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Capturing Pedagogic Change in Novice Primary Teachers of Mathematics: Development of the measuring instrument DART

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Abstract

In this thesis, I investigate factors contributing to the complex concept of quality teaching. The subjects for the study were pre-service primary school teachers in initial teacher education courses who had varying degrees of maths anxiety. The “big question” was how these people would teach when they were alone in classrooms. Would they show versatility, see the teachable moment, or analyse the difficulty their students were having?

There are many factors essential to quality teaching, such as knowing the subject matter, and there is much research about what teachers should know to teach mathematics effectively. Using a mixed methodology, data were gathered from the participants before they began teaching, as they started in the classroom, and again two years later. Through my research and trialling different instruments, I was unable to find one that captured effectively and accurately what was seen on the videos of these novice teachers teaching mathematics in the Numeracy Development Projects (NDP) style. As a result I developed a suitable data capture tool to encode NDP-style sessions, which I have called the DART (Dynamic Analysis Reflection Tool) framework.

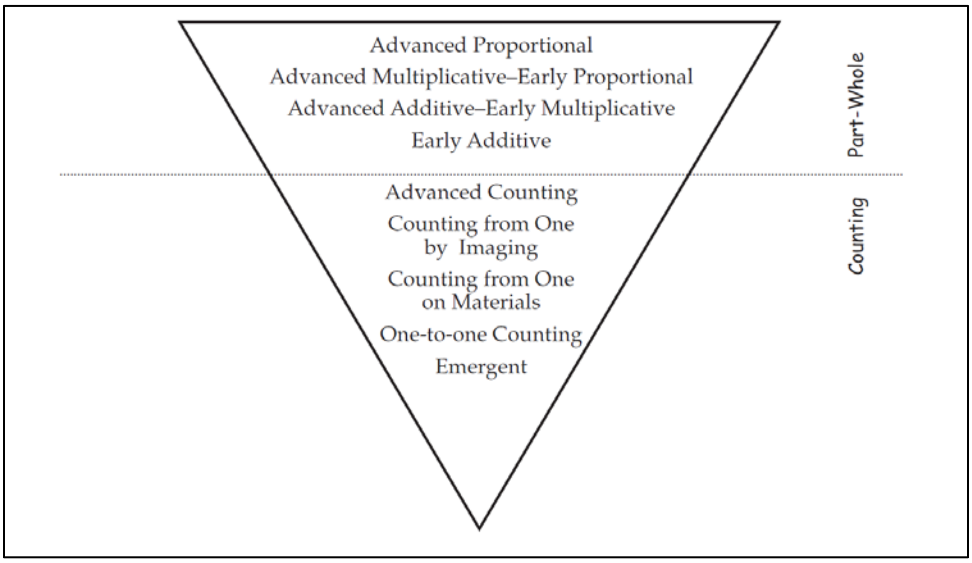
Data included demographic information of the participants, their maths anxiety levels, mathematics for teaching ability measures, and more. Evidence of the quality teaching of the seven participants is presented in two full case studies with five vignettes for contrast and verification.

The results confirm that quality teaching is complex. The DART framework has allowed the identification of increases and diminution of some of the factors involved in quality teaching. These factors are dynamic, changing as teachers’ experience intensifies, their confidence grows, and their ability to identify the teachable moment expands. Evidence was captured of novice teachers overcoming their negative feelings about mathematics and their negative projections about the difference they could make; they exhibited improved confidence and security about their teaching of mathematics, and they showed determination to do the best for their students. Quality teaching is something teachers assert that they know when they see it; using the DART instrument provides evidence that they can be sure what they are seeing is quality teaching.

Dedication

For my mentor, supervisor and friend, Gregor Lomas.

Preface



Emergent

Students at this emergent stage are unable to consistently count a given number of objects because they lack knowledge of counting sequences and/or the ability to match things in one-to-one correspondence.

The Number Framework

This thesis is an account of my journey which in some way mirrors the journey that young children take within the Numeracy Development Projects' Number Framework. Each chapter starts with a short definition that characterises the students' stage. These definitions follow the journey a child takes from an emergent counter to a fully formed proportional reasoner. This is also the story of my research journey, from my emergence into the world of research as a raw and inexperienced beginner to becoming a more informed reasoner. There are other journeys too, those of the participants of the research, which was also a journey of discovery and change.

Acknowledgements

I would like to acknowledge the special place in my journey of my supervisors, Gregor Lomas, Michael Thomas and Barbara Kensington-Miller. Thank you for sharing your wealth of knowledge with me, for keeping me on the rails and for travelling with me to the terminus. Also I want to acknowledge the editing of this thesis by Janet Rivers janet.rivers@actrix.co.nz.

My thanks, respect and love go to the seven participants who accompanied me on my journey of discovery. You showed fortitude, bravery and commitment above and beyond the call of duty of a teacher. You are the bedrock of our teaching profession, I am privileged to have known and worked with you. I wish you all, every happiness as you continue your own journey to expert teacher status.

Lastly I want to thank my family, Arthur, Adam, Kathryn, Alma and Gordon, for their support and love during the stops and starts of my journey; they truly are marvellous, splendid people.

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Introduction: Multiple Journeys

One-to-one Counting
This stage is characterised by students who can count and form a set of objects up to ten but cannot solve simple problems that involve joining and separating sets, like 4 + 3.

The Number Framework

Having emerged, I no longer lack knowledge, but my understanding is limited. I continue my journey with determination and persistence.

It was the tears of my third-year pre-service primary student teachers that sent me on this exploratory journey. When I first arrived at university to teach mathematics education courses, I learnt to temper my behaviour in the mathematics classroom, to look over students' heads and not to look into their eyes, because they were scared I would put them on the spot and ask them a question about mathematics. At first, this could not be discussed with them, it caused more distress, so traumatised were they by their experiences in school, and their fear of failure in mathematics sessions. When students finally opened up to me and shared their stories, I found they had been disrespected, told they were stupid and many of the female students were informed by their teachers that mathematics wasn't for girls! Some of these people were so badly maths anxious that, in the end, they could not become teachers of primary-aged children. Those that stayed the course, and really made efforts to learn enough mathematics to pass, took their maths anxiety, (maths anxiety, as opposed to mathematics anxiety, is a term in common usage among researchers), into the classroom with them.

These students were reluctant to engage in the problem-solving activities that are used to illustrate pedagogy and designed to help them reconfigure their mathematical knowledge from that of personal understanding to one which could facilitate the teaching of mathematics. They found great difficulty in developing strategies to solve these problems, preferring to stick to known (but little understood and often misapplied) rules and procedures. These students frequently expressed low confidence and reservations about their own ability to learn mathematics, revealing their high levels of maths anxiety.

One previous finding (Frankcom, 2006) was that regardless of how successful students were in the pre-service teacher mathematics education courses, their negative attitudes towards mathematics were largely unaffected, and their maths anxiety remained at high levels. My master's thesis addressed the problem of how strong the maths anxiety was and what relationship other factors, such as self-efficacy, played in this. The aim of this new study is to look at the way that students actually teach when they are in their first two years prior to registration and whether they overcome their maths anxiety or how it affects their teaching.

My concern for my students and the future of mathematics teaching led me to centre my master's thesis on maths anxiety. In that research, using the Maths Anxiety Rating Scale (MARS), I discovered how maths anxious the students really were; however, I had no way of knowing how this would affect their teaching of mathematics once they went into the real world of school. If they believed "maths myths" (Boaler, 2013; Franks, 1990) about how difficult mathematics was to learn and that only those with mathematical, logical minds could learn mathematics, then how could they visualise their own students as powerful learners of mathematics? Ideas from growth mind-set research (Dweck, 2006) about how brains have the capacity to change upon meeting different stimuli, informs us that brains are not set, and maths myths only serve to restrict learners of mathematics. This beginning part of my journey appraised me of the composition of maths anxiety, how it developed, the effect it had on people's lives, and its resistance or otherwise to mitigation. My journey had only just begun.

In this thesis, the journey continues. When I began my research, I wanted to answer the question of how the quality teaching of mathematics teachers developed over the course of their two-year registration period and how their maths anxiety affected this development. Over the years of this journey, the emphasis changed, and the questions became more about what quality teaching in the style of the Numeracy Development Projects (NDP) might look like, and how it might change over time.

The journey of the novice teachers

The seven participants had their own journey; the next story brings this into focus. Those who completed all the data collection phases answered a questionnaire to measure their maths anxiety, and they analysed three different teaching situations designed to assess their mathematical knowledge for teaching (MKT). Subsequent to that they were interviewed regarding their experiences with learning mathematics, their feelings and attitudes towards mathematics, and what their ideal mathematics lesson might look like. After graduation, these novice teachers were followed into a range of Auckland primary schools where they taught at various levels as generalists. They were videoed at the beginning in their first year of employment (2009) teaching mathematics lessons centred on the NDP "pink books". Near the end of their second year (2010), they were again videoed teaching an NDP-style lesson, and they responded to a questionnaire specifically designed to measure their mathematical knowledge for teaching (MKT).

The novice teachers who joined this study were brave to continue through all the phases. For any novice teacher to have me in their classroom, videoing mathematics lessons, would have taken courage; for a maths anxious person to open themselves up to such scrutiny says a lot about their determination to improve mathematics teaching. They were self-aware—their interviews are testament to the comprehension of their own lack of mathematical understanding. From their mathematics education courses they knew what was required to teach mathematics well, and they knew that they lacked some important factors. The journey of these people to this point had been fraught. Almost to a person they talked of the difficulties they had had at school in learning mathematics. They spoke of lessons filled with rote learning and algorithms they did not understand. They were frank about what they might lack, they feared they might miss the teachable moment, not be flexible enough about their teaching, but they had plenty of wonderful hopes and dreams for their future selves too. I am in awe of these people, and grateful for their trust. This is their journey too.

My journey

The first huge step on my journey was from my Auckland school to the university to teach mathematics education courses, but this was just the beginning. After my master's degree work, I began the next stage of the journey: How would novice teachers develop in their pre-registration two years of teaching? I journeyed to America to become qualified in the use of the Mathematical Quality of Instruction (MQI) instrument developed by the Learning Mathematics for Teaching (LMT) group (Hill & Ball, 2006), and I arrived home keen to code my videos of the novice teachers' mathematics teaching. Try as I might, I could not make the MQI instrument fit for my purpose: it did not capture the essence of NDP-style lessons. As a result, I developed my own measuring instrument expressly for this purpose, as none other existed that could capture the NDP-style lesson effectively. Using a video produced specifically to illustrate quality teaching under NDP, I developed the Dynamic Analysis Reflection Tool (DART) framework that could capture group dynamics, use of materials or manipulatives, and the instructional components of numeracy lessons.

Rationale for this research into Quality Teaching

There is little existing research about the quality of the lessons taught under the umbrella of the Numeracy Development Projects (NDP) (a collection of in-school professional development initiatives discussed in more detail in Chapter 2). There is also little international research on quality teaching in numeracy teaching in general. From a mathematics reform perspective, and from an NDP stance, research is discussed in Chapter two about the New Zealand response to calls for improvement in numeracy teaching. These calls came after publication of the TIMSS (1997) study which indicated that the mean numeracy levels of New Zealand school children was below the average mean of all the countries in the study. The response from New Zealand was the NDP, or at least the initial pilot which became a country-wide innovation. This study investigated the changes in various factors evident in the quality of teaching of novice numeracy teachers over the two year period of the pre-registration. This was a direct response to the gap in the literature in New Zealand and abroad.

In order to discover the important factors involved in quality teaching, this research followed seven teachers through the two years of their pre-registration, videoing their teaching in NDP style lessons, measuring their initial maths anxiety to see what effect this might have, and measuring their changes in these factors.

Research Questions

My primary research question which relates to novice teachers teaching in NDP-style was:

How could an instrument be designed to capture the development of quality teaching?

Subsidiary questions were:

- What is the nature of quality teaching evident in the teaching practice of novice teachers over their first two years in the profession?
- How does the teaching of novice teachers change over two-year period of their provisional registration?

- How does initial maths anxiety affect the development of quality teaching?

Organisation of thesis

The journey through this thesis is straightforward. The following chapter discusses the literature on quality teaching. It is organised according to Shulman's 1986 framework of three "big ideas" for teacher development: subject, pedagogy, and context. Chapter 3 discusses the research methodology, which uses mixed methods and case study to investigate the complexities of quality teaching. Chapter 4details the development of the measuring instrument framework DART: how it grew out of the MQI into something with utility and sensitivity to NDP-style teaching. Chapter 5, second results chapter, examines the results from two major case studies and five vignettes. The discussion in Chapter 6 brings this journey to a resolution by answering the research questions and investigating the implications of the research. The thesis is completed with consideration of the limitations, implications and my considerations for the future.

Finally…

Finally, at the beginning of my journey, I will start by sharing some profound words from Williams (1988). I have lived by these words, and tried to actualise this for my student teachers, and for teachers that I intersect with, who are already in the work force. I have this on my office door, so that everyone passing, or coming to see me, can read how people with maths anxiety can help to break the cycle of poor mathematics teaching producing poor teachers of mathematics.

Tell me mathematics and I will forget;

Show me mathematics and I will remember;

Involve me…and I will understand mathematics.

If I understand mathematics, I will be less likely to have maths anxiety.

And if I become a teacher of mathematics, I can thus begin a cycle that will produce less maths-anxious students for generations to come.

W. V. Williams, 1988, p. 101

Chapter 2

Quality Teaching: Literature Review

Counting from One on Materials

Given a joining or separating of sets problem, students at this stage rely on counting physical materials, like their fingers. They count all the objects in both sets to find an answer, as in "Five lollies and three more lollies. How many lollies is that altogether?"

The Number Framework

This stage is crucial to understanding how numbers work, which is reflected in the necessity of seeing in the literature what research has gone before, and having the understanding of how things in research work.

Quality teaching is a catch-all phrase that has been in common usage within education circles for approximately 50 years. It is used to describe something ephemeral and elusive about which there is little consensus. What quality teaching is, how new teachers develop it, and how the current teacher workforce enacts it, are all areas with a large research base. However, this is not so where the Numeracy Development Projects are concerned, nor in numeracy teaching in general. This chapter will examine the many-faceted attribute called quality teaching with reference to previous research, both in New Zealand and internationally, and how the term is currently used in the world of teaching. In an effort to get as well-rounded an idea about what quality teaching is, this is not confined to mathematics or Numeracy.

The seminal work of Lee Shulman (1986) provides a framework comprising three areas of importance in the pursuit of quality teaching: subject knowledge; pedagogical knowledge and contextual knowledge. Shulman's framework of these three "big ideas" will be used to illuminate the contributions of others such as Deborah Ball (e.g., 2000) to notions of quality teaching. These ideas will be connected to the work of this thesis and to New Zealand's Numeracy Development Projects (NDP). The style of teaching within NDP is specified in its large number of booklets, supporting material, and web site (NZMaths, at http://nzmaths.co.nz/numeracy-projects), some of which will feature in this chapter. Since all the participants in this research taught lessons which conform to this NDP style, the style is central to the consideration of quality teaching of mathematics (in non-NDP contexts, this might be termed reform mathematics). Thus, there will be an emphasis on research on aspects of quality teaching within mathematics generally and a specific focus on research featuring NDP.

What is quality teaching?

Ask anyone what a quality teacher is or quality teaching looks like and the answer will almost certainly be that you know it when you see it, though you may not be able to say exactly what it is. There appears to be little consensus about what specific attributes are contained in quality teaching (Alton-Lee, 2003; Looney, 2011; Parrish, 2016) but, that there is a need for quality teaching is generally acknowledged as being central to improving learning outcomes (Cochran-Smith, 2003). However people categorise it, there is no doubt that it is desired by all facets of the education world. This chapter addresses this question of “What is quality teaching?” There are many aspects to teaching that might have a bearing on whether it can be categorised as of a particular quality. There is the person who is the teacher. What are their original qualifications? What did they bring with them to the teacher education courses they took? How long was the course and what did it contain? What characteristics such as culture, social mores, or expectations do they have? How extensive is their understanding of the subject or subjects they will teach? The school in which they teach also has a role to play in the quality of their teaching. Are they well supported and are the children ready to learn? Also, while this list is by no means complete, if the factors involved in quality teaching are to be defined, these will eventually need to be tempered by the New Zealand context. This rapid scan through attributes which may contribute to quality teaching makes it apparent how multi-faceted the subject is.

Policy makers around the world have pursued the virtual holy grail of quality teaching, as a way of improving student achievement (Akiba, LeTendre, & Scribner, 2007; Alton-Lee, 2003; Ell, 2011; Gauthier & Dembélé, 2004; Gore & Bowe, 2015; Hattie, 2003; Looney, 2011; OECD, 2005; Winheller, Hattie & Brown, 2013). Quality teaching has been pursued globally both in mathematics and in non-curriculum based studies using various methods; for example, some countries have instituted more rigorous standards and higher level certifications or qualifications of their teachers and have coupled this with changes to teacher induction programmes (Akiba et al., 2007; Baumert et al., 2010; New Zealand Teachers Council, 2009; Young-Loveridge, Bicknell & Mills, 2012). Others have sought policy changes which attract high quality candidates to teaching (Akiba et al., 2007) while attempting to improve the retention rates of highly effective teachers (Cameron, 2007).

Another policy angle has been through the teacher evaluation systems; these are coupled with professional development opportunities which aim to improve the quality of pedagogy (Anthony & Walshaw, 2007; Gauthier & Dembélé, 2004; Gore & Bowe, 2015; Hanushek & Rivkin, 2006; Looney, 2011; Stigler & Hiebert, 2009; Wang, Haertel, & Walberg, 1993). These ideas of how to effect changes in quality teaching will be expanded upon in subsequent sections. The discussion starts with an examination of what quality teaching might mean in the New Zealand context.

Quality teaching in the New Zealand context

Though research on quality teaching in New Zealand is sparse, the evidence from "*Quality Teaching for Diverse Students in Schooling: Best Evidence Synthesis*" (BES) (Alton-Lee, 2003) brings together research that focuses on student outcomes as a quality assurance method. This synthesis uses research evidence on quality teaching drawn from international and New Zealand studies, with the evidence from New Zealand seen as vital to contextualise any findings. The synthesis used the Nuthall and Alton-Lee (1998, p. 1) definition of quality teaching, which states that it comprises:

> Pedagogical practices that facilitate for heterogeneous groups of students their access to information and ability to engage in classroom activities and tasks in ways that facilitate learning related to curriculum goals.

The synthesis presented evidence that teachers are responsible for about 59% of the variation in student achievement (Alton-Lee, 2003). This is a large effect, notwithstanding that the research information comes from another meta-analysis—one that looked at what matters and what makes a difference in teaching. This evidence indicates that almost any innovation or change in practice will have some effect on student outcomes (Hattie, 2003, 2009). These effects are often attributed to either the Hawthorne effect or the Pygmalion effect (Fößl, Ebner, Schön & Holzinger, 2016). Studies completed on increasing worker productivity (Hawthorne effect), in the 1920s and 30s, found that as long as the evaluation continued, productivity remained raised, but at the cessation of the study, the productivity returned to normal or base rates. The Pygmalion effect is cited when the high expectations of teachers has an effect over and above the teaching and the learning (Friedrich, Flunger, Nagengast, Jonkmann & Trautwein (2015). Furthermore, Hattie (2003) writes that teaching is the one factor which stands out above all the rest and that quality teaching is responsible for more of the variance in achievement than any of the other attributes in his study excepting, perhaps, the student themselves.

The children in the classrooms of New Zealand are diverse in attributes such as ethnicity and socioeconomic background (Alton-Lee, 2003). In the best evidence synthesis, there was a perceived need for teaching to ensure equity of outcomes for all students. Meeting diverse learning needs was complex and hence could be difficult to manage. In response to this diversity issue, 10 characteristics of quality teaching were extracted and identified. These focused on pedagogy, student learning, and achievement, as would be expected. The synthesis asserts that quality teaching should elicit high expectations from the diverse New Zealand population while fostering an inclusive and caring environment within learning communities and enabling strong links to develop between the school, home, and cultural centres. The synthesis also suggests that, to be effective, quality teaching requires teachers to have a wide knowledge of the learner and to be responsive to perceived needs while creating ample opportunity for learning in numerous learning rotations, which need to be mediated through appropriate scaffolding and feedback. Knowledge of the curriculum and the integration of information and communications technology (ICT) into an appropriate curriculum for the diversity of the students, and student autonomy and metacognition together with goal-oriented assessment in which the students are fully involved, complete these 10 characteristics of quality teaching (Alton-Lee, 2003).

As stated before, research in New Zealand about quality teaching is sparse. This section has briefly outlined the position of Alton-Lee’s (2003) best evidence synthesis, which incorporated both international and local research.

Quality teaching in the international context

Internationally, there is a substantial body of research which has a bearing on the focus of this thesis, that of quality teaching. Teacher qualifications, teacher evaluation and teacher remuneration schemes, appear in many areas of research, which will be investigated to discover factors that contribute to quality teaching.

A brief illustration of the vastness of the research on quality teaching comes from the work of American researchers who used the 2003 Trends in International Mathematics and Science Study (TIMSS) data to define teacher quality (Akiba et al., 2007). They argued that the defining attributes of quality teaching were to be found in higher teacher qualifications, since previous studies had shown that these were linked with higher student achievement (Druva & Anderson, 1983; Monk & King, 1994; Whitehurst 2002). They compared teacher quality around the world by comparing the proportion of teachers with various levels of qualifications and certification to teach in over 46 countries. Their conclusions range from the impossibility of closing the “opportunity gap” (Akiba et al., 2007, p. 369) to the need for further research regarding the role that teacher professional development might play (Knight & Duschl, 2015). The idea of the impossibility of closing the opportunity gap comes from the way that teachers have a tendency, and this happens all over the world, to move from lower socioeconomic schools to higher ones (Hanushek & Rivkin, 2006). Teachers who are able, relocate into the private sector, where pay structures reward people with higher qualifications. Many lower socioeconomic area schools are unable to compete. A personal preference for teachers to be teaching in their own local area (Reininger, (2012) has also to be taken into account. This leaves gaps in schools to be filled by teachers who are less qualified and less experienced, often without a major in mathematics. The idea of closing the opportunity gap will returned to in a later section.

A report that looked at existing research in order to see what the evidence could tell them about quality teaching, was one by Whitehurst (2002), the era of No Child Left Behind (Cochran-Smith, 2003). Whitehurst (2002) looked at the evidence for and against the prevailing ideas of what quality teaching is. He found that teachers do vary in quality, and that quality teaching is dependent on teacher factors. While their level of qualifications do matter, it is their cognitive ability that was the most important of these aspects. Secondly, came experience and content knowledge. Hammond (2008) also found in her study on quality teaching initiatives that intellectual quality was crucial in quality teaching. Whitehurst (2002) similarly found that the in-service professional development opportunities for teachers were critical, they needed to be focused and reform-centred. He further realised that being highly qualified (masters) or certificated teachers, was not simply connected with an increase in quality teaching. The evidence was equivocal. The biggest difficulty was that many studies used aggregate or average teacher certification and knowledge, and that hid the effect of individual teachers with differing qualifications. He went on to state that there were three routes to decreasing the large amount of variation in the quality of teachers, the first was to select those with higher cognitive ability, improve their teacher education, both pre-and in-service so that teachers are well supported and get specific courses focused on content and ensure that teachers are supported in schools.

Lovat (2009) examined quality teaching in an Australian initiative regarding values education, and dismissed the instrumental (Skemp, 1976) definitions he found as too simplistic, holding no understanding of the complexity of the term. When trying to define quality teaching he highlighted the intellectual quality as the key component (Lovat, 2009, p. 4)

> …it is not just the surface factual learning so characteristic of education of old that is to be superseded but it is surface learning in general that is to be surpassed in favour of a learning that engages the whole person in depth of cognition, social and emotional maturity, and self-knowledge.

Here he defines quality teaching in terms of quality learning. He goes on to cite Hattie's, contention (2003) that the teacher and teaching is the greatest source of variance in students' attainment, and that teaching is about capturing hearts and minds, in other words, the relationships that teachers forge with their students is the most important aspect of quality teaching.

Other researchers contend that it is not what is inside a teacher that is of primary important, but the performances of teachers, and certainly it might be simpler to watch the exterior productions of teachers, than to measure some inside, cognitive feature (Schacter and Thum, 2003). The large amount of American research they drew on led them to two conclusions: the first was that individual teachers were capable of producing widely different outcomes in terms of children's learning; and of those who were viewed as highly effective teachers, higher qualifications appeared not to be a factor. The authors presented the argument that to improve quality teaching you must concentrate on teacher performance. They contended that teacher practice standards needed to be developed, against which teachers could be assessed, and they recommended that teachers who perform well on these standards should be paid more than those who performed poorly. Their research found a strong effect that more learning happens in a classroom where there is quality teaching, as defined by the standards that they proposed. Their twelve teaching performance standards were: 'teacher content knowledge, lesson objectives, presentation, lesson structure and pacing, activities, feedback, questions, thinking, grouping students, motivating students, classroom environment, and teacher knowledge of students'. Many of these qualities are to be found in the work of Shulman (1986) which is presented later in this chapter.

The idea that the superiority of quality teachers could be measured against explicit standards have been cited by a number of other researchers including Hill & Grossman (2013) who concluded that though teacher evaluations had a part to play in improving quality teaching, they cautioned that many people reduced these evaluation systems, be they standards or observational studies, to a generalised and therefore more wieldy form. But doing this reduced the fidelity of the measures; consequently their results did not stand scrutiny. It has also been cautioned that using standards to determine remuneration is fraught with difficulties, not the least of which would be the lack of collaboration among teachers, leading, perhaps, to more isolated and autonomous teachers.

In the USA, teacher saleries have been dropping relative to other graduates of similar age and gender composition for over half a century (Hanushek & Rivkin, 2006) which is cited for a reduction in quality teaching. Interestingly this study also observed that entry-level qualifications were dropping as more roles in high-paid careers opened up for women (and some men). This was cited as a reason for a drop in quality teaching too, as the applicants for initial teacher education had lesser quality entry qualifications.

Widening the international discussion, in China there have been multiple attempts by government educationalists to move to more modern pedagogies, and what was seen as a consequent increase in quality teaching. However, this has led to resistance from traditionalist teachers, and exam-oriented parents (Dello-Iacovo, 2009). These reforms have proved ineffectual for the most part because the examination-orientated system has proved resistant to change, shored-up as it is by teachers and parents. One problem that has been identified is that while there are enough primary school places for the population of children, this is not so for the secondary school area, where approximately half of all secondary students are not in school (Lin and Zhang, 2006). So there is great competition for places, which means that children are under pressure to perform, which feeds into more traditionl types of teaching and learning. The teachers have found themselves under-supported, with poor text books and little time to teach in more child-centred ways, which included discussions, group work, and inquiry activities (Dello-Iacovo, 2009). The ambivalence of the teachers was preventing many reforms being instituted, with teachers continuing to teach as they always had (Marton, 2006). It was seen that change was both illusory and fragmented, with most change being identified in a few private schools. So, despite the aspirations of the educational establishment, quality teaching is proving elusive in Chinese schools.

The wide research about the use of teacher evaluations to improve quality teaching was examined by Hallinger, Heck & Murphy (2014) to see if indeed, an improvement in quality teaching could be found concomitant with an increase in growth in student learning. They concluded that in all the studies they looked at, there was still a long way to go before there would be an improvement in quality teaching through the implementation of teacher evaluations. Most of the evaluative schemes lacked rigour, and cost too much for the small returns, in terms of financial outlay and extra burdensome teacher workload. While still open to a system being built which would give the increase in growth of student learning, Hallinger, Heck & Murphy (2014) thought that schools might be better advised to go for non-evaluative systems and they identified four domains where there was research evidence of improvement: timely and actionable feedback; professional learning communities; support during teaching and professional learning opportunities.

In the same year that Schacter and Thom (2003) were disseminating their research, the New South Wales, Department of Education (2003) published their Quality Teaching Framework. This framework grew out of extensive pedagogical research that showed which aspects of pedagogy led to greater outcomes for students (e.g.Ladwig & King, 2003). This framework has three strands: intellectual quality, quality learning environment and significance for students. The first is about teachers having deep, connected, knowledge of their teaching subject, so that their students might engage in higher-order thinking and be challenged to actively construct the knowledge they were acquiring. The second strand, quality learning environment, emphasises that the environment needs to be supportive and safe for students, and should encompass their home environment too. The third strand is about the importance of the work students are doing, in a collaborative and cooperative manner, with the other students, and with the teacher, who aims to connect all concepts, topics and subjects that are being taught. This framework explicitly helps teachers move towards a more inclusive teaching style, and it highlights the need for teachers to continually reflect on, and refine, their teaching practice (Bowe & Gore, 2003). Teachers in and around Sydney, found the framework very useful in scaffolding their own reflections, supporting them to change the nature of the teaching in their classrooms (Hammond 2008). The detailed elements raised their expectations of all their students, but particulary what was termed the middle-ground, those who were not particularly good, nor those that were particularly bad at the subject being taught.

While it may be difficult to provide improvements in quality teaching which might increase the probability of extending equal opportunities to all students as Akiba et al. 2007 contended elsewhere, an Australian study using the Quality Teaching Framework, found that it was very possible to close the achievement gap (Amosa, Ladwig, Griffiths, & Gore 2007). By using authentic quality assessment tasks, they found that the closing of the gap between Aboriginal and Torres Strait Islander students and the non-indigenous students in their targeted schools was certainly possible. The tasks were rated using the Quality Teaching Framework (NSW Department of Education, 2003). The authentic quality assessment tasks resonated with the lower achieving students, and their results showed that the achievement gap was closed when the assessment tasks held genuine meaning for the students lives.

In this section the research that contributes to the meaning of quality teaching has been examined. The ideas behind teacher qualifications and what makes a difference, has been examined. The teaching central to this thesis is done under the auspices of NDP, numeracy teaching. In the following section, the numeracy initiatives around the world will be briefly highlighted.

Quality teaching in the context of numeracy acquisition.

From the time of the Cockcroft Report (1982), in the U.K., the meaning of numeracy has been accepted to be more than simply facility with computation. The other strands of the curriculum, geometry and statistics for instance, were also included. And not just for school children, numeracy was a goal for all citizens, in their normal lives as well as situations of specific mathematics use. Since that time, there has been a push to increase numeracy similar to that of the NDP in New Zealand. A country-wide effort to raise the content knowledge and connectivity of mathematics in primary school teachers grew out of this study (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997). The National Numeracy Project (1996-1998) was for students in years 1-6 in the U.K., and it led, at that time, to major changes in the Mathematics Curriculum. With an emphasis on oral and mental work, and an edict to disallow the use of calculators, the National Numeracy Project in the U.K. could be seen as dictatorial as it also stated in which week, what topic would be taught and for how long, etc. The same content was to be taught to every student regardless of attainment, and there was no room for what might be considered best practice, which might take note of the diversity in the classroom and act on it (Brown, Askew, Baker, Denvir & Millett, 1998).

Donaghue (2002) listed attempts at defining numeracy from around the globe. He found it surprising that there was no universally accepted definition, and one has not become more ubiquitous than any other. There are some countries (non-English speaking) that do not distinguish between numeracy and other types of mathematics knowledge, such as Germany and Austria (Donaghue, 2002). In the U.S.A, the term numeracy is not common, mathematical literacy is equivalent (Brown, Askew, Baker, Denvir & Millett, 1998).

The Australian drive for numeracy, in many way mirrors that of New Zealand. It was one of the Australian initiatives, Count me in Two Programme (Mulligan, Bobis, & Francis, 1999) that drove the development of NDP, so it would be expected that there might be much that these two countries have in common with their numeracy schemes. However Australia's federated states meant that each state has a separate education department, and so only in New Zealand has the NDP programme been implemented country-wide. The U.S.A. also suffers from the same plurality as Australia. The No Child Left Behind (NCLB) federal programme was written into law and therefore should have been implemented across all states. The law told the people that every child would have a highly qualified teacher but this has not proved possible, and there are many questions about the quality of the teachers needed to fulfil this dream, as well as be a national panacea for all the problems in society (Cochran-Smith, 2003).

This section had a brief look at how numeracy might be conceived in countries other than New Zealand. In the next section, the inspirational work of Lee Shulman, who developed some "big ideas" about quality teaching, will be introduced and discussed, with a view to providing a framework for this thesis about what constitutes and contributes to quality teaching.

Shulman's three "big ideas"

One of the big ideas regarding quality teaching is the role that subject knowledge plays, a much researched area over many years. Shulman (1986, 1987) is an acknowledged education expert (Baumert et al., 2009; Kersting, Givvin, Thompson, Santagata & Stigler, 2012) in this area. In his seminal work, Shulman (1986) discusses the pendulum swing from an over-importance of content knowledge against pedagogical issues in the mid-nineteenth century to the opposite state in the last quarter of the twentieth century. The current (as it was to him) emphasis on knowing *how* to teach over *what* to teach meant that there was now what he termed a “blind spot” (1986, p. 7) regarding content in both teaching and research. This missing content led him to ask (1986, p. 6):

> Why this sharp distinction between content and pedagogical process? Whether in the spirit of the 1870s, when pedagogy was essentially ignored, or in the 1980s when content is conspicuously absent, has there always been a cleavage between the two? Has it always been asserted that one either knows content and pedagogy is secondary and unimportant, or that one knows pedagogy and is not held accountable for content?

Shulman saw an undue emphasis on aspects of teaching such as classroom management that had little to do with what the teacher knew about their subject. He divided different aspects of subject knowledge into three “big ideas”: content knowledge, pedagogical content knowledge, and curricular knowledge.

Shulman defined *content knowledge* as the knowledge the teacher has about their subject. However it was not merely about knowing facts and figures, it was about structure and constructs, connectivity, and validity, which Shulman saw as in need of being deeply understood by teachers more than by non-teacher experts in the same domain or discipline.

Pedagogical content knowledge (PCK) was defined by Shulman as going beyond the aforementioned content knowledge into the realm of multiple representations and models used by the teacher to convey the deep concepts, connections, and intricacies to others, the learners. He invoked the spirit of “teachability” and the knowledge of misconceptions to illustrate the nexus of pedagogical content knowledge. Shulman’s seminal ideas on pedagogical content knowledge have been used in many research areas. They are evident in reports and articles in areas ranging from notions of quality teaching, from culture to technology, from second language teaching to distributed leadership, and they have been widely taken up by those researching in mathematics education.

In defining what he meant by *curricular knowledge* (or *contextual* knowledge), Shulman used the analogy of the highly effective physician needing to know the full range of possible treatments when diagnosing a sickness and the consequent possible treatments. From this, he argued that teachers need to know all the resources available for teaching to be effective teachers. These ranged from software through to alternative text books. He also thought that knowing other subjects’ curriculum was important, especially to primary teachers, to enhance connectivity of ideas being privileged within the classroom.

These three "big ideas" of content (or subject) knowledge, pedagogical content knowledge, and curricular (or context) knowledge are discussed in more detail in the following sections.

The subject knowledgeable teacher

The subject knowledgeable teacher is the first of Shulman's three big ideas about what constitutes and contributes towards quality teaching. The following sections will explore what it means to know a subject well enough to teach it, and what is meant by mathematical knowledge for teaching; consider the quality and qualifications of initial teacher education candidates, including their level of mathematics education; and discuss how initial teacher education courses prepare their students to become subject knowledgeable teachers.

What does it mean to be subject knowledgeable?

To be subject knowledgeable is not only to know about a subject or subjects. Shulman (1986) states that the subject knowledge of a teacher is different to that of the non-teacher expert in that the knowledge is more organised and the underlying structures are more deeply understood. Similarly, Ball, Thames, and Phelps (2008) have highlighted the importance of specialised content knowledge and its centrality in quality teaching. For a secondary teacher, this may mean knowledge of only one subject; however, as primary teachers (the focus of this research) will be teaching all eight subjects in the New Zealand curriculum, what are the ramifications for them? The level at which this knowledge is held could also be important. How many levels above the level being taught does the knowledge need to be? How is the knowledge held? Is it held in a way which will allow the deep understanding that effective teachers have of their subjects to teach the big ideas, underlying concepts, and structures (Ma, 1999; Masters, 2009)? The next sections will explore the prerequisite knowledge that people entering into the teaching profession at primary level (Years 1-8) might possess.

Entry requirements for initial teacher education courses in New Zealand

Primary teachers, who in New Zealand schools teach Years 1–8, or ages 5 to 13 years, are required to teach all the subject areas. These are mathematics & statistics, English, science, physical education, social studies, languages, art and music, and technology. Applicants for initial teacher education in the primary sector in New Zealand can opt for a three-year bachelor's degree in education (e.g., B. Ed degree) or take a three-year general subject degree (e.g., BA or BSc degree) and then complete a year at postgraduate level (Ell, 2011;

New Zealand Teachers Council, 2010). Both these pathways in initial teacher education lead to two years of provisionally registered teaching in schools that is supervised and mentored, and has a reduced teaching load. After this two-year period, teachers apply for full registration from the New Zealand Teachers Council (now Education Council).

The first pathway, the bachelor’s degree in education, requires a university entrance qualification in New Zealand although the subjects in the final years of schooling which feed this qualification are not subject specific. Also, although pre-service student teachers need a university entrance qualification to go directly into an initial teacher education programme from school, mature adults do not need the same school-based qualifications for university entry.

Candidates for initial teacher education courses do not form a homogeneous group when it comes to intellectual capacity or educational experience. Some candidates only just meet the criteria and others are way beyond what is necessary for entry. Next, the overall quality and ability or intellectual capacity of teacher education candidates will be investigated, as this relates to the subject knowledgeable teacher.

Quality of initial teacher education candidates

All those who want to be a teacher in New Zealand must meet a set of criteria, some national and some provider-specific. Applicants are interviewed and are expected to have dispositions for teaching. These dispositions include pertinent teacher knowledge, beliefs, values, and ethics. The Graduating Teacher Standards of New Zealand (New Zealand Teachers Council, 2007) gives guidance on appropriate dispositions; for example, “have the knowledge and dispositions to work effectively with colleagues, parents/caregivers, families/whanau and communities” or “demonstrate high expectations of all learners, focus on learning and recognise and value diversity”. Generally, however, there is little agreement worldwide on which dispositions are essential and which are only desirable in pre-service teacher candidates (Borko, Lister, & Whitcomb, 2007). Part of the lack of agreement is that dispositions cannot be reliably measured or discerned since they are characteristics of the novice teacher that develop as experience is gained (Johnson, Johnson, Farenga, & Ness, 2005).

As well as being interviewed, teacher applicants may also have their communications and literary skills tested through an external screening process and where this is a requirement, it would apply to all candidates regardless of whether they are a native English speaker or not. There would be a numeracy screening test too (Ell, 2011). To add to these entry standards, there are programmes of exit standards of literacy, numeracy, and ICT skills to attest to. These are often shown through a personal learning e-portfolio which addresses the graduating teacher standards (New Zealand Teachers Council, 2007).

Candidates' level of mathematics education

The majority of students come into teacher education courses in New Zealand at the beginning of the three-year bachelor's degree in education with no mathematics education themselves past Year 11 (age 16 years) compulsory mathematics (Biddulph, 1999). A later study indicates that not much has changed. Young-Loveridge et al. (2012) showed that even when students began their ITE with their NCEA Numeracy Credits, and University Entrance (New Zealand qualifications), they were unable to successfully solve the type and level of mathematics questions that the teachers would be encountering when teaching in the primary area. Tellingly, 17 per cent of the 567 students involved in their study, were unable to solve half of these tasks. Older applicants for current teacher education programmes need not even have Year 11 mathematics and indeed, it was shown that at least 50% of the applicants had not achieved any formal mathematics qualification at all (Grootenboer, 2003). Those applicants who did study further than Year 11 were seen to have very weak mathematics, with just 8% achieving a pass at a reasonable grade (Thomas, 1998). Norton (2010) found a similar pattern of low previous mathematics qualifications in a cohort of an Australian one year pre-service primary teacher programme. Many students had the lowest possible school qualifications and when the candidates did have a higher mathematics qualification; it was a relatively small portion of the cohort. Only about 20% of the cohort of 131 had qualifications that would have gained them entrance to study university science or mathematics.

Another study of Australian pre-service student teachers showed that the status of teacher education is low when compared with other choices in university education (Fitzsimons, 2002). This is cited as a reason for the subsequent lack of quality of personal mathematics content knowledge of applicants whose mathematics knowledge was judged through observation to be insecure, and their understanding of mathematical concepts to be mainly algorithmically bound (Fitzsimons, 2002). Such lack of mathematical knowledge impinges upon all of the core practices of teaching mathematics, from choosing the appropriate level and curriculum topic to developing the most apposite representations and models or "identifying and choosing high leverage practices" (Ball, Sleep, Boerst, & Bass, 2009, p.

460). Lack of deep conceptual knowledge will hamper teachers' efforts to analyse the misunderstandings of their students. Their ability to diagnose where the difficulties lie and what sense their students are making at a point in time about the mathematics is paramount. The algorithmically bound pre-service teachers will need to develop a more profound understanding of fundamental mathematics (Ma, 1999; Rowland & Turner, 2008) to achieve a greater understanding of how to divide fractions, for example, then just being able to "do" them. Teaching fraction division requires both the knowledge of how to work with fractions and deep conceptual understanding about what is happening and what prior knowledge learners will require. The next section will further illustrate this complexity, introducing the work of Skemp (1979): His seminal ideas on there being two types of mathematics (Relational and Instrumental) gets to the heart of the problem that many learners (and, indeed, teachers) of mathematics have in understanding and retaining new ideas, and this will underpin the subsequent discussion on what Ball, Hill, and Bass (2005) have termed mathematical knowledge for teaching (MKT).

Instrumental knowledge versus relational knowledge

Skemp (1976, 1979) wrote about two types of mathematical understanding, which he termed instrumental and relational. He saw these two types of mathematical understanding as so different that initially, he did not categorise instrumental understanding as being understanding of any sort. He looked at the rote learning and algorithmically bound students in many classrooms as having no understanding of mathematics. He observed that there were essentially two types of mathematics being taught and learnt in schools. These were not just variants of the teaching, with one emphasising the rules and the other the understanding. He saw them as subjects that were altogether different, regardless of the fact that they were both called mathematics.

Instrumental learning is bound up with a traditional style of instruction (Young-Loveridge et al., 2012). This could be categorised as rule-teaching, rote-learning, and symbolic manipulation, with potentially little understanding of concepts or the interconnectivity of relationships. This is portrayed in recent research as a gulf between the process and the concept (Mason, 2012). There are also aspects of teacher-tell type instruction where there is a preponderance of teacher presentation which is followed by the student doing practice exercises from a textbook. This type of instruction, with low-mastery aspirations, leads students to see mathematics as a series of rules to be learned and regurgitated at the next test (Skemp, 1979). A consequence of this is that students believe they either can or cannot do mathematics and they have little power to change the situation and affect their own learning (Furner & Duffy, 2002; Grootenboer, 2008; Turner et al., 2002).

Relational knowledge, however, is associated with "knowing both what to do and why" (Skemp, 1976, p. 20). The learning of relational mathematics is a goal in itself; it is all about connectivity and adaptability of ideas. It is easier to remember, though harder to assimilate in the first place and paradoxically, though it may take more time to learn, the learning is more entrenched and therefore relearning time is reduced. The learning of relational mathematics is associated with an alternative mode of instruction which might be termed a more child-centred and investigational or discovery style. This would be typified by problem-solving activities and investigations, perhaps working in groups, as the basis for classroom work. The teacher might act as a facilitator and the process of doing mathematics, discussing strategies, and finding the connections would be emphasised over the right method or simply getting the right answer (Levine, 1995; Turner et al., 2002).

Skemp (1976) provides some reasons in favour of instrumental mathematics, while typifying them as faux amis. When learning instrumental mathematics, it is far easier to acquire some limited understanding. Instrumental mathematics is presented simply and isolated skills are often easy to pick up due to the way it is presented. But this simplicity of skills learning has to be taken together with the fact that generally the length of a typical instrumentally-based course is longer than the typical relationally-based course (Pesek & Kirshner, 2000). The emphasis on practice of skills concentrates the students' efforts on repeating those basic skills with progressively more difficult questions which do not stray from an accepted, and expected, format. The rules are represented to students in bite-sized pieces which are easily swallowed and easily regurgitated. Instrumental mathematics can also be very rewarding as the practice exercises can be neatly and cleanly assessed with a red tick, and a page of red ticks can be extremely affirming, making the student comfortable with their level of attainment. Skemp also admits that sometimes thinking instrumentally can help one reach an answer more speedily than by using other methods and that even seasoned relational thinkers will adopt instrumental thinking when it suits them.

As an antidote to this, Skemp (1979) found "four advantages (at least) in relational mathematics" (p. 23). Relational mathematics is seen as much more adaptable to new ideas and tasks. The connectedness that develops from relational teaching and learning is transferable across both ideas and topics, making it easier to remember, though it can be harder to teach and learn.

> There is more to learn – the connections as well as the separate rules – but the result, once learnt, is more lasting. So there is less re-learning to do, and long-term the time taken may well be less altogether. (p. 23)

The learning of mathematics relationally is shown by Skemp to be intrinsically rewarding in itself; it is self-motivating, reducing the need for extrinsic rewards. It also tends towards having an organic quality in that having learnt something relationally, the learner will try to learn subsequent material relationally and will try to "seek out new material and explore new areas" (p. 24).

Skemp (1979) believed that there was something very wrong with children's mathematical knowledge, which he attributed to the prevalence of instrumental teaching in schools. Although instrumental and relational mathematics encompasses the same content, they are by no means equal. This has significant ramifications for our teacher applicants; they have been through a system which is more likely to exemplify the former than the latter. So the question now becomes, how is subject knowledge characterised in, and by, pre-service teachers? In the next section, the type of knowledge needed for effective teaching of mathematics is further elucidated through the work of Deborah Ball, Heather Hill and others in Michigan on the important role that mathematical knowledge for teaching (MKT) plays in the teaching of relational mathematics.

Mathematical knowledge for teaching (MKT)

Since the formative work of Coleman et al. (1966) and Shulman (1987) researchers have been investigating what type of knowledge is important in the teaching of mathematics. While there is little doubt that teachers' content knowledge has a large part to play in student achievement (Hill, Rowan, & Ball, 2005) it is by no means clear what this really means. The Learning Mathematics for Teaching (LMT) group, comprising members of the Mathematics Teaching and Learning Project and the Learning Mathematics for Teaching Project, posited that there was a special kind of content knowledge which was specific to effective teaching, and this they term mathematical knowledge for teaching (MKT) (Ball et al., 2008).

The term mathematical knowledge for teaching has also been used by other researchers at different times (e.g., Lampert, 1990; Thompson & Thompson, 1996) to describe activities such as lesson planning and the analysis of misconceptions, but the usage as discussed in this section is specifically about the work that teachers do when engaged in teaching mathematics. While it includes activities across the whole of the teaching environment, here its relevance to content is emphasised.

Through a survey using specifically designed and piloted items (Ball, Hill, & Bass, 2005), the LMT group found that while there are many factors important to teacher quality, they also identified a particular factor disconnected from pedagogy, that of mathematical knowledge for teaching (MKT). This is used only by teachers of the subject and comprised a wider and deeper understanding of mathematics than either lay persons or pure mathematicians held. So the importance of mathematical knowledge for teaching is the way that the teacher holds and can use their mathematical knowledge. It must be possible for a teacher to represent mathematical concepts to children using different ideas, so that something within the child can connect with something in the teacher (Hill, Shilling, & Ball, 2004), and the more varied these representations are, the more likely this is to happen. While it is true that mathematicians use proof and consideration of errors, this is not the same as the work a teacher does. The teacher must not only know that something is wrong, but must be able to posit where the error is in the child's thinking and then further analyse the situation to discover the most efficacious way of enabling a child to overcome their misconception. Within the latter, novice teachers must choose the particular set of numbers or use examples that are at just the right level to help the child. This knowledge is further broken down by the LMT group into common content knowledge (CCK) and specialised content knowledge (SCK). Common content knowledge is what non-teachers may well have but specialised content knowledge is found only in teachers. Indeed specialist content knowledge may be undesirable in non-teaching situations since it is not about the mathematics and solving a problem, but rather teasing out the many meanings and this would only serve to obfuscate the situation when, for instance, one simply wants to calculate something in the real world, away from teaching situations.

An example of this specialised knowledge is understanding division of fractions. Division by a common fraction, such as a third divided by a half, often proves to be a sticking point in the deep understanding of mathematics (Ball et al., 2008). Many people will invoke the rule of 'turn the divisor upside down and multiply' without worrying about why that will work, and for others the idea that the answer gets bigger can be very perplexing. Often the rule-bound respondent will have little idea why the rule works, never having to think about it. Teachers, however, must not only understand why it works; they must hold a versatile repertoire of different representations and know which is most appropriate to the situation and the child—that is, demonstrate specialised content knowledge (Hill et al., 2004). Should she use a diagram? Should she use circular or rectangular representations, or a number line? This is knowledge that only teachers need or know. This is where the difficulty lies in the selection of candidates for teacher education courses: What distinguishes quality candidates from merely knowledgeable ones? Who will be able to complete this complex task? For example, a sensitive approach to the initial question of what is a third divided by a half would probably entail the use of a model. A teacher with specialised content knowledge might perhaps use a model of paper-folding. This entails a piece of paper being folded first in thirds and then, at right angles, the paper is folded in half. The resulting folds show that the answer is that a third divided by a half is two thirds of a whole.

Here, the qualities of the pre-service teacher have been examined, and the way they need to hold their content knowledge questioned. The next sections will discuss the issue of subject knowledge within initial teacher education courses and their efficacy in regards to increasing subject knowledge.

Subject knowledge within initial teacher education courses

Shulman (1986) argued that content knowledge, however it is defined, was all important in teacher preparation. What is the position of initial teacher education providers now? How much knowledge is required of teachers in the 21st century? Researchers have looked at the knowledge base required for modern-day teachers and found that this is increasing all the time (McGee, Cowie, & Cooper, 2010), putting teachers under intensifying pressure. The *New Zealand Curriculum* subject areas include English, the arts, health and physical education, mathematics and statistics, learning languages, science, social sciences and technology. Within this document, there are hundreds of content and achievement objectives arranged in eight levels over 13 years of schooling (Ministry of Education, 2007). Primary teachers need to know all these content areas for Years 1 to 6. This is obviously an immense undertaking and problems associated with this have been little researched. How much a primary teacher could reasonably know and understand across all eight learning areas is debatable, regardless of the changing face of teachers from purveyors of knowledge to facilitators of learning (McGee et al., 2010; Norton, 2010).

So, too, there is a disjunction between the amount of knowledge new teachers could possibly acquire in their initial teacher education and the needs of the job of teaching. There is much debate about what must be included in these teacher education courses without risking superficial coverage due to forcing in more and more curriculum material in all school subjects (McGee et al., 2010). Many teacher preparation courses in New Zealand have three distinct strands. These are typically curriculum studies, pedagogical studies, and praxis or practicum studies. In the case of the one-year primary graduate pathway, completed after an initial undergraduate degree, approximately half the year spent at university will be in curriculum courses. With eight subject areas to cover, there is a strain on the time allowed to become sufficiently subject knowledgeable. The amount of time given to each curriculum course in initial teacher education does not match the time given to those subjects in class time. Time given to mathematics in teacher training can vary from about 10% to about 20%, depending on the programme. Most primary-aged children are learning mathematics every day, mostly in the mornings, and the portion of class time spent varies from about 20% to 25% (Norton, 2010; Walshaw, 2004). The overcrowded curriculum is a concern for many countries, and in the United Kingdom in particular, the added pressure on teachers to be knowledgeable across all the subject areas is seen as a great encumbrance and as a particular reason for the drop in the numbers of teacher candidates (Galton & MacBeath, 2008).

Although many candidates for teaching have low-level school qualifications, the New Zealand one-year graduate pathway in primary teacher preparation is predicated on pre-service teachers being well-versed in mathematics, or if they are not, that they have the ability to remedy any deficiencies. Norton (2010), researching similar students in Australian initial teacher education courses, found that time constraints meant that opportunities for students to remediate their mathematical deficiencies were limited. These courses do offer some opportunities to acquire more subject knowledge as students go through their curriculum-focused courses. Though content may not be taught in an explicit manner, the work done on remodelling knowledge gives candidates an opportunity to revise and reconfigure their knowledge by working outside the programmes. There is evidence to suggest, through the exit standards, that content knowledge may be noticeably improved over the years of study undertaken. For instance, in Germany, in three-year teacher education courses, much bigger improvements in subject knowledge were seen than in subsequent teaching years (Kleickmann et al., 2012). There is also, however, evidence that through initiatives like the New Zealand Numeracy Development Projects (NDP), the mathematics content knowledge of teachers currently in the work force can be improved (Higgins & Parsons, 2009).

Shulman's first "big idea" (1986) has been used here to frame an inquiry regarding what it means to be subject knowledgeable. The initial teacher education courses which teachers pass through on the way to becoming teachers were examined to see what strength they possess to change subject knowledge. This brought into focus the enormity of the task facing new teachers—that of knowing all the facts and connections of eight content areas, of which mathematics is just one. The quality of these candidates was examined briefly, and with regard to their mathematics knowledge, it was seen that there was a weakness in students coming into initial teacher education from schools where the teaching had been mostly of the chalk and talk exposition type. The work of Skemp (1979) was used to illuminate what is important about subject knowledge, and how teachers need something far more complex than most people needed for every-day use of mathematics.

The next sections present the manner in which the second big idea of Shulman allows pedagogy and pedagogical knowledge for teaching mathematics to be the subject of the inquiry.

The pedagogically knowledgeable teacher

Shulman (1987) defines a pedagogically knowledgeable teacher as one with the knowledge needed to powerfully represent different aspects of mathematical concepts to students. Shulman talks of the transformation of personal understanding into some format that will aid the transmission of that comprehension to someone else. This section looks at the development of the notion of the pedagogically knowledgeable teacher. There has been an emphasis on distinguishing between the mathematics skills a teacher possesses, such as the ability to divide fractions, and the knowledge about the mathematics they need to turn that skill into effective teaching that is comprehensible to students. The discussion will first consider the meaning of pedagogy, and then examine what is meant by effective pedagogy to teach mathematics. This will be followed by a discussion of the philosophy behind New Zealand's Numeracy Development Projects (NDP), keeping in mind the way these illuminate understanding of quality teaching.

The meaning of pedagogy

Pedagogy is the science or theory of teaching practice (OED, 2016). It arises from the Greek, meaning to lead the child. The word pedagogy is used in many ways; it might be used in a cultural sense, as a generic coverall, and in a particular sense, perhaps about the pedagogy of something such as cooperative learning. The most prevalent use is almost synonymous with the word teaching, as though they are interchangeable. Whereas Straesser (2007) describes pedagogy as the struggle teachers have to bring their understanding of mathematics to learners, the human struggle in the narrow confines of schools, perhaps a more fruitful idea is to see pedagogy as the relationship between teaching and learning and learning and teaching, as each influences and shapes the other (Loughran, 2010). This emphasises pedagogy as a two-way process and echoes notions of the relational quality of teaching and learning mathematics.

Pedagogy as the theory of teaching has been central to discussions about teaching since the time of Dewey, who in 1904 discussed the theory and practice of teaching (see Shulman, 1987). He identified the chasm between the theory of teaching and the practice of teaching and the difficulty of bridging this gap (Ball, 2000; Shulman, 1987). For Shulman, quality teaching rests on pedagogical reasoning and pedagogical actions; it has an intellectual component which is central to his ideas about pedagogical content knowledge, which will be addressed in the next segment.

Pedagogical content knowledge

Shulman's (1987) pedagogical content knowledge grew from consideration of teacher effectiveness and the then state of current research into the theory of teaching and learning from both an empirical and a philosophical perspective. Schulman emphasised the interchange of ideas between the teacher and the learner, and the fluid nature of teaching. Alongside notions of necessary knowledge of the content, teachers needed to know how to represent their subject knowledge to learners in ways that would allow the learner further insight, together with knowledge about what makes the learning of various topics easy or hard.

Effective teachers of mathematics have many ways of representing mathematical concepts to their students. For instance they may be able to view or represent a problem in terms of graphical or algebraic representation, or with a physical model which students could manipulate. Their knowledge of the situation, the topic being studied, and the child's prior knowledge allows them to be highly sensitive to the needs of the student and to choose just the right representation at just the right time to enable deeper understanding to develop in the recipient, the child. This versatile thinking (Thomas, 2006), also categorised as flexible thinking (Graham, Pfannkuch, & Thomas, 2009) has been shown to be vital in the search for effective teaching (Baumert et al., 2009; Ma, 1999). The development of this versatile thinking comes from being in command of the subject, a connected knowing about the subject coupled with "knowledge-in-action (Graham et al., 2009) lead to the teacher becoming an adaptive expert (Timperley, 2011). Teachers need to be adaptive experts, but how can this happen if, as suggested earlier, the candidates for initial teacher education lack content knowledge? New Zealand's Numeracy Development Projects, a teacher professional development initiative, were designed to aid this development, as seen below.

Quality teaching and the Numeracy Development Projects

The following will examine the history of the Numeracy Development Projects (NDP), its pedagogy and how it works as professional development, and the pedagogical tools it uses. Current research on NDP is limited, but what there is will be examined, together with reports commissioned by New Zealand's Ministry of Education. This exposition is crucial to this thesis as it addresses the question "What is quality teaching?" with a particular focus on lessons conducted in the NDP-style.

