Dependence Modeling with Copulas. By H. Joe. Boca Raton, Florida: CRC Press. 2015. 480 pages. £57.99 (hardback). ISBN 978-1-4665-8322-1.

Over the last decade, rapid developments have been made in the statistical theory and practice of copulas, as the modeling of dependence structures in high dimensions has become increasingly important and more flexible alternatives to the classical multivariate normal distributions have been sought. Extensive applications of copula-based statistical models can be found for instance in quantitative finance, survival analysis in biostatistics, ecology, reliability analysis in engineering, and meteorology. To devise a book that gives a systematic and complete overview of all the latest developments seems an extremely ambitious goal but that is exactly what has been accomplished with this exhaustive monograph. And it fills a major gap since recent books by Nelsen (2006), Balakrishnan & Lai (2009), Kurowicka & Joe (2011), and Mai & Scherer (2012) have a much narrower focus on mathematical properties, bivariate copulas, vine copulas, and simulation of copulas, respectively.

The book's main contributions and focal points are: a systematic, encyclopedic overview of parametric copula families, their dependence properties and tail behaviours, general methods for constructing copulas in high dimensions, statistical inference including model selection and diagnostics, algorithms for numerical optimization and simulation of copulas and last not least applications and examples of dependence modeling with copulas for longitudinal data, time series, ordinal item response variables, multivariate financial returns and extreme values.

Even though the coverage of research topics in this book is fairly comprehensive, the focus is on likelihood-based statistical inference. Parametric Bayesian approaches are only briefly referenced and brushed aside with the remark that the posterior distribution is asymptotically equivalent to the distribution of maximum likelihood estimator for large sample sizes. Arguably, this is a very narrow view of the relevance of Bayesian inference. Recent nonparametric Bayesian approaches to copula estimation as for instance by Guillotte & Perron (2012) and Burda & Prohorov (2014) are not mentioned at all.

Chapter 1 starts with an introduction to terminology, a historical review of dependence modeling using non-Gaussian distributions, the main construction principles for copulas, Sklar's theorem, and illustrations of dependence modeling, likelihood analysis and model comparison using data examples. Chapter 2 examines properties of multivariate distributions such as tail dependence, tail heaviness, tail asymmetry, multivariate dependence concepts and measures of bivariate association and asymmetry. General construction methods for copulas in high dimensions that build on the (bivariate) parametric families are presented in Chapter 3, whereas Chapter 4 then gives a systematic review of parametric copula families and their properties. Those chapters provide an invaluable guide to model selection for data analysts. Parametric and nonparametric likelihood-based inference is the focus of Chapter 5, while Chapter 6 describes algorithms for likelihood maximization, numerical quadrature, interpolation, and simulation. These algorithms are conveniently presented as pseudo-code. Chapter 7

is devoted to data analysis examples using copula-based models which will certainly be appreciated by any applied statistician. Chapter 8 lists and proves important theorems and properties of copulas.

This monograph is an essential compendium for any researcher working with copulas, and I am sure that it will become the primary reference for anything "copula". It is mathematically rigorous with consistent notation and attention to detail in every respect, but dense and terse like mathematical texts. It is not designed to be a textbook for teaching purposes but is geared towards statisticians with a solid background in mathematical and statistical theory. Anybody looking for an easy and lecture-like introduction to copulas that explains concepts using didactically enhanced illustrations and examples would be ill-advised with this book. As the author states, it is not meant to be read and studied in a linear fashion. But in each chapter, researchers will find a comprehensive and precise description of a specific research topic with many invaluable references to relevant recent publications. Although not a pocket reference, it is a "handbook" in its very sense, providing an all-embracing treatise and reference work on copulas. A must-have on the bookshelf of any statistician interested in multivariate modeling!

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