Detailed Theoretical Models of Digital Communication Systems with Imperfect Sequence Synchronization

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ABSTRACT

Direct Sequence Spread-Spectrum Systems (DS-SSS) and Code Division Multiple Access (CDMA) Systems have a wide range of applications in wireless communication systems. They are well investigated under assumption that the spreading sequences are perfectly synchronized. However there are not many references presenting their operations under assumption the spreading sequences are not perfectly synchronized, which can have significant consequences on the probability of error properties of these systems. This Report describes design of these systems under assumption that there is not perfect sequence synchronization. The analysis and results are presented for binary and non-binary sequences like chaotic and random sequences.

It was found that the signals in the systems can be represented and precisely mathematically described in discrete time domain. However, in that case, a random delay between imperfectly synchronized sequences needs to be expressed in discrete form, which opened a new problem of deriving discrete probability density functions as necessity for the statistical characterization of this delay between the received and locally generated (reference) sequence. Furthermore, the delay has a finite value, i.e., it is limited due to the nature of synchronization, being at most equal to the duration of a chip. Therefore, the derived density functions need to be defined and derived in the form of truncated discrete density functions. Also, the problem of choosing these densities is to be separately solved. Namely, the density functions need to follow real variations of the delay in these kinds of systems.

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1. INTRODUCTION

This Report presents theoretical analysis of binary and non-binary CDMA systems with interleaver and deinterleaver blocks used to mitigate effects of fading in Rayleigh fading channel. Most of the work related to direct sequence spread-spectrum (DS-SSS) and code-division multiple access systems (CDMA) was dedicated to the analysis of the system operations of binary and chaos-based systems [1]-[12] and issues related to the sequence synchronization problems [13]-[22]. Most of the work presented in these references was based on the assumption that the communication channel in CDMA system was a flat fading channel and there is a perfect synchronization between the received and locally generated reference sequence [13]-[22].

Majority of published papers are targeted to the analysis of chaos-based base-band and asynchronous systems [23, 24]. A baseband DS-CDMA system with a wide-band channel was analyzed in [25, 26], and systems with wide-band channel estimators were presented in [27, 29]. There are two papers published lately related to chaos communications [30]- [32]. In the first paper [30] a base-band system in the presence of AWGN is analyzed and then extended to a pass-band system. In the second paper [31] a multicarrier system with WB channel, characterized by delays and fading coefficients, is analyzed. In [32] detailed derivative of the probability of error are presented both for binary and non-binary spreading sequences used in CDMA systems with a wide-band channel. It was confirmed that the probability of error can be improved when the number of multipath channel increases. Chip interleaving techniques are efficiently used to mitigate fading in multi-user and CDMA systems operating in flat fading channels [33] – [35].

In this Report the mathematical model of the system is made by presenting all signals in discrete time domain and using the theory of discrete time stochastic processes. In order to investigate the influence of delay between the sequences, due to imperfect synchronization, each chip is interpolated by S samples. This interpolation is also done for the sake of future analysis that will include modulation and demodulation of the carrier. In that case, for discrete time signal processing, the chip samples will be multiplied with the corresponding samples of the carrier [36]-[38]. The number of samples will depend on the number of carrier oscillations that is required in a chip interval. It is important to note that the applied interpolation does not increase the bandwidth requirements of the analyzed system.

The system is analyzed for a general case and then formal expressions for the probability of bit error (*Pe*) are derived for both binary and chaotic sequences. Based on the *Pe* expressions derived, assuming presence of both interleaver and deinterleaver blocks in the system, the analytical analysis of theoretical expressions was conducted. It was proved that the already known expressions for the probability of error in systems with perfect synchronization are special cases of the general formulas which are derived in this paper. This analysis is partially related to the results published in a number of references [39-42]. Also the procedure of the design of the systems and generation of chaotic sequences, published in [43-48] can be applied for the system with imperfect synchronization as presented in this Report.

The paper consists of six Chapters. Chapter 2 presents the CDMA system structure and its basic operation including interleaver and deinterleaver blocks and the delay inside the synchronization block. In the case of a single user signal transmission the analyzed system is a Direct Sequence Spread Spectrum System (DS-SSS). The mathematical model of the system with Rayleigh fading is presented in Chapter 3 under assumption of imperfect synchronization between sequences. The channel with Additive White Gaussian Noise (AWGN) and related calculations of the probability of bit error are presented in Chapter 4. Analytical analysis of the system with interleavers and imperfect sequence synchronization is presented in Chapter 5. Basic conclusions are presented in Chapter 6.

2. SYSTEM OPERATION

A multiuser system structure is presented in Fig. 1. If only one branch in the transmitter is used the system becomes DSSS system.

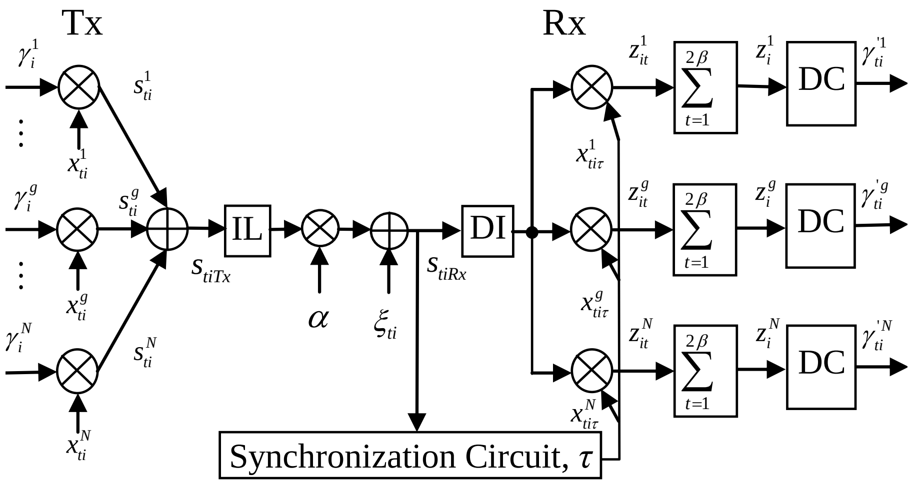


Figure 1 Block diagram of a digital communication system

The i-th message bit $\gamma_i^{(g)}$ of the g-th user is encoded (spread) with a unique spreading sequence $x_t^{(g)}$ (where g = 1, 2, …, N, i = 1, 2, 3, … is the order of the message bits in a message sequence and t = 1, 2, 3, …, S is the chips order in the i-th message bit). The uniqueness of the spreading sequence is specified by its orthogonal nature or by its initial value if a non-binary sequence is used. The i-th bits of all users are added to each other resulting in the transmitter signal expressed as

$$s_{tiTx} = \sum_{g=1}^{N} s_{ti}^{g} = \sum_{g=1}^{N} \gamma_i^{g} x_{ti}^{g} , \tag{1}$$

where the g-th user sequence is expressed as

$$s_{ti}^{g} = \gamma_i^{g} x_{ti}^{g} = \begin{Bmatrix} -1 \cdot x_{ti}^{g} & \gamma_i^{g} = -1 \\ +1 \cdot x_{ti}^{g} & \gamma_i^{g} = +1 \end{Bmatrix} . \tag{2}$$

One way of extracting message signal in the receiver is to produce exactly the same reference sequence as the spreading sequence of the transmitter and then take out a single-user message bits using a correlator, as shown in Fig. 1. If a different initial value is used in the receiver to generate a particular spreading sequence, then the message signal, due to very low cross-correlation, is not going to be successfully extracted. The design of a spreading sequence generator with, we say, “good correlation properties”, is essential in the designing a single use or a multiple user communication system.

In the system shown in Fig. 1 the transmitted chip is affected by flat fading, represented by Rayleigh coefficient α, and the additive white Gaussian noise (AWGN). Therefore, the received t-th chip of the i-th bit for the g-th user can be expressed as

$$s_{tiRx} = \alpha s_{tiTx} + \xi_{ti} = \alpha \sum_{g=1}^{N} \gamma_i^g x_{ti}^g + \xi_{ti} \,. \tag{3}$$

Each chip has a finite duration T_c represented in discrete time domain by S interpolation samples. The received sequence is correlated with the local (reference) sequence. For the system's proper operation the two sequences are supposed to be perfectly synchronized. However, that is not the case in real systems and the delay between them exists. We will assume that the delay is random and is associated to the local sequence, which does not change the generality of explanation. On the other hand, in practice, the local sequence is shifted in respect to the received sequence, as shown in Fig. 2, until the synchronization is achieved. Therefore, the received chip can be expressed in this form

$$z_t = \sum_{s=1}^{S} (\alpha s_{ti}^{Tx} + \xi_{ti}) x_{ti\tau}^g = \sum_{s=1}^{S} (\alpha \sum_{g=1}^{N} \gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti\tau}^g \,, \tag{4}$$

where S identical chip samples are multiplied with the delayed chip samples of the local delayed sequence and added to each other.

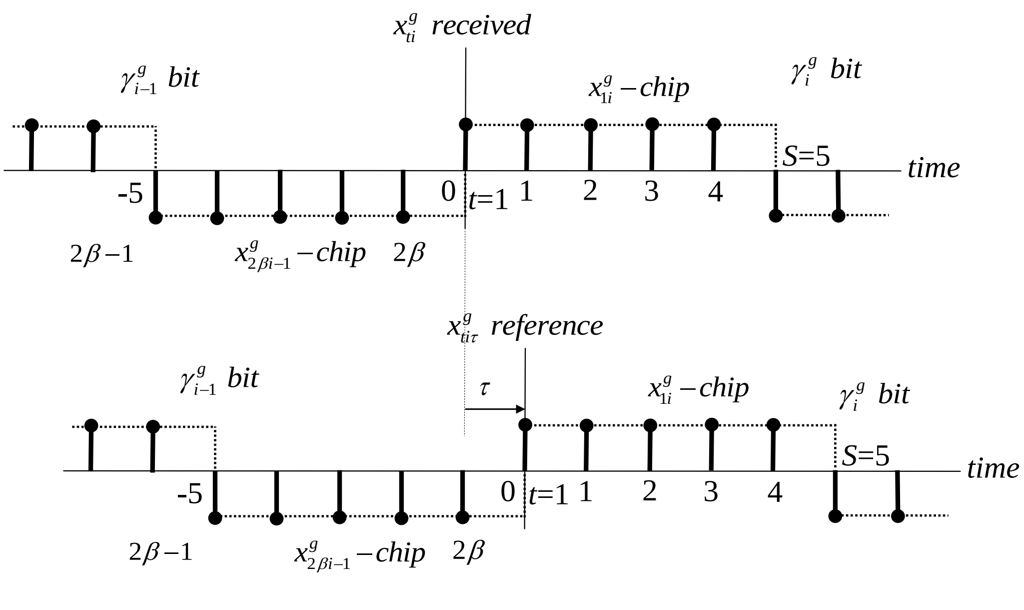


Figure 2 Delayed reference sequence of the receiver and the received sequence

Due to the delay between the sequences, (S - $|\tau|$) samples of the delayed chip will be aligned to the received chip samples, i.e., $x_{ti\tau}^g = x_{ti}^g$, and $|\tau|$ samples will be beyond the interval of the received chip, i.e., $x_{ti\tau}^g \neq x_{ti}^g$. Therefore, inside the correlation receiver, the received chip value can be expressed as

$$z_t = \sum_{s=|\tau|+1}^{S} (\sum_{g=1}^{N} \alpha\gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti}^g + \sum_{s=1}^{|\tau|} (\sum_{g=1}^{N} \alpha\gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti\tau}^g . \quad (5)$$

