

The Irradiance Error and its Effect in Photometric Stereo

Karsten Schlüns*

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This paper discusses some essential aspects on evaluating the three-source photometric stereo method (PSM). PSM is a shading based 3D shape recovery approach that calculates a dense set of surface orientations from three input images taken by changing the illumination direction without moving the optical sensor. A subsequent step can be used to convert the surface gradients into a dense height map by means of an integration method. In a previous paper [2] we carried out evaluations of integration approaches. Here we show how the resolution of surface orientations depends on perturbations in the image irradiances (intensities). Previous methods considered only single light source configurations or particular image irradiance triples, hence no general predictions could be made. We give a simple geometrical interpretation to estimate upper bounds of angular deviations with respect to expected errors in image irradiances. Such predictions are necessary for the practical application of the photometric stereo method.

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Abstract

This paper discusses some essential aspects on evaluating the three-source photometric stereo method (PSM). PSM is a shading based 3D shape recovery approach that calculates a dense set of surface orientations from three input images taken by changing the illumination direction without moving the optical sensor. A subsequent step can be used to convert the surface gradients into a dense height map by means of an integration method. In a previous paper [2] we carried out evaluations of integration approaches. Here we show how the resolution of surface orientations depends on perturbations in the image irradiances (intensities). Previous methods considered only single light source configurations or particular image irradiance triples, hence no general predictions could be made. We give a simple geometrical interpretation to estimate upper bounds of angular deviations with respect to expected errors in image irradiances. Such predictions are necessary for the practical application of the photometric stereo method.

Keywords: *photometric stereo method (PSM), 3D shape recovery, evaluation, error analysis.*

1. Introduction and Related Work

It is obvious that performance characterizations of known and new approaches play an important role in current Computer Vision research. This includes the development of suitable quality measures and test strategies for three-dimensional shape recovery methods. The number of publications that evaluate shading based approaches like shape-from-shading, PSM [9, 10], and photometric sampling is still rather small. In the case of the PSM just a few publications exist that address accuracy aspects. The investigations made in [1, 3, 7, 9] for the three-source PSM and in [4, 5] for the two-source PSM show a rough impression of the resolution of recovered surface orientations. General predictions of the resolution depending on errors in image irradiances have not yet been done. This paper gives a

general statement about the achievable accuracy with respect to the introduced irradiance error.

2. Contributing Types of Errors

The error sources of the PSM can be divided into three classes:

- calibration errors,
- measurement errors, and
- model errors.

Calibration errors can be further subdivided into deviations that arise during the photometric calibration procedure of the sensor and the determination of the illumination parameters (illumination directions and light source strengths). These errors can be avoided in certain

cases if a look-up-table realization of the PSM is employed. Measurement errors are caused by the sensor and the digitization unit. Under controlled conditions noise effects in the image acquisition system predominate other error sources. Errors that arise in particular in uncontrolled image acquisition environments are discussed in [6]. Errors from the third group depend partly on the object scene, hence they are the most difficult ones to handle. Therefore it is not possible to estimate their influence without any knowledge about the characteristics of the objects. Model errors can be classified into deviations from the illumination model, deviations from the reflection model, and deviations from the projection model. In case of the common PSM [9] (the one we are dealing with) deviations appear for insufficient parallel radiating light sources and non-uniform image irradiance distributions of the light sources. Effects that also belong to model errors are shadows and mutual illuminations. Deviations from the reflection model occur if the (usually employed) reflection analysis [8] cannot entirely eliminate the specular reflection component.

Nearly all of the above listed factors contribute in an additive manner to the total error amount. In the following we consider the sum of these perturbations in the irradiances. We will call the sum the *irradiance error*. The influence of light source calibration errors cannot be considered as being a component of the irradiance error, therefore they must be treated separately.

3. Effects of the Irradiance Error on a Surface Normal

For a point \mathbf{P} on a diffuse reflecting (Lambertian) surface the relationship between the irradiance vector

$$\mathbf{E} = (E_1, E_2, E_3)^T$$

and a surface normal¹ \mathbf{n}° can be described as a linear mapping $\mathbf{n}^\circ = \mathbf{S}^{-1}\mathbf{E}$. The rows of the 3×3 illumination matrix \mathbf{S} are formed by the illumination directions $\mathbf{s}_1, \mathbf{s}_2$ and \mathbf{s}_3 . The vector \mathbf{n}_r° denotes the correct (true) surface normal that is

expected in an ideal system. \mathbf{E}_t is the true irradiance vector, respectively. Obviously it holds $\mathbf{n}_t^\circ = \mathbf{S}^{-1}\mathbf{E}_t$. The measured irradiance vector \mathbf{E}_r is an additive composition of the vector \mathbf{E}_t and an *irradiance error vector* $\Delta\mathbf{E}$:

