Performance Analysis of Amplify-and-Forward Wireless Relay Networks

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering

THE UNIVERSITY OF AUCKLAND
NEW ZEALAND

Department of Electrical and Computer Engineering
The University of Auckland

Muhammad I. Khalil

2017

THE UNIVERSITY OF AUCKLAND
SCHOOL OF GRADUATE STUDIES

Co-Authorship Form

Graduate Centre
The ClockTower – East Wing
22 Princes Street, Auckland
Phone: +64 9 373 7599 ext 81321
Fax: +64 9 373 7610
Email: postgraduate@auckland.ac.nz
www.postgrad.auckland.ac.nz

This form is to accompany the submission of any PhD that contains published or unpublished co-authored work. **Please include one copy of this form for each co-authored work**. Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements. Co-authored works may be included in a thesis if the candidate has written all or the majority of the text and had their contribution confirmed by all co-authors as not less than 65%.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

Chapter 2 of this thesis is based on our published :

Khalil, M. I., Berber, S. M., & Sowerby, K. W. (2016). Bit error rate performance analysis in amplify-and-forward relay networks. Wireless Networks, 1-11. doi:10.1007/s11276-016-1196-0

Nature of contribution by PhD candidate	Majority of work
Extent of contribution by PhD candidate (%)	80%

CO-AUTHORS

Name	Nature of Contribution
Stevan Berber	reviewing
Kevin Sowerby	reviewing

Certification by Co-Authors

The undersigned hereby certify that:

- ❖ the above statement correctly reflects the nature and extent of the PhD candidate's contribution to this work, and the nature of the contribution of each of the co-authors; and
- ❖ that the candidate wrote all or the majority of the text.

Name	Signature	Date
Stevan Berber	Berber	24/04/2017
Kevin Sowerby	Kevin Sowerby	24/04/2017

THE UNIVERSITY OF AUCKLAND
SCHOOL OF GRADUATE STUDIES

Co-Authorship Form

Graduate Centre
The ClockTower – East Wing
22 Princes Street, Auckland
Phone: +64 9 373 7599 ext 81321
Fax: +64 9 373 7610
Email: postgraduate@auckland.ac.nz
www.postgrad.auckland.ac.nz

This form is to accompany the submission of any PhD that contains published or unpublished co-authored work. **Please include one copy of this form for each co-authored work**. Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements. Co-authored works may be included in a thesis if the candidate has written all or the majority of the text and had their contribution confirmed by all co-authors as not less than 65%.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

Chapter 3 of this thesis is based on our published:

Khalil, M. I., Berber, S. M., & Sowerby, K. W. (2016). Error Rate Analysis of AF-Relay Wireless Networks Under Different SNR levels. Physical Communication, 19, 11-17. doi:http://dx.doi.org/10.1016/j.phycom.2016.06.002

Nature of contribution by PhD candidate	Majority of work
Extent of contribution by PhD candidate (%)	80%

CO-AUTHORS

Name	Nature of Contribution
Stevan Berber	reviewing
Kevin Sowerby	reviewing

Certification by Co-Authors

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate's contribution to this work, and the nature of the contribution of each of the co-authors; and
- that the candidate wrote all or the majority of the text.

Name	Signature	Date
Stevan Berber	Berber	24/04/2017
Kevin Sowerby	Kevin Sowerby	24/04/2017

THE UNIVERSITY OF AUCKLAND
SCHOOL OF GRADUATE STUDIES

Co-Authorship Form

Graduate Centre
The ClockTower – East Wing
22 Princes Street, Auckland
Phone: +64 9 373 7599 ext 81321
Fax: +64 9 373 7610
Email: postgraduate@auckland.ac.nz
www.postgrad.auckland.ac.nz

This form is to accompany the submission of any PhD that contains published or unpublished co-authored work. **Please include one copy of this form for each co-authored work**. Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements. Co-authored works may be included in a thesis if the candidate has written all or the majority of the text and had their contribution confirmed by all co-authors as not less than 65%.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

Chapter 4 of this thesis is based on our published:

Khalil, M. I., Berber, S. M., & Sowerby, K. W. (2016, 4-6 June 2016). Energy efficiency and spectrum efficiency balance of wireless relay networks. Paper presented at the 2016 8th IEEE International Conference on Communication Software and Networks (ICCSN).

Nature of contribution by PhD candidate	Majority of work
Extent of contribution by PhD candidate (%)	80%

CO-AUTHORS

Name	Nature of Contribution
Stevan Berber	reviewing
Kevin Sowerby	reviewing

Certification by Co-Authors

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate's contribution to this work, and the nature of the contribution of each of the co-authors; and
- that the candidate wrote all or the majority of the text.

Name	Signature	Date
Stevan Berber	Berber	24/04/2017
Kevin Sowerby	Kevin Sowerby	24/04/2017

THE UNIVERSITY OF AUCKLAND
SCHOOL OF GRADUATE STUDIES

Co-Authorship Form

Graduate Centre
The ClockTower – East Wing
22 Princes Street, Auckland
Phone: +64 9 373 7599 ext 81321
Fax: +64 9 373 7610
Email: postgraduate@auckland.ac.nz
www.postgrad.auckland.ac.nz

This form is to accompany the submission of any PhD that contains published or unpublished co-authored work. **Please include one copy of this form for each co-authored work**. Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements. Co-authored works may be included in a thesis if the candidate has written all or the majority of the text and had their contribution confirmed by all co-authors as not less than 65%.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

Chapter 5 of this thesis is based on our published:

Khalil, M. I., Berber, S. M., & Sowerby, K. W. High SNR Approximation For Performance Analysis of Two-way Multiple Relay Networks. Physical Communication, 2017

Nature of contribution by PhD candidate	Majority of work
Extent of contribution by PhD candidate (%)	80%

CO-AUTHORS

Name	Nature of Contribution
Stevan Berber	reviewing
Kevin Sowerby	reviewing

Certification by Co-Authors

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate's contribution to this work, and the nature of the contribution of each of the co-authors; and
- that the candidate wrote all or the majority of the text.

Name	Signature	Date
Stevan Berber	Berber	24/04/2017
Kevin Sowerby	Kevin Sowerby	24/04/2017

THE UNIVERSITY OF AUCKLAND
SCHOOL OF GRADUATE STUDIES

Co-Authorship Form

Graduate Centre
The ClockTower – East Wing
22 Princes Street, Auckland
Phone: +64 9 373 7599 ext 81321
Fax: +64 9 373 7610
Email: postgraduate@auckland.ac.nz
www.postgrad.auckland.ac.nz

This form is to accompany the submission of any PhD that contains published or unpublished co-authored work. **Please include one copy of this form for each co-authored work**. Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements. Co-authored works may be included in a thesis if the candidate has written all or the majority of the text and had their contribution confirmed by all co-authors as not less than 65%.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

Chapter 6 of this thesis is based on our published:

Muhammad I. Khalil, Stevan M. Berber, and Kevin W. Sowerby. Precise error rate analysis of wireless relay networks. Wireless Personal Communications, pages 1–16. doi:.10.1007/s11277-017-4146-y

Nature of contribution by PhD candidate	Majority of work
Extent of contribution by PhD candidate (%)	80%

CO-AUTHORS

Name	Nature of Contribution
Stevan Berber	reviewing
Kevin Sowerby	reviewing

Certification by Co-Authors

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate's contribution to this work, and the nature of the contribution of each of the co-authors; and
- that the candidate wrote all or the majority of the text.

Name	Signature	Date
Stevan Berber	Berber	24/04/2017
Kevin Sowerby	Kevin Sowerby	24/04/2017

Graduate Centre
The ClockTower – East Wing
22 Princes Street, Auckland
Phone: +64 9 373 7599 ext 81321
Fax: +64 9 373 7610
Email: postgraduate@auckland.ac.nz
www.postgrad.auckland.ac.nz

THE UNIVERSITY OF AUCKLAND
SCHOOL OF GRADUATE STUDIES

Co-Authorship Form

This form is to accompany the submission of any PhD that contains published or unpublished co-authored work. **Please include one copy of this form for each co-authored work**. Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements. Co-authored works may be included in a thesis if the candidate has written all or the majority of the text and had their contribution confirmed by all co-authors as not less than 65%.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

Chapter 7 of this thesis is based on our published:

Khalil, M. I., Berber, S. M., & Sowerby, K. W. (2016, 19-23 Sept. 2016). Energy Efficiency and Spectrum Efficiency trade-off over Optimal Relay Location in bidirectional relay networks. Paper presented at the 2016 IEEE-APS Topical Conference on Antennas and Propagation in Wireless Communications (APWC).

Nature of contribution by PhD candidate	Majority of work
Extent of contribution by PhD candidate (%)	80%

CO-AUTHORS

Name	Nature of Contribution
Stevan Berber	reviewing
Kevin Sowerby	reviewing

Certification by Co-Authors

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate's contribution to this work, and the nature of the contribution of each of the co-authors; and
- that the candidate wrote all or the majority of the text.

Name	Signature	Date
Stevan Berber	Berber	24/04/2017
Kevin Sowerby	Kevin Sowerby	24/04/2017

Abstract

Wireless relay technology plays an important role in the next-generation mobile and wireless networks. It has been proposed as a cost effective solution to addressing a growing demand for high data rates and low power consumption. In the relay communication model, a source communicates to a destination through a wireless node(s) called a relay. There are two main relaying techniques: Decode-and-Forward (DF) and Amplify-and-Forward (AF). AF is the focus of this thesis. The AF relay networks provide considerable performance improvement in link reliability, Spectral Efficiency (*SE*), Energy Efficiency (*EE*) and Bit Error Rate (BER) performance. They also offer a low-complexity solution for practical relay networks that have critical energy constraints, as they enable the received signal to be amplified without the decoding process at the relays.

This thesis proposes new methods of reducing the energy consumption and evaluating the error performance of AF relay networks, when the relay is either one-way (unidirectional) or two-way (bidirectional). The analysis is commenced by formulating the actual Signals-to-Noise Ratio (*SNR*) level, which corresponds to a low *SNR* level, in order to calculate the Exact Bit Error Rate (EBER) performance. Then, the approximated *SNR*, which refers to a high *SNR* level, is expressed and adopted to evaluate the Asymptotic Bit Error Rate (ABER). Another *SNR* level is presented in this work and defined as an optimal *SNR* level, which is obtained by optimal balancing of the *SE* and *EE*. The optimal *SNR* level allows evaluation of the ABER and the EBER of AF relay networks under optimal *SNR* conditions. Accordingly, a unifying BER evaluation at low, high, and optimal levels of AF relay networks is provided in this study. This thesis focuses on a high *SNR* level to derive an effective method for evaluating the ABER and EBER, whether in optimal or sub-optimal AF relay networks. The effectiveness of such a BER evaluation method is demonstrated by enhancing the ABER of AF relay networks at all *SNR* levels (i.e., low, high, optimal *SNR* levels). This is shown when disparity between the ABER and the EBER at

a low *SNR* level is reduced, and this helps to calculate the accurate BER of AF relay networks under any *SNR* levels using either the ABER or EBER.

The proposed methods are examined under different channels environments, as in practice the channel of each hop of an AF relay network can be located in a different environment. When both hops are located in the same environment, the fading channels of the hops are equal, while for different environments, the fading channels of the hops are different. Furthermore, relay location is also investigated for the aforementioned ABER and EBER, and a new method for determining the relationship between the *EE* and EBER with relay location is derived.

Acknowledgment

First and foremost, I must acknowledge and thank the Almighty Allah SWT for blessing, protecting, and guiding me throughout this period.

I wish to express my sincere appreciation to those who have contributed and supported me on the journey of my PhD. I would like to thank my supervisors, Dr Stevan Berber and Prof. Kevin Sowerby for supporting me during my study. Both of my supervisors provided much guidance in shaping out the final product of this research and they have constantly available to listen, to clarify my doubts and to give advice. They were always there to proofread and mark up my papers, and to ask me challenging questions to help me think through my problems.

Words are short to express my deep sense of gratitude towards early mom prayer in order to complete my PhD study and all family members; my wife and my children who have been with me all these years, also my sisters and my brother for their love, patience and understanding. This last word of acknowledgment goes to my dear father, who supported my study. Without his support, I could not be able to study overseas in New Zealand.

Contents

List of Figures

List of Abbreviations and Acronyms

AF	Amplify and forward
DF	Decode and Forward
SE	Spectrum Efficiency
EE	Energy Efficiency
SER	Symbol error rate
CDF	Cumulative distribution function
ABER	Asymptotic bit error rate
MGF	Moment Generating Function
O-BER	Optimal bit error rate
MRC	Maximum Ratio Combiner
BPSK	Binary phase shift keying
EBER	Exact bit error rate
CBER	conventional bit error rate
SNR	Signal to noise ratio
TS_1	First time slot
TS_2	Second time slot
MPSK	M-ary Phase Shift keying modulation schemes
FER	Frame error rates
$(\overline{EE})$	Average Energy Efficiency
$(\overline{SE})$	Average Spectrum Efficiency
CCDF	Complementary cumulative distribution function
$erfc$	Error function
MAFR	Multiple AF Relay
MAF-RS	Multiple AF Relay with Relay Selection strategy
BLER	Block error rate ,
pdf	Probability density function
AvSNR	Average SNR
U-BER	Unified bit error rate
HW-SNR	Highest Worse Signal-To-Noise Ratio
PER	Probability Error Rate

Chapter 1

Introduction

1.1 Overview

Recently, the world has seen dramatic growth in the number of small wireless devices such as cell phones and laptops. This growth has caused a significant increase in demands for high data rates and low energy requirements to deliver such services. Thus, the data rate and energy consumption are two vital parameters that need to be considered in the design of future generations of wireless networks. Furthermore, the impact of such parameters on the network performance is another factor that should be taken into account.

Relay networks have been proposed as a promising solution for the next generations of wireless systems [1–4]. The main concept of the relay network is that the direct communication (or one-hop) link between a source and destination is divided into two or more (multiple-hop) links through nodes called relays [5]. The relay element is commonly classified as one of two techniques: Amplify-and-Forward (AF) and Decode-and-Forward (DF). Both techniques can be operated as one-way (unidirectional) or two-way (bidirectional) links. Relay networks have been shown to improve the coverage, link reliability, Spectral Efficiency (SE) [6], Energy Efficiency (EE) [7] and Bit Error Rate (BER) performance [8] of one-hop links. All such advantages can be achieved by deploying relay nodes between sources and destinations without incurring the associated high costs of adding extra base stations. Thus, it is important for future wireless standards to include relay schemes if the above advantages are to be realised [9, 10].

1.2 Networks Relay Techniques

Classical wireless communications are based on one-hop links, i.e., only two wireless nodes are involved in the communication of data. These two nodes are, e.g., the mobile station and base station in a cellular network, or the access point and portable device in a wireless local area network. In wireless relay networks, however, an additional node (or nodes) is used to help two wireless nodes in the communication of data. In such networks, the source node broadcasts signals to a destination node through set of relay nodes, as shown in Fig 1.1.

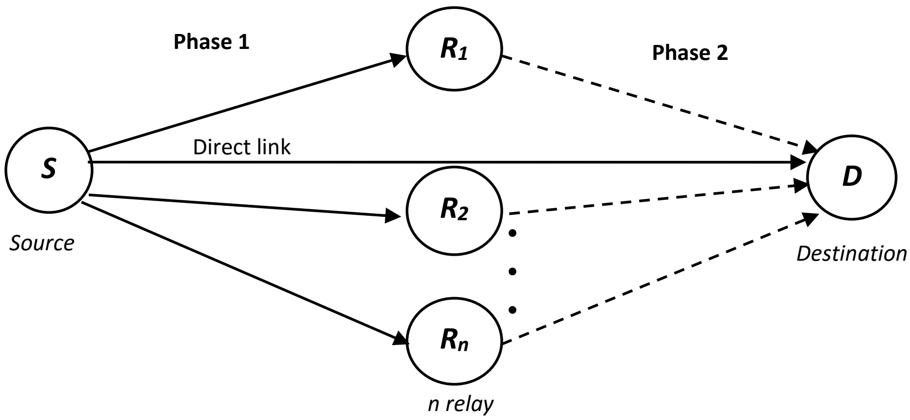

Figure 1.1: Relay Networks

The relay node commonly processes the received signal according to the type of relay technique. In the AF relay, the received signal is amplified and then forwarded to the destination without further processing. On the other hand, in the DF relay, the received signal is demodulated, channel decoded, encoded and modulated again, before being delivered to its destination [11, 12].

The relayed signals and possible direct-link signal are then received at their destination in multiple, independently-faded copies of the same signal. The availability of such multiple signal copies at the destination is termed diversity [13]. Commonly, diversity methods include; time diversity, frequency diversity and spatial diversity [14]. In time diversity, multiple copies of a signal are sent by a source node at different time instants. In frequency diversity, multiple copies of a symbol are broadcast through channels having different carrier frequencies [11]. Spatial diversity can be achieved by providing multiple antennas at the transmitter or receiver, or both. The use of multiple antennas at the transmitter is termed transmit spatial diversity; the use of multiple antennas at the destination is termed receive spatial diversity. Spatial diversity is recognized as an important method of combating fading without expanding the bandwidth of the transmitted signal [13].

At the destination, all the received signals are combined to retrieve the transmitted message. Such combining of signals is in general achieved by one of the following techniques: selection combining, equal gain combining or maximal ratio combining. These combining methods are elaborated in [15]. Overall, selection combing is the least complex for implementation comparison with other techniques. Thus, this thesis focuses on the selection combining. This combining scheme is based on the premise that all signals coming out of diversity branches have different amplitude values at the same time. As a result, the diversity branch with the strongest signal is chosen and all other signals are neglected.

As mentioned earlier, the main relay techniques are AF relay and DF relay. DF relay is more complex than AF relay, as DF relay needs full processing including decoding, re-modulating and re-transmitting the received signal. The DF relay also requires sophisticated power control [16], which is unnecessary in AF relay. AF relays therefore is considered in this thesis as they can potentially be implemented with lower complexity.

1.3 AF Relay Technique

Amplify-and-Forward (AF) is the principle technique used in relay networks that have critical power constraints. The basic principle of AF relay is that the relay receives the transmitted signal in the first phase, amplifies it (by the amplification factor), then forwards it to the destination in the second phase. Two-phase communication scenario therefore allows relay element to operate without self-interference, as shown in Fig. 1.2.

The amplification factor is an important design issue in AF relay networks. It depends on channel fading conditions between the source and relay. Generally, the amplification factor is classified as either variable and average [11]. The variable amplification factor is a function of the instantaneous channel conditions (short-term), while the average factor is a function of the average channel conditions (long-term).

The AF relay technique has been recognized by [17] as practical, low-complexity solution for relay networks that have critical energy constraints. Its complexity is low because it avoids decoding process used in the DF networks and so eliminates propagation of decoding errors at the relay [18]. Another advantage of AF relay is that its relative design simplicity means less hardware is needed [19]. Its main drawbacks, however, are noise amplification and interference propagation, since the relay amplifies noise

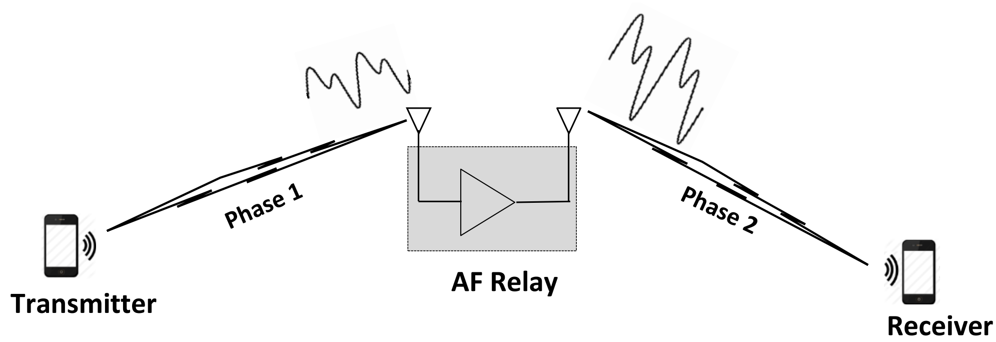

Figure 1.2: Operating profile of AF relay scheme

as well as the original signal and no interference cancellation technique is applied [5]. But even with this weakness, AF relay achieves a higher diversity order compared to other relay types [20].

The AF relay scheme is expected to be employed in future cellular networks, in which the base station (or equivalent to source node) can broadcast signals to a mobile phone (i.e., destination) through relay nodes. Accordingly, this thesis aims to develop multiple AF relay networks in which the relay run is either unidirectional or bidirectional. This development is focused on BER and *EE* perspectives.

1.3.1 BER performance of AF Relay networks

The performance of wireless digital communication has been evaluated using the probability of detection error, or bit error rate (BER). Generally, BER is a function of the transmitter power, the types of wave forms used to broadcast the signals over the channel, the code characteristics, the method of demodulation, and the characteristics of the transmission channel (i.e., the nature of the interference, the quantity of noise) [13]. In this thesis we consider BER with respect to Signal-to-Noise Ratio (*SNR*), which is a function of two factors: the network's resource powers (i.e., source and relay powers) and characteristics of the transmission channels. In this regard, many techniques have been proposed in the literature to determine the effect of channel characteristics on error rate performance. For instance, in [21], multi-hop and multi-relay networks were involved in the analysis of the Symbol Error Rate (SER) of AF relay networks. It was found that SER performance is improved by increasing the number of relays. Similar studies in [22–24] have also investigated SER performance of multi-relay AF relay networks and they confirm that relay networks are powerful in reducing the error rate of wireless networks. However, all of the above SER studies require further analysis to determine BER, which is a more useful metric than SER [25]. Thus, other studies such as [21, 26–31] have analysed BER in respect to channel characteristics and have proposed different techniques for calculating asymptotic BER based on a high *SNR* level.

These studies, however, have lacked accuracy in evaluating asymptotic BER, in particular at low *SNR* level. Other researchers, such as [22–24, 32–34], obtained the exact BER by using the actual *SNR* analysis, but such BER methods were dedicated to evaluating the performance of AF relay networks under sub-optimal network conditions only.

1.3.2 Energy efficiency of Relay Networks

Relay networks not only allow the BER in a wireless communication system to be reduced, as mentioned in the previous section, but also can reduce Energy Consumption (EC) in wireless networks [7, 35]. An early study in [36] has proved that a relay network with a number of shorter links, instead of one longer, direct link, can reduce the overall energy consumption, hence lowers transmission power assigned to the source and relays, as presented for two-hop transmission in [37] and for multi-hop in cellular networks in [38].

Wireless energy consumption and throughput are commonly expressed via an Energy Efficiency (EE) metric, which is expressed as: total data delivered/total energy consumed (bit/Joule) [39]. This metric is generally used when seeking to accomplish the same communication task but with less energy [20]. To increase the value of EE (i.e., decreasing the EC), the transmitted data and power source should be designed optimally. In this context, recent EE research in [39] has identified four fundamental frameworks for improving the EE of wireless networks. These frameworks are the balancing (trading off) of different network metrics, as follows:

- EE versus SE.
- Deployment efficiency versus EE.
- Bandwidth versus network power.
- Delay versus network power.

The above balancing frameworks are demonstrated in [40, 41], where it was found that balancing the EE and SE is a very important factor in designing wireless networks, particularly in diverse network environments. Thus, this thesis focuses on developing the balance between EE and SE for AF relay networks. It is important here to note that the term "optimal AF relay networks" henceforth refers to the optimal EE versus SE balance case.

SE is commonly defined as throughput per unit BW (bits/sec/Hz) [42]. It is a main criterion when optimizing a wireless network, while improving the *EE* inevitably implies decreasing the *SE*, as proved in many studies, such as [41, 43, 44].

In this thesis we develop a new method of increasing the *EE* with the least sacrifice of *SE*. The analysis of *EE-SE* is expressed in term of: instantaneous $EE-SE$ and the average *EE-SE* by using short-term and long-term conditions, respectively. Such analyses are desirable in communication system applications. For example, applications like adaptive channel assignment and handoff prefer long-term, whereas applications like adaptive interference cancellation, etc., prefer instantaneous $EE-SE$. Furthermore, the BER of such developed *EE-SE* balance is calculated. It is worth mentioning here that the calculation of BER for an *EE-SE* balance in AF relay networks has not yet been proposed in the literature, as current BER methods are not appropriate to calculating BER of optimal AF networks.

1.4 Relaying Strategies

As already mentioned, relay can act by one of the following strategies: unidirectional or bidirectional.

1.4.1 Unidirectional Strategy

In unidirectional relay networks, a source (*S*) node communicates to its destination (*D*) node via one or multiple relay nodes using one of the following strategies: non-orthogonal, two-hop and orthogonal. In the non-orthogonal strategy, the node *S* transmits continuously to node *D* [45, 46]. In the two-hop strategy, the source node *S* transmits signals to one or multiple relays in the first phase, and in the second phase, the relay(s) forwards the signals received from the source to the destination node *D* and the *S* remains silent, as shown in Fig.1.1. Such a two-hop scheme has been investigated in many studies such as [27, 28, 46–48]. It was found that this strategy can be implemented easily in practice to enhance the coverage of the wireless network. Actually, it has already been considered as a standard design for the next generation of mobile networks [2, 3]. Moreover, the two-hop scheme is the only alternative when the direct channel, between the source and destination, is subjected to propagation conditions such as severe shadowing. The availability of a direct-link transmission for the node *D* beside the relay link is referred to as the orthogonal strategy. This strategy has been demonstrated by several studies such as [9, 45, 49–52]. It was found that this strategy is useful for the two-hop strategy involving only one relay.

The orthogonal strategy is considered in this thesis, but we mainly focus on two-hop strategy with multiple relays, which is shown in Fig. 1.3. In this strategy, multiple independent paths are available at the destination to receive multiple copies of the same transmitted signal. These signal copies are combined at the receiver using one of the several combiner techniques. This helps increase the reliability of the data transfer by reducing the risk of deep channel fade [53]. Thus, increasing the number of relays reduces the probability of bit error, as demonstrated by many studies such as [28, 30, 54].

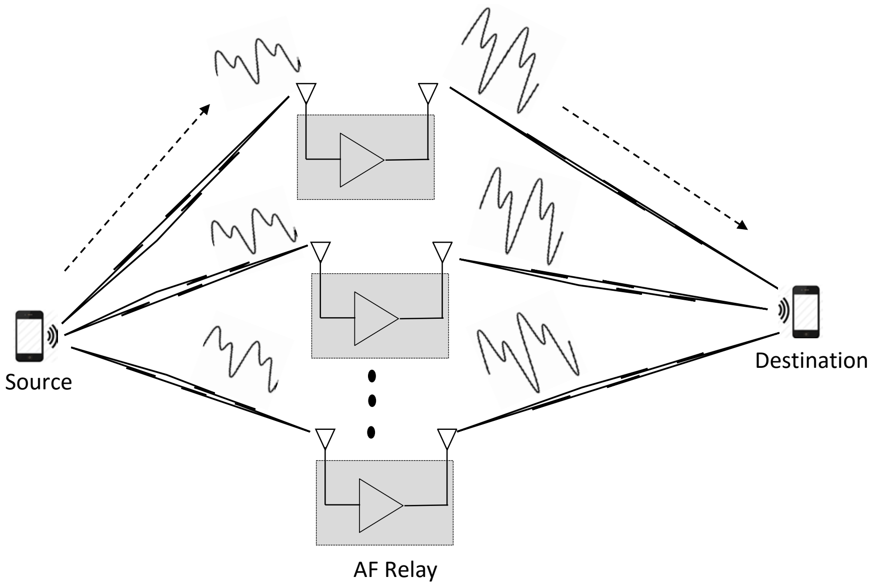

Figure 1.3: Two-hop strategy with multiple relays

1.4.2 Bidirectional Strategy

Bidirectional relaying has generally been proposed to avoid wasting channel bandwidth, which is an issue in the unidirectional strategy [55]. In bidirectional relaying, two wireless nodes (S_a and S_b) can exchange information with each other simultaneously at the same transmission rates and through one or a set of relays, as shown in Fig. 1.4. Each relay in the bidirectional strategy is able to receive and transmit signals at the same time and using the same frequency channel, and hence achieves a higher *SE* [56, 57]. However, the big difference in power levels between the received and transmitted signals makes bidirectional relaying more complex than unidirectional relaying.

A number of studies, such as [44, 58–60], have investigated the relationship between *EE* with *SE* in bidirectional relaying. It is found that reducing the *SE* in bidirectional relaying can increase the *EE* of networks. Such studies, however, have ignored the BER metric in their analyses, although the BER is decreased by the reduction in the *SE*. Also, these studies have employed a system model without considering the impact of relay location in developing the balance between the *EE* and *SE* of the networks, although relay location is a very important factor in practical wireless networks. Accordingly, this thesis considers the BER and the distances between the source, relay and destination to develop the *EE* in bidirectional relay networks.

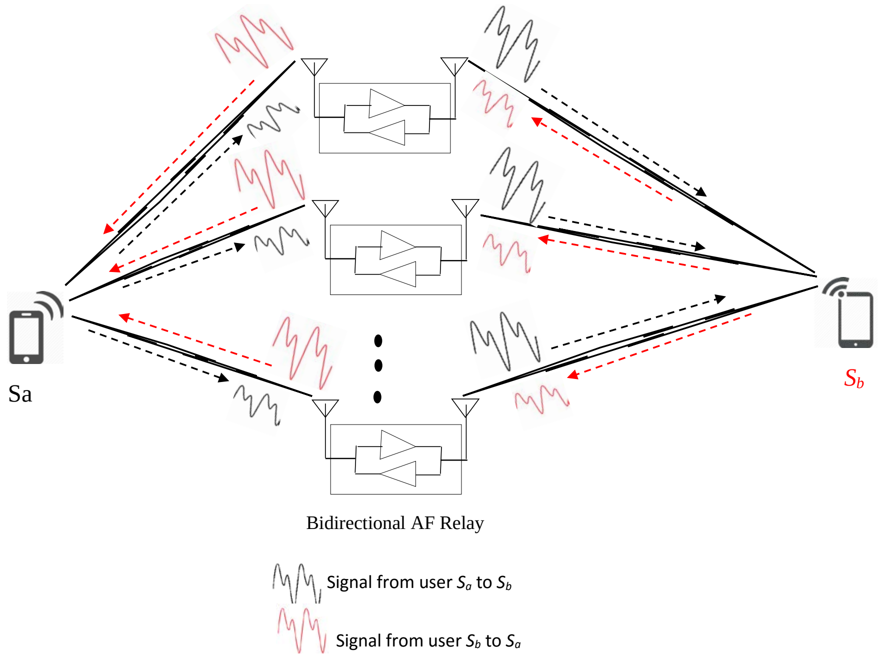

Figure 1.4: Multiple bidirectional relays Network

1.5 Objectives and Contribution of the Thesis

As noted previously, future wireless relay networks will require low error rates, minimum power consumption and high spectral efficiency. Motivated by these requirements, the first objective of this thesis is to provide a new method of reducing power consumption by balancing *EE* and *SE* optimally. The second objective is to define a method for evaluating the BER of AF relay networks, regardless of whether the balance between *EE* and *SE* is optimal or sub-optimal, and considering the relay locations.

The proposed BER for evaluating sub-optimal networks is found to be more accurate than the existing approaches. Furthermore, the proposed BER for evaluating the optimal balance between *EE* and *SE* in AF relay networks has not been previously presented. These contributions apply when AF relay networks operate either unidirectionally or bidirectionally. The main contributions of the thesis are summarized by the following goals:

1. Formulating low and high *SNR*s levels of unidirectional AF relay networks and to adopt them in evaluating the ABER and EBER of sub-optimal AF relay networks.
2. Formulating an optimal *SNR* level, by optimizing the balance between the *EE* and *SE* of unidirectional AF relay networks. Such optimal *SNR* level is used to evaluate the ABER and EBER in an optimal, unidirectional AF relay network.
3. Deriving methods that allow the ABER result to converge to the EBER result for goals 1 and 2.
4. Considering relay location and re-evaluating the ABER and EBER for goals 1, 2 and 3.
5. Considering bidirectional AF relay networks and reanalyzing goals 1, 2, 3 and 4.
6. Deriving a method to determine the relationship between *EE* and BER when considering relay location for unidirectional and bidirectional AF relay networks.

1.6 Thesis Outline

There are eight chapters in this thesis. Figure 1.5 shows the thesis's structure and the interconnections between the chapters. The outline of the thesis is as follows.

In chapter 2, two methods, namely Unified BER (U-BER) and Optimal BER (O-BER), are proposed for calculating the ABER performance of two-hop AF relay networks. Bit error rate expressions for both methods are derived by assuming that the channel of each hop is located in two different environments. It has been found that the U-BER method unifies the BER performance analysis for both the AF relay network and the one-hop communication system. The O-BER method is an appropriate tool for estimating the BER performance in optimal AF relay networks. The proposed methods provide efficient ways of calculating the ABER when the AF relay networks are operating in sub-optimal or optimal conditions.

Enhancing the ABER of Chapter 2 is presented in Chapter 3, which also evaluates the exact BER of AF relay networks. The method in Chapter 3 shows that an ABER analysis of an AF relay network can be reduced to the traditional BER analysis of a one-hop communication network. Also, the ABER analysis can be used directly to determine the EBER performance. This is because the ABER expressions have been derived with respect to one-hop *SNR*. The optimal *SNR* level is achieved by balancing *EE* and *SE* optimally. For high, low and optimal *SNR* levels, BER expressions are derived, and validated by simulation, in which it was found that the ABER analytical results align with the EBER results. This allowed calculation of the accurate BER of AF relay networks under any *SNR* level using either ABER or EBER.

Chapter 4 presents a new method for reducing both the energy consumption and the BER of AF relay networks. The method combines BER and relay location with the optimal balancing of *EE* and *SE*. The proposal allows evaluation of the energy consumption, in which a selected relay can be placed at any point on the line connecting the transmitter and its destination. Furthermore, the minimum evaluated energy is used to obtain the optimal balance between the *EE* and *SE*. This balance enables the *EE* to increase significantly with the least loss in *SE*. Such a balance is then expressed with respect to the BER. Results derived from the analytical expressions were simulated numerically. It was found that the optimal balance between the *EE* and *SE* allows an increase in the *EE* and decreases BER, but such an increase in the *EE* should remain within small limits.

All of the above chapters address the *EE* and BER issues for unidirectional relay networks under different conditions. In Chapter 5, a new method for analysing the asymptotic BER performance of bidirectional AF relay networks is presented. Analytical expressions for calculating the ABER expressions under high and optimal *SNR* levels are presented. These analyses use both the Arithmetic and Geometric Mean inequality (AGM) and the Harmonic Mean ($\mathscr{HM}$). It was found that the ABER analysis using AGM or $\mathscr{HM}$ gives the same results at high and optimal *SNR* domains. However, at low *SNR* domain, AGM gives a better ABER result than $\mathscr{HM}$. It was also found that the optimal *SNR* obtained from a balanced network has a lower asymptotic BER than sub-optimal networks.

To increase the accuracy of the ABER of bidirectional AF relay networks covering a wide range of applications, Chapter 6 investigates a new method of increasing the accuracy of the ABER evaluation under low and high *SNR* levels. Further, the ABER is analyzed under two different geographic environments, as the channel of each hop of a bidirectional AF relay network is located in a different environment. Moreover, the ABER is used to calculate the EBER, which is more complex than the ABER. Thus, the BER of a bidirectional AF relay network can be evaluated under low and high *SNR* levels by using ABER or EBER.

In Chapter 7, a new method of reducing both the energy consumption and the BER of bidirectional AF relay networks is presented. The method integrates the BER and the relay location with the optimal balancing of the *EE* and *SE*. The proposal allows the evaluation of the energy consumption, in which a selected relay can be placed at any point on the line connecting the transmitter and its destination. Furthermore, the minimum evaluated energy is used to obtain the optimal balance between the *EE* and the SE. This balance enables the *EE* to increase significantly with the least loss of SE. Such a balance is then expressed with respect to the BER. It was found that the optimal balance between the *EE* and the *SE* allows the *EE* to be increased and decreases BER of a bidirectional AF relay network. Increasing the EE, however, should remain within small limits.

Chapter 8 concludes this thesis, summarizing the key research findings and making suggestions for potential further work.

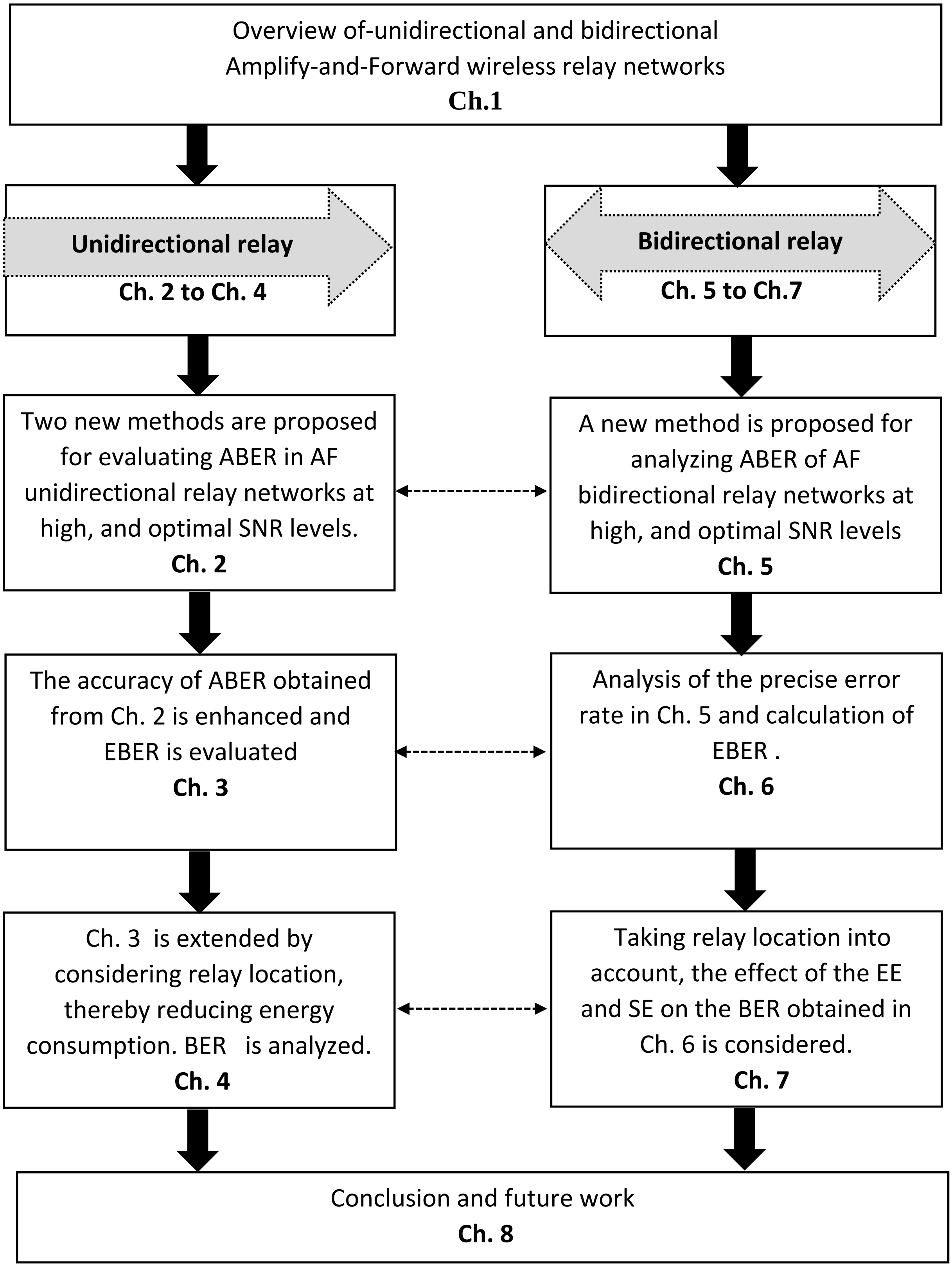

Figure 1.5: Thesis writing structure

Chapter 2

ABER Analysis in Unidirectional Relay Networks

2.1 Introduction

Wireless relay networks have been proposed as a key concept underlying future wireless systems capable of providing high network reliability and significant error rate reduction. However, in such network paradigms, two major challenges have to be overcome: the effect of the wireless medium and the selection strategy. Amplify-and-Forward (AF) relay network, which is one of the main relay networks as mentioned in chapter 1, typically uses two wireless channels to transfer information from a source to a destination, while the main function of the AF-relay node is to amplify the dispatched signal. Such a process, however, reduces the Signal-to-Noise Ratio (*SNR*) as a result of amplifying the noise [50]. Thus, noise poses a major problem that affects the error rate performance of AF-relay networks [5]. Another challenge for such a network is that it typically uses a selection strategy to choose the relay branch with the best *SNR* in order to enhance the Energy Efficiency (EE), Spectrum Efficiency (SE) and (Bit Error Rate) BER performance [43,61]. However, this strategy requires the *SNR* of each branch to be monitored continuously and forces the receiver to switch continually between branches as the branch with the best *SNR* changes [62]. Thus, a new mathematical challenge for error rate performance analysis is posed.

Various methods have been proposed for investigating the BER performance of AF-relay networks. Many of these investigations [21, 26–31] have analyzed asymptotic BER based on approximated *SNR*, while others [22–24, 32–34] have examined the exact BER estimation using the actual *SNR*.

A general method of calculating asymptotic BER performance using various diversity techniques was presented in [26]. In this study, an approximation for the *SNR* of fading channels was analyzed using advanced statistical methods to obtain the BER performance. This analysis shows that the average BER can be characterized by diversity and coding gains at a high *SNR* value, and also that the high *SNR* approximations are useful for wireless performance analysis. A similar system was applied by [27, 28] to compare the BER performances of AF-relay and DF-relay networks. They presented a new statistical method that depends on the harmonic mean theorem. The results show that the DF-relay system has a better performance at low *SNR* values than the AF-relay system, while at higher *SNR* values both the AF-relay and DF-relay systems have similar BER performances. Similarly, in [21], the Symbol Error Probability (SEP) for diversity links under an *SNR* approximation was used to analyse the performance for any number of hops and relays in the AF-relay system. The simulation shows that the performance improves as the number of network relays increases. The *SNR* approximation of AF-relays was also used in [29] for analyzing asymptotic outage probability and to derive the error rate performance in different systems, including multi-hop, multi-relay and multi-relay-multi-hop systems. It was shown that a multi-hop relay can perform better than other systems

Authors in [30] proposed a new method of calculating the asymptotic BER for the AF-relay selection strategy by using the harmonic mean of two variables. They determined the cumulative distribution function (CDF) and the probability density function (pdf) for the selected *SNR* relay. Their results demonstrate that increasing the number of relays enhances the performance of the system. A similar AF-relay system has also been investigated in [34] using a low-complexity relay selection scheme, and accurate approximations of both the BER and the outage probability were obtained. A selection-switching threshold technique was employed to determine the relay with the best *SNR* value using the CDF, pdf and Moment Generating Function (MGF) distributions of the *SNR*. This was subsequently used to derive expressions for the BER and outage probability. Results of this proposed method showed that the selection-switching threshold reduces the complexity of the AF-relay network compared with existing schemes. The study in [31] analyzed the BER performance of an AF-relay network using the approximation and exact *SNR* for cascaded and unintegrated slowly fading Rayleigh channels. The CDF, pdf, Meijer G-function and other tools were used for the analysis in this study. The study showed how the BER estimation can be used for optimal power allocation in AF-relay networks with a slowly fading channel using maximum likelihood estimation. This study showed that the BER expressions used for the estimation of cascaded channels are more complex than those for unintegrated channel estimation.

For exact BER calculation, the study in [22] obtained the SER performance over a Rayleigh-fading channel, for parallel relays, using a number of approaches such as the MGF of half the harmonic mean. This study showed that the AF-relay and DF-relay networks achieve full diversity order. A similar system in [32] derived closed-form expressions for the BER performance in AF-relay and DF-relay networks. The BER calculation in this study comprises three stages. First, a closed-form expression is derived for the pdf of the *SNR* values for the best relay selection. Second, an expression is obtained for the MGF of the *SNR*. Finally, the MGF expression is used to derive expressions for the BER performance. In [33] a new method is presented for computing the exact BER over Rayleigh, Nakagami-m and Rician fading channels, based on the pdf and CDF of the *SNR* of an opportunistic network. It was observed that the selection scheme improved the BER for the AF-relay system. The harmonic mean was used in [23] to calculate the upper and lower bounds for the exact performance. This study shows that the difference between the actual *SNR* and the harmonic mean approximation is less than 0.5 in any fading environment. In [24] the asymptotic and actual Frame Error Rate (FER) performances were analyzed to compare the relay selection criteria for AF-relay hybrid cooperative networks. This study derived the FER expression using the Laplace transform of the CDF and pdf of the end-to-end *SNR*. The results indicate that the selection of one optimum relay achieves better performance than the selection of multiple relays when the transmitting power is constrained.

Evaluating the BER performance of AF-relay selection networks from the perspective of EE has been investigated by several studies such as [19, 63, 64]. In [63] the Symbol Error Rate (SER) performance of a single AF-relay network is investigated. This study compared the SER performance using the optimum power allocation with that of the systems using the equal-power scheme. It showed that the SER performance using the optimum power technique was slightly better than that of the equal-power technique. Another study in [64] adopted the BER as a performance metric to investigate the optimum relay network from a battery energy efficiency perspective.The results showed that, owing to the extra relay circuit consumption, relaying does not necessarily increase the system EE. Most other studies that focused on analyzing the optimal AF-relay network did not consider the BER behaviour. It needs to be noted that the above studies used AF-relay networks on the assumption that there is no direct transmission link between the source and destination. This is because introducing a direct link would require calculating two different *SNR* values for the BER analysis.

Following on from the above discussion, this chapter investigates two methods for evaluating Asymptotic BER (ABER) in AF-relay networks. The first method, namely Unified BER (U-BER), can be used to evaluate the ABER for different channel conditions. It can be also used to standardize the BER analysis for both two-hop AF relays and one-hop (or direct link) communication networks. This method enables analysis of the BER in one direction whether the direct line exists or not. It is worth mentioning here that combining a direct link with relay links provides a low BER for the AF-relay network [53]. The second method is referred to as Optimal BER (O-BER) method. This method is appropriate for analyzing optimal AF-relay networks, particularly when the EE and *SE* are optimized in a balanced way, which is achieved by maximizing the EE, which in turn increases the overall *SNR*. Thus, the BER is minimized, as the maximization of the overall received *SNR* is equivalent to the minimization of the BER [30].

The analytical expression for the asymptotic BER performance of both (U-BER) and (O-BER) are derived. An expression for the exact BER for (U-BER) is also obtained. All these expressions were validated by simulations, and it was found that the proposed methods provide useful and efficient tools for analyzing the BER performance of AF-relay networks.

