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Hierarchies, Ties and Power in Organizational Networks: Model and Analysis

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Abstract An organizational structure consists of a network where employees are connected by working and social ties. Analyzing this network, one can discover valuable insights into information flow within the organization. We develop this idea and propose a simple network model that is consistent with management theory, and that captures main traits of large corporations. The carcass of the model is an organizational hierarchy. We extend it by allowing additional types of connections such as collaboration, consultation, and friendship. Having both formal and informal interpersonal ties, our model supports a multilevel approach to social networks. Using a centrality-based definition of power, we are able to identify important individuals in the network. Our model provides novel insights into a range of organizational properties: 1) Organizations have limited hierarchy height. 2) Flattening, the process when a business changes from a tall to a flat hierarchy by delayering, is intimately related to changes in the power of employees. 3) Informal relations significantly impact power of individuals. 4) Leadership styles could be reflected and analyzed through understanding weights on the ties in an organizational network. We implement our model and tools in a stand-alone application CORPNET, which provides functions for generating synthesized organizational networks, analyzing and visualizing interpersonal relations, and computing network measures.

Keywords organizational networks · formal and informal ties · power · flattening · leadership styles

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1 Introduction

The rapid progress of information technology in the last half-century greatly improved communication and productivity in corporations, facilitating them to grow into giant enterprises. Clearly, the bigger the company gets, the more incentive there is to identify hidden information inside its structure – the more sense it makes to study how decisions pass from the top levels to the bottom, how individuals interact with each other, and which are the most important positions. Chaotic growth can lead to inefficient management, and, hence, loss of money. This is why corporations are willing to pay large sums of money not only on hiring talented managers who define the firm's direction, but also on costly business intelligence that guarantees all the layers are on the same wavelength (Chaudhuri et al., 2011).

In behavioral management theory, one usually considers how different tenuous aspects – such as motivation, personality, expectation, and conflicts – define the productivity of an individual. Alternatively, a structural theory moves away from these personal traits, and rather views an organization as patterned and repeated interactions among social actors within the organization (Weick, 1979). In this work, we focus on the structural theory. More precisely, we represent an organizational structure as a network where agents are connected with each other by interpersonal relations. Using this network, one can analyze how the flow of information circulates within the organism of the firm and uncover properties invisible at first glance. Moreover, centrality measures are capable of detecting the most powerful positions in the network, and hence, provide a rigorous analysis of *power* in organizations.

Power has been a key notion in sociology, politics, and management theory; The term is often interpreted as the capability of a person to affect the behaviors of others (Martinez et al., 2012), and is closely related to notions such as influence and authority (Section 3.2 contains a brief discussion of their differences). Organizational power may come from multiple bases such as personal traits, ranks, skills as well as interpersonal ties French and Raven (1959). While we acknowledge the importance of behavioral factors in power, the focus of this paper is solely on power that arises from interpersonal ties.

The reporting relation is the strongest indicator of power in a company; ranks, titles, and uniform clearly define privilege of individuals. On the other hand, a network perspective of power posits that informal social ties and communication also grant power. For example, Brass in his work (1984) suggested that individual power in organizations comes from a structural perspective, which includes both formal and informal communication.

The aim of this work is to analyze organizational structures from a network perspective. More specifically, the main contribution of this paper is three-fold.

Firstly, by integrating different interpersonal relations in the same network model, we suggest a uniform approach to perform *organizational network analysis* (ONA) (Cross et al., 2002; Cross and Parker, 2004; Ehrlich and Carboni, 2005; Bryan et al., 2007). Our model is consistent with management theory, and captures main traits of large corporations. More specifically, we define the structure of a firm as a network where employees are connected to their managers and each other by working ties. The carcass of the model is an organizational hierarchy. We extend it by allowing additional types of connections between two employees (e.g. collaboration, friendship, family relations and others), and introduce the notion of an *organizational network*. Having both reporting and non-reporting relationships, our model supports a multiplex approach to organization structures.

Secondly, we define a notion of power based on a centrality measure for individuals in an organization. This notion not only enriches the mathematical management theory (Bonacich, 1987; Bonacich and Lloyd, 2001) but also enables formal analysis of concepts specific to organizations such as stability and flattening. Comparing to existing centrality notions, our definition of power is novel in the following aspects: 1) the model takes into account three types of interactions: the interaction between a manager and her subordinates, the mutual interaction effect between two employees connected by a non-reporting relation, and the *backflow* effect from a subordinate to her manager. 2) the model enables a natural interpretation of the "loss of control" of a manager: the more connections a manager maintains, the less her power depends on each of her neighbors' power (Meagher, 2003).

Thirdly, based on our model, we design and implement a novel business intelligence software tool, CORPNET, to provide automated and accurate decision support. The prototype implements statistical and stability analysis, community detection, synthesizing networks, and visualization. Using a range of parameters, the software not only allows identification of personal power in a company but also reasoning about leadership styles and strategies.

Based on our network model and definition of power, we are able to formally study multiple important phenomena relevant to organizational management. In particular, we build our theory around the following issues:

1. *Bounded height*: A management hierarchy typically involves a bounded number of levels, regardless of the individual capabilities. A common belief is that a tall hierarchy reduces the effectiveness of communication. Using a natural measure on the stability of a network, we provide an alternative explanation: as a company creates more and more levels in its hierarchy, it will eventually become unstable, i.e. employees at lower levels possess more power than those at the higher levels. See Section 4.2.
2. *Flattening*: Flattening is a well-known phenomenon of organizational change when a company acquires a new structure with fewer hierarchical levels. The alleged benefits of flattening include empowering employees, increasing flexibility, pushing down decision making, and improving information flow (Kubheka et al., 2013). We provide a somewhat paradoxical view on flattening through computation: Although flattening reduces average power in the company, the majority of employees gain more power. See Section 5.
3. *Workplace homophily*: Homophily refers to the tendency of individual to be associated and linked to others who are similar to themselves. In the workplace, this principle translates to the fact that employees tend to associate with people

in the same unit (i.e. department, office, etc.) as well as the same level (McPherson et al., 2001; Castilla, 2011). Clustering of the formal tie hierarchy alone does not reveal this tendency in an organizational network. Hence we provide a benchmark for informal ties that is in line with the observed homophily principle. See Section 6.1.

4. *Importance of informal ties*: As argued by numerous studies, informal ties significantly impact on organizations (Cross et al., 2002). We analyze this phenomenon from the point of view of power consistency: A network is more likely to be destabilized by social links in taller hierarchies than in flattened hierarchies. On the other hand, the gap between the power of upper and lower levels can be diminished with the presence of informal ties. See Section 6.
5. *Leadership styles*: Leadership styles refer to ways in which a manager leads by setting directions, carrying out plans and communicating with subordinates. It is generally agreed in management studies that leadership styles play a decisive role in shaping the working atmosphere and effectiveness of an organization (Tannenbaum and Schmidt, 1973; Hall, 1972). Here we deviate from the traditional, behavioral approach to analyzing leadership styles, but provide network-oriented angle using parameters in the definition of power. See section 7.

Through these analyses above, we demonstrate that our theoretical framework can be used to reflect general properties of organizations. This novel, structural approach to organization analysis provides us new insights, explanations and potentially predictive guidelines for organizational decision making.

Related works. The concepts introduced in the current paper are rooted in the classical and notable work by Chester Barnard (1938), who laid most of the foundation of the structural theory of organizations. According to Barnard, formal organizations coexist with informal organizations within the same entity. Barnard defines formal organizations as dictated by a set of rules and policies. An informal organization by Barnard is the personal contacts and interactions between workers that form into s-mall groups; these informal groups of workers form their own organization in the larger organization. Power thus arises out of the amalgamation of the formal and informal organizations.

There are two major technical ingredients in this paper: (1) We follow Barnard's view that any model of an organization must contain the dual-structure of formal and informal relations. (2) We define power from a structural perspective based on interpersonal ties. In the following, we present an overview of related works along each of these directions.

Barnard's dual-structural approach to organization studies has been revisited many times. For example, Emerson claims in (1962) that power is a property of the social relations, and resides largely in the dependence between social actors in a network. Brass in (1984) focused on an informal network that unifies workflow, communication and friendship relations, and displayed correlations between powerful nodes and central positions in this network. In a similar vein, Krackhardt and Hanson in (1993) drew an analogy between a company and a human body: the formal structure of a company is the skeleton, while the informal structure is the central nervous system.

Informal networks are more flexible and adaptive, formal structure is static. Krackhardt then related power to *cognitive accuracy* of an individual surrounding network in (1990). Cross et al. in (2002) adopted a computational approach and argued that even though informal networks are invisible, they are more reflective than the formal organizations. The authors defined scenarios where social network analysis is useful to assess informal networks and facilitate effective collaboration. We also mention a number of works that show how informal networks can be used to uncover the reporting hierarchy, revealing interesting insight of an organization (Fire et al., 2015; Tyler et al., 2003).

The network aspect of power and influence flourished in the last 5-10 years (Brass and Krackhardts, 2011). For example, social networks play a central role in studies on important problems such as structural holes (Ahuja, 2000), group cohesion (Bothner et al., 2004) and resource allocation (Bothner et al., 2011). The first work discusses a dual-structure within an organization, which consists of *direct ties* and *indirect ties*. However, different to our notions of formal and informal ties, direct ties refer to edges between nodes, and indirect ties correspond to paths in the network. The second work used a mathematical model to simulate the interactions between newly hired employees and relates cohesion with managerial autonomy. The third paper applies a mathematical, centrality-based approach to study two strategies in designing status-based competitions. Similar to the current paper, Bothner et. al. in (2010) also applied Bonacich power to social network analysis; the difference with our work is that they emphasized on individuals' "statues" in a social network which does not take into account hierarchical levels. More recently, Franceschet and Bozzo (2015) provided a definition of power that is motivated from negative exchange networks: a node gains power by connecting to nodes with low power.

Paper organization. The rest of the paper is structured as follows: Section 2 presents our organizational network model. Section 3 introduces our definition of power in the organization and demonstrates the terminology through a real-world case study (Krackhardt and Hanson's network (1993)). Section 4 focuses on formal tie hierarchies of organizations and discusses the relation between stability and height. Section 5 discusses the phenomenon of flattening and tries to explain it from a network point of view by focusing on power distribution. Section 6 focuses on informal ties in an organization. Section 7 applies our model to the analysis of leadership styles. Section 8 discusses the software CORPNET and finally Section 9 concludes with future works.

Remark. The current paper expands from (Liu and Moskvina, 2015), from which the model of organizational networks and the notion of power are extended. The introduction to the software tool CORPNET, the algorithms for random networks simulation, discussion on leadership styles and numerous experimentations and simulations are novel and give significant new insights into the implication of our mathematical models.

2 Organizational networks

An organizational structure is often defined as a set of positions, groups of positions, reporting relationships, and interaction patterns (Barney and Griffin, 1992). We use the network approach and propose a model that captures main traits of a company. On the one hand, our model delineates the organizational hierarchy of a firm by featuring reporting relationship. On the other hand, we enrich the model by also including non-reporting relations. Indeed, as we will show later, these non-reporting relations can significantly affect a company as a whole.

Definition 1 (Organizational network) An *organizational network* is a structure $\mathscr{G} = (V, r, E_{\mathsf{fml}}, E_{\mathsf{inf}})$, where V is a set of *nodes*, $E_{\mathsf{fml}}, E_{\mathsf{inf}} \subseteq V^2$ are edge relations such that

1. $r \in V$ is called the *root* and $(r, r) \in E_{\mathsf{fml}}$;
2. the pair (V, E_{fml}) forms a directed acyclic graph (ignoring the edge (r, r)), where every node apart from r has an incoming edge from another node;
3. the pair (V, E_{inf}) forms an undirected graph.

Informally, the set V denotes the individuals (or work positions) in the network. The root r is the top manager, i.e. r does not report to anyone else. The edge set E_{fml} represents the *reporting relation* on members of the network; if $(u, v) \in E_{\mathsf{fml}}$ then v reports to u and is called a *subordinate* of u. By the definition above, any nodes in the network may play the roles of *managers* and *subordinates*. Clearly, any node $v \neq r$ reports to its manager u and thus is a subordinate of u; at the same time, u may also have its subordinates. A node that has no subordinates is called an *operative*.

The edge set E_{inf} represents the undirected dyadic *non-reporting relation*. This could be collaborations, advice relations, or friendship between employees, etc. We will refer to edges in E_{fml} as *formal ties* since reporting relations are usually more important. We will call undirected edges in E_{inf} *informal ties*. For simplicity, we assume that any two nodes (u, v) can be connected either by a formal tie or a informal tie, but not both. In fact, this can be justified intuitively: any reporting relation presumes some social interaction between a manager and her subordinates.

To define a "well-built" structure, we accompany the definition above with two principles:

Firstly, *unity of direction* refers to the principle that there are one leader and one plan for business activities; It has been a fundamental criterion for an effective organization (Fayol, 1917). Translating this principle to our model, we assert that each person should have exactly one manager in the formal tie hierarchy.

Principle 1: Unity of Direction. Each node has exactly one incoming directed edge, which represents relationship with its manager, i.e., for all $u \in V$ there is a unique $v \in V$ with $(v, u) \in E_{\mathsf{fml}}$.

Principle 1 requires the directed graph (V, E_{fml}) to form a tree structure, which we call the *formal tie hierarchy* (or *reporting hierarchy*) of $\mathscr{G}$. The managers of the hierarchy are all the internal nodes of the tree (V, E_{fml}) and the operatives are the leaves. The top (level 0) of the hierarchy contains only the root r. We will use the following terminology.

Definition 2 The *level* of any node v in $\mathscr{G}$ is the length of the path from r to v in the reporting hierarchy. The *height* of the hierarchy is the number of levels.

Secondly, one may notice that a person can maintain only a limited number of interpersonal relations, due to limited time and effort. In fact, all social networks emerge under the constraint of limited resources. For example, in the context of online social networks, the number of formal ties (mutual communication during some period) for networks of more than 500 nodes on Facebook varies from 10 to 20 (Easley and Kleinberg, 2010).

In defining the notion of *capacity* of individuals, we distinguish the formal and informal ties in regarding how much resource each of them consumes. Let Δ be an abstract quantity that defines the maximum amount of resources (working hours, for instance) that a person can distribute between his or her ties. For simplicity, we assume that each individual in the network has the same amount of resources Δ. We also assume that a person needs f resources and i resources to maintain a formal and an informal tie, respectively. The root node also spends f resource on some exogenous factors, which are represented by the loop (r,r). Therefore, for any node v, if $|E_{\mathsf{fml}}(v)|$ is the number of directed edges (including self-loop), and $|E_{\mathsf{inf}}(v)|$ is the number of undirected edges, then A spends $|E_{\mathsf{fml}}(v)| \times f + |E_{\mathsf{inf}}(v)| \times i \leq \Delta$ resources to maintain all his connections. Let $\delta := \frac{i}{f}$ be called the *correlation coefficient*.

Definition 3 The *relative degree* of a node $v \in V$ is defined as $\mathsf{deg}(v) = |E_{\mathsf{fml}}(v)| + |E_{\mathsf{inf}}(v)| \times \delta$, where $|E_{\mathsf{fml}}(v)|$, $|E_{\mathsf{inf}}(v)|$ are the numbers of formal ties (including both incoming and outgoing edges) and informal ties v maintains, respectively.

Clearly, if $\delta = 1$, then we assume that maintaining a informal tie requires the same amount of resources as maintaining a formal tie; in this case, the relative degree $\mathsf{deg}(v)$ is the conventional degree notion in graph theory. Such assumption may be reasonable when the organization contains equipotent members who have respective expertise (e.g., a research team). The lower δ is, the greater distinction there is between formal and informal ties.

The *relative capacity* of a node v is a given number that defines the upper bound on its relative degree $\mathsf{deg}(v)$. In other words, it defines the total available resources for a person to maintain all ties.

Principle 2: Maximal Relative Capacity. There is a constant relative capacity c for any node $v \in V$.

Management theory defines the *span of control* of a manager as the number of her direct subordinates. If we only consider the reporting relation, Principle 2 guarantees that the span of control of every individual is limited, and, thus, refers to the "limited managerial attention", a phenomenon in hierarchy theory (Geanakoplos and Milgrom, 1991). The loop (r,r) guarantees that the root must not have more direct subordinates than all the other managers and, hence, make our approach uniform.

Definition 4 An organizational network is called *well-built* if it satisfies the principles 1 and 2.

In the rest of the paper, we assume that all organizational networks are well-built without explicit mention. Given its simplicity, the model of organizational networks above has natural limitations, which we explain in the remarks below:

Remark 1. We remark that the requirement that there is a single root of the network may seem too restrictive. Indeed, large corporations tend to have a board of directors. Nevertheless, we argue that this simplified model be still reasonable as the board of directors normally perform as a whole by hiring a CEO. The loop $(r,r) \in E_{\mathsf{fml}}$ indicates that the root makes decisions by herself. Another reason why we need this loop is technical – as we will show later, it makes the capacity of nodes uniform.

Remark 2. In this model, we eliminated the functional differences between individuals in the organization. Such a restriction again may seem like a departure from real life; Indeed, people in a corporation perform vastly different tasks, and it is these tasks that give their positions real "meaning". Nevertheless, we argue that the model still encapsulate meaningful interpretation: Firstly, the interpersonal ties in the model capture in a certain sense the channels of information/resource flow within an organization, irrespective of the duties of individuals. Secondly, regardless of the tasks of individuals, any person who has at least one subordinate will need to act in the role of a manager, which involves a type of decision-making process or directive actions that are common to all managers. Thirdly, our goal is to use a general model that captures any types of organizational structure which may perform very different functions (e.g. a university department or a bank). It is very difficult to fix tasks for all kinds of roles in such a general setting.

3 A measure of power

3.1 A network perspective of power

Power is a multiplex concept that is affected by behavioral, cognitive as well as social factors. Among these factors, the social network has been identified as the most significant one; as pointed out in (Pfeffer, 1981), "power is first and foremost a structural phenomenon, and should be understood as such". Following a social network approach to organization analysis (Brass and Krackhardts, 2011), we focus on power that emerges from the organizational network.

Example 1. Consider the organizational network as described in Figure 1. The directed edges form the set E_{fml} of formal reporting relations, and undirected edges form the set E_{inf} of informal ties. It is natural to believe that r would enjoy a high power in this organization as r is on the top level of the formal tie hierarchy. Comparing the managers A, B and C on level 1, we see the following differences: A has two direct subordinates, but he does not maintain any informal tie; C has three subordinates, but only one of them is a child, while B does not have any outgoing directed edges; nevertheless, she has collaboration with all nodes except A. Several natural

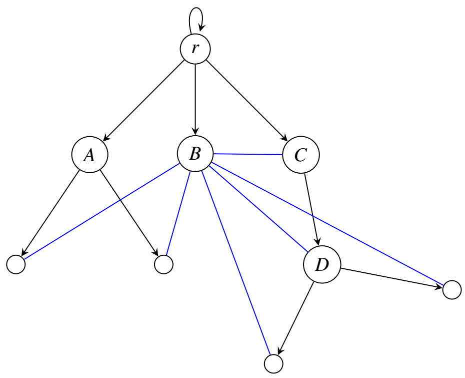


Fig. 1 Defining power of A, B and C

questions arise: which position is the 'best' among A, B, and C? How much power does each node has? Does the link between B and C affects B's power the same way as the link between B and D does? All of these questions originate from a social network perspective of power, which we elaborate below:

- The network structure defines the formal tie hierarchy of the network, and hence expresses certain *legitimate power* in the organization (Raven, 1992). For example, r naturally has power since as top manager, r has a responsibility to make decisions.
- The network structure also implies a type of *referent power* (French and Raven, 1959). For example, having extensive and broad interpersonal ties (e.g. manager B) also means that the individual is capable of developing statues and building loyalty.
- Viewing power as a product of the competition for resources, one may regard interpersonal ties as sources for resources, i.e., they serve as access points for resources such as information and skills (human resources) (Raven, 1965). For example, A has access to information that may be passed from his subordinates, while B's informal ties provide her with information across diverse departments. Both of these cases empower the particular individuals.
- From a social exchange theory perspective, interpersonal ties provides people alternatives during negotiations and hence enhance one's power (Baldwin, 1978). For example, the fact that A has two subordinates means that A is at a more advantageous position when he assigns tasks to the subordinates, i.e., the competition between the two subordinates may allow A to exercise more control. On the other hand, B is also in an advantageous position when seeking advice from his peers as he has informal connections across wide parts of the network.

3.2 Power, influence and authority

Marketing and management studies customarily compare power with other notions such as authority and influence. Indeed, it is tempting to use these terms interchangeably as they all imply the ability to affect others and infer leadership. Before we proceed with a formal definition of power, it is necessary to clarify the differences between these notions.

Influence is a measure of the ability of one person to affect another person's perception, attitude, and thought. In management studies, influence commonly relies on skillful tactics to alter the other person's point of view (Kipnis et al., 1980). In marketing, influence is often associated with the word of mouth in consumer decisions (Katona et al., 2011). A major line of research concerns the use of social networks to analyze the individuals' connections to harness *influence* word of mouth by identifying influencers and predicting adoption probabilities. More recently, efforts have been focused on the spread of influence through a physical diffusion model, and influential individuals in a complex network that maximally spread influence in this model (Morone and Makse, 2015)(Kempe et al., 2015)(Anagnostopoulos et al., 2008).

Authority refers to the right given to a person to achieve the objectives of the organization. In other words, it is an entitlement of the individual and thus is predominantly a positional concept (Grimes, 1978). For example, the top manager of an organization has the authority to make decisions about the future directions of the company largely thanks to her position. Hence authority refers to a certain form of privilege.

Power of an individual is defined by three intuitive factors: The first is the person's proximity to the root of the reporting hierarchy. The second is the number of ties the individual maintains – more connections provide more sources of information. Finally, the span of control indicates how many subordinates a person has, and, hence, how much involved he or she is in making decisions over the network.

The notion of power is distinct from both influence and authority in the following aspects. Firstly, power refers to the overall ability of the person to define the entire course of the organization. Thus, power is considerably distinct with influence, which in principle relates to the capacity to affect the behavior of one's neighbors. In this sense, there is an overlap between power and the *spread* of influence. However, the spread of influence is the outcome of a physical process Katona et al. (2011), while power is a fixed attribute that is defined by the network structure. Secondly, authority denotes the type of power that is accepted within an organization and is derived from the formal roles. While formal ties affect power, informal ties also play a crucial role, which is not captured by authority. Hence power is also inherently different from authority.

3.3 A definition of power

To capture the difference between formal and informal ties, we assign a weight of 1 to all formal ties and a weight of k between the values 0 and 1 to all informal ties to represent the strength of the tie. We call the parameter k *interaction effect*. In some sense, interaction effect measures the capability to affect the neighbors and is therefore similar to the notion of influence. However, here we keep a uniform weight for all informal ties as the focus is not on the influence of individual ties, but rather the overall power of nodes. Furthermore, we introduce a weight ρ to the self-loop on the root, which measures the "self-assertiveness" of the root. A weight ρ of 1 suggests that it has the same effect as all the other formal ties, while 0 means that these edges only affects the capacity of the root node but not the power. We will show later that the weight ρ is useful for defining *leadership styles*: A larger ρ indicates a more "autocratic" style of management. For simplicity, we will assume that $\rho = 1$ if without explicit mention. More formally, we define the *weight function* μ which depends on the two parameters k and ρ:

- *Formal ties*: If $e \in E_{\mathsf{fml}}$, we define the following:

$$\mu_{\rho,k}(e) = \begin{cases} 1 & \text{if any end point } e \text{ is a non-root node} \\ \rho & \text{if } e = (r,r) \end{cases}$$

- *Informal ties*: If $e \in E_{\mathsf{inf}}$, we assign $\mu_{\rho,k}(e) = k$. The range of k guarantees that directed edges are more important than undirected. An edge from A to B can be interpreted as the interaction effect between A and B, ranked as the weight of this edge.
- *Backflow*: It is natural to assume that the interaction effect between an employee and her manager is not one-way: While the manager is empowered by having subordinates, the subordinate also gains power from her manager through social interaction. On one hand, as a manager acquires subordinates, the manager has increased her span of control and hence becomes more powerful. On the other hand, the subordinate also increases power through support and patronage of the manager. Hence we can assume an informal tie from the subordinate back to the manager, i.e., a backflow. Therefore we set $\mu_{\rho,k}(e) = k$ where $e = (u,v)$ whenever u reports to v.

Based on the definition above, we regard any organizational network as a *weighted interaction graph*; an example of this is shown in Figure 2.

Definition 5 (Weighted interaction graph) Let $\mathscr{G} = (V, r, E_{\mathsf{fml}}, E_{\mathsf{inf}})$ be an organizational network. The weighted interaction graph of $\mathscr{G}$ is

$$W(\mathscr{G}) = (V, r, E_{\mathsf{fml}}, E_{\mathsf{inf}}, k, \rho, \mu_{\rho,k})$$

where the parameters $k, \rho \in [0,1]$ and the weight function $\mu_{\rho,k}$ are defined as above.

