Multi-echelon Supply Chain Flexibility Enhancement through Detecting Bottlenecks

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Abstract This study suggests a supply chain design deploying a novel idea from production planning. The idea of capacity bottlenecks is used to improve flexibility in a multi-echelon multi-product supply chain. We suggest an optimization model that focuses on optimal capacity allocations to bottleneck points in order to enhance overall flexibility. The proposed mixed-integer linear programming model minimizes the total cost of facility establishment as well as their utilization and transportation cost. The performance of suggested model is investigated by several test problems with uncertainty in demand, cost, capacity, and product specifications. The results indicate the superiority of the suggested model to the previous flexibility formulation method. According to the numerical results, the proposed model decreases the total supply chain cost by up to 16% on average. Another advantageous feature of the proposed model is its capability of solving previously insoluble test problems by optimizing flexibility levels.

Keywords Supply Chain . Network Design . Flexibility . Bottleneck . Uncertainty . Multi-echelon

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1. Introduction

Supply Chains (SCs) are integrated networks of several business entities such as suppliers, manufacturers, distributors, retailers, and customers (Simchi-Levi 2000). Nowadays, supply chains are the main means of support in global logistics and their network design has been focused by scholars using the mathematical models and algorithms designed to increase productivity within an extended scope. As a supply chain is in charge of operations including but not limited to material procurement, components transportation, and product distribution, it requires approaches to optimize objectives such as total cost, service level, lead time, inventory cost, production time, and so forth.

SC network design usually has substantial impacts on long-term performance of the included business entities as well as the whole logistic system. Although designs for the SC network address challenges such as facility location, capacity planning, entities allocation, and material flow, it might need a reconfiguration whenever emerging issues shape a new strategy for the network. For instance, innovative product development necessitates changes in capacity planning and material flow. As an attempt to deal with such problems, flexibility as a new measure is being currently focused on by the contemporary researchers (Seebacher and Winkler 2015). Creating potential and unutilized capacities in the SC's capacity constrained entities enhances its overall performance in terms of time and cost by making it flexible to inevitable planning changes (Calantone and Dröge 1999). Having potential capacity established in SC key points, decrease in production time facilitates an agile introduction of new products to the market which in turn gains competitive advantage (Swafford, Ghosh, and Murthy 2006).

Flexibility concepts are diverse, as they can be looked over from different perspectives (Esmaeilikia et al. 2014b). One may find facility capacity an appropriate platform for implementing flexibility practices, while others implement flexibility towards product delivery (Cheshmehgaz, Desa, and Wibowo 2013), sourcing (Yu, Zeng, and Zhao 2009), sharing inventory risk (Lai, Debo, and Sycara 2009), transportation system (Baffo, Confessore, and Stecca 2013), and volume and product mix (Hallgren and Olhager 2009; Hasuike and Ishii 2009; Fernandes, Gouveia, and Pinho 2012). Flexibility enhancement has positive effects on other main objectives of SC planning. For instance, Dell, the high-tech manufacturer, has gained a higher level of service by decreasing delivery time as a consequence of flexible delivery practices (Thatte 2007).

As facility capacity is influenced by different factors such as, demand, level of technology, and safety stock, it is too uncertain not to be reconfigured after being planned based on the initial network parameters. As a matter of fact, capacity planning is one of the challenging decisions in SC network design. Flexible capacity planning is a technique with a focus on the reduction of reconfiguration impact by allocating unutilized capacities to the critical facilities within an SC network.

Bottleneck points, or bottlenecks for short, are metaphors for referring to components that are determinant of holistic system parameters such as throughput, capacity, and cycle time as they receive more input than their constrained capacity. Simply speaking, from a line balance point of view, if the sum of processing durations on average exceeds the time between two consequent arrival of parts to a workstation, waiting time and number of works in process will increase while the parts make the workstation crowded. Identifying bottlenecks and increasing their capacity to the optimal value are the main steps of adapting theory of constraints to facility capacity planning. This study aimed to design a four level SC by minimizing the total cost of facility establishment as well as their utilization and transportation cost while taking bottlenecks flexibility into account to achieve a productive as well as a flexible SC network design. Incorporating mathematical constraints into the model to detect bottlenecks and optimize their capacities, the overall supply chain capacity is increased by investing on the minimum additional capacities it needs to reach a certain flexibility level. The main motivation behind this research is addressing one of the key challenges raised by contemporary supply chain researchers arguing that supply chain flexibility planning should take heed of achievable/available flexibility options in other network entities (Esmaeilikia et al. 2014a).

In this research a multi echelon multi product supply chain network design, consisting of suppliers, manufacturers, warehouses, and customers which each product is made of some Work in Process (WIP) and each WIP can be produced by some manufacturers equipped with different production modules is considered. For flexible designing of this SC network a new methodology of flexibility of capacities including the allocation potential unutilized capacities in entities by considering and identifying bottleneck points is used and flexibility of capacities of the whole supply chain is increased to the optimal values.

2. Supply Chain vs. Flexibility Concepts

Most of the supply chain network design studies take the parameters of the problem to be deterministic. In contrast, network design is a strategic decision which calls for considering parameter variation in the long-term (Baghalian, Rezapour, and Farahani 2013). This issue necessitates taking uncertainty into account, as it is undeniably an essential factor in today's business (Corominas 2013). In the previous research, some models, in which most of the parameters are uncertain, have replaced traditional modeling approaches. One of the most crucial parameters of a network design problem is customers' demand which behaves stochastically (Schütz, Tomasgard, and Ahmed 2009; Petridis 2013). In what follows, the current stochastic network design studies are briefly reviewed and then studies of flexible manufacturing and flexibility in supply chains are discussed, respectively.

Let us start by discussing different modeling approaches used in network design problems. SC researchers have taken different approaches to deal with optimization in stochastic environment including but not limited to minimizing expected value, minimizing deviations from goals, minimizing maximum costs, and optimization with soft constraints. These approaches can be divided into three main approaches comprising stochastic programming, fuzzy programming, and stochastic dynamic programming (Ben-tal and Nemirovski 2000). Stochastic programming includes four different types of optimization models known as linear stochastic, integer stochastic, non-linear stochastic, and robust stochastic. It is noteworthy that the variation of parameters is argued to be accounted for by sensitivity analysis within deterministic approaches. However, there are others who argue that sensitivity analysis is merely a technique to question the quality of solution and is not capable of creating robust solutions. Moreover, the counter argument is stronger as sensitivity analysis is not possible for models with a high number of variable parameters due to the large number of combinatorial scenarios.

2000s was the period of groundbreaking research in supply chain and uncertainty, during which two revolutionary papers were published focusing on product uncertainty and bullwhip effect, a topic that interested many SC researchers (Lee 2002; Lee, Padmanabhan, and Whang 2004). Afterwards, Santoso et al. developed a stochastic programming model in which a decomposition method was deployed to deal with uncertainty in facility planning and design in an SC network (Santoso et al. 2005). They contributed to the approach towards SC modeling as well as novelty in solution methodology. Their proposed decomposition technique can be used to obtain high quality solutions for real-life scenarios.

