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Composite Habits and International Transmission of Business Cycles

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Abstract

This paper shows how internal habit formation defined over a composite of consumption and leisure can bring a two-country business cycle model closer to the data. Conditional on productivity shocks, our model reconciles with the data by closing the gap between cross-country correlations of consumption and output ('quantity anomaly'). It also predicts positive international correlations of investment and employment of the magnitude observed in the data ('international comovement puzzle'). In other words, a rather parsimonious departure from a canonical two-country, two-good model goes a long way toward addressing two long-standing puzzles in international macroeconomics.

Keywords : Quantity anomaly; International comovement puzzle; Composite habits; Wealth effects

JEL Codes: E32; F41; G15

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1 Introduction

International business cycle models have been widely used to analyze the transmission of macroeconomic disturbances across national borders. Early contributions established that a two-country versions of the real-business-cycle (RBC) model with complete markets fail to account for several important properties of the data (Backus et al., 1992, 1994; Baxter and Crucini, 1995). The model predicts that consumption is strongly correlated across countries, while output is not. Examining data for the U.S., in conjunction with some proxy for its industrialized trading partners, suggests that the opposite is true. Backus et al. (1992) dubbed this discrepancy the 'quantity anomaly'. Another problematic prediction of the model concerns cross-country correlations of investment and employment. While the model predicts negative comovement of investment and employment, the data suggest the opposite. Baxter (1995) branded this discrepancy as the 'international comovement puzzle'.

These discrepancies with the data stem from several features of the canonical model's transmission mechanism for technology shocks. To understand these transmission channels, suppose for a moment, that two countries produce a single homogenous good. Now, suppose a shock makes the home country relatively more productive. Three effects will affect international comovements. First, a persistent differential in marginal products of capital induces capital reallocation to the most productive economy. This effect, which Baxter (1995) refers to as "make hay where sun shines", translates into the negative comovement of domestic and foreign investments. Second, the wealth effect compels agents abroad to consume more of every normal good. Since unchanged foreign real wages produce no substitution effect to counter the wealth effect, foreign agents end up with more consumption and leisure. In other words, workers in the unlucky country go on a "paid vacation", whereas workers in the lucky country work more hours. The negative comovement of domestic and foreign employments partly offsets the positive comovement of productivity leaving little synchronization in the output levels. Third, complete markets require that marginal utilities of consumption be equated across countries. Time-separable preferences translate this equality of marginal utilities into a high correlation of consumption. Consequently, the model predicts highly correlated consumptions and little correlated outputs, whereas data suggest the opposite.

All three effects could be ameliorated by terms of trade (TOT) movements in two-good economies. A relative abundance of goods produced in the more productive country coupled with home bias in consumption leads to deterioration in the domestic TOT. This change in the TOT reduces the difference in marginal products of capital, weakening the "make hay where sun shines" effect. It increases the foreign real wage, which compels foreign workers to substitute away from relatively more expensive leisure, and this weakens the "paid vacation" effect. Finally, a fluctuating TOT drives a wedge between marginal utilities of consumption at home and abroad. This makes the ratio of marginal utilities change over time, leading to lower correlation of domestic and foreign consumption levels. As a result, two-good models are often better able to account for the two puzzles than outlined their single-good counterparts. Nonetheless, these two puzzles still constitute a challenge for many two-country models.

This paper analyses how a departure from the assumption of separability of preferences over time helps a prototypical two-country model to account for the pattern of international comovements observed at business cycle frequencies. A departure from time-separable preferences simultaneously suppresses these three problematic transmission channels. First, our time-nonseparable preferences break the link between the proportionality of marginal utilities and strong consumption correlations. Second, they eliminate the "paid vacation" phenomenon by reducing wealth effects on labor supply to

an arbitrarily low level. Third, in conjunction with capital adjustment costs, our time-nonseparable preferences diminish the "make hay where sun shines" channel. By suppressing the problematic transmission channels, our preference specification brings the model closer to the data by improving its ability to account for the 'quantity anomaly' and 'international comovement puzzle.'

Often, deviation from time-separable preferences is achieved by introducing habit formation or a recursive preference structure.[1] Habits are typically defined over an aggregate consumption good as in Boldrin et al. (2001), individual varieties of goods as in Ravn et al. (2006), or leisure as in Lettau and Uhlig (2000). By contrast, we consider a preference ordering, introduced by Jaccard (2011, 2014), that defines internal habit formation over a composite of consumption and leisure.

Under standard preferences, agents care about smoothing the level of the composite good comprised of consumption and leisure. Defining habit formation over the composite compels the agents to care not only about smoothing its level, but also about smoothing its growth rate. Intuitively, the agents become very averse to rapid simultaneous decreases in consumption and leisure. In other words, they are very reluctant to increase their working hours after a sudden fall in consumption. As in Heathcote and Perri (2002), we consider the two-country, two-good environment of Backus, Kehoe, and Kydland (1994) (henceforth, BKK) under different assumptions about market incompleteness. We augment the model with composite habits and estimate its preference parameters using the simulated method of moments (SMM).

The results of our simulations show that introducing composite habits improves the empirical performance of the model along several dimensions. First, the model matches the data by delivering positive cross-country correlations of employment and investment. Second, the model predicts cross-country consumption and output correlations of the same magnitude as each other. By contrast, an identically parameterized model with time-separable preferences is at odds with the data: it counterfactually predicts negative international correlations of employment and investment ('international comovement puzzle') and international consumption correlation much in excess of the output correlation ('quantity anomaly'). Getting investment comovement right in our model right does not come at a cost of counterfactual implications for the trade balance.[2] Our model is successful at predicting countercyclical net exports and the S-shaped pattern of cross-correlations between the trade balance and the TOT. In addition, if we introduce stochastic trends in productivity, our model contributes to explaining the 'volatility puzzle' by delivering a realistically high volatility in the terms of trade. Finally, our model is consistent with balanced growth.

The intuition behind our results relates to several properties of composite internal habits. First, the degree of habit formation governs the wedge between wealth elasticity and Frisch elasticity of labor supply. For a given level of Frisch elasticity, increasing the intensity and reducing the persistence of habits scales down the wealth effect to a sufficiently low level. A lower wealth elasticity of labor supply contributes to explaining the positive international comovement of employment and output, that is, it helps the model address both puzzles.

Second, composite habits reduce the Edgeworth substitutability between consumption and leisure. A lower substitutability between consumption and labor contributes to obtaining a lower cross-country correlation of consumption despite strongly linked marginal utilities of consumption. Hence, composite habits make consumption levels less synchronized while increasing the comovement of outputs. This

[1] See Backus et al. (2008) and references therein.

[2] Models that are successful at accounting for positive international comovement of investment often fail to account for countercyclical trade balance and vice versa. For instance, Kehoe and Perri (2002), Dmitriev and Roberts (2012), and Dmitriev and Krznar (2012) get investment comovement right at the cost of predicting procyclical net exports. On the other hand, Raffo (2008) obtains countercyclical net export only to end up with negative investment correlations.

helps the model to address the 'quantity anomaly'.

Third, under internal habits, consumers not only dislike changes in the composite of consumption and leisure but also changes in its growth rate. Following a positive productivity shock, domestic consumers are reluctant to rapidly increase their consumption and leisure simultaneously. As a result, output growth exceeds consumption growth, leading to a large increase in savings. These savings raise investment in both countries due movement in the TOT and sluggish capital adjustment. Thus, composite habits help us to explain the positive comovement of investment and employment across countries, which addresses the 'international comovement puzzle'. Fourth, and finally, composite habits are consistent with balanced growth, which allows us to consider an extension of our model with cointegrated productivity shocks. As in Rabanal et al. (2011), cointegrated shocks in our model generate realistic volatility in the terms of trade for a narrow range of trade elasticity.

The properties of composite habits which we used to explain international comovement have been shown to be instrumental in improving the performance of various macroeconomic models in a range of ways. First, the ability to reduce the wealth elasticity of labor supply proved important in many applications, including those which focus on the propagation of investment specific technology shocks (Greenwood et al., 1988; Mandelman et al., 2011), demand shocks Wen (2007), news shocks (Jaimovich and Rebelo, 2009), and government spending shocks (Monacelli and Perotti, 2008) [3]. Second, in contrast to the popular GHH preferences of Greenwood et al. (1988), composite habits are consistent with balanced growth. This allows researchers to consider the models with both deterministic and stochastic trends in forcing processes[4]. Third, composite habits alter Edgeworth substitutability between consumption and leisure. For instance, Bilbiie (2011) highlights the role of Edgeworth substitutability in understanding the effects of fiscal policy shocks on private consumption. Fourth, and finally, under composite habits agents dislike changes in the growth rate of consumption of the composite good. This might translate into the hump-shaped impulse responses of consumption and employment often documented in VAR studies.

Composite habits have been shown to improve the empirical performance of closed economy DSGE models along several dimensions. Together with building restrictions, composite habits provide an explanation for high volatility in house prices (Jaccard, 2011). Christoffel et al. (2013) show that augmenting a standard New Keynesian model with slow-moving composite habits generates a sizable bond premium without deteriorating cyclical properties of consumption. Habit formation in the composite of consumption and leisure helps simultaneously to account for asset pricing and business cycle facts in a standard RBC model (Jaccard, 2014). In a recent paper, Jaccard (2016) shows that composite habits contribute to understanding the 2008-2009 recession in the United States. Habits strengthen the internal propagation mechanism of DSGE models with financing frictions. In this environment, a small short-lived shock to the economy's pledgeable collateral is capable of generating severe and long-lasting recessions. Despite their success in addressing macroeconomics phenomena in closed economies, to the best of my knowledge, this is the first paper that explores implications of composite habits in an international setting.

Our work builds upon and contributes to several strands of the literature which explores the international dimension of business cycles. This paper is closely related to other studies that investigate the role of nonstandard preferences within two-country business cycle models. For instance, recent work by Colacito and Croce (2011), Gourio et al. (2013), Backus et al. (2016), and Kollmann (2015a,b)

[3]This partially explains the popularity of GHH preferences as well as appearance of alternative utility specifications that modify the wealth effect on labor supply (e.g. Jaimovich and Rebelo, 2009; Monacelli and Perotti, 2008).

[4]For instance, in order to use GHH preferences in an economy with non-stationary investment specific technology shocks Mandelman et al. (2011) are forced to introduce a trending random variable into the utility function.

investigate how Epstein and Zin (1989) preferences interact with different sources of stochastic disturbances under various market structures. Devereux et al. (1992) and Raffo (2008) examine the role of GHH preferences introduced by Greenwood et al. (1988), which eliminate the wealth effect on labor supply. Engel and Wang (2011) and Dmitriev and Roberts (2012) consider versions of GHH preferences featuring nonseparability over time due to the presence of durables or habits. Dmitriev and Krznar (2012) discuss the role of linear consumption habits.

This paper is related to the literature which proposes alternative explanations of international business cycles puzzles. Following RBC tradition, many pioneering studies in the literature have relied on models driven by stationary productivity shocks. By contrast, some recent papers focus on the role of alternative macroeconomic disturbances. For instance, Beaudry et al. (2011) consider cross-country effects of news shocks. Mandelman et al. (2011) examine the quantitative importance of investment-specific technology shocks in generating international business cycles. Wen (2007) explores whether international comovements could be explained by demand shocks, while Corsetti et al. (2014) consider the interaction of demand and productivity shocks. Alessandria et al. (2015) employ stochastic disturbances to the preferences for imported goods to account for the cyclical properties of imports and exports. Kollmann (2016) shows that output volatility shocks help to explain the empirical behavior of real exchange rates. It is largely a consideration of these aforementioned shocks that led researchers to resort to alternative preference specifications. This paper contributes to this line of research.

Our work is also related to the papers that address the international transmission of business cycles by departing from the assumption of perfect risk sharing. Pioneering studies in this strand of literature, which include Kollmann (1996) and Baxter and Crucini (1995), considered exogenously incomplete international markets. Kehoe and Perri (2002) examined the role of endogenously incomplete markets in international comovements, whereas Heathcote and Perri (2002) focused on the role of financial autarky. More recently, Gao et al. (2014) and Kollmann (2012) emphasize the role of limited asset market participation. Several recent studies highlight the importance of banking and financial factors in propagation of macroeconomic disturbances across borders (Faia, 2007; Olivero, 2010; Devereux and Yetman, 2010; Kollmann et al., 2011; Kollmann, 2013).

This paper links to the literature which analyses other features of the propagation mechanism in order to understand cross-country comovement of macroeconomic aggregates. Some papers from this strand of literature emphasize the role of durables (Engel and Wang, 2011), non-traded goods (Stockman and Tesar, 1995), household production (Karabarbounis, 2014), capital maintenance (Deli, 2016), and fixed costs of capital adjustment Bayer and Tjaden (2016).

This paper naturally relates to the literature on habit formation. Incorporating habits into macroeconomic modeling has improved the ability of DSGE models to account for both business cycle and asset pricing phenomena. A question which necessarily arises in connection with habit formation is thus: at what level are the habits of consumers formed? Are habits formed at an individual level or at some societal level? Do consumers form their habits at the level of individual goods or at the level of some aggregate good? Our paper contributes to this discussion by focusing on internal habits defined over a composite good that comprises consumption and leisure.

The early literature concerned with habit formation exclusively modeled such behavior at the level of a consumption aggregate. By contrast, Ravn et al. (2006) considered the possibility that consumers form habits at the level of individual goods, a preference specification they called "deep habits". They showed that deep habits help explain the cyclical character of wages and consumption, through the mechanism of counter-cyclical mark-ups. Since this initial contribution, deep habits have enhanced

the empirical performance of macroeconomic models along several important dimensions. Ravn et al. (2012) demonstrate that deep habits can improve empirical efficiency of DSGE models that include a shock to government spending. Heyerdahl-Larsen (2014) shows that the inclusion of deep habits helps to account for observed asset pricing and real exchange rate data. Di Pace and Faccini (2012) illustrate that deep habits can account for the volatility in vacancies and unemployment present in U.S. data.

The idea that an agent's consumption of a certain good affects her future utility and demand for that good is intuitively appealing. Deep habits are consistent with this notion only if they are formed internally. Unfortunately, all the applications cited above rely on deep habits which are formed externally. Under external habits, agents derive utility from consumption relative to the economy average. Hence, consumers do not internalize the effect of their consumption on their future utility. However, internal habit formation on the level of individual goods would render the firms' pricing problem time-inconsistent (Ravn et al., 2006).

There are two reasons for relying on internal habits: one empirical and one theoretical. First, empirical studies that use asset prices to statistically distinguish between internal and external habits tend to conclude in favor of internal habits. For instance, Chen and Ludvigson (2009) and Grishchenko (2010) find that the data on asset returns are consistent with internal habits and that the degree of habit formation is high. Second, linear internal habits have established axiomatic foundations, whereas external habits do not (Rozen, 2010). These considerations compel us to rely on the internal mechanism of habit formation, and this precludes us from the use of deep habits.

The rest of the paper is organized as follows. Section 2 reviews some empirical regularities associated with international business cycles. Section 3 describes our model economy. Section 4 presents our parameterization of the model. It describes the minimum distance approach that we use to estimate the unknown habit parameters. Our approach relies on the simulated method of moments (SMM) that optimizes the predictions of the model for international comovements. Section 5 presents our quantitative results and explains how each feature of the model contributes to reproducing the observed features of the data. Section 6 discusses several extensions to the baseline model, including incomplete markets, internal and external consumption habits, preference shocks, and cointegrated productivity shocks. Section 7 concludes the paper.

2 Empirical regularities

This section briefly reviews some empirical regularities associated with international business cycles and how they have changed over time. Table 1 presents historical business cycle statistics for the U.S. and an aggregate of 15 European economies for the post-Bretton woods period 1973Q1-2007Q4. Prior to computing statistics all time series except for net exports have been logged and filtered using HP filter with a smoothing parameter 1600. Standard errors in parenthesis are computed using a moving block bootstrap with 10.000 replications[5]. The source of the data is OECD Quarterly National Accounts and OECD Economic Outlook. Further details on the data sources and definitions are available in Appendix A.

[5]We rely on the bootstrap for computing the standard errors for the business cycle statistics following the work of Chistiano and den Haan (1996). They show that using asymptotic inference for computing business cycle statistics from HP-filtered series could be misleading for available post-WWII sample sizes. Because our series for the cyclical components of the macroeconomic variables show high degree of autocorrelation we rely on the moving block bootstrap technique of Kunsch (1989).

Table 1: Historical Business Cycle Statistics

	1973Q1-2007Q4		1984Q1-2007Q4	
	U.S.	EU-15	U.S.	EU-15
(A) Cross-country correlations				
$corr(GDP_1, GDP_2)$	0.51		0.31	
	(0.11)		(0.20)	
$corr(C_1, C_2)$	0.36		0.32	
	(0.14)		(0.21)	
$corr(I_1, I_2)$	0.39		0.28	
	(0.12)		(0.19)	
$corr(N_1, N_2)$	0.46		0.36	
	(0.11)		(0.18)	
(B) Volatilities				
$\sigma(GDP), \%$	1.47	0.90	0.85	0.75
	(0.20)	(0.10)	(0.10)	(0.08)
$\sigma(C)/\sigma(GDP)$	0.81	0.84	0.88	0.83
	(0.05)	(0.07)	(0.07)	(0.06)
$\sigma(I)/\sigma(GDP)$	2.89	2.61	3.05	3.03
	(0.12)	(0.20)	(0.23)	(0.15)
$\sigma(N)/\sigma(GDP)$	0.67	0.70	0.79	0.77
	(0.05)	(0.07)	(0.07)	(0.07)
$\sigma(TOT), \%$	2.60	1.56	1.34	1.08
	(0.39)	(0.22)	(0.12)	(0.14)
$\sigma(RER), \%$	3.49	n.a.	3.06	n.a.
	(0.47)		(0.34)	
(C) Correlations with own GDP				
$corr(C, GDP)$	0.84	0.81	0.81	0.81
	(0.04)	(0.05)	(0.06)	(0.05)
$corr(I, GDP)$	0.94	0.88	0.89	0.91
	(0.02)	(0.03)	(0.03)	(0.04)
$corr(N, GDP)$	0.83	0.78	0.73	0.87
	(0.04)	(0.06)	(0.07)	(0.04)
$corr(NX, GDP)$	−0.49	−0.40	−0.40	−0.55
	(0.09)	(0.11)	(0.13)	(0.10)
(D) Correlations with own productivity				
$corr(GDP, Z)$	0.94	0.92	0.88	0.92
	(0.02)	(0.08)	(0.03)	(0.03)
$corr(C, Z)$	0.84	0.65	0.71	0.65
	(0.04)	(0.08)	(0.06)	(0.09)
$corr(I, Z)$	0.86	0.73	0.77	0.80
	(0.04)	(0.05)	(0.05)	(0.06)
$corr(N, Z)$	0.59	0.48	0.33	0.60
	(0.09)	(0.09)	(0.13)	(0.10)
$corr(NX, Z)$	−0.46	−0.25	−0.36	−0.47
	(0.09)	(0.12)	(0.12)	(0.11)

Note: Empirical statistics are shown for the U.S. and an aggregate of 15 European countries (EU-15). European data cover the following 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. The data are at quarterly frequency, seasonally adjusted and aggregated at the source. The sample include gross domestic product (GDP), consumption (C), investment (I), employment (N), terms of trade (TOT) for both U.S. and EU-15, as well as real exchange rate (RER) for the U.S. Productivity (Z) refers to Solow residuals defined as $\log(Z_t) = \log(y_t) - (1-\alpha)\log(N_t)$, where y_t is linearly detrended real GDP, and N_t denotes total employment. Labor share of income, $1-\alpha$, is assumed to be 0.36 as in BKK. All series except for NX have been logged and filtered using HP filter with a smoothing parameter 1600. Standard error in parenthesis are computed using moving block bootstrap with 10.000 replications. The sourse of the data is OECD Quarterly National Accounts and OECD Economic Outlook. Further details on the data sources and construction are available in Appendix A.