Bonacich power, introduced in (Bonacich, 1987), is a widely-adopted eigenvector centrality measure in social networks. The basic idea is that the power of any individual depends on the power of those it is connected to; the difference between Bonacich power and the usual eigenvector centrality is the inclusion of a parameter β, which affects the meaning of centrality.

Definition 6 (Bonacich power) Let R be the adjacency matrix of the network (here we implicitly mean there is an indexing of all nodes in the matrix as natural numbers $1,\ldots,n$), and $R_{i,j}$ denotes the (i,j)-entry of R. The *Bonacich power* of $i = 1,\ldots,n$ is

$$p_i = \sum_{j=1}^{n} (\alpha + \beta p_j) R_{i,j} \tag{1}$$

where α, β are scalar constants. In matrix form, the vector of Bonacich power $\mathbf{p} = (p_1,\ldots,p_n)$ is

$$\mathbf{p} = \alpha (I_n - \beta R)^{-1} R \mathbf{e}_n \tag{2}$$

where I is the $n \times n$ identity matrix, and $\mathbf{e}_n$ is the column vector of ones with length n.

It is clear that different values of α and β result in different centrality measures. Here α only serves as a normalizing factor; It is selected such that the norm $||\mathbf{p}||$ equals $\sqrt{n}$. Thus, the most "evenly distributed" case is when $p_i = 1$ for every $i = 1,\ldots,n$.

For the matrix $I_n - \beta R$ to be invertible, the parameter β can be any value on the interval $[-\frac{1}{\lambda}, \frac{1}{\lambda}]$ where λ is the dominating eigenvalue of R. In some sense, it captures the contribution of ties of a node to its power. When $\beta = 0$, Bonacich power coincides with degree centrality. When $\beta > 0$, a node becomes more powerful as its neighbors become more powerful. In contrast, when β is negative, nodes become more powerful as their neighbors become less powerful[1].

Intuitively, when $\beta > 0$, it specifies how much the power of a person depends on the power of her neighbors. Thus the parameter β also corresponds to a managerial reality. The principle of *loss of control* states that as an individual acquires more social ties, the less her power depends on each of her neighbor's power Meagher (2003). We can reflect this principle by setting a range for β. In particular, since the capacity c indirectly indicates how much effect a person spends with each of their subordinates, friends or collaborator, we require β to be inversely proportional to the capacity minus one (the "minus one" is for the relation with its manager):

$$\beta < \begin{cases} \frac{1}{\lambda} & \text{if } \lambda > 1, \\ \frac{1}{c-1} & \text{otherwise} \end{cases} \tag{3}$$

To derive a measure of power in an organizational network, we adopt Bonacich power on the interaction graph of the network.

Definition 7 Let $i = 1,\ldots,n$ be a node in $\mathscr{G}$. Let Fml_i denote the set $\{j \mid 1 \le j \le n, (i,j) \in E_{\mathsf{fml}}\}$ of all subordinate of i, let Inf_i denote the set $\{j \mid 1 \le j \le n, (i,j) \in E_{\mathsf{inf}}\}$ of nodes connected from i by informal ties, and let μ_i be the node such that $(\mu_i, i) \in E_{\mathsf{fml}}$. We define the *power* p_i of i as discussed above, i.e., by (1) it is

$$p_i = \sum_{s \in \mathsf{Fml}_i} (\alpha + \beta p_s) + k \sum_{w \in \mathsf{Inf}_i \cup \{\mu_i\}} (\alpha + \beta p_w) \tag{4}$$

[1] A negative value of β implies a negative exchange power where connections to nodes with smaller power results in a bigger power. See e.g. (Bonacich and Lloyd, 2001) and (Franceschet and Bozzo, 2015)

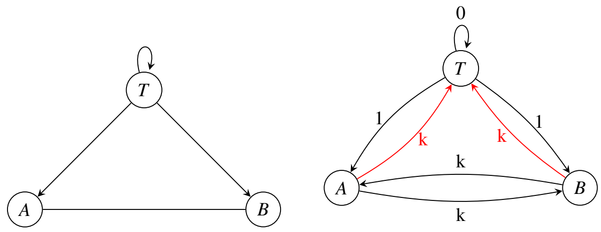


Fig. 2 An organizational network (on the left) and its weighted interaction graph (on the right)

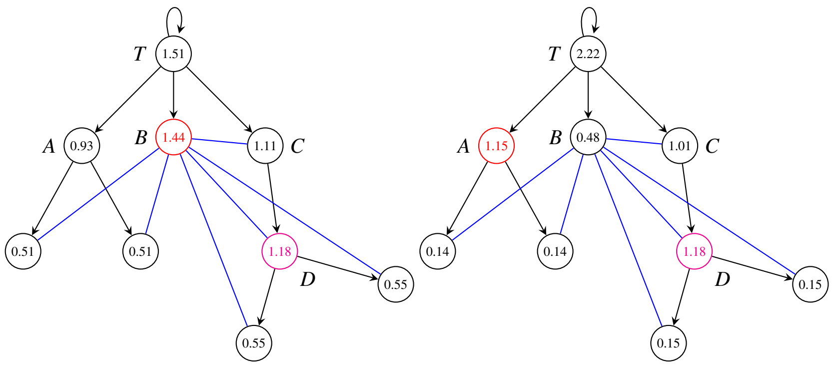


Fig. 3 Individual power with $k = 0.5$ (left) and $k = 0.1$ (right)

Now we can answer the questions stated in Example 1. Let the correlation coefficient $\delta = 0.5$. Assume that capacity of each node is 4, and $\beta = 0.3 < \frac{1}{3}$. Figure 3 shows the resulting power of each node when $k = 0.5$ (left) and $k = 0.1$ (right). When $k = 0.5$, even though B does not have a single subordinate, she is almost as powerful as the top manager while A and C possess similar power. However, when $k = 0.1$, A and C are much more powerful than B. Hence k captures in some sense the "importance" of informal ties.

Note also that the power of D, who has two subordinates (through formal ties) and an informal tie with B, exceeds her manager C in both cases above. We interpret this situation as follows: Since our notion of power aims to capture a node's ability to promote the node's ideas and decisions to others, it denotes in a sense a level of "real power". In the case of D, the real power is higher than her "nominal power", which is indicated by her level formal position. This may imply a form of "inconsistency" within the structure, as D may seek more formal recognition (say, in the form of promotion). Furthermore, C may experience certain loss of control over D's subordinates, as communication may not effectively pass down from C to these nodes. Such inconsistency gives the network a potential to change. Thus we say that in this case the organizational network is *unstable*. We stipulate that in a *stable* network, the levels in the formal tie hierarchy should truthfully reflect the actual power of individuals. In other words, the power of nodes is consistent with their respective levels in the reporting hierarchy.

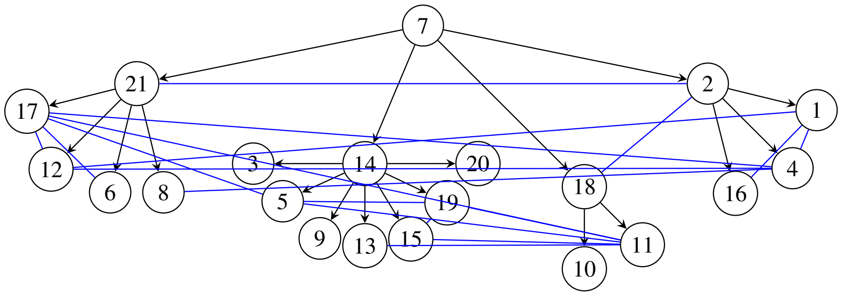


Fig. 4 Krackhardt and Hanson's hierarchy with 21 nodes.

Definition 8 An organizational network $\mathcal{G}$ is *stable* if for any nodes $i, j \in V$, $\mathsf{lev}(i) < \mathsf{lev}(j)$ implies that $p_i > p_j$ where $\mathsf{lev} : V \to \mathbb{N}$ maps every node to its level in the hierarchy of E_{fml}. We say that $\mathcal{G}$ is *unstable* if it is not stable.

This definition allows us to formally analyze several phenomena, which we elaborate in the subsequent sections.

3.4 Case study: Krackhardt and Hanson's network

Krackhardt and Hanson in (1993) studied a high-tech company with 21 managers. They analyzed the reporting hierarchy in the company, as well as reconstructed two types of social links on the same group of employees through a series of interviews – one type of social link is the advice relation (based on the interview question "*To whom do you go for advice?*") and the other is friendship (based on the question "*Who are your friends?*"). This data provides a real world case study for testing our model. In (Krackhardt and Hanson, 1993), the friendship links are directed; to fit our model we make them undirected by keeping only mutual friendship connections. The reporting hierarchy of the network is depicted in Figure 4: there is one top manager (7), four departments, managed by 2, 14, 17, and 21, respectively.

We considered separately a reporting hierarchy and a "hybrid" organizational network that contains both formal and informal ties. The results are listed in Table 1. From the results we draw two conclusions:

(1) There is no correlation between power and the age, nor years of service of employees.
(2) By taking into consideration the informal ties, the power of individuals on the bottom (leaves) increases while those on higher levels lose some of their power.

Note further that this network is unstable by our definition as 14 has more power than 7 in all the cases. We suggest two possible ways to interpret this fact:

- A high power of a manager may suggest high capability and performance, as well as a high workload. This could be used as a rigorous basis for certain rewards to the particular employee in the form of, for instance, bonuses or promotion. Such bonuses would increase the loyalty of the employee, and, as a result, decrease possible risks.

Table 1 Power in Krackhardt and Hanson's network, $\beta = 0.1$

	Attribute			Hierarchy			Hybrid		
ID	Dept	Age	YoS	$k = 0.1$	0.5	0.75	$k = 0.1$	0.5	0.75
1	4	33	9	0.06	0.25	0.34	0.18	0.68	0.84
2	4	42	20	1.35	1.41	1.42	1.43	1.55	1.48
3	2	40	13	0.07	0.32	0.45	0.11	0.41	0.5
4	4	33	8	0.06	0.25	0.34	0.22	0.85	1.06
5	2	32	3	0.07	0.32	0.45	0.2	0.77	0.97
6	1	59	28	0.06	0.27	0.37	0.1	0.39	0.49
7	-	55	30	2.39	2.13	1.96	2.33	1.67	1.33
8	1	34	11	0.06	0.27	0.37	0.1	0.38	0.47
9	2	62	5	0.07	0.32	0.45	0.07	0.24	0.28
10	3	37	9	0.05	0.23	0.31	0.05	0.18	0.21
11	3	46	27	0.05	0.23	0.31	0.26	1	1.24
12	1	34	9	0.06	0.27	0.37	0.19	0.72	0.9
13	2	48	0	0.07	0.32	0.45	0.11	0.42	0.51
14	2	43	10	3.06	2.98	2.88	2.99	2.34	1.96
15	2	40	8	0.07	0.32	0.45	0.16	0.59	0.73
16	4	27	5	0.06	0.25	0.34	0.1	0.36	0.45
17	1	30	12	0.06	0.27	0.37	0.27	1.03	1.29
18	3	33	9	0.92	1.03	1.07	0.96	1.04	1
19	2	32	5	0.07	0.32	0.45	0.24	0.89	1.09
20	2	38	12	0.07	0.32	0.45	0.07	0.24	0.28
21	1	36	13	1.77	1.8	1.78	1.9	1.68	1.53

- The node 14 is overwhelmed, as it has too many direct subordinates. To reduce this number and, therefore, to "stabilize" the structure, certain structural changes can be done. One of the possible solutions is to promote two of 14's most powerful direct subordinates (5 and 19) and distribute the rest (3, 9, 13, 15, 20) between them.

4 Stability and height

We continue in this section to study the relation between the notion of stability introduced above and the height of an organizational network. As the goal is to explore the formal tie hierarchy, throughout this and the next section, we assume the set of informal ties $E_{\mathsf{inf}} = \varnothing$.

4.1 Chain networks

Consider a network $\mathscr{C}_n$ consisting of n nodes $1, \ldots, n$ such that $E_{\mathsf{fml}} = \{(i, i+1) \mid 1 \leq i \leq n-1\}$; this is a *chain* of n nodes. A chain network does not appear as a typical management structure; clearly, a large number of nodes connected in a chain structure leads to ineffective communication as the top node will find it difficult to pass down her power to the bottom of the chain. In the following, we show that the notion of stability provides us a formal evidence for this ineffectiveness of the chain network.

By (4), the power of node i is

$$p_i = \begin{cases} \alpha + \beta p_2 & \text{if } i = 1 \\ k(\alpha + \beta p_{n-1}) & \text{if } i = n \\ \alpha + \beta p_{i+1} + (\alpha + \beta p_{i-1})k & \text{if } 2 \leq i \leq n-1 \end{cases} \tag{5}$$

Lemma 1 *If $0 < \beta \leq k < 1$, $\mathscr{C}_n$ is unstable for any $n > 2$.*

Proof By (5) we get the following derivation

$$\begin{aligned} p_1 &= \alpha + \beta p_2 \\ &= \alpha + \beta(\alpha + \beta p_3 + (\alpha + \beta p_1)k) \\ &= \alpha + \alpha\beta + \alpha\beta k + \beta^2 p_3 + \beta^2 k p_1 \end{aligned}$$

In other words, $p_1 = \dfrac{\alpha + \alpha\beta + \alpha\beta k + \beta^2 p_3}{1 - \beta^2 k}$.

Similarly, by (5) we get

$$\begin{aligned} p_2 &= \alpha + \beta p_3 + (\alpha + \beta p_1)k \\ &= \alpha + \alpha\beta k + \alpha k + \beta p_3 + \beta^2 k p_2 \end{aligned}$$

In other words, $p_2 = \dfrac{\alpha + \alpha k + \alpha\beta k + \beta p_3}{1 - \beta^2 k}$.

Combining the above, we get

$$p_1 - p_2 = \frac{\alpha\beta + \beta^2 p_3 - \alpha k - \beta p_3}{1 - \beta^2 k}$$

Since $1 - \beta^2 k > 0$ for any positive $\beta, k < 1$, $p_1 - p_2$ is negative whenever $\alpha\beta + \beta^2 p_3 < \alpha k + \beta p_3$. Clearly, since α is positive and $\beta^2 p_3 < \beta p_3$, $\beta \leq k$ implies $p_1 < p_2$ for any $n > 2$ □

Lemma 2 *The chain $\mathscr{C}_n$ is stable if and only if $p_1 > p_2$.*

Proof We only need to prove the "only if" direction. Suppose $p_1 > p_2$. Then by (5), $\beta p_2 > \beta p_3 + (\alpha + \beta p_2)k$. Since $(\alpha + \beta p_2)k > 0$, $p_2 > p_3$. Consequently, we have $\beta p_2 > \beta p_4 + (\alpha + \beta p_2)k$, and hence $p_2 > p_4$. Inductively, we may show that $p_2 > p_i$ for any $i = 3, \ldots, n$.

We now prove that $p_i > p_{i+1}$ for any $i = 3, \ldots, n-1$. Suppose on the contrary that $i > 2$ is the smallest such that $p_{i+1} \geq p_i$. Then by (5), we have $p_{i+2} + kp_i \geq p_{i+1} + kp_{i-1}$. Since $p_{i-1} > p_i$, it must be that $p_{i+2} \geq p_{i+1} \geq p_i$. Iterate the same argument we conclude $p_n \geq p_i$. However, by (5) again this would mean that

$$\alpha + k\beta p_{n-1} \geq \alpha + \beta p_{i+1} + k(\alpha + \beta p_{i-1}) > \alpha + k\beta p_{n-1}$$

A clear contradiction. Hence such an i does not exist and we conclude $p_1 > p_2 > \cdots > p_n$. □

Combining the two lemmas above, we get the following theorem.

Theorem 1 *Fix k and β such that $0 < k < 1$ and $0 \leq \beta < 1$. There is some $n \geq 1$ such that $\mathscr{C}_m$ is unstable for any $m \geq n$.*

Proof Lemma 1 shows the statement holds when $\beta \leq k$ (where $n = 3$). Suppose $\beta > k$, by Lemma 2 we need to find n such that $p_1 < p_2$ holds in $\mathscr{C}_n$. Iteratively applying (5), we get that

$$\begin{aligned} p_1 &= \alpha + \alpha\beta + \cdots + \alpha\beta^{n-2} + k(\beta(\alpha + \beta p_1) + \beta^2(\alpha + \beta p_2) + \\ &\quad \cdots \beta^{n-1}(\alpha + \beta p_{n-1})) \\ p_2 &= \alpha + \alpha\beta + \cdots + \alpha\beta^{n-3} + k((\alpha + \beta p_1) + \beta(\alpha + \beta p_2) + \\ &\quad \cdots \beta^{n-2}(\alpha + \beta p_{n-1})) \end{aligned}$$

Subtracting the first equation with the second, we get

$$\begin{aligned} p_1 - p_2 &= \alpha\beta^{n-2} - \alpha k \left(1 + \beta + \beta^2 + \cdots + \beta^{n-2}\right)(1-\beta) - \\ &\qquad \beta k \left(p_1 + \beta p_2 + \beta^2 p_3 + \cdots + \beta^{n-2} p_{n-1}\right)(1-\beta) \\ &= \alpha\beta^{n-2} - \alpha k \frac{1-\beta^{n-1}}{1-\beta}(1-\beta) - (1-\beta)\beta k \sum_{i=0}^{n-2} \beta^i p_{i+1} \\ &= \alpha\beta^{n-2} - \alpha k(1-\beta^{n-1}) - (1-\beta)\beta k \sum_{i=0}^{n-2} \beta^i p_{i+1} \end{aligned}$$

Since $0 \leq \beta < 1$, $p_1 < p_2$ if $\alpha\beta^{n-2} \leq \alpha k(1-\beta^{n-1})$. We solve this inequality and get

$$n \geq \left\lceil \log_\beta \frac{k}{1+k\beta} \right\rceil + 2$$

Thus the theorem is proved. □

Theorem 1 justifies that the chain networks are not suitable for organizations from the point of view of stability: The network will become unstable as the number of people (and thus levels) increases.

Remark 3. The above example also provides a mathematical explanation for the use of backflows in the model. Recall that a backflow represents the reciprocal interaction effect from a subordinate to the supervisor in a formal relation, which means that the subordinate is empowered by the supervisor through support and privilege, and it is given a weight of k in the weighted interaction graph. If such weight is not given, then the adjacency matrix R of the weighted interaction graph of a chain network will be $R_{ij} = 1$ if $j = i-1$ and 0 otherwise. The corresponding Bonacich power vector will be

$$\mathbf{p} = \begin{pmatrix} \alpha + \alpha\beta + \cdots + \alpha\beta^{n-3} + \alpha\beta^{n-2} \\ \alpha + \alpha\beta + \cdots + \alpha\beta^{n-3} \\ \cdots \\ \alpha + \alpha\beta \\ \alpha \\ 0 \end{pmatrix}$$

In this case the power is strictly decreasing from the top of the chain to the bottom and the structure will remain stable regardless of the size of the chain, which does not meet with our intuition. Thus the weight added to backflows in our model is necessary.

4.2 Perfect tree networks

With a similar but more involved technical analysis, we can generalize Theorem 1 to perfect tree networks.

Definition 9 Fix $d > 1$. A *perfect d-ary tree network* is an organizational network where the formal ties E_{fml} form a tree in which every non-leaf node has exactly d children and all leaves are at the same level in the tree. We use $\mathscr{D}_h^d$ to denote a perfect d-ary tree network of height h. The number d is called the *arity* of the tree.

Note that a unary perfect tree is simply a chain network. The arity d in the perfect tree network equals to the capacity c minus one, and therefore we get $d\beta < 1$ by our earlier assumption (3) that $\beta < \frac{1}{c-1}$. Similarly to (5), the power of node i is computed by

$$p_i = \begin{cases} d(\alpha+\beta p_2) & \text{if } i=1 \\ k(\alpha+\beta p_{h-1}) & \text{if } i=h \\ d(\alpha+\beta p_{i+1})+(\alpha+\beta p_{i-1})k & \text{if } 2 \le i \le h-1 \end{cases} \tag{6}$$

The following is a lemma that generalizes Lemma 1.

Lemma 3 *If $\beta \le \frac{k}{d^2}$, then any perfect d-ary tree network $\mathscr{D}_h^d$, with $d \ge 1$ and height $h > 2$, is unstable.*

Proof By (6) we derive the following equations:

$$p_1 = \frac{d\alpha + d^2\alpha\beta + d\alpha\beta k + d^2\beta^2 p_3}{1 - d\beta^2 k}$$

and

$$p_2 = \frac{d\alpha + \alpha k + d\alpha\beta k + d\beta p_3}{1 - d\beta^2 k}$$

Combining the above we get

$$p_1 - p_2 = \frac{d^2\alpha\beta + d^2\beta^2 p_3 - \alpha k - d\beta p_3}{1 - d\beta^2 k}$$

By assumption we get $d\beta < 1$. Thus $1 - d\beta^2 k > 0$ for any positive $k < 1$. Therefore, $p_1 - p_2$ is negative whenever

$$d^2\beta - k < \frac{d\beta p_3(1-d\beta)}{\alpha}$$

This clearly holds for any $\beta \le \frac{k}{d^2}$ □

The next theorem generalizes Theorem 1 to d-ary perfect trees. Lemma 3 handles the case when $\beta \leq \frac{k}{d^2}$. The case when $\beta > \frac{k}{d^2}$ can be proved similarly to Theorem 1.

Theorem 2 *For any arity $d \geq 1$, there is a constant $c_d \in \mathbb{R}$ such that any perfect tree network $\mathscr{D}_h^d$ is unstable if*

$$n \geq c_d + \log_{d\beta}(1/d), \tag{7}$$

where an upper bound for the constant c_d is defined as $c_d \leq \log_{d\beta} k/(1+k\beta)+2$

Proof Lemma 3 shows that the statement holds when $\beta \leq k/d^2$ (where $h \geq 3$). Suppose $\beta > k/d^2$. Iteratively applying (6), we get that

$$\begin{aligned} p_1 &= d\alpha + d\alpha(d\beta) + d\alpha(d\beta)^2 + \cdots + d\alpha(d\beta)^{h-2} + \\ &\quad\; d\beta k(\alpha+\beta p_1) + (d\beta)^2 k(\alpha+\beta p_2) + \cdots + (d\beta)^{n-1}k(\alpha+\beta p_{h-1})) \\ &= d\alpha \sum_{j=0}^{h-2}(d\beta)^j + k\sum_{r=1}^{h-1}(d\beta)^r(\alpha+\beta p_r) \\ p_2 &= d\alpha + d\alpha(d\beta) + d\alpha(d\beta)^2 + \cdots + d\alpha(d\beta)^{h-3} + \\ &\quad\; k(\alpha+\beta p_1) + (d\beta)k(\alpha+\beta p_2) + \cdots + (d\beta)^{h-2}k(\alpha+\beta p_{h-1})) \\ &= d\alpha \sum_{j=0}^{h-3}(d\beta)^j + k\sum_{r=1}^{h-1}(d\beta)^{r-1}(\alpha+\beta p_r) \end{aligned}$$

Subtracting the first equation by the second, we obtain

$$\begin{aligned} p_1 - p_2 &= d\alpha(d\beta)^{h-2} - \alpha k\left(1-(d\beta)^{h-1}\right) - \\ &\quad\; \beta k(1-d\beta)\left(p_1 + d\beta p_2 + (d\beta)^2 p_3 + \cdots + (d\beta)^{h-2}p_{h-1}\right) \\ &= d\alpha(d\beta)^{h-2} - \alpha k\left(1-(d\beta)^{h-1}\right) - (1-d\beta)\beta k\sum_{i=0}^{h-2}(d\beta)^i p_{i+1} \end{aligned}$$

Since $0 \leq \beta < 1/d$ by our assumption, $p_1 < p_2$ if $d\alpha(d\beta)^{h-2} \leq \alpha k(1-(d\beta)^{h-1})$. Solving this inequality we get

$$h \geq \left\lceil \log_{d\beta}\frac{k}{1+k\beta}\right\rceil + 2$$

Thus the theorem is proved. □

Remark 4. The proof of Theorem2 gives us an upper bound for the constant c_d which only depends on d:

$$c_d \leq \log_{d\beta} k/(1+k\beta)+2 \tag{8}$$

The inequality (8) provides a theoretical upper bound on the number of levels for a perfect d-ary tree to stay stable. Note that this bound may be much larger than the minimum value for such c_d. For example, using UCINET (Borgatti et al., 2002), we

Table 2 Stable d-ary tree networks: theoretical bound on the number of layers computed as $n = \left\lfloor \log_{d\beta} \frac{k}{d(1+k\beta)} \right\rfloor + 2$

span of control	$\beta < \frac{1}{d}$	k =0.1	k =0.5	k =0.75
1	0.9	25	13	10
2	0.45	31	18	15
3	0.3	35	21	18
4	0.225	38	23	20
5	0.18	40	25	22
6	0.15	42	27	23
7	0.128571	43	28	25
8	0.1125	44	29	26
9	0.1	45	30	27
10	0.09	46	31	28

computed the actual limits on numbers of hierarchy levels with $k = 0.5$: for $d = 2$, it is 5; for $d = 3$, it is 8 (the theoretical bounds are 18 and 21, respectively.) Furthermore, by increasing the span of control (i.e., d) of nodes, the theorem implies a logarithmic growth on the bounds on the number of levels. Theoretical bounds for small values of d can be found in Table 2.

We now interpret the main result (Theorem 2) of the section. A general and significant organizational change trend in the last 50 years is the shift from *tall hierarchies* with many levels to *flat hierarchies*, where the number of levels is kept bounded. Research has found that most large companies changed their structures to the flattened ones in the past 3-4 decades (Wulf, 2012), e.g., back in 1950s companies had up to twenty layers in their hierarchies while by the end of the twentieth century they had been trimmed to five or six. We conjecture that this delayering process implies some fundamental truth regarding organizational networks. The well-known theory of "six degrees of separation" has been extensively studied and verified in the social network analysis community(Newman, 2008). This theory states that six is a natural bound in the acquaintance relation on the distance between two people in the world. Analogously, it seems that for organizational networks, a bound on the number of levels of the hierarchy also exists. Moreover, this upper bound is natural as it allows the top manager to maintain control over the hierarchy. Theorem 2 provides an evidence of the existence of such a bound: *As the arity d is bounded (by capacity of individuals), the maximum height for a perfect tree network to maintain power consistency is bounded.* It will be an interesting future work to study the exact value of such a bound.

5 Flattening – workplace democratization or power concentration?

Flattening (or delayering) is the phenomenon that an organization acquires a new structure by decreasing the number of hierarchy levels. It reflects a notable trend in organizational structure in the last half a century. Researches show that in last few decades most large companies changed their structures to the flattened ones. For instance, as it is shown in (Wulf, 2012), the average number of those who reported directly to the CEOs in large companies was 4.7 in 1980, and 9.8 in 1999.

The alleged reason for flattening include empowering staffs with decision making entitlement, increasing flexibility of employees, pushing down decision making and improving information flow and, thus, enabling faster decision. In general, flattening is viewed as a strategy for democratization of the work place. However, some researchers also argue that flattening leads to the opposite effect – more control and decision making is concentrated on the top in the flattened organization, and hence it is also a strategy of strengthening controls by the top managers (Teubner, 2001; Wulf, 2012).

In this section, we analyze the flattening process from the point of view of power. Based on our organizational network model, we argue that most employees indeed obtain more power through flattening, although the average power decreases. Moreover, the upper level managers are the ones whose power improves considerably.

Example 2. Consider a network $\mathscr{A}$ representing a perfect tree network without informal ties, such that there are 31 nodes and the capacity of each node is 3. Suppose that the span of control is exactly two for all nodes. This network can be represented as a perfect binary tree of height 5 as in Figure 5. Suppose we increased the span of control of each node to 5, and the total number of nodes is kept the same, the network becomes a 5-ary perfect tree with height 3; See Figure 6. We ran several tests computing the individual power with different parameters and list results in the Table 3, where n is the number of nodes and ℓ is a hierarchy level. One can see that the average power in $\mathscr{A}$ is strictly greater than the average power in $\mathscr{B}$. Similarly, the figures for the case $k = 0.1$ shows that flattening negatively impact power of individuals in the network: only *four* nodes increase their power while 11 others become less powerful and 16 stay the same. However, when we even slightly increase k to 0.15, a majority of nodes increase their power. Moreover, when $k = 0.8$, the network $\mathscr{A}$ becomes unstable while $\mathscr{B}$ is still stable.

In (Kubheka et al., 2013), the authors carried out a survey in a company after introducing a new flat structure. The survey showed that 65.9% of employees were very happy, 26.3% were not happy, and 7.8% were not concerned about the change. This correlates very well with the results we obtain: the computation reveals that 64.5% (20 out of 31) of nodes when $k = 0.15$ become more powerful.

Through this example, we argue the following rather paradoxical aspect of flattening in an organizational network *(flattening paradox): Flattening decreases the average power in the company, but empowers most employees.*

The above displays a complicated relation between organizational power and structural properties of the formal tie hierarchy. To develop a better understanding of this relation, we perform a series of experiments.

Experiment 1. [Perfect tree hierarchies] We first focus on perfect tree hierarchies. Here the goal is to investigate the distribution of power in perfect trees of different

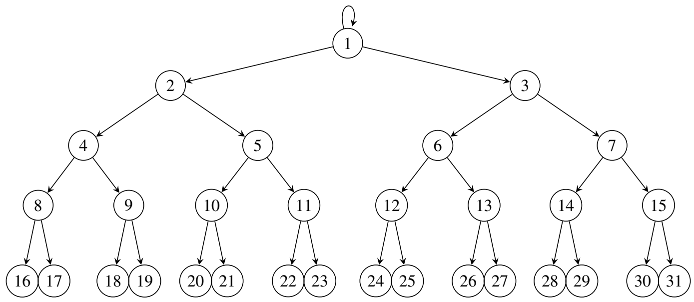


Fig. 5 Network $\mathscr{A}$ with 31 nodes

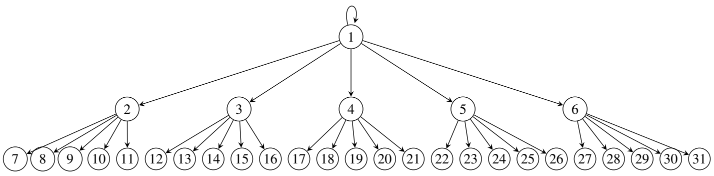


Fig. 6 Network $\mathscr{B}$ with 31 nodes

Table 3 Comparing individual power in networks $\mathscr{A}$ and $\mathscr{B}$ (tests performed using UCINET (Borgatti et al., 2002))

	$\mathscr{A}: d = 2, \beta = 0.45$				
	n	k=0.1	k=0.15	k=0.5	k=0.8
$l = 0$	1	2.613	2.553	2.193	1.949
$l = 1$	2	2.175	2.181	2.181	2.149
$l = 2$	4	1.522	1.527	1.556	1.587
$l = 3$	8	0.819	0.828	0.864	0.882
$l = 4$	16	0.070	0.100	0.252	0.323
max		2.613	2.533	2.193	2.149
min		0.070	0.100	0.252	0.323
avrg		0.668	0.685	0.765	0.800
	$\mathscr{B}: d = 5, \beta = 0.18$				
$l = 0$	1	3.503	3.450	3.056	2.733
$l = 1$	5	1.929	1.941	1.966	1.936
$l = 2$	25	0.070	0.103	0.306	0.437
max		3.503	3.450	3.056	2.733
min		0.070	0.103	0.306	0.437
avrg		0.481	0.507	0.662	0.753

heights and arities. Using CORPNET we generate perfect tree hierarchies of various heights and set the parameter $k = 0.1$ and $\rho = 0$. The arity of the trees is set between 2 and 9, while the height h is between 2 and 7. When the tree has more than three levels and the sum of the arity and height exceeds 9, the tree becomes too large for the software to handle. So we only provide results for the remaining cases. In particular, Table 4, Table 5 and Table 6 list the power of the roots of the trees, the average power of all nodes, and its variance, respectively.

Expectedly, when arity stay fixed, as the tree becomes taller, the root gets more powerful; at the same time, the average power among all nodes drops, which means that the distribution of power becomes more uneven. This observation gives the impression that when we fix individuals' capability, power is distributed more evenly in flattened hierarchies than taller hierarchies. However, when taking into account of possible changes in capability, the situation is rather different. As shown in the tables below, if the height of the tree is fixed, as arity increases (that is, as people's span of control increases), the root becomes significantly more powerful, while the average power drops and variance increases.

We then plot the distribution of power across all levels of perfect tree hierarchies with arity ranging in 2,3,4 and heights $3 \leq h \leq 8$; See Fig. 7. In each plot, as the hierarchy flattens, the power of any level strictly decreases; At the mean time, when we consider the distance of nodes from the leaves, for almost any k, the kth last level of the trees gains power slightly. When the arity increases, the difference between upper levels and lower levels becomes increasingly visible. One can interpret this intuitively: when managers gain more subordinates, there is a wider power gap between an upper and a lower level.

Table 4 The Power of Top-level managers (roots) in perfect trees of varying heights and arities

arity\height	8	7	6	5	4	3
2	3.5684	3.2424	2.922	2.6126	2.317	2.031
3		5.0269	4.4021	3.7703	3.1529	2.5671
4			5.7099	4.8204	3.9219	3.0557
5				5.7602	4.6223	3.5034
6					5.2635	3.9172
7					5.8549	4.3025
8					6.4049	4.6639
9					6.9198	5.0049

Table 5 The average power of nodes in perfect trees of varying heights and arities

arity\height	8	7	6	5	4	3
2	0.5865	0.5663	0.5625	0.6503	0.6795	0.6883
3		0.5620	0.5652	0.5710	0.5806	0.5923
4			0.5080	0.5109	0.5172	0.5277
5				0.4679	0.4722	0.4807
6					0.4376	0.4445
7					0.4100	0.4156
8					0.3873	0.3918
9					0.3680	0.3726

Table 6 Variance of power of nodes in perfect trees of varying heights and arities

arity\height	8	7	6	5	4	3
2	0.5876	0.5840	0.5759	0.5538	0.5383	0.5263
3		0.6841	0.6810	0.6740	0.6629	0.6492
4			0.7277	0.7052	0.6686	0.6561
5				0.7811	0.7771	0.7690
6					0.8085	0.8024
7					0.8319	0.8273
8					0.8500	0.8465
9					0.8646	0.8613

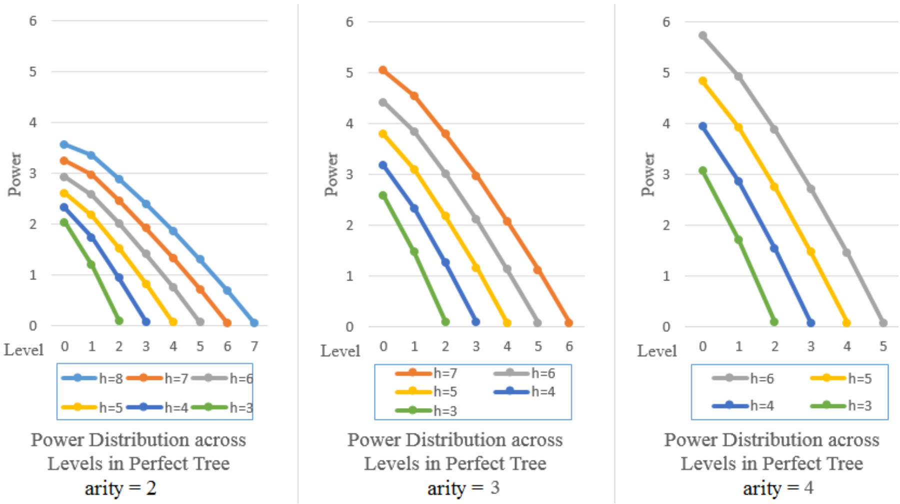


Fig. 7 The power of three perfect tree networks with arities 2,3, and 4. The plots show the power across all levels of the hierarchies.

Experiment 2. [Random tree hierarchies] Flattening may refer to two types of structural changes in an organization: The first type reduces the height of an organization by removing nodes, while not changing the capacity of its managers. An effect of this process may be conceptually revealed in the results of Experiment 1. The second type reduces the height of a hierarchy by improving the capacity of nodes, while not changing too much the number of members of the organization. We now focus on this type of changes. Here perfect tree hierarchies no longer apply as they tend to have very different sizes when arities and heights differ. For this experiment, we simulate formal tie hierarchies using a random tree model. Procedure 1 is a simple procedure that produces trees with a given height h, where the degrees of all internal nodes are taken from a normal distribution with mean d and standard deviation.

Procedure 1 has been implemented by CORPNET. We generate 10 random trees for each height between 3 and 7 while setting $s = 1$ and $k = 0.1$. We then compute the average power of nodes in each level and take the average over all trees of the same height. The details of the generated trees are listed in Table 7. Starting from random

Procedure 1 RandomTree(d,h,s) where $h \in \mathbb{Z}$, $d,s \in \mathbb{R}$

Initialize an empty tree and add a root
$\mathsf{lev} := 0$
while $\mathsf{lev} < h$ **do**
  for every node u at level lev of the current tree **do**
    Randomly generate a number m in a normal distribution with mean d and standard deviation s
    Create m children for the node u
  end for
end while
Return the constructed tree

trees with height 7 in the top row, we can imagine a flattening process that reduces the height of the hierarchy iteratively, while expanding the expected arity. Hence the first row corresponds to the tallest hierarchy where the last row corresponds to the flattest. The third column lists the average number of nodes of trees for each height; as the hierarchy flattens, the number of nodes reduces slightly.