The chapter is organized as follows. Section 2.2 outlines the system model; Section 2.3 describes the expression used to calculate the *SNR*; Section 2.4 presents the U-BER method for analyzing asymptotic and exact BER performance while the O-BER method is presented in Section 2.5. The attributes of both methods are discussed in the Sections 2.6 and 2.7; Section 2.8 illustrates analysis and simulation results, and finally conclusions are presented in Section 2.9.

2.2 SYSTEM MODEL

A communication between a source node (S) and a destination node (D) through Multiple AF Relay (MAFR) network is considered. This network model is illustrated in Fig. 2.1. Each of the nodes (i.e S, D and relays) is equipped with one antenna. The availability of a direct link between S and D is subject to propagation conditions and transmission power limitations of the source node (The existing direct-link case is investigated in Section 2.6). In the case of no direct link between source and destination, the node S transmits symbol x using Binary Phase Shift Keying (BPSK) modulation with average power p_s to all relays over Rayleigh fading channels specified by the Independent and Identically Distributed (i.i.d.) random variables.

The received signal by any relay (y_i) can be calculated as

$$y_i = \sqrt{p_s} x h_i + z_{ri}, \tag{2.1}$$

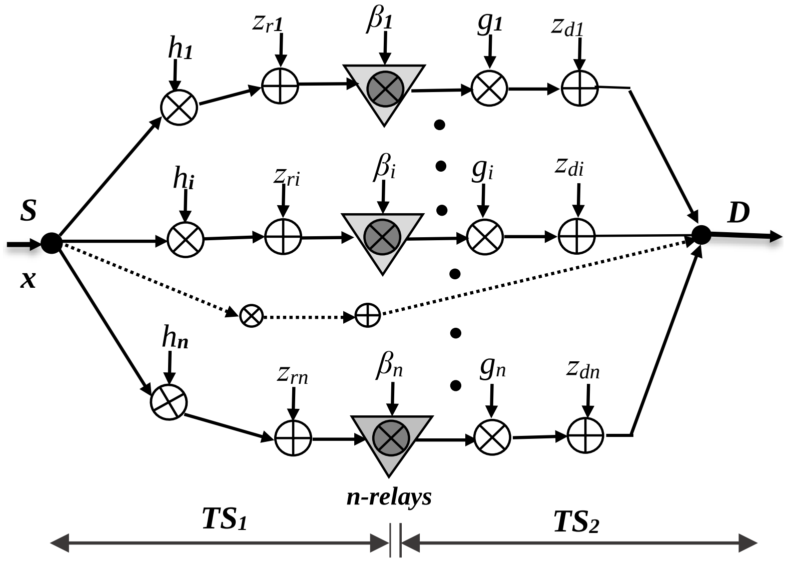

Figure 2.1: System model

where h_i is a fading coefficient between S and relay (r_i), z_{ri} is Gaussian noise with zero mean and variance σ^2. It is assumed that the node D knows all channel conditions. This node also includes a selection combining scheme that select one relay with the highest *SNR*.

The output signal for a selected relay is amplified by the amplification factor (β_i). It can be calculated as

$$\beta_i = \sqrt{\frac{p_{ri}}{|h_i|^2 p_s + \sigma^2}}, \tag{2.2}$$

where p_{ri} is the relay power.

The amplified signal from the selected relay is forwarded to the node D through another Rayleigh-fading channel, which includes a fading coefficient (g_i) and random Gaussian noise (z_{di}) with zero mean and variance (σ^2). The received signal by node D is

$$y_{di} = \beta_i y_i g_i + z_{di}. \tag{2.3}$$

where y_{di} is the received signal.

2.3 SIGNAL-TO-NOISE-RATIO

The signal to noise ratio (*SNR*) for an MAFR network can be obtained by substituting (2.1) and (2.2) into (2.3). This results in

$$y_{di} = \sqrt{\frac{p_{ri}}{|h_i|^2 p_s + \sigma^2}}(xh_i\sqrt{p_s} + z_{ri})g_i + z_{di}. \tag{2.4}$$

By using the expected value of (2.4) and assuming $E\left\{x^2\right\} = 1$, we can obtain the end-to-end *SNR* as follows

$$SNR = \frac{\gamma_{hi}\gamma_{gi}}{\gamma_{hi} + \gamma_{gi} + 1} = \gamma_i, \tag{2.5}$$

where γ_{hi} and γ_{gi} are the instantaneous channel gain-to-noise ratios (CNR) in the source-to-relay and the relay-to-destination channels respectively; both are defined in [30] as

$$\gamma_{hi} = \frac{p_s}{\sigma^2}|h_i|^2, \tag{2.6}$$

and

$$\gamma_{gi} = \frac{p_{ri}}{\sigma^2}|g_i|^2. \tag{2.7}$$

Equation (2.5) can be approximated as follows

$$SNR = \frac{\gamma_{hi}\gamma_{gi}}{\gamma_{hi} + \gamma_{gi}} = \gamma_{xi}. \tag{2.8}$$

The γ_{xi} expression is obtained under the assumption that the channel CNRs $\gg 1$. It has been used to calculate asymptotic BER of MAFR networks, as in [24, 30]. Equation (2.8) is adopted in the following sections to derive bit error rate expressions by assuming that the CNR channel of each hop is located in two different environmental scenarios, that are: $\gamma_{hi} = \gamma_{gi}$ and $\gamma_{hi} \neq \gamma_{gi}$.

2.4 Asymptotic and Exact Performances of U-BER Method

In this section, the asymptotic BER expression for the MAFR network is derived. Also, the exact BER performance will be provided in the last part of this section. The expression of BER of one-hop communication using BPSK follows $P_b(\gamma) = \int_0^\infty Q(\sqrt{2\gamma})\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}d\gamma$, where $P_b(\gamma)$ is a rate of error probability, $\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}$ is the probability density function of γ (γ is the CNR of source-to-destination channel), and $Q(\sqrt{2\gamma})$ is the Gaussian Q-function defined as $Q(\sqrt{2\gamma})=\frac{1}{\sqrt{2\pi}}\int_{x=\sqrt{2\gamma}}^{\infty}\exp^{\frac{-x^2}{2}}dx=\frac{1}{2}\text{erfc}(\sqrt{\gamma})$ [13]. Evaluating BER in one-hop gives $P_b(\gamma)=\frac{1}{2}\left(1-\sqrt{\bar{\gamma}(1+\bar{\gamma})^{-1}}\right)$, where $\bar{\gamma}$ is the average value for $SNR=p_s\left|h\right|^2/\sigma^2$, being determined by [13,65,66] as $\bar{\gamma}=E\left\{\gamma\right\}=\frac{p_s}{\sigma^2}E\left\{\left|h\right|^2\right\}=\frac{E_b}{No}$, where $E\left\{\left|h\right|^2\right\}$is the expected value of the fading channel which equals one. The term E_b/No is the energy per bit to the spectral noise density ratio [13].

The performance of the one-hop scheme is equivalent to the first time slot (TS_1) in the MAFR network as shown in Fig. 2.1. The MAFR network includes another time slot (TS_2) from relay to destination [67]. To obtain the asymptotic BER performance for the MAFR network, the same procedures are followed as for the analysis of the BER of a one-hop network. This can be realized by assuming that two-hop channel is equivalent to a one-hop channel, as the errors at the D node can occur either when the S to r_i transmission is received correctly while the r_i to D transmission is received in error; or, vice versa. Hence, the error probability at the node D can be equivalent to one-hop. This gives

$$P_b(\gamma_{xi})=P_b(\gamma_{xi(\gamma)})=\frac{1}{\bar{\gamma}_{xi(\bar{\gamma})}}\int_{0}^{\infty}Q(\sqrt{2\gamma_{xi(\gamma)}})\,e^{-\frac{\gamma_{xi(\gamma)}}{\bar{\gamma}_{xi(\bar{\gamma})}}}d\gamma, \tag{2.9}$$

where $P_b(\gamma_{xi(\gamma)})$ is a probability of error for a single relay network with respect to γ, and $Q(\sqrt{2\gamma_{xi(\gamma)}})$ is the total complement of the CDF corresponding to a Gaussian random variable [68].

By assuming that the MAFR network has one relay node, the total Q function can be calculated as for [69]:

$$Q(\sqrt{2\gamma_{x1(\gamma)}})=\frac{1}{2}\text{erfc}(\sqrt{\frac{\gamma_{x1(\gamma)}}{2}}), \tag{2.10}$$

where $\gamma_{x1(\gamma)}$ is the approximated SNR for one relay, which is obtained from (2.8) as

$$\gamma_{x1}=\gamma_{x1(\gamma)}=\frac{\gamma_{h1}\gamma_{g1}}{\gamma_{h1}+\gamma_{g1}}, \tag{2.11}$$

where γ_{h1} and γ_{g1} are the CNRs for TS_1 and TS_2, respectively.

The fading random variables for γ_{h1} and γ_{g1} (i.e $|h_1|^2$ and $|g_1|^2$) are independent and identically distributed (i.i.d) and they both have the same expectation values [70, 71] that are $E\left\{|h_1|^2\right\}=1$ and $E\left\{|g_1|^2\right\}=1$. Accordingly, the average values for γ_{h1} and γ_{g1} can be determined from (2.6) and (2.7), respectively as

$$\bar{\gamma}_{h1} = \frac{p_{s1}}{\sigma^2} E\left\{|h_1|^2\right\}, \tag{2.12}$$

$$\bar{\gamma}_{g1} = \frac{p_{r1}}{\sigma^2} E\left\{|g_1|^2\right\}. \tag{2.13}$$

$\bar{\gamma}_{g1}$ and $\bar{\gamma}_{g1}$ are the average values of γ_{h1} and γ_{g1}.

By substituting (2.12) and (2.13) into (2.11), assuming $p_{r1} = p_{s1}$, gives rise to the following equation

$$\bar{\gamma}_{x1} = \bar{\gamma}_{x1(\bar{\gamma})} = \bar{\gamma}(2)^{-1}. \tag{2.14}$$

Another parameter $\frac{1}{\bar{\gamma}_{xi(\bar{\gamma})}} e^{-\frac{\gamma_{xi(\gamma)}}{\bar{\gamma}_{xi(\bar{\gamma})}}}$ in (2.9) should be obtained. This parameter represents the *SNR* probability density function (pdf) of double-cascaded Rayleigh channels. Many studies [11, 22, 27, 72–74] have suggested different methods to compute such pdf. In our study we suggest obtaining the pdf with respect to $\bar{\gamma}$. Since the *SNR* in i.i.d Rayleigh fading channels (i.e.$|h_i|$ and $|g_i|$) follows an exponential distribution [75], such variables can be expressed as shown in [76]. Moreover, the pdf derivation is based on assuming $\gamma_{hi} = \gamma_{gi}$ i.e. a fixed relay gain [11,30]. Thus, the pdf is expressed as

$$\rho(\bar{\gamma}_{x1}) = \frac{1}{\bar{\gamma}_{x1(\bar{\gamma})}} e^{-\frac{\gamma_{x1(\gamma)}}{\bar{\gamma}_{x1(\bar{\gamma})}}} = (0.5\bar{\gamma})^{-1} e^{-\frac{\gamma}{\bar{\gamma}}}, \tag{2.15}$$

where $\rho(\bar{\gamma}_{x1})$ is the pdf of γ, where one relay node exists in the MAFR network.

By substituting (2.10) and (2.15) into (2.9), and then evaluating the integration, the asymptotic BER of the MAFR network, which includes one relay, is expressed as

$$P_b(\gamma_{x1}) = 1 - \sqrt{1 - (1 + \bar{\gamma}_{x1})^{-1}}. \tag{2.16}$$

Equation (2.16) can be used to analyze one relay in the MAFR network, so, to extend this analysis for multiple relays in MAFR network, the order statistic of the *SNR* which was demonstrated in [77] can be applied at the selection combining scheme. In this order statistic, the output of several i.i.d. random variables (here represented by *SNR*s) can be ordered as $\gamma_{x(i)} = max\{\gamma_{x(1)}, \gamma_{x(2)}, \cdots, \gamma_{x(n,)}\}$

where $\gamma_{x(i)}$ is the i_{th} order statistic of the *SNR*, $\gamma_{x(n)}$ that represents the largest order statistic, $\gamma_{x(1),}$ is the smallest value. The CDF of such i_{th} order statistic is expressed as

$$\gamma_{xi}(\gamma) = \left[\text{Prob}\left(\gamma_{x(i)} \leq \gamma\right)\right]^n = \left[\mathscr{F}_{\gamma_{xi}}(\gamma)\right]^n. \tag{2.17}$$

where $\left[\mathscr{F}_{\gamma_{xi}}(\gamma)\right]^n$ is the CDF of the order statistic [77].

By applying the property of (2.17) into (2.16) , the BER of n relays can be expressed as

$$P_b(\gamma_{xi}(\gamma)) = n\left(1-\sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2n}}\right)^n, \tag{2.18}$$

where $P_b(\gamma_{xi}(\gamma))$ is the asymptotic BER performance of the MAFR network which includes n relays.

Equation (2.18) is obtained when $\gamma_{hi} = \gamma_{gi}$. It can be also derived in the case of a different channel condition i.e., $\gamma_{hi} \neq \gamma_{gi}$. Here we assume that the ratio between γ_{hi} and γ_{gi} is given as

$$\zeta = \frac{\gamma_{hi}}{\gamma_{gi}}. \tag{2.19}$$

Substituting (2.19) for (2.8) results in

$$\gamma_{xi(\gamma)} = \frac{\zeta\gamma_{gi}}{\zeta+1}, \tag{2.20}$$

where $\gamma_{xi(\gamma)}$ is the approximated *SNR* for the MAFR network with respect to γ_i.

The average value of $\gamma_{xi(\gamma)}$ is expressed as

$$\bar{\gamma}_{xi(\gamma)} = \frac{\bar{\zeta}\,\frac{p_{r1}}{\sigma^2}\,E\left\{|g_1|^2\right\}}{\bar{\zeta}+1}, \tag{2.21}$$

where $\bar{\zeta}$ is the average value of ζ, calculated as $\bar{\zeta} = \frac{\bar{\gamma}_{hi}}{\bar{\gamma}_{gi}}$, and the values $\bar{\gamma}_{hi}$and $\bar{\gamma}_{gi}$ are calculated from (2.12) and (2.13), respectively.

Following similarly procedures in (2.18) and considering (2.21), the ABER for n relays can be expressed as

$$p_b(\gamma_{i(\gamma)}) = n\left(1-\sqrt{1-(1+\bar{\gamma}_{ix(\gamma)})^{-1}}\right)^n, \tag{2.22}$$

where $p_b(\gamma_{i(\gamma)})$ is the ABER performance for the MAFR network where $\gamma_{hi} \neq \gamma_{gi}$, and $\bar{\gamma}_{xi}$ is the average value of the *SNR,* expressed as

$$\gamma_{xi(\gamma)} = n^{-1}\left(\frac{\bar{\zeta}}{\bar{\zeta}+1}\right). \tag{2.23}$$

2.5 Asymptotic Performance for O-BER Method

In this section, the ABER expression for the MAFR network is derived using the O-BER method. In this analysis we want to determine the largest value of the probability density function (pdf) from n relays, and then we will use the arithmetic-geometric mean (AM–GM) inequality to obtain the average value of the *SNR*. Hence, we can apply the general BER expression (i.e., $P_b(\gamma) = \int_0^\infty Q(\sqrt{2\gamma})\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}d\gamma$) for the calculation of asymptotic BER performance.

From the derivation of (2.17), the pdf of $\gamma_{xi}(\gamma)$ can be expressed as

$$p_{\gamma_{xi}max=}n\left(\mathscr{F}_{\gamma_{xi}}(\gamma)\right)^{n-1}\rho\left(\gamma_{xi}\right), \tag{2.24}$$

where $p_{\gamma_{xi}max}$ is the pdf of $\gamma_{xi}(\gamma)$.

The mean value of γ_{xi} can be determined by one of the generalized mean theorems, such as harmonic mean and AM–GM inequality. Here, we use the AM–GM inequality to calculate the total expectation value of (2.8). This gives

$$\bar{\gamma}_{xi} = \frac{1}{\frac{1}{\bar{\gamma}_{hi}}+\frac{1}{\bar{\gamma}_{gi}}} = \frac{1}{2}\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}, \tag{2.25}$$

where and $\bar{\gamma}_{hi}$ and $\bar{\gamma}_{gi}$ can be defined as (2.12) and (2.13) respectively.

Based on (2.25), the pdf $\rho(\gamma_{xi})$ for the proposed system is expressed as

$$\rho\left(\gamma_{xi}\right) = \frac{2}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}. \tag{2.26}$$

Also, the CDF of the Rayleigh random variable can be obtained by using the alternative expression of CDF, which is the Q-function [13]. This is expressed as $Q(\bar{\gamma}_{xi}) = \left(1-e^{-\frac{2\gamma_{xi}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}\right)$. By substituting this $Q(\bar{\gamma}_{xi})$ function and (2.25) into the general BER expression and assuming $n=1$ as

$$p\left(b\right) = \int_0^\infty \frac{2}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}\left(1-e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}\right)d\gamma. \tag{2.27}$$

The evaluation of (2.27), as in (2.16), results in the following expression

$$p(b) = \frac{1}{2}\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}\left(1-\sqrt{\frac{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}{\bar{\gamma}_{hi}\bar{\gamma}_{gi}+4}}\right). \tag{2.28}$$

where $p(b)$ in (2.28) represents the ABER performance of the MAFR network when $n = 1$.

In the case of n relays, (2.24) can be determined as follows

$$p_{\gamma_{xi}max} = n\left(1-e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}\right)^{n-1}\frac{2}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}. \tag{2.29}$$

Substituting (2.29) and (2.26) into the aforementioned general BER expression as

$$p_x(b) = \int_0^\infty n\left(1-e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}\right)^{n-1}\left(1-e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}\right)\frac{2}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}e^{-\frac{2\gamma_{xi.}}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}}d\gamma, \tag{2.30}$$

the evaluation of (2.30) results in the following expression

$$p_x(b) = \mathrm{n}\left(\sqrt{\frac{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}{4}}\left(1-\sqrt{\frac{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}{\bar{\gamma}_{hi}\bar{\gamma}_{gi}+4n^2}}\right)\right)^n, \tag{2.31}$$

where $p_x(b)$is the ABER performance of MAFR networks.

2.6 U-BER Method Attributes

The U-BER method which is presented in Section 2.4 can be used to unify BER calculation of MAFR two-hop and one-hop communication networks. In the principle operation of MAFR networks, the relay works when the direct link, between the source and the destination, is unavailable due to severe fading [78]. Thus, BER performance should be analyzed with and without a direct link. In this context, many studies such as [79–81] have divided the performance analysis into two parts, each part requiring a certain *SNR*. The first part considers the direct transmission link between the source and the destination to determine the *SNR*, while the second requires the calculation of a new *SNR* without a direct link. Then, from each *SNR*, the BER is obtained.

By applying the U-BER method, the BER performance for both direct communication links and two-hop MAFR networks can be formulated in one equation. This is achieved by considering the relationship between a number of relays n and the number of hops ($\mathscr{H}$), as follows:

$$\mathscr{H} = \begin{cases} 2n & Two-hop \\ 1 & One-hop \end{cases}. \tag{2.32}$$

The asymptotic BER for (2.18) and the one-hop BER performance can be unified in one equation by considering (2.32) as

$$p_b(\gamma_{x\mathscr{H}}) = \mathscr{H}2^{-1}\left(1-\sqrt{\bar{\gamma}(\mathscr{H}+\bar{\gamma})^{-1}}\right)^{\lceil \mathscr{H}2^{-1}\rceil}, \tag{2.33}$$

where $\lceil \mathscr{H}2^{-1}\rceil$ is the ceiling function, and $p_b(\gamma_{x\mathscr{H}})$ is the standardized equation for estimating the approximated BER performance in the dual-hop and single-hop communication system.

2.7 O-BER Method Attributes

The O-BER method can be utilized for estimating the BER performance when *EE* and *SE* of MAFR networks are optimized. We first briefly analyze the relationship between *SE* and *EE*, and then one expression to combine *EE* and *SE* is obtained. The model system presented in Section 2.2 is adopted for optimizing *SE* and *EE* in a balanced way by considering the energy consumption at the relay circuit. Afterwards, we apply (2.31) to evaluate BER performance.

A general definition for *EE* in wireless relays networks is a measure of energy consumed in a transmitting bit during transmission across a network. It is formulated as

$$EE = \frac{\text{Total data delivered } (\Re_i)}{\text{Total energy consumed } (p_{ti})} \text{ (bit/Joule)}, \tag{2.34}$$

where $\Re_i$ is the throughput for unit bandwidth (bits/sec/Hz) which is defined in MAFR networks as $\Re_i = \frac{1}{2}\log_2(1+\gamma_{xi})$ [39] and p_{ti} is the total energy consumption by relay nodes. This energy consumption includes the power consumption of the signal processing and amplification circuits [82]. It can be presented as

$$p_{ti} = (\xi(p_s + p_{ri}) + (p_{ci} + \rho\Re_i)), \tag{2.35}$$

where ξ is the constant associated with the amplifier power efficiency, and p_{ci} is the static circuit power consumption (further information about this power is presented by [83]), and $\rho\Re_i$ is the dynamic power per-unit data rate.

Now, to balance *SE* and *EE* optimally, the highest *SNR* is obtained first by first minimizing total energy consumption under a fixed throughput, and next by maximizing the ratio of EE. Minimizing the total energy consumption for a fixed throughput can be achieved by minimizing total transmitted power as follows

$$p_{s(\gamma_{xi})} = \gamma_{xi}\left(\frac{1}{\gamma_{hi}} + \frac{1}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right) \tag{2.36}$$

$$p_{ri(\gamma_{xi})} = \gamma_{xi}\left(\frac{1}{\gamma_{gi}} + \frac{1}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right) \tag{2.37}$$

where $p_{s(\gamma_{xi})}$ and $p_{ri(\gamma_{xi})}$ are the minimum powers for the source and relay respectively, both with respect to γ_{xi}.

Substituting (2.36) and (2.37) into 2.34 metrics as

$$EE_{(\gamma_{xi})} = \frac{\log_2(1+\gamma_{xi})}{2\,\gamma_{xi}\,\xi\left(\frac{1}{\gamma_{gi}} + \frac{1}{\gamma_{hi}} + \frac{2}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right) + 2\,p_{ci} + \rho\,\log_2(1+\gamma_{xi})} \tag{2.38}$$

where $EE_{(\gamma_{xi})}$ is the EE with respect to γ_{xi}.

By taking the derivative of equation (2.38) with respect to γ_{xi}, the optimal *SNR* value of the MAFR network is obtained as follows

$$\overset{*}{\gamma}_{xi} = \frac{\frac{p_{ci}}{\xi} - \left(\frac{2}{\sqrt{\gamma_{hi}\gamma_{gi}}} + \frac{1}{\gamma_{hi}} + \frac{1}{\gamma_{gi}}\right)}{\left(\frac{2}{\sqrt{\gamma_{hi}\gamma_{gi}}} + \frac{1}{\gamma_{hi}} + \frac{1}{\gamma_{gi}}\right) w\left(\frac{\frac{p_{ci}}{\xi} - \left(\frac{2}{\sqrt{\gamma_{hi}\gamma_{gi}}} + \frac{1}{\gamma_{hi}} + \frac{1}{\gamma_{gi}}\right)}{\left(\frac{2}{\sqrt{\gamma_{hi}\gamma_{gi}}} + \frac{1}{\gamma_{hi}} + \frac{1}{\gamma_{gi}}\right)\mathrm{e}}\right)} - 1 \tag{2.39}$$

where $\overset{*}{\gamma}_{xi}$ is the optimal value of (2.8) under the optimal *SE* and EE, *e* is the base of the natural logarithm and *w* is the omega function.

From (2.39), the optimal data rate can be determined as $\overset{*}{\Re}_i = 0.5\,\log_2(1+\overset{*}{\gamma}_{xi})$, which can be rewritten as

$$\overset{*}{\gamma}_{xi} = 2^{2\overset{*}{\Re}_i} - 1\,. \tag{2.40}$$

By using (2.12), (2.13), (2.39) and (2.40) while considering the relationship between energy bit E_b and bit rate $\Re_i$ described in [84] and assuming $\gamma_{hi} = \gamma_{gi}$, one can find the optimal average value of γ_{xi} ($\overset{*}{\bar{\gamma}}_{xi}$) as $\overset{*}{\bar{\gamma}}_{xi} = \frac{\Re_i E_b}{\overset{*}{\Re}_i 2No}$.

In the case of $\gamma_{hi} \neq \gamma_{gi}$, the optimal average value is obtained as follows

$$\overset{*}{\bar{\gamma}}_{xi} = \frac{\Re_i \sqrt{P_s P_{ri}}}{\overset{*}{\Re}_i 2No}. \tag{2.41}$$

Accordingly, following the same procedures for (2.31) using the optimal *SNR* value in (2.41), the optimal BER performance is determined as follows

$$p_{x(b)} = \mathrm{n} \left(\frac{\Re_i}{\overset{*}{\Re}_i 2No} \sqrt{P_s P_{ri}} \left(1 - \sqrt{\frac{1}{1 + \frac{4\overset{*}{\Re}_i^2 n^2}{\left(\Re_i^2 P_s P_{ri}\right)}}} \right) \right)^n. \tag{2.42}$$

where $p_{x(b)}$ is the asymptotic BER for the MAFR networks in the case of optimal *SE* and *EE* metrics.

2.8 Analysis and Simulation Results

Analytical results for the proposed technique are presented in this Section. These results are compared to the simulation and analytical results presented in [30]. All simulations are performed on the system model in Section 2.2, assuming three relay nodes in the network.

Fig. 2.2 shows the ABER performance for the proposed technique from (2.5). It is clear from this figure that the BER performance of the MAFR network improves by increasing the number of relays. Further, the analysis results approach the simulation results as the *SNR* increases. Also, by comparing Fig. 2.2 with the analytical results in [30], it is clear that there is a good match between both analyses in the high *SNR* region. On the other hand, our analytic results for low *SNR* values are relatively close to the simulation results as compared to [30] and [24]. Thus, our asymptotic BER expression gives slightly more accurate results.

To validate the BER performance in (2.31), Figs. 2.3 and 2.4 were produced to show the results of both cases $\bar{\gamma}_{hi} = \bar{\gamma}_{gi}$ and $\bar{\gamma}_{hi} = 5\,dB + \bar{\gamma}_{gi}$ and then compared to the existing results in [30]. The figures confirm that the O-BER method gives excellent match results at high *SNR* values as compared to the existing study result.

For BER performance in an optimal MAFR network, Equation (2.42) is used in Fig. 2.5 assuming $\overset{*}{\bar{\gamma}}_{hi} = \overset{*}{\bar{\gamma}}_{gi}$. It demonstrates that the BER is enhanced as compared to Fig. 2.3 under similar conditions.

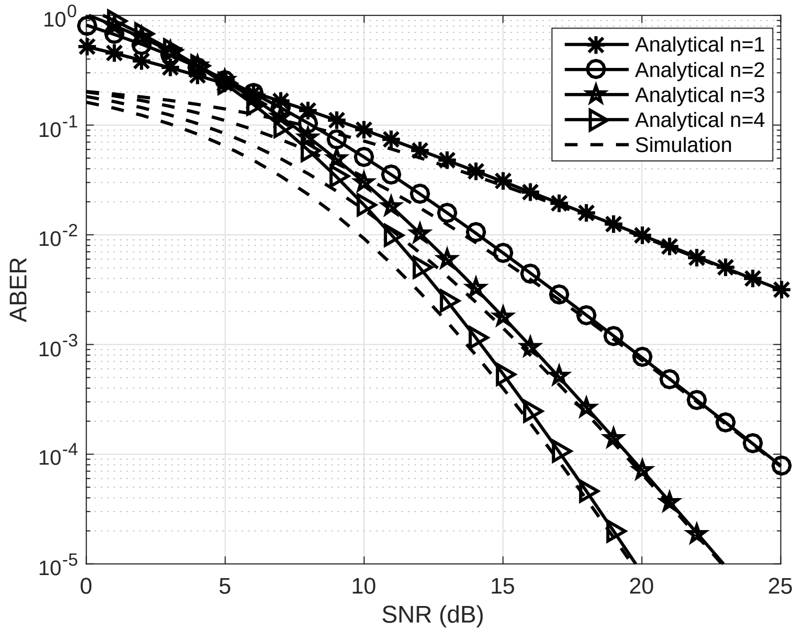

Figure 2.2: ABER performance in an AF relays network, $\bar{\gamma}_{hi} = \bar{\gamma}_{gi}$

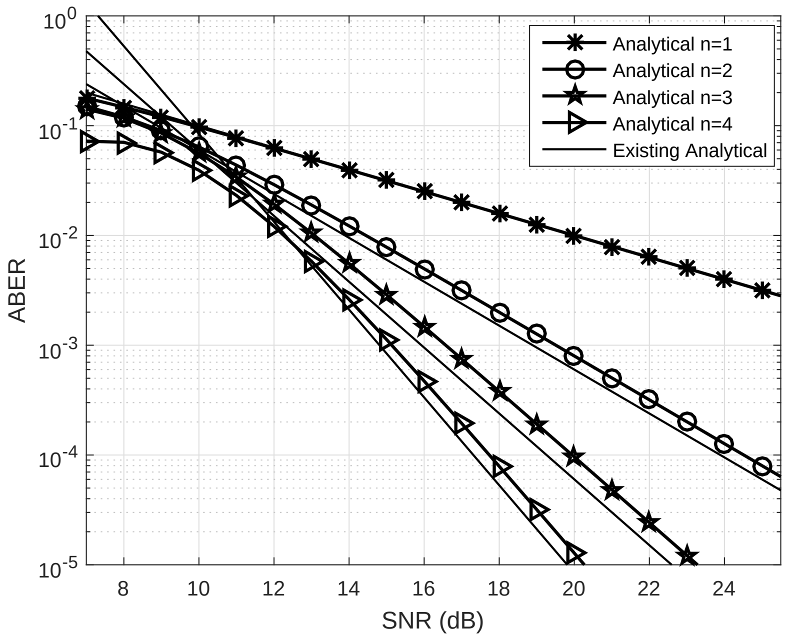

Figure 2.3: ABER performance in an AF relays network, $\bar{\gamma}_{hi} = \bar{\gamma}_{gi}$

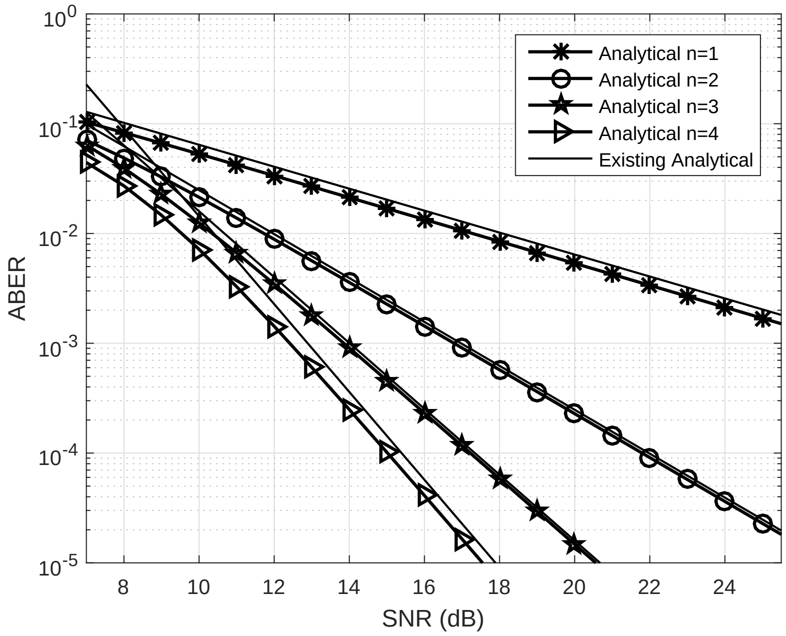

Figure 2.4: ABER performance in an AF relays network, $\bar{\gamma}_{hi} = 5\,dB + \bar{\gamma}_{gi}$

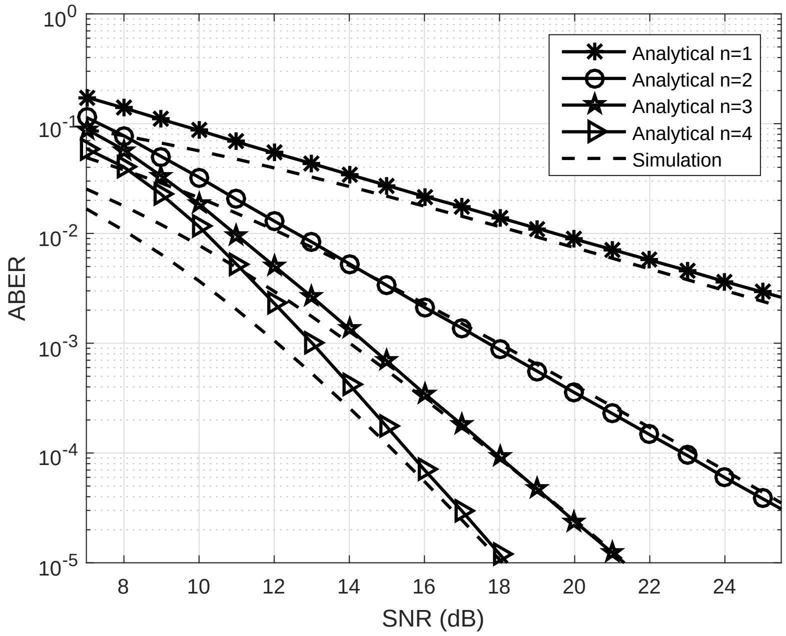

Figure 2.5: ABER performance in an optimal AF relays network, $\overset{*}{\bar{\gamma}}_{hi} = \overset{*}{\bar{\gamma}}_{gi}$

2.9 SUMMARY

In this chapter, two methods, namely U-BER and O-BER, are proposed for calculating the ABER performance of two-hop MAFR networks. Bit error rate expressions for both methods are derived by assuming that the CNR channel of each hop is located in two different environmental scenarios: $\gamma_{hi} = \gamma_{gi}$ and $\gamma_{hi} \neq \gamma_{gi}$. It has been found that the U-BER method unifies the BER performance analysis for both the two-hop MAFR network and the one-hop communication system. The O-BER method is an appropriate tool for estimating the BER performance in optimal MAFR networks. The derived expressions for both methods have been validated by simulation and comparing their corresponding results to the existing analytical results. It is shown that there is a reasonably good match between the analytical and the simulation results. The proposed methods provide efficient ways for calculating the ABER when the MAFR networks operate in optimal or sub-optimal conditions.

In the next chapter, the above bit error rate methods are enhanced. Further, the approximated BER of O-BER method is extended to the exact BER analysis.

Chapter 3

Accurate BER Analysis Under Different *SNR* Levels

3.1 INTRODUCTION

The Approximated Bit Error Rate (ABER) of Multiple AF Relay with relay selection scheme (MAFR) networks has been examined in Chapter 2, and was found to be a reasonably accurate measure of the error rate performance under high and optimal *SNR* levels. While Chapter 2 focused on ABER, this chapter focuses on increasing the accuracy of the ABER in covering a wide range of applications. Furthermore, the Exact Bit Error Rate (EBER) is analyzed based on the ABER in order to unify the calculation of BER of MAFR networks under low, high and optimal *SNR* levels.

Various methods have been proposed for analyzing the BER performance of wireless networks. An early method was investigated by [26] to calculate the ABER of a one hop communication system using various diversity techniques. In this study, a high *SNR* level for fading channels was adopted and different M-ary Phase Shift keying (MPSK) modulation schemes were used. It was found that the high *SNR* approximations are very useful for analyzing wireless performance.

The error rates for two hop AF relay and DF relay networks were analyzed by [27, 28]. Both studies presented a new statistical method that depends on one kind of Pythagorean mean. Their results demonstrated that a DF-relay network performs better at low *SNR* values than an AF-relay network, while at higher *SNR* values, both demonstrate similar BER performances. However, the BER analysis in both studies was limited to one relay networks.

In [21], multi-hops and multi-relay networks were involved in the analysis of the error rate of AF relay networks. This study used a high *SNR* level to calculate the Symbol Error Probability (SER) performance. It found that the SEP performance improves as the number of relays increases. Similar studies [22–24] have investigated the SER performance of MAFR networks, the authors in [22] obtaining the exact SER performance of AF parallel relays over a Rayleigh-fading channels. They used the Moment Generating Function (MGF) of half the harmonic mean to calculate the SER performance. In [23], the harmonic mean approximation was used to calculate the upper and lower bounds for exact SER and outage probability performances. Further, Laplace transforms of the Cumulative Distribution Function (CDF) and Probability Density Function (pdf) of the exact *SNR* were used in [24] to obtain asymptotic and actual Frame Error Rate (FER) performance. The results of [24] revealed that the selection of one optimum relay achieves better FER performance than multiple relay selection under transmitting power constraints. However, all of the above SER studies require more analysis to determine the BER, which is a more useful metric than SER [25]. Moreover, calculating the FER depends on the required BER [85].

The authors in [29] considered different network scenarios including: multi-hop, multi-relay, and multi-relay multi-hop, to analyze the ABER and outage probability using a high *SNR* level, which was also adopted by [30] to calculate the ABER of an AF-relay network selection scheme using the harmonic mean of two variables. The study of [30] found that increasing the number of relays with a high *SNR* level enhances the ABER performance. A similar system was also investigated in [34] using a low-complexity relay selection scheme to obtain accurate approximations for both BER and outage probability. Study [31] analyzed the BER performance of AF relay networks using approximated and exact *SNRs* for cascaded and unintegrated slowly fading Rayleigh channels. The above [29–31, 34]. studies, however, gave accurate results when calculating the BER at high *SNR* region, but did not consider the BER at low or intermediate *SNR* regions. Thus, the ABER of these studies was limited when computing the error rate at *SNR* domains that were too high.

The optimal *SNR* for a given BER was analyzed by [19,64] to achieve maximum EE. Similar studies, such as [43,44], have also maximized EE, but without considering error rate performance. Another study in [63] has evaluated SER performance of AF-relay networks at an optimal *SNR* level. This SER is used to determine an optimal power allocation between the source and the relays

The aforementioned studies indicate that the error rate analysis of AF relay networks is limited at low, high or optimal *SNR* level. Given the complexity associated with error rate analysis at low and optimal *SNR*s, most of these studies have assumed a high *SNR* level to simplify the calculation. However, each of the above error rate analyses is limited to particular types of applications, e.g., the error rate analysis at high *SNR* levels is not appropriate for applications working at low *SNR* levels. Therefore, each application associated with a particular *SNR* level may require a separate error rate analysis.

Accordingly, this chapter proposes a unified method that allows the calculation of the BER of MAFR networks under low, high and optimal *SNR* levels. To the best of the authors' knowledge, this idea has not yet been proposed in the literature.

In the proposed method, the BER is calculated approximately (i.e. ABER) at a high *SNR* level. Further, the EBER can be calculated using the same analysis as for the ABER. The proposed ABER method is unlike the existing ABER techniques in [24,30,86,87], because it provides the least disparity between the ABER and EBER. The method thus provides an accurate bit error rate analysis for MAFR networks by using either ABER or EBER. This is because the proposed scheme is achieved by considering the Conventional one-hop BER (CBER) analysis. Thus, the BER evaluation of two hop MAFR at various *SNR* levels (i.e low high and optimal) is roughly the same as in a one hop communication network.

Furthermore, our proposal presents a new approach to optimizing the *SNR* by balancing the average Energy Efficiency ($\overline{EE}$) and the average Spectrum Efficiency ($\overline{SE}$). This approach allows us to calculate the BER performance at an optimal *SNR* level. In the literature, a number of studies, such as [43,44,88], have investigated an optimal *SNR* based on *EE* and *SE* without considering error rate analysis, owing to the complexity in their optimal *SNR* outcomes. Such investigations are incomplete, since error rate analysis is an important parameter in evaluating the performance of many wireless network applications [89].

The chapter is organized as follows; Section 3.2 outlines the system model and an expression used to calculate the *SNR*; Section 3.3 presents the proposed method to analyze two-hop system performance with one relay; Section 3.4 analyzes the asymptotic BER performance of multi-relay systems; Section 3.5 presents the analysis of EBER under the optimal *SNR* level. In Section 3.6 simulation results are demonstrated and, finally, conclusions are drawn in Section 3.7.

3.2 SYSTEM MODEL

We consider two wireless nodes S and D, which are communicating via relay node r_i among n relays as shown in Fig. 3.1. The node r_i which has the highest *SNR* is chosen by the selection scheme. All the nodes (i.e S, D and r_i) are equipped with one antenna. The D node is considered to be out of the S

node coverage area. In this model, the Binary PSK (BPSK) modulation is adopted by the S node which broadcasts symbol x with average power p_s to all relays.

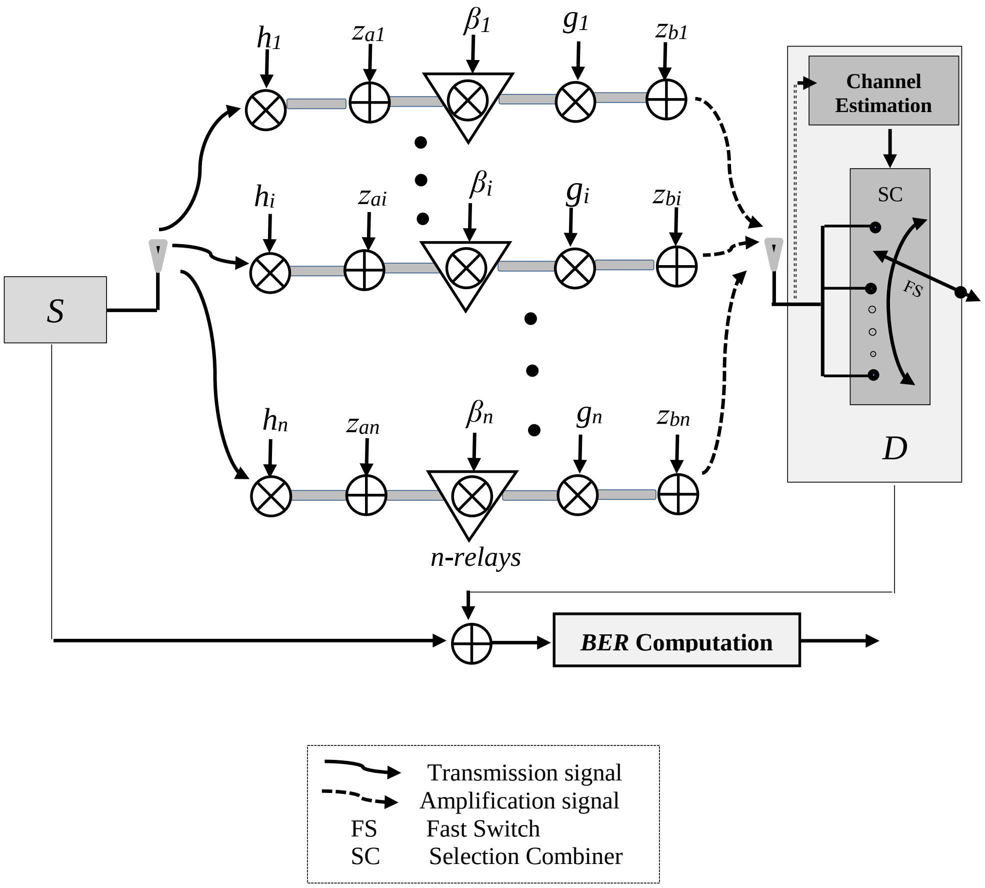

Figure 3.1: System model

Now, assuming that the transmitted symbol from S to r_i passes through a Rayleigh fading channel with a coefficient of h_i. In this case the signal received by r_i can be calculated as $y_i = \sqrt{p_s} x h_i + z_{ri}$, where y_i is the received signal at the i^{th} relay, z_{ri} is Gaussian noise with zero mean and variance σ^2. It is also assumed that the D node knows all channel conditions and is able to select one relay using a selection mechanism which includes a comparator and a fast switch [90].

The output signal for a selected relay, is amplified by the amplification factor (β_i), which can be calculated as $\beta_i = \sqrt{\frac{p_{ri}}{|h_i|^2 p_s + \sigma^2}}$, where p_{ri} is the power allocated to the relay. The amplified signal from the relay is then forwarded to the D node through another Rayleigh-fading channel which includes a fading coefficient (g_i) and random Gaussian noise (z_{di}) with zero mean and variance (σ^2). All relay channels are assumed to be independent and identically distributed (i.i.d.) random variables.

The received signal at the D-node (y_{di}) can be calculated as (2.4). To calculate the signal power of y_{di}, the concept of expected value can be used by assuming $E\left\{x^2\right\}=1$, as

$$E\left\{y_{di}\right\}^2=\frac{p_{ri}}{\sigma^2}|g_i|^2\left(\frac{p_s}{\sigma^2}|h_i|^2+1\right)+\frac{p_s}{\sigma^2}|h_i|^2+1. \tag{3.1}$$

where $E\left\{y_{di}\right\}^2$ is the expected value for the received power signal.

By letting $\text{G}=\frac{p_{ri}}{\sigma^2}|g_i|^2\left(\frac{p_s}{\sigma^2}|h_i|^2+1\right)+\frac{p_s}{\sigma^2}|h_i|^2$, expression (3.1) is rewritten as

$$E\left\{y_{di}\right\}^2=\text{G}+1, \tag{3.2}$$

Determining the actual end-to-end *SNR* from (3.2) as follows

$$SNR=\frac{\gamma_{hi}\gamma_{gi}}{\gamma_{hi}+\gamma_{gi}+1}=\text{ɣ}_i, \tag{3.3}$$

where γ_{hi} and γ_{gi} are the instantaneous channel gain-to-noise ratios (CNR) of the source-to-relay and the relay-to-destination respectively; both are defined in [30] as (2.6) and (2.7) .