The reported statistics based on the entire post-Bretton woods sample confirm several well-established international business cycle facts:

i) Consumption levels are less correlated across countries than outputs (0.36 vs. 0.51);

ii) International correlations of investment and employment levels are positive (0.39 and 0.46 respectively);

iii) Trade balance is countercyclical for the U.S. and Europe (-0.49 and -0.40 respectively);

iv) U.S. real exchange rate is more volatile than its terms of trade (3.49 vs. 2.60).

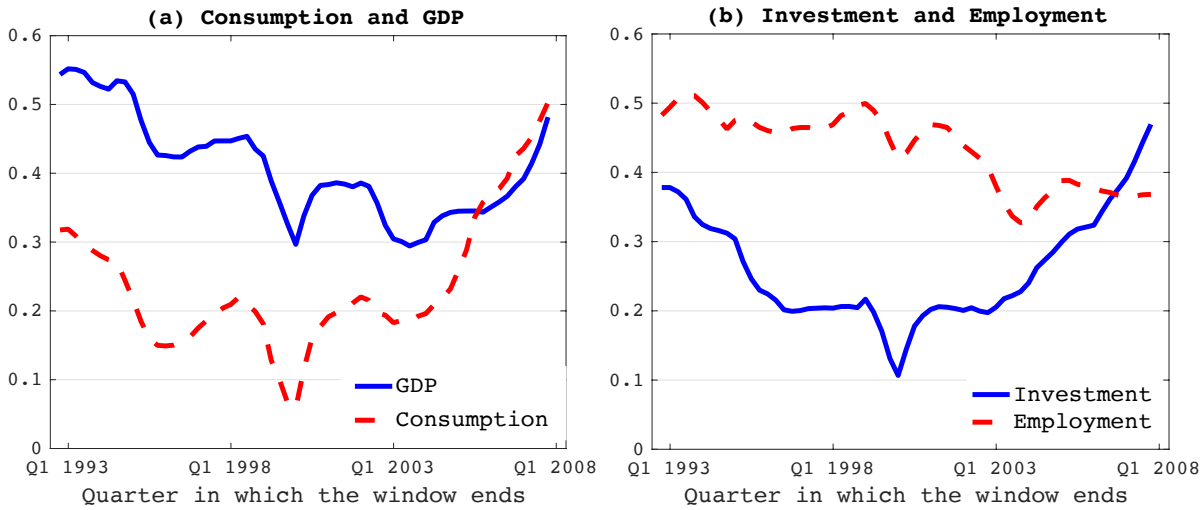
These features of the data are similar to those reported in the pioneering international business cycle papers (e.g. BKK, Baxter and Crucini (1995); Kollmann (1996); Stockman and Tesar (1995)). But have these regularities been invariant over time?

Heathcote and Perri (2003) raise this question and document that U.S. business cycles have become less synchronized over time with the an aggregate of EU-15 and Japan. Yoon (2005) calls their conclusion into question due to the fact that heteroscedasticity in the data can cause bias in correlation coefficients. Several studies have documented that reduction in US output volatility known as 'The Great Moderation' that occurred around mid-1980s. For instance, McConnell and Perez-Quiros (2000) provide strong evidence that a structural break in the volatility of U.S. GDP growth occurred in the first quarter of 1984. Due to the availability of data, all the most recent samples used by Heathcote and Perri (2003) include the structural break of 1984. To deal with the potential bias in the correlation coefficients we consider empirical moments for the subsample 1984Q1-2007Q4 shown in Table 1.

Comparing the 'Great Moderation' subsample to the full post-Bretton woods sample reveals the following. First, cross-country correlation of outputs has decreased whereas correlation of consumption has remained almost unchanged. In the the 'Great Moderation' subsample correlation of outputs are roughly the same as the correlation of consumption (0.31 vs 0.32). Second, volatilities of output and the terms of trade have substantially decreased, whereas relative volatilities have remained largely unchanged. Finally, correlations of most macroeconomic variables with economy's own GDP or productivity have remained virtually the same. The exception is the cyclical behavior of employment that substantially varies across samples.

To trace the evolution of the international comovement we also present cross-country correlations of cyclical components of GDP, consumption, investment and employment over 80 quarters rolling windows (Figure 1). The last 17 windows cover exclusively post-1983 observations. A few observations are worth noting. First, we replicate the result of Heathcote and Perri (2003) that prior to 2003, the U.S. macroeconomic aggregates were becoming less correlated with their European counterparts. Second, international correlations for GDP, consumption, investment seem to be increasing in the post-1983 data. Finally, cross-country correlations of consumption and output seem to be about the same order of magnitude in the post-1983 data. This makes the 'quantity anomaly' less stark but it still remains present.

Figure 1: Cross-country correlations over 80 quarters rolling windows

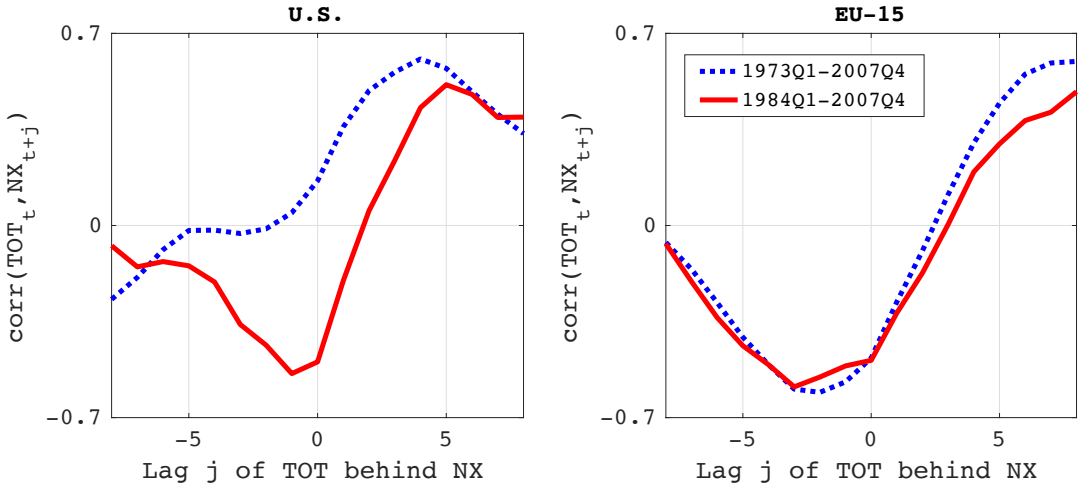


Note: International correlation over 80 quarters rolling windows are shown for the US and an aggregate of 15 European countries (EU-15). All the statistics are based on logged and HP filtered data with a smoothing parameter 1600. The first window covers the period 1973Q1-1992Q4. The last window covers the period 1988:Q1-2007:Q4.

We conclude our brief review of the international business cycle facts by examining the relationship between the trade balance (NX) and the terms of trade (TOT). As noted by Backus (1993) this relationship central to important questions in international macroeconomics[6]. Figure 2 plots cross-correlation functions for $NX(t+k)$ and $TOT(t)$ for k ranging from -8 to 8 quarters. Our samples covers the U.S. and the composite of 15 European countries (EU-15). As shown by BKK, the pattern of cross-correlations between TOT and NX might depend on the sampling period. Once again, we consider the sample covering 1973Q1-2007Q4 and its subsample covering the 'Great Moderation' period, 1984Q1-2007Q4.

[6]For instance, Backus (1993, p. 375) states that "One of the central questions of international economics concerns the relation between the trade balance and the terms of trade: what features of an economy determine whether an increase in the relative price of imports is associated with improvement or deterioration in the balance of trade?"

Figure 2: Cross-correlation function of the trade balance and the terms of trade



Note: The figure presents the cross-correlation function between leads and lags of net exports and the terms of trade for the U.S and 15 European countries (EU-15). Data are from the OECD Quarterly National Accounts. NX denotes the ratio of Net Exports to GDP measured in current prices. The terms of trade (TOT) are defined as the logarithm of the ratio of the import deflator to the export deflator as in BKK. All series are HP-filtered with a smoothing parameter of 1600. European data (EU-15) cover the following 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. The data are at quarterly frequency, seasonally adjusted and aggregated at the source.

With the exception of the U.S. sample that covers the OPEC oil crises of the 1970s, the comovement between the trade balance and the terms of trade exhibits the S-shaped pattern described by BKK[7]. These regularities confirm the results documented by Senhadji (1998) and, more recently, by Bahmani-Oskooee and Ratha (2007). In addition, Enders and Müller (2009) document both conditional and unconditional S-curves for recent US data.

3 The Model Economy

The baseline model economy mimics an environment of BKK with complete markets. The world consists of two countries, home and foreign. Each country is populated by a continuum of identical infinitely lived individuals. The same parameters describe technology and preferences in both countries.

Labor is immobile across countries. In each period t, the world economy experiences an event s_t drawn from the countable set of events, S. Let $s^t = (s_0, s_1, ..., s_t) \in S^t$ be the history of events from time 0 to time t. The probability at time 0 of any given history s^t is denoted by $\pi(s^t)$.

Our specification of preferences and technology ensures that the model is consistent with balanced growth. Labor augmenting technological progress, Γ_t, grows at a constant rate of

$$g = \Gamma_{t+1}/\Gamma_t > 1.$$

Following King et al. (1988, 2002), we rescale all trending variables by dividing them with Γ_t. This implies an adjustment to the discount factor and the equations of motion for the state variables. Throughout the paper, lowercase letters denote detrended variables.

[7]Using TOT series that eliminates petroleum products from the data virtually eliminated differences for the U.S. cross-correlation function across samples. Both exhibit the S-shaped pattern.

In section 6 we show that our main results do not hinge on assumptions that imply market completeness and a trend-stationary stochastic process for total factor productivity (TFP). We consider exogenously incomplete markets as in Heathcote and Perri (2002), and cointegrated TFP shocks as in Rabanal et al. (2011).

3.1 Composite Habit-Forming Consumers

Consumers have their preferences defined over stochastic sequences of consumption $C_j\left(s^t\right)$, composite habits $H_j\left(s^{t-1}\right)$, and leisure $L_j\left(s^{t-1}\right)$

$$U=\sum_{t=0}^{\infty}\beta^t\sum_{s^t\in S^t}\pi\left(s^t\right)u(C_j\left(s^t\right),H_j\left(s^{t-1}\right),L_j\left(s^t\right)), \tag{1}$$

where $\beta\in(0,1)$ is the discount factor, and $C_j\left(s^t\right)$ denotes household consumption in country $j\in\{1,2\}$ after realization of history s^t. The way we model habit formation has four distinct features.

First, consumer's utility depends on the stock of habits, $H_j\left(s^{t-1}\right)$, defined over the composite of consumption and leisure, $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$. The instantaneous utility function defined over the habit-adjusted composite good, $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)-\chi H_j(s^{t-1})$, belongs to the CRRA family

$$u(C_j\left(s^t\right),H_j\left(s^{t-1}\right),L_j\left(s^t\right))=\frac{\left[C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)-\chi H_j(s^{t-1})\right]^{1-\sigma}}{1-\sigma},$$

where $\sigma>0$ is the curvature parameter, and $\Phi\left(L\right)\equiv\psi+L^{\nu}$. The parameter $\chi\in[0,g)$ denotes the intensity of habit formation and introduces time nonseparability of preferences. Coefficients ψ and ν jointly control the steady-state level of employment and the Frisch elasticity of labor supply. The time endowment in each period is normalized to 1 and is divided between hours worked, $N_j\left(s^t\right)=1-L_j\left(s^t\right)$, and leisure.

Second, habits are persistent. The stock of habits depends on the entire past history rather than the previous period only. We define the stock of habits, $H_j\left(s^t\right)$, with which the agent enters period $t+1$ as a convex combination of her current stock of habits, $H_j(s^{t-1})$, and her current consumption of the composite good, $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$:

$$H_j(s^t)=\lambda H_j(s^{t-1})+\left(1-\lambda\right)C_j\left(s^t\right)\Phi\left(L_j(s^t)\right). \tag{2}$$

Under this specification, the habit stock depreciates at a constant rate as in Ferson and Constantinides (1991). The parameter $\lambda\in[0,1]$ determines the degree of habit persistence. The lower the λ, the more weight an agent places on recent lifestyle history relative to the past. That is, as λ declines, so does the persistence of habits. When $\lambda=0$, the next period's habit stock is simply the current standard of living.

Third, habits are internal. The agent cares about her consumption of composite good relative to her own past history. Consumers internalize the effect their current behavior has on their future felicity. This feature makes marginal utility of consumption forward-looking.

Finally, habits are additive. The agent smooths the quasi-difference between the composite of her consumption and leisure, $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$, and her current stock of habits, $H_j\left(s^{t-1}\right)$. This helps to ensure that the consumer's objective function remains concave.

This specification of preferences nests several well-known special cases. First, when $\chi=1$, it reduces to the preference used by Jaccard (2014) to show that composite habits help a stochastic

growth model simultaneously explain asset pricing and business cycle facts in a closed-economy setting. Second, when $\chi = 0$, it reduces to the time-separable specification of King et al. (1988), which satisfies restrictions necessary for balanced growth (henceforth, KPR preferences). Third, if labor supply becomes inelastic, the preference structure will feature persistent habit formation in consumption as in Ferson and Constantinides (1991). If, in addition, $\lambda = 0$, the preference ordering reduces to non-persistent consumption habits popularized by Constantinides (1990).

Could a more parsimonious formulation of habits help the canonical two-country, two-good business cycle model explain the pattern of international comovements? To answer this question Section 6, considers both external and internal consumption habits as alternatives to our composite habits. When consumption habits are external, consumers derive utility from their consumption relative to a reference level determined by past or current aggregate consumption. They fail to internalize the effect their current consumption has on their future marginal utility. Under internal habits, consumers take into account the effect of their current actions of their own future marginal utility. While the two types of consumption habits produce somewhat different business cycle statistics, they share the main implications for international comovements.

Both types of consumption habits help little to account for the quantity anomaly. They leave the cross-country consumption correlation excessively high while decreasing correlation of outputs. Outputs become more disconnected because habits enhance the negative comovement of employments. The assumption of intratemporal non-separability of consumption and leisure compels agents to smooth their consumption of the composite good. In addition, habit formation makes agents more averse to changes in their consumption levels following a shock. This leaves consumers no option but to rely heavily on leisure to absorb shocks.

Consumption habits have the potential to deal with investment comovement, which is determined by the interplay between habits and capital adjustment cost. In the absence of adjustment cost, increasing habit intensity leads to higher volatility of investment, leaving international comovement unaffected. The inclusion of a combination of habits and a sluggish adjustment of capital has the ability to change this. Habits reduce the instantaneous response of consumption to a productivity shock, whereas adjustment cost reduces the instantaneous response of investment. As a result, a positive shock in the domestic economy increases its savings by more than its investment. In the presence of complete international markets, these savings are channeled abroad to increase foreign investment. However, the synchronization of investment might come at a cost of excessive volatility of investment or counterfactually procyclical net exports. An increase in exports of relatively cheaper domestic goods might reverse the sign of nets export, depending on whether quantity or price effects dominate.

3.2 Producers

There are two types of firms in this environment: intermediate good producers and final good producers.

3.2.1 Intermediate Good Producers

Intermediate good firms in each country specialize in the production of a single country-specific good. Country 1 produces good a, whereas country 2 produces good b. Households in country j own the domestic capital stock, $K_j\left(s^t\right)$, and rent it to domestic intermediate good firms. Installed capital is

immobile across countries. It evolves according to the law of motion

$$K_j\left(s^t\right) = (1-\delta)K_j\left(s^{t-1}\right) + \phi\left(\frac{I_j\left(s^t\right)}{K_j\left(s^{t-1}\right)}\right)K_j\left(s^{t-1}\right), \text{ for } j \in \{1,2\}, \tag{3}$$

where δ is the depreciation rate, and $I_j\left(s^t\right)$ is gross investment. Capital accumulation is subject to convex adjustment costs described in Hayashi (1982).[8] The adjustment cost function ϕ satisfies $\phi\left(\cdot\right) > 0$, $\phi'\left(\cdot\right) > 0$, and $\phi''\left(\cdot\right) < 0$. Let i/k denote the investment-to-capital ratio along the deterministic BGP, and then the restrictions $\phi\left(i/k\right) = i/k$ and $\phi'\left(i/k\right) = 1$ will ensure that incorporation of the adjustment cost does not affect the deterministic BGP of the model.

Households supply labor to the intermediate firms at the wage $W_j\left(s^t\right)$ and capital at the rental price $R_j\left(s^t\right)$. The production function for the intermediate goods is Cobb-Douglas with labor-augmenting productivity growth:

$$Y_j(s^t) = z_j\left(s^t\right)K_j^{\alpha}\left(s^{t-1}\right)\left(\Gamma_t N_j\left(s^t\right)\right)^{1-\alpha}, \text{ for } \text{ for } j \in \{1,2\}. \tag{4}$$

Production is subject to country-specific exogenous random shock, $z_j\left(s^t\right)$, to total factor productivity (TFP). The TFP shocks follow a stationary vector autoregressive process in logs:

$$\begin{bmatrix} \log(z_1\left(s^t\right)) \\ \log(z_2\left(s^t\right)) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} \log(z_1\left(s^{t-1}\right)) \\ \log(z_2\left(s^{t-1}\right)) \end{bmatrix} + \begin{bmatrix} \varepsilon_1\left(s^t\right) \\ \varepsilon_2\left(s^t\right) \end{bmatrix}.$$

Diagonal elements of the transition matrix, ρ_{11}, determine the degree of persistence in productivity within each country. Off-diagonal elements, ρ_{12}, determine the speed with which productivity innovations spill over national borders. The innovations to the productivity process are serially uncorrelated bivariate normal random variables with zero mean and the contemporaneous covariance matrix

$$E\left[\varepsilon_t\varepsilon_t'\right] = \sigma_{\varepsilon}^2 \cdot \begin{bmatrix} 1 & \rho_{\varepsilon} \\ \rho_{\varepsilon} & 1 \end{bmatrix}.$$

3.2.2 Final Good Producers

The final good firms in country 1 use as inputs the quantities $A_1\left(s^t\right)$ and $B_1\left(s^t\right)$ of the two intermediate goods. They produce an amount $G\left(A_1\left(s^t\right), B_1\left(s^t\right)\right)$ of the final good, using a constant returns-to-scale technology

$$G\left(a,b\right) = \left(\omega a^{1-\eta} + (1-\omega)\, b^{1-\eta}\right)^{\frac{1}{1-\eta}},$$

where $1/\eta$ is the elasticity of substitution between intermediate inputs. The constant $\omega \in (1/2, 1)$ reflects the extent of home bias. In the same way, the firms in country 2 produce an amount $G\left(B_2\left(s^t\right), A_2\left(s^t\right)\right)$ of the final good using $A_2\left(s^t\right)$ and $B_2\left(s^t\right)$ of the intermediate inputs.