There is a group of studies concerning performance enhancement under environmental uncertainty. One of the first studies of this group goes back to 1987 when Swamidass and Newell evaluated a number of manufacturers in the U.S. and argued that those with higher flexibility can perform better under uncertainty (Swamidass and Newell 1987). Almost the same conclusion was drawn in (Ward et al. 1995) whose target manufacturers were located in Singapore. Five years later, almost the same team of researchers analyzed the impact of environmental uncertainty on flexibility (Ward and Duray 2000). Moreover within the same context, it was concluded by (Chase, Aquilano, and Jacobs 2001) that outsourcing and customized manufacturing require manufacturers to search for flexible approaches to satisfy the demand. In 2005, Sánchez and Pérez studied 126 automobile spare parts manufacturers in Spain and found a strong relationship between environmental uncertainty and potential increase in performance due to adapting flexible practices (Sánchez and Pérez 2005). Twenty years after the publication of the first study, Avittathur and Swamidass evaluated 26 production plants in India and proved that the flexibility of supply chain entities leads to increase in the performance of production plants (Avittathur and Swamidass 2007).

Flexible Supply chain network design require innovative business practices (Sushil 2012). An innovative practice can be the adoption of manufacturing and production planning techniques to enhance overall flexibility. Vokurka and O'Leary-Kelly reviewed the manufacturing flexibility studies and classified them into 15 divisions with distinctive definitions based on their scopes and dynamics. They argued that there were four element including strategy, environmental factors, organizational features, and technology that drove industries into planning and designing under flexibility concepts (Vokurka and O'Leary-Kelly 2000). There are three other studies concerning manufacturing flexibility taxonomy suggested for further investigation (D'Souza and Williams 2000; Koste and Malhotra 1999; Tat Leung and Sheen 1993).

Although manufacturing flexibility is essential, flexibility in supply chains should have been investigated beyond the scope of manufacturing. Supply chain flexibility was first defined by Lau as the production plants capability of encountering uncertainty by their inner components and knowledge (Lau 1996). The study also proposed a framework for designing flexible supply chains. This study can be regarded as the pioneer of supply chain flexibility. Fisher compared and contrasted supply chain flexibility and optimization and introduced the nature of demand as the determinant factor for deciding upon strategic network design.

According to his findings, if the future demand were predictable, then strategic design should optimize the capacity. On the contrary, flexible capacity was necessary when the future demand was unpredictable (Fisher 1997). Five strategic flexible SC design models were introduced by Calantone and Dröge which defined flexibility concept in five different areas including customized production, capacity, New Product Development (NPD), distribution, and quick responding to market (Calantone and Dröge 1999).

Moreover, the main argument of some agile supply chain studies conceptually matches the concepts in question while there is no trace of the term "flexibility". For instance, Mason-Jones et al. argued that designing a supply chain can be an integration of optimization and agility to face unpredictable circumstances (Mason-Jones, Naylor, and Towill 2000). Discussing agility and flexibility, one may refer to another study which argues that agility is required for establishing a supply chain with a high level of responsiveness. In this study, the author stated that agility was composed of four elements including market demand response, virtual capacity, process integration, and cooperation (Christopher 2000).

Integration of speed and flexibility as two ingredients of supply chain agility was introduced by (Prater, Biehl, and Smith 2001). According to the author's discussions, as manufacturers conceived that simple optimization of costs will not make them competitive in the current uncertain markets, they decided to take heed of finding the appropriate level of flexibility while lowering the production time (Duclos, Vokurka, and Lummus 2003). Besides, they realized that satisfying customers' demand requires flexibility within all the entities if the products are frequently transported among them (Gunasekaran, Patel, and Tirtiroglu 2001).

Conceptual models of flexible supply chains were categorized by (Duclos, Vokurka, and Lummus 2003). In what follows, six different types of flexibility concepts in supply chains including flexibility in operation systems, market, logistics, supply, organization, and information systems are discussed briefly.

Flexibility of operation systems is defined as reconfiguration capabilities of supply chain entities in the case of change in product quantity and/or its features. Agile operation, as a value-adding capability, was suggested by (Anderson and Lee 1999) to tackle unpredictable changes of customers' requests. This concept was also noticed by (Radjou 2000) after assessing 50 production plants in terms of the capability of responding to dynamic conditions and delivering products to customers in a timely manner.

Market flexibility is one of the most essential elements of gaining competitive advantage by designing and introducing new products based on the information gathered from different levels of supply chain about customers' needs, potentially available materials, and technology. This concept was also defined by (Fisher 1997) as successful responding to market demand.

Logistics flexibility focuses on the optimality of costs in sending and receiving while supplies and customers are in change. The scope of this process encompasses packaging, transportation, documentation, contracting, warehouse and inventory management, reverse logistics, and routing (Ricker and Kalakota 1999). Flexible logistics provides different requests with different processes to satisfy volatile demands of customers (Fuller, O'Conor, and Rawlinson 1993).

Supply flexibility demonstrates connection to different suppliers to request in the case of change in product material and components. It enhances capability of manufacturers in finding new providers and using different levels of their capacities (Jordan and Michel 2000).

Organizational flexibility has been investigated by many researchers including (Hall and Parker 1993; Lau 1996; Miles 1989; Vokurka and O'Leary-Kelly 2000). It introduces the capability of converging personnel skills to satisfy customers' demand.

Finally, flexibility of information systems integrates informational entities to improve the process of customers' demand satisfaction at the time of change. This concept has been investigated by a number of researchers including (Lee and Hong 2002; Reddy and Reddy 2002).

Although there are many studies focusing on flexibility concepts and the related theoretical discussions, lack of flexibility modeling and analysis can be noticeable in the current literature (Esmaeilikia et al. 2014a). The theory and concepts of flexible capacity planning were first introduced by (Slack 1987). Despite the success of supply chain literature in increasing productivity from a holistic viewpoint, concerns have been reported regarding lack of analytical models in the SC literature which take flexibility issues into account (Lau 1996; Fisher 1997; Calantone and Dröge 1999; Gunasekaran et al. 2001; Duclos et al. 2003). It seems that modeling complex SC networks, though necessary, has prevented researchers from simple prospective considerations like flexible capacity planning. Singh and Acharya define SC flexible capacity planning as the capability to increase the whole system capacity as needed (Singh and Acharya 2013). This concept is also referred to as supply chain expansion flexibility (Singh and Acharya 2013; Tiwari, Tiwari, and Samuel 2015). A broader definition for the same concept is suggested by Sahu et al. as the overall ability to satisfy a variable market without excessive cost, delay, interruptions, or performance losses (Sahu et al. 2015). Such considerations are essential as the manufacturers in common types of SCs deal with diverse demands of products with short life-cycle in an uncertain competitive environment. This issue calls for prospective approaches towards SC network design capable of meeting new challenges beyond the economic efficiency whether obtained in a deterministic environment or a stochastic one.

To the best of our knowledge only two research studies consider flexibility of the capacities which is represented by capacity slacks of operational resources (Voudouris and Consulting 1996; Sabri and Beamon 2000).