In the late 1990s, academics from throughout New Zealand, notably university academics Peter Hughes in Auckland, Vincent Wright in Waikato, and Gill Thomas in Dunedin, were engaged in damage control regarding fallout from the 1997 Third International Mathematics and Science Study (TIMSS) results. These results showed New Zealand mathematics was in a parlous state, with the mean attainment levels of New Zealand children significantly below those of international means (Mullis, 1997; Garden, 1997). Hughes was made national director of the NDP pilot study (ministry funded in-school professional development programme) in 2000, which used the Australian Count Me in Two programme (Mulligan, et al., 1999), with the aim to improve the learning outcomes in mathematics for Years 1–3 in primary schools. In 2001, this was superseded by the Early Numeracy Project, and this was also the year that the NDP teaching model was developed (Hughes, 2002), based on work of Pirie and Kieren (1989). The Early Numeracy Project was rapidly joined by the Advanced Numeracy Project in 2002, along with the Senior Numeracy Project, which was for Years 9 and 10. The small pilot study of 2,000 participants had rapidly become a whole of New Zealand project, and all primary schools, teachers and principals, had the opportunity to join in this professional development programme (Wright, 2014). Eventually, tens of thousands of participating students, teachers, schools, and regions joined these initiatives, ultimately named the Numeracy Development Projects (NDP).

The NDP had many reports about its efficacy but little full-scale independent research. There are some 20 reports for the Ministry of Education (for example, Ministry of Education, 2004). These reports have been conceptualised as important in the development of NDP, and from these, NDP was viewed as having a positive effect on raising teacher content knowledge and improving pedagogical practices, with proven effect on children's learning outcomes (Higgins & Parson, 2009). The evidence for this could be seen as largely anecdotal and there is a vital need for research to bridge the gaps. However the Ministry of Education continued to fund NDP until 2011, and it continues to the present but with a much reduced workforce since the initial school-based phase concluded in 2011.

The NDP as professional development

The NDP was always categorised by its champions as a professional development opportunity for teachers of mathematics. Through changing the ways that mathematics was taught in schools, it was hoped to raise achievement (Anthony & Walshaw, 2007), at first to show improvement in subsequent TIMSS tests but also to raise expectations and to implement the curriculum better. Not only were there concerns regarding TIMSS but also about the level of performance of New Zealand children when compared with OECD countries results (Timperley, Wilson, Barrar, & Fung, 2007). While those children at the top were seen to be performing well, New Zealand was perceived as having a very long "tail" of underachievement. It was through professional development that it was hoped to remedy this situation.

Professional development itself has been widely researched and, generally, the good characteristics of effective professional development are: extended time; being involved with expert others; strong teacher engagement; challenges current thinking; participation in a community of practice; in line with current research; and active school leadership participation (Clarke, 1994; Timperley et al., 2007). How the NDP professional development measures up against each of these attributes will now be addressed.

The characteristic of *extended time* for professional development is not a new phenomenon (Clarke, 1994). Clarke listed 10 important principles necessary for effective professional development, which are reflected in those developed in the New Zealand best evidence synthesis (Timperley et al., 2007), where time in teacher professional learning and development opportunities was seen as "necessary but not sufficient" (p. xxvii) in that most professional development opportunities were usually more than one-off sessions. However, extending the number of sessions further did not necessarily result in better outcomes (Timperley et al., 2007). Research indicates that sustained teacher change takes a long time to develop (Anthony & Walshaw, 2007). And so it was, that after an initial one-year pilot study, NDP instituted a one or two-year professional development programme aimed at initiating and supporting teacher professional growth. This included release time, mentoring by an expert teacher (facilitator), and the chance to watch the facilitator modelling teaching in groups.

Typically, each teacher was given release time to individually interview all the students in their class. This task-based interviewing of each student had many purposes. An overt purpose was to find out what the students actually knew and to place them on the appropriate level on the Number Framework (Ministry of Education, 2005a). This process involved listening to the answers of a set list of questions from the diagnostic interview (Ministry of Education, 2005b), which allowed a teacher to discover that children have different ways of conceptualising their knowledge and strategies. This information was not available in classrooms where students were not having one-on-one time with the teacher. After the knowledge of what happened during individual assessment had been assimilated by teachers, it was hoped that teachers would continue to assess individuals whilst they worked in groups. This was an ambitious goal, (Young-Loveridge, 2010; Young-Loveridge & Mills, 2010) most likely to be seen in experienced and knowledgeable teachers. It also insures that a more accurate assessment might be made of the stage of mathematics learning of the student. However, a more covert outcome is that teachers see the effects of their teaching, and perhaps, from hearing from the students themselves how they hold their mathematical concepts, learn to be more flexible in their own mathematical thinking. After some period of teaching, typically at the end of that teaching year or the beginning of the next, the diagnostic interview is repeated, and change in levels on NDP framework are noted and decisions made about where to next (Bobis et al., 2005)

There was further evidence of the attribute of extended time when time with the facilitator was taken into consideration (Cheeseman, 2006). The facilitator (expert other) would demonstrate different teaching strategies within the teacher's own classroom and give feedback on teacher practice. This was usually over one or two school years, with extra input possible through various professional bodies and universities. This length of time was determined by the need to allow complete coverage, or as near as possible, of all primary and intermediate teachers of mathematics (Young-Loveridge, 2010) over a period of 10 years.

The *challenges to current thinking* were arguably quite simple; teachers of primary-aged children could see for themselves that their current teaching of mathematics was problematic, and often ineffective (Bobis et al., 2005; Clarke, 1994). From the diagnostic interview, teachers perceived that their students were not at the level they should have been according to their age groups and this was often contrary to the teacher's previous assessments. This occasioned dissonance (Loucks-Horsley & Matsumoto, 1999) and the facilitator was able to help the challenged teacher make the necessary changes to the classroom pedagogy and/or organisation, thereby increasing quality teaching.

The *expert others* in NDP were the facilitators who were chosen for having extensive teacher knowledge and content knowledge of mathematics. Facilitators worked closely with teachers in their own classrooms, providing opportunities for modelling group teaching, feedback, and ideas. Though this varied greatly around the country, many of the original people involved in facilitation were taking papers at university (Hughes & Petersen, 2003), so their interest in improving the pedagogy of teachers of mathematics was already known. Later these facilitators, through personal recommendation, were able to tap into the group of teachers they worked with to grow the necessary experts to continue to work with other schools as NDP was implemented throughout New Zealand.

Strong teacher engagement is central to effective professional development (Timperley et al., 2007). NDP has had strong teacher buy-in, with many teachers taking further opportunities for professional development in mathematics education, or moving up to master's degree study. These teachers were working with children who showed obvious progress (Timperley et al., 2007) and they wanted this to continue. For that to happen, they needed to change their ways of teaching and their organisation of the classroom, moving from whole-class teaching to streamed groups (Knight, 2008). While not universally successful, the teachers who underwent professional development in NDP have been more positive than not about the project (Knight, 2008).

These facilitators were in the teachers' environment and the school-wide nature of the professional development meant that there was a greater chance of a *community of practise* developing within schools (Jaworski, 2014; Putnam & Borko, 2000; Wenger, 1998). Identified as an important factor in successful professional development, the community of practise has the capacity to effect long-term pedagogical change in teacher practice and is therefore highly desirable (Alton-Lee, 2006; Timperley et al., 2007). When teachers are supported and given opportunities to participate with a group of fellow teachers for discussions and serious, extended engagement, then the outcomes for students are enhanced.

Numeracy research in mathematics education prior to the year 2000, when the first of NDP projects was initiated, was focused on improving numeracy through teachers working in small streamed-achievement groups. Therefore the pilot and subsequent schemes of the NDP were in line with policy research (identified as important and good characteristics of effective professional development) in countries such as the United Kingdom and Australia, though the research was thin in mathematics itself. Such aims were supported by national professional bodies such as New Zealand Association of Mathematics Teachers (NZAMT). If the implementation of the NDP had waited for solid evidence from research, these projects may never have taken place.

For professional development to be effective, to increase the likelihood of quality teaching, the systemic change must come from *school-wide participation*. In the NDP there has been a strong emphasis on all teachers in a school participating in the projects. School principals have a dynamic and strategic role to play in supporting a shared vision for what effective mathematics teaching might look like. Without this support the outlook for long term change is questionable (Timperley et al., 2007). The development of the NDP has been characterised as ideal professional development, in that it was sustained, whole school, and concentrated on enhancing teachers' pedagogy for teaching mathematics.

Next, the pedagogical tools which underpin the NDP will be discussed, with the aforementioned in mind.

Pedagogical tools or approaches in the Numeracy Development Projects

The NDP has particular recommendations for teacher practice to increase quality teaching. These range from being numerate oneself to becoming a diagnostician of children's misconceptions. *Book 3: Getting Started* (Ministry of Education, 2008c) identifies three teaching tools: the Number Framework; the diagnostic interview; and the strategy teaching model. These are seen as not only central to the NDP and its propensity to effect teacher change, but also as central to many ideas discussed previously, such as subject and pedagogical content knowledge, (Higgins & Parsons, 2009). These three tools essentially scaffold the NDP's purpose. The Number Framework allows a teacher to place students at development stages using the diagnostic interview and then the model helps the decision making involved in the press (Kazemi & Stipek, 2001) for moving up the levels (Stevens, 2010). Each of these three pedagogical tools will be looked at in turn, with particular reference to quality teaching.

The Number Framework

The first pedagogical tool is the Number Framework and this is found in *Book 1: The Number Framework* (Ministry of Education, 2008a). This depicts NDP in a way that teachers, and parents, can understand in words which illustrate the levels of the framework through diagrams and exemplars. The framework is divided into knowledge and strategies sections. This division was justified as being necessary to highlight the different teaching required for each section (Hughes, 2002). The knowledge section focuses on what needs to be known at any particular level and the corresponding strategies detail the types of strategic thinking a student needs to problem solve.

Each strategy section has eight global stages moving from less advanced mathematical thinking to more sophisticated and connected ways of thinking. This is about how children acquire mathematical thinking abilities and it frames the teachers' pedagogical decisions about where to next. The framework supported the change in pedagogy occasioned by the implementation of the NDP. Implementation of this pedagogical change was complex and fraught with difficulties but teachers found the explicit levels and explanation of behaviours in the framework led to more purposeful, quality teaching. They were more certain about the learning trajectories they were planning for their students (Cheeseman, 2006; Higgins & Parson, 2009). Knowing about how children learnt mathematics, the progressions and stages, has had a powerful effect on how and what is taught in New Zealand schools (Stevens, 2010; Young-Loveridge, 2010).

The diagnostic interview

The second pedagogical tool is found in *Book 2: The diagnostic interview* (Ministry of Education, 2008b) which presents the interview questions and all required resources. Its purpose is to build up the teachers' understanding about the child's current level of thinking. The scripted, tasked interview is conducted between the teacher and one child, with the teacher leading this with dialogue designed to elicit the child's thinking. This helped teachers structure the necessary dialogue and meant that the interviews were roughly similar. The book includes all the instructions and necessary materials for children to use when solving various problems.

The types of questions in the interview help teachers recognise when a child is using knowledge to answer a question. Typically recall will be swift and accurate, or if using strategies, then the response time may be more prolonged. Children's thinking is probed and they are questioned about *what they did* and *how they did it*. This helps make their thinking apparent (von Glasersfeld, 1992). The teacher also watches what the child does with materials and synthesises all the information to make a judgement of achievement level. The questions themselves suggest the types of activities that may be useful in moving the child on to the next stage. The questions also deepened teachers' understanding and connected thinking of the concepts. The diagnosis involved in this process was considered to be the turning point for many teachers, increasing their identification with the professional development and occasioning change in their pedagogy towards a more child-centred learning, and gave quality teaching a further chance to develop. For many teachers it caused some disquiet, because it turned on its head their previous practice and showed them what children truly understood, providing impetus to truly focus on the needs of the child (Higgins & Parson, 2009).

The strategy teaching model

The third pedagogical tool is that of the strategy teaching model, found in *Book 3: Getting Started* (Ministry of Education, 2008c). This details the important and ubiquitous strategy teaching model (Hughes, 2002; MOE, 20008c) shown in Figure 2.1. This was influenced by the Pirie and Kieren (1989) model, retaining their idea of folding back while focusing on key stages. The Pirie and Kieren model was seen as a descriptor of what happens when children learn mathematics. It details the growth of understanding in a dynamic way, and is a recursive model, in that it expects learners to regress to previous, simpler, stages when a hiatus occurs in the learning.

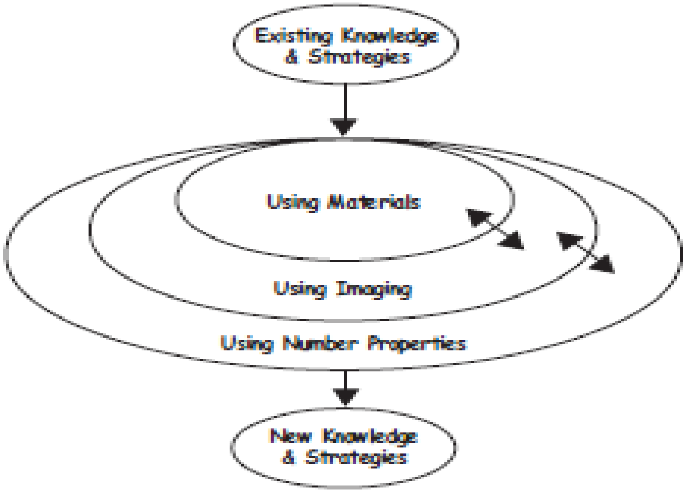

Figure 2.1. The strategy teaching model (Hughes, 2002)

However, it was not developed as a teaching tool, but as a research model, one which was over-complicated for using with teachers in their classrooms.

The ideas behind the NDP strategy teaching model, are that conceptual understandings in mathematics (for example the "noticing then using properties of numbers" stage) are built from manipulating materials that allow the development of images, with movement back and forth between layers (recursion) fundamental to that conceptual understanding (Britt & Irwin, 2008; Hughes, 2002; Hughes & Peterson 2003; Ministry of Education 2008c; Wright, 2014).

The model begins with prior knowledge, or in the words of the Pirie and Kieren model, primitive knowing. This is termed existing knowledge and strategies. The model starts with the child "using materials" which exemplify the concept in question. One example might be when developing notions of place value, the material, tens frames and counters are used to represent numbers. Once the child begins to complete the task without reliance on the equipment, the teacher will explicitly require the child to image or think about the materials to solve a question—"using imaging"—but not to manipulate the material any further. This stage assumes some abstraction is taking place (Hughes, 2002). And once this abstraction becomes firmer in the child's mind, they will move into the "using number properties" stage, which can be provoked by using larger numbers, hence superseding the previous reliance on materials and then imaging.

The strategy teaching model is used throughout the stages in the NDP framework, and it has been seen as demanding of teachers (Cheeseman, 2006) as it stretches them to change pedagogy to accommodate these new processes. The model is also seen as supporting teachers, as it requires them to make moment-by-moment decisions about the needs of the learner, while supporting them with defined stages and progressions that scaffold their teaching decisions (Cheeseman, 2006; Hughes, 2002; Stevens, 2010). The model is explicitly referred to throughout all of NDP resource books, from 1–8, all the teaching resources, and all the lesson plans. So now the question must be, how does the model support the development of quality teaching? This question will be examined next.

Quality teaching and the pedagogy in the NDP

The three pedagogical tools that underpin the NDP, the Number Framework, the diagnostic interview and the strategy teaching model, highlight the changed pedagogy necessary to teach well under the new system. In this section, the ideas about quality teaching, explicit in the NDP, will be examined.

As well as the three pedagogical tools, there are other very important characteristics which are evident in the teaching of the NDP. These range from running an inclusive classroom through using problem-centred activities to having high expectations of the students (Stevens, 2010). The change in pedagogy necessary to institute the reforms under NDP is considerable. *Book 3: Getting Started*, (Ministry of Education, 2008c) identifies seven dimensions to be found in quality teaching: inclusive classroom climate; focused planning; problem-centred activities; responsive lessons; connections; high expectations; and equity. Each of these dimensions will be taken in turn to highlight quality teaching and the NDP.

In an *inclusive classroom environment*, everyone is expected to take an active part in lessons, listening to others and being prepared to take risks while they contribute to discussions (Wenger 2000). This echoes the ideas about communities of practise; for instance, Wenger talks of the rights of participation in the social systems that children are in, such as the school, and this inclusive classroom environment exemplifies these ideas. Socio-mathematical norms (Yackel & Cobb, 1996) need to be established which make this apparent to all students. Diversity of students, both in their mathematical understanding and in other aspects of diversity, including ethnicity and culture, are valued in an overt manner. Having such a classroom climate is surely a part of quality teaching.

Focused planning is vital to enable the most apposite learning trajectory to be developed both for and with the student. Quality teaching in NDP uses many different assessment methods, both formal and informal, which the teacher synthesises into a coherent whole.

Thus, students are encouraged to know, and be responsible for, their learning goals, which both the teacher and student continually reassess in the light of further knowledge gathered regarding the students' mathematical understandings. This is maintained through high-level conversations and highly focused planning for learning.

In a classroom focused on *problem-centred activities*, students are expected to be active learners, working in groups and as an individual, to solve problems in a context. The cooperative learning model (Cheng, 2011) is encouraged; ensuring students have a voice in this learning process. High performing countries, according to international studies such as PISA and TIMSS (OECD, 2010), spend much of their class time problem-solving, so it is the preferred teaching style of NDP, which speaks to quality teaching in the classroom. If traditional classroom instruction remains the preferred teaching style, then use of NDP pedagogical tools and teaching methods are not likely to be enacted.

Producing *responsive lessons* and being responsive to students is a complex undertaking. Teachers need to be aware of how all the children in the target group are responding, while still being aware of the rest of the class. Minute-by-minute decisions are being made which allow tasks to be attuned to fit the newly realised needs of the students. This analysis includes the scaffolding the teacher uses in the task and the feedback given to the student, both of which need to be sensitive and appropriate to that moment. This propensity to be flexible and change to capture and use the teachable moment (Kahan, Cooper, & Bathea, 2003) is central to the ideas behind quality teaching.

Teachers need to help students make *connections* between concepts they are learning, and so the teacher needs to be able to recognise that those connections exist; this can only happen if they understand the mathematics in a connected way. Teachers need to represent to the students different models and ways of thinking, ways which are powerful and sensitive to the students' nascent understandings. Without this connectedness, there will be little going on in the mathematics classroom that could be labelled quality teaching.

High expectations are part of quality teaching in NDP style. Teachers are expected to convey to students their high expectations (Irwin & Woodward, 2005), but not just to expect their charges to learn but to use higher order thinking skills such as justifying and synthesising. High expectations are conveyed to the student through careful questioning and the types of activities the teacher anticipates the students to be successful in and problems they are expected to solve. The configuring of the student as an independent learner is central to this idea of quality teaching in NDP teaching.

Equity is much more about responding to what each child needs (Ministry of Education, 2008c), rather than about equal shares of time or resources. Equity, a dimension of quality teaching, is not just about working where the child is at, but it is about being sensitive to the requirements of the students who have different needs, and it is about encouraging all the class to be sensitive to others needs too.

It is well to note here that overlaying all these seven dimensions of quality teaching in the NDP is the absolute need for teachers to fully know and understand the content they are attempting to teach (Ward, Thomas, & Tagg, 2007). Only then can they understand any misconceptions or find the most appropriate path to the learning goal.

This discussion of the facets of what quality teaching might be under NDP-style teaching has presented the strategy teaching model and other characteristics. The final exposition here regarding the pedagogically knowledgeable teacher will discuss mathematics discourse, both within NDP teaching in New Zealand and in mathematics classes in other countries where research has been focusing on teacher–student interactions.

Quality teaching and Mathematics discourse

Discourse is about the way human beings interact and communicate. Though by no means centred exclusively on language, the use of language is the focus for this section. What is of interest here is the language that teachers use in mathematics lessons to provoke more discourse from their students, and ultimately to occasion learning. Mathematics discourse is about the complexity and connectivity of images, ideas, and symbols that mathematics teachers use to provoke learning in their students (Sfard, 2008). This connects with NDP pedagogy requiring deep understanding and knowledge of the concepts in mathematics to make apparent this connectivity. There is little research on mathematics as a discourse, but what is found are notions of discourse communities in classrooms (Ryve, 2011).

Mathematics talk in the classroom is now conceptualised as students talking to other students in groups, replacing the traditional “chalk and talk” type pedagogy with less of the “sage on the stage” and more of the “guide on the side” type (King, 1993, p. 30). The talk about mathematics between children is seen as essential to deep concept development, especially if the talk is about defending an answer and discussing the argument behind it (Ministry of Education, 2005; Sfard, Foreman, & Kieran, 2001). Part of this reasoning is that if children are talking about mathematics then they must, on some fundamental level, be engaged with the mathematics and it is this engagement that is seen as central to learning. Therefore, quality teaching, it seems, must have quality discourse to complement and be an integral part of quality pedagogy. Teachers need to see themselves as listeners rather than simple imparters of information. They need to observe and actively listen to their charges, moving them towards sense-making of mathematics rather than concentrating on mere procedures and rote learning of rules (Walshaw & Anthony, 2008). So how much talking is compatible with quality teaching?

There is strong research evidence (Boaler, 2002; Lampert, 2001; Staples, 2008) that in classrooms where teachers maintain a strong hold on and authority over, the lesson, the children become passive learners. According to Boaler (2002), in classrooms where the authority is more distributed, students develop new, positive dispositions towards the learning of mathematics and become more independent learners. It has been recommended that in a mathematics problem-solving context, the teacher should inhabit no more than 30% of the time with talking, which will help ensure that the children are active, and talking themselves (Pennant, 2013).

Changing to an inclusive pedagogy that encourages mathematical discourse between children has been shown to be both important to try but difficult to achieve (Walshaw & Anthony, 2008). It is one aspect of NDP that the teacher encourages the discourse of the children (Ministry of Education, 2008c). This is achieved principally by working in small groups with children who are at similar stages on the framework. The teacher encourages them to verbalise their thinking so that others may extend their own thinking. The principal pedagogical framework that the early developers of NDP used (Irwin & Woodward, 2005) was that of Fraivillig, Murphy, and Fuson (1999). This framework, Advancing Children's Thinking (ACT) is the same framework central to the development of DART, the lesson coding tool which is detailed in Chapter 4. In the ACT framework there are three strands of teaching strategies for quality teaching and discourse encouragement: *eliciting*, *supporting*, and *extending*. In the following discussion, words in italics are taken directly from the framework of Fraivillig et al. (1999, p. 155).

Eliciting describes the teacher's efforts to encourage the students to talk, to describe their thinking. Questions that help students express their current understandings and perhaps use these understandings to provide the trajectory of the lesson, may be as simple as "Tell me what you are thinking" and "Does anyone else have another way?" It is what is done with the responses that is seen as so central to a truly interactive mathematical discourse. Having *elicited all the strategies,* and *having listened and given time for students to respond*, the teacher encourages the students *to elaborate their solutions* while conveying an *accepting attitude* of both correct and wrong solutions. This *orchestration of classroom discussions***,** while *promoting collaborative problem solving,* allows the teacher to use the discourse to propel the level of the work towards greater depth of understanding. While the teacher *monitors the students' levels of engagement,* the needs that various members of the group have to speak and air their understandings are managed, while at the same time, the teacher makes in-the-moment decisions about the strategies that need to be privileged. This eliciting phase is seen as vital to understanding where-to-next in the learning trajectory of any particular group.

Having elicited the children's responses, quality teaching means *supporting describer's thinking* by various means. These include *reminding students of conceptually similar problem situations,* while *providing background knowledge* to help individuals and the group clarify *own solution methods.* Teacher actions may include *instant replays* and *demonstrations of the methods* while accepting all solutions. While *recording symbolic representations of these different solution methods,* the teacher might *ask a different student* to explain another's methods, and although the teacher will help individuals, these students are encouraged to ask for teacher help *only when needed.* These supporting actions were seen as vital to support the talkers and the listeners within the groups and to keep quality mathematical discourse at the centre of the lesson.

These two strands of the framework, eliciting and supporting, are most often noted in the lessons analysed by Fraivillig et al. (1999), with the supporting strategy being seen much more than eliciting; nonetheless, although these are seen as basic building blocks to quality teaching, it is in the extending phase where children are moved from current thinking into new understandings. However, extending does more than move children through or beyond their current level of understanding or proximal zone of development (Vygotsky, 1978); it also shows that in quality teaching the teacher *has high standards and expectations* of every student regardless of level of achievement and that the teacher believes that students have the potential to become powerful learners. The extending phase does this through expecting all learners to *solve difficult problems, to reflect on solution methods.* This carries assumptions that students will *analyse, compare and generalise mathematical* *concepts* and *discuss interrelationships among concepts.* The teacher pushes the group *beyond initial solution methods*, and the individual is encouraged to try *alternative solution methods* then *more efficient solution methods* are promoted. Having used the responses of the students *as the core lesson*, the teacher ensures that they have *cultivated a love of challenge.* This use of the children's responses to drive the lesson speaks more to quality teaching than merely being desirable. By placing the children's work at the heart of the lesson, the teacher privileges the current thinking of the children, and builds on this the necessary scaffolding to move them to the next level.

This concludes the exposition on the second "big idea", that of the pedagogically knowledgeable teacher. As evidenced, there is a wide definition of what it means to be pedagogically knowledgeable. This ranges from the mathematical knowledge of teachers (MKT) of Ball (Ball et al., 2005) to the work on discourse (Walshaw and Anthony, 2008) and, specifically, quality teaching within NDP. The development of the NDP was discussed and NDP pedagogy central to quality teaching explored, followed by a discussion of the importance of discourse and the use of the ACT framework (Fraivillig et al., 1999). The third division on the Shulman's framework (1989), the contextually knowledgeable teacher, will be discussed next.

The contextually knowledgeable teacher

The third big idea in identifying quality teaching is that of contextual knowledge. Teachers need what Shulman (1987) defines as curricular knowledge to encompass all the paraphernalia of teaching. He included different forms of curriculum, different ways of teaching, and all the subject curricula children are taught, which are of special importance to primary teachers as these are the tools of their trade. This enabled, in Shulman's view, the interconnectivity of all subjects to be made apparent to school children. He also thought that there was a wider context that the teacher needed to know. These he called propositions and they included the norms of a teacher: fairness, equity, justice and other "philosophical commitments" (1989, p. 11) that are part of the wider, contextual knowledge of the teacher. This section will explore and expand on Shulman's reasoning that teachers require much more knowledge than mere practice. He contends that they require a deep understanding that goes beyond the subject matter and the pedagogical practice. The playwright George Bernard Shaw envisioned a teacher as a person who really does not know their subject well enough to "do" it. This is rejected by Shulman who declares that this "infamous aphorism", *He who can, does, he who cannot, teaches,* "is a calamitous insult to our profession" (p. 4). Having asserted in his presidential address to the 1985 annual meeting of the American Educational Research Association that the insult was well and truly refuted, he finished the address by saying:

> We reject Mr Shaw and his calumny. With Aristotle we declare that the ultimate test of understanding rests on the ability to transform one's knowledge into teaching. Those who can do, those who understand, teach.

It is this wide understanding of the context that quality teaching requires which will be considered here.

The context of the quality teacher is complicated because each aspect of context will be operating within and across other aspects of context. Goodwin (2010) argues against trying to find *the* one way to enact quality teaching, the one thing that will magically make every act in a school one of quality. This thinking is reverting to a time when linear or single-strand solutions were sought, and asks for quality teaching to have only one route from a teacher's education to the classroom learning situation. It harks back to that time when educators thought the huge complexity that is the act of teaching could be summed up in a few sentences. The act of quality teaching will undoubtedly look different depending on the context, and the interactions between various aspects. Unquestionably the other two big ideas from Shulman, subject and pedagogical knowledge, will be required. What other aspects of quality teaching are in the context zone? These must include: the teacher; the classroom; the curriculum; the school; and all the diversity of cultural aspects.

Context of the teacher

This thesis has at its heart the context of the teacher of students in the primary years, ages 5 to 13 years. Primary teachers in New Zealand are required to teach mathematics everyday as part of their daily teaching, since in general there is no expert available to take the mathematics lessons. Though primary school teachers will in most instances be part of a team or syndicate, nonetheless they are completely in charge of the day-to-day mathematics education of the students in their classes. They will ideally be working from the documents from both the New Zealand curriculum and the NDP, but unpacking these and tailoring them to the needs of the children.

Teachers at the primary teaching level have their own classroom and remain in it for most of the day, so their immediate environment includes the classroom, and what the children bring in to this environment from their families and communities. Making the classroom an inclusive and welcoming place is obviously important, but more than that, the classroom must accept all the diversity that the students bring and, even more, celebrate the diversity that is the population of New Zealand (Alton-Lee, 2003). Diversity in the classroom comprises attributes which range from gender and ethnicity to culture and language. Socioeconomic factors and the interests of the children also have to be seen as part of the context of the classroom (Rubie-Davies, Flint & McDonald, 2012). The children and what they bring are among many other factors of which the teacher needs to be contextually aware. This is coupled with not just the here and now but includes the history of all participants plus the history and politics of teaching up to this point (Goodwin, 2010). This social-cultural context is interacting with the digital age, forcing re-evaluation on a continual basis of what might be the most efficacious way to help children learn. Understanding how all these different forces may be acting within a classroom is a complex undertaking, and central to ideas of quality teaching.

Context of the curriculum

The curriculum that the teacher enacts can be very different from the written curriculum, which again can differ from the received or intended curriculum (Dole, Makar, & Gillies, 2012). While the curriculum that the teacher delivers probably has a great deal to do with the teacher's knowledge and confidence in their own mathematics and their pedagogical practice, the curriculum that the child receives is mitigated by the context of the learner, their prior knowledge and learning, and their readiness to learn that particular concept. Here the curriculum will be taken to mean the intended curriculum, such as is contained in *The New Zealand Curriculum* (Ministry of Education, 2007).

The curriculum in *The New Zealand Curriculum* is different from the curricula that preceded it in that, although it is a statement of what should be the subject matter of mathematics lessons, there is a much broader intention and a determination to embrace diversity (Cowie et al., 2009). Subject areas of previous curricula were reviewed, revised, and published separately, whereas this latest version has all subject areas, and levels, reviewed and published together, giving a fully integrated document. This is a support for schools to "design and review their curriculum" (Ministry of Education, 2007, p. 6) one which takes account of local needs and mores, which might meet the needs of the community, and has an emphasis on effective pedagogy which, through inquiry learning, might improve outcomes (Ell, 2011). The emphasis in the document is on a vision for learners involving three pathways: values, key competencies, and learning areas. The curriculum document comprises 44 pages, of which more than half are devoted to an explanation of the vision, principles, values, and beliefs underpinning the curriculum. The outline of the subject content areas (called learning areas), which does not start until nearly half-way through the document, is set out over 18 pages.

In 2009, the Ministry of Education published *national standards* in reading, writing, and mathematics, and required schools to implement them in 2010. This was a controversial move, with the teaching profession protesting that national standards were counter to good pedagogical practice and had the capacity to move teachers away from teaching interconnected mathematics lessons into teaching-to-the-test type lessons (Ell, 2011). The introduction of the standards was seen as shifting the focus from improving mathematics teaching to the assessment of achievement and reporting to parents and the country (Young-Loveridge, 2010).

Overlaying the *New Zealand curriculum* and its statements about content, and the national standards issue, is the NDP and how this is perceived to contain the curriculum in numeracy for young children. In some countries, such as Australia, the emphasis on numeracy and practical numeracy rather than straight mathematics has led to their curriculum review reflecting the expert view that numeracy should be the emphasis in the mathematics curriculum up to the Australian Year 10 (New Zealand's Year 9) (Anderson, White, & Sullivan, 2012). In New Zealand, the tenets of the NDP have been incorporated within the curriculum achievement objectives of the curriculum under the number and algebra strand. The language of the NDP is used in the New Zealand Curriculum to describe the concepts of mathematics at different levels.

Context of the school

What content is taught and how it is taught in schools is influenced by the wider socio-political forces that have been a factor since schooling first began. In New Zealand, schools may be public, private, integrated, or of special character; large or small; and urban or rural. Schools differ almost as much as the students who inhabit them. Quality teaching is situated within both the classroom and the school. How might school factors affect quality teaching? In many of the 10 characteristics of quality teaching identified in the best evidence synthesis (Alton-Lee, 2003), there is an emphasis on whole-school implementation. For example, "There is whole school alignment and coherence across policies and practices that focus on resources and support quality teaching for diverse students" (Alton-Lee, 2003, p. ix).

So what of the schools themselves? What forces acting on a school provide different contexts for learning? Schools in New Zealand are rated (and funded) by decile, which is based on the socioeconomic status of the local community. The lowest decile schools, decile 1, receive the greatest targeted funding from the Ministry of Education and the highest decile schools, decile 10, get the standard funding. This is an attempt to provide greater resources for students who may be disadvantaged. Discussion of decile seems to be central to discussions of inequality between schools and of poor outcomes for children at low decile schools. There is a large difference in attainment between high and low decile schools, commonly conceived as "the gap" (see, for instance, Snook & O'Neill, 2010; Thrupp & Alcorn, 2011). Schools are generally seen as powerless to change deeply entrenched views of children from disadvantaged groups and the society which relegates these children to the bottom of the achievement statistics (Snook & O'Neill, 2010). The concensus seems to be that it would be *possible* for the schools to change, to allow the poor or disadvantaged child to progress successfully through the school system, but that the changes required would need to be more than just school-wide—they would need to be society-wide too (Snook & O'Neill, 2010).

Though care needs to be extended to any deterministic view of the likelihood of children from low decile schools being relegated to failure and low achievement (Chenoweth, 2007), studies and reports on the NDP have shown that children from high decile schools are performing at higher levels on the NDP framework than children at lower decile schools (Young-Loveridge, 2010). These findings are consistent between 2004 and 2009. Why might this be? Could the teachers in these schools be different in some way?

Some schools have problems in recruiting quality teachers. The more educated, experienced, and effective a teacher is, the more likely they are to be employed in higher decile schools (Ritchie, 2004). It has been noted through the ages and across countries that as teachers move from novice to expert status, they also move from low socioeconomic schools into higher socioeconomic schools. The lower the status of the school, the more likely the teachers are to be older and less educated, and the longer they teach at a lower status school the less desirable they appear to higher status schools. The reasons for this are many and varied. For instance, the proximity of middle-class schools to where the predominantly middle-class teachers reside could result in more concentration of those teachers in those schools. Also some schools have better working conditions than others, such as a high decile school with a small number of perceived difficult students as compared to a lower decile school (Amrein-Beardsley, 2012). In New Zealand, there is little evidence that there is currently any differential in salary to explain this phenomenon, except, perhaps, in private schools. So if the school by virtue of its position or status or decile is less desirable, then it may be hard to attract quality teachers (Snook & O'Neill, 2010).

Cultural context

New Zealand is sometimes typified as a bicultural nation, with indigenous Māori on one side of what might be seen as a cultural divide, and all other groups of immigrants, among them New Zealanders of European descent who have been here for many generations. In recent years there has been a large influx of Pacifika and Asian immigrants, and now the constitution of the New Zealand population is roughly 14% Māori; smaller, but similar percentages for each of the Pacifika and Asian groupings, while more than 65% are European. This mix of ethnic groups has seen a push for New Zealand to be termed a multicultural country. Whatever term is used, these statistics hide the reality of the numbers of children in each ethnic group in our schools. About 25% of children identify themselves as Māori, and these children make up a large but diminishing group in the population. In 1971, children comprised more than 30% of the population, but this proportion is projected to be a little as 18% in 2021 (Statistics New Zealand, 2002). If the children are more ethnically diverse than the adult population, there may be a considerable mismatch between them and the ethnic make-up of their teachers. Couple this mismatch with the socioeconomic mismatch and there may be many reasons for concern regarding connections and expectations.

For many people, New Zealand is not conceptualised as bi- or multicultural at all: It is seen by the dominant ethnic group of people who identify as European (Henderson, 2013) as monocultural. There have been many teaching initiatives from the government (for example, Te Kotahitanga, an initiative to raise Māori achievement) which should have the capacity to improve outcomes for those who have not been at the top of the achievement statistics. However, there has been little improvement in the achievement for those students at the bottom of the statistics (Snook and O'Neill, 2010). While it is possible for teachers to be ignorant of the needs of some students and to have low expectations of them, (Anthony & Walshaw, 2007), the arrogance of racism has been posited as one reason for the lack of movement on these measures of student achievement (Henderson, 2013). The cultural context of our teachers becomes extremely important in the achievement of our students and the mismatch of gender, ethnicity, socioeconomic status, and class make up of our teachers is something which needs to be taken into consideration when the diversity within the teacher population is discussed, as it will be in the following section.

Cultural context of the teacher

The teachers in New Zealand primary schools are predominantly female and white (Hope, 2004), but they are also products of their own upbringing, and they have beliefs about mathematics and their students which can be resistant to change. Couple this with the prevalence of young women in the applicants for teaching and the prejudice that some members of society have towards teaching and teachers, and reasons for the low status of teaching as a career becomes clearer. Professions that become more "feminised" generally become low status after a time, with all the concomitant lowering of desirability of the profession, as well as, in some cases, a lowering of salary (Elbaz-Luwisch, 2011). And if not an actual lowering of salary, it is known that women generally in the New Zealand population earn less than men. It can be seen from the latest census that the median salary of men was 1.6 times that of the median salary of women (Statistics New Zealand, 2013). The demographics of the national population of New Zealand pre-service student teachers in 2004 showed that the median age of entrants was 30, and 80 percent of the cohort was female (Hope, 2004). This feminisation of teaching is seen in countries where universal schooling is in effect but not in developing countries, where women are underrepresented in the teacher workforce (Moller, Mickelson, Stearns, Banerjee, & Bottia, 2013). Moller et al. (2013) also found in their study that students who might be termed part of the "tail" of underachievement were advantaged if their teachers had a strong sense of collaboration and affirmed the culture of their school. In that study, about boosting mathematical attainment, it was found that race, ethnicity, and socioeconomic standing of both teachers and students was not the over-riding factor in boosting attainment as long as the teachers were strongly connected to each other. It was found that "a collective pedagogical teacher culture boosts student achievement and reduces gaps in achievement" (p. 188). A collective pedagogical teacher culture is one that involves teachers in professional communities and teacher collaboration. Barriers to these types of communities developing to the betterment of schools and the closing of the gaps might include non-professionalisation of teachers,

teachers' preference for planning as individuals, and teachers having a change of organisation forced upon them.

Another problem that has arisen since the feminisation of the teacher population is that male role models and male ways of working are not available to the male learners, when the majority of the teachers are female. This lack of male teachers, especially in the primary school, is variously reported at roughly 16% of the primary teacher workforce. This topic is mostly to be seen in popular media, where calls are made for more male teachers, but the news media is less concerned with learning than with finding reasons for young males to be in need of a role model to fix whatever the perceived problem is in their lives, and that is entwined with the lack of male figures in their home lives too. Many children are being reared in women only households, while undoubtedly in the context of the teacher (Martino, 2008), this adds to the mismatch between many teachers and their students.

Context of teacher education and teacher educators

Teacher education takes place mainly in universities in New Zealand, while there are a handful of courses which teach small numbers of teachers within the school setting. In the main, teacher educators made their way from teaching in schools to university teaching through master's degree study, and had little if any education regarding teaching people to be teachers. This is the same pathway followed in much of the world: that teacher educators are assumed to be able to teach the art of teaching simply by the fact of being, possibly, an exemplary teacher themselves. Furthermore, teacher educators in universities must also be researchers, so they have two strands to their work, and they will most likely be novices at both when they arrive. How they learn to be good teacher educators is not strongly researched (Llinares & Krainer, 2006), but the monolingual and predominantly female and white teacher population from which they come does not, of itself, bode well for teachers being able to relate to cultural diversity and to foster a classroom climate that embraces and celebrates cultural diversity.

This mismatch of cultural factors could mean that teachers are not connecting with their students in the best way to occasion good learning opportunities, so the teaching in these classrooms may not be of the highest quality. If quality teaching is the important issue, what are the effects on quality of the sorts of mismatches discussed above? Next, teacher beliefs are briefly examined.

Teacher beliefs

Beliefs are central to who a person is, since “the beliefs that individuals create and develop and hold true about themselves form the very foundation of human agency and are vital forces in their success or failure in all endeavours” (Pajares, 2002, p. 1). So too the beliefs that teachers hold about mathematics are fundamental to the way that they teach mathematics, often teaching as they were taught (Beswick, 2005; Gourgey, 1992; Hannula, Kaasila, Laine, & Pehkonen, 2005; Rubie-Davies et al., 2012; Uusimaki & Kidman, 2004). Beliefs about mathematics are said to be “strongly held and deeply ingrained through continual reinforcement over many years of schooling” (Lomas, 2004, p. 26).

Teachers have varied beliefs about the ability of their students to succeed (Kim, Kim, Lee, Spector, & DeMeester, 2013), about the efficacy of the NDP, and about all sorts of matters pertaining to the teaching of mathematics. The more positive and optimistic a teacher is about these aspects, the more likely that positive outcomes will result (Boaler, 2013). Their beliefs about the nature of mathematics are varied too, and there are many who believe in “maths myths” (Boaler, 2013; Franks, 1990) such as that there is a “maths brain”, and if you do not have a “maths brain” you cannot learn maths. This is just one example where beliefs constrain teachers and their actions. Boaler (2013) and many others talk of the “mindset revolution” and the plasticity of the brain. If you have a growth mindset, then you believe that intelligence is not fixed, that brains can grow in response to different stimuli. People with this growth mindset persist with tasks and try to understand difficult concepts, whereas those with a fixed mindset do not, and find it impossible to see failure as an opportunity to learn (Dweck, 2006). When students have been encouraged to move from a fixed to a growth mindset, by being given messages about “study making a difference”, and “study can change the brain”, they have achieved at a higher level (Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2006; Good, Aronson, & Inzlich, 2003).

In their study involving Greek mathematics teachers, Barkatsas and Malone (2005) found that even though the teachers espoused what could be termed a constructivist view of teaching, a majority of the teachers remained immersed in the more traditional or transmission model of teaching and learning. Their beliefs in the primacy of the traditional model overcame their new learning and hence the study showed that beliefs can be an impediment to change. Beliefs influence how students learn mathematics, since, for example, if they believe that mathematics is a group of learned skills which are unrelated, it follows that they may fail to see that there is a different way to learn mathematics (Carroll, 1994; Gourgey, 1992).

Pre-service student teachers' beliefs are made up of those they bring with them into the teacher education courses and those that they encounter, and make their own, within the courses (Grootenboer, Lomas, & Ingram, 2008). Existing beliefs filter new information and overlay the new information with their original beliefs. Because "beliefs exist in a tacit or implicit form and are difficult to articulate" (Perry, Way, Southwell, White, & Pattison, 2005, p. 625), then not only are beliefs robust and resistant to change but they are hard to express, and therefore may remain outside the influence of many forces for change within teacher education courses. However, reflection has been seen as a way to engender change. Kaasila (2001) showed in her study on pre-service students in Finland that by reflecting on their school days, the students can "enter into a dialogue with his or her former self and may redefine his or her mathematical past in a more positive manner than earlier" (p. 33) and some change was engendered. Nevertheless, she still cautions that the type of change they underwent and the type of teacher these students would become was by no means certain.

This resistance to change was found throughout studies on pre-service student teachers (Barkatsas & Malone, 2005; Perry et al., 2005). In these studies it was speculated that poor attainment by the pre-service student teachers was due to incomplete understanding of the mathematical concepts they were expected to teach and that this coloured their beliefs about what mathematics is and how it should be taught. One primary pre-service student teacher's beliefs were investigated by Mewborn (2000), who found that the student was able to change her beliefs about children and the learning and teaching of mathematics, but that this change was only possible because her behaviour had changed, and she saw the consequences of that change. In that study, they posit two explanations for this change. Either this teacher's belief's had fallen in line with this behaviour or she had kept her original belief, that teaching mathematics was very difficult and that she would not make a good job of modelling an interest in mathematics, but was now able, through her behaviour, to see mathematics positively. Again, resistance to change was found in a study by Hannula et al. (2005), who investigated the beliefs of student teachers about mathematics and their ability to succeed at doing mathematics. They found a small group within the original 269 student teachers in their study who were very negative in their beliefs both about mathematics and their own ability to do mathematics. They believed that they could not succeed at mathematics and were labelled hopeless by their lecturers as their view of themselves was so negative that there seemed no hope for change.

So it would seem reasonable that teachers' beliefs will have an effect on quality teaching, and some of those entrenched views about the impossibility of understanding mathematics are bound up with maths anxiety, the next topic in this discussion of the contextually knowledgeable teacher.

Maths anxiety

Many studies have found maths-anxious people from all walks of life, not just students (Burns, 1998). It is commonly accepted by the general public that approximately 10% of students can "learn" mathematics because it is so "hard" (Furner & Duffy, 2002). When questioned as to their experiences in mathematics learning, only about 7% of pre-service primary student teachers were able to say that their experiences had all been uniformly positive (Jackson & Leffingwell, 1999).

The situations in which people will exhibit maths anxiety lie along an extended continuum, from the classroom, representing for some the most anxiety-provoking situations, through to manipulating numbers in a seemingly benign place, such as paying money in a shop or deciding how much of an ingredient to put into a recipe that has been halved (Levine, 1995). The situation does not need to be overtly evaluative, or even be under the watch of a third person; the maths-anxious person's perception of the situation is the major factor in causing the reactions that constitute maths anxiety (Newstead, 1998; Zeidner, 1998). People lie along this continuum virtually unaffected by reality (how poor they are at mathematics), or their overall intelligence level. In fact, it appears that intelligence and maths anxiety are only very slightly connected, and research has attributed most of the small negative correlation (-0.17) to the fact that IQ tests contain mathematical elements, which necessarily lead to a poorer performance (Ashcraft, 2002). So if intelligence is not a strong factor, and if people are maths anxious in many different mathematical situations—during tests and while doing homework for example,—then how that makes people feel should be a consideration. A later section will explore what causes maths anxiety to arise in the first place and to be a prevailing factor in people's lives; next I will consider the physical consequences of maths anxiety.

The physical reactions to anxiety that people exhibit, such as shaking, sweating, and a racing heart, are the same as for a physical threat (Maloney, Schaeffer, & Beilock, 2013) such as, for example, being watched by a big, hungry tiger through the leaves in the jungle. It may seem incredible that mathematics could evoke such strong anxiety. However, not for the two-thirds of the American public whom Burns (1998) reported as having such severe maths anxiety that it approached maths phobia. For many students, mathematics was a subject to be feared in school, and for some it affected, and curtailed, career choices and their important life decisions (Beilock & Maloney, 2015; Furner & Berman, 2003; Hembree, 1990; Stodolsky, 1985; Trujillo & Hadfield, 1999; Zettle & Houghton, 1998).

Causes of maths anxiety

Maths anxiety becomes evident in students at about eight or nine years of age; before this, children are often reported as very positive about mathematics, citing it as their favourite subject (Attard, 2010; Stodolsky, 1985; Turner et al., 2002). In a New Zealand study of 319 ITE students, Loveridge, et al. (2012) found that over three quarters of them were most positive towards their primary school mathematics, but they were less positive towards their secondary school mathematics, with only half reported as positive. Generally, by about the middle school years, attitudes, though still positive, have changed so that mathematics is less likely to be a student's best-liked or favourite subject. By the time students in their last two years at school, aged about 17, are asked, mathematics becomes the least liked subject, and the number of students who do not choose mathematics as a school subject in their senior years is testament to the change from a positive to a negative perspective (Dowker, Sarkar, & Looi, 2016; Hembree, 1999; Ma, 1999; Stodolsky, 1985; Walmsley & Muniz, 2003; Wolodko, Willson, & Johnson, 2003).

It may not be too much of a surprise that negative feelings are found in increasing strengths as the school years progress, but it is a surprise that students enrolled in mathematics courses at university also display a negative attitude towards mathematics (Townsend, Lai, Lavery, Sutherland, & Wilton, 1999). Stronger even than these are the negative feelings held by a large number of students who become pre-service primary school teachers (Trujillo & Hadfield, 1999).

One explanation for antipathy towards mathematics has centred on the influence that parents and significant others have on a young child's predisposition towards mathematics. Primary school students in a Greek study were found to have a generalised fear of mathematics, which, it was contended, the students had contracted from their parents and the society in which they lived (Furner & Duffy, 2002; Ufuktepe & Özel, 2002). In an American study of 133 parents and their children, Maloney, Ramirez, Gunderson, Levine, and Beilock (2015) found that parents' maths anxiety was socially transmitted to their children only if they frequently helped with their homework. This resulted in low maths achievement and high maths anxiety. Parents seem all too keen to inform teachers that they themselves strongly disliked mathematics and are not at all surprised that their children are failing too. This opinion is not covert; on the contrary, it seems socially acceptable for parents and other adults to state their loathing of mathematics, with no feelings of shame or blame attached (Burns, 1998).

Factors that have been found to be implicated in the rise of maths anxiety are as diverse as the negative attitudes of parents, over-reliance on text books, or the dogmatic attitudes about the right way to do mathematics. The idea that pre-service student teachers hold negative beliefs about the universality of mathematics ability, and that research "showed that these future teachers shared many of the mathematical beliefs held by severely math-anxious people enrolled in math-anxiety clinics" (Franks, 1990 p. 10) is disquieting. More about maths anxiety and schooling follows.

Maths anxiety and school

What are the major factors in schooling that could affect the change from a positive attitude toward mathematics to a negative one? There are the teachers, and the attitudes they bring to their classes. There is much evidence that the development of maths anxiety is to be found in the teaching and in the teachers (Boyd, Foster, Smith & Boyd, 2014), and that teachers teach as they were taught (Anderson, et al., 2005; Grootenboer, 2008; Tooke & Lindstrom, 1998). Consequently it may be hard for teachers as adults to envisage a different mathematics teaching from that they received when they were children (Wood, 2002). That is, if they came from a classroom where there was an emphasis on the rote learning of rules, then that person is more likely to teach in that way (Barkatsas & Malone, 2005; Wilson & Thornton, 2005; Wolodko et al., 2003). Many of the students who were reported in the studies spoke of disassociation in the classroom and about the nature of mathematics, how it was a "hard" subject. Also they talked of how there was a set of rules for finding the right answer, and that there was a right and a wrong way of achieving those answers. It seemed to them that some teachers taught as though they wanted the school students to read the teacher's mind to find the one right answer. Even when the teacher education received had emphasised different and new ways of teaching from that experienced in their own classroom days, many teachers were shown to preserve their basic attitudes towards mathematics from their school days (Boyd et al., 2014; Ingleton & O'Regan, 1998; Wolodko et al., 2003) There is a circularity of maths-anxious students being taught by maths-anxious teachers (Furner & Berman, 2003; Hawera, 2004; Tooke & Lindstrom, 1998), so perhaps something has to happen to break the circle and shift mathematics teaching and learning on to some other more productive path. Studies which take as their focus teacher and pre-service teacher maths anxiety will be scrutinised next.

Some studies have cited the type of instruction received as the major cause of the rise of maths anxiety. Students who had a more traditional-style of instruction were found to be more maths anxious than those whose style of instruction was more like what might be termed reform-based mathematics, an alternative approach (Clute, 1984; Newstead, 1998; Young-Loveridge et al., 2012). A traditional style of instruction could be categorised in the extreme as being bound up with rote learning and symbolic manipulation with little understanding of concepts. There are also aspects of teacher-tell type instruction where there is a preponderance of teacher exposition, often categorised as "chalk and talk", followed by the student doing practice exercises from a text book. This type of instruction with low-mastery aspirations, leads students to see mathematics as a series of rules to be learned and regurgitated at the next test (Skemp, 1979). A consequence of this is that students believe they either can or cannot do mathematics and they have little power to change the situation and affect their own learning (Furner & Duffy, 2002; Levine, 1995; Stodolsky, 1985; Turner et al., 2002). An alternative mode of instruction might be termed a more child-centred and investigational or discovery style. Skemp (1976) calls this relational teaching, where the interconnecting of concepts and ideas is paramount. This would be typified by problem-solving activities and investigations as the basis for classroom work. The teacher might act as a facilitator and the process of doing mathematics would be emphasised over the right method or simply getting the right answer (Levine, 1995; O'Brien, 2010; Turner et al., 2002). Furthermore it has been recognised as a basic tenet of good teaching that, when teaching mathematics, teachers ought to be sensitive to the needs of their students and to adopt teaching strategies which will minimise maths anxiety (Levine, 1995). These would include informing the children as to how the anxiety may have arisen in the first place (Beilock & Maloney, 2015), encouraging the writing about how they feel about mathematics (Wilson, 2013), and involving the students in problem-solving activities which have the effect of creating active learners (Furner & Berman, 2003; O'Brien, 2010).

Maths anxiety and the consequences of mathematics avoidance

The prevalence of mathematics avoidance, due to some disinclination to continue with mathematics education, is indicative of how attitudes towards mathematics education can shape and mould the career and life choices of students at all levels (Faust, Ashcraft, & Fleck, 1996; Zettle & Raines, 2000). There are school students who avoid mathematics by making option choices which remove any further contact with mathematics education going into Year 12. Then there are other school students who limit the number of mathematics courses they choose (Ashcraft, 2002; Ma, 1999). Even where higher level mathematics courses cannot be avoided, such as the obligatory statistics courses which pervade approximately 50% of programmes at many universities in New Zealand, people who are reluctant mathematicians often still manage to restrict further courses where possible (Beilock & Maloney, 2015; Townsend et al., 1999; Uusimaki & Kidman, 2004).

The consequences of mathematics avoidance strategies are apparent early on in a school student's life, in early primary school. These students can be seen to handicap themselves in various ways. They may withdraw from the class, bodily or in their minds, disengaging from the lesson. They often fail to ask for help when it is obviously required, deflecting attention from themselves and their perceived low ability. This can become a self-fulfilling prophesy, bound to succeed, because children who resort to these avoidance tactics are often found to be mathematics drop-outs (Dowker et al., 2016; Turner et al., 2002). These students also resist novel ways to learn; they are disaffected and non-compliant students and are often difficult to motivate in the classroom even when the activities are captivating to other students. So now the question becomes, is this maths avoidance a problem in New Zealand? The evidence that it is may be provided by the pre-service teachers themselves, discussed in the following section.

Pre-service teachers and maths anxiety

The preparation of primary teachers in New Zealand requires them to be ready to teach mathematics as part of their everyday teaching, since in general there is no expert ready to step in and take the mathematics lessons. Though mostly they work in a team or syndicate, primary school teachers, teaching from Year 0 to Year 8, are completely in charge of the day-to-day mathematics education of the students in their classes and so pre-service education has courses designed to enable this teaching. Usually there are two or three courses at different levels in teacher education degree programmes, and they are not only compulsory (as are all other seven subject areas in the national curriculum) but they must be passed before the degree can be conferred. There may also be optional papers offered as a way to lift student teachers' personal content knowledge of the mathematics curriculum and to prepare their epistemological and pedagogical knowledge (Hawera, 2004). There are also graduate pathways for people who already possess a degree, whereby they take a one-year course where again, they must cover all areas of the curriculum.

The requirements of entry to teacher education programmes are the same as any university entrance requirements in New Zealand. This is achieved through the National Certificate for Educational Achievement (NCEA) which is the national qualification offered by the New Zealand Qualifications Authority (NZQA). Concern has been expressed about the ability of pre-service primary students to achieve basic competency in mathematics on teacher education courses. Certainly their entry qualifications leave a great deal of uncertainty as to the likelihood of these people becoming excellent teachers of mathematics. (Cameron & Baker, 2004). The majority of students come in at the beginning of the three-year degree with no mathematics past the Year 11 compulsory mathematics (Level 1 NCEA) (Biddulph, 1999). Older applicants for teacher education programmes need not even have Year 11 mathematics and indeed it was discovered that at least 50% of the applicants had not achieved any formal mathematics qualification at all (Grootenboer, 2003; Young-Loveridge et al., 2012).

In studies to investigate the attitudes that New Zealand pre-service student teachers hold toward mathematics, it was found that more than 50% of students had negative attitudes. (Biddulph, 1999). Others in New Zealand have found similar negative attitudes with what they term apprehension about their ability to teach mathematics (Grootenboer, 2003). The range of reasons given by pre-service student teachers about why they are reluctant to engage and participate in mathematics classes range from seeing mathematics as a maze from which they can never escape (Wolodko et al., 2003) to how they would wake at night in a cold sweat, dreaming that they had been caught for ever in a mathematics nightmare (Mewborn, 2000). Such descriptions are typical anxiety symptoms.

Are pre-service teachers more prone to maths anxiety than other groups of students? In America, pre-service primary student teachers were the group with almost the highest level of maths anxiety; the only higher group were those already in a course to overcome maths anxiety (Kelly & Tomhave, 1987). In a Turkish study of 167 elementary pre-service teachers (Bekdemir, 2010), levels of maths anxiety were described as persistent during their pre-service studies The data suggest that pre-service student teachers are more maths anxious than the general population, which raises the issue, that if this is so, how might it affect their ability to teach mathematics in the primary school?

Maths anxiety and the novice teacher

In New Zealand, a beginning teacher is one whose only teaching experience is likely to have been the practicum encountered through their teacher education period, whether it was a three-year degree or the one-year graduate course. Beginning teachers are given provisional registration for a two-year induction period. The government acknowledges beginning teachers' lack of expertise by providing extra funding that enables schools to reduce the programme of teaching in teachers' first year and provide mentors and support within the school. Mostly beginning teachers have an 80% programme and their extra non-contact time is often used to work with their mentor, to prepare and reflect on their lessons (Cameron & Baker, 2004). There is scant research on this passage from pre-service student to fully fledged teacher, and very little on maths anxiety and the effects on the subsequent mathematics teaching in the primary school.

