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# Digital Surface Curvature

PhD Thesis  
by John Rugis

Submitted in Partial Fulfillment of the Requirements of the  
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## Abstract

*The theoretical basis for this thesis can be found in the subject of differential geometry where both line and surface curvature is a core feature. We begin with a review of curvature basics, establish notational conventions, and contribute new results (on  $n$ -cuts) which are of importance for this thesis. A new scale invariant curvature measure is presented.*

*Even though curvature of continuous smooth lines and surfaces is a well-defined property, when working with digital surfaces, curvature can only be estimated. We review the nature of digitized surfaces and present a number of curvature estimators, one of which (the 3-cut mean estimator) is new.*

*We also develop an estimator for our new scale invariant curvature measure, and apply it to digital surfaces. Surface curvature maps are defined and examples are presented. A number of curvature visualization examples are provided.*

*In practical applications, the noise present in digital surfaces usually precludes the possibility of direct curvature calculation. We address this noise problem with solutions including a new 2.5D filter.*

*Combining techniques, we introduce a data processing pipeline designed to generate surface registration markers which can be used to identify correspondences between multiple surfaces. We present a method (projecting curvature maps) in which high resolution detail is merged with a simplified mesh model for visualization purposes.*

*Finally, we present the results of experiments (using texture projection merging and image processing assisted physical measurement) in which we have identified, characterized, and produced visualizations of selected fine surface detail from a digitization of Michelangelo's David statue.*



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To Anna,  
Beauty,  
and Truth.

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