3.3 Markets

Agents have access to a complete set of state-contingent claims traded internationally. The claims are denominated in terms of the intermediate good a. Let $Q\left(s^t, s_{t+1}\right)$ be the price of a claim sold after

[8] We choose the capital adjustment cost over investment adjustment cost à la Christiano et al. (2005) for the following reason. Dmitriev and Roberts (2013) show than international RBC models with capital adjustment costs à la Hayashi (1982) tend to deliver S-shaped cross-correlation function between the trade balance and the terms of trade. On the other hand, the models with investment adjustment costs à la Christiano et al. (2005) fail to do so. Yet, Enders and Müller (2009) present ample evidence in favor of the S-curve, both unconditional and conditional on the technology shocks.

realization of s^t, that delivers a unit of good a at time $t+1$ provided that the state s_{t+1} is realized. Let $B_2(s^t, s_{t+1})$ be the quantity of such claims held by the residents of country 2 after history s^t. Then, the consumers in country 2 face the following budget constraint:

$$\begin{aligned} & C_2\left(s^t\right) + I_2\left(s^t\right) + q_2^a\left(s^t\right) \sum_{s_{t+1} \in S} Q\left(s^t, s_{t+1}\right) B_2\left(s^t, s_{t+1}\right) \qquad (5) \\ = \; & q_2^b\left(s^t\right) \left[R_2\left(s^t\right) K_{2t}\left(s^{t-1}\right) + W_2\left(s^t\right) \cdot N_2\left(s^t\right) \right] + q_2^a\left(s^t\right) B_2\left(s^{t-1}, s_t\right), \end{aligned}$$

where $q_2^a(s^t)$ and $q_2^b(s^t)$ denote the prices of good a and b respectively, in terms of the final good of country 2. The agents in country 1 face a similar constraint.

3.4 Equilibrium

The equilibrium consists of the state-contingent sequences of factor prices $R_j(s^t), W_j(s^t)$; intermediate good prices $q_j^a(s^t), q_j^b(s^t)$; bond prices $Q(s^t)$; and allocations $A_j(s^t)$, $B_j(s^t)$, $C_j(s^t)$, $I_j(s^t)$, $N_j(s^t)$, $K_j(s^t)$, $\{B_j(s^t, s_{t+1})\}$ that satisfy the following conditions. Given the prices:

(i) Consumers in country j choose state-contingent sequences $\{C_j(s^t)\}_{t=0}^{\infty}$, $\{N_j(s^t)\}_{t=0}^{\infty}$, $\{I_j(s^t)\}_{t=0}^{\infty}$ and bond holdings $\{B_j(s^t, s_{t+1})\}_{t=0}^{\infty}$ for all $s^t \in S^t$, and all $s_{t+1} \in S$ to maximize expected utility in equation (1) subject to the flow budget constraint in equation (5), the laws of motion in equations (3) and (2), and the initial conditions.

(ii) Intermediate firms in country j choose $N_j(s^t)$ and $K_j(s^{t-1})$ to maximize profits

$$Y_j(s^t) - R_j(s^t)K_j(s^{t-1}) - W_j(s^t)N_j(s^t),$$

subject to the technological constraint in equation (4) and the non-negativity constraints $N_j(s^t) \geq 0$ and $K_j(s^{t-1}) \geq 0$.

(iii) The final good producers in country j choose $A_j(s^t)$ and $B_j(s^t)$ to maximize profits $\Pi_j(s^t)$ given by

$$\Pi_j\left(s^t\right) = \begin{cases} G\left(A_j\left(s^t\right), B_j\left(s^t\right)\right) - q_j^a(s^t)A_j(s^t) - q_j^b(s^t)B_j(s^t), & \text{for} \quad j = 1; \\ G\left(B_j\left(s^t\right), A_j\left(s^t\right)\right) - q_j^a(s^t)A_j(s^t) - q_j^b(s^t)B_j(s^t), & \text{for} \quad j = 2. \end{cases}$$

The prices ensure that for all $t \geq 0$ and for all $s^t \in S^t$:

(iv) Intermediate good markets clear

$$\begin{aligned} A_1\left(s^t\right) + A_2\left(s^t\right) &= Y_1(s^t), \\ B_1\left(s^t\right) + B_2\left(s^t\right) &= Y_2(s^t). \end{aligned}$$

v) Final good markets clear

$$\begin{aligned} C_1\left(s^t\right) + I_1\left(s^t\right) &= G\left(A_1\left(s^t\right), B_1\left(s^t\right)\right), \\ C_2\left(s^t\right) + I_2\left(s^t\right) &= G\left(B_2\left(s^t\right), A_2\left(s^t\right)\right). \end{aligned}$$

vi) Asset markets clear

$$B_1(s^t, s_{t+1}) + B_2(s^t, s_{t+1}) = 0, \text{ for all } s^t \in S^t, s_{t+1} \in S.$$

3.5 Other Variables of Interest

We measure GDP in country 1 as

$$GDP_1(s^t) = q_1^a(s^t) Y_1(s^t).$$

The real exchange rate for country 1 is defined as

$$RER\left(s^t\right) = q_1^a(s^t)/q_2^a(s^t),$$

and the terms of trade are defined as

$$TOT\left(s^t\right) = q_1^b(s^t)/q_1^a(s^t).$$

We define the trade balance as the net exports for country 1 as a fraction of GDP for country 1, both measured in current prices

$$NX\left(s^t\right) = \frac{q_1^a(s^t) A_2(s^t) - q_1^b(s^t) B_1(s^t)}{GDP_1(s^t)}.$$

4 Model Calibration, Estimation, and Solution

This section describes our empirical approach that combines the elements of calibration and structural estimation. It also outlines our numerical solution approach.

4.1 Calibration

To solve the benchmark model numerically, we use the parameter values reported in Tables 3, 4, and 5. The first group includes parameters calibrated to match long-run averages in the U.S. and European data, reported in Table 2. To construct long-run ratios we use novel comparable estimates of the

Table 2: Historical long-run averages

	U.S.	EU-15	Average (U.S.-EU15)
Capital-to-GDP ratio, k/gdp	12.652	16.608	14.630
Investment-to-GDP ratio, i/gdp	0.190	0.213	0.202
Import share in GDP, im	0.097	0.217	0.157
GDP per capita trend growth rate, % p.a.	1.995	1.646	1.820

Note: The table displays historical averages for the U.S. and an aggregate of 15 European countries (EU-15) for the period 1973-2007. Annual data on capital stock and real GDP are from Penn World Tables 9.0. The rest of the data are from OEDC QNA.

real stock of capital proceeding from the Total Economy Database recently included in Penn World Tables 9.0 (Feenstra et al., 2015). Our sample consists of annual data covering the period 1973 to

2007. Given the symmetry of the model we calibrate its parameters using the averages between the US and European statistics.

We interpret one period as a quarter and set the gross trend growth rate g to 1.0045. This reflects the fact that the average trend growth rate of real GDP per capita was 1.82% per annum. The quarterly depreciation rate, δ, follows from the equation of motion for capital at the steady state:

$$\delta = \frac{i/gdp}{k/gdp} - (g-1).$$

Given the value of the trend growth rate, the quarterly depreciation rate is set at 0.0093 to ensure that the steady-state investment-to-GDP ratio, i/gdp, is 0.202, and the capital-to-GDP ratio, k/gdp, is 14.63. This value for depreciation differs from the traditional value of 2.5% per quarter assumed in early business cycle literature (e.g. BKK). However, in the absence of secular growth, this calibration corresponds to quarterly depreciation rate of 1.38%, which is similar to 1.5% used in Heathcote and Perri (2013). The growth-adjusted discount factor, β^*, follows from the consumer's Euler equation in the steady state

$$\beta^* = \frac{g}{\alpha\,(gdp/k) + 1 - \delta}.$$

The capital income share, α, takes the value 0.36 which is common in international business cycle literature (see e.g. BKK or Kehoe and Perri (2002)). Given the values for g, α, δ, and the capital-to-GDP ratio, we set the adjusted discount factor, β^*, to 0.9894. The degree of home bias, ω, is chosen to match the observed long-run import share of GDP, $im = 0.157$. Because we consider a symmetric equilibrium, we compute the home bias parameter as

$$\omega = \frac{(1/im - 1)^{\eta}}{1 + (1/im - 1)^{\eta}},$$

where the elasticity of substitution between intermediate traded goods $1/\eta$ to 1.5 as in BKK or Yakhin (2007).

Table 3: Calibrated parameters

Notation	Definition	Value	Target /Source
g	gross trend growth rate	1.0045	GDP per capita trend growth rate
α	capital income share	0.36	BKK, Heathcote and Perri (2002)
δ	depreciation rate	0.0093	Capital/GDP and investment/GDP ratios
$1/\eta$	trade elasticity	1.5	BKK, Yakhin (2007)
ω	home bias parameter	0.7541	Import share in GDP
β^*	adjusted discount factor	0.9894	Capital/GDP and investment/GDP ratios

Note: The two countries share the same parameterization. One time period is a quarter. Trade elasticity refers to the elasticity of substitution between domestic and foreign inputs.

Utility function parameters ψ and ν are chosen to match the following two features of labor supply: (i) consumers spend on average one-third of their time endowment working; (ii) the Frisch elasticity of labor supply is 1.5. For a given ν, Frisch elasticity varies with habit intensity, χ, and habit persistence, λ. Hence, for a given choice of the habit parameters we set ν to keep Frisch elasticity constant across parameterizations.

The capital adjustment cost function, $\phi(x)$, is parameterized to match the second moments of the

data. Its functional form follows from Boldrin et al. (2001):

$$\phi(x) = \frac{\Omega_1}{1 - 1/\xi} (x)^{1-1/\xi} + \Omega_2.$$

Parameters Ω_1, Ω_2 must satisfy $\phi'(i/k) = 1$ and $\phi'(i/k) = i/k$ to ensure that the introduction of adjustment costs does not affect the BGP of the model. Therefore, it follows that

$$\Omega_1 = (i/k)^{-1/\xi},$$

and

$$\Omega_2 = \frac{i/k}{1 - \xi}.$$

Because the elasticity parameter ξ controls cyclicality of trade balance, its baseline value set to equate the correlation of net exports and GDP predicted by the model to that in the data.

4.2 Estimation

In the absence of econometric estimates for composite habit parameters, we rely on the simulated method of moments (SMM). As shown by Ruge-Murcia (2007), SMM is generally more robust to misspecification than maximum likelihood is. This is important in our context because to highlight the role of composite habits our model abstracts from other important features of the economy. The moments we target reflect the international comovements and cyclical behavior of net exports. Given the changes in the targeted moments reported in Table 1, we conduct the moment matching exercise for the entire post-Bretton wood sample and its 'Great Moderation' subsample.

A few comments on the magnitude of the estimated habit parameters are in order. First, the habit intensity value $\chi = 0.97$ corresponds to a 96.6 percent steady-state habit-composite ratio due to the presence of secular growth. This value is close to the empirical estimates by Chen and Ludvigson (2009) who conclude that "the habit is a substantial fraction of current consumption−about 97 percent on average-echoing the specification of Campbell and Cochrane (1999) in which the steady-state habit-consumption ratio exceeds 94 percent." Using quarterly returns on Treasury bills, bonds, and portfolios of common stocks, Ferson and Constantinides (1991) also estimate the habit intensity to be as high as 0.97. Second, our minimum distance approach picks up a specification with non-persistent habits. This is similar to the results of Boldrin et al. (2001), who conclude that persistent habit memory adds very little to their asset pricing model with production. Christoffel et al. (2013), who rely on Bayesian methods to estimate a New Keynesian DSGE model augmented with composite habits, also doubt on whether persistence is identified. Their estimate of composite habit intensity is as low as 0.841. However, Christoffel et al. (2013) estimate a very simple New Keynesian model.

4.2.1 Estimated Productivity Process

We estimate the parameters of the productivity process as in Heathcote and Perri (2002) and Gao et al. (2014). We compute Solow residuals for the U.S. and an aggregate of 15 European countries for the period 1973Q1-2007Q4. We identify log of productivity in economy j as

$$\log\left(z_j\left(s^t\right)\right) = \log\left(y_j\left(s^t\right)\right) - (1-\alpha)\log\left(N_j\left(s^t\right)\right),$$

where $y_j(s^t)$ represents linearly detrended real GDP, and $N_j(s^t)$ describes total employment in the economy. Labor share of income, $1-\alpha$, is assumed to be 0.36 as in BKK or Heathcote and Perri (2002). We estimate a first order vector autoregressive (VAR(1)) process for the logs of productivity using seemingly unrelated regressions (SURE). The symmetry restrictions $\rho_{11} = \rho_{22}$, and $\rho_{21} = \rho_{12}$ on the transition matrix $[\rho_{ij}]$ are imposed during the estimation. Prior to estimation we test for stationarity the individual series of (log) productivity using a KPSS test (Kwiatkowski et al., 1992). The test fails to reject the null of stationarity at conventional levels of significance.

Table 4: Estimated Productivity Process

Notation	Definition	Value
ρ_{11}	persistence of the TFP shocks	0.976 (0.012)
ρ_{12}	TFP spillover parameter	0.019 (0.009)
σ_{ϵ_1}	standard deviation of the innovations to the TFP process	0.0066
σ_{ϵ_2}		0.0042
ρ_{ε}	contemporaneous correlation of the innovations	0.1887

Note: We estimate the following VAR(1) process for TFP shocks using seemingly unrelated regressions (SURE): $\begin{bmatrix} \log(z_1(s^t)) \\ \log(z_2(s^t)) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} \log(z_1(s^{t-1})) \\ \log(z_2(s^{t-1})) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(s^t) \\ \varepsilon_2(s^t) \end{bmatrix}$ with the symmetry restrictions imposed, $\rho_{11} = \rho_{22}$, and $\rho_{21} = \rho_{12}$. Standard errors are in parenthesis.

Our estimation results presented in Table 4 are almost identical to the estimates in Heathcote and Perri (2002) and Gao et al. (2014). The shocks are very persistent and cross-country spillovers are small but statistically significant. The minor differences in the estimates could potentially be attributed to the differences in the sample period or the definition of the proxy for the U.S. trading partners.

4.2.2 Simulated Method of Moments

In this section, we use the simulated method of moments to estimate three preference parameters. Hence, we choose habit intensity χ, its persistence λ, utility curvature σ to maximize the model's ability to account for five moments. The remaining preference parameters are calibrated to match long-run averages (β^*, ψ) and Frisch elasticity of labor supply (ν).

The moments we target reflect the international comovement of quantity aggregates as well as cyclical behavior of the trade balance. In particular, let $\mathbf{m}$ denote a (5×1) vector of cross-country correlations of GDP, consumption, investment, and employment and a correlation of an economy's net export with its own GDP

$$\mathbf{m} = \Big(\; corr(\widetilde{gdp}_1, \widetilde{gdp}_2) \quad corr(\widetilde{c}_1, \widetilde{c}_2) \quad corr(\widetilde{i}_1, \widetilde{i}_2) \quad corr(\widetilde{n}_1, \widetilde{n}_2) \quad corr(\widetilde{nx}_1, \widetilde{gdp}_1) \; \Big)',$$

where tildes indicate that we deal with cyclical components of the variables obtained by removing a trend with an HP filter. International correlations for the United States and an aggregate of 15 European countries in $\mathbf{m}_{data}$ are reported in the Table 1. The approach selects the preference parameters $\theta \equiv (\chi, \lambda, \sigma)$ by minimizing a distance $L(\theta)$

$$\theta = \arg\min_{\theta \in \Theta} L(\theta),$$

between empirical moments $\mathbf{m}_{data}$ and their simulated counterparts $\mathbf{m}_{sim}(\theta)$, obtained after solving the model for given parameter vector θ:

$$L(\theta) = (\mathbf{m}_{data} - \mathbf{m}_{sim}(\theta))' \mathbf{W} (\mathbf{m}_{data} - \mathbf{m}_{sim}(\theta)). \quad (6)$$

In the equation above, Θ denotes an admissible range for the estimated preference parameters and $\mathbf{W}$ is a diagonal weighting matrix with inverse estimated variances of the empirical moments $\mathbf{m}_{data}$ on the main diagonal. To perform the estimation, we discretize Θ into an equally spaced grid and perform maximization over it. During the estimation the habit parameters are restricted to the range from 0 to 0.99 to ensure the stability of the numerical solution[9]. Utility curvature is restricted to takes values from 1 to 3, which is considered in the literature as a plausible range (e.g. Heathcote and Perri (2013)). Our estimates are reported in Table 5.

Table 5: Estimated preference parameters

Notation	Definition	Sample Period	
		1973Q1-2007Q4	1984Q1-2007Q4
χ	intensity of composite habits	0.97	0.99
λ	persistence of composite habits	0.00	0.00
σ	utility curvature	1.00	1.30

Note: Reported parameters (χ, λ, σ) minimize $(\mathbf{m}_{data} - \mathbf{m}_{sim}(\chi, \lambda, \sigma))' \mathbf{W} (\mathbf{m}_{data} - \mathbf{m}_{sim}(\chi, \lambda, \sigma))$ over an admissible range Θ. Empirical moments $\mathbf{m}_{data}$ and their simulated counterparts $\mathbf{m}_{sim}(\chi, \lambda, \sigma)$ include cross-country correlations of GDP, consumption, investment, and employment as well as a correlation of an economy's net export with its own GDP. $\mathbf{W}$ is a diagonal matrix whose main diagonal is populated with the inverse variances of the empirical moments.

Several remarks are in order. First, the estimates are very close to those chosen by Jaccard (2014) to match the observed equity premium and the risk-free rate in a closed-economy RBC model. Indeed, he relies on unitary utility curvature and assumes a habit intensity of one. Second, we find no evidence of long memory in habit formation. However, this might be due to the lack of identification as the distance function (6) has very little curvature along the persistence dimension. Third, our baseline estimate for habit intensity, $\chi = 0.97$, implies 96.6 percent steady-state habit-composite ratio due to the presence of secular growth. It is virtually identical to the estimates of Chen and Ludvigson (2009) and Ferson and Constantinides (1991) who use GMM to estimate Euler equations implied by internal consumption habits. Forth, and finally, both utility curvature estimates are close to one. Moreover, when we apply the same SMM approach to the model with time-separable preferences, we also obtain an estimate for the utility curvature of one. This is a value recently used by Heathcote and Perri (2013) to address the international diversification puzzle. In section 6.5, we examine the sensitivity of our key results to the choice of utility curvature, σ, and habit parameters, χ and λ.

4.3 Solution Methods

We solve the baseline model numerically using a version of the generalized stochastic simulation algorithm (GSSA) proposed by Judd et al. (2011). It is classified by Maliar et al. (2011) as belonging to the

[9]Because numerical solution contains approximation errors and because the model is subjected to stochastic disturbances we have to restrict the range for habit intensity from above. The exact feasible range that ensures non-negativity of the objective of the CRRA aggregator depends both on the parameters of the shocks and accuracy of the numerical solution. The upper bound on the range was chosen experimentally as the largest number that ensured non-negativity of habit adjusted composite for a given parameterization of the shocks. This issue is also discussed in Boldrin et al. (2001) in the case of linear consumption habits.

class of the ergodic set methods. An advantage of this approach is that it allows us to avoid computing solutions in areas of the state space that are never visited in equilibrium. For instance, many of the exogenous grid methods will cover the points in the state-space where $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)-\chi H_j(s^{t-1})$ will be negative off the equilibrium path.[10] It also enables us to avoid the approximation errors associated with linearization of every optimality condition. The algorithm parameterizes decision rules derived from the first order conditions using a polynomial approximation function of the current state variables. It then uses a fixed point iteration method to compute fixed-point values of the polynomial coefficients, evaluating conditional expectations using a combination of Monte-Carlo integration and regression.

