CDMTCS Research Report Series

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CDMTCS-486 (UCNC'2015 Full Poster)
August 2015

Centre for Discrete Mathematics and
Theoretical Computer Science

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Abstract

This paper briefly presents advances in development of efficient Evolutionary Algorithms (EA) for a wide class of large non-linear constrained optimization problems. In particular, two important engineering applications are taken into account, namely residual stress analysis in railroad rails, and vehicle wheels, as well as a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental data. However, the main objective of this research is to develop various means of significant acceleration of the EA-based approach for large optimization problems, and to provide ability to solve problems when standard EA procedure fails. The efficiency of speed-up techniques proposed was examined using several simple but demanding benchmark problems. Results obtained so far are very encouraging and indicate possibilities of further development of acceleration techniques proposed.

Keywords: Evolutionary Algorithms, computation efficiency increase, large non-linear constrained optimization problems.

1 Introduction

Development of several new acceleration techniques for Evolutionary Algorithms (EA) is briefly discussed in this paper. Though our approach is very general, the target practical objective of this research is development and application of the improved EA to chosen problems of computational mechanics, including residual stress analysis in railroad rails and vehicle wheels [4, 12, 13, 18], as well as a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental and/or numerical data [7, 13]. The PBA is a hybrid method allowing for simultaneous use of the whole experimental, theoretical, and heuristic knowledge of the analyzed problems, taking into account their likelihood. Moreover, the improved EA may be also applied to a wide class of other engineering and scientific problems formulated in terms of large non-linear constrained optimization.

In contrast to majority of deterministic methods, the EA may be successfully applied with similar efficiency to both the convex and non-convex problems [3, 9]. However, the standard EA are generally rather slowly convergent methods. Therefore, this research is focused, first of all, on significant acceleration of the EA-based solution process. Moreover, the improved EA should provide possibility of solving such problems, when the standard EA methods fail.

2 General optimization problem formulation

In considered optimization problems a function given in the discrete form, e.g. expressed in terms of its nodal values, is sought. These problems may be posed in a following general way:

find a function $u = u(\mathbf{x}), \mathbf{x} \in \mathbb{R}^N$, that yields the stationary point of a functional $\Phi(u)$ satisfying the equality $\mathbf{A}(u) = 0$, and inequality constraints $\mathbf{B}(u) \leq 0$. After discretization we seek a vector $\mathbf{u} = \{u_i\}$ consisting of nodal values u_i, $i = 1, 2, ..., n$. These nodal values are defined on a mesh formed by arbitrarily distributed nodes. Here, N is the dimension of the physical space (1D, 2D or 3D), and n is a number of decision variables.

In the particular case of the PBA approach [7], the functional to be optimized and related constraints consist of the weighted experimental, and theoretical parts. The experimental part is defined as the weighted averaged error resulting from discrepancies between the measured data and its approximation. The theoretical part is based on a known theory (e.g. energy functional in mechanics), and/or on a heuristic principle (e.g. smoothness requirement).

3 EA acceleration techniques

The EA are understood here as real-value coded genetic algorithms consisting of selection, crossover, and mutation operators [3, 9]. Our long-term research includes various ways for increasing efficiency of the EA. Recently, we have proposed several new acceleration techniques, including solution smoothing and balancing, a'posteriori error analysis and related techniques, an adaptive step-by-step mesh refinement, as well as non-standard parallel and distributed computations [5, 14, 16]. These general ideas may be applied in various ways. Some basic concepts are described below.

3.1 Smoothing

In the problems considered unknowns (decision variables) mostly are discretized function values (e.g. nodal values). When using the EA approach they usually present together quite rough solution. Therefore, we have proposed two various approaches for smoothing of raw EA results. One of them is based on the Moving Weighted Least Squares (MWLS) technique [8, 14, 17]. The second approach uses the mean solution curvature taken together with the fitness function evaluating individuals in a population [5].

When using the extra smoothing procedure based on the MWLS technique, it is very useful to introduce the following weighting function [7]:

$$w_i^2 = \left(h_i^2 + \frac{g^4}{h_i^2 + g^2} \right)^{-p-1} \tag{1}$$

where h_i is a distance between nodes, p is the local approximation order, and g is a smoothing intensity parameter. Such type of weighting function allows to control intensity of solution smoothing by adjusting the parameter g.

Smoothing techniques are applicable to any optimization problems where a smooth (at least in subdomains) function is sought. However, application of smoothing to raw EA solution may result in violating of constraints. For instance, in problems of mechanics, it may cause global equilibrium loss of a considered body. The equilibrium is restored in a series of standard EA iterations after smoothing. However, it may also be restored at once by means of an artificial balancing of global body forces, performed directly after the smoothing. More general approach for any optimization problems uses elitism strategy. Smoothing is applied to all individuals in population, but the best individual from unsmoothed population is additionally preserved unaltered. It guarantees that at least one individual always satisfies constraints.

