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Water Wave Scattering by Floating Elastic Plates with Application to Sea-Ice

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Alison L. Kohout
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Supervised by
Dr. Michael H. Meylan

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Abstract

This thesis considers the scattering of small amplitude water waves, obliquely incident on a set of floating elastic plates occupying the entire water surface. The problem is two-dimensional and assumes invariance in the width of the plates. All non-linear physical effects are neglected. The plates are floating on a body of water of finite depth and each plate has uniquely defined properties. The problem is formulated by imposing boundary conditions on the eigenfunction expansion of Laplace's equation. A set of transmission and reflection coefficients is generated, which is solved by applying the edge conditions and matching at each plate boundary. We label this solution method the Matched Eigenfunction Expansion Method (MEEM). The problem is solved for a variety of edge conditions including free, clamped, sliding, springed and hinged. To verify the MEEM results, the problem is also solved using a Green Function Method. The convergence of the two methods is compared and found to be almost identical. The MEEM is used to simulate wave–ice interaction in the Marginal Ice Zone (MIZ). The model removes the resonance effects and predicts that the transmitted energy is independent of floe length, provided the wavelength is more than three times the floe length. The model predicts an exponential decay of wave energy with distance of propagation through the MIZ, which agrees with experimental findings. The results have been summarised in a graph with the attenuation coefficient expressed as a function of period for various floe thicknesses. We also provide an estimate of the attenuation coefficient using an approximation theory. The displacements of the MEEM are compared against a series of laboratory experiments performed in a two-dimensional wave-tank and show good agreement. The attenuation model results are compared against a series of field experiments carried out in the Arctic and off the West Antarctic Peninsula. Generally, the decay rates of the model agree well with the field experiments in diffuse ice. We suggest that factors other than wave scatter are relevant in models of wave-attenuation in non-diffuse ice.

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Nomenclature

a	Attenuation coefficient	90
$\breve{a}$	Attenuation coefficient from field Experiments	120
$\tilde{a}$	Approximated attenuation coefficient	108
A	Wave amplitude	99
α	Frequency squared	23
β	Stiffness constant	23
$\mathcal{C}$	Term appearing in the Green's Function	48
C	Concentration of ice	120
CI	Confidence Interval	121
D	Rigidity constant	20
$\mathcal{D}$	Energy balance coefficient	72
E	Transmitted energy	80
η	Displacement	18
η^s	Shallow water displacement	42
η^I	Incident displacement	50
$\eta^+(x'_n)$	Right edge of the n^{th} discontinuity	65
$\eta^-(x'_n)$	Left edge of the n^{th} discontinuity	65
f	Rayleigh's distribution	82
g	Gravitational constant	19
G	Free-Surface Green Function for a floating elastic plate	47
$\mathcal{G}$	Simplifying term in the Green Function	50
GFM	Green Function Method	1
γ	Mass constant	22
H_s	Significant wave height	102
ω	Frequency	18
h	Water depth	18
I	Incident wave amplitude in potential	28
$\Im$	The imaginary part of a complex number	72
k	Wave number in the z direction	26
κ	Wave number in the x direction	26
k_y	Wave number in the y direction	18
k^f	Free surface wave number in the z direction	27

l	Left edge of the plate	20
l'	Actual floe length	82
L	Length of the plate	81
$\mathcal{L}$	Scaling length parameter	22
Λ	Number of plates	28
M	Number of positive real roots of the dispersion equation	28
MEEM	Matched Eigenfunction Expansion Method	1
MIZ	Marginal Ice Zone	5
μ	Plate number	19
$\bar{\mu}$	Mean floe length	82
ν	Poisson's constant	20
ω	Frequency	18
ω_m	Peak frequency	97
p	Rayleigh distribution probability	82
P	Pressure at the water surface	19
Φ	Time dependant velocity potential	18
ϕ	Time independent velocity potential	18
ϕ^s	Shallow water velocity potential	41
r	Right edge of the plate	20
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R_{wp}	The reflected coefficient from open water to a plate	104
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S_T	Spectral strain	100
S_E	Strain envelope	100
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s_v	Vertical spring constant	21
σ	Peak width	97
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T	Transmitted potential coefficient	28
$\tilde{T}$	An approximation of the transmitted coefficient	103
T_{pw}	The transmitted coefficient from a plate to open water	103
T_{wp}	The transmitted coefficient from open water to a plate	104
T_{av}	The average of the transmitted coefficient over one period	107
$\mathcal{T}$	Incident wave period	80
$\mathcal{T}_m$	Peak period	101
τ	Plate thickness	19
θ	Incident angle	27
$\mathcal{U}$	Area of the modelled region	48

Υ	Power Spectrum of waves	97
υ	Peak enhancement factor	97
VLFS	Very Large Floating Structures	15
x	Horizontal co-ordinate of the problem	18
x'	Shifting variable in the Green function	48
x'_n	Position of the n^{th} discontinuity	49
y	Horizontal co-ordinate of the problem	18
Y	Effective Young's modulus	20
z	Vertical co-ordinate of the problem	18
$[F]$	Denotes the jump in the function F	50
F^*	Denotes the conjugate of F	104