# Estimating 3D Flow for Driver Assistance Applications

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**Abstract.** This paper proposes a technique for estimating 3D flow vectors, by combining a KLT tracker with subsequent scale-space analysis of tracked points. A tracked point defines a 2D vector, which is mapped into 3D space based on ratios of maxima of scale-space characteristics. The approach is tested for night-vision sequences as recorded (at Daimler A.G., Germany) for driver assistance projects. Those image sequences (at 25Hz) are characterized by being slightly blurry and of low contrast.

Key words: motion analysis, motion vector fields, 3D motion, driver assistance

# 1 Introduction

The estimation of dense motion fields is still a challenging task for vision-based driver assistance systems (DAS), where motion vectors are often relatively long even if sequences are taken at a frame rate of more than 30 Hz. This paper suggests a way to derive 3D directions of observed 2D motion vectors, which allows a more consistent interpretation of the motion field.

Note that a 3D direction of a motion vector is not yet defining its pose, which would also require to identify its position (e.g., via stereo analysis). Algorithms for identifying the 3D pose of projected motion vectors (known as *range flow* or *scene flow*) have been studied, for example, in [10] (and subsequent papers by the same authors). However, the applied methodology differs from the one suggested in this paper.

The methods used crucially depend on whether a sparse or a dense representation is desired, or whether motion is assumed to be rigid or not. In case of sparse representations, this involves some kind of spatio-temporal feature matching where, for the monocular case, this is accomplished by methods as known from structure-from-motion (SfM), which usually assumes a rigid motion of the whole scene [9]. When more than one view is available, as in the stereo case, the computation of scene flow relies on integration of depth and temporal information in some cooperative way [12]. For the case of dense representations, this involves the minimization of energies in a variational framework in order to 2 Jorge A. Sanchez, Reinhard Klette, Eduardo Destefanis

add some smoothness constraint needed to deal with the aperture problem [11]. Our approach tries to use the information provided by observed changes in size (scale) of local image regions over time, when a single camera moves relatively to the scene. This is known to be a very important source of information for the visual perception of motion, and we show how to integrate this into computational vision.

# 2 Estimation of 3D Directions

We consider a 3D point P, tracked between frames  $I_t$  and  $I_{t+1}$ , and propose a possible way for calculating the 3D direction of the observed motion.

# 2.1 Update Equation

Consider a disk of radius  $\rho$  moving towards an ideal pinhole-type camera of focal length f. Without loss of generality, let the radius move parallel to the Y-axis of the XYZ-camera coordinate system (i.e.,  $r = Y_c - Y_e$ , for center  $P_c$ and an edge point  $P_e$  of the disk). A 3D point P = (X, Y, Z) in the world (in camera coordinates) projects into a point p = (x, y, f) in the image plane, with  $x = f \frac{X}{Z}$  and  $y = f \frac{Y}{Z}$ . Point  $P_c$  projects into  $p_c = (x_c, y_c, f)$ , and  $P_e$  projects into  $p_e = (x_e, y_e, f)$ . The moving disk is at time t at distance  $Z_t$ , and projected into image  $I_t$  as a disk of radius  $r_t$ . We obtain the following for the area of this projected disk:

$$A_t = \pi r_t^2 = \pi \left(y_c - y_e\right)^2 = f \frac{\pi}{Z_t^2} \left(Y_c - Y_e\right)^2 = \pi f \frac{\rho^2}{Z^2}$$

Radius  $\rho$  of the disk is constant over time, thus, the product  $A_t Z_t^2 \sim \rho^2$  will also not change over time.

We consider projections of the disk at times t and t + 1. Because the ratio of square roots of areas is proportional to the inverse of the ratio of corresponding Z-coordinates of the disk, we are able to define a z-ratio



**Fig. 1.** Two projections of a moving disk, at times t and t + 1.

either by area or Z-values.

Such a z-ratio can also be defined just for a pair of projected points  $P_t = (X_t, Y_t, X_t)$  and  $P_{t+1} = (X_{t+1}, Y_{t+1}, Z_{t+1})$  (just by the ratio of Z-coordinates). Using the central projection equations for both projected points, we obtain for their x-ratio and y-ratio the following:

$$\mu_x = \frac{X_{t+1}}{X_t} = \frac{Z_{t+1}}{Z_t} \cdot \frac{x_{t+1}}{x_t} = \mu_z \frac{x_{t+1}}{x_t} \tag{2}$$

$$\mu_y = \frac{Y_{t+1}}{Y_t} = \frac{Z_{t+1}}{Z_t} \cdot \frac{y_{t+1}}{y_t} = \mu_z \frac{y_{t+1}}{y_t}$$
(3)