GSSA suffers from several shortcomings that make it poorly suited for solving the model repeatedly, and hence required simulation-based estimations. First, GSSA is rather slow for a problem with six or seven state variables when the degree of approximation is sufficiently high. Second, it is much more unstable than the methods based on a fixed grid. To circumvent this problem, we compare the model's solutions with GSSA, with the results of first-, second- and third-order perturbation algorithms implemented in DYNARE 4.4.3. The algorithms produce virtually identical business cycle moments.[11] Hence, all simulations reported in the paper rely on first-order perturbation. In addition, we check that the non-negativity constraint never binds in simulations.

5 Results

This section compares empirical performance of our model with the international business cycles regularities discussed in Section 2. It provides some intuition about the results by focusing on the role played by the composite habits in transmission of the productivity shocks.

5.1 Quantity Anomaly and International Comovement Puzzle

We start by reviewing the puzzles. The first row of Table 6 displays correlations between cyclical components of the quantity aggregates of the United States and an aggregate of 15 European countries.

Predictions of the model with time-separable KPR preferences are reported in the second row. The model has difficulty in matching empirical moments along two dimensions. First, it predicts negative cross-country correlations in investment and hours worked when these are positive in the data (international comovement puzzle). Second, in the data, international correlations of output and consumption are of similar magnitude, with the former slightly exceeding the latter. Relative to the data, the model predicts a too high consumption correlation and too low output correlation. The fact that consumption correlation is far in excess of output correlation is often referred to as the quantity anomaly.

[10] The present discounted value of a sequence of composite goods $\{C_t\Phi\left(L_t\right)\}$ is not necessarily increasing in the composite good consumed in any particular period. This stems from the fact that while period t utility is increasing in $C_t\Phi\left(L_t\right)$, utility at $t+1$ is decreasing in it. Under additive specification of habits, a habit-adjusted composite might turn out negative in simulations. We check that in all simulation reported in the paper the non-negativity constraint never binds.

[11] The details are presented in the supplement available in the online version at http://dx.doi.org/10.1016/j.jedc.xxxx.xx.xxx

Table 6: International comovements

	Cross-Country Correlations			
	GDP	Consumption	Investment	Employment
(A) Sample Period: 1973Q1-2007Q4				
Data (U.S.-Europe)	0.51	0.36	0.38	0.46
Standard Preferences (KPR)	0.29	0.59	−0.02	−0.08
	(0.15)	(0.11)	(0.16)	(0.16)
Composite Habits	0.39	0.41	0.27	0.37
	(0.14)	(0.14)	(0.15)	(0.14)
(B) Sample Period: 1984Q1-2007Q4				
Data (U.S.-Europe)	0.31	0.32	0.28	0.36
Standard Preferences (KPR)	0.28	0.60	−0.05	−0.08
	(0.15)	(0.11)	(0.16)	(0.16)
Composite Habits	0.37	0.39	0.23	0.36
	(0.14)	(0.14)	(0.15)	(0.16)

Note: International statistics in the Data row refer to the averages between the U.S. and an aggregate of 15 European countries (EU-15). The model's statistics refer to averages and standard error (in parenthesis) of 10,000 simulations of 140 periods each (96 periods each in panel B). All the statistics are based on logged and HP-filtered data with a smoothing parameter of 1600.

The introduction of composite habits helps our model address both discrepancies between the data and the model's predictions. It accounts for what Baxter (1995) calls the international comovement puzzle. It matches the data by predicting positive cross-country correlations of investment and hours worked. The model moves in the right direction in accounting for the quantity anomaly by predicting a realistic international correlation of consumptions (0.41 vs. 0.36 in the data) and outputs (0.39 vs. 0.51 in the data). Whereas the standard model counterfactually predicts that consumptions are twice more correlated than outputs, our benchmark eliminates the gap between consumption and output correlations. While both correlations are well within one standard deviation of their empirical counterparts, the precision of the estimates is limited by the sample size. Conditional on productivity shocks, the model still fails to reverse the order of the correlations. However, as shown in Section 6.1, augmenting the model with very small preference shocks corrects the order of correlations.

5.2 Intuition

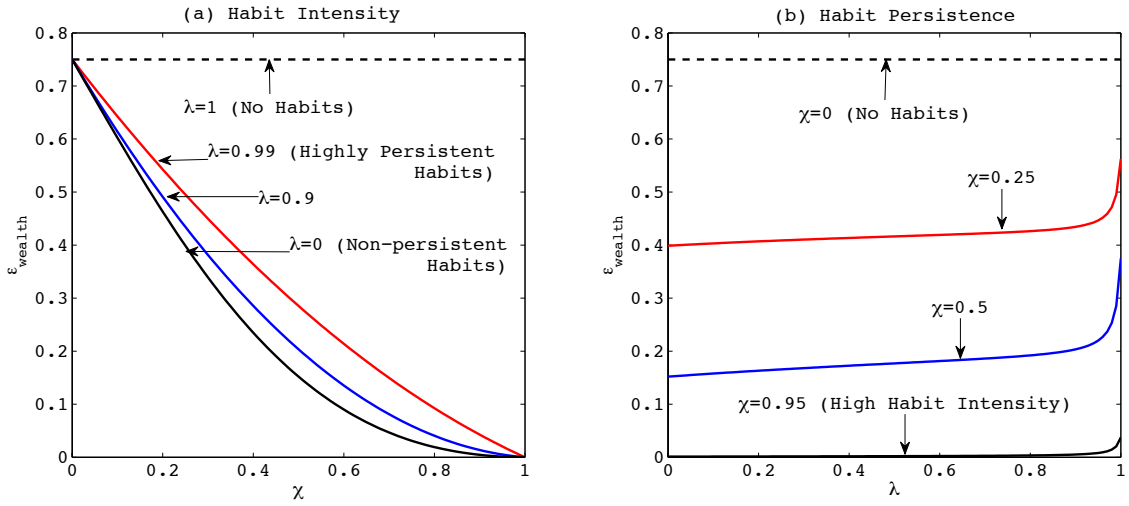
In this section, we discuss why augmenting the model with composite habits improves the predictions of the model for the cross-country correlation of quantity aggregates.

5.2.1 Consumption Correlation

To understand why our model delivers a realistic consumption correlation, we first consider an extreme case. Suppose that the terms of trade (TOT) are constant and that utility is separable in consumption and leisure. In this case, consumption and leisure are neither Edgeworth complements nor substitutes. Under complete markets, the risk-sharing condition implies that all agents' marginal utilities are equated for every time and state. Hence, consumption is perfectly correlated across counties.

Departing from separability in consumption and leisure and allowing variation in the TOT somewhat reduces consumption correlation. Because the marginal utilities of consumption now depend on

Figure 3: How wealth elasticity of labor supply is governed by habit parameters



Note: Both panels display how the wealth elasticity of labor supply varies for a given level of Frisch elasticity. Wealth elasticity is decreasing in habit intensity and increasing in habit persistence.

leisure, equality of the former no longer implies equality of consumption. Under KPR preferences, consumption and leisure become complements in the Edgeworth sense. As a result, a model with KPR preferences predicts a cross-country consumption correlation of 0.59 rather than 1. In other words, reducing Edgeworth substitutability between consumption and leisure lowers the consumption correlation.

Introducing composite habits and increasing their intensity further reduces the substitutability between consumption and leisure.[12] The reason for this is the adjacent complementarity in the agent's composite good. A higher level of the composite today increases the marginal utility of consumption and leisure in the near future. After a positive productivity shock, consumers do their best to avoid simultaneously increasing their consumption and leisure. An additional reduction in Edgeworth substitutability between consumption and leisure is partially responsible for moving consumption correlation toward its empirical counterpart (0.41 vs. 0.36 in the data). The other reason for the reduction in consumption correlations is the positive cross-country comovement of leisure, which will be discussed in the next section.

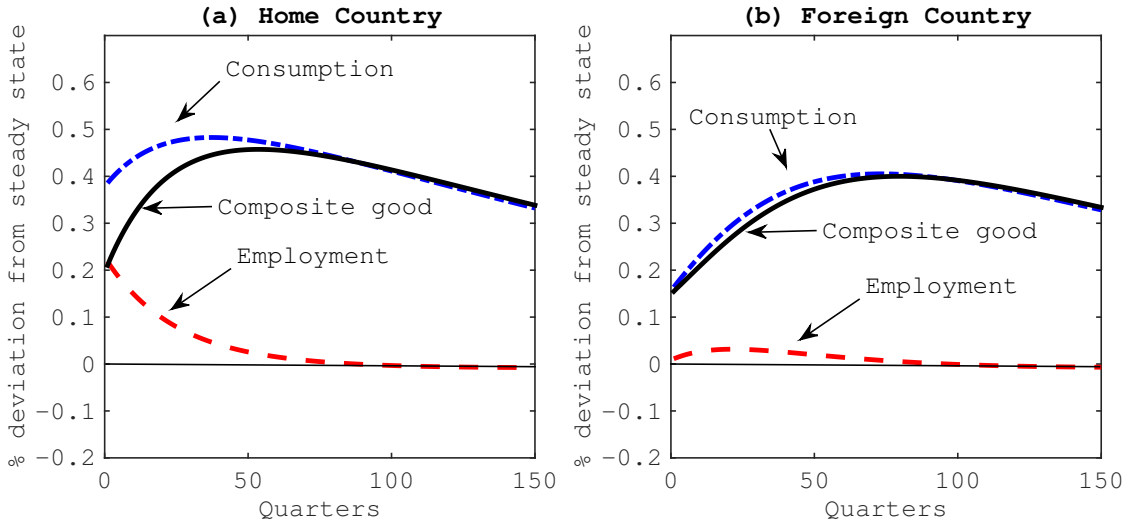
5.2.2 Employment Correlation

The introduction of composite habits reduces the wealth effect on labor supply for a given level of Frisch elasticity. How important is this result quantitatively? Figure 3 shows how habit intensity and habit persistence affect wealth elasticity of labor supply, given our choice of β, g, and ϵ_{frisch}.

Increasing habit intensity makes the wealth effect on labor supply arbitrarily small irrespective of habit persistence. The converse is not true. Reducing habit persistence lowers wealth elasticity to a level determined by intensity of habit formation. Habit intensity, rather than its persistence, is the dimension more quantitatively important for the magnitude of the wealth effect. For any given

[12]To quantify the degree of Edgeworth substitutability, we can use what Frisch (1959) refers to as utility acceleration. In Dmitriev (2016), we show that utility acceleration is negative for KPR preferences, and this implies that consumption and leisure are Edgeworth complements. Increasing the intensity of composite habit formation further decreases utility acceleration.

Figure 4: Responses of consumption and employment to a TFP shock at home: KPR preferences



Note: The figure reports responses of consumption $C_j\left(s^t\right)$, employment $N_j\left(s^t\right)$, and a composite of consumption and leisure $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$ to a one-standard-deviation TFP shock at home. All variables are expressed in terms of percentage deviation from their respective deterministic steady-state values.

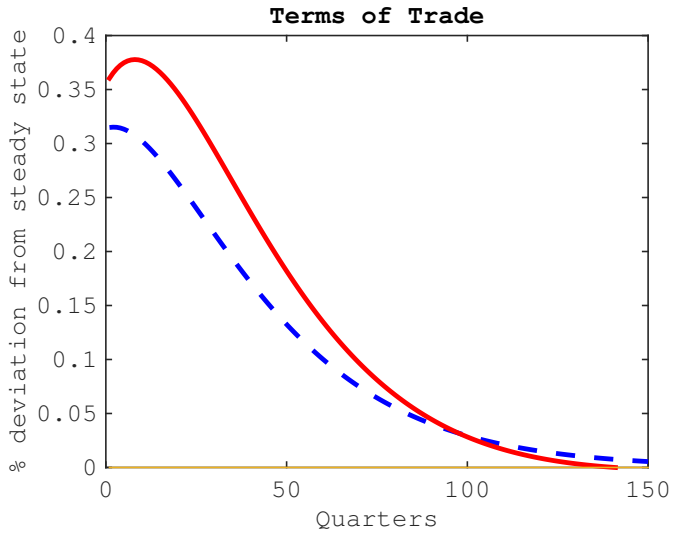
level of intensity, χ, changing depreciation of habits from 100 percent ($\lambda = 0$) to 1 percent ($\lambda = 0.99$) reduces wealth elasticity by less than 15 percent of its initial value.

Now consider the effect of a one standard deviation increase in TFP at home (Figures 4-6). The home country becomes more productive in manufacturing good A. The domestic real wage in terms of the intermediate good A, $w_1\left(s^t\right)$, goes up. Domestically produced goods are now relatively more abundant, but they are only imperfect substitutes for imported goods. The price of good A relative to good B falls, which corresponds to a jump in the terms of trade, $TOT\left(s^t\right)$ (see Figure 5). Despite the deterioration in the terms of trade, domestic wages in terms of the domestic final good, $q_1^a(s^t)w_1(s^t)$, go up. While the substitution effect compels the consumer to increase the hours worked, the wealth effect makes her demand more leisure. The relative magnitude of the two forces determines the response of employment to the shock.

Under standard preferences, wealth effects are quite substantial (dashed line in Figure 3). At home, the substitution effect dominates and domestic employment rises. Abroad, the terms-of-trade effect increases the wages measured in terms of foreign final goods, $q_2^b(s^t)w_2(s^t)$. The substitution effect compels the agents to increase their hours worked. However, the wealth effect dominates, and employment abroad falls, causing negative cross-country comovement of hours worked (Figure 4). This mechanism is known as "make hay where the sun shines."

Composite habits improve the model's predictions for comovement of hours through two channels. First, they affect the magnitude of the substitution effect by increasing the volatility of the terms of trade. A more pronounced fall in the relative price of domestic goods further discourages domestic workers from supplying more labor (see Figure 5). On the other hand, a more pronounced hike in the price of imported goods compels foreign workers to increase their labor supply. Second, composite habits reduce the wealth elasticity of labor supply. This prevents workers in both countries from demanding more leisure after a productivity improvement. In the home country, the wealth effect and TOT effect move employment in opposing directions. As shown in Figures 4 and 6, the response of employment in the immediate aftermath of the shock almost doubles. Abroad, the wealth effect and

Figure 5: Responses of the terms of trade to a TFP shock: composite habits versus KPR preferences



Note: The figure reports responses of the terms of trade, $TOT\left(s^t\right) = q_1^b(s^t)/q_1^a(s^t)$, to a one-standard-deviation TFP shock at home. The variable is expressed in terms of percentage deviation from its deterministic steady-state value.

terms-of-trade effect move employment in the same direction. The two forces make foreign employment rise after a positive productivity shock at home.

5.2.3 Output Correlation

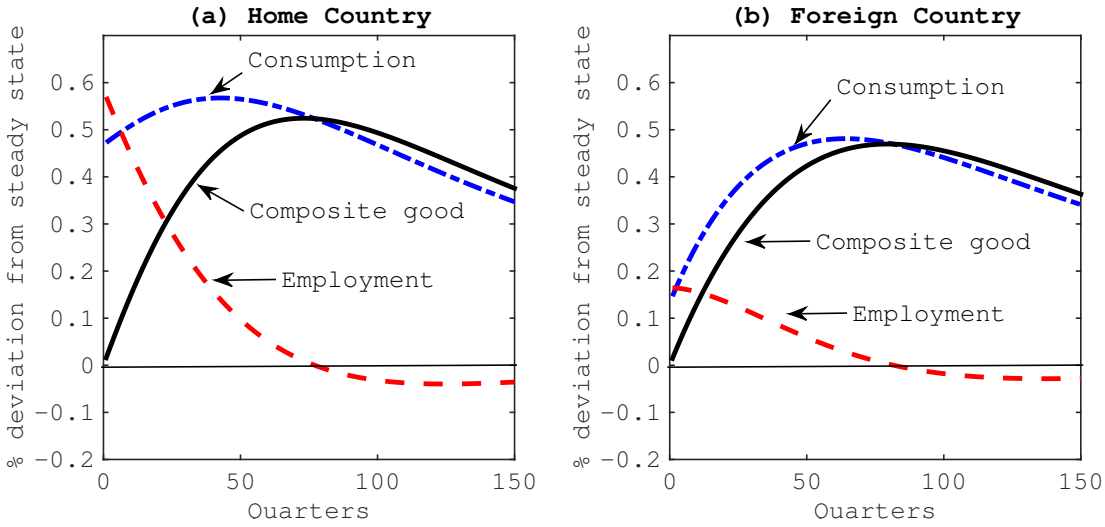
Getting comovement of employment right directly translates into a plausible cross-country correlation of GDP. Under standard preferences, positive comovement of productivity and negative comovement of intermediate good prices are partially offset by a counterfactual negative correlation of hours worked (-0.08 vs. 0.46 in the data). As a result, the model predicts output correlations that are too low relative to the data (0.29 vs. 0.51 in the data). When the employment correlation approaches its empirical counterpart (0.37 vs. 0.46 in the data), so does the cross-country correlation of GDP (0.39 vs. 0.51 in the data).

5.2.4 Investment Correlation

To understand international comovement of investment, consider how the dynamics of world savings and world investment differs across models. Notice that under standard preferences, households dislike changes in the composite of consumption of leisure, $c_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$. Under composite habits, they also dislike changes in the growth rate of the composite good, which makes the optimal response of the composite good to a productivity shock hump-shaped. In contrast, the optimal response of the composite good under standard preferences is a jump (solid lines in Figure 4 vs. Figure 6).

The desire to smooth the growth rate of the composite good under composite habits translates into a hump-shaped response of consumption to the productivity shock (top row of Figure 7). On impact, consumption in both countries does not jump as much as it does under standard preferences. What happens to worldwide savings after the shock? A lower wealth effect on the labor supply increases the response of GDP to higher productivity in both countries (row 2 of Figure 7). The positive response of world GDP is exacerbated, whereas the positive response of world consumption is dampened.