The approximation of *SNR* in (3.3) is given, however, when $\text{G}\gg 1$ as

$$SNR=\frac{\gamma_{hi}\gamma_{gi}}{\gamma_{hi}+\gamma_{gi}}=\gamma_i\,. \tag{3.4}$$

The γ_i expression has been used to calculate ABER performance in the MAFR network at high *SNR* as [24, 30], while (3.3) is applied to obtain EBER performance. The variables γ_{hi} and γ_{gi} are statistically independent, and identically distributed (i.i.d). Thus, the average value of each variable is given as $\bar{\gamma}_{hi}=\frac{p_s}{\sigma^2}\text{E}\{|h_i|^2\}$, and $\bar{\gamma}_{gi}=\frac{p_{ri}}{\sigma^2}\text{E}\{|g_i|^2\}$ respectively, where $E(.)$ denotes the expectation value. $E\left\{|h_1|^2\right\}$ and $E\left\{|g_1|^2\right\}$ are given as the Rayleigh fading channel, and each has an expectation value equal to one [66]. Thus the average value of (3.4) is obtained as follows

$$(\bar{\gamma}_i)^{-1}=\frac{1}{\frac{p_s}{\sigma^2}\text{E}\{|h_i|^2\}}+\frac{1}{\frac{p_{ri}}{\sigma^2}\text{E}\{|g_i|^2\}} \tag{3.5}$$

3.3 ABER and EBER Analysis For One Relay Network

In this section, we derive ABER and EBER expressions for a MAFR network assuming a single relay node (i.e $i = 1$). The proposed expressions are derived using the existing BER analysis of the BPSK one-hop communication which is defined by [13] as : $P_b(\gamma) = \int_0^\infty Q(\sqrt{2\gamma})\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}d\gamma$, where $P_b(\gamma)$ is the rate of error probability, γ is one-hop *SNR* equates to multiplying source power (p) by $\sigma^{-2}|h|^2$, the term $\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}$ is the probability density function of γ, and $Q(\sqrt{2\gamma})$ is the complementary cumulative distribution function (CCDF) of a standard Gaussian random variable. The CCDF can also be defined as the Q-function. Evaluating $P_b(\gamma)$ gives: $P_b(\gamma) = \frac{1}{2}\left(1 - \sqrt{\bar{\gamma}(1+\bar{\gamma})^{-1}}\right)$, where $\bar{\gamma}$ is the average value of *SNR*. Determining $\bar{\gamma}$ is given as $\bar{\gamma} = E\{\gamma\} = \frac{p}{\sigma^2}E\left\{|h|^2\right\} = \frac{Eb}{No}$, where $E\left\{|h|^2\right\}$is the expected value of the fading channel which equals to one, Eb/No is the energy per bit to the spectral noise density ratio [13].

The performance of the one-hop scheme is equivalent to the first time slot (TS_1) in the MAFR network as shown in Fig. 3.1. The MAFR network, however, includes another time slot (TS_2) from the relay to destination. To obtain asymptotic BER performance for this type of network, the same procedures for the BER of a one-hop network are followed

$$P_b(\gamma_{i(\gamma)}) = \frac{1}{\bar{\gamma}_{i(\bar{\gamma})}}\int_0^\infty Q(\sqrt{2\gamma_{i(\gamma)}})\, e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}d\gamma, \tag{3.6}$$

where $P_b(\gamma_{i(\gamma)})$ is the bit error rate for a single relay network, $Q(\sqrt{2\gamma_{i(\gamma)}})$ is the total CCDF corresponding to a Gaussian random variable and $\frac{1}{\bar{\gamma}_{i(\bar{\gamma})}}e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}$ is the probability density function (pdf) for the double-cascaded Rayleigh channels, all (3.6) parameters are with respect to γ.

The term $\frac{1}{\bar{\gamma}_{i(\bar{\gamma})}}e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}$ can be calculated from both equations (3.4) and (3.5). Since $|h_i|$ and $|gi|$ are i.i.d random variables, and each *SNR* of those variables follows an exponential distribution [75] , the joint distribution of such *SNR*s also follows an exponential distribution as shown in [76]. Thus, the total pdf of the above MAFR network can be yielded as

$$\varphi_i(\gamma_i) = \frac{1}{\bar{\gamma}_{i(\bar{\gamma})}}e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}, \tag{3.7}$$

where $\varphi_i(\gamma_i)$ is total pdf in respect to $\gamma/2$.

By integrating (3.7) relative to γ_i in a similar manner to that used by [91], the cumulative distribution function of $\varphi_i(\gamma_i)$ is derived as

$$\mathscr{F}_i(\gamma_{i(\gamma)}) = 1 - e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}, \tag{3.8}$$

From (3.8) and the relationship between the Q-function and the error function (erfc) as presented in [13, 92], the value of $Q(\sqrt{2\gamma_{i(\gamma)}})$ in (3.6) is obtained as follows

$$Q(\sqrt{2\gamma_{i(\gamma)}}) = \frac{1}{2}\text{erfc}\left(\sqrt{\gamma_{i(\gamma)}}\right). \tag{3.9}$$

Substituting (3.7) and (3.9) into (3.6) as: $\int_0^\infty \text{erfc}\left(\sqrt{\gamma_{i(\gamma)}}\right) \frac{1}{\bar{\gamma}_{i(\bar{\gamma})}} e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}} d\gamma$. The evaluation of such an integral results in the following expression

$$P_b(\gamma_{i(\gamma)}) = 1 - \sqrt{\frac{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}{(\bar{\gamma}_{hi}+1)\bar{\gamma}_{gi}+\bar{\gamma}_{hi}}}. \tag{3.10}$$

To calculate the EBER performance of (3.6), we need to follow the same procedure for (3.10). However, the actual *SNR* in (3.3) is required with its average. The average $\bar{\text{Ɣ}}_i$ is obtained from substituting (3.5) into (3.2), as the term G in (3.2) is a set of independent random variables and the linearity of the expectation property in [93] is applied for this analysis. Thus, the value of $\bar{\text{Ɣ}}_i$ can be further written as:

$$\bar{\text{Ɣ}}_{i(\bar{\gamma})} = \frac{\frac{p_s}{\sigma^2}\text{E}\{|h_i|^2\}\left(\frac{p_{ri}}{\sigma^2}\text{E}\{|g_i|^2\}+1\right)+\frac{p_{ri}}{\sigma^2}\text{E}\{|g_i|^2\}}{\frac{p_s}{\sigma^2}\text{E}\{|h_i|^2\}+\frac{p_{ri}}{\sigma^2}\text{E}\{|g_i|^2\}} \tag{3.11}$$

where $\bar{\text{Ɣ}}_{i(\bar{\gamma})}$ is the total average value of actual *SNR* of (3.3).

Following the same procedure for (3.10) with respect to (3.3) and (3.11), the exact BER is obtained as

$$P_b(\text{Ɣ}_{i(\gamma)}) = 1 - \sqrt{\frac{\bar{\gamma}_{hi}(\bar{\gamma}_{gi}+1)+\bar{\gamma}_{gi}}{\bar{\gamma}_{hi}\bar{\gamma}_{gi}+2(\bar{\gamma}_{hi}+\bar{\gamma}_{gi})}}. \tag{3.12}$$

where $P_b(\text{Ɣ}_{i(\gamma)})$ is the EBER performance of a one relay MAFR network.

3.4 ABER and EBER Performances For MAFR Network

The ABER and EBER performance of the MAFR network which includes a single relay node is derived in section 3.3. In a typical MAFR network, among *n* relays, a single relay with the highest *SNR* is selected to forward data symbols from the sender to the receiver. Implementing such a selection at the receiver requires knowledge of order statistics. In this section, we briefly introduce a basic order statistical tool, which will be used for the BER performance analysis.

Using a selection strategy, the *SNR's* output from the MAFR network can be ordered statistically as: $\gamma_{(i)} = max\{\gamma_1, \gamma_2, ..., \gamma_n\}$, where $\gamma_{(i)}$ is the i_{th} order statistic of *SNR*, γ_n representing the largest order statistic and γ_1 is the smallest one [94]. The *SNR* in a statistical order is based on CDF as follows: let the CDF of γ_n be denoted as $\mathscr{F}_{(i)}(\gamma_n)$, and then applying the order statistical analysis to $\gamma_{(i)}$ as presented by [95] results in: $\mathscr{F}_{(i)}(\gamma_n) = \text{Prob}\left(\gamma_{(i)} \leq \gamma_n\right) = \mathscr{F}_{(1)}(\gamma_1)\mathscr{F}_{(2)}(\gamma_2)...\mathscr{F}_{(n)}(\gamma_n) = \left[\mathscr{F}_{(n)}(\gamma_n)\right]^n$. Thus for n relays, the order statistics of γ_n based on CDF can be expressed as

$$\mathscr{F}_{(i)}(\gamma_n) = \left(\mathscr{F}_{(n)}(\gamma_n)\right)^n, \tag{3.13}$$

where n is the relay number.

Differentiating (3.13) relative to γ_n by using a similar approach to that in (2.26) , derives the total pdf as

$$\varphi_i(\gamma_n) = \frac{n}{\bar{\gamma}_{i(\bar{\gamma})}} e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}} \left(\mathscr{F}_{(n)}(\gamma_n)\right)^{n-1} \tag{3.14}$$

Equation (3.14) is further analyzed by using the relationship between CDF and pdf as in [96], and it is finally shown as

$$\varphi_i(\gamma_n) = \frac{n}{\bar{\gamma}_{i(\bar{\gamma})}} \left(e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}\right)^n \tag{3.15}$$

Now, substituting (3.15) into (3.6) and using (3.7) to produce the Q-function as (3.9) assuming the pdfs of n are as in [97, 98]. Thus, the ABER expression for n relays is

$$P_b(\gamma_{i(\gamma)}) = \frac{n^2}{\bar{\gamma}_{i(\bar{\gamma})}} \int_0^\infty \text{erfc}\left(\sqrt{\gamma_{i(\gamma)}}\right) \left(e^{-\frac{\gamma_{i(\gamma)}}{\bar{\gamma}_{i(\bar{\gamma})}}}\right)^n. \tag{3.16}$$

By evaluating the integral of (3.16), the general form of ABER performance of the Amplify-and-forward two-hop relay selection is computed as

$$P_b(\gamma_{i(\gamma)}) = n\left(1 - \sqrt{\frac{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}{\bar{\gamma}_{hi}\bar{\gamma}_{gi} + n\left(\bar{\gamma}_{hi+}\bar{\gamma}_{gi}\right)}}\right)^n, \tag{3.17}$$

where n is the total number of relays in a MAFR network.

From (3.17), the average value for n relays is shown as (3.5)

$$\bar{\gamma}_i = \frac{1}{n}\left(\frac{1}{\frac{p_s}{\sigma^2}\text{E}\{|h_i|^2\}} + \frac{1}{\frac{p_{ri}}{\sigma^2}\text{E}\{|g_i|^2\}}\right)^{-1} \tag{3.18}$$

To calculate the EBER performance, we need to calculate the average of the actual *SNR* for n relays, here we can analyze (3.18) in a similar way to obtain (3.11). This results in

$$\bar{\mathrm{ɣ}}_{i(\bar{\gamma})} = \left(\frac{\frac{1}{n}\bar{\gamma}_{hi}\,\bar{\gamma}_{gi} + \bar{\gamma}_{hi} + \bar{\gamma}_{gi}}{\bar{\gamma}_{hi} + \bar{\gamma}_{gi}} \right) \tag{3.19}$$

where $\bar{\mathrm{ɣ}}_n(\bar{\gamma})$ is the average value of the actual *SNR* with respect to $\bar{\gamma}$.

Accordingly, the EBER performance is calculated by following a similar procedures for (3.17) with respect to (3.19). Thus, the EBER for n relay $P_b(\mathrm{ɣ}_i(\gamma))$ is expressed as:

$$P_b(\mathrm{ɣ}_i(\gamma)) = n\left(1 - \sqrt{\frac{\frac{1}{n}\bar{\gamma}_{hi}\,\bar{\gamma}_{gi} + \bar{\gamma}_{hi} + \bar{\gamma}_{gi}}{\frac{1}{n}\bar{\gamma}_{hi}\,\bar{\gamma}_{gi} + 2\left(\bar{\gamma}_{hi} + \bar{\gamma}_{gi}\right)}}\right)^{n}. \tag{3.20}$$

It should be noted that the expression (3.20) has been expressed in the context of BPSK modulation in which M=2 in the M-ary PSK modulation techniques. To calculate $P_b(\mathrm{ɣ}_i(\gamma))$ using higher orders of M (e.g. M: 4, 16, 64, 128, . . .), the procedure used to derive (3.20) can be followed, providing that the average energy per bit in each symbol, as presented in [99], is considered. However, the case of M > 2 is commonly used when one needs to increase the bandwidth efficiency of a communication system [100].

When compared to other M-ary modulation schemes, such as M-ary Amplitude-shift keying (MASK) and Frequency-shift keying (MFSK), BPSK and Quadrature Phase Shift Keying (QPSK) are widely used in the communication industry [101]. BPSK is the most error resistant M-ary modulation scheme and is thus the main focus of this thesis.

3.5 Analysing EBER Under Optimal *SNR* level

The BER analysis in the previous section was performed based on the *SNR* which is analyzed in equation (3.4). This *SNR* has been optimized by studies such as [43, 44] to reduce power consumption in MAFR networks. However, these studies have not considered BER behaviour, because such optimal *SNR* is commonly more complex than the *SNR* in (3.4). Thus, we aim in this section to extend the results of those studies with a new approach to optimize *SNR*, and derive an expression which allows the calculation of BER performance at an optimal *SNR* level. It should be mentioned here that the optimal *SNR* can provide the desired BER for some communication networks as presented in [102]. The optimal *SNR* in equation (3.4) is usually achieved by using one of two main techniques: Optimizing power allocated between relay and the transmitter as in [30, 86], or balancing *EE* and *SE* optimally as in [43, 44, 88]. In the technique balancing *EE* and *SE*, the optimal *SNR* is obtained by first minimizing total energy consumption for a fixed data rate, and then maximizing *EE*.

In this chapter, balancing $\overline{EE}$ and $\overline{SE}$ is adopted to calculate the optimal value for equation (3.5). According to [103], the average *EE* is given as

$$\overline{EE} = \bar{\zeta} = \frac{\text{Average of total data delivered } (\bar{\Re}_i)}{\text{Average of total energy consumed } (\bar{p}_{ti})} \text{ (bit/Joule)}, \tag{3.21}$$

where $\bar{\zeta}$ is the average *EE* [103].

The term $\bar{\Re}_i$ in (3.21) can be expressed as

$$\bar{\Re}_i = E\{0.5\log_2(1+\gamma_{ai})\} = \frac{1}{2}\int_0^{\infty} \log_2(1+\gamma_{ai})\, pdf_{\gamma_{ai}}(\gamma_{ai})\, d\gamma_{ai} \tag{3.22}$$

where 0.5 number is due to the transmission of data during two time slots [39], and $E\{.\}$ is the expectation value and $pdf_{\gamma_{ai}}$ is the total pdf of the Rayleigh fading channels, which is obtained in (3.15). By using the approximated throughput analysis of Rayleigh fading channel in [104, 105], expression (3.22) can be evaluated as

$$\bar{\Re}_i \approx 0.5\log_2(1+\bar{\gamma}_{ai}) \tag{3.23}$$

The average power consumption $(\bar{p}_{ti})$ in (3.21) is represented by the power consumption of signal processing and amplification circuits. It is given as

$$\bar{p}_{ti} = \xi(\bar{p}_s + \bar{p}_{ri}) + (p_{ci} + \rho\bar{\Re}_i), \tag{3.24}$$

where ξ is the constant associated with amplifier power efficiency, p_{ci} is the static circuit power consumption and $\rho\bar{\Re}_i$ is the dynamic power per average data rate unit. Minimizing total energy consumption for a fixed average data rate can be achieved by minimizing total transmitted power as presented in Chapter 2. However, this chapter will use average *EE* and this is required to calculate average powers of source and relay as follows

$$\bar{p}_{s(\bar{\gamma}_i)} = \bar{\gamma}_i\bar{\gamma}_{hi}^{-1} + \bar{\gamma}_i(\bar{\gamma}_{gi}\bar{\gamma}_{hi})^{-1/2} \tag{3.25}$$

$$\bar{p}_{ri(\bar{\gamma}_i)} = \bar{\gamma}_i\bar{\gamma}_{gi}^{-1} + \bar{\gamma}_i(\bar{\gamma}_{gi}\bar{\gamma}_{hi})^{-1/2} \tag{3.26}$$

where $p_{s(\bar{\gamma}_i)}$ and $p_{ri(\bar{\gamma}_i)}$ are the minimum values of the average powers for the source and relay, respectively, and both are with respect to $\bar{\gamma}_i$.

Now, substituting (3.23), (3.25) and (3.26) into (3.21) as

$$\bar{\zeta}_{(\bar{\gamma}_i)} = \frac{\log_2(1+\bar{\gamma}_i)}{2\,\bar{\gamma}_i\,\xi\,(\frac{1}{\bar{\gamma}_{hi}}+\frac{1}{\bar{\gamma}_{gi}}+\frac{2}{\sqrt{\bar{\gamma}_{gi}\bar{\gamma}_{hi}}})+2\,p_{ci}+\rho\,\log_2(1+\bar{\gamma}_i)} \tag{3.27}$$

where $\bar{\zeta}_{(\bar{\gamma}_i)}$ is the average *EE* with respect to $\bar{\gamma}_i$.

By taking the derivative of equation (3.27) with respect to $\bar{\gamma}_i$, the average optimal *SNR* value of a MAFR network is obtained as follows

$$\mathring{\bar{\gamma}}_i = \exp^{1+w\left(\left(p_{ci}-\xi(\frac{1}{\bar{\gamma}_{hi}}+\frac{1}{\bar{\gamma}_{gi}}+\frac{2}{\sqrt{\bar{\gamma}_{gi}\bar{\gamma}_{hi}}}(\frac{1}{\bar{\gamma}_{hi}}+\frac{1}{\bar{\gamma}_{gi}}+\frac{2}{\sqrt{\bar{\gamma}_{gi}\bar{\gamma}_{hi}}})\right)/\xi\left(\frac{1}{\bar{\gamma}_{hi}}+\frac{1}{\bar{\gamma}_{gi}}+\frac{2}{\sqrt{\bar{\gamma}_{gi}\bar{\gamma}_{hi}}}\right)\mathrm{e}\right)} - 1 \tag{3.28}$$

where $\mathring{\bar{\gamma}}_i$ is the optimal value of equation (3.5), e is the base of the natural logarithm and $w(.)$ is the Lambert w function [106].

Equation (3.28) can be expressed in respect to the data rate as

$$\mathring{\bar{\gamma}}_i = 2^{2\mathring{\bar{\Re}}_i} - 1\,, \tag{3.29}$$

where $\mathring{\bar{\Re}}_i = \frac{1}{2}\log_2(1+\mathring{\bar{\gamma}}_i)$.

By using (3.29) as well as considering the relationship between bit energy E_b and the bit rate described in [84], the optimal average *SNR* can be further expressed as

$$\mathring{\bar{\gamma}}_i = \left(\frac{\bar{\gamma}_{hi}\,\bar{\gamma}_{gi}(\bar{\Re}_i/\mathring{\bar{\Re}}_i)(\bar{\gamma}_{hi}+\bar{\gamma}_{gi})}{n\,(\bar{\gamma}_{hi}+\bar{\gamma}_{gi})}\right). \tag{3.30}$$

The optimal *SNR* in expression (3.30) is adopted to calculate EBER performance, and this can be achieved by following the same procedure for (3.20). This results in

$$\mathring{P}_b(\mathring{\bar{\gamma}}_i) = n\left(1-\sqrt{\frac{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}+n\left(\mathring{\bar{\gamma}}_{hi}+\mathring{\bar{\gamma}}_{gi}\right)}{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}+n\,2\left(\mathring{\bar{\gamma}}_{hi}+\mathring{\bar{\gamma}}_{gi}\right)}}\right)^n. \tag{3.31}$$

where $\mathring{\bar{\gamma}}_{hi} = \bar{\gamma}_{hi}\bar{\Re}_i/\mathring{\bar{\Re}}_i$, $\mathring{\bar{\gamma}}_{gi} = \bar{\gamma}_{gi}\bar{\Re}_i/\mathring{\bar{\Re}}_i$ and $\mathring{P}_b(\mathring{\bar{\gamma}}_i)$ is the exact BER performance of the MAFR networks under optimal *SNR* level.

3.6 SIMULATION RESULTS

This section presents the accuracy of the analytical method by comparing it to the simulation results and some existing results. All simulations are performed on the system model in Section 3.2. However, for this evaluation we are limiting the number of relays to four (i.e, $n = 4$).

Fig. 3.2 shows the ABER performance in Eq. (3.17) when $p_{ri} = p_s$. It is clear that the analysis results show a high match with the simulation results. For more validation, we compared our results with references [30, 48], and also found a high match with them. The same figure illustrates that the ABER performance has improved by increasing the number of relays and the analysis results approach the simulation result as the *SNR* increases. In the low *SNR* region, our results are slightly different to the simulation results, but this difference is still better than those in studies [30, 48, 86] and in more recent works [24, 87]. Overall, the ABER expression in Eq. (3.17) gives more accurate results particularly in the low *SNR* region. Fig. 3.3 illustrates the theoretical and simulation results for the EBER expression in (3.20) when $p_{ri} = p_s$. It is evident in this figure that the proposed analysis provides highly accurate results for all *SNR* regions. Fig. 3.4 corresponds to the optimal MAFR network in equation (3.31), we observe that the EBER performance of the optimum *EE* and *SE* at $p_{ri} = p_s$ is better than the suboptimal case in Fig. 3.3, as optimizing the overall received *SNR* is equivalent to BER minimization.

Fig. (3.5) shows the analytic result of ABER performance presented in equation (3.17) with relay power 5 db higher than source power. The results are compared with the simulation result, and it exhibits a high degree of accuracy between the simulation and the proposed analysis at the high *SNR* region with a small difference being seen in the low *SNR* region. Further it is observed that the ABER tends to achieve better results than ABER in Fig. (3.2) as the increases in the overall received *SNR* at the destination will decrease the ABER. Fig. (3.6) also illustrates the analytic result of EBER in (3.20) with relay power 5 db more than source power. Compared with the simulation results, the EBER analysis achieves a high qualitative match to the simulation results. This result proves the accuracy of the EBER analysis. Fig. (3.7) shows that the analytic results align with the simulation result for the optimal *SNR* level as presented in Fig. (3.4). However, Fig. (3.7) provides further improvement upon optimizing the value of *SNR*.

3.7 SUMMARY

In this chapter, a new method for analyzing the BER performance of a MAFR network over flat fading channels and having a selection scheme has been presented. The method shows that an ABER analysis of a MAFR network can be reduced to a traditional BER analysis of a one hop communication link. Also, the ABER analysis can be used directly to determine the EBER performance. This is because the ABER expressions have been derived with respect to a one-hop *SNR* and the analysis makes fewer assumptions. The effectiveness of the method is demonstrated by unifying the BER evaluations under low, high and optimal *SNR* conditions. The optimal *SNR* level is achieved by optimal balancing of the $\overline{EE}$ and $\overline{SE}$. For each *SNR* level, the BER expression is derived, and is validated by simulation. It was found that the ABER analytical results align with the EBER results. This allowed us to calculate the accurate BER of MAFR networks under any *SNR* level using either ABER or EBER.

The next chapter evaluates the above BER methods when considering relay location in MAFR networks. Further, such BER methods are extended in Chapter 6 to involve the evaluation of BER in bidirectional AF relay networks.

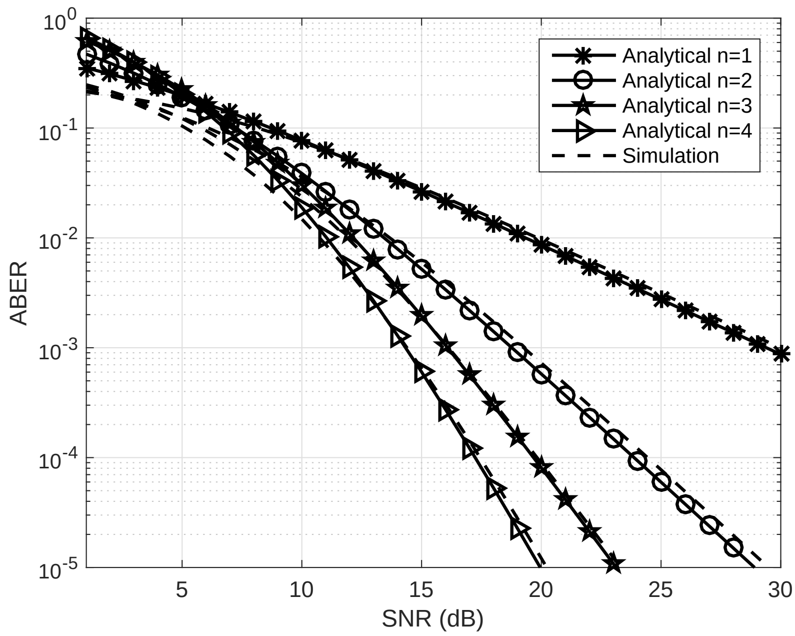

Figure 3.2: ABER performance in an AF relays network; $p_{ri} = p_s$

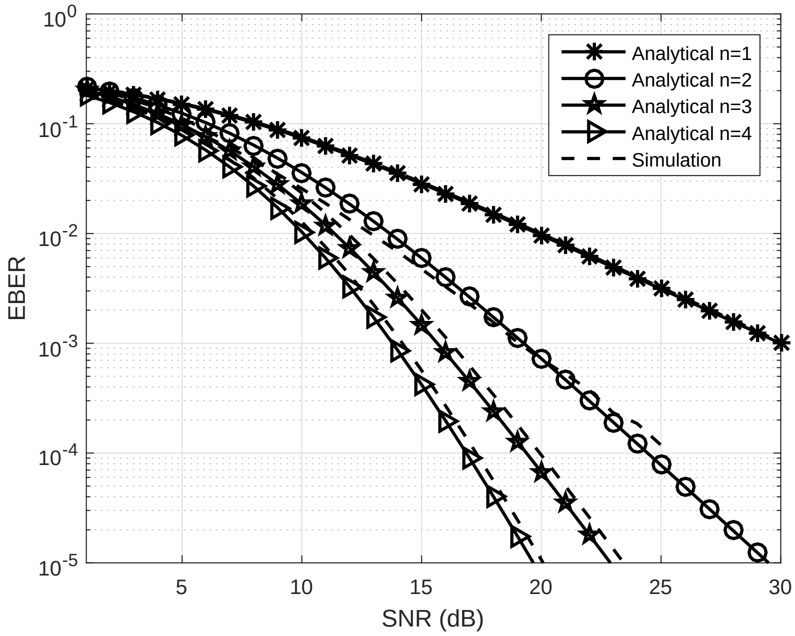

Figure 3.3: EBER performance in an AF relays network; $p_{ri} = p_s$

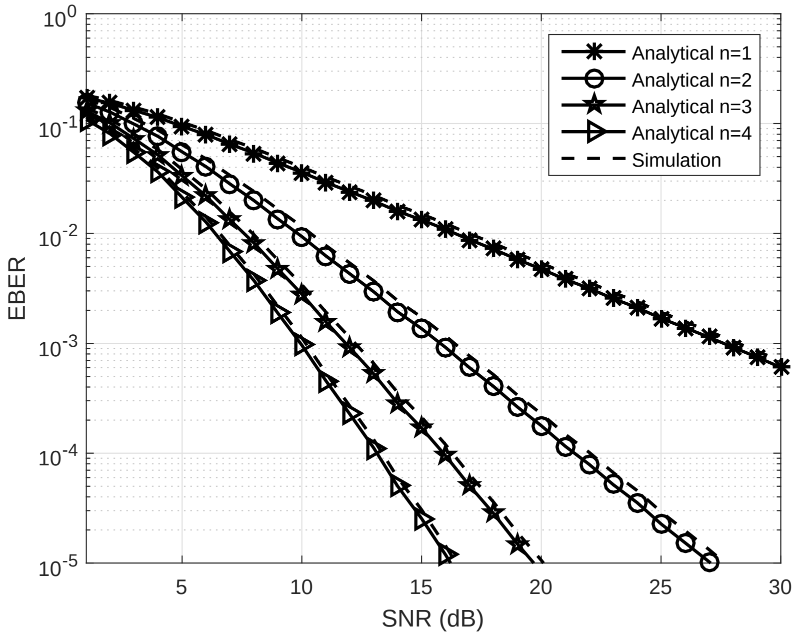

Figure 3.4: EBER performance in an Optimal AF relays network; $p_{ri} = p_s$

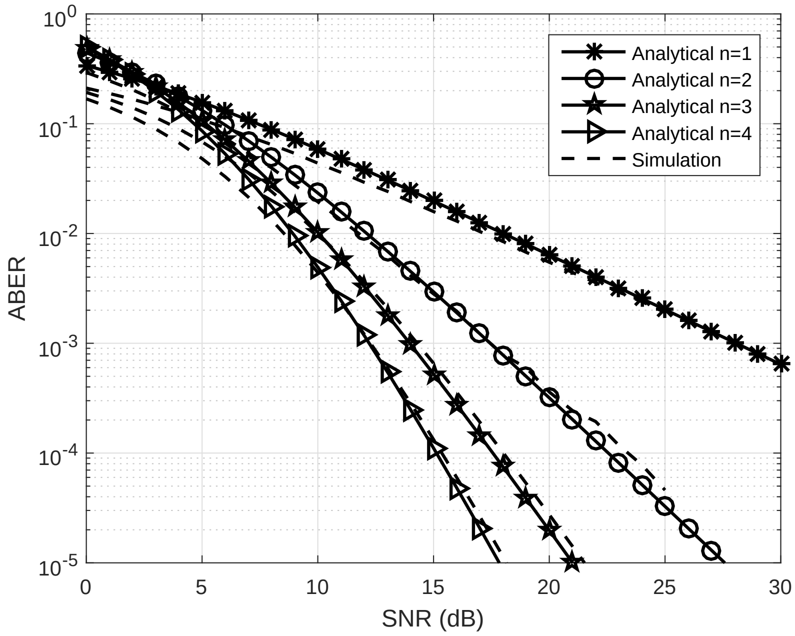

Figure 3.5: ABER performance with high relay power

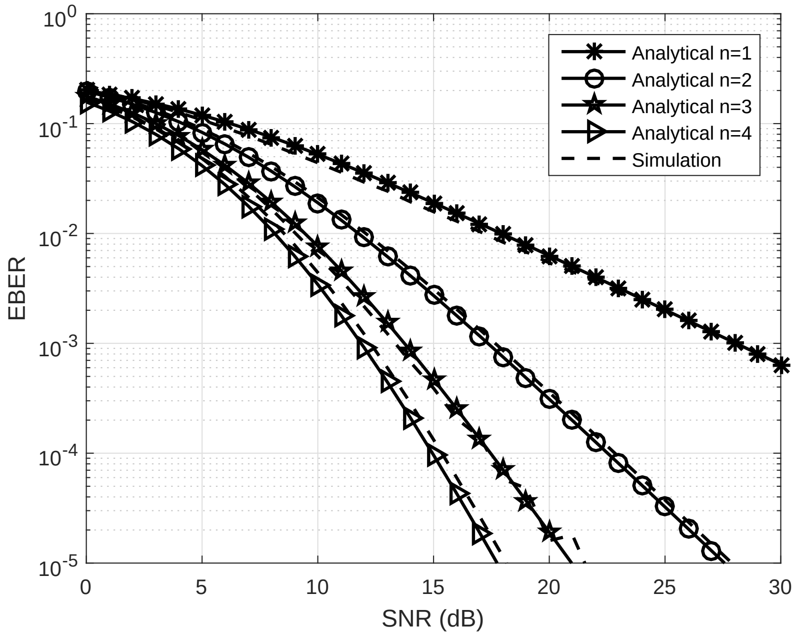

Figure 3.6: EBER performance, relay power more than source power

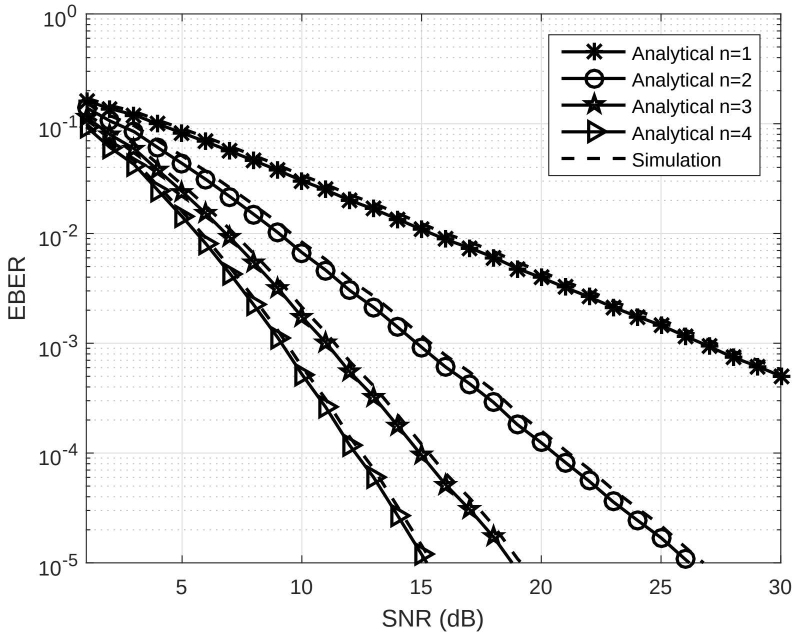

Figure 3.7: EBER performance in an Optimal AF relays network

Chapter 4

The Effect of the *EE* and *SE* on the BER of MAFR networks

4.1 Introduction

In the previous chapters, we analyzed new methods of evaluating the asymptotic and exact BER of MAFR networks under exact, high and optimal *SNR* levels. The high *SNR* was employed to calculate exact *SNR*, and the optimal *SNR* was obtained by balancing energy efficiency and spectral efficiency. These *SNR* levels were adopted to provide an accurate BER analysis of MAFR networks. On the other hand, in order to implement such solutions in actual MAFR networks, it is important to consider relay location in the *SNR* analysis. Thus, this chapter aims to calculate the *SNR* when considering also the relay location. Furthermore, a new method of increasing the *EE* and decreasing the BER is discussed.

Energy consumption is commonly evaluated by the *EE* metric, which is defined as the number of transmitted data bits per unit of transmitted power (bit/Joule) [82]. This definition indicates that a high *EE* value (i.e., decreasing energy consumption) is achieved when the bit rate is maximized or the power per unit (i.e., Joule) is minimized. The bit rate, referred to as Spectrum Efficiency (*SE*), is quantified using the unit bits per second (bits/sec) [42].

With the aim of increasing *EE*, different methods have been proposed. Wireless relay networks are one of the most promising means of achieving high *EE* [107]. In such networks one or many intermediate relay nodes increase the quality of communication between source and its destination

Energy efficiency in relay networks has been investigated by many researchers. An early study by [108] found that relay networks achieve better *EE* than direct transmission when the Relay Located

Power (RLP) is unlimited, while increasing the *EE* with limited RLP can be achieved only when the *SE* is low or when the relay destination distance is short. The relay location is considered by [107], who observed that the trade-off between *EE* and *SE* is possible when a relay selection scheme is adopted. However, these studies [107, 108], did not consider circuit energy (CE) consumption, which includes electronic devices and signal processing. This limitation was overcome in [109] and [110], in which the CE consumption in addition to other network consumption was analyzed for a given capacity of multiple-hop networks over Rayleigh fading channels.

Recently, some studies have investigated combining the *EE* and *SE* to decrease network energy consumption and ensure a desirable *SE*. Reference [111] investigated a transmission scheme for reducing the power consumed per transmitted bit for both unidirectional and bidirectional relay networks by combining the power allocation and relay selection. Similar results were achieved by [112, 113]] by including the CE in the analysis. In some research, such as [103, 114], the average channel status was adopted to calculate the average *EE* and *SE*, but these studies did not consider relay location. Most other researchers, such as [43, 44], have achieved a high *EE*, but such studies have not considered error behaviour.

The error rate of a two-hop AF relay has also been investigated by many studies. In [21], multi-hop and multi-relay networks were involved in the analysis of the error rate of AF-relay networks. This study used a high *SNR* level to calculate the Symbol Error Probability (SER) performance. It found that the SEP performance was improved by increasing the number of relays. Similar studies [22–24] have investigated the SER performance of MAFR networks, the authors in [22] obtained the exact SER performance of AF parallel relays over a Rayleigh-fading channel. They used the Moment Generating Function (MGF) of half the harmonic mean to calculate the SER performance. In [23], the harmonic mean approximation was used to calculate upper and lower bounds for exact SER and outage probability performances. Further, the Laplace transform of the Cumulative Distribution Function (CDF) and Probability Density Function (pdf) of the exact *SNR* were used in [24] to obtain asymptotic and actual Frame Error Rate (FER) performance. The results of [24] showed that the selection of one optimum relay achieves better FER performance than multiple relay selection under transmitting power constraints. However, all of the above SER studies require further analysis to determine the BER, which is a more useful metric than SER [25]. Moreover, calculating FER depends on the required BER [85].

Evaluating error rate performance of AF-relay networks under increasing *SNR* has been investigated by studies such as [19, 63]. In [63], the SER performance of a single AF-relay network was investigated for two cases; the first considered equal source and relay powers, while the second optimized both powers. It found that the SER performance using optimum powers was slightly better than that of the equal-powers case. The best *SNR* for a given BER was calculated by [19, 64]. Both studies aimed to reduce the energy consumption in relay networks while constraining the error rate to a certain value.

The above studies, however, have presented various techniques for developing the *EE* and *SE* metrics, jointly or individually, without considering the relationship between these metrics and the BER, although error rate analysis is an important parameter for evaluating the performance of many wireless network applications [89]. Furthermore, most of the error rate expressions allow the evaluation of error rate performance of only suboptimal networks. To overcome this issue, we propose a new method which aims to:

a) Define the minimum power consumption in an AF-relay network, considering the best relay location.

b) Use the minimum power consumption in (a) to optimize *EE* and *SE* in a balanced manner.

c) Analyze the BER of the balanced scheme in (b) and investigate the following question: does increasing the *EE* lead to a positive effect on the BER?

Combination of (a) to (c) in an AF relay network has not been presented previously. While [103] found that the relay location is an important determinant of any network parameter, such as *EE* and BER, to the best of the authors' knowledge, the idea of associating the optimal balancing of *SE* and *EE* with unconstrained BER has not yet been proposed in the literature, whether for full-duplex AF relays or other wireless networks.

The remainder of the chapter is organized as follows: Section 4.2 outlines the system model; Section 4.3 introduces a new method to combine *EE* and *SE* with relay location; Section 4.4 discusses BER performance in AF relay networks; Section 4.5 examines the relationship between BER performance and *EE*; Section 4.6 presents the simulation results; and finally, conclusions and remarks are presented in Section 4.7.

4.2 SYSTEM MODEL

The system model of this chapter also considers two wireless nodes; the *S*-node which transmits signals to the *D*-node through a network of parallel AF-relays. The *S*-node adopts binary phase shift keying (BPSK) modulation and broadcasts symbols x with average power p_s to all relays over flat fading channels which have coefficients $h_1, h_2.., h_n$. The symbols are then received by a group of parallel relays $R_1, R_2...R_n$ as shown in Fig 4.1. The best relay with the highest *SNR* is selected by the destination node using a selection scheme.

The received signals by the i^{th} relay (R_i) can be calculated as in equation (2.1). Such signals are amplified by the amplification factor (β_i), which can be calculated as (2.2).

The signal amplified by R_i is then forwarded to the D-node through another Rayleigh-fading channel which includes a fading coefficient (g_i) and random Gaussian noise (z_{bi}) with zero mean and variance (σ^2).

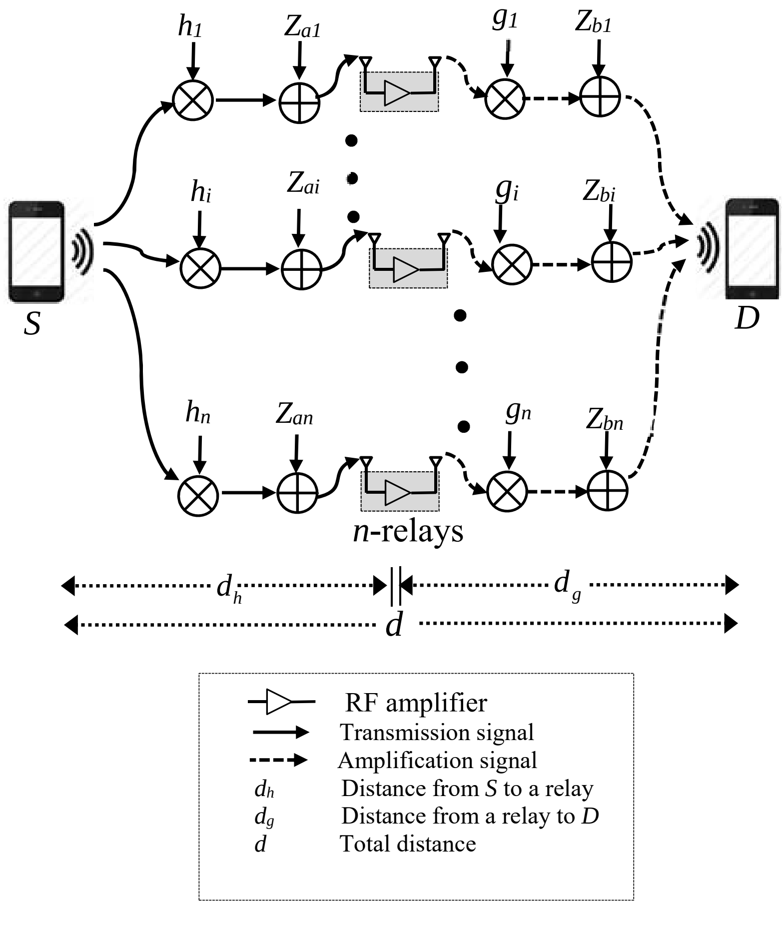

Figure 4.1: System model

The received signal at the D-node from R_i can be expressed as (2.4), and its expected value evaluated as (3.1). Thus, the actual end-to-end *SNR* is expressed as

$$SNR = \frac{\gamma_{hi}\gamma_{gi}}{\gamma_{hi} + \gamma_{gi} + 1} = \gamma_i, \tag{4.1}$$

where γ_{hi} and γ_{gi} are the instantaneous channel gain-to-noise ratios (CNR) of source-to-relay and relay-to-destination respectively; both are defined as $\gamma_{hi} = \frac{p_s}{\sigma^2}|h_i|^2$ and $\gamma_{gi} = \frac{p_{ri}}{\sigma^2}|g_i|^2$.

Equation (3.1) can also be approximated, when $\frac{p_{ri}}{\sigma^2}|g_i|^2\left(\frac{p_s}{\sigma^2}|h_i|^2 + 1\right) + \frac{p_s}{\sigma^2}|h_i|^2 > 1$, as

$$SNR = \frac{\gamma_{hi}\gamma_{gi}}{\gamma_{hi} + \gamma_{gi}} = \gamma_i\,. \tag{4.2}$$

Expression (4.2) is valid to calculate any parameter such as bit error rate, *EE* and *SE* at high *SNR* level [8, 30].

The variables γ_{hi} and γ_{gi} are statistically independent and identically distributed (i.i.d). Thus, the average value of each variable is given as $\bar{\gamma}_{hi} = \frac{p_s}{No}E\{|h_i|^2\}$, and $\bar{\gamma}_{gi} = \frac{p_{ri}}{No}\mathrm{E}\{|g_i|^2\}$ respectively, where $E(.)$ denotes the expectation value and $No = \sigma^2$. Each of $E\left\{|h_1|^2\right\}$ and $E\left\{|g_1|^2\right\}$ is given as a Rayleigh fading channel, and each has expectation value equal to one [66]. Thus the total average value of (4.2) is obtained as follows

$$\frac{1}{\bar{\gamma}_i} = \left(\frac{p_s}{No}\mathrm{E}\{|h_i|^2\}\right)^{-1} + \left(\frac{p_{ri}}{No}\mathrm{E}\{|g_i|^2\}\right)^{-1} \tag{4.3}$$

4.3 Balancing *EE* and *SE*

In this section, we use the balance of average Energy Efficiency ($\overline{EE}$) and Spectrum Efficiency ($\overline{SE}$) as presented in Chapter 3 considering relay location. Here, equation (3.21) can be also used as

$$\bar{\zeta} = \frac{\bar{\Re}_i}{\bar{p}_{ti}}\,, \tag{4.4}$$

where $\bar{\zeta}$ is the average *EE* and $\bar{\Re}_i$ defined in (3.23) as

$$\bar{\Re}_i \approx 0.5\log_2(1 + \bar{\gamma}_i) \tag{4.5}$$

To consider relay location with $\bar{\Re}_i$, we can use the data rate model presented in [40, 115]. In such a model, the average *SNR* can be computed by (4.3). Further, considering a constant attenuation on the transmitted signal, $\mathrm{E}(|h_i|^2)$ and $E(|g_i|^2)$ can be defined as

$$\mathrm{E}(|h_i|^2) = \frac{k}{d_h^{\alpha}} \tag{4.6}$$

$$E(|g_i|^2) = \frac{k}{d_g^{\alpha}} \tag{4.7}$$

where k is the path loss coefficient (in decibel scale (dB)) at the reference distance equal to one, α is the path loss exponent which is commonly estimated from 2 to 6 [116], d_h is the distance between nodes S and the relay, and d_g is the distance from node D to a relay as illustrated in Fig. 4.1.

By substituting (4.6) and (4.7) into (4.2), we can obtain new terms for the average SNRs for the D-node in terms of distance as

$$\frac{1}{\bar{\gamma}_i} = \frac{No}{k}\left(\frac{d_h^{\alpha}}{p_s} + \frac{d_g^{\alpha}}{p_{ri}}\right) \tag{4.8}$$

To calculate both distances d_h and d_g as a function of relay location and total distance between the source and the destination (d), we assume that the relay is allocated at position δ, which can be specified as $0 < \delta < 1$. Thus, the distance d_h^{α} can be calculated as

$$d_h^{\alpha} = (d\,\delta)^{\alpha}\,, \tag{4.9}$$

and for d_g^{α} as

$$d_g^{\alpha} = (d\,(1-\delta))^{\alpha}\,. \tag{4.10}$$

Substituting (4.9) and (4.10) into (4.8) results in

$$\frac{1}{\bar{\gamma}_i} = d^{\alpha}\frac{No}{k}\left(\frac{\delta^{\alpha}}{p_s} + \frac{(1-\delta)^{\alpha}}{p_{ri}}\right) \tag{4.11}$$

To express $\bar{p}_{ti}$ in (4.4), the power consumption of the signal processing and the amplification circuits should be considered as described in [82]. This can be presented as

$$\bar{p}_{ti} = \left(\varsigma\,(\xi_{ti}) + \left(\jmath\bar{\mathfrak{R}}_i + p_{ci}\right)\right)/2 \tag{4.12}$$

where ς is a constant associated with the amplifier power efficiency, ξ_{ti} is the total energy per bit ($\xi_{ti} = p_{s+}\,p_{ri}$), p_{ci} is the static circuit power and $\jmath\bar{\mathfrak{R}}_i$ is the dynamic power per-unit data rate. The term of $\left(\jmath\bar{\mathfrak{R}}_i + p_{ci}\right)$ is related to the AF relay circuit power consumption, (further information about this power is presented by [103, 117]).