Altogether, this may also be expressed by the following *update equation*:

$$\begin{pmatrix} X_{t+1} \\ Y_{t+1} \\ Z_{t+1} \end{pmatrix} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \begin{pmatrix} X_t \\ Y_t \\ Z_t \end{pmatrix}$$
(4)

with  $\mu_x$ ,  $\mu_y$ , and  $\mu_z$  as in Equations (2), (3), and (1) respectively. In other words, knowing  $\mu_z$  and ratios  $\frac{x_{t+1}}{x_t}$  and  $\frac{y_{t+1}}{y_t}$  allows to update the position of point  $P_t$ into  $P_{t+1}$ . Assuming that  $P_t$  and  $P_{t+1}$  are positions of one tracked 3D point P, from time t to time t + 1, we only have to solve two tasks: (1) decide for a technique to track points from t to t + 1, and (2) estimate  $\mu_z$ . If an initial position  $P_0$  of a tracked point P is known then we may identify its 3D position at subsequent time slots. Without having an initial position, we only have a 3D direction  $P_t$  to  $P_{t+1}$ , but not its 3D position.

### 2.2 3D Direction of Projected Motion

Consider a mobile platform moving on a planar surface, as illustrated in Figure 2. The relative motion of a point in 3D space can be expressed (with respect to the camera coordinate system) by the following increments:

$$\Delta X = X_{t+1} - X_t = (\mu_x - 1)X_t$$
  

$$\Delta Y = Y_{t+1} - Y_t = (\mu_y - 1)Y_t$$
  

$$\Delta Z = Z_{t+1} - Z_t = (\mu_z - 1)Z_t$$

The ratios

$$\frac{\Delta X}{\Delta Z} = \left(\frac{\mu_x - 1}{\mu_z - 1}\right) \frac{X_t}{Z_t} = \left(\frac{\mu_x - 1}{\mu_z - 1}\right) \frac{x_t}{f} \tag{5}$$

$$\frac{\Delta Y}{\Delta Z} = \left(\frac{\mu_y - 1}{\mu_z - 1}\right) \frac{Y_t}{Z_t} = \left(\frac{\mu_y - 1}{\mu_z - 1}\right) \frac{y_t}{f} \tag{6}$$

of those increments are the tangents of the *navigation angles*  $\Phi_{zx}$  and  $\Phi_{zy}$  (see Figure 2), respectively, that represent the 3D direction of motion (between two subsequent frames) for a tracked 3D point.

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Fig. 2. A tilted camera translating along a plane (top left), motion angles on the ZY-plane (bottom left), and on the ZX-plane (right).

# 3 Feature Tracking and Test Data

The determination of those navigation angles relays on the detected projected motion of tracked points in the image plane. Tracking methods for estimating a dense 2D motion field are known as *optical flow techniques*.

Actually, the used tracking method is not essential for presenting the basic idea of our approach for estimating 3D directions; however, it is, of course, important for obtaining reliable results of the proposed approach.

For this paper we simply used an open implementation [7] of the Lucas-Kanade [5] feature tracker, with initial points selected as in [8].

Regarding test sequences, we decided for Set 1 (seven night vision stereo sequences, provided by Daimler A.G.) as available on [1]. Those will be called



Fig. 3. Optical flow computed for Sequences 1 and 2.



Fig. 4. Optical flow computed for Sequence 3 and 4.

'Sequence 1', 'Sequence 2', and so forth, as on this website and described in [13]. Additionally, also one 'Desktop sequence' was generated and used for performance analysis. This desktop sequence was generated by translating a calibrated camera along an optical bench, with constant 3D viewing angles relatively to the surface of the desktop.

Figures 3 to 6 illustrate tracking results for Sequences 1 to 7, and the Desktop sequence. By using relatively high thresholds, only relatively sparse motion fields are shown. The used coloring is based on the length of the optic flow vectors.

# 4 Scale Ratio for a Tracked Point

The idea behind the presented approach is as follows: instead of directly tracking image regions (such as disks, as discussed in Section 2), single feature points are tracked, but an 'area of influence' is assigned to such a point, basically taking the role of a tracked disk.



Fig. 5. Optical flow computed for Sequence 5 and 6.

 $\mathbf{6}$ 



Fig. 6. Optical flow computed for Sequence 7 and the Desktop sequence.

For tracked points, a measure is computed for the 'extension of the local image structure' in a local (or semi-local) neighborhood. Such measures, computed independently for each pair of points (i.e., a 2D flow vector between time t and t + 1), are then used to determine a scale ratio of associated intensity profiles 'surrounding' those feature points, which is finally used as an estimate of the z-ratio  $\mu_z$ .

