

# Accuracy Improvement in Camera Calibration

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## Abstract

Camera calibration is a necessary and critical step in 3D object analysis. The accuracy of calibration results will affect the object's position in world coordinates, especially for 3D object tracking. In this paper, we present a new camera calibration approach, and discuss its accuracy. We use 3D marks instead of 2D marks for calibration. Our experimental results show that our approach has the potential to improve the calibration accuracy.

**Keywords:** 3D reconstruction, camera calibration.

## 1 Introduction

In the context of three-dimensional machine vision, camera calibration is a process for determining the internal geometric and optical camera characteristics (intrinsic parameters), and the 3D position and orientation data of the camera frame relative to a defined world coordinate system (extrinsic parameters). A calibration technique is based on known 3D coordinates of geometrically configured points. Here the 3D coordinates are usually referenced to the world coordinate system. The configured points are commonly referred to as *calibration points* which are physically realized by *calibration marks* on a *calibration object*.

Common calibration methods are DLT (Direct Linear Transform) and Tsai's method [1, 2]. The latter one also models lens distortion coefficients and the mapping of sensor elements to an image buffer matrix; it requires at least seven accurately detected calibration points in an arbitrary but known geometric configuration. Using Tsai's calibration method, we can transfer 3D world coordinates into image coordinates. But even using the same calibration method, differences in image acquisition environments may affect the accuracy, such as distances between camera and object, the size and number of calibration marks, or the size of calibration objects. We evaluated Tsai's method with respect to such variations, where 3D marks are used instead of the common 2D calibration marks. Section 2 reports about improvements in calibration accuracy; Section 3 specifies this new alteration of Tsai's method; Section 4 presents further experimental results, and Section 5 contains our conclusions.

## 2 Accuracy Improvement

Calibration accuracy will affect calculations of object's position and tracking parameters. We summarize four methods for evaluating camera calibration accuracy:

1. *Image Coordinates Error Statistics:* This measurement method analyzes the distorted error statistics on the image plane. Steps are: convert world coordinates into camera coordinates, then into undistorted sensor plane coordinates, then into distorted sensor plane coordinates, then into image coordinates. After all these conversions, determine errors between ideal image coordinates and actual locations of data points.
2. *Undistorted Image Plane Error Statistics:* This measurement method analyzes the undistorted error statistics on the image plane. Steps are: convert world coordinates into camera coordinates, then into undistorted sensor plane coordinates; convert from 2D image coordinates into distorted sensor plane coordinates, then into undistorted sensor plane coordinates. After converting, determine the error between ideal and actual location of the data point.
3. *Camera Coordinates Error Statistics:* This measurement method analyzes the error statistics in object space. Steps are: convert world coordinates into camera coordinates; convert from 2D image coordinates into distorted sensor plane coordinates, then into undistorted sensor plane coordinates, then into 3D camera coordinates. After converting, the error is defined by distances of closest points to ideal projection rays in 3D camera coordinates space.

Method 3 is also known as Normalized Calibration Error [5]. We explored four different schemes for improving calibration accuracy. Methods 3 and 4 (in the order below) proved to be efficient and of practical use.

## 2.1 Subset Search

It is common to use all the calibration marks to produce the calibration data file. In practice, some of the image coordinates of projected calibration marks deviate from their true values. Using different numbers of marks will lead to different calibration accuracies. If the “bad marks” (which deviate from their true values more than others do) are removed, then the calibration accuracy will be improved.

The idea of a subset search method can be described as follows: suppose that there are  $n$  calibration marks in the calibration data file. As well known in computer vision,  $n$  should be greater than or equal to 7. Let  $7 \leq k \leq n$ , where  $k$  is an integer. Let  $S$  be a set of  $n$  marks. Then we can search all subsets of  $S$  to determine such a subset which minimizes the calibration error. We estimate how many subsets need to be considered. For a set of size  $n$ , there are

$$m = nk = \frac{n!}{k!(n-k)!}$$

subsets of size  $k$ , where  $0 \leq k \leq n$ . Then we have

$$\lg m = \frac{1}{\ln 10} \left( \sum_{i=1}^n \ln i - \sum_{i=1}^k \ln i - \sum_{i=1}^{n-k} \ln i \right).$$

For example, let  $n=27$ ,  $7 \leq k \leq 27$ . Then the values of  $\lg(\cdot)$  are between 0 and 7.4. The average of these values is 5.58. Therefore there are quite a lot of subsets to be considered. On the other hand, if we can already calculate from the image that some marks are distorted, then it is better to delete these first. The advantage of this method is to improve calibration accuracy by searching and removing bad marks, but its disadvantage are the computational costs.

## 2.2 2D Neighbour Search

We can also consider neighbors of projected marks in the image plane as possible replacements, in order to improve the calibration accuracy.