A high general background stress level has been reported for all teachers over all ages and experience ranges. However, beginning teachers have been reported as suffering from elevated levels of stress in their first term (Grudnoff & Tuck, 2004). If this is so then mixing the high level of general stress with specific anxieties about teaching mathematics could make things particularly difficult for these beginning teachers. The question is how best to help teachers overcome their maths anxiety and become more effective teachers of mathematics. The need to improve the skills of mathematics teachers in the primary schools has been well documented (Bailey, 2014; Carroll, 2005), and teachers who are maths anxious have a tendency to pass this on to their students (Bekdemir, 2010; Furner & Berman, 2003; Williams, 1988). There is little doubt, then, that most primary teachers will exhibit some maths anxiety when they have come from the cohorts of pre-service student teachers who exhibit maths anxiety to a large degree.

What is of utmost interest, however, is how to influence the practice of current teachers to lessen the transmission of maths anxiety to the next generation. Changing these practices is enormously difficult (Bailey, 2014; Furner & Duffy, 2002). What must be taken into consideration is the way that a teacher learns to teach. As noted earlier, teachers have a tendency to teach as they were taught (Grootenboer, 2008), and changing this is problematic; this is partly because most teachers will have experienced one way of being taught mathematics for approximately 11 years. Confronted with the rigours and stresses of teaching in their own classroom, they often abandon new practices discovered (and appreciated) in their teacher education course, and revert to what they know and are comfortable with (Hiebert, Morris, & Glass, 2003). This suggests that it is at the pre-service level that the cycle must be broken. Teachers must stop reproducing the type of teaching that they received as schoolchildren which is likely to have added or caused their own maths anxiety. Then there might be an improvement in teaching outcomes and a reduction in maths anxiety.

Maths anxiety has been looked at from the view of children, pre-service students, and teachers. While it is true that some level of maths anxiety can provide stimulus and impetus (Chaman & Callingham, 2013), for the majority of sufferers it has a debilitating effect. Also discussed here is what can be done to ameliorate this, such as sensitive teaching styles, and changing of mindset. However, maths anxiety has been researched and debated for many decades, and there is little sign of improvement. Having discussed the context of the teacher at some length, now it is the turn of the context of the learner.

Context of the learner

Children are a diverse group. This has been recognised for at least a century (Ballard, 1915). While they may be homogeneous on a small aspect when grouped, they can look different when the lens is shifted to take in a different concept. Further, children, who seem similar today, may look different in a different context, even from morning to afternoon (Alton-Lee, 2003). Other areas within diversity are ethnicity, gender, background, talents or propensities, home language, culture, and religion. Being able to deal with diversity, including developmental level, is a large aspect of being a contextually knowledgeable teacher. Trying to teach each child according to their needs is complex (Alton-Lee, 2003), and requires a great deal of knowledge of sociocultural aspects which may be difficult for the monocultural and mono-racial group that constitutes the teaching population in New Zealand to know and understand. However, the child is not a central part of this thesis and so will not feature further here.

Summary

Quality teaching is a means to an end, that of quality learning and this review has presented the literature on quality teaching using the three "big ideas" in Shulman's (1987) framework: those of the subject knowledgeable teacher, the pedagogically knowledgeable teacher, and the contextually knowledgeable teacher. These three are summarised below.

The subject knowledgeable teacher

The first "big idea" was the specialist mathematical knowledge that teachers need if they are to be effective. The lack of formal qualifications in mathematics that many candidates in initial teacher education bring with them was presented, together with how deep the conceptual roots had to go in order to see the teachable moment, and develop an effective learning trajectory. The instrumental versus relational teaching of Skemp (1979) was used to frame the argument for the type of mathematical knowledge required. Though a definite answer was not reached, it was generally agreed that teaching by rote and being strongly rule-bound or algorithmic would present huge challenges to quality teaching. Finally, the idea of whether a primary teacher, who is expected to teach in eight different curriculum areas, could have the expertise required of them, to teach mathematics well, was considered.

The pedagogically knowledgeable teacher

The second big idea, that of the pedagogically knowledgeable teacher, builds on the first; the subject knowledge mentioned in the first big idea is transformed into pedagogical knowledge that enables deep and powerful representations to be communicated to learners. For this to happen, the teacher must be a versatile thinker (Thomas, 2006) and an adaptive expert (Timperley, 2011). The professional development—the NDP—was considered at length, the seven dimensions of quality teaching identified central ideas, and then the three pedagogical tools of the NDP—the framework, the diagnostic interview and the strategy teaching model—were scrutinised to see how well these dimensions were reflected in others' conceptualisations of quality teaching, such as the 10 dimensions of quality teaching from Alton-Lee's (2003) best evidence synthesis.

The contextually knowledgeable teacher

The third big idea, that of the contextually knowledgeable teacher, gets less of an emphasis than the other two in the original work of Shulman (1987), but the ideas contained in here are probably more important than all those in the first two. This is the big idea of the five

Cs: classroom; collaboration; curriculum; culture; and community. Within the school context, how the teachers work together and whether that is in a spirit of cooperation or competition is central to quality teaching. What is taught, and how it is taught is an obvious inclusion, with the ideas of being culturally responsive to the children. However, the monocultural, monolingual and monoracial group that is the majority of teachers (put together with the skew in gender numbers, predominantly women) is less likely to understand the child's context, but this is the status quo. The final areas discussed under context were beliefs and maths anxiety, and it is hard to say which is the more important when it comes to quality teaching. There is a terrible circularity to maths anxious teachers teaching mathematics to children who will, in turn, develop maths anxiety.

Lastly in this literature review, a re-stating of the research questions and a short exposition of the purpose of this research.

Research Questions

My primary research question which relates to novice teachers teaching in NDP-style was:

How could an instrument be designed to capture the development of quality teaching?

Subsidiary questions were:

- What is the nature of quality teaching evident in the teaching practice of novice teachers over their first two years in the profession?
- How does the teaching of novice teachers change over two-year period of their provisional registration?
- How does initial maths anxiety affect the development of quality teaching?

This literature review has presented previous research and ideas, concerning the nature of quality teaching, examining both the New Zealand context and the wider international mathematics education community. Notions of quality teaching outside of mathematics was also discussed.

Putting all this together, completes the circle and reinforces the proverb-like saying from Williams (1988), which was presented at the end of Chapter 1:

Tell me mathematics and I will forget;

Show me mathematics and I will remember;

Involve me…and I will understand mathematics.

If I understand mathematics, I will be less likely to have maths anxiety.

And if I become a teacher of mathematics, I can thus begin a cycle that will produce less maths-anxious students for generations to come.

W. V. Williams, 1988, p. 101

Chapter 3

Research Methodology

Counting from One by Imaging

This stage is also characterised by students counting all of the objects in simple joining and separating problems. Students at this stage are able to image visual patterns of the objects in their mind and count them.

The Number Framework

Counting all is analogous to ensuring that all the machinations and underlying assumptions have been laid bare for all to see. Children have a very concrete vision of number in this stage, and I have had to stop my journey to make decisions and justify them, seeing all the nuts and bolts of the research laid before me.

This chapter presents the research methodology and method in this investigation into quality teaching. The chapter discusses the methodological approach of case study and mixed qualitative and quantitative methods; provides details of the participants, the instruments used, data collection and methods of data analysis; and discusses ethical considerations. The various data collection methods included face-to-face interviews, questionnaires, videoing and coding of Numeracy Development Projects' (NDP) lessons, and tests of mathematical knowledge for teaching (MKT).

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Methodology

In order to answer these research questions, data on the nature of quality teaching, together with the change found over two years provisional teaching was required, so the methodology used in this study was that of case study involving mixed methods comprising both qualitative and quantitative data. A case study approach was adopted because this form of research design allows for deep understanding to develop about a particular phenomenon, in this case the phenomenon of quality teaching, and allows the voice of the teachers to come to the fore (Yin, 2012). The following sections discuss the reasons for using case study methodology and the rationale for a mixed methods approach.

Case study

Case study design has become increasing popular over the past few decades. Yin (2014) cites three areas that show this increase: research using case study; reference works about case study; and the broad range of disciplines, fields, and professions which involve case study. This growing popularity has seen types of case study described variously as intrinsic or instrumental, single or multiple, holistic or embedded. My study used a multiple case study design. Case study is particularly recommended when low-interference (Gast & Ledford, 2014) is required, or where there is a reluctance to engineer an intervention (Yin, 2014). Used to investigate complex social phenomena (Yin, 2014), the case study approach allows quality teaching to be investigated through the "purposeful sampling" (Merriam, 2002) of participants to highlight and contextualise factors involved in quality teaching. In my study, this allowed "purposeful sampling" of the participants, as the factors that each possessed led to two particular teachers being further studied in depth, resulting in two full case studies, while the other five provided vignettes to offer supplementary understanding about the phenomenon of quality teaching.

Mixed methods

Just as case study methodology has increased in popularity, so research shows an increase in the popularity of mixed methods research in the education field (e.g., Onwuegbuzie, 2004; Silver, 2004; Teddlie &Tashakkori, 2003; Zbiek, 2011) as more evidence-based teaching practice is called for. For example, in a study of methods from 710 mathematics education research articles from 1995 to 2005, it was found that only 29% used mixed methods (Hart, Smith, Swars, & Smith, 2009) compared with 50% using qualitative methods only. It would seem that the mixed methods approach was not as favoured within the mathematics education community at that time. However, since the year 2000, there has been an upsurge in the use of mixed methods, with a reported increase of over 200% in an eight-year period (Ivankova & Kawamura, 2010), and the trend continues in an upward direction (Bergman, 2011) in social and related sciences.

When using mixed methods, as with all methodological paradigms, the intentions of the researcher with regard to the various methods used needs to be clearly stated (Bryman, 2007; Greene & Caracelli, 1997). To this end, Greene and Caracelli (1997) discuss three possible stances the researcher can adopt with regard to mixed methods evaluations: the purist, the dialectical, and the pragmatic. Briefly, the purist maintains that mixed methods research is not possible because of fundamental differences in qualitative and quantitative data being incompatible across paradigms. The dialectical stance is the most complex of the three in that, in adopting mixed methods, the choices made are deliberate, the types of data collection or analyses are chosen to complement each other, and they interact with each other in some synergistic way. The pragmatic stance is one where the separateness of various paradigms is accepted and they remain separate, yet are able to be "mixed and matched" (Greene & Caracelli, 1997, p. 8) to describe the various pieces of data better; being pragmatic is about what works best. A purely qualitative or purely quantitative approach would seem to deny the complexity of human beings. To reveal this complexity and achieve worthwhile insights, requires both qualitative and quantitative tools. The concept of worthwhileness—that any outcomes have importance and gravitas—is what makes research worth doing; this can be achieved with mixed methods research through triangulation.

Triangulation is a term which has long been used to describe the process of validation of data from two or more differing viewpoints. Just as the land surveyor needs at least two fixed points in space to find an unknown point, so the triangulator in mixed methods research uses at least two different phenomena to fix the fluid nature of social research (Mertens & Hesse-Biber, 2012). Fielding (2012) conceptualises triangulation as "convergent validation" (p. 124), highlighting the fact that no social phenomenon can be measured exactly twice, and he cautions about complications inherent in qualitative and quantitative data integration. Integrating what are often different data streams, say Likert-scale results and interview data, can be fraught with difficulties, particularly giving rise to a pseudo-validity of agreement, giving an appearance of convergent triangulation when none exists. This applies especially if this pseudo-validity is seen solely from the perspective of the researcher and does not arise from the subjects of the research (Fielding, 2012).

My study used a separate component design option (Greene & Caracelli, 1997), the ramifications of which are predominantly that all the components are distinct from each other in the data-collection phase and only brought together for analysis at the conclusion of the study, Sale, Lohfield, and Brazil (2002) support this separation of qualitative and quantitative data collection and analysis in their solution to the difficulties in mixed methods design. This means that the separate components are not acting on nor influencing any of the other data collection methods. While it is best if decisions are made before data collection starts, the nature of the research and the changeability of longer term research means that these decisions will need to have flexibility, and be able to change where necessary. This allows each component to be somewhat independent of every other. The multiple sources of evidence allow increased validity when the researcher begins to analyse the data for theme lines. Any similar ideas and themes emerging during the analysis phase are more likely to be reliable and robust, with less contamination of researcher expectations (Teddlie & Tashakkori, 2003). One issue considered in the design process for my study was that of how using mixed methods would give a rounded picture of quality teaching. These researchers and others emphasise the complementarity inherent in mixed methods, leading to results which are more complex and have a deeper meaning (DeCuir-Gunby, Marshall, & McCulloch, 2011).

Relevant literature on the data collection methods

This section presents an overview of the literature regarding the different data collection methods used in this study. Taken in chronological order they were: a measure for maths anxiety (MARS); Mathematics Knowledge for Teaching (MKT_1); Interviewing, videoing of two lessons and the subsequent coding of the data collected from the lessons, and the final MKT_2 test.

Mathematics Anxiety Rating Scale (MARS)

The Mathematics Anxiety Rating Scale (MARS) (Betz, 1978; Richardson & Suinn, 1972) is a 5-point Likert-scaled, 10-item test; with responses ranging from strongly agree at one end to strongly disagree at the other. The MARS has been used in many different situations since its inception in 1972, and assessments of its reliability have generally reported reliability as good (Richardson & Suinn, 1972; Zettle, 2003). Wilson (2013) found it had both high reliability and validity in her study on rural pre-service teachers in Australia. The original 98-point scale was soon amended to contain just the 10 items in the MARS, and indeed, it has been found in other research (e.g., Ashcraft, 2002) that the 10-question scale is as valid as the one containing 98 questions.

MKT$_1$

Each of the three multiple-choice questions on mathematical knowledge for teaching (the MKT$_1$) taken from the question banks at the University of Michigan (Hill & Ball, 2006) used a teaching scenario that had to be analysed. The Michigan group researched the mathematical knowledge needed for teaching by writing and piloting problems which reflected real mathematical situations in classrooms. These range from explaining common mathematical concepts to diagnosing the difficulty students may have regarding a mathematical procedure. The answers to these types of questions are often complex in that some answers are more right than others, just as in real life. The Learning Mathematics for Teaching group has generally sequestered its questions, so the questions chosen in this first part of the research were from the small number of questions they have allowed to be used in other research. The three chosen were close to the types of situations that the participants might have met in NDP-style, or reform mathematics teaching.

Interviewing

Interviews are often used when seeking data about the experiences of others and this was central to this study. The complexity of the stories of the participants can be uncovered by interview and at times the telling of the journey in mathematics can have a cathartic effect, and lead to further opening up to the interviewer (Doody & Noonan, 2013). Interviews may be structured, semi-structured or unstructured, with each having a different set of advantages and disadvantages. The most common of these, in qualitative studies, according to Holloway and Wheeler (2010), is the semi-structured interview, where the fluid nature of the questions allows for a more conversational narrative, out of which arise the rich and complex understanding that people hold. The one-to-one nature of the interview, and the open-ended nature of the questions, can alleviate some of the anxiety that participants might feel on being asked about their thoughts and feelings regarding mathematics.

Previous studies have shown that pre-service primary student teachers often have negative attitudes towards, or lack of confidence about, teaching mathematics (Biddulph, 1999; Grootenboer, 2003; Uusimaki & Nason, 2004). An interview is therefore useful to access some of the attitudes of the respondents with regard to mathematics and mathematics teaching.

Videoing

Capturing the complexity of an NDP-style group session would have been difficult to do by observation alone. The richness of both qualitative and quantitative data would be difficult to note down, even for experienced observers. Videoing allows the repeated viewing of sessions, deeper analysis, and provides the comparisons needed when seeking change over time (Han, 2016). The videoed lessons also become a permanent record of the lesson. Drawbacks of videos mostly relate to privacy issues, and the implicit assumption that they capture all the rich tapestry of the lesson, even though there are aspects of teaching that cannot be captured by video, such as dispositions and beliefs. In addition, the fact that videoing lessons must change the behaviour of both the teachers and the students has to be considered.

Coding of videos

Videos capture a large amount of data and they enable analysis beyond the actual time of the lesson. This large amount of data has the potential to be analysed using a coding system, and perhaps, a data capture sheet. There are coding protocols developed, which range hugely in complexity. For instance, in professional learning development areas, simple coding systems might ask for teachers just to reflect on what they see, and another coding system might specify 15 or so reflections from each video (Tripp & Rich, 2012). If a coding system is very detailed, it might prove very difficult for inexperienced teachers to use. It might also be hard for teachers to code for everything when reflecting on their videod lesson. So coding protocols have been developed both for research such as the Mathematical Quality of Instruction (MQI) study (Hill & Ball, 2006), and for teacher reflections, keeping in mind that inexperienced teachers need a simpler coding system (Prusak, Dye, Graham, & Graser, 2010).

The MQI, a coding system used to capture important aspects of mathematics teaching in the USA, and the one employed in this research, was itself developed from two previous coding protocols: the Reformed Teaching Observation Protocol (RTOP) Sawada & Pilburn, 2000 and the Inside the Classroom Observation and Analytic Protocol (Horizon Research, 2000) both cited in Learning Mathematics for Teaching (2006).

Method

In this section, beginning with the research design, the discussion will centre on how the different tools listed above were actually used in this study.

Research design

The purpose of this study arose from previous research (e.g., Beilock, Gunderson, Ramirez, & Levine, 2010; Frankcom, 2006; Hembree, 1990) into maths anxiety among pre-service and pre-registration primary teachers. In my master' degree I found that many pre-service student teachers left their initial teacher education establishment to become teachers with their maths anxiety unabated. Hence the next question to consider was what happened to these maths-anxious people in their first school position as daily teachers of mathematics, and how were they developing as teachers, particularly with regard to their teaching quality. To address this issue, the current study considered novice teachers' level of maths anxiety, their mathematical knowledge for teaching, and the quality of their teaching,. The question central to the research was whether an instrument could be developed to capture quality teaching. This is addressed in Chapter 4.

The following sections introduce the participants, the research instruments, how the data was analysed, and ethical issues.

Participants

The targeted group of participants were all due to complete their teacher education courses in 2008. These potential participants attended a New Zealand university faculty of education for their pre-service studies in the primary sector. There were approximately 500 such students who were approached to take part in the study. Each of these students was given a named envelope which included information about the purpose of the study, information sheets for participants, a consent form, and a questionnaire (See Appendix A). These envelopes were distributed to each student by their mathematics education lecturer. Six envelopes were not claimed and were returned by the lecturers. The people who completed and returned the paperwork were a self-selected group who originally numbered 28, of whom 26 indicated they were prepared to proceed to the interview stage. This self-selected group is by definition not a representative sample

Of the 28 respondents who began the first phase of the study, seven completed all the research activities. Achieving a larger group of willing participants was not possible despite further distributions of information, and making personal and persuasive contact.

Acquiring a large group for research projects can be problematical especially when it involves mathematics and maths anxiety, areas that many potential participants may shy away from (Frankcom, 2006). It is possible to speculate about some of the specific reasons behind this lack of participants. Previous experience, responses during the interview stage from the previous study (Frankcom, 2006), and discussing the problem with lecturers of these students, suggested that this was a recurring and expected phenomenon. Any study involving anxiety is by its nature likely to result in some maths-anxious people displaying avoidance techniques. The students were informed from the outset that they would be followed into schools and videoed in their classrooms teaching mathematics. This required a great deal of the students and it was entirely reasonable that it would make many diffident or nervous.

Of the original 26 who replied and completed the questionnaire, two did not go into teaching because of pregnancy. This is an ever-present factor associated with the youth and gender of the majority of the respondents, and indeed in the workforce of teachers. Four wanted to continue, and completed the interview stage but they were refused permission to continue with the study by their schools. This was an unexpected outcome, but both schools' boards of trustees who replied to inquiries were sure that the business of their schools was to educate their students, not to be involved in research projects. Two people refused to continue once they had a post in a school, saying they were too busy to join in. This was an expected outcome. Both were apologetic but adamant that being videoed in their classroom was out of the question. They cited work overload and a wish for no exterior distractions from the job of teaching. One person did not continue because she obtained a job too far away for her to take part in the research, and another because she could not get a job. Of the rest of the original 26 respondents, four have not been contactable despite repeated efforts through e-mail, letters, and phone calls. This too was expected, since many students move back to their homes once they have completed their qualifications, or out of student accommodation and into other living situations, often not informing the registry of their new contact details. The seven participants who remained in this study are all female, and of similar ethnic group, that of European. Their ages range from 20 to 40 years.

Procedure

The data were gathered over a two-year period. The student teachers, approximately 500, were approached to take part in the study towards the end of their initial teacher education courses in 2008, and the data collection was completed towards the end of 2010. Table 3.1 outlines the chronology for the study.

Table 3.1
Time Line for Data Gathering and Subjects' Participation

Year	Data gathering	Subjects' participation
2008	First questionnaire (demographics, MARS, MKT_1) and consent forms completed (28)	Students become participants
		Students answer first questionnaire
		Students complete and pass teacher education course
	Students' agreement for interview obtained (26)	Students complete interview
	Students' interviewed (10)	Students find teaching posts
2009	First video recorded (8)	These new teachers approached once they were working
		Principals contacted and consent from them and school boards sought
		These now first-year teachers videoed teaching NPD-type lessons
2010	Second video recorded (7)	These teachers, now nearing the end of their second year teaching, videoed teaching NPD-type lessons
	Second questionnaire (MKT_2) completed (7)	These now second-year teachers answer second questionnaire (MKT_2)

Note: The numbers in parenthesis indicate the numbers of participants who completed that phase.

Research instruments

This section provides an overview of the various research instruments used in the study. These are discussed in the order they were administered: The initial questionnaire was in the envelope distributed to potential participants, and contained demographic questions, the measure of maths anxiety (MARS), and three questions involving situated teaching (the MKT_1) to test their mathematical knowledge for teaching. Next, the participants were interviewed (see Appendix B); then, over two years, their teaching of mathematics was captured on videos; and, last, they completed a second questionnaire, on situated teaching (the MKT_2).

Initial questionnaire

The questionnaire was designed to be multi-purpose. Data were gathered to establish baselines of the respondents' maths anxiety levels, their mathematical content knowledge, and their mathematical knowledge for teaching. Personal demographic items were placed at the beginning of the questionnaire. These included name, gender, and age of respondents, and the level they had reached in their mathematics education at school and university before their mathematics education (teaching) courses.

The questionnaire contained the 10 items of the Mathematics Anxiety Rating Scale (MARS) (Betz, 1978; Richardson & Suinn, 1972), and three multiple-choice situated teaching questions to assess mathematical knowledge for teaching (MKT_1). These two measures are discussed in more detail below.

Mathematics Anxiety Rating Scale (MARS)

A typical question for this scale would be:

> My mind goes blank and I am unable to think clearly when working out mathematics problems.

The data from the MARS comprised a number between 1 and 5 for each question and a total was achieved for each participant by adding together their score for each question. Half of the 10 questions were positively worded and half negatively worded, which meant half of the questions' scores had to be reversed to allow for totalling. This gave a score in the range of 10–50, with scores close to 10 indicating low maths anxiety and a score near to 50, high maths anxiety. The mid-point, which would indicate a neutral stance, was around 30, which would equate to someone answering all 10 questions as "uncertain" (at three marks for each question).

Situated teaching questions: MKT_1

As an example, the final question of the three given was:

> Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4?
> (Mark ONE answer)

(a) Four is an even number, and odd numbers are not divisible by even numbers.

(b) The number 100 is divisible by 4 (and also 1,000, 10,000, etc.).

(c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

(d) It only works when the sum of the last two digits is an even number.

This question implies that each of the possible answers has some validity and so the answers were ranked 1 to 4 for being closest to the divisibility rule. As the multiple-choice answers to these questions are all correct to some degree, but are in a hierarchy of partially correct to more correct, they needed to be scored accordingly. In this question, (b) is the closest to a complete answer and so was scored 4, while (a) scored 1, (c) scored 3 and (d) scored 2.

The results for the three questions were totalled to give a score out of 10 (the questions had variable numbers of answer options).

Interviews

The purpose of the semi-structured interviews in this study was to gather data about the participants' attitudes to mathematics learning and teaching. Where possible, the interviews were conducted before the students completed their teacher education courses, and if this was not possible, the interview was carried out close to the start of their teaching careers. The provocations or questions contained in the protocol of the semi-structured interviews encouraged the pre-service teachers to talk about their experiences as a learner in school, both primary and secondary, together with their experiences at university on their mathematics teacher education courses. They were asked to contrast their learning of mathematics with the way they were encouraged to conceptualise the teaching of mathematics at university, using reform-based mathematics teaching. This meant using materials and manipulatives in conjunction with the strategy teaching model (Hughes, 2002). They speculated on the attributes of an ideal teacher of mathematics, and on what would be going on in an ideal lesson. Finally they were asked how they felt about mathematics. These questions were used to elicit the information that might illuminate the predispositions, beliefs, and understandings preservice teachers held about mathematics teaching.

Videos

The seven pre-registration teachers who completed all phases in this study were videoed teaching mathematics lessons (all in the NDP-style) early in their first year of teaching and again near the end of their second year, before they became eligible for registration. Any use of materials, either by teacher or children, was captured, along with writing, and the voices of the children and the teacher. The videoing was always done from over the back of the children to keep their faces off the film as much as possible, as no ethics permission to film them was sought, and the children's voices, movements of materials, discussions, and so on are used in this research only as they reflect or explain what the teacher is doing.

These videos were coded for teaching attributes such as group work, using the Dynamic Analysis Reflection Tool (DART), the coding system I developed specifically for this purpose. The DART coding system was developed from an NDP exemplar video that demonstrated the features of an excellent numeracy lesson. The experienced teacher on this NDP video teaches eight children for about 40 minutes. Her actions, the way she reflects the children's thinking back to them, and her listening and questioning skills, among many other attributes, were coded, using the Advancing Children's Thinking (ACT) framework (Fraivillig et al., 1999). All the details about the development of DART and its use in this study are found in Chapter 4.

With the aid of a summer scholarship research assistant, I carried out the coding over approximately 2 weeks. Each of us worked independently, watching and coding the video in one-minute segments. In the event of disagreement, a consensus was reached through discussion. This enabled a measure of reliability to be established in the coding with agreement ranging between 80-90% for all codings.

Second questionnaire: MKT_2

The final measure of mathematical knowledge for teaching was given to each participant after the videoing of their final session. This written questionnaire, MKT_2 was completed and returned by post. Participants had no conditions imposed on them as to length of time, and so on, that they may take when answering, other than the written instructions for them to answer naturally and quickly. They were also asked to complete details of the classes they had taught over this pre-registration period.

Data analysis

There is a range of data types collected in this study, from demographic information through the quantitative data of tests of mathematical knowledge for teaching to the qualitative data of interviews, as expected in a mixed methods study. This section discusses the analysis of the various types of data, followed by a critical look at the issues of validity, reliability, and generalisability.

Analysis of interview data

In all the interviews, I followed the semi-structured interview protocol. The interview data were recorded and transcribed. The stimulations in the protocol followed a similar pattern, in roughly the same order, depending on how the narrative or discussion progressed. These questions were deliberate choices, in an appropriate order, to enable what was thought would be a large number of usable transcripts to be analysed. However, only the seven transcripts of the participants who completed all phases of the research were analysed. This reduced the necessity of using a computer control system such as QSR NVIVO 9 data management program, which was the original intent.

In many studies, coding is a somewhat ad hoc process where researchers will read the transcripts and look for patterns without any particular categories in mind at the start. This process is deceptively simple, but there are many pitfalls for the unwary user of such ad hoc methods. Critics of the qualitative woolliness that can result from loose coding (e.g., Antaki, Billig, Edwards, & Potter, 2002) argue for the need for more rigour in discourse analysis. They criticise the propensity for human failure during the coding phase. For instance, they point to the partiality of the researcher as a large obstacle, because the coding could be biased and only those themes that fit with the researcher’s expectation might be included. They also cite the under-analysis inherent in the researcher who presents many quotations but little synthesis when coding.

The interview data were coded using some of the concepts contained in thematic analysis (Boyatzis, 1998; Braun & Clarke, 2006; Fereday & Muir-Cochrane, 2006) because this theoretical approach fitted well with my wish for both flexibility and rigour. The process of thematic analysis has been gaining acceptance in the coding of qualitative data since the early 1990s and its strength is the explicit way the data is used to give rise to themes. It is the embodiment of the process that many researchers had already been using, allowing a purely data-driven approach: areas of interest are suggested, which then causes the researcher to look for more instances of that theme. Making the coding explicit allows the data to be more rigorously mined (Fereday & Muir-Cochrane, 2006) for themes that develop from a theory-driven approach.

Analysis of video data

All videos were encoded using the DART coding system (explained in Chapter 4) by two parallel encoders, the summer scholarship student and me. Issues of reliability and researcher ratings were to the fore in this process, mostly because the DART is a new tool and lacked any external validity at this time. Applying the DART coding required us to watch one-minute sections of the video many times—perhaps as many as 10. Any codes which differed were minutely examined, and a compromise reached. Surprisingly, there were few instances where the codes differed wildly. One such would be where one of us saw the teacher talking to the group, and the other saw the teacher talking to an individual. That minute was watched and we discussed what we were seeing. Once this was actioned, all codings were agreed.

Analysis of questionnaires and other data

This is the final section regarding forms of analysis for the data in this study, and discusses the MARS, the MKT questions (MKT_1 and MKT_2), and demographic data.

Participants marked their responses to the 10 questions in the MARS on a five-point Likert-type scale, as described earlier. The score thus produced would be high for those exhibiting high maths anxiety—these would be results close to 50—and low for those exhibiting low maths anxiety—results close to 10.

MKT_1 comprised three questions set in a teaching context, as described earlier. These questions were answered at the same time as the consent forms and the MARS questionnaire were completed. The answers to the MKT_1 were marked and numbers scored, added, and then converted to a percentage.

The results from $MKT_{2,}$ the final activity for participants in this study, were marked according to the answers provided by the Learning Mathematics for Teaching group (2005). Participants' scores were added and turned into a percentage.

The demographic data was used to find commonalities within and between participants. For instance, the level of mathematics education prior to their university mathematics education courses was considered as the case studies and vignettes unfolded.

Ethical considerations

Before any research was carried out, approval was sought and was gained from the University of Auckland Human Participants Research Committee (2007), reference 2007/158, on 27 May, 2007 (See appendix A). Permission was first requested from within the university faculty from the dean for site access and assurances to participants that agreeing, or not, to take part in the study would not affect their studies or results in any way. As the population of possible participants were all students and potentially a student of the researcher, it was vital to reassure everyone that their best interests would not be compromised. These issues will be examined here.

Students were sent details of the study, a participant's information sheet and a consent form to complete in the envelope which also contained the first questionnaire. All those who returned the questionnaire completed the consent form too. Consent was voluntary, as was continuation in the project. All participants agreed to be audio-recorded during the interview and to be videoed while teaching mathematics lessons in their future employment in school. They were assured that they could withdraw at any time and that their data could also be withdrawn up to the conclusion of the data collection phase. It was also decided not to analyse, total, or in any way interact with the results of the MARS until after the analysis of the interviews and coding of the videos was complete, so that knowledge of maths anxiety levels would not influence or bias the researcher during videoing and during video-coding.

Once participants were teaching in a school, their principal was sent a principal information sheet and consent form. When this was completed, access to the classroom was requested and mutually acceptable dates and times were agreed between the teacher and researcher for videoing of mathematics lessons. Schools are not identified.

No individual consent forms were given to the children in the classes being videoed because they were not the subject of the video. Their interactions with their teacher were not the principle concern of the study. There was little analysis applied to their interactions with the teacher which could have been attributed to individuals. These were captured in a general way, with no names attached. Any videoing of students was viewed as incidental, with the researcher avoiding videoing faces. All the principals of the schools consented to this.

The transcriber, who transcribed the interviews and the audio signal on the videos, signed a confidentiality agreement, as did the summer research student who coded the videos using the DART framework (See Appendix C).

Summary

In this chapter the methodology used in this study, including a description of the methodological design, the method and the research instruments, were presented. Also the participants, the procedure, and the data analysis were detailed. Finally, ethical considerations were examined.

Chapter 4

Results 1: Developing DART,

Dynamic Analysis Reflection Tool

Advanced Additive-Early Multiplicative

Students at this stage are learning to choose appropriately from a repertoire of part-whole strategies to solve and estimate the answers to addition and subtraction problems. They see numbers as whole units in themselves but also understand that "nested" within these units is a range of possibilities for subdivision and recombining.

The Number Framework

In this stage children know that a number holds much information, developing the DART was very like this idea of understanding that codes and symbols hold important, vital information. These codes are a recombination of the information which led to DART, the repository of quality teaching change.

This chapter details the development of the Dynamic Analysis Reflection Tool (DART) Framework. The development of this tool grew from a need to code for quality teaching in NDP-style lessons when it had become apparent there was no suitable tool available that could capture all the nuances of NDP-style teaching.

I knew from the beginning that this study would require a system for coding videos that would capture quality teaching in the classroom. The Mathematical Quality of Instruction instrument (MQI), an instrument from the Learning Mathematics for Teaching (LMT) project (Hill & Ball, 2006), looked to be a suitable video coding system, and I travelled to America for training on how to use it. However, when I started using the MQI instrument for my study, it soon became apparent it was not a good match for NDP-style lessons as it was designed specifically to capture quality teaching in whole class teaching, and was not sensitive to the characteristics of NDP-style teaching. This chapter addresses the main research question:

Research Questions

My primary research question which relates to novice teachers teaching in NDP-style was:

How could an instrument be designed to capture the development of quality teaching?

This chapter starts with a brief discussion of the features, development, and uses of the MQI. Unfortunately, the MQI proved inappropriate for coding the videos in my study, which led to the development of the DART framework, outlined in the second section. The final section analyses the uses of the DART framework and discusses the fidelity of the instrument.

Mathematical Quality of Instruction instrument (MQI)

The coding instrument Mathematical Quality of Instruction (MQI), was the intended coding tool for the teacher videos in this study. The MQI had been developed by researchers at University of Michigan and Harvard University to measure the work that teachers do in classrooms when teaching children mathematics (Hill & Ball, 2006). The Learning Mathematics for Teaching (LMT) project's video codes (Hill & Ball, 2006) were originally designed by the LMT group for American schools, and whole-class mathematics lessons taught there, to investigate quality teaching in mathematics.

All the lesson videos in my New Zealand study followed the NDP-style, and the teachers all adopted the same pattern of using small groups of five to seven students during their lessons. Generally this meant that in a one-lesson period, three or four groups, in sequence, would be the focus of the teacher's attention, while the other students all worked on pre-arranged activities.

It was only when the MQI was used to code the first of the videos in this study that the inappropriateness of the MQI became apparent. It was discovered that whole pages of MQI had few coding entries and some none at all. In addition, the MQI did not code for the behaviours apparent in a typical reform-based or NDP classroom lesson. One reason may be that MQI was perhaps too generalised, and did not capture important aspects of a reform-based or NDP-style lesson, such as who was manipulating materials, who was speaking, and so on. The discovery that MQI was unsuitable for the types of lessons being taught in numeracy classes in primary schools in New Zealand meant another framework was needed. The first idea was to supplement the MQI with pages that would capture the essential data. However this led to an even more complex and thoroughly unwieldy tool, which lead to the development of the DART Framework. In the following section the various coding parts of the DART framework will be discussed. DART consists of two pages for coding (referred to as page 1 and page 2), with an additional insert (an extension of page 1) if coding for lessons longer than 10 minutes is required. Each of the two pages which comprise DART will be discussed separately in the following sections.

Features of DART page 1

The first feature of DART is its name, Dynamic Analysis Reflection Tool. It has been named thus because any analysis of teaching is about the movement and actions within the lesson and it was planned that the lesson analysis would be carried out using this data-capture form in real time. In this study, it has been used to code important aspects related to quality teaching in NDP-style lessons. Outside of this research, it is hoped that it could be used by teachers to analyse their own teaching, and here the reflection tool aspect of the name becomes even more appropriate.

Part of the development of DART was deciding on the unit of time to be used for the coding form. In MQI, coding was originally in 10-minute groups, but moved to five-minute groups once the coders realised they could not keep in mind everything that happened in that period. It was a pragmatic choice for the researchers developing MQI to settle on five minutes for coding a possible 60-minute lesson. Others have gone to the other end of the time continuum, and used a 15 second interval (Alton-Lee & Nuthall, 1992) Typically, NDP-style sessions would be 10–15 minutes in duration and within each minute there would be much to code for. When, in the development phase, I tried to code the NDP exemplary teacher video using the DART, one-minute groups worked best, and so a pragmatic decision was made that in DART, a one-minute group would be used.

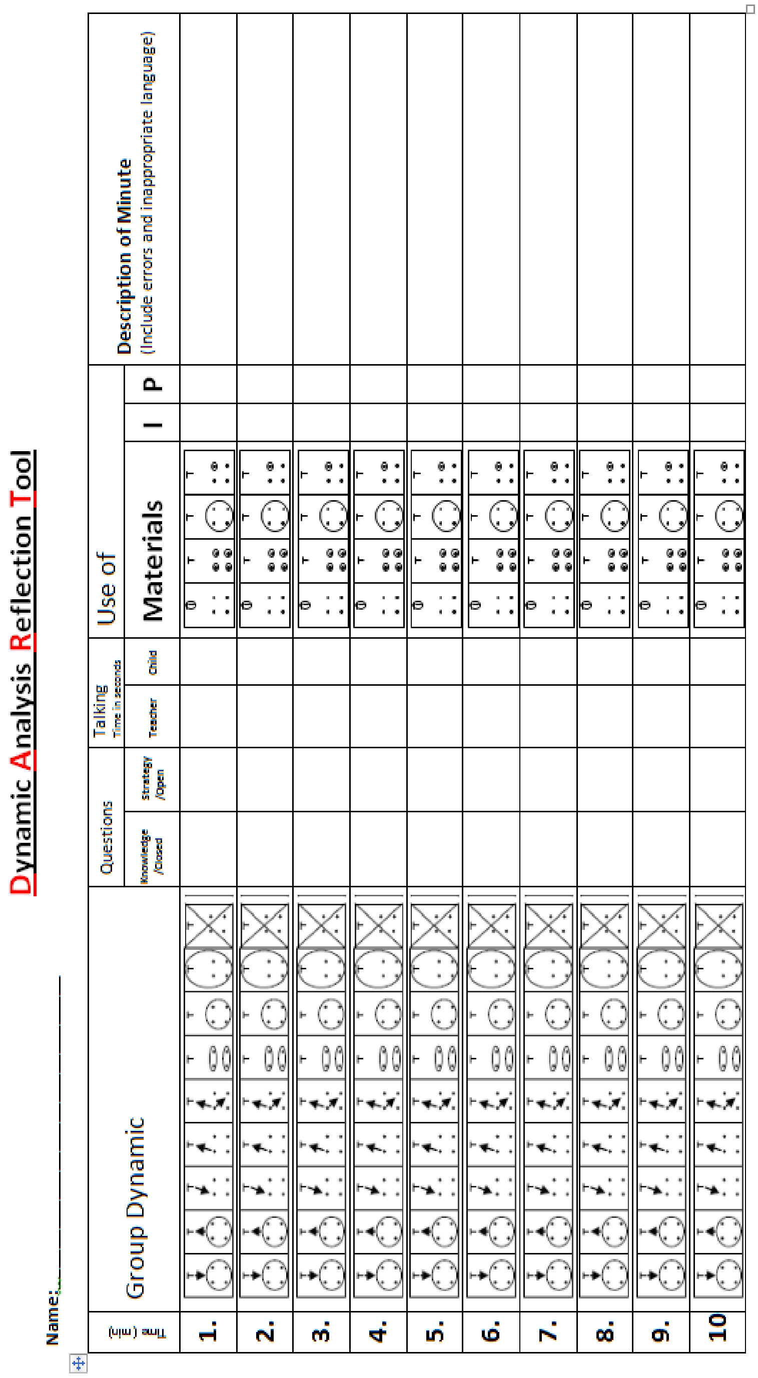
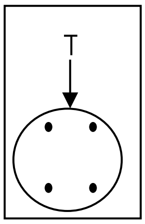


Figure 4.1. The first page of the DART lesson-coding instrument.

It can be seen from Figure 4.1 that each minute of the videoed lesson has the following aspects to be coded horizontally: group dynamic; questions; talking; use of materials, including columns for imaging (I) and number properties and description of that minute. Each of these will be discussed in turn below. Some of the coding sections are designed to be coded in real time, as the video shows the actions, and others are intended to be coded after repeated viewings of the video, such as the timings of teacher talk.

Group dynamic



The following will explain the development and uses of the DART icons—the small diagrams that show the dynamic of the group at any particular time. One example of these are the G-icons (an example is shown here) which represent the teacher and a group of children. The "T" in the icon is the teacher and the four dots represent a group of children. These are held in a rectangular outline and arrows and circles are added to represent all the actions taking place. There are nine G-icons in the group dynamic section and these are displayed horizontally on page 1 and repeated 10 times vertically down the page (see Figure 4.1).

The ways of working in an NDP-style lesson are both prescribed and varied. The different configurations of teacher and children that occur, from the teacher setting the scene to children manipulating materials, are all likely to happen in one session. These ways of working, though varied, are predictable, and hence nine different G-icons were developed to capture the differing configurations (see Table 4.1). For example, it is expected that the teacher will talk to the whole group at the beginning of the lesson, setting the scene and explaining what is going to happen, or asking questions. The icon labelled G-1 (see Table 4.1) was developed to capture this action. In this, the teacher is represented by the "T" and the arrow indicates that she is talking to the whole group. The four dots represent the group of children, who could number more or less than four since these dots are intended to be a general representation of a group. The circle shows they are all involved in the same action; in this case listening or paying attention to the teacher.

Table 4.1
G-icons and Their Meanings and Use in DART

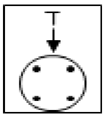
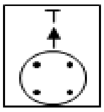
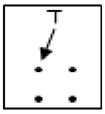
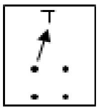
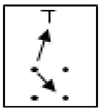
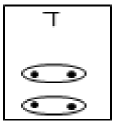
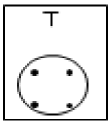
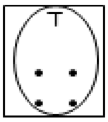
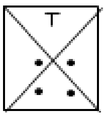
Dynamic Analysis Reflection Tool (DART)				
Icons	**Diagram**	**Label**	**Explanation**	**Example**
G-1		Teacher-Group	Teacher talks to the whole group	Today we will be looking at addition of two two-digit numbers
G-2		Group-Teacher	The whole group interacts with the teacher	Group agrees they all know one and two digit numbers
G-3		Teacher-Student	Teacher talks to a student	Teacher checks a student who may seem insecure
G-4		Student-teacher	Student talks to teacher	Student responds to teacher re above
G-5		Student-group	Student takes centre stage	Student talks or demonstrates to everyone
G-6		Sets working	Students work in pairs or similar without teacher input	Two or more students work on aspect of lesson, teacher listens and observes
G-7		Group working	Students work together, cooperatively	Whole group discuss and problem-solve together teacher listens and observes
G-8		Altogether working	Teacher and students engage in common task	Everyone engaged in a task, basic facts, counting down, etc.
G-9		Nobody working	Teacher and students not on task	Possibly interrupted by school admin, or students outside group

Figure 4.2 shows a close-up of the nine G-icons and the order they appear in the horizontal strip of icons used in DART to code for actions that take place in an NDP-style lesson.

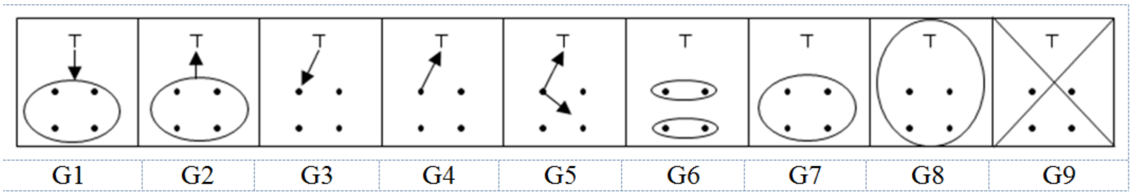


Figure 4.2. The line of G-icons in a horizontal strip, as seen on page 1 of DART.

The coder uses the G-icons to capture the different group actions taking place during each minute of the lesson by shading the appropriate icon using a highlighter pen and after each minute, the coder moves on to the next row.

The placement of the G-icons in the order seen in Figure 4.2 was decided upon after piloting the icons with videos of real classroom interactions. It was found that the actions to the left (or lower numbered G-icons) are more likely to take place early in the lesson, and the other icons are all likely to follow as the lesson progresses (with G-9 used only when no mathematics is evident). Although some moving backwards and forwards between icon representations is expected, this order minimises the extent of the movement. The coder is able to capture multiple actions in each minute in the group dynamic section by highlighting more than one icon. After the teacher has initiated the session, the group may ask for clarification, or respond to questions, represented by G-2. In turn, the third and fourth icons, G-3 and G-4, are about individual communication between a teacher and student, which is likely to occur as clarification is given by the teacher or is sought by individuals. Next, G-5 represents the time in an NDP class when a student might take centre stage, demonstrating a solution path, or a strategy, or perhaps giving information to the group and the teacher. G-6 denotes when students are working in pairs, or individually, and G-7 shows children working as a whole group. G-8 codes for when teacher and students are all working together on some common task and G-9 is a catch-all icon which is used when the teacher and group are not directly engaged in mathematics.

Questions

The questions section of DART, seen in Figure 4.1, is a teacher-only section since knowledge about the questions the students ask was not being sought in this study. This section has sub-headings dividing the questions into two types: knowledge/closed and strategy/open. In each minute the teacher may ask a number of questions of the students, which are counted using a tally system. Questions which are really instructions, such as “Have you got your pencil?”, and questions which are capable of being answered with a one-word answer, such as “Three and what makes ten?”, were considered to be knowledge questions. The type of questions which ask for explanation, elucidation or multiple solution paths would be considered strategy or open questions and tallied accordingly. An example of one of these might be: “John says ‘I know 5 and 6 is 5 + 5 + 1 and as 5+5 is 10 then the answer must be 11’. How could you use John’s method to work out 7 plus 8?” Another might be “Here are 20 counters. Using these counters, find as many pairs of numbers as you can that add to 20”.

Talking

The talking section of DART (Figure 4.1) has a coding place for the total teacher talk time and a place for the students’ talk time. Using a stop watch, the coder accumulates the time a teacher talks during each minute, and records the number of seconds. If the students talk, their time is also accumulated and noted. In an NDP lesson, the amount of teacher talk is expected to change as the lesson progresses; for example, teacher talk time might increase as the session is concluded. The children’s talk time, too, will change according to the task.

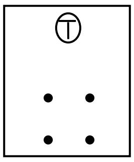
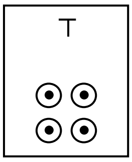
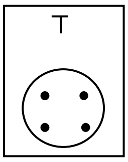
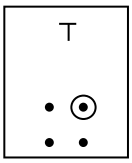
Use of materials, imaging, and number properties

The next section to the right on the DART page 1 is used for coding the stage from the NDP strategy teaching model (Hughes, 2002) being used in this lesson. This section is coding for use of material or imaging or working in number properties and refers explicitly to the ways of working in NDP lessons.

Materials

When introducing a new concept or stage, a teacher would usually begin with concrete materials, move on to imaging (I) and finish at the number properties (P) stage, although these will not necessarily be evident in every lesson. These shifts, in practice, are coded under the materials section using the M-icons shown in Table 4.5. There are four M-icons to represent the different configurations for using materials in sessions. As with the G-icons, “T” represents the teacher and teacher actions, and the students are represented by the group of four dots.

Table 4.2
Showing the M-icons, Their Meaning and Their Usage

Dynamic Analysis Reflection Tool (DART)				
Icons	**Diagram**	**Label**	**Explanation**	**Example**
M-1		Teacher materials	Teacher manipulates the materials	Please bundle these 27 sticks using the rubber bands into groups of 10 and see how many are left over, like this
M-2		Student materials Individual	Students work on own set of materials	Individual students bundle the sticks using the rubber bands and count those left over
M-3		Students materials shared	Students work together as a group on materials	Students work together cooperatively bundling the one set of 27 sticks into bundles of 10
M-4		Student demonstrating materials	Student showing the group a strategy or method	One student uses the sticks to show how to bundle into tens using rubber bands to show others in group

Once again, these four M-icons are arranged horizontally for each minute (see Figure 4.2). The M-icon 1 (M-1) represents the teacher manipulating the material and this is shown by a circle around the T. M-2 shows children using their own set of materials, as the circles around each individual shows. Similarly, M-3 indicates the children all working together on one set of the materials and the fourth, M-4, shows one student using the materials, demonstrating to the group, usually encouraged by the teacher who want to showcase a particular or desirable strategy.

By highlighting the appropriate M-icon, the video is coded for use of materials in each minute.

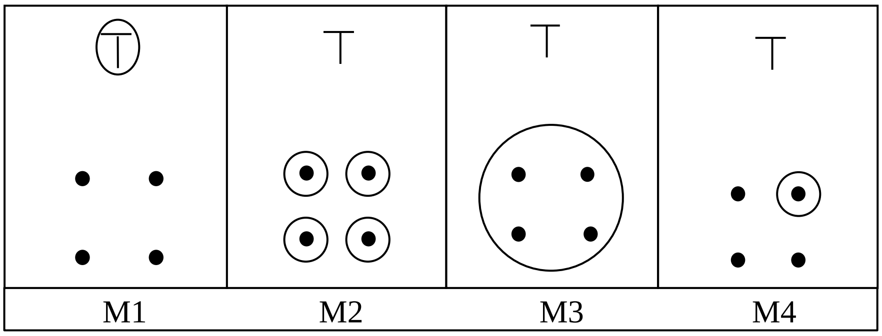


Figure 4.3. The 4 M-icons in the order they are placed in DART.

These M-icons are arranged from left to right because this arrangement is more likely to follow the NDP model lesson. Trialling of the DART showed that of all the M-icons, M-2 is the most likely to be seen as, and indeed is, more desirable, because within NDP, students are encouraged to manipulate the materials themselves to begin concept development. It may be that in some classrooms one may see all the students working together with one lot of equipment if the equipment is large or scarce in that classroom.

I and P

The two columns in DART headed "I" and "P" stand for imaging and number properties. The coder uses a highlighter to show whether either of these strategies is being used. If a particular minute of a session includes asking students to image materials, then that sector is highlighted for that minute, and this continues for as long as the students are imaging. When number properties are being introduced by the teacher and accessed by the students, this section would be highlighted in a similar way to the "I" section. An example that would signify that students were working in number properties would be the use of higher-magnitude numbers than were used previously, and a non-reliance on the materials or imaging.

Description of minute

The final section on the right of the first page of the DART coding system (Figure 4.7) is provided to code any action that needs noting which has not already been encoded earlier on the line. It is a narrative coding and the coder will try to capture things such as what magnitude or type of numbers are being used, how the teacher may be changing between materials and imaging, or student actions. Examples that may be written here include features such as high excitement from the students or perhaps a change of learning trajectory occasioned by the student's misconceptions. Also any teacher actions that were notable would be inscribed here; for instance, if a teacher failed to address a student misconception.

Thus this first page has the capacity to encode 10 minutes of video time, which in the NDP is a probable amount of time for one group. Should that group lesson last longer than 10 minutes, there is a supplementary page on which to continue the coding.

Features of DART page 2

The information gathered on the second page of DART (Figure 4.4) is no longer bound by the minute-by-minute coding of the first page. Instead this page is provided for one-off data capture and the holistic judgement of the coder. There are two parts to the second page: the first (Part A) is the general information-gathering section related to the content and structure of the NDP lesson; the second (Part B) is for coding teacher behaviour, using Fraivillig et al.'s (1999) Advancing Children's Thinking (ACT) framework.

Numeracy Lesson

	Book No	Page No	WALT	Prior Knowledge Check	Context	Number Choices	Materials
Book							
Lesson							

Instructional components of ACT Framework								
Eliciting (Solution Methods)	✓	**When**	**Supporting (Conceptual Understanding)**	✓	**When**	**Extending (Mathematical Thinking)**	✓	**When**
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods			Provides background knowledge			Encourages mathematical reflection		
Encourages elaboration of students' responses			Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts			Assists individual students in clarifying their own solution methods			Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking			Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement			Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed			Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions					
			Encourages students to request assistance (only when needed)					

Figure 4.4. The second page of DART.

	Book Number	Page Number	WALT	Prior knowledge check	Context	Number choices	Materials
Book							
Lesson							

Figure 4.5. Part A on page 2 DART, which captures the NDP lesson information.

Part A, seen in Figure 4.5, captures information about the source of the lesson (the "book" line) and records what happens in the lesson (the "lesson" line).

In the book line, the first two boxes are used to record the NDP book from which the lesson may have come and the page number of the lesson. Not all NDP-style lessons come from these books but many do. If the lesson is from a different source and it is known to the coder, it could also be noted in these two boxes. The next box is for the WALT, which is shorthand for "What we are learning today", also known as a learning intention or learning objective. This is shared with the students and often written in the recording or modelling book. The final four boxes in the book line are strongly recommended ways of working and are also contained within each lesson in NDP books. First, the prior knowledge check, is used to ensure that the group in front of the teacher is ready for the lesson about to start. Next, a context is seen as vital to understanding in mathematics and indeed the 2007 curriculum states before every topic and at all year levels that the mathematics being taught should be set in a "range of meaningful contexts" (Ministry of Education, 2007). The next box is used to note the choice of numbers as, in the NDP books, recommendations are given about suitable numbers for use at a particular stage, and we want to know if these are being followed. The final box, materials, is used to list the suitable materials for the lesson, since the intended lessons from NDP books often have more than one material that would suit the work at hand.

The second line, the lesson line, is used to record what is enacted during the lesson. The coder would note any WALT statements used in the lesson, any knowledge check, and what, if any, context is applied, the types of numbers being chosen to exemplify the new concept being learnt, and the name or description of any materials being used.

Instructional components of ACT Framework								
Eliciting **(Solution Methods)**	✓	When	**Supporting** **(Conceptual Understanding)**	✓	When	**Extending** **(Mathematical Thinking)**	✓	When
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods			Provides background knowledge			Encourages mathematical reflection		
Encourages elaboration of students' responses			Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts			Assists individual students in clarifying their own solution methods			Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking			Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement			Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed			Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions					
			Encourages students to request assistance (only when needed)					

Figure 4.6. Part B on page 2 of DART, for capturing teacher actions from ACT

This encoding of the table should make apparent any differences between the recommendations of the NDP book and the enacted lesson.

The second table on page 2 of the DART framework (Part B) shown in Figure 4.6 uses most of the strategies that appear in Fraivillig et al.'s (1999) work on developing children's conceptual understanding—the Advancing Children's Thinking (ACT) Framework. This work is discussed next, along with the reasons some of their ideas were considered superfluous to the needs of my study.

The work of Fraivillig et al. (1999) followed the reform movement which emanated from the mathematics standards of the National Council of Teachers of Mathematics (1989, 1991) in the United States. This reform movement was concerned with the change in pedagogy from the traditional classroom where teachers instructed and students practised, to a student-centred classroom where the work focused on helping students develop personally meaningful understanding of mathematical concepts. In their ACT framework that arose from the research, Fraivillig et al. (1999) used the actions they observed in lessons conducted by excellent teachers to inform them as to important teacher work. In particular they observed excellent teachers in minute detail and categorised teacher work in the classroom into three areas where they interacted to facilitate children's thinking. These they labelled *eliciting*, *supporting*, and *extending*. Under each heading they teased out the various behaviours they saw when they observed the teachers in their study. The diagram in Figure 4.6 encapsulates the successful teaching of mathematics in the ACT study and it makes apparent the strategies the researchers saw between the three areas of eliciting, supporting, and extending, which were viewed as overlapping and complementary but essentially separate teacher actions.

These intersections and others were gathered by Fraivillig et al. in a table (1999, p. 155) which show examples of teacher strategies and behaviours. It was the ACT table that enabled the items in table two of DART (Figure 4.6) to be developed. The components of those ACT behavours that were not included in the DART table are the three headings, since the strategies and behaviours included already reflect their content and therefore made them redundant. The three within the body that were left out were "Promotes collaborative problem solving", "Records symbolic representation of each solution methods on the chalkboard", and "Encourages students to consider and discuss interrelationships among concepts". When the NDP exemplary teacher video (Hughes,

2002) was watched and coded against the ACT original table, these three strategies were not seen explicitly in the lessons as separate entities. Instead, they were embedded throughout the lesson and so, as they were built into the lesson style, it was decided these were surplus to requirements and were excluded.

How the DART framework was developed

The content and nature of the DART framework has been established; I now turn to the way DART was developed. After viewing the NDP exemplary teacher video, I realised that all the actions of ACT teacher strategies and behavours seem to be exemplified by the teacher. I started to think about how to code for NDP use of materials and other teacher and student behaviours. I started to draw icons to represent all the teacher and student actions that might be seen in an NDP-style classroom. At this time I had a summer research assistant and I used her to code with me, as a way of ensuring whatever was being coded for, it was being seen by both of us. This research assistant was an invaluable part of my development phase, her enthusiasm and pleasure in taking part in this development was infectious, and her efforts at coding all the participant videos is very much appreciated.

Having drawn up the icons into a grid, similar to the final version, we both coded the NDP exemplary teacher video, and afterwards we compared our codings. There was a high degree of agreement, and where our coding differed, we re-ran that minute and agreed on the coding for what we were seeing. This process took months of development, where, after discussions, an unused icon might be delated, or where a particular teacher action was first seen, another icon might have to take its place. Icons were reordered to highlight where teacher centred teaching and child-centred teaching could be easily viewed on the coded document. So the exemplary teacher video propelled me to develop DART, and it became the video that assured me that the icons were coding for important actions.