$$\mathbf{E}_r = \mathbf{E}_t + \Delta\mathbf{E} .$$

Because of the irradiance error vector $\Delta\mathbf{E}$ we reconstruct the distorted surface normal

$$\mathbf{n}_r = \mathbf{S}^{-1}\mathbf{E}_r = \mathbf{S}^{-1}\mathbf{E}_t + \mathbf{S}^{-1}\Delta\mathbf{E}$$

which is not necessarily a unit vector. The true surface normal \mathbf{n}_t° is distorted by the normal error vector $\Delta\mathbf{n}$, with

$$\mathbf{n}_r = \mathbf{n}_t^\circ + \Delta\mathbf{n} .$$

The irradiance error vector $\Delta\mathbf{E}$ and the normal error vector $\Delta\mathbf{n}$ are related by

$$\Delta\mathbf{E} = \mathbf{S}\Delta\mathbf{n} .$$

Without loss of generality we may assume that $\Delta\mathbf{E}$ has a constant length $\varepsilon = \|\Delta\mathbf{E}\|$ as we want to describe maximal angular deviations from the true surface normal. We will call ε the (scalar) *irradiance error*. For the squared irradiance error it holds

$$\varepsilon^2 = \|\Delta\mathbf{E}\|^2 = \Delta\mathbf{E}^T \Delta\mathbf{E} = (\mathbf{S}\Delta\mathbf{n})^T (\mathbf{S}\Delta\mathbf{n}) .$$

After reordering we get the quadratic form

$$\Delta\mathbf{n}^T \mathbf{B} \Delta\mathbf{n} = \varepsilon^2 ,$$

with the symmetric, positive definite matrix $\mathbf{B} = \mathbf{S}^T \mathbf{S}$. Hence the quadratic form describes an ellipsoid. In [11, 10] the complementary form

$$\mathbf{E}^T \mathbf{C} \mathbf{E} = 1, \text{ with } \mathbf{C} = \mathbf{S}^{-1T} \mathbf{S}^{-1}$$

is employed to develop the PSM without knowledge of the light source parameters.

3.1 General Illumination Configs

The quadratic form describes the set of potential normal error vectors $\Delta\mathbf{n}$ for a given irradiance vector ε . It is well-known that the shape of the ellipsoid is described by the Eigen system of the matrix \mathbf{B} . We will call that ellipsoid the *irradiance error ellipsoid*. The orientations of

¹ Unit vectors are denoted by superscript $^\circ$.

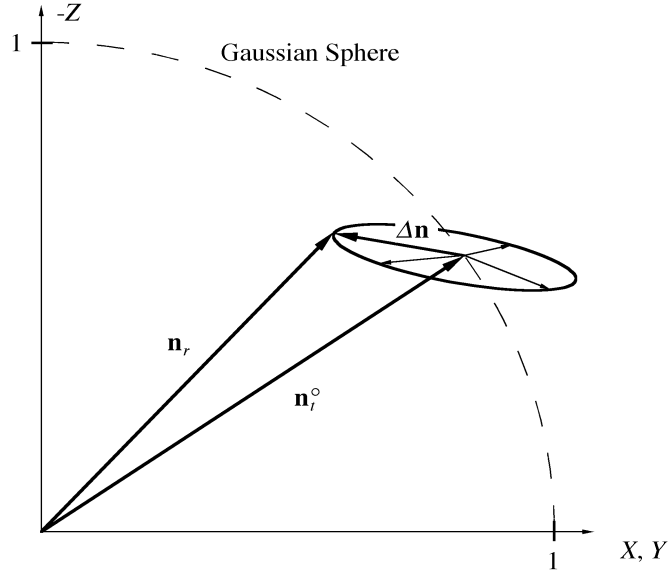


Figure 1: Cross-section through the Gaussian sphere to sketch an irradiance error ellipsoid

its semiaxes are equal to the eigenvectors of matrix \mathbf{B} . The lengths of the semiaxes are given by the reciprocal square roots of the eigenvalues of \mathbf{B} . Fig. 1 shows a cross-section through the Gaussian sphere which is used to represent the true surface normals \mathbf{n}_i^o . The maximal angular deviation $\angle(\mathbf{n}_i^o, \mathbf{n}_r)$ which is possible for a constant irradiance error is

$$\max_{\Delta \mathbf{n}} \angle(\mathbf{n}_i^o, \mathbf{n}_r) = \max_{\Delta \mathbf{n}} \angle(\mathbf{n}_i^o, \mathbf{n}_i^o + \Delta \mathbf{n}) .$$

Note that the shape of the irradiance error ellipsoid does not depend on the true surface normal \mathbf{n}_i^o .

