Computer Science Department of The University of Auckland
CITR at Tamaki Campus (http://www.citr.auckland.ac.nz/)

CITR-TR-127 Apr 2003

ACCURATELY MEASURING THE SIZE OF THE PUPIL OF THE EYE

Xiang Lin, Jennifer Craig, Simon Dean, Gisela Klette and Reinhard Klette

Abstract

A new method is developed to accurately analyze the elliptical character of the contour of human's pupil in images captured by a special infrared CCD video camera. The method is an effective combination of edge detection algorithms, labelling method and least square fitting technique, which makes it robust to the eyes of different human races and easy to trace the random movements of the pupil. It even solved the problem of Purkinje phenomenal in images, which was previously only avoided by adjusting the light direction [1]. The parameters of a fitted ellipse, including the length of both axes, the orientation and the center position, are output as result for further analysis.

ACCURATELY MEASURING THE SIZE OF THE PUPIL OF THE EYE

Xiang Lin, Jennifer Craig, Simon Dean, Gisela Klette and Reinhard Klette

CITR Tamaki and Medical School
University of Auckland

ABSTRACT

A new method is developed to accurately analyze the elliptical character of the contour of human's pupil in images captured by a special infrared CCD video camera. The method is an effective combination of edge detection algorithms, labelling method and least square fitting technique, which makes it robust to the eyes of different human races and easy to trace the random movements of the pupil. It even solved the problem of Purkinje phenomenal in images, which was previously only avoided by adjusting the light direction [1]. The parameters of a fitted ellipse, including the length of both axes, the orientation and the center position, are output as result for further analysis.

Keywords: ellipse fitting, Purkinje, pupil size measuring.

1. INTRODUCTION

The size of the human pupil is an important parameter in different studies in Vision and Ophthalmology as well as in other areas such as Psychology, Psychiatry, etc. Especially in excimer laser refractive surgery, accurate measurement of the pupil size of the eye in darkened conditions is an important key to the successful operation performance. A special infra-red pupillometer (the Dean Pupillometer) is developed by Dr. Craig and Dr. Dean at the University of Auckland to overcome the problem. The image seems clear and simple. The dark pupil with obvious size is the only object in the image (Fig. 1).

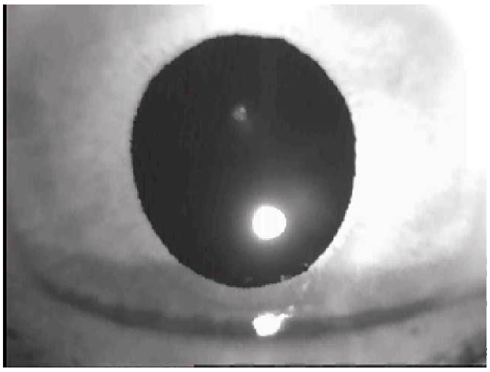

Figure 1: Original image.

Unfortunately, there is still no article on how to accurate measuring of the pupil size. Most pupil-related articles [2,3,4] are interested in the position of the pupil which reflects the orientation of eye-gaze. Some other articles involved in the size of pupils just treat pupil as a circle [5] and develop some simple algorithm [1] to roughly estimate the size.

When we analyze the size and the color of pupil in original images, we found it is not difficult to separate the pupil from background just by thresholding the whole image with suitable threshold (Fig. 2).

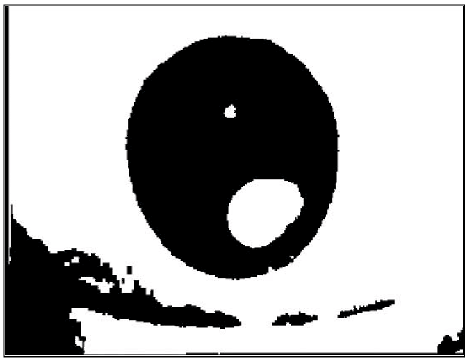

Figure 2: Image after thresholding.