If the g-th user receives the signal, the t-th received chip of that user can be expressed in this form

$$z_{tg} = \sum_{s=|\tau|+1}^{S} \left(\alpha\gamma_i^g x_{ti}^{g2} + \sum_{n=1, n\neq g}^{N} \alpha\gamma_i^n x_{ti}^n x_{ti}^g + \xi_{ti} x_{ti}^g \right) + \sum_{s=1}^{|\tau|} \left(\alpha\gamma_i^g x_{ti}^g x_{ti\tau}^g + \sum_{n=1, n\neq g}^{N} \alpha\gamma_i^n x_{ti}^n x_{ti\tau}^g + \xi_{ti} x_{ti\tau}^g \right). \quad (6)$$

The first sum contains the processed samples that are perfectly synchronized with the received chip. The first addend in this sum is signal part, the second term is inter-user interference and the third one is the noise part. The second sum contains the samples that are not synchronized producing inter-chip interference, which has tree addends. The first addend is inter-chip interference related to the g-th user chip. The second term is interference from other users and the third term is the noise related to the delayed samples. Because the interpolation samples inside any chip are identical, the chip value can be expressed as

$$\begin{aligned} z_{tg} &= (S-|\tau|)\alpha\gamma_i^g x_{ti}^{g2} + |\tau|\alpha\gamma_i^g x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|) \sum_{n=1, n\neq g}^{N} \alpha\gamma_i^n x_{ti}^n x_{ti}^g + |\tau| \sum_{n=1, n\neq g}^{N} \alpha\gamma_i^n x_{ti}^n x_{ti\tau}^g \\ &+ (S-|\tau|)\xi_{ti} x_{ti}^g + |\tau|\xi_{ti} x_{ti\tau}^g \end{aligned} \quad (7)$$

The final output of the correlator is a random function that represents the i-th bit decision variable expressed as

$$\begin{aligned} z_i^g &= (S-|\tau|)\cdot\gamma_i^g \sum_{t=1}^{2\beta} \alpha x_{ti}^{g2} + |\tau|\cdot\gamma_i^g \sum_{t=1}^{2\beta} \alpha x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot \sum_{t=1}^{2\beta} \sum_{n=1, n\neq g}^{N} \alpha\gamma_i^n x_{ti}^n x_{ti}^g + |\tau|\cdot \sum_{t=1}^{2\beta} \sum_{n=1, n\neq g}^{N} \alpha\gamma_i^n x_{ti}^n x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot \sum_{t=1}^{2\beta} \xi_{ti} x_{ti}^g + |\tau|\cdot \sum_{t=1}^{2\beta} \xi_{ti} x_{ti\tau}^g \\ &= A + B + C + D + E + F \end{aligned} \quad (8)$$

Supposing that the i-th bit transmitted is +1 and the intrleaver/deinterleaver blocks are used, the decision variable can be expressed in this general form

$$
\begin{aligned}
z_i^g &= (S-|\tau|)\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g\,2} + |\tau|\sum_{t=1}^{2\beta}\alpha_t x_{ti}^g x_{ti\tau}^g \\
&+ (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\alpha_t\gamma_i^n x_{ti}^n x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\alpha_t\gamma_i^n x_{ti}^n x_{ti\tau}^g \\
&+ (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti\tau}^g \\
&= A+B+C+D+E+F
\end{aligned}, \qquad (9)
$$

as will be proved in Section 5.

3. PROBABILITY OF ERROR DERIVATION FOR NOISY FADING CHANNEL

Following explanation presented in Section 4, the decision variable (8) may be expressed as a sum of six random variables. The mean and variance of these random variables, conditioned on fading and delay variables, α and τ respectively, can be found as follows. The means are

$$E\{A|\alpha\tau\} = E\left\{(S-|\tau|)\alpha\sum_{t=1}^{2\beta} x_{ti}^{g2}\right\} = \left\{(S-|\tau|)\alpha\sum_{t=1}^{2\beta} x_{ti}^{g2}\right\}\sum_{t=1}^{2\beta} E\left\{x_{ti}^{g2}\right\} = (S-|\tau|)\alpha 2\beta P_c = \eta_{Zi} \quad (10)$$

$$E\{B|\alpha\tau\} = E\left\{|\tau|\alpha\sum_{t=1}^{2\beta} x_{ti}^{g} x_{ti\tau}^{g}\right\} \underset{iid}{=} |\tau|\alpha\sum_{t=1}^{2\beta} E\left\{x_{ti}^{g} x_{ti\tau}^{g}\right\} = 0 \quad (11)$$

$$E\{C|\alpha\tau\} = E\{(S-|\tau|)\alpha\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{2\beta} \gamma_i^n x_{ti}^n x_{ti}^g\} = (S-|\tau|)\alpha\sum_{t=1}^{2\beta}\sum_{\substack{n=1\\ n\neq g}}^{2\beta} E\{\gamma_i^n x_{ti}^n x_{ti}^g\} = 0 \quad (12)$$

and, finally, due to the fact the mean of the noise is zero, we may have

$$E\{D|\alpha\tau\} = E\{E|\alpha\tau\} = E\{F|\alpha\tau\} = 0\,. \quad (13)$$

The variances of A conditioned on α and τ is

$$\begin{aligned} \sigma^2(A|\alpha\tau) &= E\left\{\left[(S-|\tau|)\alpha\sum_{t=1}^{2\beta} x_{ti}^{g2}\right]^2\right\} - E^2(A|\alpha\tau) \\ &= (S-|\tau|)^2\alpha^2\left\{2\beta E\left\{x_{ti}^{g4}\right\} + 2\beta(2\beta-1)P_c^2\right\} - (S-|\tau|)^2\alpha^2 4\beta^2 P_c^2 \\ &= (S-|\tau|)^2\alpha^2 2\beta(P_{x4} - P_c^2) = (S-|\tau|)^2\alpha^2 2\beta P_c^2(\psi-1) \end{aligned} \quad (14)$$

where $\psi = P_{x4}/P_c^2$ is a spreading sequence factor that depends on the statistical properties of a particular spreading sequence. The other variances are as follows

$$\sigma^2(B|\alpha\tau) = E\left\{\left[|\tau|\alpha\sum_{t=1}^{2\beta} x_{ti}^{g} x_{ti\tau}^{g}\right]^2\right\} = |\tau|^2\,\alpha^2 E\left\{\sum_{t=1}^{2\beta} x_{ti}^{g2} x_{ti\tau}^{g2}\right\} \underset{iid}{=} |\tau|^2\,\alpha^2 2\beta P_c^2\,, \quad (15)$$

$$\sigma^2(C|\alpha\tau) = E\{[(S-|\tau|)\alpha\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{2\beta} \gamma_i^n x_{ti}^n x_{ti}^g]^2\} \underset{iid}{=} (S-|\tau|)^2\alpha^2 2\beta(N-1)P_c^2\,, \quad (16)$$

and, in analogous way, we may have

$$\sigma^2(D|\alpha\tau) = |\tau|^2\,\alpha^2 2\beta(N-1)P_c^2\,, \quad (17)$$

$$\sigma^2(E \mid \alpha\tau) = (S - |\tau|)^2 2\beta\sigma^2 P_c \text{ , and} \tag{18}$$

$$\sigma^2(F \mid \alpha\tau) = |\tau|^2 \, 2\beta\sigma^2 P_c \text{ .} \tag{19}$$

The probability of error, conditioned on the fading factor and delay, can be calculated as

$$\begin{aligned} P_e(\tau) &= \frac{1}{2} erfc \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2} \right)^{-1/2} = \frac{1}{2} erfc \left(\frac{2}{\eta_{Zi}^2} \sum_{x=A}^{F} \sigma^2(x \mid \alpha\tau) \right)^{-1/2} \\ &= \frac{1}{2} erfc \left(\frac{(\psi-1)}{\beta} + \frac{1}{\beta} X + \frac{N-1}{\beta}(1+X) + (1+X) \frac{1}{\alpha^2} \left(\frac{E_b}{N_0} \right)^{-1} \right)^{-1/2}, \end{aligned} \tag{20}$$

where X depends on the delay of the chip and the fraction number of interpolated samples as follows

$$X = \frac{|\tau|^2}{(S - |\tau|)^2} = \frac{|\tau / S|^2}{(1 - |\tau| / S)^2} \text{ .} \tag{21}$$

These general expressions can be reduced to already known results obtained for the systems with perfect synchronization.

SPECIAL CASE 1: Suppose the delay is eliminated. Then, we have $X = 0$ and the expression for the probability of error becomes

$$Pe = \frac{1}{2} erfc \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2} \right)^{-1/2} = \frac{1}{2} erfc \left(\frac{(\psi-1)}{\beta} + \frac{N-1}{\beta} + \frac{1}{\alpha^2} \left(\frac{E_b}{N_0} \right)^{-1} \right)^{-1/2} , \tag{22}$$

which is equivalent to the already known expression for the probability of error in the presence of flat fading in the channel.

SPECIAL CASE 2: For CASE 1, suppose the fading is additionally eliminated. Therefore, $\alpha = 1$ and the expression for the probability of error becomes

$$Pe = \frac{1}{2} erfc \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2} \right)^{-1/2} = \frac{1}{2} erfc \left(\frac{(\psi-1)}{\beta} + \frac{N-1}{\beta} + \left(\frac{E_b}{N_0} \right)^{-1} \right)^{-1/2} , \tag{23}$$

which is equivalent to the already known expression for the probability of error in the presence of flat fading in the channel. In Fig. 3 the graphs of this probability, for the spreading factor ψ as parameter, are presented.