To maximize (4.4), we can first define the minimization problem of each power node (i.e., *min* (ξ_{ti})) under the required minimum constraint $\bar{\mathfrak{R}}_i$ as follows

$$max \quad \bar{\zeta} = \frac{\bar{\mathfrak{R}}_i}{\bar{p}_{ti}} \qquad i_s \in \{S_a,\, S_b\, r_i\} \tag{4.13}$$

$$s.t \quad \bar{\mathfrak{R}}_{ab} = \bar{\mathfrak{R}}_{ba} \tag{4.14}$$

where$\bar{\mathfrak{R}}_{ba}$ and $\bar{\mathfrak{R}}_{ab}$ are the average throughput from users S_b and S_a respectively.

To calculate the power allocation of all nodes (i.e, users and relay) in terms of ξ_{ti}, we can define

$$p_s = \sigma\, \xi_{ti}\,, \tag{4.15}$$

$$p_{ri} = (1-\sigma)\, \xi_{ti}\,, \tag{4.16}$$

where σ is the energy control factor for the node-D , it is specified in the range of : $0<\sigma<1$ [118].

To optimize the value of ξ_{ti} based on energy control factor, we can substitute equations (4.15) and (4.16) into (4.8). This results in

$$\xi_{ti} = \frac{No}{k}\left(\frac{d_h^{\alpha}}{\sigma} + \frac{d_g^{\alpha}}{1-\sigma}\right)\left(\frac{n\left(1-\left(\frac{P(\phi)}{n}\right)^{\frac{1}{n}}\right)^2}{1-\left(1-\left(\frac{P(\phi)}{n}\right)^{\frac{1}{n}}\right)^2}\right) \tag{4.17}$$

By taking the derivative of (4.17) with respect to the σ and equating it to zero, we obtain

$$\mathring{\sigma}^{-1} = \sqrt{\frac{d_g^{\alpha}}{d_h^{\alpha}}} + 1 \tag{4.18}$$

where $\mathring{\sigma}$ is the optimal value of σ.

Substituting (4.9) and (4.10) into (4.18) results in

$$\mathring{\sigma} = \left(d^{\frac{\alpha}{2}}\sqrt{\frac{(1-\delta)^{\alpha}}{\delta^{\alpha}}} + 1\right)^{-1} \tag{4.19}$$

By replacing the termσ in (4.17) by (4.19). This results in

$$\mathring{\xi}_{ti} = \frac{d^{\alpha} No}{k} \left(\frac{\delta^{\alpha}}{\mathring{\sigma}} + \frac{(1-\delta)^{\alpha}}{1-\mathring{\sigma}} \right) \bar{\gamma}_i \tag{4.20}$$

where $\mathring{\xi}_{ti}$ is the optimal value of ξ_{ti} in respect to optimal of $\mathring{\sigma}$.

Now, substituting equations (4.5), (4.11) and(4.20) into (4.13) gives

$$\bar{\zeta} = \frac{\log_2(1+\overline{\gamma}_i)}{\varsigma \left(\frac{d^{\alpha} No}{k} \left(\frac{\delta^{\alpha}}{\mathring{\sigma}} + \frac{(1-\delta)^{\alpha}}{1-\mathring{\sigma}} \right) \overline{\gamma}_i \right) + \log_2(1+\overline{\gamma}_i)\,\mathrm{ɲ} + p_{ci}}. \tag{4.21}$$

By deriving equation (4.21) with respect to $\overline{\gamma}_i$ and equating it to zero, we obtain the best relay location which gives the highest *EE*

$$\mathring{\overline{\gamma}}_i = e^{w\left(\frac{1}{e}\left(\frac{K p_{ci}}{\varsigma\left(\sqrt{((1-\delta)*d)^{a}}+\sqrt{(\delta*d)^{a}}\right)^{2} No)}-1\right)\right)+1} - 1 \tag{4.22}$$

where $\mathring{\overline{\gamma}}_i$ is the optimal of $\overline{\gamma}_i$, e is the base of the natural logarithm and $w(.)$ is the omega function [106].

The optimal trade-off between *SE* and *EE* can be obtained by substituting (4.22) into (4.21) as

$$\mathring{\bar{\zeta}} = \frac{\log_2\left(1+\mathring{\overline{\gamma}}_i\right)}{\varsigma \psi \mathring{\overline{\gamma}}_i + \log_2\left(1+\mathring{\overline{\gamma}}_i\right)\mathrm{ɲ} + p_{ci}} \tag{4.23}$$

where $\mathring{\bar{\zeta}}$ is maximum *EE* associated with lowest sacrifice of *SE* and $\psi = \frac{d^{\alpha} No}{k}\left(\frac{\delta^{\alpha}}{\mathring{\sigma}} + \frac{(1-\delta)^{\alpha}}{1-\mathring{\sigma}}\right)$. Now equation (4.3) can be formulated in respect to (4.22) as

$$\mathring{\overline{\gamma}}_i = \frac{\mathring{\overline{\gamma}}_{hi}\mathring{\overline{\gamma}}_{gi}}{\mathring{\overline{\gamma}}_{hi} + \mathring{\overline{\gamma}}_{gi}} \tag{4.24}$$

where $\mathring{\overline{\gamma}}_{hi}$ and $\mathring{\overline{\gamma}}_{gi}$ are the arguments of the function $\mathring{\overline{\gamma}}_i$.

Equations (4.24) is employed in the analysis of the next sections to calculate the BER performance of 4.23.

4.4 BER Performance in AF relay networks

The previous section presented a method to calculate the maximum *EE* in an AF relay network. To study the BER behavior of such a network (i.e., AF relay networks after *EE* is maximized), this section considers a new method to analyze the BER of AF-relay networks. We start by assuming that an AF-relay network includes one relay, and extend BER calculation for n relays, as discussed in the next section.

In a two-hop relay network, the transmitted signal from a source to the destination passes through cascade channels, as shown in Fig.4.1. Each hop includes a Rayleigh fading channel and, in such a channel, *SNR* follows an exponential distribution. The total *SNR* obtained from i.i.d Rayleigh fading channels follows an exponential distribution [75]. Here, the distribution of the harmonic mean of two i.i.d. gamma random variables defined in [48] can be employed to find the total pdf. We first define Rayleigh fading pdf from the S-node to a relay, i.e. first hop, as

$$\underset{S\to R_i}{\varphi_{hi(\phi)}} = (\mathring{\bar{\gamma}}_{hi})^{-1}\exp^{-\phi(\mathring{\bar{\gamma}}_{hi})^{-1}}, \tag{4.25}$$

and similarly pdf of the second hop results in

$$\underset{R_i\to D}{\varphi_{gi(\phi)}} = (\mathring{\bar{\gamma}}_{gi})^{-1}\exp^{-\phi(\mathring{\bar{\gamma}}_{gi})^{-1}} \tag{4.26}$$

where ϕ is the harmonic mean (denoted by $\phi = \mu_H(\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi})$) according to a gamma distribution [47], $\underset{S\to R_i}{\varphi_{hi(\phi)}}$ and $\underset{R_i\to D}{\varphi_{gi(\phi)}}$are the pdfs of γ_{hi} and γ_{gi}, respectively.

Given two i.i.d. exponential random variables as (4.25) and (4.26), the modified harmonic mean defined in [119] can be used to joint them as follows

$$\varphi_{ai}(\phi) = \frac{\exp^{-\phi\left(\frac{\mathring{\bar{\gamma}}_{hi}+\mathring{\bar{\gamma}}_{gi}}{\mathring{\bar{\gamma}}_{hi}\mathring{\bar{\gamma}}_{gi}}\right)}}{\left(\sqrt{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}}\right)^3}\left(4\sqrt{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}}\,k_0\left(\frac{2\phi}{\sqrt{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}}}\right)+2\phi\,(\mathring{\bar{\gamma}}_{hi}+\mathring{\bar{\gamma}}_{gi})\,k_1\left(\frac{2\phi}{\sqrt{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}}}\right)\right) \tag{4.27}$$

where $\varphi_i(\phi)$ is the total pdf, $k_0(.)$ and $k_1(.)$ are the first and the second order modified Bessel function of the second kind.

By applying the modified Bessel function properties, which are $k_0(\phi) \to 0$ and $k_1(\underset{\phi\to 0}{\phi}) \to 1/\phi$, to (4.27), the total pdf obtained is

$$\varphi_i(\phi) = \frac{1}{\mathring{\bar{\gamma}}_i} e^{-\frac{\phi}{\mathring{\bar{\gamma}}_i}}, \tag{4.28}$$

Integration of (4.28) relative to ϕ, similar to (3.7), results in

$$\mathscr{F}_i(\phi) = 1 - e^{-\frac{\phi}{\mathring{\bar{\gamma}}_i}} \tag{4.29}$$

where $\mathscr{F}_i(\phi)$ is the cumulative distribution function for $\varphi_i(\phi)$.

From (4.29) and the relationship between the Q-function and the error function (erfc) as presented in [92], the term of $\mathscr{F}_i(\phi)$ can be obtained as follows

$$Q(\sqrt{2\phi}) = \frac{1}{2}\text{erfc}\left(\sqrt{\phi}\right). \tag{4.30}$$

According to error probabilities with random variables defined in [13], the average BER with the random variables $\mathring{\bar{\gamma}}_{hi}$ and $\mathring{\bar{\gamma}}_{gi}$ can be expressed by using (4.28) and (4.30). This is expressed as

$$\bar{P}_e = \frac{1}{\mathring{\bar{\gamma}}_i} \int_0^\infty \text{erfc}\left(\sqrt{\phi}\right) e^{-\frac{\phi}{\mathring{\bar{\gamma}}_i}} d\phi, \tag{4.31}$$

where $\bar{P}_e$ is the average BER.

It is worth noting that the expression of (4.31) is consistent with the result of one hop BER obtained by [13].

By evaluating (4.31), the BER of MAFR network is obtained as follows

$$\bar{P}_e = 1 - \sqrt{\frac{\mathring{\bar{\gamma}}_i}{\mathring{\bar{\gamma}}_i + 1}}. \tag{4.32}$$

Equation (4.32) is analyzed based on approximated *SNR* presented in equation (4.2). To calculate the exact BER performance, we can apply the linearity of the expectation property in [93] for the term

$\frac{p_{ri}}{\sigma^2}|g_i|^2\left(\frac{p_s}{\sigma^2}|h_i|^2+1\right)+\frac{p_s}{\sigma^2}|h_i|^2$ in equation (3.1) as this term is a set of independent random variables, while the second term in the same equation is a constant equal to one. Thus, the outcome of $\bar{\mathring{\gamma}}_i$ can be easily adapted to employ in the calculation of actual average *SNR*. This results in

$$\left(\bar{\mathring{\mathrm{x}}}_i\right)^{-1}=\frac{\bar{\mathring{\gamma}}_{hi}\bar{\mathring{\gamma}}_{gi}+\bar{\mathring{\gamma}}_{hi}+\bar{\mathring{\gamma}}_{gi}}{\bar{\mathring{\gamma}}_{hi}\bar{\mathring{\gamma}}_{gi}} \tag{4.33}$$

where $\bar{\mathring{\mathrm{x}}}_i$ is the total average value of actual *SNR* in respect to $\bar{\mathring{\gamma}}_{hi}$ and $\bar{\mathring{\gamma}}_{gi}$.

Following same procedure for (4.32) with respect to (4.33), the exact BER is obtained as

$$\bar{P}_e=1-\sqrt{\left(\frac{2\bar{\mathring{\gamma}}_{hi}\bar{\mathring{\gamma}}_{gi}+\bar{\mathring{\gamma}}_{hi}+\bar{\mathring{\gamma}}_{gi}}{\bar{\mathring{\gamma}}_{hi}\bar{\mathring{\gamma}}_{gi}+\bar{\mathring{\gamma}}_{hi}+\bar{\mathring{\gamma}}_{gi}}\right)^{-1}}. \tag{4.34}$$

Equations (4.34) represent the exact average BER of a MAFR network, assuming that the network includes one relay.

4.5 Relationship of BER with other network parameters

The previous section presented the BER performance of the AF relay network for one relay node. In this section, the general form for n relays is derived. Then the relationships between BER and other network parameters, including (4.20) and (4.23), are expressed.

For an AF relay network with a selection strategy, among n relays, a single relay with the highest *SNR* is selected to forward data symbols from a sender to a receiver. Implementation of such selection at the receiver requires knowledge of order statistics. In this section, we briefly introduce a basic order statistical tool, which will be used for the BER performance analysis.

Using a selection strategy, the *SNRs* output of an AF realy network can be ordered statistically as: $\mathrm{x}_i=max\{\mathrm{x}_1,\mathrm{x}_2,....,\mathrm{x}_n\}$, where x_i is the i_{th} order statistic of the *SNR*s, x_n representing the largest order statistic and x_1 is the smallest one. Here, we can apply such order in respect to ϕ as follows

$$\phi_i=max\{\phi_1,\phi_2,....,\phi_n\}, \tag{4.35}$$

The *SNRs* in a statistical order is dependent on the following CDF analysis: let the CDF of ϕ_i be denoted as $\mathscr{F}_i(\phi_n)$, and then apply the order statistical analysis for ϕ_i as presented in [95]. This results in

$$\mathscr{F}_i(\phi_n) = Prob\,(\phi_1 \leq \phi_n) = \mathscr{F}_1(\phi_1)\,\mathscr{F}_2(\phi_2)...\mathscr{F}_n(\phi_n) = [\mathscr{F}_n(\phi_n)]^n\,, \tag{4.36}$$

where $\mathscr{F}_i(\phi_n)$ is the CDF order statistics.

For i.i.d channels, equation (4.36) can be written as $\mathscr{F}_i(\phi_n) = n\mathscr{F}_n(\phi_n)$ [66]. By substituting one relay CDF of (4.29) into (4.36), we obtain

$$\mathscr{F}_i(\phi_n) = n\,\mathrm{erfc}\left(\sqrt{\phi}\right) \tag{4.37}$$

By taking the derivative of (4.36) relative to ϕ, using a similar approach in [120], the total pdf is calculated as

$$\varphi_i(\phi) = \frac{n}{\mathring{\bar{\gamma}}_i}\, e^{-\frac{\phi}{\mathring{\bar{\gamma}}_i}}\,(\mathscr{F}_n(\phi_n))^{n-1} \tag{4.38}$$

Equation (4.38) is further analyzed can be presented as

$$\varphi_i(\phi) = \frac{n}{\mathring{\bar{\gamma}}_i}\,\left(e^{-\frac{\phi}{\mathring{\bar{\gamma}}_i}}\right)^n \tag{4.39}$$

By substituting (4.37) and (4.39) into (4.31), the ABER expression for *n* relays becomes

$$\bar{P}_e = \frac{n^2}{\mathring{\bar{\gamma}}_i}\int_0^\infty \mathrm{erfc}\left(\sqrt{\phi}\right)\left(e^{-\frac{\phi}{\mathring{\bar{\gamma}}_i}}\right)^n d\phi. \tag{4.40}$$

Evaluating (4.40), the general form of ABER performance of the MAFR network is obtained as follows

$$\bar{P}_e = n\left(1 - \sqrt{\frac{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi}}{\mathring{\bar{\gamma}}_{hi}\,\mathring{\bar{\gamma}}_{gi} + n\,(\mathring{\bar{\gamma}}_{hi} + \mathring{\bar{\gamma}}_{gi})}}\right)^n, \tag{4.41}$$

It is also important to mention here that the expression (4.41) can be analyzed by using (4.3) to obtain the BER of a suboptimal network. Now, from (4.41), the average *SNR* of (4.24) can be determined in respect to BER as

$$\mathring{\bar{\gamma}}_i = \frac{n\left(1-\left(\frac{\bar{P}_e}{n}\right)^{\frac{1}{n}}\right)^2}{1-\left(1-\left(\frac{\bar{P}_e)}{n}\right)^{\frac{1}{n}}\right)^2} \tag{4.42}$$

Equation (4.42) is very useful as it allows us to obtain many relationships between BER and other parameters within AF relay networks, and this provides the opportunity to study the behavior of BER for each parameter developed in an AF relay networks.

Thus, by substituting (4.42) into (4.20), an expression which combines BER with $\mathring{\xi}_{ti}$ is obtained as follows

$$\bar{P}_e = n\left(1-\sqrt{\frac{\mathring{\xi}_{ti}}{n\psi+\mathring{\xi}_{ti}}}\right)^n \tag{4.43}$$

.

Equation (4.43) gives the BER behavior for delivering a bit from an *S*-node to a *D*-node in an AF relay network with respect to relay location and distances from source to relay and relay to destination.

Also, substituting (4.42) into (4.21) and using some manipulation, we get BER associated with maximum *EE* which was obtained in (4.23). This is expressed as

$$\bar{P}_e = n\left(1-\left(1+\left(\frac{\frac{1}{\mathring{\zeta}}-\beta}{n\,\psi\varsigma\ln(2)}\right)1/\,w\left(-\frac{2^{\left(\frac{p_{ci}-\psi\varsigma}{\frac{1}{\mathring{\zeta}}-\beta}\right)}\psi\varsigma\ln(2)}{(\frac{1}{\mathring{\zeta}}-\beta)}\right)\right)^{0.5}\right)^n \tag{4.44}$$

where $w(.)$ is the omega function described [106].

4.6 Simulation Results

This section provides simulations to verify expressions derived through our theoretical analysis. The expressions are plotted in two cases: the first shows the suboptimal design based on $\bar{\gamma}_i$ of equation (4.41); in the second case, the $\mathring{\bar{\gamma}}_i$ of equation (4.24) was used for the optimal analysis. Both cases are combined

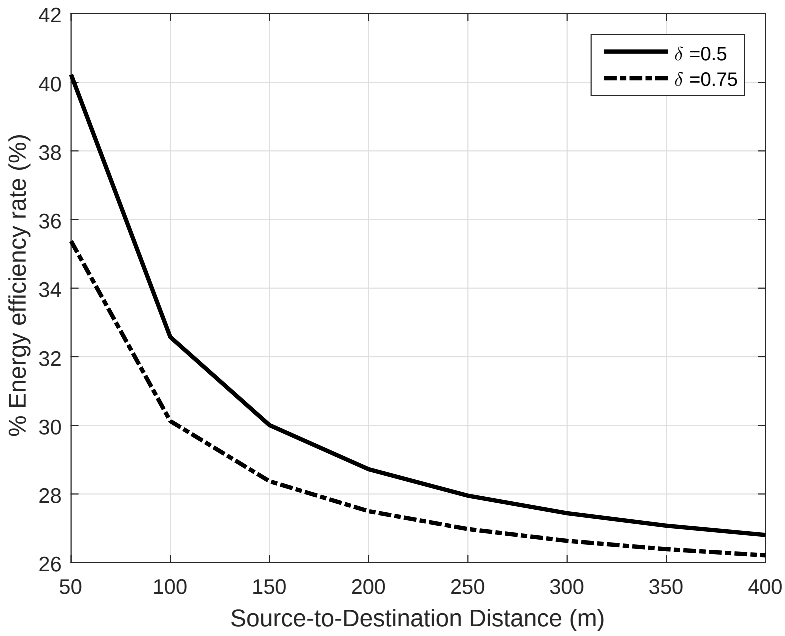

Figure 4.2: Power consumption for suboptimal network

and presented in a single figure, using the following parameters: $n = 4$, $d = 400$, $\alpha = 2.5$, $k = 10^{-3}$, $\varsigma = 2$, $p_{ci} = 0.5W$, ɲ $= 0.2W/bit/s$, .

Fig. 4.2 illustrates that *EE*, which is defined in (4.4), reduces with increasing distance between the source and the destination. The figure also compares two different values for the relay position, namely, $\delta = 0.5$ and $\delta = 0.75$. It is clear that *EE* is higher when the relay is close to the destination and this result corresponds with the previous result obtained by [103].

Similar parameters were adopted in Fig. 4.3 to present the optimized *EE* evaluated in (4.21). A comparison of Fig. 4.2 and Fig. 4.3 reveals that the proposed method provides higher *EE* than the suboptimal network in Fig. 4.2. Furthermore, energy efficiency for different relay locations in Fig. 4.3 is roughly equal.

Optimal *EE* in (4.21), with respect to (4.22), is shown in Fig. 4.4. It can be seen that *EE* increases with the rise of average *SNR*, as the *EE* design uses lower transmit power with increasing *SNR*. Furthermore, it shows that the proposed scheme outperforms the suboptimal one and the *EE* gap becomes larger when the *SNR* increases. Here, increasing *EE* comes at a small cost to the data rate, as shown in Fig. 4.5. This means that the proposal can achieve a better balance between *EE* and *SE*.

Fig. 4.6 illustrates the BER performance of suboptimal AF relay networks. It can be seen that the BER decreases as the number of relays increase from $n = 1$ to $n = 4$. These results are compared with

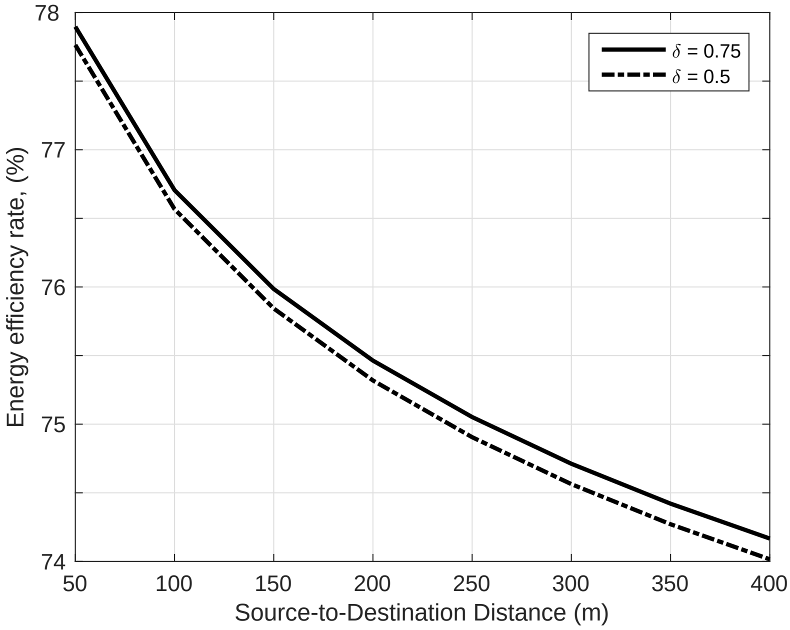

Figure 4.3: Power consumption for optimal network

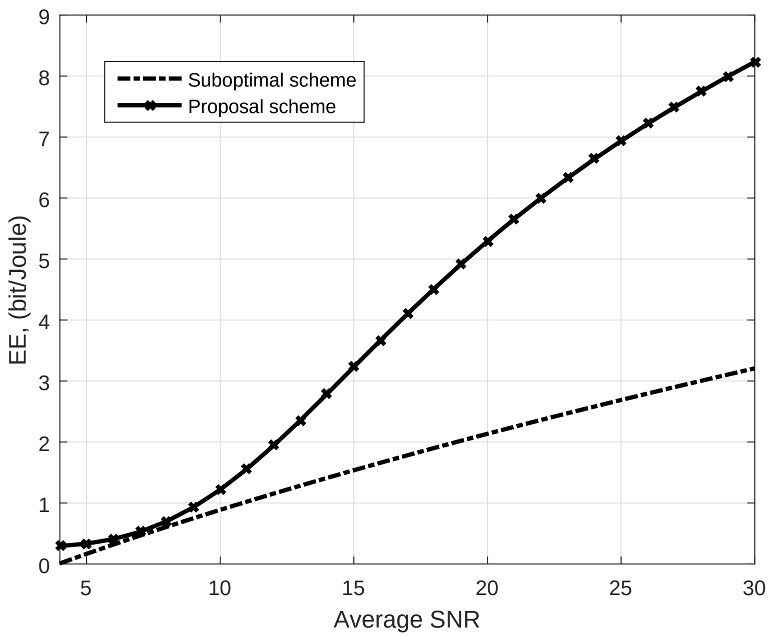

Figure 4.4: Proposal *EE* scheme versus suboptimal network

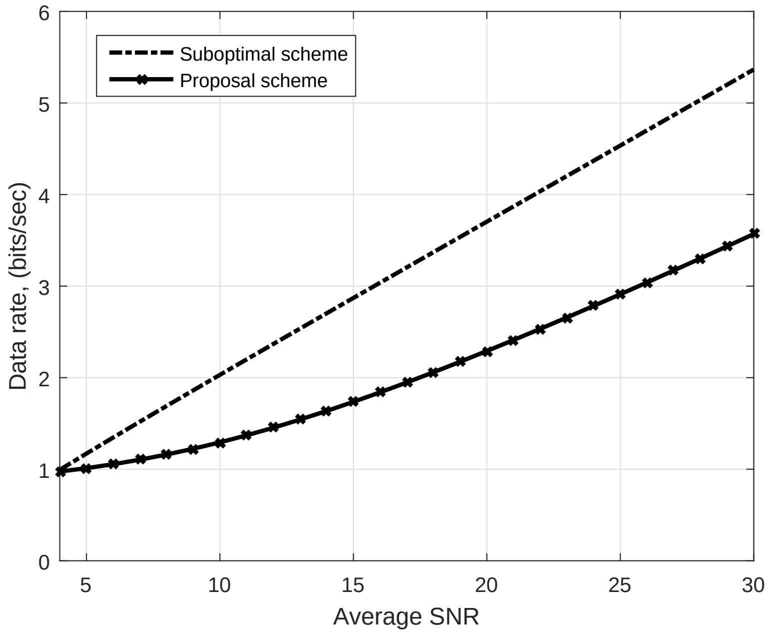

Figure 4.5: Proposal *SE* scheme versus suboptimal network

the results of the same network condition, but take into account the optimum $\mathring{\bar{\gamma}}_i$ proposed in (4.22). As shown, the optimizing *EE* allows a further reduction in BER results as the received *SNR* increases at the destination. Similar results are obtained in Fig. 4.7, when the relay power becomes $p_{ri} = p_s + 5$. This figure illustrates a further reduction in BER as the power amplification increases.

The BER behaviour of (4.44) is illustrated in Figs. 4.8 to 4.10. Fig. 4.8, shows the relationship between BER and *EE* of a suboptimal AF relay network. It is clearly seen that the high *EE* value increases the BER result, and this result confirms the theoretical analysis, as high *EE* indicates low power consumption. The figure also shows that the system can achieve lower BER by increasing the number of relays. Increasing *EE* while keeping BER at the same level can be obtained by using equation (4.23), and this confirms that the BER of an AF relay is reduced by balancing *EE* and *SE*, as shown in Fig. 4.9.

Further, by increasing the relay power to $p_{ri} = p_s + 5$, a considerable reduction in BER is achieved, as depicted in Fig. 4.10. However, the results of Figs. 4.8 to 4.10 reveal that the increase in *EE* increased BER slightly, this is because *EE* is based on reducing power consumption. This means that the increase in *EE* should be remained within narrow limits.

Fig. 4.11 shows the effect of relay location on Transmission Energy per bit which is evaluated in equation (4.20). It can be shown that the allocated energy reaches a minimum value uniformly when the

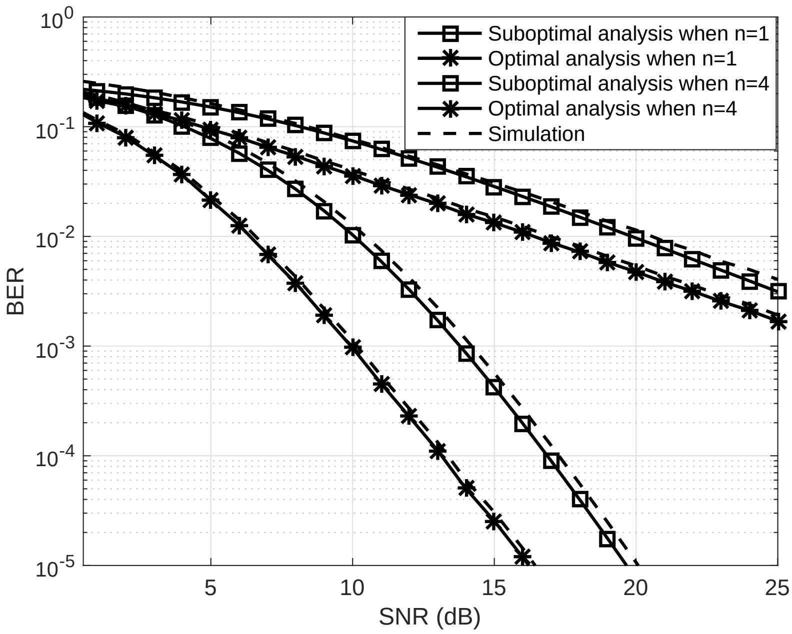

Figure 4.6: Comparing optimal BER scheme with suboptimal network at $p_{ri} = p_s$

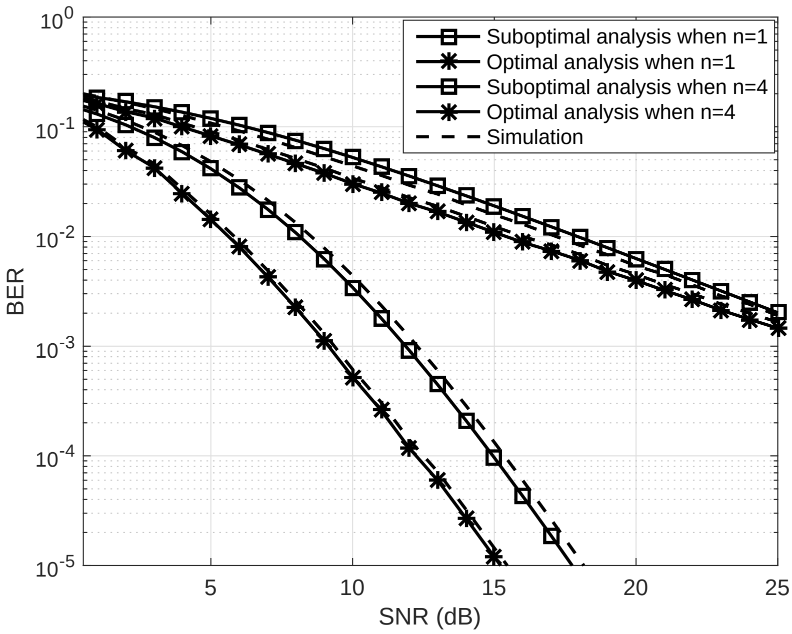

Figure 4.7: Comparing optimal BER scheme with suboptimal network at $p_{ri} = p_s + 5$

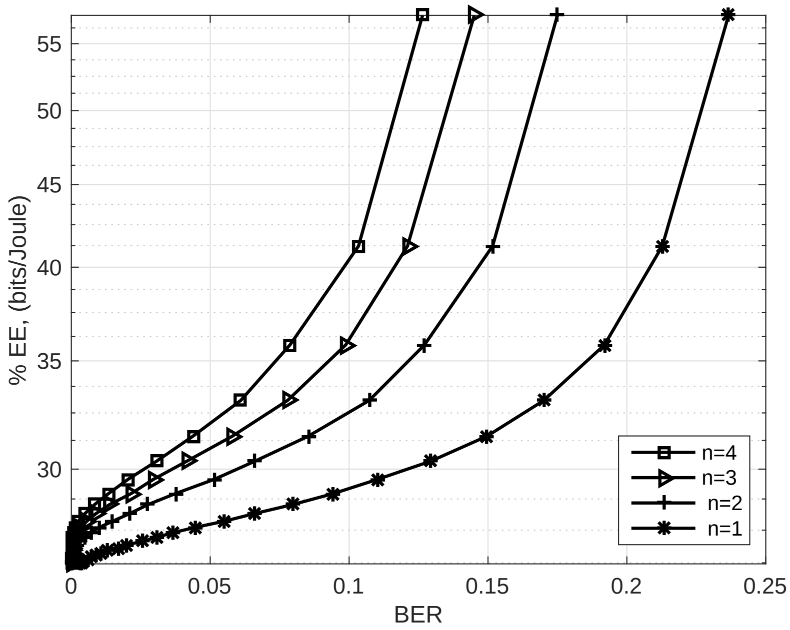

Figure 4.8: BER behavior in term of *EE*, at $p_{ri} = p_s$

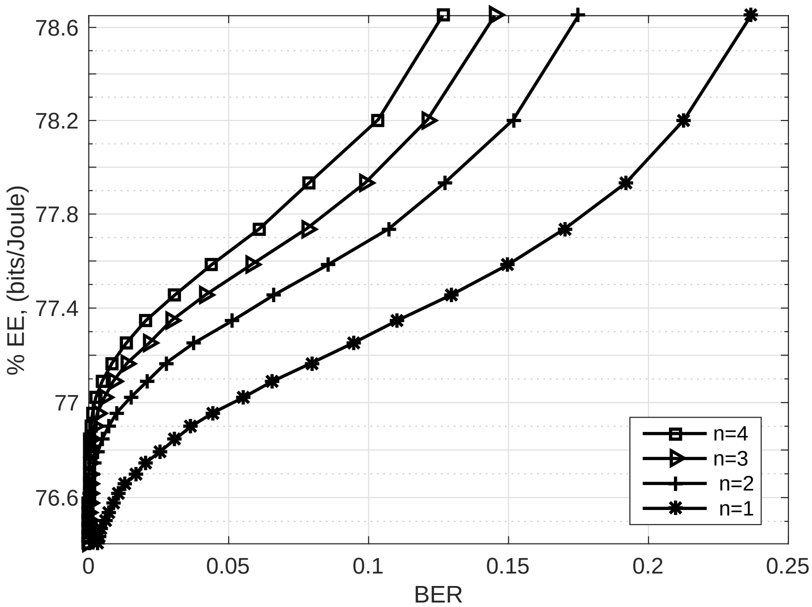

Figure 4.9: BER behavior in term of optimal *EE* scheme at $p_{ri} = p_s$

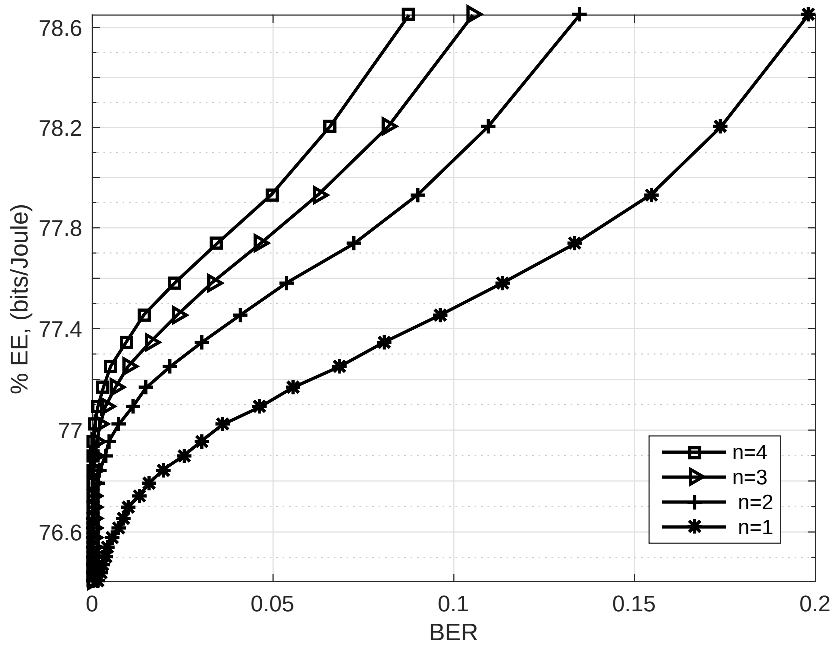

Figure 4.10: BER behavior in term of optimal *EE* scheme at $p_{ri} = p_s + 5$

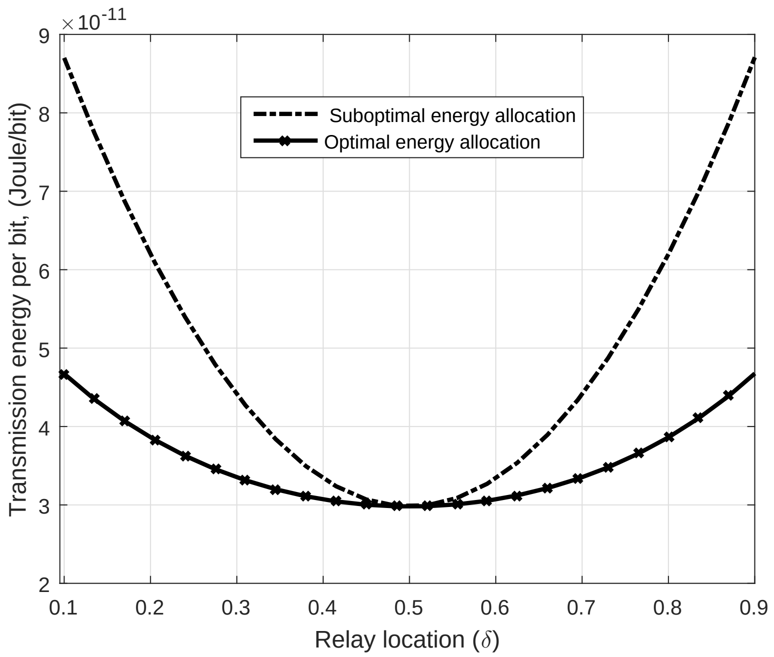

Figure 4.11: Relay location impact on transmission power

relay is located close to the midpoint between the transmitter and receiver nodes. This result agrees with the previous result obtained for the relay location in [121].

4.7 SUMMARY

This chapter presents a new method for reducing both the energy consumption and BER of multiple AF-relay networks. The method combines the BER and relay location with the optimal balancing of the *EE* and *SE*. A flat-fading channel and a relay selection scheme are used in this work. The proposal allows evaluation of the energy consumption, in which a selected relay can be placed at any point on the line connecting the source and its destination. Furthermore, the minimum evaluated energy is used to obtain the optimal balance between the *EE* and *SE*. This balance enables the *EE* to increase significantly with the least loss in *SE*. Such a balance is then expressed with respect to the BER. The results derived from the analytical expressions were simulated numerically. It was found that the optimal balance between the *EE* and *SE* increases the *EE* and decreases the BER of AF-relay networks. However, the increase in *EE* is likely to be within small limits.

The above combination of the BER and relay location with the optimal balancing of the *EE* and *SE* enhances the *EE* and BER in unidirectional AF-relay networks. In chapter 7, we adopt this idea to improving the *EE* and BER in bidirectional AF-relay networks. But first, we start with developing the ABER and EBER for such networks in chapters 5 and 6, respectively.

Chapter 5

Asymptotic BER analysis of Bidirectional Relay Networks

5.1 Introduction

In the previous chapters, we addressed *EE* and BER issues for several unidirectional relay network conditions. We now turn our attention to bidirectional relay networks. As presented in Chapter 1, the main principle of a bidirectional relaying scheme is the ability of a relay to transmit and receive, simultaneously, using the same frequency channel and hence achieve a higher *SE*. However, the big difference in power levels of the transmit and receive signals increases the power consumption which in turn affects the error rate performance. Thus, this chapter aims to provide a new method of analyzing the BER of Bidirectional Amplify-and-Forward Multiple Relay (BAF-MR) networks. Further, a new method of reducing the power consumption (i.e., increasing *EE*), which allows us to study the BER of BAF-MR networks under optimal conditions, is presented.

Several studies proposed means of developing the Probability Error Rate (PER), namely: Bit Error Rate (BER), Block Error Rate (BLER), Packet Error rate, or Symbol Error Rate (SER) [122]. One of the early methods, proposed by [123], used a high *SNR* to quantify the ABER of wireless transmission over fading channels. It showed that the high *SNR* approximation is useful, specifically for analyzing the ABER performance of wireless communications under severe fading conditions. The authors in [48] examined an approximated *SNR* with the harmonic mean for analyzing the BER and outage probability for both AF-relay and DF-relay networks. Their results showed that DF-relay gives better performance than AF relay at low *SNR*. However, at high *SNR*, the two systems are essentially equivalent in terms of the BER and outage probability. The BER performance of an AF-relay network was also analyzed by [74] using the inequality between the harmonic and geometric means of random variables to determine the upper bound for the *SNR*. Similarly, the upper bound of *SNR* was used in [124] to analyze the BLER of the BAF-MR relaying protocol over different flat fading channels. It adopted the harmonic mean of two independent gamma-distributed variables to calculate the *SNR*.

The asymptotic SER performance of BAF-MR networks has been investigated in [125], which also considered the high *SNR* region to obtain the SER expression. It incorporated advanced analysis techniques to develop relay selection schemes for enhancing SER performance. A similar selection scheme was adopted by [126] to analyze error probability performance for BAF-MR networks in the low and high *SNR* regions. The high *SNR* region was used to estimate the asymptotic error probability expression. However, at low *SNR* more complex mathematical forms have been considered in calculating the error probability performance. Similarly, reference [127] considered the exact SER performance in BAF-MR network analysis. It calculated the SER in a closed form using the Moment Generating Function (MGF) approach for a more beneficial analysis [122]. For the same reason, the authors in [128] derived an asymptotic SER expression at a high *SNR* using the same MGF approach.

In [129], the BLER performance was derived based on the Highest Worse Signal-To-Noise Ratio (HW-*SNR*) of the BAF-MR networks. It adopted the approximation cumulative distribution function (CDF) of the HW-*SNR* to obtain the BLER performance. The authors in [130] also used an approximated CDF to evaluate the BER performance of BAF-MR networks. They assumed the availability of high source and relay transmitting power to simplify the analysis. The authors in [131] based their estimates of BER and outage probability performances of relaying networks having the HW-*SNR*. This included both AR and DF relays. The study assumed a high *SNR* for evaluating both CDF and pdf to reduce the complexity of the analysis.

The BER and outage probability performances of BAF-MR networks were also investigated by [132]. Their study found that assuming a high *SNR* is crucial to simplifying the analysis of two different fading channel models, namely, the log-normal shadowing and generalized-K fading channels.

In the studies by [133, 134], both relay selection and power allocation of BAF-MR networks were optimized in analyzing the SER. Both studies found that the network with combined optimal relay selection and optimal power performed better than the network with optimal power allocation only. Similar results were presented by [135] when the relay selection was combined with network coding in BAF-MR networks. The authors in [136] also found similar performance enhancement, but this time BER performance. They used a high *SNR* region for evaluating BER when combining relay selection and power allocation conditions.

The above optimal BAF-MR studies focused on the PER performance in terms of either optimizing the power allocation or combining relay selection with optimal power allocation. However, the PER estimation for other types of optimal schemes, such as balanced *EE* and *SE*, have not been considered, whereas many wireless applications commonly demand that the probability of error remains below a specific threshold [137]. Therefore, we propose a method of evaluating the ABER performance of BAF-MR networks, and then applying it to calculate the ABER for an BAF-MR network satisfying both optimal *SE* and optimal *EE* objectives, which is referred to an optimal BAF-MR network. The ABER is derived from the *SNR* approximation at the high *SNR* region, since this region is useful for ABER analysis [123].

To make ABER estimation possible for both optimal and sub-optimal BAF-MR networks, a new form of the average *SNR* (Av*SNR*) is obtained by using both the Arithmetic and Geometric Mean inequality (AGM) and Harmonic Mean ($\mathscr{HM}$), as the Av*SNR* is a common term for both ABER and *EE* in the wireless channel [122, 137]. By using AGM and $\mathscr{HM}$ approaches, two average *SNR* analyses are provided. These enable the best approach to be selected, thus giving an accurate result for the ABER performance. It is worth mentioning here that many studies in the literature that have analyzed the Av*SNR* of relay networks have used AGM or $\mathscr{HM}$ e.g. [30, 48, 74], but the Av*SNR* of such studies is only suitable for suboptimal networks. The proposed method is applied to BAF-MR networks involving two users communicating through flat-fading channels and a set of parallel relays. Each user node includes a relay selection scheme to choose the highest *SNR* relay from among multiple relays.

The rest of the chapter is organized as follows: Section 5.2 outlines the system model; Section 5.3 describes the expressions used to calculate the approximation *SNR*; Section 5.4 gives the analysis of the ABER proposed method; Section 5.5 presents the proposed method for an optimal BAF-MR network. The simulation results are presented in Section 5.6. Finally, conclusions and remarks are discussed in Section 5.7.

5.2 System Model

This chapter considers multiple Amplify-and-Forward relay nodes assisting two wireless users (S_a and S_b) to exchange information simultaneously. Each relay is allocated between two-hop cascade channels as shown in Fig.5.1. These channels are assumed to be Rayleigh fading specified by independent and identically distributed (i.i.d.) random variables. All the nodes (i.e., users and relays) are equipped with a single antenna. The transmitted signal is modulated using Binary Phase Shift Keying (BPSK), and we assume that both users are transmitting to all relays at the same time slot T_{s1}, without a direct link between S_a and S_b.

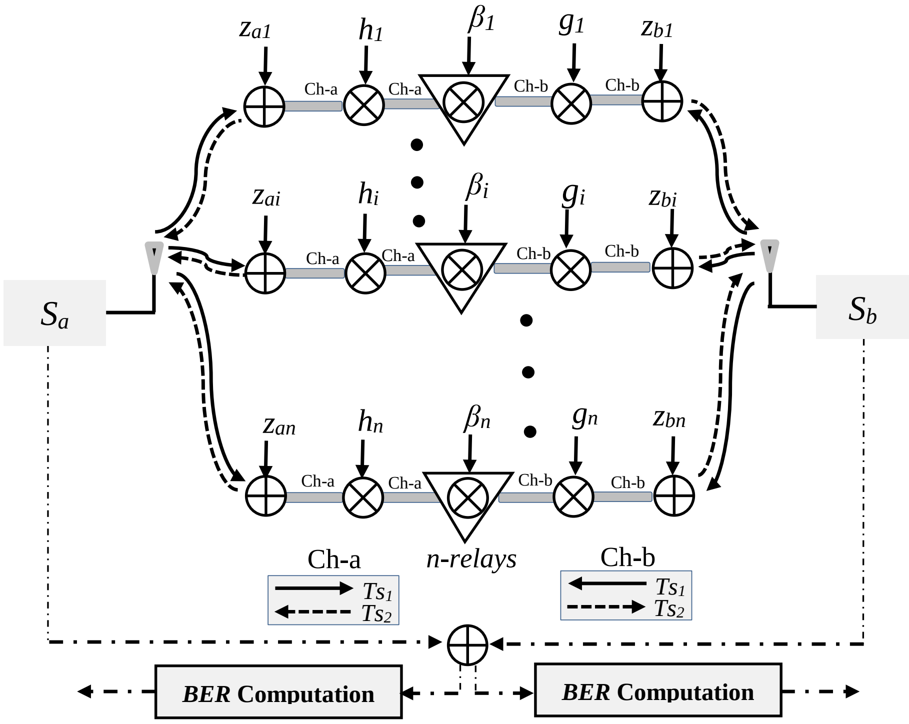

Figure 5.1: Proposed System Model

The received signal at any relay is expressed as

$$y_i = \sqrt{p_a} x_a h_i + \sqrt{p_b} x_b g_i + z_{ri} \tag{5.1}$$

where, y_i is the received signal at the i^{th} relay, P_a, P_b are the signal powers for user S_a and S_b respectively, x_a and x_b are information symbols from each user, h_i is the flat fading channel coefficient between S_a and the relay (R_i), g_i is the flat fading channel coefficient between S_b and R_i, and z_{ri}~$N(0,\sigma^2)$ is zero-mean complex Gaussian noise at the relay R_i [131, 138].