Table 7 Random trees generated by Procedure 1 with height h and expected arity d. The third column shows the average number of nodes of the generated trees. The subsequent columns show the average power of nodes across all levels. The last column shows the average power of all nodes for height h.

h	d	Avg size	Level 0	1	2	3	4	5	6	Avg
7	1.7	84	1.804	1.785	1.232	1.293	1.355	0.878	0.071	0.647
6	2	76.4	1.873	1.997	1.738	1.358	0.953	0.070		0.647
5	2.5	67.2	2.279	2.106	1.649	1.247	0.071			0.611
4	3.5	65.4	2.930	2.400	1.567	0.067				0.536
3	7.5	65.2	4.314	2.445	0.056					0.399

We then plot the average power of nodes across all levels for each height; each curve shows changes to the average power of nodes at a particular level as the organization flattens. We also illustrate the change of power in the last level, and in the second-to-last level; See Fig. 8. From these plots, one can identify the following pattern: As the hierarchy is flattened, the root (level 0) gains the most power, while the other levels will tend to lose power as it gets closer to the last level. In particular, the leaves (operatives) loses power as a result of flattening, while levels that are above the last level tend to gain power. Overall, this flattening process reduces the average power in the hierarchy.

These results are consistent with our observation in Example 2, where flattening gives more power to upper levels of the hierarchy. In general, flattening empowers managers in the organization, and therefore a large number of individuals would prefer a flat hierarchy than a taller one. However, the process also reduces the power of the operatives.

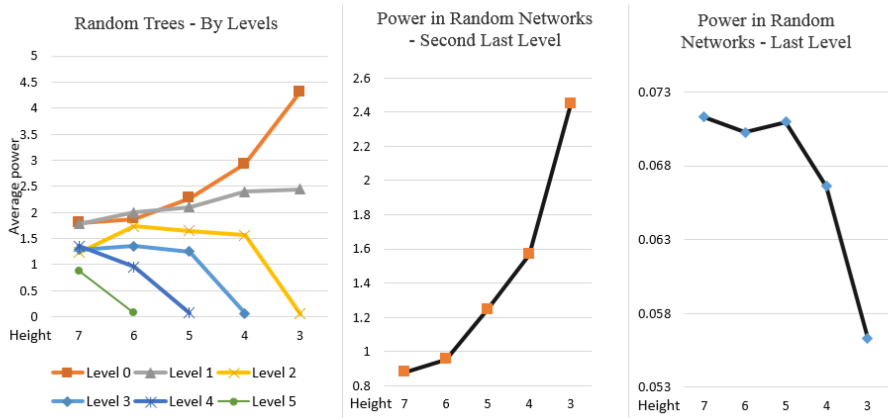


Fig. 8 (left) The average power of nodes across all levels in different random trees as the hierarchy flattens. Each curve corresponds to a particular level, and the horizontal axis refers to the different trees. (center) average power of nodes in the last level of these trees. (right) average power of nodes in the second-to-last level.

6 Understanding informal ties

Our analysis so far discusses only formal tie hierarchies, where no undirected informal tie is maintained. A long argument in management studies addresses the importance of informal social relations among members of organizations. Such relations, such as collaboration, advice or friendship, play important roles in the cohesion and effectiveness of the organizational structure (Krackhardt and Hanson, 1993; Cross et al., 2002). It is, therefore, crucial to incorporate informal ties into our analysis. In this section, we no longer assume that $E_{\mathsf{inf}} = \varnothing$ and aim to find how these informal ties affect the power of individuals.

6.1 A benchmark for organizational networks

To correctly predict the impacts of informal ties on the network, it is imperative to adopt a reasonable benchmark for generating random social links. Popular benchmarks models such as planted ℓ-partition, relaxed caveman graphs, and the LFR graphs (Fortunato, 2010) are not suitable for organizational networks as the informal ties generated by these models will be independent of the formal tie hierarchy of the organization. Naturally, the establishment of informal ties in an organizational network is significantly affected by the position of individuals in the reporting hierarchy.

Homophily is a recurring theme in social network studies which means that individuals have a natural tendency to bond with others that are similar to themselves. Numerous management studies also observe a kind of homophily in the workplace: Employees in an organization are more likely to establish social connections within a certain “circle”, such as departments, offices. Moreover, individuals tend to establish personal ties with those that are at the same or similar levels in the hierarchy

(McPherson et al., 2001; Castilla, 2011). No social network model so far has been defined taking into account this multiplex view of an organization. Hence we provide a new benchmark graph; our view is that the new benchmark graph shall exhibit community structure that reflects the observed homophily phenomenon in an organization.

We adopt a distributed approach where each node randomly chooses to set up informal ties with other, in such a way that closer nodes (in distance) enjoy a higher "probability" of an informal tie. The procedure is described in Procedure 2.

Procedure 2 RandomInformalTies(T,γ,p) where $T=(V,E_{\mathsf{fml}})$ is a formal tie hierarchy, $\gamma\in\mathbb{N}$, $p\in[0,1]$

Initial a set of undirected edges $E_{\mathsf{inf}} := \varnothing$
for Every node $u\in V$ **do**
    Compute the level $\ell(u)$ of u in T
end for
for Every node $u\in V$ **do** $\triangleright$ Generate a probability distribution of all nodes in $V\setminus\{u\}$ by setting for any node $v\neq u$ a probability $\mathsf{Pr}_u(v)$
    for $v\in S_u$ **do**
        Compute the *lowest common ancestor* of u and v, that is, a node w such that w is an ancestor of both u and v, and $\ell(w)$ is maximal.
        Set $\Delta := \max\{\ell(u),\ell(v)\}-\ell(w)$
        Set $\mathsf{Pr}_u(v) := p^{\Delta}$
    end for
    Randomly select γ nodes that are not u based on the probability Pr_u
    For each selected node x, add to E_{inf} an edge (u,x) if it is not in E_{inf} already
end for
Return the constructed set E_{inf} of undirected edges

To generate a random organizational network with both formal and informal ties, we first apply a procedure that constructs a formal tie hierarchy $T=(V,E_{\mathsf{fml}})$, and then apply Procedure 2 with the parameters T, γ, p to derive the set of informal ties. The resulting network not only captures main characteristics of social networks (such as community structure), but also entails reporting hierarchy of the network; see Fig. 9 for a generated network visualized using a force-directed method. The community structure clearly resembles departments and reflect hierarchical levels in an organization.

Example 3. To further validate our benchmark, in Figure 10(a), we consider a perfect 3-ary tree hierarchy with no social ties on it. We perform Newman's spectral graph clustering algorithm on this network and show the identified cluster (i.e. community) of each node, as indicated by its color (Fortunato, 2010). In this case, we can see that clusters reflect departments, but not the levels of hierarchy. However, when we enrich this hierarchy with a generated social network, we get a quite different picture in Figure 10(b). The resulting clustering clearly indicates the following pattern:

1. Clusters typically reflect departments: people in the same department tend to form a cluster.

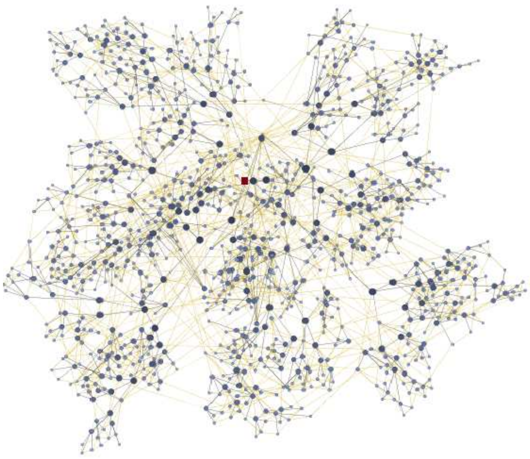
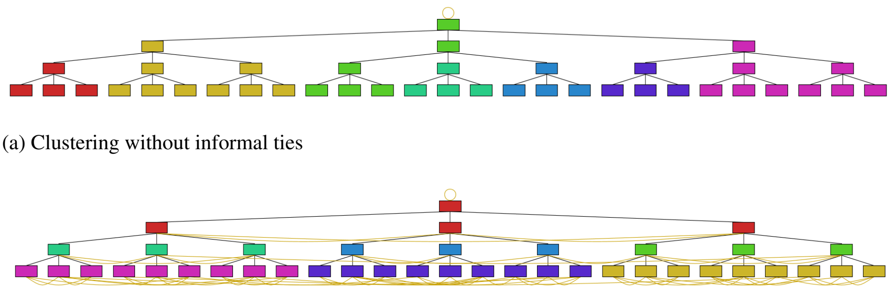


Fig. 9 A randomly generated network for $d = 3$ and 7 levels. Blue and yellow lines are formal and informal ties, resp. The root is the brown square. Sizes of nodes indicates their power. The graph is generated and visualized by CORPNET.



(a) Clustering without informal ties

(b) Clustering with informal ties

Fig. 10 Power of informal ties on community formations in organizations. Clusters are indicated by different colors. The graphs and their clustering are computed by CORPNET

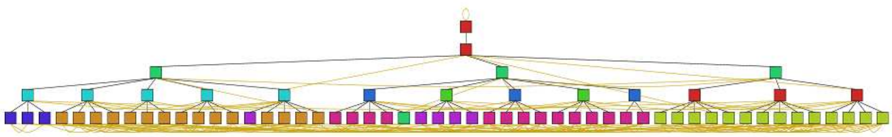


Fig. 11 Random tree and random social network. The graph is generated and visualized by CORPNET

2. Clusters also reveal levels: managers in the same level tend to form a cluster.

Clustering of the same nature is observed for another network, with randomly generated formal tie hierarchy using Procedure 1; See Figure 11.

Table 8 Power Distribution in Two Perfect Tree Hierarchies

β	0.3		0.07	
d	3		10	
n	1093		1111	
Level:	k = 0.1	k =0.5	k =0.1	k=0.5
0	5.03	4.8	6.16	5.66
1	4.53	4.79	4.86	4.7
2	3.77	3.9	2.89	2.85
3	2.95	2.91	0.05	0.22
4	2.06	1.96	-	-
5	1.1	1.07	-	-
6	0.06	0.24	-	-

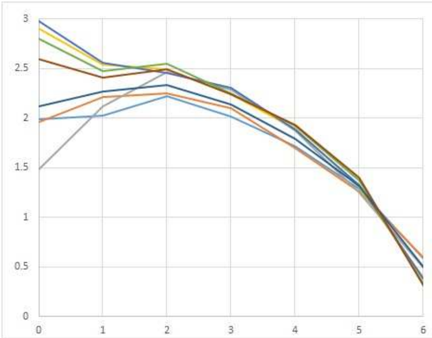
6.2 The importance of informal ties

We use two experiments to test how informal ties affect power in an organizational network. The first goal is to compare the power distribution before and after informal ties are introduced to a formal tie hierarchy. The second goal is to see how the formal tie hierarchy plays a part in power with the presence of informal ties. The third goal is to see how changing *density* of informal ties in the network affects structural properties and power. All experiments are carried out using CORPNET.

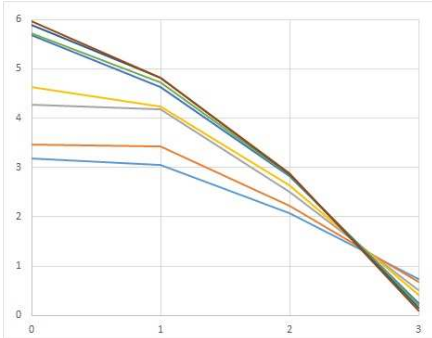
Experiment 3. [Stability and Informal Ties] We consider two perfect trees: one has the span of control $d = 3$ and 7 levels, the other one has the span of control $d = 10$ and 4 levels. The resulting values of individual power in both networks are listed in Table 8. Note that both hierarchies are stable. We then generate random informal ties with $\gamma = 8$ and different parameter p over the formal tie hierarchies. In Figure 12, we plot the distribution of average values of power at each hierarchy level in eight randomly generated social networks over the tall and the flat organizations. In 12(a), one may see that only two out of eight generated networks are stable. However, as shown in Figure 12(b), in the flat organizations the non-reporting relations does not change the power distribution: all the networks stay stable.

As the result shows, the taller hierarchy's power consistency is very fragile – adding informal ties in all experiments makes the network unstable. On the other hand, the flattened hierarchy stays stable in most of our experiments with $k = 0.1$ and the probability $p = 0.5$ of existing friendship between two nodes which have the same direct manager. When the probability is small, corporate networks stay stable even with $k = 0.5$. Thus, this experiment justifies that: *As an organizational hierarchy has more levels, it is much more likely to be destabilized by non-reporting connections.*

Experiment 4. [Perfect Tree Networks with Social Ties] We generate three perfect tree formal tie hierarchies. The first is a tall hierarchy with arity 2 and height 8; There are 255 nodes in the tree. The second is a flat hierarchy with arity 6 and height 4; There are 259 nodes in the tree. The third one is a hierarchy in between the previous



(a) Distribution of power in the tall organization



(b) Distribution of power in the flat organization

Fig. 12 Average values of power at each hierarchy level in randomly generated social networks (a) in the tall organization, and (b) in the flat organization. The different lines indicate differences in "density" of the informal ties; in general, a denser informal relation causes a more even distribution of power across levels, hence a "flattened" (less-steep) curve.