SPECIAL CASE 3: Suppose the system analyzed is a single user DSSS system. Then, $\psi = 1$ and $N = 1$ and the expression for the probability of error is

$$Pe = \frac{1}{2} erfc \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2} \right)^{-1/2} = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}, \tag{24}$$

which is expected expression for the probability of error for the BPSK system in the presence of AWGN. The graphs for this case are presented by star lines in all Figures in this Report for the sake of comparison. For the same reason, the graphs for the probability of error for fading channel are presented in all figures by black solid graph.

For a constant number of users, N = 1 in Fig. 3a), the probability of error increases when the sequence factor ψ increases from 1 to 3. For a fixed probability of error $P_e = 10^{-5}$, the system with random spreading, ψ =3, requires 1 dB higher signal-to-noise ratio in respect to the system with binary spreading, ψ = 1.

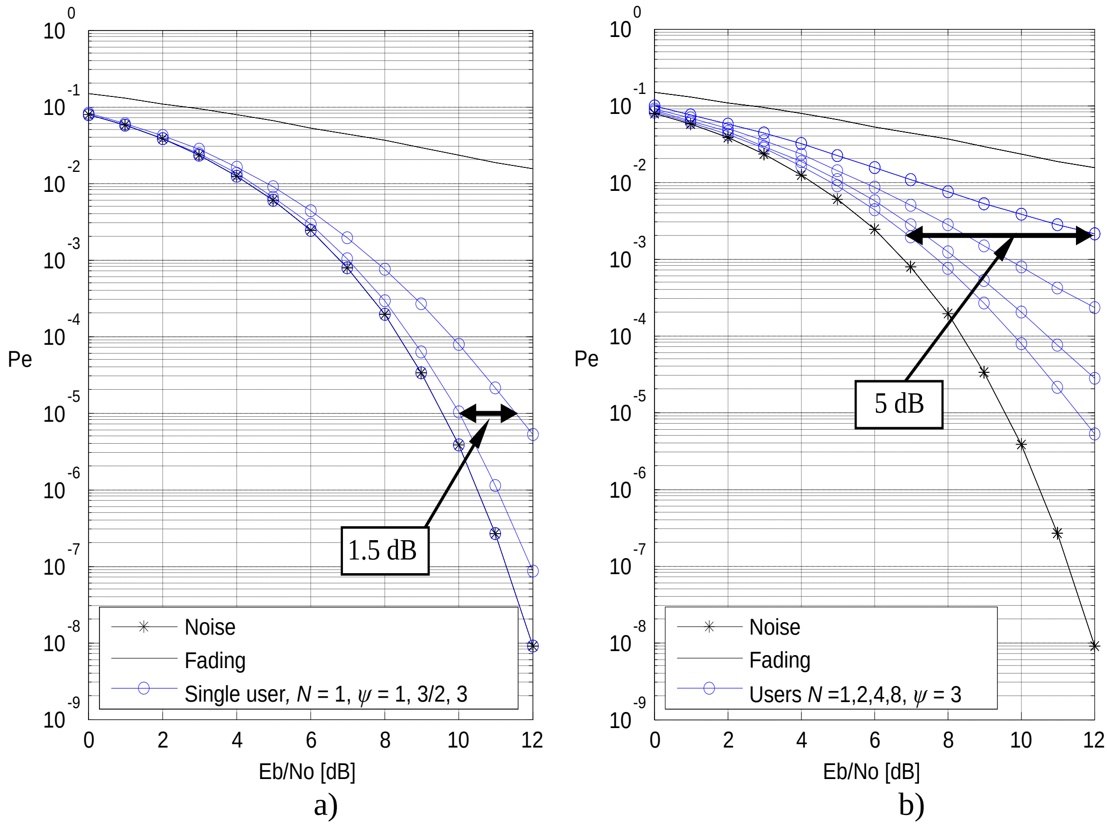


Figure 3 Probability of error P_e performance for different spreading sequences and different number of users.

a) Different spreading sequences defined by ψ = 1 (binary), ψ = 3/2 (chaotic) and ψ = 3 (Gaussian) from bottom to top, and for a single user N = 1, unit power P_c = 1 and processing gain 2β = 100.
b) Different number of users N = 1, 2, 4, 8 (from bottom to top), fixed spreading factor ψ = 3 (Gaussian), and for unit power P_c = 1 and processing gain 2β = 100.

For a constant sequence factor the probability of error increases when the number of user increases. Fig. 3b) presents P_e curves for constant sequence factor ψ =3 and different number of users. For a fixed probability of error $P_e = 2 \cdot 10^{-3}$, the required signal-to-noise ratio in the system with one user increases for 5 dB in respect to the system with 8 users, as can be seen from Fig. 3b).

Probability of error P_e performance for different processing gain and different spreading factors. General conclusion is, the probability of error increases when both the processing gain and sequences factor decreases, as can be seen from Fig. 4a) and 4b).

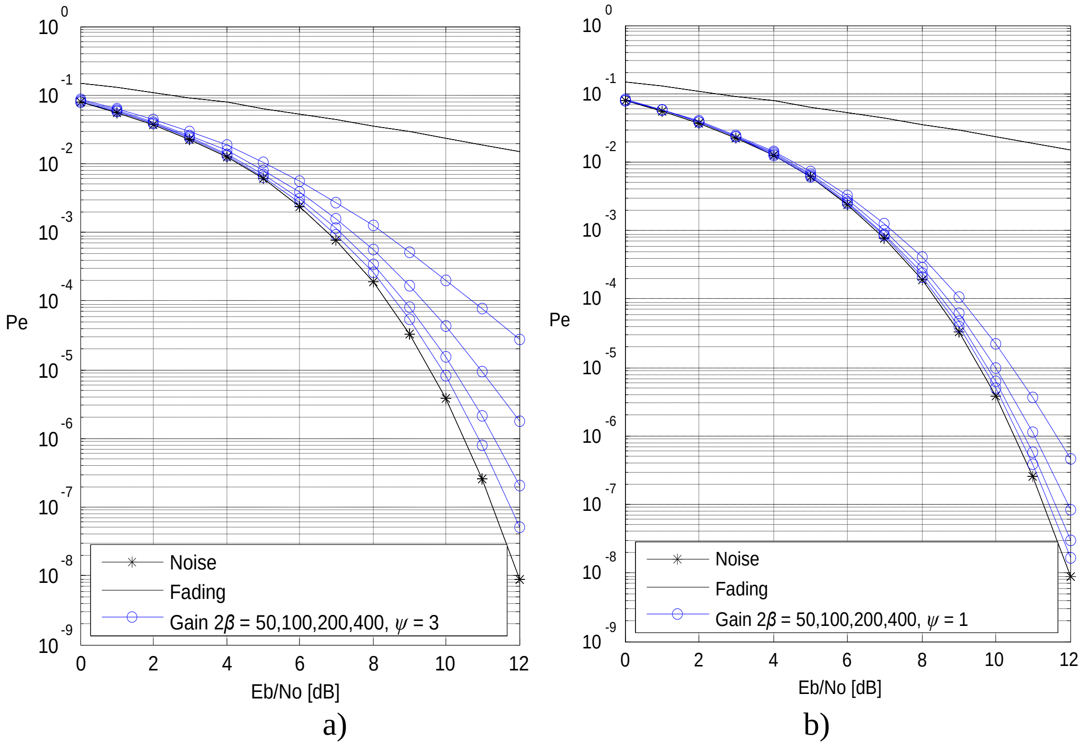


Figure 4 Probability of error P_e performance for different spreading sequences factors.

a) Different processing gains 2β = 50, 100, 200, 400 (*from top to bottom*), spreading sequences defined by $\psi = 3$ (Gaussian), and for a 2-user system, $N = 2$, and unit power, $P_c = 1$.
b) Different processing gains 2β = 50, 100, 200, 400 (*from top to bottom*), spreading sequences defined by $\psi = 1$ (binary), and for a 2-user system, $N = 2$, and unit power, $P_c = 1$.

4. PROBABILITY OF ERROR DERIVATION WITH FINITE DELAYS

The expression for the probability of error in a fading channel for $\alpha = 1$ becomes the expression for P_e the AWGN channel having this form

$$P_e(\tau) = \frac{1}{2} erfc \left(\frac{(\psi - 1)}{\beta} + \frac{N-1}{\beta} + X\frac{N}{\beta} + (1+X)\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2} . \tag{25}$$

The expressed probability depends on the sequence factor ψ that represents statistical characteristics of spreading sequences. For the sequences of our interest this factor can have the following values

$$\psi = \begin{Bmatrix} 1 & \text{binary with power } P_c = 1 \\ 1/2 & \text{chaotic with power } P_c = 1/2 \\ 3/2 & \text{chaotic with power } P_c = 1 \\ 3 & \text{random Gaussian for } P_c = 1 \end{Bmatrix}, \tag{26}$$

with the corresponding expressions for the probability of error:

$$P_e(\tau) = \frac{1}{2} erfc \left(\frac{N-1}{\beta} + X\frac{N}{\beta} + (1+X)\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2}$$, for binary sequence;

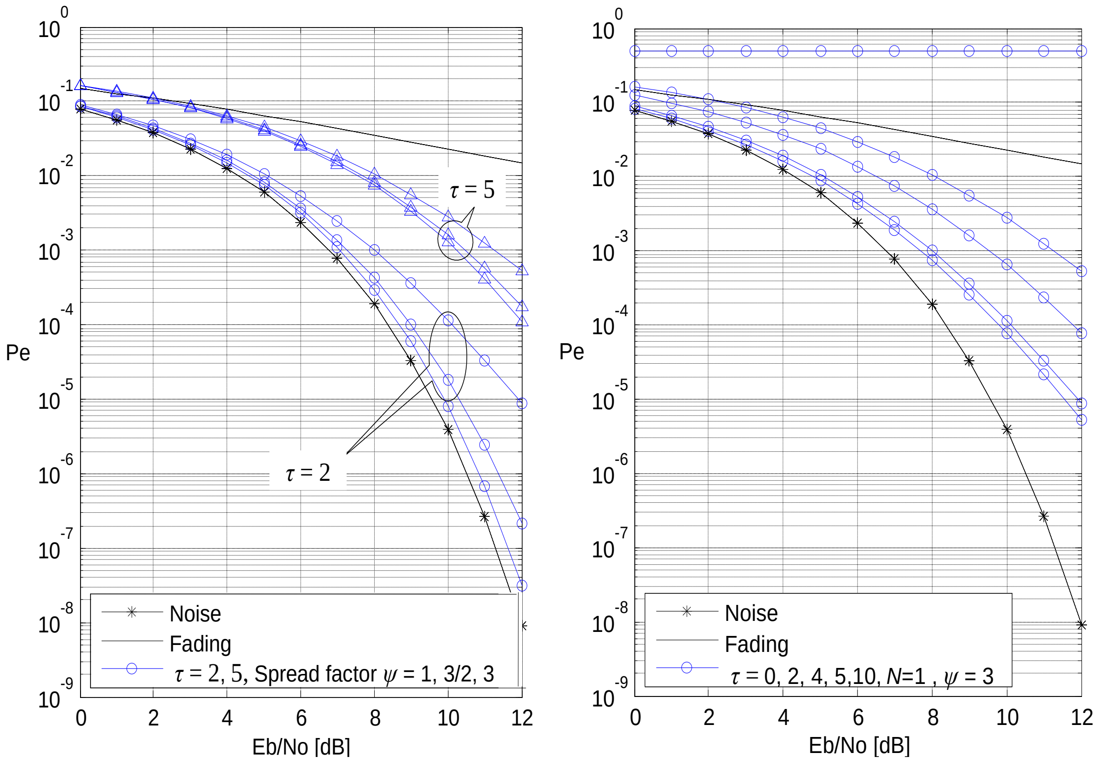
$$P_e(\tau) = \frac{1}{2} erfc \left(\frac{2N-1}{2\beta} + X\frac{N}{\beta} + (1+X)\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2}$$, for chaotic sequence with a chip power $P_c = 1$;