From (5.1), the amplification factor for the i^{th} relay can be expressed as

$$\beta_i = \sqrt{\frac{p_{ri}}{|h_i|^2 p_a + |gi|^2 p_b + \sigma^2}}, \tag{5.2}$$

where, β_i is the amplification factor for the i^{th} relay, p_{ri} is the allocated power for R_i [139], σ^2 is the variance of z_{ri} (we assumed that all noises are to be i.i.d. Gaussian with zero-mean and σ^2 variance).

The signals, once amplified at the relay, are then forwarded to their target users in the second time slot T_{S2} as shown in Fig. 5.1. It is assumed here that the channel conditions are known at the receiving ends, and the users are able to remove self-interference from the received signal completely. At the each user, the signal with the highest *SNR* is chosen using a selection scheme mentioned in [90].

The received signal for users S_a, S_b can be expressed as

$$y_{ai} = y_i h_i \beta_i + z_{ai}, \tag{5.3}$$

$$y_{bi} = y_i g_i \beta_i + z_{bi}, \tag{5.4}$$

where, y_{ai} is the received signal for S_a, y_{bi} is the received signal for S_b, z_{ai} and z_{bi} are the Gaussian noises at S_a and S_b respectively.

5.3 Signal-to-Noise Ratio (*SNR*)

In this section, the approximate *SNR and* Av*SNR* for a BAF-MR network are presented. The Av*SNR* is calculated by using the Arithmetic and Geometric Mean inequality (AGM).

The exact *SNR* can be calculated for node a using Equations (5.1) and (5.3) as follows:

$$y_{ai} = \left(\sqrt{p}_a x_a h_i + \sqrt{p}_b x_b g_i + z_{ri}\right) h_i \beta_i + z_{ai}, \tag{5.5}$$

By substituting (5.2) into (5.5) and then applying the expectation value on the outcome, the exact *SNR* for the node S_a can be expressed as:

$$\Upsilon_{ai} = \frac{p_{ri}\, p_b \gamma_{hi} \gamma_{gi}}{p_{ri}\gamma_{hi} + p_a \gamma_{hi} + p_b \gamma_{gi} + 1}, \tag{5.6}$$

where Υ_{ai} is the exact *SNR*, γ_{hi} and γ_{gi} are the instantaneous channel gain-to-noise ratios (CNRs) of the first and second hop respectively.

The CNRs can be defined as [54]:

$$\gamma_{hi} = |h_i|^2 \sigma^{-2} \tag{5.7}$$

$$\gamma_{gi} = |g_i|^2 \sigma^{-2}, \tag{5.8}$$

Both (5.7) and (5.8) are independent and identically distributed (i.i.d). Furthermore, $|h_i|^2$ and $|g_i|^2$have the same expectation value, equal to one [70, 71].

Following the similar method to calculate *SNR* at node S_a, the exact *SNR* for node S_b can be expressed as

$$\Upsilon_{bi} = \frac{p_{ri} p_a \gamma_{hi} \gamma_{gi}}{p_a \gamma_{hi} + p_{ri} \gamma_{gi} + p_b \gamma_{gi} + 1}, \tag{5.9}$$

The approximations of Equations (5.6) and (5.9) give the approximate *SNR* for users S_a and S_b as

$$\text{ɣ}_{ai} = \frac{p_{ri} p_b \gamma_{hi} \gamma_{gi}}{p_{ri} \gamma_{hi} + p_a \gamma_{hi} + p_b \gamma_{gi}} \tag{5.10}$$

$$\text{ɣ}_{bi} = \frac{p_{ri} p_a \gamma_{hi} \gamma_{gi}}{p_a \gamma_{hi} + p_{ri} \gamma_{gi} + p_b \gamma_{gi}} \tag{5.11}$$

where ɣ_{ai}, ɣ_{bi} are the approximate *SNR* for user S_a and S_b, respectively.

In this calculation, we assume the allocated power of user S_a takes the form as below

$$p_{ri} = \zeta \, p_{a,}, \tag{5.12}$$

where ζ is the ratio of different power between the relay and user S_a

Substituting (5.12) into (5.10) results in

$$\text{ɣ}_{ai} = \zeta / \left(\frac{\zeta + 1}{p_b \gamma_{gi}} + \frac{1}{p_a \gamma_{hi}} \right) \tag{5.13}$$

A similar analysis can be applied for user S_b by assuming the same way to consider ζ with γ_{bi} i.e., $p_{ri} = \zeta\, p_{b.}$Thus, equation (5.11) becomes

$$\gamma_{bi} = \zeta / \left(\frac{1}{p_b \gamma_{gi}} + \frac{\zeta + 1}{p_a \gamma_{hi}} \right) \tag{5.14}$$

To find the average value of (5.13) and (5.14), we first need to obtain the average value of each independent variable of γ_{hi} and γ_{gi}. Here, the average value of γ_{hi} evaluated as $\bar{\gamma}_{hi} = \frac{1}{N_o} E\left\{ |h_i|^2 \right\}$ and for γ_{gi} as $\bar{\gamma}_{gi} \; = \frac{1}{N_o}\left\{ |g_i|^2 \right\}$, where $N_o = \sigma^2$.

Both $\bar{\gamma}_{hi}$ and $\bar{\gamma}_{gi}$ are used to obtain the total average value of γ_{ai} by applying the formula of AGM to (5.13), using a similar way to calculate total expectation in [140–142]. This gives the average value of γ_{ai} as: $\bar{\gamma}_{ai} = \frac{\zeta}{2\sqrt{\zeta+1}} \sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}$. Similarly, the average value of γ_{bi} can be calculated as in $\bar{\gamma}_{ai}$ considering (5.14) , and this results in $\bar{\gamma}_{bi} = \frac{\zeta}{2\sqrt{\zeta+1}} \sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}$ which is equal to $\bar{\gamma}_{ai}$. Accordingly, a common expression for $\bar{\gamma}_{ai}$ and $\bar{\gamma}_{bi}$ is expressed as

$$\bar{\gamma}_i = \frac{\zeta}{2\sqrt{\zeta+1}} \sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}\,, \tag{5.15}$$

where $\bar{\gamma}_i = \bar{\gamma}_{ai} = \bar{\gamma}_{bi}$,$\bar{\gamma}_{ai}$ =$p_a\, \bar{\gamma}_{hi}$ and $\bar{\gamma}_{bi}$= $p_b\, \bar{\gamma}_{gi}$.

Expression (5.15) reveals that the same average *SNR* results, for the different γ_{ai} and γ_{ai}, can be realized for users S_a and S_b.

Another assumption for ζ can be expressed as

$$p_{ri} = \zeta\,(p_a + p_b), \tag{5.16}$$

where ζ is the ratio of selected relay power to both users S_a and S_b.

Substituting (5.16) into (5.10) results in

$$\gamma_i = \left(\frac{\zeta + 1}{\zeta\, \gamma_{bi}} + \frac{\zeta + 1}{\zeta\, \gamma_{ai}} \right)^{-1}, \tag{5.17}$$

where

$$\gamma_{ai} = (p_a + p_b)\gamma_{hi}\,, \tag{5.18}$$

and

$$\gamma_{bi} = \frac{(p_a + p_b)\, p_b \gamma_{gi}}{p_a}. \tag{5.19}$$

To calculate average value of (5.17) based on (5.16), we can use the Harmonic Mean ($\mathscr{H}m$) presented in [47] . This results in

$$\bar{\gamma}_i = \frac{1}{2}\mathscr{H}m\left(\frac{\zeta\,\gamma_{ai}}{\zeta+1}, \frac{\zeta\,\gamma_{bi}}{\zeta+1}\right). \tag{5.20}$$

where $\bar{\gamma}_i$ is the average value of (5.17).

5.4 Analysing ABER of BAF-MR network

This section presents the proposed ABER analysis for two-hop cascade BAF-MR networks. In this cascade fashion, the source signal passes through two hops. Each hop includes a Rayleigh fading channel, and such a channel has a *SNR* that follows an exponential distribution [75], Therefore, the destination node that receives the total *SNR* of two i.i.d Rayleigh fading channels also follows an exponential distribution. Here, the distribution of the harmonic mean of two i.i.d. gamma random variables defined in [48] can be employed to find the pdf in each hop. Such pdfs are expressed as

$$\underset{S_b \to R_i}{\varphi_{gi(\phi)}} = \frac{1}{\bar{\gamma}_{gi}} \exp^{\frac{-\phi}{\bar{\gamma}_{gi}}} \tag{5.21}$$

$$\underset{R_i \to S_a}{\varphi_{hi(\phi)}} = \frac{1}{\bar{\gamma}_{hi}} \exp^{\frac{-\phi}{\bar{\gamma}_{hi}}} \tag{5.22}$$

where ϕ is the harmonic mean (denoted by $\phi = \mu_H(\gamma_{hi}, \gamma_{gi})$) according to a gamma distribution [47], $\underset{S_b \to R_i}{\varphi_{gi(\phi)}}$ and $\underset{R_i \to S_a}{\varphi_{hi(\phi)}}$ are the pdfs of ϕ for source to relay and relay to destination channels, respectively.

Now, the modified harmonic mean defined by [119] can be used to join both (5.21) and (5.22) as follows

$$\varphi_{ai}(\phi) = \frac{\exp^{-\phi\left(\frac{\bar{\gamma}_{hi}+\bar{\gamma}_{gi}}{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}\right)}}{\left(\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}\right)^3} \left(4\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}\, k_0\left(\frac{2\phi}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}\right) + 2\phi\,(\bar{\gamma}_{hi}+\bar{\gamma}_{gi})\, k_1\left(\frac{2\phi}{\sqrt{\bar{\gamma}_{hi}\bar{\gamma}_{gi}}}\right)\right) \tag{5.23}$$

where $\varphi_{ai}(\phi)$ is the total pdf, $k_0(.)$ and $k_1(.)$ are the first and the second order modified Bessel function of the second kind.

By applying the modified Bessel function properties, which are $k_0(\phi) \to 0$ and $k_1(\underset{\phi\to 0}{\phi}) \to 1/\phi$, to (5.23), the total pdf obtained is

$$\varphi_{ai}(\phi) = \frac{2\sqrt{\zeta+1}}{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}} e^{-\phi \frac{2}{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}(\sqrt{\zeta+1})} \tag{5.24}$$

where $\varphi_{ai}(\phi)$ is the total pdf of two-cascaded Rayleigh fading channels.

Integration of (5.24) relative to γ_{ai} , yields the cumulative distribution function for $\varphi_{ai}(\phi)$ [91]. Thus, the CCDF of (5.24) is then given as

$$\mathscr{F}_{ai}(\phi) = 1 - e^{-\phi \frac{2}{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}(\sqrt{\zeta+1})}, \tag{5.25}$$

Using the relationship between the Q-function and (5.25) as in [92], the term of $\mathscr{F}_{ai}(\phi)$ can be rewritten as

$$Q(\phi) = \left(\sqrt{2\phi \frac{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}{2\sqrt{\zeta+1}}}\right). \tag{5.26}$$

The Q-function can be given in terms of complementary error function (erfc) as in [13]. Thus, the relationship between (5.26) and the erfc functions can be summarized as

$$Q(\phi) = \frac{1}{2}\operatorname{erfc}\left(\frac{\phi}{\sqrt{2}}\right). \tag{5.27}$$

Substituting(5.26) into (5.27) results in

$$Q(\phi_{ai}) = \frac{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}{2\sqrt{\zeta+1}} \operatorname{erfc}\left(\sqrt{\phi \frac{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}{2\sqrt{\zeta+1}}}\right) \tag{5.28}$$

Both (5.24) and (5.28) will allow us to create an ABER expression for BAF-MR networks, as defined by [13]. This is expressed

$$P_{\mathrm{e}}(\phi) = \int_0^\infty e^{-\phi \frac{2}{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}(\sqrt{\zeta+1})} \operatorname{erfc}\left(\sqrt{\phi \frac{\zeta\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}{2\sqrt{\zeta+1}}}\right) d\phi, \tag{5.29}$$

By evaluating (5.29), the ABER of BAF-MR network is obtained in case of one relay network. This results in

$$P_{\mathrm{e}}(\phi)=\zeta\sqrt{\frac{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}{4\zeta+4}}\left(1-\sqrt{\frac{\zeta^{2}\bar{\gamma}_{ai}\bar{\gamma}_{bi}}{\zeta^{2}\bar{\gamma}_{ai}\bar{\gamma}_{bi}+4\zeta+4}}\right) \tag{5.30}$$

Expression (5.30) is derived for a single relay branch (i.e, $i=1$). In a BAF-MR network, however, several signals are received through independent relay branches, and then these signals are combined using a combining strategy. The selection combiner is adopted in this work to select the highest instantaneous *SNR* among several available relays. Thus, the higher order statistics technique in [95] is applied to analyze *SNR* as: $\gamma_{ai}=max\{\gamma_{a1},\gamma_{a2},....,\gamma_{an}\}$. Here, we can apply such order in respect to ϕas

$$\phi_{ai}=max\{\phi_{a1},\phi_{a2},....,\phi_{an}\}, \tag{5.31}$$

where ϕ_{an} represents the largest order statistic and ϕ_{a1}be the smallest one.

The order in (5.31) requires to obtain CDF of $\phi_{(ai)}$. In order to do that, we can apply the order statistical analysis presented by [95] to obtain

$$\mathscr{F}_{ai}(\phi)=Prob\,(\phi_{ai}\leq\phi_{an})=\mathscr{F}_{a1}(\phi_{a1})\mathscr{F}_{a2}(\phi_{a2})...\mathscr{F}_{an}(\phi_{an})=[\mathscr{F}(\phi_{an})]^{n} \tag{5.32}$$

where $\mathscr{F}_{ai}(\phi)$ is the CDF of ϕ_{ai}.

Since the channels are specified as i.i.d, expression (5.32) can be defined as in [66]. This results in $\mathscr{F}_{ai}(\phi_{an})=n\mathscr{F}(\phi_{an})$.

Now, substituting (5.28) which assumed $i=n=1$ into $\mathscr{F}_{ai}(\phi_{an})$ expression considering ϕ distribution, the total CCDF at S_a is expressed as

$$\mathscr{F}_{(ai)}(\phi)=n\frac{\zeta\sqrt{\bar{\gamma}_{an}\bar{\gamma}_{bn}}}{2\sqrt{\zeta+1}}\,\mathrm{erfc}\left(\sqrt{\phi\frac{\zeta\sqrt{\bar{\gamma}_{an}\bar{\gamma}_{bn}}}{2\sqrt{\zeta+1}}}\right). \tag{5.33}$$

By taking the derivation of (5.32) in relative to $\phi_{(an)}$, the total pdf is obtained as follows

$$\Im_{(ai)}(\phi)=\frac{n}{\bar{\gamma}_{(an)}}\,e^{-\frac{\phi_{(an)}}{\bar{\gamma}_{(an)}}}\left(\mathscr{F}(\phi_{(an)})\right)^{n-1}, \tag{5.34}$$

where $\Im_{(ai)}(\phi)$ is the total value of pdf at S_a.

Further analysis for (5.34) results in

$$\Im_{(ai)}(\phi)=\frac{n}{\bar{\gamma}_{(an)}}\left(e^{-\frac{\phi_{(an)}}{\bar{\gamma}_{(an)}}}\right)^{n}. \tag{5.35}$$

Now, ABER of n relay in a BAF-MR network is obtained by using (5.33) and (5.35) as in (5.29). This gives

$$P_e(\phi_{(an)}) = \frac{n^2}{\bar{\gamma}_{(an)}} \int_0^{\infty} \left(e^{-\frac{\phi_{(an)}}{\bar{\gamma}_{(an)}}}\right)^n \frac{\zeta\sqrt{\bar{\gamma}_{an}\bar{\gamma}_{bn}}}{2\sqrt{\zeta+1}} \operatorname{erfc}\left(\sqrt{\phi_{(an)}\frac{\zeta\sqrt{\bar{\gamma}_{an}\bar{\gamma}_{bn}}}{2\sqrt{\zeta+1}}}\right) d\phi_{(an)} \tag{5.36}$$

Evaluating (5.36) gives

$$P_e(\phi_{(an)}) = \left(\frac{\zeta}{2}\sqrt{\frac{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}{\zeta+1}}\left(1-\sqrt{\frac{1}{1+\frac{(\zeta+1)4n}{\zeta^2\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}}\right)\right)^n, \tag{5.37}$$

where $P_e(\phi_{(an)}))$ is the ABER of BAF-MR networks for n relays when (5.15) is used for analysis.

Now, to calculate ABER performance based on the Harmonic mean in (5.20), we employ the error probability performance of BAF-MR networks which was presented in [119] as

$$P_e(\gamma_i) = \int_0^{\infty} \frac{e^{-\gamma_i}}{\sqrt{\gamma_i 4\pi}} \mathscr{F}(\gamma_i)\, d\gamma_i, \tag{5.38}$$

where $\mathscr{F}(\gamma_i)$ is the Cumulative Distribution Function (CDF) of the output SNR, $\frac{e^{-\gamma_i}}{\sqrt{\gamma_i 4\pi}}$ is the probability density function (pdf) of two random variables in the cascade form. Such pdf can be determined by following the procedure described in (5.21) and (5.22) and following the similar approach demonstrated in for unidirectional relay networks [47]. The value of $\mathscr{F}(\gamma_i)$ is then expressed as:

$$\mathscr{F}(\gamma_i) = 1 - \frac{2\gamma_i \exp^{-\gamma_i(\frac{\zeta+1}{\zeta\bar{\gamma}_{ai}}+\frac{\zeta+1}{\zeta\bar{\gamma}_{bi}})}}{\frac{\zeta}{\zeta+1}\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}} k_1\left(\frac{2\gamma_i}{\frac{\zeta}{\zeta+1}\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}\right), \tag{5.39}$$

where, k_1 is the modified first-order Bessel function which can be approximated as $k_1(\gamma_i) \rightarrow 1/\gamma_i$. This leads to

$$\mathscr{F}(\gamma_i) \approx 1 - \exp^{-\gamma_i(\frac{\zeta+1}{\zeta\bar{\gamma}_{ai}}+\frac{\zeta+1}{\zeta\bar{\gamma}_{bi}})}. \tag{5.40}$$

Expression (5.40) is derived for a single relay branch (i.e, $i=1$). In a multiple BAF-MR network, however, several signals are received through independent relay branches, and then these signals are combined by one of the combining strategies. The selection combiner is adopted to select the highest instantaneous *SNR* among several available relays. Thereby, a higher order statistics technique is applied to analyze *SNR* as presented in equation (5.32).

Thus, for n relay, equation (5.40) can be expressed, based on (5.32), as

$$\mathscr{F}_{(i)}(\gamma_n) \approx \left(1 - \exp^{-\gamma_i\left(\frac{\zeta+1}{\zeta\bar{\gamma}_{ai}} + \frac{\zeta+1}{\zeta\bar{\gamma}_{bi}}\right)}\right)^n. \tag{5.41}$$

The first order expansion of (5.41) results in

$$\mathscr{F}_{(i)}(\gamma_n) \approx \left(\exp^{-\gamma_i\left(\frac{\zeta+1}{\zeta\bar{\gamma}_{ai}} + \frac{\zeta+1}{\zeta\bar{\gamma}_{bi}}\right)}\right)^n. \tag{5.42}$$

Substituting (5.42) into (5.38), and evaluating the integral. This results in

$$P_{\mathfrak{e}}(\gamma_n) \approx \frac{2^n}{\sqrt{4\pi}}\Gamma(n+0.5)\left(\frac{\zeta+1}{\zeta}\left(\frac{1}{\bar{\gamma}_{ai}} + \frac{1}{\bar{\gamma}_{bi}}\right)\right)^n, \tag{5.43}$$

where $P_{\mathfrak{e}}(\gamma_i)$ is the proposed ABER which is adopted in (5.20)

5.5 Analysing ABER of an Optimal BAF-MR level

The optimal *SNR* in relay networks can be achieved by optimizing the power allocated between the relay and the transmitter as in [30]. Another strategy is to balance *EE* and and *SE* (BES) as in [43, 44, 88]. In this section we consider the second strategy (i.e., BES) to analyse BER in BAF-MR networks.

To calculate optimal *SNR* by using the BES strategy, we adopted *EE* metrics defined as the number of transmitted data bits per Joule of energy. As the definition of *EE* includes data rate/powers consumption, thus, the steps of BER calculation are as follows:

1. Minimizing total powers consumption under fixed average data rate.

2. Maximizing *EE* based on maximized *SE*.

3. Obtaining optimal *SNR* from 1 and 2.

In this chapter we use average Energy Efficiency ($\overline{EE}$) to calculate average optimal *SNR* by following the above steps.

According to (3.23), $\overline{EE}$ can be expressed as

$$\bar{\zeta} = \frac{\bar{\Re}_i}{\bar{p}_{ti}} \,, \tag{5.44}$$

where $\bar{\Re}_i$ is the average data rate, and $\bar{p}_{ti}$ is the average power consumption. In BAF-MR networks the value of $\bar{\Re}_i$ is equal to sum rates of the first and the second hops as $\bar{\Re}_i = \bar{\Re}_{ba} + \bar{\Re}_{ab}$, where$\bar{\Re}_{ba}$ and $\bar{\Re}_{ab}$ are the average throughputs from users S_b and S_a respectively. Each of $\bar{\Re}_{ba}$ and $\bar{\Re}_{ab}$ is calculated as in (3.23). as

$$\bar{\Re}_i \approx \frac{1}{2}\log_2(1+\bar{\gamma}_{ai}) + \frac{1}{2}\log_2(1+\bar{\gamma}_{bi}) \tag{5.45}$$

From equation (5.15) , expression $\bar{\Re}_i$ can be given as

$$\bar{\Re}_i \approx \log_2(1+\bar{\gamma}_i) \tag{5.46}$$

The total average energy consumption $\bar{p}_{ti}$ in BAF-MR networks includes signal processing of power amplification and the circuit's energy consumption [82]. It can be presented as

$$\bar{p}_{ti} = (\xi\,(\,\bar{p}_a + \bar{p}_b + \bar{p}_{ri}) + \beta\,\bar{\Re}_i + p_{ci})/2 \tag{5.47}$$

where ξ is a constant associated with the amplifier power efficiency, p_{ci} is static circuit power and $\beta\Re_i$ is the dynamic power per-unit data rate [43]. The term of $(\beta\Re_i + p_{ci})$ is related to static circuit power consumption, (further information about the relay power is presented by [103, 117]).

Now to minimize the total energy consumption for the given average data rate, we can use the following equation

$$Min\, f_a(\bar{\Re}_i) + f_b(\bar{\Re}_i) + f_{ri}(\bar{\Re}_i) \tag{5.48}$$

The powers $f_a(\bar{\Re}_i)$ and $f_b(\bar{\Re}_i)$ can be obtained from (5.15) as

$$p_a = f_a(\bar{\Re}_i) = \frac{1}{\bar{\gamma}_{hi}\, p_b\, \bar{\gamma}_{gi}} \left(2\bar{\gamma}_i \frac{\sqrt{\zeta+1}}{\zeta} \right)^2 \tag{5.49}$$

$$p_b = f_b(\bar{\Re}_i) = \frac{1}{\bar{\gamma}_{hi}\, p_a\, \bar{\gamma}_{gi}} \left(2\bar{\gamma}_i \frac{\sqrt{\zeta+1}}{\zeta} \right)^2 \tag{5.50}$$

Also relay power in (5.12), is expressed as a function of $\bar{\Re}_i$ as follows

$$p_{ri} = f_{ri}(\bar{\Re}_i) = \zeta\, \bar{p}_{a,}, \tag{5.51}$$

By substituting (5.49), (5.50) and (5.51) into (5.48) and taking the outcome derivation, the minimum S_a, S_b and relay powers are obtained as follows

$$f_a(\bar{\Re}_i) = f_b(\bar{\Re}_i) = \frac{2\bar{\gamma}_i}{\zeta\, \sqrt{\bar{\gamma}_{hi}\, \bar{\gamma}_{gi}}} \tag{5.52}$$

$$f_{ri}(\bar{\Re}_i) = \frac{2\bar{\gamma}_i}{\sqrt{\bar{\gamma}_{hi}\, \bar{\gamma}_{gi}}} \tag{5.53}$$

Substituting (5.46), (5.52) and (5.53) into $\overline{EE}$ expression as

$$\overline{EE} = \frac{2\, \log_2(1+\bar{\gamma}_i)}{\frac{\bar{\gamma}_i}{\sqrt{\bar{\gamma}_{hi}\, \bar{\gamma}_{gi}}}\xi(\frac{1}{\zeta}+1) + \beta\, \log_2(1+\bar{\gamma}_i) + p_{ci}} \tag{5.54}$$

By taking the derivation of (5.54) with respect to $\bar{\gamma}_i$, the optimal *SNR* value based on balancing *SE* and *EE* is obtained as follows

$$\overset{*}{\gamma}_{ai} = \frac{p_{ci} - \Gamma}{\Gamma\, Wo\left(-\frac{e^{-\frac{p_{ci}}{\Gamma}-1}(\Gamma - p_{ci})}{\Gamma} \right)}, \tag{5.55}$$

where $\overset{*}{\gamma}_{ai}$ is the optimal value of γ_{ai} , e is the base of the natural logarithm, $\Gamma = \frac{\xi}{\sqrt{\bar{\gamma}_{hi}\, \bar{\gamma}_{gi}}}(\frac{1}{\zeta}+1)$ and $Wo(.)$ is the real branch of omega function [106].

From (5.55), the optimal data rate can be determined as $\overset{*}{\Re}_i = \log_2(1+\overset{*}{\gamma}_{ai})$, which can be rewritten as $\overset{*}{\gamma}_{ai} = 2^{\overset{*}{\Re}_i} - 1$.

Estimating ABER of BAF-MR networks based on (5.55) requires following the same procedures as for (5.37). However, the average value in (5.15) is subjected to the relation of energy per bit to noise power spectral density ratio (i.e., E_b/N_o) as presented in [84]. This gives $\frac{E_b}{N_o}\frac{\zeta\, \Re_i}{\sqrt{\zeta+1}\overset{*}{\Re}_i}$. Accordingly, the optimal value of (5.15) is expressed as

$$\overset{*}{\gamma}_{ai} = \sqrt{\frac{\zeta^2}{\zeta+1}\frac{\Re_i\sqrt{\bar{\gamma}_{ai}\bar{\gamma}_{bi}/4}}{\overset{*}{\Re}_i}}. \tag{5.56}$$

Using (5.56) and following the same procedures for (5.37), ABER of the optimal BAF-MR network is expressed as

$$\mathring{P}_e(\phi_{an}) = \left(\overset{*}{\gamma}_{ai}\left(1-\sqrt{\frac{\Re_i}{\Re_i+\frac{\overset{*}{\Re}_i(\zeta+1)4n}{\zeta^2\bar{\gamma}_{ai}\bar{\gamma}_{bi}}}}\right)\right)^n \tag{5.57}$$

Equation (5.57) is analyzed based on (5.15). Now, ABER of the optimal BAF-MR network employs average *SNR* value in (5.20) can be analyzed as follows:

The total average energy consumption $\bar{p}_{ti}$ in (5.47) can be presented as: $p_{ti} = \frac{(\zeta+1)\xi}{2}(p_a+p_b) + (\beta\,\Re_i + p_{ci})$. Thereby, we can follow the procedure of (5.55) to obtain optimal *SNR* of balancing *SE* and *EE*. This given as

$$\mathring{\gamma}_i = \exp^{1+Wo\left(\frac{p_{ci}}{\Gamma(\zeta+1)\xi e}-\frac{1}{e}\right)} - 1, \tag{5.58}$$

where $\mathring{\gamma}_i$ is the optimal value of γ_i, $\Gamma = \frac{1}{\gamma_{hi}} + \frac{1}{\gamma_{gi}} + \frac{2}{\sqrt{\gamma_{hi}\gamma_{gi}}}$.

From (5.58), the optimal data rate ($\mathring{\Re}_i$) can be determined as $\mathring{\Re}_i = \log_2(1+\mathring{\gamma}_i)$, which can be expressed as

$$\mathring{\gamma}_i = 2^{\mathring{\Re}_i} - 1 \tag{5.59}$$

The optimal powers for both S_a and S_b users can be obtained also by following the same procedure of (5.49) and (5.50) . This is expressed as

$$\mathring{p}_a = \left(2^{\mathring{\Re}_i} - 1\right)\left(\frac{1}{\gamma_{hi}} + \frac{1}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right) \tag{5.60}$$

$$\mathring{p}_b = \left(2^{\mathring{\Re}_i} - 1\right)\left(\frac{1}{\gamma_{gi}} + \frac{1}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right) \tag{5.61}$$

where $\mathring{p}_a$ and $\mathring{p}_b$ are the optimal powers of p_a and p_b respectively.

The optimal values of (5.18) and (5.19) are derived as

$$\mathring{\gamma}_{ai} = \left(2^{\mathring{\Re}_i} - 1\right)\left(\frac{1}{\gamma_{hi}} + \frac{1}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right)\gamma_{hi} \tag{5.62}$$

and

$$\mathring{\gamma}_{bi} = \left(2^{\mathring{\Re}_i} - 1\right)\left(\frac{1}{\gamma_{gi}} + \frac{1}{\sqrt{\gamma_{gi}\gamma_{hi}}}\right)\gamma_{gi}. \tag{5.63}$$

where $\mathring{\gamma}_{ai}$ and $\mathring{\gamma}_{bi}$ are the optimal value of γ_{ai} and γ_{bi} respectively.

By using (5.62) and (5.63) in (5.20) , the optimal value of average *SNR* is calculated as

$$\mathring{\mathfrak{Y}}_i = \frac{1}{2}\mathscr{H}m\left(\frac{\mathring{\gamma}_{ai}}{(\zeta+1)/\zeta}, \frac{\mathring{\gamma}_{bi}}{(\zeta+1)/\zeta}\right). \tag{5.64}$$

where $\mathring{\mathfrak{Y}}_i$ is the optimal average value.

Following the same procedures of (5.43), ABER of the optimal BAF-MR network, using (5.20), is expressed as follows

$$\mathring{P}_{\mathfrak{e}}(\mathring{\mathfrak{Y}}_i) = \frac{1}{\sqrt{4\pi}}\left(\frac{\zeta+1}{0.5\,\zeta}\right)^n \Gamma(n+0.5)\left(\frac{1}{\mathring{\gamma}_{ai}} + \frac{1}{\mathring{\gamma}_{bi}}\right)^n. \tag{5.65}$$

5.6 Simulation Results

The results of the proposed methods applied to the system model described in Section 5.2 are presented in this section. However, for this analysis we are limiting the number of relays to four (i.e, $n = 4$). Further, we consider two values of ζ as 1 and 2 for validating the results. For every value a comparison between simulated and analytic results is presented.

Fig. 5.2 shows the ABER performance of equation (5.37) assuming $\zeta = 1$ (i.e., equal power allocation for all nodes). It can be observed that the ABER performance improves as the number of relays increases in the network. The proposed results approach the simulation results especially at the high *SNR* region. However, there are considerable differences between them in the low *SNR* region. This trend is related to the fact that the analysis is limited to the high *SNR* region.

A similar result is shown in Fig. 5.3 when equation (5.43) is adopted. This result confirms that the ABER analysis whether by using (5.15) or (5.20) gives a similar result when user powers are equal.

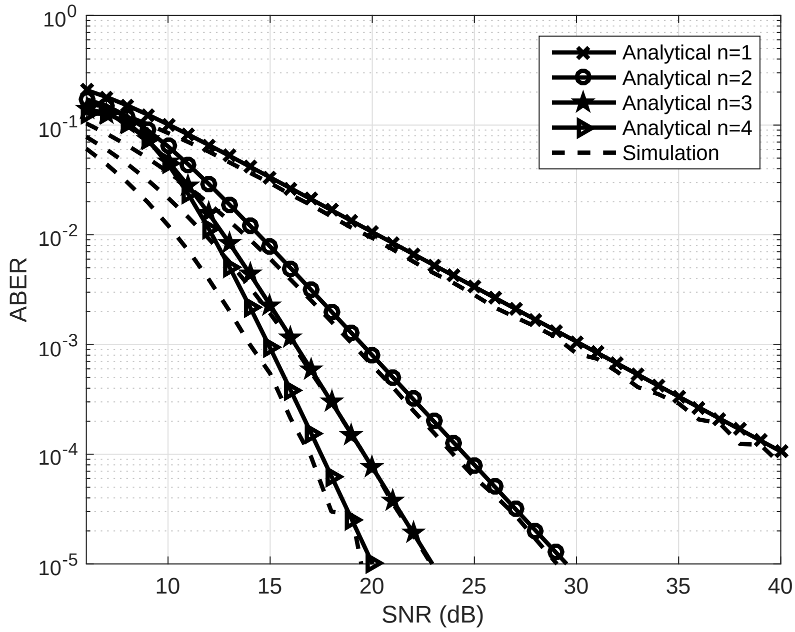

Figure 5.2: ABER performance in (5.37) assuming $\zeta = 1$

By increasing ζ to 2, the result of ABER gives more enhancement, as the higher ζ increases overall received *SNR* and this leads to lower BER [30], and such a result is illustrated in Figs. (5.4) and (5.5) for equations (5.37) and (5.43), respectively. However, Fig. (5.5) illustrates more ABER enhancement at low *SNR* level when AGM is used, and this result agrees with [143] who compared between AGM with HM.

Figs. 5.2 to (5.5) correspond to the suboptimal network. The ABER results pertaining to optimal networks are shown in Figs. 5.6 to 5.9. Figs. 5.6 compares the ABER analytic and simulation results to verify equation (5.57) assuming $\zeta = 1$. It can be seen from the figure a high match exists between the analytic and simulation results. It also shows that the ABER performance is enhanced compared to Fig. 5.2. This proves that a considerable performance can be achieved by an optimal balance between *EE* and *SE*. Similar result depicts in Fig. 5.7 to demonstrate expression (5.65).

Further ABER enhancement is illustrated in Figs. 5.8 and 5.9 when $\zeta = 2$. It can be seen from these figures that the asymptotic analytical results approach the simulation results at high *SNR*. Both figures prove that the ABER performance is enhanced when optimal *SNR* defined in (5.57) and (5.65) is adopted.

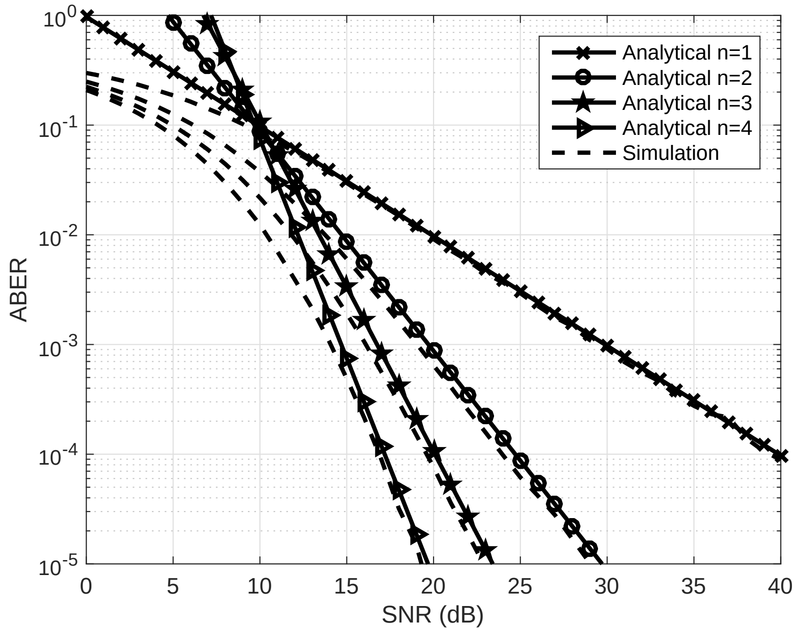

Figure 5.3: ABER performance in (5.43), assuming $\zeta = 1$

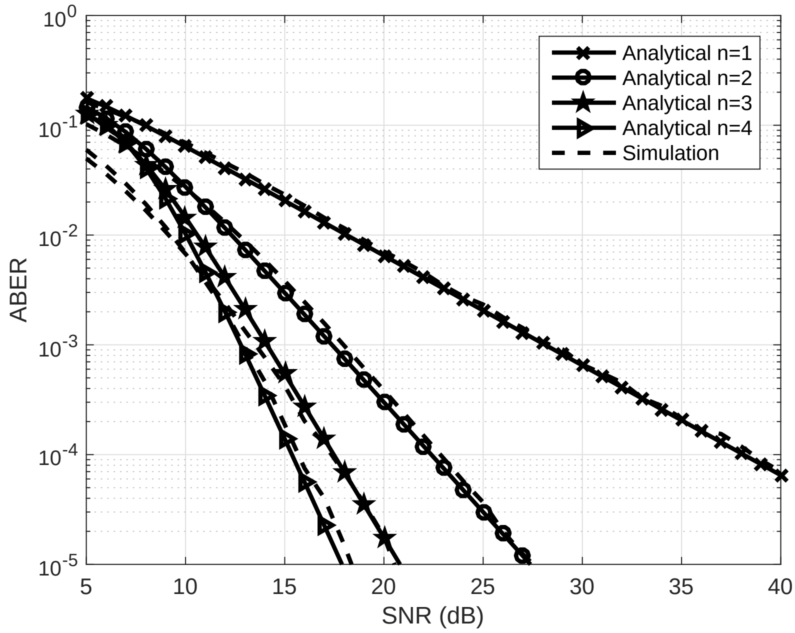

Figure 5.4: ABER performance in (5.37) assuming $\zeta = 2$

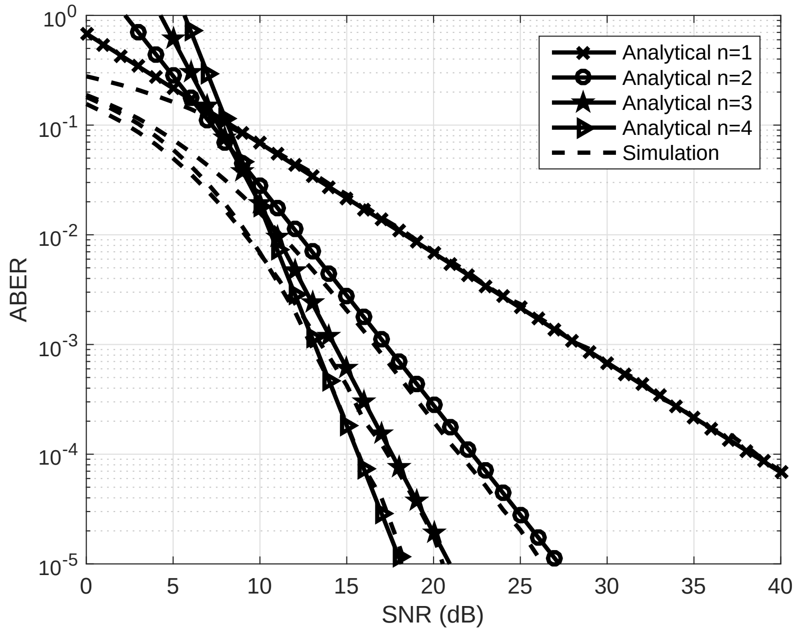

Figure 5.5: ABER performance in (5.43), assuming $\zeta = 2$

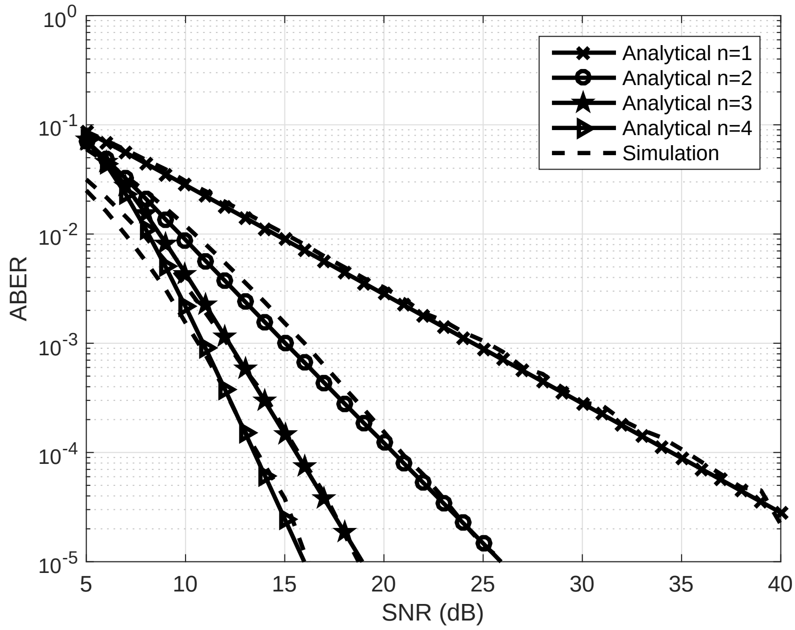

Figure 5.6: ABER performance assuming $\zeta = 1$ and network is an optimal

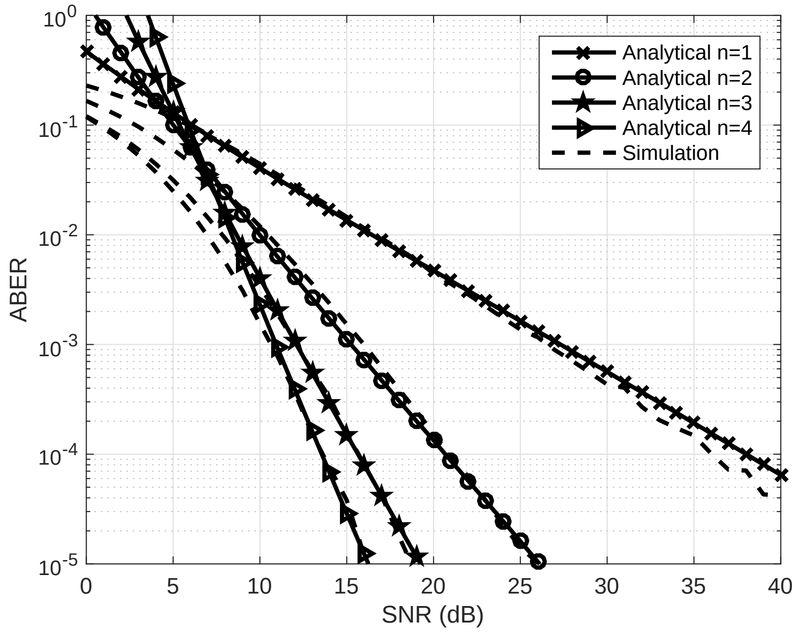

Figure 5.7: ABER performance of an optimal network, assuming $\zeta = 1$

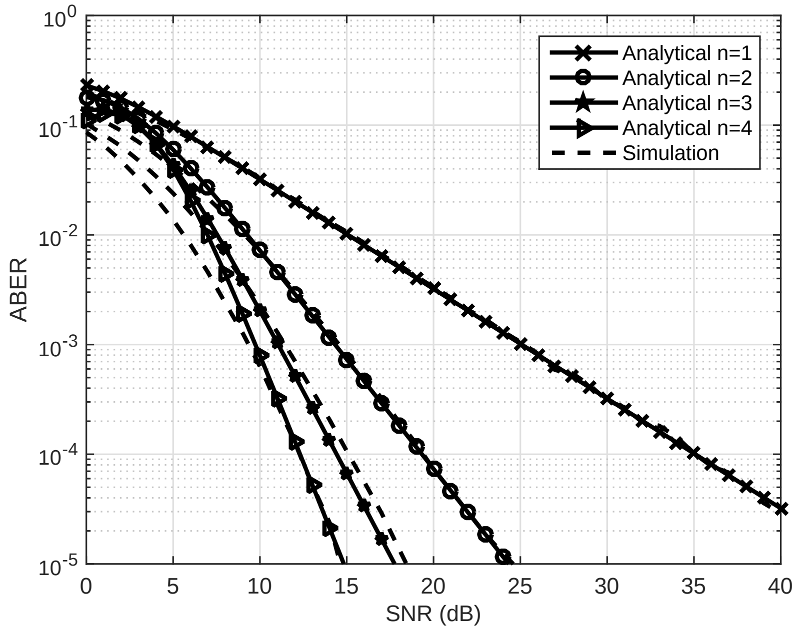

Figure 5.8: ABER performance assuming $\zeta = 2$ and network is an optimal

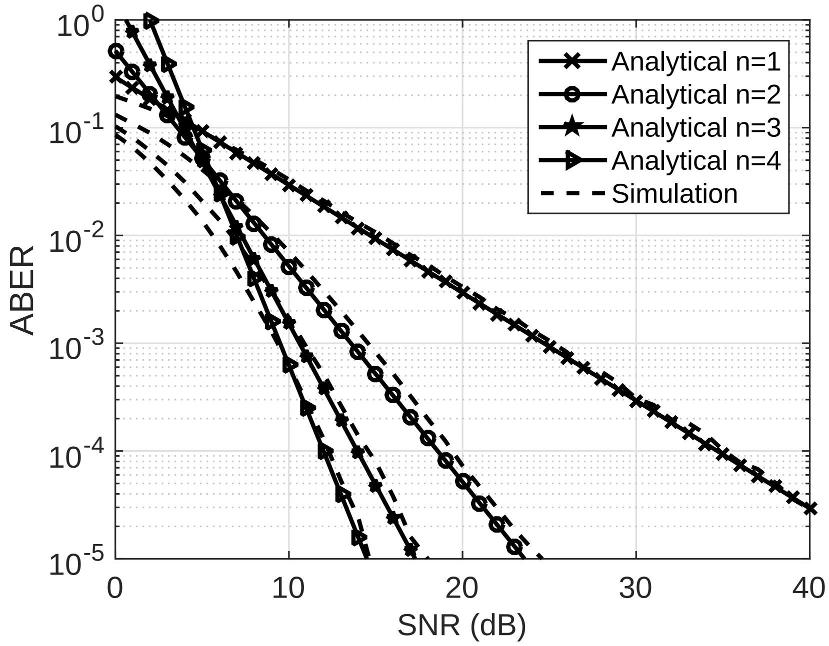

Figure 5.9: ABER performance of an optimal network, assuming $\zeta = 2$

5.7 SUMMARY

A new method has been presented in this chapter for analysing the asymptotic BER performance of a BAF-MR network that uses flat fading channels and a relay selection scheme. Analytical expressions calculating the ABER under high and optimal *SNR* domains have been presented. These analyses use both AGM and $\mathscr{HM}$. The results were verified by simulation, and it was found that both analyses gave the same ABER result at high and optimal *SNR* domains. However, in the low *SNR* domain, AGM gives a better ABER result than $\mathscr{HM}$, particularly with high ζ. The effectiveness of the method was demonstrated by estimating the asymptotic BER of a BAF-MR network under two different *SNR* domains, namely, high and optimal, a result that has not previously been presented. It was found that the optimal *SNR* obtained from a balanced network has a lower asymptotic BER than suboptimal networks.

In the next chapter, the accuracy of the above asymptotic BER is increased, and the work is extended to analyze the exact BER performance.

Chapter 6

Precise Error Rate Analysis of BAF-MR Networks

6.1 Introduction

In Chapter 5, we demonstrated a result from evaluating the Asymptotic BER (ABER) performance under a high *SNR* level. To increase the accuracy of the ABER of BAF-MR networks for a wide range of applications, this chapter investigates a new method of increasing the accuracy of ABER evaluation under low and high *SNR* levels. Further, the ABER is analyzed for two different geographic environments, as the channel of each hop in BAF-MR networks is located in a different environment. Moreover, the ABER is used to calculate the Exact Bit Error Rate (EBER), which is more complex than the ABER. Thus, the BER of MAF-RS networks can be evaluated under low and high *SNR* levels using either ABER or EBER.

Bit error rate and related performance analyses of relay networks have been investigated by many researchers. In an early study, [123] employed a high *SNR* approximation to express the ABER of wireless transmission over flat fading channels under severe fading conditions. The study, however, was limited to only the ABER. The approximated *SNR* using the harmonic mean was investigated by authors [48], who analyzed the BER of AF-relay networks and then compared it to that of DF-relay networks. They found that DF-relay networks outperform AF-relays at low *SNR*, while at high *SNR* the AF and DF have similar BER performances. Similar results have been obtained by [144]. However, [47, 123, 144] studies did not consider a bidirectional AF relay scheme.

For BAF-MR network, study [145] adopted a high *SNR* region for analyzing different BER conditions with a relay selection scheme, including the best relay selection, best worst-channel selection and maximum-harmonic-mean selection. The results showed that the BER improves in the high *SNR* region. This study, however, used symmetrical transmission channel coefficients, while the relay network channels are commonly passing through different geographic environments.

Authors in [125, 127] investigated the asymptotic Symbol Error Rate (SER) performance of a BAF-MR network by considering a high *SNR* region and the moment generating function (MGF) approach to make their calculations easier [122]. In fact, the SER requires further analysis to determine the BER, which is a more useful metric than SER [25]. Thus, studies [87, 125, 126, 128] analyzed asymptotic BER in a high *SNR* region. However, all these studies show a big gap between the ABER and EBER at both low and middle *SNR* regions. Accordingly, the ABER of these studies are limited for the highest *SNR* domain applications.

In [129], the Block Error Rate (BLER) performance was derived based on the Highest Worse Signal-to-Noise Ratio (HW-SNR) of the BAF-MR networks. The derivation adopted the approximated cumulative distribution function (CDF) of the HW-SNR to obtain the BLER performance, in order to simplify the analysis. However, [146] observed that evaluation error rate using BLER may not led to proper error rate evaluation because, the channel variability and bursts of errors may occur only in certain transport blocks.

For the BAF-MR with co-channel interference, [130] used an approximated CDF to evaluate the ABER performance. This study assumed high source and relay transmitting powers to simplify the analysis. Similarly [136], calculated the ABER expression for high SNRs using the probability density function (PDF) of high *SNR*. Both the ABER schemes in [130, 136], presented a substantial gap between the ABER and EBER at the low *SNR* region. The BER performances of general relay selection schemes for log-normal shadowing and generalized-K fading channels were investigated by [132], who showed that assuming a high *SNR* is crucial to simplifying the analysis of the channels.

The above studies presented various techniques for evaluating asymptotic and exact error rate performance of BAF-MR networks. Given the complexity of the analysis of exact error rate performance, most of these studies considered approximated *SNR* to simplify the analysis of asymptotic error rate performance. However, this comes at the cost of not being able to evaluate the BER at a low *SNR* level, when in fact several wireless applications operate at such a level [137, 147]. Thus, the presented asymptotic error rate analyses, as they currently stand, are not usable for such applications. Therefore, we propose a new method to enhance the ABER analysis, specifically designed to address the ABER analysis at the low *SNR* region. The proposed method enables calculation of the ABER performance of two-hop BAF-MR networks using the conventional BER analysis of one-hop communication. In other words, a two-hop BAF-MR network is equivalent to a one-hop link. So, the one-hop *SNR*'s can roughly unify the ABER analysis at high and low *SNR* values. Furthermore, using the equivalent one-hop link helps to reduce the assumptions made in the BER analysis compared with the above studies. Accordingly, the proposed method provides an accurate ABER result, especially at the low *SNR* region, as is confirmed by the simulation results. Further, the exact BER performance can be directly obtained from the ABER, and this provides a standard solution for the BER analysis of bidirectional two-hop relay networks. Last but not least, the ABER and BER are expressed for two different geographic environments to overcome the shortcomings of previous studies.

The rest of the chapter is organized as follows: Section 6.2 outlines the system model; Section 6.3 describes the expressions used to calculate the *SNR*; Section 6.4 briefly reviews the BER for direct link performance and discusses the proposed method for analyzing the BAF-MR network performance assuming one relay in the network; Section 6.5 analyzes ABER performance in multi-relay systems using relay selection strategy; Section 6.6 presents the EBER performance; Section 6.7 shows the simulation results. Finally, conclusions and remarks are discussed in Section 6.8.

6.2 System Model

This chapter considers two wireless users: S_a and S_b who are communicating with each other through multiple AF-relay nodes as shown in the block diagram in Fig. 5.1. All nodes (i.e, users and relays) are equipped with a single antenna. The transmitted signal for each user is modulated using binary phase shift keying (BPSK) scheme.

It is also assumed that each user has a selection scheme which includes a comparator and a fast switch [90]. In a selection scheme, the received signals are continuously monitored so that the best signal can be chosen based on a particular criterion. In theory, the highest *SNR* criterion is commonly adopted, but this is difficult to obtain in practice. Thus, the strongest signal with its noise is selected. Using the selection scheme at the receiver, the signals must be monitored at a rate faster than that of the fading occurrence, and full branches information should be available at the destination [148]. Fig. 6.1 shows the principles of selection schemes employed in BAF-MR networks.

We also assume that S_a and S_b simultaneously broadcast their signal over Rayleigh fading channels in the first time-slot T_{s1}. Thus, the signal received by any relay can be obtained as in (5.1), and the amplification factor for the i^{th} relay is obtained as in (5.2). Each relay amplifies transmitted signals by β_i factor. The amplified signals are then forwarded to their target users in the second time slot T_{S2} as shown in Fig. 6.1. It is assumed here that the channel conditions are known at the receiving ends. Each user in the network receives signals from all relay branches but the signal with the highest *SNR* is chosen using a selection scheme, which includes a comparator and a fast switch [90]. Thus, the signal received by users S_a and S_b can be expressed as (5.3) and (5.4), respectively.

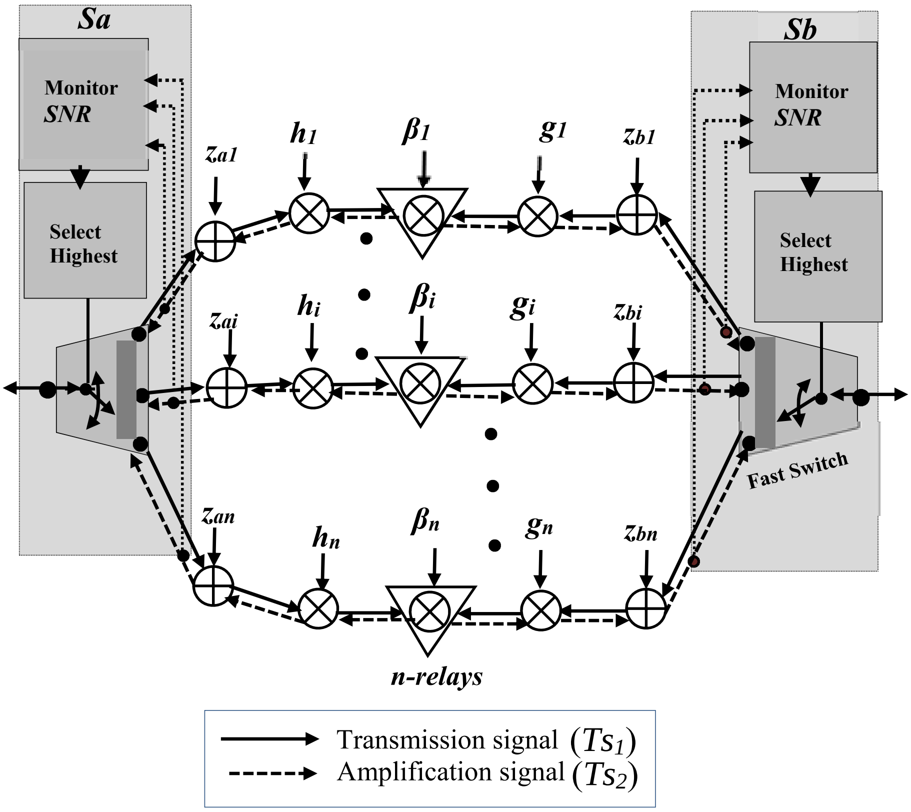

Figure 6.1: System Model

6.3 Signal-to-Noise Ratio

In this section, exact and approximated *SNRs* for multi AF-relay network are formulated. The exact *SNR* is formulated as in (5.5), and this can be expressed as

$$y_{ai} = \left(\sqrt{p_a}x_a h_i + \sqrt{p_b}x_b g_i + z_{ri}\right) h_i \beta_i + z_{ai}, \tag{6.1}$$

The S_a node removes its own transmitted signal via self-interference cancellation [55]. By substituting (5.2) into (6.1) and assuming both $E\{x_a\}$ and $E\{x_b\}$ equal one, we can obtain the power signal as follows

$$E\left\{y_{ai}^2\right\} = \frac{|h_i|^2 p_{ri}}{\sigma^2}\frac{|g_i|^2 p_b}{\sigma^2} + \frac{(p_a + p_{ri})}{\sigma^2}|h_i|^2 + \frac{p_b}{\sigma^2}|g_i|^2 + 1. \tag{6.2}$$

where $E\left\{y_{ai}^2\right\}$ is the signal power for S_a obtained by removing its own transmitted signal from the return signal coming from the relay using self-interference cancellation [55] .

By letting $\Bbbk = \frac{|h_i|^2 p_{ri}}{\sigma^2}\frac{|g_i|^2 p_b}{\sigma^2} + \frac{(p_a+p_{ri})}{\sigma^2}|h_i|^2 + \frac{p_b}{\sigma^2}|g_i|^2$, equation (6.2) can be rewritten as

$$E\left\{y_{ai}^2\right\} = \Bbbk + 1. \tag{6.3}$$

Determining the exact end-to-end *SNR* from (6.3) results in (5.6). Similarly, the exact *SNR* for S_b is defined as (5.9).

By assuming that $\Bbbk \gg 1$, we obtain the following expression

$$\text{ɣ}_{ai} = \frac{p_{ri} p_b \gamma_{hi} \gamma_{gi}}{p_{ri}\gamma_{hi} + p_a \gamma_{hi} + p_b \gamma_{gi}}, \tag{6.4}$$

where ɣ_{ai} is the *SNR* approximation for S_a. Similarly, *SNR* of S_b can be expressed as

$$\text{ɣ}_{bi} = \frac{p_{ri} p_a \gamma_{hi} \gamma_{gi}}{p_{ri}\gamma_{hi} + p_a \gamma_{gi} + p_b \gamma_{gi}} \tag{6.5}$$

ɣ_{bi} is the *SNR* approximation for S_b.

Equation (6.4) and (6.5) can be used to analyze asymptotic error rate performance at high *SNR* region as [35, 87, 131].

Based on the above *SNR* approximations, asymptotic and actual BER analysis are derived by considering two scenarios: the *first scenario* considers similar CNRs as $\gamma_{hi} = \gamma_{gi}$, while the *second scenario* assumes different environments i.e. $\gamma_{hi} \neq \gamma_{gi}$. It is worth mentioning, that the channels in each hop of a relay network are typically located in different environments.

6.4 ABER Performance For One Relay Network

In this section, the ABER of the BAF-MR network is calculated by assuming that the network has one relay (i.e, $i = n = 1$) . The ABER calculation is based on the one-hop (direct link) BER analysis which assumes BPSK modulation and a flat channel. This analysis was presented by [13] as

$$P_b(\gamma) = \int_0^\infty Q(\sqrt{2\gamma})\,\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}\,d\gamma, \tag{6.6}$$

where $P_b(\gamma)$ is the rate of error probability, $\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}$ is the probability density function of γ, and $Q(\sqrt{2\gamma})$ is the complement of the cumulative distribution function (CCDF) related to the Gaussian random variable which is defined as $Q(\sqrt{2\gamma}) = 0.5\,\mathrm{erfc}\sqrt{\gamma}$ [13]. By evaluating the integral of (6.6), BER performance for a direct link is $P_b(\gamma) = 0.5\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$, where $\bar{\gamma}$ is the average*SNR*defined as

$$\bar{\gamma} = E\{\gamma\} = p\sigma^{-2}E\left\{|h|^2\right\} = Eb/No, \tag{6.7}$$

where $E\left\{|h|^2\right\}$ is the expected value of the fading channel coefficient which is equal to one, p is the transmitter power and Eb/No is the energy per bit to the spectral noise density ratio.

By following a similar approach for calculating BER of a direct link, ABER performance of a BAF-MR network can be analyzed as described below:

Let equation (6.6) defined as a function of γ as

$$P_a(ɣ_{ai(\gamma)}) = \frac{1}{\bar{ɣ}_{ai(\bar{\gamma})}} \int_0^\infty Q\left(\sqrt{2ɣ_{ai(\gamma)}}\right) e^{-\frac{ɣ_{ai(\gamma)}}{\bar{ɣ}_{ai(\bar{\gamma})}}}\,d\gamma, \tag{6.8}$$

where $P_a(ɣ_{ai(\gamma)})$ is the probability of error, $\bar{ɣ}_{ai(\bar{\gamma})}$ is the average *SNR*, $Q(\sqrt{2ɣ_{ai(\gamma)}})$ is the CCDF and $\frac{e^{-\frac{ɣ_{ai(\gamma)}}{\bar{ɣ}_{ai(\bar{\gamma})}}}}{\bar{ɣ}_{ai(\bar{\gamma})}}$ is the total pdf. All these variables are expressed with respect to γ.

Calculating $\bar{ɣ}_{ai(\bar{\gamma})}$ requires obtaining the average value of (6.4), and this is achieved by evaluating the average value of γ_{hi} and γ_{gi} as [25], accordingly, the average values of both γ_{hi} and γ_{gi} are expressed as

$$\bar{\gamma}_{hi} = E\left\{|h_i|^2\right\}/\sigma^2, \tag{6.9}$$

$$\bar{\gamma}_{gi} = E\left\{|g_i|^2\right\}/\sigma^2. \tag{6.10}$$

Both $|h_i|^2$ and $|g_i|^2$ are independent, identically distributed (i.i.d) random variables and they have the same expectation value equal to one [70,71].

Now, the average value of (6.4) can be calculated by assuming that the distribution powers among selected relay and users are subjected to the first scenario mentioned in Section 6.3, namely:

$$p_a = p_b = p_{ri}, \tag{6.11}$$

Thus, the value of $\bar{\gamma}_{ai(\bar{\gamma})}$ is obtained by substituting (6.9), (6.10) and (6.11) into (6.4). This results in

$$\bar{\gamma}_{ai(\bar{\gamma})} = 2\bar{\gamma}/3, \tag{6.12}$$

The other term in (6.8) represents the total probability density function (pdf) for double-cascaded Rayleigh channels. It can be obtained from (6.4) and (6.12), as $|h_i|$ and $|gi|$ are i.i.d random variables and their *SNRs* follow an exponential distribution [75]. Such independent variables can be expressed as shown in [76], to give the pdf as

$$\Im(\gamma_{ai(\gamma)}) = \frac{e^{-\frac{\gamma_{ai(\gamma)}}{\bar{\gamma}_{ai(\bar{\gamma})}}}}{\bar{\gamma}_{ai(\bar{\gamma})}} = (2\bar{\gamma}/3)^{-1} e^{-\frac{\gamma}{\bar{\gamma}}} \tag{6.13}$$

where $\Im(\bar{\gamma}_{a1}(\bar{\gamma}))$ is the total pdf value.

Integration of (6.13) relative to $\gamma_{ai(\gamma)}$ by following similar way in [91], results in the following cumulative distribution function ($\mathscr{F}_{ai}(\gamma_{ai(\gamma)})$) as

$$\mathscr{F}_{ai}(\gamma_{ai(\gamma)}) = 1 - e^{-\frac{\gamma_{ai(\gamma)}}{\bar{\gamma}_{ai(\bar{\gamma})}}}, \tag{6.14}$$

By using both (6.14) and the relationship between the Q-function with the error function (erfc) as demonstrated in [92], the term of $Q(\sqrt{2\gamma_{ai(\gamma)}})$ in (6.8) becomes

$$Q(\sqrt{2\gamma_{ai(\gamma)}}) = 2(3)^{-1}\text{erfc}\,(\sqrt{\frac{2\gamma}{3}}), \tag{6.15}$$

Substituting (6.13) and (6.15) into (6.8) results in

$$P_a(\gamma_{ai(\gamma)}) = \frac{1}{\bar{\gamma}} \int_0^{\infty} erfc\,(\sqrt{2\gamma/3})\, e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma, \tag{6.16}$$

Evaluating (6.16), the ABER performance of BAF-MR network is obtained as follows

$$P_a(\gamma_{ai(\gamma)}) = 1 - \sqrt{1 - (1 + \frac{2\bar{\gamma}}{3})^{-1}} \tag{6.17}$$

Equation (6.17) represents ABER solution for the first scenario (i.e. $\gamma_{hi}{=}\gamma_{gi}$). To derive ABER performance for a two-hop BAF-MR network under the second scenario, we assume that the first hop channel (i.e., γ_{hi}) is located in a different environment fromγ_{gi}. Here, the disparity between both channels can be explained as

$$\gamma_{hi} = \mathbb{C}\gamma_{gi} \tag{6.18}$$

where $\mathbb{C}$ is a constant number ($\mathbb{C} \in \mathbb{R}$).

Substituting (6.18) into (6.4), and following the same procedures used for obtaining (6.12) results in

$$\ddot{\bar{\mathfrak{y}}}_{ai(\gamma)} = \frac{2\mathbb{C}}{\mathbb{C}+2}\bar{\gamma}, \tag{6.19}$$

where $\ddot{\bar{\mathfrak{y}}}_{ai(\gamma)}$ is the average *SNR* value under the second scenario.

The total probability density function for the second scenario is determined as in (6.13). This results in

$$\ddot{\mathfrak{I}}(\ddot{\mathfrak{y}}_{ai(\gamma)}) = \frac{(\mathbb{C}+2)}{2\mathbb{C}\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}, \tag{6.20}$$

where $\ddot{\mathfrak{I}}(\ddot{\mathfrak{y}}_{ai(\gamma)})$ is the system's pdf in respect to $\bar{\gamma}$.

Also, the Q-function under the second scenario is obtained by following (6.15) and this results in

$$Q(\sqrt{2\ddot{\mathfrak{y}}_{ai(\gamma)}}) = \frac{2\mathbb{C}}{\mathbb{C}+2}\,\mathrm{erfc}\left(\sqrt{\frac{2\mathbb{C}}{\mathbb{C}+2}\gamma}\right). \tag{6.21}$$

By substituting (6.20) and (6.21) into (6.8) and then evaluating the integration, ABER performance of one relay BAF-MR network under the second scenario is calculated as

$$P_a(\ddot{\mathfrak{y}}_{ai(\gamma)}) = 1 - \sqrt{1-(1+\frac{2\mathbb{C}}{\mathbb{C}+2}\bar{\gamma})^{-1}} \tag{6.22}$$

6.5 ABER Performance for Multiple Relays

The previous section presented ABER performance of the BAF-MR network for one relay node. In this section, the general form for *n* relays is derived.

The BAF-MR network with a selection strategy allows the selection of the highest *SNR* output among all relays branches and this can significantly improve system performance. Implementation of such selection at the receiver requires knowledge of order statistics. Here, we briefly present the basic order statistical tool, which is used for our analysis.

SNR output can be ordered statistically as $\gamma_{a(i)} = max\{\gamma_{a1}, \gamma_{a2}, ..., \gamma_{an}\}$, where $\gamma_{a(i)}$ is the i_{th} order statistic of *SNR*, γ_{an} represents the largest order statistic and γ_{a1} is the smallest [94, 128]. The *SNRs* in a statistical order are based on CDF as follows: let the CDF of γ_{an} be denoted as $\mathscr{F}_{(i)}(\gamma_{an})$, and then apply the order statistical analysis as presented by [95]. This results in $\mathscr{F}_{(i)}(\gamma_{an}) = Prob\left(\gamma_{a(i)} \leq \gamma_{an}\right) = \mathscr{F}_{(1)}(\gamma_{a1})\mathscr{F}_{(2)}(\gamma_{a2})...\mathscr{F}_{(n)}(\gamma_{an}) = \left[\mathscr{F}(\gamma_{a(i)})\right]^n$. Thus for n relays, the order statistics of γ_{an} based on CDF is expressed as

$$\mathscr{F}_{(i)}(\gamma_{an}) = \left(\mathscr{F}_{a(n)}(\gamma_n)\right)^n, \tag{6.23}$$

where n is the relay number.

In case of i.i.d channels, equation (6.23) can be written as $\mathscr{F}_{a(i)}(\gamma_n) = n\left(\mathscr{F}_{a(n)}(\gamma_n)\right)$ [66]. By substituting (6.15) into (6.23), the total CCDF at S_a is calculated as

$$\mathscr{F}_{a(i)}(\gamma_n) = \frac{2n}{3}\operatorname{erfc}\left(\sqrt{\frac{2\gamma}{3}}\right) \tag{6.24}$$

By taking the derivative of (6.23) relative to γ_n using similar approach in [120], the total pdf of (6.24) is obtained as

$$\Im_{a(i)}(\gamma_n) = \frac{n}{\bar{\gamma}_{ai(\bar{\gamma})}} e^{-\frac{\gamma_{ai(\gamma)}}{\bar{\gamma}_{ai(\bar{\gamma})}}} \left(\mathscr{F}_{(n)}(\gamma_n)\right)^{n-1} \tag{6.25}$$

where $\Im_{a(i)}(\gamma_n)$ is the total pdf signals at S_a

Further analysis for (6.25) results in

$$\Im_{a(i)}(\gamma_n) = \frac{n}{\bar{\gamma}_{ai(\bar{\gamma})}} \left(e^{-\frac{\gamma_{ai(\gamma)}}{\bar{\gamma}_{ai(\bar{\gamma})}}}\right)^n \tag{6.26}$$

Substituting (6.24) and (6.26) into (6.8) gives

$$P_a(\gamma_{ai(\gamma)}) = \frac{2n^2}{3\,\bar{\gamma}_{ai(\bar{\gamma})}} \int_0^\infty \operatorname{erfc}\left(\sqrt{\frac{2\gamma}{3}}\right)\left(e^{-\frac{\gamma_{ai(\gamma)}}{\bar{\gamma}_{ai(\bar{\gamma})}}}\right)^n d\gamma. \tag{6.27}$$

Evaluating (6.27) in respect to γ, the general form of ABER performance of the BAF-MR network is obtained as follows

$$P_a(\gamma_{ai(\gamma)}) = n\left(1 - \sqrt{\left(\frac{2\bar{\gamma}.}{3n + 2\bar{\gamma}.}\right)}\right)^n, \tag{6.28}$$

where n is the total number of relays in the network.

To express ABER in the second scenario (i.e., different CNRs), we can use the same procedures of equations (6.19) to (6.22) considering n relays. This results in

$$\ddot{P}_a(\gamma_{ai(\gamma)}) = n\left(1 - \sqrt{\frac{\bar{\gamma}}{n(1+1/2\mathbb{C})+\bar{\gamma}}}\right)^n, \tag{6.29}$$

where $\ddot{P}_a(\gamma_{ai(\gamma)})$ is the ABER performance under the second scenario.

6.6 Analyzing EBER Performance

In this section, the EBER performance of a BAF-MR network is expressed by using the same analysis of ABER discussed in the previous section. However, the *SNR* for the EBER requires different analysis. Here, we can reanalyze term $\mathbb{k}$ in (6.3), which involves a set of independent random variables, by using the linearity of expectation property in [93]. Accordingly, the average value of actual *SNR* is calculated as

$$\bar{\gamma}_{ai(\bar{\gamma})} = 2n^{-1}\left(\frac{n}{2}+\frac{\bar{\gamma}}{3}\right), \tag{6.30}$$

where $\bar{\gamma}_{ai(\bar{\gamma})}$ is the total average value of actual *SNR* as a function of $\bar{\gamma}$.

Following the same analysis of (6.28) as well as considering (6.30), the EBER performance for the first scenario is obtained as follows

$$P_a(\gamma_{ai}) = n\left(1 - \sqrt{\frac{\left(\frac{\bar{\gamma}3^{-1}}{n}\right)+\frac{1}{2}}{\left(\frac{\bar{\gamma}3^{-1}}{n}\right)+1}}\right)^n. \tag{6.31}$$

For the second scenario, we follow the steps of (6.29) to calculate the EBER performance of the BAF-MR network as

$$\ddot{P}_a(\gamma_{ai}) = n\left(1 - \sqrt{1-(2+2\mathbb{C}\frac{\bar{\gamma}}{n(2\mathbb{C}+1)})^{-1}}\right)^n. \tag{6.32}$$

where $\ddot{P}_a(\gamma_{ai})$ is the EBER performance under the second scenario.

6.7 Simulation Results

Evaluating asymptotic and exact BER performance using the proposed method are presented in this section. The system described in Section 6.2 is adopted, but with limiting relay numbers to four (i.e, n=4). All analytic results are compared to the baseline system (i.e., the simulation) in order to validate the result.

Fig. 6.2 shows ABER performance in Eq. (6.28). It is clear from this figure that the ABER performance improves as the number of relays increases in the network. Further, our results align with the simulation results especially in the high *SNR* region. In the low *SNR* region, our results are slightly different to the simulation results, but still far better than other systems operating in this region, such as [87, 125, 126, 128, 131]. Overall, our ABER expression gives more accurate results particularly in the low *SNR* region. Fig. 6.3, illustrates the proposed EBER results from (6.31) approaching the simulation results. A comparison between figures 6.2 and 6.3 at the low *SNR* region shows a slight difference between ABER and EBER results.

Figs. 6.2 and 6.3 represent the first scenario, which assumes equal CNRs channels. Fig. 6.4 shows results of ABER expression in (6.29), when $\mathbb{C}$=2. It is clearly seen in this figure that the ABER performance improves as the *SNR* increases. Further, the figure shows a high degree of accuracy between the simulation and the proposed analysis in the high *SNR* region with small differences in the low *SNR* region.

Fig. 6.5 also illustrates the second scenario, but this time using the EBER expression in (6.32). Still, our results match the simulation baseline for the entire range of *SNR*s. The BER performance has reduced upon decreasing the value of $\mathbb{C}$ to 3/4, and this is confirmed in Figs. 6.6 and 6.7 which depict the results of asymptotic and exact BER performance respectively. Furthermore, Fig. 6.6 shows a significant match between the simulated and the proposed analysis at the high *SNR* region. However, the differences in the low *SNR* region are slightly worse than in the above ABER results, as the high value of $\mathbb{C}$ reduces the total *SNR* value.

6.8 SUMMARY

New asymptotic and exact BER expressions for two-hop BAF-MR network are presented in this chapter. Each hop includes a flat fading channel and a relay selection scheme at each destination terminal. The

ABER and EBER expressions are derived by assuming that the channel of each hop is located in a particular environment.

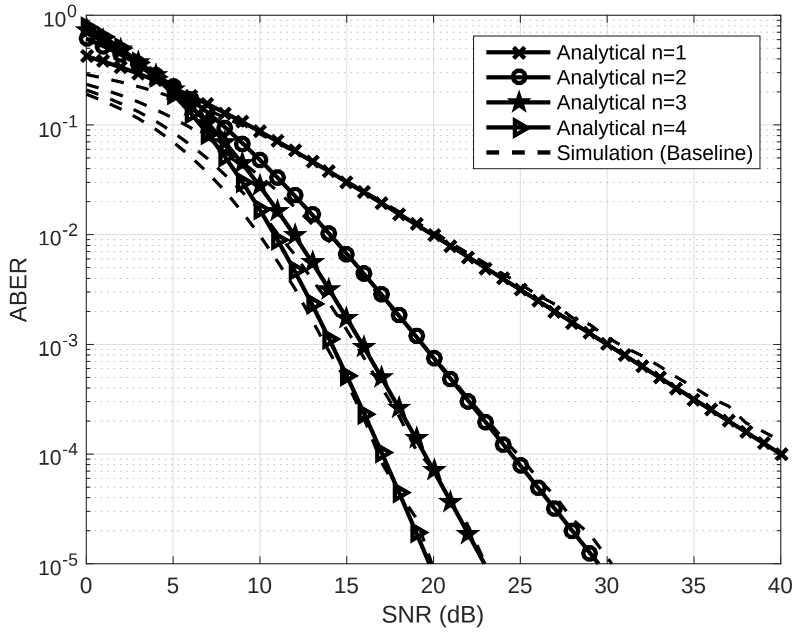

Figure 6.2: Asymptotic-BER performance

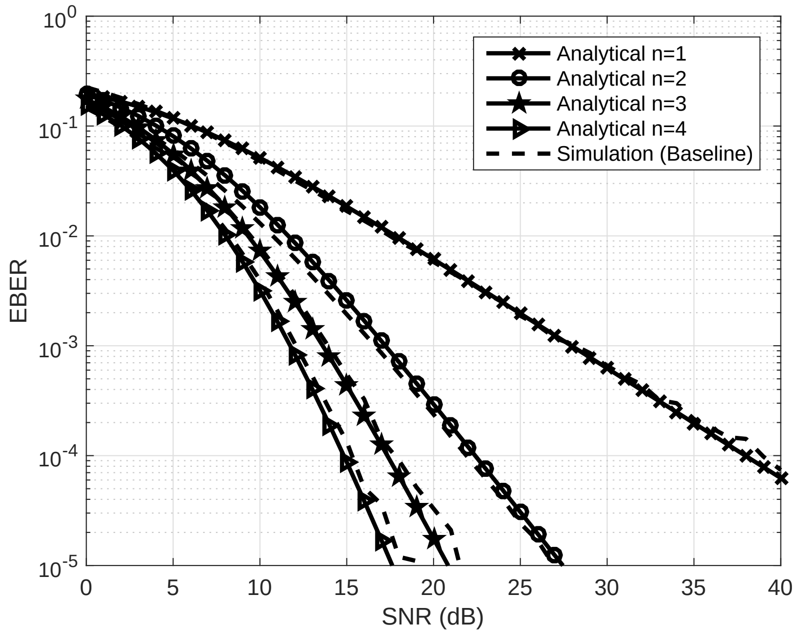

Figure 6.5: Exact-BER Performance at $\mathbb{C}$=2

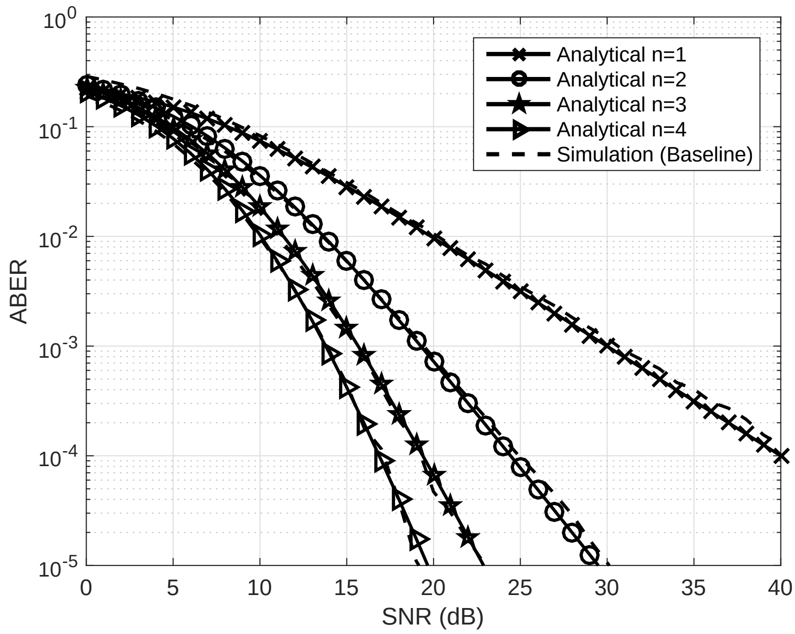

Figure 6.3: Exact-BER performance

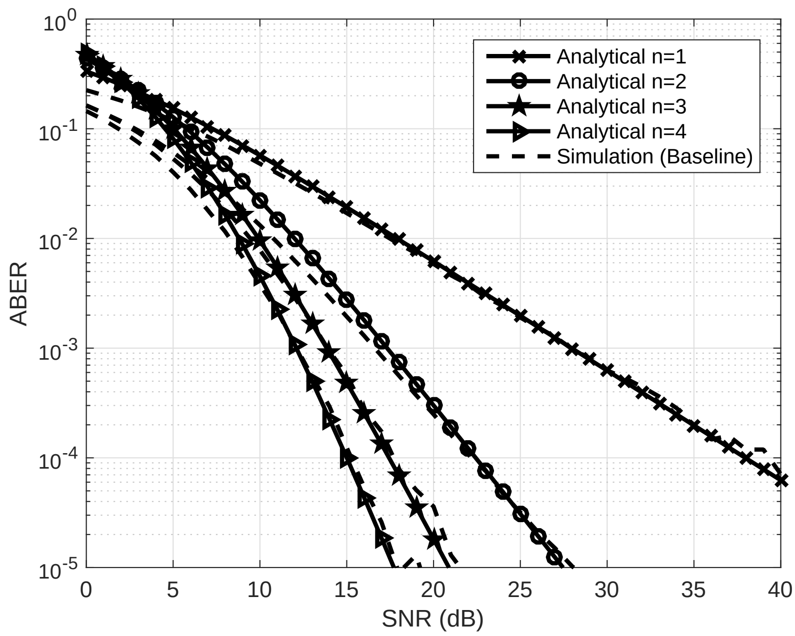

Figure 6.4: Asymptotic-BER performance $\mathbb{C}$=2

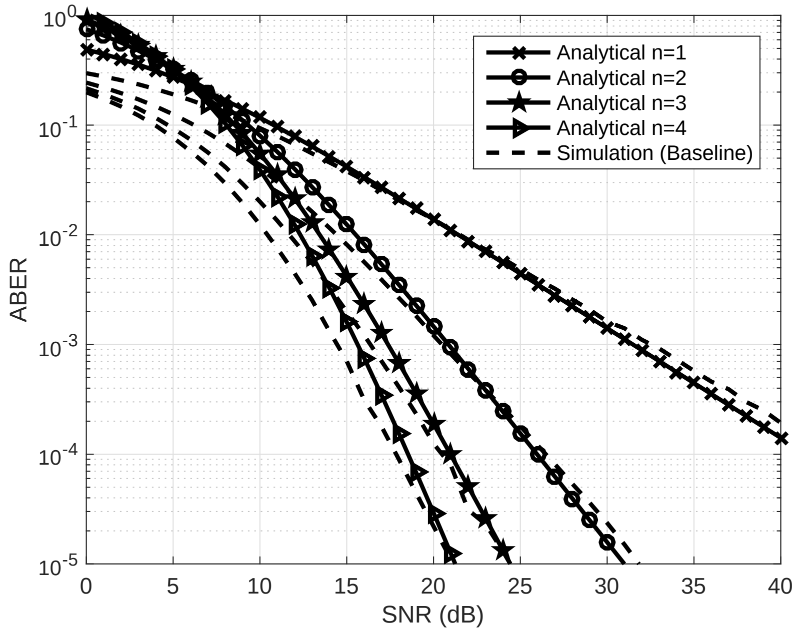

Figure 6.6: Asymptotic-BER Performance at ℂ=3/4

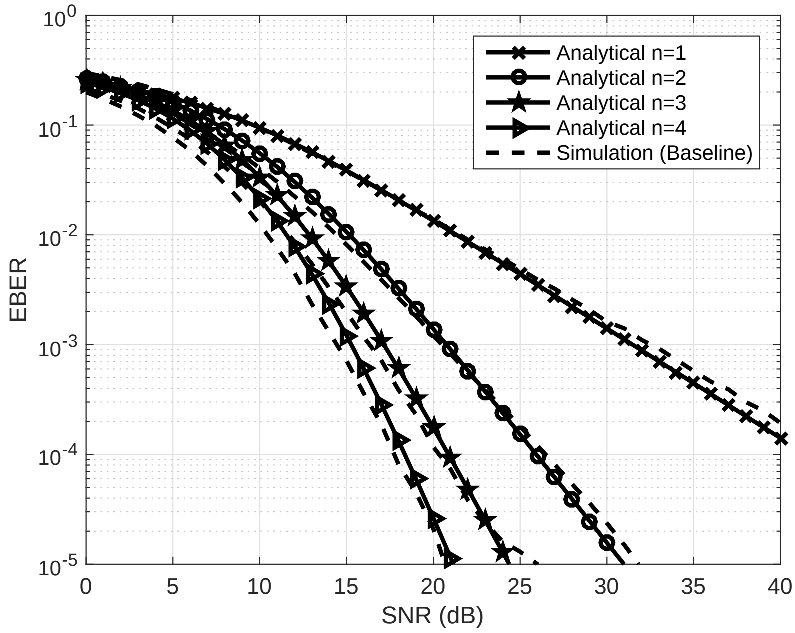

Figure 6.7: Exact-BER Performance at ℂ=3/4

When both hops are located in the same environment, the fading channels of the hops are i.i.d, while different environments make the fading channels of the hops different. This enables the evaluation of the BER of BAF-MR networks under different environments. The proposed method also shows that the ABER analysis of a two-hop BAF-MR network can be reduced to a conventional BER analysis of a one-hop communication network. Further, the ABER analysis can be used directly to determine the EBER performance.

The analytical results have been found to align with the simulation results and to show that the difference between the ABER and EBER is marginal. This makes it possible to use either of them to obtain precise BER results for BAF-MR networks. Accordingly, the proposed ABER provides a more accurate result than the existing ABER techniques.

The above BER method is investigated, considering also the relay location, in the next chapter.

Chapter 7

Effect of the *EE* and *SE* on the BER of BAF-MR Networks

7.1 Introduction

In the previous chapters, we addressed the BER analysis of BAF-MR networks under different *SNR* levels. This chapter presents a new method of increasing the *EE* and decreasing the BER. In this method, the *EE* and *SE* is derived in a balance scheme, and the relay location is considered. Such a balance between *EE* and *SE* is achieved by minimizing power consumption.

Enhancing the *EE* for relay networks was proposed by [108]. That study found that relay networks achieve better *EE* than direct transmission when there is unlimited relay-located power, and that further *EE* enhancement is obtained with power allocation. But this is achieved when the *SE* is low or the relay location is close to the destination. The relay location is also considered in [107], which observed that balancing the *EE* and *SE* is possible when a relay selection scheme is adopted. However, those studies [107, 108], did not consider circuit energy (CE) consumption, which includes electronic devices and signal processing. This limitation is overcome in [109] and [110], in which the CE consumption is analyzed in conjunction with other network consumptions for a given capacity in multiple-hop networks over Rayleigh fading channels.

For a bidirectional AF relay network, which was proposed by [149], [150] investigated optimizing the *EE* when both the *SE* and the transmission power are constrained, while considering the effect of self-interference cancellation on the *EE*. The study found that bidirectional AF-relay networks can achieve higher *SE* than unidirectional relay networks, but at the cost of a lower optimal *EE*. A similar result was obtained by [39], when the CE consumption was also considered. The study demonstrated that optimizing both *EE* and *SE*, when also considering CE, is a more complicated issue in practical relay networks. However, the low *EE* presented by [39] and [150] appeared because their analyses were limited to an increase in *SE* at the cost of *EE*.

Other studies [103, 111–114] investigated combining the *EE* and *SE* to decrease the network energy consumption while ensuring a desirable *SE*. Reference [111] investigated a transmission scheme for reducing the power consumed per transmitted bit for both unidirectional and bidirectional AF relay networks by combining power allocation and relay selection. Similar results were achieved by [112] and [113] when CE was considered in the analysis. However, [113] considered an OFDM system that combined the *EE* and *SE*. In some studies, such as [103, 114], the average channel status was adopted in calculating the average *EE* and *SE*, but such studies did not consider relay location.

Regarding the analysis of BER performance for bidirectional AF relay networks, many studies, such as [131, 132, 136], proposed different methods of calculating the error rate performance, but most were focused on the BER calculation of sub-optimal networks. Thus, the analyses are unsuitable for calculating the BER of optimal networks. It is worth mentioning here that the term "optimal" refers to optimized sources and relay powers.

From the above literature, it was observed that studies that increased the *EE* ignored the effect of increasing *EE* on the BER. Similarly, the studies that analyzed the BER performance did not consider how the BER is calculated if one of the network parameters, e.g. *SNR*, has changed, although the BER is an important parameter for evaluating the performance of many wireless network applications [89]. Motivated by such problems, we propose a new method of increasing the *EE* with the least sacrifice in *SE*. The method considers CE and the relay allocation. It is achieved as follows:

1. Defining the minimum power consumption in a BAF-MR network, considering the best relay location.

2. Using the minimum power consumption in (1) to optimize the *EE* and *SE* in a balanced scheme.

3. Analyzing the BER of the balanced scheme in (2), and investigating the following question: does increasing the *EE* lead to a positive effect on BER?

Integration of the *EE*, *SE*, relay location and BER has not been presented previously. While [103] recognized that relay location is an important determinant of any network parameter, such as *EE* and

BER, to the best of the authors' knowledge, the idea of associating the optimal balancing of *SE* and *EE* with unconstrained BER has not yet been proposed in the literature, whether for BAF-MR networks or other wireless networks.

The remainder of the chapter is organized as follows: Section 7.2 outlines the system model; Section 7.3 describes analysis of average *SNR*; Section 7.4 discusses the proposed method for combining *EE* and *SE* with relay location; Section 7.5 discusses BER of AF relay networks; Section 7.6 presents the relationship between BER and *EE*; Section 7.7 shows the simulation results; and finally, conclusions and remarks are presented in Section 7.8.

7.2 System Model

This chapter considers two wireless users (S_a and S_b) exchanging information simultaneously with each other at the same transmission rates and through a set of parallel relays (n: $n = 1, 2, ..N$) in a BAF-MR network.

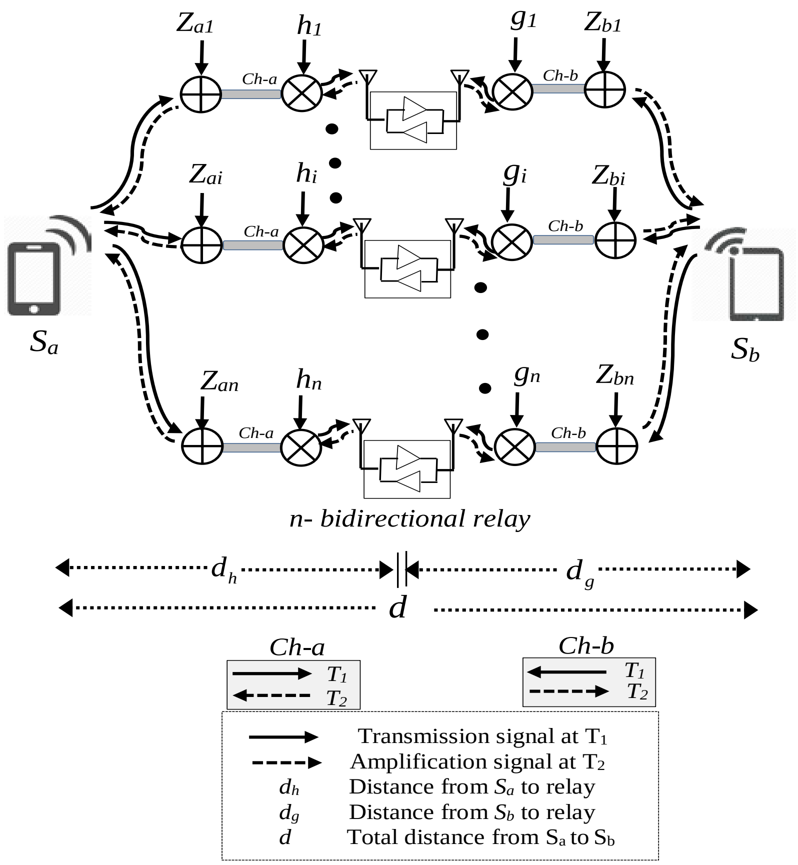

Figure 7.1: System Model

In this model, the transmitted signal is modulated using a binary phase shift keying (BPSK) scheme, and it is sent through Rayleigh fading channels. We assume that all the nodes (users and relays) are equipped with a single antenna, and the users S_a and S_b are transmitting to all relays in the first time slot T_1.

The signal received by any relay (y_i) is expressed as (5.1). The relay signal y_i has an amplification relay factor calculated as follows: $\beta_i = \sqrt{\frac{p_{ri}}{|h_i|^2 p_a + |gi|^2 p_b + \sigma^2}}$, where β_i is the amplification factor, and p_{ri} is the allocated power to relay r_i. Each relay then amplifies the signals by β_i factor, and forwards it to their target users during the second time slot T_2 as shown in Fig. 7.1. It is assumed here that channel conditions are known at the receiving ends. Each user in the network receives signals from all relay branches but the signals with the highest *SNRs* are chosen by a selection scheme, which includes a comparator and a fast switch [90]. Thus, the received signal by user S_a is given as $y_{ai} = y_i h_i \beta_i + z_{ai}$, where y_{ai} is the received signal by S_a , and z_{ai} is Gaussian noise with a zero mean and variance σ^2. Likewise, the received signal by S_b can be obtained as $y_{bi} = y_i g_i \beta_i + z_{bi}$, where z_{bi} is Gaussian noise for S_b with a zero mean and variance σ^2.