3.2 Standard Illumination Configuration

Now we discuss the maximal angular deviations for the Standard Illumination Configuration (SIC) with a SIC-angle α (see Appendix). We get the quadratic form

$$\frac{3}{2} \cdot \sin^2(\alpha) \cdot \Delta n_x^2 + \frac{3}{2} \cdot \sin^2(\alpha) \cdot \Delta n_y^2 + 3 \cdot \cos^2(\alpha) \cdot \Delta n_z^2 = \varepsilon^2 ,$$

with vector $\Delta \mathbf{n} = (\Delta n_x, \Delta n_y, \Delta n_z)^T$. For the SIC the irradiance error ellipsoid is defined by the equation

$$\left(\frac{\Delta n_x}{a}\right)^2 + \left(\frac{\Delta n_y}{b}\right)^2 + \left(\frac{\Delta n_z}{c}\right)^2 = 1 .$$

The lengths a , b and c of the semiaxes are

$$a = b = \frac{\sqrt{2}}{\sqrt{3}} \varepsilon \cdot \frac{1}{\sin(\alpha)} \quad \text{and}$$

$$c = \frac{1}{\sqrt{3}} \varepsilon \cdot \frac{1}{\cos(\alpha)} .$$

Fig. 2 shows all three existing shapes of irradiance error ellipsoids (ellipsoids of revolution) for SIC-angles

$$0^\circ < \alpha < \arctan(\sqrt{2}) \quad (\text{left}),$$

$$\alpha = \arctan(\sqrt{2}) \quad (\text{middle}) \quad \text{and}$$

$$\arctan(\sqrt{2}) < \alpha < 90^\circ \quad (\text{right}).$$

Only if

$$\alpha = \arctan(\sqrt{2})$$

the maximal angular deviation does not depend on the true surface normal \mathbf{n}_i^o . In this case it holds

$$\max \angle(\mathbf{n}_i, \mathbf{n}_r) = \arcsin(\varepsilon),$$

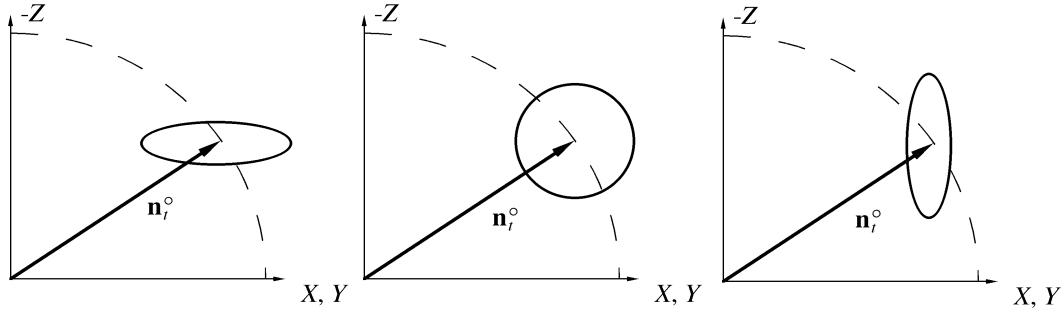


Figure 2: Irradiance error ellipsoids belonging to the Standard Illumination Configuration for different SIC-angles

since the maximal deviation occurs when the reconstructed surface normal \mathbf{n}_r is oriented in tangential direction with respect to the irradiance error ellipsoid. Otherwise the maximal angular deviation depends on \mathbf{n}_r^o . Because of our goal to find upper bounds, we search for normals \mathbf{n}_r^o that maximize the maximal angular deviation. The angular deviation for all physically possible surface normals is maximized, if $\Delta\mathbf{n} \perp \mathbf{n}_r$. The maximized maximal error occurs for a set of surface normals, since the ellipsoids are rotational symmetric.

4. Results

For all SIC-angles $0^\circ < \alpha \leq \arctan(\sqrt{2})$ we get $\max \angle(\mathbf{n}_r^o, \mathbf{n}_r) = \arcsin(\sqrt{2/3} \cdot \varepsilon \cdot \sin(\alpha)^{-1})$ as

an upper bound of the angular deviations. This formula can be calculated from the Eigen system of matrix \mathbf{B} and taken to estimate the upper bounds with respect to the angular deviations of the surface normals determined by the PSM. The formula depends on the SIC-angle α and the scalar irradiance error ε . Fig. 3 shows the maximal deviations (in degrees) for different irradiance errors ε over the SIC-angle α .