In mathematical terms, assume that there are  $k$  (e.g.  $k = 27$ ) marks. Let  $r$  be a positive real number,

$$W(x_0, y_0) = \{(x, y) \in Z : |x - x_0| \leq r, |y - y_0| \leq r\}.$$

$$S_i = \{(X_i, Y_i, Z_i, x_i, y_i) : (x_i, y_i) \in W(x_0, y_0)\},$$

where  $Z$  is the set of integers.  $X_i, Y_i, Z_i$  are the world coordinates of the  $(i+1)$ th mark,  $(x_0, y_0)$  are the image coordinates of it, and  $i = 0, 1, \dots, k-1$ .

Like the subset search method, using  $S_i, i = 0, \dots, k-1$ , we can create a number of calibration data files and compute their calibration errors. Then, we compare these calibration errors and choose one calibration data file such that the corresponding calibration error is minimal.

In defining  $W(x_0, y_0)$ , we could also consider non-integer coordinates so as to obtain more accurate result. The disadvantage of this method is again its computational complexity. If we consider all 4-adjacent grid points as possible alternatives, then we have a search space of size  $5^k$ , which already indicates inefficiency of this approach.

## 2.3 Least-Squares Error

Assume that there are  $k = n \times m$  marks, which means there are  $n$  rows and  $m$  columns, where  $n, m \geq 3$ . For every row, there are  $m$  corresponding image coordinates, denoted by  $P_i(x_i, y_i)$ , where  $i = 1, \dots, m$ . From this, we can obtain a least-squares error line, denoted by  $RL_i$ , where  $i = 1, \dots, m$ . Similarly, for every column we can obtain a least-squares error line, denoted by  $CL_j$ , where  $j = 1, \dots, n$ . Then we can compute the intersection point of the line  $RL_i$  with the line  $CL_j$ , where  $i = 1, \dots, m, j = 1, \dots, n$ . Replace the original image coordinates by the corresponding intersection points. Our experiments show that this method is very effective.

## 2.4 Sufficient Image Coordinates

In practice two calibration points can produce  $n+1$  image coordinates by dividing the line between these two points using  $n$  points. Let  $P_i(X_i, Y_i, Z_i)$  be the world coordinates of these two marks, where  $i=1,2$ . The corresponding image coordinates of them are denoted by  $P'_i(x'_i, y'_i)$ , where  $i=1,2$ . Then for every positive integer  $n$ , we can compute  $n+1$  calibration points and their corresponding image coordinates as follows:

$$X_i = X_1 + \frac{X_2 - X_1}{n} i,$$

$$Y_i = Y_1 + \frac{Y_2 - Y_1}{n} i,$$

$$Z_i = Z_1 + \frac{Z_2 - Z_1}{n} i,$$

$$x'_i = x'_1 + \frac{x'_2 - x'_1}{n} i, \text{ and } y'_i = y'_1 + \frac{y'_2 - y'_1}{n} i,$$

where  $i = 0, \dots, n$ . Our experiments show that this method is effective. See Table 1.

### 3 A New Calibration Method

Camera calibration can be subdivided into four steps (capture images, pre-process images, prepare data file, and produce parameters), and we describe the new method along this line.

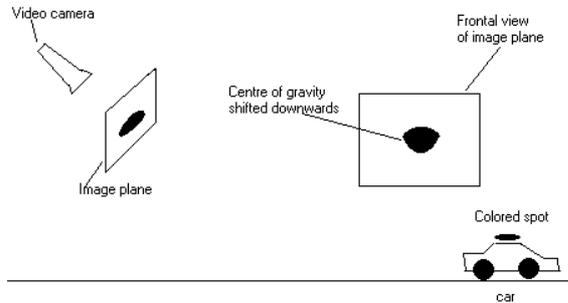


Figure 1: Distortion of a circle's center of gravity under perspective distortion. The center of gravity is shifted downwards as shown in the frontal view of the image plane [7].

We use the right hand system in defining the world coordinates. 3D balls are chosen as calibration marks instead of 2D marks, because 2D marks cause perspective distortion (see Fig. 1). Although it is efficient, but moments of a circle are distorted when projected onto the image plane. 3D marks can avoid this problem: a sphere produces a disk in any viewing direction (see Fig. 2). Assume that Tsai's calibration method is used. The main procedures are detailed as follows:

**Step 1.** Capture images of the calibration object composed of  $k$  spheres in measured positions. For example, we used two planes as the calibration object. We arranged 16 black 3D marks on each plane such that the distance between any two neighboring marks is 114 mm and the distance between the two planes is 11 mm.

**Step 2.** Pre-process the images from step 1 by combining two processes:

1. cut off the background, and
2. reduce the noises and shadows.

We achieved this by applying a suitable threshold to each pixel to reduce the noise. Finally we can get noiseless images as shown in Figures 2.

**Step 3.** Produce a data file which contains the world coordinates of every calibration mark and their corresponding image coordinates. This can be done by combining the following two processes:

1. For every calibration mark, find out its center (image coordinates).

2. Next, number the 16 marks, so as to match world coordinates to image coordinates. We slightly modified 16 marks and then applied it respectively to the four pre-processed images from the previous step.

**Step 4.** Run the software [8] to produce the camera parameters file which is needed, e.g., for object tracking or 3D analysis.

