \quad \text{and} \quad T = G(s)K(s)(I + G(s)K(s))^{-1} \]  

(J.5)

In the equation (J.4), system performance can be specified in the following way:

\[ W_1(s) \] - bounds the system speed of response, disturbance rejection and additive model uncertainty.

\[ W_2(s) \] - bounds the magnitude of control signals.

\[ W_3(s) \] - bounds the reference tracking, measurement noise rejection and multiplicative model uncertainty.

The condition for robust stability (RS) is:

\[ RS \iff |T(j\omega)| < 1 / |W_3(j\omega)|, \forall \omega \]  

(J.6)

and the condition for nominal performance (NP) is:

\[ NP \iff |S(j\omega)| < 1 / |W_1(j\omega)|, \forall \omega \]  

(J.7)

By using the system formulation from (J.2-J.4), the \( H_\infty \) control algorithm will return the values for the controller matrix \( K(s) \), such that the condition (J.1) is satisfied.
Appendix
PUBLICATIONS CLOSELY RELATED TO THIS THESIS:

Journals:

Conference Proceedings:

Technical Reports:

Patent Documents: