An investigation into the effects of weight constraints, estimation error and taxes on the benefits from international diversification.

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Abstract

This thesis investigates the effect of allocation weight constraints, estimation error and taxes on the potential benefits from international diversification. The empirical results presented in this thesis address gaps in the literature concerning the statistical significance of the potential gains from international diversification and whether these potential benefits can be realized ex-ante. Weight constraints on market allocations used with static in-sample optimized portfolios are found to reduce the potential benefits of international diversification to statistically insignificant levels versus the domestic portfolio for the majority of investors in developed markets for the 1993 to 2014 investment period. The naive global market capitalization weighted (1/M) portfolio, the most strongly constrained portfolio, provides statistically significant positive gains for only 2 of the 34 markets studied. Estimation error reduction strategies designed to improve ex-ante optimization are not found to outperform the naive domestic market portfolio or the naive 1/M portfolio for investors in the majority of countries measured. And the unequal taxation of overseas equity income is reported to reduce the potential benefits of international diversification and increase the efficiency of a home bias for New Zealand investors indirectly investing in equities held in a portfolio

investment entity (PIE). These empirical findings suggest that a home bias may exist in part because allocation constraints, estimation error and taxes on overseas equity investments can make overweighting the domestic market a statistically efficient investment decision.

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Chapter 1

Introduction

Modern portfolio theory and the mutual fund theorem upon which the international capital asset pricing model (CAPM) is framed suggest that there are benefits from international diversification. Despite the potential to reduce portfolio risk through naive diversification, or improve portfolio efficiency with optimization, investors continue to hold large portions of their wealth in domestic assets. Research into the equity home bias puzzle has not found a single factor that explains this arguably inefficient investment behaviour. Rather, numerous factors have been reported as contributing towards an equity home bias. This thesis conducts empirical studies on the cross-country effects of allocation weight constraints, estimation error and taxes on the potential benefits from international diversification. The results address gaps in the literature regarding the significance of these factors in reducing the benefits from diversification and, as a result, their potential influence on the equity home bias.

This thesis investigates whether investors can expect to achieve positive benefits from international diversification, or whether the theoretical gains from diversification are eliminated or reduced to statistically insignificant levels because of weight constraints on allocations to international markets, estimation error in the optimization solution and taxes. This is done by empirically testing a series of questions. The first question is whether investors in all 34 markets studied can expect to achieve benefits from naive diversification into a naive global market capitalization weighted (1/M) portfolio? If naive diversification does not always provide significant gains beyond an investor's local market portfolio a home bias may be a reasonably efficient investment decision given the uncertainty of future benefits. Second, does relaxing allocation weight constraints provide potentially significant performance improvements beyond the 1/M portfolio to justify optimization? It seems logical to this author that a rational investor will choose to diversify into an optimized portfolio of international markets only if there exists the possibility of significant positive gains beyond a naive investment strategy. If diversification into optimized portfolios with weight constraints does not provide statistically significant gains beyond an investor's local market portfolio, again a home bias may be a reasonably efficient investment decision given the uncertainty of capturing these benefits ex-ante. Next, do estimation error reduction strategies recently presented in the literature outperform the naive domestic market portfolio and the naive 1/M portfolio? If there exist potential gains from diversification, but active optimization strategies available in the public domain do not capture these gains, it seems reasonable to assume that many investors will also have difficulty capturing these potential benefits. Finally, what are the effects of taxes on the potential benefits from diversification? Given the difficulties in capturing the potential gains from diversification with imperfect ex-ante information, taxes are likely to reduce these gains further. The empirical results presented in this thesis suggest that while there can be potential benefits from international diversification, weight constraints on the optimization solution, estimation error in ex-ante optimization and taxes can reduce the statistical significance of these benefits, and as a result, increase the relative efficiency of a home bias.

This thesis consists of an introduction, a literature review, four empirical studies and a summary of conclusions. This chapter introduces the empirical studies presented in this thesis, their contributions and results. Chapter 2 provides a brief background of the literature on the benefits of international diversification and the equity home bias puzzle related to the studies conducted in this thesis.

Chapter 3 investigates the potential benefits of international diversification for U.S. investors holding static portfolios with weight constraints and no short sales. It is a preliminary study that introduces some of the data, methods and statistical testing used in the chapters that follow. The primary objective of the chapter is to identify adjustments to the data, methods and perspective presented in Chiou (2008) that result in lower measured benefits for the 1988-2004 holding period than previously reported. It extends Chiou (2008) to report the potential benefits achieved by diversifying into the naive 1/M portfolio, which is the most strongly weight constrained portfolio presented. An additional contribution is the use of the Ledoit and Wolf (2008) bootstrapping methods to test the significance of the benefits achieved by the optimized portfolios versus each national market portfolio. The results find that diversifying out of the U.S. market and into the 1/M portfolio for the 1988-2004 and 1988-2014 periods results in reduced portfolio efficiency. Relaxing the weight constraints on the maximum return-to-risk portfolio (MRRP) does not provide statistically significant return-to-risk (RR) improvements beyond the domestic U.S. market portfolio for both periods studied. The minimum variance portfolio (MVP) with relaxed weight constraints does provide lower volatility levels than the the U.S. market portfolio that is statistically different. However, these MVPs do not provide RR ratios that exceed the U.S. market portfolio.

Chapter 4 conducts a cross-country examination of the effect of weight constraints on the potential benefits from international diversification during the 1993-2014 investment period. Similar data, methods and statistical testing as presented in chapter 3 are used. A Bayesian approach is also applied to the data which adds robustness to the results. The chapter extends the single country perspective that much of the existing literature on the benefits of international diversification with trading constraints present (De Roon et al., 2001; Li et al., 2003; Fletcher and Marshall, 2005; Chiou, 2008). It also extends the cross-country results presented in Driessen and Laeven (2007) which only considers short selling restrictions into the MRRP. I find that the 1/M portfolio formed using the market capitalization weights at the start of the sample period does not provide an RR ratio improvement over the domestic market for investors in the majority of developed markets for the 1993 to 2014 investment period. These portfolios provide statistically significant positive RR gains for only 2 of the 34 countries studied. Relaxing the weight constraints on the MRRP and the MVP increases the positive potential gains from diversification. However, these portfolios fail to provide statistically significant positive RR benefits for many investors. The results do not find conclusive evidence that there exist statistically significant positive gains from diversification for all investors from static optimized portfolios with or without weight constraints and no short sales during the 1993-2014 investment period.

Chapter 5 conducts a cross-country evaluation of the effectiveness of estimation error reduction strategies recently presented in the literature in providing benefits from international diversification for investors in the 34 markets studied. The out-of-sample performance of the Ledoit and Wolf (2003b, 2004, 2003a) sample covariance matrix shrinkage strategies and recent allocation weight constraint strategies (DeMiguel et al., 2009a; Levy and Levy, 2014a) are compared to the naive domestic portfolio, the naive 1/M portfolio and the naive 1/N portfolio from the perspective of investors in each of the countries studied.

This study contributes to the literature regarding the effectiveness of estimation error reduction strategies in capturing the potential benefits of international diversification in several ways. First, this is the only study that this author is aware of to report the results of the Levy and Levy (2014a) strategy applied to the optimization of international portfolios. This study extends Levy and Levy (2014a) to report the relative performance of their model against other shrinkage and constraint strategies. Also, this paper proposes the use of the cross-validation technique presented in DeMiguel et al. (2009a) to determine the alpha constraint used in the Levy and Levy

(2014a) strategy. This extends Levy and Levy (2014a) by providing a method with which to dynamically set alpha and utilize the strategy ex-ante. Another contribution is the testing of the results for statistical significance versus the naive benchmarks which Levy and Levy (2014a) does not do.

Chapter 5 also extends the results of Jacobs et al. (2014) which reports the effectiveness of some estimation error reduction strategies applied to international diversification. That paper compares common optimization strategies versus heuristic portfolio selection from the perspective of European investors diversifying amongst the four regional equity indices of North America, Europe, Asia and emerging markets. Chapter 5 includes more of the Ledoit and Wolf (2003a) strategies and implements short sales in the optimization solution. Jacobs et al. (2014) only implements the constant correlation strategy and restricts short sales in the minimum variance optimization solution. Also, chapter 5 reports the optimization results using 34 market indices rather than the 4 naive regional indices used in Jacobs et al. (2014). Using individual markets may improve the effectiveness of the estimation error strategies as the information these models are designed to retain, and subsequently use in creating diversification benefits, may be lost with the use of naive regional portfolios. Chapter 5 also conducts optimization using 4 naive regional indices. This is done to provide robustness to the results and allows for the analysis of the effectiveness of the optimization strategies in reducing estimation error as the number of assets is decreased but information is lost with naive regional diversification.

Finally, chapter 5 reports the results from the perspective of investors in 34 countries and presents the performance of each naive domestic market portfolio. Jacobs et al. (2014) only reports the performance of internationally optimized portfolios for European investors and does not report the local market performance. Chapter 5 reports the performance of the naive local portfolio for all investors so that a comparison of the benefits from optimization can be more robustly determined. Also, presenting the estimation error reduction strategy results from the perspective of investors in each country assists in determining whether the results are consistent across all countries, or if there are differences in the magnitude of the benefits between countries.

The results presented in chapter 5 do not identify an optimization strategy that consistently outperforms the naive domestic portfolio or statistically outperforms the 1/M benchmark for all investors. Some optimization strategies have relatively large short positions and turnover rates which will introduce additional costs that may further reduce their effectiveness versus the naive benchmark portfolios. The use of regional indices is found to reduce the effectiveness of the strategies versus the 1/M benchmark.

Chapter 6 measures the effect of taxes on the potential benefits of international diversification for New Zealand investors indirectly invested in equities held in a portfolio investment entity (PIE). For New Zealand tax residents there exist unequal tax frictions between dividends and capital gains earned in New Zealand versus those realized overseas. This is the only study that this author is aware of that investigates the effect of taxes on the potential benefits of international diversification for New Zealand investors holding indirect equity investments in PIEs. The results of this study are of particular importance to investors in New Zealand given the recent introduction of the KiwiSaver retirement scheme. The KiwiSaver program began in 2007. As of

June 2015, New Zealand has a resident population of roughly 4.5 million and there are 2,530,919 active KiwiSaver accounts[1]. All of the default KiwiSaver funds are PIEs.

Reporting the potential effect of taxes on the gains from international diversification contributes to the literature in two ways. First, the results provide evidence of the significance of taxes in influencing the retirement balances of KiwiSaver investors which previous literature has not considered. For example, MacDonald et al. (2014) suggests that heavier KiwiSaver allocations to international equities might improve investor retirement outcomes, but the paper does not include taxes in the analysis. Chapter 6 reports that the combination of allocation weight constraints on overseas markets and the unequal taxation rates applied to international equity investment returns makes it unlikely that internationally diversified KiwiSaver portfolios can provide statistically significant positive return-to-risk improvements versus the untaxed domestic New Zealand market portfolio. Second, the empirical results extend previous literature that has identified taxes as an explanatory factor in the documented home bias that exists with international investment funds in New Zealand (e.g., Chan et al., 2005). The results presented here find that the tax effect is more pronounced when optimizing a portfolio to maximize the RR performance than when minimizing portfolio variance.

The results find that weight constraints on the optimization solution and taxes reduce the benefits of international diversification and increase the efficiency of heavier allocations to the domestic market. Relaxing the weight

[1]source: http://www.rbnz.govt.nz/statistics/tables/t43/ (accessed December 5, 2015).

constraints on the MRRP reduces the home bias, while taxes increase the home bias. In the case of the MVP, taxes do not effect asset allocations significantly, but they reduce the RR performance of these portfolios which reduces the benefits of diversification. The significance of the positive gains achieved from optimization into taxed internationally diversified portfolios without weight constraints and with no shorts are not found to be statistically different from the New Zealand market portfolio. Strengthening the constraints on the weights allocated to international markets and taxes can eliminate the positive gains from diversification into taxed portfolios and increases the efficiency of a larger domestic market allocation.

The results presented in this thesis contribute to the literature investigating the benefits of international diversification. The results find that neither naive international diversification or in-sample optimized static portfolios with weight constraints necessarily provide investors with statistically significant diversification benefits over extended holding periods. Also, estimation error reduction strategies recently presented in the literature are not found to consistently and statistically outperform the domestic market portfolio or the naive 1/M portfolio out-of-sample. Lastly, the taxation of overseas equity assets held by New Zealand investors in PIEs is reported to reduce the potential benefits from international diversification to a statistically insignificant level and increases the efficiency of a home bias. It seems that it is the theory that international diversification can provide potential improvements to portfolio efficiency which makes the documented home bias a puzzle. The empirical evidence presented in this thesis suggests that for many investors a home bias may just be a case of home is best.

Chapter 2

Literature Review

The documented bias investors have for domestic assets over international assets (French and Poterba, 1991) is a puzzle because of the theoretical and potential benefits from international diversification reported in the literature. The mutual fund theorem (Sharpe, 1964; Lintner, 1965) and the CAPM extended to an international setting (Solnik, 1974a; Sercu, 1980) assumes investors can improve portfolio efficiency by simply diversifying into the naive global market capitalization weighted (1/M) portfolio. Modern portfolio theory predicts that investors will choose to optimize their portfolio to capture the risk reduction benefits of weakly correlated international markets and to improve the risk-to-return performance resulting from the selection of markets with superior returns. Tobin's separation theorem also suggests that an optimized tangent portfolio combined with a risk-free asset can provide further gains beyond the efficient frontier of equity assets. This chapter provides a brief overview of some of the literature on the potential benefits of international diversification and the equity home bias that is related to the empirical studies presented in this thesis and to which these studies seek to contribute.

2.1 The benefits of diversification

Adding weakly correlated assets to a portfolio can reduce total portfolio risk. Early literature documents the weak correlations between markets and advises investors to diversify internationally in order to capture potential benefits from diversification (Grubel, 1968; Levy and Sarnat, 1970; Solnik, 1974c,b; Lessard, 1976; Grauer and Hakansson, 1987). Recent studies find correlations between markets have oscillated over the twentieth century. Goetzmann et al. (2007) and Quinn and Voth (2008) report that correlations between markets peak at the end of the 19th century, the Great Depression and the late 20th century. The strengthening correlations at the end of the 20th century, which are the strongest correlations over the period studied, are inferred to be the result of globalization (Goetzmann et al., 2007) and the openness of markets (Quinn and Voth, 2008). Despite strengthening correlations between markets, diversification continues to be reported as beneficial in reducing total risk (Odier and Solnik, 1993; Malkiel, 2002). Levy and Levy (2014b) propose that the strengthening correlations between markets magnify the asymmetric information costs and higher variance in returns that result from currency risk. As a result, they posit that despite lower international transaction costs over time, higher correlations offset these gains and the home bias will persist. Other literature reports that correlations between markets can strengthen during periods of extreme market drops (Roll, 1988;

Solnik et al., 1996; Butler and Joaquin, 2002) which reduces the risk reduction benefits of diversification that investors are likely seeking during these periods of higher than normal volatility.

2.1.1 Measuring the benefits ex-post

Early literature reporting the potential benefits from international diversification generally uses historical data ex-post to measure the potential benefits from diversification (Grubel, 1968; Levy and Sarnat, 1970; Solnik, 1974c,b; Lessard, 1976). Grubel and Fadner (1971) show that potential benefits can exist over different investment horizons. Costs associated with investing in international markets can reduce these benefits (Black, 1974; Stulz, 1981). These costs include taxes, informational disadvantages, restrictions on ownership and restrictions on market access.

Trade constraints that reduce the benefits of international diversification can increase the relative efficiency of the local market. Empirical studies find the potential benefits of international diversification are reduced with restrictions on short sales (De Roon et al., 2001; Driessen and Laeven, 2007). A Bayesian approach that measures the probable gains that can be expected given the stochastic nature of markets also finds that the removal of short sales reduce, but do not eliminate, international diversification benefits for U.S. investors (Li et al., 2003) and U.K. investors (Fletcher and Marshall, 2005).

Classical mean variance optimization can result in weight allocations many times the size of a market that may be impractical to implement in practice. If all investors follow the extreme weights optimization can set, large capital inflows into overweighted small markets will likely increase market volatility and reduce future returns. Constraints on asset allocations can reduce estimation error in the optimization solution and improve portfolio performance ex-ante (Jagannathan and Ma, 2003; DeMiguel et al., 2009a). Good portfolio governance will likely impose allocation limits to manage these risks.

Chiou (2008) measures the effect of weight constraints on the potential benefits of diversification for U.S. investors. The potential gains from diversifying out of each market and into optimal portfolios with market allocations constrained by various multiples of the markets' capitalization weight are measured. The largest gains in the return-to-risk (RR) premium from investing in the maximum return-to-risk portfolio (MRRP) and the greatest reductions in volatility that results from investing in the minimum variance portfolio (MVP) are generally found to occur for investors diversifying out of emerging markets. The weakest benefits are realized by investors diversifying out of developed markets. The results suggest that short restrictions and weight constraints together result in reductions to the benefits of diversification, but do not eliminate them.

2.1.2 Measuring the benefits ex-ante

While the results from in-sample optimization find that there are potential benefits from diversification, DeMiguel et al. (2009b) reports that ex-ante optimization strategies generally do not outperform a naive 1/N investment strategy. Early literature identifies the weak ex-ante performance of classical mean-variance optimization used in international diversification with historical estimates (Jorion, 1985). Return estimation error is large and results in poor out-of-sample optimization performance relative to the naive investment strategy of buying a market weighted portfolio. The use of minimum-variance portfolio optimization to remove return error from the optimization process is advised. Jorion (1985) reports that the use of Bayes-Stein shrinkage on the estimates improves the out-of-sample performance relative to the naive benchmark.

The minimum-variance portfolio optimized with no shorts and using a sample covariance matrix is reported by Jagannathan and Ma (2003) to perform as well as a covariance matrix estimate based on factor models, shrinkage estimators, or higher frequency data. They explain that weight constraints reduce estimation error when using the sample covariance matrix. Constraints on asset weights is equivalent to constructing an unconstrained minimum variance portfolio using the shrunk covariance matrix derived using Lagrange multipliers from the constraints. Imposing weight constraints or the use of a shrunk covariance matrix can reduce the estimation error in the optimization solution and improve the efficiency of the out-of-sample portfolio performance.

Ledoit and Wolf (2003b,a, 2004) present a shrinkage strategy which seeks the optimally weighted average of a sample covariance matrix and a target covariance matrix that minimizes the mean squared error between the shrunk covariance matrix and the true covariance matrix of stock returns. The three target matrices these papers present are the one-factor covariance matrix, the constant correlation matrix and the Identity matrix. The strategy reduces the extreme values in the sample covariance matrix towards the target matrix values with the objective of reducing estimation error. The strategies are applied to U.S. equities and generally show improved ex-ante performance over the non-shrunk models or industry factors. Increasing the number of assets reduces the relative performance of these strategies.

DeMiguel et al. (2009a) present a 1-norm and an A-norm weight constraint framework with which estimation error is reduced. The results suggest that these norm-constrained portfolios can produce higher Sharpe ratios than the strategies in Jagannathan and Ma (2003), Ledoit and Wolf (2003b, 2004), 1/N and other common strategies in the literature. While the constraint strategies provide improved performance over naive portfolios, the results are not statistically significant for all the data sets of U.S. equities considered.

Behr et al. (2013) present a constrained minimum-variance model using a shrinkage theory framework which is reported to outperform naive portfolios for most U.S. equity datasets studied. The model uses a bootstrap technique to create sample portfolios from which a direct search method is applied to determine the minimum and maximum constraints that minimise the mean squared error of the corresponding Lagrange multiplier modified sample covariance matrix. The model is tested against the three shrinkage targets of Ledoit and Wolf (2003b, 2004, 2003a), the DeMiguel et al. (2009a) 1-norm and 2-norm weight constraint framework and some common optimization models. The p-values from the Ledoit and Wolf (2008) bootstrap tests find that the model outperforms the naive portfolio, but is not significantly different from the other recent estimation error reduction methods tested.

Levy and Levy (2014a) present a variance-based constraint method to reduce estimation error which imposes more stringent weight constraints on assets with relatively higher standard deviations than the average standard deviation of the assets. The principle behind the method is to reduce the potential estimation error associated with high volatility assets. The model is tested using portfolios of U.S. equities. The significance of the results are not tested. Nor is the method tested against other recent models designed to reduce the estimation error in the optimization inputs.

The literature measuring the effectiveness of these estimation error reduction strategies has largely focused on measuring performance when used with stocks in a single market. Jacobs et al. (2014) is the only paper that this author is aware of that reports the effectiveness of some of these strategies applied to international diversification. The paper considers the effectiveness of common optimization strategies versus heuristic portfolio selection from the perspective of European investors diversifying amongst the four regional equity indices of North America, Europe, Asia and emerging markets. Classical mean-variance optimization models, as well as the DeMiguel et al. (2009a) and Ledoit and Wolf (2003a) strategies are found to provide no significant improvement over naive allocation strategies consisting of regional equity indices only, or regional equity indices, European bonds and world commodities. The paper does not report the results of the local market portfolio versus the optimized portfolios or the results achieved by investors in countries outside Europe. It is not clear whether the use of naive regional indices in Jacobs et al. (2014) improves the optimization results or reduces the effectiveness of the shrinkage and constraint models as individual market information can be lost with the use of naively diversified regional indices.

2.2 The equity home bias puzzle

Despite the reported benefits from international diversification, investors have a bias for domestic equities. This bias is identified in the early literature investigating the potential benefits from international diversification (e.g., Levy and Sarnat, 1970; Solnik, 1974b,c). French and Poterba (1991) highlight the extent of this bias across countries. Investors are increasing the size of foreign asset positions over time but the home bias persists (Tesar and Werner, 1995; Stulz, 2005).

Various factors and explanations have been presented in the literature as contributing to a home bias[1]. These include domestic risk hedging (Cooper and Kaplanis, 1994; Sercu and Vanpe, 2008), foreign investment trade costs (Tesar and Werner, 1995; Warnock, 2002), information asymmetries (Chan et al., 2005; Coval and Moskowitz, 1999), behavioural biases (French and Poterba, 1991; Li, 2004), and exchange rate volatility (Fidora et al., 2007; Levy and Levy, 2014a). Chapter 6 of this thesis seeks to contribute to the literature and report the potential effect of taxes on the increased efficiency of a home bias that can result for New Zealand investors holding equities indirectly in portfolio investment entities (PIEs).

[1] For a more complete survey of the literature on the equity home bias puzzle refer to Cooper et al. (2012).

2.2.1 Measuring the home bias

There are several approaches used to measure the size of the home bias. The 'model-based' approach will use the optimal weights from an international asset-pricing model such as the world market portfolio (Solnik, 1974a; Sercu, 1980). Dahlquist et al. (2003) suggest that the world float portfolio - the market capitalization weighted portfolio of freely floated shares - should be used in the 'model-based' approach rather than the 1/M portfolio. A 'data-based' approach will use the data to determine the optimal weights. This approach commonly uses mean-variance optimization to determine the optimal weights. Other strategies can be used such as a Bayesian approach (Pastor, 2000) or a multi-prior approach (Garlappi et al., 2007).

Model misspecification can contribute to a measured home bias. Simplifying assumptions used in portfolio models can reduce the degree of the measured home bias. Glassman and Riddick (1996) report that relaxing the assumption that PPP holds reduces the bias. Glassman and Riddick (2001) report that transaction costs, perceived riskiness of foreign assets and omitting other asset classes that can effect correlations do not individually explain the home bias. However, consideration of these various factors together has a more promising effect in reducing the magnitude of the measured home bias. Baele et al. (2007) compares 5 different approaches to measure home bias. The model-based approach is reported to over estimate the home bias. The paper reports that a Bayesian approach that distrusts the model can produce a bias that is as much as 22% lower than a model-based approach.

2.2.2 Taxes

Taxes reduce investment returns and this affects portfolio optimization solutions and performance. In many countries, such as the United States, capital gains are taxed when assets are sold. Investors have an incentive to realize capital losses and defer capital gains (Constatinides, 1984) in order to minimize these taxes. As a result, capital gains taxes can create a capitalization effect that reduces demand and a lock-in effect that can reduce supply (Dai et al., 2008). Trading and holding investments for purely tax reasons can be sub-optimal over time if an investor is seeking benefits associated with diversification. As a result of such taxes, optimal portfolio allocations can be different between investors because of the different trade frictions incurred by investors.

Taxes are reported to influence portfolio selection amongst asset managers. Chan et al. (2005) examines how mutual funds in 26 different countries allocate assets locally and internationally. The evidence finds that a domestic bias is influenced by market development and familiarity variables. The foreign bias is effected by economic development, capital controls and withholding tax variables. The paper reports that in the 1999 to 2000 period the international New Zealand funds used in the study in aggregate allocate 74.93% of assets to New Zealand. The New Zealand market represents .07% of the world market in the paper. The paper notes that the New Zealand funds have a foreign bias towards its nearest geographical neighbour Australia and allocates 14.4% of assets to that market. The Australian market represents 1.2% of the world market in the paper.

Australian superannuation funds have a home bias that may be partially explained by factors such as anchoring formed from legacy investment, industry peers (Warren, 2010) and overseas taxes (Mishra and Ratti, 2013; Mishra, 2014). Christoffersen et al. (2005) report that Canadian fund managers consider the tax preferences of investors when setting investment policy. While taxes are reported to be a factor influencing the foreign bias, gaps exist in the literature concerning the measured effect of taxes on both the potential benefits of international diversification and the equity home bias.

Chapter 3

Measuring the benefits of international diversification: re-examining the effect of market allocation constraints

Abstract

This chapter makes adjustments to the data, methods and perspective presented in Chiou (2008) to report lower measured benefits from international diversification for the previously presented 1988-2004 investment period. Diversification out of the U.S. market and into the market capitalization weighted (1/M) portfolio does not provide positive return-to-risk (RR) gains or volatility reducing benefits. The optimized maximum return-to-risk portfolio (MRRP) with weakened weight constraints and no shorts can provide

positive RR improvements but it is not found to be statistically different from the U.S. market portfolio for both periods measured. Weakening the weight constraints on the optimized minimum variance portfolio (MVP) improves the statistical significance of the volatility reduction benefits. These MVPs do not provide RR ratios that exceed the U.S. market for the extended 1988-2014 holding period.

3.1 Introduction

Quantifying the potential gains from international diversification is useful for assessing the magnitude of the home bias puzzle. Trade constraints that reduce the benefits from international diversification can increase the relative efficiency of the local market. Restrictions on short sales in the optimization solution have been shown to reduce the potential benefits from diversification into developed and emerging markets for U.S. investors (De Roon et al., 2001; Li et al., 2003), U.K. investors (Fletcher and Marshall, 2005) and investors in other countries (Driessen and Laeven, 2007). The potential benefits from diversification for U.S. investors are reduced further when weight constraints on market allocations are considered (Chiou, 2008). This chapter makes adjustments to the data, methods and perspective used in Chiou (2008) and reports lower benefits from diversification for the previously presented period of 1988-2004. The benefits for a U.S. investor diversifying out of the U.S. market are reported to weaken further into the extended 1988-2014 period.

Early literature measures the weak correlations between markets and concludes that there are benefits from international diversification (Grubel, 1968;

Levy and Sarnat, 1970; Lessard, 1973). Recent studies find correlations between markets have oscillated over the twentieth century (Goetzmann et al., 2007; Quinn and Voth, 2008). Correlations peak at the end of the 19th century, the Great Depression and the late 20th century. The strengthening correlations at the end of the 20th century, which are the strongest correlations over the period studied, are inferred to be the result of globalization (Goetzmann et al., 2007) and the openness of markets (Quinn and Voth, 2008). Levy and Levy (2014b) report that the strengthening correlations between markets magnify the asymmetric information costs and higher variance in returns that result from currency risk. As a result, they posit that despite lower international transaction costs over time, higher correlations offset these gains and the home bias will persist as a result.

Critical to evaluating the benefits from international diversification derived from the less than perfect correlations between markets are the methods used to measure and test the potential gains. Trading constraints have been shown to reduce these benefits. Short sale constraints are reported to reduce the potential benefits of diversification with in-sample optimized portfolios of international markets (De Roon et al., 2001; Li et al., 2003; Fletcher and Marshall, 2005; Driessen and Laeven, 2007). The largest reductions can occur in markets which have the greatest potential for diversification gains such as developing markets.

Classical mean variance optimization can result in weight allocations many times the size of a market that may be impractical to implement in practice. If all investors follow the extreme weights optimization can set, large capital inflows into overweighted small markets will likely increase market volatility and reduce future returns. Constraints on asset allocations can reduce estimation error in the optimization solution and improve portfolio performance out-of-sample (Jagannathan and Ma, 2003; DeMiguel et al., 2009a). Good portfolio governance will likely impose allocation limits to manage these risks.

Chiou (2008) investigates the effect of weight constraints on the potential benefits of international diversification from the perspective of investors measuring returns in U.S. dollars. The paper measures the gains that result when diversifying from a single market into in-sample optimized portfolios with weight constraints and no short sales. The greatest gains from diversification are reported to occur for U.S. investors diversifying out of developing markets. The weakest benefits are for investors diversifying out of developed markets. The results report that short sale restrictions and weight constraints together result in reductions to the potential benefits from diversification, but do not eliminate them.

This chapter contributes to the literature investigating the potential benefits of international diversification with no short sales and constraints on weight allocations in several ways. First, adjustments to the data, methods and perspective used in Chiou (2008) are identified which result in lower benefits from diversification than previously presented for the 1988-2004 period. The results for the extended period of 1988 to 2014 report a further reduction in the benefits from diversification for a U.S. investor diversifying out of the U.S. market.

Next, this study reports the potential benefits achieved by diversifying into the 1/M portfolio. The mutual fund theorem (Sharpe, 1964; Lintner,

1965) and the CAPM extended to an international setting (Solnik, 1974a; Sercu, 1980) assumes investors can improve portfolio efficiency by simply diversifying into the naive global 1/M portfolio. In this chapter the 1/M portfolio is the most strongly weight constrained portfolio presented. Chiou (2008) does not report the potential benefits from investing in the 1/M portfolio. This chapter addresses this omission and reports that the 1/M portfolio captures a majority of the potential unconstrained diversification gains for U.S. investors diversifying out of most individual markets. The exception is an investor diversifying out of the U.S. market. Diversifying out of the U.S. market and into the 1/M portfolio for the 1988-2004 and 1988-2014 periods results in reduced portfolio efficiency.

An additional contribution is the use of the Ledoit and Wolf (2008) bootstrapping methods to test the significance of the benefits achieved by the optimized portfolios versus the local market. These tests are designed to address the non-normality of returns and fat-tail events that occur with historical financial data. Using a bootstrap technique, regressions are performed on paired data points of a given block size between the monthly returns of two portfolios to provide a p-value measuring the significance of the hypothesis that the difference between the two portfolios is zero.

Given that the optimal portfolio results presented in this chapter are formed using the data in-sample, the measured benefits from diversification will likely be greater than those achievable by an investor forming optimal portfolios ex-ante. As a result, the use of the Ledoit and Wolf (2008) bootstrap testing methods assists in determining at what level of relaxed weight constraints an optimized portfolio has the potential to offer statistically significant positive diversification benefits to justify an investor's attempt to capture the diversification benefits from optimization. The test results find that the 1/M portfolio and all optimal MRRPs with and without weight constraints do not provide statistically significant return-to-risk (RR) improvements beyond the domestic U.S. market portfolio for both periods studied. The MVPs with relaxed weight constraints do provide volatility levels that are statistically different from the U.S. market. However, these MVPs do not provide RR ratios that exceed the U.S. market.

This chapter is an introductory study that identifies adjustments to the data, methods and statistical testing used in Chiou (2008) that more accurately measure the potential benefits of diversification and which are subsequently used in the more substantive chapters that follow. Chapter 4 performs a cross-country analysis of the potential benefits from diversification with weight constraints. The study extends previous literature that reports only the U.S. investor perspective (Chiou, 2008) or the cross-country results into regional portfolios without weight constraints and no short sales (Driessen and Laeven, 2007). A Bayesian method is also used to report the probable gains from diversification that can be expected given the stochastic nature of market returns. This adds robustness to the results and extends the single country perspective presented in the literature which use the same method to measure the effect of short sale constraints on the potential benefits from diversification (e.g., Li et al., 2003; Fletcher and Marshall, 2005). Chapter 5 reports the out-of-sample performance of recent estimation error reduction strategies (Ledoit and Wolf, 2003b, 2004, 2003a; DeMiguel et al., 2009a; Levy and Levy, 2014a) presented in the literature relative to the naive domestic market portfolio and the naive 1/M portfolio benchmarks for investors in 34 countries. The results extend Jacobs et al. (2014) which only reports the performance of internationally optimized regional portfolios for European investors and which does not report the local market performance. The chapter also contributes to Levy and Levy (2014a) by proposing a method with which the global variance-based constraint strategy presented in their paper can be used ex-ante. Finally, chapter 6 investigates the potential effect of taxes and allocation constraints on the home bias for equities held in a portfolio investment entity (PIE) in New Zealand. The chapter addresses gaps in the literature regarding the measurable effect of taxes on the benefits of international diversification for New Zealand investors and the relative efficiency of a home bias that may result.

This chapter is divided into three more sections. Section 3.2 presents the data and methods used to measure the benefits of international diversification. Adjustments to the perspective, methods and data used in Chiou (2008) and implemented in this study are identified. The average correlation of returns between the 34 markets studied for the 1988-2014 period are also reported. Section 3.3 reports the returns and the standard deviation of returns used in this study for the 34 markets. The MRRP and MVP optimized using total return index data for the 1988-2004 and 1988-2014 periods are presented. Section 3.3 then presents the results of this study and provides analysis of the results. Section 3.4 concludes with suggestions for further study.

3.2 Data and Methods

The monthly price and total return MSCI equity index data in U.S. dollars for the 21 developed and 13 emerging markets presented in Chiou (2008) are used in this study. The sample period covers December 31, 1987 to December 31, 2014. The price index data is used to replicate the results presented in Chiou (2008) and identify the methods used in that paper. The total return data, which is adjusted for reinvested gross dividends, is used to report the adjusted results for the 1988-2004 period presented in Chiou (2008) and the extended 1988-2014 period presented in this chapter.

Annual market capitalization data in U.S. dollars for 1988 to 2012 is from two sources: the World Bank and the World Federation of Exchanges. The World Bank data is available for 33 of the 34 markets from 1988 to 2012. The data is incomplete for Korea and Ireland. Korea is missing the 1988 capitalization and Ireland is missing the 1988 to 1994 capitalization. This incomplete data is calculated using changes to the MSCI total return data to backward fill the missing capitalization values from the first available market capitalization data point. The Taiwanese market capitalization data is retrieved from the World Federation of Exchanges. The 1988 Taiwanese exchange capitalization data is not available and is calculated using the same method applied to the missing data for Korea and Ireland. The 2002 market weights presented in Chiou (2008) are used to replicate the results presented in that paper.

3.2.1 Measuring the benefits of international diversification

Investors can choose to maximize the return of their portfolio while seeking the same volatility obtained in the local market. Similarly, they can choose to minimize the volatility of their portfolio. They can choose to allocate funds in international markets where the investment opportunities of those markets can be stated as a vector of multivariate Gaussian stochastic returns of N assets, $R^T = [r_1, r_2, ..., r_n]$. The mean of asset returns for these markets can be expressed as a vector μ. The variance-covariance of asset returns can be expressed as a positive definite matrix V. Let S be the set of all real vectors w that define the weights such that $w^T 1 = w_1 + w_2 + ... + w_n = 1$, where 1 is an N vector of ones. Using the methods developed by Markowitz (1952), the efficient frontier of global investments can be formed when the objective function and restrictions are combined to find the efficient portfolio that minimizes volatility at every level of expected return:

$$\min_{\{w,\phi,\eta\}} \Xi = \frac{1}{2} w^T V w + \phi(\mu_p - w^T \mu) + \eta(1 - w^T 1) \tag{3.1}$$

where μ_p is the expected return of the portfolio, and the shadow prices ϕ and η are two positive constants. The quadratic programming solution of assets in a portfolio spanning w_p can be obtained by the first-order conditions of equation (3.1).

This chapter proposes adjustments to the methods presented in Chiou (2008) to assess the benefits of diversification out of a single market and used to measure the increment of mean-variance efficiency brought by international diversification. When there are no investment restrictions, the maximum return-to-risk (MRR) achieved by the MRRP is obtained by:

$$MRR = \max_{\{w_p\}} \left\{ \frac{w_p^T \mu}{(w_p^T V w_p)^{1/2}} \middle| w_p^T \in S \right\} \tag{3.2}$$

where w_p is the vector of weights[1] of assets held in the MRRP. The MRR[2] measures the return achieved by the MRRP compared to the standard deviation of the MRRP.

Chiou (2008) measures the greatest increment of unit-risk performance between the internationally diversified MRRP and the single market as:

$$\delta' = 1 - \frac{RR_i}{MRR} \tag{3.3}$$

where RR_i is the return-to-risk performance of the market portfolio in country i. This measure reports the percentage reduction in the MRR when moving from the optimal MRRP to the local portfolio. The larger the δ' (delta prime), the greater the benefits of diversification. While δ' approaching zero represents diminishing benefits from diversification. Section 3 reports the δ' results using the adjusted data and methods presented in this chapter for the 1988-2004 period previously presented in Chiou (2008).

This chapter proposes an alternative measure of the greatest increment of unit-risk performance, δ (delta), that measures the gains from diversifying

[1]To ensure feasible allocations, the sum of the portfolio weights must equal 1. Setting the condition that market weights must be positive restricts short sales.

[2]The MRR equation printed in Chiou (2008) is incorrect. Equation 3.2 reported in this chapter was used to replicate the results reported in that paper. Chiou (2009) and Chiou et al. (2009) use equation 3.2 presented in this chapter and refer to it as the maximum Sharpe ratio (MSR).

out of a single market and into the internationally diversified portfolio that maximizes the return-to-risk ratio:

$$\delta = \frac{MRR}{RR_i} - 1 \tag{3.4}$$

This measures the percentage change in the return-to-risk ratio the investor achieves when moving out of the local portfolio and into the optimally diversified international portfolio that will maximize the return-to-risk performance. This δ is an intuitive measure of the percentage improvement achieved beyond the single market portfolio and is a similar relative measure of the gains from diversification as ε (epsilon) that follows.

This chapter measures the reduction in volatility that can result from diversification out of a local market and into the MVP as presented in Chiou (2008) such that,

$$\varepsilon = 1 - \left[\frac{w_{MVP}^T V w_{MVP}}{V_i}\right]^{1/2} \tag{3.5}$$

where w_{MVP} is the vector of weights of assets held in the MVP and V_i is the variance of the local market. A positive value represents the percentage reduction in portfolio volatility when diversifying out of a single market portfolio and into the optimal internationally diversified MVP. A result approaching zero represents diminishing benefits from diversification.

3.2.2 Adjustments to the perspective and data presented in Chiou (2008)

This section identifies adjustments to the perspective and data presented in Chiou (2008) that are used in this chapter to re-examine the potential benefits from international diversification. First, the perspective presented in Chiou (2008), and subsequently reported in this chapter, is that of a U.S. investor interested in returns measured in the currency of the United States. It is not 'for domestic investors in various countries' as stated in Chiou (2008). The MSCI market data for each country is adjusted to U.S. dollars. At no point are portfolio values converted back to the currency of the domestic market. The comparative measures are therefore reported from the perspective of an investor who is interested in the value of the investment in the currency in which they will utilize its value and which, in this study, is the U.S. dollar. Such an investor would most commonly be a U.S. investor.

Next, the data used in Chiou (2008) is incomplete as it does not include dividends. Using total return index data changes the expected returns for the markets studied. These changes affect the RR results for each market and shifts the efficient frontier. The adjusted unconstrained MRRP and MVP used in this study for the 1988-2004 period are presented in section 3.3. The increase in market returns realized by the inclusion of dividends has a greater effect on the δ' results than the ϵ results. The increased returns result in different δ' results for the various weight constrained MRRPs presented in Chiou (2008) and generally lower δ' results for the adjusted MRRPs reported in this study.

Table 3.1: Markets, weights of capitalization and average coefficients of correlation for the period 1988 to 2014.

Market	w(%) 1988	w(%) 2002	Region	Average Correlation Coefficient DM	EM	All	NA	LA	CW	NE	SE	EA	OC
Panel A: Developed Markets													
Australia	1.44	1.73	OC	0.61	0.43	0.54	0.67	0.43	0.59	0.63	0.38	0.48	0.72
Austria	0.09	0.15	CW	0.59	0.40	0.51	0.54	0.34	0.65	0.56	0.47	0.41	0.55
Belgium	0.62	0.12	CW	0.63	0.37	0.53	0.62	0.34	0.71	0.59	0.41	0.38	0.50
Canada	2.53	2.59	NA	0.62	0.45	0.55	0.77	0.46	0.60	0.65	0.40	0.49	0.62
Denmark	0.32	0.35	CW	0.62	0.36	0.52	0.62	0.34	0.68	0.65	0.39	0.37	0.50
Finland	0.32	0.63	NE	0.53	0.33	0.45	0.59	0.32	0.53	0.61	0.36	0.34	0.50
France	2.56	7.00	CW	0.68	0.40	0.57	0.69	0.38	0.75	0.68	0.46	0.41	0.54
Germany	2.63	3.12	CW	0.67	0.41	0.57	0.67	0.36	0.74	0.68	0.47	0.42	0.54
H.K.	0.78	2.11	EA	0.51	0.44	0.48	0.60	0.40	0.48	0.51	0.31	0.53	0.53
Ireland	0.13	0.27	CW	0.58	0.34	0.49	0.60	0.31	0.63	0.58	0.37	0.37	0.52
Italy	1.41	2.17	CW	0.59	0.35	0.50	0.57	0.32	0.66	0.61	0.44	0.35	0.48
Japan	40.86	9.41	EA	0.45	0.28	0.38	0.45	0.23	0.45	0.44	0.24	0.34	0.45
Holland	1.19	0.56	CW	0.71	0.42	0.59	0.72	0.38	0.77	0.70	0.46	0.44	0.60
N.Z.	0.14	0.10	OC	0.51	0.38	0.46	0.51	0.32	0.49	0.52	0.36	0.43	0.72
Norway	0.15	0.31	NE	0.64	0.41	0.55	0.67	0.43	0.67	0.62	0.41	0.43	0.61
Singapore	0.25	0.46	EA	0.57	0.50	0.54	0.64	0.44	0.53	0.55	0.39	0.59	0.61

Spain	0.95	2.10	CW	0.64	0.41	0.55	0.61	0.41	0.69	0.65	0.46	0.40	0.58
Sweden	1.05	0.81	NE	0.66	0.42	0.56	0.68	0.39	0.68	0.68	0.46	0.44	0.62
Suisse	1.47	2.49	CW	0.61	0.35	0.51	0.59	0.31	0.67	0.58	0.39	0.38	0.54
U.K.	8.06	8.19	CW	0.68	0.38	0.56	0.71	0.36	0.71	0.68	0.41	0.43	0.62
U.S.	29.16	50.27	NA	0.63	0.42	0.55	0.77	0.43	0.64	0.65	0.39	0.46	0.56
Panel B: Emerging Markets													
Argentina	0.02	0.008	LA	0.22	0.23	0.22	0.28	0.30	0.22	0.22	0.26	0.18	0.27
Brazil	0.34	0.58	LA	0.38	0.31	0.36	0.43	0.36	0.37	0.43	0.33	0.30	0.41
Chile	0.07	0.23	LA	0.41	0.38	0.40	0.50	0.41	0.38	0.42	0.32	0.41	0.38
Greece	0.04	0.30	SE	0.47	0.31	0.41	0.41	0.30	0.54	0.45	0.39	0.28	0.42
Indonesia	0.003	0.14	EA	0.35	0.34	0.34	0.39	0.28	0.31	0.31	0.25	0.42	0.42
Korea	1.46	0.98	EA	0.42	0.33	0.39	0.47	0.28	0.38	0.44	0.26	0.43	0.47
Malaysia	0.24	0.56	EA	0.35	0.34	0.35	0.40	0.29	0.30	0.33	0.23	0.46	0.35
Mexico	0.14	0.47	LA	0.45	0.38	0.43	0.57	0.42	0.42	0.46	0.31	0.41	0.44
Philipines	0.04	0.08	EA	0.38	0.37	0.38	0.44	0.31	0.34	0.32	0.25	0.49	0.45
Portugal	0.07	0.10	CW	0.56	0.32	0.47	0.49	0.31	0.62	0.55	0.47	0.32	0.49
Taiwan	1.35	1.19	EA	0.36	0.33	0.35	0.40	0.33	0.33	0.35	0.23	0.40	0.34
Thailand	0.09	0.21	EA	0.42	0.41	0.42	0.49	0.35	0.37	0.36	0.26	0.54	0.50
Turkey	0.01	0.16	EA	0.33	0.29	0.32	0.37	0.30	0.33	0.37	0.39	0.26	0.32

The percentage of market capitalization for each market is reported as the percent of market capitalization of all 34 markets combined in U.S. dollars at the end of the year 1988 and 2002. The 2002 weights are the weights reported in Chiou (2008). The region that the country is in and the equal weighted average of coefficients of correlation using total return index data for each market with developed markets (DM), emerging markets (EM), all markets (All) and the markets in each region (North America (NA), Latin America (LA), Central & Western Europe (CW), Southern Europe (SE), Northern Europe (NE), East Asia (EA) and Oceania (OC)) are reported.

Finally, the use of 2002 market weights as the weight constraints used in Chiou (2008) introduces a hindsight bias into the results that can increase the measured benefits from diversification. Markets that do well ex-post can be expected to have heavier weights in the optimization solution. Such markets would also be expected to have larger capitalizations over time as a result of this growth. For example, in 1988 Japan is roughly 40% of the total global market capitalization and the U.S. is just over 29%. In 2002 Japan represents 9.41% of the total market capitalization of the 34 markets in this study. The United States is over 50%. The emerging markets combined are 3.89% of global capitalization in 1988. They expand to 5.01% in 2002. When using market capitalization weights to set the weight constraints in the optimization solution, it seems reasonable to this author that the market weights from the beginning of the period should be used to avoid hindsight bias. This chapter uses the 1988 market capitalization weights in the optimization solutions to reflect the optimization decision facing an investor at the beginning of the 1988-2004 and 1988-2014 periods reported in this study.

Table 3.1 reports the markets, their percentage of world capitalization in 1988 and 2002, the region the market is in, and the average coefficients of correlation between each market and other groups of markets categorized by development stage and geographical region as calculated using the total return index data from 1988 to 2014. The 1988-2004 correlation results using the total return data are not presented as they are very similar to those reported in Chiou (2008). The correlations between all markets and all regions increased from the 1988-2004 period to the 1988-2014 period. While the use of total return index data affects the returns markets are estimated to provide, it does not significantly change the standard deviation of these returns or correlations between markets. As a result, the MVP ϵ results are expected to be similar whether total return data or price index data is used.

3.3 Results

This section reports the results of re-examining the benefits of international diversification with weight constraints using the data and methods presented in section 3.2. Analysis of the results seeks to determine whether optimized static international portfolios with or without weight constraints and no short sales can offer statistically significant efficiency gains for a U.S. investor. The δ' results for the 1988-2004 period are reported first and show lower potential benefits from diversification for the weight constrained portfolios than presented in Chiou (2008). The δ and ϵ results for the 1988-2004 period and the extended 1988-2014 period are then presented and used to examine the time-varying nature of the potential benefits of diversification. The results from the Ledoit and Wolf (2008) Sharpe ratio and volatility tests are presented with the δ and ϵ results respectively. The p-values from these tests report the significance of the hypothesis that the difference between the monthly returns of the market portfolio and the optimized portfolio is zero. The subsection that follows measures the diversification gains that emerging markets provide a U.S. investor during the 1988 to 2014 period.

The geometric annual returns, the standard deviation of returns and the RR ratios for the markets used in this study are presented in Table 3.2. The use of total return data, which includes dividends, increases the RR ratios compared to those reported in Chiou (2008). Table 3.2 reports the MRRPs and MVPs optimized with no shorts and no weight constraints used in this study. These are referred to as unconstrained portfolios in this study. Panel C of Table 3.2 reports the annual returns, the standard deviation of returns and the MRR ratios of the unconstrained MRRP and MVP for both periods.

Changes to market returns that result from using total return data rather than price data have a greater effect on the optimal weights of the MRRP than the MVP. The MRRP for the 1988-2004 period has allocations to 3 more markets than in Chiou (2008): Australia, Belgium and Greece. The inclusion of these markets reduces the 50% weighting Denmark and Switzerland have in Chiou (2008) to roughly 30% here. The use of total return data does not change the standard deviation of market returns and correlations between markets significantly. As a result, the unconstrained MVP presented in this chapter for the 1988-2004 period is similar to that reported in Chiou (2008). The number of markets held in the MRRP and MVP in the 1988-2014 period is lower than the 1988-2004 period. This change to the portfolio weights between periods exemplifies the time-varying nature of portfolio efficiency and the difficulty that an investor will have predicting future market performance over extended periods. For example, it would be a confident investor to select a 6.57% weight in the MRRP for Greece in 1988 when it only represents .04% of the world market capitalization at that time. It would be an extraordinary investor to accurately predict in 1988 the impressive rise and dramatic fall of the Greek market to 2014 which warrants the exclusion of Greece from the MRRP optimized for the 1988-2014 holding period.

The market and portfolio characteristics reported in Table 3.2 provide the backdrop from which the benefits of international diversification for a U.S. investor are measured. An investor choosing a single market in which to invest would achieve both the highest RR and the lowest standard deviation of returns by investing in the domestic U.S. market. As the holding period is extended, the market returns and volatility are not constant. As a result, the optimal portfolios change. The MRRP and MVP market allocations in the later period are similar, more consolidated portfolios from the earlier sub-period. An investor choosing a buy and hold strategy must have confidence that they can align the selected efficient portfolio with their holding duration and that their risk tolerance over the extended holding period will remain as static as the chosen portfolio.

An investor who diversifies internationally must identify the efficient portfolio that will provide a higher MRR or a lower standard deviation than the local portfolio to meet their risk tolerance objectives. Critical to achieving efficiency gains is the accurate prediction of future market returns and volatility. It might be reasonable to think that many investors will have difficulty doing this. As a result, an investor may be interested in diversifying into the naive 1/M portfolio that the international Capital Asset Pricing Model (CAPM) suggests is optimal. Then, following the principles of the separation theorem, a risk adverse investor can adjust the size of the equity position with the risk free asset to achieve their preferred level of volatility. This chapter measures the gains the 1/M portfolio provides an investor in relation to the perfect hindsight required of the optimized MRRPs and MVPs.

Table 3.3 reports the benefits of diversification into the MRRP as measured by δ' for the 1988-2004 period using the data and methods presented in this study. The first column reports the measured benefits of diversifying from each single market into the no shorts restricted optimal portfolio constrained to the 2002 market capitalization weights (wx1). The next five columns report the measured benefits from diversifying out of each market and into the no shorts restricted optimal portfolios constrained by the 1988 market capitalization weights at various weight constrained limits: one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraints (nw). The final four columns report the portion of the maximum possible gains achieved by the MRRP with no weight constraints that are captured by each of the various weight constrained MRRPs using the 1988 market capitalization weights.

The first column reports the 2002 weight constrained results. It is presented to contrast the results reported for the 1988 market capitalization weight constrained portfolio. The MRRP optimized with no shorts and weight constraints set to equal the market capitalization weight of each market is the 1/M portfolio. These results show that using the 2002 weights introduces a hindsight bias that can magnify the potential measurable benefits from diversification. For example, a U.S. investor measuring the diversification benefit of the 1/M portfolio versus the Australian market will observe a 19.33% benefit when using the 2002 weights. The use of the 1988 weights results in a negative 44.05% benefit. Only the U.S. and Chilean markets provide negative benefits from the 1/M portfolio when calculated using the 2002 market capitalization weights. The 1988 weighted results report negative benefits for sixteen developed markets and three emerging markets. While not presented here, the size of the hindsight bias on the results weakens

Table 3.2: Market characteristics and optimal portfolio weights for 1988 to 2004 and 1988 to 2014.

	1988-2004					1988-2014				
Market(i)	r	sd	RR	w_i (MRRP)	w_i (MVP)	r	sd	RR	w_i (MRRP)	w_i (MVP)
Panel A: Developed Markets										
Australia	0.1162	0.1868	0.6220	0.0463	0.1057	0.1045	0.2116	0.4940	.	.
Austria	0.1035	0.2289	0.4519	.	0.0931	0.0433	0.2724	0.1590	.	.
Belgium	0.1303	0.1820	0.7156	0.0730	0.0790	0.0929	0.2145	0.4331	.	.
Canada	0.1031	0.1758	0.5864	.	.	0.0949	0.1925	0.4933	.	.
Denmark	0.1404	0.1855	0.7569	0.1467	0.0551	0.1330	0.2019	0.6586	0.1584	.
Finland	0.1169	0.3370	0.3468	.	.	0.0905	0.3149	0.2873	.	.
France	0.1185	0.1970	0.6017	.	.	0.0896	0.2093	0.4283	.	.
Germany	0.1004	0.2274	0.4417	.	.	0.0901	0.2337	0.3856	.	.
H.K.	0.1367	0.2729	0.5008	.	.	0.1200	0.2530	0.4743	.	.
Ireland	0.1111	0.1995	0.5571	.	.	0.0444	0.2293	0.1937	.	.
Italy	0.0827	0.2338	0.3536	.	.	0.0430	0.2460	0.1746	.	.
Japan	-0.0043	0.2339	-0.0185	.	0.0625	0.0062	0.2084	0.0299	.	0.1451
Holland	0.1224	0.1733	0.7062	.	.	0.1016	0.1950	0.5212	.	.
N.Z.	0.0718	0.2340	0.3070	.	0.0059	0.0662	0.2304	0.2874	.	0.0116
Norway	0.1158	0.2354	0.4920	.	.	0.0940	0.2696	0.3484	.	.

Singapore	0.0846	0.2506	0.3377	.	.	0.0922	0.2449	0.3764	.	.
Spain	0.1139	0.2202	0.5173	.	.	0.0887	0.2416	0.3672	.	.
Sweden	0.1469	0.2587	0.5680	.	.	0.1241	0.2573	0.4825	.	.
Suisse	0.1302	0.1727	0.7541	0.1501	0.0540	0.1147	0.1708	0.6713	0.2462	0.1857
U.K.	0.0988	0.1584	0.6235	.	0.0423	0.0791	0.1669	0.4737	.	0.0110
U.S.	0.1247	0.1439	0.8667	0.2319	0.4196	0.1073	0.1459	0.7355	0.3151	0.5481
Panel B: Emerging Markets										
Argentina	0.1821	0.6371	0.2859	0.0039	0.0097	0.1176	0.5709	0.2059	.	.
Brazil	0.1738	0.5641	0.3081	.	.	0.1448	0.4910	0.2949	.	.
Chile	0.1972	0.2538	0.7770	0.2008	0.0492	0.1498	0.2447	0.6122	0.1849	0.0570
Greece	0.1875	0.3501	0.5356	0.0657	.	0.0602	0.3656	0.1647	.	.
Indonesia	0.0799	0.5136	0.1555	.	.	0.1107	0.4521	0.2449	.	.
Korea	0.0556	0.3926	0.1416	.	.	0.0675	0.3556	0.1899	.	.
Malaysia	0.0689	0.3344	0.2059	.	0.0029	0.0828	0.2849	0.2906	.	0.0414
Mexico	0.2373	0.3440	0.6900	0.0815	.	0.1877	0.3113	0.6030	0.0955	.
Philipines	0.0320	0.3381	0.0946	.	.	0.0874	0.3047	0.2867	.	.
Portugal	0.0487	0.2274	0.2142	.	0.0209	0.0137	0.2345	0.0582	.	.
Taiwan	0.0627	0.3995	0.1569	.	.	0.0645	0.3466	0.1861	.	.
Thailand	0.0582	0.4198	0.1385	.	.	0.0840	0.3727	0.2253	.	.
Turkey	0.1136	0.6091	0.1865	.	.	0.1014	0.5388	0.1882	.	.
Panel C: MRRP and MVP characteristics										

Return	.	.	.	0.1559	0.1163	.	.	.	0.1287	0.0946
Standard Deviation	.	.	.	0.1422	0.1263	.	.	.	0.1515	0.1362
MRR	.	.	.	1.0964	0.9215	.	.	.	0.8494	0.6947

This table reports the geometric annual returns (r), the standard deviation of returns (sd) and the return-to-risk ratios (RR) for each market for the periods 1988-2004 and 1988-2014 using total return index data in U.S. dollars. The market weights for the MRRP and MVP that restrict short sales to zero and that have no weight constraints are reported for each period. Panel C reports the annual returns, the standard deviation of returns and the maximum return-to-risk (MRR) ratio for each MRRP and MVP.

Table 3.3: Delta prime results for the 1988 to 2004 holding period.

	2002w	1988w					benefit gained			
Market	wx1	wx1	wx2	wx5	wx10	nw	wx1	wx2	wx5	wx10
Panel A: Developed Markets										
Australia	0.1933	-0.4405	0.2946	0.3353	0.3627	0.4328	-1.02	0.68	0.77	0.84
Austria	0.4139	-0.0466	0.4875	0.5170	0.5370	0.5879	-0.08	0.83	0.88	0.91
Belgium	0.0719	-0.6574	0.1884	0.2352	0.2667	0.3474	-1.89	0.54	0.68	0.77
Canada	0.2394	-0.3582	0.3349	0.3733	0.3991	0.4652	-0.77	0.72	0.80	0.86
Denmark	0.0182	-0.7531	0.1415	0.1910	0.2244	0.3096	-2.43	0.46	0.62	0.72
Finland	0.5502	0.1967	0.6067	0.6293	0.6446	0.6837	0.29	0.89	0.92	0.94
France	0.2196	-0.3936	0.3176	0.3569	0.3834	0.4512	-0.87	0.70	0.79	0.85
Germany	0.4271	-0.0229	0.4991	0.5280	0.5474	0.5972	-0.04	0.84	0.88	0.92
H.K.	0.3504	-0.1599	0.4320	0.4648	0.4868	0.5432	-0.29	0.80	0.86	0.90
Ireland	0.2774	-0.2902	0.3682	0.4046	0.4292	0.4919	-0.59	0.75	0.82	0.87
Italy	0.5413	0.1809	0.5989	0.6220	0.6376	0.6775	0.27	0.88	0.92	0.94
Japan	1.0239	1.0427	1.0209	1.0197	1.0189	1.0168	1.03	1.00	1.00	1.00
Holland	0.0840	-0.6356	0.1991	0.2452	0.2764	0.3559	-1.79	0.56	0.69	0.78
N.Z.	0.6018	0.2890	0.6519	0.6719	0.6854	0.7200	0.40	0.91	0.93	0.95
Norway	0.3619	-0.1394	0.4420	0.4742	0.4959	0.5513	-0.25	0.80	0.86	0.90
Singapore	0.5620	0.2179	0.6170	0.6391	0.6540	0.6920	0.31	0.89	0.92	0.95
Spain	0.3291	-0.1980	0.4134	0.4472	0.4700	0.5282	-0.37	0.78	0.85	0.89

Sweden	0.2633	-0.3156	0.3558	0.3929	0.4180	0.4819	-0.65	0.74	0.82	0.87
Suisse	0.0219	-0.7466	0.1447	0.1940	0.2273	0.3122	-2.39	0.46	0.62	0.73
U.K.	0.1913	-0.4441	0.2928	0.3336	0.3611	0.4313	-1.03	0.68	0.77	0.84
U.S.	-0.1241	-1.0073	0.0171	0.0737	0.1119	0.2096	-4.81	0.08	0.35	0.53
Panel B: Emerging Markets										
Argentina	0.6292	0.3379	0.6758	0.6945	0.7071	0.7393	0.46	0.91	0.94	0.96
Brazil	0.6004	0.2865	0.6506	0.6708	0.6843	0.7190	0.40	0.90	0.93	0.95
Chile	-0.0078	-0.7996	0.1188	0.1696	0.2038	0.2914	-2.74	0.41	0.58	0.70
Greece	0.3053	-0.2405	0.3925	0.4275	0.4512	0.5115	-0.47	0.77	0.84	0.88
Indonesia	0.7982	0.6397	0.8236	0.8338	0.8406	0.8581	0.75	0.96	0.97	0.98
Korea	0.8164	0.6721	0.8394	0.8487	0.8549	0.8709	0.77	0.96	0.97	0.98
Malaysia	0.7329	0.5231	0.7665	0.7799	0.7890	0.8122	0.64	0.94	0.96	0.97
Mexico	0.1051	-0.5980	0.2175	0.2626	0.2930	0.3707	-1.61	0.59	0.71	0.79
Philippines	0.8773	0.7808	0.8927	0.8989	0.9030	0.9137	0.85	0.98	0.98	0.99
Portugal	0.7221	0.5038	0.7570	0.7710	0.7805	0.8046	0.63	0.94	0.96	0.97
Taiwan	0.7965	0.6366	0.8220	0.8323	0.8392	0.8569	0.74	0.96	0.97	0.98
Thailand	0.8203	0.6792	0.8429	0.8519	0.8581	0.8737	0.78	0.96	0.98	0.98
Turkey	0.7581	0.5680	0.7884	0.8006	0.8089	0.8299	0.68	0.95	0.96	0.97

This table reports the delta prime benefits at each level of market weight constrained MRRP with no shorts. The first column reports the 1/M portfolio using the 2002 market capitalization weights which has no shorts and market weights constrained to one times the market weight (wx1). The following five columns report the results using no shorts and the 1988 market capitalization weights at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. The percent, in decimal form, of the unconstrained benefits captured at each constraint level using the 1988 market capitalization weights are reported in the last four columns.

Table 3.4: Delta results for the 1988 to 2004 holding period.

						benefit gained			
Market	wx1	wx2	wx5	wx10	nw	wx1	wx2	wx5	wx10
Panel A: Developed Markets									
Australia	-0.3058 (0.532)	0.4177 (0.224)	0.5044 (0.180)	0.5691 (0.106)	0.7629 (0.048)	-0.40	0.55	0.66	0.75
Austria	-0.0445 (0.984)	0.9512 (0.194)	1.0705 (0.128)	1.1596 (0.096)	1.4263 (0.028)	-0.03	0.67	0.75	0.81
Belgium	-0.3966 (0.400)	0.2322 (0.424)	0.3075 (0.338)	0.3638 (0.246)	0.5322 (0.124)	-0.75	0.44	0.58	0.68
Canada	-0.2637 (0.644)	0.5036 (0.106)	0.5955 (0.082)	0.6642 (0.060)	0.8697 (0.014)	-0.30	0.58	0.68	0.76
Denmark	-0.4296 (0.284)	0.1649 (0.468)	0.2361 (0.424)	0.2893 (0.304)	0.4485 (0.084)	-0.96	0.37	0.53	0.64
Finland	0.2449 (0.828)	1.5423 (0.094)	1.6978 (0.060)	1.8139 (0.082)	2.1614 (0.038)	0.11	0.71	0.79	0.84
France	-0.2824 (0.538)	0.4654 (0.078)	0.5550 (0.066)	0.6219 (0.034)	0.8222 (0.010)	-0.34	0.57	0.67	0.76
Germany	-0.0224 (0.950)	0.9964 (0.030)	1.1185 (0.046)	1.2096 (0.012)	1.4826 (0.004)	-0.02	0.67	0.75	0.82
H.K.	-0.1379 (0.860)	0.7606 (0.202)	0.8683 (0.148)	0.9487 (0.110)	1.1894 (0.040)	-0.12	0.64	0.73	0.80
Ireland	-0.2249 (0.746)	0.5828 (0.148)	0.6796 (0.124)	0.7518 (0.064)	0.9682 (0.030)	-0.23	0.60	0.70	0.78
Italy	0.2209 (0.664)	1.4932 (0.052)	1.6457 (0.036)	1.7595 (0.014)	2.1004 (0.010)	0.11	0.71	0.78	0.84
Japan**1	-24.3931 (0.002)	-48.7725 (0.006)	-51.694 (0.006)	-53.8752 (0.006)	-60.4059 (0.004)	0.40	0.81	0.86	0.89
Holland	-0.3886 (0.278)	0.2485 (0.256)	0.3249 (0.164)	0.3819 (0.076)	0.5526 (0.018)	-0.70	0.45	0.59	0.69
N.Z.	0.4065 (0.546)	1.8723 (0.060)	2.0480 (0.046)	2.1791 (0.026)	2.5718 (0.010)	0.16	0.73	0.80	0.85
Norway	-0.1224 (0.904)	0.7923 (0.166)	0.9019 (0.118)	0.9837 (0.110)	1.2287 (0.020)	-0.10	0.64	0.73	0.80
Singapore	0.2786 (0.668)	1.6111 (0.066)	1.7708 (0.034)	1.8900 (0.040)	2.2469 (0.002)	0.12	0.72	0.79	0.84
Spain	-0.1653 (0.826)	0.7046 (0.064)	0.8089 (0.044)	0.8867 (0.024)	1.1197 (0.006)	-0.15	0.63	0.72	0.79

Sweden	-0.2399 (0.576)	0.5523 (0.094)	0.6472 (0.096)	0.7181 (0.058)	0.9303 (0.008)	-0.26	0.59	0.70	0.77
Suisse	-0.4274 (0.282)	0.1692 (0.436)	0.2408 (0.288)	0.2941 (0.142)	0.4540 (0.080)	-0.94	0.37	0.53	0.65
U.K.	-0.3075 (0.546)	0.4141 (0.090)	0.5006 (0.100)	0.5651 (0.066)	0.7585 (0.026)	-0.41	0.55	0.66	0.75
U.S.	-0.5018 (0.084)	0.0174 (0.494)	0.0796 (0.120)	0.1261 (0.128)	0.2651 (0.056)	-1.89	0.07	0.30	0.48
Panel B: Emerging Markets									
Argentina	0.5103 (0.936)	2.0843 (0.192)	2.2729 (0.146)	2.4137 (0.076)	2.8354 (0.030)	0.18	0.74	0.8	0.85
Brazil	0.4016 (0.950)	1.8622 (0.192)	2.0373 (0.118)	2.1680 (0.088)	2.5592 (0.020)	0.16	0.73	0.80	0.85
Chile	-0.4443 (0.336)	0.1348 (0.744)	0.2042 (0.630)	0.2560 (0.548)	0.4111 (0.152)	-1.08	0.33	0.50	0.62
Greece	-0.1939 (0.780)	0.6462 (0.260)	0.7469 (0.190)	0.8220 (0.158)	1.0471 (0.036)	-0.19	0.62	0.71	0.79
Indonesia	1.7757 (0.540)	4.6685 (0.074)	5.0152 (0.046)	5.2740 (0.048)	6.0489 (0.012)	0.29	0.77	0.83	0.87
Korea	2.0499 (0.348)	5.2284 (0.044)	5.6093 (0.042)	5.8937 (0.032)	6.7451 (0.010)	0.30	0.78	0.83	0.87
Malaysia	1.0969 (0.514)	3.2823 (0.072)	3.5441 (0.066)	3.7397 (0.042)	4.3251 (0.010)	0.25	0.76	0.82	0.86
Mexico	-0.3742 (0.426)	0.2779 (0.648)	0.3561 (0.578)	0.4144 (0.420)	0.5891 (0.156)	-0.64	0.47	0.60	0.70
Philipines	3.5626 (0.352)	8.3176 (0.032)	8.8874 (0.022)	9.3128 (0.004)	10.5865 (0.002)	0.34	0.79	0.84	0.88
Portugal	1.0153 (0.428)	3.1155 (0.030)	3.3671 (0.016)	3.5551 (0.010)	4.1177 (0.004)	0.25	0.76	0.82	0.86
Taiwan	1.7514 (0.474)	4.6189 (0.058)	4.9625 (0.054)	5.2190 (0.042)	5.9871 (0.020)	0.29	0.77	0.83	0.87
Thailand	2.1168 (0.476)	5.3649 (0.056)	5.7542 (0.050)	6.0448 (0.040)	6.9149 (0.014)	0.31	0.78	0.83	0.87
Turkey	1.3146 (0.766)	3.7269 (0.112)	4.0160 (0.074)	4.2318 (0.050)	4.8780 (0.014)	0.27	0.76	0.82	0.87

This table reports the delta benefits from diversification out the local market and into optimized MRRPs with no shorts and the 1988 market capitalization weight constraints at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. The p-value results from the Ledoit and Wolf (2008) Sharpe ratio tests using a block size of 5 and 499 iterations are presented in brackets. The percent, in decimal form, of the unconstrained benefits captured at each constraint level using the 1988 market capitalization weights are reported in the last four columns.

Note **1: The Japan delta results have a negative sign, but are positive gains. The return-to-risk ratio of the Japanese market is negative during the 1988-2004 period which makes the result negative, although the MRRPs provide positive benefits.

Table 3.5: Delta results for the 1988 to 2014 holding period.

Market	wx1	wx2	wx5	wx10	nw	benefit gained			
						wx1	wx2	wx5	wx10
Panel A: Developed Markets									
Australia	-0.2059 (0.562)	0.4055 (0.206)	0.5420 (0.180)	0.5810 (0.130)	0.7195 (0.028)	-0.29	0.56	0.75	0.81
Austria	1.4672 (0.300)	3.3665 (0.018)	3.7906 (0.012)	3.9120 (0.010)	4.3422 (0.002)	0.34	0.78	0.87	0.90
Belgium	-0.0942 (0.892)	0.6032 (0.192)	0.7588 (0.150)	0.8034 (0.110)	0.9614 (0.042)	-0.10	0.63	0.79	0.84
Canada	-0.2047 (0.600)	0.4075 (0.098)	0.5442 (0.094)	0.5834 (0.070)	0.7220 (0.006)	-0.28	0.56	0.75	0.81
Denmark	-0.4043 (0.186)	0.0543 (0.800)	0.1566 (0.598)	0.1859 (0.472)	0.2898 (0.098)	-1.40	0.19	0.54	0.64
Finland	0.3656 (0.706)	1.4168 (0.070)	1.6515 (0.056)	1.7187 (0.050)	1.9568 (0.034)	0.19	0.72	0.84	0.88
France	-0.0841 (0.944)	0.6211 (0.036)	0.7785 (0.046)	0.8236 (0.016)	0.9833 (0.004)	-0.09	0.63	0.79	0.84
Germany	0.0174 (0.922)	0.8006 (0.036)	0.9755 (0.034)	1.0255 (0.028)	1.2029 (0.006)	0.01	0.67	0.81	0.85
H.K.	-0.1728 (0.712)	0.4640 (0.336)	0.6061 (0.232)	0.6468 (0.172)	0.7911 (0.052)	-0.22	0.59	0.77	0.82
Ireland	1.0256 (0.248)	2.5849 (0.008)	2.9330 (0.008)	3.0327 (0.002)	3.3859 (0.002)	0.30	0.76	0.87	0.90
Italy	1.2471 (0.232)	2.9769 (0.010)	3.3631 (0.006)	3.4737 (0.002)	3.8655 (0.004)	0.32	0.77	0.87	0.90
Japan	12.1254 (0.004)	22.2296 (0.008)	24.4855 (0.004)	25.1314 (0.006)	27.4201 (0.004)	0.44	0.81	0.89	0.92
Holland	-0.2474 (0.440)	0.3320 (0.124)	0.4613 (0.084)	0.4984 (0.064)	0.6296 (0.004)	-0.39	0.53	0.73	0.79
N.Z.	0.3649 (0.542)	1.4157 (0.038)	1.6503 (0.044)	1.7174 (0.024)	1.9555 (0.012)	0.19	0.72	0.84	0.88
Norway	0.1259 (0.852)	0.9926 (0.074)	1.1861 (0.092)	1.2415 (0.050)	1.4378 (0.008)	0.09	0.69	0.82	0.86
Singapore	0.0422 (0.888)	0.8445 (0.084)	1.0236 (0.074)	1.0749 (0.042)	1.2567 (0.006)	0.03	0.67	0.81	0.86
Spain	0.0682 (0.818)	0.8906 (0.066)	1.0741 (0.034)	1.1267 (0.030)	1.3130 (0.014)	0.05	0.68	0.82	0.86

Sweden	-0.1869 (0.600)	0.4391 (0.130)	0.5788 (0.108)	0.6188 (0.074)	0.7606 (0.016)	-0.25	0.58	0.76	0.81
Suisse	-0.4156 (0.162)	0.0343 (0.736)	0.1348 (0.526)	0.1635 (0.428)	0.2654 (0.130)	-1.57	0.13	0.51	0.62
U.K.	-0.1719 (0.814)	0.4656 (0.046)	0.6079 (0.026)	0.6487 (0.018)	0.7931 (0.004)	-0.22	0.59	0.77	0.82
U.S.	-0.4666 (0.026)	-0.0560 (0.880)	0.0357 (0.088)	0.0620 (0.076)	0.1550 (0.166)	-3.01	-0.36	0.23	0.40
Panel B: Emerging Markets									
Argentina	0.9052 (0.828)	2.3718 (0.180)	2.6992 (0.130)	2.7930 (0.128)	3.1252 (0.036)	0.29	0.76	0.86	0.89
Brazil	0.3303 (0.940)	1.3545 (0.260)	1.5831 (0.208)	1.6486 (0.164)	1.8806 (0.058)	0.18	0.72	0.84	0.88
Chile	-0.3592 (0.430)	0.1341 (0.766)	0.2443 (0.644)	0.2758 (0.550)	0.3875 (0.214)	-0.93	0.35	0.63	0.71
Greece	1.3819 (0.424)	3.2155 (0.062)	3.6248 (0.036)	3.7421 (0.032)	4.1574 (0.016)	0.33	0.77	0.87	0.90
Indonesia	0.6019 (0.764)	1.8351 (0.146)	2.1104 (0.124)	2.1892 (0.112)	2.4685 (0.034)	0.24	0.74	0.85	0.89
Korea	1.0663 (0.378)	2.6570 (0.050)	3.0122 (0.040)	3.1139 (0.026)	3.4742 (0.010)	0.31	0.76	0.87	0.90
Malaysia	0.3499 (0.722)	1.3890 (0.180)	1.6210 (0.146)	1.6875 (0.102)	1.9229 (0.062)	0.18	0.72	0.84	0.88
Mexico	-0.3494 (0.288)	0.1514 (0.804)	0.2632 (0.638)	0.2952 (0.600)	0.4087 (0.240)	-0.86	0.37	0.64	0.72
Philipines	0.3684 (0.758)	1.4218 (0.176)	1.6570 (0.128)	1.7244 (0.078)	1.9630 (0.052)	0.19	0.72	0.84	0.88
Portugal	5.7371 (0.102)	10.9235 (0.010)	12.0814 (0.008)	12.413 (0.004)	13.5878 (0.004)	0.42	0.80	0.89	0.91
Taiwan	1.1080 (0.468)	2.7309 (0.082)	3.0932 (0.070)	3.1969 (0.054)	3.5645 (0.018)	0.31	0.77	0.87	0.90
Thailand	0.7409 (0.626)	2.0811 (0.116)	2.3803 (0.094)	2.4660 (0.070)	2.7696 (0.022)	0.27	0.75	0.86	0.89
Turkey	1.0846 (0.808)	2.6894 (0.176)	3.0477 (0.118)	3.1503 (0.114)	3.5138 (0.042)	0.31	0.77	0.87	0.90

This table reports the delta benefits from diversification out the local market and into optimized MRRPs with no shorts and the 1988 market capitalization weight constraints at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. The p-value results from the Ledoit and Wolf (2008) Sharpe ratio tests using a block size of 5 and 499 iterations are presented in brackets. The percent, in decimal form, of the unconstrained benefits captured at each constraint level using the 1988 market capitalization weights are reported in the last four columns.

Table 3.6: Epsilon results for the 1988 to 2004 holding period.

										benefit gained			
Market	wx1		wx2		wx5		wx10		nw		wx1	wx2	wx5	wx10
Panel A: Developed Markets														
Australia	0.1789	(0.006)	0.2937	(0.002)	0.3068	(0.002)	0.3141	(0.002)	0.3242	(0.002)	0.55	0.91	0.95	0.97
Austria	0.3300	(0.002)	0.4236	(0.002)	0.4343	(0.002)	0.4403	(0.002)	0.4485	(0.002)	0.74	0.94	0.97	0.98
Belgium	0.1574	(0.084)	0.2751	(0.004)	0.2886	(0.004)	0.2961	(0.004)	0.3064	(0.004)	0.51	0.9	0.94	0.97
Canada	0.1276	(0.100)	0.2496	(0.002)	0.2635	(0.002)	0.2712	(0.002)	0.2820	(0.002)	0.45	0.89	0.93	0.96
Denmark	0.1729	(0.002)	0.2885	(0.002)	0.3017	(0.002)	0.3091	(0.002)	0.3193	(0.002)	0.54	0.9	0.95	0.97
Finland	0.5448	(0.002)	0.6084	(0.002)	0.6157	(0.002)	0.6197	(0.002)	0.6253	(0.002)	0.87	0.97	0.98	0.99
France	0.2212	(0.002)	0.3300	(0.002)	0.3424	(0.002)	0.3494	(0.002)	0.3590	(0.002)	0.62	0.92	0.95	0.97
Germany	0.3254	(0.002)	0.4197	(0.002)	0.4305	(0.002)	0.4365	(0.002)	0.4448	(0.002)	0.73	0.94	0.97	0.98
H.K.	0.4379	(0.002)	0.5165	(0.002)	0.5255	(0.002)	0.5305	(0.002)	0.5374	(0.002)	0.81	0.96	0.98	0.99
Ireland	0.2311	(0.002)	0.3386	(0.002)	0.3508	(0.002)	0.3577	(0.002)	0.3671	(0.002)	0.63	0.92	0.96	0.97
Italy	0.3439	(0.002)	0.4356	(0.002)	0.4461	(0.002)	0.4519	(0.002)	0.4600	(0.002)	0.75	0.95	0.97	0.98
Japan	0.3441	(0.002)	0.4358	(0.002)	0.4463	(0.002)	0.4521	(0.002)	0.4602	(0.002)	0.75	0.95	0.97	0.98
Holland	0.1147	(0.200)	0.2385	(0.002)	0.2526	(0.002)	0.2605	(0.002)	0.2714	(0.002)	0.42	0.88	0.93	0.96
N.Z.	0.3444	(0.002)	0.4360	(0.002)	0.4465	(0.002)	0.4523	(0.002)	0.4604	(0.002)	0.75	0.95	0.97	0.98
Norway	0.3483	(0.002)	0.4394	(0.002)	0.4498	(0.002)	0.4556	(0.002)	0.4636	(0.002)	0.75	0.95	0.97	0.98
Singapore	0.3879	(0.002)	0.4735	(0.002)	0.4832	(0.002)	0.4887	(0.002)	0.4962	(0.002)	0.78	0.95	0.97	0.98
Spain	0.3034	(0.002)	0.4008	(0.002)	0.4119	(0.002)	0.4181	(0.002)	0.4267	(0.002)	0.71	0.94	0.97	0.98

Sweden	0.4070	(0.002)	0.4899	(0.002)	0.4994	(0.002)	0.5047	(0.002)	0.5120	(0.002)	0.80	0.96	0.98	0.99
Suisse	0.1118	(0.012)	0.2359	(0.002)	0.2501	(0.002)	0.2580	(0.002)	0.2689	(0.002)	0.42	0.88	0.93	0.96
U.K.	0.0315	(0.482)	0.1669	(0.004)	0.1823	(0.004)	0.1910	(0.002)	0.2029	(0.004)	0.16	0.82	0.90	0.94
U.S.	-0.0657	(0.224)	0.0833	(0.004)	0.1003	(0.002)	0.1098	(0.002)	0.1229	(0.004)	-0.53	0.68	0.82	0.89
Panel B: Emerging Markets														
Argentina	0.7592	(0.002)	0.7929	(0.002)	0.7967	(0.002)	0.7989	(0.002)	0.8018	(0.002)	0.95	0.99	0.99	1.00
Brazil	0.7281	(0.002)	0.7661	(0.002)	0.7704	(0.002)	0.7729	(0.002)	0.7762	(0.002)	0.94	0.99	0.99	1.00
Chile	0.3955	(0.002)	0.4800	(0.002)	0.4897	(0.002)	0.4950	(0.002)	0.5025	(0.002)	0.79	0.96	0.97	0.99
Greece	0.5618	(0.002)	0.6231	(0.004)	0.6300	(0.002)	0.6339	(0.002)	0.6393	(0.002)	0.88	0.97	0.99	0.99
Indonesia	0.7013	(0.002)	0.7431	(0.002)	0.7478	(0.002)	0.7505	(0.002)	0.7542	(0.002)	0.93	0.99	0.99	1.00
Korea	0.6092	(0.002)	0.6639	(0.002)	0.6701	(0.002)	0.6736	(0.002)	0.6784	(0.002)	0.90	0.98	0.99	0.99
Malaysia	0.5413	(0.004)	0.6054	(0.002)	0.6127	(0.002)	0.6168	(0.002)	0.6224	(0.002)	0.87	0.97	0.98	0.99
Mexico	0.5541	(0.002)	0.6164	(0.002)	0.6235	(0.002)	0.6275	(0.002)	0.6330	(0.002)	0.88	0.97	0.99	0.99
Philipines	0.5464	(0.002)	0.6098	(0.002)	0.6170	(0.002)	0.6211	(0.002)	0.6266	(0.002)	0.87	0.97	0.98	0.99
Portugal	0.3255	(0.002)	0.4198	(0.002)	0.4306	(0.002)	0.4366	(0.002)	0.4449	(0.002)	0.73	0.94	0.97	0.98
Taiwan	0.6161	(0.002)	0.6697	(0.002)	0.6759	(0.002)	0.6793	(0.002)	0.6840	(0.002)	0.90	0.98	0.99	0.99
Thailand	0.6346	(0.002)	0.6857	(0.002)	0.6915	(0.002)	0.6948	(0.002)	0.6993	(0.002)	0.91	0.98	0.99	0.99
Turkey	0.7482	(0.002)	0.7834	(0.002)	0.7874	(0.002)	0.7896	(0.002)	0.7927	(0.002)	0.94	0.99	0.99	1.00

This table reports the epsilon benefits from diversification out the local market and into optimized MVPs with no shorts and the 1988 market capitalization weight constraints at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. The p-value results from the Ledoit and Wolf (2008) volatility tests using a block size of 5 and 499 iterations are presented in brackets. The percent, in decimal form, of the unconstrained benefits captured at each constraint level using the 1988 market capitalization weights are reported in the last four columns.

Table 3.7: Epsilon results for the 1988 to 2014 holding period.

											benefit gained			
Market	wx1		wx2		wx5		wx10		nw		wx1	wx2	wx5	wx10
Panel A: Developed Markets														
Australia	0.2733	(0.002)	0.3437	(0.002)	0.3492	(0.002)	0.3532	(0.002)	0.3563	(0.002)	0.77	0.96	0.98	0.99
Austria	0.4356	(0.002)	0.4902	(0.002)	0.4945	(0.002)	0.4976	(0.002)	0.5000	(0.002)	0.87	0.98	0.99	1.00
Belgium	0.2831	(0.008)	0.3525	(0.002)	0.3580	(0.002)	0.3619	(0.002)	0.3650	(0.002)	0.78	0.97	0.98	0.99
Canada	0.2012	(0.002)	0.2785	(0.002)	0.2846	(0.002)	0.2890	(0.002)	0.2924	(0.002)	0.69	0.95	0.97	0.99
Denmark	0.2386	(0.002)	0.3123	(0.002)	0.3181	(0.002)	0.3223	(0.002)	0.3255	(0.002)	0.73	0.96	0.98	0.99
Finland	0.5118	(0.002)	0.5591	(0.002)	0.5628	(0.002)	0.5655	(0.002)	0.5675	(0.002)	0.90	0.99	0.99	1.00
France	0.2653	(0.002)	0.3364	(0.002)	0.3420	(0.002)	0.3461	(0.002)	0.3492	(0.002)	0.76	0.96	0.98	0.99
Germany	0.3421	(0.002)	0.4058	(0.002)	0.4108	(0.002)	0.4144	(0.002)	0.4172	(0.002)	0.82	0.97	0.98	0.99
H.K.	0.3924	(0.002)	0.4512	(0.002)	0.4558	(0.002)	0.4592	(0.002)	0.4618	(0.002)	0.85	0.98	0.99	0.99
Ireland	0.3295	(0.002)	0.3945	(0.002)	0.3995	(0.002)	0.4032	(0.002)	0.4061	(0.002)	0.81	0.97	0.98	0.99
Italy	0.3752	(0.002)	0.4357	(0.002)	0.4404	(0.002)	0.4438	(0.002)	0.4465	(0.002)	0.84	0.98	0.99	0.99
Japan	0.2623	(0.002)	0.3338	(0.002)	0.3393	(0.002)	0.3434	(0.002)	0.3466	(0.002)	0.76	0.96	0.98	0.99
Holland	0.2116	(0.002)	0.2880	(0.002)	0.2939	(0.002)	0.2983	(0.002)	0.3016	(0.002)	0.70	0.95	0.97	0.99
N.Z.	0.3327	(0.002)	0.3973	(0.002)	0.4024	(0.002)	0.4061	(0.002)	0.4089	(0.002)	0.81	0.97	0.98	0.99
Norway	0.4298	(0.002)	0.4850	(0.002)	0.4894	(0.002)	0.4925	(0.002)	0.4949	(0.002)	0.87	0.98	0.99	1.00
Singapore	0.3723	(0.002)	0.4330	(0.002)	0.4378	(0.002)	0.4413	(0.002)	0.4439	(0.002)	0.84	0.98	0.99	0.99
Spain	0.3637	(0.002)	0.4253	(0.002)	0.4301	(0.002)	0.4336	(0.002)	0.4363	(0.002)	0.83	0.97	0.99	0.99

Sweden	0.4025	(0.002)	0.4603	(0.002)	0.4649	(0.002)	0.4682	(0.002)	0.4707	(0.002)	0.86	0.98	0.99	0.99
Suisse	0.0999	(0.012)	0.1871	(0.002)	0.1939	(0.002)	0.1988	(0.002)	0.2027	(0.002)	0.49	0.92	0.96	0.98
U.K.	0.0790	(0.014)	0.1682	(0.002)	0.1751	(0.002)	0.1802	(0.002)	0.1841	(0.002)	0.43	0.91	0.95	0.98
U.S.	-0.0539	(0.172)	0.0482	(0.016)	0.0561	(0.004)	0.0620	(0.004)	0.0665	(0.002)	-0.81	0.72	0.84	0.93
Panel B: Emerging Markets														
Argentina	0.7307	(0.004)	0.7568	(0.002)	0.7588	(0.002)	0.7603	(0.002)	0.7615	(0.002)	0.96	0.99	1.00	1.00
Brazil	0.6869	(0.002)	0.7172	(0.002)	0.7196	(0.002)	0.7213	(0.002)	0.7227	(0.002)	0.95	0.99	1.00	1.00
Chile	0.3717	(0.002)	0.4325	(0.002)	0.4373	(0.002)	0.4408	(0.002)	0.4434	(0.002)	0.84	0.98	0.99	0.99
Greece	0.5795	(0.002)	0.6202	(0.002)	0.6234	(0.002)	0.6257	(0.002)	0.6275	(0.002)	0.92	0.99	0.99	1.00
Indonesia	0.6599	(0.002)	0.6929	(0.002)	0.6954	(0.002)	0.6973	(0.002)	0.6988	(0.002)	0.94	0.99	1.00	1.00
Korea	0.5677	(0.002)	0.6095	(0.002)	0.6128	(0.002)	0.6152	(0.002)	0.6170	(0.002)	0.92	0.99	0.99	1.00
Malaysia	0.4604	(0.004)	0.5126	(0.002)	0.5167	(0.002)	0.5197	(0.002)	0.5220	(0.002)	0.88	0.98	0.99	1.00
Mexico	0.5061	(0.002)	0.5539	(0.002)	0.5576	(0.002)	0.5603	(0.002)	0.5625	(0.002)	0.90	0.98	0.99	1.00
Philipines	0.4955	(0.002)	0.5443	(0.002)	0.5482	(0.002)	0.5509	(0.002)	0.5531	(0.002)	0.90	0.98	0.99	1.00
Portugal	0.3444	(0.002)	0.4078	(0.002)	0.4128	(0.002)	0.4164	(0.002)	0.4192	(0.002)	0.82	0.97	0.98	0.99
Taiwan	0.5564	(0.002)	0.5994	(0.002)	0.6027	(0.002)	0.6052	(0.002)	0.6071	(0.002)	0.92	0.99	0.99	1.00
Thailand	0.5875	(0.002)	0.6274	(0.002)	0.6305	(0.002)	0.6328	(0.002)	0.6346	(0.002)	0.93	0.99	0.99	1.00
Turkey	0.7147	(0.002)	0.7423	(0.002)	0.7444	(0.002)	0.7460	(0.002)	0.7472	(0.002)	0.96	0.99	1.00	1.00

This table reports the epsilon benefits from diversification out the local market and into optimized MVPs with no shorts and the 1988 market capitalization weight constraints at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. The p-value results from the Ledoit and Wolf (2008) volatility tests using a block size of 5 and 499 iterations are presented in brackets. The percent, in decimal form, of the unconstrained benefits captured at each constraint level using the 1988 market capitalization weights are reported in the last four columns.

as the weight constraints are relaxed. At the two times weight constraint level the effect is largely dissipated. For example, the Canadian δ' benefit at the two times constraint level using the 2002 weights is 35.83%. At the five times level it is 39.25%. The 1988 weights result in improvements of 33.49% and 37.33% respectively.

A comparison of the benefits presented in table 3.3 with those reported in Chiou (2008) confirm that the data and methods presented in this chapter change the size of the reported benefits. The inclusion of dividends effects the magnitude of potential gains from diversification. The size of the effect is different for each market as the size of the dividends paid by these markets are different. For example, the unconstrained MRRP provides a 43.28% benefit in the Australian case using the data and methods reported in this study. It is reported to provide a 54.28% benefit in Chiou (2008). However, in the case of Austria, the benefit reported here is 58.79% and 58.34% in Chiou (2008). The size of the reported benefits adjust downwards as weight constraints are strengthened. Chiou (2008) reports that the optimized portfolio constrained to a ten times weight constraint level does not lose any diversification benefits compared to the unconstrained MRRP. However, the results presented in table 3.3 report that the ten times weight constrained portfolio is not as efficient as the unconstrained portfolio. In the case of emerging markets, the ten times weight constrained MRRP captures as much as 98 percent of the unconstrained MRRP gains. It only captures 53% of these potential gains when diversifying out of the U.S. market.

Table 3.4 reports the δ benefits from diversification for the 1988-2004 investment period. This method measures the percentage change in the return-to-risk ratio an investor achieves when they diversify out of the local market portfolio and into the MRRP. These results provide a different perspective of the benefits as reported by the δ' results from table 3.3 which measure the percentage reduction to the MRR when an investor moves from the MRRP to the local portfolio. In the case of Japan the results for δ are negative. This is an anomaly that results from the negative RR ratio that Japan has for the 1988-2004 period. For example, the MRR of the 1/M portfolio is .4295 and the Japan market RR ratio is -0.0185. Moving to the MRRP provides a positive benefit to the investor that is reported as a negative value because of the negative RR ratio of the Japanese market used in the δ equation.

Table 3.4 also reports the results from the Ledoit and Wolf (2008) Sharpe ratio tests. The p-values from these tests are presented in parentheses. The results report the significance of the hypothesis that the RR difference between the monthly returns of the market portfolio and the optimized portfolio is zero. A 10% result is treated as statistically significant. In the 1988-2004 period, only the unconstrained MRRP provides a positive diversification benefit that is statistically significant when compared to the domestic U.S. market portfolio. In the case of the MRRPs with allocation constraints, the weights assigned to the U.S. market in these portfolios is larger than the U.S. market capitalization weight for 1988. As a result, these MRRPs may be considered portfolios that are approaching the U.S. market. This may explain why the p-values for these portfolios are above 10% when diversifying out of the U.S. domestic market portfolio.

Table 3.5 reports the δ benefits for the 1988-2014 period. While the benefits are different than those for the 1988-2004 period, there are some patterns that hold during both periods. First, a U.S. investor does not benefit from diversifying out of the domestic U.S. portfolio and into the naive 1/M portfolio formed using the market weights at the beginning of the sample period. Next, as weight constraints are relaxed the potential benefits from diversification increase. For many markets the two times weight constraint level captures the majority of potential gains from diversification offered by the MRRP unconstrained by market weights. For the majority of emerging markets, over 70% of the maximum potential gains are captured by the two times market weight constrained portfolio. An investor diversifying out of developed markets tends to capture the majority of potential non-weight constrained gains at the two times weight constraint level. The size of these gains are not constant. In the first period, an investor diversifying out of the U.S. market achieves a small positive 1.74% improvement of the RR ratio at the two times constraint level. In the 1988-2014 period the investor does not achieve a positive improvement at the two times constraint level.

For the 1988-2014 period, the 1/M portfolio compared to the domestic U.S. market portfolio has a p-value below 5%. However, the 1/M portfolio does not provide a positive diversification benefit. The 1/M portfolio has an MRR of .3923 while the U.S. RR for the period is .7355. While relaxing weight constraints on the MRRP to the five times level provides positive potential gains from diversification, none of the MRRPs provide gains that are statistically different than the domestic U.S. market portfolio for both the 1988-2004 and the 1988-2014 holding periods.

Table 3.6 reports the benefits of diversification into the MVP for the 1988-2004 period. As with the MRRP, the unconstrained MVP provides potential benefits when diversifying out of any market. The smallest gains come from diversifying out of the U.S. market. In the case of the 1/M portfolio, diversifying out of the U.S. market results in a loss in efficiency. Similar to the MRRP, the majority of the potential unconstrained gains are generally captured by the two times market weight constrained MVP. The smallest gains come from diversifying out of the U.S. market. The two times weight constrained MVP volatility reduction out of the U.S. market is 8.33%. This is 67.77% of the maximum potential unconstrained gains from diversifying away from the U.S. market. At the two times market constraint level all markets achieve positive diversification benefits that are statistically different from the local portfolio. While the RR ratios for the MVPs are not presented, in the case of diversifying out of the U.S. market portfolio, only the MVP with weights relaxed to 10 times market capitalization and the non-weight constrained MVP provide improved RR ratios. The 10 times market capitalization restricted MVP has an MRR of .8849 which exceeds the local U.S. market RR for the 1988-2004 period of .8667. The non-weight constrained MVP has an MRR of .9215.

Table 3.7 reports the benefits of diversification into the MVP for the 1988-2014 period. The results are similar to those presented in the previous table. The 1/M portfolio does not show statistically different results versus the domestic U.S. portfolio. Weakening the weight constraints results in p-values below the 1% level. None of the optimized MVPs in the 1988-2014 period have RR ratios that exceed the local U.S. market portfolio.

The delta and epsilon results for the 1988-2004 and 1988-2014 periods suggest that the single best market for the U.S. investor to be invested in is the domestic U.S. market for both periods studied. Diversification into the naive 1/M portfolio does not provide efficiency gains over the domestic U.S. market portfolio when trying to improve the RR ratio or reduce volatility. Relaxing the weight constraints on the MRRP and MVP does increase the potential gains from improved RR performance and the volatility reducing benefits from diversification. The optimal portfolios with market allocations constrained at two times market weight generally capture a majority of the potential unconstrained gains from diversification. A U.S. investor diversifying out of the domestic U.S. market portfolio and into optimized MRRPs does not achieve positive gains from diversification that are statistically significant in both periods. Diversification out of the domestic U.S. market portfolio and into optimized MVPs does achieve statistically significant volatility reducing benefits. However, none of the optimized MVPs in the 1988-2014 period have RR ratios that exceed the local U.S. market portfolio.

3.3.1 The benefits of emerging markets

This section investigates the potential benefits of diversification that emerging markets provide a U.S. investor diversifying out of the domestic U.S. market during the 1988 to 2014 holding period. Emerging markets are a growing portion of the global equities market. In 1988, the 13 emerging markets in this study represent roughly 3.9% of the total market value of the 34 markets. In 2002, these markets represent 5.75% of global market capitalization. By 2012 their combined capitalization increases to 13.4%.

Table 3.8 reports the potential benefits from diversification out of the

U.S. market and into spanning MRRPs consisting of only developed markets (DM), developed markets with Asian emerging markets (DM+A), developed markets with European emerging markets (DM+E), developed markets with Latin American emerging markets (DM+LA) and all 34 markets (All). The percent of the maximum all market unconstrained portfolio gains that each weight constrained spanning optimized portfolio captures are also reported.

Table 3.8 shows that adding emerging markets to a portfolio of developed markets does not always improve MRRP performance. In 1988, the developed markets were 96.11% of the total market capitalization of the 34 markets used in this study. Asian, European and Latin American markets were 3.19%, 0.13% and 0.57% of global market capitalization respectively. When market allocation constraints on the optimized spanning portfolios are relaxed, most Asian and European emerging market positions are reduced or dropped from the optimization solutions. As a result, the spanning DM+A and DM+E portfolio market allocations approach the allocation weights in the DM portfolio. Without weight constraints the portfolios have the same market allocations which is evident by the 0.0861 delta benefit achieved by the three portfolios.

The Latin American emerging markets of Chile and Mexico offer the greatest potential efficiencies to a portfolio of developed markets held during the 1988-2014 investment period. As a result, as weight constraints are relaxed on the DM+LA portfolio, the Chilean and Mexican allocations are maximized and RR improvements beyond the DM portfolio are achieved. The -0.0481, 0.0367 and 0.0635 delta benefits obtained by the DM+LA portfolio at the wx2, wx5 and wx10 constraint levels are larger than the All

Table 3.8: Spanning MRRP delta results for the 1988-2014 period.

						benefit gained				
Span	wx1	wx2	wx5	wx10	nw	wx1	wx2	wx5	wx10	nw
DM	-0.4707	-0.0512	0.0313	0.0530	0.0861	-3.04	-0.33	0.20	0.34	0.56
DM+A	-0.4705	-0.0587	0.0305	0.0518	0.0861	-3.04	-0.38	0.20	0.33	0.56
DM+E	-0.4709	-0.0516	0.0312	0.0529	0.0861	-3.04	-0.33	0.20	0.34	0.56
DM+LA	-0.4663	-0.0481	0.0367	0.0635	0.1550	-3.01	-0.31	0.24	0.42	1.00
All	-0.4666	-0.0560	0.0357	0.0620	0.1550	-3.01	-0.36	0.23	0.40	1.00

This table reports the delta benefits at each level of market weight constrained MRRP with no shorts versus the U.S. market for the 1988-2014 period. The results for spanning portfolios optimized with only developed markets (DM), developed markets with Asian emerging markets (DM+A), developed markets with European emerging markets (DM+E), developed markets with Latin American emerging markets (DM+LA) and all 34 markets (All) are presented. The percent, in decimal form, of the maximum all market unconstrained MRRP gains that each weight constrained spanning portfolio captures is also reported.

Table 3.9: Spanning MVP epsilon results for the 1988-2014 period.

						benefit gained				
Span	wx1	wx2	wx5	wx10	nw	wx1	wx2	wx5	wx10	nw
DM	-0.0480	0.0483	0.0537	0.0575	0.0581	-0.72	0.73	0.81	0.87	0.87
DM+A	-0.0526	0.0480	0.0555	0.0609	0.0630	-0.79	0.72	0.83	0.92	0.95
DM+E	-0.0479	0.0484	0.0538	0.0577	0.0581	-0.72	0.73	0.81	0.87	0.87
DM+LA	-0.0493	0.0484	0.0543	0.0587	0.0638	-0.74	0.73	0.82	0.88	0.96
All	-0.0539	0.0482	0.0561	0.0620	0.0665	-0.81	0.72	0.84	0.93	1.00

This table reports the epsilon benefits at each level of market weight constrained MVP with no shorts versus the U.S. market for the 1988-2014 period. The results for spanning portfolios optimized with only developed markets (DM), developed markets with Asian emerging markets (DM+A), developed markets with European emerging markets (DM+E), developed markets with Latin American emerging markets (DM+LA) and all 34 markets (All) are presented. The percent, in decimal form, of the maximum all market unconstrained MVP gains that each weight constrained spanning portfolio captures is also reported.

portfolio benefits. This occurs because Chile and Mexico have larger relative global weights in the DM+LA portfolio than the All portfolio. As a result, the Chilean and Mexican positions are larger in the DM+LA portfolio than the All portfolio, which produces larger RR gains versus the DM portfolio. Despite the potential RR gains that Latin American emerging markets offer, weakening the weight constraints on the wx2 DM portfolio can potentially provide larger diversification benefits than holding weight constraints constant and including emerging markets.

Table 3.9 shows that the addition of emerging markets to the MVP of developed markets can improve performance slightly. As with the MRRP results, weakening the weight constraints on a portfolio of developed markets can offer potentially larger gains than holding weight constraints constant and including emerging markets.

3.4 Conclusion

This chapter identifies adjustments to the data, methods and perspective presented in Chiou (2008) that result in lower potential benefits from international diversification than previously reported. The U.S. market is the single best market for a U.S. investor to invest in during the 1988-2004 and 1988-2014 holding periods and, as a result, it provides a U.S. investor with the smallest gains from diversification of all the markets measured. The Ledoit and Wolf (2008) bootstrapping methods designed to test the difference in return-to-risk ratios and volatility between two portfolios are used to test the significance of the benefits achieved by the optimized portfolios versus each local market. The results find that the 1/M portfolio and all optimal MRRPs with and without weight constraints do not provide statistically significant return-to-risk improvements beyond the local U.S. market portfolio for both periods studied. The MVPs with relaxed weight constraints provide levels of volatility that are statistically different than the local U.S. market portfolio. However, none of these MVPs provide a RR ratio that exceeds the naive local U.S. market portfolio during the 1988-2014 period.

It is unclear from these results whether the magnitude and the significance of the potential gains from diversification are similar for investors in other countries. Alternative approaches to measure the potential gains may provide insight into the probable level of diversification benefits given the time varying behaviour of market returns. While the results presented in this chapter suggest that weakening weight constraints may provide potential gains in portfolio efficiency, this study does not investigate whether such optimization can be done successfully ex-ante. Further investigation into the effect of taxes on the benefits from diversification may find additional reductions to the potential gains reported in this study. The chapters that follow will address these points.

Chapter 4

The empirical benefits of international diversification: a cross-country examination

Abstract

This chapter examines the effect of weight constraints on the potential benefits from international diversification for investors in 34 countries for the single investment period of 1993 to 2014. The naive global market capitalization weighted (1/M) portfolio does not provide a statistically significant return-to-risk (RR) ratio improvement over the domestic market portfolio for investors in the majority of developed markets studied. Relaxing the weight constraints on the maximum return-to-risk portfolio (MRRP) and the minimum volatility portfolio (MVP) increases the potential gains from diversification. These portfolios fail to provide statistically significant positive RR

benefits for many investors.

4.1 Introduction

Determining whether international diversification provides gains to portfolio efficiency is of theoretical and practical importance. The mutual fund theorem (Sharpe, 1964; Lintner, 1965) and the CAPM extended to an international setting assumes investors can improve portfolio efficiency simply by diversifying into the naive global market capitalization weighted (1/M) portfolio. Modern portfolio theory predicts that investors will choose to optimize their equity portfolio away from the local market to capture the risk reduction benefits of weakly correlated international markets and to improve the risk-to-return (RR) performance resulting from the selection of markets with superior returns. Tobin's separation theorem also suggests that an optimized tangent portfolio combined with a risk-free asset can provide further gains beyond the efficient frontier of equity assets. French and Poterba (1991) document the extent of the equity home bias across countries which raises the question of whether investors are unaware of the potential benefits of international diversification available from naive diversification and active optimization, or if they know that these benefits are not as large as theory suggests.

In this chapter I investigate whether short sale restrictions and market weight allocation constraints reduce the potential benefits from international diversification and can make a home bias a statistically efficient portfolio choice. Specifically, I measure the statistical significance of the potential gains achieved from diversifying into static in-sample optimized portfolios with various levels of weight constraints during the 1993 to 2014 investment period for investors in 34 countries. The significance of these benefits are tested using the bootstrap Sharpe and volatility testing techniques of Ledoit and Wolf (2008). A Bayesian approach is then applied to the data to measure the probable gains that can be achieved given the stochastic nature of market returns. The empirical results find that many investors do not achieve positive benefits from diversification that are statistically different from their domestic market portfolio when restricting short sales and imposing weight constraints on allocations to international markets.

The cross-country results presented in this chapter contribute to the literature in several ways. First, reporting the potential benefits from diversifying into the various weight constrained MRRPs and MVPs for investors in all 34 countries studied extends the single country perspective that much of the existing literature presents on this topic (De Roon et al., 2001; Li et al., 2003; Fletcher and Marshall, 2005; Chiou, 2008). Measuring the potential benefits from diversifying into the MVP, as well as consideration of market constraints, extends the cross-country results presented in Driessen and Laeven (2007) which only considers short selling restrictions into the MRRP.

Next, this chapter tests the significance of the potential benefits by using the bootstrap Sharpe and volatility testing techniques of Ledoit and Wolf (2008). These tests are designed to address the non-normality of returns and fat tail-events that occur with historical financial data. Using a bootstrap technique, regressions are performed on paired data points of a given block size between the monthly returns of two portfolios to provide a p-value measuring the significance of the hypothesis that the difference between the two portfolios is zero. Given that the optimal portfolio results presented in this chapter are formed using the data in-sample, the measured benefits from diversification will likely be greater than those achievable by an investor forming optimal portfolios ex-ante. As a result, the use of the Ledoit and Wolf (2008) bootstrap testing methods assists in determining at what level of relaxed weight constraints an optimized portfolio has the potential to offer statistically significant positive diversification benefits to justify an investor's attempt to capture the gains from optimization. The results identify countries for which international diversification with weight constraints does not provide benefits that are statistically different from the naive local market portfolio.

Finally, the Bayesian method presented in Wang (1998) and employed by Li et al. (2003) and Fletcher and Marshall (2005) is used to provide robustness to the results and measure the probable level of diversification benefits investors might achieve given the stochastic nature of market returns. Using this method to report the effects of weight constraints on the probable benefits of diversification for investors in each of the 34 countries studied extends the U.S. and U.K. results presented in Li et al. (2003) and Fletcher and Marshall (2005) respectively. The Bayesian results find that investors in markets with the largest reported potential gains from diversification, such as emerging markets, are the most likely to achieve significantly lower gains given the stochastic nature of market returns.

Early literature investigating the benefits of international diversification focuses on the weak correlations between markets to posit that benefits from diversification exist (Grubel, 1968; Levy and Sarnat, 1970; Lessard, 1973). Recent studies report that correlations between markets have oscillated over the twentieth century (Goetzmann et al., 2007; Quinn and Voth, 2008). Correlations peak at the end of the 19th century, the Great Depression and the late 20th century. The strengthening correlations at the end of the 20th century, which are the strongest correlations over the period studied, are inferred to be the result of globalization (Goetzmann et al., 2007) and the openness of markets (Quinn and Voth, 2008). Levy and Levy (2014b) propose that strengthening correlations between markets magnify international diversification costs such as the asymmetric information costs and higher variance in returns that result from currency risk. As a result, they posit that despite lower international transaction costs over time, higher correlations offset these gains and the home bias will persist.

Short sale constraints have been shown to reduce the potential benefits of diversification into international markets. Individual investors and institutional investors may be restricted from short sales for reasons which include legal constraints and risk management policy. Some markets, such as emerging markets, with high levels of government control in the financial market may restrict short sales. De Roon et al. (2001) finds short sale restrictions reduce the measurable benefits from diversification into emerging markets for U.S. investors. Li et al. (2003) investigate the effect of short sale constraints on the benefits for U.S. investors diversifying into emerging markets using a Bayesian approach. The paper finds that short constraints reduce, but do not eliminate the potential gains from diversification. Fletcher and Marshall (2005) use the same Bayesian approach to investigate the effect of short-sale constraints on the benefits for U.K. investors and reports similar results. Driessen and Laeven (2007) measure the diversification benefits available between regional and global portfolios consisting of regional indices. The results find that short-sale constraints reduce, but do not eliminate, the potential benefits from diversification for investors in the 52 markets studied. The largest reductions generally occur in markets which have the greatest potential for diversification gains such as developing markets.

Chiou (2008) measures the potential benefits of international diversification from the perspective of investors measuring returns in U.S. dollars when restrictions on short sales and constraints on market weights are considered. Developing markets may create restrictions on foreign investment which can limit the ability of foreign investors to overweight investment in these markets. Also, if all investors follow the extreme weights mean-variance optimization may set for small markets, large capital inflows into these markets will likely increase market volatility and reduce future returns. Good investment governance will likely impose limits on allocation weights to reduce estimation error in the optimization solution (Jagannathan and Ma, 2003).

Chiou (2008) measures the potential gains available to U.S. investors diversifying out of each individual market and into optimal portfolios with market allocations constrained by various multiples of the markets' capitalization weight. The benefits are measured using the increase in return-to-risk premium from investing in the MRRP and the reduction in volatility that results from investing in the MVP. The greatest gains from diversification generally occur for investors diversifying out of emerging markets. The weakest benefits are realized by investors diversifying out of developed markets. The results suggest that short restrictions and weight constraints together result in reductions to the benefits of diversification, but do not eliminate them.

This study seeks to address gaps in the literature regarding the potential benefits of international diversification with restrictions on short sales and constraints on allocation weights by empirically testing three questions. First, can investors in all 34 markets studied expect to achieve benefits from naive international diversification? If naive diversification does not provide statistically significant gains beyond an investor's local market portfolio, a home bias may be a reasonably efficient investment decision given the uncertainty of future benefits. Second, does relaxing allocation weight constraints provide potentially significant performance improvements beyond the 1/M portfolio to justify optimization? It seems logical to this author that a rational investor will choose to diversify into an optimized portfolio of international markets only if there exists the possibility of significant positive gains from diversification. If diversification into optimized portfolios with weight constraints does not provide statistically significant gains beyond an investor's local market portfolio, a home bias may be a reasonably efficient investment decision given the uncertainty of capturing these benefits ex-ante. Third, what are the probable gains from diversification on weight constrained portfolios given the stochastic nature of markets? As the empirical results presented in this paper are representative of the historical sample, the Bayesian approach provides robustness to the results and reports the probable level of potential gains given alternative market outcomes.

The results presented in this chapter find that naive diversification into the 1/M portfolio formed using the market weights at the beginning of the investment period does not provide statistically significant RR benefits for investors in the majority of developed countries. While weakening the weight constraints on market allocations can increase the potential gains from diversification, these gains remain statistically insignificant for investors in many countries. Furthermore, the Bayesian analysis finds that investors in countries that have the greatest potential gains from diversification, such as emerging markets, are likely to achieve significantly lower gains due to the stochastic nature of market returns.

This chapter is divided into three more sections. Section 4.2 reviews the data and methods used to measure the potential gains from diversification. Section 4.2 also presents the risk and return characteristics of the markets used in this study for the 1993 to 2014 period. Section 4.3 reports the results of this empirical study. Section 4.4 concludes with a summary of the main findings and suggestions for further study.

4.2 Data and Methods

The monthly total return MSCI equity index data for 21 developed and 13 emerging markets are used in this study. The data for all 34 markets is collected in the local currency of each of the 34 countries. The sample period covers December 31, 1992 to December 31, 2014. The MSCI equity index data is retrieved from Datastream.

Market capitalization data in U.S. dollars from 1993 to 2012 is from two sources: the World Bank and the World Federation of Exchanges. The World Bank data is available for 33 of the 34 markets. The 1993 Ireland capitalization is not available. This incomplete data is calculated using the annual change to the MSCI index to backward fill the missing 1993 capitalization value from the 1994 Ireland market capitalization data. The Taiwanese market capitalization data is retrieved from the World Federation of Exchanges.

Table 4.1 reports the market capitalization of each market used in this study in U.S. dollars as a percent of all 34 markets combined at the end of 1993 and 2012. The geometric annual returns, the standard deviation of returns and the RR ratios of these markets as measured in the domestic currency for the 1993 to 2014 holding period are presented. The characteristics of the 1/M portfolio formed from the market capitalization weights for 1993, and the MRRP and the MVP optimized with no shorts and no weight constraints in the local currency of each of the 34 countries are also reported. The 1/M portfolio is the most strongly weight constrained portfolio with market allocations restricted to equal the 1993 market capitalization weight of each market. The MRRP and MVP optimized with no shorts and no weight constraints are referred to as unconstrained portfolios in this study.

The market and portfolio characteristics reported in Table 4.1 provide preliminary results from which the benefits of international diversification for investors in these countries can be measured. At the beginning of the investment period, an investor might choose to diversify out of their local market portfolio and into the naive 1/M portfolio that the mutual fund theorem suggests is optimal. A different investor may choose to optimize their portfolio to minimize volatility or maximize the RR ratio. Comparing the

Table 4.1: The local market portfolio, the 1/M portfolio, the unconstrained MRRP and the unconstrained MVP characteristics for each country for the 1993 to 2014 holding period.

	w_i(%)		Local			1/M			MRRP			MVP		
Market(i)	1993	2012	r	sd	RR	r	sd	RR	r	sd	RR	r	sd	RR
Panel A: Developed Markets														
Australia	1.51	2.89	0.0981	0.132	0.74	0.0661	0.119	0.56	0.1038	0.111	0.94	0.0762	0.101	0.75
Austria	0.21	0.24	0.0270	0.227	0.12	0.0745	0.153	0.49	0.1203	0.146	0.83	0.0773	0.132	0.58
Belgium	0.58	0.67	0.0830	0.192	0.43	0.0748	0.153	0.49	0.1205	0.145	0.83	0.0782	0.132	0.59
Canada	2.41	4.53	0.1004	0.155	0.65	0.0698	0.121	0.58	0.1071	0.122	0.88	0.0750	0.110	0.68
Denmark	0.31	0.51	0.1321	0.185	0.72	0.0737	0.153	0.48	0.1194	0.146	0.82	0.0781	0.133	0.59
Finland	0.17	0.36	0.1428	0.311	0.46	0.0714	0.152	0.47	0.1175	0.145	0.81	0.0737	0.131	0.56
France	3.37	4.10	0.0754	0.184	0.41	0.0737	0.153	0.48	0.1195	0.145	0.82	0.0771	0.132	0.58
Germany	3.43	3.34	0.0883	0.212	0.42	0.0745	0.153	0.49	0.1202	0.146	0.83	0.0771	0.132	0.58
H.K.	2.85	2.49	0.0933	0.257	0.36	0.0746	0.152	0.49	0.1133	0.151	0.75	0.0803	0.137	0.59
Ireland	0.14	0.25	0.0453	0.215	0.21	0.0773	0.152	0.51	0.1235	0.146	0.84	0.0826	0.133	0.62
Italy	1.01	1.08	0.0640	0.217	0.30	0.0786	0.151	0.52	0.1248	0.146	0.86	0.0816	0.133	0.62
Japan	22.18	8.28	0.0186	0.186	0.10	0.0726	0.180	0.40	0.1209	0.193	0.63	0.0639	0.162	0.40
Holland	1.34	1.46	0.0933	0.186	0.50	0.0744	0.151	0.49	0.1201	0.144	0.84	0.0777	0.131	0.59
N.Z.	0.19	0.18	0.0746	0.163	0.46	0.0543	0.130	0.42	0.0918	0.123	0.75	0.0638	0.111	0.57
Norway	0.20	0.57	0.1034	0.224	0.46	0.0785	0.147	0.53	0.1248	0.142	0.88	0.0847	0.129	0.66

Singapore	0.98	0.93	0.0642	0.227	0.28	0.0642	0.133	0.48	0.1012	0.135	0.75	0.0673	0.118	0.57
Spain	0.88	2.24	0.1184	0.215	0.55	0.0835	0.154	0.54	0.1299	0.147	0.88	0.0877	0.135	0.65
Sweden	0.79	1.26	0.1363	0.223	0.61	0.0796	0.137	0.58	0.1254	0.130	0.97	0.0900	0.118	0.76
Suisse	2.01	2.43	0.0935	0.155	0.61	0.0558	0.169	0.33	0.1002	0.156	0.64	0.0650	0.144	0.45
U.K.	8.52	6.79	0.0737	0.138	0.53	0.0732	0.150	0.49	0.1184	0.154	0.77	0.0690	0.130	0.53
U.S.	37.98	41.99	0.0956	0.149	0.64	0.0746	0.152	0.49	0.1134	0.152	0.75	0.0804	0.137	0.59
Panel B: Emerging Markets														
Argentina	0.33	0.08	0.1340	0.388	0.35	0.1842	0.228	0.81	0.2351	0.235	1.00	0.1827	0.215	0.85
Brazil	0.74	2.77	0.5189	0.408	1.27	0.4366	0.329	1.33	0.5079	0.326	1.56	0.4627	0.316	1.47
Chile	0.33	0.70	0.1042	0.193	0.54	0.0975	0.139	0.70	0.1329	0.132	1.01	0.1009	0.123	0.82
Greece	0.09	0.10	0.0322	0.318	0.10	0.0879	0.150	0.59	0.1330	0.143	0.93	0.0919	0.131	0.70
Indonesia	0.24	0.89	0.1621	0.334	0.48	0.1659	0.260	0.64	0.2028	0.236	0.86	0.1627	0.218	0.75
Korea	1.03	2.66	0.0898	0.293	0.31	0.0910	0.158	0.58	0.1298	0.160	0.81	0.0941	0.143	0.66
Malaysia	1.63	1.07	0.0750	0.256	0.29	0.0888	0.180	0.49	0.1245	0.176	0.71	0.0899	0.156	0.58
Mexico	1.49	1.18	0.1718	0.230	0.75	0.1531	0.159	0.96	0.1911	0.151	1.27	0.1602	0.143	1.12
Philipines	0.30	0.59	0.0813	0.268	0.30	0.1063	0.153	0.69	0.1414	0.152	0.93	0.1063	0.138	0.77
Portugal	0.09	0.15	0.0490	0.203	0.24	0.0805	0.153	0.53	0.1268	0.146	0.87	0.0847	0.133	0.64
Taiwan	1.43	1.65	0.0688	0.270	0.26	0.0853	0.135	0.63	0.1221	0.135	0.90	0.0882	0.122	0.72
Thailand	0.97	0.86	0.0523	0.352	0.15	0.0874	0.156	0.56	0.1240	0.158	0.79	0.0849	0.139	0.61
Turkey	0.28	0.60	0.4481	0.454	0.99	0.3866	0.192	2.02	0.4356	0.183	2.38	0.4092	0.180	2.28

This table reports the geometric annual returns (r), the standard deviation of returns (sd), and the return-to-risk (RR) ratios for the local portfolio, the 1/M portfolio, and the MRRP and the MVP optimized with no shorts and no weight constraints for each country for the 1993 to 2014 holding period using total return index data in each country's local currency. The weight of each country's market capitalization (w_i(%)) is reported as the percent of market capitalization for all 34 markets combined in U.S. dollars at the end of 1993 and 2012.

characteristics between each of the domestic markets and the alternative portfolios highlights the difficulty in assessing the universality of benefits from international diversification. Over long holding periods markets do not provide the same return-to-risk ratios. These differences in the returns and standard deviation of returns between markets may give investors in markets with relatively high RR ratios lower benefits from diversification than investors in markets with lower RR ratios which may reduce the incentive to diversify.

Figure 4.1 presents the difference in the annual returns provided by the naive global portfolio and the local US market portfolio for varying holding periods starting from January 1988 and ending in December 2012. The returns from the naive global portfolio are calculated using the market capitalization weights for each year against the returns the markets provided each month. From each month, starting at January 1988, the returns for the local and naive global portfolios are calculated for each possible monthly holding period to the end of 2012. Starting from January 1988 there are 300 measurable one month holding periods and a single measurable 300 month holding period. The maximum and minimum difference in outcomes, along with the 75%, 50% and 25% quantiles, are shown in figure 4.1. The graph starts from the 60 month holding period to reduce the scale of the graph that occurs using shorter holding periods.

The results from figure 4.1 show that the naive global portfolio does not always outperform the local U.S. market portfolio. For the investor with shorter holding periods the local portfolio more often outperformed the naive global portfolio. At the 60 month holding period, the 50% quantile perfor-

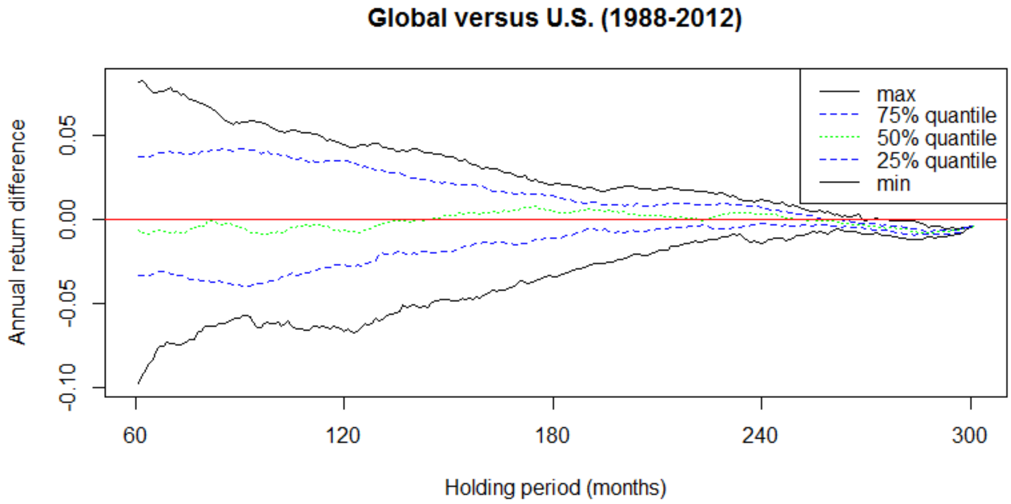

Figure 4.1: *This figure shows the annual return difference between the naive global portfolio in U.S. dollars and the local US market for all holding periods between 1988 to 2012. The naive global portfolio's market weights are updated each January and are based on the market capitalization weights in U.S. dollars for that year.*

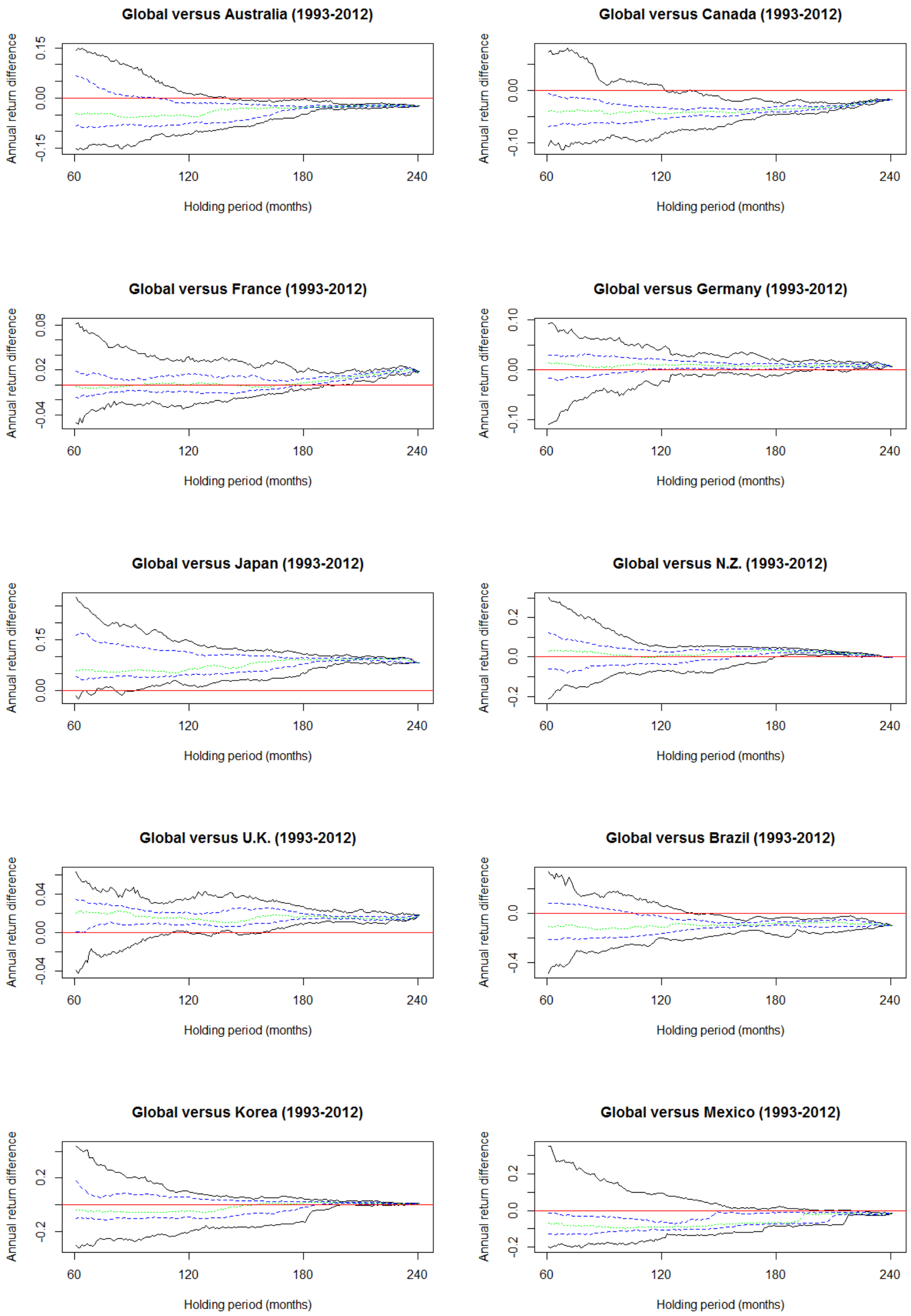

Figure 4.2: *Graphs of the annual return difference between the naive global portfolio and various domestic markets in the local currency for all monthly holding periods between January 1993 to December 2012.*

mance of the naive global portfolio minus the local portfolio is negative. An investor that held the naive portfolio for 15 to 20 years had better returns with the naive global portfolio only slightly more than 50% of the time. As the holding period is extended to 20 or more years, the number of measurable sample periods is small. Over the sample period, the naive global portfolio underperformed the local portfolio over longer investment periods. These results suggest diversification into the naive global portfolio can provide potential excess returns beyond the local portfolio, but that these gains are not consistent across time.

The graphs presented in figure 4.2 report the naive global portfolio performance versus the local market of 10 countries for all possible 60 month to 240 month holding periods from 1993 to 2012. A positive difference occurs when the naive global portfolio outperforms the local market portfolio. The results show that there is no country for which the naive global portfolio always provides superior return results for investors. While there are countries for which the naive global portfolio can provide greater returns than the local market much of the time, there are also markets that more often outperform the naive global portfolio. The size of the distribution in possible return differences between the naive global portfolio and the domestic portfolio is different for each country.

Should an investor believe that the assumptions underlying the mutual fund theorem hold, that there are no significant transaction costs and markets are perfectly transparent, then an investor might seek to invest in the mutual fund in order to capture the naive diversification benefits available from the less than perfectly correlated markets. Table 4.1 reports that the volatility of the 1/M portfolio formed using the market capitalization weights at the start of the sample period is lower than the domestic market for most investors. Only the U.S., the U.K. and Suisse markets have a lower standard deviation of returns than the 1/M portfolio. Although the volatility of the 1/M portfolio is generally lower, the returns are not necessarily larger than the local market. As a result, not all investors achieve a higher RR ratio moving from the local portfolio to the naive 1/M portfolio. Several developed markets have RR ratios that are greater than the 1/M portfolio: Australia, Canada, Denmark, the Netherlands, New Zealand, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Of interest is the effect of a common currency shared by European Union countries on the 1/M characteristics. The Euro became the accounting currency from January 1, 1999. It appears that the introduction of a common currency has allowed investors in EU countries to share risk more equally as can be seen from the similar returns, standard deviation of returns and RR ratios of the 1/M portfolio held by those investors.

In this study, an investor who seeks to maximize their optimized portfolio RR ratio will invest in the MRRP optimized with no shorts and no market allocation weight constraints. The benefits of diversification into the optimized portfolios with no weight constraints and using short sales are also reported in order to add robustness to the results and to extend the literature that reports the effect of short sale constraints from a single country perspective (De Roon et al., 2001; Li et al., 2003; Fletcher and Marshall, 2005; Chiou, 2008). An investor who seeks to minimize their optimized portfolio volatility will invest in the MVP with no shorts and no market allocation weight constraints. The MRRP and MVP optimized with no shorts and no market allocation weight constraints are referred to as unconstrained portfolios in this paper.

While not presented here, the unconstrained MRRPs and MVPs often hold only a few, heavily weighted markets. Some of the overweighted markets can have small market capitalizations such as Denmark and Switzerland. This paper does not investigate how an optimal unconstrained MRRP or MVP to be held over 21 years is determined successfully ex-ante. Strengthening the weight constraints on the optimized solutions may reduce ex-ante estimation error (Jagannathan and Ma, 2003) and will likely reduce the potential benefits of diversification as well. This paper reports the potential benefits available from optimized portfolios with weight constraints. This paper will also use a Bayesian approach to investigate whether the historical results are representative of the expected probable outcomes given the stochastic nature of market returns.

4.2.1 Measuring the benefits of international diversification

Investors can choose to maximize the return of their portfolio while seeking the same volatility obtained in the local market. Similarly, they can choose to minimize the volatility of their portfolio. They can choose to allocate funds in international markets where the investment opportunities of those markets can be stated as a vector of multivariate Gaussian stochastic returns of N assets, $R^T = [r_1, r_2, ..., r_n]$. The mean of asset returns for these markets can be expressed as a vector μ. The variance-covariance of asset returns can be expressed as a positive definite matrix V. Let S be the set of all real vectors w that define the weights such that $w^T 1 = w_1 + w_2 + ... + w_n = 1$, where 1 is an N vector of ones. Using the methods developed by Markowitz (1952), the efficient frontier of global investments can be formed when the objective function and restrictions are combined in order to find the efficient portfolio that minimizes volatility at every level of expected return:

$$\min_{\{w,\phi,\eta\}} \Xi = \frac{1}{2} w^T V w + \phi(\mu_p - w^T \mu) + \eta(1 - w^T 1) \tag{4.1}$$

where μ_p is the expected return of the portfolio, and the shadow prices ϕ and η are two positive constants. The quadratic programming solution of assets in a portfolio spanning w_p can be obtained by the first-order conditions of equation (4.1).

An investor may choose to diversify out of the domestic market and into the maximum return-to-risk portfolio (MRRP). When there are no weight restrictions, the maximum return-to-risk (MRR) achieved by the MRRP is obtained by

$$MRR = \max_{\{w_p\}} \left\{ \frac{w_p^T \mu}{(w_p^T V w_p)^{1/2}} \middle| w_p^T \in S \right\} \tag{4.2}$$

where w_p is the vector of weights[1] of assets held in the MRRP. The MRR measures the return achieved by the MRRP compared to the standard deviation of the MRRP.

For the investor investing in country i, adjusting the portfolio out of that

[1]To ensure feasible allocations, the sum of the portfolio weights must equal 1. Setting the condition that market weights must be positive restricts short sales.

single market and into the internationally diversified portfolio that maximizes the return-to-risk ratio, the greatest increment of unit-risk performance is measured as,

$$\delta = \frac{MRR}{RR_i} - 1 \tag{4.3}$$

where RR_i is the RR ratio of market i. Here, δ (delta) represents the percentage change in the return-to-risk ratio the investor achieves when moving out of the local portfolio and into the optimally diversified international portfolio that will maximize the return-to-risk performance.

This chapter measures the reduction in volatility that can result from diversification out of the domestic market and into the MVP as presented in Chiou (2008):

$$\varepsilon = 1 - \left[\frac{w_{MVP}^T V w_{MVP}}{V_i}\right]^{1/2} \tag{4.4}$$

where w_{MVP} is the vector of weights of the assets held in the MVP and V_i is the variance of the local market. A positive value for ϵ (epsilon) represents the percentage reduction in portfolio volatility when diversifying out of the single domestic portfolio and into the optimal internationally diversified MVP. A result approaching zero represents diminishing benefits from diversification.

4.2.2 A Bayesian approach to measuring the benefits

The Bayesian inference method presented in Wang (1998) is used to analyse the effect of weight constraints with no short-sales on the probable potential benefits from international diversification. Li et al. (2003) and Fletcher and

Marshall (2005) use this method to measure the effects of short-constraints on the benefits of international diversification for U.S. and U.K. investors respectively. This chapter extends those results to report the probable gains at the 95 and 99 percent level for the optimized weight constrained MRRPs and MVPs with no short sales using the Bayesian approach with a non-informative prior.

Li et al. (2003) argues that the Bayesian approach is easier to implement than classical inference methods. Once the sample means and covariance matrix are drawn, the measured gains from diversification are straightforward to calculate. Statistical methods to test whether two mean-variance frontiers span each other can become complex when considering investment constraints. This Bayesian approach makes this testing relatively easy. Also, this method provides an inference as to the magnitude of the benefits, rather than the classical inference results of testing the null hypothesis that the measured diversification benefits between two portfolios are not statistically different.

A four-step approach is used to measure the approximate posterior distribution of optimized portfolio results and the resulting benefits from diversification. First a random sample covariance matrix (Σ) is drawn from the inverse Wishart (TV,T-1) distribution with $T - 1$ degrees of freedom, where T is the number of observations, V is the covariance matrix of the N risky assets drawn from a multivariate normal distribution, and TV is the parameter matrix. Next, using Σ, a random sample of expected returns are drawn from the multivariate normal (e,(1/T) Σ) distribution, where e is the sample estimates of the expected returns. Then, using the new random sample means and covariance matrix, the optimization problem with constraints is solved and the benefits measured. These steps are repeated 10,000 times to determine the approximate posterior distribution of the potential gains from diversification into the MRRP and MVP optimized with varying levels of weight constraints.

These results are used to estimate the size and the statistical significance of the benefits from diversification. The average values from the posterior distribution give the average benefits. For the diversification benefits to be statistically significant from the local market, the average values should be 2 standard deviations from zero. The 5 and 1 percentile results taken from the posterior distribution represents a posterior 95 to 99 percent probability that benefits of that size will occur. While not presented in this study, an added benefit of this method is that the market weights of the optimized portfolios that provide these results can also be easily approximated.

4.3 Results

This section reports the results of examining the potential benefits of international diversification with weight constraints using the data and methods presented in section 4.2. First, the delta and epsilon results for the 1993-2014 investment period are presented. The benefits of diversification are tested for statistical significance using the Ledoit and Wolf (2008) bootstrap testing methods designed to test whether the difference between two portfolio strategies is significantly different from zero. Next, the results from the Bayesian approach are reported. Analysis of these results seeks to determine whether the naive 1/M portfolio can be expected to provide positive diversification benefits for all investors and whether optimized portfolios with or without weight constraints offer statistically significant efficiency gains beyond the naive domestic portfolio.

Table 4.2 reports the delta benefits of diversifying out of an investor's domestic market and into the no shorts restricted optimal MRRP constrained by the 1993 market capitalization weights at various weight constraint limits: one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraints (nw). The portfolio constrained to one times the market capitalization weights is the 1/M portfolio. The benefits from diversifying into the MRRP optimized with no weight constraints and with short sales (nws) is also reported. The last five columns report the portion of the potential gains achieved by the MRRP with no weight constraints and no shorts sales captured by the different MRRPs. The results from the Ledoit and Wolf (2008) Sharpe ratio tests are presented in parentheses. The p-values report the significance of the hypothesis that the difference between the RR ratio of the monthly returns of the domestic portfolio and the optimized portfolio is zero. A 10% result is treated as statistically significant.

The results presented in table 4.2 find that the 1/M portfolio does not provide improved return-to-risk performance over all domestic portfolios for the extended holding period. Investors in 10 of the 21 developed markets have reduced return-to-risk performance with the 1/M portfolio. Diversification out of emerging markets and into the 1/M portfolio provides benefits. In the case of Brazil, these benefits are small. The RR ratio only improves by 4.47%. Developed markets have relatively lower levels of volatility than the emerging markets. Diversification into the 1/M portfolio, while reducing the total level of risk resulting from the less than perfect correlations between markets, provides relatively lower levels of risk reduction for investors in less volatile markets. Given that the 1/M portfolio return is determined by the weighted sum of market returns, some markets have returns that are greater than the 1/M portfolio. As a result, when the local market has relatively low volatility and high returns, naive diversification into the 1/M portfolio does not necessarily result in a positive RR benefit. Of the 24 countries reported to have positive potential diversification benefits with the 1/M portfolio, only Japan and Turkey have gains that are statistically different than the domestic portfolio.

As weight constraints are relaxed the potential benefits from diversification increase for all investors. Despite this, investors in three countries do not benefit from optimization at the two times weight constraint level: Australia, Denmark and Switzerland. The United States only achieves a 1.19% improvement of the RR ratio. For most countries the two times weight constraint level captures the majority of potential gains from diversification offered by the MRRP with no short sales and unconstrained by market weights. For most emerging markets, over 70% of the unconstrained potential gains are captured by the two times market weight constrained portfolio. Of the emerging markets, Brazil captures the lowest portion of the maximum potential gains at 63 percent. At the two times weight constraint level only Japan and Turkey have positive potential benefits from diversification that are statistically different than the domestic portfolio.

The unconstrained MRRP, which is optimized with no weight constraints

Table 4.2: MRRP delta results in percent for 1993 to 2014.

							benefit gained (%)				
Market	wx1	wx2	wx5	wx10	nw	nws	wx1	wx2	wx5	wx10	nws
Panel A: Developed Markets											
Australia	-25.01 (0.598)	-6.18 (0.960)	2.41 (0.792)	12.45 (0.502)	26.45 (0.118)	106.87 (0.036)	-95	-23	9	47	404
Austria	309.64 (0.140)	421.39 (0.096)	451.11 (0.038)	487.55 (0.010)	594.30 (0.004)	1093.87 (0.016)	52	71	76	82	184
Belgium	13.37 (0.770)	44.34 (0.562)	52.55 (0.412)	62.54 (0.330)	92.40 (0.100)	230.09 (0.052)	14	48	57	68	249
Canada	-11.43 (0.920)	17.11 (0.530)	23.58 (0.378)	29.32 (0.184)	35.46 (0.152)	137.90 (0.014)	-32	48	66	83	389
Denmark	-32.59 (0.292)	-13.63 (0.630)	-8.42 (0.838)	-3.01 (0.966)	14.64 (0.346)	98.74 (0.080)	-223	-93	-58	-21	674
Finland	1.95 (0.996)	30.37 (0.678)	38.46 (0.540)	48.25 (0.388)	75.83 (0.252)	208.53 (0.028)	3	40	51	64	275
France	17.59 (0.438)	49.88 (0.182)	58.45 (0.068)	68.88 (0.024)	100.1 (0.006)	245.20 (0.018)	18	50	58	69	245
Germany	17.23 (0.570)	49.22 (0.252)	57.75 (0.150)	68.17 (0.032)	98.68 (0.014)	241.69 (0.020)	17	50	59	69	245
H.K.	35.97 (0.418)	79.68 (0.288)	86.57 (0.260)	93.13 (0.190)	106.52 (0.162)	299.20 (0.018)	34	75	81	87	281
Ireland	141.20 (0.156)	206.38 (0.048)	222.84 (0.046)	242.71 (0.028)	300.72 (0.010)	585.46 (0.010)	47	69	74	81	195
Italy	76.88 (0.260)	122.09 (0.100)	134.65 (0.064)	149.73 (0.044)	190.38 (0.020)	394.38 (0.010)	40	64	71	79	207
Japan	304.93 (0.070)	411.71 (0.066)	430.19 (0.046)	450.32 (0.042)	527.75 (0.044)	1247.22 (0.008)	58	78	82	85	236
Holland	-1.70 (0.814)	25.16 (0.402)	32.32 (0.208)	41.07 (0.076)	66.15 (0.004)	185.22 (0.022)	-3	38	49	62	280
N.Z.	-9.22 (0.986)	20.78 (0.702)	30.78 (0.534)	42.42 (0.374)	63.14 (0.162)	196.97 (0.014)	-15	33	49	67	312
Norway	15.66 (0.716)	46.86 (0.448)	55.26 (0.304)	63.97 (0.272)	90.41 (0.096)	213.52 (0.038)	17	52	61	71	236
Singapore	71.13 (0.292)	126.55 (0.140)	135.92 (0.156)	147.62 (0.088)	166.44 (0.100)	393.52 (0.008)	43	76	82	89	236
Spain	-1.18 (0.898)	23.38 (0.536)	29.95 (0.386)	38.25 (0.264)	60.67 (0.116)	171.78 (0.026)	-2	39	49	63	283

Sweden	-5.29 (0.998)	19.64 (0.558)	27.33 (0.302)	35.91 (0.170)	58.18 (0.076)	154.15 (0.020)	-9	34	47	62	265
Suisse	-45.56 (0.216)	-26.11 (0.428)	-21.27 (0.526)	-15.45 (0.708)	5.83 (0.446)	117.19 (0.068)	-781	-448	-365	-265	2009
U.K.	-8.81 (0.910)	17.54 (0.262)	21.78 (0.146)	27.11 (0.100)	43.78 (0.048)	164.18 (0.022)	-20	40	50	62	375
U.S.	-23.50 (0.438)	1.19 (0.548)	5.05 (0.202)	8.77 (0.218)	16.42 (0.324)	125.26 (0.038)	-143	7	31	53	763
Panel B: Emerging Markets											
Argentina	133.47 (0.282)	167.12 (0.230)	171.59 (0.160)	176.30 (0.162)	189.96 (0.114)	391.29 (0.014)	70	88	90	93	206
Brazil	4.47 (0.894)	14.12 (0.616)	15.89 (0.500)	17.47 (0.412)	22.44 (0.128)	88.84 (0.022)	20	63	71	78	396
Chile	30.39 (0.454)	59.87 (0.284)	66.10 (0.230)	73.06 (0.216)	86.82 (0.064)	197.77 (0.022)	35	69	76	84	228
Greece	479.57 (0.104)	612.85 (0.056)	650.18 (0.022)	697.37 (0.016)	818.04 (0.006)	1403.09 (0.002)	59	75	79	85	172
Indonesia	31.44 (0.906)	50.95 (0.684)	56.16 (0.596)	60.51 (0.494)	77.47 (0.162)	223.00 (0.014)	41	66	72	78	288
Korea	88.43 (0.306)	130.56 (0.174)	142.77 (0.112)	153.31 (0.050)	165.30 (0.036)	368.52 (0.012)	53	79	86	93	223
Malaysia	68.63 (0.570)	115.31 (0.376)	123.49 (0.316)	130.92 (0.226)	142.25 (0.210)	394.17 (0.014)	48	81	87	92	277
Mexico	29.04 (0.454)	49.81 (0.188)	55.95 (0.098)	62.90 (0.032)	69.82 (0.020)	155.28 (0.004)	42	71	80	90	222
Philipines	128.37 (0.168)	175.95 (0.114)	184.00 (0.084)	193.07 (0.058)	206.09 (0.060)	421.79 (0.004)	62	85	89	94	205
Portugal	117.85 (0.172)	174.00 (0.096)	188.63 (0.050)	206.90 (0.032)	260.11 (0.006)	508.07 (0.004)	45	67	73	80	195
Taiwan	147.16 (0.130)	206.81 (0.088)	217.46 (0.048)	230.60 (0.060)	253.64 (0.044)	494.53 (0.006)	58	82	86	91	195
Thailand	277.02 (0.224)	366.34 (0.112)	380.89 (0.110)	397.25 (0.096)	428.17 (0.076)	874.31 (0.004)	65	86	89	93	204
Turkey	104.37 (0.006)	121.08 (0.002)	126.43 (0.002)	131.79 (0.002)	140.94 (0.002)	219.96 (0.002)	74	86	90	94	156

This table reports the delta benefits from diversification out of each domestic market and into MRRPs optimized with no shorts and the 1993 market capitalization weight constraints at the one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraint (nw) level. The results for the MRRP with no weight constraints and short sales (nws) are also presented. The p-value results from the Ledoit and Wolf (2008) Sharpe ratio tests using a block size of 5 and 499 iterations are presented in brackets. The percent of the unconstrained benefits captured at each constraint level are reported in the last five columns. The percent of the benefits the nw constraint portfolio captures are not presented as it captures 100% of its own benefits.

Table 4.3: MVP epsilon results in percent for 1993 to 2014.

Market	wx1	wx2	wx5	wx10	nw	nws	benefit gained (%) wx1	wx2	wx5	wx10	nws
Panel A: Developed Markets											
Australia	10.17 (0.082)	15.14 (0.016)	19.18 (0.004)	21.45 (0.002)	23.48 (0.002)	36.44 (0.002)	43	64	82	91	155
Austria	32.65 (0.002)	37.04 (0.006)	39.69 (0.002)	40.73 (0.002)	41.60 (0.002)	53.54 (0.002)	78	89	95	98	129
Belgium	20.55 (0.180)	25.67 (0.092)	28.85 (0.038)	30.12 (0.026)	31.20 (0.022)	45.10 (0.006)	66	82	92	97	145
Canada	21.46 (0.004)	27.53 (0.002)	28.71 (0.002)	28.74 (0.002)	28.74 (0.002)	45.21 (0.002)	75	96	100	100	157
Denmark	17.28 (0.002)	22.63 (0.002)	25.73 (0.002)	26.96 (0.002)	27.97 (0.002)	42.21 (0.002)	62	81	92	96	151
Finland	50.95 (0.002)	54.14 (0.002)	56.35 (0.002)	57.21 (0.002)	57.81 (0.002)	65.70 (0.002)	88	94	97	99	114
France	16.97 (0.002)	22.37 (0.002)	25.67 (0.002)	26.96 (0.002)	27.96 (0.002)	42.45 (0.002)	61	80	92	96	152
Germany	28.06 (0.002)	32.74 (0.002)	35.55 (0.002)	36.67 (0.002)	37.64 (0.002)	50.35 (0.002)	75	87	94	97	134
H.K.	41.15 (0.002)	46.39 (0.002)	46.69 (0.002)	46.86 (0.002)	46.86 (0.002)	62.54 (0.002)	88	99	100	100	133
Ireland	29.19 (0.004)	33.69 (0.002)	36.48 (0.002)	37.42 (0.002)	37.96 (0.002)	50.00 (0.002)	77	89	96	99	132
Italy	30.50 (0.002)	35.03 (0.002)	37.56 (0.002)	38.38 (0.002)	38.85 (0.002)	51.29 (0.002)	79	90	97	99	132
Japan	3.46 (0.358)	12.20 (0.010)	12.70 (0.004)	13.19 (0.006)	13.30 (0.004)	33.06 (0.002)	26	92	96	99	249
Holland	18.90 (0.002)	24.27 (0.002)	27.46 (0.002)	28.66 (0.002)	29.57 (0.002)	43.94 (0.002)	64	82	93	97	149
N.Z.	19.91 (0.012)	23.76 (0.002)	26.77 (0.002)	28.41 (0.002)	31.41 (0.002)	41.58 (0.002)	63	76	85	90	132
Norway	34.34 (0.002)	38.37 (0.002)	41.40 (0.002)	42.23 (0.002)	42.47 (0.002)	51.89 (0.002)	81	90	97	99	122
Singapore	41.54 (0.002)	46.32 (0.002)	47.44 (0.002)	47.83 (0.002)	47.85 (0.002)	59.80 (0.002)	87	97	99	100	125
Spain	28.67 (0.002)	33.17 (0.002)	35.72 (0.002)	36.70 (0.002)	37.44 (0.002)	49.81 (0.002)	77	89	95	98	133

Sweden	38.37 (0.002)	42.53 (0.002)	45.28 (0.002)	46.38 (0.002)	46.97 (0.002)	56.07 (0.002)	82	91	96	99	119
Suisse	-9.59 (0.100)	-2.61 (0.666)	1.60 (0.766)	3.83 (0.466)	6.94 (0.024)	26.97 (0.004)	-138	-38	23	55	389
U.K.	-8.79 (0.056)	0.07 (1.000)	4.88 (0.118)	5.65 (0.062)	5.65 (0.038)	31.34 (0.002)	-156	1	86	100	555
U.S.	-1.98 (0.572)	7.13 (0.004)	7.65 (0.008)	7.92 (0.004)	7.92 (0.004)	35.03 (0.002)	-25	90	97	100	442
Panel B: Emerging Markets											
Argentina	41.13 (0.322)	43.49 (0.354)	43.84 (0.294)	44.11 (0.296)	44.58 (0.282)	55.01 (0.218)	92	98	98	99	123
Brazil	19.45 (0.116)	21.30 (0.142)	21.92 (0.104)	22.27 (0.092)	22.70 (0.022)	28.31 (0.004)	86	94	97	98	125
Chile	28.25 (0.004)	31.96 (0.002)	33.06 (0.002)	33.67 (0.002)	36.64 (0.002)	48.66 (0.002)	77	87	90	92	133
Greece	52.90 (0.002)	55.92 (0.002)	57.55 (0.002)	58.26 (0.002)	58.87 (0.002)	67.27 (0.002)	90	95	98	99	114
Indonesia	22.12 (0.760)	24.43 (0.706)	26.70 (0.632)	28.13 (0.600)	34.68 (0.362)	43.90 (0.026)	64	70	77	81	127
Korea	46.21 (0.076)	48.62 (0.070)	49.80 (0.074)	50.88 (0.020)	51.13 (0.010)	58.69 (0.002)	90	95	97	100	115
Malaysia	29.77 (0.372)	33.69 (0.336)	35.71 (0.306)	38.03 (0.254)	39.16 (0.142)	52.46 (0.002)	76	86	91	97	134
Mexico	30.93 (0.406)	33.74 (0.340)	35.67 (0.308)	37.26 (0.266)	37.89 (0.148)	48.76 (0.006)	82	89	94	98	129
Philipines	42.73 (0.002)	46.18 (0.002)	47.26 (0.002)	47.84 (0.002)	48.47 (0.002)	60.30 (0.002)	88	95	97	99	124
Portugal	24.63 (0.002)	29.45 (0.002)	32.37 (0.002)	33.51 (0.002)	34.39 (0.002)	47.46 (0.002)	72	86	94	97	138
Taiwan	49.87 (0.002)	53.73 (0.002)	54.46 (0.002)	54.76 (0.002)	54.78 (0.002)	65.41 (0.002)	91	98	99	100	119
Thailand	55.65 (0.002)	58.19 (0.002)	59.39 (0.002)	60.21 (0.002)	60.36 (0.002)	67.89 (0.002)	92	96	98	100	112
Turkey	57.78 (0.002)	58.82 (0.002)	59.25 (0.002)	59.65 (0.002)	60.38 (0.002)	64.32 (0.002)	96	97	98	99	107

This table reports the epsilon benefits from diversification out of each domestic market and into MVPs optimized with no shorts and the 1993 market capitalization weight constraints at the one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraint (nw) level. The results for the MVP with no weight constraints and short sales (nws) are also presented. The p-value results from the Ledoit and Wolf (2008) volatility tests using a block size of 5 and 499 iterations are presented in brackets. The percent of the unconstrained benefits captured at each constraint level are reported in the last five columns. The percent of the benefits the nw constraint portfolio captures are not presented as it captures 100% of its own benefits.

Table 4.4: MVP delta results in percent for 1993 to 2014.

Market	wx1	wx2	wx5	wx10	nw	nws	benefit gained (%)				
							wx1	wx2	wx5	wx10	nws
Panel A: Developed Markets											
Australia	-25.01(0.582)	-26.68(0.588)	-19.06(0.702)	-10.59(0.896)	1.52(0.698)	19.90(0.720)	-1640	-1750	-1250	-694	1305
Austria	309.64(0.136)	311.05(0.154)	334.31(0.092)	372.51(0.080)	390.25(0.062)	450.03(0.194)	79	80	86	95	115
Belgium	13.37(0.768)	13.93(0.806)	20.99(0.710)	31.08(0.614)	36.98(0.552)	53.64(0.872)	36	38	57	84	145
Canada	-11.43(0.912)	1.44(0.858)	4.46(0.708)	4.86(0.698)	4.85(0.702)	38.53(0.704)	-236	30	92	100	794
Denmark	-32.59(0.292)	-31.93(0.272)	-26.94(0.328)	-22.21(0.446)	-17.88(0.610)	-7.97(0.316)	182	179	151	124	45
Finland	1.95(0.992)	1.22(0.928)	6.76(0.986)	16.24(0.884)	22.28(0.754)	42.44(0.956)	9	5	30	73	191
France	17.59(0.440)	17.39(0.484)	24.90(0.368)	34.87(0.248)	41.80(0.154)	58.05(0.720)	42	42	60	83	139
Germany	17.23(0.574)	17.63(0.648)	24.24(0.554)	35.22(0.408)	39.99(0.332)	57.38(0.784)	43	44	61	88	143
H.K.	35.97(0.440)	49.10(0.452)	53.58(0.358)	61.93(0.296)	61.93(0.328)	95.27(0.732)	58	79	87	100	154
Ireland	141.20(0.174)	147.57(0.138)	161.46(0.146)	180.19(0.088)	194.14(0.076)	230.36(0.262)	73	76	83	93	119
Italy	76.88(0.212)	79.76(0.280)	84.92(0.234)	98.56(0.216)	108.63(0.180)	145.31(0.260)	71	73	78	91	134
Japan	304.93(0.060)	245.32(0.084)	251.69(0.044)	277.95(0.034)	296.84(0.026)	369.16(0.364)	103	83	85	94	124
Holland	-1.70(0.832)	-0.69(0.922)	4.73(0.746)	13.40(0.574)	18.20(0.388)	35.64(0.988)	-9	-4	26	74	196
N.Z.	-9.22(0.978)	-14.64(0.926)	-6.82(0.988)	9.29(0.708)	24.70(0.410)	24.78(0.972)	-37	-59	-28	38	100
Norway	15.66(0.716)	21.68(0.746)	27.73(0.604)	37.27(0.558)	42.32(0.494)	52.61(0.752)	37	51	66	88	124
Singapore	71.13(0.292)	78.87(0.336)	84.77(0.248)	97.37(0.208)	101.23(0.164)	119.09(0.510)	70	78	84	96	118
Spain	-1.18(0.900)	0.94(0.912)	6.16(0.832)	13.75(0.644)	18.34(0.508)	42.64(0.838)	-6	5	34	75	232

Sweden	-5.29(1.000)	-2.02(0.986)	3.66(0.858)	12.59(0.684)	24.54(0.466)	47.11(0.664)	-22	-8	15	51	192
Suisse	-45.56(0.208)	-51.59(0.122)	-48.12(0.110)	-43.45(0.104)	-25.37(0.260)	-27.48(0.026)	180	203	190	171	108
U.K.	-8.81(0.874)	-6.21(0.986)	-1.29(0.722)	-0.87(0.734)	-0.87(0.744)	19.68(0.734)	1013	715	149	100	-2263
U.S.	-23.50(0.434)	-15.94(0.458)	-13.39(0.608)	-8.72(0.830)	-8.72(0.862)	10.07(0.422)	269	183	153	100	-115
Panel B: Emerging Markets											
Argentina	133.47(0.294)	142.66(0.278)	145.34(0.308)	147.05(0.262)	145.87(0.210)	182.37(0.164)	91	98	100	101	125
Brazil	4.47(0.898)	13.21(0.666)	12.61(0.638)	13.03(0.566)	15.36(0.438)	34.21(0.224)	29	86	82	85	223
Chile	30.39(0.552)	39.85(0.484)	43.31(0.412)	49.69(0.354)	52.73(0.198)	77.61(0.368)	58	76	82	94	147
Greece	479.57(0.094)	487.92(0.110)	516.20(0.088)	562.97(0.062)	593.45(0.058)	715.01(0.112)	81	82	87	95	120
Indonesia	31.44(0.910)	32.37(0.888)	38.54(0.782)	41.52(0.640)	53.73(0.232)	57.01(0.524)	59	60	72	77	106
Korea	88.43(0.342)	88.15(0.340)	96.20(0.246)	112.04(0.108)	114.50(0.078)	111.28(0.226)	77	77	84	98	97
Malaysia	68.63(0.556)	81.85(0.576)	91.25(0.460)	96.24(0.362)	97.02(0.198)	130.92(0.454)	71	84	94	99	135
Mexico	29.04(0.444)	38.61(0.318)	42.82(0.196)	47.96(0.092)	50.13(0.044)	79.97(0.058)	58	77	85	96	160
Philipines	128.37(0.260)	138.10(0.214)	146.45(0.142)	157.32(0.122)	153.91(0.056)	198.78(0.124)	83	90	95	102	129
Portugal	117.85(0.188)	122.24(0.192)	135.10(0.158)	152.24(0.114)	163.39(0.092)	205.57(0.182)	72	75	83	93	126
Taiwan	147.16(0.138)	156.68(0.138)	164.45(0.112)	179.00(0.138)	183.54(0.124)	222.94(0.140)	80	85	90	98	121
Thailand	277.02(0.210)	274.81(0.250)	294.55(0.186)	310.96(0.126)	309.82(0.150)	360.41(0.188)	89	89	95	100	116
Turkey	104.37(0.002)	114.96(0.002)	119.04(0.002)	124.16(0.002)	130.53(0.002)	173.77(0.002)	80	88	91	95	133

This table reports the delta benefits from diversification out of each domestic market and into MVPs optimized with no shorts and the 1993 market capitalization weight constraints at the one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraint (nw) level. The results for the MVP with no weight constraints and short sales (nws) are also presented. The p-value results from the Ledoit and Wolf (2008) Sharpe ratio tests using a block size of 5 and 499 iterations are presented in brackets. The percent of the unconstrained benefits captured at each constraint level are reported in the last five columns. The percent of the benefits the nw constraint portfolio captures are not presented as it captures 100% of its own benefits.

and no short sales, provides positive diversification benefits for investors in all countries. The unconstrained MRRPs provide positive statistically significant benefits for investors in 12 developed markets and 9 emerging markets. Only when short sales are permitted do the MRRPs for all investors offer statistically significant potential diversification gains.

Table 4.3 reports the epsilon benefits of diversification into the MVP. The results from the Ledoit and Wolf (2008) volatility tests are presented in parentheses. The p-values report the significance of the hypothesis that the difference between the volatility of the monthly returns of the domestic portfolio and the optimized portfolio is zero. As with the MRRP, diversification into the 1/M portfolio does not provide positive benefits for all investors. The domestic Suisse, U.K., and U.S. markets have lower levels of volatility than the 1/M portfolio. The majority of markets do have positive potential gains from diversification. The 1/M portfolio's volatility reduction benefits are significant for 17 developed countries and 8 emerging markets. Weakening the weight constraints on the MVP provides increased benefits when diversifying out of all the markets studied. The smallest gains come from diversifying out of the U.S., U.K., and Switzerland. For investors in most countries, the majority of the unconstrained MVP benefits are captured by the 1/M portfolio.

Table 4.4 reports the delta performance of the weight constrained MVPs. As previously presented in table 4.2, investors in 10 of the 21 developed markets have reduced return-to-risk performance with the 1/M portfolio. As the weight constraints on the MVP are relaxed the number of countries that achieve positive RR improvements increases. Although the volatility reducing benefits of diversification are significant for many investors, the weak RR ratios of these portfolios do not necessarily support the argument for diversification. For example, in the case of a U.S. investor diversifying into the MVP with market allocations constrained to two times the market capitalization, volatility is reduced by 7.13% and is reported to be statistically significant. However, the RR ratio of this MVP is 15.93% lower than the domestic market RR. At the two times market capitalization constraint level 8 developed countries do not achieve MVPs with higher RR ratios than the domestic market. The unconstrained MVPs do not achieve higher RR ratios than the domestic market for investors in Denmark, Switzerland, the United Kingdom and the United States. Only investors in 2 developed countries and six emerging markets achieve positive RR benefits from diversification into the unconstrained MVP that is statistically different from their domestic market portfolio.

These results suggest that not all investors benefit from diversification into a naive or optimized MVP. Statistically lower volatility alone does not support the argument that investors will achieve positive benefits from diversification into optimized MVPs consisting of international equity markets. While diversification can reduce volatility compared to the local market, the RR ratios are not necessarily greater than the domestic market. As a result, an investor might find that investing locally and holding a portion of their portfolio in the risk free asset may achieve a level of volatility equivalent to the MVP, while realizing higher returns. Even portfolios with weakened weight constraints that achieve positive RR benefits over the domestic portfolio are not necessarily statistically different.

4.3.1 The Bayesian approach

This section examines the potential benefits of international diversification with weight constraints using the Bayesian approach presented in section 4.2. Given the varying nature of market correlations and the stochastic behaviour of market returns it remains to be determined whether the magnitude of the potential gains reported in the previous section are highly probable outcomes or only representative of the sample period used. The posterior distribution produced by the Bayesian approach gives the range of benefits that might be expected given alternative sampled outcomes. For diversification benefits to be statistically significant, the results should be 2 standard deviations from zero. The 5 and 1 percentile results from the posterior distribution are reported in this section and represent a posterior 95 to 99 percent probability that potential benefits of at least that size may occur.

Table 4.5 reports the posterior distribution statistics of the benefits from diversifying out of the investor's local market and into the constrained and unconstrained MRRPs optimized in the local currency. The Monte Carlo samples drawn from the historical distribution are reported for each MRRP at the 1% and 5% levels, along with the mean and the standard deviation of results. In the case of the 1/M portfolio, investors in 11 developed countries do not achieve positive RR gains with a posterior probability of 99% or 95%. Another 3 developed markets, and one emerging market have posterior gains of less than 10% at the 99% level. Most investors in emerging markets can expect to achieve diversification gains from the 1/M portfolio. However, Brazilian investors only achieve a 2.48% improvement in return-to-risk ratios at the 95% level. Chilean, Indonesian and Mexican investors only improve RR ratios by roughly 20 percent. These measured benefits are lower than the historical results presented in table 4.2.

Relaxing the weight constraints does not immediately result in positive benefits of diversification for all investors. At the two times weight constraint level, the historical potential benefits presented in table 4.2 are negative for 3 countries. The Bayesian results report negative potential gains at the 95 percent level for the same three countries. Five other countries achieve gains that are less than 15 percent. The countries with the largest potential gains from diversification are emerging markets. It seems that the relatively higher levels of domestic market volatility associated with emerging markets can be diversified away with international diversification. However, the Bayesian results suggest that the size of the potential gains is generally more variable than with developed markets.

Removing all weight constraints results in the smallest potential gains of 11.51%, 3.62% and 12.68% for Danish, Swiss and American investors respectively. Given the stochastic nature of market returns, these Bayesian results suggest that naive diversification into the 1/M portfolio or optimised MRRPs with relaxed weight constraints do not provide positive potential RR gains with certainty over extended holding periods for investors in all countries.

Table 4.6 reports the posterior distribution of the benefits of diversifying out of the investor's local market and into constrained and unconstrained MVPs optimized in the local currency. The 1/M portfolio provides volatility reduction benefits for 31 of the 34 markets at the 99% level. Investors in all emerging markets achieve positive volatility reduction benefits from diversification into the 1/M portfolio.

Removing all weight constraints results in volatility reducing benefits from diversification for all investors. The marginal improvement in performance as weight constraints are relaxed is arguably small for most markets. For example, a German investor relaxing weight constraints completely can improve the volatility reduction benefits of 27.63% at the 95% level for the 1/M portfolio to 37.26% at the 95% level with the MRRP with no weight constraints and no shorts. It is unclear from these results whether the small potential reduction in volatility at the 95% level warrants the risk of potential loss from estimation error that can result from the ex-ante optimization required to capture these benefits in practice.

Table 4.7 reports the posterior distribution of the improved RR ratios for the various MVPs. 10 developed markets and all emerging markets achieve positive RR gains at the 99% level with the 1/M portfolio. Only 4 of the 21 developed markets provide positive RR gains in excess of 15% at the 95% level. Twelve of the 13 emerging markets provide positive RR gains in excess of 15% at the 95% level. While the 1/M portfolio provides volatility reduction benefits for many markets, the RR ratios are not necessarily higher than the local market. In such a case, an investor may be inclined to hold the local market and diversify into a risk free asset to achieve a level of volatility similar to the 1/M portfolio while potentially achieving higher returns.

Weakening the weight constraints on the MVP generally improves the RR performance. With no weight constraints and no short sales, 5 developed markets still do not achieve positive RR gains at the 95% level. Another 6 developed markets achieve RR improvements of less than 20%. Investors in

Table 4.5: Posterior MRRP delta benefits in percent.

	wx1				wx2				wx5				wx10				nw				nws			
Country	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd
Panel A: Developed Markets																								
Australia	-30.14	-28.68	-25.00	2.27	-11.75	-10.13	-6.07	2.51	-3.00	-1.38	2.55	2.43	7.10	8.76	12.61	2.39	22.06	23.38	26.59	2.01	96.97	99.96	107.51	4.74
Austria	209.46	232.65	318.96	64.39	290.82	320.93	434.08	84.89	313.68	345.43	464.65	89.49	341.81	375.26	501.87	95.32	420.30	461.21	611.28	113.65	770.57	847.35	1128.79	213.96
Belgium	4.64	7.03	13.43	3.99	34.11	37.00	44.54	4.76	41.97	45.08	52.81	4.91	51.63	54.61	62.82	5.21	79.35	82.90	92.72	6.20	200.94	209.14	231.65	14.19
Canada	-16.49	-14.96	-11.43	2.17	11.51	13.22	17.26	2.51	18.14	19.82	23.69	2.42	24.40	25.86	29.47	2.26	29.71	31.34	35.68	2.70	124.65	128.50	138.71	6.40
Denmark	-37.14	-35.77	-32.57	1.92	-18.15	-16.84	-13.56	1.99	-12.83	-11.56	-8.35	1.94	-7.39	-6.08	-2.88	1.95	11.51	12.42	14.74	1.49	89.35	92.14	99.37	4.57
Finland	-6.40	-3.87	2.06	3.72	20.83	23.69	30.61	4.40	28.78	31.64	38.76	4.51	38.25	41.18	48.56	4.69	63.25	66.70	76.27	6.07	183.27	191.06	210.22	12.30
France	10.11	12.29	17.61	3.34	40.66	43.11	50.04	4.36	49.15	51.61	58.68	4.47	59.06	61.67	69.14	4.72	86.80	90.45	100.48	6.39	214.10	222.71	246.92	15.52
Germany	9.66	11.82	17.35	3.46	39.84	42.51	49.47	4.42	48.11	50.88	58.05	4.57	58.00	60.86	68.48	4.86	85.13	89.02	99.06	6.45	210.54	219.10	243.29	15.52
H.K.	25.27	28.24	36.18	5.00	64.49	68.83	80.06	7.21	70.64	75.09	86.98	7.63	76.73	81.25	93.60	7.94	88.26	93.39	107.09	8.78	259.13	270.24	301.39	20.30
Ireland	107.66	116.36	142.65	17.70	163.35	174.14	208.47	23.15	177.18	188.76	225.13	24.59	193.95	206.17	245.12	26.35	241.19	256.71	303.65	32.19	470.40	499.80	592.69	63.51
Italy	57.22	62.44	77.22	9.45	97.16	103.93	122.74	12.20	108.80	115.82	135.36	12.71	122.70	129.90	150.47	13.37	158.39	166.73	191.31	16.00	331.64	348.14	397.50	32.02
Japan	199.25	223.85	317.61	75.20	267.48	301.65	429.28	102.73	280.90	316.16	448.66	106.61	295.88	332.21	470.03	111.07	349.27	389.05	550.20	129.45	839.98	928.12	1303.43	303.26
Holland	-7.00	-5.43	-1.68	2.32	19.04	20.88	25.26	2.77	26.30	28.01	32.46	2.83	34.79	36.56	41.22	2.96	58.12	60.21	66.31	3.82	164.04	170.01	186.27	10.30
N.Z.	-18.48	-16.02	-9.12	4.29	9.50	12.58	21.01	5.27	19.45	22.61	31.11	5.36	30.83	34.09	42.78	5.47	52.14	55.14	63.51	5.37	173.03	179.95	198.45	11.80
Norway	6.67	9.18	15.81	4.11	35.85	39.10	47.14	5.02	44.51	47.56	55.56	5.01	53.26	56.20	64.39	5.18	77.70	81.13	90.78	6.12	188.52	195.51	214.92	12.36
Singapore	53.01	57.88	71.52	8.93	101.22	108.20	127.33	12.74	109.43	116.98	136.94	13.41	119.83	127.60	148.52	14.06	135.39	143.72	167.59	15.90	330.80	347.36	396.85	33.14
Spain	-7.60	-5.70	-1.08	2.86	16.03	18.26	23.57	3.33	22.75	24.95	30.19	3.29	31.06	33.27	38.47	3.27	51.54	54.21	60.92	4.19	153.61	158.92	172.86	8.77
Sweden	-10.71	-9.18	-5.23	2.41	13.24	15.21	19.74	2.80	21.25	23.03	27.48	2.72	30.10	31.74	36.08	2.68	50.18	52.63	58.43	3.62	138.94	143.48	155.00	7.31
Suisse	-50.62	-49.15	-45.57	2.14	-30.88	-29.46	-26.08	2.05	-25.71	-24.45	-21.24	1.94	-19.35	-18.20	-15.35	1.73	3.62	4.20	5.91	1.10	104.48	108.42	118.07	6.10
U.K.	-13.17	-11.92	-8.81	1.89	12.96	14.24	17.57	2.10	17.47	18.68	21.95	2.12	22.44	23.72	27.31	2.30	37.40	39.15	43.99	3.07	145.90	151.26	165.22	8.84
U.S.	-26.60	-25.69	-23.53	1.30	0.20	0.51	1.21	0.43	4.16	4.40	5.10	0.44	7.26	7.69	8.82	0.70	12.68	13.72	16.50	1.77	112.54	116.34	126.01	6.05
Panel B: Emerging Markets																								
Argentina	109.41	115.83	134.11	11.78	139.36	146.53	167.90	13.68	143.65	150.87	172.52	13.92	147.82	155.16	177.24	14.20	159.76	167.64	190.97	15.04	337.77	352.81	393.86	26.93
Brazil	1.60	2.48	4.47	1.23	11.11	12.00	14.13	1.31	12.99	13.84	15.90	1.26	14.69	15.53	17.50	1.22	20.06	20.77	22.48	1.06	83.66	85.13	89.06	2.41
Chile	19.90	23.20	30.52	4.62	47.76	51.41	60.11	5.48	53.87	57.56	66.39	5.57	60.41	64.23	73.36	5.75	74.65	78.27	87.12	5.63	177.05	183.02	198.74	9.99
Greece	311.62	351.95	499.74	120.14	406.76	453.90	638.58	150.35	433.75	483.35	677.39	158.02	467.12	520.27	726.19	167.82	549.73	612.80	851.58	194.66	943.11	1051.43	1464.81	337.47
Indonesia	18.60	22.29	31.64	5.85	37.01	40.95	51.21	6.46	42.45	46.29	56.50	6.43	46.81	50.76	60.90	6.43	65.29	68.78	77.85	5.80	198.02	204.85	224.17	12.21
Korea	63.87	70.50	89.16	12.08	100.57	108.50	131.58	14.84	112.25	120.42	143.83	15.06	123.19	131.20	154.47	14.98	134.41	142.34	166.81	15.83	310.64	326.80	371.95	29.48
Malaysia	45.81	52.07	69.24	11.17	86.86	94.77	116.20	14.01	95.17	102.99	124.47	14.07	103.41	110.91	131.93	13.81	113.48	121.02	143.48	14.74	331.15	348.25	397.68	32.64
Mexico	21.65	23.80	29.14	3.35	41.78	44.13	49.99	3.68	48.06	50.42	56.18	3.64	55.07	57.41	63.08	3.58	62.07	64.50	70.03	3.54	142.32	146.26	155.94	6.10
Philipines	100.43	107.96	129.04	13.59	142.83	151.31	176.83	16.36	150.39	158.97	185.06	16.75	158.68	167.55	194.19	17.19	169.80	179.31	207.42	18.23	357.78	375.08	424.86	32.31
Portugal	88.33	96.27	119.01	15.19	136.39	146.28	175.72	19.54	149.68	160.18	190.55	20.35	166.07	177.12	208.93	21.48	211.44	224.75	262.55	25.40	414.94	439.85	514.37	49.78
Taiwan	114.70	123.09	148.58	16.59	165.10	176.19	208.81	21.33	174.67	185.73	219.74	22.20	185.64	197.35	232.90	23.19	204.52	217.34	256.22	25.45	408.44	431.65	500.18	45.32
Thailand	195.67	215.88	283.13	47.70	264.77	289.85	374.40	60.31	276.77	302.42	389.68	62.00	289.09	316.40	406.35	64.16	311.17	340.86	437.91	69.27	650.14	707.34	894.92	134.00
Turkey	95.18	97.86	104.39	4.08	111.41	114.18	121.15	4.35	116.60	119.42	126.49	4.42	121.80	124.73	131.85	4.46	131.40	134.24	141.02	4.26	206.89	210.73	220.22	5.90

This table reports the distribution of potential delta benefits at each level of market weight constrained MRRP with no shorts using the Bayesian approach described in section 4.2. The 1 percentile, the 5 percentile, the mean and the standard deviation of results are reported for the MRRPs with no shorts and various levels of weight constraints: one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraints (nw). The results for the MRRP with no weight constraints and short sales (nws) are also presented.

Table 4.6: Posterior MVP epsilon benefits in percent.

	wx1				wx2				wx5				wx10				nw				nws			
Country	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd
Panel A: Developed Markets																								
Australia	6.68	9.64	16.09	3.93	11.84	14.63	20.72	3.72	16.04	18.70	24.50	3.54	18.39	20.97	26.62	3.44	20.50	23.02	28.52	3.35	33.96	36.06	40.62	2.78
Austria	30.15	32.22	37.08	2.90	34.71	36.64	41.18	2.71	37.45	39.30	43.65	2.59	38.53	40.35	44.63	2.55	39.43	41.22	45.44	2.51	51.82	53.25	56.60	2.00
Belgium	17.57	20.00	25.73	3.44	22.89	25.16	30.52	3.22	26.18	28.36	33.49	3.08	27.50	29.64	34.68	3.03	28.62	30.73	35.69	2.98	43.05	44.72	48.68	2.38
Canada	18.52	20.90	26.58	3.42	24.82	27.01	32.25	3.15	26.04	28.19	33.35	3.10	26.07	28.23	33.39	3.10	26.07	28.23	33.39	3.10	43.16	44.81	48.78	2.38
Denmark	14.20	16.61	22.73	3.63	19.76	22.01	27.74	3.39	22.97	25.14	30.63	3.26	24.25	26.38	31.78	3.20	25.30	27.40	32.72	3.16	40.06	41.75	46.02	2.53
Finland	49.11	50.64	54.19	2.15	52.42	53.85	57.17	2.01	54.72	56.07	59.23	1.91	55.60	56.93	60.03	1.88	56.23	57.55	60.60	1.85	64.42	65.48	67.96	1.50
France	13.84	16.29	22.36	3.62	19.45	21.74	27.42	3.39	22.88	25.07	30.50	3.24	24.21	26.36	31.70	3.19	25.24	27.37	32.64	3.14	40.28	41.98	46.19	2.51
Germany	25.50	27.63	32.75	3.12	30.34	32.33	37.12	2.92	33.25	35.16	39.75	2.80	34.41	36.29	40.80	2.75	35.41	37.26	41.70	2.71	48.58	50.05	53.58	2.16
H.K.	38.86	40.81	45.03	2.55	44.30	46.08	49.92	2.33	44.62	46.39	50.21	2.31	44.79	46.55	50.36	2.31	44.79	46.55	50.36	2.31	61.08	62.33	65.01	1.63
Ireland	26.44	28.66	33.81	3.09	31.12	33.20	38.03	2.89	34.01	36.01	40.63	2.77	34.98	36.95	41.50	2.73	35.55	37.50	42.02	2.71	48.05	49.62	53.26	2.18
Italy	27.90	30.11	35.10	3.02	32.60	34.66	39.33	2.82	35.22	37.20	41.69	2.71	36.07	38.03	42.46	2.68	36.56	38.50	42.90	2.66	49.46	51.01	54.51	2.12
Japan	0.19	3.05	9.83	4.14	9.24	11.84	18.00	3.77	9.75	12.34	18.47	3.75	10.25	12.82	18.92	3.72	10.37	12.93	19.02	3.72	30.80	32.78	37.48	2.87
Holland	15.96	18.50	24.25	3.50	21.52	23.90	29.27	3.27	24.83	27.10	32.24	3.14	26.07	28.31	33.36	3.08	27.01	29.22	34.21	3.04	41.91	43.67	47.64	2.42
N.Z.	17.00	19.32	25.19	3.46	20.99	23.20	28.79	3.29	24.10	26.22	31.59	3.16	25.81	27.88	33.13	3.09	28.91	30.90	35.93	2.96	39.46	41.15	45.43	2.52
Norway	31.57	33.78	38.59	2.90	35.78	37.86	42.37	2.72	38.94	40.91	45.20	2.58	39.80	41.75	45.98	2.55	40.05	41.98	46.20	2.54	49.87	51.49	55.01	2.12
Singapore	39.35	41.23	45.39	2.53	44.31	46.04	49.85	2.32	45.47	47.16	50.90	2.28	45.87	47.56	51.26	2.26	45.90	47.58	51.28	2.26	58.30	59.60	62.45	1.74
Spain	26.11	28.28	33.38	3.10	30.78	32.80	37.58	2.90	33.42	35.37	39.96	2.79	34.43	36.35	40.88	2.75	35.20	37.09	41.57	2.72	48.01	49.54	53.12	2.18
Sweden	36.01	37.94	42.41	2.70	40.32	42.12	46.29	2.52	43.18	44.90	48.87	2.40	44.32	46.00	49.89	2.35	44.93	46.59	50.44	2.32	54.38	55.76	58.95	1.93
Suisse	-13.66	-10.29	-2.36	4.82	-6.42	-3.27	4.16	4.51	-2.05	0.97	8.10	4.33	0.26	3.22	10.18	4.23	3.48	6.34	13.08	4.09	24.26	26.50	31.79	3.21
U.K.	-13.02	-9.45	-1.69	4.73	-3.81	-0.52	6.60	4.34	1.19	4.31	11.09	4.13	1.99	5.09	11.81	4.10	1.99	5.09	11.81	4.10	28.67	30.93	35.82	2.98
U.S.	-5.89	-2.71	4.67	4.47	3.57	6.47	13.19	4.07	4.11	6.99	13.68	4.05	4.39	7.27	13.93	4.04	4.39	7.27	13.93	4.04	32.54	34.57	39.27	2.85
Panel B: Emerging Markets																								
Argentina	38.87	40.67	45.00	2.57	41.32	43.05	47.21	2.46	41.68	43.40	47.54	2.45	41.96	43.66	47.78	2.44	42.45	44.14	48.23	2.42	53.28	54.66	57.97	1.96
Brazil	16.46	18.97	24.74	3.47	18.37	20.83	26.47	3.39	19.01	21.45	27.04	3.37	19.38	21.80	27.37	3.35	19.82	22.23	27.77	3.33	25.65	27.88	33.02	3.09
Chile	25.52	27.72	32.96	3.13	29.37	31.45	36.42	2.97	30.50	32.56	37.45	2.93	31.15	33.18	38.02	2.90	34.23	36.17	40.80	2.77	46.71	48.28	52.03	2.24
Greece	51.07	52.62	55.98	2.06	54.21	55.66	58.80	1.92	55.90	57.29	60.32	1.85	56.64	58.01	60.99	1.82	57.27	58.62	61.55	1.80	66.00	67.08	69.41	1.43
Indonesia	19.32	21.51	27.27	3.41	21.71	23.83	29.43	3.31	24.06	26.12	31.55	3.21	25.55	27.57	32.89	3.15	32.33	34.16	39.00	2.86	41.89	43.46	47.61	2.46
Korea	44.18	45.81	49.71	2.34	46.69	48.25	51.98	2.23	47.92	49.44	53.08	2.18	49.03	50.52	54.08	2.13	49.29	50.77	54.32	2.12	57.13	58.38	61.38	1.80
Malaysia	27.37	29.34	34.37	3.03	31.42	33.28	38.03	2.86	33.51	35.31	39.92	2.77	35.91	37.64	42.08	2.67	37.08	38.78	43.14	2.62	50.84	52.17	55.58	2.05
Mexico	28.57	30.55	35.49	2.99	31.48	33.37	38.11	2.87	33.47	35.31	39.91	2.79	35.12	36.91	41.40	2.72	35.77	37.54	41.99	2.69	47.01	48.47	52.14	2.22
Philipines	40.60	42.37	46.47	2.50	44.18	45.84	49.70	2.35	45.30	46.93	50.71	2.30	45.90	47.51	51.25	2.28	46.56	48.15	51.84	2.25	58.83	60.05	62.90	1.73
Portugal	21.71	24.27	29.56	3.27	26.71	29.11	34.06	3.06	29.75	32.05	36.80	2.94	30.93	33.19	37.86	2.89	31.84	34.07	38.68	2.85	45.43	47.21	50.90	2.28
Taiwan	48.01	49.54	53.14	2.17	52.01	53.43	56.75	2.00	52.77	54.17	57.43	1.97	53.09	54.47	57.71	1.96	53.11	54.49	57.73	1.96	64.13	65.19	67.67	1.50
Thailand	54.06	55.33	58.52	1.93	56.69	57.89	60.90	1.82	57.94	59.10	62.02	1.77	58.79	59.93	62.79	1.73	58.95	60.08	62.93	1.73	66.74	67.66	69.97	1.40
Turkey	56.05	57.49	60.56	1.85	57.14	58.54	61.54	1.81	57.59	58.97	61.94	1.79	58.00	59.37	62.31	1.77	58.77	60.11	63.00	1.74	62.86	64.07	66.67	1.56

This table reports the distribution of potential epsilon benefits at each level of market weight constrained MVP with no shorts using the Bayesian approach described in section 4.2. The 1 percentile, the 5 percentile, the mean and the standard deviation of results are reported for the MVPs with no shorts and various levels of weight constraints: one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraints (nw). The results for the MVP with no weight constraints and short sales (nws) are also presented.

Table 4.7: Posterior MVP delta benefits in percent.

	wx1				wx2				wx5				wx10				nw				nws			
Country	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd	1%	5%	mean	sd
Panel A: Developed Markets																								
Australia	-30.14	-28.68	-25	2.27	-31.95	-30.43	-26.67	2.3	-24.16	-22.67	-19.04	2.24	-15.52	-14.1	-10.58	2.16	-2.57	-1.4	1.54	1.78	14.4	15.94	19.96	2.47
Austria	209.46	232.65	318.96	64.39	210.07	233.56	320.44	64.77	228.88	252.89	344.19	68.04	257.03	283.59	383.34	74.51	270.3	297.89	401.49	77.7	308.55	341.26	463.41	91.61
Belgium	4.64	7.03	13.43	3.99	5.19	7.66	13.98	3.95	12.37	14.85	21.04	3.91	22.22	24.69	31.15	4.1	27.46	30.23	37.08	4.37	41.03	44.59	53.78	5.79
Canada	-16.49	-14.96	-11.43	2.17	-3.84	-2.33	1.46	2.34	-0.74	0.79	4.48	2.29	-0.33	1.18	4.88	2.31	-0.33	1.17	4.87	2.31	30.76	32.92	38.6	3.49
Denmark	-37.14	-35.77	-32.57	1.92	-36.48	-35.07	-31.91	1.91	-31.3	-29.96	-26.92	1.85	-26.42	-25.15	-22.18	1.8	-22.21	-20.86	-17.84	1.85	-13.19	-11.76	-7.94	2.34
Finland	-6.4	-3.87	2.06	3.72	-7.16	-4.62	1.34	3.73	-1.89	0.58	6.88	3.92	7.09	9.78	16.38	4.18	12.67	15.45	22.43	4.36	29.42	33.18	42.71	6.01
France	10.11	12.29	17.61	3.34	9.87	12.09	17.4	3.32	17.16	19.28	24.93	3.5	26.73	28.96	34.93	3.73	33.30	35.66	41.89	3.90	45.21	48.59	58.19	6.06
Germany	9.66	11.82	17.35	3.46	9.88	12.09	17.75	3.58	16.16	18.42	24.37	3.75	26.57	29	35.37	4.03	31.08	33.63	40.14	4.15	43.51	47.45	57.60	6.46
H.K.	25.27	28.24	36.18	5.00	36.63	40.02	49.35	5.87	41.03	44.40	53.84	5.97	48.72	52.2	62.21	6.35	48.72	52.20	62.21	6.35	75.54	80.88	95.72	9.55
Ireland	107.66	116.36	142.65	17.70	113.89	122.25	149.05	18.10	126.19	134.79	163.02	19.09	141.67	151.33	181.88	20.77	152.55	163.32	195.94	22.21	180.07	192.07	232.56	27.39
Italy	57.22	62.44	77.22	9.45	59.57	65.07	80.10	9.62	65.01	70.40	85.25	9.56	77.44	83.02	98.94	10.25	86.12	92.09	109.05	10.97	116.42	123.94	145.93	14.13
Japan	199.25	223.85	317.61	75.2	162.85	182.05	255.22	58.84	167.79	187.21	261.80	60.05	186.03	207.02	289.07	66.07	199.00	221.25	308.70	70.48	241.51	271.21	384.31	91.63
Holland	-7	-5.43	-1.68	2.32	-5.87	-4.38	-0.67	2.31	-0.22	1.16	4.76	2.24	8.29	9.7	13.44	2.32	13	14.35	18.23	2.41	26.24	28.69	35.7	4.36
N.Z.	-18.48	-16.02	-9.12	4.29	-23.78	-21.35	-14.56	4.23	-15.95	-13.62	-6.73	4.29	-0.25	2.28	9.4	4.46	16.57	18.72	24.8	3.79	15.23	17.97	24.87	4.34
Norway	6.67	9.18	15.81	4.11	12.24	14.86	21.84	4.31	18.33	20.94	27.89	4.31	27.45	30.1	37.45	4.55	32.24	34.98	42.51	4.7	40.67	43.85	52.82	5.66
Singapore	53.01	57.88	71.52	8.93	58.74	64.11	79.31	9.97	64.71	70.14	85.21	9.93	75.89	81.71	97.86	10.7	79.3	85.09	101.74	10.97	91.5	99.03	119.72	13.68
Spain	-7.6	-5.7	-1.08	2.86	-5.5	-3.56	1.04	2.87	-0.56	1.52	6.27	2.96	6.89	8.91	13.88	3.08	11.49	13.51	18.46	3.06	33.13	35.76	42.81	4.41
Sweden	-10.71	-9.18	-5.23	2.41	-7.64	-6.13	-1.95	2.55	-2.16	-0.56	3.74	2.63	6.55	8.16	12.69	2.77	17.99	19.84	24.65	2.97	37.55	40.4	47.27	4.31
Suisse	-50.62	-49.15	-45.57	2.14	-56.88	-55.33	-51.61	2.23	-53.16	-51.65	-48.14	2.12	-48.17	-46.71	-43.47	1.96	-28.43	-27.53	-25.38	1.29	-32.5	-31.02	-27.47	2.16
U.K.	-13.17	-11.92	-8.81	1.89	-10.84	-9.48	-6.21	1.99	-4.74	-3.72	-1.30	1.47	-3.63	-2.81	-0.88	1.19	-3.63	-2.81	-0.88	1.19	12.71	14.66	19.73	3.15
U.S.	-26.6	-25.69	-23.53	1.30	-18.5	-17.75	-15.96	1.09	-16.01	-15.25	-13.41	1.12	-11.42	-10.62	-8.74	1.17	-11.42	-10.62	-8.74	1.17	4.09	5.82	10.07	2.64
Panel B: Emerging Markets																								
Argentina	109.41	115.83	134.11	11.78	117.16	123.92	143.35	12.46	119.8	126.54	146.03	12.49	121.71	128.36	147.74	12.42	122.07	128.32	146.51	11.64	153.28	161.18	183.16	14.15
Brazil	1.60	2.48	4.47	1.23	10.19	11.08	13.21	1.31	9.64	10.52	12.61	1.28	10.16	11.01	13.04	1.24	12.79	13.55	15.36	1.11	30.96	31.94	34.22	1.4
Chile	19.9	23.2	30.52	4.62	28.52	32.03	40.00	5.01	31.96	35.42	43.46	5.02	38.20	41.6	49.83	5.12	43.12	46.08	52.85	4.22	65.66	69.00	77.74	5.47
Greece	311.62	351.95	499.74	120.14	317.84	357.95	508.44	122.19	338.82	380.06	537.77	128.04	372.16	416.20	586.29	138.09	393.96	439.99	617.92	144.68	471.54	527.95	745.06	176.49
Indonesia	18.60	22.29	31.64	5.85	19.42	23.27	32.58	5.87	25.77	29.43	38.76	5.84	29.01	32.62	41.74	5.74	43.64	46.56	53.89	4.58	45.94	49.06	57.17	5.13
Korea	63.87	70.50	89.16	12.08	63.54	70.18	88.87	12.16	71.52	78.07	96.94	12.22	86.94	93.55	112.81	12.45	90.23	96.66	115.24	12	85.7	93.07	112.04	12.35
Malaysia	45.81	52.07	69.24	11.17	57.06	63.86	82.55	12.16	66.17	73.15	91.96	12.27	72.53	79.03	96.92	11.64	75.67	81.52	97.62	10.46	103.59	110.91	131.74	13.54
Mexico	21.65	23.8	29.14	3.35	30.74	32.99	38.73	3.57	34.99	37.27	42.94	3.52	40.56	42.71	48.07	3.34	43.44	45.37	50.23	3.04	71.08	73.72	80.10	3.99
Philipines	100.43	107.96	129.04	13.59	108.42	116.33	138.83	14.43	116.88	124.7	147.18	14.52	126.99	134.86	158.08	14.91	125.80	133.17	154.62	13.83	163.96	172.93	199.73	17.29
Portugal	88.33	96.27	119.01	15.19	92.35	100.3	123.44	15.48	103.57	111.91	136.37	16.28	118.66	127.53	153.63	17.37	129.61	138.62	164.82	17.51	163.34	174.65	207.46	22.03
Taiwan	114.7	123.09	148.58	16.59	122.4	131.42	158.20	17.54	129.30	138.31	166.02	18.13	141.66	151.33	180.69	19.23	145.53	155.28	185.27	19.58	177.33	189.53	225.05	23.36
Thailand	195.67	215.88	283.13	47.7	193.61	214.14	280.91	47.47	209.60	230.60	300.97	49.92	222.72	244.57	317.59	51.59	222.77	244.01	316.36	51.02	256.88	282.99	368.35	60.45
Turkey	95.18	97.86	104.39	4.08	105.31	108.13	114.98	4.28	109.35	112.19	119.07	4.32	114.33	117.25	124.19	4.35	121.22	124.00	130.56	4.11	162.43	165.86	173.82	4.97

This table reports the distribution of potential delta benefits at each level of market weight constrained MVP with no shorts using the Bayesian approach described in section 4.2. The 1 percentile, the 5 percentile, the mean and the standard deviation of results are reported for the MVPs with no shorts and various levels of weight constraints: one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraints (nw). The results for the MVP with no weight constraints and short sales (nws) are also presented.

emerging markets have greater certainty in achieving potential benefits from diversification into MVPs with or without weight constraints. Interestingly, weakening the weight constraints on the 1/M portfolio can result in lower probable RR ratios for investors in 5 countries. For example, a New Zealand investor achieves a 19.32% reduction in volatility at the 95% level with the 1/M portfolio. Reduce the weight constraints to the two times level and the volatility reducing benefit is 23.20% at the 95% level. However, the RR ratio of the 1/M portfolio at the 95% level is minus 16.02% and this weakens to minus 21.35% with the MVP with weights constrained at the two times level. These MVP delta results are lower than the historical results of minus 9.22% for the 1/M portfolio and minus 14.64% with the optimized MVP at the two times weight constraint level.

4.4 Conclusion

This chapter reports the potential benefits from international diversification available to investors in 34 countries diversifying into weight constrained MR-RPs and MVPs optimized in the local currency during the 1993 to 2014 investment period. The naive 1/M portfolio does not provide improved return-to-risk performance or lower volatility for all investors. While weakening the weight constraints imposed on the MRRP does increase the potential gains from diversification for all investors, the improved RR performance provided by the optimized portfolios is not statistically different from many of the domestic market portfolios. There are potential volatility reduction benefits from diversification into MVPs with relaxed weight constraints, but these

MVPs do not necessarily provide higher RR ratios than the local market. The size of the probable gains an investor may expect to realize from international diversification is further reduced when a Bayesian approach with a non-informative prior is used.

This study extends the single country perspective that much of the existing literature presents on this topic (De Roon et al., 2001; Li et al., 2003; Fletcher and Marshall, 2005; Chiou, 2008). This chapter also extends the cross-country results presented in Driessen and Laeven (2007) which only considers short selling restrictions into the MRRP. The use of the Ledoit and Wolf (2008) bootstrap Sharpe and volatility testing techniques also contributes to the literature by providing a method to determine the level of relaxed weight constraints at which an optimized portfolio can potentially offer statistically significant positive diversification benefits. Finally, the use of the Bayesian method to measure the probable level of diversification benefits investors might achieve given the stochastic nature of market returns provides robustness to the results and extends the U.S. and U.K. perspective results presented in Li et al. (2003) and Fletcher and Marshall (2005) respectively which only consider short sale restrictions.

It is unclear whether the potential gains from diversification achieved by the static portfolio solutions considered in this chapter are similar for dynamic ex-ante optimization strategies using incomplete time-varying risk and return estimates prone to estimation error. Also, this chapter does not consider the effect of taxes on dividends and capital gains which may be different for each country. An examination of the effect of taxes on the potential gains from international diversification may contribute further to an accurate measurement of the potential benefits of international diversification and the resulting significance of the equity home bias.

Chapter 5

An empirical evaluation of estimation error reduction strategies applied to international diversification

Abstract

This chapter evaluates the performance of estimation error reduction strategies recently presented in the literature versus the domestic market portfolio and the naive global market capitalization weighted (1/M) portfolio from the perspective of investors in 34 countries. The results do not identify a naive benchmark or an optimization strategy that is certain to provide all investors with efficiency gains beyond their domestic market portfolio. The estimation error reduction strategies can provide lower levels of volatility than the naive

benchmarks that are statistically significant. These strategies do not provide positive return-to-risk (RR) benefits over the naive benchmarks that are statistically significant for investors in as many as 32 countries. Some optimization strategies have relatively large short positions and high turnover rates which will introduce additional costs that may further reduce their effectiveness versus the passive portfolio benchmarks.

5.1 Introduction

The equity home bias puzzle is framed by the assumption of risk sharing in the capital asset pricing model and the efficient portfolio optimization envisaged by modern portfolio theory. Factors such as the time-varying return distribution of assets can reduce the effectiveness of classical mean-variance optimization out-of-sample (Jorion, 1985). Shrinkage of the sample covariance matrix and constraints on optimization weights can reduce the estimation error related to the covariance between assets and improve the effectiveness of classical mean-variance optimization (Jagannathan and Ma, 2003). Recent strategies designed to reduce estimation error through sample covariance matrix shrinkage (Ledoit and Wolf, 2003b, 2004, 2003a) or constraints on the allocation weights in the optimization solution (DeMiguel et al., 2009a; Behr et al., 2013; Levy and Levy, 2014a) have been reported to provide inconsistent ex-ante performance improvements over naive portfolios of U.S. equities. This chapter reports the effectiveness of estimation error reduction strategies in optimizing internationally diversified portfolios relative to the domestic market portfolio and the naive global market capitalization weighted (1/M) portfolio for investors in 34 countries.

The results in chapter 4 report that there can exist potential benefits from international diversification into static portfolios optimized with perfect ex-post information. In practice, investors are unlikely to hold static portfolios over extended holding periods, nor have the perfect market estimates needed to achieve the efficiency gains reported in chapter 4. In this chapter I seek to determine whether a home bias may exist in part because international diversification out-of-sample is not certain to provide positive diversification benefits that are statistically different from the local market portfolio. Specifically, I ask whether investors in all 34 markets studied can expect to achieve positive benefits from naive diversification into the time-varying 1/M portfolio? If naive diversification does not provide significant gains beyond the local market portfolio for all investors, a home bias may be a reasonably efficient investment decision given the uncertainty of future benefits. Next, I ask whether estimation error reduction strategies recently presented in the literature outperform the naive domestic portfolio and the 1/M portfolio? If active optimization strategies available in the public domain do not consistently provide significant benefits beyond both benchmark portfolios, it seems reasonable to assume that many investors will also have difficulty achieving efficiency gains beyond a naive benchmark that would justify active optimization.

Early literature investigating the benefits of international diversification reports weak correlations between markets (Grubel, 1968; Levy and Sarnat, 1970; Lessard, 1973) and concludes that benefits from diversification exist. Recent studies find that correlations between markets have oscillated over the twentieth century with correlations peaking at the end of the 19th century, the Great Depression and the late 20th century (Goetzmann et al., 2007; Quinn and Voth, 2008). Levy and Levy (2014b) propose that strengthening correlations between markets magnify international trade frictions such as the asymmetric information costs and higher variance in returns that result from currency risk. As a result, despite lower international transaction costs over time, higher correlations offset these gains and the home bias will persist.

Empirical studies measuring the potential benefits of international diversification find benefits are reduced with restrictions on short sales (De Roon et al., 2001; Driessen and Laeven, 2007) and weight allocation constraints (Chiou, 2008). These studies generally use perfect ex-post information to form optimal portfolios from which the potential benefits of diversification are measured. A Bayesian approach that uses static portfolios to measure the probable gains that can be expected given the stochastic nature of markets also finds that the removal of short sales reduce, but do not eliminate, international diversification benefits for U.S. investors (Li et al., 2003) and U.K. investors (Fletcher and Marshall, 2005).

DeMiguel et al. (2009b) reports that ex-ante optimization strategies generally do not outperform a naive 1/N investment strategy. Early literature identifies the weak ex-ante performance of classical mean-variance optimization used in international diversification with historical estimates (Jorion, 1985). Return estimation error is large and results in poor out-of-sample optimization performance relative to the naive investment strategy of buying a market weighted portfolio. Jorion (1985) suggests the use of minimum-variance portfolio optimization to remove return error from the optimization process. The use of Bayes-Stein shrinkage on the estimates is also reported to improve the out-of-sample performance relative to the naive benchmark.

Jagannathan and Ma (2003) reports that minimum-variance portfolio optimization with no shorts and using a sample covariance matrix performs as well as a covariance matrix estimate based on factor models, shrinkage estimators, or higher frequency data. They explain that weight constraints reduce estimation error when using the sample covariance matrix. Constraints on asset weights is equivalent to constructing an unconstrained minimum-variance portfolio optimization using the shrunk covariance matrix derived using Lagrange multipliers from the constraints. Imposing weight constraints or the use of a shrunk covariance matrix can reduce the estimation error in the optimization solution and improve the efficiency of the out-of-sample portfolio performance.

Ledoit and Wolf (2003b,a, 2004) present a shrinkage strategy which seeks the optimally weighted average of a sample covariance matrix and a target covariance matrix that minimizes the mean squared error between the shrunk covariance matrix and the true covariance matrix of stock returns. The three target matrices these papers present are the one-factor covariance matrix, the constant correlation matrix and the Identify matrix. The strategy reduces the extreme values in the sample covariance matrix towards the target matrix values with the objective of reducing estimation error. The strategies are applied to U.S. equities and generally show improved ex-ante performance over the non-shrunk models or industry factors. Increasing the number of assets reduces the relative performance of these strategies.

DeMiguel et al. (2009a) present a 1-norm and an A-norm weight constraint framework with which estimation error is reduced. The results suggest that these norm-constrained portfolios can produce higher Sharpe ratios than the strategies in Jagannathan and Ma (2003), Ledoit and Wolf (2003b, 2004), 1/N and other common strategies in the literature. While the strategy provides improved performance over naive portfolios, the results are not statistically significant for all the data sets of U.S. equities considered.

Behr et al. (2013) present a constrained minimum-variance model using a shrinkage theory framework which is reported to outperform naive portfolios for most U.S. equity datasets studied. The model uses a bootstrap technique to create sample portfolios from which a direct search method is applied to determine the minimum and maximum constraints that minimise the mean squared error of the corresponding Lagrange multiplier modified sample covariance matrix. The model is tested against the three shrinkage targets of Ledoit and Wolf (2003b, 2004, 2003a), the DeMiguel et al. (2009a) 1-norm and 2-norm weight constraint framework and some common optimization models. The p-values from the Ledoit and Wolf (2008) bootstrap tests find that the model outperforms the naive portfolio, but is not significantly different from the other recent estimation error reduction methods tested.

Levy and Levy (2014a) present a variance-based constraint method to reduce estimation error which imposes more stringent weight constraints on assets with relatively higher standard deviations than the average standard deviation of the assets. The principle behind the method is to reduce the potential estimation error associated with high volatility assets. The model is tested using portfolios of U.S. equities. The significance of the results are not tested. Nor is the method tested against other recent models designed to reduce the estimation error in the optimization inputs.

The literature measuring the effectiveness of these estimation error reduction strategies has largely focused on measuring performance when used with stocks in a single market. Jacobs et al. (2014) considers the effectiveness of common optimization strategies versus heuristic portfolio selection from the perspective of European investors diversifying amongst the four regional equity indices of North America, Europe, Asia and emerging markets. Classical mean-variance optimization models, as well as the DeMiguel et al. (2009a) and Ledoit and Wolf (2003a) strategies are found to provide no significant improvement over naive allocation strategies consisting of regional equity indices only, or regional equity indices, European bonds and world commodities. The paper does not report the results of the local market portfolio versus the optimized portfolios or the results achieved by investors in countries outside Europe. It is not clear whether the use of naive regional indices in Jacobs et al. (2014) improves the optimization results or reduces the effectiveness of the shrinkage and constraint models as individual market information is lost with the use of naively diversified regional indices.

This chapter addresses gaps in the literature regarding the effectiveness of estimation error reduction strategies in capturing the potential benefits of international diversification. This is the first study that this author is aware of to report the results of the Levy and Levy (2014a) strategy applied to the optimization of international portfolios. This chapter also extends the results of Jacobs et al. (2014) to include more of the Ledoit and Wolf (2003a) strategies and implements short sales in the optimization solution. Jacobs et al. (2014) only implements the constant correlation strategy and restricts short sales in the minimum-variance optimization solution.

This study extends Levy and Levy (2014a) to report the relative performance of their model against other shrinkage and constraint strategies. Also, this chapter proposes the use of the cross-validation technique presented in DeMiguel et al. (2009a) to determine the alpha constraint used in the Levy and Levy (2014a) strategy. This extends Levy and Levy (2014a) by providing a method with which to dynamically set alpha and utilize the strategy ex-ante. Another contribution is the testing of the results for statistical significance versus the naive benchmarks which Levy and Levy (2014a) does not do.

Next, this chapter reports the optimization results using 34 market indices rather than the 4 naive regional indices in Jacobs et al. (2014). Using individual markets may improve the effectiveness of the estimation error methods in the optimization solution as the information these models are designed to retain, and subsequently use in creating diversification benefits, may be lost with the use of naive regional portfolios. This chapter also conducts optimization using 4 naive regional indices. This is done to provide robustness to the results and allows for the analyse of the effectiveness of the optimization strategies in reducing estimation error as the number of assets is decreased but information is lost with naive regional diversification.

This chapter also reports the results from the perspective of investors in 34 countries as well as the performance of each naive domestic market portfolio. Jacobs et al. (2014) only presents the performance of internationally optimized portfolios for European investors and does not report the local market performance. This paper reports the performance of the naive local portfolio for all investors so that a comparison of the benefits from optimization can be more robustly determined. Also, presenting the estimation error reduction strategy results from the perspective of investors in each country studied assists in determining whether the results are consistent across all countries, or if there are differences in the magnitude of the benefits between countries.

This study also contributes to the line of literature that reports that there are potential benefits from international diversification, which generally uses perfect ex-post information to form optimal portfolios in-sample from which the potential benefits of diversification are measured (Grubel, 1968; Levy and Sarnat, 1970; Lessard, 1973; De Roon et al., 2001; Li et al., 2003; Fletcher and Marshall, 2005; Driessen and Laeven, 2007; Chiou, 2008). This study extends these results to report the effectiveness of optimization strategies in capturing diversification benefits ex-ante.

The documented persistence of a home bias amongst investors (Tesar and Werner, 1995; Stulz, 2005) makes analysis of the diversification gains shrinkage and constraint strategies provide relative to the naive local portfolio and the naive global portfolio of interest. Reporting whether optimization strategies can provide benefits that are consistently better than the naive domestic portfolio and capture statistically positive efficiency gains over the naive global 1/M portfolio may contribute towards an understanding of the significance of the home bias puzzle. Identifying countries for which these strategies improve optimization performance may also lead to further research investigating the factors that contribute to their effectiveness.

The results presented in this chapter do not identify a strategy that consistently outperforms the naive local market portfolio for all investors or which provides statistically significant improvements beyond the naive global 1/M portfolio. The shrinkage strategies which allow heavy short sale allocations generally achieve the lowest volatility levels that are significantly different from the 1/M portfolio. The low volatility associated with heavier short positions can result in lower portfolio returns and lower return-to-risk (RR) ratios compared to the constrained optimization strategies. The Ledoit and Wolf (2008) bootstrap tests find that the optimization strategies provide improved RR ratios that are statistically different from the naive portfolio benchmarks for at most 3 of the 34 countries studied. The large short positions and turnover rates of some strategies will likely introduce additional costs that may further reduce the effectiveness of the strategies versus the naive benchmarks.

This chapter is divided into three more sections. Section 5.2 reviews the data and methods used in this study. Section 5.3 reports the results of this study. Section 5.4 concludes with a summary of the main findings.

5.2 Data and Methods

The monthly total return MSCI equity index data for 21 developed and 13 developing markets are used in this study. The index data for all 34 markets is collected in the local currency of each of the 34 countries. The sample period covers December 31, 1992 to December 31, 2012. The total return MSCI equity index data, which is adjusted for reinvested gross dividends, is

Table 5.1: Portfolio strategies

#	Description	Abbreviation
Panel A. Benchmark strategies		
1	Naive local portfolio	Local
2	Market capitalization weighted portfolio	1/M
2	Equally weighted portfolio	1/N
Panel B. MVP strategies		
3	Mean-variance with no constraints	MV
4	Mean-variance with market as one-factor	MV1F
5	Mean-variance with no shorts	MVNS
6	Mean-variance with no shorts and max. 2x weight	MVWC
7	Constant correlation shrinkage	LWCC
8	One-factor model shrinkage	LW1F
9	Identity matrix shrinkage	LWI
10	1-norm constraint with variance calibration	1NVC
11	One-factor-norm constraint with variance calibration	1FNVC
12	2-norm constraint with variance calibration	2NVC
13	Global variance-based constraint with variance calibration	GVBC

retrieved from Datastream.

Annual market capitalization data in U.S. dollars for 1993 to 2012 is from two sources: the World Bank and the World Federation of Exchanges. The World Bank data is available for 33 of the 34 markets from 1993 to 2012. The 1993 and 1994 data is incomplete for Ireland. This incomplete data is calculated using changes to the MSCI total return data to backward fill the missing capitalization values from the first available market capitalization data point. The Taiwanese market capitalization data is retrieved from the World Federation of Exchanges.

This study measures the performance of optimization strategies presented in the literature against the naive local and naive global portfolios. Optimization is performed at the beginning of each year using the preceding 120 months of data as an estimation window. The resulting static optimized minimum-variance portfolio (MVP) is used to calculate the monthly returns for the year. The portfolio performance is measured by the reduction in volatility and change in return-to-risk (RR) ratios compared to the benchmarks. Bootstrap testing methods presented in Ledoit and Wolf (2008) are used to measure the statistical significance of the difference in a portfolio's variance and RR ratios versus the domestic market portfolio and the 1/M benchmark. The level of portfolio turnover and size of short positions are also reported. The optimization strategies implemented in this study are listed in table 5.1. The subsections that follow provide descriptions of these strategies.

5.2.1 Naive benchmarks

The performance of the optimization strategies are compared to the naive domestic market portfolio and the naive market capitalization weighted (1/M) portfolio. The naive equally weighted (1/N) portfolio performance results are also reported. The weights for the 1/M portfolio are set at the start of each year from the market capitalization data. Then, the 1/M weights are updated each month to reflect the returns achieved in the portfolio by the individual markets. The 1/N portfolio weights are static throughout the entire sample period.

5.2.2 Mean-variance strategies

The classical mean-variance [MV] strategy of Markowitz (1952) is the first model to consider. The sample covariance matrix calculated from the sample data is used to determine the minimum-variance portfolio. Short sales are allowed and no weight constraints are placed on the optimization solution. The global market as a single factor in the classical mean-variance optimization solution will also be considered [MV1F]. Using a factor model will impose structure on the sample covariance matrix.

The Jagannathan and Ma (2003) strategy of no short sales [MVNS] will be an extension of the classical mean-variance strategy. Not using short sales may be a restriction investors enforce because of risk management and legal requirements (Li et al., 2003). An extension of Jagannathan and Ma (2003) that uses no shorts and constrains the maximum asset allocation to two times the global market weight will be considered [MVWC]. As with short sale restrictions, investors might be restricted by the levels of investment they can make into markets because of legal constraints or the difficulties and risks of investing in new, small capitalization markets (Chiou, 2008). It is also of interest to measure the effectiveness of easy to implement restrictions on market weights in controlling estimation error in comparison to the more complex shrinkage and constraint strategies.

5.2.3 Covariance matrix shrinkage

The shrinkage strategy presented in Ledoit and Wolf (2003b, 2004, 2003a) will be used with the three shrinkage targets presented in those papers: the single factor variance-covariance matrix (LW1F), the constant correlation matrix (LWCC) and the identity matrix (LWI). The shrinkage strategy seeks the optimally weighted average of the sample covariance matrix and the target covariance matrix that minimizes the mean squared error between this shrunk covariance matrix and the true covariance matrix of stock returns. The shrinkage estimator, α is determined using the quadratic loss function:

$$L(\alpha) = \|\alpha F + (1-\alpha) S - \Sigma\|^2 \tag{5.1}$$

where:

F = The target covariance matrix;

S = The sample covariance matrix;

Σ = The true covariance matrix; and

α = The shrinkage estimator.

Given that an infinite sample of returns is not obtainable, acceptable estimators to use given a sample size of T are presented in the papers, such that the quadratic function can be re-arranged and a reasonable estimate of α can be derived as:

$$\hat{\alpha} = \frac{(\hat{\pi} - \hat{\rho})}{\hat{\gamma}} \tag{5.2}$$

where:

$$\hat{\pi} = \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{\pi}_{ij}$$

$$\hat{\pi}_{ij} = \frac{1}{T} \sum_{t=1}^{T} \{(y_{it} - \bar{y}_i)(y_{jt} - \bar{y}_j) - s_{ij}\}^2$$

$$\hat{\rho} = \sum_{i=1}^{N} \hat{\pi}_{ii} + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{\hat{r}}{2} \left(\sqrt{\frac{s_{jj}}{s_{ii}}} \hat{\vartheta}_{ii,ij} + \sqrt{\frac{s_{ii}}{s_{jj}}} \hat{\vartheta}_{jj,ij} \right)$$

$$\hat{\vartheta}_{ii,ij} = \frac{1}{T}\sum_{t=1}^{T}\{(y_{it} - \bar{y_i})^2 - s_{ii}\}\{(y_{it} - \bar{y_i})(y_{jt} - \bar{y_j}) - s_{ij}\}$$

$$\hat{\vartheta}_{jj,ij} = \frac{1}{T}\sum_{t=1}^{T}\{(y_{jt} - \bar{y_j})^2 - s_{jj}\}\{(y_{it} - \bar{y_i})(y_{jt} - \bar{y_j}) - s_{ij}\}$$

$$\hat{\gamma} = \sum_{i=1}^{N}\sum_{j=1}^{N}(f_{ij} - s_{ij})^2$$

where:

y_{it} is the return of asset y at time t,

s_{ij} is the sample covariance matrix entry,

f_{ij} is the target covariance matrix entry, and

r̂ is the average correlation of the sample assets.

The shrinkage estimator, $\hat{\alpha}$, shrinks the sample covariance matrix towards the target. The larger the estimator, the heavier the weight towards the target structure. The value $\hat{\alpha}$ is a function of the 3 estimators in equation (5.2). Considered in isolation, changes to one estimator will increase or decrease the shrinkage factor which will result in shrinkage towards or away from the target matrix. $\hat{\pi}$ is an estimator of the sum of the asymptotic variances of the sample covariance entries. Higher volatility in the covariance between assets in the sample will result in a higher $\hat{\pi}$ value, which will increase $\hat{\alpha}$, which increases the shrinkage towards the target matrix. $\hat{\rho}$ is an estimator of the sum of asymptotic covariances of the sample covariance entries and the target covariance entries. Higher results will give a lower $\hat{\rho}$ value, which will decrease $\hat{\alpha}$, which decreases the shrinkage towards the target matrix. $\hat{\gamma}$

is an estimator of the misspecification of the shrinkage target. The larger the sum of squared differences between the sample covariance entries and the target covariance entries, the larger $\hat{\gamma}$ will be, reducing the value of $\hat{\alpha}$. The shrunk covariance matrix determined by the shrinkage estimator is used in the optimization solution.

DeMiguel et al. (2009b) does not test the effectiveness of the Ledoit and Wolf (2003b, 2004, 2003a) models. They argue that the Jagannathan and Ma (2003) no short-sales model addresses the covariance matrix shrinkage that these models perform. The results presented in Behr et al. (2013) show that the no short-sales strategy does not perform as well as the more theoretically refined Ledoit and Wolf (2003b, 2004, 2003a) strategies. This paper will test these strategies in an international context which will extend the results presented in the current literature.