3.2 A'posteriori error analysis

A'posteriori solution error analysis is based on the assumption that we are able to generate reference solutions of sufficient quality for the error estimation [1]. We have already proposed such new technique for the reference solutions generation, based on a stochastic nature of the EA [16]. We have also improved evolutionary operators (mutation, crossover and selection) to take into account all information about estimated local and global solution errors [5, 16].

Other related techniques include weighted solution averaging, generation of population of representatives, as well as non-standard distributed and parallel computations [16].

3.3 Adaptive step-by-step mesh refinement

The approach using step-by-step mesh refinement starts the analysis from a coarse mesh, where a solution is obtained much faster than in the fully dense mesh case. However, such solution is usually not precise enough. The precision of the solution is increased by inserting new nodes in the best possible locations. Such process is repeated until errors in all nodes are smaller than their admissible level. The iterative solution procedure is as follows:

1. Evaluation of solution on a coarse mesh.

2. Smoothing of the above rough solution (e.g. using MWLS method).

3. Mesh refinement and the best approximation (or interpolation) of initial function values at inserted nodes.

4. Repetition of this procedure until a sufficiently dense mesh is reached.

Furthermore, step-by-step mesh refinement may be combined with a'posteriori error analysis and additional smoothing techniques [5, 16].

3.4 Other techniques

In our research we also consider other acceleration techniques, well known in the field of evolutionary computation, including efficient constraint handling techniques [2], hybrid algorithms combining EA with deterministic methods [6], as well as parallel and distributed algorithms [11]. So far we have shortly investigated efficient approaches based on penalty functions for constraint handling [14]. Their influence on the solution process was sometimes found significant. Recently, we have also studied techniques proposed for estimation of the convergence point of populations [10]. Preliminary results are encouraging, but wider research is needed.

4 Sample benchmark problems

The efficiency of the proposed techniques was examined by using simple but demanding benchmark problems, including ones resulting from the PBA approach. Most of the considered benchmarks may be analyzed as either 1D or 2D problems, and allow to choose for testing almost any number of decision variables involved (large problems). Our set of benchmark problems includes residual stress analysis in chosen elastic-perfectly plastic bodies (prismatic bar, thick-walled cylinder) under various appropriate cyclic loadings (such as bending moment, internal pressure, torsion, tension) [5, 14, 16]. We also investigated several benchmarks using the PBA approach and simulated pseudo-experimental data, including smoothing of beam deflections, and reconstruction of residual stresses [5, 14, 15]. Smoothing of real experimental data obtained by the vision measurement system was successfully applied as well [15].

Two simple chosen benchmark problems are formulated in a more detailed way below.

4.1 Residual stress analysis in a cyclically bending bar

Considered is residual stress analysis in a bar subject to pure cyclic bending. Assumed is an elastic-perfectly plastic material, and a rectangular cross-section ($b \times 2H$) of the bar. This problem may be posed as the following optimization:

Find stresses $\sigma = \sigma(y, z)$ minimizing the total complementary energy

$$\min_{\sigma(y,z)} \int_{-b/2}^{b/2} \int_{-H}^{H} \sigma^2(y, z) \, \mathrm{d}z \, \mathrm{d}y \tag{2}$$

and satisfying constraints:

- global self-equilibrium equation

$$M = \int_{-b/2}^{b/2} \int_{-H}^{H} \sigma(y, z) z \, \mathrm{d}z \, \mathrm{d}y = 0 \tag{3}$$

- yield condition for the total stresses

$$|\sigma + \sigma^e| \leq \sigma_Y, \tag{4}$$

where σ_Y is the yield stress (plastic limit), and σ^e is the purely elastic solution of the same problem.

Due to symmetry, only a half of the cross-section ($b \times H$) may be considered. After discretization, where the sought stress $\sigma = \sigma(y, z)$ is replaced by the piecewise linear function spanned over the nodal values σ_i, the following formulation is obtained:

find stresses $\sigma_1, \sigma_2, ..., \sigma_n$ satisfying

$$\min_{\sigma_1, \sigma_2, ..., \sigma_{n-1}} \frac{h^2}{9} \left(\sum_{k=1}^{n} \sigma_k^2 \alpha_k \right), \quad \sigma_n = \frac{-1}{z_n \alpha_n} \sum_{k=1}^{n-1} \sigma_k z_k \alpha_k \ , \tag{5}$$

where α_k are the Simpson integration coefficients. The following inequality constraints

$$|\sigma_k + \sigma_k^e| \leq \sigma_Y, \quad k = 1, 2, ..., n. \tag{6}$$

have to be also satisfied. Though simple Simpson method for numerical integration is used in the above formulation, in real calculations any other effective method may be applied.