Voudouris considers flexibility of capacities as a performance measure of a chemical industry SC’s flexibility while Sabri and Beamon deployed the theory that was first introduced by Slack in 1987. It was a secondary objective function in an SC optimization model to maximize the sum of potential capacities of entities while making the network design as cost-effective as possible. Although such a model results in a more flexible network design, the potential capacities are all naively allocated to the entities with the least cost of capacity upgrade. The most essential consideration which is missing in the current literature is bottleneck points and how increasing their capacity results in achieving higher capacity for the whole system.

The necessity of modeling and analyzing flexibility stems from three different motivations. First, costs of reconfiguration for a distinctive product needs to be minimized in the current markets where customers request their specially designed products (Gilmore and Pine 2nd 1996). Second, there are industries, usually possessing high levels of technology, in which manufacturers should be able to quickly increase (decrease) their production size up to 20% which makes capability of adapting to different levels of production essential (Fisher 1997). Finally, manufacturers permanently deal with NPD issues. From a strategic management point of view, flexibility enables manufacturers to handle NPD more successfully, which in turn enhances their competitive advantage in the market (Swafford, Ghosh, and Murthy 2006).

Designing flexible supply chains is an active area of research with practical applications already in demand in high-tech and automotive industries.

A flexible supply chain is the one which has additional unutilized capacity allocated to its key entities that make it adjustable to uncertainty factors such as fluctuations in diverse demands, changes in corporate strategy, short life-cycle products, new product development, and so forth. Implementing flexibility practices in SCs is a fundamental stage in adaptive and agile system management, as it embodies all major activities of the product life-cycle from raw material processing to the product delivery.

3. Problem Statement

Consider a four level SC, consisting of suppliers, manufacturers, warehouses, and customers, which is designed to meet the customers' demand for multiple products via warehouses. Each product is made up of a number of Works in Process (WIPs). The suppliers are responsible for providing the manufacturers with material and components which are converted and assembled into WIPs by the manufacturers equipped with different production modules. Each module is capable of performing processes to produce a number of products but not all. In other words, different production modules are required to manufacture a product. As establishing production modules such as assembly lines is expensive, the manufacturers do not necessarily have all the modules required for manufacturing a certain product. This issue necessitates deciding upon the establishment of production modules besides the manufacturing plant itself while considering transportation of WIPs within the level of manufacturers in the network.

Having manufacturing processes completed, products are then transported to the warehouses whose locations also make another decision variable. Allocation of network entities to each other is another decision to be made which includes supplier-to-manufacturer, manufacturer-to-manufacturer, manufacturer-to-warehouse, and finally warehouse-to-customer allocations. Moreover, the establishment of each entity in the network and each module in an entity involves fixed and variable types of costs. Concerning optimization objectives, the first objective function of the optimization problem is total cost including not only transportation, but also facility and module establishment as well as their utilization. New constraints are also introduced to deal with the main challenge of this study about flexibility of capacities. Prior to the allocation of unutilized capacities, a set of constraints is to be developed as means of identifying bottleneck points. Afterwards, the flexibility of the whole supply chain is increased by allocating potential unutilized capacities to such bottleneck points to their corresponding optimal values based on the utilized and nominal capacities and structure of the network.

4. Mathematical Modeling

In this section, two different optimization models are introduced. First, a basic model with the flexibility formulation approach of (Sabri and Beamon 2000) is developed which consists of the objective function outlined in (4.1) to (4.4) and constraints to be discussed one by one in (4.6) to (4.37). Then, a more sophisticated optimization model is presented embodying the concept of flexibility enhancement by detecting bottlenecks. The objective function of this model is modified to match the additional constraints discussed in the final part of this section in (4.38) to (4.64).

4.1. Notation

In this section, the notation used in mathematical modeling is delineated. Shorthand used includes # for number and O.W. for otherwise. Indices, parameters, and variables are as follows:

Sets and indices

$m \in M$	Set of potential locations for establishing production plants
$n \in N$	Set of production modules to be established
$t \in T$	Set of available suppliers (providers)
$p \in P$	Set of products based on production modules
$r \in R$	Set of parts
$q \in Q$	Set of WIPs
$f \in F$	Set of potential locations for establishing warehouse
$e \in E$	Set of customers
$s \in S_w$	Set of possible demand scenarios

Parameters

C_m	Fixed cost for production plant establishment in location m ∈ M
${L_{nm}}^{\phi}$	Cost of establishing module n ∈ N with capacity ϕ in production plant at m ∈ M
${CD_f}^{k}$	Fixed cost for capacity level k warehouse establishment in location f ∈ F
CR_{rtm}	Cost of transporting component r ∈ R from supplier t ∈ T to the production plant at m ∈ M
$CQ_{qmm'}$	Cost of transporting WIP q ∈ Q from production plant at m ∈ M to the production plant at m' ∈ M
CP_{pmf}	Cost of transporting product p ∈ P from production plant at m ∈ M to the the warehouse at f ∈ F
CF_{pfe}	Cost of transporting product p ∈ P from warehouse at f ∈ F to customer e ∈ E
G_{pm}	Cost of producing a unit of product p ∈ P at the production plant at m ∈ M
J_{qn}	Cost of producing a unit of WIP q ∈ Q by module n ∈ N
H_{rt}	Cost of providing a unit of component r ∈ R by supplier t ∈ T
W_{qn}	Capacity of module n ∈ N utilized by WIP q ∈ Q
ZT_{tr}	Capacity of supplier t ∈ T in providing component r ∈ R
${ZK_n}^{\phi}$	Capacity of type ϕ module n ∈ N
${ZD_f}^{k}$	Capacity of type k warehouse at f ∈ F
WP_p	Capacity utilized by product p ∈ P at the warehouse
D_{ep}^{s}	Demand of customer e ∈ E for product p ∈ P based on scenario s ∈ S_w
Pr_s	Probability of each demand scenario s ∈ S_w
$\psi 1_{rp}$	# of components r ∈ R required for producing a unit of product p ∈ P
$\psi 2_{pq}$	# of WIPs q ∈ Q required for producing a unit of product p ∈ P
$\psi 3_{rq}$	# of components r ∈ R required for producing a unit of WIP q ∈ Q
UP_{pq}	Equal to one if producing product p ∈ P requires WIP q ∈ Q, O.W. equals zero
V_{qn}	Equal to one if producing WIP q ∈ Q requires module n ∈ N, O.W. equals zero

Decision Variables

X_{rtm}	Quantity of component r ∈ R transported from supplier t ∈ T to the production plant at m ∈ M
Y_{pm}	Quantity of product p ∈ P produced at production plant at m ∈ M
$Y2_{qnm}$	Quantity of WIP q ∈ Q produced by module n ∈ N at production plant at m ∈ M
A_{pmf}	Quantity of product p ∈ P transported from production plant at m ∈ M to warehouse at f ∈ F
λ_{pfe}	Quantity of product p ∈ P transported from warehouse at f ∈ F to customere ∈ E
$B_{qmm'}$	Quantity of WIP q ∈ Q transported from production plant at m ∈ M to the production plant at m′ ∈ M
U_m	Equal to one if a production plant is established at m ∈ M, O.W. equals zero
$U2_{nm}{}^{\phi}$	Equal to one if production plant at m ∈ M is equipped with capacity level ϕ module n ∈ N , O.W. equals zero
$U3_f{}^k$	Equal to one if a capacity level k warehouse is established at f ∈ F , O.W. equals zero
$U4_{tm}$	Equal to one if supplier t ∈ T is connected to production plant at m ∈ M, O.W. equals zero
$U5_{mf}$	Equal to one if production plant at m ∈ M is connected to warehouse at f ∈ F, O.W. equals zero
$U6_{fe}$	Equal to one if warehouse at f ∈ F is connected to customere ∈ E, O.W. equals zero
$U7_{mm'}$	Equal to one if production plant at m ∈ M is connected to the production plant at m′ ∈ M, O.W. equals zero