5.2.4 Constraining portfolio norms

DeMiguel et al. (2009a) presents a framework with which constraints on the allocation weights, rather than the moments of asset returns, are the object of shrinkage meant to reduce estimation error and improve out-of-sample performance. The first case consists of the 1-norm constraint [1N*] on the vector of portfolio weights be smaller than a threshold δ. The second case is the A-norm of the portfolio-weight vector w be smaller than the threshold δ such that:

$$w^T A w \leq \delta \tag{5.3}$$

where A is the Identity matrix when performing the 2-norm constraint strategy [2N*] and A is the one-factor covariance matrix when applying the single-factor constraint strategy [1FN*].

DeMiguel et al. (2009a) presents two methods to determine the threshold δ. The first method uses a cross validation approach that seeks the threshold that minimizes the variance in optimized bootstrapped sample returns using a 10 year estimation window [1NVC, 1FNVC, 2NVC]. The cross validation method implemented in this study is performed as follows:

1. Start with a value for δ[1].

2. Given an estimation window of 120 months, for each t from 1 to 120, delete the t^{th} data point from the sample and create a sample covariance matrix with the remaining data.

3. Find the portfolio weights that minimize the estimated variance while meeting the norm constraint.

4. Measure the estimated return that this portfolio will provide using the t^{th} sample returns.

5. Measure the variance of all these sample returns.

6. Repeat the previous steps with a new, less constrained δ value until a threshold is found that minimizes the cross validated variance.

[1]This study starts each search with the δ value set at the strictest constraint level which is the no short sales constraint. Subsequent iterations in the cross validation method relax the lower bound constraint on short sales.

The second method seeks to maximize the MVP portfolio return using the returns from the last period in the sample window. The weights for this return maximizing MVP are used to set δ which is then used as the constraint to find the optimized minimum-variance portfolio. This method attempts to exploit the positive autocorrelation in portfolio returns. DeMiguel et al. (2009a) implements monthly optimization. This paper does not utilize this method because optimization is performed once a year which will arguably reduce the strength of the autocorrelation in portfolio returns between periods.

5.2.5 Differential variance-based constraints

Levy and Levy (2014a) present a variance-based constraint strategy to reduce estimation error that imposes more stringent weight constraints on assets with relatively higher standard deviations than the average standard deviation of all assets. This strategy is meant to reduce the potential estimation error associated with high volatility assets.

Levy and Levy (2014a) tests the strategy results against 8 common optimization methods such as the naive 1/N portfolio, classical mean-variance optimization with and without shorts and a Bayes model. The strategies are tested using 5, 20 and 100 stock portfolios consisting of the 100 largest U.S. firms for the period of 1992 to 2011. The strategies are also tested using the Fama-French industry portfolios for the period 1969 to 2011. The results are ranked by their Sharpe ratios and level of short positions. While the variance-based constraint method ranks highly, the Sharpe ratio is not much different than the naive portfolio or other common methods. The results are not tested with any statistical method. Nor are the strategies tested against other estimation error reduction strategies presented in the literature.

Levy and Levy (2014a) present two strategies: a variance-based constraint (VBC) strategy and a global variance-based constraint (GVBC) strategy. The VBC strategy imposes a weight constraint on each individual asset and the GVBC imposes a weight constraint on the sum of the assets. This chapter will only implement the GVBC. This strategy, as presented in Levy and Levy (2014a) imposes a single global positive constant constraint α (alpha) on the vector of portfolio weights to determine the portfolio weights of the MVP such that:

$$\sum_{i=1}^{N}(x_i - \frac{1}{N})^2 \frac{\sigma_i}{\bar{\sigma}} \leq \alpha \tag{5.4}$$

where, N is the number of assets, σ_i is the volatility of asset i, and $\bar{\sigma}$ is the average volatility of all assets. This strategy uses the 1/N portfolio as a benchmark from which the optimized portfolio is determined. This chapter implements the GVBC strategy without the 1/N benchmark so that:

$$\sum_{i=1}^{N}(x_i)^2 \frac{\sigma_i}{\bar{\sigma}} \leq \alpha \tag{5.5}$$

Without the 1/N benchmark, the GVBC is similar to the DeMiguel et al. (2009a) 2-norm strategy with an extension that adjusts the individual asset allocation based on its relative volatility compared to the other assets. Higher levels of volatility, in theory, may be a source of greater estimation error which the GVBC strategy seeks to reduce. Thus, this strategy should give heavier allocations to assets that will more reliably contribute to portfolio performance.

Levy and Levy (2014a) does not provide a method to determine α ex-ante. I propose that the minimum-variance search algorithm presented in DeMiguel et al. (2009a) be used to determine α. Using the same minimum-variance cross validation approach as the norm-constrained strategies will allow for an analysis of performance gains achieved with the GVBC versus the 2-norm strategy.

5.2.6 Measuring performance

This chapter uses a rolling horizon approach to measure the effectiveness of the optimization strategies. Portfolio weights for each strategy are determined at the beginning of each year. A rolling sample window of $\tau = 120$ months of index returns ending the preceding December is used as input for each optimization strategy to determine the MVP weights for the year from January. The first optimization occurs in January 2003 and uses the 10 year sample window of returns from January 1993 to December 2002. The out-of-sample monthly returns for the year are calculated using the static MVP weights determined by the optimization strategy. This process is repeated each January and results in $(T - \tau)$ monthly out-of-sample returns, where T equals the 240 months of sample return data from January 1993 to December 2012.

Four measures are used to evaluate the out-of-sample performance of the optimization strategies: the annual standard deviation of returns, the annual return-to-risk ratio, the average annual turnover rate and the average annual short position. These measures are calculated from the out-of-sample monthly returns and are used to analyse the effectiveness of the optimization strategies relative to the naive local portfolio and the naive 1/M portfolio. The average annual turnover rates and short positions provide a measure of the potentially non-negligible costs associated with these metrics such as transaction fees, taxes and management expenses.

The Ledoit and Wolf (2008) circular block bootstrap volatility and Sharpe testing methods are used to test the statistical significance of the reduced variance and increased Sharpe ratios that result from the optimization strategies compared to the naive 1/M portfolio. These methods test whether the difference in volatility or Sharpe ratios between the benchmark and the optimization strategy's monthly returns is different from zero. The bootstrapping technique employed by the tests are designed to control for the autocorrelation and volatility clustering that can occur with financial data. The bootstrap tests use 499 iterations with the block length set to 5. In this study, p-value results below 0.1 will be interpreted as statistically significant.

Turnover

The average annual turnover for each optimization strategy is measured. Each optimization strategy's monthly portfolio return is calculated using the static weights determined for the strategy at the beginning of the year. Monthly returns for each market will be different, and because of this, the market weights in the portfolio will change. These market positions must be adjusted at the end of each month to rebalance the portfolio back to the static allocation weights determined by the optimization strategy. Then, at the beginning of the next year, the portfolio weights must be adjusted to the new optimized weights for that year.

Turnover is calculated as the average percentage of the portfolio that will be traded each year for the $(T-\tau)/12$ years in the estimation period, where T is the number of months in the entire sample and τ is the 120 month estimation window used in the optimization strategy.

$$Turnover = \frac{1}{T-\tau-1} \times 12 \sum_{t=\tau}^{T-1} \|\hat{w}_{t+} - \hat{w}_{t+1}\| \tag{5.6}$$

Here, $\hat{w}_{t+}$ is the vector of portfolio weights before rebalancing at the end of period t and $\hat{w}_{t+1}$ is the sample estimate of optimal portfolio weights at time $t+1$.

Shorts

The average annual short position held by each optimization strategy is measured to assess the practical non-negligible costs that may be associated with the greater level of trading management needed to implement short positions in a portfolio. The size of the short positions held in the portfolio each year for the $(T-\tau)/12$ years in the estimation period is calculated, where T is the number of months in the entire sample and τ is the 120 month estimation window used in the optimization strategy.

$$Shorts = \frac{1}{T-\tau-1} \times 12 \sum_{t=\tau}^{T-1} \frac{\|\hat{w}_t\| - 1}{2} \tag{5.7}$$

Here, $\hat{w}_t$ is the portfolio weights at time t.

5.2.7 Regional indices

The optimization strategies are also run using the four regional indices of Europe, North America, the Pacific region and emerging markets. The regional indices are calculated from the market indices. The market capitalization weights of each market for each year from 1993 to 2012 are used to calculate the regional index monthly returns for each month in each year. This is done to add robustness to the results and to investigate whether the regional results reported in Jacobs et al. (2014) may have understated the potential benefits of international diversification. In theory, reducing the number of assets in the covariance solution can reduce estimation error, which may improve ex-ante performance. However, naively diversifying markets into regional indices may result in the loss of information that may be useful in mean-variance optimization.

5.3 Results

This section presents the performance results of the optimization strategies described in section 5.2 versus the benchmark portfolios. First, the annual volatility results and the RR ratios of the the naive benchmarks and the MVPs determined using the optimization strategies are reported. The results from the Ledoit and Wolf (2008) volatility and RR ratio bootstrap tests are also presented. Finally, the short positions and turnover rates are reported. Analysis of the results seeks to determine whether the estimation error reduction strategies consistently outperform both the naive local portfolio and the naive 1/M portfolio for investors in 34 countries. The subsection that follows reports whether the use of regional indices, rather than individual market indices, can improve the performance of the estimation error reduction strategies.

Table 5.2 presents the annual standard deviation results of the optimization strategies for the 2003 to 2012 annual rolling window investment period. Investors from almost all countries realize lower volatility with the 1/M portfolio versus their naive local market portfolio. The exceptions are Swiss, U.K. and U.S. investors. The 1/N portfolio has higher levels of volatility than the 1/M portfolio for all countries. The 1/N portfolio fails to provide lower volatility than the domestic portfolio for investors in Canada, New Zealand, Switzerland, the United Kingdom, the United States, Chile and Malaysia. If the size of the market is representative of the proportion of global investors in that country, then arguably the majority of investors do not achieve lower volatility than their domestic market portfolio from diversification into the naive internationally diversified benchmarks.

Panel C of table 5.2 reports the number of countries for which an optimization strategy provides a lower volatility level than the naive domestic benchmark. The optimization strategies generally achieve lower volatility levels than the domestic market with similar frequency as the 1/M portfolio. The classical mean-variance optimization strategy and the one-factor constraint strategy provide lower volatility levels for all markets except Brazil. Table 5.2 also presents the p-values for the test that the variance in the monthly returns of a portfolio strategy is different than the domestic market portfolio. The p-values are presented in parentheses and values below the 10% level are treated as significant. Panel D reports the number of countries for which an optimization strategy provides a lower volatility level than the naive domestic benchmark that is statistically significant. As previously noted, Swiss, U.K. and U.S. investors do not realize reduced portfolio volatility diversifying out of the domestic portfolio and into the 1/M portfolio. New Zealand, Chilean and Malaysian investors do not realize statistically significant volatility reduction benefits diversifying out of the domestic portfolio and into the 1/M portfolio. As a result, the 1/M portfolio provides volatility reducing benefits versus the domestic market portfolio that are statistically significant for 28 of the 34 markets measured. Only classical mean-variance optimization and the one-factor constraint strategy provide significant volatility reducing benefits for more countries than the naive 1/M benchmark.

Panel E of table 5.2 reports the number of countries for which each optimization strategy provides a lower volatility level than the naive 1/M benchmark. There is no strategy that provides lower volatility for all investors. The classical mean-variance optimization strategies provide varying levels of volatility. The MV strategy generally provides the lowest volatility within the group and the MVWC generally provides the highest levels of volatility. The use of short sales in the MV strategy seems beneficial in realizing lower volatility. Constraints on weight allocations seems to limit the potential for volatility reduction. Not all investors achieve lower volatility with all the mean-variance strategies. Investors in Argentina, Brazil, Indonesia, Korea, Mexico, and Turkey achieve lower volatility with the naive 1/M portfolio than any of the classical mean-variance strategies.

The LWCC, LW1F and LWI strategies provide lower volatility than the 1/M portfolio for all developed market investors and for investors in 6 of the

13 emerging markets. These strategies use unconstrained short sales and generally provide the lowest volatility levels of the various strategies. Referring to table 5.4 it is apparent that the shrinkage strategies have short positions that are greater than most other strategies. The norm constrained strategies of DeMiguel et al. (2009a) provide lower volatility than the 1/M portfolio less frequently and at higher levels than the Ledoit and Wolf (2003b, 2004, 2003a) shrinkage strategies. The GVBC strategy of Levy and Levy (2014a) provides lower volatility than the 1/M portfolio for the same countries as the 2NVC strategy. The GVBC strategy provides lower standard deviation results than the 2NVC strategy for all countries. The GVBC strategy is similar to the 2-norm constraint and variance calibration strategy but has an additional feature that reduces the assets in the optimized solutions which have relatively greater volatility than the other assets.

Panel F reports the number of countries for which an optimization strategy provides a lower volatility level than the naive 1/M benchmark that is statistically significant. The strategies that use short sales tend to have lower volatility and lower p-values for more countries than those strategies that constrain allocation weights or restrict short sales. In particular, the MV and LW strategies achieve lower volatility levels than the local and the 1/M portfolios for 28 and 27 markets respectively. The MV strategy results are significantly lower than both the local and the 1/M portfolios for 24 countries. The LW strategy results are significantly lower than both the local and the 1/M portfolios for as many as 21 countries. The volatility reduction can be relatively large compared to the other strategies. For example, in the case of the U.S., the local and the 1/M portfolios provide standard deviation re-

Table 5.2: Volatility results

Country	Local	1/M	1/N	MV	MV1F	MVNS	MVWC	LWCC	LW1F	LWI	NC1V	NC1FV	NC2V	GVBC
Panel A: Developed Markets														
Australia	0.1343	0.1114	0.1331	0.0766	0.1060	0.1025	0.1204	0.0829	0.0809	0.0862	0.1005	0.0861	0.1005	0.0997
	(1.000)	(0.042)	(0.842)	(0.002)	(0.028)	(0.022)	(0.262)	(0.002)	(0.004)	(0.002)	(0.026)	(0.002)	(0.032)	(0.022)
Austria	0.2587	0.1341	0.1638	0.1008	0.1365	0.1416	0.1682	0.1117	0.1066	0.1076	0.1331	0.1382	0.1316	0.1311
	(1.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.006)	(0.020)	(0.002)	(0.002)	(0.004)	(0.016)	(0.002)	(0.012)	(0.004)
Belgium	0.2076	0.1341	0.1638	0.1011	0.1372	0.1412	0.1682	0.1118	0.1068	0.1080	0.1338	0.1378	0.1329	0.1313
	(1.000)	(0.066)	(0.106)	(0.038)	(0.102)	(0.058)	(0.302)	(0.062)	(0.05)	(0.052)	(0.102)	(0.072)	(0.118)	(0.132)
Canada	0.1419	0.1199	0.1573	0.0834	0.1169	0.1141	0.1251	0.0899	0.0861	0.0891	0.1107	0.1059	0.1096	0.1078
	(1.000)	(0.094)	(0.060)	(0.036)	(0.246)	(0.136)	(0.274)	(0.030)	(0.034)	(0.038)	(0.162)	(0.058)	(0.156)	(0.130)
Denmark	0.1861	0.1339	0.1643	0.1027	0.1396	0.1423	0.1686	0.1137	0.1085	0.1098	0.1357	0.1389	0.1345	0.133
	(1.000)	(0.002)	(0.006)	(0.008)	(0.016)	(0.026)	(0.364)	(0.012)	(0.002)	(0.014)	(0.034)	(0.008)	(0.03)	(0.044)
Finland	0.2507	0.1341	0.1638	0.1022	0.1412	0.1416	0.1680	0.1137	0.1084	0.1100	0.1342	0.1382	0.1327	0.1311
	(1.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.008)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
France	0.1697	0.1341	0.1638	0.1019	0.1382	0.1413	0.1678	0.1125	0.1074	0.1086	0.1337	0.1379	0.1322	0.1306
	(1.000)	(0.002)	(0.666)	(0.002)	(0.020)	(0.092)	(0.918)	(0.004)	(0.004)	(0.006)	(0.032)	(0.064)	(0.036)	(0.032)
Germany	0.2019	0.1341	0.1639	0.1008	0.1368	0.1416	0.1683	0.1118	0.1068	0.1077	0.1348	0.1382	0.1334	0.1313
	(1.000)	(0.002)	(0.058)	(0.004)	(0.010)	(0.020)	(0.100)	(0.004)	(0.002)	(0.004)	(0.004)	(0.008)	(0.010)	(0.008)
Hong Kong	0.2177	0.1688	0.2100	0.0874	0.1223	0.1232	0.1391	0.0976	0.0918	0.0956	0.1229	0.1075	0.1229	0.1229
	(1.000)	(0.008)	(0.636)	(0.002)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Ireland	0.2250	0.1341	0.1638	0.1011	0.1412	0.1429	0.1685	0.1123	0.1075	0.1087	0.1327	0.1395	0.1315	0.1301
	(1.000)	(0.002)	(0.002)	(0.004)	(0.004)	(0.008)	(0.052)	(0.006)	(0.002)	(0.002)	(0.010)	(0.004)	(0.016)	(0.016)
Italy	0.1952	0.1341	0.1638	0.1007	0.1424	0.1416	0.1647	0.1127	0.1073	0.1084	0.1328	0.1382	0.1327	0.1318
	(1.000)	(0.002)	(0.066)	(0.002)	(0.008)	(0.024)	(0.08)	(0.002)	(0.002)	(0.002)	(0.002)	(0.006)	(0.002)	(0.002)
Japan	0.3348	0.2330	0.2373	0.1002	0.1351	0.1329	0.1864	0.1135	0.1054	0.1055	0.1329	0.1095	0.1329	0.1325

	(1.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Netherlands	0.1808	0.1341	0.1638	0.0989	0.1346	0.1378	0.1639	0.1088	0.1041	0.1053	0.1292	0.1343	0.1279	0.1267
	(1.000)	(0.002)	(0.078)	(0.002)	(0.006)	(0.006)	(0.33)	(0.004)	(0.002)	(0.002)	(0.016)	(0.004)	(0.014)	(0.01)
New Zealand	0.1403	0.1320	0.1556	0.0843	0.1169	0.1086	0.1304	0.0928	0.0896	0.0908	0.1054	0.099	0.1054	0.1033
	(1.000)	(0.484)	(0.212)	(0.002)	(0.118)	(0.036)	(0.560)	(0.008)	(0.006)	(0.004)	(0.022)	(0.014)	(0.020)	(0.026)
Norway	0.2380	0.1295	0.1579	0.1046	0.1429	0.1400	0.1592	0.1135	0.1101	0.1099	0.1364	0.1400	0.1360	0.1347
	(1.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.006)	(0.002)	(0.002)	(0.002)	(0.006)	(0.002)	(0.002)	(0.004)
Singapore	0.1962	0.1394	0.1780	0.0850	0.1174	0.1162	0.1326	0.0955	0.0902	0.0930	0.1162	0.0892	0.1162	0.1170
	(1.000)	(0.002)	(0.258)	(0.006)	(0.052)	(0.026)	(0.068)	(0.012)	(0.012)	(0.012)	(0.030)	(0.016)	(0.014)	(0.044)
Spain	0.2040	0.1341	0.1638	0.1082	0.1413	0.1448	0.1709	0.1175	0.1136	0.1142	0.1382	0.1417	0.1371	0.1354
	(1.000)	(0.014)	(0.07)	(0.004)	(0.004)	(0.014)	(0.066)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.006)	(0.006)
Sweden	0.1924	0.1181	0.1479	0.0955	0.1342	0.1311	0.1517	0.1071	0.1019	0.1028	0.1260	0.1287	0.1241	0.1240
	(1.000)	(0.002)	(0.006)	(0.004)	(0.004)	(0.004)	(0.048)	(0.002)	(0.004)	(0.004)	(0.002)	(0.004)	(0.008)	(0.004)
Switzerland	0.1351	0.1592	0.1886	0.1058	0.1471	0.1522	0.1799	0.1187	0.1121	0.1133	0.1453	0.1365	0.1453	0.1436
	(1.000)	(0.026)	(0.004)	(0.034)	(0.342)	(0.300)	(0.012)	(0.222)	(0.074)	(0.088)	(0.488)	(0.946)	(0.486)	(0.562)
U.K.	0.1388	0.1447	0.1768	0.0924	0.1264	0.1319	0.1562	0.1014	0.0969	0.1015	0.1281	0.1053	0.1281	0.1281
	(1.000)	(0.454)	(0.002)	(0.006)	(0.330)	(0.596)	(0.196)	(0.008)	(0.006)	(0.006)	(0.390)	(0.056)	(0.394)	(0.386)
U.S.	0.1483	0.1693	0.2105	0.0881	0.1227	0.1238	0.1395	0.0982	0.0925	0.0962	0.1238	0.1081	0.1238	0.1238
	(1.000)	(0.004)	(0.002)	(0.018)	(0.196)	(0.120)	(0.632)	(0.044)	(0.020)	(0.022)	(0.148)	(0.034)	(0.140)	(0.138)
Panel B: Emerging Markets														
Argentina	0.3597	0.1673	0.2052	0.1900	0.3273	0.3147	0.3598	0.3552	0.2025	0.242	0.3031	0.3205	0.3034	0.3036
	(1.000)	(0.002)	(0.002)	(0.002)	(0.590)	(0.658)	(0.998)	(0.944)	(0.050)	(0.392)	(0.622)	(0.696)	(0.628)	(0.624)
Brazil	0.2275	0.1372	0.1560	0.3992	0.4494	0.4858	0.5014	0.4452	0.4096	0.4412	0.4845	0.4406	0.4846	0.4842
	(1.000)	(0.002)	(0.002)	(0.060)	(0.006)	(0.016)	(0.008)	(0.008)	(0.016)	(0.018)	(0.004)	(0.020)	(0.014)	(0.024)
Chile	0.1606	0.1431	0.1701	0.0880	0.1260	0.1170	0.1343	0.0973	0.0930	0.0974	0.1170	0.0967	0.1168	0.1163
	(1.000)	(0.326)	(0.562)	(0.002)	(0.020)	(0.004)	(0.05)	(0.002)	(0.002)	(0.002)	(0.006)	(0.002)	(0.012)	(0.004)

Greece	0.3326	0.1341	0.1638	0.1006	0.1406	0.1415	0.1649	0.1115	0.1075	0.1076	0.1332	0.1381	0.1324	0.1326
	(1.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Indonesia	0.2622	0.1374	0.1699	0.1959	0.2725	0.2949	0.4179	0.2614	0.2114	0.2436	0.2943	0.1967	0.2940	0.2933
	(1.000)	(0.002)	(0.004)	(0.108)	(0.786)	(0.676)	(0.398)	(0.996)	(0.238)	(0.76)	(0.664)	(0.078)	(0.712)	(0.684)
Korea	0.2155	0.1225	0.1530	0.1280	0.1968	0.1710	0.2032	0.1528	0.1419	0.1430	0.1710	0.1467	0.1710	0.1710
	(1.000)	(0.002)	(0.012)	(0.312)	(0.712)	(0.526)	(0.814)	(0.444)	(0.436)	(0.406)	(0.496)	(0.306)	(0.494)	(0.528)
Malaysia	0.1451	0.1425	0.1815	0.1277	0.1984	0.2054	0.2423	0.1869	0.1431	0.1606	0.2054	0.1395	0.2054	0.2045
	(1.000)	(0.804)	(0.054)	(0.276)	(0.040)	(0.168)	(0.266)	(0.224)	(0.892)	(0.482)	(0.180)	(0.732)	(0.184)	(0.190)
Mexico	0.1771	0.1209	0.1573	0.1298	0.2096	0.1779	0.2177	0.1545	0.1469	0.1474	0.1779	0.1536	0.1779	0.1766
	(1.000)	(0.004)	(0.030)	(0.244)	(0.554)	(0.980)	(0.586)	(0.644)	(0.510)	(0.520)	(0.986)	(0.472)	(0.982)	(0.996)
Philipines	0.2143	0.1508	0.1887	0.1127	0.1505	0.1526	0.1700	0.1234	0.1202	0.1202	0.1517	0.1174	0.1501	0.1500
	(1.000)	(0.006)	(0.214)	(0.002)	(0.018)	(0.010)	(0.054)	(0.002)	(0.004)	(0.002)	(0.016)	(0.002)	(0.008)	(0.014)
Portugal	0.1747	0.1341	0.1638	0.1053	0.1387	0.1430	0.1697	0.1149	0.1105	0.1115	0.1360	0.1397	0.1350	0.1334
	(1.000)	(0.002)	(0.406)	(0.004)	(0.016)	(0.094)	(0.774)	(0.004)	(0.006)	(0.014)	(0.044)	(0.048)	(0.036)	(0.026)
Taiwan	0.2191	0.1460	0.1851	0.0897	0.1235	0.1177	0.1329	0.0982	0.0933	0.0954	0.1167	0.0922	0.1167	0.1166
	(1.000)	(0.002)	(0.128)	(0.002)	(0.002)	(0.002)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Thailand	0.2476	0.1529	0.1891	0.1158	0.1541	0.1556	0.1760	0.1307	0.1248	0.1262	0.1556	0.1158	0.1556	0.1556
	(1.000)	(0.002)	(0.040)	(0.004)	(0.070)	(0.070)	(0.142)	(1.000)	(0.006)	(0.006)	(0.068)	(0.002)	(0.080)	(0.068)
Turkey	0.3208	0.1356	0.1548	0.1749	0.2330	0.2197	0.2367	0.1857	0.1841	0.1929	0.2156	0.1757	0.2156	0.2156
	(1.000)	(0.002)	(0.002)	(0.004)	(0.152)	(0.026)	(0.124)	(0.024)	(0.016)	(0.014)	(0.024)	(1.000)	(0.026)	(0.022)
Panel C: Number of countries for which the optimization strategy provides a lower sd than Local.														
	-	31	27	33	29	29	27	32	33	32	29	32	29	30
Panel D: Number of countries for which the optimization strategy provides a statistically lower sd than Local.														
	-	28	19	29	21	24	14	27	29	28	23	28	23	23
Panel E: Number of countries for which the optimization strategy provides a lower sd than 1/M.														
	3	-	0	28	12	11	7	27	27	27	18	14	21	23

Panel F: Number of countries for which the optimization strategy provides a statistically lower sd than 1/M.														
	2	-	0	24	3	4	0	17	21	21	4	10	5	4

This table reports the annual standard deviation of returns for the naive benchmarks and each optimization strategy for the out-of-sample period from January 2003 to December 2012 for investors in each of the 34 countries measured. The p-value results from the Ledoit and Wolf (2008) test that the variance in the monthly returns of a portfolio strategy is different than the domestic market portfolio are presented in parentheses. A block size of 5 and 499 iterations is used in the test. The Ledoit and Wolf (2008) test results that the variance in the monthly returns of a portfolio strategy is different than the 1/M portfolio are not presented, but a summary of the results are reported in Panel F.

Table 5.3: Return-to-risk results

Country	Local	1/M	1/N	MV	MV1F	MVNS	MVWC	LWCC	LW1F	LWI	NC1V	NC1FV	NC2V	GVBC
Panel A: Developed Markets														
Australia	0.7632	0.3892	0.5870	1.0595	0.8908	0.9509	0.8038	1.0092	0.9654	0.9951	0.9487	1.1288	0.9486	0.9521
	(1.000)	(0.212)	(0.352)	(0.664)	(0.850)	(0.728)	(0.940)	(0.674)	(0.744)	(0.700)	(0.748)	(0.552)	(0.776)	(0.734)
Austria	0.3264	0.6495	0.7657	0.5793	0.6578	0.6504	0.5781	0.5445	0.5382	0.5508	0.6757	0.5189	0.6708	0.6496
	(1.000)	(0.232)	(0.024)	(0.674)	(0.614)	(0.614)	(0.686)	(0.76)	(0.718)	(0.726)	(0.602)	(0.732)	(0.558)	(0.62)
Belgium	0.3510	0.6494	0.7656	0.5949	0.6599	0.6576	0.5824	0.5541	0.5483	0.5622	0.6820	0.5268	0.6689	0.6774
	(1.000)	(0.280)	(0.042)	(0.680)	(0.684)	(0.648)	(0.716)	(0.758)	(0.762)	(0.748)	(0.638)	(0.782)	(0.592)	(0.614)
Canada	0.7269	0.5019	0.6256	0.7154	0.9230	0.7950	0.7861	0.8243	0.7993	0.8242	0.8241	0.7715	0.7854	0.8065
	(1.000)	(0.388)	(0.588)	(0.992)	(0.738)	(0.872)	(0.922)	(0.880)	(0.882)	(0.856)	(0.876)	(0.934)	(0.928)	(0.892)
Denmark	0.8525	0.6580	0.7671	0.5686	0.6300	0.6397	0.5579	0.5256	0.5195	0.5316	0.6551	0.5088	0.6603	0.6679
	(1.000)	(0.404)	(0.636)	(0.680)	(0.744)	(0.720)	(0.614)	(0.622)	(0.604)	(0.578)	(0.742)	(0.570)	(0.760)	(0.782)
Finland	0.1622	0.6496	0.7658	0.5958	0.6007	0.6060	0.5396	0.5170	0.5228	0.5331	0.5951	0.4746	0.5939	0.6058
	(1.000)	(0.102)	(0.058)	(0.398)	(0.444)	(0.366)	(0.484)	(0.486)	(0.464)	(0.472)	(0.428)	(0.556)	(0.436)	(0.424)
France	0.4266	0.6496	0.7656	0.5506	0.6358	0.6388	0.5689	0.5213	0.5120	0.5258	0.6642	0.5067	0.6590	0.6779
	(1.000)	(0.246)	(0.188)	(0.796)	(0.728)	(0.684)	(0.776)	(0.844)	(0.852)	(0.850)	(0.692)	(0.912)	(0.730)	(0.678)
Germany	0.6046	0.6493	0.7655	0.5802	0.6559	0.6506	0.5777	0.5440	0.5375	0.5521	0.6662	0.5202	0.6613	0.6477
	(1.000)	(0.766)	(0.448)	(0.984)	(0.920)	(0.936)	(0.982)	(0.912)	(0.926)	(0.946)	(0.888)	(0.886)	(0.896)	(0.954)
Hong Kong	0.7446	0.6965	0.7675	0.3314	0.7003	0.5993	0.5625	0.4854	0.4680	0.4595	0.6123	0.5246	0.6122	0.6123
	(1.000)	(0.890)	(0.922)	(0.512)	(0.972)	(0.840)	(0.796)	(0.666)	(0.702)	(0.638)	(0.826)	(0.710)	(0.846)	(0.832)
Ireland	-0.1033	0.6494	0.7657	0.6357	0.6758	0.6654	0.6122	0.5852	0.5762	0.5898	0.7011	0.5323	0.7001	0.6838
	(1.000)	(0.004)	(0.010)	(0.184)	(0.230)	(0.152)	(0.222)	(0.234)	(0.246)	(0.240)	(0.154)	(0.230)	(0.144)	(0.168)
Italy	0.1401	0.6494	0.7656	0.6879	0.6890	0.7112	0.6414	0.6152	0.6178	0.6301	0.7353	0.5798	0.7421	0.7350
	(1.000)	(0.030)	(0.036)	(0.30)	(0.38)	(0.290)	(0.364)	(0.380)	(0.374)	(0.314)	(0.314)	(0.438)	(0.286)	(0.278)
Japan	0.5933	0.5217	0.5621	0.2162	0.3573	0.3079	0.6099	0.1693	0.2173	0.2689	0.3079	0.3457	0.3079	0.3037

	(1.000)	(0.716)	(0.910)	(0.568)	(0.726)	(0.570)	(0.928)	(0.480)	(0.532)	(0.590)	(0.626)	(0.720)	(0.604)	(0.566)
Netherlands	0.4165	0.6494	0.7659	0.6028	0.6653	0.6644	0.5866	0.5609	0.5535	0.5663	0.6849	0.5290	0.6799	0.6927
	(1.000)	(0.130)	(0.098)	(0.738)	(0.672)	(0.660)	(0.774)	(0.802)	(0.778)	(0.730)	(0.664)	(0.838)	(0.638)	(0.590)
New Zealand	0.5750	0.4716	0.6329	0.7969	0.6142	0.7100	0.5330	0.6629	0.6496	0.7243	0.7086	0.8181	0.7086	0.7051
	(1.000)	(0.726)	(0.828)	(0.672)	(0.940)	(0.788)	(0.948)	(0.870)	(0.856)	(0.786)	(0.798)	(0.638)	(0.760)	(0.816)
Norway	0.7336	0.6749	0.793	0.4387	0.5479	0.5699	0.5042	0.438	0.4238	0.466	0.5813	0.5699	0.5854	0.6011
	(1.000)	(0.922)	(0.746)	(0.638)	(0.790)	(0.800)	(0.730)	(0.662)	(0.640)	(0.690)	(0.838)	(0.826)	(0.836)	(0.846)
Singapore	0.7186	0.5369	0.6451	0.5070	0.6080	0.5984	0.5842	0.4999	0.5210	0.5329	0.5984	0.5167	0.5984	0.6141
	(1.000)	(0.39)	(0.666)	(0.704)	(0.910)	(0.846)	(0.860)	(0.752)	(0.818)	(0.758)	(0.844)	(0.784)	(0.888)	(0.886)
Spain	0.4766	0.6494	0.7656	0.7062	0.7661	0.7702	0.6935	0.6705	0.6591	0.6732	0.7354	0.6409	0.7280	0.7154
	(1.000)	(0.578)	(0.374)	(0.638)	(0.598)	(0.624)	(0.704)	(0.704)	(0.750)	(0.706)	(0.682)	(0.758)	(0.646)	(0.688)
Sweden	0.7687	0.6605	0.7805	0.8242	0.7910	0.7894	0.6774	0.7438	0.7554	0.7549	0.7754	0.7212	0.7888	0.7888
	(1.000)	(0.640)	(0.910)	(0.874)	(0.942)	(0.958)	(0.932)	(0.994)	(0.988)	(0.996)	(0.950)	(0.960)	(0.946)	(0.962)
Switzerland	0.5546	0.4483	0.5835	0.4022	0.4578	0.4786	0.4267	0.3406	0.3530	0.3667	0.4656	0.4075	0.4656	0.4716
	(1.000)	(0.558)	(0.944)	(0.774)	(0.884)	(0.914)	(0.826)	(0.756)	(0.758)	(0.740)	(0.850)	(0.774)	(0.868)	(0.886)
U.K.	0.6501	0.7793	0.8646	0.4426	0.5885	0.5133	0.4579	0.4352	0.4174	0.4541	0.5319	0.5111	0.5319	0.5319
	(1.000)	(0.248)	(0.166)	(0.746)	(0.918)	(0.830)	(0.690)	(0.702)	(0.698)	(0.738)	(0.824)	(0.820)	(0.812)	(0.828)
U.S.	0.5715	0.6991	0.7695	0.3251	0.6936	0.5961	0.5561	0.4799	0.4620	0.4549	0.5961	0.5207	0.5961	0.5961
	(1.000)	(0.344)	(0.404)	(0.712)	(0.864)	(0.964)	(0.982)	(0.900)	(0.894)	(0.866)	(0.970)	(0.952)	(0.948)	(0.962)
Panel B: Emerging Markets														
Argentina	0.6122	0.9607	0.9980	0.5836	0.8178	0.7809	0.7389	0.7312	0.8023	0.8341	0.8080	0.7130	0.7999	0.7984
	(1.000)	(0.328)	(0.246)	(0.992)	(0.730)	(0.756)	(0.836)	(0.854)	(0.722)	(0.654)	(0.732)	(0.852)	(0.718)	(0.726)
Brazil	0.9901	0.3896	0.5689	2.3048	2.6163	2.7014	2.5543	2.5031	2.5115	2.5207	2.6808	2.4691	2.6783	2.6675
	(1.000)	(0.238)	(0.226)	(0.148)	(0.070)	(0.092)	(0.128)	(0.092)	(0.054)	(0.094)	(0.090)	(0.100)	(0.072)	(0.108)
Chile	1.1262	0.4829	0.625	1.1198	1.3000	1.1484	1.0633	1.2783	1.2397	1.2169	1.1484	1.0451	1.1770	1.1920
	(1.000)	(0.158)	(0.140)	(0.938)	(0.712)	(0.930)	(0.948)	(0.758)	(0.780)	(0.800)	(0.916)	(0.926)	(0.898)	(0.880)

Greece	-0.1146	0.6495	0.7657	0.8134	0.8282	0.8590	0.7655	0.7527	0.7595	0.7548	0.8820	0.7193	0.8771	0.8844
	(1.000)	(0.050)	(0.024)	(0.060)	(0.082)	(0.114)	(0.090)	(0.062)	(0.070)	(0.082)	(0.074)	(0.158)	(0.096)	(0.090)
Indonesia	1.3154	0.8801	0.9480	0.9118	0.1366	0.7973	0.7380	0.6906	0.8273	0.8316	0.8130	0.7962	0.8088	0.8060
	(1.000)	(0.436)	(0.356)	(0.478)	(0.096)	(0.372)	(0.264)	(0.300)	(0.394)	(0.366)	(0.342)	(0.416)	(0.380)	(0.346)
Korea	0.7810	0.8110	0.9013	0.6398	0.5827	0.6856	0.6238	0.6026	0.6153	0.7192	0.6856	0.3169	0.6856	0.6856
	(1.000)	(0.820)	(0.648)	(0.826)	(0.680)	(0.872)	(0.816)	(0.740)	(0.742)	(0.926)	(0.86)	(0.370)	(0.870)	(0.854)
Malaysia	0.9707	0.6297	0.7185	0.5654	0.5556	0.5464	0.5636	0.5881	0.5203	0.5930	0.5464	0.6302	0.5464	0.5343
	(1.000)	(0.372)	(0.440)	(0.508)	(0.516)	(0.448)	(0.486)	(0.528)	(0.534)	(0.568)	(0.498)	(0.600)	(0.492)	(0.462)
Mexico	1.3666	1.1119	1.1123	1.1732	1.0593	1.0927	1.0716	1.1126	1.1668	1.1979	1.1060	1.0404	1.1060	1.1075
	(1.000)	(0.440)	(0.326)	(0.780)	(0.608)	(0.666)	(0.558)	(0.662)	(0.762)	(0.812)	(0.666)	(0.586)	(0.688)	(0.670)
Philipines	1.0708	0.5706	0.6716	0.8779	0.9173	0.9561	0.9817	0.9130	0.9372	0.9468	0.9376	1.2576	0.8901	0.9045
	(1.000)	(0.280)	(0.350)	(0.772)	(0.828)	(0.892)	(0.926)	(0.828)	(0.864)	(0.858)	(0.882)	(0.642)	(0.792)	(0.816)
Portugal	0.2559	0.6495	0.7656	0.6720	0.7357	0.7335	0.6543	0.6356	0.6276	0.6442	0.7020	0.6026	0.6955	0.6895
	(1.000)	(0.242)	(0.074)	(0.436)	(0.410)	(0.462)	(0.442)	(0.450)	(0.502)	(0.510)	(0.406)	(0.542)	(0.418)	(0.456)
Taiwan	0.4656	0.6459	0.7316	0.7740	0.8650	0.8542	0.7770	0.8007	0.8307	0.8472	0.8450	0.8792	0.8451	0.8303
	(1.000)	(0.492)	(0.382)	(0.480)	(0.440)	(0.434)	(0.524)	(0.518)	(0.458)	(0.440)	(0.422)	(0.416)	(0.450)	(0.452)
Thailand	0.9253	0.5086	0.6236	0.7652	0.6059	0.7576	0.7693	0.6890	0.7229	0.7731	0.7576	0.7652	0.7576	0.7576
	(1.000)	(0.146)	(0.340)	(0.812)	(0.580)	(0.812)	(0.848)	(0.708)	(0.746)	(0.822)	(0.790)	(0.800)	(0.796)	(0.812)
Turkey	0.9438	0.8870	1.0192	4.2506	3.7049	3.8303	3.5186	4.1792	4.1828	4.0817	3.8790	4.3344	3.8790	3.8790
	(1.000)	(0.972)	(0.754)	(0.002)	(0.002)	(0.002)	(0.006)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Panel C: Number of countries for which the optimization strategy provides a higher RR than Local.														
	-	16	22	16	21	21	17	18	18	18	21	18	21	21
Panel D: Number of countries for which the optimization strategy provides a statistically higher RR ratio than Local.														
	-	3	8	2	3	2	2	3	3	3	3	1	3	2
Panel E: Number of countries for which the optimization strategy provides a higher RR ratio than 1/M.														
	18	-	34	16	22	21	15	13	13	13	22	12	23	22

Panel F: Number of countries for which the optimization strategy provides a statistically higher RR ratio than 1/M.														
	0	-	0	2	2	2	2	2	2	2	2	2	2	2

This table reports the annual return-to-risk (RR) ratios for the naive benchmarks and each optimization strategy for the out-of-sample period from January 2003 to December 2012 for investors in each of the 34 countries measured. The p-value results from the Ledoit and Wolf (2008) test that the RR ratio of the monthly returns of a portfolio strategy is different than the domestic market portfolio are presented in parentheses. A block size of 5 and 499 iterations is used in the test. The Ledoit and Wolf (2008) test results that the RR ratio of the monthly returns of a portfolio strategy is different than the 1/M portfolio are not presented, but a summary of the results are reported in Panel F.