$$P_e(\tau) = \frac{1}{2} erfc \left(\frac{2N-3}{2\beta} + X\frac{N}{\beta} + (1+X)\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2}$$, for chaotic sequence with a chip power $P_c = 0.5$;

$$P_e(\tau) = \frac{1}{2} erfc \left(\frac{N+1}{\beta} + X\frac{N}{\beta} + (1+X)\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2}$$, for Gaussian sequence and a chip power $P_c = 1$.

The graphs, representing these probabilities, are shown in the following figures. In these figures, the graphs for the BPSK system with WGNC and Rayleigh fading channel are always presented for the sake of comparison. Fig. 5a) presents P_e graphs for two delays, i.e., $\tau = 2$ (circles) and 5 (triangles), and different spreading sequences for each of them, defined by sequence factors $\psi = 1$, 3/2, and 3.

The system with binary sequences has the best P_e performances, which deteriorates and become worst in the system with Gaussian random sequences. The deterioration is approximately 1 dB for $P_e = 10^{-3}$ and for the delay $\tau = 2$. When the delay increases the influence of the sequence factor becomes smaller, which is obvious comparing curves with the delay $\tau = 2$ (circles) with the curves with the delay the delay $\tau = 5$ (triangles).



a) b)

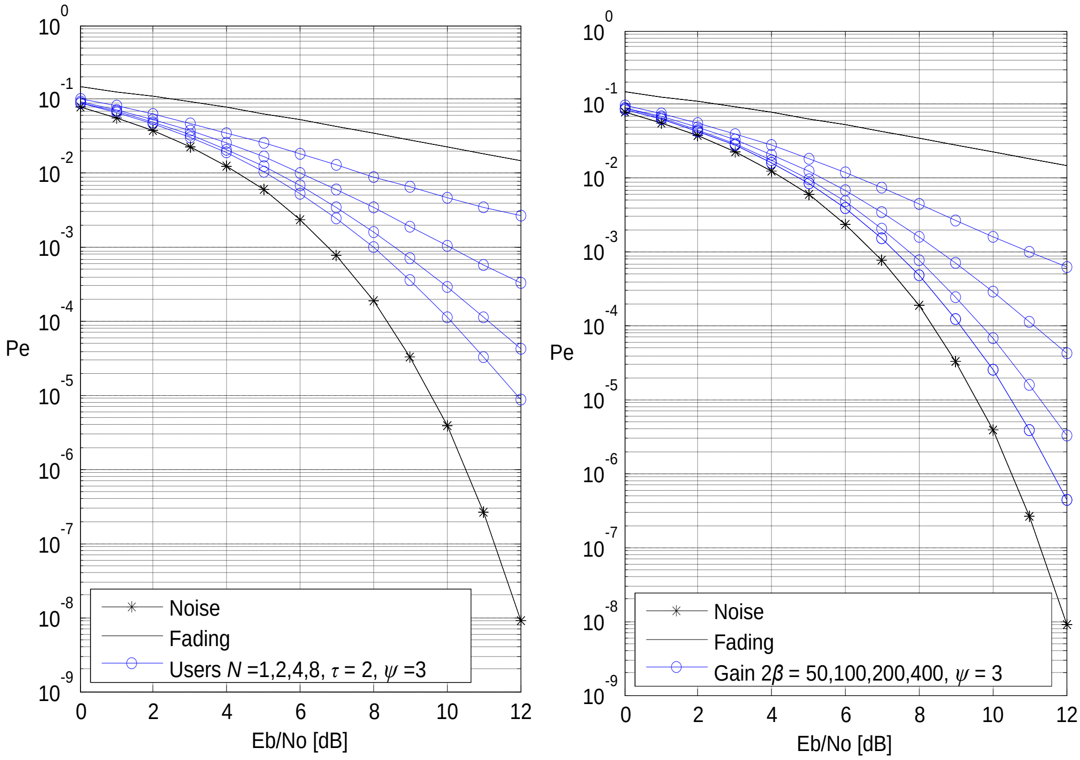
Figure 5 P_e performance for different spreading sequence and different delay values.

a) Different spreading sequences defined by $\psi = 1$ (binary), $\psi = 0.5$, $\psi = 3/2$ (chaotic) and $\psi = 3$ (Gaussian). The chip duration is $S = 10$, $P_c = 1$, a single user system, $N = 1$, $2\beta = 100$ and the constant delay is $\tau = 2$ (circles) and $\tau = 5$ (triangles).

b) Delays $\tau = 0, 2, 4, 5, 10$, spreading sequences defined by $\psi = 3$ (Gaussian). The chip duration is $S = 10$, $P_c = 1$, $2\beta = 100$ and the number of users $N = 1$.

For a constant delay the probability of error increases when the number of users increases because of the increase in inter-user interference of the system. Figure 6a) presents P_e curves for constant delay $\tau = 2$ and variable number of users $N = 1, 2, 4, 8$. For the probability of error $P_e = 3\cdot10^{-3}$, the signal-to-noise ratio improves 5 dB in a single-user system in respect to a eight-user system, as can be seen from Fig. 6a).

For a constant delay the probability of error increases when the processing gain reduces. Fig. 6b) presents P_e curves for constant delay $\tau = 2$ and variable processing gain $2\beta = 50$, 100, 200, 400. For the probability of error $P_e = 6\cdot10^{-3}$, the signal-to-noise ratio improves more than 4 dB in the system with $2\beta = 400$ in respect to the system with $2\beta = 50$, as can be seen from Fig. 6b).



a) b)

Figure 6 P_e performance for different number of users and different processing gains.

a) Number of users N = 1, 2, 4, 8 (from bottom to top), the constant delay $\tau = 2$, fixed spreading factor $\psi = 3$ (Gaussian), unit power $P_c = 1$, spreading factor $2\beta = 100$ and the chip duration $S = 10$.

b) Different processing gains 2β = 50, 100, 200, 400 (***from top to bottom***), $\tau = 2$, spreading sequences defined by $\psi = 3$ (Gaussian), and for a 2-user system $N = 2$ and unit power $P_c = 1$.

5. SYSTEM WITH BLOCK INTERLEAVER/DEINTERLEAVER

5.1 General Expression for the Probability of Error

Suppose a block interleaver is used to interleave the chips of the generated message signal at the transmitter side. The interleaver depth is equal to the spreading factor. The received chips are affected with independent Rayleigh coefficients. The *t*-th received chip can be expressed as

$$s_{itRx} = \sum_{g=1}^{N} \alpha_t \gamma_i^g x_{ti}^g + \xi_{ti} , \tag{27}$$

where α_t is the fading coefficient that affects the *t*-th chip and has different values, due to interleaving, for all chips inside the received *i*-th bit. Each chip has a finite duration T_c represented by S samples in discrete time domain. The received sequence is correlated with the local (reference) sequence. These sequences are supposed to be perfectly synchronized. However that is not the case in our analysis and the delay τ between them exists. We will assume that this delay is random and is defined inside the local reference sequences, which does not change the generality of explanation. On the other hand, in practice, the local sequence is being shifted in respect to the received sequence, as shown in Fig. 2, until the synchronization is achieved. Therefore, the received chip, correlated with the delayed reference chip $x_{ti\tau}^g$, can be expressed in this form

$$z_t = \sum_{s=1}^{S} (\alpha_t s_{ti}^{Tx} + \xi_{ti}) x_{ti\tau}^g = \sum_{s=1}^{S} (\sum_{g=1}^{N} \alpha_t \gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti\tau}^g , \tag{28}$$

where S identical chip samples are multiplied with the delayed chip samples of the local delayed sequence and added to each other.

Due to the delay between the sequences, (S - $|\tau|$) samples of the delayed chip will be aligned to the received chip samples, i.e., it will be $x_{ti\tau}^g = x_{ti}^g$, and $|\tau|$ samples will be beyond the interval of the received chip. Therefore, the received chip value can be expressed as

$$z_t = \sum_{s=|\tau|+1}^{S} (\sum_{g=1}^{N} \alpha_t \gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti}^g + \sum_{s=1}^{|\tau|} (\sum_{g=1}^{N} \alpha_t \gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti\tau}^g . \tag{29}$$

If the *g*-th user receives the signal, the *t*-th received chip of that user can be expressed in this form

$$z_{tg} = \sum_{s=|\tau|+1}^{S} \left(\alpha \gamma_i^g x_{ti}^{g\,2} + \sum_{n=1, n\neq g}^{N} \alpha \gamma_i^n x_{ti}^n x_{ti}^g + \xi_{ti} x_{ti}^g \right) + \sum_{s=1}^{|\tau|} \left(\alpha_t \gamma_i^g x_{ti}^g x_{ti\tau}^g + \sum_{n=1, n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti\tau}^g + \xi_{ti} x_{ti\tau}^g \right) . \tag{30}$$