The videos were viewed at the same time, by both coders but coded on our own. There was multiple viewing of each minute on the video, to code for page 1. First, the “group dynamic” G-icons were coded. As an action was viewed, it was coded for on the sheet by highlighting the appropriate icons, with multiple icons being highlighted in any one minute. In the same minute, the M-icons were also coded using highlighting in the areas “use of” columns of “materials”, “I”, and “P”. Then the minute was re-run to listen explicitly to the number and types of questions being asked, and to code for these. There are two types of relevant questions here: those that are, broadly, knowledge or closed, and those that are strategy or open. The numerical data captured were a simple tally count, shown here as a number. After this coding for questions, the video was run again to capture the time the teacher talked and the time students talked; the stopwatch started when the teacher began talking, paused when the teacher stopped, started when the teacher began again, and so on, for the whole minute. Thus this accumulated the time the teacher spoke in any 60 second interval was gathered. Then the children’s talk, taken altogether, was timed in a similar fashion, with all the children coded together. At the end of all the re-runs of the video minute, the final column, “description of minute” at the far right, was completed. At this point, we knew the minute very well, having viewed it at least four times, and were able to complete this column without further re-runs.

The NDP exemplary teacher video was 38 minutes long; the coding of DART page 1 of the first 10 minutes is shown in Figure 4.7.

Dynamic Analysis Reflection Tool

Name: Expert Teacher: A lesson using the Numeracy Teaching Model

Time (min)	Group Dynamic	Questions: Knowledge /Closed	Questions: Strategy /Open	Talking (Time in seconds): Teacher	Talking (Time in seconds): Child	Use of Materials	I	P	**Description of Minute** (Include errors and inappropriate language)
1.			1	3 8	23				T: Setting the scene, affirming students in what they can do and sets up the challenge, "Who Would...."
2.			1	33	23				Personalises the problem "Why would you ..." Introduced the large number problem
3.		2		49	7				Cost of plants problem, counting back too hard for 143-98. Checks for prior knowledge, hands on knees those who know
4.		6		45	3				T: sets up the 10's frames, look but don't touch. Non-verbal responses from students
5.		6		43	11				Everyone practices subtraction from 10, showing answers on hands Knowledge questions, a warm up
6.		2		45	10				Rules of ways of working in groups discussed, elicited from students, students move into groups
7.			3	40	17				Open questions about working in groups. Teacher repeats or clarifies answers
8.		3		56	3				More instructions from T, having set the scene and completed the knowledge check. Goals made explicit, success criteria defined
9.			1	56	4				10's frame and coloured counters. Everyone working on 13-7 in their thinking groups. They explain their thinking to each other
10		1		6	46				Continues on 13-7 problem, students describing and justifying their answers, still within their thinking groups

Figure 4.7. The coding of the first 10 minutes of the NDP exemplary teacher video.

The coding of the first 10 minutes of the exemplary teacher video (Figure 4.7) shows the setting of the scene with the students. This was an authentic context that any of these children could have experienced at the weekend. The teacher knew her class and the children's backgrounds well enough to say that the buying of plants was a weekend activity in which they could have been involved. These 10 minutes take the class from the authentic set-up, though using tens frames to help understand the beginnings of part–whole subtraction, to beginning to image the subtractions instead of using the materials of tens frames.

The coding can be read holistically—what is the bigger picture, what are the overarching themes of the lesson—or, much more, on the micro level, what is actually happening with the materials for instance, how are they being manipulated. Also the coding form is designed to be read either horizontally, minute by minute, or vertically, to see how things are changing. Holistically, for instance, it can be seen from the shading of the G-icons that the teacher is heavily involved in running and organising this 10-minute stage of the lesson. Looking vertically, it is not until the tenth minute that she no longer features as talking for at least some of the minute to the whole group. Looking horizontally, it can be seen that in the tenth minute she speaks for a single digit number of seconds whereas in all the preceding minutes she has spoken for more than half of the sixty seconds. The M-icons show that materials are being used by the teacher with little student involvement, and the "descriptions of minute" narrative provides the reason for the teacher manipulating the materials: the teacher wants the children to imagine scooping away the counters which are to be subtracted.

The tally of questions asked can also be looked at minute by minute or vertically to see how these types and numbers of types change in the first 10 minutes. There are 20 knowledge questions asked against six strategy questions. At another point in the lesson, this may change markedly. The "time talking" sections show that there is a lot of student talk as well as teacher talk in the first 10 minutes. The ratio of teacher to student talk can also be calculated and in these first 10 minutes the teacher talked for over two-thirds of the time and the students for roughly a quarter of the time available. Investigating each individual minute can be illuminating for a teacher, especially when times, such as minute 8 shows, that this teacher talked for nearly the whole of the time. The narrative from "description of minute" shows minute 8 was an instruction from teacher time.

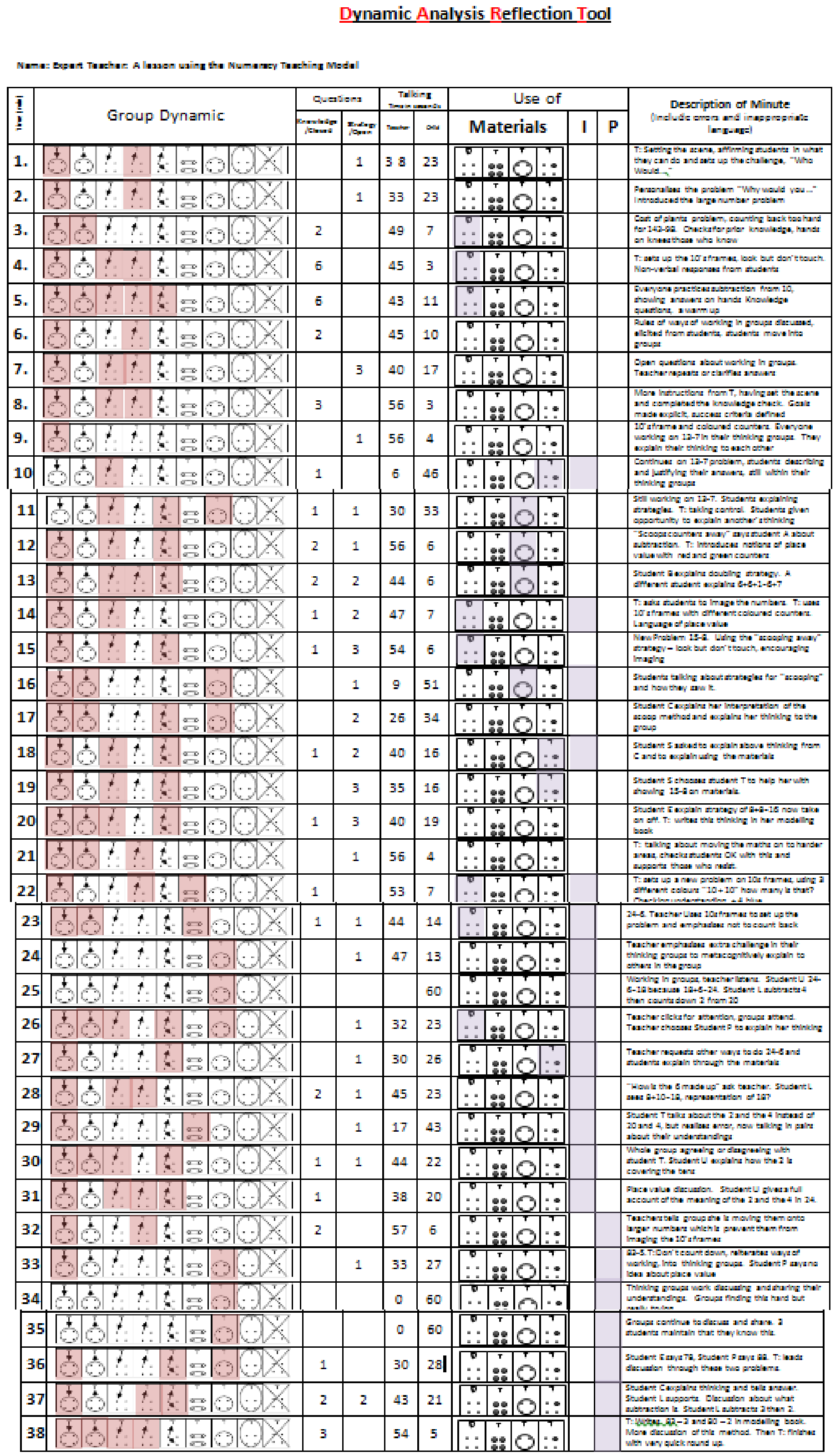


Figure 4.8. The coding of the whole lesson of the NDP exemplary teacher video, to be examined holistically, not read closely.

Figure 4.8 provides a representation of the whole 38-minute exemplary teacher lesson. Under “group dynamic” there are some unused icons. Icons G-8 and G-9 were not used at all: G-8 would be used when the teacher and the students were all working together on the same thing, something not seen in this lesson, and G-9 was not highlighted because the whole lesson has maths activity throughout the 38 minutes. Looking at the left-most G-icons, the teacher seems to be active in bursts, with a few minutes in between these where she does not talk so much. An interesting facet of this exemplary lesson is the number of time students are seen talking to the whole group, almost teaching them. Certainly they are demonstrating their thinking on many occasions. And there are longer and longer stretches of time where the students are working within their groups and the teacher is to the side of the action, listening. This is shown by the shading in G-7. The “use of materials” section shows that the lesson progresses through materials and imaging into number properties, staying in number properties for the final seven minutes of the lesson. This is fully expected of an NDP-style lesson and is something to be analysed extensively, perhaps over many 10-minute lessons, which are more the norm of the NDP-style lesson.

In my study, teachers were videoed at the beginning of their first year in teaching and again at the end of their second year. With the two videos, it is possible to make many different comparisons regarding any of the features on page 1 of DART. For instance, perhaps a very new teacher talks a great deal in the first lesson, and asks more closed than open questions. This can be compared with their lesson videoed two years later. Their talk and their questions can be investigated to see how they may have changed.

This brief discussion of potential analysis is intended to be indicative only; many other types of analysis could be carried out on the data from this first page, as will become apparent in Chapter 5, which discusses the results from my study.

Numeracy Lesson

	Book No	Page No	WALT	Prior Knowledge Check	Context	Number Choices	Materials
Book	5	29	Add by splitting numbers into parts	no	Word problems Fruit or money	8+6, 5+6, 9+7	Tens frames, bundled sticks, beans in canisters
Lesson			Subtract by splitting numbers into parts	Basic facts to 10, adding to 10 and subtraction from 10	Money remaining after buying plants	13-7, 83-5	Ten frames, fingers

Instructional components of ACT Framework								
Eliciting (Solution Methods)		**e.g. min**	**Supporting (Conceptual Understanding)**		**e.g. min**	**Extending (Mathematical Thinking)**		**e.g. min**
Elicits many solution methods for one problem from the entire class	✓	11	Reminds students of conceptually similar problem situations	✓	3	Asks all students to attempt to solve difficult problems and to try various solution methods	✓	18
Waits for and listens to students' descriptions of solution methods	✓	18	Provides background knowledge	✓	3	Encourages mathematical reflection	✓	30
Encourages elaboration of students' responses	✓	19	Directs group help for an individual student	✓	29	Encourages students to analyse, compare, and generalize mathematical concepts	✓	30
Conveys accepting attitude towards students' errors and problem-solving efforts	✓	18	Assists individual students in clarifying their own solution methods	✓	6	Lists all solution methods on chalkboard to promote reflection	✓	18
Orchestrates classroom discussions	✓	12	Supports listeners' thinking	✓	12	Goes beyond initial solution methods	✓	25
Uses students' explanations for lesson's content	✓	29	Provides teacher-led instant replays	✓	7	Pushes individual students to try alternative solution methods for one problem situation	✓	25
Monitors students' levels of engagement	✓	3	Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method	✓	18	Promotes use of more efficient solution methods for all students	✓	23
Decides which students need opportunities to speak publicly or which methods should be discussed	✓	11	Supports describer's and listeners' thinking	✓	19	Uses students' responses, questions, and problems as core lesson	✓	20
			Asks a different student to explain peer's method	✓	19	Cultivates love of challenge	✓	1
			Supports individuals in private help sessions	✓	24			
			Encourages students to request assistance (only when needed)	✓	18			

Figure 4.9. Coding on the second page of DART for the NDP exemplary teacher video.

The coding on the top of page 2 shows that the chosen lesson was a variation on one in the NDP *Book 5*: *Teaching Addition, Subtraction and Place Value*. The WALT from the book is "I am learning to add by splitting numbers into parts" (Ministry of Education, 2008, p. 29) and the enacted WALT used in the lesson was "I am learning to subtract by splitting numbers into parts". There was no prior knowledge check in the book but the lesson contained knowledge checks on basic facts to 10, adding to and subtracting from 10. The context recommended was fruit, apples, and oranges; in the lesson, how much change should be given in the context of buying plants. The number choices in the book start with 8 + 6 while using materials, and move through 42 + 9 in the imaging phase, and finish with 94 + 7 in using number properties. In the lesson, the students' numbers began with 13 - 7 (materials), 24 - 6 (imaging) and concluding with 85 - 5 (number properties). The recommended materials in the book were tens frames and counters, bundled sticks, or beans in canisters. Tens frames were used in the lesson, along with fingers.

The second and larger of the two tables on page 2 of DART, which uses components of the ACT framework (Fraivillig et al., 1999), shows that every one of the components is ticked. The exemplary teacher video was of a long NDP-style lesson, in which the teacher managed to display every teacher behaviour on DART. This was interesting because the ACT framework, developed in 1999, pre-dates NDP-style teaching, so either the original developers of NDP were using ACT, or the framework encapsulates everything that is good in a quality teaching and an effective mathematics lesson. Either way, more than 200 research papers have cited the 1999 study of Fraivillig et al. showing that other researchers have also found it useful.

This completeness of teacher behaviours from the exemplary teacher video made the DART page 2 a good fit for coding teacher behaviours. However, lessons of new teachers would be unlikely to carry all of these features and it is the comparisons between first and second codings that will tell the story of any change in teaching.

Quality teaching and DART

One way of determining the veracity of the DART encodings is to see if DART could be completed to show different levels of quality teaching, by coding the same lesson topic taught at different times and where the quality of the teaching varies. The following will set out four exemplar hypothetical codings, qualitatively different encodings for the same lesson topic, and discuss what they show, and how they distinguish between the levels, with level 1 being the highest quality and level 4 the lowest. The exemplar codings are presented here in their entirety (Figures 4.10 to 4.13), and a table of comparisons and commonalities among the four (Table 4.3) is presented prior to a discussion to discover what the codings reveal about quality teaching.

Dynamic Analysis Reflection Tool

Name: Exemplar for level 1 response

Time (min)	Group Dynamic	Questions: Knowledge /Closed	Questions: Strategy /Open	Talking (Time in seconds): Teacher	Talking (Time in seconds): Student	Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
1.		4	3	48	5				T sets up double 10s frames & counters for 8+8 (Marbles context). Asks for hands on knees if S knows the answer, looks around, quick fire doubles.
2.		1	1	43	11				T: "Can you see how.. 8+4 (context)" S_1 responds shows all group on materials. T asks S_2 to respond using S_1 method. All S watch and agree.
3.		0	2	21	22				Groups work on materials, 9+7, 8+4 (given in context) discussing how to do it, sharing their understandings. T watches and listens intently.
4.		0	3	0	43				Ditto with q 6 + 9, 3+8, 8+5 etc. S talk to each other, checking answers and strategies. T continues to listen.
5.		2	0	13	37				Talk in groups, S_3 asks for help is directed to group, T listens to this group. Reassures with smiles doesn't talk except to S_3
6.		1	2	37	20				T:"I have seen S_4 doing..." S_4 demonstrates imaging method (not using materials) and T replays it back to whole group.
7.		1	1	0	46				Everyone tries this method of splitting 7 into 2 and 5 with 35 + 7 without the materials. They work in their thinking groups, achieving consensus.
8.		1	1	0	43				Continue to use new method with 42 + 9 some students move into Number Properties such as 9 + 2 gives 11 so I add 40+11=51.
9.		0	0	36	24				Student shows own method for 42+9 as "when I add 9 I go up one on the tens and down one on the units". S_1 moves back to M as numbers too big.
10		0	0	53	6				T rounds up making it OK to still be using materials. Talks about tomorrow and how T will differentiate the work, those in M will..., those in I, P will......
	A reasonable number of questions evenly split	10	13	251	257	Roughly same amount of teacher talk as student talk			

1

DART

	Book No.	Page No.	WALT	Prior Knowledge Check	Context	Number Choices	Materials
Book	5	29	Add by splitting numbers into parts	x	Word stories for objects such as fruit, etc.	(M) 8+6, 9+7, (I) 6+35,42+9 (P)75+7, 9+89	Double Tens frames drawn on a magnetic white board, bundled sticks, beans in canisters.
Enacted Lesson			Add by splitting numbers into parts	Quick fire doubles	Word stories about pirates and their gold	(M) 8+6, 9+7, (I) 6+35,42+9 (P)75+7, 9+89	Double Tens frames drawn on a magnetic white board for T, + different colour counters. S have own paper version + counters

Instructional components of ACT Framework								
Eliciting (Solution Methods)		**When**	**Supporting (Conceptual Understanding)**		**When**	**Extending (Mathematical Thinking)**		**When**
Elicits many solution methods for one problem from the entire class	✓		Reminds students of conceptually similar problem situations	✓		Asks all students to attempt to solve difficult problems and to try various solution methods	✓	
Waits for and listens to students' descriptions of solution methods	✓		Provides background knowledge			Encourages mathematical reflection	✓	
Encourages elaboration of students' responses	✓		Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts	✓	I	Assists individual students in clarifying their own solution methods			Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking	✓		Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays	✓		Pushes individual students to try alternative solution methods for one problem situation	✓	
Monitors students' levels of engagement	✓		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed	✓		Supports describer's and listeners' thinking	✓		Uses students' responses, questions, and problems as core lesson	✓	
			Asks a different student to explain peer's method	✓		Cultivates love of challenge	✓	
			Supports individuals in private help sessions					
			Encourages students to request assistance (only when needed)			# = too numerous to mention?		

Figure 4.10. Completed DART pages 1 and 2 showing level one hypothetical teaching

Dynamic Analysis Reflection Tool

Name: Exemplar for level 2 response

Time (min)	Group Dynamic	Questions: Knowledge /Closed	Questions: Strategy /Open	Talking (Time in seconds): Teacher	Talking (Time in seconds): Child	Use of Materials	I	P	**Description of Minute** (Include errors and inappropriate language)
1.		2	1	56	4				T talks to group setting the scene. She shows the materials, reminds them they have used these before, context q 11 + 9, whole groups responds
2.		1	1	41	12				Look but don't touch, T poses 9 + 7, no context, no group discussion, S thinking alone and responding together, only 2 children get answer right.
3.		3	1	46	7				T moves on to 15+7 no context, no account taken of those having problems. S_1 says 20, T gives materials to S_1 , all move counters around
4.		3	2	42	18				T gives 24+7. Some S start to talk in pairs. S_1 talks with T about difficulties, uses materials, and fails to answer. T gives 33 + 9 even though many failing.
5.		0	1	54	0				T demonstrates again the splitting of the number into tidy +, then sets 42+9, no imaging, into properties, no formal groupings, S silent.
6.		0	0	19	23				S continue to work on problems in own groupings. S_2 decided to show the group own understandings on paper.
7.		0	0	15	29				S continue to work, checking answer with teacher, and some failing. No new questions, T watches, some S continue to struggle.
8.		0	1	23	15				Some S doing nothing, T watches, talks with individuals, groupings have broken up. T sets 33 + 6 as no carries and hopes for better response.
9.		0	3	10	35				S take up challenge and discuss in groups, agree on answer. T sets another of the same type 41+8, S succeed. T sets 55+4 and 55+5 (good idea) watches
10		0	0	32	17				T continues to watch and listen, asks what they have learnt, group is quiet, T says we continue tomorrow, but we will use materials.
	Evenly split number of questions	9	10	338	160	Twice as much teacher talk as student talk			

1

DART

	Book No.	Page No.	WALT	Prior Knowledge Check	Context	Number Choices	Materials
NDP Book	5	29	Add by splitting numbers into parts	x	Word stories for objects such as fruit, etc.	(M) 8+6, 9+7, (I) 6+35, 42+9 (P) 75+7, 9+89	Double tens frames drawn on a magnetic white board, bundled sticks, beans in canisters
Enacted Lesson			Add by splitting numbers into parts	x	Begins with word stories stops at 5min	(I)11+9, 9+7, 15+7 (P) 42+9, 33+5	Double tens frames drawn on a magnetic white board for teacher

Instructional components of ACT Framework								
Eliciting (Solution Methods)		When	**Supporting** (Conceptual Understanding)		When	**Extending** (Mathematical Thinking)		When
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations	✓		Asks all students to attempt to solve difficult problems and to try various solution methods	✓	
Waits for and listens to students' descriptions of solution methods	✓		Provides background knowledge			Encourages mathematical reflection		
Encourages elaboration of students' responses	✓		Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts			Assists individual students in clarifying their own solution methods			Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking	✓		Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement	✓		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed	✓		Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge	✓	
			Supports individuals in private help sessions	✓				
			Encourages students to request assistance (only when needed)			# = too numerous to mention?		

2

Figure 4.11. Completed DART pages 1 and 2 showing level 2 hypothetical teaching.

Dynamic Analysis Reflection Tool

Name: Exemplar for level 3 response

Time (min)	Group Dynamic	Questions: Knowledge /Closed	Questions: Strategy /Open	Talking (Time in seconds): Teacher	Talking (Time in seconds): Student	Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
1.		1	1	55	0				T sets first problem in a marbles context with 14+8. Teacher repeats story and writes vert. algorithm in modeling book.
2.		0	0	41	15				S ask for help, they are not encouraged to discuss or work in groups, they ask T, who interacts privately, says "imagine…"
3.		0	0	36	7				No one succeeds. T shows the situation on ten frames, look but don't touch. Group is still not succeeding, T repeats q. No context.
4.		1	0	47	7				T now moves the counters to model the situation, S agree they can now do the q. T sets 33+5 which is inappropriately hard
5.		0	0	41	9				T removes materials, repeats q. "Come on, we did this yesterday!!" Scaffolds strongly, some S manage it.
6.		1		29	5				T takes this as all can do it. T: 24+7 inappropriate as has carry digit, no context. S sit quietly chewing pencils.
7.		1	0	41	4				T shows S how to do the q with the vertical algorithm, not using language of place value. Some S say "ah yes"! T sets 6+38 written in book
8.		0	0	0	0				S work individually, hiding work from each other. No one talks. Many S off target. T Attention drawn to others in room
9.		4	0	25	29				T: "Right, who's done it?" T asks S_1 but everyone clamours to give the answer, mostly wrong. T questions S about why they think they are right
10		0	0	14	5				Still no discussion in groups. S who understand make up some more of their own, the rest wait sleepily for end of lesson. They play with materials
	No strategy questions	8	0	329	81	4 times more teacher talk than student talk.			

1

DART

	Book No.	Page No.	WALT	Prior Knowledge Check	Context	Number Choices	Materials
NDP Book	5	29	Add by splitting numbers into parts	×	Word stories for objects such as fruit, etc.	(M) 8+6, 9+7, (I) 6+35,42+9 (P)75+7, 9+89	Double tens frames drawn on a magnetic white board, bundled sticks, beans in canisters
Enacted Lesson			Add by splitting numbers into parts	×	First problem in context only	(P) 14+8, 33+5,	

Instructional components of ACT Framework								
Eliciting (Solution Methods)		**When**	**Supporting (Conceptual Understanding)**		**When**	**Extending (Mathematical Thinking)**		**When**
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations	✔		Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods			Provides background knowledge	✔		Encourages mathematical reflection		
Encourages elaboration of students' responses	✔		Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts			Assists individual students in clarifying their own solution methods			Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking			Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement	✔		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students	✔	
Decides which students need opportunities to speak publicly or which methods should be discussed			Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions	✔				
			Encourages students to request assistance (only when needed)			# = too numerous to mention?		

Figure 4.12. Completed DART pages 1 and 2 showing level 3 hypothetical teaching.

Dynamic Analysis Reflection Tool

Name: Exemplar for level 4 response (bottom)

Time (min)	Group Dynamic	Questions Knowledge /Closed	Questions Strategy /Open	Talking (Time in seconds) Teacher	Talking (Time in seconds) Student	Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
1.		2	0	51	2				T asks if S's remember yesterday's lesson on adding two digit numbers by parts, they all agree they do. 23+92 (This is not appropriate as carries involved)
2.		2	0	48	8				T writes q in modelling book (algorithm), and one child says the answer, quickly and unbidden. This is repeated with same child answering quickly.
3.		6	0	49	7				T writes different questions (algorithm) on each child's pad of paper, and sets them to work. No discussion between S's. Extra q added if S right.
4.		0	0	45	9				T works with individuals, instructing them to "remember to carry the ones". S's use algorithm to add e.g. 62 +19. No vocalisation. No discussion.
5.		1	0	43	9				Way of working continues. S_1 gets wrong answer, (87+69=146) teacher takes pen and paper off S_1 and writes in correct ans, then sets 65 + 25
6.		2	0	45	10				T reminds the group about how to add vertically and carry. T does not attempt to take S_1 back to materials, or imaging. No discussion of error
7.		3	0	48	7				T works with S_1 giving another q: 35 + 22, but even with no carries S cannot achieve success. T tells him how to do it. "I told you before…" S off task
8.		1	0	56	3				T watches the moves S_2 is making, correcting without explanation when wrong. Q: 35+45 when S_2 cannot add 5 to 5 T does not remind of doubles.
9.		0	0	36	4				T talks to whole group about carries again as so many failing with the algorithm, even those who can do it have to listen. T and S off task
10		0	0	31	6				Teacher watches as S cont. to fail to solve q tells S how to do it, demonstrating and S copies. End of time, "we will continue this tomorrow".
	No strategy questions	17	0	452	65	Nearly 7 times as much teacher talk as student talk			

1

DART

	Book No.	Page No.	WALT	Prior Knowledge Check	Context	Number Choices	Materials
NDP Book	5	29	Add by splitting numbers into parts	x	Word stories for objects such as fruit, etc.	(M) 8+6, 9+7, (I) 6+35,42+9 (P)75+7, 9+89	Double tens frames drawn on a magnetic white board, bundled sticks, beans in canisters
Enacted Lesson			Add by splitting numbers into parts	x	None All just number problems	(p)23+92,35+28 Using algorithmic layout	None

Instructional components of ACT Framework								
Eliciting (Solution Methods)		When	**Supporting** (Conceptual Understanding)		When	**Extending** (Mathematical Thinking)		When
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods			Provides background knowledge	✓		Encourages mathematical reflection		
Encourages elaboration of students' responses			Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts			Assists individual students in clarifying their own solution methods			Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking			Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement	✓	#	Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed			Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions	✓	#			
			Encourages students to request assistance (only when needed)			# = too numerous to mention		

2

Figure 4.13. Completed DART pages 1 and 2 showing level 4 hypothetical teaching.

If all four exemplars in Figures 4.10 to 4.13 are evaluated, it can be seen that some factors, such as children using materials, are seen less in the level 4 exemplar than the level 1 exemplar. Other factors that change, such as the time spent talking, are coded, and in the level 4 exemplar it can be seen that the time is more or less completely that of the teacher talking. Table 4.3 provides both comparisons and commonalities of these four exemplar DART codings, which are discussed below.

Table 4.3
A Comparison of the Hypothetical Exemplar DART Coding across the Four Levels

Level	Action	1	2	3	4
G-Icon use	G1->G4 : G5->G8	10:11	20:6	18:0	18:0
Questions	Closed : Open	10:13	9:10	8:1	17:0
Talking time	Teacher% : Student%	42:43	56:27	57:12	75:11
Materials	M-1/M-2/M-3/M-4	1/3/0/2	2/0/1/1	2/0/1/0	0/0/0/0
Imaging	/10	3	3	2	0
Number Properties	/10	3	6	8	10
Description	Example of problems comment	No problems	No context	Wrong magnitude number	Teaching algorithms
NDP Book	Book lesson matches enacted lesson	Wholly	Only lacks context	Materials used sporadically	Algorithmic -not in book
Act Framework	ACT Score (*x*/28)% *x* = Number of different behaviours	57	32	21	11

In these exemplars, the G-icons reveal the way the groups are working. There is a continuum in the way groups work, from sitting in a group but not interacting in any way with the other students in the group through to working collaboratively with the group or in pairs, discussing and sharing understandings.

A teacher who understands NDP group dynamics would score a ratio close to 1:1 as in the level 1 exemplar, but a teacher who does not encourage the community of learning to develop would score more like the teacher in the level 4 exemplar, highlighting G-icons only on the left-hand-side.

Questions in DART are categorised as open or closed. The more closed questions there are, the fewer opportunities there will be for students to think deeply about the mathematics in front of them. Therefore more open questions than closed are desirable in this area. Looking from the level 1 through to the level 4 exemplars (Table 4.3), it can be seen that the number of open questions diminishes rapidly, and the number of closed questions grows to take their place.

The third measure of quality teaching in DART is how time talking is divided up. The more the students talk, the more likely it is that the teacher will hear their thinking, and analyse for misconceptions. The more the teacher talks, the less likely this is to occur; therefore here, quality teaching is represented by an equality of time for teacher and student. These numbers do, however, hide minute-by-minute differences which should be explored, perhaps with the use of graphs.

In NDP quality teaching, ideally materials should be available to fall back to if students are failing to progress well in the imaging or number properties stage. In NDP, it is also highly desirable that students manipulate the equipment themselves. So, in this line of the Table 4.3, analysis could include a count of the number of times each M-icon is highlighted. The results reveal materials use, and who is using them, and these exemplars do feature this characteristic. As we move from left to right on Table 4.3, it can be seen that in the exemplars, materials use ultimately vanishes as teaching quality diminishes.

In the material, imaging and number properties section, a simple total of the highlighting has been made. These counts represent the use of the strategy teaching model (Hughes 2002), and a lesson that stays resolutely in the number properties area is probably not going to be of the highest quality, unless the students are having success, which could be ascertained by reading the narrative in the final section. If a vertical view over materials, imaging and number properties on page 1 of DART is taken, it becomes evident that quality teaching in NDP resides in a more distributed pattern.

Finally on page 1 of DART, the descriptions of the four levels of quality teaching should capture problems and make comments made about inappropriate number use, or, as in level 4, the lack of success in using algorithms to teach double-digit addition to students and the fact that the group was not taken back to materials. A point should be made here that the level one lesson is not perfect; it would be wrong to assert that perfection is the goal in teaching. The important point is that the teachable moment is noticed, and the occasional use of a number of the wrong magnitude, or demonstrating instead of allowing students to manipulate materials for themselves, is not a sign that the lesson is not of high quality.

The first section on page 2 of DART allows comparisons to be made between the resource lesson in the NDP "pink" books (Ministry of Education, 2005) and the lesson that is produced by the teacher. Looking at this line in Table 4.3, it is apparent that the written and the enacted lessons are further apart as we move from exemplar 1 to 4. Quality teaching in NDP resides in the resource book lesson and hence there should be some consistency between the two.

The figures in the final part of Table 4.3 are arrived at by noting which behaviours are observed, and dividing by the total number of possible behaviours For example, in exemplar 1, there were 16 observed and coded behaviours, and if we divide by 28 and turn this into a percentage, it gives 57%. This is labelled the ACT measure for use in the results chapter. These teacher behaviours are all desirable, but there are 28 of them, and in a ten-minute lesson, it is unlikely that all of them would appear, and, perhaps, quality teaching does not require them all to appear. However the exemplars above do highlight the reduction in the number of desirable behaviours as we move from exemplar 1 through to 4.

What does this exposition of the four exemplars levels tell us about DART? Firstly is shows that DART does have fidelity, and it is sensitive to factors considered important under NDP. Secondly, the four levels themselves were able to be distinguished by scanning the four completed DARTs. Thirdly, further coding, such as is seen in Table 4.3, enables the apparent differences between the levels to be evaluated.

Summary

This chapter has outlined how the MQI and the ACT framework were used to develop the DART framework and how the DART may be used to analyse teaching quality. The video of an experienced and skilled exemplary teacher provided in NDP was both the catalyst for the development of DART and used to affirm the validity and reliability of DART. Lastly, four hypothetical exemplars ranging from level 1 to level 4 were drawn up to purposely show what different coding might look like for different levels of quality teaching.The next chapter will present the results of using DART to analyse videos of the teacher participants in my study. This is followed by a discussion of the potential future uses of DART, including how its use might be expanded so it can be used by a teacher or a group of teachers to improve their own teaching quality.

Chapter 5

Results 2: Use of DART to measure pedagogic change in the participants

Early Additive

At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called part-whole thinking.

The Number Framework

In this chapter, my understanding of the factors involved in quality teaching is beginning to develop, coalesce into wholes which can be partitioned and reconfigured. The complexity of the situation is being understood.

This chapter presents the results of what quality teaching looks like as it develops in novice teachers. In examining this development of quality teaching, data captured by DART, such as use of materials and talk time, are discussed through the use of case study. The chapter begins with an overview of the seven novice teachers, describing their demographics and their ages and other attributes, to illustrate the differences between the participants in the study. Next, two case studies highlight different findings as examples of development. The chapter ends with vignettes of the remaining five participants to further demonstrate how quality teaching in novice teachers develops.

Research Questions

My primary research question which relates to novice teachers teaching in NDP-style was:

How could an instrument be designed to capture the development of quality teaching?

Subsidiary questions were:

- What is the nature of quality teaching evident in the teaching practice of novice teachers over their first two years in the profession?
- How does the teaching of novice teachers change over two-year period of their provisional registration?
- How does initial maths anxiety affect the development of quality teaching?

The participants

The seven participants are all female and of similar ethnicity, namely New Zealand European. They all began teaching in 2008, four in February and three in June of that year (see Table 5.1). This research spans a period of two years, 2008 to 2010, the videos were recorded near the beginning of their first year, and close to the end of their second year. Some of the novice teachers taught the same age group for each of these years, while others taught multiple age groups in this time. All the names used here are pseudonyms.

Table 5.1

*Demographic and other Attributes of the Seven Participants (two case study participants *)*

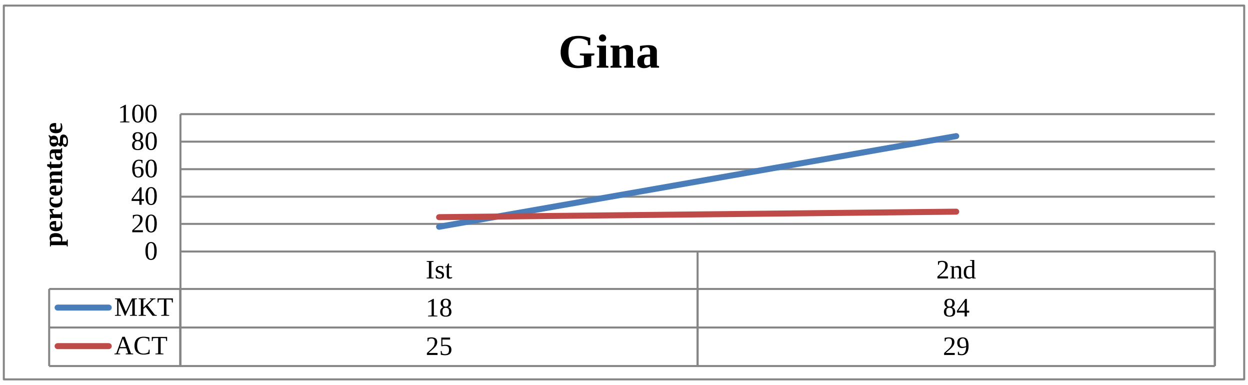
Name	Age	Began teaching 2008	Year levels taught	Level of maths	MARS	MKT_1/MK_2 %	ACT_1/ACT_2 %
Alison	40	February	2, 5, 6	>Y13	13	82/84	36/39
Barbara	21	June	5	Y12	15	63/73	54/50
Carly	21	February	7	Y13	18	36/86	29/79
Debbie*	20	June	4 ,5, 6	Y13	29	63/48	79/39
Evie	26	February	2	>Y13	29	63/55	39/43
Fiona	28	June	New entrant, 1	>Y13	40	36/91	14/29
Gina*	27	February	2	>Y13	45	18/84	25/29

Table 5.1 indicates that Alison had two years of varied teaching experience with Years 2, 5, and 6, representing either end of the primary years. Debbie also had experienced teaching with three different year groups, which were all within the same class, and the other five people remained teaching at the same levels. Carly was the only novice teacher who taught at the intermediate level. At the other end of this spectrum, Fiona taught the very youngest students in this study. In Table 5.1 it can be seen that all participants received mathematics education beyond the compulsory school Year 11, with four of them having studied mathematics subsequent to Year 13, at university level. The last two columns show the participants' scores

for the mathematical knowledge for teaching measure (MKT) and the ACT measure, from page 2 of DART.

Table 5.1 also presents the MKT and ACT scores and the relationships between them. The measures were taken approximately two years apart. Points of interest are the differences in the measures as the participants began their teaching careers, and the measures two years' later. For instance, Debbie began with both measures very high, and two years later both of these have reduced, whereas Carly's results show an opposite pattern. Alison continued to exhibit strong MKT, but her ACT score had hardly moved. Gina's MKT at the start was modest and finished very high, but her ACT began low and improved only slightly. These graphs in Figure 5.1 and Figure 5.2 facilitated my decision as to which participants would make worthy case studies.

The first case study is Gina (see Figure 5.1), the participant with the most extreme characteristics, including being the most maths anxious person in the study. This particular case study highlights the trajectory of Gina teaching as a novice and whether it developed into quality teaching. It also demonstrates how the DART analysis can be used to illuminate this.



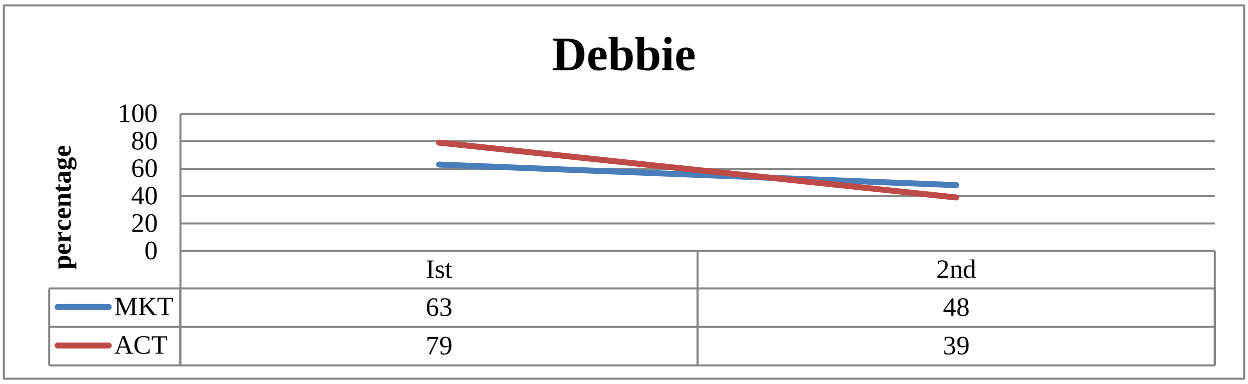


Figure 5.1. The MKT and ACT test results for full case-study participants Gina and Debbie.

The second case study was Debbie (see Figure 5.1), who had an average maths anxiety. I chose Debbie as she started with very high values on both her ACT and MKT measures and both of these scores diminished markedly. Her case study provides more evidence of how the DART framework can be used to analyse videos, developing a picture of what quality teaching looks like. The characteristics of the remaining five participants will be discussed, as vignettes, after these two in-depth case studies, highlighting some of the characteristics seen with Gina and Debbie, and providing some interesting contrasts with each other.

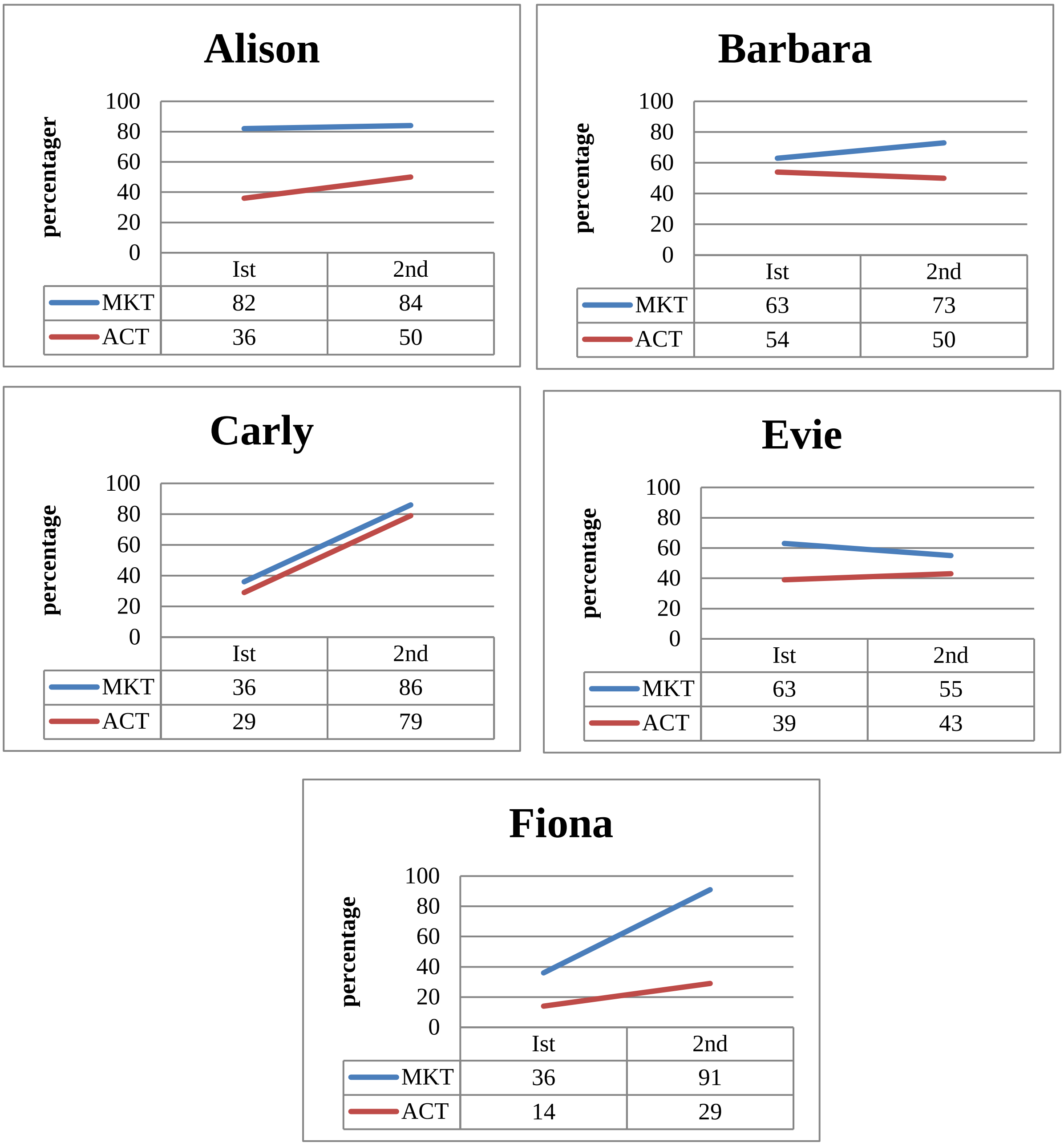


Figure 5.2. The MKT and ACT test results for each of the vignettes participants.

Case study 1: Gina

Gina was selected as she showcases a number of extreme characteristics. Of the seven participants, Gina had the highest university qualifications. In her undergraduate degree, she majored in biology, but completed mathematics papers as part of this. She also holds a master's degree in biology (genetics). Surprisingly, she is also the most maths anxious, scoring 45 on the MARS scale, out of a possible 50. She strongly ageed with the statement: 'Mathematics makes me feel uncomfortable and nervous', and strongly disagreed with the statement: 'I have usually been at ease during maths tests'. The results show that her marked antipathy towards mathematics led to both career and employment restrictions. Gina's attitude to mathematics is discussed using data from the initial interview, from the two videos of her lessons collected two years apart, and her results from the questionnaires and tests. This will illuminate the complexity of a pre-registration primary teacher's journey over two years, through to full teacher status, and highlight the quality of the mathematics teaching Gina developed during that time.

Attitude to mathematics

Gina's antipathy towards mathematics restricted her career choices, funnelling her away from her original wish to study medicine and then to pursue forensics. It also influenced her choice of which age children she would choose to teach. This antipathy continued in to her teaching, restricting her experience with other age groups, though Gina talks in her interview about her desire to widen this. Her perceived lack of mathematical ability removes, in her eyes, choices of teaching at different levels in the primary school.

Gina professed to hating mathematics, more so than any other participant; she could not remember anything positive about mathematics lessons she had received in the past. When pressed for recollections, she said of her primary school days:

> Well, I know we had to do times table tests and we had to do things where we all had to stand up and they'd ask heaps of questions and you'd sit down, and I was always like, I couldn't do it because I hated learning times tables because I couldn't see the point of it, and I didn't understand what I was doing anyway. So it was like, well, why am I rote learning something that's complete nonsense.

When asked directly how she felt about doing mathematics during her school days, she recalled:

> Yeah, I just never got it, but then I also just didn't get it at all, like I've never found maths, that I can recall, particularly easy to remember. Like, if someone asked me to spell something, I can see the word in my head. If someone tells me to do maths, I get a complete blank and I have to make my brain calm down and think about what's actually being asked before I can do it, even if it's really simple stuff.

Despite a master's degree in genetics, Gina felt that she virtually knew no mathematics and did not feel competent in this area at all. However, there is evidence to the contrary, that she actually knew much more mathematics than she gave herself credit for. In her preliminary and final MKT tests, which provide information about knowing mathematics well enough to teach it, the questions centre on the mathematics of children, and the teacher is asked to identify appropriate responses to particular classroom scenarios. As Figure 5.1 shows, Gina scored 18% on her first MKT test, which was the lowest of the group (group average 52%), but on her second she scored 84%, above the average of 74% (see Table 5.1) indicating, despite Gina's protestations, that her capacity for improving her mathematics knowledge for teaching was very high.

Although the tests indicate Gina appears to know a great deal about mathematics for teaching, there is an assumption that within this measure her mathematics content knowledge will also be high.

Here I have discussed the way that Gina's maths anxiety may have affected her capacity to change over her two years of pre-registered teaching. Her marked increase in MKT scores contrasts sharply with the static nature of the ACT scores. Gina was on a different career path until she realised that her lack of mathematical ability would hold her back. I will now discuss the way that this attitude to mathematics affected Gina's career choices.

How Gina's maths anxiety influenced her career choices

Gina was 27 years of age when the study began, coming to primary teaching after years of study and working in her chosen field of biology, where she had to use mathematics in her studies, research and work, but where she encountered many difficulties:

> I did my masters in biology and a lot of the work I did with that was manipulating numbers and things, just because it was DNA analysis, so you were having to get tiny quantities of liquid and when you do a DNA analysis you've got these minuscule quantities so you usually do five or six or 10 or 50 samples. So you have to multiply the quantities by that much and if you get it wrong then it totally stuffs it up, and I think some of the time I probably did stuff it up because I wasn't getting the (right

> answer). Even though I had a calculator, I wasn't getting it quite right but I also, it also meant that I couldn't go to Med School because, I wanted to do forensics.

Gina showed self-awareness regarding her mathematics difficulties. She had to use mathematics in her studies, but indicated she had problems; so much so that she felt she had to abandon her wish to become a doctor of medicine. This also indicates that Gina's confidence about her ability to do mathematics effectively in her work was very low, and even with quite simple calculations, Gina was sure she could not be successful. When asked if the situated learning she had done in her genetics research helped her learn and retain the more complex mathematics she required, she dismissed that notion, saying:

> No, it [mathematics] was still this big horrible incomprehensible thing. I think they had to teach me the Hardy Weinberg formula about every single year. I needed to learn it was just like a formula that went with these, for genetics you know when you cross two plants and you've got dominant traits and you put them in those things and you get a table and apparently that table translates into proportions and those proportions you can put into a formula and you can find out whether the gene frequency is changing and stuff. That was just nightmarish. I just usually just skipped that part if there was ever a question about that, that I could skip, I'd just skip it, because it was just too hard, and my flatmate, who's an engineer, was really helpful and would teach it to me in enormous detail every single year, but then I couldn't remember it next year and I could probably not remember it two days after the exam either.

Gina's teaching, during both of her pre-registration teaching years, which was the length of this study, was with Year 2 students (aged 6 years) in the same school. She said she intended to remain teaching at this level because she felt unable to teach the mathematics at a higher year level. She stated:

> But I don't think I could teach at any higher level unless I was teaching where they separated the kids who couldn't do maths and the kids who were really good at it and I took some lower end kids, I just couldn't do it, I don't think.

As a safeguard, Gina restricted her applications for teaching posts to schools where she could be sure of teaching at the younger end. She talked about her career progression thus:

> I like the little kids but like sometimes I think it would be nice to work with the higher level kids especially with the science and things that I want to be able to do.

> I mean I guess I could probably stick with the Year 2s until I'm really confident with that and then try Year 3 and move my way up.

Gina identified the need for science teaching in the upper primary area, and expressed her wish to be able to use the skills she possessed, but believed she was realistic about the way her lack of knowledge in mathematics was holding her back.

In summary, Gina found mathematics difficult to learn and retain, and the antipathy she felt towards mathematics deeply affected her ability to pursue her chosen career path. Her wish to work in forensics was thwarted by her inability to understand mathematics well enough to go to Medical School, and she deliberately restricted her job applications to those schools needing a teacher at the level of very young children in the primary school. In the following passage, Gina's interview and other data will be examined to see what effect, if any, participating in teacher education courses at university had had on her ability to do mathematics, and on her attitude to it.

Understanding versus rote learning

Gina had shown, through her degree courses, that she had the ability to study at a high level. However, she contended that the majority of her mathematical knowledge was held in a superficial manner. This was due, she maintained, to the remarkable amount of rote learning she had been prepared to do to pass the mathematical sections in any tests and examinations.

One of the first things Gina said, quite forcibly, about her mathematics knowledge, was that in examinations and tests she could do quite well because she "sat down and I learnt those formulas, so I knew them backwards and inside out and that was it, yeah, rather than actually understanding it". Gina seemed resigned to learning in this fashion until she was introduced to the style of teaching expected in the NDP. She sounded both amazed and astonished over what she learned.

> When I found out that you could split numbers like, if you added 7 and 5 you could do 7 plus 3 makes 10 and 2, that was like this big mind blowing experience and I went home and told everyone about it and they were like, yeah. I was like but, but, but, but.

This strategy for adding numbers, called partitioning or part–whole strategy, is one of the strategies NDP posits all children need to go through to achieve numeracy. However, it was no universal panacea for all her problems, since she still did not have a complete understanding, stating "but it was still really hard to do. It's just one of those things, and multiplication, anything multiplicative, I find very hard". Later in the interview Gina was asked if the two mathematics education courses in her teacher studies had helped with her understanding. She explained:

> I remember discovering how the numeracy project worked in the equal splitting of numbers and was thinking it was just amazing it was this weird thing that I'd never come across before. I couldn't understand anything beyond that, like I found the rest of the stuff with the multiplicative strategies and things they were just really confusing and proportional, I could kind of get my head around it because I was used to dealing with proportion of mils and things from my master's and stuff but I still didn't really understand what was happening with the strategies. I think I found maths easier than a lot of the people from the course, but I think it's because I spent the time trying to figure out exactly what I was trying to achieve with it, because I kept, I'd sort of stay up for hours trying to figure it out rather than just going oh this is too hard and giving up.

This illustrates the difficult nature of learning mathematics for Gina. When she found that her lack of understanding could prevent her from becoming a teacher, she worked very hard to improve, and her understanding of mathematics did advance, but not to the extent that she would have liked.

It appears that Gina had not learnt mathematics in her own education years, and while in her teacher education courses there were revelations, she did not attain much more real understanding. Gina considered that she "found maths easier than a lot of people from the course", which is disturbing indeed to think that her knowledge represented a more complete understanding than that of her colleagues.

In her interview, she continued trying to explain her lack of mathematical understanding:

> I always, my entire life, I've always counted on as long as I can ever remember so actually having to think about it I guess I mean it's made a difference to me because I now know what I don't know how to do, but I don't know how to do it still.

There is more than a hint of desperation in Gina's words. She had succeeded in the complex field of genetics, yet being multiplicative, really understanding how multiplication and division work, still eluded her. Considering that the level of mathematics she was comfortable using would be associated with children aged about seven years old, Gina cuts a sad figure, seeing no way around her problems.

Gina was not often pushed out of her comfort zone when teaching mathematics to Year 2 students during her two pre-registration years. However, she credited this time as consolidating her mathematics at this level and moving her on from being a “counter”. Gina described how she had tried hard to master multiplication before she went teaching, but found her lack of times table knowledge held her back. Asked if teaching mathematics every day meant that she had no option but to learn at that level, she said:

> Yeah, yeah, I have and it has helped in like, I’m having to learn like while I’m teaching the kids, I’m having to learn it as well so that has helped a bit, but I mean this year is the first year that I’ve had kids who are working at part–whole at the beginning of the year, last year I didn’t. So I’ve got these kids who are already ahead of me in maths and I’m having to keep up with them and be one step ahead of them kind of thing, which is tricky, because it makes it harder to teach it.

This particular excerpt from Gina’s initial interview demonstrates the self-awareness that Gina possesses. She knows some of her students, aged about 7 years old, are working at stage five or six on the numeracy framework, and are already more able at some aspects of numeracy than she is. Some are working at one or even two stages higher than her self-professed ability. Teaching mathematics every day, as Gina did, must be very difficult indeed.

So, as we have seen, Gina has struggled for understanding in mathematics, but in her words at the initial interview, it eluded her. She is aware that she can learn mathematics by rote, this had been necessary to do all her school life, but she also knew that her lack of basic knowledge of times tables, for example, was holding her back. Next, I examine her two years of teaching to see how they may have helped her develop a better understanding of, and attitude to, mathematics.

Maths anxiety and attitude to mathematics

Maths anxiety in this study is measured through the MARS instrument (Richard & Suinn, 1984), which has items, a minimum score of 10, and a maximum score of 50. Gina scored 45 on her maths anxiety test, the highest of all the participants (see Table 5.1). Being maths anxious can manifest itself in an acute dislike of mathematics, as well as an inability to use mathematics. In the following discussion, Gina’s words from her interview illuminate her level of maths anxiety. However, through the use of good NDP activities, it will be shown that Gina is very self-aware and determined to mitigate her problems with mathematics by planning lessons that move her students through the Number Framework stages.

Maths anxiety is a well-researched phenomenon, with the effect of maths anxiety usually being viewed as negative, causing people who are highly maths anxious to freeze during tests, go blank when problem solving, and the such like (Ashcraft, 2002). But there is also a motivating side to maths anxiety, when, knowing there is a problem, people will work hard to overcome their difficulties. This is seen in the studies of mastery (Goddard, Hoy, & Hoy, 2004) with self-efficacy. As self-efficacy increases, maths anxious people can take more control of the problem and, with determination and hard work, they can overcome it, thereby making their maths anxiety work for them. Important in this is authenticity and valuing the progress made. Gina appears to have leveraged off her maths anxiety and moved through to mastery. The question now must be, did Gina value the progress she had made, did she feel her maths anxiety was improving? Responses from her interview illuminate this.

Further on in the interview, Gina was asked if she ever became maths anxious when she was teaching:

> Sometimes but more often with the kids that I've got, it's because I can't explain it to them in a way that they can understand. It's like I know what I'm trying to teach them and I know what they're getting stuck on but I can't figure out how to make it, to bridge the gap between the two of us so that they can understand what I'm trying to tell them, or trying to help them find out.

Gina wanted to find the right things to do and say, the right activities to enlighten and assist her students to progress in mathematics, as they do in other learning areas. However, she lamented that she was unable to action this in the learning area of mathematics.

It would appear that Gina's maths anxiety had continued into her teaching of mathematics as she has not moved far past her own difficulties with mathematics in primary and secondary school. However, although her comprehension didn't improved, she had endeavoured to teach well, and her organisation of her mathematics resources and games show that she was using her intellect, together with knowledge of NDP, to provide worthwhile learning experiences for her students. When asked to describe a good teaching sequence that she had taught, she discussed at great length the activities she had given to her students who needed to move from "counting all" to "counting on" a level of mathematics that she admited was her ceiling.

> There's a lesson I've got about teaching, getting kids to move from imaging to counting on using number tiles which is exceptionally complicated and lasts for like a whole term. 'Cause it, you start off with the number tiles all out in a row and

> getting them to recognise all the tiles are there so you turn them over, so you leave them there and say point to one and say how many tiles up to here, so they're having to go well that's eight so there must be eight so they're having to, you know they start off by counting one, two, three, four, five, six, seven, eight and go oh eight and then eventually they go oh well you haven't taken any away so it must be eight.

The excerpt above shows that Gina believes she was having success in her mathematics teaching sessions. She continued:

> It's very [hard], it takes ages but they, it seems to work, they get the counting on thing quite well after doing that and it deals with the whole, they're anxious about the numbers not being there anymore.

These sections of her interview illustrated Gina's determination to do the very best she could for her students. She did have a vision for herself as a teacher, this is shown through the next section in the interview, in which Gina was asked about her vision of what an ideal mathematics teacher might be like:

> Well I think, I'd need to know how to do it myself so that I could be able to if someone came to me with a question about how to do it or if some kids were stuck, or so that I could look at the book and say no I think that lesson isn't going to help the kids understand it but I think that the concept in it is important so I'd be able to modify the way that it was taught, rather than just having to look at the book and go oh I'll teach them that one because that's the next one in the book or the one in the book that tells them, teaches that concept. I could take the concept and modify it to make it so that it works for the kids. So I could try the lesson and it doesn't work, or I could just try it again and do it differently but using the same concepts so they can understand it I think, but also I'd actually know the answers so that when the kids, you know when you give kids a question you know the answer straight away rather than having to work it out while the kids are doing it.

And having been pressed, Gina managed to enunciate what she considered to be the important aspects of a mathematics lesson, showing she had aspirations, even if these things were still beyond her abilities at this point. Here she illustrated her understanding of the NDP, and the model for teaching, which is crucial to understanding numeracy development:

> Yes, ah yeah. There would be materials, there'd probably be an actual real, real life context for it as well. I can't seem to be able to figure out for numeracy. Like, I've got a lesson that hopefully will work on Friday where, because we've just got a new

> playground, I'm going to draw, a footprint and the kids have to prove to Mr Davies that it wasn't them that stood in the wet concrete. So, like you know, having a real [context]. I've just been to a science conference where it was all about this sort of stuff, real context.

This "Bigfoot" activity Gina refers to here is one of the teaching activities that she encountered in her teacher education courses. However, she does not appear to remember that she worked on this previously, which might again speak to her retention and ability to access prior experiences. Also interesting from this excerpt is that Gina has been reminded, through the science professional development she attended, that a real life context was very important in teaching and central to new, strong concept development, which is another area her courses would have emphasised, since contexts are central to NDP. Perhaps as she progresses through her teaching years, ideas which were discussed as central to quality teaching in her initial teacher education courses are becoming more real to her and therefore, when she is reminded of them through professional development, she has found them easier to implement. Investigating this possibility was an idea outside the scope of this study, but nevertheless, it would be an interesting avenue for further research.

The self-awareness that Gina exhibited regarding her maths anxiety was remarkable. She was very open about her difficulties in teaching mathematics, and when asked about how she would feel if she was asked to do some mathematics then and there in the interview, Gina's emphatic response demonstrates her anxiety:

> Very uncomfortable. It makes me feel a bit queasy. Then, I'm sort of thinking, well, maybe, how hard, what kind of maths. Is it going to be stuff I can do? Is it going to be stuff that I don't understand at all? What kind of thing do you want me to do?

Her level of anxiety rose considerably at this point and her voice rose in consternation, indicating that Gina was probably thinking she was going to be asked to do something she knew she could not do. She was reassured by me that she was not going to have to produce mathematics during the interview, to which she replied about her maths anxiety:

> It makes me, yeah, it makes me get sort of butterflies kind of thing and also my brain goes a bit blank. Like it just goes, I can't do that. No, no, I can do it, there are some bits I can do so I have to tell me that I can do it and then I sometimes can.

In the analysis above, Gina's interview has been quoted extensively to illustrate her feelings and attitudes towards mathematics. She was cognisant of her shortcomings in her ability to understand where her students are on the NDP framework, and how to move them on to the next level. She feared that she would not notice when the teachable moment arrived, but she compensateed by designing engaging and interesting activities, using real-life contexts when she was able. She was concerned that her students knew more than she did, and it is this self-knowledge that prevented her from career choices other than teaching, and continued to restrict her teaching to that of young children.

Next, I examine the two videos of Gina teaching mathematics, which were taken nearly two years apart and coded using my analytical tool, DART. I also discuss the factors that influence and contribute to quality teaching, and measure the change that occurred in those two years.

Quality teaching

The two results for Gina's ACT test, 25% and 29% (presented in Figure 5.1) are at a lower level than the average for the group, and remained at about the same point. The ACT measure counts the number of different behaviours and questioning strategies used by the teacher, as identified from videos of the two lessons and analysed using the second page of DART. These behaviours and strategies arise from the Fraivillig et al. (1999) research and are seen on the second page of DART (see Chapter 4). During the coding of the two lessons (see Figure 5.3), an index is generated by counting the different behaviours in the eliciting, supporting and extending sections.

The ACT measure of quality teaching is a global one, calculated by adding up the number of times a particular behaviour is seen at some point in the video of the lesson. This count is then expressed as a percentage of the total of the possible behaviours, by dividing by the number of possible desirable behaviours captured on this sheet. Gina's score moved from 25% to 29% over the period of the study, which is a small increase over the two-year period between the two video recordings of her mathematics teaching. Although a global measure is used in the graph (see Figure 5.3), the three areas comprising the measure, namely eliciting, supporting, and extending are all desirable (Fraivillig et al., 1999). The third section on Figure 5.3 with the heading "Extending" is considered important by Fraivillig, et al. (1999) since it captures whether the students' thinking has broadened beyond that already known so that new learning might occur. This facet was rarely seen in the teachers involved in this research, (nor in the original 1999 research by Fraivillig et al.) and that was also the case with Gina's teaching.

In both videos, Gina's totals for eliciting and supporting remain constant over the two recording times, two years apart, and no extending behaviours were captured. This lack of improvement is notable because it could indicate a ceiling-effect that occurs when a teacher has high maths anxiety and lacks confidence in his or her ability to teach it, as seen in the case of Gina. However, she knows the mathematics that is being taught. An example of this is seen on the second video where Gina is teaching “counting on”. Here she teaches a session using appropriate resources and the strategies advised in Book 5 of the NDP “pink books” series (Ministry of Education, 2007, p. 18). She led the group through the idea of knowing that the number in the count is the number of articles counted, turned over the cards, and asked the students if she showed the card with 16 on it, how many tiles would be below it. The students all seemed to understand this part, and two of the children are sent away to play a game that aims to consolidate this idea. This would indicate that, in Gina’s opinion, these children did not need more input from her as the teacher. However, it was noticeable on the video that at no time did Gina facilitate the students into establishing new ideas, or encourage them into number properties, which is where NDP activities are always heading, as advised in Book 5.

The results of the analysis of the two videos (V_1 and V_2) are presented to provide evidence of whether Gina’s teaching could be considered quality teaching of mathematics. The analytical tool DART (Dynamic Analysis Reflection Tool - see chapter 4), captures many aspects of what are considered to be measures of quality teaching. DART was developed specifically to code the types of NDP lessons that all the participants in this study taught. There are many elements encoded; these will be examined to see if Gina was able to overcome her maths anxiety.

Dynamic Analysis Reflection Tool

Name: Naomi **First Video (V_1)** **First Group** **First Page**

Time (min)	Group Dynamic	Questions: Knowledge /Closed	Questions: Strategy /Open	Talking (Time in seconds): Teacher	Talking (Time in seconds): Student	Use of Materials	I	P	Description of Minute (Include errors and inappropriate language) T = teacher, CH = children
1.				7	33				Gives out cards labelled 1 to 20 to sort into number line, CH discuss the task. They complete the task and tell T.
2.		/		27	18				T asked students are they sure they have it right, and is then interrupted by a child from a different group.
3.		//		25	15				How many squares? T repeats the question. Hides one number, then turns over all the other below it. Now how many are there?
4.		///		16	28				Turns over the number 14, CH answers "14" and another CH explains why there are 14. Turns over all other tiles to show correct.
5.		/	/	38	19				Square tiles turned over and the same activity with a different number.
6.		/	/	26	25				T asks Do you think she is right? CH respond with explanations. New Q in context of lollies
7.		/		34	11				Six lollies plus 5 lollies, how many altogether? CH think independently, no discussion
8.		//	/	18	30				CH says 11. T asks who else thinks its 11? Asks CH to explain, then another to check, sends two CH away to work
9.				5	1				CH organising line, very slow, confusion between orientation of 6 and 9. T watches patiently, only interferes when wrong.
10		//		22	16				Ch decided 6 and 9 are different. T elicits answers, then gives counters of particular
11		///		15	20				CH places counters in correct positions along the number line, after instructions of what T wants him to do.
12		//		17	12				T gives another group of counters of a different number. CH counts them and puts them on number line. Tidies away.

	Book No.	Page No.	WALT	Prior Knowledge Check	Context	Number Choices	Materials
Book	5	5/29	Ordering numerals				Cards 1-10
Enacted Lesson				Used lines of cards	Lollies at times	Within range	Cards 1-20

Instructional components of ACT Framework								
Eliciting (Solution Methods)	✓	When	**Supporting (Conceptual Understanding)**	✓	When	**Extending (Mathematical Thinking)**	✓	When
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods	✓		Provides background knowledge			Encourages mathematical reflection		
Encourages elaboration of students' responses	✓		Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts	✓		Assists individual students in clarifying their own solution methods	✓		Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking	✓		Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement	✓		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed			Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions					
			Encourages students to request assistance (only when needed)					
TOTALS	4			2				

Figure 5.3. Gina: V_1 first group encoding using the DART framework.

The first video, (V_1), was recorded at the beginning of Gina's first year of teaching, and the second, (V_2), was recorded close to the end of her second year, so they were separated by about 21 months. Each video is approximately an hour long, and both lessons involved teaching two separate groups of students.

Encoding in DART

This segment will briefly examine the manner in which DART captured information on Gina's teaching. The DART encoding for Gina's first group on the first video (V_1) are shown in Figure 5.3. On the first page, DART codes lesson activities in one-minute intervals, and the second page derived from the ACT framework (Fraivillig et al., 1999) that captures teacher actions. One such action seen in Gina's first video, was that of "Encourages elaboration of students' responses".

The coding of the lesson that Gina taught can be seen in Figure 5.3. This was similar to the "counting on" lesson she talked about in her interview. Here, the group was taught by Gina for 12 minutes - represented on DART under the "group dynamic" heading - questions used, time spent talking, and materials/stages of the teaching model (Ministry of Education, 2005a). I will now examine each of these sections in detail to see what evidence there is regarding quality teaching.

Group dynamic (V_1)

The "group dynamic" analysis on the DART recording sheet from the first video (V_1), filmed at the beginning of the study, shows Gina, for the most part, talking to the group during the first eight minutes, with the group making responses. Between the third and fifth minute, the children in the group were working on problems set by the teacher, who was still talking to them, but by the eighth minute a student is talking to the group and the teacher, showing his/her solution to the current problem, which was adding the numbers 5 and 6. It was at this point Gina sent some children away to work on their own, which, as mentioned earlier, appeared to have been a lost opportunity to press for understanding and "extend" the mathematics. For instance, Gina could have seized this teachable moment and asked the group to use the same strategy that this student talked about to solve the problem. Had she done so, this would have been classified as extending behaviour under the ACT framework. This observation connects with evidence from the interview where Gina feared she might miss the important parts of lessons where she could extend students' understanding, the teachable moments.

For the rest of 12 minutes for this group, Gina treated the two remaining children separately, and talked to each of them in turn, setting them a problem to complete on their own, and finally, after another sending off, only one student remained. Gina used a typical NDP teaching strategy of sending off successive sets of children from the group, to work away from her, because they have shown that they have assimilated this current knowledge, and so they were sent to consolidate it. Evidence supporting this conclusion is seen in the "description of the minute" section on the far right of Figure 5.3. However, in reality, there was little "group" work going on in the lesson. The children were grouped by current mathematics learning need, but there is little encouragement for these students to behave as a group. What would be classified as effective group work under NDP might include students discussing answers together, explaining solution pathways, and argumentation. None of the behaviours typically ascribed to groups was encouraged by Gina in this lesson.

A similar pattern continued throughout the second group seen in V_1 of that initial recording session. There is one minute in which a student talks to the whole group, again showing his/her method of solving the question, the subtraction 19 – 6. For about half the time (10 minutes) Gina talked to the children, and they spoke as a group back to her, giving her answers to closed questions, or counting back with her, and for the other half she spoke to individuals that she identified as in need of further practice.

In summary, in the early months of Gina's teaching career she used the NDP way of group teaching. She taught groups of students combined together because they exhibited particular strategy stages in common. However, she failed to harness the power of the group to help teach each other, and to have more mathematics spoken about between the people in the groups. This is a basic tenet of NDP teaching, and in what follows, the "group dynamic" section on the second video (V_2) will be examined to see if there has been discernible progress on these quality teaching aspects.

Group dynamic (V_2)

The second video (V_2), recorded nearly two years later (see Figure 5.4) has many similarities to the first as Gina again teaches in groups; the first group for 12 minutes and the second for 15 minutes. This first group was learning to add numbers such as 9 + 5, rolling over a decade, which indicates that the answer will need students to understand what happens with trading up as the answer moves into double digits. She is teaching the second group to progress from subtracting by counting backwards through to subtracting using known number facts. One problem Gina used in this section was the subtraction 34 – 3.

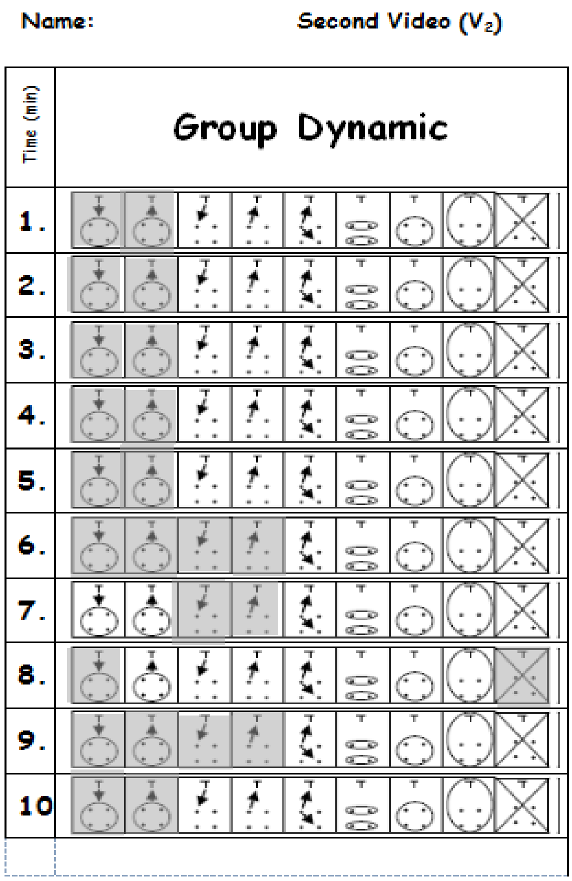


Figure 5.4. Gina: V_2 first group encoding using DART.

In the group dynamic section of the DART tool, for the first group there is little discernible difference from the first video. Gina's behaviour echoed her previous way of working when she first started teaching. For the first group (V_2), the DART highlighting for her lesson is all over to the left, as it was in the first video, indicating that she was talking to the whole group, and on occasion to individuals, who also talked back, mostly answering her queries. For the second group (see Figure 5.5) there are more times when Gina had the students working together as a group, explaining their thinking to others, and working as a group to solve one problem.

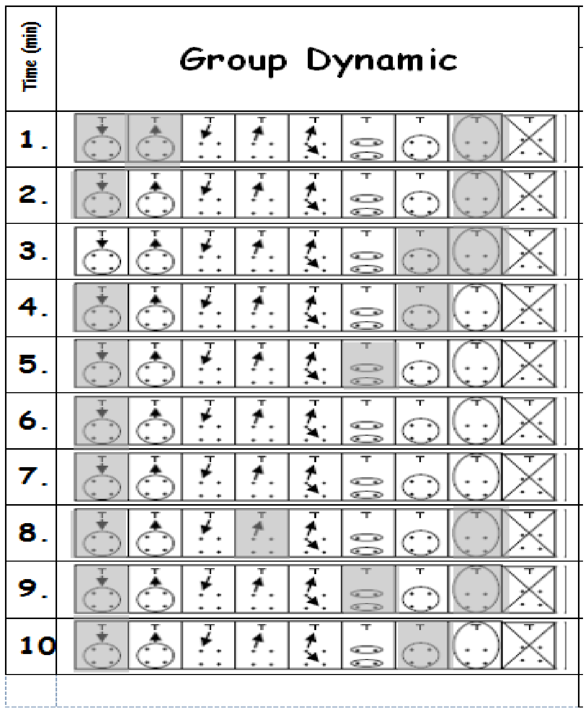


Figure 5.5. Gina: V_2 second group encoding using DART.

In the fifth minute she instructed the students to practice in pairs. This was evidence that she has progressed when teaching groups, especially in this second group where she asked for the strategy used by a student for the subtraction: 17 – 5 =12. The student explained that she imagined a number line and jumped back along it to find the answer. The next subtraction, of 34 – 3, elicited 3 different answers, and Gina then asked each of the students to explain their thinking. Also, with this second group on V_2, Gina asked the students to check their correct answer by using a different strategy, suggesting they take one that they had not previously used. So progress into quality teaching is definitely seen in this area, Gina is using the strength of the group to increase the mathematical talk, and peer teaching aspects, which were missing from V_1.

In summary, the DART framework has captured some changes to the quality of Gina's teaching over the two years, illustrating the different ways she used the strength of the group to move the mathematics forward. Instead of directing all the teaching and learning through the teacher, she organised the groups to work more collaboratively, sharing some understandings. Further sections of DART analysis will be used in a similar way, to tease out whether there were more increases in quality teaching aspects and to assess the value of the framework.

Questions: closed/open

In the questions section in both videos, V_1 and V_2, Gina asked many questions, as would be expected during this type of group work, with the majority of them being of the closed type, such as “How many now?” or “Can you tell me…?” These types of questions are essential to quality teaching of mathematics; they elicit answers, often one word answers, which allow the teacher to quickly assess the current understanding or prior knowledge of the students. What Gina did not do, however, and this is crucial in developing deep conceptual connectivity, is to extend the students’ thinking. This requires questions that are more open (Mason, 2000) and take the format of “Tell me about that”, or “Explain your reasoning”, giving students much more scope for argumentation and discussion. Mason (2012) posits that the ultimate open question or meta-question is “What question am I going to ask you?” On these recordings, Gina is never observed asking a meta-question such as this.

Table 5.2
Gina: Number of Types of Questions used on V1 and V2 Encoding using DART.

		Questions	
		Knowledge/closed	**Strategy/open**
Video 1	**Group 1**	18	3
	Group 2	10	3
Video 2	**Group 1**	21	4
	Group 2	7	1

As can be seen in Table 5.2, in V_1, group 1, Gina asked 18 closed and 3 open questions, with one of these open questions being “How are we going to work that out?” It is not possible to answer this question with a one-word answer, and it is more likely to elicit a more lengthy response, with, perhaps, evidence of strategies. When, in the eighth minute, Gina asked a student to explain his answer, she provided an opportunity for an extended piece of dialogue, in which the student will often re-think their solution path. It is self-evident that if a student is only listening and never talking, then the understanding the student holds is not available to the teacher. The teacher must ask questions; it is the type of question, and how much teachers actively listen to the answers or ensuing discussion that will feed into the quality of the teaching (Hunter & Anthony, 2011). So next I examine the questions asked in V_2.

In $V_{2,}$ group 1, Gina asked 21 closed questions and 4 open questions, but in the second group on V_2 she asked only 7 closed questions and one open question. This is quite different to the previous V_1 groups, and is probably a result of the types of questions she asked. She instructed group 2 more, but she also gave more time for the groups to think and discuss among themselves. This is indicative of a change to more quality teaching, showing that Gina was more ready to encourage the group to work together and to listen to each other. The difficulty with asking more open questions is that you can never be sure what the answer will be. Gina appeared to have eased her grip on the ways of working in the groups, allowing the responses to more varied and, when she needed to respond in novel situations, Gina was more relaxed about her ability to respond appropriately. This is a factor in quality teaching which the DART framework is capturing and highlighting.

Next, where DART captures the number of seconds talked by the teacher and students in each minute, is examined to see if these changes in types of questions are reflected in the timings.

Talking: Time in seconds

The talking time section of DART captures the number of seconds in each minute that the teacher speaks, and also the same for the students. This is considered an important aspect of a quality lesson because if a teacher is talking, then generally speaking, the students will not be. And if students are not talking, then many teaching opportunities may be lost.

In Figure 5.6, the graph shows that during $V_{1,}$ the first group of students, collectively, talked less than the teacher, with 250 seconds in total for Gina and 228 seconds for all the students collectively. There were times when the teacher talked less than the students, but this was only in the first minute, when the students worked as a group to sort out the equipment, and three other single minute occasions, all of which involve one student explaining their thinking to the group or to the teacher. In the other sections, Gina was directing the lesson during the handing out and collecting in of equipment. It is evident from the types of questions she asked and the lack of latitude allowed to the students, that Gina would have been in charge, imposing her authority on the group.

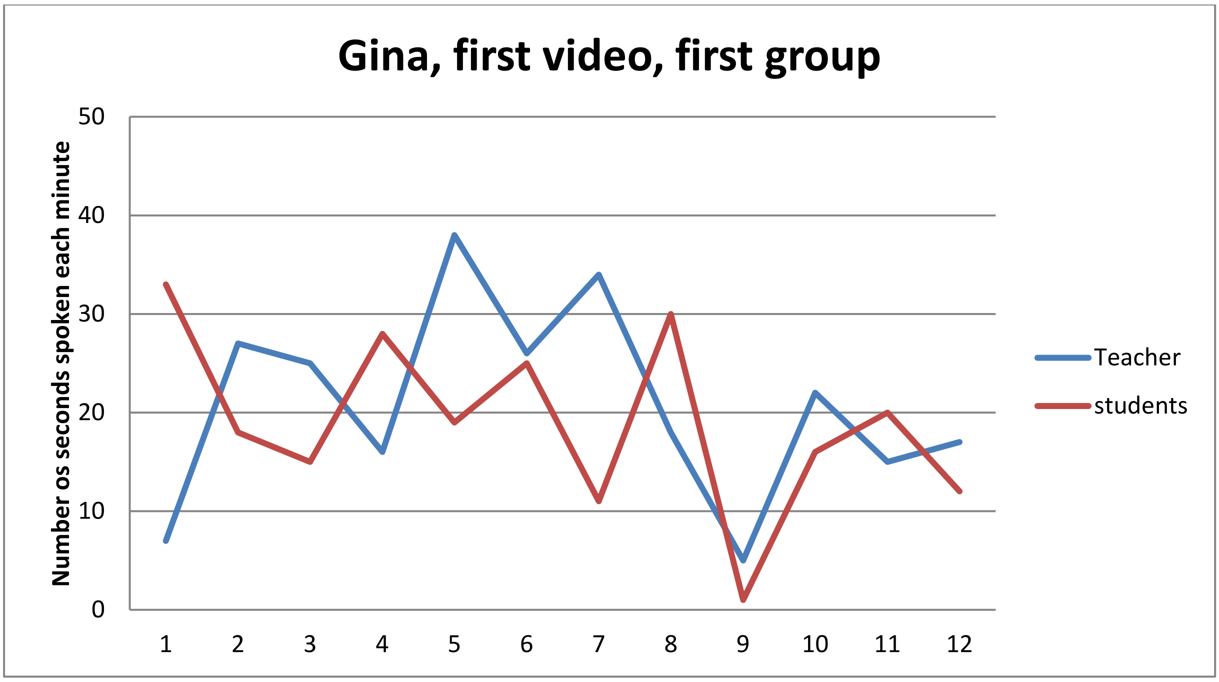


Figure 5.6. Gina's first video, V_1, first group, time talking in each minute, teacher and students.

There is robust research evidence (Boaler, 2002; Lampert, 2001; Staples, 2008) to show that in classrooms where teachers maintain a strong hold on, and authority over, the lesson, the students become passive learners, and indeed this is what was observed in much of Gina's teaching. It was probably to be expected given her declarations in the initial interview about working at the edge of her knowledge, and wanting to keep up with the students. It would appear that Gina restricted the working space of the students to maintain control over the situation. According to Boaler (2002), in classrooms where the authority is more distributed, students develop more positive dispositions towards the learning of mathematics and become more independent learners.

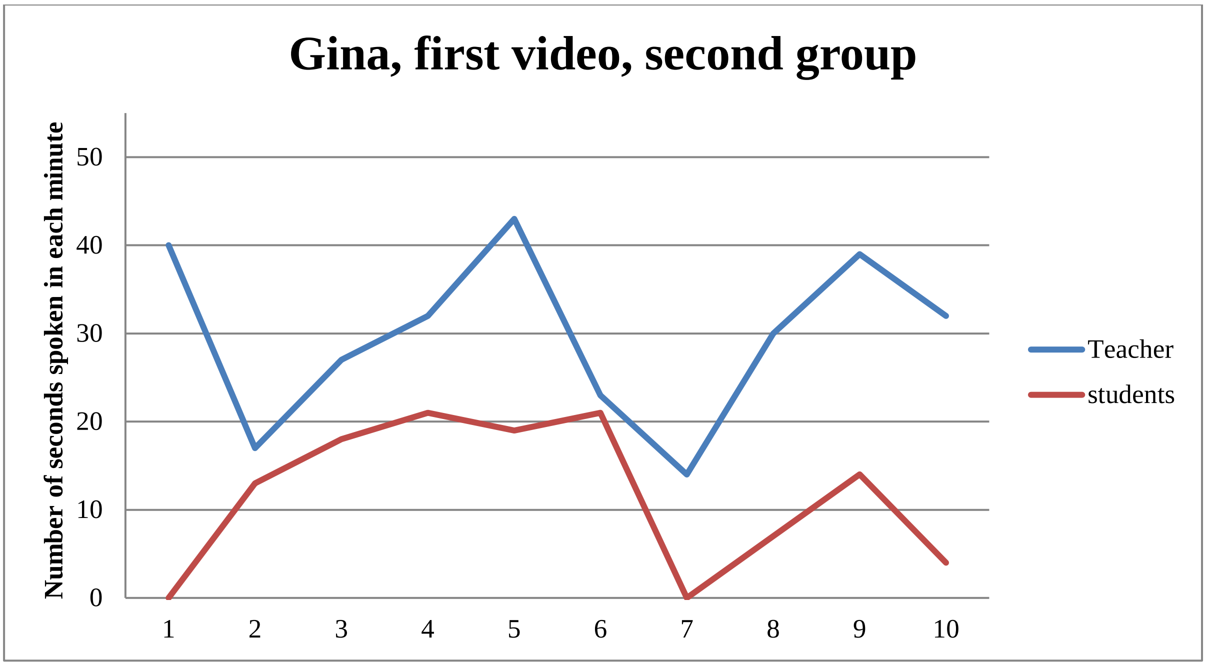


Figure 5.7. Gina's first video, V_1, second group, time talking in each minute, teacher and students.

The graph in Figure 5.7 shows that Gina was even more dominant during her second group seen in V_1. In these 10 minutes she managed to talk over two and a half times as much as the students: Gina, 297 seconds, and the students, 117 seconds. The content of this lesson was higher on the NDP framework and therefore the mathematics was at a more difficult conceptual level than the previous group work, so perhaps it is not unreasonable, given her self-professed difficulties with the more advanced content, that she becomes ever more controlling.

This section on the amount of teacher talk versus student talk is significant because it speaks to the likely quality of the instruction. Gina appeared to have a tendency not to listen to her students and nor use the knowledge gained while listening to organise the trajectory of the lesson. Indeed, the lesson plan she had, suggests she feared to deviate in case, in her words, the children asked her something to which she did not know the answer. The second video (V_2) will now be examined to see if any change in quality teaching may be inferred.

Figure 5.8 indicates that, nearly two years' later, there is very little difference in V_2, the first group, to the previous DART analysis. Gina talked for 269 seconds, more than twice the students' rate of 132. However, the graph in Figure 5.9, from V_2, the second group, shows

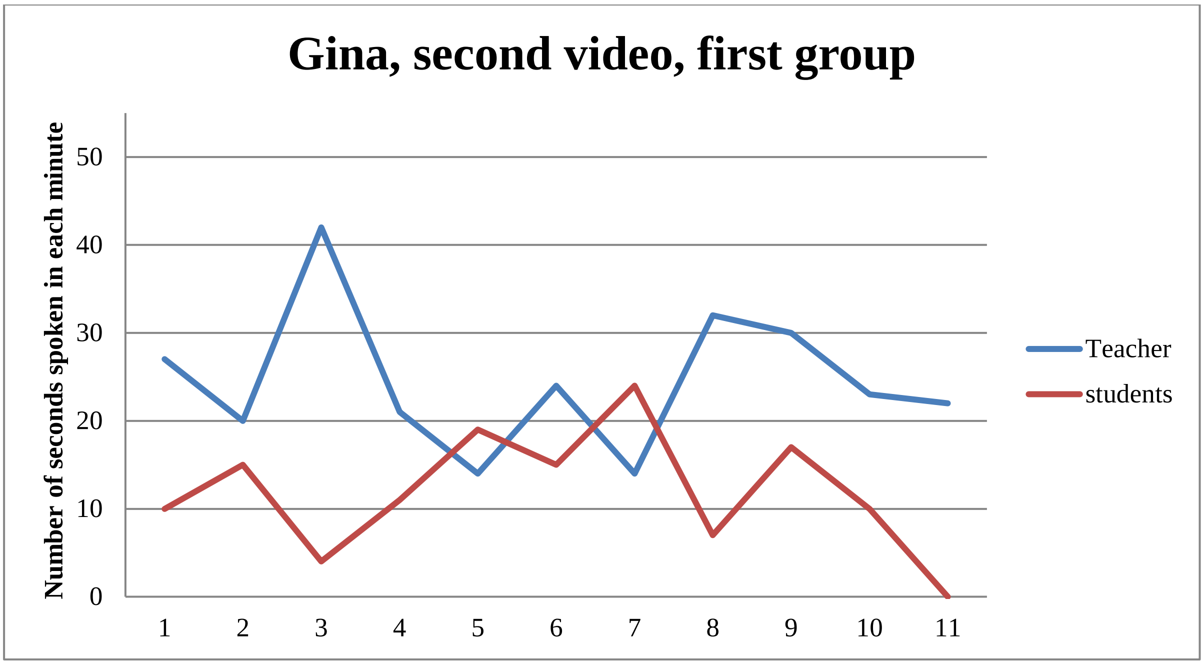


Figure 5.8. Gina's second video, V_2, first group, time talking, in each minute, teacher and students.

some remarkable differences. For close to half of the session the students talked more than Gina, and the total number of seconds spoken is almost the same for both, with Gina talking for 473 seconds and the students for 446 seconds.

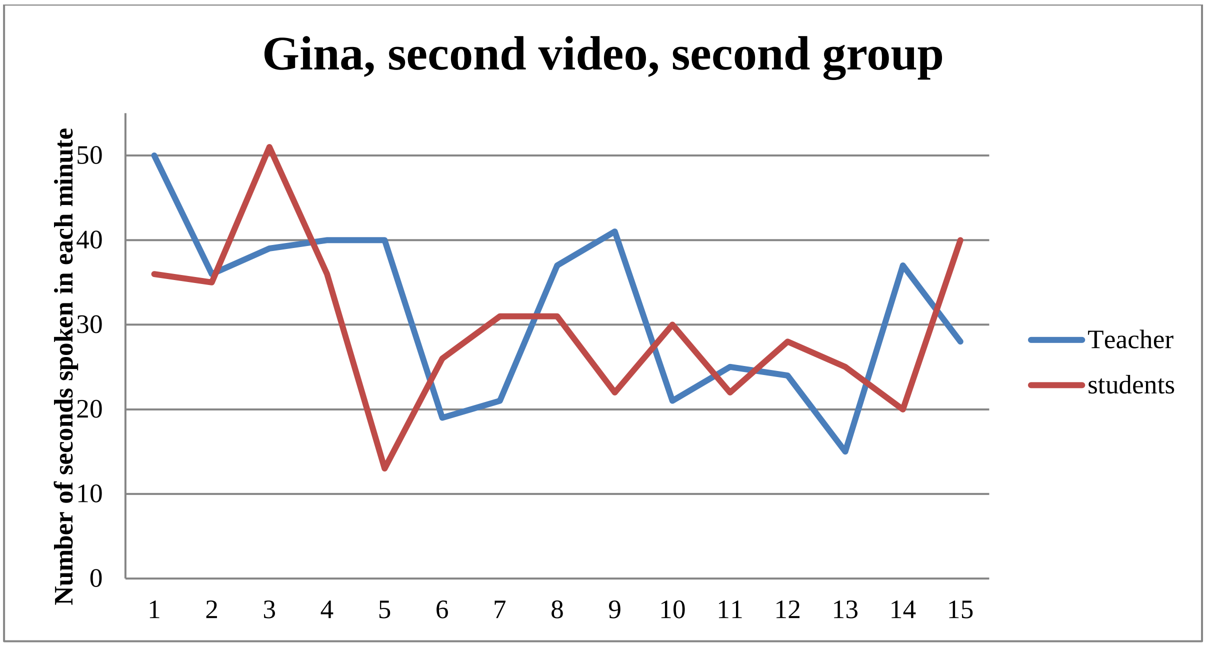


Figure 5.9. Gina's second video, V_2, second group, time talking in each minute, teacher and students.

The graph in Figure 5.9 illustrates the difference in talking on V_2 for group 2. The students were seen to be speaking and interacting more often, with less teacher talk. When Gina made a mistake in the fourth minute, she showed poise. Instead of becoming flustered as she had two years previously, Gina looked calm and carried on. She apparently knew that although her question asked the students to subtract across a decade boundary, which was not supposed to be part of this lesson, she continued on to see what would happen, and she was rewarded with the students carrying on as normal and answering the question as though the level of difficulty had not been increased. It is this, coupled with the previous analysis of V_1 in DART, which is important, as it indicates an increase in quality teaching.

The second group in the second video shows evidence of an increase in quality teaching, through the use of more open questions and less dominance of teacher talk. Gina appeared more relaxed. In her first teaching sequence she was very controlling and loath to depart from the script. She appeared to stay with her teaching plan regardless of what student responses were, but in the second group, second video, Gina noticed that the wrong question in terms of difficulty has been asked, but this time, instead of appearing flustered, and changing her mind about how to proceed, she showed a confidence not seen before.

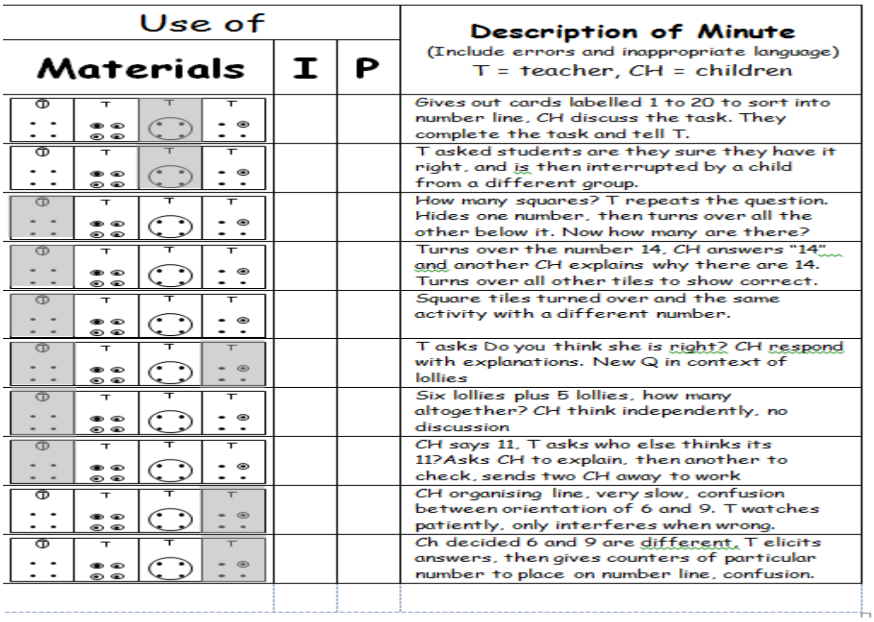
Next, this analysis continues with the sections to the right of the first page of DART. Evidence from both videos are used to compare changes that may be attributable developing quality teaching, as captured by the DART framework.

Use of the NDP teaching model: Materials, imaging and number properties

The NDP teaching model is a simplified version of the layers of growth in mathematical understanding that was first hypothesised by Pirie and Kieren (1989). The NDP teaching model is captured in the materials, imagining and number properties section of DART.

On the right side of the DART, the section that captures details about the use of materials, whether the lesson is encouraging imaging, or if the session is focused more about the properties of numbers being worked on. These three sections - materials, imaging and properties - reflect the NDP nature of the lessons being taught, which in turn reflect the NDP teaching model (Hughes, 2002).

In V_1, group 1 (see Figure 5.10) it can be seen that Gina was working exclusively with materials. She was using individual tiles with numerals on them, and for the most part, she was the one manipulating them. Within the first two minutes of the lesson, the students were organising the number line with these tiles, under Gina's instructions. From the second to the eighth minute, Gina manipulated the materials to set the questions for the students to answer. For the subsequent two minutes, the students manipulated the line, using the materials to try to answer the question.



Use of Materials	I	P	Description of Minute (Include errors and inappropriate language) T = teacher, CH = children
			Gives out cards labelled 1 to 20 to sort into number line, CH discuss the task. They complete the task and tell T.
			T asked students are they sure they have it right, and is then interrupted by a child from a different group.
			How many squares? T repeats the question. Hides one number, then turns over all the other below it. Now how many are there?
			Turns over the number 14, CH answers "14" and another CH explains why there are 14. Turns over all other tiles to show correct.
			Square tiles turned over and the same activity with a different number.
			T asks Do you think she is right? CH respond with explanations. New Q in context of lollies
			Six lollies plus 5 lollies, how many altogether? CH think independently, no discussion
			CH says 11, T asks who else thinks its 11?Asks CH to explain, then another to check, sends two CH away to work
			CH organising line, very slow, confusion between orientation of 6 and 9. T watches patiently, only interferes when wrong.
			Ch decided 6 and 9 are different, T elicits answers, then gives counters of particular number to place on number line, confusion.

Figure 5.10. Gina's first video, V_1, first group, use of materials, imaging and number properties with description of the teaching sequence.

Gina was the one doing most of the manipulation of the materials, and when the students were manipulating them, it was at the request of Gina. Again it is seen that Gina controlled the lesson. The recommendation from NDP is that students often have their own set of the equipment and will manipulate it for themselves, but at other times the use of demonstration by the teacher followed by working together, as exhibited in this first group of V_1, is recommended. To see whether the use of materials reported here is appropriate, the next section will look at the lesson Gina sourced from the NDP resources *Book 5: Teaching Addition, Subtraction, and Place Value (MOE, 2007, p. 18)*

NDP: Resource books

In the first video, V_1, first group, Gina used materials that embodied the concept of "counting on" and which are recommended in the NDP Book 5 (p. 18). By both choosing to use and adhere closely to the lesson in the NDP resources book, Gina could be sure of adding quality to her teaching, because the lesson was developed and trialled by expert teachers. In this way, as all the decisions about the lesson are made for her, Gina would tap into the expertise embedded in the activities, and, carried out to a fair level of completeness, this lesson would have a semblance of a quality lesson. The extent to which Gina adhered to this lesson is now examined.

Table 5.3

Continuum of Adherence to NDP Lessons in the Resources Books

NDP resource books	
Used as an overlay	**Used as central to session**
Uses mathematical idea (e.g., adding unit fractions) and little else	Context, story shell, actions
No materials or sometimes materials but teacher manipulates them	Appropriate materials, manipulated by students
Recommendations for magnitude of numbers ignored	Uses numbers as shown and makes up more in same magnitude
No use of NDP Model (materials/imaging/number properties)	Uses NDP model
Stays determinedly within a modality, regardless of responses	Moves back and forth between modalities as required

Table 5.3 shows the factors of quality teaching from NDP resource books (the "pink books"), moving along the continuum, from using the lessons as an overlay through to strong adherence. Using tenets of NDP as overlay, and paying only lip-service to the factors negates the aspects of quality lessons mentioned above. For the resource books to deliver on their promise of quality lessons for quality teaching, the teaching model and the recommendations for materials and so on have to be used. The material that is recommended for V_1 Group 1 is cards/tiles labelled from 1 to 20. As can be seen in Figure 5.11, DART captured evidence of Gina's use of these cards, but for the most part, she does not use a context when setting the problems for the students to solve. This is a significant departure from the NDP methodology, which will be discussed further below. The first contextualised problem is:

> Fetu has nine lollies in one bag and two lollies in another bag. How many lollies does Fetu have altogether?

The use of context is absolutely central both to NDP and statements within the curriculum document (Ministry of Education, 2007), as it makes the mathematics students are learning more meaningful, and intrinsically more interesting (Alton-Lee, 2003). Contexts bring mathematics alive for students, giving them something real to aid their thinking (Watson, 2004), and increases the likelihood of students choosing to abandon a wrong answer when the context informs them that it does not match well with the problem.

NDP lessons in the resource books often give one context as an example, and then state that "word stories" are to be used. In the curriculum document, at the top of every page that shows the content of the various levels, there is the following statement:

> In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to…

Gina does not use the given context or any other context in the first video, V_1, until the seventh minute, when she introduces a problem about lollies that uses the numbers 6 and 5, and from then on the use of a context is no longer seen in Gina's lesson. Gina also departs from instructions regarding appropriate numbers to use: the instructions for the lesson state that the second number being added should be less than 5, and read as:

> Examples (second number is 5 or less): Word problems **[make up a suitable context]** and recording for:
>
> 4 + 2. 7 + 2. 12 + 2. 7 + 3. 12 + 3. 16 + 3…
>
> [these are the suggested numbers]

These suggested numbers recommend that the adding-on numbers are much smaller than 5, while the starting number increases from 4 to 16, and upwards. Gina begins with 6, adding on 5 and then did not follow the recommendations by increasing the first number; instead, she went back to checking that the students understood that the final number in the count means that number of tiles is in the line. At the beginning of the lesson Gina asked the students to turn the number tiles over, to be face down. This is something found in the "using imaging" section, which is a step on from the "using materials" section. It seems that Gina confused the progression in the lessons, and often looked to be dragging numbers out of the air since she often changed her mind once she had said them.

The question is, can we see any differences in the data captured by DART in the second video? Allowing for her inexperience in teaching mathematics in V_1, how have the markers of quality teaching changed, and in which direction?

Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
			Prior knowledge warm up. Counting backwards from 64, to the decade, counting down using 100 square
			T: 52. S continue to count down, stop at 50. Now repeats but starts at 100 to 90, uses 100 square. Now 38 back to 30 backwards
			Counting backwards continues using the 100 square. Then T turns 100 square over, continue without.
			T makes a mistake crossing the decade, without the 100 square to help. But S respond positively. Warm up finishes
			T says to practice in pairs, subtraction problem 17 to 5 without seeing the 100 square (imaging), using counting down
			T says 17-5 = 12. Asks S to tell her how they did it by showing on the number line.
			T writes on modelling book 34-3. No instructions to visualise, so moving into number properties.
			T asks for answers and gets 30, 32 then 31. T picks a S and send him off, saying she knows he knows, play a game.
			S all count together, using their fingers (materials). Next problem 27-5. Fingers on nose when they have the answer
			Continue 27-5 counting down and checking. S says the right answer of 22.

Figure 5.11. Gina's second video, V_2, second group, use of materials, imaging and number properties with description of the teaching sequence.

In V_2, (see Figure 5.11), Gina taught two groups. With the first group, Gina used NDP Book 5, page 20, and with the second group she used NDP Book 5, page 22 (Ministry of Education, 2007). These successive pages illustrate that she was teaching these two groups in a differentiated manner, but was she using the lessons as overlay or as central to session, or as something in between on the continuum (see Table 5.3). The first thing to notice is that Gina persisted in teaching without a context where previously context was shown to be important. DART captures this, as can be seen in "description of minute". In the resource books, both these lessons have an engaging context: the first is about bears going into a cave, where an opaque container is used to hide the number of bears, but the bears to be added on are on the outside and visible to students; the second context is about a child having a number of sweets, eating some, and then finding out how many are left. The materials that are used by Gina are appropriate and so was the use of hiding the materials, by turning them face down, to force the use of imaging. Again, as in V_1, Gina confused the order of numbers, and ignored the progression of the numbers, but in V_2 group 2, this changed. She showed more confidence in her choice of numbers, as discussed earlier. Rather than pulling numbers out of the air and continually changing her mind, she now chose the numbers with confidence, she remained with her choice, even when she noticed she may have made a mistake in pushing the difficulty of the mathematics along too quickly; the indecision seemed to have left her. This is a positive development and I would posit that evidence of an increase in quality teaching is apparent at this point. Another thing to notice in V_2, group 2, is that Gina moved in and out of the imaging and number properties areas, augmented by the use of materials to support students' thinking. Gina was able to analyse what the students needed to consolidate the mathematics; for instance, when three different answers were given by the students to the subtraction, 34 – 3, (minute 8, on DART, Figure 5.12), she did not ignore this difficulty but embraced it, not just moving through the mistake which she would, by her own admission in the interview, have done earlier.

The evidence from the second page of DART, that which gave rise to the ACT measure, will now be analysed to see if it supports this idea that Gina has increased her quality teaching over the two years she has been a pre-registration teacher.

Instructional components of ACT framework

The second page of DART, using components of the ACT (Advancing Children's Thinking) framework (Fraivillig et al., 1999), was completed and analysed separately for both groups in both V_1 and V_2.

Instructional components of ACT Framework								
Eliciting (Solution Methods)	✓	When	**Supporting (Conceptual Understanding)**	✓	When	**Extending (Mathematical Thinking)**	✓	When
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods	✓		Provides background knowledge			Encourages mathematical reflection		
Encourages elaboration of students' responses	✓		Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts	✓		Assists individual students in clarifying their own solution methods	✓		Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking	✓		Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays			Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement	✓		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed			Supports describer's and listeners' thinking			Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions					
			Encourages students to request assistance (only when needed)					
TOTALS	4			2				

Figure 5.12. Gina's first video, V_2, first group, instructional components of ACT framework.

An example of this page can be seen in Figure 5.12, encoded for the first video. When behaviour is seen on the video, the coder ticks to say the behaviour is present.

When all the data on the four pages of encoding in DART are accumulated, the data show that Gina used the eliciting (solution methods) and supporting (conceptual understanding) zones but never the extending (mathematical thinking) one. Fraivillig et al. (1999) report that the teachers in their study were often found to use supporting in classrooms, but less often the eliciting and extending components. The reasons posited in the original research by Fraivillig et al. (1999) for this imbalance are that each of these three areas requires different kinds of pedagogical skills. For instance, to support students' thinking, teachers often break down a problem, or offer a demonstration, and are usually very comfortable with this more traditional level of skill. However, when eliciting and extending students' solution methods, teachers require a more complex understanding of the mathematics, and its progression, coupled with the need to "relax intellectual control" (p. 169), which teachers with less than adequate understanding of mathematics often lack.

To some extent, Gina followed this pattern. Her use of these components will now be looked at in more depth, to see if any further inferences may be made.

Taking the two teaching groups together in V_1, Gina's pedagogical practice produced five different ticks in eliciting and four different ticks in supporting, but, as noted before, none in the extending zone. In V_2, her teaching produced four ticks in eliciting and four in supporting, and again none in the extending zone. On the one hand, it is encouraging to see Gina exhibiting behaviours under the eliciting section where few were found in the 1999 research (Fraivillig et al., 1999), but overall, between the two videoing occasions, V_1 and V_2, there has been little movement into other behaviours. Fraivillig et al. (1999) report that to have more of the extending behaviours, a change in pedagogical skill seems to be required. They describe how the move away from more traditional styles of teaching, where the teacher controls the class and the students are expected to reproduce teacher actions, into more child-centred teaching (reform teaching style) is difficult for many experienced teachers, requiring much determination on the teacher's part. I would expect a novice teacher, such as Gina, to have difficulties at first, but perhaps I might also expect to see some movement into extending behaviours, where a teacher "goes beyond initial solution methods" and "uses students' responses, questions, and problems as core lesson"—two of the behaviours identified in the ACT framework.

In summary: What evidence is there for a change in quality teaching of Gina?

There is evidence that Gina has made some progress in the first two years of her teaching. She appears more calm and confident in her ability to choose numbers of the appropriate magnitude and a little more relaxed, able to allow the students more latitude in their mathematics learning. The evidence that supports this argument includes:

- an improved MKT score
- more work in groups, with peers teaching each other
- less teacher talk, more student interaction
- rising confidence, as seen in V_2 with number choices.

Evidence of improved quality teaching still lacks a vector—what influenced this improvement? Factors of influence may be inferred from the evidence. Time is a factor, but as we do not know exactly what experiences Gina had in the two years of pre-registration, we must look at the factors that have changed with time. In the classroom Gina continued to work with young students, teaching mathematics at a low level on the curriculum, but in her MKT score, Gina can be seen to have improved in mathematics that she would not have been teaching in her classroom to her young students. All three questions contained mathematics at a higher level than she was teaching to her Year 2 students. How could this happen? I would hypothesise that her confidence in her own ability to complete mathematics problems has pushed her to a level where the harder problem-solving and teacher-focused questions are no longer a mystery. My argument would be thus: In the three questions representing MKT taken at the start of this study, Gina scored 2 out of eleven. The first question is about dividing two fractions, the second requires sophisticated understanding of the multiplication algorithm and its different forms, and the third is the divisibility by 4 rule. In the exit MKT test, Gina demonstrates that she understands how division of fractions works, and she analysed wrong answers from students to say why they were wrong. In questions requiring the analysis of the use of an algorithm, in the second MKT she showed she understood how multiplication works and was able to say why the students were wrong. While there was no strong match to the third question of the divisibility rule, there was one on other rules, which apart from a problem with negative numbers, she was able to answer correctly, showing, for instance, that she knows that adding a zero to a number to multiply by 10 does not always work in all types of numbers.

Another factor that could be influencing her is maths anxiety; Gina scored the highest of all the participants at 45 out of 50, where 50 is the highest level of maths anxiety. Gina was well aware of her maths anxiety, she spoke about in her interview, and cited it as the reason she had to give up a medical career. It could be inferred that her maths anxiety may have abated throughout these two years. She would have had a calm two years' teaching at a low curriculum level, a level at which she said she was comfortable. And she might have met more mathematics at a higher level through interactions at the school, through other professional development (she had an in-school professional development session on place value) and she mixed with teachers and children who were engaged in that higher level. Whatever happened, I would infer that her level of maths anxiety had abated in real terms, but whether she would still score highly is debatable, because she could still feel highly maths anxious; however, her level of mathematics ability is attested to by her high MKT level.

From this case study, the evidence shows (see Figure 5.13) that Gina has developed a more differentiated style of teaching in that she now appears more confident and less controlling. She remains teaching in the younger age group, but her use of appropriate numbers in group work and her encouragement of more effective behaviours in her students give reason to be optimistic about Gina's ability to develop into the sort of teacher she wanted to be. In her interview she stated that an ideal mathematics teacher would be able to know mathematics so well that when students had a problem that teacher could analyse the problem and know exactly what to do next, rather than rely on the text book. The next question is, how could Gina develop into that kind of teacher? This will be discussed further in Chapter 6.

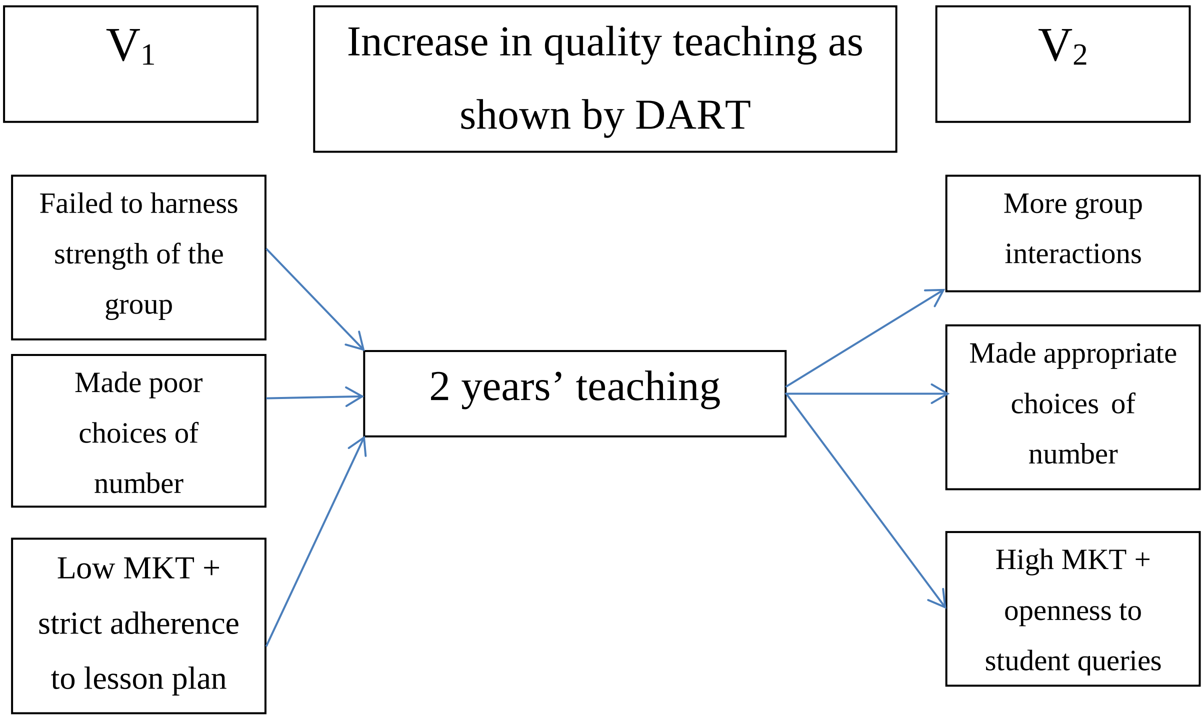


Figure 5.13. Diagram illustrating positive change in Gina's quality teaching

Summary: Gina's journey

So far in this chapter we have considered a case study of one interesting, pre-registration teacher, Gina. As a new teacher, Gina was likely to have multiple areas that required development in her role as a teacher of Year 2 students. Her honesty regarding her mathematical ability was, at times, astonishing, and she had no hesitation in allowing me to hear all the details of her lack of mathematical ability and confidence. Gina developed a more open teaching style in the two years of the study; this allowed more power to devolve to her students, and she made better use of the dynamics inherent in group work. Her much increased MKT score demonstrated her improving mathematical knowledge, something she gave voice to in her interview; she had confidence at the beginning that she could improve it, and the MKT is evidence that she has.

Using the DART system specifically designed to analyse NDP lessons, content and pedagogical methods were captured and discussed in this case study.

A second case study is now presented, that of Debbie; data is generated in a similar fashion for Debbie as was done for Gina, which provides an opportunity to compare and contrast similarities and differences between the two participants. Again aspects of teaching and pedagogy will be captured and analysed using DART. This will also allow the development of the larger picture involving quality teaching and novice teachers.

Case study 2: Debbie

Debbie is an interesting person: she began her teaching career with high scores on both the MKT and ACT measures, but after two years these had both reduced, the MKT by nearly a quarter and the ACT by over a half (see Figure 5.2). These major changes, coupled with other data, will be examined to develop further the picture of the factors that may influence quality teaching, and to continue to evaluate the efficacy of DART as a measuring instrument.

Debbie, aged 20 years at the beginning of this study, was the youngest participant; the average age for the seven participants was 26.1 years. She was close to average in her maths anxiety, while her level of mathematics taken at school was above average, having completed level 2 NCEA and half of level 3 (see Table 5.1). As with the case study of Gina, Debbie's attitude to mathematics will be discussed, through her words taken from the initial interview, from the two videos collected two years apart, and from her results in questionnaires and tests. This will further inform the complexity of a pre-registration primary teacher’s journey through to full teacher status, enabling some comparisons and contrasts between Gina and Debbie.

Attitude to mathematics

In this section, the negative attitude that Debbie held regarding the mathematics teaching she received at school will be contrasted with the more positive attitude she developed once she began mathematics education courses at university. The effect that these courses had on her, and the effects of teaching in schools, will be used to develop the picture of Debbie and mathematics. Debbie credited the experiences she had at school, at least partly, as being responsible for her wish to become a teacher and change the way mathematics was taught; she felt she could become an agent of change. This case study will centre on whether Debbie managed to come close to her wish of making those changes.

Debbie did not have a liking for mathematics at school but unlike Gina, Debbie is more moderate in her opinions, perhaps because she had wanted to be a teacher for some time, and did not view her career choices as being strongly affected by this dislike; she was prepared to accept teaching mathematics as part of the role of the teacher. However, she held strong opinions about the causes of her dislike of mathematics; this next section will highlight some of these.

In her interview, on being asked about her experiences in mathematics at school, Debbie said:

> Text books. Really that’s all I have to say about how maths was at school, its text books and extremely boring, yes. That’s what it was like, especially at high school, I don’t really remember maths at primary school, actually I don’t really remember maths but definitely intermediate and secondary it was text book work, it was teacher does a demo on the whiteboard, now you go and do the exercises by yourself and there were no group activities, there was no co-operative learning, there was not much interaction. It was watch me, do as I do, go and do the exercises, and that was it. It was extremely boring.

When pressed for recollections of primary school mathematics she said that she really did not remember much, which seems extraordinary in one so young. Perhaps these memories have been expunged. Debbie said she did not remember it being enjoyable but that it made her want to be a teacher, so that she could teach in a different way:

> I don’t really have very good memories of school, so I think it’s maybe why I want to be a teacher because I want to actually make that difference. Because I want to

> actually make school fun for kids because that's what it should be. It should be exciting to want to learn new things.

Here she is motivated by her poor start in mathematics learning. She aimed to improve what for her was a dire situation, to make a difference, to teach mathematics in a way that is not boring, which encourages students to engage with the mathematics.

Later in her interview, Debbie went on to describe how one very helpful teacher gave her extra sessions, but these were just as unengaging as the regular lessons were, still being text-book bound. When asked if she had learnt anything, she responded:

> No. I actually didn't, you know what, it wouldn't stick, it would not stick, and it was so boring and just so monotonous. It just wouldn't stick in my mind and I did so badly in my exams, so bad.