Table 5.4: Average annual shorts

Country	Local	1/M	1/N	MV	MV1F	MVNS	MVWC	LWCC	LW1F	LWI	NC1V	NC1FV	NC2V	GVBC
Panel A: Developed Markets														
Australia	0.0000	0.0000	0.0000	1.0436	0.5267	0.0000	0.0000	0.6149	0.6892	0.5615	0.0240	0.8335	0.0238	0.0404
Austria	0.0000	0.0000	0.0000	1.4625	0.7318	0.0000	0.0000	0.8326	0.9906	0.9062	0.1684	0.2674	0.2148	0.2379
Belgium	0.0000	0.0000	0.0000	1.4586	0.7297	0.0000	0.0000	0.8342	0.9878	0.9057	0.1304	0.2680	0.1972	0.2113
Canada	0.0000	0.0000	0.0000	1.1782	0.6146	0.0000	0.0000	0.7128	0.8323	0.7112	0.0727	0.3632	0.0948	0.1514
Denmark	0.0000	0.0000	0.0000	1.4649	0.7311	0.0000	0.0000	0.8307	0.9892	0.9015	0.1505	0.2654	0.2190	0.2380
Finland	0.0000	0.0000	0.0000	1.4727	0.7276	0.0000	0.0000	0.8264	0.9854	0.9018	0.0814	0.2678	0.1303	0.1906
France	0.0000	0.0000	0.0000	1.4449	0.7312	0.0000	0.0000	0.8313	0.9845	0.9006	0.1070	0.2676	0.1532	0.1889
Germany	0.0000	0.0000	0.0000	1.4638	0.7314	0.0000	0.0000	0.8333	0.9909	0.9066	0.1513	0.2676	0.1977	0.2366
Hong Kong	0.0000	0.0000	0.0000	1.5755	0.7459	0.0000	0.0000	0.8052	1.0430	0.8679	0.0219	0.6152	0.0217	0.0219
Ireland	0.0000	0.0000	0.0000	1.4322	0.7289	0.0000	0.0000	0.8155	0.9786	0.8855	0.0904	0.2677	0.1402	0.1840
Italy	0.0000	0.0000	0.0000	1.4718	0.7270	0.0000	0.0000	0.8213	0.9916	0.8974	0.0758	0.2675	0.1007	0.1235
Japan	0.0000	0.0000	0.0000	1.3911	0.8560	0.0000	0.0000	0.8168	0.9995	0.8599	0.0000	0.9497	0.0000	0.0208
Netherlands	0.0000	0.0000	0.0000	1.4484	0.7241	0.0000	0.0000	0.8224	0.9805	0.8919	0.1426	0.2678	0.1887	0.2256
New Zealand	0.0000	0.0000	0.0000	1.1096	0.6045	0.0000	0.0000	0.6540	0.7369	0.6273	0.0250	0.6621	0.0250	0.0461
Norway	0.0000	0.0000	0.0000	1.4793	0.7331	0.0000	0.0000	0.8111	0.9302	0.8620	0.0659	0.0000	0.0897	0.1360
Singapore	0.0000	0.0000	0.0000	1.2884	0.6301	0.0000	0.0000	0.7066	0.8466	0.7144	0.0000	1.0284	0.0000	0.0252
Spain	0.0000	0.0000	0.0000	1.4556	0.7285	0.0000	0.0000	0.8214	0.9830	0.8946	0.1694	0.2676	0.1942	0.2417
Sweden	0.0000	0.0000	0.0000	1.3529	0.6420	0.0000	0.0000	0.7172	0.8659	0.7812	0.0648	0.2400	0.0808	0.1010

Switzerland	0.0000	0.0000	0.0000	1.6139	0.8385	0.0000	0.0000	0.9300	1.1270	1.0234	0.0675	0.4516	0.0675	0.1529
U.K.	0.0000	0.0000	0.0000	1.4717	0.7572	0.0000	0.0000	0.8434	1.0237	0.9057	0.0500	0.8673	0.0498	0.0500
U.S.	0.0000	0.0000	0.0000	1.5820	0.7469	0.0000	0.0000	0.8071	1.0409	0.8677	0.0000	0.6158	0.0000	0.0000
Panel B: Emerging Markets														
Argentina	0.0000	0.0000	0.0000	3.3922	1.7206	0.0000	0.0000	1.2419	2.1070	0.8162	0.1530	1.7320	0.2038	0.2274
Brazil	0.0000	0.0000	0.0000	2.4869	1.3181	0.0000	0.0000	1.0217	1.4711	0.8928	0.0893	2.0910	0.0972	0.1389
Chile	0.0000	0.0000	0.0000	1.2112	0.5865	0.0000	0.0000	0.5972	0.7599	0.6094	0.0000	0.6171	0.0257	0.0856
Greece	0.0000	0.0000	0.0000	1.4655	0.7284	0.0000	0.0000	0.8432	0.9816	0.9054	0.1128	0.2658	0.1602	0.1813
Indonesia	0.0000	0.0000	0.0000	2.4795	1.1243	0.0000	0.0000	0.9767	1.4672	0.7713	0.0740	1.5103	0.0720	0.1225
Korea	0.0000	0.0000	0.0000	1.5127	0.7680	0.0000	0.0000	0.7594	0.9250	0.6663	0.0000	1.0908	0.0000	0.0000
Malaysia	0.0000	0.0000	0.0000	1.8810	0.9675	0.0000	0.0000	0.7962	1.1833	0.7295	0.0000	1.7123	0.0000	0.0261
Mexico	0.0000	0.0000	0.0000	1.5224	0.6813	0.0000	0.0000	0.8089	0.9654	0.7919	0.0250	0.5457	0.0248	0.0491
Philipines	0.0000	0.0000	0.0000	1.4369	0.7202	0.0000	0.0000	0.8244	0.9277	0.8213	0.0217	1.0240	0.0663	0.0852
Portugal	0.0000	0.0000	0.0000	1.4422	0.7270	0.0000	0.0000	0.8298	0.9802	0.8970	0.1507	0.2676	0.2405	0.2642
Taiwan	0.0000	0.0000	0.0000	1.4094	0.6386	0.0000	0.0000	0.7207	0.9012	0.7270	0.0786	0.9916	0.0788	0.1075
Thailand	0.0000	0.0000	0.0000	1.4172	0.7287	0.0000	0.0000	0.7915	0.9370	0.7663	0.0000	1.4172	0.0000	0.0000
Turkey	0.0000	0.0000	0.0000	1.8386	1.0085	0.0000	0.0000	0.9279	1.0476	0.8198	0.0452	1.2663	0.0452	0.0452

This table reports the average annual short position held in each portfolio.

Table 5.5: Average annual turnover

Country	Local	1/M	1/N	MV	MV1F	MVNS	MVWC	LWCC	LW1F	LWI	NC1V	NC1FV	NC2V	GVBC
Panel A: Developed Markets														
Australia	0.000	0.0081	0.3945	0.2881	0.1954	0.1242	0.0383	0.1696	0.1986	0.1995	0.1242	0.2438	0.1242	0.1242
Austria	0.000	0.0081	0.3944	0.3896	0.2484	0.1501	0.0488	0.2127	0.2492	0.2429	0.1501	0.1501	0.1501	0.1765
Belgium	0.000	0.0081	0.3945	0.3926	0.2461	0.1490	0.0488	0.2134	0.2493	0.2438	0.1490	0.1490	0.1490	0.1490
Canada	0.000	0.0081	0.3945	0.3253	0.2280	0.1254	0.0429	0.1683	0.2117	0.1976	0.1254	0.2735	0.1649	0.1649
Denmark	0.000	0.0082	0.3945	0.3978	0.2483	0.1507	0.0429	0.2145	0.2517	0.2470	0.1507	0.1507	0.1507	0.1507
Finland	0.000	0.0081	0.3944	0.4421	0.2426	0.1444	0.0488	0.2264	0.2745	0.2647	0.1735	0.1444	0.1731	0.1735
France	0.000	0.0081	0.3943	0.3884	0.2463	0.1523	0.0488	0.2100	0.2465	0.2416	0.1523	0.1523	0.1523	0.1523
Germany	0.000	0.0081	0.3944	0.3896	0.2482	0.1482	0.0488	0.2127	0.2492	0.2431	0.1482	0.1482	0.1482	0.1763
Hong Kong	0.000	0.0081	0.3944	0.3905	0.2892	0.1338	0.0402	0.2153	0.2553	0.2344	0.1338	0.3565	0.1338	0.1338
Ireland	0.000	0.0081	0.3942	0.4234	0.2444	0.1655	0.0479	0.2230	0.2675	0.2593	0.1655	0.1655	0.1655	0.1961
Italy	0.000	0.0081	0.3944	0.4143	0.2415	0.1581	0.0471	0.2004	0.2525	0.2452	0.1581	0.1581	0.1581	0.1581
Japan	0.000	0.0110	0.3944	0.4323	0.3049	0.1295	0.0805	0.2278	0.2809	0.2666	0.1295	0.4249	0.1295	0.1295
Netherlands	0.000	0.0081	0.3946	0.3942	0.2454	0.1550	0.0488	0.2095	0.2499	0.2424	0.1550	0.1550	0.1550	0.1550
New Zealand	0.000	0.0081	0.3943	0.3808	0.2184	0.1406	0.0452	0.1955	0.2231	0.2292	0.1406	0.3505	0.1406	0.1406
Norway	0.000	0.0081	0.3943	0.3953	0.2394	0.1090	0.0365	0.1798	0.2254	0.2352	0.1090	0.1090	0.1090	0.1090
Singapore	0.000	0.0081	0.3945	0.3476	0.2367	0.1383	0.0313	0.1914	0.2129	0.2031	0.1383	0.3353	0.1383	0.1383
Spain	0.000	0.0081	0.3944	0.3934	0.2504	0.1627	0.0492	0.2053	0.2448	0.2337	0.2372	0.1627	0.2372	0.2598
Sweden	0.000	0.0081	0.3944	0.4038	0.2202	0.1157	0.0379	0.1788	0.2311	0.2316	0.1472	0.1157	0.1469	0.1472

Switzerland	0.000	0.0081	0.3945	0.4745	0.2731	0.1309	0.0549	0.2457	0.2869	0.2858	0.1309	0.4178	0.1309	0.1606
U.K.	0.000	0.0081	0.3945	0.3760	0.2610	0.1246	0.0360	0.1868	0.2086	0.2289	0.1246	0.3222	0.1246	0.1246
U.S.	0.000	0.0081	0.3943	0.3939	0.2895	0.1337	0.0407	0.2154	0.2526	0.2333	0.1337	0.3573	0.1337	0.1337
Panel B: Emerging Markets														
Argentina	0.000	0.0081	0.3943	0.7308	0.4387	0.1422	0.0442	0.2684	0.3651	0.1761	0.2663	0.1422	0.2444	0.2444
Brazil	0.000	0.0081	0.3944	0.8570	0.5457	0.1252	0.0565	0.3920	0.5319	0.4285	0.1252	0.3992	0.1724	0.1724
Chile	0.000	0.0081	0.3944	0.3711	0.2694	0.1310	0.0424	0.2072	0.2321	0.2287	0.1310	0.3450	0.1310	0.1310
Greece	0.000	0.0081	0.3944	0.3729	0.2421	0.1363	0.0488	0.1949	0.2295	0.2247	0.1363	0.1363	0.1363	0.1363
Indonesia	0.000	0.0081	0.3943	0.7402	0.3213	0.1666	0.0363	0.3503	0.5019	0.3298	0.1666	0.1666	0.2099	0.2165
Korea	0.000	0.0081	0.3943	0.4256	0.2595	0.1693	0.0399	0.2494	0.2923	0.2265	0.1693	0.4828	0.1693	0.1693
Malaysia	0.000	0.0081	0.3945	0.4530	0.2623	0.0957	0.0296	0.2170	0.2803	0.2071	0.0957	0.4530	0.0957	0.0957
Mexico	0.000	0.0081	0.3943	0.4144	0.2677	0.1669	0.0575	0.2206	0.2903	0.2146	0.1669	0.4571	0.1669	0.1847
Philipines	0.000	0.0081	0.3943	0.3903	0.2448	0.1359	0.0359	0.2152	0.2457	0.2338	0.1786	0.3458	0.2134	0.2137
Portugal	0.000	0.0081	0.3943	0.3835	0.2476	0.1591	0.0488	0.2114	0.2452	0.2371	0.2297	0.1591	0.2294	0.2434
Taiwan	0.000	0.0081	0.3944	0.3338	0.2541	0.1373	0.0446	0.1917	0.2213	0.2014	0.1677	0.2989	0.1677	0.1915
Thailand	0.000	0.0081	0.3944	0.3574	0.2384	0.1329	0.0397	0.2324	0.2497	0.2361	0.1329	0.3574	0.1329	0.1329
Turkey	0.000	0.0081	0.3943	0.6008	0.3866	0.1172	0.0720	0.2898	0.3322	0.2904	0.1785	0.5209	0.1785	0.1785

This table reports the average annual turnover in each portfolio.

sults of 0.1483 and 0.1693 respectively. The MV strategy volatility is reported to be 0.0881 and the LW strategies provide volatility levels between 0.0925 and 0.0982. These volatility results are lower than the .1238 results of the MVNS, NC2V and GVBC strategies. These results suggest that a risk sensitive investor can achieve some of the largest volatility reduction benefits from diversification into the active MV and LW optimization strategies. This study does not consider trading costs or liquidity risks associated with short positions. It is not apparent from these results whether there exist factors that can be used ex-ante to predict the effectiveness of a strategy in improving an investor's portfolio performance versus a naive benchmark.

Table 5.3 presents the RR ratios of the benchmarks and the optimization strategies. Investors in 11 of the 21 developed markets realize larger RR ratios with the 1/M portfolio versus the naive local market portfolio. Investors in 5 of the 13 emerging markets achieve improvements to the RR ratio from diversifying out of the naive local market portfolio and into the 1/M portfolio. Investors in 16 developed markets and 6 emerging markets achieve improvements to the RR ratio from diversifying out of the local market portfolio and into the 1/N portfolio. The 1/N portfolio RR ratio outperforms the 1/M portfolio for all countries.

Panel C of table 5.3 reports the number of countries for which an optimization strategy provides a higher return-to-risk (RR) ratio than the naive domestic portfolio. Panel E reports the number of countries for which an optimization strategy provides a higher RR ratio than the 1/M portfolio. No optimization strategy outperforms the local or the 1/M portfolio for all investors. Strategies that restrict short sales and constrain allocation weights outperform both the local and the 1/M portfolio with greater frequency than the shrinkage strategies. The shrinkage strategies outperform the local and 1/M portfolios for 4 developed markets: Australia, Canada, New Zealand and Spain. The norm constrained strategies and the GVBC strategy provide greater RR ratios than the local and the 1/M portfolio for investors in as many as 11 developed markets.

Table 5.3 also presents the p-values for the test that the RR ratio of the monthly returns of a portfolio strategy is different than the domestic market portfolio. P-values below the 10% level are treated as significant. Panel D reports the number of countries for which an optimization strategy provides a higher RR ratio than the naive domestic portfolio that is statistically significant. The 1/M portfolio provides positive RR gains over the local market portfolio that are statistically significant for only 3 countries: Ireland, Italy, and Greece. The results find that the optimization strategies do not provide significant RR ratio improvements versus the local portfolio with greater frequency than the 1/M portfolio. The naive 1/N portfolio produces RR ratios that are statistically significant and greater than the local market portfolio more often than the other strategies. The 1/N portfolio RR p-values are significantly different than the local portfolio for 8 countries.

Panel F reports the number of countries for which an optimization strategy provides a higher RR ratio than the 1/M portfolio that is statistically significant. The naive 1/N portfolio fails to provide results that are statistically different from the 1/M portfolio for any country. There is no strategy that provides statistically significant positive RR improvements over the 1/M portfolio for all investors. There is no significant difference in the results between strategies that have large short sale positions and those that restrict short sales. The only statistically significant results occur for investors in Brazil and Turkey.

These results suggest that while the estimation error reduction strategies can reduce portfolio volatility to a statistically significant level versus the naive benchmarks, the resulting RR ratio improvement is unlikely to be both positive and statistically different. These results are in line with DeMiguel et al. (2009b) which reports that optimization strategies can fail to provide significant diversification benefits beyond a naively diversified portfolio of stocks.

Tables 5.4 and 5.5 present the short sale positions and turnover rates of the optimization strategies. Some optimization strategies have relatively large short positions and high turnover rates which will likely introduce additional fees and costs that may further reduce their effectiveness versus the naive benchmarks. For example, turnover may incur more frequent capital gains taxes on the sale of investments which may reduce the strategy's realized returns versus the passive local portfolio and the naive 1/M portfolio. And large short sale positions will likely incur greater administrative costs than portfolios that restrict short sales.

5.3.1 Regional optimization

This subsection describes the results of optimization using the four regional indices of Europe, North America, the Pacific region and emerging markets. This is done to add robustness to the results and to investigate whether the regional results reported in Jacobs et al. (2014) may have understated the potential benefits of international diversification available from the use of estimation error reduction strategies. Due to space considerations only an analysis of the results are presented.

In theory, reducing the number of assets in the covariance solution can reduce estimation error, which may improve ex-ante performance. However, naively diversifying markets into regional indices may result in the loss of information that may be useful in mean-variance optimization. A comparison of the standard deviation of returns between the results of market optimization and regional optimization finds that using individual market indices results in lower volatility for all optimization strategies. A comparison of the RR ratios finds that using individual market indices rather than regional indices almost always results in larger values. As a result, optimization using the individual market indices outperforms the 1/M portfolio RR ratio more often than when the regional indices are used. For example, the 2-norm constrained strategy outperforms the 1/M portfolio for 21 countries when using market indices and only outperforms for 13 countries when using regional indices. The constant correlation shrinkage strategy outperforms the 1/M portfolio for 13 countries when using market indices and outperforms for 12 countries when using regional indices.

As expected from the weaker RR ratio results, a comparison of the RR ratio p-values find that no statistically significant improvements to performance occurs when using regional indices rather than market indices. Using regional indices generally results in lower short levels. Turnover rates are only slightly lower than with the market indices. These results suggest that using individual market indices rather than naive regional indices may improve optimization performance. However, these improvements in performance do not result in statistically different results when compared to the naive portfolio benchmarks.

5.4 Conclusion

This chapter investigates whether estimation error reduction strategies recently presented in the literature consistently outperform the naive domestic market portfolio and the naive 1/M portfolio for investors in 34 countries. While the optimization strategies generally achieve lower volatility than the local market for investors in developed markets, no strategy can be identified that achieves greater RR ratios than the local portfolio or the 1/M portfolio for all investors. Strategies that provide lower volatility and larger RR ratios than the naive benchmarks are not consistently found to be statistically different than the benchmarks. While, in theory, international diversification can provide efficiency gains over an investor's local portfolio, the empirical evidence presented in this chapter suggests that naive diversification and active portfolio optimization can fail to achieve positive RR improvements beyond the naive domestic portfolio with statistical certainty.

Future research measuring the effectiveness of the estimation error reduction strategies in improving optimized maximum return-to-risk portfolio (MRRP) efficiency out-of-sample will extend the results presented in this study. This chapter neither measures the components of estimation error nor identifies factors that can be used to predict which strategies may be effective in improving portfolio performance versus a naive benchmark. It seems logical to this author that it is the difference in currency volatility between countries that accounts for the difference in optimized portfolio performance for investors in different countries. Further research into the significance of currency volatility as a component of estimation error in the optimization of internationally diversified portfolios may be useful in developing new strategies that can improve on the results presented in this study.

Chapter 6

Measuring the potential effect of taxes and weight constraints on the home bias in New Zealand PIEs

Abstract

This chapter measures the potential effect of taxes and allocation weight constraints on the efficiency of an equity home bias for New Zealand investors indirectly invested in equities held in a portfolio investment entity (PIE). Historical monthly index data for 34 markets denominated in New Zealand dollars from 1993 to 2014 is used with an in-sample data-based mean-variance optimization approach to measure the benefits of international diversification. Naive diversification into the untaxed global market capitalization weighted

(1/M) portfolio is shown to provide marginal return-to-risk (RR) benefits versus the New Zealand market portfolio during the 1993-2014 investment period. Relaxing the weight constraints on the markets held in the 1/M portfolio increases the potential benefits from diversification. Taxes are reported to reduce these benefits and increase the size of the home bias. Both weight constraints on overseas market allocations and taxes are shown to reduce the RR gains from diversification to statistically insignificant levels compared to the 100% New Zealand weighted domestic market portfolio.

6.1 Introduction

For New Zealand tax residents there exist unequal tax frictions between dividends and capital gains earned in New Zealand versus those realized overseas. A New Zealand investor receives dividend imputation credits for New Zealand company dividends which effectively makes these dividends tax free. Capital gains on New Zealand shares held by a New Zealand investor are not taxed[1]. Dividends paid by shares held overseas are often taxed by foreign governments. Additional New Zealand taxes are also applied to foreign investment fund (FIF)[2] assets exceeding $50,000 each income tax year. The unequal taxation of dividends and capital gains earned overseas can reduce the realized after-tax returns on international investment and can be expected to increase the efficiency of an equity home bias. This chapter investigates the effect of taxes on the potential benefits from international diversification on

[1]New Zealand does not have a capital gains tax. Therefore, there is no tax on the disposal of shares in New Zealand unless the shares are held as part of a profit making activity (i.e., revenue account property), or are purchased with the intention of resale.

[2]Foreign companies and foreign unit trusts are FIFs.

investments held indirectly in a portfolio investment entity (PIE).

Investigating the effect of taxes on the potential benefits from international diversification is of practical value to investors in New Zealand. The recent introduction of the New Zealand KiwiSaver savings initiative, designed to encourage New Zealanders to save for retirement, has increased the breadth of investors in the country that are concerned with optimizing their equity investment portfolio. The KiwiSaver program began in 2007 and these accounts incur taxes. PIEs are commonly used as an investment vehicle in the Kiwisaver retirement accounts. As of June, 2015, New Zealand has a resident population of roughly 4.5 million and there are 2,530,919 active KiwiSaver accounts[3]. As of March, 2015, there are roughly $28 billion in total assets under management in KiwiSaver accounts[4]. Over $8.8 billion is held in New Zealand equities and units in trusts[5]. Nearly $9.1 billion is held in overseas equities and units in trusts[6]. The market capitalization of the New Zealand stock exchange main board of equities is approximately $100 billion. This represents less than .20 of one percent of the global market capitalization in US dollars. Assets under management in KiwiSaver accounts are predicted to exceed $70 billion by the year 2020 (Heuser et al., 2015). Research measuring the effects of tax on optimal portfolio performance can be of value to retail investors selecting an indirect equity investment, managers of investment funds who are agents for investors, and pension funds interested in the aggregate portfolio performance of their clients.

[3]source: http://www.rbnz.govt.nz/statistics/tables/t43/ (accessed December 5, 2015).
[4]Ibid.
[5]Ibid.
[6]Ibid.

This is the only study that this author is aware of that investigates the potential effect of taxes on the benefits of international diversification for New Zealand investors holding indirect equity investments in PIEs. This study uses historical monthly equity index data for 34 markets denominated in New Zealand dollars from 1993 to 2014 with an in-sample data-based mean-variance optimization approach to measure the benefits of international diversification. Portfolios are optimized with various levels of weight constraints on overseas market allocations and with the different tax rates that can be applied to PIE unit holders. The significance of the benefits achieved by the optimized portfolios versus the New Zealand market portfolio are measured using the Ledoit and Wolf (2008) bootstrap testing methods. These tests are designed to address the non-normality of returns and fat-tail events that occur with historical financial data. Using a bootstrap technique, regressions are performed on paired data points of a given block size between the monthly returns of two portfolios to provide a p-value measuring the significance of the hypothesis that the difference between the two portfolios is zero.

The results find that static portfolios optimized without weight constraints, no short sales and no taxes provide potential gains from diversification for New Zealand investors using historical data from the 1993 to 2014 investment period. Weight constraints reduce the size of allocations that can be made to small markets that provide diversification benefits. Taxes reduce the returns achieved by overseas markets. Both taxation and allocation constraints on overseas investments held in a PIE reduce the size and the statistical significance of the potential diversification benefits and increase the size and efficiency of an equity home bias.

This chapter is divided into four more sections. Section 6.2 reviews the literature related to New Zealand KiwiSaver investments and the documented effect of taxes on the equity home bias. Section 6.3 outlines the dividend and capital gains tax regime for equity investments held in PIEs and used in this study. Section 6.4 describes the data and methods used in this chapter to investigate the benefits of international diversification. Section 6.5 reports the results of this investigation. Section 6.6 concludes with a summary of the main findings and suggestions for future study.

6.2 Literature Review

KiwiSaver is a recently introduced retirement scheme and the literature investigating fund performance is limited. Frijns and Tourani-Rad (2015) report that the risk-adjusted performance of KiwiSaver funds concentrated in domestic or international equity markets generally under-perform their benchmarks from September 2007 to April 2013. Heuser et al. (2015) present similar results, with under-performance more pronounced and prevalent amongst international funds. Bauer et al. (2006) reports that 143 New Zealand non-KiwiSaver domestic and international equity mutual funds from 1990 to 2003 generally under-perform their benchmarks. Funds that have positive alpha are not found to be statistically different from zero.

MacDonald et al. (2014) employ stochastic simulations to examine asset allocations that can improve the probabilities of adequate retirement balances for KiwiSaver investors. The results suggest that the legislated default funds are overly conservative and that investors may achieve better results with heavier allocations to equities. The paper reports that hypothetical funds heavily weighted in global equities have the strongest performance results. Taxes on KiwiSaver earnings and transaction fees are ignored in the study.

Taxes reduce investment returns and this affects portfolio optimization solutions and performance. In many countries, such as the United States, capital gains are taxed when assets are sold. Investors have an incentive to realize capital losses and defer capital gains (Constatinides, 1984) in order to minimize these taxes. As a result, capital gains taxes can create a capitalization effect that reduces demand and a lock-in effect that can reduce supply (Dai et al., 2008). Trading and holding investments for purely tax reasons can be sub-optimal over time if an investor is seeking benefits associated with diversification. As a result of such taxes, optimal portfolio allocations can be different between investors because of the different trade frictions incurred by investors. In the case of New Zealand investors, the annual taxation of international gains and the inability to carry forward losses would likely eliminate these trading incentives.

Taxes are reported to influence portfolio selection amongst asset managers. Chan et al. (2005) examines how mutual funds in 26 different countries allocate assets locally and internationally. The paper finds that a domestic bias is influenced by market development and familiarity variables. The foreign bias is effected by economic development, capital controls and withholding tax variables. The paper reports that the New Zealand funds used in the study in aggregate allocate 74.93% of assets to New Zealand during 1999 to 2000. The New Zealand market was .07% of world market capitalization at the time. The paper notes that New Zealand funds have a foreign bias towards its nearest geographical neighbour Australia and allocate 14.4% of equity assets in that country. The Australian market represents 1.2% of the world market in the paper. Australian superannuation funds have a home bias that may be partially explained by factors such as anchoring formed from legacy investment, industry peers (Warren, 2010) and overseas taxes (Mishra and Ratti, 2013). And Christoffersen et al. (2005) report that Canadian fund managers consider the tax preferences of investors when setting investment policy.

This chapter addresses gaps in the literature concerning the effect of taxes on the potential benefits from international diversification for New Zealand investors indirectly holding equity portfolios in PIEs. The taxation of overseas equity investments is reported to reduce the statistical significance of the diversification benefits derived from international investment and increases the efficiency of heavier allocations to New Zealand equities. The findings contribute to previous literature that has identified tax as an explanatory factor in the documented home bias that exists with international investment funds in New Zealand (Chan et al., 2005). The results presented here find that the tax effect is more pronounced when optimizing a portfolio to maximize the RR performance than when minimizing portfolio variance. Furthermore, the empirical results do not support the conclusion presented in MacDonald et al. (2014) that heavier allocations to international equities is likely to improve KiwiSaver investor retirement outcomes. Rather, this chapter reports that the combination of allocation weight constraints on overseas markets and the unequal taxation rates applied to international equity investment returns makes it unlikely that internationally diversified KiwiSaver portfolios can provide statistically significant positive return-to-risk improvements versus the untaxed domestic New Zealand market portfolio.

6.3 Tax

This section provides a brief overview of the taxes considered in this study that are applied to equities held in a portfolio investment entity (PIE). PIEs can be managed funds such as unit trusts[7]. Taxes to be paid by PIE unit holders are calculated by the PIE for each investor based on the taxable income for the investment period and the prescribed investor rate (PIR) for each investor. The four prescribed investor rates of 0%, 10.5%, 17.5% and 28% are used to measure the potential effect of taxes on the equity home bias in this study.

6.3.1 Foreign investment fund (FIF)

FIF rules were introduced from April 1, 2007. A FIF includes foreign companies and unit trusts. Australian shares listed on an approved index such as the ASX All Ordinaries are exempt from being an attributing interest in a FIF[8]. A New Zealand investor that holds less than 10% of a FIF is considered to have a portfolio position. Positions greater than $50,000 held at the beginning of each tax year must be declared.

PIEs use the fair dividend rate (FDR) method to determine the income

[7]Unit trusts are open-ended collective investments constituted under a trust deed and are similar to mutual funds.

[8]This study uses MSCI market indices to measure portfolio performance. For the purposes of this study Australian equities are treated as exempt from being an attributing interest in a FIF and only the dividends are treated as income to be taxed.

to be taxed from the FIF[9]. The FDR of 5% is multiplied by the value of assets held at the beginning of the tax year. This amount is treated as income to be taxed. Dividends received overseas are considered to be part of this amount. Taxes paid on overseas dividends are given credit to reduce the income tax on the FDR. If there is no New Zealand income tax payable on the FIF investment, the credits for foreign tax paid can not be claimed. Unused credits can't be used to reduce tax payable on other income such as Australian dividend income[10]. The FDR method ignores positions bought and sold in the tax period. These are treated separately as quick sale adjustments which are then added to the FDR amounts[11].

PIEs are generally held by many different unit holders and units in the PIE can be purchased and sold by different investors each day. For this reason PIEs generally calculate a daily closing value for existing units. The FDR is pro-rated by the number of trading days in the period being measured compared to the number of days in the tax year. The value of the PIE at the beginning of the day is multiplied by the pro-rated FDR, and this amount is

[9] New Zealand Income Tax Act 2007, section EX 53.

[10] Tax credits can be used to offset the tax payable on the FDR income associated with that attributing interest. As explained in IR 461 (Inland Revenue (2016). A guide to foreign investment funds and the fair dividend rate.): "Such foreign tax credits can only be used to reduce the income tax payable on your FIF income. If there is no New Zealand income tax payable on your FIF investment, no claim can be made for the foreign tax paid on any dividends received from the FIF. You can't use foreign tax credits to get a refund or reduce tax payable on other income. This includes other foreign income with a different nature or source, eg, dividends from companies with the Australian exemption and credits attached to United Kingdom dividends." (p.25)

[11] This study uses static portfolios. Portfolio asset weights are adjusted each month because of differences in asset returns. Quick sale adjustments are not considered in this study because all portfolio adjustments would occur on the first trading day of each month. As explained in Elliffe (2015), section 3.2.10(2): "If a fund does daily valuations, then no quick sale adjustment is required as all changes in the value of the portfolio will be picked up in the FDR formula."

taxed at the client's PIR to determine the tax owed for the trading day[12].

6.3.2 Imputation credits

Dividends paid by New Zealand companies to New Zealand tax residents receive imputation credits that effectively reduce the income tax owed on the dividends to zero[13]. The regime was introduced April 1, 1988. It is designed to avoid the double taxation of company profits. Because companies pay taxes on profits, imputation credits protect dividends paid to shareholders from being taxed again as income.

6.3.3 Taxation of foreign dividends

Dividends paid from shares held overseas generally incur non-resident withholding taxes in the source country. These dividends may also incur taxes to be paid in New Zealand. Double Taxation Agreements (DTA) provide relief from double taxation. These agreements outline the rate at which sources of income from investments are taxed in the country they are paid, and whether the country the shareholder is resident can also tax them. Thirty of the thirty-four countries studied in this paper have DTAs with New Zealand. These DTA countries tax dividends at a 15% rate. Argentina, Brazil, Greece and Portugal do not have DTAs and dividends are taxed at different rates. Table 6.1 identifies the countries that have DTAs with New Zealand and reports the tax rates applied to dividends paid in the 34 countries.

[12]For a more detailed explanation refer to Appendix A or section 3.2.10 in Elliffe (2015).

[13]In practice not all dividends receive full imputation credits. For the purposes of this study all New Zealand dividends are treated as fully imputed and are not taxed.

6.3.4 Portfolio investment entity (PIE)

From October 2007 unit trusts that meet the definition of a PIE are able to elect into tax rules under which they are not taxable on capital gains on shares in New Zealand and Australian companies listed on an approved index. Funds that do not meet the definition of a PIE are taxed on all earnings[14]. Portfolio investment entities pay tax on investment income for each investor based on that investor's PIR of 0%, 10.5%, 17.5% or 28%. Only certain trustees are able to choose a rate. All other investors have only one rate that they qualify for based on their income and tax residency status. Income in the PIE can be taxed at a lower PIR than the investor's marginal tax rate. For example, the 28% PIR is lower than the 33% marginal tax rate of investors with income in excess of $70,000. The 17.5% PIR is for investors with income between $48,000 and $70,000 who have a marginal income tax rate of 30%.

Capital gains from New Zealand shares are not taxed. Dividends from New Zealand listed companies generally receive dividend imputation credits for distributions from taxed earnings paid by the firm which effectively make dividends tax free. Capital gains from Australian shares that meet listing requirements are not taxed. Withholding taxes are not deducted in Australia on dividends paid by these firms to a PIE[15]. The dividends are treated as income by the PIE and taxes are deducted based on the client's PIR. For

[14] All KiwiSaver default schemes are PIEs.

[15] As explained in Elliffe (2015), section 2.6.5(c): "Most Australian companies pay their dividends and attach Australian franking credits. When a fully franked dividend is paid, the Australian company does not have to deduct Australian NRWT [non-resident withholding tax]. These Australian franking credits are not creditable under the New Zealand provisions." This study treats all dividends paid in Australia as fully franked.

equity held in other countries, the PIE determines the tax paid following the FDR rules.

The PIE rules address earlier differences in tax treatment between investors invested directly in New Zealand shares and those invested in shares via a New Zealand collective investment vehicles (CIV). An investor holding New Zealand shares directly was not typically taxed on capital gains. Under the old rules, an equivalent investment in a CIV was taxed on any realised New Zealand share gains. This was done because a CIV was considered a business trading in shares in order to make an income. The new PIE tax rules make indirect investments held in a PIE taxable in a similar way as directly held investments.

6.4 Data and Methods

The monthly MSCI price and total return data for 34 international markets priced in New Zealand dollars is used to investigate the effect of taxes on dividends and capital gains on the potential benefits of international diversification for New Zealand investors. The price data represents the capital gains realized by the markets. The difference between the price and the total return monthly returns represent the dividend portion of gains paid by these markets. The sample period covers December 31, 1992 to December 31, 2014. The data is collected from Datastream.

Annual market capitalization data in U.S. dollars for 1993 to 2012 is from two sources: the World Bank and the World Federation of Exchanges. The World Bank data is available for 33 of the 34 markets from 1993 to 2012.

The 1993 and 1994 data is incomplete for Ireland. This incomplete data is calculated using changes to the MSCI total return data to backward fill the missing capitalization values from the first available market capitalization data point. The Taiwanese market capitalization data is retrieved from the World Federation of Exchanges.

Table 6.1 reports market characteristics. The second column presents the market capitalization of each market used in this study in U.S. dollars as a percent of all 34 markets combined at the end of 1993. The columns that follow report the geometric annual returns, the standard deviation of returns and the RR ratios of these markets as measured in the domestic currency and the New Zealand dollar for the 1993 to 2014 holding period. The gains derived from capital gains and dividends in New Zealand dollars are also presented. Also, which countries have double taxation agreements with New Zealand and the rate of withholding taxes on dividends paid in those countries are presented.

A comparison of the market returns realized in each local currency and the returns realized in the New Zealand currency presented in table 6.1 finds that the New Zealand dollar has appreciated against all of the foreign currencies over the period studied. As a result, the RR ratios of these markets are lower in the New Zealand currency versus the local currency. A comparison of the RR values shows that only the six markets of Australia, Canada, Denmark, Sweden, Switzerland, and the United States provide a higher RR ratio than New Zealand in New Zealand dollar terms for the 1993-2014 period.

Figure 6.1 presents the distribution of the difference in annual returns between the untaxed naive global portfolio and the local New Zealand mar-

Table 6.1: Market characteristics from 1993 to 2014.

	1993	Local Currency			New Zealand Dollar							
Country(i)	w_i	r	sd	RR	r	sd	RR	cap. gain	div.	div./total	DTA	Rate
Panel A: Developed Markets												
Australia	1.51	0.0977	0.132	0.74	0.0856	0.157	0.54	0.0438	0.0418	0.49	yes	0.15
Austria	0.21	0.0269	0.227	0.12	0.0076	0.214	0.04	-0.0145	0.0221	2.90	yes	0.15
Belgium	0.58	0.0827	0.192	0.43	0.0621	0.189	0.33	0.0263	0.0358	0.58	yes	0.15
Canada	2.41	0.1000	0.155	0.65	0.0840	0.178	0.47	0.0601	0.0239	0.28	yes	0.15
Denmark	0.31	0.1316	0.185	0.71	0.1112	0.187	0.59	0.0929	0.0183	0.16	yes	0.15
Finland	0.17	0.1422	0.311	0.46	0.1240	0.304	0.41	0.0922	0.0318	0.26	yes	0.15
France	3.37	0.0751	0.184	0.41	0.0558	0.174	0.32	0.0289	0.0269	0.48	yes	0.15
Germany	3.43	0.0880	0.212	0.41	0.0676	0.202	0.33	0.0423	0.0253	0.37	yes	0.15
H.K.	2.85	0.0929	0.257	0.36	0.0723	0.230	0.31	0.0381	0.0342	0.47	yes	0.15
Ireland	0.14	0.0451	0.215	0.21	0.0228	0.208	0.11	-0.0012	0.0240	1.05	yes	0.15
Italy	1.01	0.0637	0.217	0.29	0.0398	0.223	0.18	0.0065	0.0332	0.84	yes	0.15
Japan	22.18	0.0185	0.186	0.10	0.0011	0.176	0.01	-0.0118	0.0129	11.33	yes	0.15
Netherlands	1.34	0.0929	0.186	0.50	0.0725	0.178	0.41	0.0405	0.0321	0.44	yes	0.15
N.Z.	0.19	0.0743	0.163	0.46	0.0743	0.163	0.46	0.0205	0.0538	0.72	yes	0.15
Norway	0.20	0.1030	0.224	0.46	0.0783	0.232	0.34	0.0452	0.0330	0.42	yes	0.15
Singapore	0.98	0.0639	0.227	0.28	0.0540	0.217	0.25	0.0272	0.0268	0.50	yes	0.15
Spain	0.88	0.1179	0.215	0.55	0.0879	0.215	0.41	0.0491	0.0388	0.44	yes	0.15

Sweden	0.79	0.1358	0.223	0.61	0.1092	0.229	0.48	0.0815	0.0277	0.25	yes	0.15
Switzerland	2.01	0.0932	0.155	0.60	0.0916	0.145	0.63	0.0698	0.0218	0.24	yes	0.15
U.K.	8.52	0.0734	0.138	0.53	0.0546	0.138	0.40	0.0201	0.0345	0.63	yes	0.15
U.S.	37.98	0.0952	0.149	0.64	0.0746	0.147	0.51	0.0537	0.0209	0.28	yes	0.15
Panel B: Emerging Markets												
Argentina	0.33	0.1335	0.388	0.34	0.0096	0.390	0.02	-0.0149	0.0244	2.56	no	0.00
Brazil	0.74	0.5165	0.408	1.27	0.1139	0.356	0.32	0.0726	0.0413	0.36	no	0.00
Chile	0.33	0.1038	0.193	0.54	0.0605	0.222	0.27	0.0317	0.0288	0.48	yes	0.15
Greece	0.09	0.0321	0.318	0.10	0.0003	0.311	0.00	-0.0276	0.0279	97.66	no	0.10
Indonesia	0.24	0.1614	0.334	0.48	0.0507	0.406	0.12	0.0240	0.0267	0.53	yes	0.15
Korea	1.03	0.0895	0.293	0.31	0.0529	0.330	0.16	0.0363	0.0167	0.31	yes	0.15
Malaysia	1.63	0.0747	0.256	0.29	0.0407	0.291	0.14	0.0161	0.0247	0.61	yes	0.00
Mexico	1.49	0.1711	0.230	0.74	0.0711	0.279	0.25	0.0520	0.0190	0.27	yes	0.15
Philipines	0.30	0.0809	0.268	0.30	0.0303	0.283	0.11	0.0099	0.0204	0.67	yes	0.15
Portugal	0.09	0.0488	0.203	0.24	0.0235	0.208	0.11	-0.0109	0.0344	1.47	no	0.20
Taiwan	1.43	0.0686	0.270	0.25	0.0381	0.270	0.14	0.0141	0.0241	0.63	yes	0.15
Thailand	0.97	0.0521	0.352	0.15	0.0201	0.346	0.06	-0.0088	0.0289	1.44	yes	0.15
Turkey	0.28	0.4460	0.454	0.98	0.1007	0.498	0.20	0.0721	0.0285	0.28	yes	0.15

This table reports the percentage of market capitalization for each market as the percent of market capitalization of all 34 markets combined in U.S. dollars at the end of the year 1993. The geometric annual returns (r), standard deviation of returns (sd), and the return-to-risk ratios (RR) of each market in the local currency and in New Zealand dollars is also presented. The annual capital gains (cap. gain), dividends (div.) and the percentage of gains from dividends as a percent of the total gains of these markets in New Zealand dollars (div./total) is reported. Whether a country has a double taxation agreement (DTA) with New Zealand is reported. The withholding tax rate applied to dividends paid to non-residents of each country is also reported.

ket portfolio for all possible monthly holding periods from January 1993 to December 2012.

The annual return of the domestic market portfolio is subtracted from the annual return of the untaxed naive global portfolio for each holding period. The returns from the naive global portfolio are calculated using the market capitalization weights for each year against the returns the markets provided each month. From each month, starting at January 1993, the returns for the domestic market portfolio and the naive global portfolio are calculated for each possible monthly holding period to the end of 2012. Starting from January 1993 there are 240 measurable one month holding periods and a single measurable 240 month holding period. The maximum and minimum difference in outcomes, along with the 75%, 50% and 25% quantiles, are shown in figure 6.1. The graph starts from the 60 month holding period to reduce the scale of the graph that occurs using shorter holding periods.

Figure 6.1 shows that the naive global portfolio does not always outperform the local market portfolio. For almost all holding periods presented the naive global portfolio more often outperforms the domestic portfolio. The naive global portfolio always outperformed the local portfolio for investment periods exceeding 15 years. These results suggest that for the sample period used, diversification into the untaxed naive global portfolio can provide potential excess returns beyond the local portfolio, but that the size of these gains are not consistent across time.

