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# Passive Myocardial Mechanics

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## Constitutive Laws

### and

## Material Parameter Estimation

by Holger Schmid

Supervised by Prof Peter J. Hunter & Dr Martyn P. Nash

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the degree of PhD at the University of Auckland



Bioengineering Institute,  
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New Zealand  
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# Abstract

This study investigated the performance of orthotropic constitutive laws describing the passive mechanical behaviour of the myocardium. The performance was validated against simple shear experiments of pig hearts which were available from earlier studies.

First, a homogeneous deformation model was developed which captured the main features of the deformation process. This served as the basis for a comparative study between three phenomenological material laws that had been published in the literature. Two of these laws exhibited certain limitations and two further constitutive laws were therefore developed that removed these limitations. Thus, five material laws were investigated in terms of their performance to fit the given experimental data by reducing a least-square objective function between the experimental and model data. Furthermore the consistency of the material parameters amongst experiments was investigated. As part of this study, a modified least-squares objective function was developed that decreased the computational time involved by about two orders of magnitude with comparable error.

Second, the assumption of a homogeneous deformation of simple shear was removed and the parameters were estimated using a finite element environment using an inverse estimation technique and therefore fulfilling the equations of motion that underpin continuum mechanics. It was found that

the material parameters of all laws were in the same range compared to those obtained from the homogeneous study. Relaxing the homogeneous assumptions slightly reduced the objective function error although the computational time increased by three orders of magnitude.

Third, the experimental protocol of six simple shear modes was supplemented with three uniaxial deformations modes. The material parameters for the same constitutive relations were estimated. It was possible to show that the material parameters that were associated with shear strain were very similar to those obtained from the simple shear study. The axial material parameters, however, were considerably different.

Finally, since it is recognised that phenomenological material laws do not provide insight into the underlying micro-structural mechanisms, the framework for a multi-scale constitutive relation was developed. This is based on multi-scale images of rat myocardium.

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## Dedication

O, never say that I was false of heart,  
Though absence seem'd my flame to qualify.

As easy might I from myself depart  
As from my soul, which in thy breast doth lie:

That is my home of love: if I have ranged,  
Like him that travels I return again,  
Just to the time, not with the time exchanged,

So that myself bring water for my stain.

Never believe, though in my nature reign'd

All frailties that besiege all kinds of blood,

That it could so preposterously be stain'd,

To leave for nothing all thy sum of good;

For nothing this wide universe I call,

Save thou, my rose; in it thou art my all.

**William Shakespeare:** Sonnet 109