Despite this poor start and her disappointment at not being able to do well, Debbie's negative attitude began to change as she started her teacher education mathematics courses at university. This was a surprise to her, as she had never expected to have more positive experiences in mathematics, saying:

> I thought to myself when I came here [to university], I was like oh maths is going to be my weak point because it was at school, but it's actually one of my strong points and since coming here I've absolutely loved maths, I love it. Seeing all the things that you can do and all the exciting stuff that you can do with kids and activities, it just makes it so much different and more pleasurable.

She went on to speak about how the teacher in her mathematics education courses at university made mathematics fun, and how this helped it to stick in her mind:

> Well, especially [teacher's name] class as like, whoa! This is fun, because every lesson that we had was, you know, we had like a maths joke up on the board and that was like a fun start to the day and then we had an activity where it was with groups and we were actually on our feet, you know, walking around the classroom like a bus stop activity or we had a thing outside, where we were doing dance moves, and it was something I can't remember now, but it was just fun. I just remember it being so much fun and I remember just everything he said stuck in my mind and when I did the assignment it was pages, pages, pages long and yeah, and I got full marks. It was amazing.

Debbie’s positive attitude is evident throughout that passage, to get full marks in mathematics education courses was amazing to her. Debbie’s sense of achievement is unmistakeable in all the mathematics educations courses she took at university, as she had achieved 100% in a different year level in her B.Ed studies too. When asked by her lecturers, who remarked that her recall of the content of the courses was remarkable, how she managed to achieve at such a high level, she said:

> I think all of them [lessons at university] really have been memorable; I mean also the strategies as well, because at high school they don’t actually teach you many strategies, different ways of working things out. They teach you one way, follow this way and you'll be fine.

Debbie once more laments how she found learning mathematics at school really hard, saying

> Yeah, so you’ve got to memorise that way, that particular way and it doesn’t actually teach you anything about how to do maths, it just teaches you, just go through this process but it doesn’t really teach you how to work things out for yourself.

Noticeable in the preceding passages, is Debbie’s change of attitude to mathematics in her education courses at university. She is more positive, more determined to do well, and attempting to understand the mathematics at a more complex level. This contrasts with her previous achievement, that of rote and process learning. Debbie was aware that her confidence had changed (probably partly, at least, due to success in her university courses). For Debbie, it really is true that success breeds success, (Goddard, 2001), as on her first practicum, she was already feeling really optimistic about teaching mathematics to Year 8 students.

> They were Year 8s and then that was my first year of [teacher education courses], and I was a little bit scared, but no, it was okay and I think, gradually, as it’s gone on, practicums, you know, I’ve got more confident.

Talking about her subsequent teaching experiences on practicum, Debbie was again optimistic about her own ability to understand difficulties of her students. She described helping students who could not subtract except by counting down. She believed that these students should have been further on in their learning, and Debbie identified with the difficulties they were having, she empathised with them. Having identified the problem, Debbie looked to ways to mitigate it in her teaching.

In the preceding paragraphs, Debbie has shown us how she failed at mathematics at school; her critical words show that she remembered more about the style of teaching than she did the actual content of the lessons.

Figure 5.14. Virtuous circle of Debbie (After René Descartes, 1596–1650).

In Figure 5.14, a diagram of a virtuous circle, from the philosopher René Descartes, is presented (Ackerman, 2013). Once the mathematics that Debbie was learning was perceived as fun, it fed into her motivation and confidence to do well, as in the virtuous circle above. As enjoyment rises, motivation increases and enhanced achievement follows. She worked hard at her university mathematics education courses, had surprise success and her determination to achieve became more solidified in her resolve. Debbie realised that the fun she now saw in mathematics needed to be realised in her teaching, so that she could make a tangible difference in her students' lives. So it can be seen that Debbie used her fear of mathematics (maths anxiety) and her fear of failing to propel her to teach her children to the very best of her ability, and to provide the motivation to continue to improve her ability to teach mathematics.

In the following, I will examine whether Debbie continued with this positive attitude gained from her experiences at university, and if she was able to move beyond her difficult start in mathematics and become the agent of change she envisaged earlier. I will also examine whether there is development in the quality of her teaching.

Debbie's vision for teaching mathematics

In her teacher education courses, Debbie had realised the importance of teaching mathematics well, every day. On her final practicum she was concerned that the mathematics was often pushed aside in the drive for literacy, and by the end of a week, she found she had only completed a few minutes of mathematics. About the time constraints, she said:

> I think the thing was that because they had a huge writing programme, which took up the whole middle slot of the day … there wasn't much time to do maths, it was almost 40 minutes and that wasn't very long, I didn't find it long and I just find, really at the end of the week, I thought gosh I haven't really done as much maths as I really wanted to do, because I think the real constraint was the time thing.

When Debbie was asked what her vision for her future teaching in mathematics was, she said that she was beginning to understand some of the complexity involved in teaching:

> I think I've been learning as I go, yeah, because before I came here I didn't really have much knowledge of maths and what maths really was, and I think I've sort of been learning new things about maths as I've been going through the course, and also even on practicums, I mean I'm still learning new things, but I think it is very complex. You know, you might think oh that didn't really work that well maybe I should've done this activity instead, to explain that strategy better, so next time I'll do this one instead of the one I did. So there was a lot of that going on, yeah.

Debbie's ability to reflect on her mathematics teaching and analyse what she would do to improve it for the students, shows her increasing knowledge of the complexity of teaching mathematics. She is also optimistic, and can see how continual improvement is possible. On being asked how differently she intends to teach in comparison with the teaching she received at school, she was quite emphatic about the importance of making it engaging for the students:

> I've tried to incorporate as much of the same things like the [NDP] you know, the activities trying to actually have hands-on learning, and have it more so that they're [the students] becoming engaged, you know, engaged learning.

To do this, Debbie invoked the idea of one of her university lecturers, whom she would like to emulate:

> His class, it was so much fun. We loved going to [his] class, it was yay we've got maths! Yeah, he was awesome and I used to look at him and think, if I'm anything like [him], you know even in 10 years' time, I'll be absolutely chuffed. You know, because he was awesome and we learnt so much from his classes.

Debbie was asked to expand on her vision for mathematics teaching and to describe what would be going on in an ideal lesson:

> I think I would have lots of maths activities, then, sort of like in boxes you know mixed levels and things like that, yes so I think, I think a lot of interactive things, a lot of games and work, them working together, discussing the strategies and how they've worked it out, and yeah, just really working as groups really, working together. No text books.

Debbie had a strong vision and strong ideas about what she would be doing as a mathematics teacher, and the fact that she had a vision for herself indicates that her difficult start in mathematics seems to have been mitigated through her university courses. The reasons for this appear to be:

- success in university mathematics education courses
- a vision of what an expert teacher might look like (university lecturer)
- delight at realising that mathematics could be fun and engaging (NDP).

Next I examine measures which speak to the quality of Debbie's teaching, through her results from the two MKT tests and her ACT scores.

MKT and ACT scores

As noted above, Debbie viewed her start in mathematics education at school as "extremely boring". However, the modest learning at school that she described is not apparent in her first MKT score (see Figure 5.1), completed before the interview; that score was more than the average score of all participants. Two years later, Debbie had the lowest MKT score (48%, group average 74%). Her score after two years of teaching mathematics was also lower than her score in the first MKT. Perhaps this reflected her experiences in those two years, or perhaps it indicates a diminution of her own mathematical understanding. Or perhaps it is a feature of the original MKT test which involves competence in answering situated mathematics questions, requiring knowledge of both the teaching situation and the mathematics contained within it. Having knowledge about how to teach mathematics is not something we would expect a novice student to have a strong grasp of at the beginning of their teaching careers, even though other participants might appear to. It is a factor that develops while teaching, being situated in a lesson and having to analyse the needs of the students, to choose the most apposite learning trajectory, is complex. In a question involving strategies to find the answer to the multiplication question 8 x 8 when it was not in a student's knowledge set, Debbie understood the various strategies proposed and their validity. However, she found other questions involving higher conceptual ideas, such a multiplication of common and decimal fractions, impossible to answer. Could this could be a feature of the students she had been teaching in the intervening years? Debbie had taught from Year 4 to Year 6, students aged between 9 and 11 years. In that time, she would have taught many students about fractions and decimals, but she possibly would have had many more opportunities to look at strategies for multiplication of whole numbers. It appears that Debbie may have regressed in mathematics knowledge for teaching; it would be interesting, though outside the scope of this study, to see if the measure of her MKT improves over the next few years, as she continues to teach.

If Debbie's mathematics knowledge for teaching has not substantially increased, as it did for Gina, and the test is by no means the final arbiter on this, then that could influence her ACT score too.

Figure 5.2 shows Debbie's ACT scores have reduced between the two occasions of videoing. ACT scores are established by assessing the different teacher behaviours seen on the second sheet of DART. At the beginning, Debbie would use eliciting, supporting, and extending behaviours, such as "Ask all students to attempt to solve difficult problems and to try various solution methods". This is interesting because, according to Fraivillig et al. (1999), few teachers are seen to do this, even those with many years of experience. Fraivillig et al. theorised that to develop facility with these behaviours, teachers need to know the mathematics very well and have high confidence in their own ability to understand the connections and concepts.

In the second video, Debbie's behaviour has changed. She no longer displays a large selection of behaviours in her teaching skill set, and she does not show any extending behaviours. This is most likely related to the lack of improvement in her MKT; however, more detail from other measures is needed to understand what was happening to Debbie. The next section will illuminate, from the MARS and other measures, whether her maths anxiety is a factor militating against quality teaching.

Debbie's maths anxiety

The categorisation of Debbie's maths anxiety as mild on MARS is at odds with other evidence, since, for example, in the interview and elsewhere, she exhibits strong maths anxiety. When asked in the interview about how she would feel about doing mathematics right now, she admitted that this would make her anxious, saying:

> Well, it would probably remind me of high school, because that is what we did at high school. It would probably jog my memory a little bit … a little bit anxious, yeah.

Debbie's maths anxiety score on the MARS was 29 out of a possible top score of 50 (group average of 27, range 10–50). Debbie is close to the middle of the possible scores, which could indicate that her maths anxiety is mild, and perhaps Debbie could be in a neutral position. She did respond to the question on MARS regarding her ability to solve maths problem with a neutral U, which is labelled as uncertain, and she answers U to two other questions. As MARS is a self-reporting Likert scale, there are difficulties with interpreting the meanings of these responses. Her answer of "uncertain" to the final question "Mathematics makes me feel uneasy and confused" would appear to be at odds with the above quotation from the interview about doing maths. Her confusion may well mask a higher level of maths anxiety, and it can be seen that this is possible, that in fact Debbie is very much affected by her maths anxiety. Debbie agrees with the second question on MARS "It wouldn't bother me to take more maths courses", and she may have had in mind the types of mathematics education courses she talked about earlier, where her favourite lecturer made mathematics fun and engaging, rather than the "extremely boring" maths she experienced at school. Debbie was acutely aware of her difficulties with mathematics, but her desire to teach mathematics in a more student-centred way was unquestionable. In her interview, she described her ideal lesson as "I think a lot of interactive things, a lot of games … them [the students] working together, discussing the strategies, and how they've worked it out, and yeah, really working as groups"—not like her received schooling, but very much like her university courses.

Debbie said she thought she would still be anxious about mathematics, even when it was suggested to her that she would be more competent now that she had had success in her mathematics education courses. Her ambivalence is reflected in the next passage, as she talks about doing fractions and decimals as fun:

> Yeah, maybe I'm not sure. Yeah probably would be yeah. Cause I haven't had any anxiety doing any of my assignments that I've done, it's been fine and we've even done [teacher name], we did fractions and decimals and at the beginning I was like oh no fractions and decimals you know but yeah it was good, and he made it really fun and really interesting and I actually enjoyed fractions and decimals.

Debbie sounded less anxious when she considered her performance in class and on assignments but still scored in the middle of the range on the MARS, so there would appear to be some conflict there. She was well aware of how learning mathematics as an adult has shown she could do mathematics, but she seemed not to believe in it, even when she saw how a different teaching style enabled her to learn fractions and decimals. She showed this lack of confidence in the veracity of the change both when answering questions on the MARS and when discussing it in the interview.

How this conflict resolved itself over the next two years, with Debbie teaching mathematics virtually every day of the school year, is at the very heart of this case study.

Debbie's optimism regarding teaching mathematics

Here evidence will be put forward that will help to construct a picture of Debbie's attitude to teaching mathematics. Firstly, from her interview, it can be seen that Debbie was very clear about her understanding of what mathematics was like for her in her primary and secondary schooling, and she gave many reasons why she wanted to change the way that mathematics was taught.

When she arrived at her teacher education courses she was sure that mathematics would be her weakest subject but to her surprise "it's actually one of my strong points and since coming here [to university] I've absolutely loved maths, loved it. Seeing all the things that you can do and all the exciting stuff that you can do with kids and activities, it just makes it so much different and more pleasurable".

Further in her interview, Debbie went on to discuss how her school experiences motivated her entry to the teaching profession, to change things, and to make mathematics exciting and engaging. Debbie felt she had the means to make mathematics fun, and to improve mathematics lessons for the children in her classes: "I think it's maybe why I want to be a teacher, because I actually want to make that difference". So despite her antipathy towards mathematics in school, Debbie felt a need to push herself to embrace mathematics, and drive herself to become an excellent teacher of it. It is known that maths anxiety can have the effect of depressing self-confidence and agency (Hembree, 1990). For example, people with high maths anxiety have been seen to avoid mathematics courses, as was seen in the case study of Gina, and to make career choices where they avoided mathematics (Ashcraft, 2002). On the other hand, some with high maths anxiety have been able to use that anxiety as a lever to propel themselves further into mathematics, taking more courses, and putting maximum effort into understanding and overcoming their mathematics deficiencies (Hunsley, 1987). This latter scenario explains what might have happened to Debbie. It certainly fits the picture that she paints in her interview. She finally discovered that mathematics could be compelling, engaging, and fun, and this seemed to be due to her tertiary teachers. For instance, when she talked about her favourite teacher on her pre-service teacher courses, she was complimentary, noting that she would be pleased to be able to teach like him.

When she described her ideal lesson, Debbie portrayed an interactive one, with lots of activities and games, in keeping with the ideals behind the NDP (Ministry of Education, 2007). In this lesson, students would be discussing the mathematics, working in groups, and teaching each other the different strategies they had developed, a very strong NDP principle. Debbie had spoken at length earlier in the interview about what it was about her mathematics lecturer that really engaged her, and her wish to teach in a way which would engender those feelings in the students she taught.

Debbie exhibited strong optimism regarding her capacity to grow into a teacher who would be able to teach highly engaging mathematics, emulating her university lecturer. She believed she could overcome her difficulty with mathematics and teach in a way which could “make that difference”. But she was finding it difficult and more complex to teach in a way which was in line with the NDP, which it is not so surprising in a novice teacher

In this section, excerpts from Debbie’s interview have illustrated her dislike of the mathematics taught in school, contrasting with her delight and optimism at how she was taught to teach mathematics in her university courses. She expressed her delight at meeting a teacher who she wished to emulate, and how this fitted in very well with why she came into teaching and how she wanted to make mathematics more engaging. Following on from this, the two videos which captured sections of Debbie’s teaching will be discussed with regard to the teaching quality evidenced. This will illustrate something of her learning-to-teach trajectory, and, how she attempts to teach mathematics differently.

Quality teaching within the video evidence

Here the two videos (V_1 and V_2) of Debbie teaching mathematics are analysed to see if there is evidence of quality student-centred teaching. The first video, V_1, was recorded at the beginning of Debbie's first year of teaching, and the second, V_2, was recorded close to the end of her second year. They were both approximately an hour long, and were encoded using DART, the Dynamic Analysis Reflection Tool, developed specifically to code these types of NDP lessons.

Encoding in DART

How the DART captures information on teaching can be seen in Chapter 4. To aid this exposition, I will show and explain parts of the encoding of the first video. Then I will analyse Debbie's actions, as captured by DART, with regard to quality teaching.

The first page of the DART (seen in Figure 5.15), encodes, in one-minute intervals, various aspects of Debbie's first 10 minutes of video V_1. This coding captures the way that Debbie has taught in this group, counts the questions she asked the children and identifies which type they were, and times the number of seconds that Debbie and the children spoke.

Group dynamic icons

The group dynamic icons describe the actions of the teacher and students and relate to the NDP - style. (Details of this can be found in Chapter 4.)

A teacher who is doing all the work and essentially has the children sitting passively watching her, will have most of the blocking on the left in this section. A teacher who constructs his or her teaching around a group discussion and peer-to-peer teaching, allowing the students to take the main action in the session, will have more blocking in the middle icons. In this video it can be clearly seen (see Figure 5.15) that Debbie's coding remained mostly over to the left, with some movement into the middle, and it is the same for all the encoding of this first video. The evidence from the two videos will now be examined in greater depth.

Time (min)	Group Dynamic	Questions: Knowledge /Closed	Questions: Strategy /Open	Talking (Time in seconds): T%OMT	Talking (Time in seconds): [illegible]	Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
1.		/		52	4				Gets groups together, reminds about multiplying by 10 etc., talks about big numbers Example 6 x 30
2.		/		44	14				40 x 3 forty lots of 3 is tricky says T 3 x 40 using containers of sweets, but when things get hard does not use materials
3.		/		26	30				4 x 30 5 x 50 going into the hundreds column 6 x 30 invokes 'we just know' and Ch do not seems to respond.
4.		/		25	28				Lots of interactions with Ch suggestions of strategies, ways of answering.
5.		//	/	25	25				Asks a particular Ch for response, and reminds everyone that the answer is in the hundreds. Lots of discussion
6.				18	29				Particular children asked again, now reminded that answers could go into the thousands, but this fact not elicited.
7.		/	/	17	13				7 X 500? T reminds again that the answer is more than one thousand. Again T does not try to elicit this from Ch. Ch respond.
8.		/	/	29	20				70 x 50 How will the Ch work this one out? Ch still not asked to discuss or help each other.
9.		/		22	25				20 x 60 More time for talking back to the teacher, explanations, Ch do not interact with each other.
10				25	25				T talking with one individual, rest of group listening. T reminds Ch this is practice of the idea learnt from previous lesson.

Figure 5.15. Coding of Debbie's first videoed lesson, V_1 in DART.

Evidence from the first videoed lesson, V_1

Debbie taught two groups in this first video. In the first group, Debbie began the class, sets the classwork, and asked questions of individuals. She gave each child a "multiplying by 10" question to judge whether they are ready to move to multiplying by 100. This group lesson lasted for 12 minutes and only in the final two minutes did it move away from the teacher and her control. In the eleventh minute, students were working together to solve a problem (G6) and in the twelfth, children were demonstrating to the group (G5) and helping each other with strategies (G6) to solve the multiplication problems.

In the second group, during the same teaching session, Debbie was again teaching multiplication, with children given a question such as 18 x 3 and asked to solve it using a strategy of their choice. This session lasted for 16 minutes. From the sixth minute the children worked in pairs to solve problems (G6), though this only lasted for one minute. By the tenth minute there was showcasing (G5), where one student was talking to the group about answers and strategies, and then in (G6), the children worked on the problem in pairs, sharing their knowledge and strategies.

Debbie was using the NDP ideal lesson template when teaching these groups. So within NDP parameters there is some measure of quality teaching here. Considering that Debbie had only been teaching for a few weeks, this is a very positive start. Her determination to teach mathematics well and being in a school where the NDP is prevalent, means that these factors come together to enable Debbie to make a good quality teaching effort early on in her teaching career. Next, I will discuss the actual lessons that Debbie teaches, with reference to NDP ideal.

It is by no means always the case that NDP lessons are taught straight from the NDP resource books exactly as they were intended. However, there are basic ideas within the lessons that are the same for all lessons, and there are strategies that can be generalised across lessons. There is also the question of magnitude of numbers used at different points; these are usually well-signposted. If the part of the lesson being coded is the beginning, middle or end of a sequence, then NDP tenets would specify the magnitude of numbers that are appropriate at each of the materials/imaging/number properties stages.

The first group in the first video are being taught to recognise when multiples send the numbers into the hundreds or thousands column. The closest lesson in the NDP *Book 6: Teaching Multiplication and Division* (Ministry of Education, 2007) is called Sherpa (Tensing), on page 43. The intended learning outcome is:

> I am learning to find out what happens to numbers as they are multiplied or divided by ten, one hundred, one thousand, and so on.

The materials that are identified as useful are dotty number arrays, and numbers such as 10 x 37 are recommended as these have been piloted and tested and seen as appropriate for this stage. In the imaging phase, the materials shift to use of diagrams to prove the correctness of the answer previously thought to be right is in fact right, and the numbers are still in the order of 10 x 29. In the "using number properties" section, numbers in the order of 20 x 55 are recommended. Through all phases the lesson is grounded in a context, such as "There are 10 students and each of them have 37 marbles, how many marbles are there altogether?" These dotty number arrays are considered central to the development of these ideas, and this lesson instructions runs to over five pages, illustrating the necessity of getting the use of them correct, to allow students to extract the necessary generalisations from them. Debbie's lesson was grounded in the number properties area, but there was only one context used, in the second minute, to emphasise that it is sometimes easier to imagine 3 lots of containers with 40 sweets in each. The lesson started well, with a reminder of work done the previous day; however, Debbie continued to scaffold them all the way through this 12-minute lesson, using phases such as "We just know" and "Remember the number must be bigger than…" or "It goes into thousands", with the children showing limited understanding. Debbie did not try to elicit this knowledge from the children, so she cannot be sure that they know it. Following the reminders, they have some limited success, but with every new multiplication, they needed reminders from Debbie. At this point Debbie would be advised by the NDP to revert to materials to help the understanding develop more fully, but she not only did not do this, she did not ask the students to discuss what is going on, and she basically uses the lesson as a time to interact with individuals. In the eleventh minute, Debbie gave the question 60 x 200 and sat back to watch her students, who begin to talk to each other a little, trying to solve this big number. In the final minute, Debbie asked those students who understood to help those who did not.

This lesson has been dissected in detail to illustrate that Debbie has a veneer of mathematical understanding, and the overlay of NDP-style lessons, but she undermined the lesson by removing all the support that NDP lessons have built in. Had she been teaching the children using the materials, then she could have reverted back to them, but there was no attempt to do this, and therefore the lesson becomes unattractive, with no engaging context. It is very dry, because the teacher is giving out a set of bald multiplications and the lesson becomes repetitive.

This analysis of Debbie's enactment of the NDP style lesson showed there was a mixture of a good pedagogy with some less than ideal choices of numbers; perhaps this was a lesson that could be expected from someone just beginning to teach. Next, I consider the second video, taken two years later, to ascertain if there is any change in quality teaching.

Evidence from the second videoed lesson, V_2

Nearly two years later, Debbie was again videoed teaching two groups, the first for 10 minutes and the second for 12 minutes (see Figure 5.16). From the second minute in V_2, as can be seen in Figure 5.18, the children demonstrated their strategies, adding together fractions which add to one and have the same denominator, such as $\frac{2}{9} + \frac{7}{9}$. The evidence from the DART, as seen from the fifth minute, indicated the children were either working in sub-groups or demonstrating to others in the group. Here was a lesson very much in the style of NDP. Debbie encouraged the students to discuss the answers with their partner, and after a few minutes, Debbie was seen to use materials (Unifix cubes) to press the mathematics through the imaging stage and into number properties, where the children work for the majority of the lesson. Her questioning diminished, as would be expected in a lesson where the students were working well, in groups, discussing the work.

Time (min)	Group Dynamic	Questions		Talking Time in seconds		Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
		Knowledge /Closed	Strategy /Open	Teacher	Student				
1.		1	3	34	14				Using Unifix cubes. Group of students explain why $\frac{3}{7}$ is represented by showing 3 cubes from a wrapped bar of 7
2.		3		27	15				Students use white boards to work on, T speaks the problem $\frac{3}{7}+\frac{4}{7}=\frac{7}{7}=1$
3.		2		19	12				What must be added to two ninths to make a whole? S. write ans on white boards. T watches intently, No materials used.
4.		2		18	17				$\frac{4}{11}$ (used from Pink book 4, page 8), T actually reads from this book to the students, no context though there is one in the book.
5.		2		15	20				More questions adding to the whole, still no context.
6.		2		7	24				More questions, good complete explanations from students.
7.		1		4	31				Students discuss different meanings of the fractions, diff explanations, T listens.
8.		1		30	15				Teacher writes the WALT 'I am learning to create the whole from the fraction part I am given'.
9.				7	13				T listens while the students work on more questions of same type.
10		1	1	20	15				T now explaining what to do for the next session of their work as she moves to another group.

Figure 5.16. Coding of Debbie's second videoed lesson, V_2, in DART.

This lesson was taken from *Book 4: Teaching Number Knowledge*, page 8, (Ministry of Education, 2005). Debbie adhered strongly to most aspects of this lesson, and during the lesson, as can be seen in the video, her book was open by her side. The intended learning outcome as published is:

> I am learning to create the whole from a fraction part I am given.

It is this exact wording that Debbie gave to her students in the eighth minute of the session. Debbie used the fractions advised by the book, only departing once, and her choice fitted well with the recommended ones.

From the DART V_2 in Figure 5.16, it was evident that Debbie was teaching a more student-centred lesson than in her first video, first group, substantiated by the movement of the highlighting into the centre of the group dynamic section. From the second minute, children began to talk to the teacher and the whole group demonstrated their answers. From the fifth minute students were working in groups to discuss answers, and they continued sharing answers with the whole group.

Analysis of talking time

In the first video, Debbie spoke for nearly half the available time, (see Figure 5.17), and the students for about a third, whereas in the second video Debbie talked for less than a third of the time (30%), and the students for a similar percentage of the time. Together with information from DART, Debbie now talked less than she did, and the percentage of talking between teacher and students is more equal.

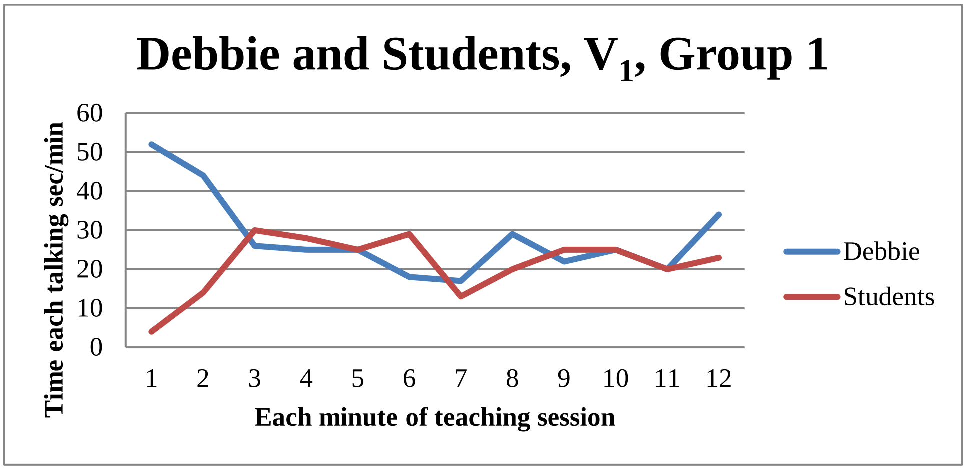


Figure 5.17. Debbie's first video, V_1, first group, time talking in each minute, teacher and students.

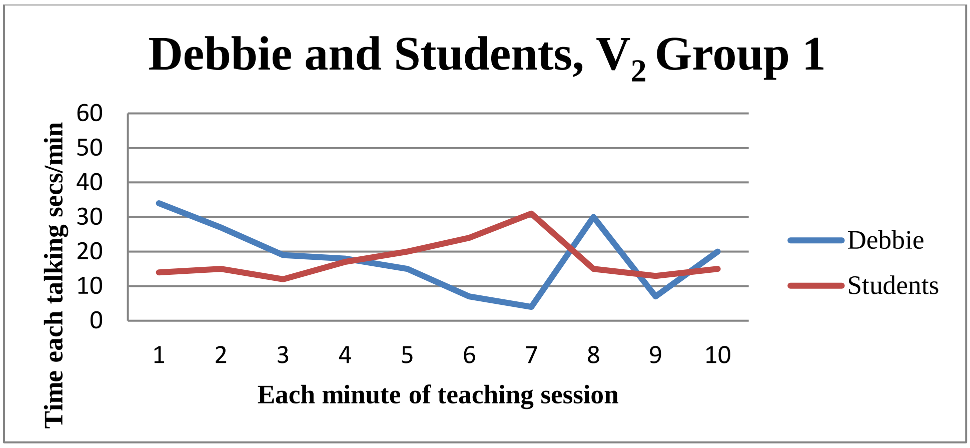


Figure 5.18. Debbie's second video, V_2, first group, time talking in each minute, teacher and students.

In V_2 (see Figure 5.18), Debbie was listening while the students spoke, which was not prevalent in V_1. The more the blue teacher line is below the red student line, the more likely the lesson is to centre the work on the students.

In this session, Debbie used materials to demonstrate the first question but after that she did not show materials to them again, and so they did not manipulate them. She had moved into a more formal phase, which perhaps indicated that she could see the need to extend her students. However, the "Packet of Lollies" title to this lesson indicated that the advice is, that materials should be used, and that it is the children who should have these cubes. Debbie was the only one who touched the materials, so she did not follow this instruction in the book but she was seen watching and listening intently as the students spoke, none of whom seemed to have difficulty answering the questions and justifying their answers. In this lesson, Debbie knew these students were in need of extending in this knowledge area, and she adapted the lesson to her teaching needs.

Comparing evidence from each of the two videos, it is interesting that some measures move up and some move down. On the ACT framework, Debbie used a greater selection of desirable behaviours in the first video than she did nearly two years later. It might look as though Debbie is taking a retrograde step in using a smaller set of behaviours in V_2 than she did in V_1. However, in V_2 she can be seen to be teaching in a more child-centred way, talking less and listening more. Debbie is still no more likely to use a context than she was two years previously, but she did demonstrate with the material, and it is an appropriate type for that lesson. Next, I will examine the final sections in DART to see if there is any further suggestion of an increase in teaching quality.

Use of materials

The use of materials is also captured by encoding videos with DART. Under the NDP and the model teaching a new concept should involve a rich context and manipulatives which support that context and the mathematical concept being taught. Students use the materials to solve problems in that context, until they no longer need the material, since ideas of imaging and number properties, in that order, take over. The "use of materials, imaging and number properties" is a section used to capture the use of materials, if any, and the purposeful progression through development of a new mathematical concept.

Debbie was the one manipulating the materials; no children were asked to touch or change them, or to explain what was going on while using them, through pointing and suchlike. The materials were meant to be in the domain of the student, but Debbie kept them for herself. Throughout all the videos, when materials were used, Debbie was the only one who manipulated them, not allowing the children to explore mathematical concepts with them.

No materials were used in the teaching on V_1, though there were certainly parts of the lessons that would have been improved with their use. The DART has a section for the coder to complete called “description of minute”, and coders are instructed to include errors and inappropriate language. These descriptions can flesh out the lesson in ways which the icon-capture cannot. Indeed, it is this section that illustrates the lack of materials when they were very much required. For instance, when the students had difficulty multiplying by 100 (V_1), and the class insisted on characterising multiplying by a factor of ten or a hundred as “adding noughts”, Debbie could have decided to employ a material such as multiplication arrays. There is a learning experience in Book 6 of the NDP supporting resources, in which children use the arrays to calculate, say, 10 x 37 and then place the original number, 37, and the number multiplied by ten, 370, together in a place value chart. Then the students are directed to notice, and they should do so eventually, after manipulating enough numbers and enough multiplied numbers, and realise that the numbers move to the left and the “adding noughts” scenario is not invoked to explain how 60 x 30 can be accomplished. Moving into imaging and number properties is to be expected, but the NDP teaching model states that children may need to move in between these areas, gradually forming the complex connections which enable them to understand at a generalisable level rather than learn something very specific that is not open to generalisation or usable across other sets of numbers. The use of an exciting and engaging context also needs to be added, but Debbie did not employ any context of any kind in the lessons she taught in V_1 and V_2, perhaps indicating her lack of understanding about the importance of the same.

Hence it could be argued that with the passing of nearly two years, Debbie had moved on to using materials herself to demonstrate, even if, however, these were not used by the children during concept development. This indicates an increased quality of teaching, so on these particular measures she has improved over time. This illustrates the difficulty with deciding if teaching quality improves, reverses, or remains stagnant. Different measures show that in some areas Debbie’s teaching has become more liable to allow students to share their understandings, and that the group discussions were more prevalent on the later video. On the earlier one, there was no sign of materials use, though it would have been appropriate, and the later video showed some improvement on this measure. Materials are now present but the teacher does not give them to the students; she does all the manipulating. These factors will be looked at again in the summary to this case study.

Further to the difficulty of deciding if there has been a change in quality teaching, can the vision that Debbie voiced about engaging her students be seen in DART? Debbie talked at length in her interview about her wish for making mathematics lessons engaging and fun for her students. She had a vision of herself being able to enthuse children, to help them really engage with mathematics, not just teaching from the text book, but choosing activities which really capture the imagination, and, in her words, it should not be like this: “It was watch me, do as I do, go and do the exercises, and that was it. It was extremely boring”. It seems fair to say that the vision that Debbie had for her teaching was not much in evidence through these two videos, on the measures investigated thus far. The second page of DART might still have something to add to this debate. The following analysis of the second DART page is the final section in this case study.

Instruction components of ACT framework (ACT score)

The ACT framework showed the outcomes of the types of encouragement that teachers employed. They were mostly seen to support the understanding of children and elicit this understanding, but more rarely are they seen to extend the understanding of their students. The second page of DART uses this ACT framework table to capture the work of the teacher; the results of this encoding of Debbie’s sessions are discussed below.

Debbie would have been immersed in the work of the ACT framework in her university courses, both because NDP used the framework to underpin the theory of what quality teaching is, and because the pre-service teacher courses are also underpinned by the same theory. This awareness could account for the first video (V_1) showing that Debbie had a strong notion of what the three areas of the framework—eliciting, supporting and extending—were all about. In $V_{1,}$ Debbie started with a reasonable spread over the three areas, with her use of the eliciting, supporting and extending way of working seen in the counts of the three areas:

- Eliciting (solution methods): Debbie exhibited seven of the statements out of a possible eight in V_1. For instance, she used “Monitors students’ levels of engagement” extensively, but never “Uses students’ explanation for lesson’s content”. This is interesting because it develops the picture of Debbie as a teacher on a teaching trajectory from which she does not deviate; this is, perhaps, the result of uncertainty about changing within the lesson, which any beginning teacher would find difficult.

- Supporting (conceptual understanding): Debbie was shown to address six out of a possible 11 statements in V_1. She “Supports listeners’ thinking” but does not use “Asks a different student to explain peer’s method”. This is particular behaviour found in NDP, and again, a beginning and uncertain teacher may not ask a student to bring in new understandings, in case these were not well understood by the teacher.
- Extending (mathematical thinking): In this coding section, Debbie used six out of a possible nine statements. She was seen encouraging mathematical reflection, but not “Goes beyond initial solution methods”. Again, she did not extend either herself or the students.

This spread across the three areas of the ACT framework is exemplary and, according to the research, rarely seen in experienced teachers. As a beginning teacher, Debbie can be seen as extraordinary, in that she managed to enrich her students’ experiences despite her youthful inexperience. These results on the ACT score point explicitly to a level of teaching quality not reflected in the second video. Earlier it was suggested that Debbie started teaching, with strong notions of eliciting, supporting and extending, because it was so prevalent in the university courses that she had excelled in. Two years later, this is no longer the case:

- Eliciting (solution methods): Debbie used six of the statements out of a possible eight in V_2. This is only slightly less than in V_1, however this is the section in which most teachers are strong.
- Supporting (conceptual understanding): Debbie was shown to address three out of a possible 11 statements in V_2.
- Extending (mathematical thinking): In this coding section, Debbie used two out of a possible nine statements.

Debbie appears to have moved backwards into a less varied pattern, displaying these highly desirable behaviours to a far less degree. It is a reasonable assumption that beginning teachers will develop more varied behaviours over time, not less, so this result is difficult to understand. But it is possible to speculate on the causes:

- New teachers have anxieties about teaching in general (Grudnoff, 2012).
- Teaching as was taught is to be expected in novices (Goodwin, 2010).
- It may be a way of coping with too much work and not enough time.

- It may be the enculturation in the school—what the others do.

And again the changes in different aspects of the teaching quality of Debbie, with some attributes moving up and others down, makes any assumption of increase in teaching quality hard to substantiate.

Is there an increase in the quality of Debbie's teaching?

Taking the videos in a holistic fashion, the evidence shows that Debbie has improved her teaching of mathematics under NDP-style lessons: the quality of teaching, overall, has improved. This is most evident in V_2, where Debbie was seen using the power of group work, encouraging students to work in groups. This presents evidence of the developing community of mathematics learners, which is desirable under NDP. The use of materials here is more evident too, and though the children are not manipulating these materials, they can now see them, and this vicarious experience can still contribute towards strong concept development (Goddard et al., 2004). More evidence of Debbie's development is seen in how she was more likely to adhere to the types of numbers recommended by NDP resource books, she was also showing signs of being more likely to choose appropriate numbers herself, and this is an improvement on V_1. Her talking time abated from V_1 to V_2, and she asked more open questions, which provided room for stronger student responses. However, there is also the matter of the reducing ACT score, and achieving a lower second MKT score after having been above average for the first. These metrics are presenting an alternative story or developing a different part of the picture of Debbie's teaching. These contradictory changes, summarised in Figure 5.19, are hard to reconcile, but seeing the interactions with her students on V_2 and the improvements captured by page one of DART, I am inclined to see an overall development of quality teaching. Debbie may not yet have got away from "It was so boring, it would not stick" into her desired state of "fun and engaging", but she is making efforts, and so this raises the question of how she will continue, and whether she will perhaps speed up, the move into more engaging and fun mathematics teaching. This theme will be returned to in Chapter 6.

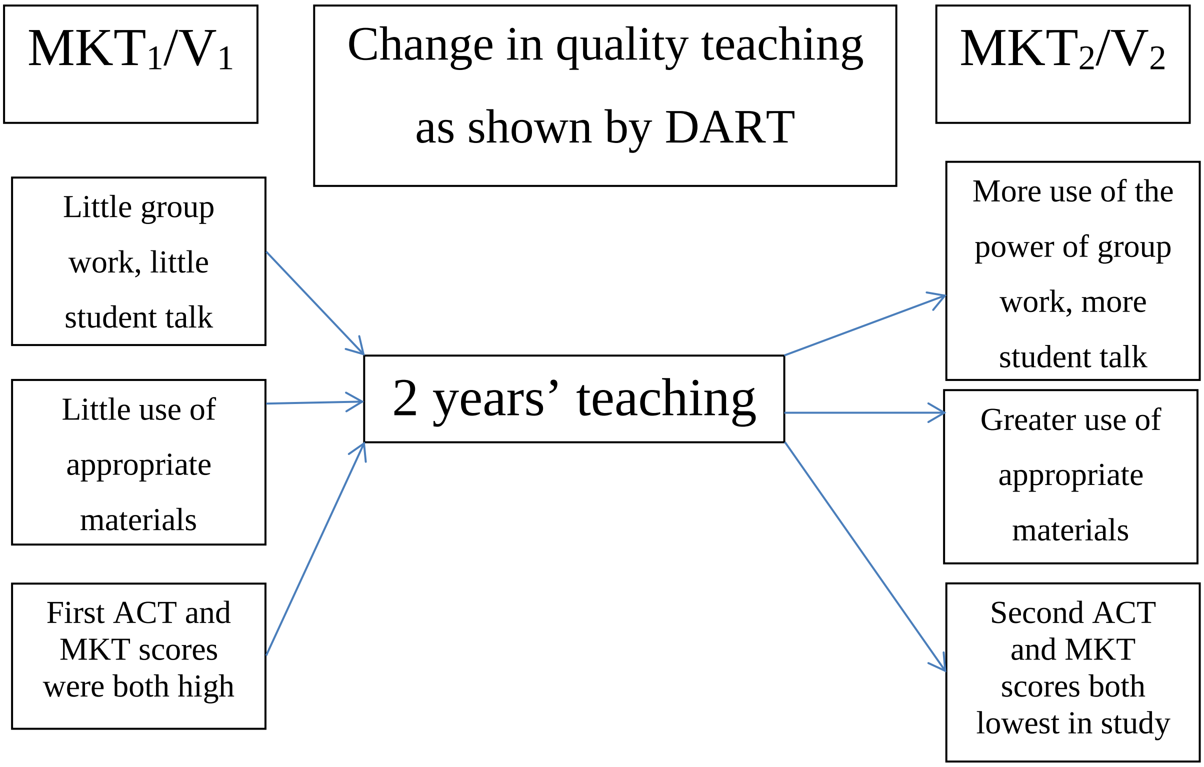


Figure 5.19. Diagram illustrating changes in Debbie's quality teaching.

Summary: Debbie's journey

Debbie had two years as a beginning teacher and, as a beginner, she was extremely brave to allow me to video her mathematics lessons. It takes courage to be open, and to be watched and ultimately evaluated by someone from the university. Her interview showed that she was unsure about her ability to teach mathematics before she started her university courses, but by the end of her three-year teaching degree, she showed more confidence, and determination that her teaching of mathematics would make her an agent for change. Her lessons have changed over these two years: she has reduced her talking, but also reduced her ACT behaviours, where once she was exemplary—she is now showing a very restricted set. But this as a positive part of the overall quality teaching picture, Debbie has the ability, as shown previously, to extend her students, perhaps in her future teaching this will again come to the fore. Debbie has more facets to her teaching, as seen in these two snapshots; in time, she may be capable of teaching with all the facets available, and hopefully she would also develop more areas.

More journeys: Five vignettes

There are five other participants who took part in this investigation into quality teaching, and their voice has not yet been heard. Each of these teachers has data that could further illuminate the continuing struggle to identify factors which affect quality teaching. Therefore, their data and attributes, which is summarised in Table 5.4, will now be examined in five vignettes.

Table 5.4

Summary of all the Remaining Five Vignette Participants Detailing Attributes used to Identify Further Factors of Quality Teaching.

Name	Age	MARS	What was noticed	Factors addressed	Evidence source
Alison	40	13	MKT and ACT measures vary little	Adherence to NDP-style teaching	Interview, MKT, ACT, DART
Barbara	21	15	Strong NDP-style teaching	Adherence to NDP-style teaching	Interview, DART
Carly	21	18	Strong NDP-style teaching and strong increases in MKT and ACT	MKT and ACT measure increase strongly	MKT, ACT, DART
Fiona	28	40	Strong increase in MKT with low increase in ACT	Highly maths anxious and increase in MKT	MARS, interview, MKT
Evie	26	29	Direct instruction of algorithms for Year 2	Moving away from NDP-style teaching	Interview, DART

Alison

Alison was the oldest participant and the least maths anxious of all the participants (see Table 5.4). She scored highly on both MKT tests, possibly meeting ceiling effects, and there is no doubt that she was confident in her mathematical ability in her teacher education courses and her personal mathematical content knowledge.

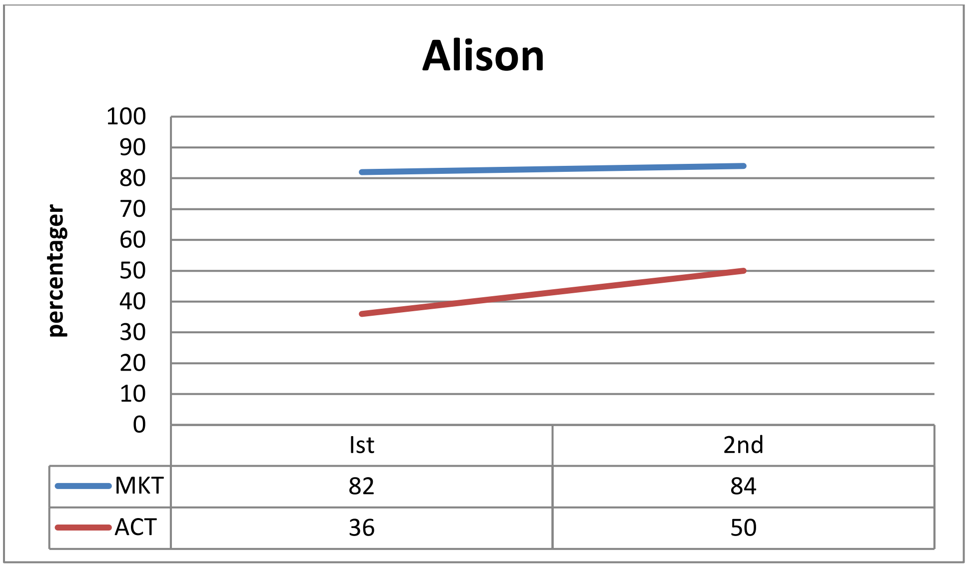


Figure 5.20. Alison's results on the two MKT tests and ACT scores.

Alison's first ACT score was not high (see Figure 5.22), being just below the group average, but the second ACT score showed some increase, and then she scored above average for the group. The data captured by DART from the two videos will be analysed, but first, some comments from Alison's interview, to illustrate the confidence she had in her own ability to understand the mathematics needed for teaching.

Alison remembers not being very good at mathematics at school, and strongly remembers statistics as being boring. She said in her interview:

> I found statistics, even though I took it in the 7th form, really boring. I took it at university level and it was really, really boring. So you know basically at university Stage 1, I basically, I failed. It was the only paper I got a D on my transcript, D just because I didn't enjoy it and I just didn't see purpose in it I think, whereas I loved algebra, I loved calculus.

She went on to say that the NDP which she encountered in her pre-service teacher courses at university had been really good for her own understanding of mathematics:

> It [NDP] actually helped my own personal maths and I can now actually do things in my head that I wouldn't have been able to attack before, because I've now got some strategies. Interesting isn't it? So, you know, all of a sudden, obviously it's something that has clicked in me like, oh yeah well, why don't you do it that way.

Alison had loved mathematics at high school, and had found the skills that were needed in her first chosen career of accountant to be at a very simple level, and she found it hard to understand why other people found such things difficult.

> I found at uni doing the B.Ed., the maths was really basic and simple. Like in Stage 3 you have to do all those maths tests in order to pass Stage 3. I remember the first day [in her teacher education mathematics course] we had to do that. There were 16 questions and you had eight minutes and I'm racing through this form going I've only got, oh no, only eight minutes and I'm racing through this form and you know writing down all the answers and then I looked up at the board and there was three minutes [gone] and I'm thinking I've just finished.

Alison found it hard to think like a child and to be empathetic with adults who could not succeed at her level and pace. She thought everyone should be able to do the basic maths, saying "I can understand that people find it difficult but I just thought some of that maths was actually really quite basic and it's just a matter of understanding". However, she explained, it was her encounter with the teaching style and the model of NDP that got her thinking about how to understand the difficulties her students had.

> Being exposed to NDP was really good for me, not just because it improved my own maths but it also showed how maths can be broken down into steps, learning steps for students because, one of the things I really personally struggle with, because I have a high understanding of mathematics, is, how do you then break that down, why do you not understand 9 + 7, how do you not know that you can break down seven into one and six and get 16. The NDP has been really good where that's concerned because it's actually allowed me to see what the steps should be and because of that, I can actually look at a child and go, ah I know what you're missing. You're missing this building block here and I need to work on that building block.

When confronted with the reality of teaching in NDP style, Alison had this to say about the difficulty of working in groups with a new class, and keeping everyone working purposefully:

> I've got a class that has got some really interesting challenges in it as far as children are concerned and this is the ideal of NDP, this is the ideal of what you should be teaching and then you've got the reality of the dynamics of the classroom and you've got the reality of children not getting on and children being sort of wilful.

When asked about a lesson in which the mathematics and style of teaching was different from that which Alison had received herself in her youth, she said:

> I have tried to make it fun for the kids to do whereas I don't think it was ever presented to us as a fun thing to do. It was never really presented to us in context sort of, it was just here's 5 + 5 go learn it. Whereas now I do still sort of 5 + 5 but I do say what's 3 + 2 is 5, well what if I've got, what's three counters and two counters and they all go five and if I've got three teddy bears and I've got two teddy bears, how many have I got, five. So it's just like bringing it into context and sort of like that you know, the things are all the same, the numbers are all the same so I think that's probably different. Fun activities like that which have got a maths element to them, a sort of investigation, thinking about what the different patterns are and things like that, I don't remember ever doing anything like that at school.

When asked about the things that happened in her best lesson, Alison answered:

> Come and ask me in a year's time. I mean the only lessons that I have been really happy with have been my stats lesson. I haven't been happy with my NDP lessons at all.

Asked to imagine the best lesson, Alison said:

> The peachiest lesson, if all the kids were actually really enjoying their maths, where they had those "aha moments" where I get it, oh yeah if I do that and if I take that counter from this frame over here and I've got seven now and I change one over and I make this nine into a 10 then I've got 10 and 6, you know that "aha moment", ah I get it. That would be my ideal.

Alison talked at length about how wonderful NDP was for helping her really think about where her students are at, as regards stages and "where to next", and exemplifying the ways of working which make mathematics lesson more fun. I will now turn to lessons that Alison taught for the two video recordings, encoded by DART, to see if her espoused beliefs about teaching, match with the enactment of her lessons.

Looking at her first video (V_1), first group (16 minutes), her teacher actions were all about Alison talking and the students responding. At no time did the encoding in DART move across to the right where more child-centred teaching would be noted. She did not use the power of the groups to get the children talking, they behave independently of each other, and there was no cross talk. Alison was giving a lesson on subtraction; she used a context and no material, and talked about imagining the bears and the ones hiding. This lesson is very like one from Book 5, page 12, but Alison used bigger numbers than those advised under the imaging section. After 5 minutes of trying to get them to answer correctly about the number of bears left, and the students having no success, she gave instructions to use their fingers, but no bears were presented. This is not considered the most efficacious way to work in the NDP style. This lesson continued on, and the students still did not have success; at times Alison instructs so minutely that she was giving away the answer.

Teaching continued on the first video with the second group for the next 20 minutes. This group was learning “to add tens to a number by counting on in tens or adding the tens together”. Alison used the numbers that the book advises. Again, this second group of students was not having a lot of success. They did have place-value equipment and the lesson progressed as the book suggests, but as the students were not having success, it would be better, in NDP terms, to revert to using smaller numbers. The students were working independently of each other, with their own set of materials, until the seventh minute when one student explained their understanding to the group. Alison moved into the imaging stage, which is obviously where she felt they should be, but she shielded the place-value equipment and most do not answer the question 34 + 20.

In NDP terms, this group lesson has good elements in it. It follows the NDP model and it strongly adheres to the recommendations in Book 5. The students did get to be active, using their own set of an appropriate material, and progress was seen with a few students. To improve it, Alison could have reduced the size of the numbers to aid their understanding, and not shield the equipment to move them into the imaging phase. They did not seem to be succeeding with the materials, so making it harder was not the best change at that time. Alison has some strong elements of NDP lessons already going in her lessons, just a few months into her first teaching year, though she seems to have a lesson plan firmly fixed and does not deviate much from it. The second video will now be analysed to see if evidence for change in quality teaching can be seen, two years on.

Video 2 (V_2), group one, was a 19-minute lesson, and this was a presentation of the lesson in Book 5, page 43. After the warm up (2 minutes) in which Alison got the students working in what she called “your thinking groups” to give meaning to the words, face, place, and total value. Between them, the thinking groups gave feedback to Alison and she moved on. This lesson was about introducing the standard written form for use when numbers get too big for mental computation. She soon realised that the groups were struggling, so she reverted to materials, and gave them place-value money to help them relate to the vertical algorithm. The groups spent the next 10 minutes sorting out how the money helps them solve the problem.

Then Alison asked the groups to explain their thinking to the whole group, and the lesson finished with some understanding. DART shows more coding to the right on the group dynamic area, indicating that a child-centred lesson was taking place. This is a big leap in quality teaching, and it continues into the next group on V_2.

Students were asked to work in their thinking groups to develop different strategies for adding numbers. The groups told other groups their ideas and strategies, and they worked in groups for nearly the whole 20-minute session. There was a point in the lesson where Alison realised that many children were getting the idea about knowing ten lots of tens make a hundred, being adept at using the place-value money and doing the change up and swapping for higher value notes, that she moved them swiftly through imaging and took them into the number properties area. It is positive to see Alison pressing for mathematical understanding. One child suggests using a number line, and she gave them empty ones which meant they have to organise the scale themselves, and they worked purposefully until the end of the lesson.

It seems clear that over the two-year period between video sessions, Alison has moved closer to her ideal lesson under NDP in the second video, a wish she voiced in her interview. More elements of NDP best practice ideals are present, and Alison is able to show her versatility by providing the empty number line.

These videos were encoded using the first page of DART, so what of the second page—the eliciting, supporting, and extending (ACT) page? In the first video, DART captured 10 different eliciting and supporting behaviours, but none in the extending section. In the second video, these behaviours have increased to 13 in the eliciting and supporting behaviours, and one in the extending part. Taken together with the increased adherence to NDP style of teaching, Alison showed a marked increase in her quality teaching, though on some measures, such as ACT, she could appear to have moved only a little upwards in quality teaching.

Summary of Alison

In her interview, Alison had a vision for teaching which appeared to capture some of the excitement she felt at encountering NDP-type thinking, and in her second video there is a move towards her meeting her vision, with students working in groups with materials, explaining their thinking to each other, and making progress. Her confidence in her own mathematical understanding and MKT, and in her knowledge of NDP strategies, Alison epitomises the non-maths anxious teacher.

Next, it is Barbara and a consideration of how her teaching in NDP-style varies over the two videos, V_1 and V_2.

Barbara

Barbara was 21 at the beginning of this investigation into quality teaching. She presented with low maths anxiety, and scored above average on both the MKT (average 36%) test and the ACT (average 14%) scores (Figure 5.21). Here, I will report on Barbara's ways of teaching in the two videos. There is adherence to NDP principles, and this did not waver. The encodings of the two videos are shown and features contained are discussed in detail, as regards quality teaching.

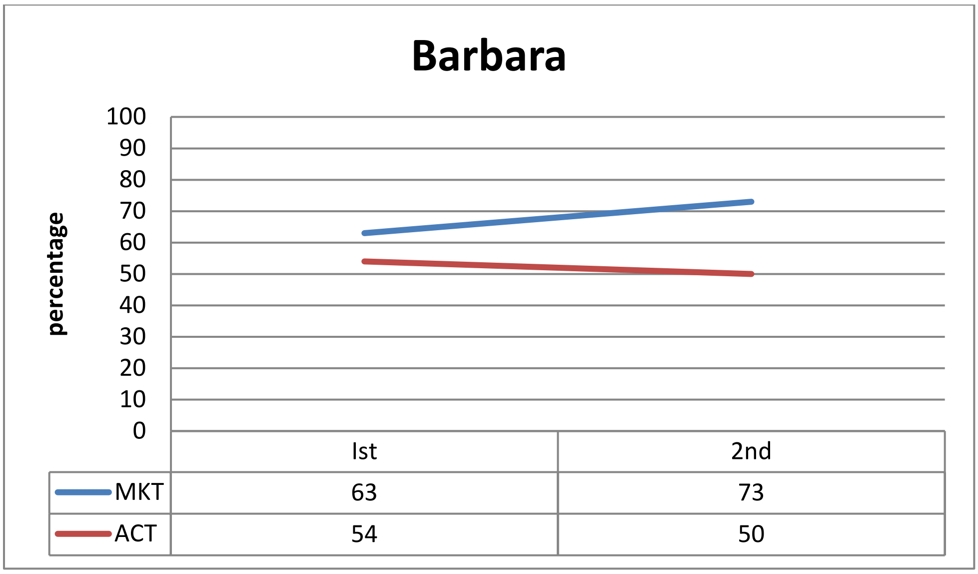


Figure 5.21. Barbara's results on the two MKT tests and ACT scores.

The encoding by DART of the group lessons, taken two years apart, is seen in Figures 5.22 and 5.23. Barbara appears to have started teaching in NDP style from the very beginning and continued in that style.

Time (min)	Group Dynamic	Questions		Talking Time in seconds		Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
		Knowledge /Closed	Strategy /Open	Teacher	Student				
1.		1	2	16	40				T: How many threes? S: work in groups, talk about their understandings, sharing strategies without being asked.
2.		1	1	16	25				T: Write down your strategies. S: talk to whole group with their strategy, wanting to share. T: How did you count in 3's?
3.		1	1	20	31				T: Did you count in 3's or 4's? How did you know that 21÷3=7? S: Talking and offering many different ways of solving problem.
4.		1	1	27	15				T: can you show us? S: Shows 5x4+1, S: describes her strategy. T: Starts to arrange blocks to illustrate problem.
5.		0	1	21	21				T: Shows piles of blocks, these are lollies etc., Asks S to say what to do to organise them, suggestions from S.
6.		1	1	24	9				Lollies arranged to show 3's. S: watch, T: watches. T: What if there were 22 lollies? Whole group think and interact with group.
7.		1	1	14	4				T: asks another S to show their way - Is it really different? S: Watch and T watches. Interruption about lost glasses.
8.		0	2	30	15				T: Poses 113 in groups of 3. S: Not enough blocks. T: How could we do it? T: Get into your groups and discuss how to do it.
9.		0	0	10	46				T: Talk in your groups. T: watches and listens to the discussions in the groups. Keeps listening, not interrupting. S: explain to each other their thinking.
10		1	1	19	30				T: What is 10X... in response to question from S. T: Keep thinking. S: Keep discussing, T listens, says Let's share now in response to progress.
		7	11	197	236				

Figure 5.22. Coding of Barbara's first videoed lesson, V_1 in DART.