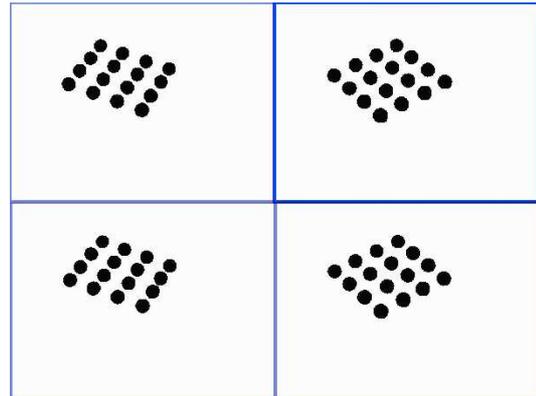


Figure 2: The top row shows the upper plane captured by left and right camera respectively. The bottom row shows the lower plane captured by left and right camera respectively.

### 4 Experimental Results

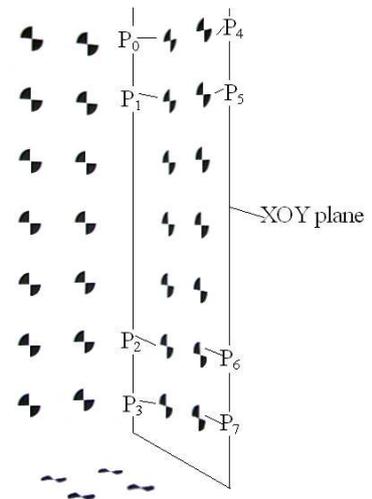


Figure 3: Calibration object with planar markers arranged in three planes.

Our experiments followed the four schemes as described in Sec. 2 for camera calibration using

$n+1$	number of calibration marks	normalized calibration error
8	32	0.426177
16	64	0.411801
32	128	0.405867
64	256	0.402175

Table 1: Relationship between numbers of calibration marks and normalized calibration errors.

evaluation method	mean	stddev	maxErr	sse
distorted error	1.37	0.58	2.30 [pix]	66.880
undistorted error	1.38	0.58	2.31 [pix]	67.263
object space error	2.15	0.89	3.52 [mm]	163.10

Table 2: Evaluation of the camera calibration parameters for the left camera.

evaluation method	mean	stddev	maxErr	sse
distorted error	1.64	0.90	3.91 [pix]	111.72
undistorted error	1.65	0.92	3.97 [pix]	113.87
object space error	2.48	1.33	5.71 [mm]	252.53

Table 3: Evaluation of the camera calibration parameters for the right camera.

3D marks. First we applied the software [8] to the calibration object with planar markers shown in Fig. 3, and the normalized calibration error we obtained is 0.652262. The following experiments showed that the method described in Sec. 2.4 is effective for reducing the normalized calibration error. We choose 8 marks from the  $XOY$  plane and denoted them as  $P_i$ , where  $i = 0, 1, \dots, 7$  (see Fig. 3). We obtained 4 line segments  $P_0P_1$ ,  $P_2P_3$ ,  $P_4P_5$  and  $P_6P_7$ . For every positive integer  $n$ , we can compute  $n+1$  calibration points and their corresponding image coordinates from every line segment.

Table 1 shows the relationship between the number of calibration marks and normalized calibration error.

Table 3 and 4 illustrate evaluations of the camera calibration parameters for left and right cameras.

Now we replaced the calibration object by an object having 3D (spherical) marks. From our experimental results, we found that using 3D marks can improve the calibration accuracy, but it depends on the calibration environment. Although 3D marks will not cause perspective distortion, it may cast shadows under nonuniform lighting situations. This shifts the centers of calibration marks and affects the final calibration accuracy.

We used three lights over calibration marks to reduce shadows, the calibration errors for using 3D marks and 2D marks are 4.2mm and 3.3mm, respectively. Here using 2D marks was better than using 3D marks, because lighting was not uniform, 3D marks were affected by shadows. Then we improved the situation by putting 3D marks under uniform lighting. The calibration error for using 3D marks and 2D marks were 2.7mm and 3.2mm, respectively. The best results we got for using 3D and 2D marks were 2.1mm and 2.9mm, respectively.

We also improved the above results by using the method described in Sec. 2.3, which is the least-squares error method. Accuracies we obtained are: the image coordinates error is less than 0.51 pixel; undistorted image plane error is less than 0.52 pixel; normalized calibration error is less than 0.34.

## 5 Conclusions

Traditional calibration marks are 2D marks. They cause perspective distortion which often affects calibration accuracy. We introduced 3D marks which can completely overcome the perspective distortion problem. But using 3D marks is restricted by environments. It requires uniform lighting, so as to have less shadows. It also requires accurate installation of 3D marks at the right position, because normally 2D marks are arranged by a plotter, and 3D marks are arranged by hands.

From our experimental results it became clear that 3D marks improved the calibration error if we work under expected lighting. Another important advantage of using 3D marks is that it is very useful in tracking of moving objects.

We also discussed four schemes to improve calibration accuracy. Among these schemes, the method of generating sufficient image coordinates and using the least-squares error method have been proved to be efficient. Our experimental results show that by using the least-squares error method, the calibration accuracy can be largely improved.

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