The first sum in this expression contains the processed chip samples that are perfectly synchronized with the received chip samples. The first addend in this sum is a signal part, the second term is an inter-user interference and the third one is the noise part. The second sum contains the samples that are not synchronized producing inter-chip interference, which has tree addends. The first addend is inter-chip interference related to the chip of the g-th user. The second term is interference for other users and the third term is the noise related to the delayed samples. Because the chip samples are identical, the chip value can be expressed as

$$\begin{aligned} z_{tg} &= (S-|\tau|)\alpha_t \gamma_i^g x_{ti}^{g\,2} + |\tau|\alpha_t \gamma_i^g x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|)\sum_{n=1,n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti}^g + |\tau| \sum_{n=1,n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti\tau}^g \; . \\ &+ (S-|\tau|)\xi_{ti} x_{ti}^g + |\tau|\xi_{ti} x_{ti\tau}^g \end{aligned} \tag{31}$$

The final output of the correlator is the i-th bit value that represents the decision variable expressed as

$$\begin{aligned} z_i^g &= (S-|\tau|)\cdot \gamma_i^g \sum_{t=1}^{2\beta} \alpha_t x_{ti}^{g\,2} + |\tau|\cdot \gamma_i^g \sum_{t=1}^{2\beta} \alpha_t x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot \sum_{t=1}^{2\beta} \sum_{n=1,n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti}^g + |\tau|\cdot \sum_{t=1}^{2\beta} \sum_{n=1,n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti\tau}^g \; . \\ &+ (S-|\tau|)\cdot \sum_{t=1}^{2\beta} \xi_{ti} x_{ti}^g + |\tau|\cdot \sum_{t=1}^{2\beta} \xi_{ti} x_{ti\tau}^g \end{aligned} \tag{32}$$

Suppose the i-th bit transmitted is γ_i^g +1, which does not have influence on the generality of this analysis. Than the decision variable can be expressed in this form

$$\begin{aligned} z_i^g &= (S-|\tau|)\sum_{t=1}^{2\beta} \alpha_t x_{ti}^{g\,2} + |\tau| \sum_{t=1}^{2\beta} \alpha_t x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot \sum_{t=1}^{2\beta} \sum_{n=1,n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti}^g + |\tau|\cdot \sum_{t=1}^{2\beta} \sum_{n=1,n\neq g}^{N} \alpha_t \gamma_i^n x_{ti}^n x_{ti\tau}^g \; . \\ &+ (S-|\tau|)\cdot \sum_{t=1}^{2\beta} \xi_{ti} x_{ti}^g + |\tau|\cdot \sum_{t=1}^{2\beta} \xi_{ti} x_{ti\tau}^g \\ &= A + B + C + D + E + F \end{aligned} \tag{33}$$

Which is equivalent to expression (9). The mean values conditioned on the delay and fading coefficients are

$$E\{A|\alpha_t \tau\} = E\left\{(S-|\tau|)\sum_{t=1}^{2\beta} \alpha_t x_{ti}^{g2}\right\} = (S-|\tau|)\sum_{t=1}^{2\beta} \alpha_t E\left\{x_{ti}^{g2}\right\} = (S-|\tau|)P_c \sum_{t=1}^{2\beta} \alpha_t = \eta_{Zi} \tag{34}$$

The remaining mean values are zero, as follows

$$E\{B|\alpha_t\tau\} = E\{C|\alpha\tau\} = E\{D|\alpha\tau\} = E\{E|\alpha\tau\} = E\{F|\alpha\tau\} = 0. \tag{35}$$

Therefore, the squared value of the mean of the decision variable can be expressed as

$$\eta_{Zi}^2 = E^2\{A|\alpha_t\tau\} = (S-|\tau|)^2 P_c^{\,2}(\sum_{t=1}^{2\beta}\alpha_t)^2, \tag{36}$$

or, in this general form,

$$\eta_{Zi}^2 = E^2\{A|\alpha_t\tau\} = E\{\left((S-|\tau|)\cdot\gamma_i^g\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g2}\right)^2\} = (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}\alpha_t^2E^2\{x_{ti}^{g2}\} + 2\sum_{t=1}^{2\beta-1}\alpha_t E\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}\alpha_j E\{x_{tj}^{g2}\}\right). \tag{37}$$

The variances of random function A, conditioned on τ and all α_t, can be calculated as follows

$$\begin{aligned}
\sigma^2(A|\alpha_t\tau) &= E\left\{\left[(S-|\tau|)\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g2}\right]^2\right\} - E^2\{A|\alpha_t\tau\} = (S-|\tau|)^2E\{\left(\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g2}\right)^2\} - E^2\{A|\alpha\tau\} \\
&= (S-|\tau|)^2E\{\sum_{t=1}^{2\beta}\alpha_t^2x_{ti}^{g4} + 2\sum_{t=1}^{2\beta-1}\alpha_t x_{ti}^{g2}\sum_{j=t+1}^{2\beta}\alpha_j x_{tj}^{g2}\} - (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}\alpha_t^2E^2\{x_{ti}^{g2}\} + 2\sum_{t=1}^{2\beta-1}\alpha_tE\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}\alpha_jE\{x_{tj}^{g2}\}\right) \\
&= (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}\alpha_t^2E\{x_{ti}^{g4}\} + 2\sum_{t=1}^{2\beta-1}\alpha_tE\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}\alpha_jE\{x_{tj}^{g2}\}\right) - (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}\alpha_t^2E^2\{x_{ti}^{g2}\} + 2\sum_{t=1}^{2\beta-1}\alpha_tE\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}\alpha_jE\{x_{tj}^{g2}\}\right) \\
&= (S-|\tau|)^2P_{x4}\sum_{t=1}^{2\beta}\alpha_t^2 - (S-|\tau|)^2P_c^2\sum_{t=1}^{2\beta}\alpha_t^2 = (S-|\tau|)^2\sum_{t=1}^{2\beta}\alpha_t^2(P_{x4}-P_c^2) \\
&= (S-|\tau|)^2P_c^2(\psi-1)\sum_{t=1}^{2\beta}\alpha_t^2
\end{aligned} \tag{38}$$

Where, as we have already said, $\psi = P_{x4}/P_c^2$ is a spreading sequence factor that depends on the statistical properties of a particular sequence. The other variances are

$$\sigma^2(B|\alpha_t\tau) = E\left\{\left[|\tau|\sum_{t=1}^{2\beta}\alpha_t x_{ti}^g x_{ti\tau}^g\right]^2\right\} = |\tau|^2 P_c^2\sum_{t=1}^{2\beta}\alpha_t^2, \tag{39}$$

$$\sigma^2(C|\alpha_t\tau) = E\{[(S-|\tau|)\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}\alpha_t\gamma_i^n x_{ti}^n x_{ti}^g]^2\} \underset{iid}{=} (S-|\tau|)^2P_c^2(N-1)\sum_{t=1}^{2\beta}\alpha_t^2, \tag{40}$$

$$\sigma^2(D|\alpha_t\tau) = |\tau|^2(N-1)P_c^2\sum_{t=1}^{2\beta}\alpha_t^2, \tag{41}$$

$$\sigma^2(E|\alpha_t\tau) = (S-|\tau|)^2 2\beta\sigma^2P_c \text{ , and} \tag{42}$$

$$\sigma^2(F\,|\,\alpha_t\tau) = |\,\tau\,|^2\; 2\beta\sigma^2 P_c\ . \tag{43}$$

The ratios of the doubled variance and the squared values of the mean can be found as follows

$$\frac{2\sigma^2(A|\alpha_t\tau)}{E^2\{A|\alpha_t\tau\}} = \frac{2(S-|\tau|)^2 P_c^2(\psi-1)}{(S-|\tau|)^2 P_c^2}\sum_{t=1}^{2\beta}\alpha_t^2(\sum_{t=1}^{2\beta}\alpha_t)^{-2} = 2(\psi-1)R_\alpha \tag{44}$$

$$\frac{2\sigma^2(B|\alpha_t\tau)}{E^2\{A|\alpha_t\tau\}} = 2XR_\alpha\,, \tag{45}$$

$$\frac{2\sigma^2(C|\alpha_t\tau)}{E^2\{A|\alpha_t\tau\}} = 2(N-1)R_\alpha\,, \tag{46}$$

$$\frac{2\sigma^2(D|\alpha_t\tau)}{E^2\{A|\alpha_t\tau\}} = 2(N-1)XR_\alpha\,, \tag{47}$$

$$\frac{2\sigma^2(E|\alpha_t\tau)}{E^2\{A|\alpha_t\tau\}} = \frac{4\beta^2}{\alpha_\Sigma^2}\frac{N_0}{E_b}\,, \tag{48}$$

$$\frac{2\sigma^2(F|\alpha_t\tau)}{E^2\{A|\alpha_t\tau\}} = X\frac{4\beta^2}{\alpha_\Sigma^2}\frac{N_0}{E_b}\,, \tag{49}$$

where X depends on the delay of the chip and the fraction number of interpolated samples, and R_α depends on fading coefficients

$$X(\tau) = \frac{|\tau|^2}{(S-|\tau|)^2} = \frac{|\tau/S|^2}{(1-|\tau|/S)^2}\,,\ \Sigma_\alpha = \sum_{t=1}^{2\beta}\alpha_t^2,\ \alpha_\Sigma^2 = \left(\sum_{t=1}^{2\beta}\alpha_t\right)^2, R_\alpha = \sum_{t=1}^{2\beta}\alpha_t^2/(\sum_{t=1}^{2\beta}\alpha_t)^2 = \frac{\Sigma_\alpha}{\alpha_\Sigma^2}\,. \tag{50}$$

The probability of error, conditioned on fading coefficients and delay, can be calculated as

$$\begin{aligned} P_e(\alpha_t,\tau) &= \frac{1}{2}erfc\ \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2}\right)^{-1/2} = \frac{1}{2}erfc\ \left(\frac{2}{\eta_{Zi}^2}\sum_{x=A}^{F}\sigma^2(x\,|\,\alpha_t\tau)\right)^{-1/2} \\ &= \frac{1}{2}erfc\ \left(2(\psi-1)R_\alpha + 2XR_\alpha + 2(N-1)(1+X)R_\alpha + (1+X)\frac{4\beta^2}{\alpha_\Sigma^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2} \end{aligned} \tag{51}$$

which can be simplified as

$$P_e(\tau) = \frac{1}{2} erfc \left(2(\psi - 1)R_\alpha + 2X(\tau)R_\alpha + 2(N-1)[1+X(\tau)]R_\alpha + \frac{4\beta^2}{\alpha_\Sigma^2}[1+X(\tau)]\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2}$$

$$= \frac{1}{2} erfc \left(2\psi R_\alpha - 2R_\alpha + 2X(\tau)R_\alpha + 2(N-1)R_\alpha + 2(N-1)X(\tau)R_\alpha + \frac{4\beta^2}{\alpha_\Sigma^2}[1+X(\tau)]\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2} .$$

$$= \frac{1}{2} erfc \left((2\psi + 2N + 2NX(\tau) - 4)R_\alpha + \frac{4\beta^2}{\alpha_\Sigma^2}[1+X(\tau)]\left(\frac{E_b}{N_0}\right)^{-1} \right)$$

However, we will use derived expression (51), because it contains meaningful terms from interference point of view.

5.2 Relation to the Previous Derivatives for the Noisy Fading Channel

Suppose the operation of the interleaver and deinterleaver are omitted. In this case we can find expression for R_α in this form

$$R_\alpha = \sum_{t=1}^{2\beta} \alpha_t^2 (\sum_{t=1}^{2\beta} \alpha_t)^{-2} = (2\beta\alpha^2)(2\beta\alpha)^{-2} = 1/2\beta , \qquad (52)$$

and the probability of error as

$$P_e(\alpha,\tau) = \frac{1}{2} erfc \left(2(\psi - 1)\frac{1}{2\beta} + 2X\frac{1}{2\beta} + 2(N-1)(1+X)\frac{1}{2\beta} + (1+X)\frac{4\beta^2}{\alpha^2}\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2}$$

$$= \frac{1}{2} erfc \left((\psi - 1)\frac{1}{\beta} + \frac{N-1}{\beta} + \frac{N}{\beta} X + (1+X)\frac{1}{\alpha^2}\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2} = \frac{1}{2} erfc \left(\frac{\psi}{\beta} + \frac{N-2}{\beta} + \frac{N}{\beta} X + (1+X)\frac{1}{\alpha^2}\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-1/2} , \qquad (53)$$

which is the same as the expression derived in previous Chapter. The average probability of error can be obtained as the mathematical expectation of the conditionalprobability of error expressed as

$$P_e = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_e(\alpha,\tau) f_{\tau\alpha}(\tau,\alpha) d\alpha d\tau . \qquad (54)$$

Due to statistical independence between the delay and the fading coefficients, this probability of error can be obtained according to this expression

$$P_e = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2} erfc \left(\frac{\psi}{\beta} + \frac{N-2}{\beta} + \frac{N}{\beta} X(\tau) + (1+X(\tau))\frac{1}{\alpha^2}\left(\frac{E_b}{N_0}\right)^{-1} \right)^{-\frac{1}{2}} f_\tau(\tau) f_\alpha(\alpha) d\alpha d\tau \qquad (55)$$

This integral, to the best of author's knowledge, cannot be solved. Therefore, the solution can be found using numerical integration.

5.3 The Probability of Error Conditioned on the Delay

Due to the CLT, the decision variable (33) can be treated as a Gaussian discrete time stochastic process and the mean values and variance of each discrete variable can be found as a function of delays only. In this way, the probability of error, which is conditioned only on the delay as a random variable, can be obtained. The decision variable (33) can be expressed in this form

$$\begin{aligned} z_i^g &= (S-|\tau|)\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g\,2} + |\tau|\sum_{t=1}^{2\beta}\alpha_t x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\alpha_t\gamma_i^n x_{ti}^n x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\alpha_t\gamma_i^n x_{ti}^n x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti\tau}^g \\ &= A+B+C+D+E+F \end{aligned} \quad . \tag{56}$$

Due to statistical independence of the fading coefficients between each other and their independence of the spreading chips, the mean values, conditioned on the delay, can be calculated as

$$E\{A|\tau\} = E\left\{(S-|\tau|)\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g2}\right\} = (S-|\tau|)\sum_{t=1}^{2\beta}E\{\alpha_t\}E\{x_{ti}^{g2}\} = (S-|\tau|)2\beta P_c b\sqrt{\pi/2} = \eta_{Zi} \tag{57}$$

The squared value of this mean is

$$E^2\{A|\tau\} = (S-|\tau|)^2 2\beta^2 P_c^2 b^2 \pi = \eta_{Zi}^2 . \tag{58}$$

The remaining mean values, as was confirmed before, are zero, i.e.,

$$E\{B|\tau\} = E\{C|\tau\} = E\{D|\tau\} = E\{E|\tau\} = E\{F|\tau\} = 0 . \tag{59}$$

The variances of A, conditioned on τ, can be calculated as follows

$$\begin{aligned} \sigma^2(A|\tau) &= E\left\{\left[(S-|\tau|)\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g2}\right]^2\right\} - E^2\{A|\alpha_t\tau\} = (S-|\tau|)^2 E\{\left(\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g2}\right)^2\} - E^2\{A|\alpha\tau\} \\ &= (S-|\tau|)^2\left(E\{\sum_{t=1}^{2\beta}\alpha_t^2 x_{ti}^{g4}\} + 2E\{\sum_{t=1}^{2\beta-1}\alpha_t x_{ti}^{g2}\sum_{j=t+1}^{2\beta}\alpha_j x_{tj}^{g2}\}\right) - (S-|\tau|)^2\left(2\beta E^2\{\alpha_t^2\}E^2\{x_{ti}^{g2}\} + 2E\{\sum_{t=1}^{2\beta-1}\alpha_t E x_{ti}^{g2}\sum_{j=t+1}^{2\beta}\alpha_j E x_{tj}^{g2}\}\right) \\ &= (S-|\tau|)^2\left(2\beta E\{\alpha_t^2\}E\{x_{ti}^{g4}\} - 2\beta E^2\{\alpha_t^2\}E^2\{x_{ti}^{g2}\}\right) = (S-|\tau|)^2 2\beta\left(2b^2 P_{x4} - b^2\pi P_c^2/2\right) \\ &= (S-|\tau|)^2 4\beta b^2 P_c^2\left(P_{x4}/P_c^2 - \pi/4\right) = (S-|\tau|)^2 4\beta b^2 P_c^2\left(\psi - \pi/4\right) \end{aligned} \tag{60}$$

Where $\psi = P_{x4}/P_c^2$ is a previously defined spreading sequence factor that depends on the statistical properties of a particular sequence. The other variances are

$$\sigma^2(B|\tau) = E\left\{\left[|\tau|\sum_{t=1}^{2\beta}\alpha_t x_{ti}^g x_{ti\tau}^g\right]^2\right\} = |\tau|^2 4\beta b^2 P_c^2 , \tag{61}$$

$$\sigma^2(C|\tau) = (S-|\tau|)^2 4\beta(N-1)b^2 P_c^2 , \tag{62}$$

$$\sigma^2(D|\tau) = |\tau|^2 4\beta(N-1)b^2 P_c^2 , \tag{63}$$

$$\sigma^2(E|\tau) = (S-|\tau|)^2 2\beta\sigma^2 P_c \text{ , and} \tag{64}$$

$$\sigma^2(F|\tau) = |\tau|^2 2\beta\sigma^2 P_c \text{ .} \tag{65}$$

The ratios of the doubled variances and the squared values of the mean can be found as follows

$$\frac{2\sigma^2(A|\tau)}{E^2\{A|\tau\}} = 2\frac{(S-|\tau|)^2 4\beta b^2 P_c^2 \left(\psi - \pi/4\right)}{(S-|\tau|)^2 2\beta^2 P_c^2 b^2 \pi} = \frac{4\left(\psi-\pi/4\right)}{\beta\pi},$$

$$\frac{2\sigma^2(B|\tau)}{E^2\{A|\tau\}} = \frac{|\tau/S|^2}{(1-|\tau|/S)^2}\frac{4}{\pi\beta} = \frac{4}{\pi\beta}X ,$$

$$\frac{2\sigma^2(C|\tau)}{E^2\{A|\tau\}} = \frac{4(N-1)R_\alpha}{\pi\beta},$$

$$\frac{2\sigma^2(D|\tau)}{E^2\{A|\tau\}} = \frac{4(N-1)}{\pi\beta}X ,$$

$$\frac{2\sigma^2(E|\tau)}{E^2\{A|\tau\}} = \frac{4}{\pi b^2}\frac{N_0}{E_b} \text{ , and}$$

$$\frac{2\sigma^2(F|\tau)}{E^2\{A|\tau\}} = X\frac{2}{\pi b^2}\frac{N_0}{E_b} ,$$

where X depends on the delay of a chip and a fraction of the number of interpolated samples, and R_α depends on fading coefficients as follows

$$X = \frac{|\tau|^2}{(S-|\tau|)^2} = \frac{|\tau/S|^2}{(1-|\tau|/S)^2} . \tag{66}$$

Now, the probability of error in closed form, conditioned on the delay only, can be calculated as

$$P_e(\tau)=\frac{1}{2}erfc\left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2}\right)^{-1/2}=\frac{1}{2}erfc\left(\frac{4(\psi-\pi/4+N-1)}{\pi\beta}+\frac{4N}{\pi\beta}X+(1+X)\frac{2}{\pi b^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2}. \quad (67)$$

The following three special cases will be analysed for various chip sequences with the same chip powers being $P_c = 1$.

SPECIAL CASE 1: Binary spreading sequences, $\psi = 1$, and the probability of error is

$$P_e(\tau)=\frac{1}{2}erfc\left(\frac{4(N-\pi/4)}{\pi\beta}+\frac{4N}{\pi\beta}X+(1+X)\frac{2}{\pi b^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2} \quad (68)$$

SPECIAL CASE 2: Chaotic spreading sequences, $\psi = 3/2$, and the probability of error is

$$\begin{aligned}P_e(\tau)&=\frac{1}{2}erfc\left(\frac{4(3/2-\pi/4+N-1)}{\pi\beta}+\frac{4N}{\pi\beta}X+(1+X)\frac{2}{\pi b^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2}\\&=\frac{1}{2}erfc\left(\frac{2-\pi+4N}{\pi\beta}+\frac{4N}{\pi\beta}X+(1+X)\frac{2}{\pi b^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2}\end{aligned} \quad (69)$$

SPECIAL CASE 3: Random spreading sequences, $\psi = 3$, and the probability of error is

$$\begin{aligned}P_e(\tau)&=\frac{1}{2}erfc\left(\frac{4(3-\pi/4+N-1)}{\pi\beta}+\frac{4N}{\pi\beta}X+(1+X)\frac{2}{\pi b^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2}\\&=\frac{1}{2}erfc\left(\frac{8-\pi+4N}{\pi\beta}+\frac{4N}{\pi\beta}X+(1+X)\frac{2}{\pi b^2}\left(\frac{E_b}{N_0}\right)^{-1}\right)^{-1/2}\end{aligned} \quad (70)$$

Figure 7a) presents the probability of error graphs for different spreading sequences defined by $\psi = 1$ (binary), $\psi = 0.5$, $\psi = 3/2$ (chaotic) and $\psi = 3$ (Gaussian), from bottom to top, and for the constant delays $\tau = 2$ (blue) and $\tau = 5$ (red).