A satisfying result can then be got by doing some simple labelling and filling. (Fig. 3)

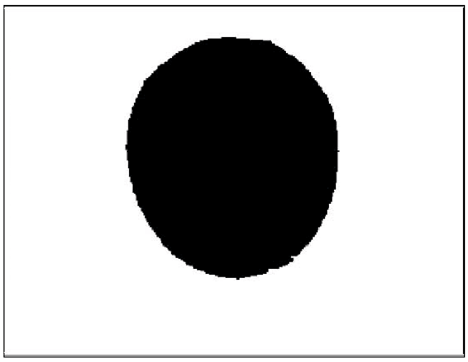

Figure 3: Image after labelling and filling.

The problem is that it is too hard to decide the value of the threshold. A low threshold may cause the difficulty of separating pupil from the around objects, especially when a part of background close to the edge is too dark. But a high threshold is also a problem. It would reduce the size of pupil and even lose a part of contour when the reflection area (Purkinje) is much close to the edge. (Fig. 4) So, the setting of threshold is mostly based on experience.

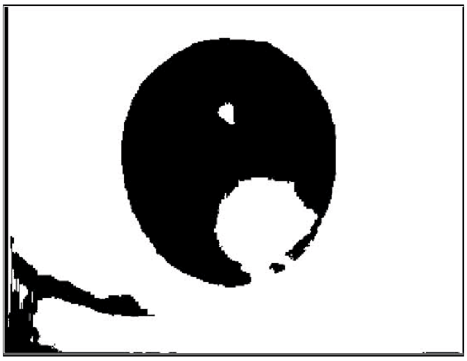

Figure 4: Image applying a high threshold.

But the program needs to be designed to deal with all kinds of eyes, blue, grey, brown or others. So, every time we have to change the threshold value. But there is no exact rule or algorithm which can be followed for the value setting.

In this paper, we employ edge detection algorithms [6] to solve this problem. It is robust to eyes with different colors because it just depends on the color changing in images, not on the color itself. We first use a fast edge detection algorithm (actually a method simple as the Sobel operator proved to be sufficient) to generate "thick edges" from the original image which allow to identify the region of pupil (by labelling). These edges are then reduced by applying a LoG operator to "thinner" contours, which rapidly improves the executing speed of subsequent fitting algorithms.

In general, there are two kinds of methods to get ellipse parameters from scatted points in an image, least square fitting [8] or Hough transform [9]. Hough transform is normally applied in "noisy images", but with the drawback that it is slow especially when the number of Hough parameters is over 3. Under many variants of least square fitting of ellipse we decided for the widely used method of "Direct Least Square Fitting of Ellipses" [8]. To ensure real-time application of the least square fitting method for captured video sequences, we calculate "ideally thin" edges first. The whole process can be subdivided into five steps:

1. Rough edge detection for all edges.
2. Separation of the edge region of pupil from other regions.
3. More accurate edge detection for pupil edge.
4. Edge thinning.
5. Fitting of ellipse for edge.

This article is structured in the order of these five steps.

2. EDGE REGION OF PUPIL

The interesting parts in our images are just the edge areas. The first step is to get all edges from images, which is the preparation to separate the pupil edge from other edges. We attempt that the edge belonging to an individual object region should be connected and also separable from other edges.

For the first step, accuracy is not very important but speed is. The Sobel edge detection method together with a relatively low threshold value proved to be sufficient to meet the requirement (Fig. 5).

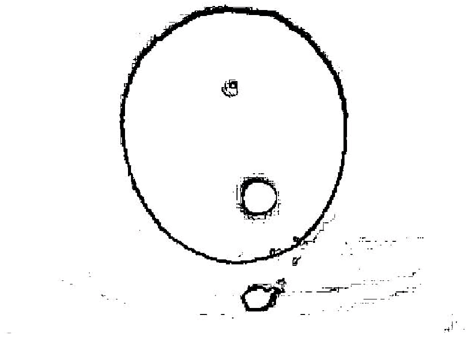

Figure 5: Sobel image (inverted image).