4.2. Model Formulation

The objective function of the basic model is to minimize total cost including supply chain network establishment cost as well as transportation cost and variable production cost. In what follows, different terms of the objective function are delineated in (4.1) to (4.4) to be joined together in (4.5). Formulation of the basic model is continued by defining different constraints (4.6), (4.9) to (4.22), and (4.29) to (4.37). The main reason of introducing such a model is to build the foundation for developing a more sophisticated model to deal with the main challenge of this study, flexible capacity allocation after identifying the bottleneck points as demonstrated by introducing additional constraints for the second model.

4.2.1. Basic Model of Flexible Supply Chain Network Design under Uncertain Demand

Let us start by considering the cost of transporting parts from suppliers to manufacturers as well as manufacturing cost in (4.1). Equation (4.2) demonstrates the sum of establishment costs of production plants and different modules, cost of manufacturing WIPs by the modules, and that of transporting WIPs among the manufacturers. Costs of products transportation to warehouses and establishment of warehouses are considered in (4.3). Finally, cost of products shipment from warehouses to the customers is demonstrated in (4.4).

$$Z1 = \sum_m \sum_t \sum_r X_{rtm} * (H_{rt} + CR_{rtm}) \tag{4.1}$$

$$Z2 = \sum_m C_m U_m + \sum_{\phi} \sum_m \sum_n U2_{nm}{}^{\phi} L_{nm}{}^{\phi} + \sum_p \sum_m Y_{pm} G_{pm} + \sum_q \sum_n \sum_m Y2_{qnm} J_{qn} + \sum_q \sum_m \sum_{m'} B_{qmm'} CQ_{qmm'} \tag{4.2}$$

$$Z3 = \sum_p \sum_m \sum_f A_{pmf} CP_{pmf} + \sum_k \sum_f U3_f CD_f{}^k \tag{4.3}$$

$$Z4 = \sum_p \sum_f \sum_e \lambda_{pfe} CF_{pfe} \tag{4.4}$$

As mentioned earlier, total cost is calculated by summing up (4.1) to (4.4) which results in (4.5).

$$Z = Z1 + Z2 + Z3 + Z4 \qquad 4.5$$

Now that the terms in the objective function are fully discussed, the constraints will be introduced one by one. Inequality (4.6) states supplier limited capacity of providing parts.

$$\sum_{m} X_{rtm} \leq \sum_{m} ZT_{rt} U4_{tm} \qquad \forall\, t, r \qquad 4.6$$

The next constraint deals with satisfying assembly lines by providing them with required parts. Inequality (4.7) belongs to the second degree nonlinear constraint type which is linearized in (4.9) and (4.10) using a condition in (4.8).

$$\sum_{p} \psi1_{rp}\, Y_{pm} + \sum_{\phi} \sum_{q} \sum_{n} U2_{nm}{}^{\phi} UV_{nr} Y2_{qnm} \psi3_{rq} \leq \sum_{t} X_{rtm} \qquad \forall\, r \in R, m \in M \qquad 4.7$$

$$if \sum_{\phi} U2_{nm}{}^{\phi} = 1 \;\; \Rightarrow \;\; Y2_{qnm} \geq 0 \qquad \forall q \in Q, n \in N, m \in M \qquad 4.8$$

$$\Rightarrow \begin{cases} Y2_{qnm} \leq M \sum_{\phi} U2_{nm}{}^{\phi} & \forall q \in Q, n \in N, m \in M \qquad 4.9 \\ \sum_{p} \psi1_{rp} Y_{pm} + \sum_{q} \sum_{n} UV_{nr} Y2_{qnm} \psi3_{rq} \leq \sum_{t} X_{rtm} & \forall r \in R, m \in M \qquad 4.10 \end{cases}$$

Limited capacity of warehouses is taken care of in (4.11). As the products transported from the manufacturing plants to the warehouses should be equal to their production quantity, (4.12) is considered as the next constraint. Similar to the previous one, in-flow and out-flow of warehouses should be balanced which generates (4.13). Moreover, Constraint (4.14) considers the satisfaction of customers' demand. Inequality (4.15) guarantees that the demands of the customers are satisfied by the established warehouses. Since establishment of at least one module is a prerequisite for establishing a production plant, (4.16) is required to be considered in the model. As an attempt to make the supply chain capable of providing all types of defined products, at least establishing one of each module is required. The corresponding constraint is stated in (4.17). Obviously as stated in (4.18), only established production plants are capable of producing a product. Similarly, (4.19) states that the products can only be sent away to the established warehouses. Each production module has a limited capacity as stated in (4.20). As each supplier is only capable of providing certain types of parts and materials, feasibility of suppliers in providing the requested is taken care of in (4.21). Customers' demands should always be satisfied, which is resolved by considering (4.22).

$$\sum_{p} \sum_{m} A_{pmf} WP_{p} \leq \sum_{k} ZD_{f}{}^{k} U3_{f}{}^{k} \qquad \forall f \in F \qquad 4.11$$

$$Y_{pm} = \sum_{f} A_{pmf} \qquad \forall p \in P, m \in M \qquad 4.12$$

$$\sum_{m} \sum_{f} A_{pmf} = \sum_{f} \sum_{e} \lambda_{pfe} \qquad \forall p \in P, m \in M \qquad 4.13$$

$$\sum_{f} \lambda_{pfe} \geq \sum_{s} Pr_{s} D_{ep}^{s} \qquad \forall p \in P, e \in E \qquad 4.14$$

$$\lambda_{pfe} \leq MU6_{fe} \qquad \forall\, p \in P, f \in F, e \in E \qquad 4.15$$

$$\sum_{\phi}\sum_{n} U2_{nm}{}^{\phi} \leq MU_m \qquad \forall\, m \in M \qquad 4.16$$

$$\sum_{\phi}\sum_{m} U2_{nm}{}^{\phi} \geq 1 \qquad \forall\, n \in N \qquad 4.17$$

$$Y_{pm} \leq MU_m \qquad \forall\, m \in M, p \in P \qquad 4.18$$

$$\sum_{m}\sum_{p} A_{pmf} \leq MU3_f \qquad \forall f \in F \qquad 4.19$$

$$\sum_{q} W_{qn} Y2_{qnm} \leq \sum_{\phi} ZK_n{}^{\phi} U2_{nm}{}^{\phi} \qquad \forall n \in N, m \in M \qquad 4.20$$

$$\sum_{m} X_{rtm} \leq MT1_{tr} \qquad \forall n \in N, m \in M \qquad 4.21$$

$$\sum_{f} U6_{fe} \geq 1 \qquad \forall e \in E \qquad 4.22$$

Based on the assumptions mentioned earlier, there are two cases for a manufacturer producing a product. It may have all the required production modules; in that case, we deal with Inequality (4.23). Otherwise, the production necessitates manufacturer-to-manufacturer transportation and the corresponding inequality is stated in (4.24) and (4.25).

