# Passive Myocardial Mechanics

—

# Constitutive Laws and Material Parameter Estimation

by Holger Schmid

Supervised by Prof Peter J. Hunter & Dr Martyn P. Nash

A thesis submitted in partial fulfilment of the requirements for the degree of PhD at the University of Auckland


Bioengineering Institute,
University of Auckland,
New Zealand

August 16, 2006

# Abstract

This study investigated the performance of orthotropic constitutive laws describing the passive mechanical behaviour of the myocardium. The performance was validated against simple shear experiments of pig hearts which were available from earlier studies.

First, a homogeneous deformation model was developed which captured the main features of the deformation process. This served as the basis for a comparative study between three phenomenological material laws that had been published in the literature. Two of these laws exhibited certain limitations and two further constitutive laws were therefore developed that removed these limitations. Thus, five material laws were investigated in terms of their performance to fit the given experimental data by reducing a least–square objective function between the experimental and model data. Furthermore the consistency of the material parameters amongst experiments was investigated. As part of this study, a modified least–squares objective function was developed that decreased the computational time involved by about two orders of magnitude with comparable error.

Second, the assumption of a homogeneous deformation of simple shear was removed and the parameters were estimated using a finite element environment using an inverse estimation technique and therefore fulfilling the equations of motion that underpin continuum mechanics. It was found that

the material parameters of all laws were in the same range compared to those obtained from the homogeneous study. Relaxing the homogeneous assumptions slightly reduced the objective function error although the computational time increased by three orders of magnitude.

Third, the experimental protocol of six simple shear modes was supplemented with three uniaxial deformations modes. The material parameters for the same constitutive relations were estimated. It was possible to show that the material parameters that were associated with shear strain were very similar to those obtained from the simple shear study. The axial material parameters, however, were considerably different.

Finally, since it is recognised that phenomenological material laws do not provide insight into the underlying micro–structural mechanisms, the framework for a multi–scale constitutive relation was developed. This is based on multi–scale images of rat myocardium.

# Acknowledgements

I am grateful to the organisations that have provided financial support for my graduate studies. In particular, the University of Auckland awarded me an International Doctoral Scholarship. Additional support for conference attendance was provided by the National Heart foundation, the Maurice Phyllis and Paykel Trust, Education New Zealand and the Research office of the University of Auckland.

I would like to thank my supervisors, Prof Peter Hunter and Dr Martyn Nash for their generous support, time and excellent guidance during my time at the Bioengineering Institute.

In addition to my supervisors, many others provided valuable input to the work covered in this thesis; Dr Alistair Young for his valuable feedback throughout my studies. Dr Ian LeGrice, Assoc Prof Bruce Smaill, Dr Paul O'Callaghan and Dr Rob Kirton for their support in organising the experiments. Vijay Rajagopal for coding the CellML background for setting up mechanical constitutive equations. Dr Greg Sands for the help in the image optimisation. Dr Oliver Röhrle for his invaluable feedback in regard to the modified objective function. Dr Cameron Walker for his advice during the setup of the algorithm for the myocyte topology. Dr Piaras Kelly for the support during my first teaching experiences.

I certainly owe a great deal to my two summer students Andy Wei Lin and

Matthew McCormick. Working with you guys was fantastic and an enriching experience.

Thanks to all in the tramping club, Lloyd, Anthea, Shane, Chantelle, Jane, Lara and all the others.

My friends Konrad, Flo, Bodo, Katrin, Marina, Linda, Alex, Wolfram, Dirk, Frithjof, Hans, Stephan, Corinna, Robert Lee Fife III, Jochen, Kohlemohr, Haubi, Raschid, Felix, Olaf, Mascha, Anke, Christian, Ann–Kathrin, Su, Jon, Vittoria, Mike, Rita, Hayley, Anita, Kilian, Daniel, Jonna, Will, Ankur, Gail, Ruth and Alice as well as Mrs Bräunig have constantly reminded me that there is a life outside the experience of a doctorate.

Nothing of this of course would have been possible without the constant support of my parents, Gisela and Thomas Schmid, my aunt and uncle Katharina and Jürgen Steuck as well as my sister Andrea Holzinger, my brother–in–law Lukas Holzinger and my little niece Svenja Holzinger with the cutest smile. They were a source of consistent refreshment and laughter.

Furthermore I am highly indebted to Dr Richard Bolstad and Fr Bernard Kiely. Their utter patience and unquestioning acceptance of myself gave me the strength I needed so often during the "thesis–blues".

Last and by no means least I would like to thank Ardass Singh Khalsa. Words cannot describe the gratitude I feel towards him. If I may try so, however, then they would be:

**Ang Sang Wahe Guru!**

# Contents

# List of Figures

# List of Tables

## Dedication

O, never say that I was false of heart,
Though absence seem'd my flame to qualify.
As easy might I from myself depart
As from my soul, which in thy breast doth lie:
That is my home of love: if I have ranged,
Like him that travels I return again,
Just to the time, not with the time exchanged,
So that myself bring water for my stain.
Never believe, though in my nature reign'd
All frailties that besiege all kinds of blood,
That it could so preposterously be stain'd,
To leave for nothing all thy sum of good;
For nothing this wide universe I call,
Save thou, my rose; in it thou art my all.

**William Shakespeare**: Sonnet 109