The simplest way to separate the edge of pupil from other edges is to do region labelling in the whole edge image, and the largest region is chosen to be the pupil's edge. Unfortunately this takes too much time, especially for images with many small and separate edges. Our method is to find one *seed point* which is located on the edge of pupil, and then label all those edge points which are connected to the seed point.

We detect a seed point on the edge of pupil by locating one special pixel (D_x, D_y) in the pupil, which allows "to avoid" the reflection areas (Purkinje) in horizontal orientation, and then we search for a seed point on the edge of pupil from (D_x, D_y) in horizontal orientation.

The pupil is the only large-size black elliptical region in the original image (Fig. 6). Compared to the background, the darkness of pupil is obvious. We divide the whole image into 8×8 blocks on experience; the center point of the darkest block must be in the pupil and we take this as (D_x, D_y). This pixel is "away" from Purkinje areas; otherwise, the block in which the point is located cannot be the darkest one.

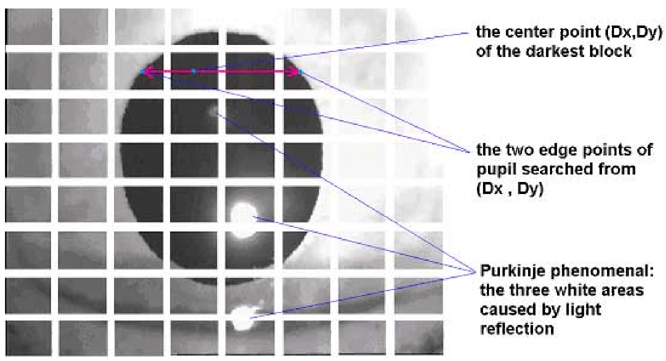

Figure 6: Center of the darkest block.

To get a seed point on the edge of pupil, we analyze the horizontal line on which pixel (D_x, D_y) is located, by analyzing image values (Fig. 7) and approximated first derivatives (Fig. 8). We evaluate the steepness of edges based on these diagrams, and choose the seed point in the middle of the steeper edge. [Note that there may be only one “column” in Fig. 8 if the pupil is close to the boundary of image.]

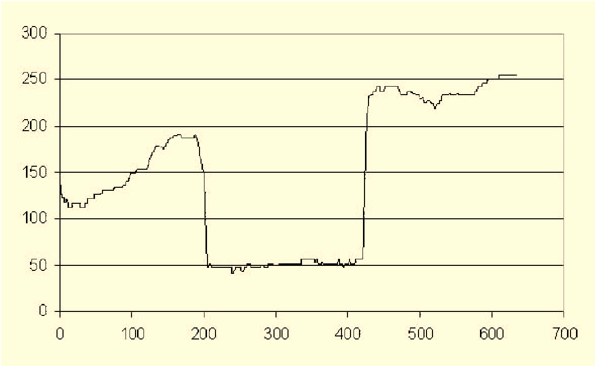

Figure 7: Curve of grey values.

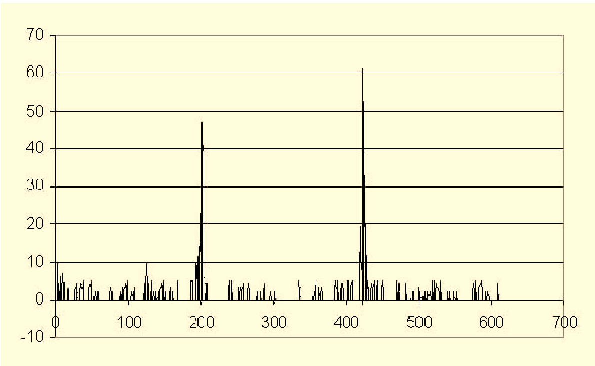

Figure 8: Curve of absolute values of first order derivatives.

The seed point allows finally a recursive fill of all the connected edge points (Fig. 9).