The approach for detecting scale-ratios follows *scale space theory* as discussed in [2, 3]. Note that this is only one option; similar to the variability when deciding for one optical flow technique, also an alternative method may be used for scaleratio estimation.

We briefly recall scale space theory. Given an image function  $I : \mathbb{R}^2 \to \mathbb{R}$ , their scale space representation  $L : \mathbb{R}^2 \times \mathbb{R}_+ \to \mathbb{R}$  can be obtained by convolutions

$$L(p,\sigma) = (g_{\sigma} * I)(p)$$

of image I with a Gauss kernel  $g_{\sigma}$ , obtained by the Gauss function  $G_{\sigma} : \mathbb{R}^2 \to \mathbb{R}$ ,

$$G_{\sigma}(p) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}p^T p}$$

parameterized by standard deviation  $\sigma \geq 1$ .

In [3], a method for automatic scale selection is proposed, based on the evolution over scales of (possibly non-linear) combinations of normalized derivatives of  $L(p, \sigma)$ . The scale level at which such a response takes a local maxima is assumed to reflect the *characteristic diameter* of the surrounding data. The operator used in our experiments for scale selection is the normalized Laplacian, which is defined by

$$\nabla_{norm} L(p,t) = \sigma^2 \left| (D_x^2 L)(p,\sigma) + (D_y^2 L)(p,\sigma) \right| \tag{7}$$

where  $D_x^2$  and  $D_y^2$  are the second order derivatives of L (at scale level  $\sigma$ ).

We use Figure 7 for visualizing the scale selection principle. In this simple example, the image on the left contains three white disks with center points  $p_1$ ,

 $p_2$ , and  $p_3$ . On the right, the figure shows the scale evolution of (7) for those three center points. In this example, the ratio between the scales, identified by maxima of the scale characteristics of those three center points, equals the ratio of areas of the corresponding white disks.

Given two consecutive frames  $I_t$  and  $I_{t+1}$  of a sequence. We calculate their scale space representations  $L_t(p, \sigma)$  and  $L_{t+1}(p, \sigma)$ , for selected scales  $\sigma$ . For each selected pair of points (as a result of the tracking algorithm), we follow their scale characteristics for the normalized Laplacian of  $L_t(p, \sigma)$  and  $L_{t+1}(p, \sigma)$ , and identify local maxima over the selected scales.

The function

$$c(\sigma) = K\sigma^p e^{-\sigma/\theta}$$

is used in order to obtain sub-scale estimates, where the parameters of this function are directly computed from the local maxima, extracted as an initial estimate, and its both neighboring values  $\sigma_1$  and  $\sigma_2$ . Applying this approach, the maximum (i.e., magnitude)

$$K = c(\sigma_1)\sigma_1^{-a}e^{\sigma_{max}/\theta}$$

is identified at a sub-scale value  $\sigma_{max} = a \cdot \theta$  The scale matching process between both projections  $p_t$  and  $p_{t+1}$  of the tracked point is then based on the magnitude K of the interpolation function at the scale level  $\sigma_{max}$ .

The determination of the scale ratio, associated to characteristic diameters of a tracked pair of points, allows to estimate the corresponding  $\mu_z$  (see Equation (1)) and thus also to estimate the navigation angles (see Equations (5) and (6)).



Fig. 7. (a) White disks of radius 25, 50 and 75 pixel. (b) Evolution on the Laplacian for center points  $p_i$ , for i = 1, 2, 3.

After the estimation of the two navigation angles (for all pairs of tracked points), histograms of these two values are computed. This allows to calculate one summarizing 3D direction, for all the detected 3D directions between images  $I_t$  and  $I_{t+1}$ . This summarizing 3D direction results from

$$\hat{\phi}_{zx} = \arg \max_{\theta} h(\theta_{zx})$$
  
 $\hat{\phi}_{zy} = \arg \max_{\theta} h(\theta_{zy})$ 

where  $h(\theta_{zx})$  and  $h(\theta_{zy})$  are the histograms of the navigation angles. Figure 8 shows an example of generated histograms.



Fig. 8. Histogram examples for Sequences 3, 5 and the Desktop sequence.



Fig. 9. Tracked points and detected scales (defining the size of the shown disks).

# 5 The Algorithm

To summarize, given a pair of consecutive images,  $I_t$  and  $I_{t+1}$ , the proposed algorithm consists of the following steps:

- 1. Compute the optical flow between images  $I_t$  and  $I_{t+1}$ , as described in Section 3.
- 2. For the same images, obtain their scale-space representation,  $L_t(p,\sigma)$  and  $L_{t+1}(p,\sigma)$ , for a given set of predefined scales  $\sigma$ .
- 3. Select characteristic scales for the corresponding points resulting from Step 1.
- 4. Compute the scale-ratio from the detected scale maxima as  $(\sigma_{t+1})_{max}/(\sigma_t)_{max}$ and compute the corresponding  $\mu$ -factors.
- 5. Obtain the motion angles as the arcus tangent of (5) and (6).