When the sequences factor ψ increases the probability of error deteriorates. For $P_e = 10^{-4}$ the loss in signal-to noise ratio is close to 2 dB for the system that uses Gaussian sequence instead of binary sequences. If the number of users increases, to $N = 4$ in Fig, 7a) (red graphs), further the probability of error increases further and the graphs are tending to the graphs corresponding to the channel with Rayleigh fading.

If the delay between sequences increases, the probability of error in the system increases. This fact can be observed in Fig. 7b) for delays $\tau = 0, 2, 4, 5, 10$ (bottom to top) and the number of users $N = 1$ (blue) and 4 (red). When delay is zero, the curve for single-user system is close to the curve obtained for the WGN channel. When the delay is equal to the chip duration, $\tau = 10$, the probability of error is 0.5, because the system is not synchronized at all.

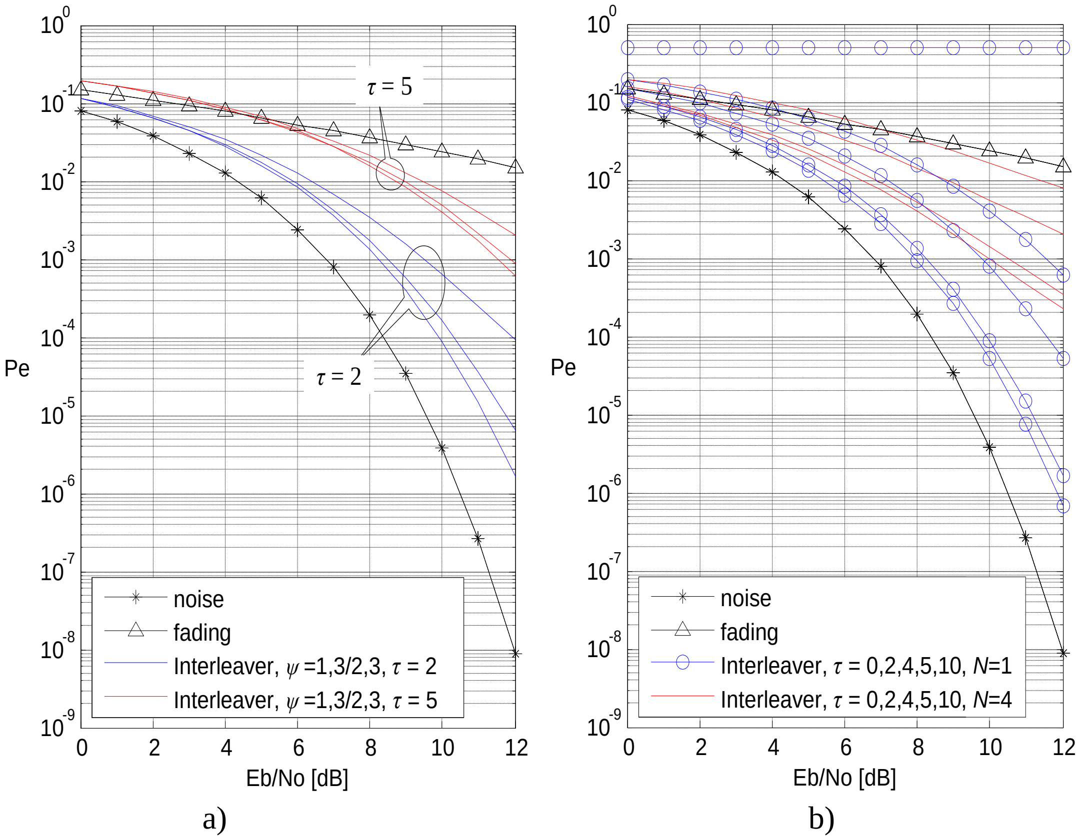


Figure 7 P_e performance for different spreading sequences and different delays, and different number of users and different delay values.

a) Different spreading sequences defined by ψ = 1 (binary), ψ = 0.5, ψ = 3/2 (chaotic) and ψ = 3 (Gaussian), from bottom to top, and the constant delays τ = 2 (blue) and τ = 5 (red). The chip duration is S = 10, P_c = 1, single user N = 1, 2β = 100

b) Different delays τ = 0, 2, 4, 5, 10 (bottom to top), number of users N = 1 (blue) and N = 4 (red), spreading sequences defined by ψ = 1 (binary). The chip duration is S = 10, P_c = 1, and 2β = 100.

6. The Probability of Error in Noisy and Flat Fading Channel Without Interleaver

6.1 Channel with White Gaussian Noise Channel

If only Gaussian noise is present in the channel, the fading coefficients in (9) can be equated by one, i.e., $\alpha_t = 1$, and the expression for the decision variable can be obtained in this form

$$\begin{aligned} z_i^g &= (S-|\tau|)\cdot\gamma_i^g\sum_{t=1}^{2\beta} x_{ti}^{g\,2} + |\tau|\cdot\gamma_i^g\sum_{t=1}^{2\beta} x_{ti}^g x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\gamma_i^n x_{ti}^n x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\gamma_i^n x_{ti}^n x_{ti\tau}^g \\ &+ (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\xi_{ti} x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\xi_{ti} x_{ti\tau}^g \\ &= A+B+C+D+E+F \end{aligned} \qquad (71)$$

Suppose the i-th bit transmitted is +1, which does not have influence on the generality of this analysis. Than the decision variable can be expressed in this form

$$\begin{aligned} z_i^g &= (S-|\tau|)\sum_{t=1}^{2\beta}\alpha_t x_{ti}^{g\,2} + |\tau|\sum_{t=1}^{2\beta}\alpha_t x_{ti}^g x_{ti\tau}^g + (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\alpha_t\gamma_i^n x_{ti}^n x_{ti}^g \\ &+ |\tau|\cdot\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\alpha_t\gamma_i^n x_{ti}^n x_{ti\tau}^g + (S-|\tau|)\cdot\sum_{t=1}^{2\beta}\xi_{ti} x_{ti}^g + |\tau|\cdot\sum_{t=1}^{2\beta}\xi_{ti} x_{ti\tau}^g \\ &= A+B+C+D+E+F \end{aligned} \quad . \qquad (72)$$

The mean values conditioned on the delay are

$$E\{A|\tau\} = E\left\{(S-|\tau|)\sum_{t=1}^{2\beta} x_{ti}^{g\,2}\right\} = (S-|\tau|)2\beta P_c = \eta_{Zi}\,. \qquad (73)$$

The remaining mean values are zero, as follows

$$E\{B|\tau\} = E\{C|\tau\} = E\{D|\tau\} = E\{E|\tau\} = E\{F|\tau\} = 0\,. \qquad (74)$$

Therefore the squared value of the mean of the decision variable is

$$\eta_{Zi}^2 = E^2\{A|\tau\} = (S-|\tau|)^2 P_c^2 (2\beta)^2\,, \qquad (75)$$

or, in this general form,

$$\eta_{Zi}^2 = E^2\{A|\tau\} = E\{\left((S-|\tau|)\cdot\gamma_i^g\sum_{t=1}^{2\beta}x_{ti}^{g2}\right)^2\} = (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}E^2\{x_{ti}^{g2}\}+2\sum_{t=1}^{2\beta-1}E\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}E\{x_{tj}^{g2}\}\right). \quad (76)$$

The variances of random function A, conditioned on τ, can be calculated using this general expression

$$\sigma^2(A|\tau) = E\left\{\left[(S-|\tau|)\sum_{t=1}^{2\beta}x_{ti}^{g2}\right]^2\right\} - E^2\{A|\tau\} = (S-|\tau|)^2 E\{\left(\sum_{t=1}^{2\beta}x_{ti}^{g2}\right)^2\} - E^2\{A|\tau\} \quad (77)$$

which can be simplified as follows

$$\begin{aligned}\sigma^2(A|\tau) &= (S-|\tau|)^2 E\{\sum_{t=1}^{2\beta}x_{ti}^{g4} + 2\sum_{t=1}^{2\beta-1}x_{ti}^{g2}\sum_{j=t+1}^{2\beta}x_{tj}^{g2}\} - (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}E^2\{x_{ti}^{g2}\}+2\sum_{t=1}^{2\beta-1}E\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}E\{x_{tj}^{g2}\}\right)\\ &= (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}E\{x_{ti}^{g4}\}+2\sum_{t=1}^{2\beta-1}E\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}E\{x_{tj}^{g2}\}\right) - (S-|\tau|)^2\left(\sum_{t=1}^{2\beta}E^2\{x_{ti}^{g2}\}+2\sum_{t=1}^{2\beta-1}E\{x_{ti}^{g2}\}\sum_{j=t+1}^{2\beta}E\{x_{tj}^{g2}\}\right) \quad (78)\\ &= (S-|\tau|)^2 P_{x4}2\beta - (S-|\tau|)^2 P_c^2 2\beta = (S-|\tau|)^2 2\beta(P_{x4}-P_c^2)\\ &= (S-|\tau|)^2 2\beta P_c^2(\psi-1)\end{aligned}$$

where, as we have already said, $\psi = P_{x4}/P_c^{\,2}$ is a spreading sequence factor that depends on the statistical properties of a particular sequence. The other variances are

$$\sigma^2(B|\tau) = E\left\{\left[|\tau|\sum_{t=1}^{2\beta}x_{ti}^g x_{ti\tau}^g\right]^2\right\} = |\tau|^2 P_c^2 2\beta, \quad (79)$$

$$\sigma^2(C|\tau) = E\{[(S-|\tau|)\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{2\beta}\gamma_i^n x_{ti}^n x_{ti}^g]^2\} \underset{iid}{=} (S-|\tau|)^2 P_c^2(N-1)2\beta, \quad (80)$$

$$\sigma^2(D|\tau) = |\tau|^2 (N-1)P_c^2 2\beta, \quad (81)$$

$$\sigma^2(E|\tau) = (S-|\tau|)^2 2\beta\sigma^2 P_c, \text{ and} \quad (82)$$

$$\sigma^2(F|\tau) = |\tau|^2 2\beta\sigma^2 P_c. \quad (83)$$

Having in mind these relations hold $R_\alpha = \sum_{t=1}^{2\beta}\alpha_t^2(\sum_{t=1}^{2\beta}\alpha_t)^{-2} = (2\beta\alpha^2)(2\beta\alpha)^{-2} = 1/2\beta$ and $\alpha_\Sigma^2 = 4\beta^2$, the ratios of the doubled variance and the squared values of the mean can be found as follows

$$\frac{2\sigma^2(A|\tau)}{E^2\{A|\tau\}} = \frac{2(S-|\tau|)^2 P_c^2(\psi-1)}{(S-|\tau|)^2 P_c^2}R_\alpha = \frac{(\psi-1)}{\beta} \quad (84)$$

$$\frac{2\sigma^2(B|\tau)}{E^2\{A|\tau\}} = \frac{X(\tau)}{\beta}, \tag{85}$$

$$\frac{2\sigma^2(C|\tau)}{E^2\{A|\tau\}} = \frac{(N-1)}{2}, \tag{86}$$

$$\frac{2\sigma^2(D|\tau)}{E^2\{A|\tau\}} = \frac{(N-1)}{\beta} X(\tau), \tag{87}$$

$$\frac{2\sigma^2(E|\tau)}{E^2\{A|\tau\}} = \frac{N_0}{E_b}, \tag{88}$$

$$\frac{2\sigma^2(F|\tau)}{E^2\{A|\tau\}} = X(\tau)\frac{N_0}{E_b}, \tag{89}$$

where X depends on the delay of the chip and the number of interpolated samples and R_α depends on fading coefficients

$$X(\tau) = \frac{|\tau|^2}{(S-|\tau|)^2} = \frac{|\tau/S|^2}{(1-|\tau|/S)^2}, \quad R_\alpha = 1/2\beta. \tag{90}$$

