Buckling and Growth of Disbonds in Honeycomb Sandwich Structure

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I have no special talents. I am only passionately curious.

Albert Einstein

ABSTRACT

The response of honeycomb sandwich structure to disbond damage, in a compressive stress field, has been assessed. Two types of disbond were considered; those extending through the full width of a panel (through-width) and those enclosed within the panel boundaries (embedded). For each type of disbond the failure process was established through four-point bend testing of 56 sandwich specimens. For both types of disbond, failure was found to be governed by buckling-driven disbond growth and, consequently, models of buckling and disbond growth were developed.

All of the models decoupled the face-sheets of the sandwich and treated the core as a spring foundation having a stiffness determined from equilibrium of a two-dimensional orthotropic solid.

A linear Winkler beam model was used to predict buckling of a through-width disbond. The model buckling loads agreed with specimen test results with an average difference of 1.7%. A non-linear Winkler beam model was then developed to predict post-buckling behaviour and the initiation of disbond growth, through a fracture mechanics analysis. A characteristic growth curve, defining the work input required to initiate disbond growth, was developed and agreed with specimen test results with an average difference of 3.3%. The model also verified that disbond growth occurs in discrete increments approximately equal to the diameter of the honeycomb cells.

A linear Winkler plate model was used to predict buckling of an embedded disbond. The model buckling loads agreed with specimen test results with an average difference of 3.7%. A non-linear Winkler plate model was then developed to predict post-buckling behaviour of a sandwich panel containing an embedded disbond. The model considered contact conditions and modelled disbond growth by releasing fractured nodes during load incrementation. Disbond growth initiation loads agreed with specimen test results with an average difference of 15.8%. Failure loads consistently over-predicted specimen test results by an average of 13.9%. It was concluded that the growth initiation loads should be used as a conservative estimate of failure.

The models developed may be used to assess the criticality of disbond damage in sandwich structure having thin-gauge, composite face-sheets.

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List of Symbols and Abbreviations

Latin Symbol	Description
A	Component sectional area
AvF	Average frequency of transient acoustic events
C	Constant
C_E	Coefficient of Euler beam buckling
C_{PE}	Coefficient of Euler panel buckling
D_f	Face-sheet bending stiffness
D_{ij}	Row *i*, column *j* of the face-sheet bending stiffness matrix
E	Young's modulus of elasticity
E_{eff}	Homogenised (effective) Young's modulus of honeycomb in through-thickness direction
G	Shear modulus of elasticity
G_c	Critical strain energy release rate
G_{total}, G_I, G_{II}, G_{III}	Strain energy release rate; total, Mode I, Mode II and Mode III, respectively
J	Core modulus expression used in Airy's stress function
L_{cr}	Wrinkling half-wavelength (sandwich beam)
L_d	Disbond Size (length for through width disbonds, diameter for circular disbonds and edge length for square disbonds)
L_{dy}, L_{dx}	Rectangular disbond side lengths in *x* and *y* directions
L_x, L_y	*w*-displacement face-sheet wavelengths in *x* and *y* coordinates
L_X, L_Y	Panel dimensions in *x* and *y* directions
M	Number of laminae in a laminate
N_{cr}	Panel buckling load
N_{grow}	Load at which disbond growth initiates in a panel
N_{ult}	Panel failure load (ultimate load)
P	Beam Load
P_{cr}	Beam buckling load
P_{wr}	Wrinkling load of perfectly bonded sandwich beam
Q	Ratio of panel length to disbond diameter
R	Ratio of panel width to disbond diameter
S	Shear stiffness
THA	Ratio of threshold to maximum amplitude (regarding acoustic events)
U	Strain energy
U_s	Surface energy
W	Specimen width in direction transverse to loading

$X_1, X_2, X_3, \ldots$	Constants
[ ]	Defines variable within brackets as a matrix
$c_1, c_2, c_3, \ldots$	Constants
$f(\ )$	Function of the bracketed variables
h	Maximum out-of-plane face-sheet displacement (disbond buckle height)
k	Elastic foundation stiffness in z-direction
l	Length of regular honeycomb cell wall
n	Node number
n_s	Sample size
p	Binomial probability
s	Sample standard deviation
t	Thickness
u, v, w	Displacement functions in x, y and z directions, respectively

Greek Symbol	**Description**
Δ	Beam end-shortening
Δ_s, Δ_b	Beam shear and bending deflections, respectively
Δ_{sx}	Nodal spacing in s-coordinate (Lagrangian) approximated by projected length in x-coordinate (Cartesian), i.e. assuming small displacements
Δ_x	Spacing between nodes in the x-direction
$P_1, P_2, P_3, \ldots$	Dimensionless parameters
$(1-a)$	Confidence level
q, s	Lagrangian coordinates
k	Beam curvature
l_1, l_2	Constants
n	Poisson's ratio
r	Component density
s	Stress
s_s	Sample mean
s_p	Population mean
f	Airy's stress function

Subscripts	**Description**
c	Core property
f	Face-sheet property
L	Laminate property
m	m^{th} layer lamina property
x, y, z	Property refers to Cartesian coordinate
n	Node number
cw	Refers to honeycomb cell wall property

Superscripts	Description
c	Core property (used when subscript specifies x, y, z orientation)
f	Face-sheet property (used when subscript specifies x, y, z orientation)
e	End value, i.e. after testing
o	Original value, i.e. before testing

Abbreviations	Description
ADL	Allowable damage limit
AE	Acoustic emission
CDF	Cumulative density function
COV	Coefficient of variance
CSB	Cracked sandwich beam fracture test
DCB	Double cantilever beam fracture test
ENF	End-notch flexure fracture test
FAR	Federal aviation regulation
FFT	Fast Fourier transform
LEFM	Linear elastic fracture mechanics
PMF	Probability mass function
SERR	Strain energy release rate
SRM	Structural repair manual
TPBS	Three-point bend sandwich fracture test
VCCT	Virtual crack closure technique