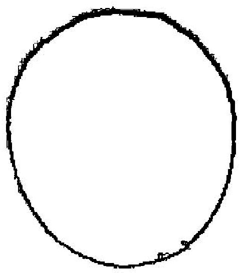

Figure 9: Rough edge region of pupil (inverted image).

3. ACCURATE EDGE DETECTION (PUPIL CONTOUR)

The contour of pupil must be in the detected edge region. The well-known LoG operator combines a Gaussian filter and the Laplacian. The Laplacian allows a calculation of edges by estimating zero-crossings of second derivatives. We just calculate LoG-values (and zero-crossings) for the points in the detected edge region. The detected zero-crossings define a subregion of the previous edge region.

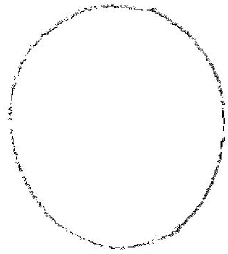

Figure 10: Edge pixels detected by LoG (inverted image).

Our subsequent algorithm of direct ellipse fitting requires that points are evenly distributed on simple digital curve. The more even the distribution is, the better the result will be. Here we employ a simple thinning algorithm to transform zero-crossings into such a simple curve.

First, the image is scanned line by line, and for every run of zero-crossings we have a leftmost and a rightmost point. We take the middle point of both, and these middle points form the final thinn edge (Fig. 11). For balanced accuracy on the whole circle, we scan it in two orientations, horizontal and vertical (i.e. dividing the “circle” into four segments, and dealing with each segment separately).

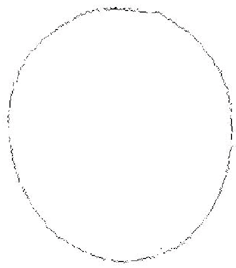

Figure 11: Edge after thinning (inverted image).

4. DIRECT ELLIPSE FITTING

Direct ellipse fitting is a robust, efficient, and easy method for fitting ellipse to scattered data. “Direct” here means that all involved points are directly employed in calculating the Least-square error, which is to find a set of parameters of ellipse that minimize some distance measure between the data points and the ellipse. Direct ellipse fitting method incorporates the elliptical constraint into the normalization factor so that it can be solved naturally by a generalized eigensystem [8].

Direct ellipse fitting consists of two parts, first a calculation of parameters of ellipse in general quadratic form with a generalized eigensystem, then a translation of the general quadratic form into centered and oriented form.

Here we use a general conic to express the general quadratic form of ellipse because ellipse can be treated as a cutting section of a conic. So, the distance of a point (x,y) to the ellipse can be expressed as

$$F(\alpha, \chi) = \alpha \cdot \chi = a\mathrm{x}^2 + b\mathrm{xy} + c\mathrm{y}^2 + d\mathrm{x} + e\mathrm{y} + f \quad (1)$$

where $\alpha = [a\ b\ c\ d\ e\ f]^T$ and $\chi = [\mathrm{x}^2\ \mathrm{xy}\ \mathrm{y}^2\ \mathrm{x}\ \mathrm{y}\ 1]^T$.

The least square error is

$$\mathcal{D}(\alpha) = \sum_{i=1}^{n} F(\alpha, \chi)^2 \quad (2)$$

for all points. To minimize it, we use a rank-deficient generalized eigenvalue system

$$\mathrm{D}^\mathrm{T}\mathrm{D}\alpha = \lambda \mathrm{C}\alpha\ , \quad (3)$$

where $\mathrm{D} = [\chi_1\ \chi_2\ \chi_3\ \cdots\ \chi_n]^T$, and a constraint matrix C which keeps $b^2 - 4ac < 0$

$$\mathrm{C} = \begin{bmatrix} 0 & 0 & 2 & \cdots \\ 0 & -1 & 0 & \cdots \\ 2 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The whole process consists of the following steps:

1. Build matrix D with all scattered points
2. Create matrix S by $\mathrm{D}^\mathrm{T}\mathrm{D}$
3. Build matrix C
4. Solve the generalized eigensystem
5. Find the only negative eigenvalue
6. Get the corresponding eigenvector as fitted parameters

Now, we have the parameters of an ellipse in general quadratic form. The next step is to transform it into the centered and oriented form [7]. Here a small modification is made for the general quadratic form:

$$a_{11}\mathrm{x}^2 + 2a_{12}\mathrm{xy} + a_{22}\mathrm{y}^2 + b_1\mathrm{x} + b_2\mathrm{y} + c = 0 \quad (4)$$

If we assume that

$$\vec{Y} = \begin{bmatrix} x \\ y \end{bmatrix};\ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix};\ \vec{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

and then assume

$$\vec{K} = \frac{1}{2(a_{12}^2 - a_{11}a_{22})} \begin{bmatrix} a_{22}b_1 - a_{12}b_2 \\ a_{11}b_2 - a_{12}b_1 \end{bmatrix},$$

we obtain

$$M = \frac{A}{\vec{B}^T A^{-1} \vec{B}/4 - c}.$$

the form in equation (4) can be written in centered and oriented form

$$(\vec{Y} - \vec{K})^T M (\vec{Y} - \vec{K}) = 1 \quad (5)$$

which can be eventually translated into the normal centered and oriented form

$$(\vec{Y} - \vec{K})^T R L R^T (\vec{Y} - \vec{K}) = 1 \quad (6)$$

where

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix};\ L = \begin{bmatrix} 1/\mathrm{a}^2 & 0 \\ 0 & 1/\mathrm{b}^2 \end{bmatrix}.$$

Parameter a is the semi-major axis of the ellipse, and b is the semi-minor axis. θ is the orientation of the major axis. A final result is shown in Fig. 12.

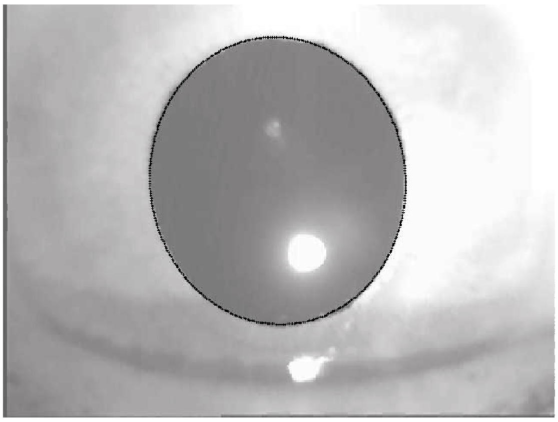

Figure 12: Ellipse fitted to the edge.

5. REQUIRED SUBPROCESSES

The above method is based on the prerequisites (i) that three reflection areas (Purkinje) mentioned before are not so close to the edge of pupil that they are separated Sobel edge regions, and (ii) that scattered point data form "circles".

5.1. Overlapping Circles

For explanation purposes, we fake an image by moving a reflection area in the original image on the edge of pupil (Fig. 13).

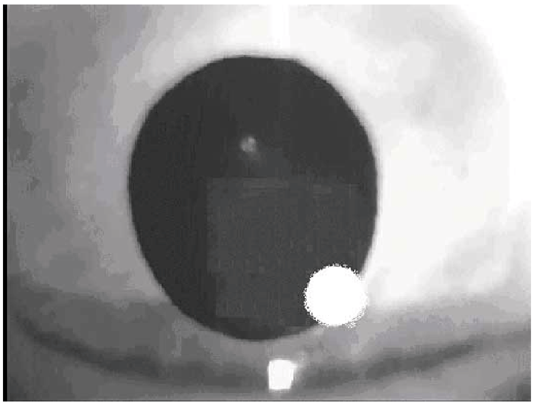

Figure 13: An image with a reflection area on the edge.