$$if \begin{cases} Y_{pm} > 0 \\ UP_{pq} = 1 \\ V_{qn} = 1 \end{cases} \qquad \begin{matrix} \forall\, m \in M, p \in P\; n \in N, q \in Q \qquad 4.23 \\ \\ \forall\, m \in M, p \in P, q \in Q \qquad 4.24 \end{matrix}$$

$$\Rightarrow \begin{cases} if \sum_{\phi} U2_{nm}{}^{\phi} = 1 \longrightarrow Y2_{qnm} \geq Y_{pm}\psi2_{pq} \quad I \\ if \sum_{\phi} U2_{nm}{}^{\phi} = 0 \longrightarrow \begin{cases} \sum_{m'} B_{qm'm} \geq Y_{pm}\psi2_{pq} \\ \sum_{m'} Y2_{qnm'} \geq Y_{pm}\psi2_{pq} \quad II \end{cases} \end{cases} \qquad \forall\, m \in M, p \in P\; n \in N, q \in Q \qquad 4.25$$

The first case can be formulated as demonstrated in (4.26).

$$I \Rightarrow if \sum_{\phi} U2_{nm}{}^{\phi} = 1 \longrightarrow Y2_{qnm} \geq Y_{pm} UP_{pq} V_{qn} \psi2_{pq} \qquad \forall\, m \in M, p \in P\; n \in N, q \in Q \qquad 4.26$$

Formulating (4.24) and (4.25) is a little tricky. First, we need to consider (4.27) and (4.28).

$$II \Rightarrow\; if \sum_{\phi} U2_{nm}{}^{\phi} = 0 \qquad \forall\, m \in M, p \in P, q \in Q \qquad 4.27$$

$$\longrightarrow \begin{cases} \sum_{m'} B_{qm'm} \geq \sum_{p} Y_{pm} UP_{pq} V_{qn} \psi2_{pq} \\ \sum_{m'} Y2_{qnm'} \geq \sum_{p} Y_{pm} UP_{pq} V_{qn} \psi2_{pq} \end{cases} \qquad \forall\, m \in M, p \in P\; n \in N, q \in Q \qquad 4.28$$

Finally, the manufacturer-to-manufacturer transportation can be formulated by considering (4.29) and (4.30) as substitutes for (4.26)-(4.28).

$$\Rightarrow \begin{cases} Y2_{qnm} \geq Y_{pm} UP_{pq} V_{qn} \psi2_{pq} & \forall\, m \in M, n \in N, q \in Q \qquad 4.29 \\ \sum_{m'} B_{qm'm} \geq \sum_{p} Y_{pm} UP_{pq} V_{qn} \psi2_{pq} & \forall\, m \in M, n \in N, q \in Q \qquad 4.30 \end{cases}$$

It should be remembered that the transportation of WIPs between production plants is possible only if they are connected to one another. Inequality stated in (4.31) takes this issue into consideration. Equations declared in (4.32) and (4.33) guarantee that facility with one capacity level at most can be established in a certain location. Transportation of products between production plants and warehouses is possible only if they are connected to one another. Inequality stated in (4.34) takes this issue into consideration.

$$\sum_{q} B_{qmm'} \leq MU7_{mm'} \qquad \forall m, m' \in M \qquad 4.31$$

$$\sum_{\phi} U2_{nm}^{\phi} = 1 \qquad \forall n \in N, m \in M \qquad 4.32$$

$$\sum_{k} U3_{f}^{k} = 1 \qquad \forall f \in F \qquad 4.33$$

$$\sum_{p} A_{pmf} \leq MU5_{mf} \qquad \forall m \in M, f \in F \qquad 4.34$$

Now, we get to defining flexibility constraints based on the formulation method proposed by (Sabri and Beamon 2000). According to the concept originally delineated in (Slack 1987), the flexibility of supply chain includes three different levels. First, flexibility of supply network defined by the difference between suppliers' nominal capacities and their corresponding utilized capacities as stated in (4.35). At the second level, flexibility of production plants is defined almost similarly by differentiating their nominal and utilized capacities as pointed in (4.36). Finally, the third term of flexibility is defined at the distribution level by subtracting utilized capacities of warehouses from the corresponding nominal capacities in (4.37). So, considering the network designer own judgment, the supply chain flexibility is adjusted to the least values defined outside the mathematical model as $fel1, fel2, fel3$.

$$\sum_{t}\sum_{r}(ZT_{tr}T1_{tr} - \sum_{m} X_{rtm}) \geq fel1 \qquad 4.35$$

$$\sum_{\phi}\sum_{n}\sum_{m} U2_{nm}{}^{\phi} ZK_{n}{}^{\phi} - \sum_{m}\sum_{n}\sum_{q} Y2_{qnm} W_{qn} \geq fel2 \qquad 4.36$$

$$\sum_{k}\sum_{f} U3_{f}{}^{k} ZD_{f}{}^{k} - \sum_{m}\sum_{p}\sum_{f} A_{pmf} WP_{p} \geq fel3 \qquad 4.37$$

Therefore, multi-echelon multi-product flexible supply chain network design under stochastic demand and its flexibility constraint inspired by flexibility formulation of (Sabri and Beamon 2000) are designed as follows.

$\min Z$

$S.t$: (4.6), (4.9), (4.10), (4.11), (4.12), (4.13), (4.14), (4.15), (4.16), (4.17), (4.18), (4.19),

(4.20), (4.21), (4.22), (4.29), (4.30), (4.31), (4.32), (4.33), (4.34), (4.35), (4.36), (4.37).

In the next section, the proposed model of this study is defined considering the concept of flexibility in bottlenecks.

4.2.2. Proposed Model of Supply Chain Flexibility Enhancement through Detecting Bottlenecks

Although according to the proposed model the same network design constraints (4.6), (4.9) to (4.22), and (4.29) to (4.34) are considered, the flexibility constraints (4.35) to (4.37) are changed to a set of nine more sophisticated constraints as stated in (4.38), (4.47) to (4.50), and (4.61) to (4.64) to take bottlenecks into account. In what follows, the substitute flexibility constraint and then the modified objective function are discussed.