The irradiance error ε is composed of calibration, measurement, and model errors as discussed in Section 2. The diagram shown in Fig. 3 can be interpreted as follows: let us assume for instance an SIC-angle $\alpha = 30^\circ$ and an irradiance error $\varepsilon = 0.05$ (5%), the maximal angular deviation with respect to this additive error is about 4.68° .

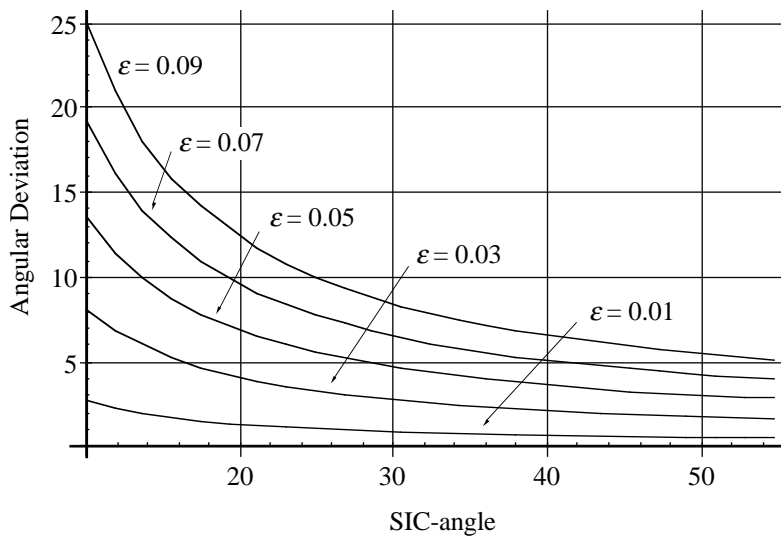


Figure 3: Upper bounds of the maximal angular deviations for different irradiance errors ε and different SIC-angles α . The angles are given in degrees

5. Conclusion and Future Work

A technique to evaluate noise robustness of the traditional three-source photometric stereo method has been developed and discussed. It enables us to describe and determine the resolution of the calculated surface normals with respect to the defined irradiance error. As a quality measure we have employed the maximum in the angular deviation from the expected surface normal. The effect of irradiance errors usually depends on the surface point being examined.

However the derived bounds are independent of a particular object surface point, since the maximal derivations in the surface normals have been calculated for the complete set of possible surface normals. That is the important difference to other published work. The upper bounds are determined for SICs by giving the SIC-angle α and the maximal irradiance error.

In future work we will generalize the analysis to non-SICs. However, in practical applications a symmetric illumination configuration is in general the best choice, if the expected surface orientations are uniformly distributed.

6. References

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Appendix

A set of three illumination directions s_1, s_2 and s_3 is called a Standard Illumination Configuration (SIC) if the angles between each pair of these illumination directions are equal and the angles between the viewing direction \mathbf{v} and each of s_1, s_2 and s_3 are equal, too (Fig. 4). We will call the angle between the illumination directions and \mathbf{v} the *SIC-angle* α , with

$$\alpha \in (0^\circ, 90^\circ).$$

The gradients \mathbf{G} of the illumination directions s_1, s_2 and s_3 are

$$\mathbf{G}(s_1) = \left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right) \cdot \tan(\alpha) ,$$

$$\mathbf{G}(s_2) = \left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right) \cdot \tan(\alpha) \text{ and}$$

$$\mathbf{G}(s_3) = (0, 1) \cdot \tan(\alpha) .$$

In the analysis we consider SICs with

$$\alpha \leq \arctan(\sqrt{2}) \approx 54.74^\circ .$$

In applied work it is not useful to take larger SIC-angles, since the area of the object which is illuminated by all three light sources would be too small.

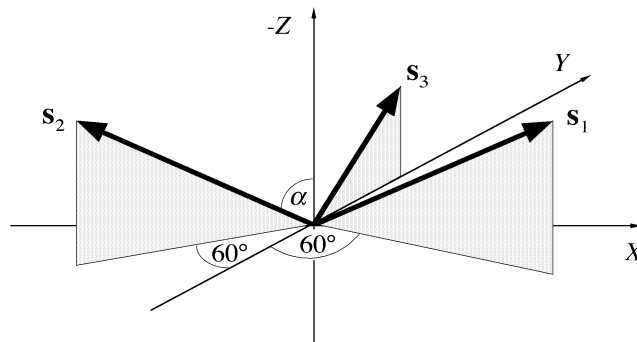


Figure 4: Standard Illumination Configuration (SIC)

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