7.3 Signal-to-Noise Ratio

In this section, *SNRs* which are used for analyzing our proposal method are expressed. The exact *SNR* at node S_a is calculated as (6.1). Further, S_a node removes its own transmitted signal via self-interference cancellation [151].

By substituting β_i into (6.1) and using the expectation value of the received x_b, as $E\{x_b\} = 1$, we can calculate the power signal as

$$E\left\{y_{ai}^2\right\} = \frac{|h_i|^2 p_{ri}}{\sigma^2} \frac{|g_i|^2 p_b}{\sigma^2} + \frac{(p_a + p_{ri})}{\sigma^2} |h_i|^2 + \frac{p_b}{\sigma^2} |g_i|^2 + 1\,. \tag{7.1}$$

where $E\left\{y_{ai}^2\right\}$ is the expectation value for the received power signal at node S_a. Equation (7.1) results in

$$SNR = \frac{p_{ri} p_b \gamma_{hi} \gamma_{gi}}{(p_{ri} + p_a)\gamma_{hi} + p_b \gamma_{gi} + 1} = \gamma_{ai}, \tag{7.2}$$

where γ_{ai} is the exact end-to-end *SNR,* γ_{hi} and γ_{gi} are the instantaneous channel gain-to-noise ratios (CNRs), which are defined as $\gamma_{hi} = |h_i|^2/\sigma^2$ and $\gamma_{gi} = |g_i|^2/\sigma^2$, respectively.

Following the same derivation of equation (7.2), the exact *SNR* for node S_b is defined as

$$SNR = \frac{p_{ri}\, p_a \gamma_{hi} \gamma_{gi}}{p_a \gamma_{hi} + (p_{ri} + p_b)\,\gamma_{gi} + 1} = \gamma_{bi}. \tag{7.3}$$

To calculate high *SNR* region, equation (7.2) can be simplified as [54]. This results in

$$\frac{1}{\gamma_{ai}} = \frac{1}{\gamma_{\text{fi}i}} + \frac{1}{\gamma_{\text{g}i}}, \tag{7.4}$$

where γ_{ai} is the approximation *SNR* for S_a , $\gamma_{\text{fi}i}$ is the instantaneous *SNR* channel between S_a and the relay. It is expressed as

$$\gamma_{\text{fi}i} = p_{ri}\gamma_{hi} \tag{7.5}$$

and $\gamma_{\text{g}i}$ is the instantaneous *SNR* channel between S_b and relay. It can be expressed as

$$\gamma_{\text{g}i} = \frac{p_{ri} p_b \gamma_{gi}}{p_{ri} + p_a} \tag{7.6}$$

The average value of (7.4) is obtained by calculating the average value of each $\gamma_{\text{g}i}$ and $\gamma_{\text{fi}i}$ directly as both variables are statistically independent identically distributed (i.i.d). Thus, the average value of (7.5) is represented as $\bar{\gamma}_{\text{fi}i} = \frac{p_{ri}}{\sigma^2}\mathrm{E}\{|h_i|^2\}$, and the average value of (7.6) is $\bar{\gamma}_{\text{g}i} = \left(\frac{p_{ri}}{p_a + p_{ri}}\right)\frac{p_b}{\sigma^2}\mathrm{E}\{|g_i|^2\}$, where $E(.)$ denotes the expectation value.

Based on $\bar{\gamma}_{\text{fi}i}$ and $\bar{\gamma}_{\text{g}i}$ equations, the average value of high *SNR* region is expressed as

$$\frac{1}{\bar{\gamma}_{ai}} = \frac{1}{\left(\frac{p_{ri}}{p_a + p_{ri}}\right)\frac{p_b}{\sigma^2}\mathrm{E}\{|g_i|^2\}} + \frac{1}{\frac{p_{ri}}{\sigma^2}\mathrm{E}\{|h_i|^2\}} \tag{7.7}$$

where $\bar{\gamma}_{ai}$ is the average *SNR* of (7.4).

Similarly, the high *SNR* region for (7.3) is obtained as follows

$$\frac{1}{\gamma_{bi}} = \frac{1}{p_{ri}\gamma_{gi}} + \frac{1}{p_a\gamma_{hi}}\left(1 + \frac{p_b}{p_{ri}}\right) \tag{7.8}$$

The average value of (7.8) is obtained in a similar way to (7.7).

7.4 Balancing *EE* and *SE* scheme

This section describes our proposed method for combining *EE* and *SE* in a balanced scheme assuming optimal relay location. The average *EE* metric is defined as a measure of energy consumed in a transmitting bit during transmission across a network [8] i.e.,

$$\bar{\zeta}= \frac{\bar{\Re}_i}{\bar{p}_{ti}}\text{(bit/Joule)}, \tag{7.9}$$

where $\bar{\zeta}$ is the average *EE*, $\bar{\Re}_i$ is the average throughput for unit bandwidth (bits/sec/Hz) and $\bar{p}_{ti}$ is the average total energy consumption [39] .

The value of $\bar{\Re}_i$ can be obtained by following the procedures of (5.45) . This gives

$$\bar{\Re}_i \approx 0.5\log_2(1+\bar{\gamma}_{ai})+0.5\log_2(1+\bar{\gamma}_{bi}), \tag{7.10}$$

To consider relay location with $\bar{\Re}_i$, we can use the data rate model used in [40, 115], which allows the evaluation of both $\text{E}(|h_i|^2)$ and $E(|g_i|^2)$ values as follows

$$\text{E}(|h_i|^2) = \frac{k}{d_h^{\alpha}} \tag{7.11}$$

$$E(|g_i|^2) = \frac{k}{d_g^{\alpha}} \tag{7.12}$$

where k is the path loss coefficient (in decibel scale (dB)) at a reference distance equal to one, α is the path loss exponent which is commonly estimated from 2 to 6 [116], d_h is the distance between S_a and the relay, and d_g is the distance from S_b to the relay as illustrated in Fig. 7.1.

By substituting (7.11) and (7.12) into expression of *SNR* in (7.7), the average *SNR*s for the user S_a as function of distance is calculated as

$$\bar{\gamma}_{ai} = \frac{k}{No}\frac{p_{ri}\,p_b}{(p_{ri}+p_a)\,d_g^{\alpha}+p_b d_h^{\alpha}} \tag{7.13}$$

Similarly, average *SNR* for user S_b in terms of distance can be determined from (7.8) as follows

$$\bar{\gamma}_{bi} = \frac{k}{No}\frac{p_{ri}\,p_a}{p_a d_g^{\alpha}+(p_{ri}+p_b)\,d_h^{\alpha}} \tag{7.14}$$

To calculate both distances d_h and d_g from the total distance (d) as shown in Fig. 7.1, we assume that the relay is allocated by the factor δ as following

$$d_h^{\alpha} = (d\,\delta)^{\alpha}, \tag{7.15}$$

where $\delta \in [0,1]$ is the allocation factor of the system adopted to characterize the boundary of relay location between between S_a and S_b.

Also, d_g^{α} is given as

$$d_g^{\alpha} = (d\,(1-\delta))^{\alpha}\,, \tag{7.16}$$

Substituting (7.15) and (7.16) into (7.13) and (7.14) results in

$$\bar{\gamma}_{ai} = \frac{k}{d^{\alpha}}\left(\frac{p_{ri}\,p_b}{(p_{ri}+p_a)\,(1-\delta)^{\alpha}+p_b\,\delta^{\alpha}}\right)\frac{1}{No} = \frac{k}{d^{\alpha}}\bar{\Phi}_{ai} \tag{7.17}$$

and

$$\bar{\gamma}_{bi} = \frac{k}{d^{\alpha}}\left(\frac{p_{ri}\,p_a}{p_a(1-\delta)^{\alpha}+(p_{ri}+p_b)\,\delta^{\alpha}}\right)\frac{1}{No} = \frac{k}{d^{\alpha}}\bar{\Phi}_{bi} \tag{7.18}$$

where $\bar{\Phi}_{ai} = (p_{ri}\,p_b/((p_{ri}+p_a)(1-\delta)^{\alpha}+p_b\,\delta^{\alpha}))/No$ and $\bar{\Phi}_{bi} = (p_{ri}\,p_a/((p_a(1-\delta)^{\alpha}+(p_{ri}+p_b)\,\delta^{\alpha}))\,/No$.

To express $\bar{p}_{ti}$ in (7.9), the power consumption of the signal processing and the amplification circuits should be considered as described in [82]. This is presented as

$$\bar{p}_{ti} = \left(\varsigma\,(\xi_{ti}) + \left(\text{ɲ}\bar{\mathfrak{R}}_i + p_{ci}\right)\right)/2 \tag{7.19}$$

where ς is a constant associated with the amplifier power efficiency, ξ_{ti} is the total energy per bit ($\xi_{ti} = p_{a+}p_{b+}\,p_{ri}$) , p_{ci} is the static circuit power and $\text{ɲ}\bar{\mathfrak{R}}_i$ is the dynamic power per-unit data rate [43]. The term of $\left(\text{ɲ}\bar{\mathfrak{R}}_i + p_{ci}\right)$ is related to the AF relay circuit power consumption, (further information about this power is presented by [103, 117]).

To maximize (7.9), we can first define the minimization problem of each power node (i.e., *min* (ξ_{ti})) under the required minimum constraint $\bar{\mathfrak{R}}_i$ as follows

$$max \quad \bar{\zeta} = \frac{\bar{\mathfrak{R}}_i}{\bar{p}_{ti}} \qquad i_s \in \{S_a,\, S_b\, r_i\} \tag{7.20}$$

$$s.t \quad \bar{\mathfrak{R}}_{ab} = \bar{\mathfrak{R}}_{ba} \tag{7.21}$$

where $\bar{\Re}_{ba}$ and $\bar{\Re}_{ab}$ are the average throughput from users S_b and S_a respectively.

To calculate the power allocation of all nodes (i.e, users and relay) in term to ξ_{ti}, we can define

$$p_a = \sigma_a \, \xi_{ti}, \tag{7.22}$$

$$p_{ri} = (1-\sigma_a) + (1-\sigma_b) \, \xi_{ti}, \tag{7.23}$$

$$p_b = \sigma_b \, \xi_{ti}, \tag{7.24}$$

where σ_a and σ_b are the energy allocation factors for users S_a and S_b respectively, their values specified in the range between greater than zero and less than one [118].

To optimize the value of ξ_{ti} based on energy allocation factor, we can substitute equations (7.22), (7.23) and (7.24) into (7.13). This results in

$$\xi_{ti} = \frac{\left((2-\sigma_b)\, d_g^{\alpha} + \sigma_b d_h^{\alpha}\right) \, \bar{\gamma}_{ai} No}{\left((2-\sigma_b)\sigma_b\right) \kappa} \tag{7.25}$$

By taking the derivative of (7.25) with respect to the sender energy allocation factor (i.e σ_b in the case of the user S_a as a receiver) and equating it to zero, we obtain

$$\mathring{\sigma}_b = \frac{-2d_g^{\alpha} - 4\sqrt{d_g^{\alpha 2} + d_g^{\alpha}(d_h^{\alpha} - d_g^{\alpha})}}{(d_h^{\alpha} - d_g^{\alpha})} \tag{7.26}$$

where $\mathring{\sigma}_b$ is the optimal value of σ_b.

By replacing the term σ_b into (7.25) by (7.26) and considering the active self-interference cancellation of node S_a to removes its own energy signal [152]. This yields

$$\mathring{\xi}_{ti} = \bar{\Phi}_{ai} No \left(\frac{(1-\delta)^{\alpha}}{\mathring{\sigma}_b} + \frac{\delta^{\alpha}}{2-\mathring{\sigma}_b} \right) \tag{7.27}$$

where $\mathring{\xi}_{ti}$ is the optimal value of ξ_{ti} with respect to the optimal value of $\mathring{\sigma}_b$.

Substituting (7.27) into (7.19) and then into (7.20) results in

$$\bar{\zeta} = \frac{\log_2(1+\frac{k}{d^{\alpha}}\bar{\Phi}_{ai}) + \log_2(1+\frac{k}{d^{\alpha}}\bar{\Phi}_{bi})}{\varsigma \bar{\Phi}_{ai} No \left(\frac{(1-\delta)^{\alpha}}{\mathring{\sigma}_b} + \frac{\delta^{\alpha}}{2-\mathring{\sigma}_b} \right) + \left(\log_2(1+\frac{k}{d^{\alpha}}\bar{\Phi}_{ai}) + \log_2(1+\frac{k}{d^{\alpha}}\bar{\Phi}_{bi})\right) \jmath + p_{ci}} \tag{7.28}$$

By deriving equation (7.28) with respect to $\bar{\Phi}_{ai}$ and equating it to zero, assuming the subtraction of the back-propagating self-interference for the transmitted symbols by node S_a (i.e $\Re_{ab}$) [55], we obtain

$$\mathring{\bar{\gamma}}_{ai} = \frac{d^{\alpha}}{k}(\exp^{1+w(\frac{1}{e}(\frac{k}{d^{\alpha}}\frac{p_{ci}}{\varsigma \eta}-1))} - 1) \tag{7.29}$$

where $\mathring{\bar{\gamma}}_{ai}$ is the optimal *SNR* value of $\bar{\gamma}_{ai}$, $\eta = No\left(\frac{(1-\delta)^{\alpha}}{\mathring{\delta}_b} + \frac{\delta^{\alpha}}{2-\mathring{\delta}_b}\right)$, e is the base of the natural logarithm and $w(.)$ is the omega function [106].

By substituting (7.29) into (7.28), the optimal balance between *SE* and *EE* is obtained as follows

$$\mathring{\bar{\zeta}} = \frac{\log_2(1+\mathring{\bar{\gamma}}_{ai}) + \log_2(1+\mathring{\bar{\gamma}}_{bi})}{\varsigma \psi \mathring{\bar{\gamma}}_{ai} + \left((\log_2(1+\mathring{\bar{\gamma}}_{ai}) + \log_2(1+\mathring{\bar{\gamma}}_{bi})\right)\mathfrak{p} + p_{ci}} \tag{7.30}$$

where $\mathring{\bar{\zeta}}$ is maximum *EE* associated with lowest sacrifice of *SE* and $\psi = \frac{d^{\alpha}}{k} No\left(\frac{(1-\delta)^{\alpha}}{\mathring{\delta}_b} + \frac{\delta^{\alpha}}{2-\mathring{\delta}_b}\right)$

Now, equation (7.7) can be formulated with respect to (7.29) as

$$\mathring{\gamma}_{ai} = \frac{\mathring{\bar{\gamma}}_{\mathfrak{h}i}\mathring{\bar{\gamma}}_{\mathfrak{g}i}}{\mathring{\bar{\gamma}}_{\mathfrak{h}i} + \mathring{\bar{\gamma}}_{\mathfrak{g}i}} \tag{7.31}$$

where $\mathring{\bar{\gamma}}_{\mathfrak{h}i}$ and $\mathring{\bar{\gamma}}_{\mathfrak{g}i}$ are the arguments of the function $\mathring{\bar{\gamma}}_{ai}$.

Equation (7.31) is employed in the analysis of the next sections to calculate BER performance of (7.30).

7.5 BER Performance in AF relay networks

The previous section presented a method to calculate the maximum *EE* in an BAF-MR network. To study the BER behavior of such such network (i.e., BAF-MR networks after the maximized *EE*), this section considers a new method to analyze the BER of this network. We start by assuming that a BAF-MR network includes one relay and extend BER calculation to n relays, as discussed in the next section.

In a two-hop relay network, the transmitted signal from a source to the destination passing through cascade channels as shown in Fig. 7.1. Each hop includes a Rayleigh fading channel, and such a channel has a *SNR* that follows an exponential distribution. Thereby, the total *SNR* obtained from i.i.d Rayleigh fading channels follows an exponential distribution [75]. Here, the distribution of the harmonic mean of two i.i.d. gamma random variables defined in [48] can be employed to find the the total pdf. We first define Rayleigh fading pdf from S-node to a relay, i.e. first hop, as

$$\underset{S_b \to R_i}{\varphi_{\mathrm{g}i(\phi)}} = (\mathring{\gamma}_{\mathrm{g}i})^{-1} \exp^{-\phi(\mathring{\gamma}_{\mathrm{g}i})^{-1}}, \tag{7.32}$$

and similarly the pdf of the second hop is expressed as

$$\underset{R_i \to S_a}{\varphi_{\mathrm{fi}i(\phi)}} = (\mathring{\gamma}_{\mathrm{fi}i})^{-1} \exp^{-\phi(\mathring{\gamma}_{\mathrm{fi}i})^{-1}}, \tag{7.33}$$

where ϕ is the harmonic mean (denoted by $\phi = \mu_H(\mathring{\gamma}_{\mathrm{fi}i}\mathring{\gamma}_{\mathrm{g}i})$) according to a gamma distribution [47], $\underset{S \to R_i}{\varphi_{hi(\phi)}}$ and $\underset{R_i \to D}{\varphi_{gi(\phi)}}$are the pdfs of γ_{hi} and γ_{gi}, respectively.

Given two i.i.d. exponential random variables as (7.33) and (7.32), the modified harmonic mean defined in [119] can be used to join them as follows

$$\varphi_{ai}(\phi) = \frac{\exp^{-\phi\left(\frac{\mathring{\gamma}_{\mathrm{fi}i}+\mathring{\gamma}_{\mathrm{g}i}}{\mathring{\gamma}_{\mathrm{fi}i}\mathring{\gamma}_{\mathrm{g}i}}\right)}}{\left(\sqrt{\mathring{\gamma}_{\mathrm{fi}i}\mathring{\gamma}_{\mathrm{g}i}}\right)^3} \left(4\sqrt{\mathring{\gamma}_{\mathrm{fi}i}\mathring{\gamma}_{\mathrm{g}i}}\,k_0\left(\frac{2\phi}{\sqrt{\mathring{\gamma}_{\mathrm{fi}i}\mathring{\gamma}_{\mathrm{g}i}}}\right) + 2\phi\,(\mathring{\gamma}_{\mathrm{fi}i}+\mathring{\gamma}_{\mathrm{g}i})\,k_1\left(\frac{2\phi}{\sqrt{\mathring{\gamma}_{\mathrm{fi}i}\mathring{\gamma}_{\mathrm{g}i}}}\right)\right) \tag{7.34}$$

where $\varphi_i(\phi)$ is the total pdf, $k_0(.)$ and $k_1(.)$ are the first and the second order modified Bessel function of the second kind.

By applying the modified Bessel function properties, which are $k_0(\phi) \to 0$ and $k_1(\underset{\phi \to 0}{\phi}) \to 1/\phi$, to (7.34), the total pdf is obtained as follows

$$\varphi_i(\phi) = \frac{1}{\mathring{\gamma}_{ai}} e^{-\frac{\phi}{\mathring{\gamma}_{ai}}}, \tag{7.35}$$

Integration of (7.35) relative to ϕ, similar to [91], results in

$$\mathscr{F}_i(\phi) = 1 - e^{-\frac{\phi}{\mathring{\gamma}_{ai}}} \tag{7.36}$$

where $\mathscr{F}_i(\phi)$ is the cumulative distribution function for $\varphi_i(\phi)$.

From (7.36) and the relationship between the Q-function and the error function (erfc) as presented in [92], the term of $\mathscr{F}_i(\phi)$ is expressed as

$$Q(\sqrt{2\phi}) = \text{erfc}\left(\sqrt{\phi}\right). \tag{7.37}$$

According to error probabilities with random variables definition in [13], the average BER with the random variables $\mathring{\bar{\gamma}}_{\text{fi}i}$ and $\mathring{\bar{\gamma}}_{\text{g}i}$ can be expressed by using (7.35) and (7.37), as follows

$$\bar{P}_e = \frac{1}{\mathring{\bar{\gamma}}_{ai}} \text{erfc}\left(\sqrt{\phi}\right) e^{-\frac{\phi}{\mathring{\bar{\gamma}}_{ai}}} d\phi, \tag{7.38}$$

where $\bar{P}_e$ is the average BER.

It is worth noting that the expression of (7.38) is consistent with the result of one hop BER obtained by [13].

By evaluating (7.38), the BER of BAF-MR network is obtained as

$$\bar{P}_e = 1 - \sqrt{\frac{1}{1 + \frac{1}{\mathring{\bar{\gamma}}_{\text{g}i}} + \frac{1}{\mathring{\bar{\gamma}}_{\text{fi}i}}}}. \tag{7.39}$$

Equation (7.39) is analyzed based on approximated *SNR* presented in equation (7.4). To calculate the exact BER performance, we can apply the linearity of the expectation property in [93] for the term $\frac{|h_i|^2 p_{ri}}{\sigma^2} \frac{|g_i|^2 p_b}{\sigma^2} + \frac{(p_a + p_{ri})}{\sigma^2} |h_i|^2 + \frac{p_b}{\sigma^2} |g_i|^2$ in equation (7.1), as this term is a set of independent random variables, while the second term in the same equation is a constant equal to one. Thus, the outcome of $\mathring{\bar{\gamma}}_{ai}$ can be easily adapted to employ in the calculation of actual average *SNR*. This results in

$$\frac{1}{\mathring{\bar{\gamma}}_i} = \left(1 + \frac{1}{\mathring{\bar{\gamma}}_{\text{g}i}} + \frac{1}{\mathring{\bar{\gamma}}_{\text{fi}i}}\right) \tag{7.40}$$

where $\mathring{\bar{\gamma}}_i$ is the total average value of actual *SNR* in respect to $\mathring{\bar{\gamma}}_{\text{fi}i}$ and $\mathring{\bar{\gamma}}_{\text{g}i}$.

Following same procedure for (7.39) and assuming (7.40), the exact BER is obtained as follows

$$\bar{P}_e = 1 - \sqrt{\frac{1 + \frac{1}{\mathring{\bar{\gamma}}_{\text{g}i}} + \frac{1}{\mathring{\bar{\gamma}}_{\text{fi}i}}}{2 + \frac{1}{\mathring{\bar{\gamma}}_{\text{g}i}} + \frac{1}{\mathring{\bar{\gamma}}_{\text{fi}i}}}}. \tag{7.41}$$

Equation (7.41) represents exact average BER of BAF-MR networks assuming that the network includes one relay.

7.6 Relationship between BER and *EE*

The previous section presented BER performance of the BAF-MR network for one relay node. In this section, the general form for n relays is derived. Then a relationship between BER and other network parameters, including (7.27) and (7.30), is expressed.

From a BAF-MR network with a selection strategy and n relays, a single relay with the highest *SNR* is selected to forward data symbols from a sender to a receiver. Implementation such selection at the receiver requires knowledge of order statistics. In this section, we briefly introduce a basic order statistical tool, which is used for the BER performance analysis.

Using a selection strategy, the *SNR* output of an AF relay network can be ordered statistically as: $\gamma_i = max\{\gamma_1, \gamma_2, ..., \gamma_n\}$, where γ_i is the i_{th} order statistic of the *SNR*s, γ_n representing the largest order statistic and γ_1 is the smallest one. Here, we can apply such order with respect to ϕas follows

$$\phi_i = max\{\phi_1, \phi_2, ..., \phi_n\}, \tag{7.42}$$

The *SNRs* in a statistical order is dependent on CDF analysis as follows:

let the CDF of ϕ_i is denoted as $\mathscr{F}_i(\phi_n)$, and then apply the order statistical analysis for ϕ_i as presented in [95]. This results in

$$\mathscr{F}_i(\phi_n) = Prob\,(\phi_1 \leq \phi_n) = \mathscr{F}_1(\phi_1)\,\mathscr{F}_2(\phi_2)...\mathscr{F}_n(\phi_n) = [\mathscr{F}_n(\phi_n)]^n\,, \tag{7.43}$$

where $\mathscr{F}_i(\phi_n)$ is the CDF order statistics.

For i.i.d channels, equation (7.43) can be written as $\mathscr{F}_i(\phi_n) = n\mathscr{F}_n(\phi_n)$ [66]. By substituting one relay CDF of (7.36) into (7.43), we obtain

$$\mathscr{F}_i(\phi_n) = n\frac{1}{2}\,\text{erfc}\left(\sqrt{\phi}\right) \tag{7.44}$$

By taking the derivative of (7.43) relative to ϕ, using a similar approach in [120], the total pdf is calculated

$$\varphi_i(\phi) = \frac{n}{\overset{\circ}{\gamma}_{ai}}\, e^{-\frac{\phi}{\overset{\circ}{\gamma}_{ai}}}\,(\mathscr{F}_n(\phi_n))^{n-1} \tag{7.45}$$

Equation (7.45) is further analyzed and can be described as follows

$$\varphi_i(\phi) = \frac{n}{\mathring{\bar{\gamma}}_{ai}} \left(e^{-\frac{\phi}{\mathring{\bar{\gamma}}_{ai}}} \right)^n \tag{7.46}$$

By considering $\mathring{\bar{\gamma}}_{ai}$ as in (3.7) and then substituting both of (7.44) and (7.46) into (7.38), the BER expression for n relays is expressed as

$$\bar{P}_e = \frac{n^2}{\mathring{\bar{\gamma}}_{ai}} \int_0^{\infty} \text{erfc}\left(\sqrt{\phi}\right) \left(e^{-\frac{\phi}{\mathring{\bar{\gamma}}_{ai}}} \right)^n d\phi. \tag{7.47}$$

Evaluating (7.47), the general form of BER performance of the BAF-MR networks is obtained as follows

$$\bar{P}_e = n\left(1 - \sqrt{\frac{\mathring{\bar{\gamma}}_{\text{fi}i}\mathring{\bar{\gamma}}_{\text{g}i}}{\mathring{\bar{\gamma}}_{\text{fi}i}\mathring{\bar{\gamma}}_{\text{g}i} + n\left(\mathring{\bar{\gamma}}_{\text{fi}i} + \mathring{\bar{\gamma}}_{\text{g}i}\right)}}\right)^n, \tag{7.48}$$

It is also important to mention here that the expression (7.48) can be analyzed by using (7.7) to obtain the BER of a suboptimal network.

Now, from (7.48), the average *SNR* of (7.31) is determined with respect to BER as

$$\mathring{\bar{\gamma}}_{ai} = \frac{n\left(1 - \left(\frac{\bar{P}_e}{n}\right)^{\frac{1}{n}}\right)^2}{1 - \left(1 - \left(\frac{\bar{P}_e)}{n}\right)^{\frac{1}{n}}\right)^2} \tag{7.49}$$

Equation (7.49) is very useful as it allows us to obtain many relationships between BER and other parameters within AF relay networks, and this provides the opportunity to study the behavior of BER for each parameter developed in BAF-MR networks.

Thus, by substituting (7.49) into (7.27), an expression included BER with $\mathring{\xi}_{ti}$ is obtained as follows

$$\bar{P}_e = n\left(1 - \sqrt{\frac{\mathring{\xi}_{ti}}{n\psi + \mathring{\xi}_{ti}}}\right)^n. \tag{7.50}$$

Equation (7.50) gives the BER behavior for delivering a bit from an a source to the destination in a BAF-MR network, when relay locations and distances from source to relay and relay to destination are also considered.

Also, substituting (7.49) into (7.28) and using some manipulation, we obtain the BER associated with maximum *EE*, which was obtained in (7.30), as follows

$$\bar{P}_e = n\left(1-\left(1+\left(\frac{\frac{1}{\varsigma}-\beta}{n\,\psi\varsigma\ln(2)}\right)1/\,w\left(-\frac{2^{\left(\frac{p_{ci}-\psi\varsigma}{\frac{1}{\varsigma}-\beta}\right)}\psi\varsigma\ln(2)}{(\frac{1}{\varsigma}-\beta)}\right)\right)^{0.5}\right)^{n} \tag{7.51}$$

where $w(.)$ is the omega function described in [106].

7.7 Simulation Results

This section provides simulation results to verify our theoretical analysis. The system model illustrated in Section 7.2 is used for the simulation. However, for this analysis we use the following parameters: $n = 4$, $d = 500$, $\alpha = 2$, $k = 10^{-3}$, $\varsigma = 2$, $p_{ci} = 0.5W$, $\eta = 0.2W/bit/s$.

Fig. 7.2 shows the relationship between power consumption and the percentage of *EE*. It can be seen that the *EE* defined in (7.9) is increased by reducing power consumption, and further enhancement of *EE* can be achieved using the proposed method in (7.30), as illustrated in Fig. 7.3.

Fig. 7.4 shows the impact of relay location on the transmission energy in the case when all of S_a, S_b and r_i have the same allocated powers. It can be clearly seen that the value δ to minimize the total power consumed by the network is equal 1/2. It has also been shown that the energy curve is symmetric with respect to the relay location. This result agrees for the equal powers allocation as demonstrated in [121]. Further the total power consumed by the network reaches a minimum value by using (7.27). Fig. 7.5 illustrates the impact of user distance on energy consumption defined in (7.19), considering three different values of δ, namely, 0.35, 0.5 and 0.75. The same δ values are adopted in Fig. 7.6 to present optimal energy consumption per bit, as evaluated in (7.28). A comparison between Fig. 7.5 and Fig. 7.6, clearly reveals that the proposed method provides lower energy consumption than the suboptimal network in Fig. 7.5. Moreover, both Figs. 7.5 and 7.6. indicate that positioning the relay at the midpoint between S_a and S_b, achieves lower power consumption than any other relay location.

The result of adopting optimal energy allocation for *EE* in (7.29) is illustrated in Fig. 7.7. It can be seen that *EE* increases with the average *SNR*, as the *EE* design uses lower transmit power with increasing

SNR. Fig. 7.7 also illustrates that the proposed scheme outperforms a suboptimal network and the performance gap in *EE* becomes larger when the *SNR* increases. Fig. 7.8 shows that the suboptimal *EE* design operates at exactly the maximum transmit power level to meet the data rate requirements. Because the proposed method has a lower data rate than the suboptimal scheme, it can achieve higher energy efficiency with less energy consumption at the cost of a slight *SE* loss. This means that the proposed method can achieve a better balance between *EE* and *SE*.

Fig. 7.9 compares the BER performance of suboptimal and optimal BAF-MR networks with respect to *EE*. As can be seen in this figure, the BER decreases in both network cases (i.e., suboptimal or optimal) as the number of relays increase from $n = 1$ to $n = 4$. A further reduction in BER results is obtained by optimizing *EE* as a result of increasing *SNR* at the destination. Similar results are obtained in Fig. 7.10 when the relay power becomes double that of the user's power (i.e., $p_a = p_b = 2p_{ri}$). This figure illustrates an additional reduction in BER as the power amplification increases.

Both Figs. 7.9 and 7.10 illustrate an enhancement in BER results when associating *EE* with BER performance. Here, the following question arises: would increase *EE* dramatically lead to reduce BER substantially?. Figs. 7.11 and 7.12 provide the answer. Fig. 7.11 shows that the increase in *EE* in a suboptimal network increases BER slightly, because *EE* is based on reducing power consumption (see Fig. 7.2).

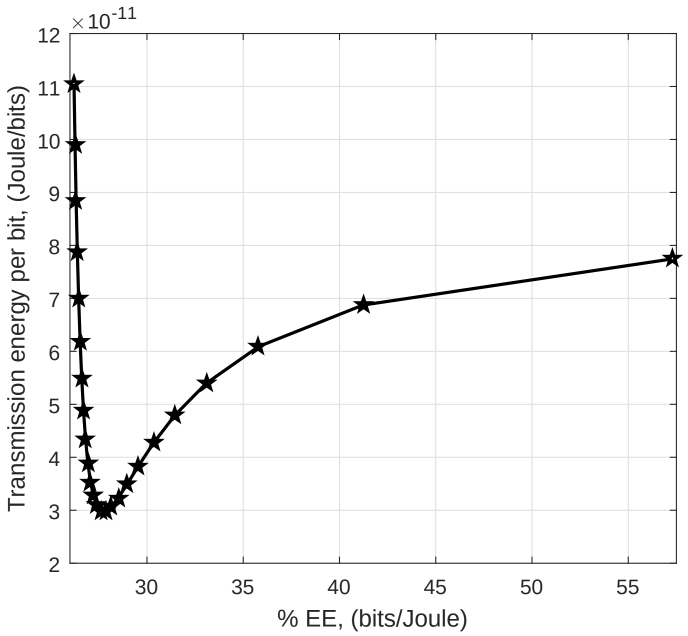

Figure 7.2: *EE* vs power consumption in suboptimal relay networks

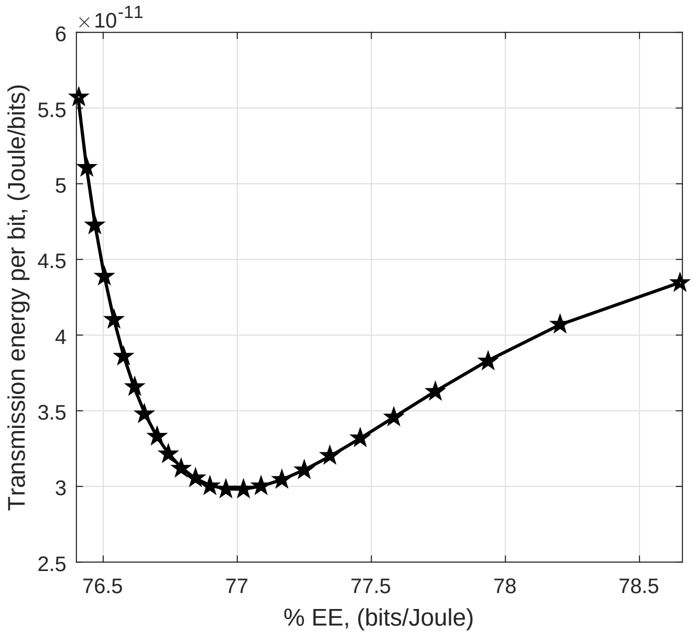

Figure 7.3: *EE* vs power consumption when the proposed scheme is adopted

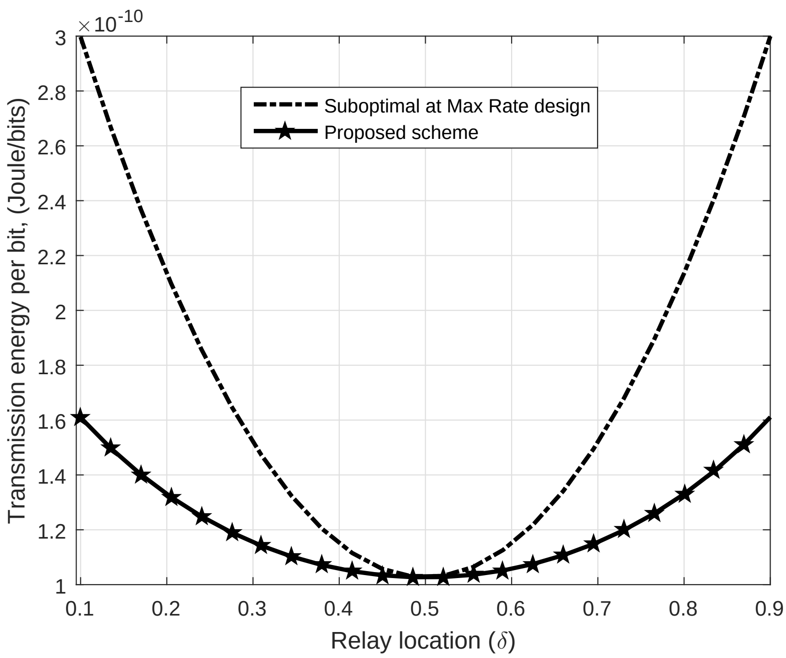

Figure 7.4: Effect of relay location on the transmission energy

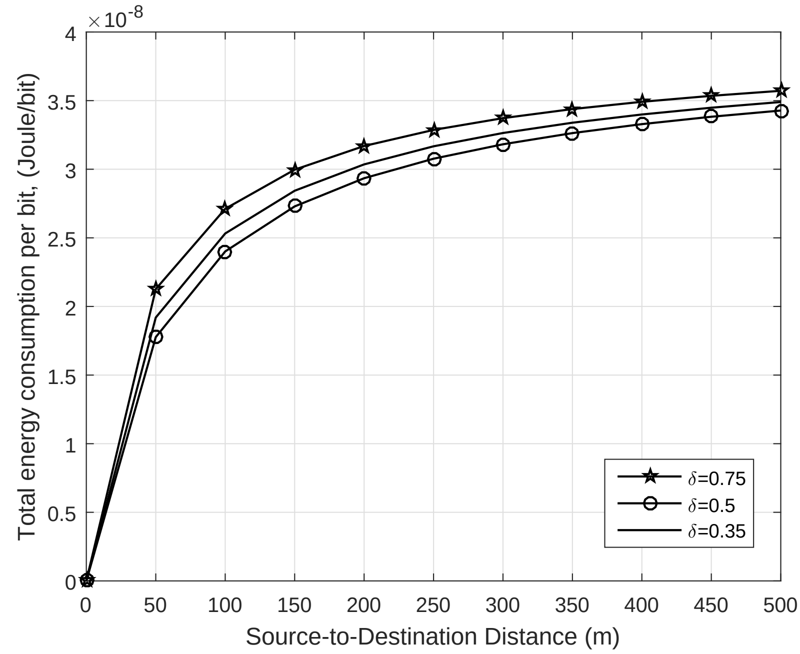

Figure 7.5: Energy consumption for the suboptimal network

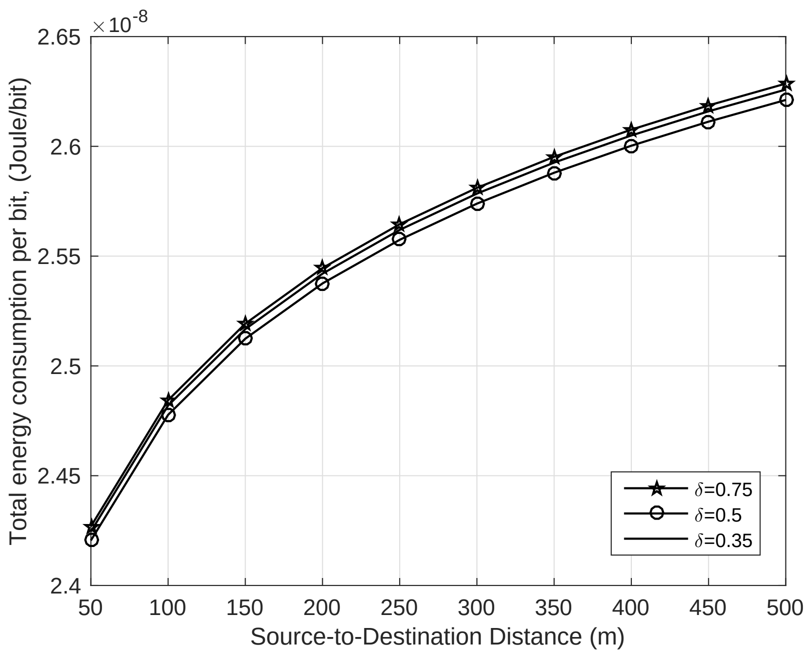

Figure 7.6: Energy consumption for the optimal network

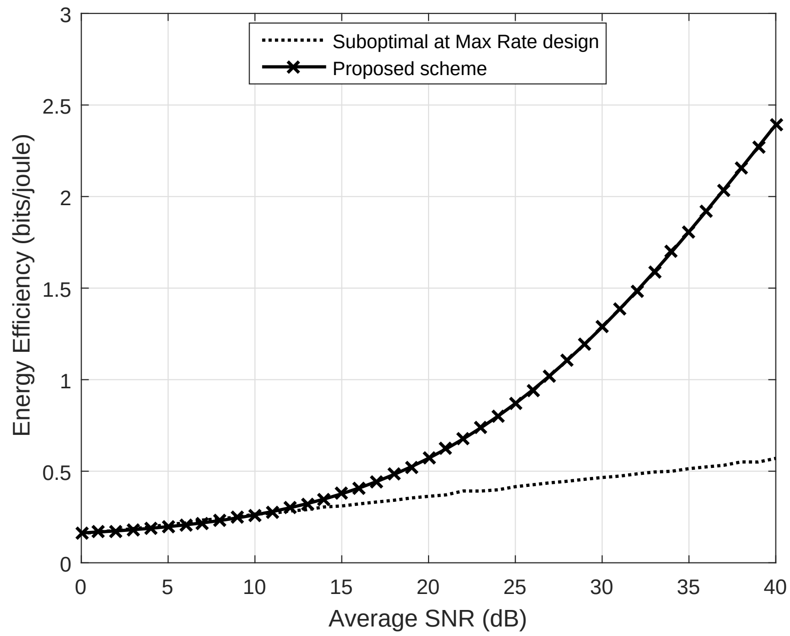

Figure 7.7: System energy efficiency

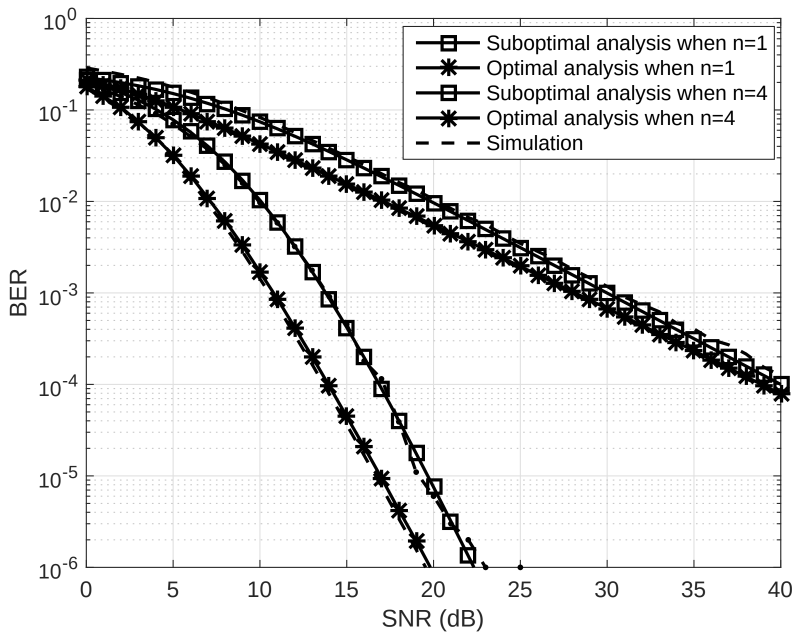

Figure 7.9: BER with equal S_a relay and S_b powers

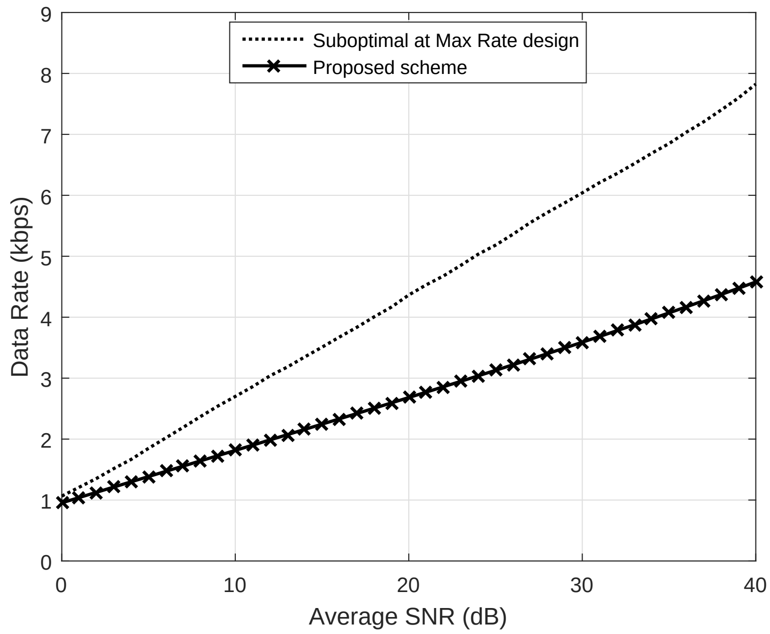

Figure 7.8: System data rate versus average *SNR*

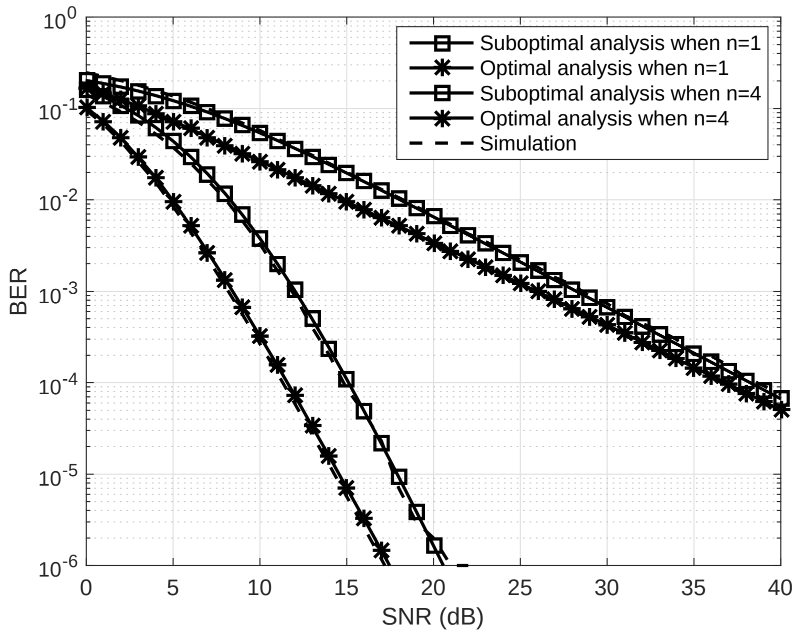

Figure 7.10: BER with equal S_a and S_b powers, while relay power is doubled

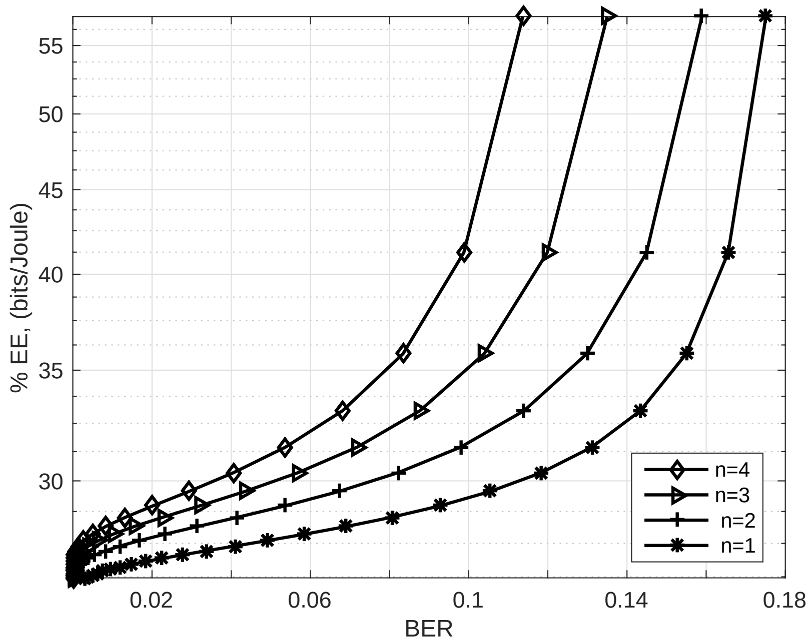

Figure 7.11: *EE* effect on suboptimal BER

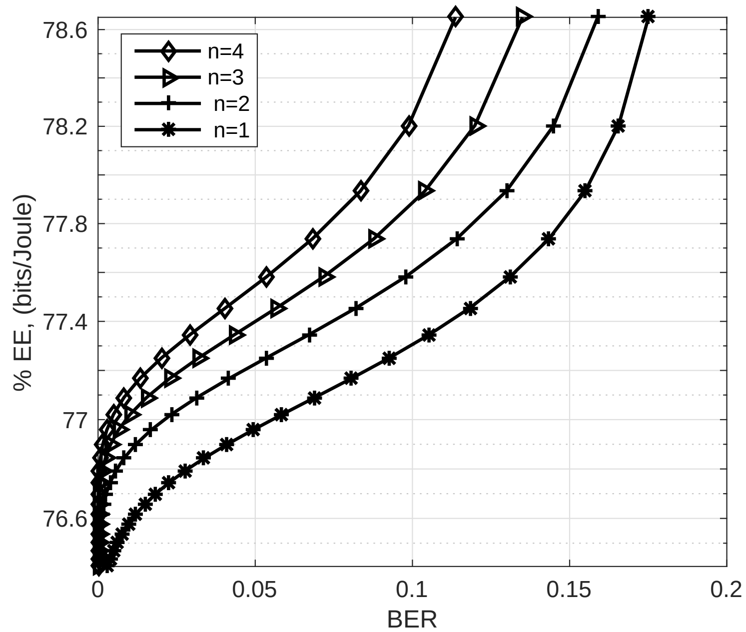

Figure 7.12: *EE* effect on optimal BER

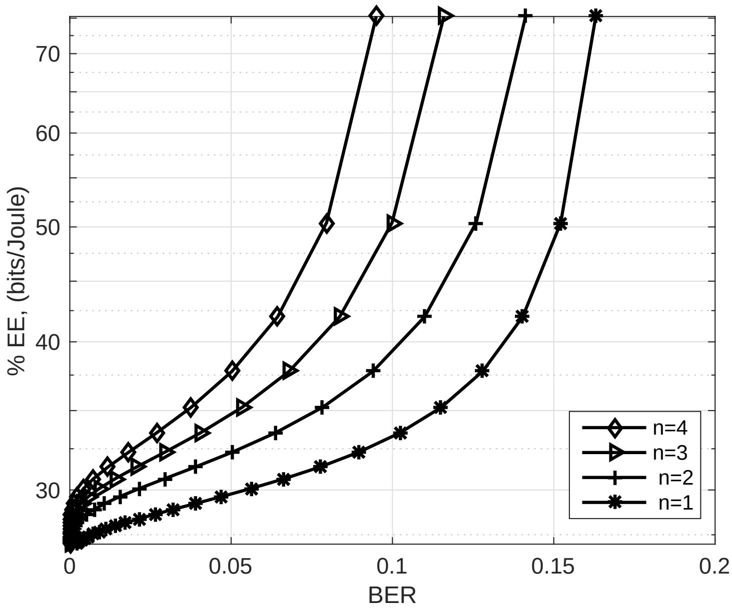

Figure 7.13: Effect *EE* on suboptimal BER, doubling p_{ri}

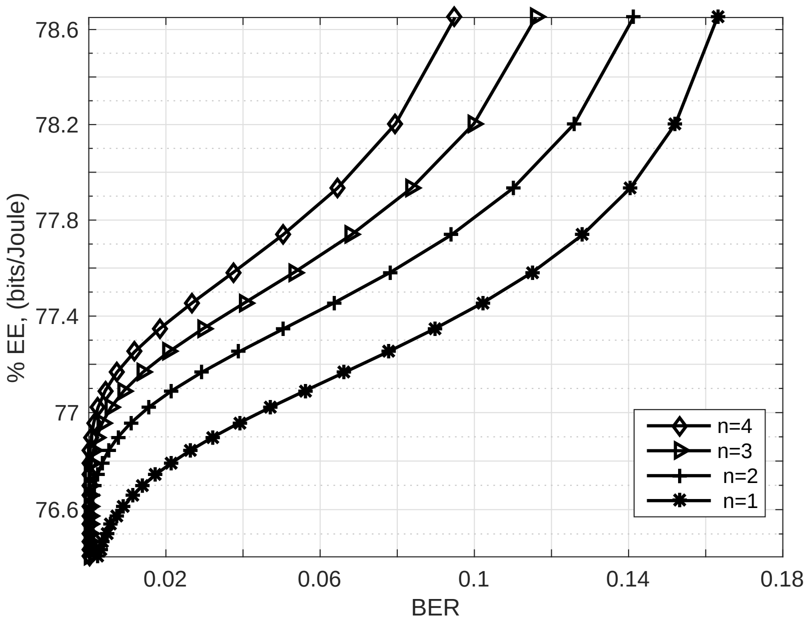

Figure 7.14: Effect *EE* on optimal BER, doubling p_{ri}

Fig. 7.11 also demonstrates that the system can achieve lower BER by increasing the number of relays. Increasing *EE* while lowering BER can be achieved by using equation 7.30, and this confirms that the BER of a BAF-MR network is reduced by balancing *EE* and *SE*, as shown in Fig 7.12. Further, by keeping $p_a = p_b$ and doubling p_{ri}, the BER is further reduced, as depicted in Figs. 7.13 and 7.14, for both suboptimal and optimal networks, respectively.

