

A broad application optimisation-based rostering model

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Abstract

This work considers the automated generation of high quality rosters for the general rostering situation. We develop a general modelling framework based on integer programming that promises to provide good descriptions of a broad range of rostering problems and we present optimisation-based solution methods that efficiently provide good solutions to the problem instances. We examine our current modelling approach in terms of expected future requirements of the system. We currently see these requirements as being an increasing demand for modelling with respect to the preferences of staff. Computational results and feedback from commercial projects demonstrating the system are provided as well as comparative results against existing rostering processes on the same rostering problem. A new solution method is reported that uses multiple constraint branching within a dive-and-fix heuristic. This method has been used with success to improve performance of the branch-and-price solution approach implemented as part of this project.

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