4.2 Smoothing of beam deflections using the PBA approach

Given free-supported beam displacements w^{exp}, experimentally measured at points x_j, $j = 1, 2, ..., m$ we seek nodal values w_i, $i = 1, 2, ..., n$ of the smoothed displacements w. The following PBA formulation of this problem is considered:

find the stationary point of the functional

$$\Phi(w) = \lambda \Phi^E(w) + (1 - \lambda) \Phi^T(w), \quad \lambda \in [0, 1] \ , \tag{7}$$

where

$$\Phi^E(w) = \frac{1}{m} \sum_{j=1}^{m} \left(\frac{\overline{w}_j - w_j^{exp}}{e_j} \right)^2 , \tag{8}$$

$$\Phi^T(w) = \frac{1}{L} \int_0^L \frac{\kappa^2}{\kappa_{ref}^2} \mathrm{d}x \approx \frac{1}{L} \int_0^L \frac{(w'')^2}{\left(w''_{ref}\right)^2} \mathrm{d}x \approx \frac{1}{n} \sum_{i=1}^{n} \frac{(w''_i)^2}{\left(w''_{ref}\right)^2} \ , \tag{9}$$

satisfying boundary conditions

$$w(0) = w(L) = 0 \ , \tag{10}$$

admissible local error constraints

$$\left|\overline{w}_j - w_j^{exp}\right| \leq e_j \ , \tag{11}$$

and admissible global error constraint

$$\sqrt{\Phi^E\left(w\right)} \leq e_E \ . \tag{12}$$

In the above formulation $\overline{w}_j$ is an approximation built upon sought values w_i at node x_j, while e_j and e_E are admissible errors, L is the beam length, κ is the mean local curvature at a point [7], κ_{ref} and κ''_{ref} are reference values, λ is a dimensionless scalar weighting factor determining a reasonable balance between the experiment and theory [7].

5 On numerical results

The main objective of numerous executed tests was to examine the efficiency of the proposed speed-up techniques on large optimization problems. Typical results illustrating our research are shown in Fig. 1-2. They were obtained for the already mentioned simple benchmark problems. Some other results of our numerical analysis have already been described in our other papers [5, 14, 16]. However, this research is still in progress. Final results of a wide analysis comparing efficiency of all proposed techniques using various benchmark problems are expected in the near future. Especially interesting may be results of interactions between particular speed-up techniques simultaneously applied.

The standard EA in our tests used rank selection, heuristic crossover (with probability $P_C = 0.9$), and non-uniform mutation (with probability $P_M = 0.1$). In Fig. 1 one may see the influence an additional smoothing has on the convergence of the mean solution error. These results were obtained for benchmark 4.1 (bending bar 2D model) involving 288 decision variables (each one corresponding to one nodal value in the cross-section). The MWLS technique with $p = 1$, and $g = 5$ was used for smoothing. For more complex problems higher order p of the local approximation may be needed. The purpose of this test was also to show, that additional cost of smoothing is relatively small when compared with advantages due to acceleration of calculations. Optimization processes shown in Fig. 1 were carried out in both cases for 5000 iterations. One may easily notice the additional time (about 15% more) needed for all extra smoothing operations, which were repeated after each 200 iterations. This extra time is not significant when compared with gains obtained. In the case of this benchmark test application of the smoothing technique based on the MWLS allowed to achieve up to about 1.8 times efficiency increase. Similar speed-up was obtained when dealing with 1088 decision variables in the same benchmark. In numerical analysis of bending bar of the same but 1D model smoothing technique using 1D MWLS approach allowed to obtain acceleration up to about 4 times [14].

