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# Mathematical and Numerical Modelling of Bacterial Colony Growth on High Nutrient Surfaces

Leonie Zandra Pipe

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# ABSTRACT

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Traditionally, growth and morphology studies of bacteria have focused on individual cells. When microbiologists subsequently became interested in bacterial colony growth, they investigated the dynamics of its height as well as those of its diameter. The models they proposed, however, were little more than empirical. In recent decades there has been a shift in focus, as researchers became interested instead in the morphology changes that colonies undergo when exposed to stressful environments such as nutrient and moisture limitation. As a result of this shift, models of colony growth in three dimensions have remained underdeveloped and rudimentary.

My first task in this thesis was to assemble a sufficiently comprehensive data set from which diameter, height and cell-number growth trends in colonies could be properly analysed. This was achieved by studying the growth of many colonies of two bacterial species, *Serratia marcescens* and *Escherichia coli*, on high-nutrient non-selective agar, over a range of incubation temperatures, over periods ranging from two hours to one week.

When graphed and analysed, colony diameter and colony height growth turned out to be most economically described as power-law in time, with exponent  $< 1$ . This contrasts with the claims of previous researchers, who had described both growth trends as linear, with diameter switching to a slower yet still linear growth after a certain time, and height growth ceasing altogether.

From my results, I proposed a simple conceptual model, an extension of a model developed by Pirt in 1967. My hypothesis was that, in colonies growing on high-nutrient surfaces, diffusion was the dominant factor in colony growth. Ron Keam transformed my conceptual model into a mathematical one, from which I have developed one-dimensional and two-dimensional numerical simulations. In all simulations to date, in both one and two dimensions, a power-law growth phase emerges as a consequence of nutrient-controlled growth, preceded by an “accelerating” phase during which colony growth overtakes diffusive processes, and succeeded by a slow transition towards growth cessation as nutrient becomes exhausted.

In addition to successful demonstration of the power laws, the model in its final form yields realistic colony profiles and exhibits other features consistent with experimental results reported in the literature.

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