Figure 6: Responses of consumption and employment to a TFP shock at home: composite habits



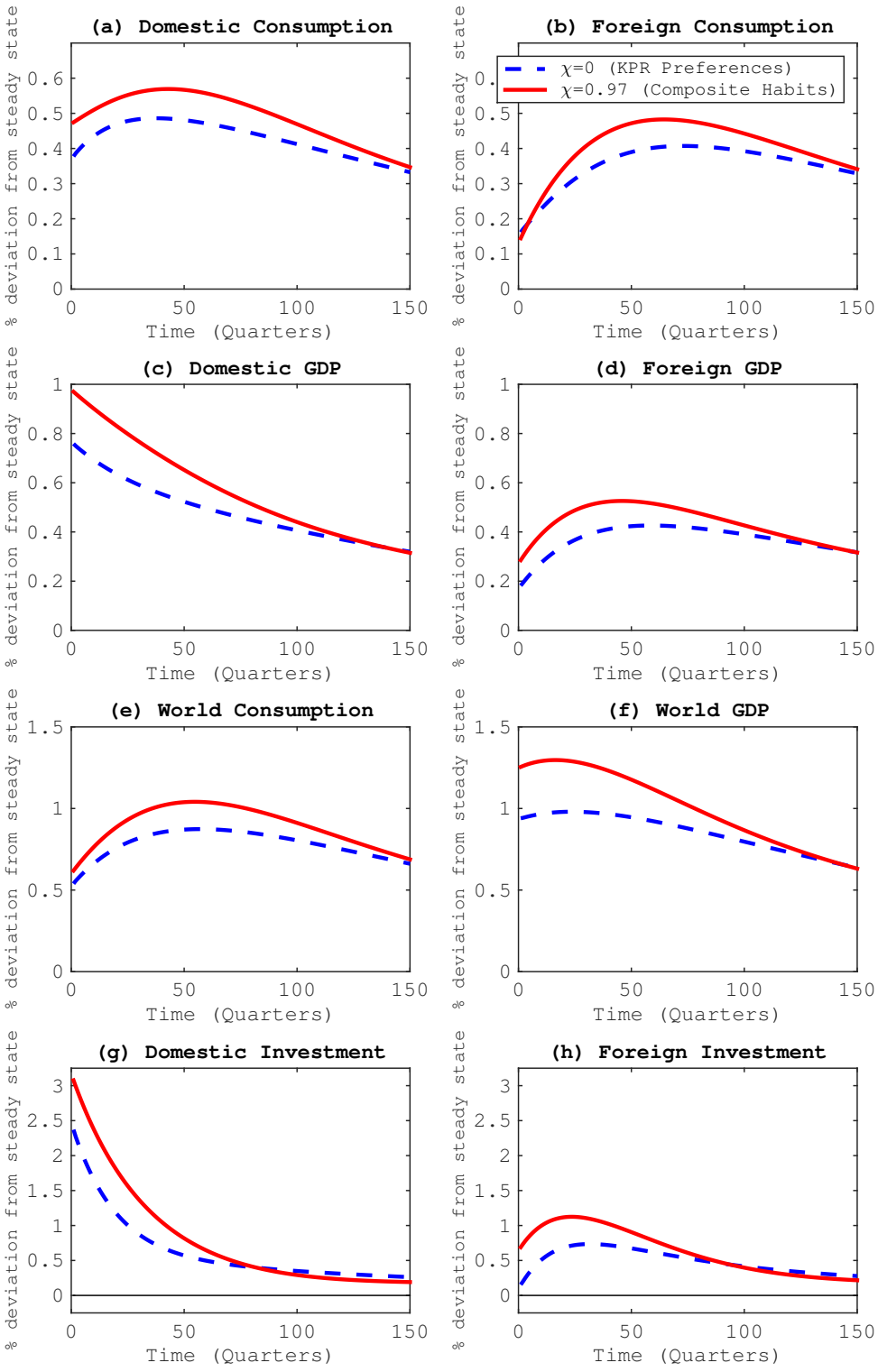
Note: The figure reports responses of consumption $C_j\left(s^t\right)$, employment $N_j\left(s^t\right)$, and the composite of consumption and leisure $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$ to a one-standard-deviation TFP shock at home. All variables are expressed in terms of percentage deviation from their respective deterministic steady-state values.

Hence, the world experiences a massive increase in savings in the immediate aftermath of the shock. Most of these extra savings are invested in the relatively more productive domestic economy. However, imperfect substitutability between domestic and foreign goods coupled with adjustment cost to capital formation ensures that foreign investment increases as well.

5.3 Other Business Cycle Regularities

Explaining international comovements does not come at a cost of deteriorating the domestic business cycle statistics reported in Tables 7 and 8. Two observations are in order. First, the model reconciles with data in predicting countercyclical trade balance. Second, the introduction of composite habits only marginally affects the volatility of the terms of trade and the real exchange rate. Still, the model's predictions fall short of their empirical counterparts.

Figure 7: Impulse responses: composite habits versus KPR preferences



Note: The figure reports responses of consumption, GDP and investment to a one-standard-deviation TFP shock at home. Dashed line corresponds to KPR preferences, whereas solid line corresponds to composite habits.

Table 7: Business cycles: volatilities

	$\sigma(GDP)$	$\frac{\sigma(C)}{\sigma(GDP)}$	$\frac{\sigma(I)}{\sigma(GDP)}$	$\frac{\sigma(N)}{\sigma(GDP)}$	$\sigma(TOT)$	$\sigma(RER)$
(A) Sample Period: 1973Q1-2007Q4						
Data (U.S.-Europe)	1.18	0.83	2.75	0.59	2.08	3.49
Standard Preferences (KPR)	0.95	0.54	2.99	0.30	0.67	0.46
	(0.11)	(0.02)	(0.08)	(0.01)	(0.08)	(0.05)
Composite Habits	1.23	0.49	3.16	0.38	0.74	0.50
	(0.14)	(0.01)	(0.04)	(0.01)	(0.09)	(0.06)
(B) Sample Period: 1984Q1-2007Q4						
Data (U.S.-Europe)	0.80	0.86	3.03	0.78	1.21	3.06
Standard Preferences (KPR)	0.96	0.53	3.07	0.30	0.65	0.45
	(0.11)	(0.02)	(0.09)	(0.01)	(0.08)	(0.05)
Composite Habits	1.24	0.49	3.21	0.60	0.71	0.49
	(0.14)	(0.01)	(0.04)	(0.01)	(0.09)	(0.06)

Note: Statistics in the Data row refer to the averages between the U.S. and an aggregate of 15 European countries (EU-15). The model's statistics refer to averages and standard errors (in parenthesis) of 10,000 simulations of 140 periods each (96 periods each in panel B). All the statistics are based on logged (except for Net Exports) and HP-filtered data with a smoothing parameter of 1600.

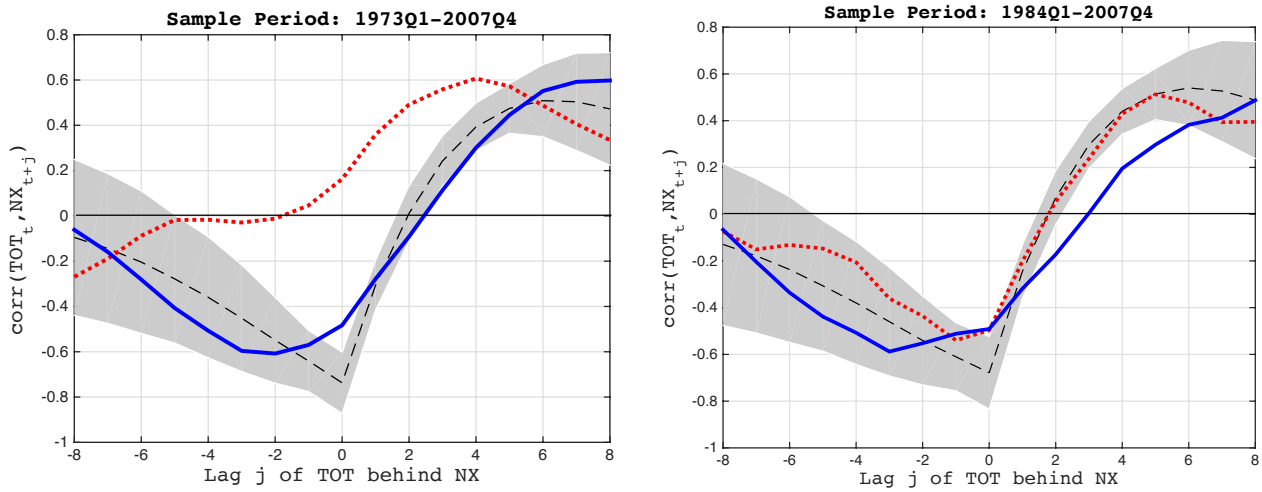
Table 8: Business cycles: correlations with own GDP

	Consumption	Investment	Employment	Net Exports
(A) Sample Period: 1973Q1-2007Q4				
Data (U.S.-Europe)	0.83	0.91	0.81	−0.44
Standard Preferences (KPR)	0.99	0.99	0.99	−0.44
	(0.01)	(0.01)	(0.01)	(0.13)
Composite Habits	0.98	0.98	0.98	−0.44
	(0.01)	(0.01)	(0.01)	(0.13)
(B) Sample Period: 1984Q1-2007Q4				
Data (U.S.-Europe)	0.81	0.90	0.80	−0.47
Standard Preferences (KPR)	0.98	0.98	0.98	−0.47
	(0.01)	(0.01)	(0.01)	(0.12)
Composite Habits	0.99	0.99	0.99	−0.47
	(0.01)	(0.01)	(0.01)	(0.12)

Note: Statistics in the Data row refer to the averages between the U.S. and an aggregate of 15 European countries (EU-15). The model's statistics refer to averages and standard errors (in parenthesis) of 10,000 simulations of 140 periods each (96 periods each in panel B). All the statistics are based on logged (except for Net Exports) and HP-filtered data with a smoothing parameter of 1600.

The model's prediction that trade net exports are countercyclical matters for two reasons. From the empirical standpoint, countercyclical trade balance is a very robust and well documented fact (see e.g. Raffo, 2008). From the modeling perspective, cyclical behavior of net exports is closely related to the model's prediction for cross-country correlation of investment. In the canonical international RBC model, investment rushes to the most productive location ('make hay where the sun shines') causing countercyclical net exports at the cost of negative cross-country investment correlation ('international comovement puzzle'). A similar scenario occurs in more elaborate international business cycle models (see e.g. Raffo, 2008). On the other hand, models that get investment comovement right often fail to account for the countercyclical net export by either imposing financial autarky (e.g Heathcote and

Figure 8: Cross-correlation function for the trade balance and the terms of trade



Note: Solid lines corresponds to the U.S. data, dotted lines to the EU-15 data. Dashed lines and the shaded areas represent the means and two-standard deviation confidence bands for the models' predictions.

Perri (2002), Yakhin (2007), and Gao et al. (2014)) or by predicting procyclical trade balance (e.g. Kehoe and Perri (2002), Dmitriev and Krznar (2012), and Dmitriev and Roberts (2012)).

Our model inherits from the BKK model some or its successes and failures in accounting for within-country business cycles. Consistent with the data, our model predicts procyclical consumption, investment, and employment. However, all three variables are more correlated with output than their empirical counterparts. Compared to the data, the model predicts too little volatility in consumption and employment. The inclusion of composite habits suppresses the wealth effect on labor supply and brings the model closer to the data by making employment more volatile. However, agent's aversion to changes in consumption growth reduces already too low volatility of consumption. In section 6.1, we show that augmenting the model with preference shocks could improve its predictions for within-country business cycles.

Another important empirical regularity discussed on the data section is the S-curve. Augmenting BKK model with alternative sources of disturbances or modifying its transmission mechanism has a potential to break the patterns of cross-correlations for the trade balance and the terms of trade. For instance, BKK show that introduction of government spending shocks breaks the S-shaped pattens. Dmitriev and Roberts (2013) show that choice of adjustment cost function affects the model's ability to deliver the S-curve.

Figure 8 compares our model's prediction (dashed lines) with the U.S. data (dotted lines) and the data for the aggregate of 15 European countries (solid line). Shaded ares correspond to the two-standard-deviation confidence bands for the model's predictions. Although, our model generate cross-correlation function conditional on the TFP shocks, Enders and Müller (2009) show that unconditional cross-correlation function display the pattern very similar to the one conditional on the technology shocks. The right panel displays the results for the subsample covering the 'Great Moderation', whereas the left panel corresponds to the entire sample.

As in the data, technology shocks in our model induce an S-shaped pattern of cross-correlations for the trade balance and the terms of trade. The match to the data is best for the 'Great Moderation' period.

6 Extensions and Sensitivity Analysis

In this section we examine how alternative specifications of habit formation, asset market structure and stochastic process for TFP affect our model's predictions. We consider whether more parsimonious specifications of habits can account for the international comovement patterns. Our extensions feature internal consumption habits as in Boldrin et al. (2001), external consumption habits as in Abel (1990), exogenously incomplete markets as in Baxter and Crucini (1995), preference shocks as in Wen (2007), and cointegrated productivity shocks as in Rabanal et al. (2011).

6.1 Preference Shocks

Our first extension augments the baseline model with preference shocks. In doing this, we follow Stockman and Tesar (1995) who were the first to consider the importance of the interaction between productivity and preference shocks in explaining international business cycle phenomena. More recent contributions include Wen (2007) and Corsetti et al. (2014). While Wen (2007) explores whether international comovements could be explained by preference shocks alone, Corsetti et al. (2014) use structural VARs with sign restrictions to identify both demand and productivity shocks.

There are many ways of modeling preference shocks in the business cycle literature. For example, shocks are variously assumed to directly affect the weight of leisure in the composite good (Stockman and Tesar (1995); Corsetti et al. (2014)), the marginal utility of consumption (Wen, 2006), and the instantaneous utility Wen (2007). We follow Wen (2007), who considered multiplicative shocks to the instantaneous utility in the context of a two-country business cycle model.

In our context, there are two advantages to this formulation. First, this specification of preference shocks interacts with the internal nature of habit formation. The shocks affect both the instantaneous and the forward looking parts of the marginal utilities of consumption. Thus, additional disturbances have the potential to further reduce comovement of consumption levels despite the fact that the risk-sharing condition requires the proportionality of marginal utilities. Uncertainty about future preference shocks increases the gap between forward looking marginal utilities and their realizations. This mechanism, therefore, has the potential to reduce the cross-country correlation of realized consumption levels. Second, this specification does not require calibration of the mean for preference shocks.

Under this specification, the consumer's objective in country j becomes

$$U = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi\left(s^t\right) \Delta_j\left(s^t\right) \frac{\left[C_j\left(s^t\right) \Phi\left(L_j(s^t)\right) - \chi H_j(s^{t-1})\right]^{1-\sigma}}{1-\sigma},$$

where preference shocks $\{\Delta_j\left(s^t\right)\}_{j=1,2}$ follow a stationary vector autoregressive process in logs:

$$\begin{bmatrix} \log(\Delta_1\left(s^t\right)) \\ \log(\Delta_2\left(s^t\right)) \end{bmatrix} = \begin{bmatrix} \rho_{11}^{\Delta} & \rho_{12}^{\Delta} \\ \rho_{12}^{\Delta} & \rho_{11}^{\Delta} \end{bmatrix} \begin{bmatrix} \log(\Delta_1\left(s^{t-1}\right)) \\ \log(\Delta_2\left(s^{t-1}\right)) \end{bmatrix} + \begin{bmatrix} u_1\left(s^t\right) \\ u_2\left(s^t\right) \end{bmatrix}.$$

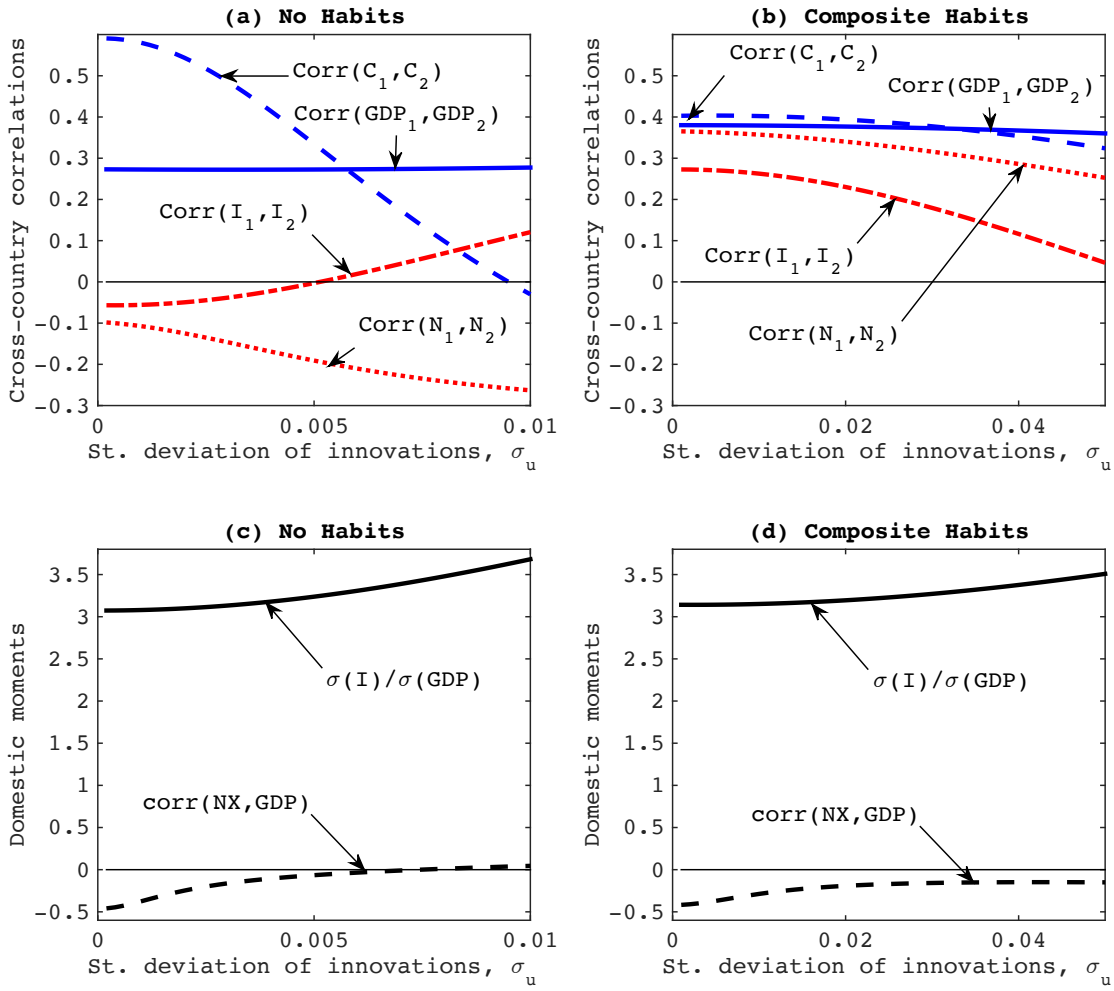
Following Wen (2007) we assume that the transition matrix is symmetric and that innovations are serially uncorrelated bivariate normal random variables with zero mean and the contemporaneous covariance matrix

$$E\left[\mathbf{u}_t \mathbf{u}_t'\right] = \sigma_u^2 \cdot \begin{bmatrix} 1 & \rho_u \\ \rho_u & 1 \end{bmatrix}.$$

To parameterize the preference shocks we follow approach of Wen (2007). We assume that prefer-

ence shocks do not spill over national borders, $\rho_{12}^{A} = 0$ and that persistence of the preference and productivity shocks is the same, $\rho_{11}^{A} = 0.976$. We also assume that contemporaneous correlations of the innovations, ρ_u, is the same as in the case of TFP shocks. These restrictions leave us one free parameter, σ_u^2, which governs the variance of the innovations and determines the relative importance of demand versus productivity shocks. Our experiments, summarized in Figure 9, explore how the magnitude of σ_u^2 affects the business cycle moments in economies with and without composite habits.

Figure 9: Preference shocks



Note: The figure shows how the volatility of preference shocks affects the business cycle moments in economies with and without composite habits. The model's predictions for cross-country correlations of cyclical components of consumption (C), employment (N), investment (I), and output (GDP) are displayed in panels (a) and (b) for different values of the standard deviation of innovations to preference shocks, σ_u. Relative volatility of investment to GDP and cyclical behavior of net exports (NX) is shown in panels (c) and (d). During the simulations the remaining model parameters were kept at the baseline values reported in Tables 3, 4, and 5.

As expected, increasing the variance of innovations, σ_u^2, reduces the cross-country correlation of consumption without affecting the international comovement of outputs. In other words, increasing the relative importance of preference shocks allows to correct the order of correlations, thus ameliorating

the 'quantity anomaly'. Depending on the specification of preferences, the cost of this improvement comes in terms of other business cycle statistics. For the model with time-separable preferences, the cost of reducing the gap between consumption and output correlations comes in the form of enhanced negative comovements of employment and procyclical net exports (Figure 9, panel (a) and (c)). In addition, investments in the two countries remain disconnected while becoming excessively volatile. Table 9 (column 5) displays the business cycle statistics for the level of volatility, σ_u^2, that eliminates the gap between cross-country correlations of consumption and output. Compared to the model with KPR preferences driven by the productivity shocks, the model with productivity and preference shocks improves domestic business cycle statistics. However, it still fails for account for the 'international comovement puzzle'.

The model with composite habits and two types of shocks exhibits somewhat better performance. Increasing the relative importance of preference shocks lowers cross-county correlations of investment and employment, while increasing volatility of investment and the correlation of investment with output. However, international comovements of investment and employment remain positive, as suggested by the data. Moreover, in line with the data, the trade balance remains countercyclical. As before, Table 9 (column 6) details statistics for the level of volatility, σ_u^2, that ensures equality of cross-country consumption and output correlations. Compared to the model with composite habits driven by TFP shocks, the model with two shocks improves domestic business cycle statistics and the volatility of prices. It predicts correct signs for international comovements, but the magnitudes of correlations move aways from their empirical counterparts. To summarize, composite habits strengthen the propagation mechanism for preference shocks.

6.2 Consumption Habits

This section examines whether a simpler specification of habits could help the canonical two-country model to account for the observed pattern of international comovements. To address this question we consider both external and internal consumption habits as alternatives to our composite habits. Both extensions are estimated using the same SMM approach as was used to estimate the baseline model.

6.2.1 Internal Consumption Habits

Our extension here features an intratemporally separable utility function with internal linear consumption habits. A member of this preference family has been used by Boldrin et al. (2001) to address asset pricing and business cycle implications of a closed production economy with elastic labor supply.[13] We consider the following instantaneous utility function

$$u(C\left(s^t\right), H(s^{t-1}), L\left(s^t\right)) = \log\left(C\left(s^t\right) - \chi H(s^{t-1})\right) - \psi \frac{\left(1 - L\left(s^t\right)\right)^{1+\nu}}{1+\nu},$$

where consumption habits evolve according to

$$H(s^t) = \lambda H(s^{t-1}) + (1-\lambda) C\left(s^t\right).$$

To make these preferences comparable with the alternatives discussed in the paper, we choose the following parameters. We set $\nu = 2/3$ to match Frisch elasticity of 1.5. The value of ψ is set to

[13] Boldrin et al. (2001) use a special case of our specification of intratemporally separable preferences, where $\psi = 1, \nu = 0, \lambda = 0$. This special case is not directly comparable with the preferences considered in this paper. It implies infinite Frisch elasticity of labor supply, whereas it is set to 1.5 in this paper. It also implies a different fixed steady-state level of hours worked. To circumvent this problem, we consider a more general specification of preferences.

ensure that $N = 1/3$ along the balanced growth path. To discuss empirical performance of the model we need to assign parameters to habits intensity and persistence. If we rely on the SMM approach used to estimate the baseline model, we end up with time-separable preferences. The reason for this is twofold. First, increasing habit intensity increases cross-country correlation of investment but decreases correlation of employment. Because the latter effect is stronger than the former, the model's ability to account for the 'international comovement puzzle' deteriorates. Second, increasing habit intensity decreases correlation of outputs due to decreased correlation of employment. At the same time, the comovement of consumption level is barely affected. Hence, increasing habit intensity also deteriorates the model's ability to address the 'quantity anomaly'. Table 9 (column 7) shows business cycle statistics for $\chi = 0.8$. Compared to the model with time-separable preferences, the model with internal habits shows deteriorated performance in terms of international comovements of all variables except for investment. Internal habits also worsen the model's predictions for volatilities of consumption and investment.

6.2.2 External Consumption Habits

In this section, we explore whether external consumption habits can help the model to account for the observed pattern of international comovements. This specification of habits, which Abel (1990) named 'Catching up with Joneses', has enjoyed a measure of success in the asset pricing and business cycle literature. Since our model abstracts from imperfect competition and price rigidities, in our framework the deep habits of Ravn et al. (2006) reduce to plain external consumption habits.

When consumption habits are external, consumers derive utility from their consumption $C\left(s^t\right)$ relative to a reference level $\overline{H}(s^{t-1})$ determined by the past history of aggregate consumption $\overline{C}\left(s^t\right)$. The instantaneous utility function is given by

$$u(C\left(s^t\right), H(s^{t-1}), L\left(s^t\right)) = \log\left(C\left(s^t\right) - \chi\overline{H}(s^{t-1})\right) - \psi\frac{\left(1 - L\left(s^t\right)\right)^{1+\nu}}{1+\nu},$$

where the reference level, $\overline{H}$, has the following law of motion

$$\overline{H}(s^t) = \lambda\overline{H}(s^{t-1}) + (1-\lambda)\overline{C}\left(s^t\right).$$

Since all consumers are identical, the equilibrium requires equality of individual and aggregate consumption levels

$$C\left(s^t\right) = \overline{C}\left(s^t\right).$$

Estimating the habit parameters using the moment matching exercise discussed in Section 4.2, we again obtain time-separable preferences. The intuition behind this finding is similar to the case of internal habits. Increasing habit intensity worsen the model's ability to account for both anomalies. Table 9 (column 8) shows business cycle statistics for $\chi = 0.8$. The model with external habits performs worse in terms of international comovements of all variables except for investment, compared to the model with time separable preferences. External habits also deteriorate the model's predictions for volatilities of consumption and investment.

Table 9. Extensions and Sensitivity Analysis

	Data	Models										
	U.S.-Europe	Complete Markets							Incomplete Markets			
	(1973Q1-	TFP Shocks			TFP and Preference Shocks		Consumption Habits		Stationary TFP Shocks		Cointegrated TFP Shocks	
	2007Q4)	No Habits	Comp. Habits		No Habits	Comp. Habits	Internal	External	$1/\eta = 1.5$	$1/\eta = 0.85$	$1/\eta = 1.5$	$1/\eta = 0.85$
		$\chi = 0$	$\chi = 0.84$	$\chi = 0.97$	$\chi = 0$	$\chi = 0.97$	$\chi = 0.8$	$\chi = 0.8$				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Cross-country correlations												
$corr(GDP_1, GDP_2)$	0.51	0.29	0.36	0.39	0.27	0.37	0.27	0.24	0.40	0.57	−0.01	0.24
$corr(C_1, C_2)$	0.36	0.59	0.44	0.41	0.27	0.37	0.70	0.67	0.41	0.58	0.21	0.31
$corr(I_1, I_2)$	0.39	−0.02	0.23	0.27	0.01	0.16	0.11	0.10	0.33	0.35	−0.15	0.19
$corr(N_1, N_2)$	0.46	−0.08	0.30	0.37	−0.20	0.30	−0.39	−0.49	0.38	0.57	0.21	0.32
B. Volatilities												
$\sigma(GDP)$	1.18	0.95	1.20	1.23	0.97	1.24	0.94	0.90	1.23	1.16	1.22	1.09
$\sigma(C)/\sigma(GDP)$	0.83	0.54	0.50	0.49	0.62	0.50	0.35	0.34	0.49	0.50	0.46	0.50
$\sigma(I)/\sigma(GDP)$	2.75	2.99	3.12	3.16	3.29	3.30	4.29	4.24	3.07	3.25	2.79	2.65
$\sigma(N)/\sigma(GDP)$	0.59	0.30	0.58	0.38	0.41	0.61	0.26	0.25	0.59	0.59	0.53	0.57
$\sigma(TOT)$	2.08	0.67	0.77	0.74	0.86	0.93	0.68	0.68	0.81	1.36	0.85	1.99
$\sigma(RER)$	3.49	0.46	0.53	0.50	0.59	0.64	0.47	0.46	0.55	0.93	0.43	1.38
C. Correlations with GDP												
$corr(C, GDP)$	0.83	0.99	0.99	0.98	0.76	0.97	0.66	0.67	0.99	0.99	0.98	0.99
$corr(I, GDP)$	0.91	0.99	0.99	0.98	0.94	0.97	0.96	0.97	0.99	0.99	0.99	0.99
$corr(N, GDP)$	0.81	0.99	0.99	0.98	0.80	0.98	0.94	0.92	0.99	0.99	0.99	0.99
$corr(NX, GDP)$	−0.44	−0.44	−0.44	−0.44	−0.04	−0.15	−0.44	−0.44	−0.44	−0.44	−0.62	−0.44

Note: International statistics in the Data row refer to the averages between the U.S. and an aggregate of 15 European countries (EU-15). The model's statistics refer to averages of 10,000 simulations of 140 periods each. All the statistics are based on logged and HP-filtered data with a smoothing parameter of 1600. The estimate of trade elasticity, $1/\eta = 0.85$, is from Heathcote and Perri (2002). The baseline value of trade elasticity of 1.5 is from BKK. The degree of habit intensity, χ, for the incomplete market models is set to the benchmark value of 0.97. For the rest of the simulations, the magnitude of habit intensity is indicated explicitly.

6.3 Incomplete Asset Markets

Under an incomplete market setting, consumers have access to one-period risk-less bonds traded internationally. The claims are denominated in terms of the intermediate good a. Let $Q\left(s^t\right)$ be the price of a claim sold after realization of s^t, which delivers a unit of good a at time $t+1$ irrespective of the realized state of nature. Let $B_1^f\left(s^t\right)$ be the quantity of such claims held by the residents of country 1 after history s^t. Then, the consumers in country 1 face the budget constraint

$$\begin{aligned} & C_1\left(s^t\right)+I_1\left(s^t\right)+q_1^a\left(s^t\right)Q\left(s^t\right)B_1^f\left(s^t\right) \\ = \; & q_1^a\left(s^t\right)\left[R_1\left(s^t\right)K_{1t}\left(s^{t-1}\right)+W_1\left(s^t\right)\cdot N_1\left(s^t\right)\right]+q_1^a\left(s^t\right)\left[B_1^f\left(s^{t-1}\right)-\Psi\left(B_1^f\left(s^t\right)\right)\right], \quad (7) \end{aligned}$$

where $q_1^a\left(s^t\right)$ denotes the price of good a in terms of the final good of country 1. As in Mandelman et al. (2011), to ensure stationarity of detrended variables, we introduce an adjustment cost for holding bonds of

$$\Psi\left(B_{1t}^f\left(s^t\right)\right)=\frac{\varsigma}{2}\Gamma_t\left(\frac{B_{1t}^f\left(s^t\right)}{\Gamma_t}\right)^2,$$

where the bond adjustment cost parameter ς is small enough to have virtually no effect on the cyclical properties of the model. We set ς to one basis point. The agents in the foreign country face a similar constraint, except for adjustment cost.

Under incomplete markets, marginal utilities are no longer equated for all times and states. Instead, stochastic discount factors are equated but only in expected terms. This promising avenue for reducing consumption correlation has delivered disappointing quantitative results so far. The reason is that stochastic discount factors turned out not to be volatile enough to make this financial friction matter. Introducing composite habits might, in principle, change this result. By increasing the elasticity of marginal utility of consumption with respect to consumption, composite habits dramatically increase volatility of stochastic discount factors for a given level of consumption volatility. Table 9 (column 9) indicates that business cycle moments are hardly affected by reduction of the span of traded securities. One possible explanation for this result lies in the nature of the stochastic process for TFP. In the next section we consider an interaction between composite habits, incomplete markets and cointegrated TPF shocks.

6.4 Cointegrated TPF shocks

In our final experiment we consider the role of the parameterization of the productivity process. As noted by Baxter and Farr (2005, p. 340) "[i]t has proved impossible to estimate the parameters of the [stochastic process for TFP] with much precision". This might raise the question as to whether predictions of our model would be affected by the specification of the productivity process. In this section we follow Rabanal et al. (2011) who provide evidence that TFP processes for the U.S. and the "rest of world" are cointegrated. We introduce cointegrated technology shocks into our model with composite habits and incomplete markets. To do so we rewrite the production function for intermediate goods as

$$Y_j(s^t)=A_j\left(s^t\right)K_j^{\alpha}\left(s^{t-1}\right)N_j\left(s^t\right)^{1-\alpha},$$

and rely on the estimates of the vector error correction model (VECM) for $A_j\left(s^t\right)$ presented by Rabanal et al. (2011). To solve the model we rescale non-stationary variables with $Z_j(s^{t-1})=A_j\left(s^t\right)^{\frac{1}{1-\alpha}}$ and

modify the adjustment cost for bond holdings following Mandelman et al. (2011)

$$\Psi\left(B_1^f\left(s^t\right)\right) = \frac{\varsigma}{2} Z_1(s^{t-1}) \left(\frac{B_1^f\left(s^t\right)}{Z_1(s^{t-1})}\right)^2.$$

Normalized equilibrium conditions in terms of rescaled variables are presented in Appendix B.

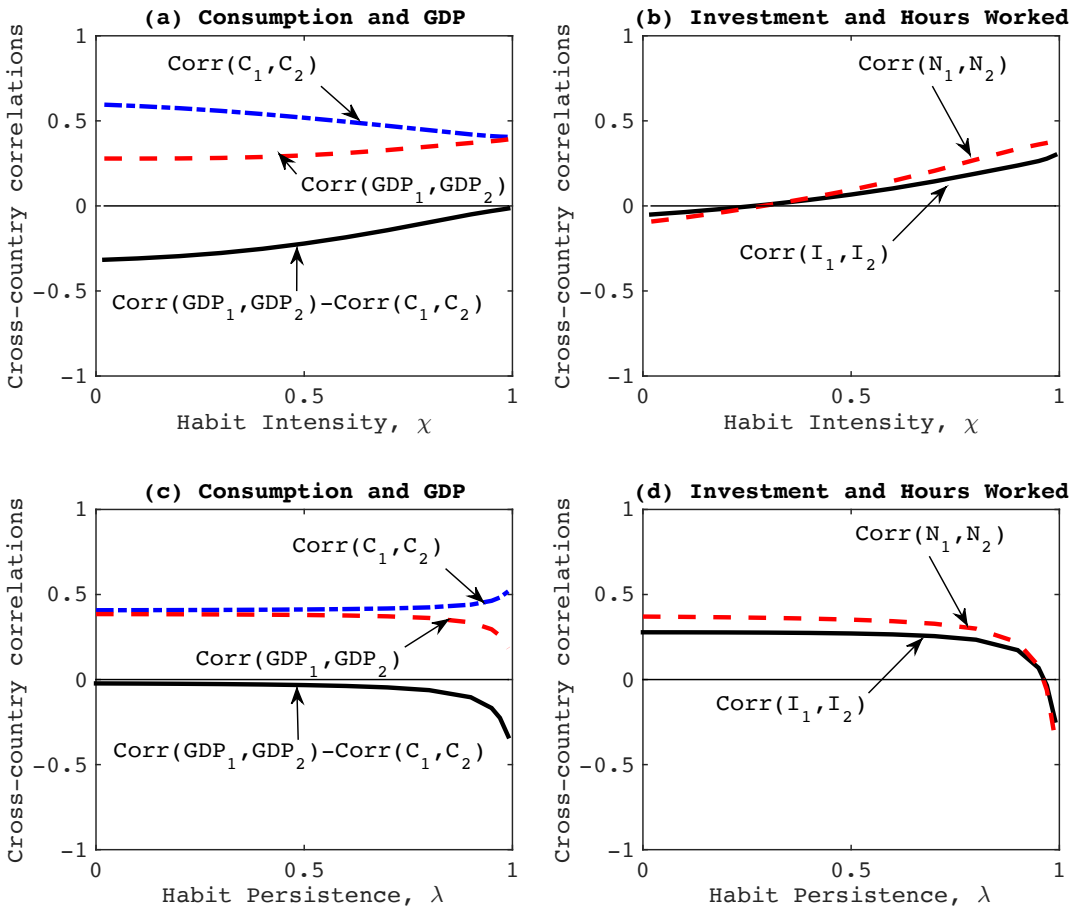
Our results presented in Table 9 (columns 11 and 12) can be summarized as follows. As in Rabanal et al. (2011), performance of our model depends on the elasticity of substitution between intermediate goods. When trade elasticity is 1.5 as in BKK, predictions of the model for international comovement are rather dismal. When we reduce the value of trade elasticity to the level estimated by Heathcote and Perri (2002), predictions of the model improve. First, the model still accounts for the comovement of quantities. It predicts positive comovement of investment and employment. Cross-country correlation of consumption is 0.31 (vs. 0.36 in the data), whereas output correlation is 0.24 (vs. 0.51 in the data). Second, the model's predictions for the volatility of prices improve. It is now capable of matching volatility of the terms of trade (1.99 vs. 2.08 in the data). Volatility of the real exchange rate almost triples, compared to our baseline model's prediction. However, it still falls short of its empirical counterpart (1.38 vs. 3.49 in the data). Third, net exports remains countercyclical. Moreover, its correlation with output matches the data exactly (-0.44 vs. -0.44 in the data).

6.5 Sensitivity Experiments

This section discusses how robust the model's predictions are to variations in the preference parameters. Figures 10 and 11 depict how cross-country correlations depend on the parameter governing the intensity and persistence of composite habits.

Degree of habit intensity. International comovements depend on the persistence and intensity of composite habit formation. Panels (a) and (b) of Figure 10 show that cross-country correlations of employment and investment become positive in our model for relatively modest habit intensity. Cross-correlations of consumptions and GDP get equalized for only for a very high degree of composite habit formation.

Figure 10: Habit Parameters and cross-country correlations

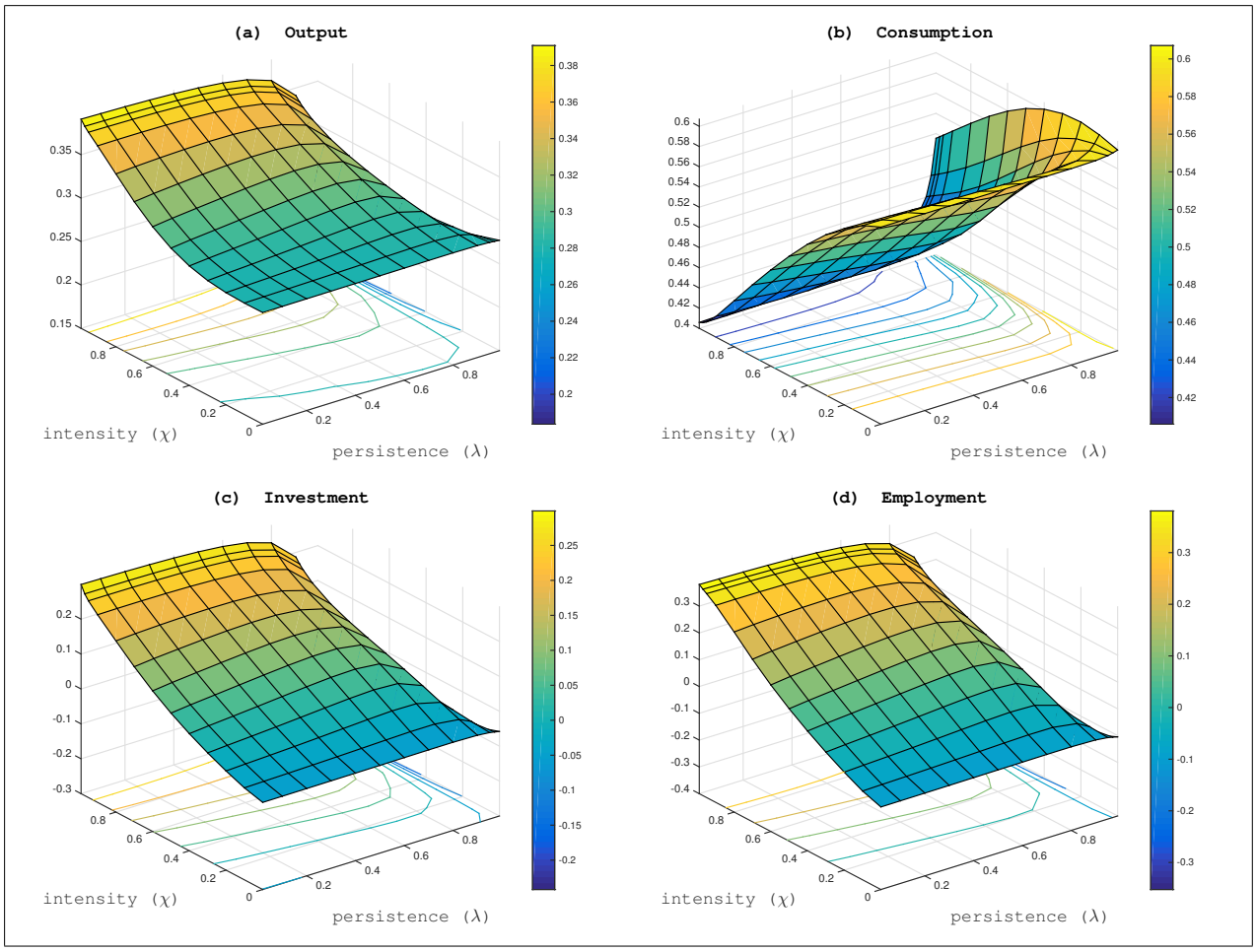


Note: The model's predictions for cross-country correlations of cyclical components of consumption (C), employment (N), investment (I), and GDP are reported for different values of habit persistence. During the simulations the remaining model parameters were kept at the baseline values reported in Tables 3, 4, and 5.

Our estimates of habit intensity exceed that of Christoffel et al. (2013), who rely on Bayesian methods to estimate a New Keynesian DSGE model augmented with composite habits. Table 9 (column 3) illustrates the extent to which the performance of our model worsens when the level of habit intensity is reduced to 0.841, as documented by Christoffel et al. (2013). Our model still accounts for the positive international comovement investment and employment. The gap between cross-country correlations of consumption and output increases. However, composite habits still constitute an improvement upon time-separable preferences.

Persistence of memory. Panels (c) and (d) of Figure 10 illustrate that, for a given level of habit intensity, variation in the persistence of habits has little effect on international comovements. The model's predictions barely change as long as the current composite has sufficient importance (more than 10%) in updating the stock of habits. For extreme levels of persistence, the stock of habits barely changes over time and the model behaves similar to one with time-separable preferences.

Figure 11: Interaction of habit parameters and cross-country correlations

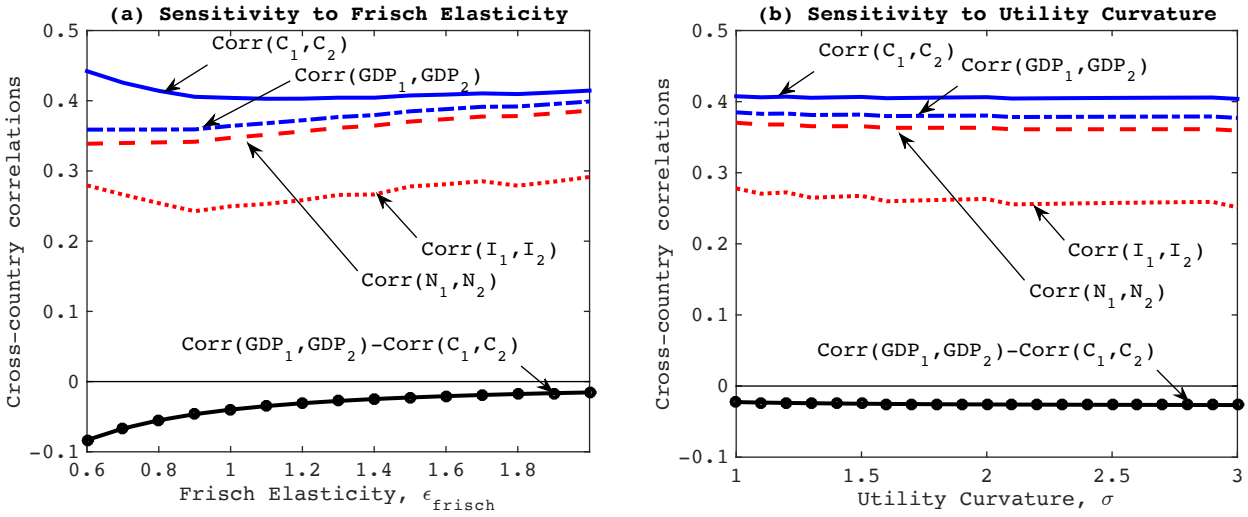


Note: The model's predictions for cross-country correlations of cyclical components of consumption, employment, investment, and output are displayed for different values of habit intensity, χ, and its persistence, λ. During the simulations the remaining model parameters were kept at the baseline values reported in Tables 3, 4, and 5.

Figure 11 displays how interaction of the intensity and persistence parameters affect predicted patterns of comovement. It also shows the contour lines for each of the four cross-country correlations. Our sensitivity results seem to be related to the experiments reported in Figure 3, where we show how the wealth elasticity of labor supply depends on habit parameters. Both emphasize the prominent role of habit intensity and minor role of habit persistence on wealth effects and international comovements.

Other preference parameters. Panel (a) of Figure 12 shows how cross-country correlations change as we vary Frisch elasticity of labor supply, ϵ_{frisch}. Changing ϵ_{frisch} does not substantially change the pattern of international comovements. Very low levels of Frisch elasticity produce slightly higher levels of consumption and investment comovement. While the higher level of consumption correlation marginally deteriorates matching ability of our model, the the higher levels of investment correlation slightly improves it. Panel (b) of Figure 12 shows very little sensitivity of the international correlations to the curvature of the utility function.

Figure 12: Sensitivity analysis



Note: The model's predictions for cross-country correlations of cyclical components of consumption (C), employment (N), investment (I), and GDP are reported for different values of habit persistence. During the simulations the remaining model parameters were kept at the baseline values reported in Tables 3, 4, and 5.

7 Conclusion

This paper shows that the introduction of internal habit formation defined over a composite of consumption and leisure can bring a two-country business cycle model closer to the data. Conditional on productivity shocks, our model reconciles with the data by closing the gap between cross-country correlations of consumption and output. It also predicts positive international correlations of investment and employment of the magnitude observed in the data. In other words, a rather parsimonious departure from a canonical two-country, two-good model goes a long way toward addressing two long-standing puzzles: the 'quantity anomaly' and the 'international comovement puzzle'.

In order to highlight the role of the preference specification in the international transmission of shocks, our analysis has abstracted from a number of issues which may also play a quantitatively important role in international comovements. We ignore alternative sources of macroeconomic disturbances, such as news shocks, uncertainty shocks, rare disasters or investment specific technology shocks. Analysis of such shocks has often compelled researchers to deviate from standard time-separable preferences, in favor of more exotic alternatives[14]. Because composite habits imply low wealth effects on labor supply and forward looking marginal utilities of consumption, they might produce potentially interesting changes in the international transmission mechanism for such shocks. We leave an analysis of these issues for future research.

References

Abel, A. B., 1990. Asset prices under habit formation and catching up with the Joneses. The American Economic Review 80 (2), 38–42.

[14]Popular alternatives include GHH preferences, Epstein and Zin (1989) recursive preferences, and Jaimovich and Rebelo (2009) time-nonseparable preferences.

Alessandria, G., Choi, H., Kaboski, J. P., Midrigan, V., 2015. Microeconomic uncertainty, international trade, and aggregate fluctuations. Journal of Monetary Economics 69, 20–38.

Backus, D., Coleman, C., Ferriere, A., Lyon, S., 2016. Pareto weights as wedges in two-country models. Journal of Economic Dynamics and Control, forthcoming.

Backus, D. K., 1993. Interpreting comovements in the trade balance and the terms of trade. Journal of International Economics 34 (3-4), 375–387.

Backus, D. K., Kehoe, P. J., Kydland, F. E., 1992. International real business cycles. Journal of Political Economy 100 (4), 745–775.

Backus, D. K., Kehoe, P. J., Kydland, F. E., 1994. Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? American Economic Review 84 (1), 84–103.

Backus, D. K., Routledge, B. R., Zin, S. E., 2008. Recursive preferences. In: Durlauf, S. N., Blume, L. E. (Eds.), The New Palgrave Dictionary of Economics. Palgrave Macmillan, Basingstoke.

Bahmani-Oskooee, M., Ratha, A., 2007. The S-curve Dynamics of US Bilateral Trade. Review of International Economics 15 (2), 430–439.

Baxter, M., 1995. International trade and business cycles. 1st Edition. Vol. 3 of Handbook of International Economics. Elsevier, North-Holland, pp. 1801–1864.

Baxter, M., Crucini, M. J., 1995. Business Cycles and the Asset Structure of Foreign Trade. International Economic Review 36 (4), 821–854.

Baxter, M., Farr, D. D., 2005. Variable Capital Utilization and International Business Cycles. Journal of International Economics 65 (2), 335–347.

Bayer, C., Tjaden, V., 2016. Large open economies and fixed costs of capital adjustment. Review of Economic Dynamics 21, 125–146.

Beaudry, P., Dupaigne, M., Portier, F., 2011. Modeling News-Driven International Business Cycles. Review of Economic Dynamics 14 (1), 72–91.

Bilbiie, F. O., 2011. Nonseparable preferences, frisch labor supply, and the consumption multiplier of government spending: One solution to a fiscal policy puzzle. Journal of Money, Credit and Banking 43 (1), 221–251.

Boldrin, M., Christiano, L. J., Fisher, J. D. M., 2001. Habit persistence, asset returns, and the business cycle. American Economic Review 91 (1), 149–166.

Campbell, J. Y., Cochrane, J. H., 1999. Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. Journal of Political Economy 107 (2), 205–251.

Chen, X., Ludvigson, S. C., 2009. Land of addicts? An empirical investigation of habit-based asset pricing models. Journal of Applied Econometrics 24 (7), 1057–1093.

Chistiano, L. J., den Haan, W. J., 1996. Small-Sample Properties of GMM for Business-Cycle Analysis. Journal of Business & Economic Statistics 14 (3), 309–27.

Christiano, L. J., Eichenbaum, M., Evans, C. L., 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. The Journal of Political Economy 113 (1), 1–45.

Christoffel, K., Jaccard, I., Kilponen, J., 2013. Welfare and bond pricing implications of fiscal stabilization policies. Research Discussion Papers 32/2013, Bank of Finland.

Colacito, R., Croce, M. M., 2011. Risks for the long run and the real exchange rate. Journal of Political Economy 119 (1), 153 – 181.

Constantinides, G. M., 1990. Habit formation: A resolution of the equity premium puzzle. Journal of Political Economy 98 (3), 519.

Corsetti, G., Dedola, L., Leduc, S., 2014. The international dimension of productivity and demand shocks in the US economy. Journal of the European Economic Association 12 (1), 153–176.

Deli, Y. D., 2016. Endogenous capital depreciation and technology shocks. Journal of International Money and Finance, forthcoming.

Devereux, M. B., Gregory, A. W., Smith, G. W., 1992. Realistic cross-country consumption correlations in a two-country, equilibrium, business cycle model. Journal of International Money and Finance 11 (1), 3–16.

Devereux, M. B., Yetman, J., 2010. Leverage constraints and the international transmission of shocks. Journal of Money, Credit and Banking 42 (s1), 71–105.

Di Pace, F., Faccini, R., 2012. Deep habits and the cyclical behaviour of equilibrium unemployment and vacancies. Journal of Economic Dynamics and Control 36 (2), 183–200.

Dmitriev, A., 2016. On the properties of composite habits. Working paper, Department of Economics, University of Auckland.

Dmitriev, A., Krznar, I., 2012. Habit persistence and international comovements. Macroeconomic Dynamics 16 (S3), 312–330.

Dmitriev, A., Roberts, I., 2012. International business cycles with complete markets. Journal of Economic Dynamics and Control 36 (6), 862–875.

Dmitriev, A., Roberts, I., 2013. The cost of adjustment: On comovement between the trade balance and the terms of trade. Economic Modelling 35, 689–700.

Enders, Z., Müller, G. J., 2009. On the international transmission of technology shocks. Journal of International Economics 78 (1), 45–59.

Engel, C., Wang, J., 2011. International trade in durable goods: Understanding volatility, cyclicality, and elasticities. Journal of International Economics 83, 37–52.

Epstein, L. G., Zin, S. E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 57 (4), 937–969.

Faia, E., 2007. Finance and international business cycles. Journal of Monetary Economics 54 (4), 1018–1034.

Feenstra, R. C., Inklaar, R., Timmer, M. P., 2015. The Next Generation of the Penn World Table. American Economic Review 105 (10), 3150–3182.

Ferson, W. E., Constantinides, G. M., 1991. Habit persistence and durability in aggregate consumption: Empirical tests. Journal of Financial Economics 29 (2), 199–240.

Frisch, R., 1959. A complete scheme for computing all direct and cross demand elasticities in a model with many sectors. Econometrica 27 (2), 177–196.

Gao, X., Hnatkovska, V., Marmer, V., 2014. Limited participation in international business cycle models: A formal evaluation. Journal of Economic Dynamics and Control 39, 255 – 272.

Gourio, F., Siemer, M., Verdelhan, A., 2013. International risk cycles. Journal of International Economics 89 (2), 471–484.

Greenwood, J., Hercowitz, Z., Huffman, G. W., 1988. Investment, Capacity Utilization, and the Real Business Cycle. American Economic Review 78 (3), 402–417.

Grishchenko, O. V., 2010. Internal vs. External Habit Formation: The Relative Importance for Asset Pricing. Journal of Economics and Business 62 (3), 176–194.

Hayashi, F., 1982. Tobin's Marginal q and Average q: A Neoclassical Interpretation. Econometrica 50 (1), 213–224.

Heathcote, J., Perri, F., 2002. Financial autarky and international business cycles. Journal of Monetary Economics 49 (3), 601–627.

Heathcote, J., Perri, F., 2003. Why Has the U.S. Economy Become Less Correlated with the Rest of the World? American Economic Review 93 (2), 63–69.

Heathcote, J., Perri, F., 2013. The international diversification puzzle is not as bad as you think. Journal of Political Economy 121 (6), 1108–1159.

Heyerdahl-Larsen, C., 2014. Asset prices and real exchange rates with deep habits. Review of Financial Studies 27 (11), 3280–3317.

Jaccard, I., 2011. Asset Pricing and Housing Supply in a Production Economy. The B.E. Journal of Macroeconomics 11 (1), 1–40.

Jaccard, I., 2014. Asset returns and labor supply in a production economy. Journal of Money, Credit and Banking 46 (5), 889–919.

Jaccard, I., 2016. Asset pricing and the propagation of macroeconomic shocks. ECB Working Paper 1525, European Central Bank.

Jaimovich, N., Rebelo, S., 2009. Can News about the Future Drive the Business Cycle? American Economic Review 99 (4), 1097–1118.

Judd, K. L., Maliar, L., Maliar, S., 2011. Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models. Quantitative Economics 2 (2), 173–210.

Karabarbounis, L., 2014. Home production, labor wedges, and international business cycles. Journal of Monetary Economics 64, 68 –84.

Kehoe, P. J., Perri, F., 2002. International business cycles with endogenous incomplete markets. Econometrica 70 (3), 907–928.

King, R. G., Plosser, C. I., Rebelo, S. T., 1988. Production, growth and business cycles : I. The basic neoclassical model. Journal of Monetary Economics 21 (2-3), 195–232.

King, R. G., Plosser, C. I., Rebelo, S. T., 2002. Production, growth and business cycles: Technical appendix. Computational Economics 20 (1-2), 87–116.

Kollmann, R., 1996. Incomplete Asset Markets and the Cross-country Consumption Correlation Puzzle. Journal of Economic Dynamics and Control 20 (5), 945–961.

Kollmann, R., May 2012. Limited asset market participation and the consumption-real exchange rate anomaly. Canadian Journal of Economics 45 (2), 566–584.

Kollmann, R., December 2013. Global banks, financial shocks, and international business cycles: Evidence from an estimated model. Journal of Money, Credit and Banking 45 (s2), 159–195.

Kollmann, R., 2015a. Exchange rates dynamics with long-run risk and recursive preferences. Open Economies Review 26, 175–196.

Kollmann, R., 2015b. Risk sharing in a world economy with uncertainty shocks. CAMA Working Paper 2015-44, Centre for Applied Macroeconomic Analysis, The Australian National University.

Kollmann, R., 2016. International business cycles and risk sharing with uncertainty shocks and recursive preferences. Journal of Economic Dynamics and Control, forthcoming.

Kollmann, R., Enders, Z., Muller, G. J., April 2011. Global banking and international business cycles. European Economic Review 55 (3), 407–426.

Kunsch, H. R., 1989. The jackknife and the bootstrap for general stationary observations. The Annals of Statistics 17 (3), 1217–1241.

Kwiatkowski, D., Phillips, P. C., Schmidt, P., Shin, Y., 1992. Testing the null hypothesis of stationarity against the alternative of a unit root. Journal of Econometrics 54 (1), 159–178.

Lettau, M., Uhlig, H., 2000. Can habit formation be reconciled with business cycle facts? Review of Economic Dynamics 3 (1), 79–99.

Maliar, S., Maliar, L., Judd, K., 2011. Solving the multi-country real business cycle model using ergodic set methods. Journal of Economic Dynamics and Control 35 (2), 207–228.

Mandelman, F. S., Rabanal, P., Rubio-Ramírez, J. F., Vilán, D., 2011. Investment-specific technology shocks and international business cycles: An empirical assessment. Review of Economic Dynamics 14 (1), 136–155.

McConnell, M. M., Perez-Quiros, G., 2000. Output Fluctuations in the United States: What Has Changed Since the Early 1980's? The American Economic Review 90 (5), 1464–1476.

Monacelli, T., Perotti, R., 2008. Fiscal Policy, Wealth Effects, and Markups. NBER Working Papers 14584, National Bureau of Economic Research.

Olivero, M. P., March 2010. Market power in banking, countercyclical margins and the international transmission of business cycles. Journal of International Economics 80 (2), 292–301.

Rabanal, P., Rubio-Ramírez, J. F., Tuesta, V., 2011. Cointegrated TFP processes and international business cycles. Journal of Monetary Economics 58 (2), 156–171.

Raffo, A., 2008. Net Exports, Consumption Volatility and International Business Cycle Models. Journal of International Economics 75 (1), 14–29.

Ravn, M., Schmitt-Grohe, S., Uribe, M., 2006. Deep habits. Review of Economic Studies 73 (1), 195–218.

Ravn, M. O., Schmitt-Grohe, S., Uribe, M., 2012. Consumption, government spending, and the real exchange rate. Journal of Monetary Economics 59 (3), 215–234.

Rozen, K., 2010. Foundations of intrinsic habit formation. Econometrica 78 (4), 1341–1373.

Ruge-Murcia, F. J., 2007. Methods to estimate dynamic stochastic general equilibrium models. Journal of Economic Dynamics and Control 31 (8), 2599–2636.

Senhadji, A. S., 1998. Dynamics of the trade balance and the terms of trade in LDCs: The S-curve. Journal of International Economics 46 (1), 105–131.

Stockman, A. C., Tesar, L. L., 1995. Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. American Economic Review 85 (1), 168–185.

Wen, Y., 2006. Demand shocks and economic fluctuations. Economics Letters 90 (3), 378–383.