7.8 Summary

A new method of reducing both the energy consumption and BER of BAF-MR networks is presented in this chapter. The method integrates the BER and relay location with the optimal balancing of the *EE* and *SE*. A flat fading channel and a relay selection scheme are used for this work. The proposal allows evaluation of the energy consumption when a selected relay is placed at any point on the line connecting the source and its destination. Furthermore, the minimum evaluated energy is used to obtain the optimal balance between the *EE* and *SE*. This balance enables the *EE* to increase significantly with the least loss in *SE*. Such a balance is then expressed with respect to the BER. Results derived from the analytical expressions were simulated numerically. It was found that the optimal balance between the *EE* and *SE* allows the *EE* to be increased and decreases the BER of a BAF-MR network. However, such an increase in *EE* should remain within small limits.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

Given the importance of the energy consumption of wireless networks, several aspects related to reducing it in wireless AF relay networks have been presented in this thesis. The main aspect has been the calculation of *SNR* at high and low levels. The high *SNR* level is the main level adopted for analyzing the ABER, EBER and optimally balanced *EE* and *SE*. Such a balance of the *EE* and *SE* allows the energy consumption in AF relay networks to be reduced with the minimum loss of spectrum efficiency. Also, the obtained optimal balance between *EE* and *SE* has been used to determine the optimal *SNR* level. Thus, the error rate performance of wireless AF relay networks has been evaluated for different *SNR* levels: high, low and optimal. All the mentioned analysis have been achieved for AF relay networks, when operating either unidirectionally or bidirectionally.

8.1.1 Unidirectional Scenario

We have first proposed two methods, namely U-BER and O-BER, to calculate the ABER performance of two-hop unidirectional AF-relay networks. The network model assumed that the channel hops are located in two different environments. Under this assumption, the error rate expressions for U-BER and O-BER methods are derived. It has been found that the U-BER method unifies the error rate performance analysis for both two-hop AF relays and the one-hop communication system. The O-BER method is an appropriate tool for estimating the error rate performance at the optimal *SNR* level, which is obtained by balancing the *EE* and *SE* optimally. Both methods provided efficient ways for evaluating the ABER for the MAFR networks operating in optimal or sub-optimal conditions.

The above ABER methods have been enhanced by reducing the two-hop analysis to the traditional one-hop BER analysis. Furthermore, the ABER analysis is used directly to determine the EBER performance. The enhanced ABER allowed us to calculate the accurate BER of MAFR networks under any *SNR* level (i.e., low, high and optimal *SNR* levels) using either the ABER or EBER. All of the above analysis is examined to analyze the error rate performance of MAF-RS networks under low, high and optimal *SNR* levels, either by using asymptotic or exact BER analysis. In order to implement such solutions in practical MAFR networks, it is important to consider relay location in the analysis. Thus, the thesis presents a new method of reducing the energy consumption by combining the BER and the relay location with the optimal balancing of the *EE* and *SE*. The proposal allows analysis of the energy consumption, in which a selected relay can be placed at any point on the line connecting the transmitter and its destination. Furthermore, the minimum evaluated energy is used to obtain the optimal balance between the *EE* and *SE*. This balance enables the *EE* to increase significantly with the least loss in *SE*. Such a balance is then expressed with respect to BER. It was found that the optimal balance between the *EE* and *SE* allows the *EE* to be increased and decreases the BER of AF relay networks. However, such an increase in *EE* should remain within small limits.

8.1.2 Bidirectional Scenario

In a BAF-MR network, we also started by developing a new method for analyzing the ABER performance under low, high and optimal *SNR* levels. These analyses are achieved by using both AGM and $\mathscr{HM}$. It was found that both analyses give the same results at high and optimal *SNR* domains. However, at the low *SNR* domain, AGM gives a better result than $\mathscr{HM}$ particularly with high ζ. It was also found that the optimal *SNR* obtained from a balanced network has lower asymptotic BER than suboptimal networks.

To examine a BAF-MR network when each hop is located in the same or different environments, new ABER and EBER expressions are derived using conventional BER analysis of a one-hop network. Further, the ABER analysis is used to determine the EBER performance. The analytical results have shown that the difference between the ABER and EBER is marginal. This makes it possible to use either of them to obtain precise BER results for BAF-MR networks. Accordingly, the proposed ABER provides more accurate result than the existing ABER techniques. To combine the energy consumption with the above BER analysis, considering relay location, a new method that integrates BER, relay location and the optimal balancing of the *EE* and *SE* is presented in this thesis. This method allows the evaluation of the energy consumption, in which a selected relay can be placed at any point on the line connecting the transmitter and its destination. Furthermore, the minimum energy is used to obtain the optimal balance between the *EE* and *SE*. This balance enables the *EE* to increase significantly with the least loss in *SE*. Such a balance is then expressed with respect to BER. It was found that the optimal balance between the *EE* and *SE* allows an increase in the *EE* and decreases the BER of a BAF-MR relay network. However, such an increase in *EE* should remain within small limits.

8.2 Future Work

Although this thesis has covered a wide area of developing the *EE* and BER for wireless AF relay networks, both unidirectional and bidirectional, there are still some important issues that need to be investigated. Most of the thesis's investigation of the *EE* and BER metrics assumed flat fading channels. Evaluation of both metrics for channels of a more complicated nature, such as the frequency-selective channels, needs to be considered. Furthermore, the thesis has focused on the BPSK modulation technique in which M=2 in the M-ary PSK modulation techniques.The higher orders of M (e.g. M: 4, 16, 64, 128, . . .) still require investigation. Also, most of the BPSK technique considered only single-antenna systems. This work needs to be extended to Multiple-Input-Multiple-Output (MIMO) systems, which use multiple antennas at the transmitter and receiver ends of a wireless communication system. Such MIMO systems are being adopted in communication systems to increase capacity of the networks. It is combined with Orthogonal Frequency-Division Multiple Access (OFDMA) techniques, to increase enhancing of capacity. Thus, the proposed *EE* and BER metrics for MIMO-OFDMA techniques need to be investigated. More scope for the future work can be summarized by using all the proposed methods for multiple users (more than two users) instead of the system assumption of only two users.

References

[1] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Communications Magazine*, vol. 42, no. 9, pp. 80–89, Sept 2004. 1.1

[2] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for wimax and lte-advanced mobile systems," *IEEE Communications Magazine*, vol. 47, no. 10, pp. 100–105, October 2009. 1.1, 1.4.1

[3] K. Loa, C. c. Wu, S. t. Sheu, Y. Yuan, M. Chion, D. Huo, and L. Xu, "Imt-advanced relay standards [wimax/lte update]," *IEEE Communications Magazine*, vol. 48, no. 8, pp. 40–48, August 2010. 1.1, 1.4.1

[4] C. Hoymann, W. Chen, J. Montojo, A. Golitschek, C. Koutsimanis, and X. Shen, "Relaying operation in 3gpp lte: challenges and solutions," *IEEE Communications Magazine*, vol. 50, no. 2, pp. 156–162, February 2012. 1.1

[5] L. Tianxi, S. Lingyang, L. Yonghui, H. Qiang, and J. Bingli, "Performance analysis of hybrid relay selection in cooperative wireless systems," *Communications, IEEE Transactions on*, vol. 60, no. 3, pp. 779–788, 2012. 1.1, 1.3, 2.1

[6] Asaduzzaman and H. Y. Kong, "Multi-relay cooperative diversity protocol with improved spectral efficiency," *Communications and Networks, Journal of*, vol. 13, no. 3, pp. 240–249, June 2011. 1.1

[7] K. Xie, J. Cao, X. Wang, and J. Wen, "Optimal resource allocation for reliable and energy efficient cooperative communications," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 10, pp. 4994–5007, October 2013. 1.1, 1.3.2

[8] M. I. Khalil, S. M. Berber, and K. W. Sowerby, "Error rate analysis of af-relay wireless networks under different snr levels," *Physical Communication*, vol. 20, pp. 85 – 92, 2016. 1.1, 4.2, 7.4

[9] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *Communications Magazine, IEEE*, vol. 42, no. 10, pp. 74–80, 2004. 1.1, 1.4.1

[10] Z. Ding, I. Krikidis, B. Rong, J. S. Thompson, C. Wang, and S. Yang, "On combating the half-duplex constraint in modern cooperative networks: protocols and techniques," *IEEE Wireless Communications*, vol. 19, no. 6, pp. 20–27, December 2012. 1.1

[11] M. Dohler and Y. Li, "Wireless relay channel," in *Cooperative Communications*. John Wiley & Sons, Ltd, 2010, pp. 43–139. 1.2, 1.3, 2.4

[12] K. S. Vasileios, S. Dimitrios, and D. P. Athanasios, *Cooperative Transmission Techniques and Protocols in Wireless Sensor Networks*. CRC Press, 2012, pp. 127–148. 1.2

[13] J. Proakis and M. Salehi, *Digital Communications*. McGraw-Hill Education, 2007. 1.2, 1.3.1, 2.4, 2.5, 3.3, 3.3, 4.4, 4.4, 5.4, 5.4, 6.4, 6.4, 7.5, 7.5

[14] S. N. DIGGAVI, N. AL-DHAHIR, A. STAMOULIS, and A. R. CALDERBANK, "Great expectations: the value of spatial diversity in wireless networks," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 219–270, Feb 2004. 1.2

[15] Y.-K. R. Kwok and V. K. N. Lau, *Diversity Techniques*. Wiley-IEEE Press, 2007, pp. 87–107. 1.2

[16] F. B. Abdesslem, L. Iannone, M. D. de Amorim, K. Kabassanov, and S. Fdida, "On the feasibility of power control in current ieee 802.11 devices," in *Fourth Annual IEEE International Conference on Pervasive Computing and Communications Workshops (PERCOMW'06)*, March 2006, pp. 5 pp.–473. 1.2

[17] I. Krikidis, J. Thompson, S. Mclaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Communications Letters*, vol. 12, no. 4, pp. 235–237, April 2008. 1.3

[18] S. Yang and J. C. Belfiore, "Towards the optimal amplify-and-forward cooperative diversity scheme," *IEEE Transactions on Information Theory*, vol. 53, no. 9, pp. 3114–3126, Sept 2007. 1.3

[19] O. Waqar, M. Imran, M. Dianati, and R. Tafazolli, "Energy consumption analysis and optimization of ber-constrained amplify-and-forward relay networks," *Vehicular Technology, IEEE Transactions on*, vol. 63, no. 3, pp. 1256–1269, March 2014. 1.3, 2.1, 3.1, 4.1

[20] S. lvaroRicieriCastroe and A. o. Taufik, *Interference-Limited Fixed Relaying-Aided Macrocellular CDMA Networks*. CRC Press, 2013, pp. 165–210. 1.3, 1.3.2

[21] A. Ribeiro, C. Xiaodong, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *Wireless Communications, IEEE Transactions on*, vol. 4, no. 3, pp. 1264–1273, 2005. 1.3.1, 2.1, 3.1, 4.1

[22] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a rayleigh-fading environment," *Wireless Communications, IEEE Transactions on*, vol. 3, no. 5, pp. 1416–1421, 2004. 1.3.1, 2.1, 2.4, 3.1, 4.1

[23] A. Behnad and W. Xianbin, "Accuracy of harmonic mean approximation in performance analysis of multihop amplify-and-forward relaying," *Wireless Communications Letters, IEEE*, vol. 3, no. 2, pp. 125–128, 2014. 1.3.1, 2.1, 3.1, 4.1

[24] H. Qiang, L. Tianxi, S. Shaohui, S. Lingyang, and J. Bingli, "Selective combining for hybrid cooperative networks," *Communications, IET*, vol. 8, no. 4, pp. 471–482, 2014. 1.3.1, 2.1, 2.3, 2.8, 3.1, 3.2, 3.6, 4.1

[25] F. Carden, R. Henry, and R. Jedlicka, *Telemetry Systems Engineering*, 2nd ed. Norwood, MA, USA: Artech House, Inc., 2002. 1.3.1, 3.1, 4.1, 6.1, 6.4

[26] W. Zhengdao and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *Communications, IEEE Transactions on*, vol. 51, no. 8, pp. 1389–1398, 2003. 1.3.1, 2.1, 3.1

[27] M. O. Hasna and M. S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *Wireless Communications, IEEE Transactions on*, vol. 3, no. 6, pp. 1963–1968, 2004. 1.3.1, 1.4.1, 2.1, 2.4, 3.1

[28] M. Hasna and M.-S. Alouini, “Outage probability of multihop transmission over nakagami fading channels,” *Communications Letters, IEEE*, vol. 7, no. 5, pp. 216–218, May 2003. 1.3.1, 1.4.1, 2.1, 3.1

[29] G. Farhadi and N. C. Beaulieu, “On the performance of amplify-and-forward cooperative systems with fixed gain relays,” *Wireless Communications, IEEE Transactions on*, vol. 7, no. 5, pp. 1851–1856, 2008. 1.3.1, 2.1, 3.1

[30] M. Dohler and Y. Li, “Transparent relaying techniques,” in *Cooperative Communications*. John Wiley & Sons, Ltd, 2010, pp. 141–207. 1.3.1, 1.4.1, 2.1, 2.3, 2.3, 2.4, 2.8, 3.1, 3.2, 3.2, 3.5, 3.6, 4.2, 5.1, 5.5, 5.6

[31] W. Kezhi, C. Yunfei, M. S. Alouini, and X. Feng, “Ber and optimal power allocation for amplify-and-forward relaying using pilot-aided maximum likelihood estimation,” *Communications, IEEE Transactions on*, vol. 62, no. 10, pp. 3462–3475, 2014. 1.3.1, 2.1, 3.1

[32] S. S. Ikki and M. H. Ahmed, “On the performance of cooperative-diversity networks with the nth best-relay selection scheme,” *Communications, IEEE Transactions on*, vol. 58, no. 11, pp. 3062–3069, 2010. 1.3.1, 2.1

[33] S. S. Soliman and N. C. Beaulieu, “Exact analysis of dual-hop af maximum end-to-end snr relay selection,” *Communications, IEEE Transactions on*, vol. 60, no. 8, pp. 2135–2145, 2012. 1.3.1, 2.1

[34] A. Salhab and S. Zummo, “A low-complexity relay selection scheme based on switch-and-examine diversity combining for af relay systems,” *Communications, IET*, vol. 7, no. 9, pp. 848–859, June 2013. 1.3.1, 2.1, 3.1

[35] M. I. Khalil, S. M. Berber, and K. W. Sowerby, “Energy-efficient design for combined relay selection and power allocation af-relay network,” in *The 20th Asia-Pacific Conference on Communication (APCC2014)*, Oct 2014, pp. 417–422. 1.3.2, 6.3

[36] J. N. Laneman and G. W. Wornell, “Energy-efficient antenna sharing and relaying for wireless networks,” in *Wireless Communications and Networking Confernce, 2000. WCNC. 2000 IEEE*, vol. 1, 2000, pp. 7–12 vol.1. 1.3.2

[37] J.-Y. Song, H. Lee, and D.-H. Cho, "Power consumption reduction by multi-hop transmission in cellular networks," in *Vehicular Technology Conference, 2004. VTC2004-Fall. 2004 IEEE 60th*, vol. 5, Sept 2004, pp. 3120–3124 Vol. 5. 1.3.2

[38] A. Radwan and H. S. Hassanein, "Nxg04-3: Does multi-hop communication extend the battery life of mobile terminals?" in *IEEE Globecom 2006*, Nov 2006, pp. 1–5. 1.3.2

[39] Y. Chen, S. Zhang, S. Xu, and G. Li, "Fundamental trade-offs on green wireless networks," *Communications Magazine, IEEE*, vol. 49, no. 6, pp. 30–37, June 2011. 1.3.2, 2.7, 3.5, 7.1, 7.4

[40] Y. Chen, S. Zhang, and S. Xu, "Fundamental trade-offs on the design of green radio networks," in *Green Radio Communication Networks*, E. Hossain, V. K. Bhargava, and G. P. Fettweis, Eds. Cambridge University Press, 2012, pp. 3–23, cambridge Books Online. [Online]. Available: http://dx.doi.org/10.1017/CBO9781139084284.002 1.3.2, 4.3, 7.4

[41] u. Budzisz and A. Wolisz, *Green Home and Enterprise Networks*. John Wiley & Sons, Ltd, 2015, pp. 179–198. 1.3.2

[42] C. Tou and D. Roy, "On efficient spectrum utilization from the standpoint of communication theory," *Proceedings of the IEEE*, vol. 68, no. 12, pp. 1460–1465, Dec 1980. 1.3.2, 4.1

[43] Y. Li and Z. Zheng, "Energy-efficient power allocation for two-hop relay networks," *Electronics Letters*, vol. 50, no. 2, pp. 123–125, 2014. 1.3.2, 2.1, 3.1, 3.5, 4.1, 5.5, 5.5, 7.4

[44] Y. Li, Z. Zheng, M. Zhao, Y. Chen, and C. Liu, "Energy efficient design for two-way af relay networks," *International Journal of Antennas and Propagation*, vol. 2014, p. 6, 2014. 1.3.2, 1.4.2, 3.1, 3.5, 4.1, 5.5

[45] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4152–4172, Dec 2005. 1.4.1

[46] H. Bolcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in mimo relay networks," *IEEE Transactions on Wireless Communications*, vol. 5, no. 6, pp. 1433–1444, June 2006. 1.4.1

[47] M. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over rayleigh-fading channels," *Wireless Communications, IEEE Transactions on*, vol. 2, no. 6, pp. 1126–1131, Nov 2003. 1.4.1, 4.4, 5.3, 5.4, 5.4, 6.1, 7.5

[48] ——, "Harmonic mean and end-to-end performance of transmission systems with relays," *Communications, IEEE Transactions on*, vol. 52, no. 1, pp. 130–135, Jan 2004. 1.4.1, 3.6, 4.4, 5.1, 5.4, 6.1, 7.5

[49] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2415–2425, Oct 2003. 1.4.1

[50] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *Information Theory, IEEE Transactions on*, vol. 50, no. 12, pp. 3062–3080, 2004. 1.4.1, 2.1

[51] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1099–1109, Aug 2004. 1.4.1

[52] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity. part i. system description," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1927–1938, Nov 2003. 1.4.1

[53] E. Elsheikh, K.-K. Wong, Y. Zhang, and T. Cui, "Chapter 10 - user cooperative communications," in *Cognitive Radio Communications and Networks*, A. M. W. N. T. Hou, Ed. Oxford: Academic Press, 2010, pp. 261 – 305. 1.4.1, 2.1

[54] M. I. Khalil, S. M. Berber, and K. W. Sowerby, "Bit error rate performance analysis in amplify-and-forward relay networks," *Wireless Networks*, pp. 1–11, 2016. [Online]. Available: http://dx.doi.org/10.1007/s11276-016-1196-0 1.4.1, 5.3, 7.3

[55] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 2, pp. 379–389, February 2007. 1.4.2, 6.3, 6.3, 7.4

[56] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2020–2040, June 2005. 1.4.2

[57] B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of mimo relay channels," *IEEE Transactions on Information Theory*, vol. 51, no. 1, pp. 29–43, Jan 2005. 1.4.2

[58] C. Zhang, J. Ge, J. Li, F. Gong, Y. Ji, and M. A. Farah, "Energy efficiency and spectral efficiency tradeoff for asymmetric two-way af relaying with statistical csi," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, pp. 2833–2839, April 2016. 1.4.2

[59] Q. Cui, X. Yang, J. Hämäläinen, X. Tao, and P. Zhang, "Optimal energy-efficient relay deployment for the bidirectional relay transmission schemes," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 6, pp. 2625–2641, July 2014. 1.4.2

[60] M. I. Khalil, S. M. Berber, and K. W. Sowerby, "A standard ber analysis for two-way relay networks at high and optimal snr domains," in *2016 8th IEEE International Conference on Communication Software and Networks (ICCSN)*, June 2016, pp. 166–170. 1.4.2

[61] M. Torabi, D. Haccoun, and J. F. Frigon, "Relay selection in af cooperative systems: An overview," *Vehicular Technology Magazine, IEEE*, vol. 7, no. 4, pp. 104–113, 2012, iD: 1. 2.1

[62] K. D. Wong, *Propagation*, ser. Fundamentals of Wireless Communication Engineering Technologies. John Wiley & Sons, Inc., 2011; 2011, pp. 125–155. 2.1

[63] K. J. R. Liu, A. K. Sadek, W. Su, and A. Kwasinski, "Multi-node cooperative communications," in *Cooperative Communications and Networking*. Cambridge University Press, 2008, pp. 194–237, cambridge Books Online. 2.1, 3.1, 4.1

[64] W. Zhang, D. Duan, and L. Yang, "Relay selection from a battery energy efficiency perspective," *Communications, IEEE Transactions on*, vol. 59, no. 6, pp. 1525–1529, June 2011. 2.1, 3.1, 4.1

[65] S. R. Bullock, *Transceiver and System Design for Digital Communications.* SciTech, 2014. [Online]. Available: http://ezproxy.auckland.ac.nz/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=nlebk&AN=802460&site=ehost-live&scope=site 2.4

[66] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005. 2.4, 3.2, 4.2, 4.5, 5.4, 6.5, 7.6

[67] R. Prasad and A. Mihovska, “Protocols and algorithms for ad hoc networks,” in *New Horizons in Mobile and Wireless Communications: Ad hoc networks and PANs*. Artech House, 2009, pp. 25–127. 2.4

[68] M. Simon, *Probability Distributions Involving Gaussian Random Variables: A Handbook for Engineers and Scientists*. Springer US, 2007. [Online]. Available: https://books.google.co.nz/books?id=zjJdP0CJUAYC 2.4

[69] “Linear vector spaces,” in *Advanced Linear Algebra for Engineers with MATLAB*. CRC Press, 2009, pp. 105–155. [Online]. Available: http://dx.doi.org/10.1201/b15839-4 2.4

[70] H. David and H. Nagaraja, *Order Statistics*. Wiley, 2004. 2.4, 5.3, 6.4

[71] H.-M. Kaltenbach, “Basics of probability theory,” in *A Concise Guide to Statistics*, ser. SpringerBriefs in Statistics. Springer Berlin Heidelberg, 2012/01/01 2012, ch. 1, pp. 1–27. 2.4, 5.3, 6.4

[72] J. Salo, H. M. El-Sallabi, and P. Vainikainen, “The distribution of the product of independent rayleigh random variables,” *Antennas and Propagation, IEEE Transactions on*, vol. 54, no. 2, pp. 639–643, 2006. 2.4

[73] C. S. Patel, G. L. Stuber, and T. G. Pratt, “Statistical properties of amplify and forward relay fading channels,” *Vehicular Technology, IEEE Transactions on*, vol. 55, no. 1, pp. 1–9, 2006. 2.4

[74] G. K. Karagiannidis, “Performance bounds of multihop wireless communications with blind relays over generalized fading channels,” *Wireless Communications, IEEE Transactions on*, vol. 5, no. 3, pp. 498–503, 2006. 2.4, 5.1

[75] K. Pahlavan and P. Krishnamurthy, *Physical Layer Alternatives forWireless Networks*. Wiley, 2013, pp. 99–145. 2.4, 3.3, 4.4, 5.4, 6.4, 7.5

[76] J. Komo, “Functions of random variables,” in *Random Signal Analysis in Engineering Systems*. Elsevier Science, 1987, ch. 3, pp. 89–123. 2.4, 3.3, 6.4

[77] “Chapter {II} classical variational methods,” in *Variational Methods in Statistics*, ser. Mathematics in Science and Engineering, J. S. Rustagi, Ed. Elsevier, 1976, vol. 121, pp. 16

– 45. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0076539208602209 2.4, 2.4

[78] A. Papadogiannis, G. Alexandropoulos, A. Burr, and D. Grace, "Bringing mobile relays for wireless access networks into practice - learning when to relay," *Communications, IET*, vol. 6, no. 6, pp. 618–627, April 2012. 2.6

[79] W. Cho, R. Cao, and L. Yang, "Optimum resource allocation for amplify-and-forward relay networks with differential modulation," *Signal Processing, IEEE Transactions on*, vol. 56, no. 11, pp. 5680–5691, Nov 2008. 2.6

[80] S. Mallick, P. Kaligineedi, M. M. Rashid, and V. K. Bhargava, "Radio resource optimization in cooperative cellular wireless networks," in *Cooperative Cellular Wireless Networks*, E. Hossain, D. I. Kim, and V. K. Bhargava, Eds. Cambridge University Press, 2011, pp. 205–232, cambridge Books Online. 2.6

[81] M. Zaeri-Amirani, S. ShahbazPanahi, T. Mirfakhraie, and K. Ozdemir, "Performance tradeoffs in amplify-and-forward bidirectional network beamforming," *Signal Processing, IEEE Transactions on*, vol. 60, no. 8, pp. 4196–4209, Aug 2012. 2.6

[82] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, "Energy- and spectral-efficiency tradeoff in downlink ofdma networks," *Wireless Communications, IEEE Transactions on*, vol. 10, no. 11, pp. 3874–3886, November 2011. 2.7, 4.1, 4.3, 5.5, 7.4

[83] G. Miao, N. Himayat, and G. Li, "Energy-efficient link adaptation in frequency-selective channels," *Communications, IEEE Transactions on*, vol. 58, no. 2, pp. 545–554, February 2010. 2.7

[84] E. McCune, "Common issues and signal characterization," in *Practical Digital Wireless Signals*. Cambridge University Press, 2010, pp. 23–73, cambridge Books Online. 2.7, 3.5, 5.5

[85] P. GUPTA, *DATA COMMUNICATIONS AND COMPUTER NETWORKS*. PHI Learning, 2013. 3.1, 4.1

[86] Y. Fu, L. Yang, W.-P. Zhu, and C. Liu, "Optimal relay precoding for two-hop af transmission and performance analysis over rayleigh-fading channels," *Communications, IEEE Transactions on*, vol. 59, no. 11, pp. 3072–3079, November 2011. 3.1, 3.5, 3.6

[87] O. Amin, R. Mesleh, S. Ikki, M. Ahmed, and O. Dobre, "Performance analysis of multiple-relay cooperative systems with signal space diversity," *Vehicular Technology, IEEE Transactions on*, vol. 64, no. 8, pp. 3414–3425, Aug 2015. 3.1, 3.6, 6.1, 6.3, 6.7

[88] A. Reznik, S. R. Kulkarni, and S. Verdu, "Degraded gaussian multirelay channel: capacity and optimal power allocation," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3037–3046, Dec 2004. 3.1, 3.5, 5.5

[89] M. Luo, G. Villemaud, J. M. Gorce, and J. Zhang, "Realistic prediction of ber and amc for indoor wireless transmissions," *IEEE Antennas and Wireless Propagation Letters*, vol. 11, pp. 1084–1087, 2012. 3.1, 4.1, 7.1

[90] J. Ramakrishna, "Spatial diversity for wireless communications," in *Handbook of Antennas in Wireless Communications*, ser. Electrical Engineering & Applied Signal Processing Series. CRC Press, 2001. 3.2, 5.2, 6.2, 7.2

[91] U. Madhow and U. Madhow, *Demodulation Fundamentals of Digital Communication*. Cambridge University Press, 2008. 3.3, 5.4, 6.4, 7.5

[92] U. Madhow, *Probability and random process*. Cambridge University Press, 2014, book section 5, pp. 207–304. 3.3, 4.4, 5.4, 6.4, 7.5

[93] S. Ross, "Chapter 2 - elements of probability," in *Simulation (Fifth Edition)*, S. Ross, Ed. Academic Press, 2013, pp. 5–38. 3.3, 4.4, 6.6, 7.5

[94] A. Molisch, "Diversity," in *Wireless Communications*, 2nd ed. Wiley, 2012, pp. 249–275. 3.4, 6.5

[95] H.-C. Yang and M.-S. Alouini, "Distributions of order statistics," in *Order Statistics in Wireless Communications*. Cambridge University Press, 2011, pp. 40–71, cambridge Books Online. 3.4, 4.5, 5.4, 5.4, 6.5, 7.6

[96] D. Hsu, *Appendix A: Probability Density Functions*. Wiley-IEEE Press, 1999, pp. 129–132. [Online]. Available: http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=5273272 3.4

[97] P. A. Bromiley, "Products and Convolutions of Gaussian Probability Density Functions," Tech. Rep. 3.4

[98] E. Robinson, “Multivariate distributions,” in *Probability Theory and Applications*. Springer Netherlands, 1985, pp. 125–187. 3.4

[99] I. Glover and P. M. Grant, *Digital Communications*. Prentice Hall, 1998. [Online]. Available: https://books.google.com.au/books?id=0BofAQAAIAAJ 3.4

[100] H. H. Nguyen and E. Shwedyk, “Ary signaling techniques,” in *A First Course in Digital Communications*. Cambridge University Press, 2009, pp. 302–342, cambridge Books Online. 3.4

[101] F. Xiong, *Digital Modulation Techniques, Second Edition (Artech House Telecommunications Library)*. Norwood, MA, USA: Artech House, Inc., 2006. 3.4

[102] M. Shikh-Bahaei, “Joint optimization of "transmission rate" and "outer-loop snr target" adaptation over fading channels,” *IEEE Transactions on Communications*, vol. 55, no. 3, pp. 398–403, March 2007. 3.5

[103] O. Amin, S. Bavarian, and L. Lampe, “Cooperative techniques for energy-efficient wireless communications,” in *Green Radio Communication Networks*, E. Hossain, V. K. Bhargava, and G. P. Fettweis, Eds. Cambridge University Press, 2012, pp. 125–149, cambridge Books Online. 3.5, 3.5, 4.1, 4.3, 4.6, 5.5, 7.1, 7.1, 7.4

[104] G. Farhadi and N. C. Beaulieu, “On the ergodic capacity of multi-hop wireless relaying systems,” *IEEE Transactions on Wireless Communications*, vol. 8, no. 5, pp. 2286–2291, May 2009. 3.5

[105] G. Miao and G. Song, “Asymptotic performance analysis for channel-aware scheduling,” in *Energy and Spectrum Efficient Wireless Network Design*. Cambridge University Press, 2014, pp. 126–146, cambridge Books Online. 3.5

[106] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, “On the lambertw function,” *Advances in Computational Mathematics*, vol. 5, no. 1, pp. 329–359, 1996. 3.5, 4.3, 4.5, 5.5, 7.4, 7.6

[107] R. Madan, N. B. Mehta, A. F. Molisch, and J. Zhang, “Energy-efficient cooperative relaying over fading channels with simple relay selection,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 8, pp. 3013–3025, August 2008. 4.1, 7.1

[108] Y. Yao, X. Cai, and G. B. Giannakis, "On energy efficiency and optimum resource allocation of relay transmissions in the low-power regime," *IEEE Transactions on Wireless Communications*, vol. 4, no. 6, pp. 2917–2927, Nov 2005. 4.1, 7.1

[109] C. Bae and W. Stark, "End-to-end energy bandwidth tradeoff in multihop wireless networks," *Information Theory, IEEE Transactions on*, vol. 55, no. 9, pp. 4051–4066, Sept 2009. 4.1, 7.1

[110] C. L. Chen, W. E. Stark, and S. G. Chen, "Energy-bandwidth efficiency tradeoff in mimo multi-hop wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1537–1546, September 2011. 4.1, 7.1

[111] R. Huang, C. Feng, T. Zhang, and W. Wang, "Energy-efficient relay selection and power allocation scheme in af relay networks with bidirectional asymmetric traffic," in *Wireless Personal Multimedia Communications (WPMC), 2011 14th International Symposium on*, Oct 2011, pp. 1–5. 4.1, 7.1

[112] C. Sun and C. Yang, "Energy efficiency analysis of one-way and two-way relay systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2012, no. 1, pp. 1–18, 2012. [Online]. Available: http://dx.doi.org/10.1186/1687-1499-2012-46 4.1, 7.1

[113] C. Sun, Y. Cen, and C. Yang, "Energy efficient ofdm relay systems," *Communications, IEEE Transactions on*, vol. 61, no. 5, pp. 1797–1809, May 2013. 4.1, 7.1

[114] L. Wei, R. Hu, Y. Qian, and G. Wu, "Energy efficiency and spectrum efficiency of multihop device-to-device communications underlaying cellular networks," *Vehicular Technology, IEEE Transactions on*, vol. 65, no. 1, pp. 367–380, Jan 2016. 4.1, 7.1

[115] Y. Chen, S. Zhang, and S. Xu, "Characterizing energy efficiency and deployment efficiency relations for green architecture design," in *Communications Workshops (ICC), 2010 IEEE International Conference on*, May 2010, pp. 1–5. 4.3, 7.4

[116] F. de Chaves, F. Cavalcanti, R. de Oliveira Neto, and R. Santos, "Power control for wireless networks: Conventional and qos-flexible approaches," in *Optimizing Wireless Communication Systems*, F. R. P. Cavalcanti and S. Andersson, Eds. Springer US, 2009, pp. 3–49. 4.3, 7.4

[117] S. lvaroRicieriCastroe and A. o. Taufik, *Interference-Limited Fixed Relaying-Aided Macrocellular CDMA Networks*. CRC Press, 2013, pp. 165–210, doi:10.1201/b15571-11. 4.3, 5.5, 7.4

[118] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *Information Theory, IEEE Transactions on*, vol. 55, no. 6, pp. 2499–2517, June 2009. 4.3, 7.4

[119] R. H. Louie, Y. Li, H. Suraweera, and B. Vucetic, "Performance analysis of beamforming in two hop amplify and forward relay networks with antenna correlation," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 6, pp. 3132–3141, June 2009. 4.4, 5.4, 5.4, 7.5

[120] S. L. Miller and D. Childers, "{CHAPTER} 6 - multiple random variables," in *Probability and Random Processes (Second Edition)*, second edition ed., S. L. M. Childers, Ed. Boston: Academic Press, 2012, pp. 245 – 288. 4.5, 6.5, 7.6

[121] H. Li and Q. Zhao, "Distributed modulation for cooperative wireless communications," *Signal Processing Magazine, IEEE*, vol. 23, no. 5, pp. 30–36, Sept 2006. 4.6, 7.7

[122] M. Dohler and Y. Li, *Introduction*. John Wiley & Sons, Ltd, 2010, pp. 1–41. 5.1, 6.1

[123] Z. Wang and G. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *Communications, IEEE Transactions on*, vol. 51, no. 8, pp. 1389–1398, Aug 2003. 5.1, 6.1

[124] Y. Han, S. H. Ting, C. K. Ho, and W. H. Chin, "Performance bounds for two-way amplify-and-forward relaying," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 1, pp. 432–439, Jan 2009. 5.1

[125] S. Lingyang, "Relay selection for two-way relaying with amplify-and-forward protocols," *Vehicular Technology, IEEE Transactions on*, vol. 60, no. 4, pp. 1954–1959, 2011. 5.1, 6.1, 6.7

[126] S. S. Ikki and S. Aissa, "Two-way amplify-and-forward relaying with gaussian imperfect channel estimations," *Communications Letters, IEEE*, vol. 16, no. 7, pp. 956–959, 2012. 5.1, 6.1, 6.7

[127] M. Xia and S. Aissa, "Moments based framework for performance analysis of one-way/two-way csi-assisted af relaying," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 8, pp. 1464–1476, 2012, iD: 1. 5.1, 6.1

[128] C. Hongyu, Z. Rongqing, S. Lingyang, and J. Bingli, "Relay selection for bidirectional af relay network with outdated csi," *Vehicular Technology, IEEE Transactions on*, vol. 62, no. 9, pp. 4357–4365, 2013. 5.1, 6.1, 6.5, 6.7

[129] S. Atapattu, Y. Jing, H. Jiang, and C. Tellambura, "Relay selection schemes and performance analysis approximations for two-way networks," *Communications, IEEE Transactions on*, vol. 61, no. 3, pp. 987–998, March 2013. 5.1, 6.1

[130] Y. Liang, K. Qaraqe, E. Serpedin, and M. S. Alouini, "Performance analysis of amplify-and-forward two-way relaying with co-channel interference and channel estimation error," *Communications, IEEE Transactions on*, vol. 61, no. 6, pp. 2221–2231, 2013. 5.1, 6.1

[131] K. Song, B. Ji, Y. Huang, M. Xiao, and L. Yang, "Performance analysis of antenna selection in two-way relay networks," *Signal Processing, IEEE Transactions on*, vol. 63, no. 10, pp. 2520–2532, May 2015. 5.1, 5.2, 6.3, 6.7, 7.1

[132] L. Yang, K. Qaraqe, E. Serpedin, and X. Gao, "Performance analysis of two-way relaying networks with the n th worst relay selection over various fading channels," *Vehicular Technology, IEEE Transactions on*, vol. 64, no. 7, pp. 3321–3327, July 2015. 5.1, 6.1, 7.1

[133] S. Talwar, Y. Jing, and S. ShahbazPanahi, "Joint relay selection and power allocation for two-way relay networks," *Signal Processing Letters, IEEE*, vol. 18, no. 2, pp. 91–94, Feb 2011. 5.1

[134] T. P. Do, J. S. Wang, I. Song, and Y. H. Kim, "Joint relay selection and power allocation for two-way relaying with physical layer network coding," *Communications Letters, IEEE*, vol. 17, no. 2, pp. 301–304, February 2013. 5.1

[135] Y. Li, R. H. Louie, and B. Vucetic, "Relay selection with network coding in two-way relay channels," *Vehicular Technology, IEEE Transactions on*, vol. 59, no. 9, pp. 4489–4499, Nov 2010. 5.1

[136] Z. Fang, F. Liang, L. Li, and L. Jin, "Performance analysis and power allocation for two-way amplify-and-forward relaying with generalized differential modulation," *Vehicular Technology, IEEE Transactions on*, vol. 63, no. 2, pp. 937–942, Feb 2014. 5.1, 6.1, 7.1

[137] K. Pahlavan and A. Levesque, *Wireless Information Networks*. Wiley, 2005. 5.1, 6.1

[138] M. I. Khalil, S. M. Berber, and K. W. Sowerby, "Energy efficiency and spectrum efficiency balance of wireless relay networks," in *2016 8th IEEE International Conference on Communication Software and Networks (ICCSN)*, June 2016, pp. 397–401. 5.2

[139] M. Pischella and D. Le Ruyet, "Optimal power allocation for the two-way relay channel with data rate fairness," *Communications Letters, IEEE*, vol. 15, no. 9, pp. 959–961, September 2011. 5.2

[140] R. M. Gray and L. D. Davisson, "Expectation and averages," in *An Introduction to Statistical Signal Processing*. Cambridge University Press, 2004, pp. 182–274, cambridge Books Online. 5.3

[141] D. P. Dubhashi, A. Panconesi, D. P. Dubhashi, and A. Panconesi., *Martingales and the Method of Bounded Differences Concentration of Measure for the Analysis of Randomized Algorithms*. Cambridge University Press, 2009. 5.3

[142] P. Das and N. Mehta, "Direct link-aware optimal relay selection and a low feedback variant for underlay cr," *Communications, IEEE Transactions on*, vol. 63, no. 6, pp. 2044–2055, June 2015. 5.3

[143] G. Quin, B. Pinel-Puysségur, and J.-M. Nicolas, "Comparison of Harmonic, Geometric and Arithmetic means for change detection in SAR time series," in *EUSAR. 9th European Conference on Synthetic Aperture Radar, 2012.*, Germany, Apr. 2012. 5.6

[144] R. Cao and L. Yang, "The affecting factors in resource optimization for cooperative communications: A case study," *IEEE Transactions on Wireless Communications*, vol. 11, no. 12, pp. 4351–4361, December 2012. 6.1

[145] H. Nguyen, H. Nguyen, and T. Le-Ngoc, "Diversity analysis of relay selection schemes for two-way wireless relay networks," *Wireless Personal Communications*, vol. 59, no. 2, pp. 173–189, 2011. 6.1

[146] J. Romero, O. Sallent, R. Agusti, and M. Diaz-Guerra, *Radio Resource Management Strategies in UMTS*. Wiley, 2005. 6.1

[147] M. Obaidat and S. Misra, *Cooperative Networking*. Wiley, 2011, book section 12, pp. 192–214. 6.1

[148] M. Yacoub, *Foundations of Mobile Radio Engineering*. Taylor & Francis, 1993. 6.2

[149] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," in *Vehicular Technology Conference, 2006. VTC 2006-Spring. IEEE 63rd*, vol. 2, May 2006, pp. 588–593. 7.1

[150] H. Chen, G. Li, and J. Cai, "Spectralenergy efficiency tradeoff in full-duplex two-way relay networks," *IEEE Systems Journal*, vol. PP, no. 99, pp. 1–10, 2015. 7.1

[151] M. I. Khalil, S. M. Berber, and K. W. Sowerby, "Energy efficiency and spectrum efficiency tradeoff over optimal relay location in bidirectional relay networks," in *2016 IEEE-APS Topical Conference on Antennas and Propagation in Wireless Communications (APWC)*, Sept 2016, pp. 298–302. 7.3

[152] E. Ahmed and A. Eltawil, "All-digital self-interference cancellation technique for full-duplex systems," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 7, pp. 3519–3532, July 2015. 7.4