Figure 14 shows the Sobel edge region. Obviously, the big "circle" representing the edge of pupil cannot be separated from the small circle of the reflection area following the steps described above. The resulting thinn edge introduces many incorrect edge points into the ellipse fitting step and eventually affects the result. The bigger the reflection area, the worse the result would be.

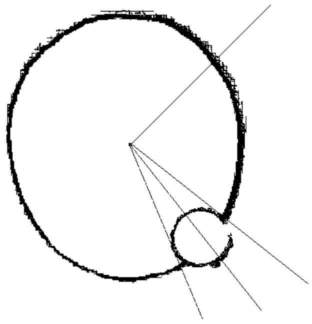

Figure 14: Sobel image for assumed case (inverted image).

To reduce the effect caused by such special cases, a good solution is to erase a small circle from the Sobel edge image. The result will be improved because the fitting accuracy is better for missing points instead of additional incorrect points. For erasing we apply four steps:

1. Get the centroid of the whole region
2. Radiate rays from the centroid and intersect with the region
3. If there is more than one run of intersection points, the ray must intersect with a small circle. All such intersection points are deleted.
4. Otherwise, the points are kept as edge points.

5.2. Scattered Points

In generalization of the previous method, we discuss the case as typically occurring in our images: scattered points do not form "circles", there can be several "overlaps" with different edge fragments. The general procedure is as follows:

1. Calculate the centroid for the Sobel (or zero-crossing) edge region.
2. Calculate the relative orientation of edge points to this centroid, for every point and save it as an integer value $(0, \ldots, 359)$.
3. Calculate the distance between the points and the centroid for every point
4. For each degree $0, \ldots, 359$, check if there are any two points with the same degree whose distance is bigger than a predefined threshold. If it is, delete all points with this degree from the image. The predefined threshold must be bigger than the "thickness" of the thickest part of the "big circle".

A result is shown in Fig. 15.

For comparing difference between ellipses with and without data cleaning, see Fig. 16 for an image with reflection area, and Fig. 17 after cutting the reflection area from the Sobel image.

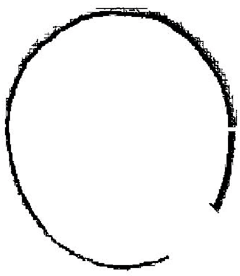

Figure 15: SOBEL of the image with no small circle (inverted image).

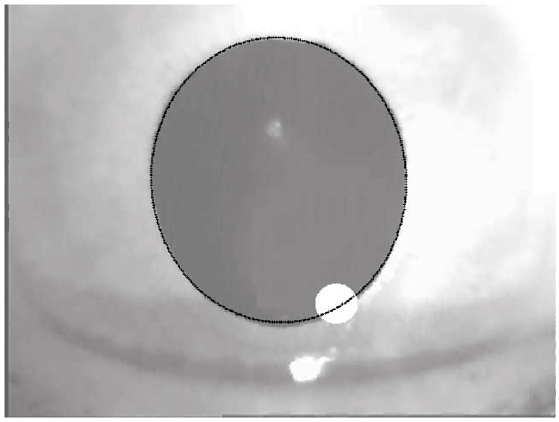

Figure 16: Ellipse from an image without cutting the circle.

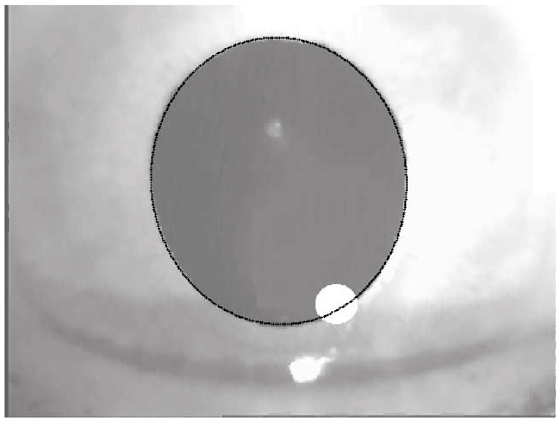

Figure 17: Ellipse from the same image after cutting the circle.