Wen, Y., 2007. By Force of Demand: Explaining International Comovements. Journal of Economic Dynamics and Control 31 (1), 1–23.

Yakhin, Y., 2007. Staggered Wages, Financial Frictions, and the International Comovement Problem. Review of Economic Dynamics 10 (1), 148–171.

Yoon, G., 2005. Has the U.S. economy really become less correlated with that of the rest of the world? Economic Modelling 22 (1), 147–158.

A Data Sources

We collect data from Penn World Table 9.0, Federal Reserve Economic Data (FRED), OECD Economic Outlook (EO), and OECD Quarterly National Accounts (QNA) for the period 1973–2007. The definitions of the variables we construct are summarized in Table A1. The EU-15 comprises the following 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom. Due to data unavailability, we compute the employment series for Europe as the weighted aggregate of 13 European countries: all the countries included in the EU-15 except Greece and Luxembourg.

Table A1. Data sources and definitions

Variable	Definition	Source
The Unites States		
Output (GDP)	Gross Domestic Product (at constant 2010 prices)	OECD QNA
Capital (K)	Capital Stock (at constant 2011 prices)	Penn World Table 9.0
Consumption (C)	Final Private Consumption (at constant 2010 prices)	OECD QNA
Investment (I)	Gross Fixed Capital Formation (at constant 2010 prices)	OECD QNA
Employment (N)	Total Employment	OECD EO
Real Exchange Rate (RER)	Real Trade Weighted U.S. Dollar Index	FRED
Import Price	Imports at current prices/imports at constant 2010 prices	OECD QNA
Export Price	Exports at current prices/exports at constant 2010 prices	OECD QNA
Terms of Trade (TOT)	Import price/export price	
Net Exports ratio (NX)	(export-TOT*import)/GDP (all at current prices)	
Europe (EU-15)		
Output (GDP)	Aggregate of 15 European Countries (PPP exchange rates in 2010)	OECD QNA
Capital (K)	Aggregate of 15 European Countries (PPP exchange rates in 2011)	Penn World Table 9.0
Consumption (C)	Aggregate of 15 European Countries (PPP exchange rates in 2010)	OECD QNA
Investment (I)	Aggregate of 15 European Countries (PPP exchange rates in 2010)	OECD QNA
Employment (N)	Population weighted aggregate of 13 European Countries	OECD EO

B Optimality Conditions: Models with Stationary TFP Shocks

Baseline Model

This section summarizes the system of optimality conditions formulated in terms of stationary variables. In what follows, $c_j(s^t) \equiv C_j(s^t)/\Gamma_t$ and $k_j(s^{t-1}) \equiv K_j(s^{t-1})/\Gamma_t$ will be denoted as c_{jt} and k_{jt-1} respectively. The same notational convention applies to the rest of the variables. The equilibrium is characterized by the following set of optimality conditions:

i) marginal utilities of consumption

$$\Lambda_{jt} = (\psi + (1 - N_{jt})^{\nu}) \left[[c_{jt} (\psi + (1 - N_{jt})^{\nu}) - \chi h_{jt-1}]^{-\sigma} + \theta_{jt} (1 - \lambda) \right], \text{ for } j = 1, 2, \quad (8)$$

ii) labor supply equations

$$c_{1t} \frac{\nu(1 - N_{1t})^{\nu-1}}{\psi + (1 - N_{1t})^{\nu}} = q^a_{1t} w_{1t}, \quad (9)$$

$$c_{2t} \frac{\nu(1 - N_{2t})^{\nu-1}}{\psi + (1 - N_{2t})^{\nu}} = q^b_{2t} w_{2t}, \quad (10)$$

iii) demand for intermediate goods

$$a_{1t} = \omega^{1/\eta} (q^a_{1t})^{-1/\eta} da_{1t}, \quad (11)$$

$$a_{2t} = (1 - \omega)^{1/\eta} (q^a_{1t}/RER_t)^{-1/\eta} da_{2t}, \quad (12)$$

$$b_{2t} = \omega^{1/\eta} \left(q^b_{2t}\right)^{-1/\eta} da_{2t}, \quad (13)$$

$$b_{1t} = (1-\omega)^{1/\eta} \left(RER_t \cdot q_{2t}^b\right)^{-1/\eta} da_{1t}, \tag{14}$$

iv) production functions for the final goods

$$da_{1t} = \left(\omega a_{1t}^{1-\eta} + (1-\omega)\, b_{1t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{15}$$

$$da_{2t} = \left((1-\omega)\, a_{2t}^{1-\eta} + \omega b_{2t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{16}$$

v) market clearing for the final goods

$$c_{jt} + i_{jt} = da_{jt}, \text{ for } j = 1, 2, \tag{17}$$

vi) market clearing for intermediate goods

$$y_{1t} = a_{1t} + a_{2t} = z_{1t} k_{1t-1}^{\alpha} N_{1t}^{1-\alpha}, \tag{18}$$

$$y_{2t} = b_{1t} + b_{2t} = z_{2t} k_{2t-1}^{\alpha} N_{2t}^{1-\alpha}, \tag{19}$$

vii) factor prices

$$R_{jt} = \alpha z_{jt} k_{jt-1}^{\alpha-1} N_{jt}^{1-\alpha}, \text{ for } j = 1, 2, \tag{20}$$

$$w_{jt} = (1-\alpha)\, z_{jt} k_{1t-1}^{\alpha} N_{jt}^{-\alpha}, \text{ for } j = 1, 2, \tag{21}$$

viii) risk sharing condition

$$\Lambda_{1t} RER_t = \Lambda_{2t}, \tag{22}$$

ix) the Euler equations

$$gm_{1t} = \beta^* E_t \left[\Lambda_{1t+1} q_{1t+1}^a R_{1t+1} + m_{1t+1} \left(1 - \delta - \frac{\Omega_1}{1-\xi} \left(\frac{i_{1t+1}}{k_{1t}}\right)^{1-1/\xi} + \Omega_2\right)\right], \tag{23}$$

$$gm_{2t} = \beta^* E_t \left[\Lambda_{2t+1} q_{2t+1}^b R_{2t+1} + m_{2t+1} \left(1 - \delta - \frac{\Omega_1}{1-\xi} \left(\frac{i_{2t+1}}{k_{2t}}\right)^{1-1/\xi} + \Omega_2\right)\right], \tag{24}$$

where $\beta^* = \beta g^{1-\sigma}$, and

$$m_{jt} \Omega_1 \left(\frac{i_{jt}}{k_{jt-1}}\right)^{-1/\xi} = \Lambda_{1t}, \text{ for } j = 1, 2. \tag{25}$$

x) the equations of motion for capital

$$gk_{jt} = (1-\delta) k_{jt-1} + \left(\frac{\Omega_1}{1-1/\xi} \left(\frac{i_{jt}}{k_{jt-1}}\right)^{1-1/\xi} + \Omega_2\right) k_{jt-1}, \text{ for } j = 1, 2. \tag{26}$$

xi) intertemporal condition for habits

$$g\theta_{jt} = \beta^* E_t \left[\lambda \theta_{jt+1} - \chi \left[c_{jt+1} \left(\psi + (1 - N_{jt+1})^{\nu}\right) - \chi h_{jt}\right]^{-\sigma}\right], \text{ for } j = 1, 2, \tag{27}$$

xii) the equations of motion for habits

$$gh_{jt} = \lambda h_{jt-1} + (1-\lambda)\, c_{jt}\left(\psi + (1-N_{jt})^{\nu}\right), \quad \text{for } j = 1,2,$$

Baseline Model Augmented with Preference Shocks

When we augment the model with preference shocks our baseline optimality conditions are respectively replaced with the following:

i) marginal utilities of consumption

$$\Lambda_{jt} = \left(\psi + (1-N_{jt})^{\nu}\right)\left[\Delta_{jt}\left[c_{jt}\left(\psi + (1-N_{jt})^{\nu}\right) - \chi h_{jt-1}\right]^{-\sigma} + \theta_{jt}\,(1-\lambda)\right], \text{ for } j = 1,2, \tag{28}$$

xi) intertemporal condition for habits

$$g\theta_{jt} = \beta^{*} E_t\left[\lambda\theta_{jt+1} - \chi\Delta_{jt+1}\left[c_{jt+1}\left(\psi + (1-N_{jt+1})^{\nu}\right) - \chi h_{jt}\right]^{-\sigma}\right], \text{ for } j = 1,2, \tag{29}$$

Model with Internal Consumption Habits

When we analyze internal consumption habits, our baseline optimality conditions are respectively replaced with the following:

i) marginal utilities of consumption

$$\Lambda_{jt} = \frac{1}{c_{jt} - \chi h_{jt-1}} + (1-\lambda)\theta_{jt}, \text{ for } j = 1,2, \tag{30}$$

ii) labor supply equations

$$\psi {N_{1t}}^{\nu} = \Lambda_{2t} q^{a}_{1t} w_{2t}, \tag{31}$$

$$\psi {N_{2t}}^{\nu} = \Lambda_{2t} q^{b}_{2t} w_{2t}, \tag{32}$$

xi) intertemporal condition for habits

$$g\theta_{jt} = \beta E_t\left[\lambda\theta_{jt+1} - \chi\frac{1}{c_{jt+1} - \chi h_{jt}}\right], \text{ for } j = 1,2, \tag{33}$$

xii) the equations of motion for habits

$$gh_{jt} = \lambda h_{jt-1} + (1-\lambda)\, c_{jt}, \quad \text{for } j = 1,2. \tag{34}$$

Model with External Consumption Habits

When consumption habits are external marginal utilities are no longer forward-looking. Because agents do not internalize the effect that their own current consumption has on utility of the others, the intertemporal condition for habits (xi) no longer holds. Our baseline optimality conditions are respectively replaced with the following:

i) marginal utilities of consumption

$$\Lambda_{jt} = \frac{1}{c_{jt} - \chi h_{jt-1}}, \text{ for } j = 1,2, \tag{35}$$

ii) labor supply equations

$$\psi {N_{1t}}^{\nu} = \Lambda_{2t} q^a_{1t} w_{2t}, \tag{36}$$

$$\psi {N_{2t}}^{\nu} = \Lambda_{2t} q^b_{2t} w_{2t}, \tag{37}$$

xi) intertemporal condition for habits is eliminated

xii) the equations of motion for habits

$$g h_{jt} = \lambda h_{jt-1} + (1-\lambda)\, c_{jt}, \quad \text{for } j = 1, 2. \tag{38}$$

Model with Incomplete Markets

When we reduce the span of internationally traded assets to a one-period risk-less bond, our baseline optimality conditions are respectively replaced with the following:

viii) risk sharing condition

$$E_t\left[\beta^* \frac{\Lambda_{2t+1}}{\Lambda_{2t}} \frac{q^a_{1t+1}}{q^a_{1t}} \frac{RER_t}{RER_{t+1}}\right] = E_t\left[\beta^* \frac{\Lambda_{1t+1}}{\Lambda_{1t}} \frac{q^a_{1t+1}}{q^a_{1t}}\right] - g\varsigma b^f_{1t}, \tag{39}$$

where $\beta^* = \beta g^{1-\sigma}$.

In addition, bond prices and quantities must satisfy the following conditions:

xiii) bond price

$$Q_t = E_t\left[\beta^* \frac{\Lambda_{1t+1}}{g\Lambda_{1t}} \frac{q^a_{1t+1}}{q^a_{1t}}\right] - \varsigma b^f_{1t},$$

xiv) equation of motion for bonds

$$Q_t b^f_{1t} = a_{2t} - \frac{q^b_{1t}}{q^a_{1t}} b_{1t} + \frac{1}{g} b^f_{1t-1} - \frac{\varsigma}{2}\left(b^f_{1t}\right)^2.$$

C Optimality Conditions: The Model with Cointegrated TFP shocks

Model with Composite Habits

Since the presence of the unit root makes the model non-stationary, we rescale the appropriate variables. Let $\widehat{C}_{jt} = \frac{C_{jt}}{Z_{jt-1}}$, $\widehat{I}_{jt} = \frac{I_{jt}}{Z_{jt-1}}$,... $, \widehat{K}_{jt-1} = \frac{K_{jt-1}}{Z_{jt-1}}, \widehat{H}_{jt-1} = \frac{H_{jt-1}}{Z_{jt-1}}$, where $Z_{jt} = A_{jt}^{\frac{1}{1-\alpha}}$. Using rescaled variables we can summarize the normalized optimality conditions as follows:

i) marginal utilities of consumption

$$\widehat{\Lambda}_{jt} = (\psi + {L_{jt}}^{\nu}) \left[\left[\widehat{C}_{jt}(\psi + {L_{jt}}^{\nu}) - \chi \widehat{H}_{jt-1}\right]^{-\sigma} + \left(\frac{Z_{jt-1}}{Z_{jt}}\right)^{\sigma} \widehat{\theta}_{jt}\right], \text{ for } j = 1, 2, \tag{40}$$

where

$$\widehat{\theta}_{jt} = -\chi\beta E_t\left[\left(\widehat{C}_{jt+1}(\psi + {L_{jt+1}}^{\nu}) - \chi \widehat{H}_{jt}\right)^{-\sigma}\right], \text{ for } j = 1, 2, \tag{41}$$

and $\widehat{\Lambda}_{jt} = \Lambda_{jt} Z^{\sigma}_{jt-1}, \widehat{\theta}_{jt} = \theta_{jt} Z^{\sigma}_{jt}$.

ii) labor supply equations

$$\widehat{C}_{1t}\frac{\nu {L_{1t}}^{\nu-1}}{\psi+{L_{1t}}^{\nu}}=q_{1t}^{a}\widehat{W}_{1t}, \tag{42}$$

$$\widehat{C}_{2t}\frac{\nu {L_{2t}}^{\nu-1}}{\psi+{L_{2t}}^{\nu}}=q_{2t}^{b}\widehat{W}_{2t}, \tag{43}$$

iii) demand for intermediate goods

$$\widehat{a}_{1t}=\omega\left(q_{1t}^{a}\right)_{1t}^{-1/\eta}\widehat{da}_{1t}, \tag{44}$$

$$\widehat{a}_{2t}\left(\frac{Z_{1t-1}}{Z_{2t-1}}\right)=(1-\omega)\left(q_{1t}^{a}/RER_{t}\right)^{-1/\eta}\widehat{da}_{2t}, \tag{45}$$

$$\widehat{b}_{2t}=\omega\left(q_{2t}^{b}\right)^{-1/\eta}\widehat{da}_{2t}, \tag{46}$$

$$\widehat{b}_{1t}=(1-\omega)\left(RER_{t}\cdot q_{2t}^{b}\right)_{1t}^{-1/\eta}\widehat{da}_{1t}\left(\frac{Z_{1t-1}}{Z_{2t-1}}\right), \tag{47}$$

iv) production functions for the final goods

$$\widehat{da}_{1t}=\left(\omega^{\eta}\widehat{a}_{1t}^{1-\eta}+(1-\omega)^{\eta}\widehat{b}_{1t}^{1-\eta}\left(\frac{Z_{2t-1}}{Z_{1t-1}}\right)\right)^{\frac{1}{1-\eta}}, \tag{48}$$

$$\widehat{da}_{2t}=\left((1-\omega)^{\eta}\widehat{a}_{2t}^{1-\eta}\left(\frac{Z_{1t-1}}{Z_{2t-1}}\right)+\omega^{\eta}\widehat{b}_{2t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{49}$$

v) market clearing for the final goods

$$\widehat{C}_{jt}+\widehat{I}_{jt}=\widehat{da}_{jt},\ \text{for } j=1,2, \tag{50}$$

vi) market clearing for intermediate goods

$$\widehat{Y}_{1t}=\widehat{a}_{1t}+\widehat{a}_{2t}, \tag{51}$$

$$\widehat{Y}_{2t}=\widehat{b}_{1t}+\widehat{b}_{2t}, \tag{52}$$

where

$$\widehat{Y}_{jt}=\left(\frac{A_{jt}}{A_{jt-1}}\right)\widehat{K}_{jt-1}^{\alpha}N_{jt}^{1-\alpha},\ \text{for } j=1,2,$$

vii) factor prices

$$\alpha N_{jt}\widehat{W}_{jt}=(1-\alpha)\,R_{jt}\widehat{K}_{jt-1},\ \text{for } j=1,2, \tag{53}$$

$$\widehat{W}_{jt}=(1-\alpha)\left(\frac{A_{jt}}{A_{jt-1}}\right)\widehat{K}_{1t-1}^{\alpha}N_{jt}^{-\alpha},\ \text{for } j=1,2, \tag{54}$$

viii) risk sharing condition

$$E_{t}\left[\beta\frac{\widehat{\Lambda}_{2t+1}}{\widehat{\Lambda}_{2t}}\left(\frac{Z_{2t-1}}{Z_{2t}}\right)^{\sigma}\frac{q_{1t+1}^{a}}{q_{1t}^{a}}\frac{RER_{t}}{RER_{t+1}}\right]=E_{t}\left[\beta\frac{\widehat{\Lambda}_{1t+1}}{\widehat{\Lambda}_{1t}}\left(\frac{Z_{1t-1}}{Z_{1t}}\right)^{\sigma}\frac{q_{1t+1}^{a}}{q_{1t}^{a}}\right]-\varsigma\widehat{B}_{1t}^{f}, \tag{55}$$

ix) the Euler equations

$$\widehat{m}_{1t}\left(\frac{Z_{1t}}{Z_{1t-1}}\right)^{\sigma}=\beta E_{t}\left[\widehat{\Lambda}_{1t+1}q_{1t+1}^{a}R_{1t+1}+\widehat{m}_{1t+1}\left(1-\delta\right)\right], \tag{56}$$

$$\widehat{m}_{2t}\left(\frac{Z_{2t}}{Z_{2t-1}}\right)^{\sigma}=\beta E_t\left[\widehat{\Lambda}_{2t+1}q^b_{2t+1}R_{2t+1}+\widehat{m}_{2t+1}\left(1-\delta\right)\right], \tag{57}$$

where

$$\widehat{m}_{jt}=\widehat{\Lambda}_{1t},\ \text{for } j=1,2. \tag{58}$$

x) the equations of motion for capital

$$\widehat{K}_{jt}\left(\frac{Z_{jt}}{Z_{jt-1}}\right)=(1-\delta)\widehat{K}_{jt-1}+\widehat{I}_{jt},\ \text{for } j=1,2. \tag{59}$$

xii) the equations of motion for habits

$$\widehat{H}_{jt}\left(\frac{Z_{jt}}{Z_{jt-1}}\right)=\widehat{C}_{jt}\left(\psi+(1-N_{jt})^{\nu}\right),\ \ \text{for } j=1,2, \tag{60}$$

xiii) bond price

$$Q_t=E_t\left[\beta\frac{\widehat{\Lambda}_{1t+1}}{\widehat{\Lambda}_{1t}}\left(\frac{Z_{1t-1}}{Z_{1t}}\right)^{\sigma}\frac{q^a_{1t+1}}{q^a_{1t}}\right]-\varsigma\widehat{B}^f_{1t}, \tag{61}$$

xiv) the equation of motion for bonds

$$Q_t\widehat{B}^f_{1t}=\widehat{a}_{2t}-\frac{q^b_{1t}}{q^a_{1t}}\widehat{b}_{1t}\left(\frac{Z_{2t-1}}{Z_{1t-1}}\right)+\widehat{B}^f_{1t-1}\left(\frac{Z_{1t-2}}{Z_{1t-1}}\right)-\frac{\varsigma}{2}\left(\widehat{B}^f_{1t}\right)^2. \tag{62}$$