In Figure 5.22, the DART coding under group dynamic is diffuse, and showed Barbara speaking and the children working in thinking groups. The lesson was in the number properties zone; however at the fifth minute Barbara saw a need for demonstrating materials to help with a particularly difficult question, 21 divided by 7, and she told the story of 21 lollies being grouped, then asked the students to arrange the lollies to show 21 divided by 7. First one child showed their understanding by moving the counters that represent the lollies, then a second student reorganised the "lollies" and showed a different but equally valid understanding. Barbara asked the whole group to discuss whether these are really different, then asksed what would happen if there were 22 lollies, then she set further question, 113 divided by 3. This question was too big to solve using the equipment, showing that Barbara had moved firmly back into number properties, where she remainsed for the rest of the session. She instructed students to get into groups and explain to each other their thinking about this piece of problem-solving and Barbara listened to the discussions. The inference from Figure 5.22, is that Barbara asks more open questions than closed and talks less in total than the students. Taken together, all these factors add up to a strong, NDP-style lesson.

Time (min)	Group Dynamic	Questions		Talking Time in seconds		Use of Materials	I	P	Description of Minute (Include errors and inappropriate language)
		Knowledge /Closed	Strategy /Open	Teacher	Student				
1.		1	1	31	20				Begins with a review of yesterday's lesson eliciting from S's how they solved the problem.
2.		0	1	20	16				T: Sets a problem in the modelling book asks 'how would you solve it'. T waits and listens then asks them to form groups.
3.		0	2	10	43				S: work and T listens. In groups S explain to each other their thinking. T: questions each group and they show to all.
4.		0	1	21	26				T elicits all the different strategies and then says "'are we all right?' What is the number was bigger? 43+18 in groups again
5.		0	0	10	54				T Listens and S solve, talking to each other. S asks T for answer-she refuses. T encourages further discussion, talking.
6.		0	0	10	31				T listens, more S working. Then each S asks to write strategy on a piece of paper in quarters on the floor, they begin to write.
7.		0	0	4	15				S: Write different strategies on their quarters, everybody quiet and concentrating. T: looks at writing, encourages explanations.
8.		0	0	13	29				S Continue to work in group and share, then explain each strategy to the group. T: Why add a zero? T questions to ensure clarity.
9.		1	0	31	12				T: elicits place value knowledge asking what is the 4 in 40? Checks they say forty, enunciates it well herself, listens to S say it back.
10		2	0	20	23				T:continues to emphasise the need for the S to write in the zeros, correcting their mistakes takes awhile but final they see, right ans, wrong method.
		4	5	170	269				

Figure 5.23. Coding of Barbara's second videoed lesson, V_2 in DART.

Figure 5.23 details DART encoding of the second video, two years later, which can be seen as similar to that of the first video. Barbara remained resolutely working in groups, with similar instructions from the first here in the second, asked fewer questions this time, but still more open than closed ones. Barbara was again talking less than the children and at one point the students asked if their answers were right. She did not directly answer, rather deciding to ask the students to persuade her that they are right. Again, this is a strongly NDP-focused session, with all the ideal elements being seen. Barbara remained teaching in a child-centred way.

The analysis of these two videos shows that it is possible to teach in NDP-style from the outset, and it is included here as evidence that it is not only expert teachers, such as the one DART was modelled on, who can teach in this way, but an absolute novice manages it, and continues to do so. Why did Barbara teach in this way from the very beginning? Perhaps we might look at her data, and her interview:

- She was young and had a very low maths anxiety (15).
- She scored well on the MKT tests (63% and 73%).
- She said: "I love maths myself and so I try and … show that I enjoy it as well".
- She was "realising how important maths really is".

I find those last two snippets from her interview expressive; they are redolent with meaning about why Barbara might teach in NDP-style so readily and sustainably.

These codings are now contrasted with those of Carly, who began and continued to teach in a very similar way to Barbara, but whose results on the second page of DART, where teacher behaviours are captured, is vastly different.

Carly

Carly was similar to Barbara on many factors. She, too, was 21 when this investigation into quality teaching began, showed low maths anxiety, and was teaching in the NDP style from the start, and continuing into the second video. Factors of interest about Carly which add to the understanding about quality teaching are that she started low on both the MKT and the ACT scores (see Figure 5.4), but finished among the highest on the MKT and the highest on ACT scores. It is these change in ACT and then MKT scores that will be discussed here.

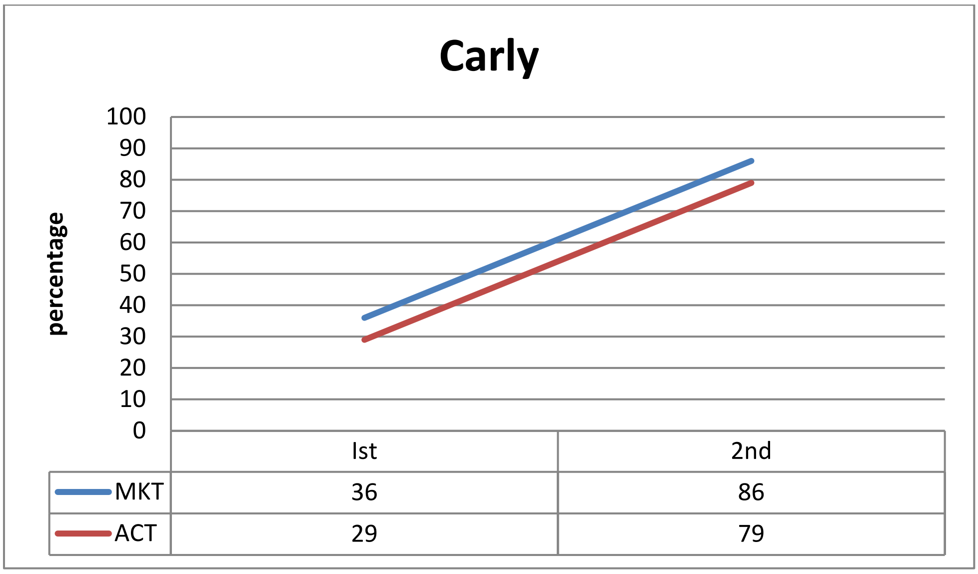


Figure 5.24. Carly’s results on the two MKT tests and ACT scores.

Figure 5.25 shows the encoding of teacher behaviours on the second page of DART. Here Carly was seen using a very small selection of behaviours taken only from the eliciting and supporting areas. While a completely novice teacher may be expected to begin with a low score, Carly showed a remarkable increase, in the second video, two years later, now scoring higher than the other participants.

Instructional components of ACT Framework								
Eliciting (Solution Methods)	✓	**When**	**Supporting (Conceptual Understanding)**	✓	**When**	**Extending (Mathematical Thinking)**	✓	**When**
Elicits many solution methods for one problem from the entire class			Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods		
Waits for and listens to students' descriptions of solution methods			Provides background knowledge			Encourages mathematical reflection		
Encourages elaboration of students' responses			Directs group help for an individual student			Encourages students to analyse, compare, and generalize mathematical concepts		
Conveys accepting attitude towards students' errors and problem-solving efforts	✓		Assists individual students in clarifying their own solution methods	✓		Lists all solution methods on chalkboard to promote reflection		
Orchestrates classroom discussions			Supports listeners' thinking	✓		Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays	✓		Pushes individual students to try alternative solution methods for one problem situation		
Monitors students' levels of engagement	✓		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method			Promotes use of more efficient solution methods for all students		
Decides which students need opportunities to speak publicly or which methods should be discussed	✓		Supports describer's and listeners' thinking	✓		Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method			Cultivates love of challenge		
			Supports individuals in private help sessions					
			Encourages students to request assistance (only when needed)			# = too numerous to mention?		

Figure 5.25. Carly: First video (V_1) first group encoding using DART.

Carly's second video encoding (Figure 5.26) coded for nearly all the desirable behaviours. With seven out of a possible nine behaviours on the extending section, Carly is quite the most extraordinary of teachers. This is the type of effect seen in the expert teacher used in the original development of DART, and no one comes close to her values in this study: the average for the second video results are 44%, whereas Carly scores 79%.

Instructional components of ACT Framework								
Eliciting (Solution Methods)	✓	**When**	**Supporting (Conceptual Understanding)**	✓	**When**	**Extending (Mathematical Thinking)**	✓	**When**
Elicits many solution methods for one problem from the entire class	✓		Reminds students of conceptually similar problem situations			Asks all students to attempt to solve difficult problems and to try various solution methods	✓	
Waits for and listens to students' descriptions of solution methods	✓		Provides background knowledge			Encourages mathematical reflection	✓	
Encourages elaboration of students' responses	✓		Directs group help for an individual student	✓		Encourages students to analyse, compare, and generalize mathematical concepts	✓	
Conveys accepting attitude towards students' errors and problem-solving efforts	✓		Assists individual students in clarifying their own solution methods	✓		Lists all solution methods on chalkboard to promote reflection	✓	
Orchestrates classroom discussions			Supports listeners' thinking	✓		Goes beyond initial solution methods		
Uses students' explanations for lesson's content			Provides teacher-led instant replays	✓		Pushes individual students to try alternative solution methods for one problem situation	✓	
Monitors students' levels of engagement	✓		Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method	✓		Promotes use of more efficient solution methods for all students	✓	
Decides which students need opportunities to speak publicly or which methods should be discussed	✓		Supports describer's and listeners' thinking	✓		Uses students' responses, questions, and problems as core lesson		
			Asks a different student to explain peer's method	✓		Cultivates love of challenge	✓	
			Supports individuals in private help sessions	✓				
			Encourages students to request assistance (only when needed)	✓		# = too numerous to mention?		

Figure 5.26. Carly: Second video (V_2) first group encoding using DART.

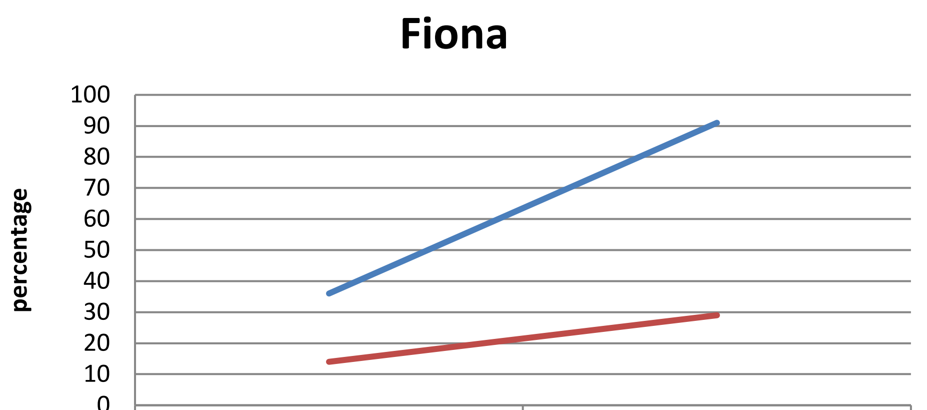
What might account for this large change? Carly had a very similar change in her MKT scores. She started with a less than average score on this measure, and two years later she scored twice the average and was second only to Fiona, who will be highlighted in the next section.

One factor that is different for Carly from every other participant is that Carly had only taught Year 7 students. This means that though she had taught only one year level, she had met a much larger range of mathematics than those who teach only very young children. By definition, when children are starting school at 5 years old, there is less variation between the levels of mathematics in the class. When children have been at school for seven years, there is a greater likelihood that there will be a wider range, and in a study by Thomas and Tagg (2005), it was shown that classes in the Year 6 to 8 range have students who straddle all the levels (1 to 8) of NDP, in varying ratios. While there may only be a small number at each end of the range—level 0-3 (counting) and level 8 (advanced proportional)—and many more in the level 5 (early additive), level 6 (advanced additives), and level 7 (advanced multiplicative), this still shows the range of mathematics Carly may have encountered in her two years' of teaching. Hence, I would argue, the large increase in MKT, is because the questions in MKT are teaching based, and they range from questions involving counting to division of fractions and decimal fractions. It shows us that, perhaps, teaching mathematics at this level had improved Carly's own mathematics knowledge.

If it is the range of mathematics that Carly taught that accounts of her wide range of teacher behaviours, how could that have happened? It is hard to know without further evidence, which is not available to us. However, it seems likely that Carly was pressed into teaching a wide range of age-related mathematics. Perhaps in the course of up-skilling her teaching content she was also pressed to increase the variety of behaviours she used with her students, to be effective. However, next we will meet Fiona, who is the only participant in this study to score a higher MKT result than Carly, to see if factors that impinge on her teaching might illuminate our quest for knowing about quality teaching.

Fiona

At 28 years of age, Fiona came to teaching after some years in another career. She attained the second highest maths anxiety score (40), and the lowest beginning ACT score (14) (see Figures 5.1and 5.27). Fiona has only taught new entrant and Year 1 students, so her huge rise in MKT cannot be explained by the range of mathematics she would have encountered, as has been suggested in the case of Carly. First Fiona's interview and her attitude to mathematics will be examined (MARS), and then the results of the MKT will be analysed to see if there are any further understanding to be gleaned.



	Ist	2nd
MKT	36	91
ACT	14	29

Figure 5.27. Fiona's results on the two MKT tests and ACT scores.

Fiona's interview mirrors that of Debbie (maths anxiety: 45) in many ways. There is the professed hatred of mathematics coupled with her own high maths anxiety of 40:

> Well every time it was maths I would be absolutely scared. Yeah it was quite a nerve-racking experience. I remember we had these books and those were the tests. You were given the tests, so it was up to you to learn them, so you kind of learnt them off by heart but then when the test was done you forget them, that's why you never understand, especially geometry and statistics I'd say, but just the basic stuff was mostly formulas and stuff like that.

And her positive experiences at her university teacher education courses mirror Debbie's experiences too:

> I just hated maths pretty much and just left it but now, doing maths here [university] I see it in a different light. I kind of go oh I understand it now. I'm not scared of it because I know why I'm doing it. I'm a very logical person so I need to know what I'm doing and why I'm doing it.

Fiona credited her teacher education courses with enabling her to understand mathematics well enough to teach it:

> Definitely. That's why I'm doing maths now. Usually I'm oh I don't want to do maths but now I'm doing maths as an extra paper and actually it's become one of my strengths in teaching.

She went on to talk about how aspects of the NDP have really helped her visualise the mathematics, saying

> I'm a visual learner and before I started doing this [learning about NDP] I didn't realise how in my head I was thinking and things and because I'm a very logical person but like I need to visually see it, in order to be able to do it.

But finally, she seemed to have discovered what this dislike of mathematics was about: being tested and being put on the spot. She says:

> I do have a problem with maths, on the spot, like the tests we did in the last paper and I think you know I knew that question, it's just the fact that you know it's a test and that word test. From past experiences it builds anxiety up for me personally but I think as a teacher you should make the students feel that they don't have to have the anxiety of maths and that it's just, you are with them but yeah I think it's just being put on the spot, that's the problem. I love doing, I love solving things and I love watching numbers because that's the favourite thing, you work with them, how did you do that, but yeah it's just being put on the spot.

In the MARS (Richardson & Suinn, (1972), Fiona was at one and the same time, highly maths anxious if it involves tests, but happy if it involves repetition of recent mathematics course-based activities. Questions about tests or testing situations she rates as "strongly agrees", or "agrees", but the statement "It wouldn't bother me at all to take more maths courses" she rates as "strongly agrees". Fiona and Debbie both agreed that their experiences at university predisposed them to enjoy the prospect of further mathematics courses. This illustrates that though both their past experiences had occasioned maths anxiety, recent experiences had been pleasant, and this may be the key to why Fiona has scored so well on the final MKT. She learnt more mathematics in both the compulsory and elective courses in mathematics that she took as part of her B.Ed. Teaching young children the first building blocks of mathematics understanding—counting—may have allowed her knowledge of the pedagogy behind teaching to awaken and inform her about the teaching situation in other mathematics levels too. In the final MKT, the only questions that Fiona got wrong were those involved with fractions, and she would not have had occasion to teach those to new entrant and Year 1 students.

So, it appears, the factors that might affect the increase in MKT are neither simple nor one-dimensional. Fiona exhibits high confidence when teaching young children, and she has developed knowledge of mathematics far beyond those she has been teaching, so I would speculate that she possessed this knowledge previous to beginning teaching. She may have developed much of it in the university mathematics courses, after which she decided that she "understand[s] it now" and mathematics had become a strength in her teaching. What she may have been lacking was the pedagogical knowledge to answer the MKT questions in a teaching context. This increase in understanding of pedagogy might have improved just by teaching. The age of the students did not particularly matter; it may be that the act of teaching has universal tenets regardless of age or level.

The final participant in this section is Evie, who's MKT score reduces over time, as did Debbie's. Like Debbie, she has a medium maths anxiety score (29), and she taught only Year 2, which is the same at Gina. With so many different factors in common with different people, what could Evie add that will help the picture develop?

Evie

While Evie was of medium maths anxiety (29), she really did not like mathematics, saying:

> Maths at school for me was quite boring. I always found it really boring . . . So yeah the basic maths I'm quite able to do and that sort of thing but anything that you don't use in everyday occurrence, you know money is easy and adding and subtracting stuff is good but anything else, I just saw as pointless really, when am I ever going to use that? I remember it being really boring and always in the afternoon which really had a big impact on how I've decided to teach maths.

Again we see the experiences at school being a catalyst for how mathematics could be taught, which is different to that they themselves received.

> What I did find when I went to university, when I started doing my maths paper and teaching the children again, the things I was teaching them, I actually re-learnt and I learnt so much maths at university because I was having to go out and do micro teaching and I was like, oh, in the middle of a lesson, I'd sort of go oh that's what it is, that's how you do it. So I've learnt a lot while I was at university.

Here Evie echoed the words of Debbie when she talked about how studying the NDP at university was good for both teaching and for her own learning, saying:

> I think we had a middle school somewhere and I was, I don't know how to do this and once I got out there and did it and using [NDP] it was just fantastic and so the next time I went out in my first teaching session it was easier.

Evie had a vision for teaching but this was quite different from her own experiences.

> I think because of what I went through with maths and what I remember and wanting to become a teacher that's like what sticks out in my mind the most, is to make maths really enjoyable and then hopefully even if they are struggling at least they're having fun and they're learning something.

She went on to highlight these differences:

> I also think I was only ever taught algorithms and now they have all these different strategies and you use whichever one is best for you but you still learn the others which are great.

Then she became very specific about what she would be using in her teaching:

> It would have hands on stuff, so that they can manipulate the materials if they need to, but also, if they don't need it, they can just have the opportunity to work it through in their heads and interaction as well. Sharing the ideas with each other and obviously the teacher so that the teacher knows what they're doing and a good follow up lesson or follow up activity that's relevant to what you're learning so they can go away and take that knowledge and apply it in different contexts.

Evidence from the DART coding will now be accessed to ascertain whether or not Evie managed to teach differently from the way she was taught mathematics, and whether she managed to use the NDP as she had hoped.

In the first video, from the DART coding, Evie was teaching her Year 2 class to add by counting on; this is a lesson from Book 5, page 18. She changed the materials, using tens frames instead of number cards, and at points she gave each student a card and counters to aid them in discovering the answer to the addition 6 + 2. At times the children were working together, but mostly they were working as individuals. As is the usual practice of NDP lessons, there was a context. This time it was about sweets in two different bags: how many sweets are there altogether? In her lesson, Evie had no context at all and she continued with all her groups in this way, using NDP Book 5, using materials, working mostly independently, but occasionally in groups. This could be considered a fair attempt so early in her teaching career. Obviously she could have improved the lesson, but it would easily fit under the NDP umbrella.

In the second video, she was working with a group of Year 2 students, again using a lesson from Book 5, page 24, subtracting two digit numbers without renaming. Evie began with the subtraction 37 - 12, without the obligatory context, in fact, never using a context at all. The lesson progressed with members of the group being sent away as Evie notices them having success, until only four people remained who did not answer her questions. At this point, instead of giving them a context, and perhaps materials to help, Evie continued to work in number properties, and took them through every subtraction, trying to ensure their learning by force of will. They hd no success, and Evie provided all the answers. She was using an algorithmic overlay here, and instead of stepping back into either imaging or materials, she continued with the lesson, irrespective of the evidence of no progress from the students. This was a very retrograde development; these are only Year 2 students who appear to be being taught a mathematical concept that was way above their expected level of achievement, and not having success.

Evie said in her interview how she wanted to make mathematics more fun; instead she has stripped it of all context, made it a non-engaging session, and kept going when it was obvious that the students were being asked to work, algorithmically, above their present level.

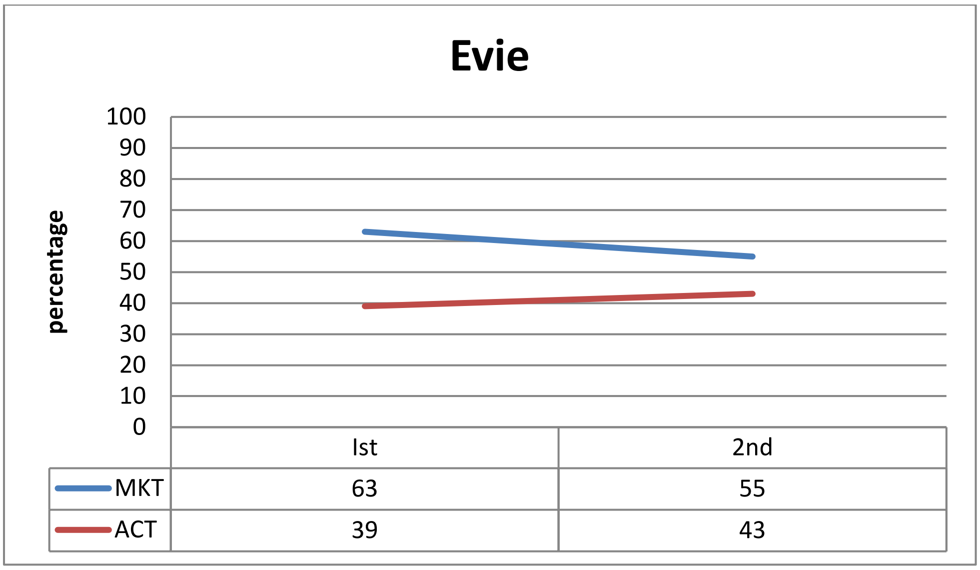


Figure 5.28. Evie's results on the two MKT tests and ACT scores.

Looking at the ACT score section of DART (see Figure 5.28), there was little change in Evie's score over the two years but what there was looked to be in the positive direction. However, when the actual behaviour that Evie invoked in the eliciting, supporting, extending areas were analysed, it became apparent that there were more high-level behaviours at the beginning of her teaching career than there were two years later. Evie's MKT score of 63% was above average for these participants, and only Debbie (48%) scored less than Evie (55%) for the second MKT test. This reduction in MKT is an indication of stalled mathematical pedagogy development, and the evidence of DART seems to back this up.

Summary: The five vignette participants

Information from each of these five people, Alison, Barbara, C, Fiona, and Evie, has been used to highlight factors of quality teaching. Some factors for each of these five participants matched or contrasted with one of the major case studies, others reinforced ideas about NDP teaching and the pedagogy used. A precis of these matches and contrasts are shown in Figure 5.29.

Name	MARS	Gina Match/Contrast		Debbie Match/Contrast	
Alison	13	Same MKT_2 scores	Very low maths anxiety	Same ACT_2 scores	Enjoyed maths at school
Barbara	15	Small change in ACT scores	Strong NDP style	Same MKT_1 scores	Strong NDP style
Carly	18	Same low maths anxiety	Opposing ACT_2 scores		Opposing ACT trajectory
Fiona	40	Similar MKT & ACT trajectory		Same high maths anxiety	Opposing ACT trajectory
Evie	29	Similar low change in ACT	Opposing MKT scores	Similar reduction in MKT	

Figure 5.29. Factors from the five vignettes which match or contrast with those of Gina and with Debbie.

These five vignettes were used to further develop the ideas regarding quality teaching, and what it looks like in NDP style. When completing Figure 5.29, I realised that some of the participants did not have contrasts or matches with the two main case studies; this was because these participants were not chosen but were serendipitously available, and therefore gaps should be expected.

Overview of results

This chapter has presented the results of the study into quality teaching. There are many factors feeding into whether the videos and other evidence presented here through data capture by DART represent quality teaching. The following framework (see Figure 5.30) shows lines of influence between factors that have been found to affect quality teaching in this investigation.

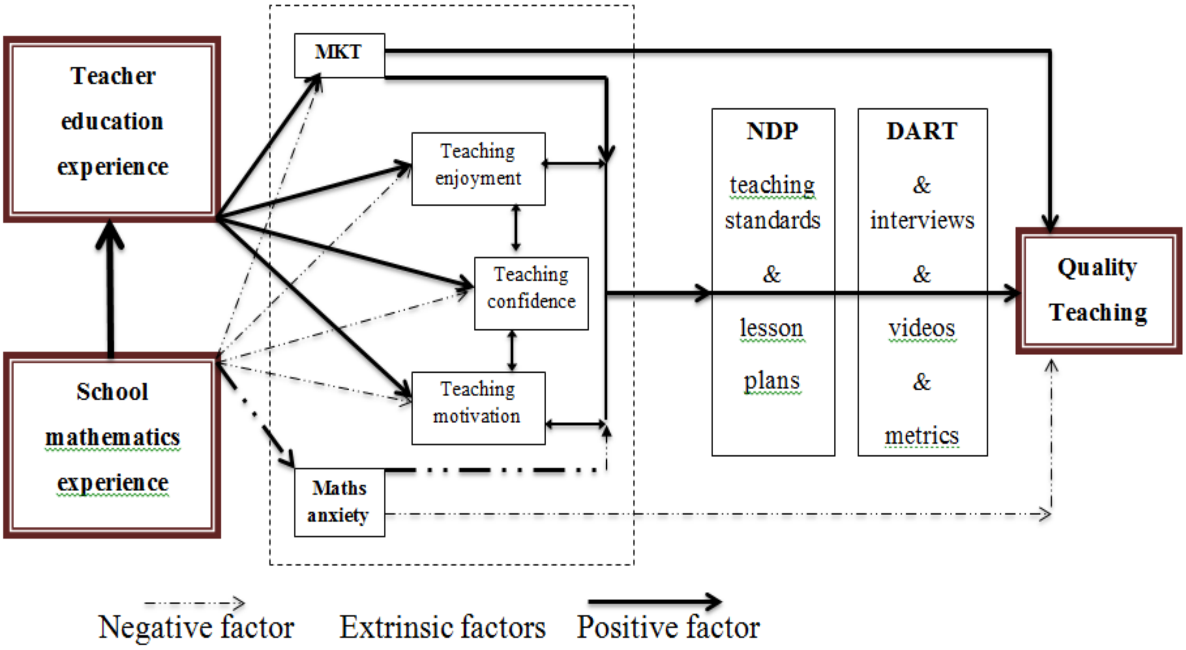


Figure 5.30. Factors involved in quality teaching.

The use of the two full case studies and five vignettes has allowed an exploration of what pre-service teachers bring with them to their initial teacher education courses, what factors they may possess when they start teaching, and how, after two years, these factors are still evident. Some factors are subsumed under another factor, or they have become neutral, now that the teachers are fledged out of their novice teacher status and on their way to becoming experienced teachers. The lines of influence emphasis the complexity of the research in this study.

The next chapter will discuss and compare the DART results and other measures for the two case studies and the five vignettes to see what, finally, one may say about quality teaching and the novice teacher. The research tool, DART, will also be discussed, to answer the main research question.

Chapter 6

General Discussion

Advanced Multiplicative-Early Proportional

Students at this stage are learning to choose appropriately from a range of part-whole strategies to solve and estimate the answers to problems involving multiplication and division. These strategies require one or more of the numbers involved in a multiplication or division to be partitioned, manipulated, and then recombined.

The Number Framework

This stage of NDP framework resonates with me because students who reach this stage in number are able to choose from a repertoire of possible solution paths, which sounds analogous to a researcher choosing their method or methodology, both about choice with knowledge of what went before (benchmarks) and then testing out the choices. This chapter is the completion of that testing.

This chapter presents a discussion of the study comprising two case studies and five vignettes. These are discussed with regards to answering the research questions, and in particular the DART instrument: what it disclosed about quality teaching in NDP and the novice teacher. In addition, DART is examined to see what application it might have with regard to quality teaching outside of NDP, outside of mathematics and in the international context. Within this discussion are embedded the factors that are important for a beginning teacher to aim at, in order to strengthen their teaching early in their career. This chapter continues with considerations of some limitations and implications of the research, discusses possible avenues for further research, and concludes with some final remarks.

Research Questions

My primary research question which relates to novice teachers teaching in NDP-style was:

How could an instrument be designed to capture the development of quality teaching?

Subsidiary questions were:

- What is the nature of quality teaching evident in the teaching practice of novice teachers over their first two years in the profession?
- How does the teaching of novice teachers change over two-year period of their provisional registration?
- How does initial maths anxiety affect the development of quality teaching?

First, to orientate the discussion, a restatement of some theoretical considerations is presented. The "big ideas" of Shulman (1987, see Figure 6.1) provided an appropriate framework for this investigation which concentrated on uncovering the qualities a new teacher requires to teach well. The three areas contained in Shulman's framework are presented on the left, with the evidence from this study on the opposing side, each feeding into the complexity which is quality teaching in NDP-style.

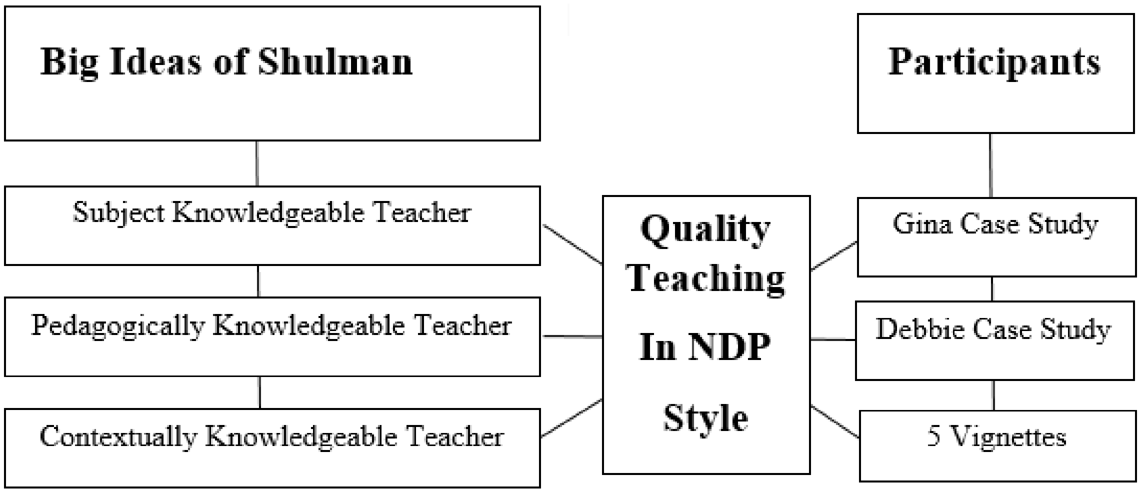


Figure 6.1. The "big ideas" of Shulman (1986) and the case studies used to illuminate aspects of quality teaching.

Evidence from the two case studies and the five vignettes illustrated the usefulness of Shulman's framework (1986), and highlighted the important aspects of teaching in NDP or a reform teaching style (Alton-Lee, 2003; Cheng, 2011). These points, along with the development of the measuring instrument DART, will be the focus of this discussion.

Factors in quality teaching

In my search for a suitable tool (Hill, 2006; Hill & Ball, 2006) to capture quality teaching (Akiba et al., 2007), I was compelled to develop the measuring framework of DART. It indicates that beginning teachers bring many different factors to their initial teaching education (Bailey, 2014)(See Figure 6.2.). The development of the factors contained in quality teaching were seen in the various data streams through two case studies, and five vignettes.

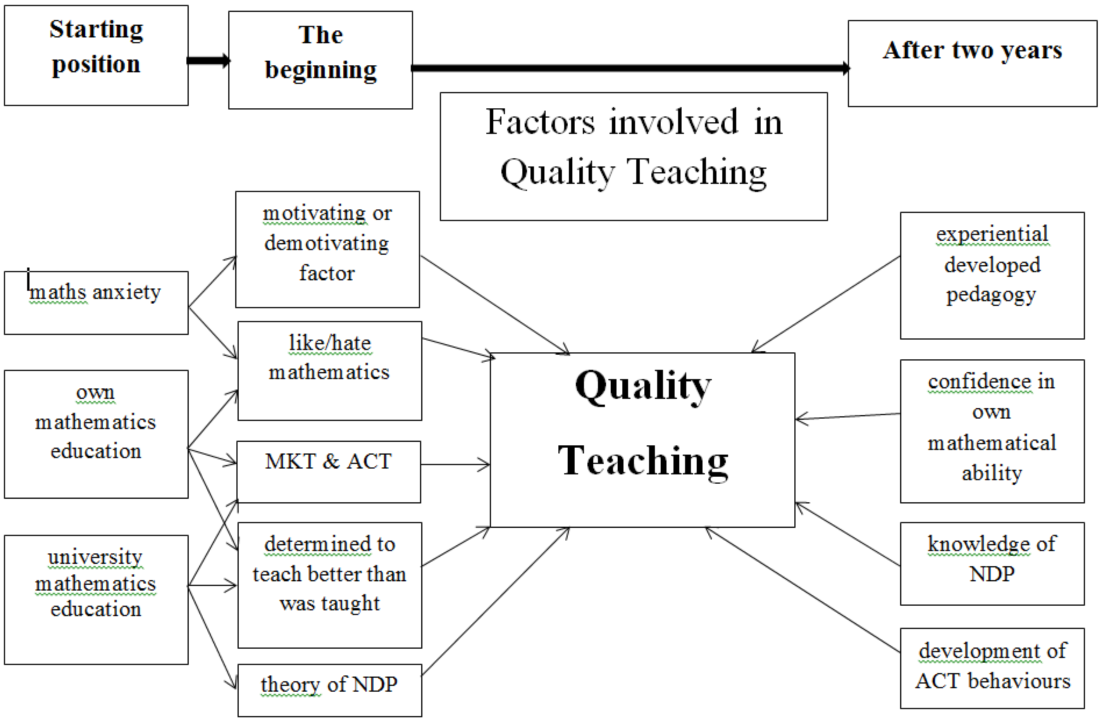


Figure 6.2. The factors that were found to affect quality teaching.

At the inception of this research, I was aware that maths anxiety might be a factor in the development of the novices' teaching (Richardson & Suinn, 1972) but what the effect was might be was unknown. However, the results show that the factors affecting teaching are more numerous and more complex than a simple view of maths anxiety might allow. The maths anxiety of the participants could have depressed their ability to develop factors involved in quality teaching, but the interviews and the DART coding show that maths anxiety can have a positive effect too (Chaman & Callingham, 2013; Maloney et al., 2013) This is evidenced where teachers, such as Gina, show determination to teach well despite their obvious deficit in factors such as mathematical knowledge for teaching (MKT). It drives them towards quality teaching because they are compelled to give the best lesson in NDP-style of which they are capable, and so what might have been a depressing factor could become an enhancing factor, as was seen in the case study of Gina.

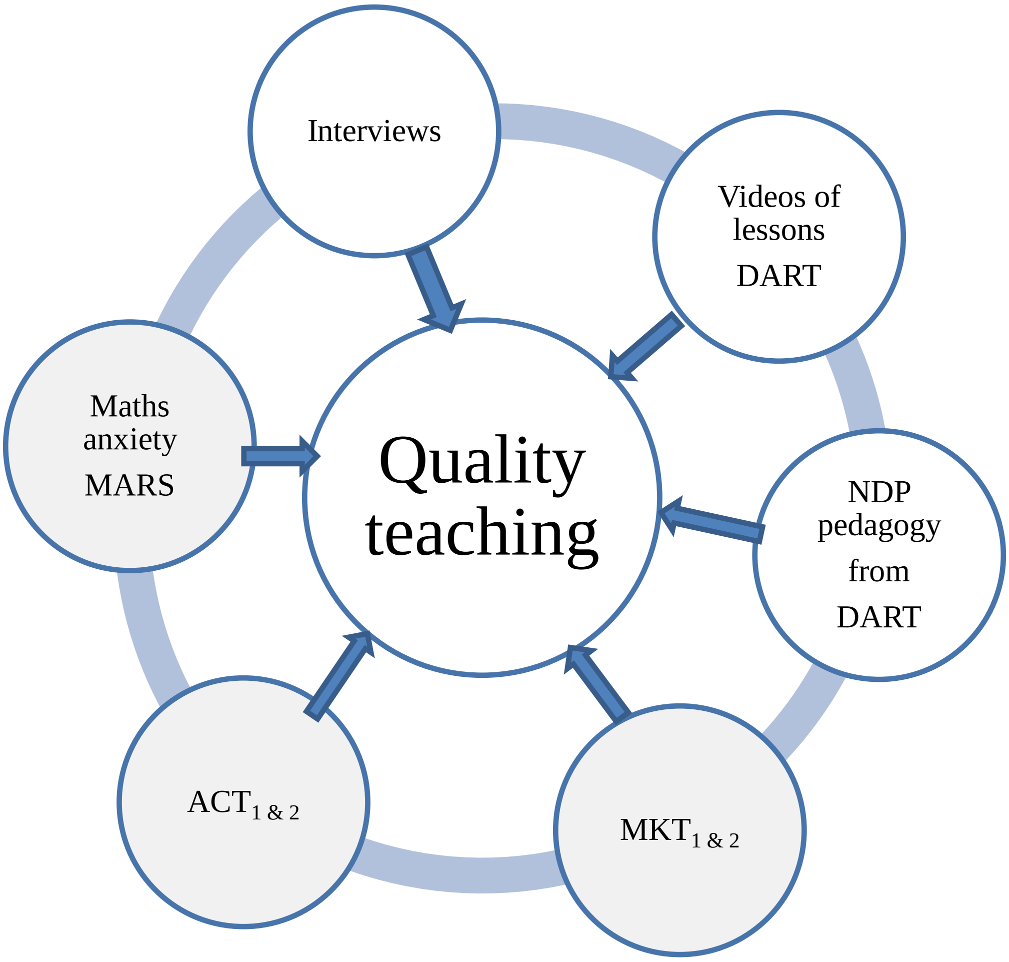


Figure 6.3. The research data collected to investigate quality teaching.

The research data as depicted in Figure 6.3, was both quantitative (MKT, ACT, MARS) and qualitative (videos, interviews, DART), both types are required to tell the story of the novice teacher moving through the first two years of teaching towards becoming an experienced teacher (Bryman, 2007). Being able to measure quality teaching through these two years was the focus and outcome of this study. In Figure 6.4 is a summary of the results of the two case studies and 5 vignettes.

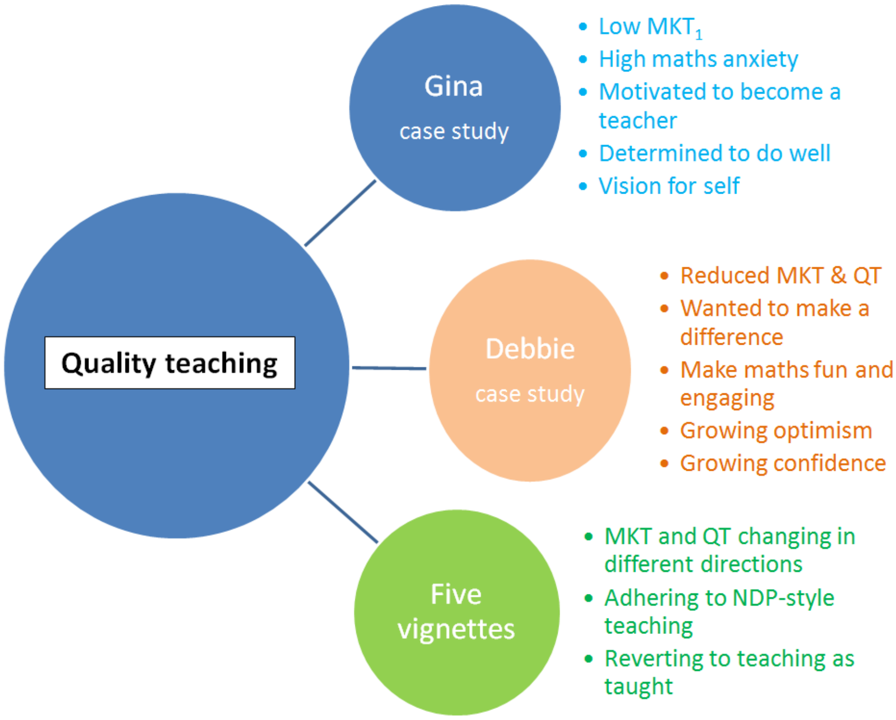


Figure 6.4. An overview of the results of the two case studies and five vignettes.

A tool to measure quality teaching of novice teachers in NDP lessons

Since the original instrument of choice to encode the videos, MQI (Hill, 2006), did not capture sufficiently well the important aspects of teaching in NDP-style, it became necessary to ask if it were possible to develop a suitable tool. Hence, the research question, asking whether such an instrument could be developed, was central to this research. The development of DART in response to this need is detailed in Chapter 4; here I present an argument regarding how DART is indeed capable of capturing quality teaching.

The motivation for the development of DART was to capture the actions of the lessons of teaching in the NDP-style, recorded by the videos. The MQI, the original tool, suffered from being unwieldy, a characteristic of other encoding methods, as Schoenfeld (2013) discovered when he and colleagues were investigating an appropriate coding instrument for their work on teacher decision-making. They too, like me, could not find an appropriate instrument, and so set about constructing one expressly for their purpose, a process they found to be a very complex and a very time consuming, but in the end they had an instrument fit for purpose. This mirrors my development of the DART instrument. I originally developed and designed DART using the three pedagogical tools of NDP: the framework; the diagnostic interview; and the strategy teaching model (Higgins & Parsons, 2009), together with the exemplary teacher video. This development was set firmly in the New Zealand context, for classes being taught in NDP style. In the next section, I will look at international research into quality teaching, and numeracy, to consider how DART might have applications in other situations outside the New Zealand NDP setting.

How useful might DART be in the international context?

New Zealand's NDP is one of many numeracy initiatives around the world. For example, Australia has had a numeracy initiative that grew from the Count Me in Two research. In the U.K. there has also been a country-wide National Numeracy Project, which began earlier than NDP, and where teachers work in different ways to NZ. The greatest difference is the dictatorial instruction to teach the class all together, a marked departure from NDP which recommends working in small groups of students, grouped for some knowledge similarity, and differentiating the curriculum for children's learning. DART will be examined to see what use it might be put to in such different numeracy contexts.

Taking our closest country first, Australia's implementation of their equivalent of N.Z.'s NDP, has not been put into action country-wide, but a quality teacher framework was implemented state-wide in New South Wales (Ladwig & King, 2003). This framework, covering all curriculum areas, not just mathematics, has three strands; intellectual quality; quality learning environment; and significance for students. DART would certainly be applicable to teachers who video'd themselves teaching because it has the capacity to capture the kinds of pedagogical change associated with the NSW quality teacher framework, where teachers need to have a deep, connected understanding of the concepts they are teaching (Hammond, 2008; Lovat, 2009; Shulman, 1986). In DART the group icons (G-icons) could be used to capture the current way that the teacher allows the students to work in their thinking groups, even if they were not working in the small groups that NDP privileges. To begin the change of pedagogy cycle, a teacher uses the first page of DART to see what portion of the lesson she is allowing students to work with each other, and how much she dominates the teaching space. The types of questions she asks and the amount of time she talks versus the students' talk-time could also be captured, together with who is using the equipment. In addition, she could fill in the final line of boxes in order to describe each minute of the teaching she captured. In fact the only area this Australian teacher might not find useful in DART could be the section that is predicated on the New Zealand Numeracy Model (Hughes, 2002), Imaging and Number properties. Another part that could be less useful is the first table on the second page of DART, first table, where books and resources specific to NDP is captured. However, the second table, labelled

Instructional components of the ACT Framework, could be employed to capture the ways of working that contribute to rich, connected pedagogy, with statements such as *Elicits many solution methods for one problem from the entire class* being an indicator of quality teaching. DART will help direct the teacher's attention to various components of quality teaching, and coding these in DART might illuminate how the teacher could effect pedagogical change. The encoding of their own videos would then allow them to capture and improve on the desired pedagogical change.

Thus, the claim made here for DART is that it is flexible enough to work outside NDP and outside mathematics, although not every aspect of the coding's two pages will necessarily be equally useful. However, repeated videoing of lessons and encoding could allow teachers, such as those in NSW, to make apparent their pedagogical change. Over time, they would see their highlighted G-icons move into the centre of that section, meaning the children will be working in their groupings, and the teacher's role has moved from one of didactic instruction to that of facilitation.

The middle strand of the NSW Quality Framework is that of quality learning environment, which emphasises that the milieu needs to be supportive and safe for students. This would be seen in DART when the G-icons are in the middle, when the totality of the children's talk vastly outweighs that of the teacher, and where the teacher's questions are open, and the extending section on page two comes into play with, perhaps, *encourages mathematical reflection,* or *encourages students to analyse, compare and generalise mathematical concepts.* This would indicate that this is an equitable classroom, with student and teacher re-configured as life-long learners, learning side-by-side (Skemp, 1976). For their part, students will be able to inhabit more of the learning space, do more of the talking, and get the opportunity to ask open, searching questions, of each other and their teacher (Mason, 2000).

The final strand in the NSW Quality Framework is about significance for students, so the work they undertake should have collaborative and cooperative ways of working, with teacher helping to connect all the important concepts. Again, the G-icons will show how the children are working, are they in groups, are they discussing and putting forward conjectures that are received by the teacher and the group and taken as an opportunity to refine and connect their understandings (Gore & Bowe, 2015). The work needs to be authentic, collaborative problem solving, as well as having rich meaning in their lives. (Akiba, et al 2007; Sheryn, Frankcom, & Ledger, 2014).

Having considered an application to the New South Wales experience of pedagogical change, I will now turn to the U.K. where numeracy pedagogical change could be said to begin with the Cockcroft Report in 1982. While not solely concerned with numeracy aspects, nevertheless, the report began the push for greater numeracy in both people's lives and at school. The National Numeracy Project (NNP) (1996-1998) centred on changing the pedagogy of the teachers (Years 1-6), paralleling NDP in NZ (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997), but the U.K. experience was very authoritarian, stipulating what was taught and that it should be taught to the whole class. The idea behind this hoped-for interactive whole class teaching was that there would be stimulating, high quality discussions with students expected to be asking questions, contributing to the class, and explaining their thinking (Smith, Hardman, Wall, & Mroz, 2004). These intentions, with high expectations of students, were not found to be easily implemented. The types of classes seen after the implementation of the NNP were very similar to normal or previous-type classes, where teachers still occupied the teaching space, using didactic methods, and funnelling students into short, expected answers (Mason, 2000). Teachers have often proved very change-resistant, and overcoming their resistance has proved difficult in many areas. This difficulty associated with instituting change, e.g. establishing good mathematics discussions, was found in the efforts of the educational establishment in China. Educational authorities found it was too difficult to elicit change in classrooms where teachers were both unwilling and unable to implement these new pedagogies. The conversations tended to be either straight questioning from the teacher, or teachers probing until they got the answer they required (Dello-Iscova, 2009). To achieve a more responsive pedagogy Smith et al. (2004) found that teachers needed reflection and on-going self-help sessions and DART has the capacity to help teachers in this manner. They could begin by videoing themselves teaching, and then encode their teaching, for further reflection. This could continue for as long as it took to effect the change they desired. The questions section could be particularly apposite here, capturing the numbers and types of questions asked (e.g., open or closed). It has been found that teachers rarely ask the sort of questions that can make a real difference in whole class teaching (Smith et al., 2004) but this could be captured by page 2 of DART, where improvement is demonstrated by an increased number of ticks in the Extending column, for instance under *Uses students' responses, questions and problems as core lesson.*

As teachers pursue their path of more interactive teaching, DART is capable of recording this change, together with the degree of talk from the students. Over time, a determined teacher could use the group dynamic section to see when the teacher was dominant (G-icons more over to the left) and when they were achieving a more authentic interaction, where students were able to discuss the work in their thinking groups, and at times explain to the group or class. This would mean that, over time, there would be more G-icons highlighted in the middle of that section, more equitable distribution of teacher and students asking questions, talking, and interacting with other students. This could be enlightening to the teacher, as Smith et al. (2004) found that students talked very little, and the underlying pedagogical change required for this type of transformation was difficult to achieve (Whitehurst, 2002). As the instructions in NNP to move from the didactic to the problem-solving class came from outside the teacher domain, and hence was perceived as top-down, (Smith et al., 2004) they proved ineffective, with teachers either not accessing the available support documentation or not realising it was available. It is known that to activate an enduring change in pedagogy, teachers must feel part of the process (Timperley et al., 2007), and using DART certainly gives them the chance to effect change for themselves The notion of facilitating a modern pedagogy with teachers engaged in empowering students to have resilience and determination, working at where their current understanding is, often through collaborative problem-solving (Sheryn et al., 2014) would be viewed as best practice.

The issue of measuring teachers' performance or practice is complex but Schacter and Thum (2003) have theorised that it would best be done by using explicit standards and that this would lead to improvement in the quality of teaching. Their twelve teaching performance standards were: '*teacher content knowledge, lesson objectives, presentation, lesson structure and pacing, activities, feedback, questions, thinking, grouping students, motivating students, classroom environment, and teacher knowledge of students'*. How could a teacher use such standards to effect pedagogical change? Could DART have a role in improving teaching against these performance standards?

This might be particularly important if the political climate meant that all teachers might be measured against objectives and standards such as these. Such a proposal could prove very difficult for more nervous, or maths anxious, teachers of mathematics who might not welcome these performance standards and the public evaluation that would inevitably accompany them (Burns, 1998). So teachers could use DART to take charge of this pedagogical change themselves. The advantage of this is that DART could be used by a teacher without outside interference, or the worry of how others might perceive the teaching they are coding, since this can all be done by, and for, the individual teacher. There is no restriction on how long DART might be used, so teachers could cycle through, looking interchangeably at one or more of the standards. In the research of Schacter and Thum (2003), a system of standards, rubrics and definitions to rate their teachers was used, but what they had yet to do, was to construct something that teachers could use and be in charge of while evaluating their own pedagogy. DART is a strong contender for use in both pre-testing against the standards and coding for being at, above, or below the standard. This is high stakes rating; it might not just be about whether a teacher progressed to the next level on a salary scale, it might be about whether the teacher should cease to teach altogether (Schacter & Thum, 2003). In the next section, I examine where DART could be of use when measuring against these standards, using the rubrics generated by Schacter and Thum (2003). While not all twelve standards might be encoded by DART, there are eight that are very explicitly captured. They are *activities, questions, feedback, grouping students, thinking, motivating students, environment and teacher knowledge of children*. By analysing the Schacter and Thum (2003) work, it is hoped that the depth that DART can capture will be made visible.

Found in the *activities* standard rubric are these words: challenging, sustain 90% of the students' attention, provide time for reflection, incorporate…manipulatives. Coding for these would be found in DART particularly in the G-icons, (G-icons in the Group Dynamic section), as these tell the way the lesson is progressing, whether students are working collaboratively, or if the teacher is taking a didactic stance, in which case, the G-icons would be mostly over to the left. This would allow a teacher to see if there is progression to a lesson where students were being active in their learning and working in groups. Whether students are investigating a situation set up by the teacher, with manipulative use, would be seen in the M-icons, and the type of manipulatives would be captured in the description of minute section. Not only would the appropriateness of manipulatives be apparent in the M-icons, but also if the manipulatives were being used by the students would be coded. Providing time for reflection could be captured on the second page of DART under, *encourages mathematical reflection* and *lists all solution methods on board to promote reflection*. Furthermore, 90% of the students' attention could be captured under the Eliciting section, *Monitors student's levels of engagement.*

In Schacter and Thum's (2003) *questions* rubric is found: Teacher questions are varied and provide a balanced mix of question types and the teacher asks both volunteers and non-volunteers, and students are expected to generate questions that lead to inquiry and self-directed learning. The first page of DART contains a section for explicitly capturing the types of questions a teachers asks. These can be accumulated and if the teacher needs to adjust the number of questions they ask or the return questions from students, the subsequent section, which captures the time the teacher talks and how long all the students combined talk, could be used to get this balance finely tuned. The ACT section on page 2 of DART contains *uses students' responses, questions and problems as core lesson*, which, when ticked, reveals that students are questioning for themselves.

Keeping to page two of DART, under the Eliciting section is found *waits for and listens to students'descriptions of solutions methods,* and *Encourages elaboration of students' responses.* The *feedback* Standard of Schacter and Thum (2003) explicitly says the 'Feedback from students is used to monitor and adjust instruction. Teacher engages students in giving specific and high quality feedback to one another'. In addition their *grouping students* standard contains instructions about grouping consistently to maximise student understanding, and this would be captured on DART page 2 under the Extending section *asks all students to attempt to solve difficult problems and to try various solution methods*. Together with the group information in the G-icons, this could give a fuller picture of the extent to which the groupings were effective.

The *thinking* standard describes four types of thinking: analytical thinking; practical thinking; creative and research thinking. Once again, in the words of Schacter and Thum, DART could 'monitor their thinking to insure that they understand what they are learning, are attending to critical information, and are aware of the learning strategies that they are using and why' (Schacter & Thum, 2003, p. 428), through use of both the first and second page. As a teacher can only be sure of student thinking when it is verbalised or written, the G-icons could capture the students working together in groups, and sections of their speech could be captured in description of minute section. Add this to the ratio of time that the students are talking, and a fuller picture would emerge. On page 2 of DART, various statements enable evidence of thinking, particularly in the Extending section *encourages students to analyse, compare, and generalise mathematical concepts* and the Supporting section is *supports listeners' thinking and supports describers; and listeners thinking.*

The standard Motivating students involves meaningful and relevant content, inqury and teachers rewarding effort. These ideas could be encoded for in the use of NDP resources, where all activities are in a range of meaningful contexts, using the inquiry method (NZC, 2007). Also the second page of DART under the Eliciting section has *waits for and listens to students descriptions of solution methods*, and *decides which students need opportunities to speak* *publicly or which methods should be discussed.* This data, among others, shows how the teachers reward students with attention and show appreciation for their efforts.

The final standard that can be most easily encoded in DART is that of *environment*, in which the teacher is seen to provide opportunities for making errors and learning from them, together with high expectations and mutual respect. Again, on the second page of DART these statements are found: *conveys accepting attitude towards students' errors and problem-solving efforts, asks all students to attempt to solve difficult problems and to try various solution methods, and goes beyond initial solution methods.* This last statement is intended to elicit data that shows how a teacher presses for mathematical understanding, showing her high expectations of her class or group.

This analysis of how DART could measure quality teaching as it relates to various bodies of research that define quality teaching in different ways, from the desirability of cognitive abilities, to the performance standards of Schater and Thum (2003), has demonstrated the various parts of the instrument that would be most strongly evidential. This application of DART in an international context has shown it potential to provide teachers with the power of self-help in their professional learning. It could also be used to initiate appraisal, or to show that individual teachers have been able, not only to consider the standards but also to provide evidence in the form of a completed DART to show that there has been movement towards a modern pedagogy, including use of manipulatives and teaching in modern learning spaces.

For some, the goal for quality teachering is a cognitive one. Whitehurst (2002) came to the conclusion that to reduce the variation in teacher quality, three things should be done. They were, firstly, to choose teacher candidates who had high cognitive ability, secondly, improve their intitial and ongoing teacher education with the support and input necessary to produce high quality teachers, and thirdly, have specific content focus in the courses in ITE and on-going education. It seems clear that DART is able to capture that high cognitive ability (Hammond, 2008; Lovat, 2009; Shulman, 1986) through teacher actions. One place it does so is on page one of DART, via the G-icons, recording the way the teacher organises her sessions with children. While aspects of this capture cognitive ability, it is more easily seen in, the ACT section. Teachers who have high cognitive ability will be able to see the variety of their ACT behaviours, and work to include more of the higher order components, especially those in the Extending (Mathematical Thinking) column. For instance, *uses students' responses, questions and problems as core lesson*, involves teaching acts rarely seen in classrooms (Fraivillig et al., 1999). Someone who is capable of acting on childrens' responses to open questions, analyse the information that provides in real time, and then synthesise it into the direction for the next part of the lesson, regardless of what might have been originally planned for, would be a special teacher indeed.

To complete this section on how DART can be used internationally outside of NDP, its intended setting, will address the interior factors that are not imposed byeducational theorists but are initiated by teachers, for themselves. Evaluative systems imposed on teachers by others were usually disliked by teachers, tend to lack intellectual rigour and give little return for the high cost of implementation (Hallinger et al., 2014). Add to this the extra teacher work-load associated with evaluation compliance, and it has been concluded that the research evidence supports more teacher-centred ideas, not something imposed from outside. There is a need for feedback that is conveyed to teachers in real time. This can be met by teachers using DART to code videos of themselves teaching and to analyse the various attributes they are attempting to improve. This would provide very timely feedback, showing improvements, and areas of shortcomings that require more effort. Crucially then, DART is not an evaluative system for external imposition on teachers, but more a tool for teachers to capture the elements in their teaching that they themselves have decided are important. Other elements identified as important for increasing teaching qualilty (Hallinger et al., 2014) were professional learning communities, support during teaching and professional learning opportunities. DART could deliver for professional learning communities through the action of groups of teachers watching and coding other teachers in their groups, leading to discussions about what ACT elements are currently being improved. While DART could be said to support teachers during their teaching, the real value of it as a supportive coding scheme occurs after the teaching has taken place. It could be used to analyse areas where teachers require some professional learning, with the DART forms giving evidence that help was needed on such diverse aspects as how to ask questions in an open manner (Mason, 2000, 2012), or how to cultivate a love of challenge (Fraivillig et al., 1999).

Having examined how useful DART could be in capturing quality teaching in international and/or non-mathematical teaching, in the next section I return to the people in this thesis, and further analysis of the way that DART has been shown to encode their changes in quality teaching.