For the construction of the scale space representations (Step 2), an exponential sampling was used, where the sigma value for scale level (n) is obtained from scale level (n-1) as

$$\sigma_n = k \sigma_{n-1} = k^n \sigma_0$$
 for  $n = 1, 2, \dots, N-1$ 

This allows the use of the *Difference of Gaussians* (DoG) operator as a fast approximation technique of the Laplacian operator, as in [4]. Here, given  $\sigma_0$  and  $\sigma_k$ , a scale space representation consisting of N levels is computed for  $I_t$  and  $I_{t+1}$ , respectively. In practice, 17 levels are computed with  $\sigma_0 = 1$  and k = 1.2.

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Given a scale selection operator, the scale selection for points in Step 3 consists of analyzing the evolution over scales of the operator's response (i.e., if we



Fig. 10. Optical flow computed for Sequences 1, 3, 7 and for the Desktop sequence.

consider the scale-space representation as a three-dimensional space with coordinates  $(x, y, \sigma)$ , the characteristic scale(s) for point  $(x_0, y_0)$  are those values of  $\sigma$  at which the magnitude of the response takes a (local) maxima). In the common case in which more than one local maxima are detected, the magnitude of the response can be used as a measure of similarity.

Figure 9 shows an example of computed flow vectors and their corresponding characteristic scales (detected for those points for which a maxima over the scales of the Laplacian was correctly found); the size of disks represents the corresponding characteristic scale.

### 6 Results for Desktop and Daimler Sequences

Figure 10 shows some of the results obtained for Sequences 1, 3 and 7, and the Desktop sequence.

The accuracy of the estimated values depends in some way on the magnitude of the relative motion. For points that remain almost static, and where the scale ratio y is close to 1, the quotients in Equations 5 and 6 are not well defined, due to noisy measurements. In our computations, points pairs with  $|\sigma_{max}/\sigma_{min} - 1| < 10^{-6}$  are discarded, where  $\sigma_{max}$  ( $\sigma_{min}$ ) denotes the largest (smallest) detected scale of tracked points. An upper limit to the  $\mu_z$ 's was also imposed, allowing only values < 5.

The Desktop sequence was generated with a calibrated camera, with translational motion on a rail with fixed navigation angles of approximately  $\Phi_{zx} = 12^{\circ}$ and  $\Phi_{zy} = -10^{\circ}$ .

In the case of the Desktop sequence, mean and standard deviation of  $\Phi_{zx}$  are equal to 13.531° and 2.93874°, respectively. For  $\Phi_{zy}$ , those values are equal to -10.3073° and 2.93874°, respectively, taken over the entire sequence. In the case of the Daimler sequences, there was no ground truth available (estimated), and they were only used as a qualitative (visual) reference. For example, for Sequence 7 (see example in Figure 10), estimated directions  $\Phi_{zx}$  correspond 'quite well' to the steering of the car over the sequence. Estimated values  $\Phi_{zy}$  remain at about 4°. In Sequence 2 (see Figure 10), we observed a low frequency oscillation in the value of  $\Phi_{zy}$ , starting about at frame 180, when the 'squirrel' (actually, a cat) crossed the street and the car made a maneuver (some breaking).

In all cases, observed noise is mainly due to the scale matching subprocess. As given, for example, projective distortions of local image patches are not taken into account, causing possibly some serious underestimations of scale factors. The proposed (non-run-time-optimized) algorithm runs at approximately 1 fps on a 3.0 GHz Intel<sup>©</sup> Core 2 Duo CPU.

# 7 Conclusions

A method for the instantaneous (frame to frame) estimation of the 3D direction of motion was proposed and studied, based on the determination of scale ratios

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between tracked points. The critical issue is the accurate scale estimation step. The MSER region extractor [6] is an alternative (to the presented choice) option and possibly a more robust way for the determination of scales (characteristic diameters), where local image patches can be represented with affine invariance, serving as a first-order approximation for (more general) projective deformations induced by the relative motion of the camera.

Besides some poor estimations for some frames, the proposed method may be recommended as a possible approach for the use of perceptually very important spatio-temporal cues induced on images as an observer moves relatively to the scene. The extracted information has the advantage of being local, and allowing robustness in the case of multiple moving objects. The same principle of scaleratio estimation could also be used for motion segmentation, or to add new constraints to multiple-view approaches of 3D motion estimation, thus further contributing to the already known coherence between optic flow vectors and image disparities.

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