The probability of error conditioned on delay can be calculated as

$$\begin{aligned} P_e(\tau) &= \frac{1}{2} erfc \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2}\right)^{-1/2} = \frac{1}{2} erfc \left(\frac{2}{\eta_{Zi}^2}\sum_{x=A}^{F}\sigma^2(x|\tau)\right)^{-1/2} \\ &= \frac{1}{2} erfc\left(\frac{\psi}{\beta} + \frac{N-2}{\beta} + \frac{N}{\beta}X(\tau) + [1+X(\tau)]\left(\frac{E_b}{N_0}\right)^{-1}\right) \end{aligned} \tag{91}$$

which can be also obtained from (51) for $R_\alpha = 1/2\beta$ and $\alpha_\Sigma^2 = 4\beta^2$.

6.2 System with Flat Fading Channel

If only Gaussian noise is present in the channel, the fading coefficients in (9) can be equated by one, i.e., $\alpha_t = 1$, and the expression for the decision variable can be obtained in this form

$$
\begin{aligned}
z_i^g &= (S-|\tau|)\cdot\alpha\gamma_i^g\sum_{t=1}^{2\beta}x_{ti}^{g2}+|\tau|\cdot\alpha\gamma_i^g\sum_{t=1}^{2\beta}x_{ti}^g x_{ti\tau}^g \\
&+(S-|\tau|)\cdot\alpha\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\gamma_i^n x_{ti}^n x_{ti}^g+|\tau|\cdot\alpha\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\gamma_i^n x_{ti}^n x_{ti\tau}^g \\
&+(S-|\tau|)\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti}^g+|\tau|\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti\tau}^g
\end{aligned}
\quad (92\ )
$$

Suppose the i-th bit transmitted is +1, which does not have influence on the generality of this analysis. Than the decision variable can be expressed in this form

$$
\begin{aligned}
z_i^g &= (S-|\tau|)\cdot\alpha\sum_{t=1}^{2\beta}x_{ti}^{g2}+|\tau|\cdot\alpha\sum_{t=1}^{2\beta}x_{ti}^g x_{ti\tau}^g \\
&+(S-|\tau|)\cdot\alpha\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\gamma_i^n x_{ti}^n x_{ti}^g+|\tau|\cdot\alpha\sum_{t=1}^{2\beta}\sum_{n=1,n\neq g}^{N}\gamma_i^n x_{ti}^n x_{ti\tau}^g \\
&+(S-|\tau|)\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti}^g+|\tau|\cdot\sum_{t=1}^{2\beta}\xi_{ti}x_{ti\tau}^g \\
&= A+B+C+D+E+F
\end{aligned}
\quad (93)
$$

The mean values conditioned on the delay are

$$
E\{A|\tau\}=E\left\{(S-|\tau|)\alpha\sum_{t=1}^{2\beta}x_{ti}^{g2}\right\}=(S-|\tau|)b\sqrt{\pi/2}2\beta P_c=\eta_{Zi}\,. \quad (94)
$$

The remaining mean values are zero, as follows

$$
E\{B|\tau\}=E\{C|\tau\}=E\{D|\tau\}=E\{E|\tau\}=E\{F|\tau\}=0\,. \quad (95)
$$

Therefore the squared value of the mean of the decision variable is

$$
\eta_{Zi}^2=E^2\{A|\tau\}=(S-|\tau|)^2b^2\pi P_c^2(2\beta)^2/2\,, \quad (96)
$$

The variances of random function A, conditioned on τ, can be calculated using this general expression

$$
\sigma^2(A|\tau)=E\left\{\left[(S-|\tau|)\alpha\sum_{t=1}^{2\beta}x_{ti}^{g2}\right]^2\right\}-E^2\{A|\tau\}=(S-|\tau|)^2E\{\alpha^2\}E\{\left(\sum_{t=1}^{2\beta}x_{ti}^{g2}\right)^2\}-E^2\{A|\tau\} \quad (97)
$$

which can be simplified as follows

$$
\begin{aligned}
\sigma^2(A|\tau) &= (S-|\tau|)^2 2b^2[2\beta P_{x4}+2\beta(2\beta-1)P_c^2]-(S-|\tau|)^2 2b^2 2\beta P_c^2\beta\pi/2 \\
&= (S-|\tau|)^2 2\beta 2b^2 P_c^2[P_{x4}/P_c^2-(2\beta-1)-\beta\pi/2] \\
&= (S-|\tau|)^2 2\beta 2b^2 P_c^2[\psi-\beta(2+\pi/2)+1]
\end{aligned}
\quad (98)
$$

where, as we have already said, $\psi = P_{x4} / P_c^{\ 2}$ is a spreading sequence factor that depends on the statistical properties of a particular sequence. The other variances are

$$\sigma^2(B\,|\,\tau) = E\left\{\left[|\,\tau\,|\,\alpha\sum_{t=1}^{2\beta} x_{ti}^g x_{ti\tau}^g\right]^2\right\} = |\,\tau\,|^2\, 2b^2 2\beta P_c^2\,, \tag{99}$$

$$\sigma^2(C\,|\,\tau) = (S - |\,\tau\,|)^2\, 2b^2 (N-1) 2\beta P_c^2\,, \tag{100}$$

$$\sigma^2(D\,|\,\tau) = |\,\tau\,|^2\,(N-1) 2b^2 2\beta P_c^2\,, \tag{101}$$

$$\sigma^2(E\,|\,\tau) = (S - |\,\tau\,|)^2\, 2\beta\sigma^2 P_c \text{ , and} \tag{102}$$

$$\sigma^2(F\,|\,\tau) = |\,\tau\,|^2\, 2\beta\sigma^2 P_c\ . \tag{103}$$

Having in mind these relations hold $R_\alpha = \sum_{t=1}^{2\beta}\alpha_t^2 (\sum_{t=1}^{2\beta}\alpha_t)^{-2} = (2\beta\alpha^2)(2\beta\alpha)^{-2} = 1/2\beta$ and $\alpha_\Sigma^2 = 4\beta^2$, the ratios of the doubled variance and the squared values of the mean can be found as follows

$$\frac{2\sigma^2(A\,|\,\tau)}{E^2\{A\,|\,\tau\}} = \frac{4[\psi - \beta(2 + \pi/2) + 1]}{\pi\beta} \tag{104}$$

$$\frac{2\sigma^2(B\,|\,\tau)}{E^2\{A\,|\,\tau\}} = \frac{4}{\pi\beta} X(\tau)\,, \tag{105}$$

$$\frac{2\sigma^2(C\,|\,\tau)}{E^2\{A\,|\,\tau\}} = \frac{4(N-1)}{\pi\beta}\,, \tag{106}$$

$$\frac{2\sigma^2(D\,|\,\tau)}{E^2\{A\,|\,\tau\}} = \frac{4(N-1)}{\pi\beta} X(\tau)\,, \tag{107}$$

$$\frac{2\sigma^2(E\,|\,\tau)}{E^2\{A\,|\,\tau\}} = \frac{2\sigma^2}{b^2\pi\beta P_c} X(\tau) = \frac{2}{b^2\pi}\frac{N_0}{E_b}\,, \tag{108}$$

$$\frac{2\sigma^2(F\,|\,\tau)}{E^2\{A\,|\,\tau\}} = \frac{2}{b^2\pi}\frac{N_0}{E_b}\,, \tag{109}$$

where X depends on the delay of the chip and the number of interpolated samples and R_α depends on fading coefficients

$$X(\tau) = \frac{|\,\tau\,|^2}{(S - |\,\tau\,|)^2} = \frac{|\,\tau / S\,|^2}{(1 - |\,\tau\,| / S)^2}\,, \quad R_\alpha = 1/2\beta\,. \tag{110}$$

The probability of error conditioned on delay can be calculated as

$$
\begin{aligned}
P_e(\tau) &= \frac{1}{2} erfc \left(\frac{2\sigma_{Zi}^2}{\eta_{Zi}^2}\right)^{-1/2} = \frac{1}{2} erfc \left(\frac{2}{\eta_{Zi}^2}\sum_{x=A}^{F}\sigma^2(x\,|\,\tau)\right)^{-1/2} \\
&= \frac{1}{2} erfc \left(\frac{4[\psi-\beta(2+\pi/2)+1]}{\pi\beta} + \frac{4}{\pi\beta}X(\tau) + \frac{4(N-1)}{\pi\beta} + \frac{4(N-1)}{\pi\beta}X(\tau) + [1+X(\tau)]\left(\frac{E_b}{N_0}\right)^{-1} \right). \qquad (111) \\
&= \frac{1}{2} erfc \left(\frac{4[\psi-\beta(2+\pi/2)+N]}{\pi\beta} + \frac{4N}{\pi\beta}X(\tau) + [1+X(\tau)]\frac{2}{b^2\pi}\left(\frac{E_b}{N_0}\right)^{-1} \right)
\end{aligned}
$$

7. CONCLUSIONS

In this Report a theoretical model of a CDMA system, which can be also applied to the analysis of DSSS systems, is present for the case of imperfect synchronisation of spreading sequences. Two cases are analysed in detail. The first system analysed assumed that the channel is characterised by the Rayleigh fading and additive white Gaussian noise. In second system it was assumed that an interleaver/deinterlever structure is incorporated into the system. The general expressions for the probability of error can be relatively easily applied for particular system analysis, as illustrated on the example of channels with Gaussian noise in Section 5.4.

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