As the suppliers' capacity cannot be controlled, the same flexibility formulation method in (Sabri and Beamon 2000) was used, in which ε_1 denotes the least value of flexibility defined by the network designer ($\varepsilon_1 = fel1$) that requires suppliers of different parts to increase their flexibility by establishing additional unutilized capacities.

$$\sum_t \sum_r (\mathrm{ZT_{tr}T1_{tr}} - \sum_m \mathrm{X_{rtm}}) \geq \varepsilon_1 \qquad 4.38$$

At the second level, the bottleneck points of the production level are first identified by (4.39), in which ε_2 denotes the lower threshold of entities' unutilized capacity at the production level. If an entity had unutilized capacity of less than ε_2 threshold, it would be regarded as a bottleneck point of production activities. Then, its corresponding capacity would be increased to the economic value. This economic increase in capacity is formulated in (4.40). It is noteworthy that, as the number of parts in a product differs for different levels of supply chain, a simple unit conversion is used in (4.40).

$$if \sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn} \leq \varepsilon_2 \sum_\phi U2_{nm}^\phi \qquad \forall n \in N, m \in M \qquad 4.39$$

$$then \sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn} \qquad 4.40$$

$$\geq min \left\{ \min_{m,n} \left(\sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn} \right), min_{r,t} \left(\frac{ZT_{tr}T1_{tr} - \sum_m X_{rtm}}{\frac{\sum_r \sum_q \psi 3_{rq}}{|R| \times |Q|}} \right), \right.$$

$$\left. min_f \left(\left(\sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \right) \times \left(\sum_q \psi 3_{pq} \;\; /(|P| \times |Q|) \right) \right) \right\}$$

The inequality accounting for economic increase in capacity is nonlinear and mathematically complex. Below, the linearization procedure is first stated in (4.41) to (4.46) and then the substitute linear constraint is formulated in (4.47) to (4.50).

$$if \sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn} \leq \varepsilon_2 \sum_\phi U2_{nm}^\phi \qquad \forall n \in N, m \in M \qquad 4.39$$

$$then \begin{cases} z \leq \min \left(\sum_\phi U2_{n1m1}^\phi ZK_{n1}^\phi - \sum_q Y2_{qn1m1} W_{qn1} \right) & \forall n, n1 \in N, m, m1 \in M \qquad 4.41 \\ z \leq \min \left(\frac{ZT_{tr}T1_{tr} - \sum_m X_{rtm}}{(\sum_r \sum_q \psi 3_{rq})/(|R| \times |Q|)} \right) & \forall t \in T, r \in R \qquad 4.42 \\ z \leq min \left(\left(\sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \right) \times \frac{\sum_q \psi 3_{pq}}{|P| \times |Q|} \right) & \forall f \in F \qquad 4.43 \end{cases}$$

As the inequalities in (4.41) to (4.43) are less than or equal to a minimum function, the minimum function is redundant and can be omitted which results in (4.44) to (4.46).

$$if \sum_{\phi} U2_{nm}^{\phi} ZK_n^{\phi} - \sum_q Y2_{qnm} W_{qn} \le \varepsilon_2 \sum_{\phi} U2_{nm}^{\phi} \qquad \forall n \in N, m \in M \qquad 4.41$$

$$then \begin{cases} z \le \sum_{\phi} U2_{n1m1}^{\phi} ZK_{n1}^{\phi} - \sum_q Y2_{qn1m1} W_{qn1} & \forall n, n1 \in N, m, m1 \in M \qquad 4.44 \\ z \le \dfrac{ZT_{tr} T1_{tr} - \sum_m X_{rtm}}{(\sum_r \sum_q \psi3_{rq})/(|R| \times |Q|)} & \forall t \in T, r \in R \qquad 4.45 \\ z \le \left(\sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \right) \times \dfrac{\sum_q \psi3_{pq}}{|P| \times |Q|} & \forall f \in F \qquad 4.46 \end{cases}$$

Subsequently, as an attempt to linearize the conditional terms, a set of new 0-1 variables is deployed and the final form of constraints is stated in (4.47) to (4.50). $\delta1_{mn}$is the new 0-1 variable for linearizing the conditional terms and $z1_{mn}$ is the additional variable which is multiplied by negative big M and then added to the objective function. The model small value is considered as the small m.

$$\sum_{\phi} U2_{nm}^{\phi} ZK_n^{\phi} - \sum_q Y2_{qnm} W_{qn} - (m-1)\delta1_{mn} \ge \varepsilon_2 \sum_{\phi} U2_{nm}^{\phi} + 1 \qquad \forall n \in N, m \in M \qquad 4.47$$

$$\sum_{\phi} U2_{n1m1}^{\phi} ZK_{n1}^{\phi} - \sum_q Y2_{qn1m1} W_{qn1} - z1_{mn} - M\delta1_{mn} \ge -M \qquad \forall n, n1 \in N, m, m1 \in M \qquad 4.48$$

$$\frac{ZT_{tr} T1_{tr} - \sum_m X_{rtm}}{(\sum_r \sum_q \psi3_{rq})/(|R| \times |Q|)} - z1_{mn} - M\delta1_{mn} \ge -M \qquad \forall t \in T, r \in R \qquad 4.49$$

$$\left(\sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \right) \times \frac{\sum_p \sum_q \psi3_{pq}}{|P| \times |Q|} - z1_{mn} - M\delta1_{mn} \ge -M \qquad \forall f \in F \qquad 4.50$$

Thus, the identification of bottlenecks in the distribution level and economic allocation of additional capacities is formulated. Similar to the discussed production level, bottlenecks of the distribution activities are identified by (4.51), in which ε_3 denotes the lower threshold of entities' unutilized capacity at the distribution level. If a warehouse had unutilized capacity of less than ε_3 threshold, it would be regarded as a bottleneck point of distribution activities. Again, a unit conversion is used in increasing the capacities of distribution bottlenecks to their corresponding economic values, as demonstrated in (4.52).

$$if \sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \le \varepsilon_3 \sum_k U3_f^k \qquad \forall f \in F \qquad 4.51$$

$$then \sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \qquad 4.52$$

$$\ge \min\left\{ min_{m,n}\left(\frac{\sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn}}{(\sum_q \sum_p \psi 2_{pq})/(|Q| \times |P|)}\right), \right.$$

$$min_{r,t}\left(\frac{ZT_{tr} T1_{tr} - \sum_m X_{rtm}}{\frac{\sum_r \sum_p \psi 1_{rp}}{|R|\times|P|} + \frac{\sum_r \sum_p \sum_q \psi 3_{rq} \times \psi 2_{pq}}{|R|\times|Q|\times|P|}}\right),$$

$$\left. min_f\left(\sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p\right)\right\}$$

Obviously, (4.51) and (4.52) are non-linear and mathematically complex. The linearization procedure is a replica of what we had for the production level. The first stage of linearization is stated in (4.53) to (4.56) and the second stage is formulated as (4.57) to (4.60).