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two, with arity 4 and height 5; There are 341 nodes in the tree. We then generate informal ties by fixing $p = 0.5$ and varying parameter $\gamma \in \{0, 1, \ldots, 10\}$.

In Figure 13, we plot the average power of nodes and its variance in each network. It is clear that as more informal ties are introduced to the network the average power in all network increases, although this increase is more evident in the flattened networks and when there are fewer informal ties. On the other hand, the variance drops significantly as more informal ties are introduced, showing that power tends to be more evenly distributed with informal ties.

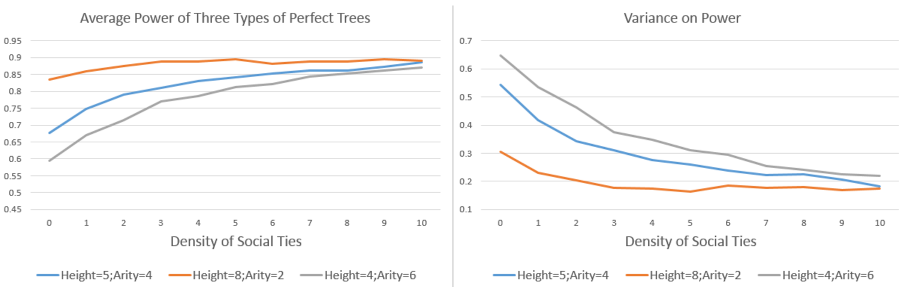


Fig. 13 Power (left) and variance (right) for three types of perfect trees with increasing density of informal ties.

We next measure the level of stability in the network as more informal ties are added. To do that, we recall Definition 8 where a network is called *stable* if power in upper levels is consistently higher than power in lower levels. We thus call an internal node (a manager) *u stable* if there is no node *v* with a lower level than *u* and whose power exceeds the power of *u*; otherwise, the internal node is called *unstable*.

Note that this definition only applies to managers (people with subordinates) of the network. In the following, we introduce a measure for the level of *instability* within a network.

Definition 10 The *instability index* ι of the network is the proportion of unstable internal nodes among the set of all internal nodes in the formal tie hierarchy.

Based on the definition above, in Figure 14 (left) we plot the instability index for all three networks as more informal ties are added. In all three formal tie hierarchies, instability increases as more informal ties are added, however, the tall hierarchy is especially unstable only after a small set of informal ties ($\gamma = 1$) is added, while in the flat hierarchy, the instability index is kept low (lower than 0.1) even when a large number of informal ties are added. This further confirms the finding in Experiment 3 about the comparisons between tall and flat hierarchies.

We then apply Newman's spectral community detection algorithm to the networks with informal ties and compute *modularity* in each case. Modularity is a standard measure for a network and indicates how "clustered" a network is, i.e., how strong the network exhibits community structure with the detected communities Newman (2006). Intuitively, the higher the modularity is, the most evident the clusters in the network become. The experiments show that Newman's spectral algorithm produces roughly the same number of communities in each case (between 18 to 22). However, there has been a considerable variation regarding modularity as more informal ties are added; as the density increases, modularity significantly drops from above 0.6 to around 0.4 (for tall networks) and 0.3 (for flat networks). In a certain sense, this result captures the fact that all members of the organization form a more cohesive and unified team with more informal ties.

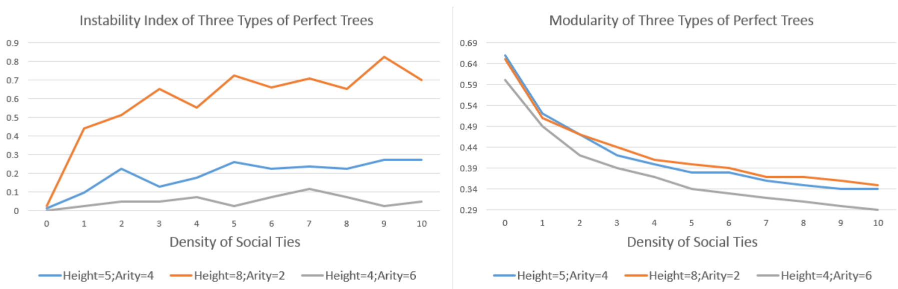


Fig. 14 (left) The instability index for three types of perfect trees with informal ties. (right) The modularity for these networks.

We further perform analysis on the distribution of power and show the results in Figure 15.

– We plot the power of roots in the networks. The plot reveals that as the density of informal ties increases, the power of roots initially drops but then lifts up when more and more informal ties are present.

– We plot the average power of the leaves in the networks. The plot shows that in general more informal ties brings higher power to the operatives in the network, and the change can be dramatic, the power of operatives almost triples when the parameter γ is changed from 0 to 10.
– We compute the ratio between the power of the root and the average power of leaves in the networks. In some sense, this ratio reflects the level of inequality of the network. In general, more informal ties makes the power among members more equal and the ratio drops. The most significant change occurs for flat networks, where without informal ties, this ratio reaches above 14, much higher than the other taller hierarchies. However, as more informal ties are added to the network, this ratio converges to about 3, which is very similar to the other hierarchies.

This experiment also gives us more insights towards the general phenomenon of flattening. Recall from Section 5, the experimental results show that flattening increases the power of the majority, but widens the gap between upper-level managers to lower levels. Here, our experiments show the importance of informal ties under this context: *One can significantly reduce the gap between power across upper and lower levels by enabling more informal ties in the organization.*

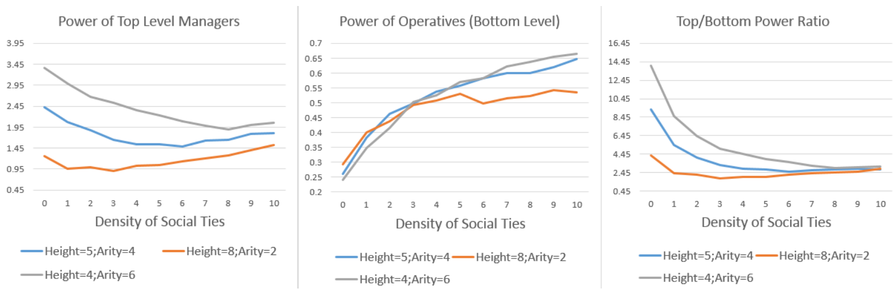


Fig. 15 The results include three types of perfect trees with informal ties. The horizontal axis for all plots is the density of informal ties in the network. (left) The average power of the root. (center) The average power of the leaves. (right) The ratio between the average power of the root against the leaves.

7 A network perspective of leadership styles

Leadership style refers to the general approach a manager sets goals, commands and motivates team members. A good leader makes sure the whole team is working towards a common goal, delivers outcomes and develop a healthy working atmosphere within the organization. Thus, the cohesion and productivity of an organization often hinge on the adoption of effective leadership styles by its top managers (Sales, 1966). Leadership is a well-studied topic in administrative and management sciences, with theoretical foundations pioneered by works of Likert (Hall, 1972) and (Hersey and

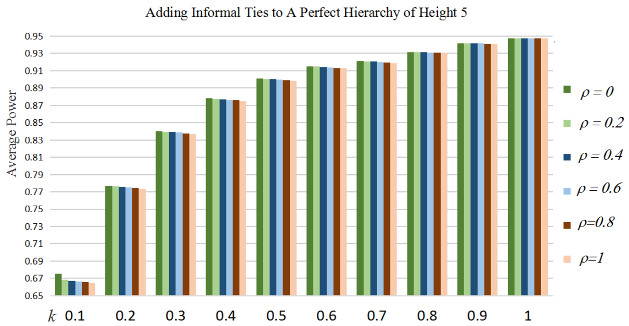


Fig. 16 Average power of perfect tree of arity of 4 and height 5 with random informal ties

Blanchard, 1977). Traditionally, management studies focus on a behavioral perspective of leadership styles and analyze important traits of good leaders. Based on traits of cognitive, social and psychological factors, management theorists and practitioners typically classify leadership styles into several well-established categories, such as autocratic, paternalistic/consultive and democratic styles (Goleman, 2000). In this section, our goal is to provide an alternative, structural angle to the categorization and analysis of management styles. We start with a detailed discussion of the factors affecting the distribution of power.

Experiment 5. We carry out this experiment based on the 4-ary perfect tree hierarchy with height 5. The hierarchy is "standard" in the sense that it is neither tall (with a low span of control) nor flat (with a large span of control) and therefore should capture an idealized typically formal tie hierarchy. We generate random informal ties in the hierarchy using the parameters $\gamma = 5$ and $p = 0.5$. Two important factors influence power in this network: the value of the parameter k, which measures the weight of informal ties as compared to formal ties, and the value of ρ, which measures the *self-assertiveness* of the root of the hierarchy. Our goal for this experiment is to see how the combination of k and ρ affects the distribution of power.

We take $k \in \{0.1, \ldots, 1\}$ and $\rho \in \{0.1, \ldots, 1\}$ and generate 10 networks for each combination of k and ρ values. We then calculate using CORPNET the average power of nodes for each combination of k and ρ and plot them in Figure 16. As k increases, the average power in the network also increases; on the other hand, a higher value of γ leads to a decrease in average power, although the difference is minor.

We then plot more statistics in Figure 17. Firstly, as shown in the plot on the left, the power of the root of the network decreases linearly on the increase in k, and increases with respective to the value of ρ. Secondly, the variance of the distribution of power drops exponentially with increasing k. Thirdly, the ratio of power between the root and leaves also drops exponentially with increasing k. This fact and the second point above indicate that the power is distributed more evenly as k increases.

Finally, we plot the modularity and instability index of the community structure identified by Newman's spectral algorithm with increasing k in Figure 18. Just like

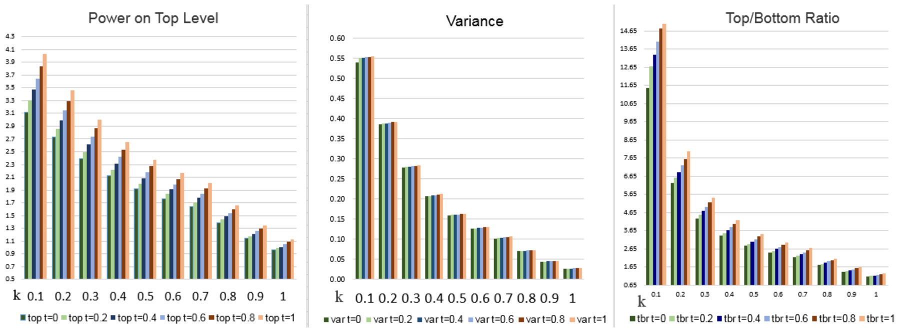


Fig. 17 Perfect Tree of height 5 with informal ties. (left) The power of the roots. (center) The variance of power among all nodes. (right) The ratio between power of roots against leaves.

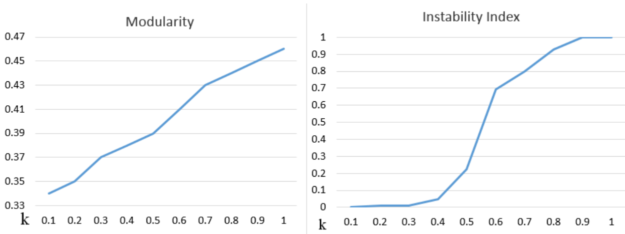


Fig. 18 Perfect Tree of height 5 with informal ties. (left) Modularity of the identified clusters by Newman's spectral algorithm. (right) Instability index of the networks.

in Experiment 4, the algorithm identified 18-22 communities in all cases, and as k increases from 0.1 to 1, the modularity increases between 0.34 and 0.46 in a linear fashion. This observation shows that the network tends to be more clustered as k increases. On the other hand, as k increases, the instability also gets larger. More interestingly, the instability index grows in different speed as k grows and roughly separate into three stages: It grows very slowly (below 0.1) when k is small (between 0.1 and 0.4), and then grows very fast when k is between 0.4 and 0.75 before slowing down again for large k (above 0.8) but at a much higher value. We also obtain the same pattern regardless of the value of ρ. Thus in the plots, we do not modify the values of ρ.