How DART captures changes in quality teaching

The following is a discussion of how DART performed when capturing the development of quality teaching of the novice teachers in this study.

In the case-study of Gina, DART was able to capture the complexities inherent in both the similarities and differences between the two videos, V_1 and V_2. In both these videos Gina teaches groups that have been formed using the diagnostic interview (Ministry of Education. 2008b), one of NDP's pedagogical tools. Gina teaches a lesson from the "pink books" (the resources from NDP); however, DART clearly shows that she teaches the group as though each student was a separate learner, not harnessing the strength of group work (Fraivillig et al., 1999). which involves participants' discussing and peer teaching. Gina is also restricted in her ability to identify the teachable moment, so her lessons adhere strongly to her original intentions; and DART shows that she did not seem capable of the flexibility (Thomas, 2006; Timperley, 2011) that quality teaching requires. In the second group on V_2, there is a change, with DART evidence that Gina teaches in a different, more NDP-style manner. She encourages students to work together on solving one big problem, to explain their thinking to each other, and she asks the group to use a different strategy to solve the problem; all these are changes not previously seen in V_1 and thus DART provides evidence of the development of quality teaching. This was portrayed by DART through the highlighting of the right-most G-icons (G-5, G-6, and G-7), indicating group work and more child-centred teaching. The teacher actions captured in DART include a reduction in teacher talk and an increase in student talk, signs that the teacher is developing her child-centred pedagogy (Fraivillig et al., 1999).Gina also displays more confidence in choosing the appropriate magnitude of numbers to use, captured in the "description of minute" section. All these factors were made evident by analysis of data captured by DART because specific NDP-type behaviours are built into it, DART has the capacity to show these clearly. Sometimes there is no need for further analysis since inspection of the two pages of DART reveals changes in aspects of quality teaching visibly and concisely. If necessary, a simple count of behaviours on the ACT (Fraivillig et al., 1999) page (the second page of DART), can show an increased variety of ways of working, implying development of quality teaching.

It may seem axiomatic that a measuring instrument built on the tenets of NDP will capture quality teaching in NDP, but that is hardly a given. Instruments must still be tested, and must demonstrate validity (Teddlie &Tashakkori, 2003), that they show what they purport to show. In this respect the DART instrument is able to capture and show changes in quality teacher over a time span, in this case, two years of teaching. Further, DART is simple, clear, easy to understand, use and employ in analysis, as its use in this investigation shows.

Next I consider the ways that DART could be used to capture quality teaching by using a mechanism of imagining a journey of professional development that Gina, and later Debbie, could undertake, in which I use the DART framework to make apparent the areas of teaching which could be changing or could be subject to change. This would be the completion of Gina's and Debbie's journey.

How DART can lead to an increase in quality teaching

DART was developed in response to a need to capture the complexities inherent in teaching in NDP-style lessons. While it has been used to analyse the videos of lessons from participants in this study, DART can also be used for self-reflection, analysis, and self-development (Whitehead, 1989). This aspect of the tool is the subject of this discussion.

There are two major applications of the DART tool envisaged here. One method involves an observer teacher, capturing information in real time, and the other would involve the teacher analysing their own practice, through self-videoing. The aim would be to capture information not easily available to a teacher because she or he is absorbed in the day-to-day act of teaching. Once the teacher reviewed their DART information, decisions could be made about which areas of pedagogy required attention, and in an action research-type endeavour (see for example, McNiff, Lomax, & Whitehead, 2003), the teacher could put forward a plan for self-review, going through iterations to achieve the desired outcome.

I propose two scenarios. First, Gina will be taken on an imagined journey in which she uses DART for professional development to enhance her progress towards quality teaching, and then I will take Debbie on a similar journey, this time highlighting different factors not apparent in the journey of Gina.

Gina's journey

As part of the journey that Gina might undertake she could video herself teaching mathematics to her groups, capturing her voice, actions, use of materials, and other factors such as how much her talking inhabits the teaching time. Having analysed her videos using DART, which she could do for herself, or with another teacher for an independent view to see if they are both observing the same things, she could then decide upon the story of the lesson. Gina may be surprised to observe the things that she does, and might be pleasantly surprised to see that she is an effective, but obviously still novice, teacher of mathematics. She could possibly also be unhappy about the fact that, although she groups children in her class, they work independently within their groups. In this way, using DART as a professional development tool (Timperley et al., 2007) she would be able to see these methods she employs, and use this to make informed decisions about her teaching. Gina might realise from the analysis that she cannot develop all the areas at once, and must judiciously choose the one or two areas to which she could reasonably expect to pay attention. A possible plan could be that Gina decides to ask more open questions, and try to “encourage mathematical reflection” (Fraivillig et al., 1999) while giving her students more time to do both, thereby possibly reducing her own talking time.

For a number of lessons, Gina would again video herself teaching mathematics, while trying to put her new plan into effect. She would need multiple lessons, since making changes to one’s pedagogy takes time. Gina may not see the qualities she needs to develop among the teachers in her own school, reinforcing the difficulty of affecting change in a short time period. However, in her interview, Gina indicated that she had a vision for herself in teaching mathematics, so she would be prepared to work on this plan.

Having taught and videoed and analysed these lessons, Gina might find that she is able to change some aspects of her teaching but not others. Her action research plan would allow her to change her strategy to accommodate what she notices, and what she now wants to transform. This circularity of iterations could be organised alone, or in conjunction with other teachers in her school. This constitutes excellent professional development, because it contains the very best of what are considered to be the primary factors in affecting change through professional development with the capacity to positively influence student outcomes (Timperley et al., 2007). There were seven effective contexts identified in this meta-study which will be discussed with reference to DART.

Extended time: There is no restriction on the length of time that this iterative process with DART might continue. It is entirely within the scope of the teacher, and will depend on how many elements they identify as useful to their development within DART. It could also be used as an audit sheet, once a year for example, to ensure that changes to practice are maintained. Timperley et al. (2007) found that the most effective professional development was between six months and two years in length.

External expertise: While Timperley et al. (2007) maintain that effecting change through professional development would probably necessitate an expert, this was not as overwhelmingly important a need as they originally thought. While providing time and place without expertise was found to be ineffective, experts by themselves could also be ineffective, especially if the extended time was not present. Using DART could be seen as a substitute for an expert. It contains elements that capture the essence of NDP lessons, and as the NDP resources carry much expert advice within them, and so DART could be used to assist a teacher in becoming an expert NDP-style teacher.

Teacher learning: Gina would maintain her stance that she voiced in her interview, that she was capable of learning more mathematics, and she did not seem to doubt that she could eventually become an effective teacher of mathematics. DART would enable Gina to engage in continued learning of mathematics, and to learn new pedagogical ways. Timperley et al. (2007) identified the level of engagement as being central to sustainability.

Changed discourse: One of the central findings in the Timperley research, (2007) was that for change to happen, the teacher needed to want to change. Teachers having higher expectations of themselves is highly contextualised and has to emanate from the teacher, it cannot be imposed upon them. This independence is inherent in DART, as teachers can use it as and when they require, and they can be their own agent for change. Two areas of change that were identified as most likely to happen through professional development was the re-imagining of students as powerful learners, and the acceptance that the teaching of mathematics requires a deep pedagogical change.

Community of practise: Identified as an important factor in successful professional development, the community of practise (Jaworski, 2014; Wenger, 1998) could be constituted through a fellow teacher who becomes a critical friend, perhaps each using DART to capture the teaching in each other's classrooms. "Communities of practise grow out of a convergent interplay of competence and experience that involves mutual engagement" (Wenger, 2000, p. 229) and this describes perfectly the interaction Gina would have with a fellow teacher on a similar learning trajectory. Not having a significant group to work with may lead to less determination to continue with any change, but having a group which was not a stimulus for change, but rather a move towards maintaining the status quo, would be more damaging. DART could work well for both groups, and for a solitary teacher.

Wider trends: NDP at different levels has been around for nearly fifteen years, and now pervades the New Zealand curriculum (Ministry of Education, 2007). NDP is the framework that is recommended by the government, the curriculum, and the national primary teachers' association (NZEI), with all three indicating the centrality of NDP. DART is specifically designed to be used during NDP-style lessons, therefore the match could not be stronger.

Effective school leadership: As DART is designed to be used by an individual or a group of teachers, there is every chance that a whole-school approach could be effective. With the expectation that school leaders would support someone such as Gina in her quest for quality teaching, DART could well be used school-wide.

Gina: Journey's end

This brief look at the factors that affect the success rate of professional development, and the way that DART embodies some of these, concludes Gina's journey. She is still teaching, and although it is mere speculation, one can only hope that the journey is continuing, and that her mathematical teaching is showing improved quality compared with what it was before.

The next section will look at the way that DART facilitated Debbie's change in quality teaching, and it will discuss a hypothetical scenario in which she uses DART to occasion further development of quality teaching. In this way, I will elaborate on the research question regarding the development of DART.

Debbie's journey

The different aspects of Debbie's teaching were examined in Chapter 5 for evidence of quality teaching. Since DART made this examination possible, I will first look at the information provided by analysis of DART about how engaging and interactive Debbie's lessons were, both in V_1 and V_2, to see if change was detectable, and whether it was the kind of change she might value.

Chapter 5, contained evidence of Debbie's wish to get away from her view of school mathematics. She said "It's so boring, it would not stick", when speaking about her difficulties with learning mathematics content in school. How could DART aid Debbie in her stated quest, to get away from teaching mathematics in a boring way, and become a teacher of really engaging and fun maths lessons, in which she wanted "A lot of interactive things, a lot of games … and working together".

An important feature of DART is that Debbie could capture and analyse her own data for the attributes that she wants in her teaching. According to one version of activities involved in action research (McNiff, Lomax, & Whitehead, 2003), Debbie could ask herself the question "How do I improve my work?" (Whitehead, 1989). Making use of iterative cycles, Debbie could gather data about her teaching, use that data to change aspects of her teaching that she had decided upon, and then use DART again to see if there had been the desired improvement. This would require periods of reflection from her, but more importantly, if she worked with one of her peers, she could ask the observer teacher to capture her teaching in all its complexity, and for that teacher to play the role of a critical friend. If Debbie was asked to reciprocate, there would be double learning, as she further reflected on the other teacher's actions. Having a data capture sheet that privileges the favoured behaviours in NDP would mean she could decide to concentrate on those behaviours that she believed would lead towards her stated intentions when teaching mathematics.

To achieve her goal of creating interesting (contextually) and engaging (mathematically) lessons, Debbie would need to make decisions about changing how she works in groups. Although she had made some progress towards this, as evidenced in her second video, V_2, how could she enhance her organisation to continue this change? The group dynamic section of DART displays explicitly if students are working as a community of learners (Wenger, 1998), working together as a thinking group, and not separately as they had been before. She would hope that under the questions area, she could move towards a more open type of question, that she would make materials more readily available, and that the students would manipulate them. Once she started to do these things she should observe that the amount of time she talked reduced further, and that the students started to take a more active and central role in their learning. Capturing her eliciting/supporting/extending behaviour during mathematics lessons would be enlightening. Debbie has strong ideas about what teachers must do to engage with learners; and commented in her interview that activities, hands-on learning and challenge was something she needed to concentrate on. In this way, DART would be able to identify if that was becoming a reality.

Analysis of her two videos, V_1 and V_2, show that Debbie was originally an exemplary teacher in her use of the different behaviours captured by ACT score on the second page of DART. Her original ACT score was 79%, compared with the average of the six other participants of 33%, and that is over five times as many behaviours as Fiona, and three times as many as Gina. This indicates that while Debbie was rated as an exemplary teacher in V_1, two years later this had changed considerably—her ACT score from V_2 reduced to 39% while the average for the other six participants rose to 45%. Interestingly, Debbie seems to have had the opposite journey to that of Carly, who started with an ACT score of 29% and finished with a score of 79%. If Debbie was aware of this information she might be dismayed, and wish to work to restore the

ACT score to its original excellence. How she might use the information contained in DART, and how she might use this as an on-going measure to restore the ACT scores, is discussed next.

The statements behind the ACT score are descriptive. In the first video, DART documented Debbie using six of the possible nine behaviours detailed in the extending section. Two years later she used only two, “Asks all students to attempt to solve difficult problems and to try various solution methods” and “Cultivates a love of challenge”. The original ACT researchers (Fraivillig et al., 1999) reported the lack of these extending behaviours of experienced teachers in their study, so seeing so many in the first video from Debbie, a tyro, is very encouraging. Two years later she still has two behaviours in that extending line, so this should give her both confidence that she can do this, and the inclination to return to her first ways of teaching. If she used DART to help her to make her lessons more engaging and interactive as she wished, then concentrating on her ACT score would facilitate this.

Debbie: Journey’s end

By her own account in her interview, Debbie has been on a very “bumpy” journey. She began the study laying out her poor start in learning mathematics at school and how it just would not stick in her brain. She felt powerless to effect change with her poor understanding and negative attitude towards mathematics until she came to university and encountered mathematics being taught in NDP-style. She was not particularly maths anxious at that stage, and, after her university mathematics education courses, she displayed a positive and optimistic attitude at the prospect of teaching mathematics, knowing she could analyse the difficulties students were having, because she understood how that feels. I am optimistic for Debbie, and indeed for all the participants of this study, as it seems that there are a number of paths to quality teaching.

The five vignettes

Although not reported in detail in Chapter 5, DART coding was completed for Alison, Barbara, Carly, Fiona, and Evie. We will consider each of the vignettes for these five participants and briefly mention one aspect that examines the versatility and fidelity of DART.

Alison found teaching in NDP-style taxing, but desirable. She said that she is not at all happy with her lessons in NDP-style. The analysis of her DART encodings confirms the gap between her wish to have an engaging lesson with many “aha” moments when a student finally understands a new concept, and the reality of her lessons. When asked “What was the best thing that had happened in her lesson”, she asked me to come back in a year, showing she had set herself high standards but did not value what she was currently doing. The professional learning tool DART would enable her to work purposefully toward her goals by making choices about what factors in her NDP-style teaching she wanted to enhance. Once that decision was made, she could continue to analyse lessons until she observed that trait as deeply embedded, enabling her to move to improving another desired trait. She had the aspiration and now DART would give her the means to affect change in her quality teaching.

Barbara was seen in her first video teaching very much in NDP-style, and two years later she was still teaching well, but at about the same level, so the two year gap does not appear to have made a huge difference to her. One area she could improve is her use of the extending behaviours within her ACT score. If she used DART Barbara could look at the list and choose a particular behaviour she felt would enhance her lessons, such as "Encourages students to analyse, compare and generalise mathematical concepts". This is an important action, as it speaks to the heart of mathematics. Mason (2000) says that a mathematics lesson without generalisation, is not a mathematics lesson at all, and Skemp (1979) would see this action as moving the lesson into the realm of relational thinking. Having made the decision, Barbara could use DART to capture her actions and check that this particular behaviour was seen and was enduring over time, because DART can be used for as long as the user wishes.

Carly is almost the poster girl in terms of improvements in both MKT (from 36% to 86%) and ACT (from 29% to 79%) scores. Her success, it was argued in Chapter 5, was likely due to teaching at Year 7 level, the only person in the study to teach at intermediate level. Let us send Carly on a hypothetical journey where she has to teach at a much younger level. The years of teaching at Year 7-level have done a great deal for her MKT, but what difficulties would she face teaching younger students? Because this is something of an unknown, Carly could decide to video herself teaching her younger class, and use DART encodings to look at what she continued to do well, and what attributes she needed to work on. Let us speculate that she conceptualises younger children as less powerful learners of mathematics, and she find that she talks too much, with too many closed questions, and feels that she is over-scaffolding her lessons so that the challenge is weakened. By analysing and reflecting on this using DART, she could research her teaching, and ensure that the desired changes were visible in her lessons. DART captures these attributes, and is the only tool she would need to affect these changes.

Fiona was highly maths anxious before her two years of teaching (MARS score: 40). She professed a great dislike for mathematics at school, but the data showed a large increase in

MKT over the two years. This was interesting because she only taught young students, suggesting her mathematical life in school was only concerned with the first of the levels of the NDP framework. In terms of ACT scores, however, she started low and continued in this fashion. Fiona knows mathematics for teaching; now she might decide that she needed to improve her ACT score, particularly in the extending area. Of course she could reflect on the different behaviours on the second page of DART, but how would she bring about change? The first page of DART holds the key: Fiona would need to analyse where students could be encouraged to reflect on their work, go beyond initial solutions and become more powerful learners of mathematics. So she could capture the behaviour in ACT, but she would need to consciously practise it and then this could be observed within the first page data of DART. When she put students in groups, she could reconfigure these as thinking groups, and let them feedback to other groups about their strategies for problem-solving. If she needed to ask questions, she would formulate them as open ones, and ensure that the students could occupy more of the talking space in her sessions. DART would capture her attempts to institute this conscious change, and she could further reflect on how she is increasing her range of extending behaviours. Since DART is sensitive to these changes, she could look back over time and see her journey, ensuring that when she chose to improve another part of her mathematics teaching, she had proof that DART worked for her.

Evie had professed a wish in her interview to make her mathematics lessons fun, but analysis of the videos showed that this was not the case at all. She had removed all context, did not use materials when they were required, and taught by rote learning, which she had hated in her own school days. If she became alert to this situation, the one powerful thing she could do would be to go back to the source, back to the NDP Pink Books and use the lessons there to reconnect with the NDP teaching model (Hughes, 2002). The top table of the second page of DART could be filled in by her to show what the lesson recommended in terms of materials, context, and so on. Then she could video herself and evaluate her adherence to the lesson, using appropriate materials, correct number magnitudes and progressing through from materials to imaging and on to number properties. At the moment she is steadfastly remaining in number properties; she would need to move back through the model, when her students encountered difficulties, using the recursive nature of the model. She might also value the fact that DART can be used by the teacher without needing to involve others, as she may realise that her teaching does not reflect the quality teaching she so fervently desired when she said of her imagined lesson:

> It would have hands on stuff, so that they can manipulate the materials if they need to, but also, if they don't need it, they can just have the opportunity to work it through in their heads and interaction as well. Sharing the ideas with each other and obviously the teacher so that the teacher knows what they're doing…

These vignettes have been used to highlight different aspects of DART use, taking each of the participants on a small imagined journey where DART would help them improve their quality teaching. All the people in this study came to teaching with a different set of attributes, but they all displayed positive dispositions for teaching. It would be a clearer, less uncertain path to quality teaching if all candidates for initial teacher education had certain abilities and attitudes from the beginning. In a subsequent section, I image what this ideal student might look like, next I will answer the subsidiary research questions.

A response to the subsidiary research questions

Subsidiary questions were:

- What is the nature of quality teaching evident in the teaching practice of novice teachers over their first two years in the profession?
- How does the teaching of novice teachers change over two-year period of their provisional registration?
- How does initial maths anxiety affect the development of quality teaching?

These questions will be answered by drawing from the two results Chapters, 4 & 5, together with considerations of the uses of DART, and both national and international definitions of quality teaching.

What is the nature of quality teaching evident in the teaching practice of novice teachers over their first two years in the profession?

As was stated at the beginning of this thesis, most people know what quality teaching (Ell, 2011) looks like when they see it, but whether they could accurately describe it is another thing entirely. In this work, the nature of quality teaching in mathematics resides in the NDP-style of teaching, but the factors involved are not exclusive to NDP, they are universal, as was shown in the international research discussed in Chapter 2. Throughout this response to the subsidiary research questions, the research found in Chapter 2 will be used to highlight the ways DART could be useful in non-NDP teaching.

Under the auspices of NDP, quality teaching begins with knowing where the student's understanding of a concept currently sits within the NDP Framework levels (emerging to level eight). This is facilitated by the diagnostic interview (Ministry of Education, 2008b), and continual intimate knowledge of the student's progress. All seven participants in this study were teaching in NDP style, and had used the knowledge gained from the diagnostic interview to group and teach their students. The videos showed that teachers' use of that information was mixed, with some checking that students needed to be in the group and then sending them off for independent work if they could demonstrate understanding, as Gina did in her group teaching (detailed in Chapter 5). Other teachers, such as Evie, seemed to have made their decision about what to teach in a session and did not deviate from that intention even when presented with evidence that either the students already knew the concept or, because of misunderstandings, were in need of backtracking, apropos the NDP teaching model (Hughes, 2002). In Evie's case, she continued to teach the concept of two-digit addition to her students who were clearly not making progress, and she did not backtrack to use of materials, such as place value money, even though this might have been the best course of action at that time.

Teacher knowledge represented here by mathematical knowledge for teaching (MKT), underpins so much of what might be termed quality teaching. In this study the ACT score was used in conjunction with the MKT measures to help make the judgement as to which students would be best included in the full case studies and which to use in the vignettes. Figure 5.1 shows the seven participants' changes in both these measures over the two-year span of the study. It is generally agreed that MKT is important in quality teaching of mathematics, but what aspects, how much, what is really central and what merely desired, are arguable (Ball, Thames, & Phelps, 2008). The ACT score is one I developed specifically for DART; the teacher behaviours in the 'extending', so desirable, but so little evident in teaching, are the type which lead to child-centred learning and relational and versatile thinking (Skemp, 1979; Thomas, 2006). These are the cornerstones of quality teaching: without knowledge of the mathematical terrain, without a pedagogy which put the work firmly into the children's domain, quality teaching is less likely to be evident.

Thus, the nature of quality teaching is fluid, informed, well-resourced (thinking of NDP resources), subject to change, in a local context, and above all, engaging. How easy is it to choose some factors required for quality teaching? How difficult is it, to put pedagogy, knowledge of students, and MKT together and emerge with a chance of producing quality teaching? In this study a few factors have been found within the teaching of the participants, and new tool has been developed to aid the development of quality teaching. Next a brief look at the final two subsidiary research questions.

How does the teaching of novice teachers change over two-year period of their provisional registration?

We know from the work of Hattie (2003, 2009) that any change is usually seen as having a positive effect on teaching and teachers usually develop and change over the two-year period of their provisional registration. In order to help teachers develop desired teaching behaviours, it is important that we should know what we are looking for. Data captured by the encodings in DART over the two videos of each participant shows there is change in the time frame. Some change is for the good, such as stronger teacher knowledge and improved pedagogical organisation. Another example might be change in teacher questioning from closed to open-type questions (Mason 2000, 2012), thereby reducing scaffolding and increasing the challenge inherent in the lesson. However, other changes found in this study were not so positive, typified by a move from child-centred teaching to a didactic, rule-bound lesson, full of missed opportunities, or moving from initially using a wealth of behaviours from ACT to, two years later, using only a few, as though the others had been found wanting, or surplus to requirements. Change is the given: What sort of change, and how we can track it is the question. The use of DART to capture the data and enable comparisons to be made over time is one way to make change visible. By comparing successive videos, and using an action-type research inquiry (McNiff, Lomax, & Whitehead, 2003), teachers can take responsibility for their own professional development, one that they are interested in. This type of inquiry would possess many of the attributes of good professional development (Timperley et al., 2007) and thus be more likely to succeed than that imposed from outside.

In view of the above, the answer to the second subsidiary research question is yes, but it depends. It depends on what the student first brings to the intial teacher education establishment; it depends on their beliefs with regard to their own ability to understand mathematics and to identify the teachable moment; and their understanding of their students as powerful mathematics learners. So many factors are involved that, without a data capture sheet such as DART, many of them and their interactions, would likely go unnoticed.

How does initial maths anxiety affect the development of quality teaching?

This is the question that started off this journey of mine: Does high maths anxiety predispose novice teachers (Ball et al., 2008) to teach in a less than adequate fashion? With such a small set of participants, one can only speculate, but the most highly maths anxious of the participants, Gina, though professing life-long hatred of mathematics, and a complete inability to understand it, managed to teach in a very competent fashion. Her mathematical knowledge for teaching improved hugely, showing that once teaching mathematics every school day, she could learn mathematics. Gina's ACT score remained low, but she maintained a fierce determination to do the best by her students, and perhaps this is the other side of maths anxiety that, once faced and accepted, it is possible to mitigate the effects, to work diligently to improve one's mathematical knowledge for teaching and, hopefully, do a competent job of teaching mathematics. Personally, I feel that Gina did better than a competent job, and given more time, I would hope that she would start to trust herself more, have greater self-belief, (Grootenboer, 2008), branch out to teaching those older students that she had hoped to do, so that all her wonderful science knowledge could be used to the betterment of her students. I am full of optimism for all the people in this study, from the moment they signed to say they would take part, allowing me to video their mathematics lessons, I felt these were exceptional people—but of course, that may apply to all teacher candidates.

In an effort to more completely answer both the main research question and the subsidiary ones, I wanted to draw all the evidence together and propose that there might be such a thing as the ideal novice teacher. In the next section, I allow my imagination to take flight, and my optimism to come to the fore as I visualise this paragon of virtue. This will allow the drawing together of the results, and the conclusion of this study.

The ideal novice teacher

DART has been examined for its use in helping novice teachers achieve their voiced wishes to become expert teachers of mathematics. Both Gina and Debbie had strong visions of how they wanted to make mathematics more engaging and effective for their own students, which was in itself a strong contrast from the style of mathematics teaching they had received in school. The ideal teacher may be a chimera, but discussions and evidence in this thesis point to desirable traits that people might develop to better teach in a quality teaching fashion. What follows looks at the issues of mathematical ability, openness to new ideas, and adherence to the NDP model. Lastly, how DART could be used to provide professional development for the ideal beginning teacher, and indeed any teacher, is discussed.

Mathematical ability of the ideal initial teacher education candidate

On entering initial teacher education courses, my vision of an ideal teacher would have strong dispositions for mathematics, and a low maths anxiety score. This person would exhibit strong cognitive ability, actively engage in mathematical problem-solving, enjoy all types of mathematical puzzles and conundrums, show curiosity about unknown mathematics and work towards understanding this. Such a person's personal content knowledge on entry would include fractions, decimals and the place value system, they would be numerate themselves, and comfortable with estimation. Their experiences all through their school days would have been uniformly positive; they would have encountered wonderful, optimistic teachers who were their role models for how they would teach. Their teachers would have encouraged persistence and learning from mistakes. This ideal teacher would know that mathematics could be understood by everyone, and in fact their motto would be "Maths for All". They would hold beliefs that were positive about their own ability to do mathematics, perceive their students as powerful learners of mathematics, and know about their students' capacity to develop those qualities which enable deep understanding of mathematics to develop (Skemp, 1976).

Teachers who are problem-solvers are seen as likely to produce quality teaching (Boaler, 2013; Hattie, 2003), because they are able to identify the teachable moment and take advantage of their own knowledge to enhance that of their students. Hattie (2003) characterised expert teachers as those who were problem-solvers themselves, who challenged their students, had high expectations of them, those who had an ability to represent the mathematics in different interconnecting ways, with models, and lastly by the way they feedback to their students. While novice teachers could not be described as expert, there are many things they can do to move towards becoming an expert, and one of those would be if they were already, prior to entering their teaching education courses, an inveterate problem-solver, a doer of crosswords, a life-long learner of all things mathematical.

The ideal teacher and initial teacher education

While at university learning to be a teacher, the ideal teacher would relish the mathematics courses as a chance to turn around their own understanding into the type that would enable them to teach effectively. They would realise that teaching requires more than simply knowing mathematics, or understanding algorithms and they would be determined to develop into the best beginning teacher of mathematics. They understand that they will need to analyse their students' responses to mathematical problem-solving, and ensure that they notice the teachable moment and respond with their best efforts as to where to next. On practicums they would seek out opportunity to teach more mathematics, first in small groups, then taking charge of the whole class, presenting their students with interesting provocations to problem-solve, and ultimately to generalise. They would work hard on their mathematics curriculum courses, passing all of them at a high level. They would also develop a community of learners in their classes, contribute fully, even when unsure of the right answer, emulating, what for them, would be their own ideal student.

The ideal candidate heading into their first position as a classroom teacher

On entering their own classroom, the ideal novice teacher would organise a mathematics area as they would a literacy one, with colourful mathematical-based reading books, inviting their students to engage with materials, puzzles, and technology based interactivities. Within their teaching sessions, they would consult and use the resources of NDP, including the "pink books" (Ministry of Education, 2005a). They would thoughtfully include appropriate materials (taking the advice of the pink books) which exemplify the concept being taught, each child, or pair, or group, as appropriate, would have access to this material, to investigate and play with, as they wish. They would be determined to ask open-ended questions most of the time, and they would deliberately not occupy all the talking space, they would leave plenty of time for their students to discuss, feedback and generally talk about what they are learning. When they saw, or heard, the teachable moment, they would seize it, when they saw a student needing scaffolding, they would give it by, perhaps, folding back to an easier modality in the NDP teaching model (Hughes, 2002; Pirie & Kieren, 1989). Whatever they do, it is through sensitive analysis, and the knowledge that failure and struggle when learning mathematics is highly desirable, not to be avoided by an overly concerned teacher who takes away all challenge. During teaching sessions and when planning for learning, they would press the mathematics forward (Kazemi & Stipek, 2001), and they would always have their eye on the main prize of algebraic thinking (Irwin & Britt, 2005).

The ideal teacher is, of course, teaching in the ideal school, where every teacher is also an ideal teacher, especially of mathematics. However, what if that was not the case? How could our ideal teacher go against the existing culture, possibly a school not so sympathetic to her ways of working mathematically? That is another story and another journey. Suffice to say that she would be armed with great self-belief, strong personal skills which she would use to get teachers to give her ideas a fair chance, and the proof needed would be in the engagement of her classes, their enthusiasm for mathematics and the joyful noise emanating from her classroom. Although the novice teacher must be part of the team, and conform to certain pedagogical expectations of the school, this does not mean that the teacher must relinquish all control over how they teach. Even with a system not so sympathetic to their preferred mathematical teaching style, our ideal teacher will still be able to ask great questions, produce appropriate materials at the most apposite time, and provide a challenging and exciting classroom atmosphere because they are aware of what is required for quality teaching.

Professional development for our ideal teacher

Our ideal teacher may identify an area within their teaching that they are not certain about. They might ask themselves if they really challenge their students. By capturing some sessions on video, they could use DART to analyse these videos for elements which represent challenge. Categorising a challenging lesson as one where the students do all the work, they could check that, under the group dynamic section on the first page of DART, they are doing more than instructing, that students are given time to work in groups, and to showcase their understandings. Under the questions and talking sections, they should notice their ratio of closed to open questions, looking to see if students have a more or less equal voice and the time is occupied by student talk, rather than that of teacher.

Under the use of materials/imaging/properties section they could see what the students were doing and reflect on appropriateness of materials. Whatever stage they are working in, they capture their own words in the description of minute area, to check for using the right magnitude of numbers, that they did not ignore some difficulties that a student voiced but the teacher did not hear. On the second page of DART, the teacher would be interested in which desirable behaviours are identified from the videos, and will look particularly at the extending column to ensure that the lessons really challenged the students. What would the ideal teacher see? Hopefully that they were teaching well under NDP, but as with everything in teaching, there is room for improvement. So they will be both satisfied with their teaching and dissatisfied, at one and the same time. And then the ideal student would make plans to ensure that developments of areas of concern, having been identified, are attended to. For instance, if the extending section on page 2 is lacking, deciding on which of the teacher behaviours they wish to privilege, and then later using videoing and again using DART to capture the use of that section, would reassure and encourage the ideal teacher that their teaching was developing in ways they desire.

I have always like the idea of the ideal teacher, even while knowing that there is no such thing as perfection, either in the people or the lessons. However, by reflecting on the concept of the ideal teacher, various elements can be argued to be highly desirable in a teacher who aspires to quality teaching. These elements are, hopefully, present in differing amounts, and by judicious professional development and reflection, classroom teachers move inexorably towards this ideal.

Limitations of this research

The results discussed in this chapter have to be seen in their context. There is the context of the participants and the nature of mathematics, and ideas about quality teaching. There are biases at every turn, both overt and covert. The following is my attempt to lay out these limitations and place them in their context.

The ultimate limitation in study such as this could be the small number of cases. This is both a strength and a weakness of this, and for any, case study research. It is a strength because the two major case study participants were able to be analysed at a deeper level (Yin, 2012), catching the uniqueness and the context of the situation. Their interviews, MKT, and the DART measures were all subject to the type of scrutiny not possible with a larger number of participants. It is a weakness because having so few participants can make it difficult to generalise from any findings. However, proponents of the case study research paradigm (Gast & Ledford, 2014; Yin 2012), point to the idea of snapshots that tell an intricate and involved story about human beings, that human beings are complex, and continually changing, and this complexity means that there is little by way of generalisation possible. Yin (2012) distinguishes between generalisations that have a statistical basis, and those that have an analytical basis. The latter are what have arisen from this study. Outcomes using case studies form a continuous chain of evidence that build on succeeding case studies and allow the complexity to stay within the study, and not be averaged out, as happens in some, more statistical, analysis.

While triangulation may be desirable as a means to establishing validity, it is hard to achieve in case study research. The sources of data are often, as in this study, separate and of a different quality. Interviews are subjective and emotive, while analysis with an instrument such as DART carries with it a surface validity that it might not merit. For instance, the way DART looks, with the small boxes showing group behaviour (G-icons) might look authoritative, but in fact not be capable of showing any change to teaching. The MKT has been extensively piloted on American teachers but this may be lost when the environment changes to a different context (group teaching and NDP-style, as opposed to mostly whole-class teaching) in a different country.

It must also be appreciated that case study as a research paradigm is still evolving and changing as more researchers use it, either as the main research paradigm or as a subsidiary one (Yin, 2012). Limitations of the research paradigm are central to any research, but all of them have different uses, in different contexts. However the choice of case study here was determined, not chosen, because achieving a large-scale study was unlikely to be possible in this situation. This context for research is discussed next.

In my master's research, completed in 2006, I approached approximately 500 students to be part of my maths anxiety study, and received only 29 replies. In this study I again approached the whole cohort of third-year initial teacher education students. Twenty-eight respondents completed the first phase of the study, but only seven fulfilled all the research activities. I knew it would be difficult to get many people prepared to pledge themselves to a two-year study involving being videoed teaching mathematics, especially as so many of them would have been maths anxious (Frankcom, 2006). Therefore the idea of case study was a forced choice, but it provided me with the opportunity to delve deeply into the data and pick up the detail using the DART instrument.

As the sole researcher, I have to be cognisant of my own biases, and guard against the tendency to see what I want to see. Over the long period of this research, I have also changed. I am arguably a better researcher than I was at the beginning, and knowing what I know now, I most certainly would have done a number of things differently. One that springs to mind would be to interview each teacher after each videoing occasion. At the time I thought this would give too much information, take too much time, and be difficult to arrange due to teachers' commitments. However, now I find I am curious about how they viewed their lessons. Anecdotally, some of the teachers would ask me for reassurance that the lesson had been a good one. It would have added to the study immeasurably to have their thoughts. I might also have asked for another MARS test to be completed to see if the maths anxiety of the participants had changed, but since I thought of it too late to include it, that will have to be for another day.

There is also bias apparent to all who read this, that I am an aficionado of NDP. When I saw how it changed the pedagogy of mathematics teaching in our schools, and gave confidence and support to previously unhappy teachers, I knew there was a lot of good in it. I was also acutely aware that there had been limited research done on the NDP, and so when I was developing DART, I developed a tool that could show change in quality teaching over time. There is bias in my development of DART. I was privileging many aspects of pedagogy when I arranged the icons, and ACT sections, and so on, and all I could do was try to be continually aware of that, and look for confirmation in other areas that I was doing the right thing. DART has potential outside of this study, which I will address in the last few sections of this chapter.

Implications of this research

This research into the nature of quality teaching and whether an instrument could be developed to capture it, has the capacity to affect professional development in many different areas, some of these are: professional development of teachers of NDP lessons; improvement of an aspect of NDP lesson; use of DART in non-NDP lessons; and development of DART. Each of these will be addressed in turn.

Professional development of teachers of NDP lessons

Most of the primary schools in New Zealand introduced NDP as consequences of government decisions to enable facilitators to spend extended time with teachers in classrooms to develop mathematics teaching professionally. It was seen that changes to pedagogy would only come about when teachers were given the means to teach differently, which meant improving the teachers' own mathematical content knowledge. The facilitators modelled the style of teaching under NDP, and teachers were given feedback in their own classrooms. This came to an end in 2009, when funding finished, though all the resources were still in place and many schools continued to teach in this way; this is positive, as sustainability was one of the hopes for NDP. Many schools have teachers joining them who are new to NDP. For example, these could be immigrants from outside New Zealand, or simply teachers who have not taught in NDP-style before. There is a need to engage new teachers (and those new to NDP) in professional development. This is usually done in-house, with help from teachers who have already been through NDP, and help that comes at a price through private providers. There is a repository of professional development tools on NZMaths http://nzmaths.co.nz/numeracy-projects which has the following introduction to their professional learning and development web site:

> The Numeracy Development Projects (NDP) were a collection of Ministry-funded, in-school professional learning and development initiatives implemented in New Zealand schools between 2000 and 2009. The goal of the NDP was to improve student performance in mathematics through improving the professional capability of teachers. The NDP knowledge and strategy frameworks (the Number Framework) strongly influenced the development of the Mathematics and Statistics

> learning area of The New Zealand Curriculum, and in particular the Number and Algebra strand.
>
> (NZMaths, n.d.)

If a school wants to improve the skills of new teachers, the web site has these resources: for example the Numeracy Development Project books (the “pink books) mentioned in this thesis; material masters, copies of worksheets and equipment used in NDP activities; equipment animations, help with how to use materials; assessment resources for use with students, such as the diagnostic interview; numeracy results database, where results are entered for the Ministry of Education; expected levels of achievement, exemplars; home school partnership handbook for parents; numeracy planning sheets; and research findings from NDP. These are all resources of different kinds which support NDP teaching. There is a lack of a professional learning instrument that can do what DART can do—that is, capture the workings of a NDP-style lesson. DART could easily be placed here, as the professional learning tool that facilitates teachers self-help. The instructions provided in Chapter 4 could easily be re-written to keep research jargon to a minimum, and then the suite of resources for NDP would be complete. Teachers would be able to include DART assessments in their classroom as and when they felt the need or a school could specify when teachers should complete a DART, as a pre-cursor to professional development discussions.

Use of DART to improve an aspect of an NDP lesson

As NDP is no longer government funded, due to first phase of implementation, there has been no development of the teaching and learning model and no development or changes made to ways of working. The inception of NDP took a decade to complete in all New Zealand schools (Years 1–8); there was even a brief foray into the secondary school arena (Years 9–10) (Secondary Numeracy Project, 2005–7), but since that time there has been little further development. As the use of NDP in schools is widespread, the web site mentioned above continues to be updated, and there are schools still requiring new teachers and teachers who missed NDP to be made familiar with it, it would seem reasonable that NDP be further developed. DART is one of the tools that could be used for this, if a research group wanted to see what was happening in NDP classrooms, using video recording and DART encoding, they could research the effectiveness of NDP teaching, and make their findings available to the Ministry of Education.

Use of DART in non-NDP lessons

DART was specifically designed to capture factors in NDP lessons. However, as discussed above, many of the encoding areas could be used to capture information in any reform mathematics lesson with parts that might not apply left blank. For instance, a video of a lesson in the collaborative learning currently undergoing piloting in South Auckland schools (NZMaths, 2015) could be encoded using DART. DART captures much that is student-centred and what appears to go on in collaborative mathematics classes is children doing most of the talking and the teacher talking less, with children discussing, writing, and problem-solving. Then, if they get stuck, the teacher can ask judicious questions of the group. It would seem that the second page of DART could be used for this, as the teacher behaviours there are universal in being appropriate for teaching mathematics in general, not just NDP.

Development of DART

It must be said that DART was developed for this investigation, but with not just NDP in mind but best practice too. Hence it would be interesting to me to see what development DART might undergo once it is used by groups of teachers or researchers and so perhaps there are improved versions of DART that could develop over the next few years. Anecdotally, teachers have told me how useful minute-by-minute capture of lessons would be, and the further encoding, such as counting question types, or ratios of teacher-to-student talk time, makes DART versatile. With the section for "description of minute" too, there is space for many ideas to be captured, not just how the class works (group dynamics), and the teaching model, but what is being said, and which children talk—there is room for all this.

Other implications

Implications of this research mostly centre on the development and use of DART. However the information discovered from the deep analysis of the two case studies and the five vignettes could have a great resonance in the years to come. For instance, the interview data showed that novice teachers might have a deficit in mathematics content, but it could be mitigated through the teacher goal of not perpetuating the cycle of rule teaching and rote learning. This study has highlighted this as one of the reasons for teachers having an acute dislike for mathematics, while at the same time provoking their determination to teach in an engaging fashion.

Further research

I hope that this exploration into quality teaching will engender and promote further research, and that this research will build on and use, the measuring instrument DART to make any changes in quality teaching apparent over time. What follows is a wish list of sorts, firmly set in the context of beginning teachers.

In this research, the case studies are set in a context, as mentioned above. However, DART opens up a raft of new opportunities for researchers to more fully understand the factors involved in quality teaching in NDP. For instance, videos of beginning teachers, coded using DART, could be compared and contrasted with other teacher evaluation tools. This could lead to notions of what factors would be more likely to develop first, or perhaps an order of such factors may be imagined to be best to work on prior to others. This would be a thought-provoking development, because, as it stands at the moment, there are no recommendations for nascent teachers to develop, for example, group work, before worrying about what type of question is asked. I find this area ripe for research, as it seems redolent with possibilities for improving teacher trajectories to quality teaching. If we could advise beginning teachers about what to work on first, and how that affects the next factor in quality teaching, we would know something fundamental about the important factors and their order of development for optimal beginning teacher development and quality teaching.

This research into quality teaching was a continuation of my master's thesis research into maths anxiety (Frankcom, 2006). These were my words in the introduction to that document:

> Primary school teachers require resilience, idealism and a love of learning, not necessarily in that order (Cameron & Baker, 2004). The pre-service student teachers that I first encountered when beginning my career as a teacher-educator had all three to varying degrees. They also showed ambivalence in the mathematics education classes. While they knew they would have to teach mathematics every day in their own classrooms, they did not have a passion or a love for mathematics. Almost to a person they had an antipathy which manifested itself as a reluctance to engage in 'doing' mathematics and for some this bordered on an aversion [maths anxiety].

As mentioned in the introduction, many pre-service teachers were so maths anxious that I had to develop a new style of teaching, where I never looked anyone in the face in case they thought I would call on them to answer, and I never drew attention to their tears, or they would simply leave the room. So, my curiosity aroused, I began this sequence of investigations (Frankcom, 2009), which had at their heart, identification of quality teaching. As this is my journey and my sequence, I will begin this exposition on implications of the research with a discussion about the implications on my continuing research.

My particular interest is in the way that DART encodes for quality teaching. There are plenty of areas that would benefit from being further researched, but my main interest is how people might use DART in their own professional development in schools, and whether it does help them increase the likelihood of them further developing quality teaching. I would like to work with teachers in schools in collaborative cycles of action research, where the teacher drives the research in a continuous cycle of change management. The teacher would reflect on what they would like to improve, and then use DART to see if they had improved. My role would be to see if DART is beneficial, and how DART actually helped them to attain their goal. I would be very interested in their views of how they had changed their teaching to better reflect their goal of quality teaching.

As described above, DART could also be used as a personal research tool for teachers in school. The teaching as inquiry (Ministry of Education, 2007, p. 35) page in the current New Zealand curriculum document, discusses the need for teachers to be researchers into their own pedagogy and ways of working, so that their effect on their students could in some way be evaluated. This is envisaged as a cyclical action research type (McNiff, Lomax, & Whitehead, 2003), and DART could be a useful tool in the manner presented above. It is important for teachers to be researching their own practice, and discover what they actually do when teaching.

If teachers are thinking that their teaching of mathematics could do with improvement on aspects of what sort of questions they ask, they could use DART, analyse videos of their teaching, or have real-time coding from a critical friend who captures the questions on DART, so that teachers can actually see where they might improve, or reassure themselves that their enacted teaching reflects their vision for teaching.

Lastly, it would be splendid if the original teachers in this study could be again part of a study, to see what the intervening years have done for their teaching. In some circles, it is thought that time produces an experienced teacher, but what of those destined to be expert teachers? As seen previously, Hattie (2003) characterised expert teachers as those who were problem-solvers themselves, who challenged their students, had high expectations of them, those who had an ability to represent the mathematics in different interconnecting ways, with models, and lastly by the way they feedback to their students. It would definitely be something to wish for.

Closing remarks: My vision for future teachers of mathematics

When I began this investigation, I wanted to see if highly maths anxious pre-service teachers made as good a teacher of mathematics as the person without maths anxiety. This was connoted by my research for my master's thesis regarding how maths anxious the cohort was, but a master's thesis did not have the longitudinal scope for assessing change over the two years of pre-registered teaching, and so I was left wondering how my maths anxious people in the master's study would fare once they were out in schools. This current thesis has gone some way to informing me about how they might develop. The development of the coding system DART allowed me to see the change in quality teaching over time. This thesis had a change of focus, and exchanged lenses from the initial study of maths anxiety alone, to consideration of quality teaching. My journey through the years of research trying to describe quality teaching has resulted in a measuring instrument, DART. The development of the instrument has the capacity to add to quality teacher studies by giving both practitioners and researchers the means to capture the change in teaching in NDP-style, and in non-mathematical studies in an international context.

I would also argue here that the factors captured by DART as being in NDP-style teaching, are factors found generally in quality teaching. Therefore, although developed especially for coding of NDP-style lessons, I am confident that DART has uses outside of NDP, though the veracity of this statement will only be known as, hopefully, others use the tool, and report back on its uses.

Advanced Proportional

Students at this stage are learning to select from a repertoire of part-whole strategies to solve and estimate the answers to problems involving fractions, proportions, and ratios. These strategies are based on finding common factors and include strategies for the multiplication of decimals and the calculation of percentages.

The Number Framework

This is the final stage in the NDP framework, and I am reflecting the final stage of this research, here, where I can select from a repertoire of new skills, learned from the twists and turns in my journey. And here at last I am at journeys end.

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Appendices

Appendix A	Information Sheets for Participants
	Consent Form
	First Questionnaire.
Appendix B	Interview Protocol
Appendix C	Information Sheets for Principal
	Confidentiality Agreement

Appendix A

THE UNIVERSITY OF AUCKLAND
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SCHOOL OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

PARTICIPANT INFORMATION SHEET FOR STUDENT TEACHERS

TITLE: Maths Anxiety: How does it affect the teaching of mathematics in the primary school?

RESEARCHER: Gillian Frankcom

TO: Student Teachers of 921.623 and EDCURRIC 608/9

My name is Gillian Frankcom. I am a doctoral student at the University of Auckland in the Faculty of Education, where I am also a Lecturer in the School of Science, Mathematics and Technology. I am writing to invite you to participate in a research project on Maths anxiety. The purpose of this research is to find out how anxious students are about mathematics, and what effect maths anxiety might have on teaching mathematics in the future. I would greatly appreciate any assistance you could give me.

This study will:

1) Investigate levels of anxiety associated with mathematics.

2) Follow students into employment in schools where the way they teach mathematics will be investigated.

I would like to invite you to participate in this study, and I would greatly appreciate any assistance that you can give me with this. I have the assurance of the Dean of the Faculty, Dr. John Langley, that your decision to participate or not to participate in this study will have no affect on your grades or your standing within the course.

If you consent you will be asked to complete a questionnaire which should take approximately 15 minutes of your time. No time will be taken from lectures. I will ask you to take the questionnaire away with you for completion and to put the completed form back into a box provided in the School of Science Mathematics and Technology, A block 2nd floor, room A223. Participation or otherwise will have no effect on any course outcomes. The questionnaires are not anonymous because I need to follow up with participants, however your name and information given will be confidential and not given to third parties.

If you agree to complete the questionnaire you may be invited to attend an interview at a time which is convenient to you. This interview with your permission will be audio recorded. Even if you agree to being recorded, you may choose to have the recorder turned off at any time. Only the person recorded will have access to the tape other than the transcriber and the researcher. The transcriber will have signed an agreement of confidentiality and the tapes will be stored within the Faculty in a secure place within the School of Science, Mathematics and Technology. They will be stored for six years and then destroyed.

You will leave the Faculty at the end of 2007 and if you take up a post teaching within the primary system somewhere in Auckland or Northland then you will be invited to participate in the second phase of this investigation. This will involve the videoing of mathematics lessons and subsequent interviews with me which with your permission will be audio recorded. Even if you agree to being recorded, you may choose to have the video or audio recorder turned off at any time. The video recordings will be coded by me. The audio recording may be transcribed by a transcriber who will have signed a confidentiality agreement. These recordings will be stored within the Faculty in a secure place within the School of Science, Mathematics and Technology. They will be stored for six years and then destroyed.

You will be free to withdraw from the research at any time and you can withdraw any information you have given up until May 2008. The attached consent form will need to be completed along with the questionnaire.

No names or identifying information will be given to any other parties and no identifying information will be used when reporting the results of the study. The information will be used for research purposes only. All information collected will be securely stored within the Faculty of Education for 6 years, and then destroyed. I hope to publish my findings in educational and psychological journals and present my results at conferences.

If you are happy to participate in this study, please fill in the consent form along with the questionnaire and place them both in the box provided.

If you have any further queries regarding this study please contact me on 623 8899 ext 48663 or write to me at:

Gillian Frankcom

School of Science, Mathematics and Technology

Faculty of Education

The University of Auckland

Private Bag 92019

Auckland.

Ph: 623 8899 ext 48663

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My supervisor is:

Dr Gregor Lomas,

Head of School

School of Science, Mathematics and Technology

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Private Bag 92019,

Auckland.

Ph: 623 8899 ext 48517

e-mail: g.lomas@auckland.ac.nz

For any enquiries regarding ethical concerns please contact: The Chair, University of Auckland Human Participants Ethics Committee, The University of Auckland, Research Office – Office of the Vice Chancellor, Private Bag, 92019, Auckland. Tel (09) 272 7999 Ext 87830

Yours Sincerely

Gillian Frankcom

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE on…21 …/…May…./…2007….

For a period of…three……….Years, From…………21…/…May……..../…..2007…….….Reference 2007/158

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STUDENT TEACHER CONSENT FORM

(This Consent Form will be held for a period of six years)

TITLE: Maths Anxiety: How does it affect the teaching of mathematics in the primary school?

RESEARCHER: Gillian Frankcom

I have been given and understood an explanation of this research project. I have had an opportunity to ask questions and had them answered. I understand that my decision to participate in the study will not affect my grades or standing in the course.

I am interested in taking part in this study and I understand that I may withdraw from the study at any time and that I can withdraw any data collected on me at any time without giving reason up to May 2008.

I understand that all collected data will be stored securely for six years in the School of Science, Mathematics and Technology and then destroyed. I am aware that the findings of the study may be published in research journals and presented at conferences.

I CONSENT TO:

- Answering the questionnaire.
- Attending an interview at my convenience which will be audio recorded.
- Having mathematics lessons video recorded in my classroom in 2008.
- Attending an interview after the lessons which will be audio recorded.

I understand that I can withdraw from the research at any time and any information I have provided can be withdrawn up until May 2008.

Signed:____

Name:____

Date:____

E-mail Address____________________ Phone Number____________________

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE on…21 May 2007…

For a period of…Three years,

From……21…/……May…..…/……2007…………..Reference…2007…/……158…

Student Questionnaire

NAME:______________________ Gender: M F

Lecturer______________________ Age………………..

Please answer the questions below and then complete the questionnaire that follows. In order for this information to be most useful, please answer as honestly and accurately as possible. Your participation is voluntary.

1. Have you studied maths beyond School Certificate? **YES NO**
2. Please indicate highest level of maths study ________________

Circle the response which most closely expresses how you feel about mathematics.

SA = Strongly Agree A = Agree U = Uncertain D = Disagree SD = Strongly Disagree

1. **Mathematics makes me feel uncomfortable and nervous. SA A U D SD**
2. **It wouldn't bother me at all to take more maths courses. SA A U D SD**
3. **I get really uptight while taking maths tests. SA A U D SD**
4. **I have usually been at ease during maths tests. SA A U D SD**
5. **I almost never get uptight while taking maths tests. SA A U D SD**
6. **My mind goes blank and I am unable to think clearly when working out mathematics problems. SA A U D SD**
7. **I get a sinking feeling when I think of trying hard maths problems. SA A U D SD**
8. **I have usually been at ease during maths courses. SA A U D SD**
9. **I usually don't worry about my ability to solve maths problems. SA A U D SD**
10. **Mathematics makes me feel uneasy and confused. SA A U D SD**

Now answer these questions please. There is no time limit.

Personal Content Knowledge questions

1	A school has 810 children. If 198 children are away on a trip, *about* how many are still at school? (Circle your answer)	**A**	400
		B	500
		C	600
		D	700
2	The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Put one digit in each box so that the answer will be as small as possible. Note that digits 1, 3, 5 and 8, have already been used and cannot be used again. Use any digit only once.	581 - 3 $\square$ $\square$ = ?	
3	A cat eats 500 g of fish in 4 days. How many grams will the cat eat in 6 days?	**A**	450 g
		B	750 g
		C	850 g
		D	950 g
		E	1050 g
4	A tank holds 500 fish. If I increase the number by 20%, how many fish will there be now in the tank? (Circle the correct answer)	**A**	520
		B	550
		C	600
		D	650
5	A student increased his exam score from 40 to 45. What percentage increase is this? Circle your answer.	**A**	5%
		B	12.5%
		C	25%
		D	45%

Sample content knowledge for teaching mathematics questions

1 Mr Lewis asked his students to divide 6/8 by 1/2. Charlie said I have an easy method Mr Lewis. I just divide numerators and denominators. I get 6/4 which is correct. Mr Lewis was not surprised by this as he had seen it before. What did he know?

a) He knew that Charlie's method was wrong, even though he happened to get the right answer for this problem.

b) He knew that Charlie's answer was actually wrong

c) He knew that Charlie's method was right, but that for many numbers this would produce a messy answer.

d) He knew that Charlie's method only works for some fractions.

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3

b) Method B	1	2	3
c) Method C	1	2	3

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

Appendix B

Questions for 2008 Interviews – before teaching

If any question has already been addressed do not repeat it, make a note of the same

Let us talk about mathematics and teaching

Firstly let us talk about your experiences with mathematics at school. Could you tell me what you recall of mathematics in school?

Secondly let us talk about your experiences with mathematics after school.

What about your experiences in your teacher education courses?

You will be teaching your own class in a few weeks; tell me how you feel about teaching mathematics.

Tell me about a really good thing that happened within your mathematics lessons on practicums.

Thinking about the things you were taught in your courses at the University, tell me about how the reality of teaching mathematics matches with the expectations of those courses.

Give me some examples of how your teaching of mathematics is different from the teaching that you received.

Tell me some things about your ideal mathematics teacher.

Thinking about the ideal mathematics lesson, tell me what sorts of things are going on in it.

Now let us talk about mathematics in general:

How does the prospect of doing mathematics make you feel?

If mathematics was a food, what would it be? Why?

If mathematics was a colour, which would it be? Why?

Please draw me a picture of a mathematician. (Hand subject piece of paper and pencil)

Appendix C

Participant information sheet for principals

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PARTICIPANT INFORMATION SHEET PRINCIPALS

TITLE: Maths Anxiety: How does it affect the teaching of mathematics in the primary school?

RESEARCHER: Gillian Frankcom

TO:

RE:

My name is Gillian Frankcom. I am a doctoral student at the University of Auckland in the Faculty of Education, where I am also a Lecturer in the School of Science, Mathematics and Technology. I am writing to inform you about a research project on Maths anxiety, and to attain your consent for this project to continue in your school. The purpose of this research is to find out how anxious students are about mathematics, and what effect maths anxiety might have on teaching mathematics in the future. I would greatly appreciate any assistance you could give me.

This study will:

1) Investigate levels of anxiety associated with mathematics.

2) Follow students into employment in schools where the way they teach mathematics will be investigated.

The students who take part in this study will have left the Faculty of Education at the end of 2007 and taken up posts as beginning teachers. If their schools are in Auckland or Northland then they may be videoed in their classrooms and interviewed about their teaching.

No names or identifying information will be given to any other parties and no identifying information will be used when reporting the results of the study. The information will be used for research purposes only. All information collected will be securely stored within the Faculty of Education for 6 years, and then destroyed. I hope to publish my findings in educational and psychological journals and present my results at conferences.

If you are happy for xxxx and your school to participate in this study, please fill in the consent form and return to me in the envelope provided. The children in the classes will not be part of the study, though they could be furnished with an information sheet for them and their parents if required.

If you have any further queries regarding this study please contact me on 623 8899 ext 48663 or write to me at:

Gillian Frankcom

School of Science, Mathematics and Technology

Faculty of Education

The University of Auckland

Private Bag 92019

Auckland.

Ph: 623 8899 ext 48663

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My supervisor is:

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For any enquiries regarding ethical concerns please contact: The Chair, University of Auckland Human Participants Ethics Committee, The University of Auckland, Research Office – Office of the Vice Chancellor, Private Bag, 92019, Auckland. Tel (09) 272 7999 Ext 87830

Yours Sincerely

Gillian Frankcom

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE on………21……/………May………./………2007………..

For a period of……three years,

From……21………/…May…….…./…..2007…….….Reference………2007/…158

SCHOOL OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

THE UNIVERSITY OF AUCKLAND
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Confidentiality Agreement

Research Project: *Maths Anxiety: How does it affect the teaching of mathematics in the primary school?*

Researcher: Gillian Frankcom

I agree to work with the audio and/or video files and understand that the information contained in the recordings is absolutely confidential and may not be disclosed to any person other than for whom it is intended.

Signed______________________________

Name ______________________________

Date ______________________________

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN PARTICIPANTS ETHICS COMMITTEE on…….….21 May 2007…….For a period of three Years,

From…21…../…May…….../………..2007……Reference…….2007…/ 158…