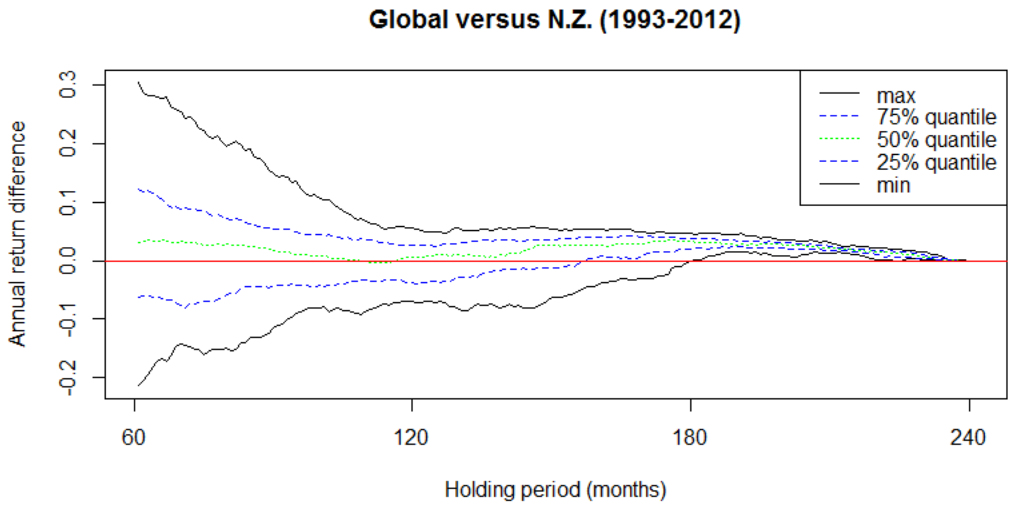

Figure 6.1: *This figure shows the difference in untaxed annual returns between the naive global portfolio in New Zealand dollars and the domestic New Zealand market portfolio in New Zealand dollars for all possible holding periods between January 1993 to December 2012.*

6.4.1 International diversification

Investors can choose to maximize the return of their portfolio while seeking the same volatility obtained in the local market. Similarly, they can choose to minimize the volatility of their portfolio. They can choose to allocate funds in international markets where the investment opportunities of those markets can be stated as a vector of multivariate Gaussian stochastic returns of N assets, $R^T = [r_1, r_2, ..., r_n]$. The estimated mean of asset returns for these markets can be expressed as a vector μ. The variance-covariance of asset returns can be expressed as a positive definite matrix V. Let S be the set of all real vectors w that define the weights such that $w^T 1 = w_1 + w_2 + ... + w_n = 1$, where 1 is an N vector of ones. Using the methods developed by Markowitz

(1952), the efficient frontier of global investments can be formed when the objective function and restrictions are combined in order to find the efficient portfolio that minimizes volatility at every level of expected return:

$$\min_{\{w,\phi,\eta\}} \Xi = \frac{1}{2} w^T V w + \phi(\mu_p - w^T \mu) + \eta(1 - w^T 1) \tag{6.1}$$

where μ_p is the expected return of the portfolio, and the shadow prices ϕ and η are two positive constants. The quadratic programming solution of assets in a portfolio spanning w_p can be obtained by the first-order conditions of equation (6.1).

For the purposes of this study, an investor may choose to diversify out of the domestic market and into the maximum return-to-risk portfolio (MRRP). When there are no weight restrictions, the MRRP can be determined using the estimated mean of asset returns and the estimated variance-covariance of asset returns to find the vector w of weights that define the estimated maximum return-to-risk (MRR) achieved by the MRRP such that:

$$MRR = \max_{\{w_p\}} \left\{ \frac{w_p^T \mu}{(w_p^T V w_p)^{1/2}} \middle| w_p^T \in S \right\} \tag{6.2}$$

where w_p is the vector of weights of the assets held in the MRRP. The estimated MRR measures the estimated return achieved by the MRRP compared to the estimated standard deviation of the MRRP.

Similarly, an investor may choose to diversify out of the domestic market and into the minimum volatility portfolio (MVP). When there are no weight restrictions, the MVP can be determined using the estimated variance-covariance of asset returns to find the vector w of weights that define the estimated minimum volatility (MV) achieved by the MVP such that:

$$MV = \min_{\{w_p\}} \left\{ (w_p^T V w_p)^{1/2} \middle| w_p^T \in S \right\} \tag{6.3}$$

where w_p is the vector of weights of the assets held in the MVP.

When an investor has constraints on the weights that can be assigned to assets held in the optimized portfolio and does not allow short sales, equations (6.2) and (6.3) can be solved such that $w_p^T \in S_c$ where $S_c = \{w_p \in S : 0 \leq w_i \leq \eta w(Cap)_i \leq 1, i = 1, 2, ..., N\}$, where η is the weight constraint multiplier greater than or equal to one, and $w(Cap)_i$ is the weight of the market value of each country i in the set of markets. This chapter analyses the changes to the benefits of diversification when constraints are relaxed and overweighting of international markets can occur with η set to 1, 2, 5 and 10. The optimized portfolio that constrains all markets by η set to 1 is referred to in this chapter as the naive 1/M portfolio. For the purposes of this study, the domestic New Zealand market allocation is not constrained by the the market capitalization weight in order to measure the potential effect of taxes on the home bias in the optimal portfolios. The optimized portfolio that constrains all markets except New Zealand by η set to 1 is referred to as the $1/M^*$ portfolio. Similarly, for the purposes of measuring the home bias as the weight constraints are relaxed, the New Zealand market is not weight constrained in any of the optimized portfolios with relaxed weight constraints. As unit trusts are likely to restrict short sales, this chapter does not consider short sales in the optimization solutions.

Estimated returns

This study uses the historical monthly returns of 34 markets denominated in New Zealand dollars from January 1993 to December 2014 to assess the potential effect of taxes on the benefits of international diversification for equity investments held in a PIE. The estimated optimal MVPs and MRRPs held in a PIE with different tax rates are determined using a vector μ of estimated mean after-tax asset returns for these markets. The estimated monthly after-tax market returns are calculated by deducting the appropriate overseas dividend withholding taxes and New Zealand taxes to be paid each month for each individual market.

New Zealand dividends and capital gains are not taxed. Australian equities are treated as exempt from being an attributing interest in a FIF. Capital gains on Australian shares are not taxed. This study treats all Australian dividends as fully franked and no withholding taxes are held. The dividends are taxed as income using the PIR. For all other markets, taxes withheld on overseas dividends are given credit to reduce the New Zealand FDR taxes. The New Zealand taxes to be paid are equal to the monthly pro-rated FDR multiplied by the PIR. The after-tax monthly returns for each market are used to calculate the estimated geometric annual market returns used in the optimization solution. The monthly after-tax returns are also converted to logarithmic monthly returns which are used to build the variance-covariance matrix used in the optimization solution.

Portfolio returns

The monthly after-tax returns achieved by the estimated MRRP and the MVP are used to measure the benefits from diversification. Using the same historical data used in the optimization stage, the after-tax monthly returns achieved by the static portfolios are calculated. A portfolio's monthly after-tax returns are used to calculate the geometric annual return and the standard deviation of returns for the portfolio. These are used to determine the RR ratio of the MRRP (RR_{MRRP}) and the variance of returns of the MVP (V_{MVP}) which are then used to measure the benefits from diversification.

Measuring the potential benefits of international diversification

For the New Zealand investor, adjusting their portfolio out of the domestic market and into the internationally diversified portfolio that maximizes the return-to-risk ratio, the greatest increment of unit-risk performance is measured as:

$$\delta = \frac{RR_{MRRP}}{RR_{nz}} - 1 \tag{6.4}$$

where δ (delta) represents the percentage improvement over the New Zealand market portfolio return-to-risk ratio (RR_{nz}) the investor achieves when moving into the internationally diversified portfolio that maximizes the return-to-risk performance (RR_{MRRP}).

This chapter measures the reduction in volatility, ε (epsilon), that can result from diversification out of the domestic market and into the MVP as

$$\varepsilon = 1 - \left[\frac{V_{MVP}}{V_{nz}}\right]^{1/2} \tag{6.5}$$

where V_{nz} is the variance of the domestic New Zealand market portfolio and V_{MVP} is the variance of the MVP. A positive value represents the percentage reduction in portfolio volatility when diversifying out of the domestic portfolio and into the optimal internationally diversified MVP. A result approaching zero represents diminishing benefits from diversification.

6.5 Results

This section reports the results of examining the potential effect of taxes and weight constraints on the equity home bias for indirect equity investments held in a PIE using the data and methods presented in section 6.4. First, the domestic New Zealand market weight assigned to the MRRPs and MVPs with different weight constraints and tax levels optimized for the 1993 to 2014 investment period are presented. Next, the geometric annual returns, standard deviation of returns and RR ratios of these portfolios are reported. Then the delta and epsilon results for these optimized static portfolios are presented. The results are tested for statistical significance using the Ledoit and Wolf (2008) bootstrap testing methods designed to test whether the difference between two portfolio strategies is significantly different from zero. The results show that while the RR ratio improvement achieved with the unconstrained MRRP with no taxes approaches a statistically significant level, none of the taxed portfolios provide statistically different return-to-risk

(RR) ratios than the New Zealand market. Weight constrained MVPs can provide volatility reducing benefits that are statistically significant. Taxes on these MVPs reduce the significance of the RR ratios versus the domestic New Zealand market portfolio. The results report that taxes and allocation weight constraints on overseas markets increase the efficiency of a home bias for equity investments held in a PIE.

Table 6.2 reports the domestic New Zealand market allocations for each of the MRRPs and MVPs with no shorts and overseas market weights constrained by the 1993 market capitalization weights at various weight constraint limits: one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraints (nw). The New Zealand market is the only market unconstrained by weight constraints in the optimization solutions. For this study, it is assumed that an investor starting with a 100% weighting in the domestic New Zealand market portfolio might choose to diversify internationally into an optimized portfolio where the New Zealand market weight is unconstrained but the international markets are constrained by the market capitalization weight of each market at the beginning of the investment period. In this chapter a portfolio with only the overseas markets constrained to a maximum of one times the country's market capitalization weight (wx1) is referred to as the $1/M^*$ portfolio. The portfolios are optimized with no taxes (No Tax), or with withholding taxes on overseas dividends and the local New Zealand FDR taxes calculated with the PIR set to 0%, 10%, 17.5% or 28%.

In the case of the $1/M^*$ portfolio with no taxes, the New Zealand allocation is 44.07%. When overseas dividend taxes are considered and a PIR of

0% is used, the New Zealand allocation increases to 48.02%. When a PIR of 10% is used, the New Zealand allocation increases to 49.37%. At the 0% PIR level, credit for withholding taxes on overseas dividends can not be used as there is no New Zealand tax owed. As the PIR is increased, these credits can be used against the New Zealand tax owed. This partially explains why the marginal change in the home bias between the 0% and 10% PIR levels is smaller than between the no tax level and the 0% PIR level. The $1/M^*$ portfolio with a PIR of 28% has the highest New Zealand allocation of the MRRPs reported at 53.05%.

When no taxes are considered, weakening the weight constraints to two times the market capitalization weight reduces the allocation to the New Zealand market to 38.34%. As the weight constraints are relaxed further, allocations to small beneficial markets can increase and the domestic allocation falls. The smallest allocation to the New Zealand market of 16.67% occurs in the unconstrained MRRP with no taxes. At each weight constraint level, because the expected returns from the overseas equity markets are reduced by higher tax rates, a home bias becomes more efficient and the New Zealand market allocations increase as the PIR is increased.

In the case of the MVP, taxes at each weight constraint level has little effect on the domestic allocation. The home bias is reduced as the weight constraints are relaxed. The largest New Zealand allocation of 32.98% is with the $1/M^*$ portfolio taxed at a 28% PIR. The New Zealand allocation in the unconstrained MVP with no taxes is 24.50%.

Chan et al. (2005) reports that in the 1999 to 2000 period, New Zealand unit-trusts in aggregate held 74.93% of equity assets in New Zealand. While

Table 6.2: Domestic market allocations.

	MRRP domestic market weight					MVP domestic market weight				
Case	wx1	wx2	wx5	wx10	nw	wx1	wx2	wx5	wx10	nw
No Tax	0.4407	0.3834	0.3338	0.2863	0.1667	0.3289	0.3001	0.2637	0.2467	0.2450
0% PIR	0.4802	0.3996	0.3564	0.3039	0.1740	0.3291	0.3001	0.2638	0.2468	0.2448
10% PIR	0.4937	0.4151	0.3717	0.3154	0.2041	0.3293	0.3004	0.2641	0.2471	0.2453
17.5% PIR	0.5181	0.4405	0.3958	0.3385	0.2305	0.3295	0.3005	0.2644	0.2473	0.2455
28% PIR	0.5305	0.4788	0.4333	0.3723	0.2713	0.3298	0.3008	0.2647	0.2477	0.2460

This table reports the domestic New Zealand market allocation held in each MRRP and MVP optimized with different tax levels and allocation weight constraints. The portfolios do not implement short sales and the domestic New Zealand market weight is unconstrained in the optimization solutions. Each overseas market weight is constrained by the country's 1993 market capitalization weight at the one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraint (nw) level. The portfolios are optimized with no taxes (No Tax), or with withholding taxes on overseas dividends and the local New Zealand FDR taxes calculated with the PIR set to 0%, 10%, 17.5% or 28%.

Table 6.3: Annual returns, standard deviation of returns and RR ratios.

Tax Rate	MRRP overseas markets constrained by ... 1/M	wx1	wx2	wx5	wx10	nw	MVP overseas markets constrained by ... 1/M	wx1	wx2	wx5	wx10	nw
Panel A: Annual Returns												
No Tax	0.063	0.080	0.083	0.087	0.088	0.098	0.063	0.065	0.065	0.067	0.070	0.071
0% PIR	0.060	0.080	0.082	0.084	0.087	0.096	0.060	0.063	0.063	0.064	0.068	0.068
10% PIR	0.058	0.078	0.080	0.082	0.084	0.093	0.057	0.061	0.061	0.063	0.066	0.067
17.5% PIR	0.054	0.077	0.078	0.080	0.082	0.090	0.054	0.059	0.058	0.060	0.063	0.064
28% PIR	0.048	0.075	0.076	0.077	0.079	0.086	0.048	0.055	0.055	0.056	0.059	0.060
NZ	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075
Panel B: Standard Deviation of Returns												
No Tax	0.130	0.121	0.122	0.121	0.118	0.122	0.130	0.115	0.113	0.112	0.111	0.111
0% PIR	0.130	0.123	0.123	0.121	0.117	0.122	0.130	0.115	0.113	0.112	0.111	0.111
10% PIR	0.130	0.123	0.123	0.121	0.117	0.122	0.130	0.115	0.113	0.112	0.111	0.111
17.5% PIR	0.130	0.124	0.123	0.121	0.118	0.122	0.130	0.115	0.113	0.112	0.111	0.111
28% PIR	0.130	0.125	0.124	0.122	0.118	0.122	0.130	0.115	0.113	0.112	0.111	0.111
NZ	0.163	0.163	0.163	0.163	0.163	0.163	0.163	0.163	0.163	0.163	0.163	0.163
Panel C: Return-to-Risk												
No Tax	0.483	0.663	0.681	0.714	0.752	0.798	0.483	0.565	0.573	0.596	0.632	0.636
0% PIR	0.458	0.647	0.669	0.697	0.737	0.784	0.458	0.547	0.553	0.574	0.612	0.615
10% PIR	0.441	0.638	0.656	0.683	0.718	0.760	0.441	0.533	0.540	0.560	0.595	0.598

17.5% PIR	0.412	0.621	0.637	0.664	0.700	0.738	0.412	0.511	0.516	0.536	0.570	0.574
28% PIR	0.370	0.600	0.612	0.637	0.670	0.707	0.370	0.479	0.482	0.500	0.534	0.537
NZ	0.459	0.459	0.459	0.459	0.459	0.459	0.459	0.459	0.459	0.459	0.459	0.459

This table reports the realized geometric annual returns, the standard deviation of returns and the RR ratios for each portfolio optimized at different tax levels and allocation weight constraints. The untaxed domestic New Zealand (NZ) market characteristics are also presented. The 1/M portfolio is the portfolio with all market weights, including the New Zealand market, set by the 1993 market capitalization weights. The MRRPs and the MVPs are optimized with no shorts and the international markets constrained by the 1993 market capitalization weights at the one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraint (nw) level. The domestic New Zealand market weight is unconstrained in the MRRP and MVP optimization solutions. The portfolios are optimized with no taxes (No Tax), or with withholding taxes on overseas dividends and the local New Zealand FDR taxes calculated with the PIR set to 0%, 10%, 17.5% or 28%.

Table 6.4: The measured benefits.

Tax Rate	1/M	Overseas market weights constrained by ... wx1	wx2	wx5	wx10	nw	benefit gained 1/M	wx1	wx2	wx5	wx10
Panel A: MRRP delta											
No Tax	0.0513 (0.982)	0.4446 (0.278)	0.4839 (0.268)	0.5547 (0.218)	0.6384 (0.140)	0.7369 (0.114)	0.07	0.60	0.66	0.75	0.87
0% PIR	-0.0023 (0.948)	0.4084 (0.286)	0.4557 (0.274)	0.5179 (0.236)	0.6051 (0.148)	0.7065 (0.128)	0.00	0.58	0.64	0.73	0.86
10% PIR	-0.0394 (0.896)	0.3883 (0.296)	0.4277 (0.302)	0.4881 (0.252)	0.5676 (0.166)	0.6560 (0.148)	-0.06	0.59	0.65	0.74	0.87
17.5% PIR	-0.1028 (0.802)	0.3513 (0.322)	0.3869 (0.336)	0.4460 (0.278)	0.5228 (0.186)	0.6079 (0.172)	-0.17	0.58	0.64	0.73	0.86
28% PIR	-0.1946 (0.680)	0.3040 (0.376)	0.3320 (0.370)	0.3872 (0.318)	0.4584 (0.216)	0.5405 (0.228)	-0.36	0.56	0.61	0.72	0.85
Panel B: MVP epsilon											
No Tax	0.1989 (0.012)	0.2906 (0.002)	0.3026 (0.002)	0.3116 (0.002)	0.3150 (0.002)	0.3151 (0.002)	0.63	0.92	0.96	0.99	1.00
0% PIR	0.1989 (0.012)	0.2906 (0.002)	0.3026 (0.002)	0.3116 (0.002)	0.3152 (0.002)	0.3152 (0.002)	0.63	0.92	0.96	0.99	1.00
10% PIR	0.1986 (0.012)	0.2904 (0.002)	0.3024 (0.002)	0.3115 (0.002)	0.3150 (0.002)	0.3150 (0.002)	0.63	0.92	0.96	0.99	1.00
17.5% PIR	0.1983 (0.012)	0.2903 (0.002)	0.3023 (0.002)	0.3112 (0.002)	0.3147 (0.002)	0.3147 (0.002)	0.63	0.92	0.96	0.99	1.00
28% PIR	0.1980 (0.012)	0.2900 (0.002)	0.3021 (0.002)	0.3110 (0.002)	0.3144 (0.002)	0.3144 (0.002)	0.63	0.92	0.96	0.99	1.00
Panel C: MVP delta											
No Tax	0.0513 (0.982)	0.2300 (0.654)	0.2478 (0.688)	0.2970 (0.550)	0.3770 (0.418)	0.3839 (0.426)	0.13	0.60	0.65	0.77	0.98
0% PIR	-0.0023 (0.948)	0.1917 (0.724)	0.2047 (0.738)	0.2499 (0.610)	0.3317 (0.496)	0.3401 (0.490)	-0.01	0.56	0.60	0.73	0.98
10% PIR	-0.0394 (0.896)	0.1616 (0.772)	0.1752 (0.780)	0.2191 (0.654)	0.2955 (0.560)	0.3029 (0.552)	-0.13	0.53	0.58	0.72	0.98
17.5% PIR	-0.1028 (0.802)	0.1132 (0.876)	0.1242 (0.868)	0.1661 (0.756)	0.2420 (0.648)	0.2491 (0.630)	-0.41	0.45	0.50	0.67	0.97

28% PIR	-0.1946 (0.680)	0.0434 (0.998)	0.0506 (1.000)	0.0881 (0.920)	0.1620 (0.784)	0.1687 (0.784)	-1.15	0.26	0.30	0.52	0.96

This table reports the delta and epsilon benefits from diversification out of the New Zealand domestic market and into the MRRPs and the MVPs optimized with no shorts and the international markets constrained by the 1993 market capitalization weights at the one times (wx1), two times (wx2), five times (wx5), ten times (wx10) and no weight constraint (nw) level. The results for the 1/M portfolio, which is the portfolio optimized with no shorts and all market weights, including the New Zealand market, constrained by the 1993 market capitalization weights, are also presented. The portfolios are optimized with no taxes (No Tax), or with withholding taxes on overseas dividends and the local New Zealand FDR taxes calculated with the PIR set to 0%, 10%, 17.5% or 28%. The p-value results from the Ledoit and Wolf (2008) Sharpe ratio and volatility tests using a block size of 5 and 499 iterations are presented in parentheses. The last five columns report the percent of the unconstrained portfolio benefits that are captured by each weight constrained portfolio.

the tax polices for that period are different than considered in this study, the results presented here suggest that taxes on overseas equity investments and weight constraints imposed on overseas allocations held in a unit-trust are factors that can contribute towards a home bias. The tax effect is more pronounced when optimizing a portfolio to maximize the RR performance than when minimizing variance.

Table 6.3 reports the realized after-tax geometric annual returns, standard deviation of returns and RR ratios for the MRRPs and MVPs optimized with different weight constraints and tax levels. The 1/M portfolio results are also presented. The 1/M portfolio is the naive market capitalization weighted (1/M) portfolio that the international CAPM suggests investors will hold in order to capture the naive diversification benefits from sharing risk. The New Zealand market allocation in this portfolio is .19%. Panel A reports the annual returns for each optimized portfolio with different tax rates and the annual returns of the domestic New Zealand (NZ) market which is not taxed. The untaxed 1/M portfolio achieves annual returns of 6.29% versus the New Zealand market portfolio return of 7.46%. Relaxing the weight constraints on the 1/M portfolio increases the returns realized by the MRRP and the MVP. At each constraint level, taxes reduce these returns.

Panel B of table 6.3 reports that the standard deviation of returns for the MRRPs and MVPS are lower than the New Zealand market for all weight constraint and tax levels. Panel C reports the RR ratios for the various portfolios. The taxed 1/M portfolios do not achieve RR ratios that are larger than the New Zealand market. The MRRPs and MVPs with relaxed weight constraints and taxes have RR ratios that are greater than the New

Zealand market.

Table 6.4 reports the delta and epsilon results for the MRRPs and MVPs with various weight constraints and tax rates. Panel A reports the percentage increase in the RR ratio when moving from the domestic New Zealand market portfolio to the MRRP. The results from the Ledoit and Wolf (2008) Sharpe ratio tests are presented in parentheses. The p-values report the significance of the hypothesis that the difference between the RR ratio of the monthly returns of the domestic portfolio and the optimized portfolio is zero. A 10% result is treated as statistically significant[16]. The last five columns in the table report the percent of the unconstrained portfolio gains that are captured by the constrained portfolio at each tax level. At each level of weight constraint, taxes reduce the diversification benefits. Taxes negate the benefits from diversifying into the naive 1/M portfolio. The $1/M^*$ portfolio with no taxes provides a positive delta benefit of 44.46%. The $1/M^*$ portfolio captures the majority of potential gains available with the unconstrained portfolio at each tax level. For example, 56% of the benefits available from the unconstrained MRRP taxed with a 28% PIR can be captured by the $1/M^*$ portfolio. As the weight constraints are relaxed the potential benefits from diversification increase. Taxes reduce the benefits from diversification which remain positive at the highest PIR of 28%. Only the unconstrained MRRP with no taxes approaches a statistically significant benefit level with a p-value of 0.114.

Panel B of table 6.4 reports the percentage reduction in portfolio volatility

[16] A 10% significance level is used in Behr et al. (2013). DeMiguel et al. (2009a) uses a 5% significance level.

when moving from the New Zealand domestic market portfolio to the optimized MVPs. The results from the Ledoit and Wolf (2008) volatility tests are presented in parentheses. The p-values report the significance of the hypothesis that the difference between the standard deviation of the monthly returns of the domestic portfolio and the optimized portfolio is zero. A 10% result is treated as statistically significant. At each tax level, the 1/M portfolio captures 63% of the potential volatility reducing benefits available from the unconstrained MVP. The $1/M^*$ portfolio can capture as much as 92% of the maximum unconstrained potential gains at each tax level. All the portfolios provide reductions in volatility that are statistically significant.

Panel C of table 6.4 reports the delta results for the various weight constrained and taxed MVPs. The results from the Ledoit and Wolf (2008) Sharpe ratio tests are presented in parentheses. As with the MRRP, taxes reduce realized returns which results in lower potential RR improvements from diversification. With no taxes, none of the optimized MVPs have positive RR improvements that are statistically significant. The unconstrained portfolio with no taxes offers the greatest potential RR improvement of 38.39% over the domestic market. The p-value for this portfolio is .426. This is greater than the .10 significance level. In the case of a PIE with a 28% PIR, taxes on the unconstrained portfolio reduce the delta gains to 16.87%. The statistical significance of the results weakens to 0.784. Strengthening the weight constraints on the MVPs reduces the delta results. Taxes at each level of weight constraint reduces the potential RR gains. While MVPs with weight constraints and taxes offer potential volatility reducing benefits that are statistically significant, the resulting RR ratios are not significantly larger than the domestic market. An investor choosing to reduce portfolio volatility may find maintaining a home bias and diversifying into a risk free asset may achieve a similar level of volatility offered by an internationally diversified MVP which does not offer statistically different RR performance.

These results do not support the suggestion by MacDonald et al. (2014) that KiwiSaver funds are likely to improve long term performance by increasing allocations to global equities. Rather, based on historical data, static optimized portfolios of international markets held in a PIE are unlikely to achieve statistically significant RR efficiency gains beyond the domestic New Zealand market portfolio when taxes are considered. Furthermore, a PIE optimizing a portfolio with imperfect market return and covariance estimates is unlikely to achieve the ex-post optimized results presented in this study. Weight constraints on assets may be used to reduce ex-ante estimation error in the optimization solution (Jagannathan and Ma, 2003; DeMiguel et al., 2009a) which will further reduce the potential benefits from diversification and increase the efficiency of a home bias.

These results suggest that an optimized MRRP or MVP with no taxes and no weight constraints held during the 1993 to 2014 investment period is unlikely to provide positive RR improvement beyond the domestic market portfolio that can be reported to be statistically different. As the PIR is increased on equity investments held in a PIE, the potential RR gains from an unconstrained portfolio falls and the efficiency of a home bias increases. Increasing the weight constraints on the optimized portfolios further reduces the potential gains of diversification and increases the home bias.

6.6 Conclusion

This chapter investigates the potential effect of taxes on the benefits from international diversification and the efficiency of an equity home bias for New Zealand investors with indirect equity investments held in a PIE. Weight constraints on overseas market allocations and taxes reduce the potential benefits of international diversification. Relaxing the weight constraints on overseas market allocations reduces the equity home bias. Taxes increase the equity home bias. While relaxing weight constraints on the optimization solution and reducing taxes does increase the potential benefits from international diversification for both the MRRP and the MVP, the RR gains from these portfolios are not statistically different than the domestic New Zealand market portfolio.

The findings presented in this chapter contribute to the literature investigating the benefits of international diversification for New Zealand investors. First, the results provide evidence of the importance of taxes in reducing the benefits from international diversification for KiwiSaver investors which previous literature has not considered in their conclusions. MacDonald et al. (2014) suggests that heavier KiwiSaver allocations to international equities may improve investor retirement outcomes. This chapter reports that the combination of allocation weight constraints on overseas markets and the unequal taxation rates applied to international equity investment returns makes it unlikely that internationally diversified KiwiSaver portfolios can provide statistically significant positive return-to-risk improvements versus the untaxed domestic New Zealand market portfolio. Second, the empirical results contribute to previous literature that has identified tax as an explanatory factor in the documented home bias that exists with international investment funds in New Zealand (Chan et al., 2005). The results presented here find that the tax effect is more pronounced when optimizing a portfolio to maximize the RR performance than when minimizing portfolio variance.

The results presented in this chapter reflect the historical benefits from diversification. The New Zealand currency strengthened against all other currencies over the sample period. It is unclear whether this currency appreciation will continue and what effect the currency will have on the potential benefits from diversification that may be available in the future. Furthermore, this study does not consider the effect of the forecast increase in ownership of the New Zealand market by KiwiSaver funds (e.g., Heuser et al., 2015) on future New Zealand market performance. Research that addresses these points may extend the results presented in this chapter.

Chapter 7

Conclusions

This thesis investigates the effects of allocation weight constraints, estimation error and taxes on the potential benefits from international diversification. The results address gaps in the literature regarding the contribution of these factors in reducing the benefits from diversification and, as a result, their potential influence on an equity home bias. Naive international diversification and in-sample optimized static portfolios with weight constraints are not reported to provide all investors in 34 countries with positive diversification benefits over extended holding periods that are statistically different than their domestic market portfolio. Estimation error reduction strategies recently presented in the literature that are designed to improve ex-ante optimization performance are not found to consistently and statistically improve the risk-to-return performance of optimized portfolios versus the domestic market portfolio or the naive 1/M portfolio for investors in as many as 32 of the 34 countries measured. Lastly, the unequal taxation of overseas equity assets versus domestic equity assets held by New Zealand investors in portfolio investment entities (PIEs) is reported to reduce the potential benefits from international diversification and increase the efficiency of a home bias. The empirical evidence presented in this thesis suggests that a home bias may be a statistically efficient portfolio decision for many investors given the difficulty of achieving significant positive benefits from international diversification because of allocation constraints, estimation error and taxes.

The documented equity home bias that persists amongst investors worldwide is considered a puzzle because international CAPM assumes investors will naively diversify into the global market capitalization weighted (1/M) portfolio and modern portfolio theory suggests mean-variance optimizing investors will diversify internationally to improve portfolio efficiency because of the risk reducing benefits of weakly correlated overseas markets. My research extends the literature on this topic in several ways. First, chapter 3 identifies weaknesses with the data, methods and perspective as reported in Chiou (2008) which overstate the potential benefits from diversification into optimized portfolios with market allocation weight constraints for U.S. investors. I also contribute to the line of literature which measures the potential benefits from international diversification by reporting the significance of these benefits using the statistical testing techniques of Ledoit and Wolf (2008). The results suggest that a home bias might not be a puzzle given that neither naive diversification or in-sample optimized portfolios with market allocation weight constraints are certain to achieve diversification benefits that are both positive and statistically significant for U.S. investors diversifying out of the domestic equity market.

Chapter 4 reports the cross-country benefits from diversification into weight constrained MRRPs and MVPs. This extends Driessen and Laeven (2007) which reports the cross-country diversification gains from portfolios optimized to increase returns and the Sharpe ratio with or without short sales. Chapter 4 also implements a Bayesian approach used to measure the significance of the potential benefits which extends the U.S. and U.K. results presented in Li et al. (2003) and Fletcher and Marshall (2005) respectively. The Bayesian results provide robustness to the findings that, while historically there may be potential benefits from international diversification, the size and the significance of these gains are conditional on the sample. The results find that portfolios optimized in-sample with perfect information do not provide positive benefits from diversification that are statistically significant for investors in many of the countries measured.

Next, chapter 5 extends the results of Jacobs et al. (2014) to report the usefulness of common mean-variance optimization strategies in providing efficiency gains versus the domestic market portfolio and the effectiveness of estimation error reduction strategies applied to international diversification for investors in 34 countries. The results do not identify an optimization strategy that consistently outperforms the naive domestic portfolio or statistically outperforms the 1/M benchmark for all investors. The use of 34 individual market indices in the optimization solutions, rather than 4 regional indices as used in Jacobs et al. (2014), is reported to improve optimization performance. However, neither the naive 1/M portfolio or the optimization strategies are found to provide significant positive efficiency gains versus the local market portfolio for as many as 32 of the 34 markets studied.

Finally, chapter 6 measures the effect of taxes on the potential benefits of international diversification for KiwiSaver investors. This is the only study to investigate the effect of taxes on the potential benefits of international diversification for New Zealand investors holding indirect equity investments in PIEs. The combination of allocation weight constraints on overseas markets and the unequal taxation rates applied to international equity investment returns are reported to increase the efficiency of a home bias. The results find that the tax effect is more pronounced when optimizing a portfolio to maximize the RR performance than when minimizing portfolio volatility. These results do not support previous literature that suggest heavier allocations to overseas equity assets will improve KiwiSaver investment performance (e.g., MacDonald et al., 2014) but which do not consider taxes.

Future research may extend the results presented in this thesis. Using exchange traded fund (ETF) data for each market, rather than the MSCI indices data used in this thesis, may more accurately measure the potential benefits that an investor might be able to achieve in practice. Consideration of trading costs such as fees and trading spreads would add further accuracy to the measured potential benefits available to investors. Of particular interest to this author is research that can identify and measure the significance of factors that contribute to estimation error. This may lead to estimation error reduction strategies that reduce the error introduced by a factor rather than the asset. In the case of international equity investment, currency risk is likely to be such a factor.

Appendix A

Calculating the FDR for a PIE

The New Zealand Income Tax Act 2007 sets out the rules governing tax obligations in New Zealand. Section EX 53 defines the rules used by a unit-valuing fund, such as a unit trust, to calculate tax owed by unit holders using the FDR method. The unit trust assigns each investor an interest (the unit) in a proportion of the net returns from the investments[1] and determines the value of the investors' units for each of a number of periods making up the income year[2]. The FDR amount is pro-rated by the number of trading days in the period being measured compared to the number of days in the tax year. The value of the PIE unit at the beginning of the period is multiplied by the pro-rated FDR, and this amount is treated as income to be taxed at the clients' PIR to determine the tax owed.

PIEs generally calculate a daily closing value for existing units. The pro-rated annual FDR of 5% for a single trading day is 1/365 of the FDR. If the unit price at the end of the previous closing is $10.00, and a client has a PIR

[1]EX53, 1(b)(ii)
[2]EX53, 1(b)(iii)

of 28% and holds 1,000 units, the unit-trust can calculate the tax owed by the client at the end of the trading day as follows:

$$1/365 \times 0.05 \times 1,000 \times \$10.00 \times 0.28 = \$0.383562 \tag{A.1}$$

For this study, the monthly after-tax portfolio returns are determined using a monthly pro-rated FDR equal to $1/12 \times .05$.

Appendix B

Excess returns versus raw returns

Modern portfolio theory assumes that investors are mean-variance optimizers who seek investments that offer a return in excess of the risk-free rate to compensate the investor for the risk associated with that asset. The Sharpe ratio measures the excess returns an asset provides per unit of risk and is often used to report risk-adjusted results in financial literature. This thesis uses 'raw' returns in the optimizations and the reported results. This appendix explains why return-to-risk (RR) ratios are used to measure the benefits from diversification and why the conclusions presented in this thesis are unlikely to change if Sharpe ratios are used.

B.1 Measuring the significance of the benefits from diversification

This thesis measures the potential benefits from international diversification available to investors in 34 countries. This is done by measuring the percentage improvement in the RR ratio between an investor's domestic equity market and the MRRP, and the reduction in the volatility between the local portfolio and the MVP. These results are used to determine whether there is the potential for positive gains to portfolio efficiency from international diversification. Of particular interest to this author are the significance of these gains as determined by the Ledoit and Wolf (2008) bootstrap testing techniques. As reported in Wolf and Wunderli (2012) using excess returns or raw returns in these tests does not tend to change the results qualitatively.

There are two primary reasons that this author chose not to use excess returns in this thesis. The first is the desire to use consistent methods to measure the benefits from diversification in each chapter. Chapter 3 introduces many of the methods used in this thesis. It reports the adjusted results presented in Chiou (2008), which does not use excess returns. The second reason is that risk-free data for all markets used in this thesis is not easily accessible from 1992[1]. In order to avoid reducing the size of the data-set available for use in chapters 4 and 5, 'raw' returns rather than excess returns are used.

[1]Driessen and Laeven (2007), which also reports cross-country diversification benefits, identifies issues related to incomplete risk-free data and high risk-free rates in some emerging markets in footnotes (11) and (12) of their paper.

B.2 Shifting the efficient frontier

Figure B.1 shows how measuring the benefits from diversification based on excess returns rather than raw returns may alter the magnitude of the reported results. Optimizing portfolios with excess returns, rather than raw returns, is like shifting the base of a mean-variance graph upwards towards the efficient frontier, or similarly like shifting the efficient frontier towards the base. As presented in figure B.1, if the same MRRP is determined using excess returns rather than raw returns, the relative change in benefits between the local portfolio and the MRRP will be larger when measured using excess returns rather than raw returns. This is because the absolute difference in returns and volatility between the local portfolio and the MRRP may be the same, but because the risk-free rate is deducted from the raw returns, the resulting Sharpe ratios will be lower than the return-to-risk (RR) ratios calculated from the raw returns. This will result in larger relative improvements to the Sharpe ratios when moving from the local portfolio to the MRRP. As a result, the reported benefits from diversification into the MRRP as measured by δ (delta) may be expected to be larger when measured using the Sharpe ratio rather than raw returns. The ϵ (epsilon) results might be expected to be the same as it measures the percentage reduction in volatility between the domestic market and the MVP.

It is possible that the MVP and MRRP on the efficient frontier determined using raw returns may not be the same portfolios determined when using excess returns. As figure B.1 shows, if the risk-free rate is relatively low (rf') then the MVP might be the same but a new MRRP' along the efficient

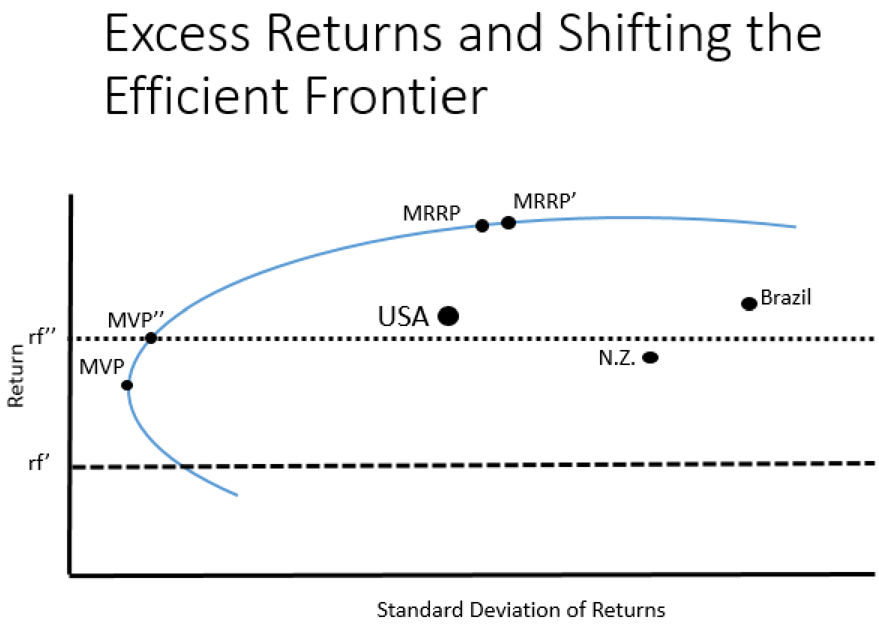

Figure B.1: *This figure shows how measuring the benefits from diversification based on excess returns rather than raw returns can alter the magnitude of the reported results.*

frontier may be found that provides a larger Sharpe ratio than the MRRP determined using raw returns. Should a country have a relatively high risk-free rate (rf"), such as an emerging market with high inflation, an alternative MVP" with a return equal to rf" might be selected.

B.3 Comparing the results

This section presents the measured benefits from diversification into the MRRP and the MVP for U.S. dollar investors diversifying out of the domestic U.S. market portfolio for the 1988-2004 investment period. The portfolios are optimized using 'raw' returns and excess returns. The 'raw' return results are the same as those reported in table 3.4 and table 3.6 in chapter 3. The monthly 3 month U.S. Treasury Bill rate data, which is used as the risk-free rate, is collected from Datastream.

Panel A of table B.1 reports the δ benefits from diversification for the 1988-2004 investment period using 'raw' returns. Panel B reports the benefits using excess returns. The significance levels of these results are reported in parentheses. Generally the excess return δ results are larger than the 'raw' return results. This is expected as the relative size of the gains are larger when measured with excess returns because the domestic portfolio's Sharpe ratio will be lower than the 'raw' RR ratio. The exception is the wx2 constraint level results. The MRRP optimized with weight constraints at the two times level does not provide positive Sharpe ratio improvements versus the naive U.S. market portfolio. While not presented here, the market allocation weights of the excess return optimized portfolios with weakened weight constraints can be slightly different than the 'raw' return portfolios. This may be due in part to a small increase in market volatility that results from the reduction of the monthly risk-free rates from the monthly market returns. A comparison of the p-values finds the significance of the positive gains are qualitatively similar.

Panel A of table B.2 reports the ϵ benefits from diversification for the 1988-2004 investment period using 'raw' returns. Panel B reports the benefits using excess returns. The significance levels are reported in parentheses. The excess return ϵ results are similar to the 'raw' return results. While not presented here, the market allocation weights in the optimized portfolios are very much the same. The deduction of the monthly risk-free rates from the

Table B.1: Delta results for the 1988 to 2004 holding period.

						benefit gained			
Market	wx1	wx2	wx5	wx10	nw	wx1	wx2	wx5	wx10
Panel A: Results using 'raw' returns.									
U.S.	-0.5018 (0.084)	0.0174 (0.494)	0.0796 (0.120)	0.1261 (0.128)	0.2651 (0.056)	-1.89	0.07	0.30	0.48
Panel B: Results using excess returns.									
U.S.	-0.7527 (0.098)	-0.0037 (0.646)	0.0856 (0.184)	0.1377 (0.192)	0.4350 (0.078)	-1.73	-0.01	0.20	0.32

This table reports the delta benefits from diversification out the domestic U.S. market and into optimized MRRPs with no shorts and the 1988 market capitalization weight constraints at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. Panel A reports the results of optimization using 'raw' returns. Panel B reports the results of optimization using excess returns. The p-value results from the Ledoit and Wolf (2008) Sharpe ratio tests using a block size of 5 and 499 iterations are presented in brackets. The percent, in decimal form, of the unconstrained benefits captured at each constraint level are reported in the last four columns.

Table B.2: Epsilon results for the 1988 to 2004 holding period.

											benefit gained			
Market	wx1		wx2		wx5		wx10		nw		wx1	wx2	wx5	wx10
Panel A: Results using 'raw' returns.														
U.S.	-0.0657	(0.224)	0.0833	(0.004)	0.1003	(0.002)	0.1098	(0.002)	0.1229	(0.004)	-0.53	0.68	0.82	0.89
Panel B: Results using excess returns.														
U.S.	-0.0709	(0.192)	0.0814	(0.004)	0.0980	(0.002)	0.1072	(0.002)	0.1205	(0.004)	-0.59	0.68	0.81	0.89

This table reports the epsilon benefits from diversification out the domestic U.S. market and into optimized MVPs with no shorts and the 1988 market capitalization weight constraints at the one times (wx1), two times(wx2), five times(wx5), ten times(wx10) and no weight constraint (nw) level. Panel A reports the results of optimization using 'raw' returns. Panel B reports the results of optimization using excess returns. The p-value results from the Ledoit and Wolf (2008) volatility tests using a block size of 5 and 499 iterations are presented in brackets. The percent, in decimal form, of the unconstrained benefits captured at each constraint level are reported in the last four columns.

market returns increases market volatility slightly, which reduces the benefits achieved by the MVP. A comparison of the p-values finds the significance of the benefits are qualitatively similar.

The results presented in table B.1 and B.2 show that while the magnitude of the MRRP diversification gains are generally larger when measured using excess returns, there is little change in the MVP results. Furthermore, the statistical significance of the reported gains do not change qualitatively when measured using excess returns rather than 'raw' returns. Therefore, while reporting the potential gains from international diversification using excess returns rather than 'raw' returns may change the magnitude of the reported diversification benefits because of a change in the perspective used, it is the opinion of this author that the significance of these gains and the conclusions presented in this thesis are unlikely to change.

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