The results shown in Fig. 2 were obtained for benchmark 4.2 and randomly generated pseudo-experimental data (with admissible error up to 20%). They present solution process for one (optimal) value of the λ parameter determining balance between experiment and theory involved. In numerical analysis of the PBA problems necessary is to solve

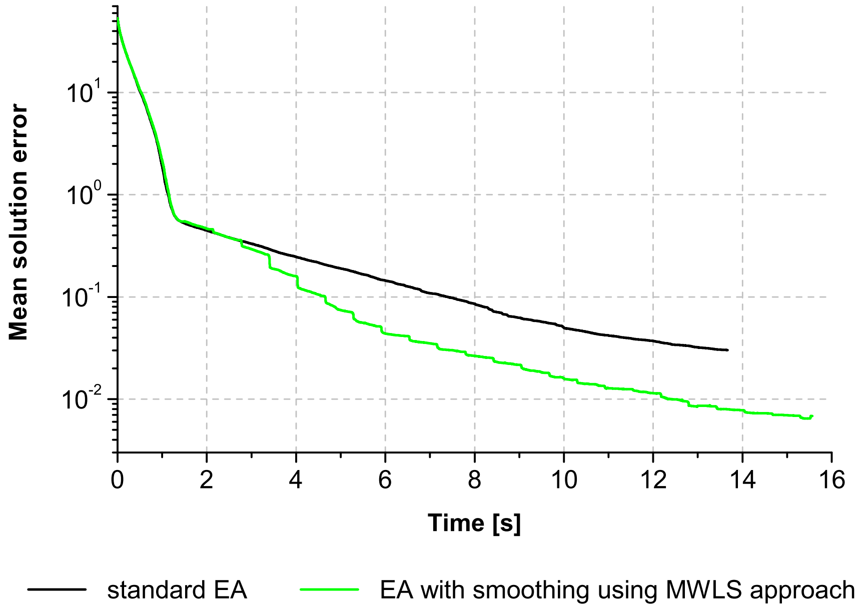

Figure 1: Convergance of the mean solution error for the standard EA and EA using smoothing approach - benchmark 4.1

optimization problem for each value of λ. Adjusting of the λ parameter may be done in various ways [7], e.g. by applying the simple bisection method. In Fig. 2 one may find comparison of the improved EA (using smoothing and step-by-step mesh refinement) with the standard EA. When using the step-by-step mesh refinement technique, the process started with 10 nodes, and after two refinements the final number of 37 nodes was reached. This strategy was also combined with a smoothing technique using the MWLS approach. In this case application of the improved EA allowed to reach a speed-up of about 300 times.

Similar approach using the step-by-step mesh refinement combined with smoothing was also evaluated for other PBA benchmarks, and involving much bigger number of decision variables. In the case of reconstruction of residual stresses in the cyclically pressurized thick-walled cylinder, the optimization process involved up to about 1200 decision variables, and the speed-up factor about 140 times was reached [14].

6 Final remarks

Results obtained so far are very encouraging and indicate possibilities of further development of speed-up techniques proposed. Each of the new speed-up techniques allowed for a significant computational efficiency increase. For more complex benchmarks considered, the speed-up factor up to about 140 times was reached so far [14].

Future research includes, inter alia, testing of further new acceleration techniques and their combinations, as well as application of the improved EA to real large complex

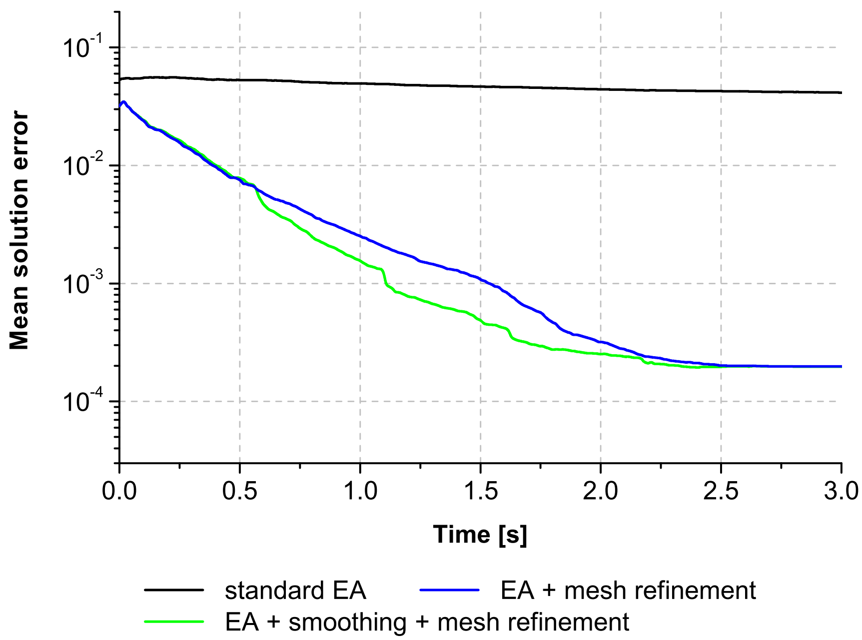

Figure 2: Convergance of the mean solution error for the standard and improved EA - benchmark 4.2

engineering problems, including broad PBA data smoothing, and residual stress analysis in railroad rails, and vehicle wheels.

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