6. CONCLUSIONS

A video clip (avi file in mpeg4 format) is typically about 6-seconds. The task is to measure the minimum, maximum and mean values of pupil size which are required to be present as the radii on both axes of fitted ellipses. Figure 18 illustrates calculated axes, and Table 6 shows values in pixels (for one sequence).

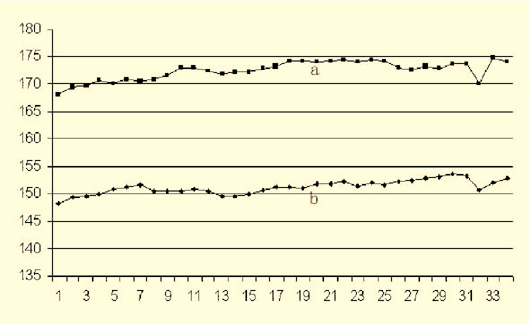

Figure 18: Results of radii in 36 images of a video clip.

Table 1: minimum, maximum and mean values

	value a	value b	average
average	172.3	151.1	161.7
maximum	174.7	153.6	164.2
minimum	167.2	148.2	157.7

The developed method is able to measure the elliptical parameters of human pupils in real time (6 images per second) by processing a sequence of images captured from 5 or 6 second video clips. The accuracy of results is at subpixel level, which is an improvement compared to other current approaches.

The robustness of the method is also verified by a group of video clips taken from persons with different colors of eyes. The method does not imply any limitations for the color of eye, the size range of pupil and the condition of illumination. The Purkinje phenomenal is also considered and attention has been paid to some special cases which will affect the size of pupils. The only requirement is that the depicted pupils are not crossing the image boundaries. For such cases we need an algorithm for ellipse fitting which is able to cope with situations where only a segment of the ellipse is provided. Nevertheless, this is not a serious limitation in practice because such crossings can be avoided.

Parameter calculations in this paper are still in pixel units. These need to be translated into standard length units for normal use. Calibration is required.

7. REFERENCES

[1] I. Miro, N. Lopez-Gil, P. Artal. “Pupil-meter and tracking system based in a fast image processing algorithm”. Proc. SPIE, 3591, pp.63-70 (1999).

[2] N. M. Taylor, R. H. Eikelboom, P. P. v. Sarloos, P. G. Reid. “Determining the Accuracy of an Eye Tracking System for Laser Refractive Surgery”. Jo of Refractive Surgery, Vol. 16, pp.643-646 (2000).

[3] Y. Ebisawa. Unconstrained pupil detection technique using two light sources and the image difference method. Visualization and Intelligent Design in Engineering and Architecture, pp.79-89, 1995.

[4] C. Lankford. ”Effective eye-gaze input into windows”. Proc. in Eye Tracking Research and Applications Symposium 2000, pp.23-27 (2000).

[5] B. Chou, B. Wachler. ”The Role of Pupil Size in Refractive Surgery.” Agarwal A, Agarwal S. Textbook of Ophthalmology. 4 eds. Vol.1. India: Jaypee Brothers, (2001).

[6] R. Klette, K. Schlüns, A. Koschan. *Computer vision : three-dimensional data from images*. Springer, Singapore, 1998.

[7] D. Eberly. “Information About Ellipses”. http://www.magic-software.com/Documentation/InformationAboutEllipses.pdf. (Accessed: 9 Dec. 2002).

[8] Andrew Fitzgibbon, Maurizio Pilu, and Robert B. Fisher. ”Direct Least Square Fitting of Ellipses”. Pattern Analysis and Machine Intelligence, VOL. 21, NO. 5, (1999).

[9] S. Inverso. ”Ellipse Detection Using Randomized Hough Transform”. http://www.cs.rit.edu/ sai6189/vision/final/ EllipseDetection.pdf (Accessed: 11 Dec. 2002).