The results of Experiment 5 shows that the distribution of power is greatly influenced by the value k and, at a much smaller scale, the value of ρ. These results suggest a network-based approach to define and assess leadership styles is possible. In general, leadership styles in an organization are classified by the level of control exercised by the top managers. For example, managers in an autocratic organization make decisions unilaterally with no initiatives from the bottom while in a democratic organization, decisions are made by majority rather than by the top managers. The value of k characterizes the amount of influence to the power of informal ties compared to formal ties; in this sense, k can be regarded as an indicator of a level of control from higher to lower level of the reporting relation. Moreover, ρ also intuitively characterizes the top manager's sense of self-determination when it comes to decision making and hence also affects leadership style.

We describe below some major classified leadership styles defined in management science (Tannenbaum and Schmidt, 1973). We also interpret each style using a combination of parameters k and ρ:

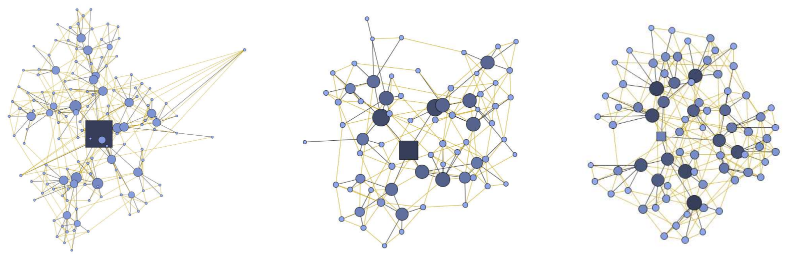
Autocratic style. This style assumes that the decisions are made from the top management unilaterally. There are few or no initiatives from the bottom of the hierarchy. The number of connections of any individual employee is relatively low because maintaining a reporting relation requires a lot of resources. Hence we say that an organizational network (with a fixed weighted interaction graph) is *autocratic* if $\rho > 1$, k is very small (i.e. within the range $[0, 0.1]$). The benefit of this style is high stability, while the negative effect is the lack of motivation of employees.

Democratic style. Here the decisions are made by the majority of the employees. There are many initiatives from the bottom of the hierarchy and collaboration requires as many resources to maintain as the reporting relations. Thus we say that an organizational network is *democratic* if $\rho = 0$, $k \in [0.5, 1)$. The benefit of this style lies in job satisfaction and quality. However, it does mean a higher level of instability and inefficiency

Paternalistic (consultative) style. This leadership style sits somewhere between autocratic and democratic styles. While the decisions are made mainly from the top managers, they take into account the best interests of the employees. The interaction is mainly one-directional (downwards), but feedbacks are encouraged. Hence we define an organizational network to be *paternalistic* if $\rho > 0$, $k \in (0.1, 0.5)$.

Chaotic style. This is a more recent style of management, which gives employees total control over decision making. Here informal ties become the dominant personal links and thus require a larger weight as their effect and a large amount of resources to maintain. We define an organizational network to be *chaotic* if $\rho = 0$, $k = 1$. One would expect that any chaotic organizational structure to be unstable.

Example 4. Based on the description above, we use CORPNET to generate organizational networks that capture each management styles above, by setting the parameters correspondingly. Figure 19 shows three typical networks with different management styles: autocratic, paternalistic, and democratic. The sizes of the nodes (drawn in force-based layout) represent their power. One can clearly identify that the distribution of power in such networks is quite different, and reflect the analysis above.



(a) A network with autocratic management styles (b) A network with paternalistic management styles (c) A network with democratic management styles

Fig. 19 Distribution of power in networks with different management styles

Table 9 Organizational networks with different leadership styles

Leadership style	ρ	k	Instability index
Autocratic	$\rho = 1$	$k \in [0, 0.1]$	low
Paternalistic	$\rho > 0$	$k \in [0.1, 0.5)$	low
Democratic	$\rho = 0$	$k \in [0.5, 1)$	high
Chaotic	$\rho = 0$	$k \in 1$	high

Experiment 6. [Leadership styles] We elaborate the discussion above by carrying out systematic experiments on random networks. Here once again we consider both a tall hierarchy (Height 7) and a flat hierarchy (Height 4). We generate random trees using Procedure 1 setting the mean arity of the trees to be 3 (for tall hierarchy) and 6 (for flat hierarchy). For each leadership style in Table 9, we generate ten networks of each type and compute average power at each level. We then plot the average power of each level in Figure 20. In both organizations, autocratic style results in the largest variation of power across levels. The difference between a tall organization and a flat one is that the flat organization is stable under autocratic style whereas middle levels of the tall organization display much fluctuation. The other three styles, on the other hand, gives a much more even distribution of power across the levels. In the tall organization, the network is stable under the paternalistic style and becomes highly unstable under chaotic style.

8 CORPNET: an ONA tool

An ONA tool is a software that provides analytics the network structures within a company. It should reveal information flows, identify potential structural holes, gaining insights into properties that are invisible at first sight. Such insights can then be used to derived beneficial business strategies such as restructuring or promotion/demotion of staffs. There are various existing ONA tools, examples of which

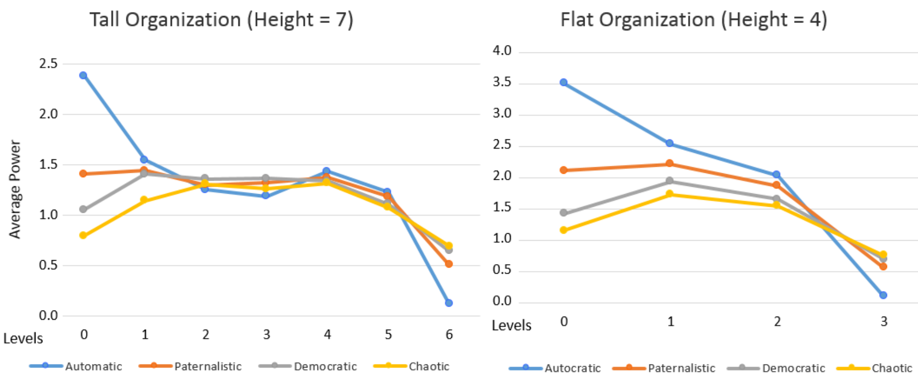


Fig. 20 The distribution of average power across all levels in randomly generated networks. The networks consist of random formal tie hierarchy and random informal ties. (left) Power in random tall organizations where the formal tie hierarchy has height 7 and mean arity 3. (right) Power in random flat organizations where the formal tie hierarchy has height 4 and mean arity 6

include InFlow[2], SYNAPP[3], and SYNDIO[4]. These software tools usually perform data visualization tasks, as well as extracting standard network measures. However, two significant limitations exist:

1) Such products are mostly commercially available which made them difficult to be adopted for management science research, and,
2) Most importantly, they do not study correlations between formal and informal structures.

The aim of CORPNET is a stand-alone software application created to perform ONA functions based on our model above. CORPNET provides interactive simulation, analysis, and visualization functionalities. It is developed using the Scala programming language and runs on the Java Virtual Machine[5].

The main features of CORPNET include instruments of statistical and stability analysis, community detection, and functions to generate random formal tie hierarchies and social networks:

Network creation and visualization The network graph can be visualized as either a *tree layout* or a *force directed layout*. The tree layout is hierarchical with respect to the directed edges only. The color of a node can represent either its power or its membership within a detected community; See Figure 21.

Power analysis. CORPNET computes Bonacich power as defined above and visualizes power in multiple ways. For both layout styles, a darker shade of blue indicates a higher power within the network. The force layout draws nodes with

[2] Retrieved from `http://orgnet.com/inflow3.html`
[3] Retrieved from `http://www.seeyournetwork.com/`
[4] Retrieved from `https://synd.io/`
[5] A prototype of CORPNET and its source code can be downloaded from `https://github.com/mourednik/corpnet`

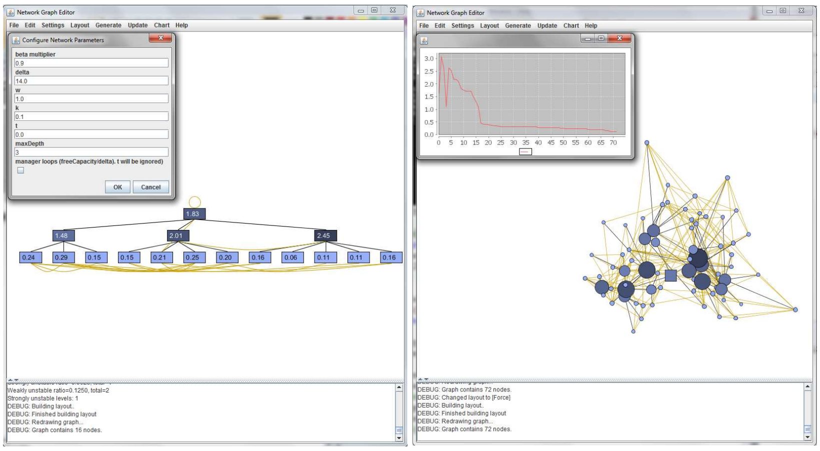


Fig. 21 CORPNET user interface: a tree layout (left) and a force directed layout with a power distribution (right)

varying sizes, such that more powerful nodes are relatively larger. Two plots are available providing statistical information regarding individual power:

– *Descending power grouped by level*: This is a scatter plot of individual node powers. The nodes are arranged from left to right in descending order of power, grouped by level such that the nodes on higher levels are to the left of nodes on lower levels.
– *Power histogram*: This is a histogram of node powers with a configurable number of bins.

See Fig. 22 for examples of both types of plots.

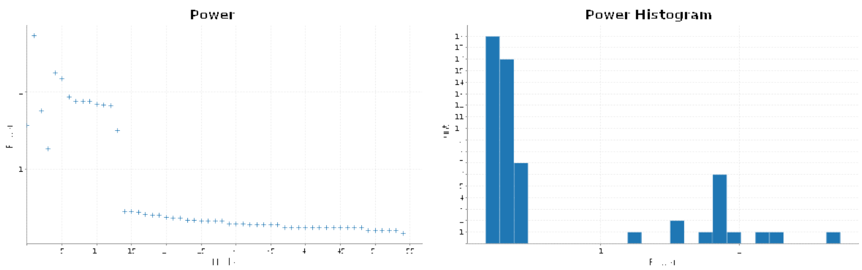


Fig. 22 Power distribution plots: Descending power grouped by level (left) and power histogram (right)

Community detection. CORPNET incorporates a module which computes clusters in the network based on formal and informal ties using Newman's spectral algorithm (Fortunato, 2010). This is visualized by node colors which represent the detected clusters. See Figure 10 and Figure 11 for examples of clusterings in the tree layout.

Random network generation. To facilitate experiments on organizational networks, CORPNET incorporates a network simulator which is able to generate synthetic benchmark organizational networks.

1. *Random tree generator.* This module generates tree using Procedure 1. The number of children is sampled from a normal distribution with a specified mean and standard deviation.
2. *Social network generator.* This module generates a benchmark social network of undirected edges over the existing hierarchical network, as described in Procedure 2.

9 Conclusion and future work

This paper is a natural continuation of the works mentioned above. We notice that

1. there is a lack of mathematical analysis on the dual-structure of formal and informal organizations; and
2. existing formal definitions of power only deal with networks whose edges have a single interpretation of social links, while not incorporating formal roles and levels.

Our aim is therefore to develop a mathematical model that sits at the confluence of the two directions above. Our network model is simple, elegant and novel in the sense that it unifies existing studies. Our experimental results are consistent with the knowledge that informal relations significantly affect individual power in an organization. Moreover, we demonstrate using the previous sections that our mathematical model has the potential to provide an explanation to complex phenomena such as flattening and leadership style.

There are several obvious ways in which the model can be enriched: 1) As argued above a company may be lead by a board of directors rather than a single person. Hence, one may allow several nodes in the network making the reporting hierarchy a forest rather than a single tree. 2) Informal ties are heterogeneous; different types of informal ties (such as friendship, and collaborations) may result in different impacts on power. Hence one may allow several informal ties (undirected or directed) with different correlation coefficients and k. 3) The capacity of individuals are different, and therefore, one may assign different capacities to different individuals. 4) The current model only applies to functional or divisional structural type of organization, while other types, such as team-based or matrix-organizations are not captured. A model that incorporate the above considerations will provide more realistic analysis. Hence the current work is a first step towards building a generally applicable organizational network analytical tool set.

Another interesting future direction is to add an extra layer of complexity to the model by incorporating different roles which specify tasks an individual perform in the organization. We note that different roles may rely on each other (with common interests) or be in conflict with each other (with conflicting interests). The type of effects that arises due to such interactions is different and hence requires a more complex definition of power.

The significance of this paper lies mainly in theoretical models, simulations and analysis. Nevertheless, we would like to mention that the model is ready to be applied to field works where the hypotheses are verified in an empirical setting. Carefully designed experimentations are required on a real organization to collecting data about formal and informal relations and analyze the internal structures. This would be a natural and crucial next step of our research.

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