$$if \sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p \le \varepsilon_3 \sum_k U3_f^k \qquad \forall f \in F \qquad 4.51$$

$$then \begin{cases} z \le \sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p & \forall f \in F \qquad 4.53 \\ z \le \min\left(\dfrac{\sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn}}{\left(\sum_q \sum_p \psi 2_{pq}\right)/(|Q| \times |P|)}\right) & \forall n \in N, m \in M \qquad 4.54 \\ z \le \min\left(\dfrac{ZT_{tr} T1_{tr} - \sum_m X_{rtm}}{\frac{\sum_r \sum_p \psi 1_{rp}}{|R|\times|P|} + \frac{\sum_r \sum_p \sum_q \psi 3_{rq} \times \psi 2_{pq}}{|R|\times|Q|\times|P|}}\right) & \forall t \in T, r \in R \qquad 4.55 \\ z \le min\left(\sum_k U3_{f1}^k ZD_{f1}^k - \sum_m \sum_p A_{pmf1} WP_p\right) & \forall f, f1 \in F \qquad 4.56 \end{cases}$$

$$then \begin{cases} z \le \sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p & \forall f \in F \qquad 4.57 \\ z \le \left(\dfrac{\sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn}}{\left(\sum_q \sum_p \psi 2_{pq}\right)/(|Q| \times |P|)}\right) & \forall n \in N, m \in M, f \in F \qquad 4.58 \\ z \le \left(\dfrac{ZT_{tr} T1_{tr} - \sum_m X_{rtm}}{\frac{\sum_r \sum_p \psi 1_{rp}}{|R|\times|P|} + \frac{\sum_r \sum_p \sum_q \psi 3_{rq} \times \psi 2_{pq}}{|R|\times|Q|\times|P|}}\right) & \forall t \in T, r \in R \qquad 4.59 \\ z \le \left(\sum_k U3_{f1}^k ZD_{f1}^k - \sum_m \sum_p A_{pmf1} WP_p\right) & \forall f, f1 \in F \qquad 4.60 \end{cases}$$

At last, the final linearized substitute constraints are stated in (4.61) to (4.64), in which $\delta 2_f$ is the new 0-1 variable for linearizing the conditional terms and $z2_f$ is the additional variable which is multiplied by negative big M and added to the objective function.

$$\sum_k U3_f^k ZD_f^k - \sum_m \sum_p A_{pmf} WP_p - (m-1)\delta 2_f \geq \sum_k U3_f^k + 1 \qquad \forall f \in F \qquad 4.61$$

$$\frac{\sum_\phi U2_{nm}^\phi ZK_n^\phi - \sum_q Y2_{qnm} W_{qn}}{(\sum_q \sum_p \psi 2_{pq})/(|Q| \times |P|)} - z2_f - M \geq -M \qquad \forall n \in N, m \in M, f \in F \qquad 4.62$$

$$\frac{ZT_{tr} T1_{tr} - \sum_m X_{rtm}}{\frac{\sum_r \sum_p \psi 1_{rp}}{|R| \times |P|} + \frac{\sum_r \sum_p \sum_q \psi 3_{rq} \times \psi 2_{pq}}{|R| \times |Q| \times |P|}} - z2_f - M\delta 2_f \geq -M \qquad \forall t \in T, r \in R \qquad 4.63$$

$$\sum_k U3_{f1}^k ZD_{f1}^k - \sum_m \sum_p A_{pmf1} WP_p - z2_f - M\delta 2_f \geq -M \qquad \forall f, f1 \in F \qquad 4.64$$

Therefore, the formulation of multi-echelon multi-product flexible supply chain network design under stochastic demand with a focus on bottlenecks is performed. The final model including the modified objective function is stated in (4.65).

$$\min Z - M \sum_m \sum_n z1_{mn} - M \sum_f z2_f \qquad 4.65$$

$S.t$: (4.6), (4.9), (4.10), (4.11), (4.12), (4.13), (4.14), (4.15), (4.16), (4.17), (4.18), (4.19), (4.20), (4.21), (4.22), (4.29), (4.30), (4.31), (4.32), (4.33), (4.34), (4.38), (4.47), (4.48), (4.49), (4.50), (4.61), (4.62), (4.63), (4.64).

5. Computational Results

In this section, the proposed model is numerically compared with the basic model in which the flexibility formulation is derived by the method proposed in (Sabri and Beamon 2000). Seven different test problems were designed to evaluate the performance of the model over a wide scope.

Having a glance over the test problems in the literature, the range for the number of customers was considered ten to two hundreds. Besides, the potential warehouses were bounded from two to five with three different capacity levels. Moreover, the number of potential production plants ranged from five to twelve and the number of production modules was from three to fifteen. Also, range of the number of potential suppliers was five to thirty. The test problems varied in the number of components from five to twenty five as well as in the number of WIPs from three to ten. Finally, the range for the number of products was three to twelve. The test problems are fully characterized in Table 1.

Table 1 Description of the seven designed test problems

Problem	# of Potential Suppliers	# of Potential Production Plants	# of Production Modules	# of Potential Warehouses	# of Customers	# of Components	# of WIPs	# of Products
1	5	5	3	2	10	5	3	3
2	10	5	6	2	20	10	3	3
3	10	5	10	2	50	10	5	5
4	20	8	10	2	50	10	5	5
5	20	10	12	3	70	20	8	10
6	20	10	12	3	100	20	8	10
7	30	12	15	5	200	25	10	12

The demand of the customers was considered by four different scenarios with equal probability of 0.25 according to the uniform 10 to 30 distribution. Other parameters, which included establishment costs, transportation costs, production costs, capacities, required modules, components and WIPs, and customers' demand, are introduced in Table 2.

Table 2 Stochastic distributions for specifying the parameters of the test problems

Description	Distribution
Fixed cost for warehouse type 1 establishment	Normal(μ=100000, σ =1000)
Fixed cost for warehouse type 2 establishment	Normal(μ=500000, σ =5000)
Fixed cost for warehouse type 3 establishment	Normal(μ=1000000, σ =10000)
Capacity of warehouse type 1	Normal(μ=2000, σ =10)
Capacity of warehouse type 2	Normal(μ=10000, σ =50)
Capacity of warehouse type 3	Normal(μ=20000, σ =100)
Demand of customers for products s in different scenarios	Uniform(10,30)
Warehouse capacities utilized by products	Normal(μ=0.5, σ =0.25)
Cost of component transportation from suppliers to manufacturers	Normal(μ=20, σ =5)
Cost of WIP transportation between manufacturers	Normal(μ=20, σ =5)
Cost of products transportation from manufacturers to warehouses	Normal(μ=20, σ =5)
Cost of products transportation from warehouses to customers	Normal(μ=20, σ =5)
Cost of producing one unit of product in the production plants	Normal(μ=5, σ =5)
Cost of producing one unit of WIP in the production modules	abs(Normal(μ=5, σ =5))
Cost of producing one unit of components by the supplier	abs(Normal(μ=5, σ =5))
Components required by the production modules	Uniform(0,1)
Production plant establishment cost	Normal(μ=10000000, σ=1000000)
Fixed cost for production module type 1 establishment	Normal(μ=1000, σ =300)
Fixed cost for production module type 2 establishment	Normal(μ=5000, σ =1500)
Fixed cost for production module type 3 establishment	Normal(μ=10000, σ =3000)
Capacity of production module type 1	Normal(μ=10000, σ =100)
Capacity of production module type 2	Normal(μ=5000, σ =1500)
Capacity of production module type 3	Normal(μ=10000, σ =3000)
Supplier capability of providing components	Uniform(0,1)
WIPs required for producing products	Uniform(0,1)
Modules required for producing WIPs	Uniform(0,1)
Module capacities utilized by WIPs	Uniform(0,1)
Components required for producing products	Uniform(1,10)
WIPs required for producing products	Uniform(1,10)
Components required for producing WIPs	Uniform(1,10)
Capacity of supplier type 1	Normal(μ=200000, σ =1000)
Capacity of supplier type 2	Normal(μ=1000000, σ =5000)
Capacity of supplier type 3	Normal(μ=2000000, σ =10000)

As mentioned earlier, the test problems were designed to investigate the basic and proposed models in terms of the quality of solutions and computation time. Sabri et al. defined $fel1, fel2, fel3$ as the designers' own judgment about levels of flexibility; i.e. the least unutilized capacity at different levels of the network. On the contrary, the proposed model of this study calculated the optimal potential capacities to be allocated to the entities. This optimal allocation of potential capacities can be used to obtain $fel1, fel2, fel3$ as the least unutilized capacities. As an attempt to make the test problems comparable, the least unutilized capacities were first calculated by the proposed model and then divided by the sum of network flow at that particular level to obtain $fel1, fel2, fel3$. If the obtained levels of flexibility were set to the basic model, the results would be comparable. Calculation of flexibility levels for three different echelons of the network is demonstrated in (5.1) to (5.3).

$$fel1 = \frac{min_{t,r}[ZT_{tr} - \sum_m X_{rtm}]}{\sum_r \sum_t \sum_m X_{rtm}} \quad 5.1$$

$$fel2 = \frac{min_{n,m}[\sum_\phi U2_{nm}{}^{\phi} * ZK_n{}^{\phi} - \sum_q Y2_{qnm} * W_{qn}]}{\sum_q \sum_n \sum_m Y2_{qnm}} \quad 5.2$$

$$fel3 = \frac{min_f[\sum_k U3_f{}^{k} * ZD_f{}^{k} - \sum_p \sum_m A_{pmf} * WP_p]}{\sum_p \sum_f \sum_e \lambda_{pfe}} \quad 5.3$$

Table 3 represents the results of the computational experiment performed by Cplex 7.0 in a notebook with Core i5 2.4 GHz CPU, 4 GB RAM and Windows 7 x86 Operation System.

Table 3 Results of the computational experiments for seven different test problems

Problem	$fel1$ %	$fel2$ %	$fel3$ %	Basic model with flexibility formulation by Sabri et al.		Proposed Model		Expected percentage of decreased cost
				Total expected cost (million $)	Computation time (s)	Total expected cost (million $)	Computation time (s)	
1	0.04	0.02	0.16	10.406	12.00	10.297	10.00	1.05
2	5.81	7.50	5.30	98.676	120.00	84.260	39.00	14.61
3	2.00	2.40	0.05	426.970	7.00	310.520	1.60	27.27
4	5.00	1.00	1.60	323.390	1.17	238.270	3.19	26.32
5	7.00	6.00	9.00	Not feasible	-	1030.000	23.00	-
6	5.00	5.00	6.00	2979.659	2.10	2622.100	24.00	12.00
7	2.00	2.00	7.00	Not feasible	-	9874800.000	2.27	-

As demonstrated in Table 3, the expected total cost was decreased for all the test problems. Besides, the computation time was improved for the first three test problems, while test problems four and six suffered from increased computation time. However, the size of the problem justifies more calculations of the CPU for finding the optimal solution.

The flexibility formulation method proposed by Sabri et al. is not capable of finding a feasible solution for test problems number five and seven, because the required additional capacity cannot be allocated to all the entities. This observation explains the importance of bottlenecks in flexible capacity planning. Introducing an example might help clarify the situation. If the optimal set of $fel1, fel2, fel3$ were 1%, 2%, and 6%, respectively, for test problem number seven, the basic model would find an optimal solution with total expected cost of 9.7×10^6 million dollars; i.e. a distinctively lower level of flexibility is obtained almost with the same total cost.

Therefore, the results indicated an advantageous performance for the proposed model, which was capable of decreasing the total expected cost up to 27.27% in the best case scenario (test problem number three) and 16.25% on average of cases that were also soluble by the dominated basic model. Moreover, the proposed model was capable of finding the solution in a timelier manner in more than half of the investigated cases. Finally, the essentiality of the whole concept of bottlenecks should be highlighted, as it strengthens the ability of model in solving cases that have been previously insoluble.

6. Discussion

This paper aimed to design a novel multi-echelon, multi-product supply chain for a production-distribution system with WIP flow between manufacturers. The novelty of this design was in its capability of optimizing flexibility levels by integrating concept of bottlenecks into flexible capacity planning. Design of the supply chain network, identifying of bottlenecks, and unutilized capacity allocation were all formulated by a mixed-integer linear programming model. The demand was assumed to be stochastic based on different scenarios with specific probabilities. The proposed model had different advantages in comparison with the only quantitative model of flexible supply chain capacity planning by Sabri and associates.

As designing supply chains is a strategic decision with long-term impacts, decision makers are willing to invest on more cost-effective designs, especially when it comes to designs with distinctive financial advantages. Capacity allocation based on identification of bottlenecks and allocating unutilized capacities has made it possible to design a supply chain with less total cost of establishment, transportation, and production for the same levels of flexibilities.

Moreover, previous models have required designers' own judgment on flexibility levels for different echelons of the network. Although this issue makes the model more adjustable, it results in sub-optimal designs, as there is not a well-argued basis for choosing flexibility levels. The proposed model in this study optimized the levels of flexibility based on different network parameters and their impacts on other terms of the total cost.

Finally yet importantly, the dominated flexibility formulation method suggest increase in capacities of all the entities for flexibility enhancement which sometimes makes the solutions infeasible due to violating constraints of potential capacity. According to the proposed model of this study, previously insoluble problems were solved to the optimality, as increasing flexibility did not require increase in the capacity of all the entities anymore.

Similar to the advantages of the model, its disadvantages should be addressed. The proposed model was undeniably more complex than the basic model demonstrated in the literature. However, it seems that the linearity of the model could compensate for the complexity resulting in decreased computation time for small- and mid-sized problems and tolerably increased computation time for large-sized instances.

7. Conclusion and Future Research

New challenges of today's logistics require sophisticated dynamic models for deciding upon network parameters with long-term impacts. Flexibility issue is a highly regarded challenge of supply chain network design concerning solutions for improved responsiveness to demand by managing the supply and logistic activities. This study proposed a new approach for designing a multi-echelon multi-product supply chain network concerning the minimization of total cost, while taking bottleneck point into account as the main vehicle of flexibility enhancement. A mixed-integer linear programming model was developed to obtain flexible network designs with significantly lower total cost of establishment, transportation, and production.

As this study proposed a novel conceptual approach for flexible network design, it offered significant research gaps for further investigation. Using the same research structure of this study, other conceptual aspects of flexibility can be considered and formulated as optimization models to investigate the effects of a certain flexibility strategy by conducting quantitative analysis. Another suggestion for future works is improving the model of this study by contributing to its comprehensiveness using additional objective functions such as green, sustainable, and so forth. From a practical point of view, it is suggested to implement the main findings of this study to real supply chain establishment or reconfiguration cases in automotive or high-tech industries owing to their inherent compatibility in terms of fluctuation of demand and products with short life-cycles. Finally, as the current model might be computationally intractable for problems with extra-large size, it is recommended to provide a problem of this kind with tailored evolutionary algorithms to investigate possible decrease in computation time. It is hoped that this study could stimulate further investigations on flexible capacity planning and supply chain network design.

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