Redistributive Policy and Endogenous TFP Disparity across Countries with a Special Focus on the US, Japan and Australia

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Abstract:

The traditional emphasis on breaking down output growth according to growth in inputs and growth in TFP is misplaced. More appropriately, we should break it down according to changes in "efficiency" and "technology". These two are equilibrium concepts, rather than purely technical properties of production functions. The influence of redistributive taxation (on growth) is nonlinear and depends crucially on the current state of the economy. How an economy responds to changes in the average marginal tax rate of income reveals its efficiency state. We establish the above three key results in a neoclassical, dynamic general equilibrium model in which the institutional and technological factors, as well as redistributive policy, endogenously determine the total factor productivity (TFP) of a country. Economists point towards cross-country differences in TFP to explain wide differences in income per capita across countries. However, econometric exercise proves to be challenging in the absences of a theoretical model which could map institutional and policy parameters into TFP and accumulated inputs as an equilibrium outcome. In this paper, we take up this challenge. The model allows us to break down TFP into two key components: "technology" - which identifies the production possibility frontier and "efficiency" - which determines the location of the economy inside that frontier. We calibrate that model to three separate economies: the US, Japan, and Australia. The numerical analysis of the data from those three

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calibrated economies highlight that TFP differences play a significant role in explaining the differences in output per capita across U.S., Japan, and Australia. We then conduct simulation experiments based on random selections of a set of parameters with suitably specified distributions to match the per capita income distribution among the countries around the world and generate data for per capita output and its various components. We do a variance decomposition of analysis of that simulated data to support our key findings.

KEYWORDS: heterogeneous agents, endogenous TFP, externality, progressive redistribution.

JEL CLASSIFICATION: E13, E25, E62, O11, O47

1 Introduction

Understanding the importance of total factor productivity (TFP) for variations in per capita income has been a longstanding issue in macroeconomics and development economics, since Solow's (1957) seminal paper. In the context of the standard neoclassical growth model, while factor inputs can play a role in these variations in the short run, TFP is the only source in the long run. Empirical work, which tries to quantify TFP's contribution to differences in per capita income across countries, has produced mixed results.[4] Interpreting these results in a consistent way requires a theory of TFP, something Prescott (1998) famously called for.

Jerzmanowski (2007) provides a useful distinction by breaking down TFP into two components: "technology" (where the frontier is) and "efficiency" (where the economy operates relative to the frontier). In general, both technology and efficiency are *equilibrium* properties of an economy, rather than properties purely of production functions. Using Jerzmanowski's classification, existing theories of TFP can be broken down into theories of efficiency and technology. Recent theories of efficiency such as Restuccia and Rogerson (2008), Alfaro *et al* (2008), Hsieh and Klenow (2009), and Bartelsman *et al* (2013), have identified the *misallocation* of capital across establishments as the key source of inefficiency, following the overwhelming empirical evidence of significant productivity differences across establishments.[5] The source of the misallocation itself, in these papers, is policy-induced distortions.

Theories of technology, on the other hand, have emphasized human capital and spillovers, following Romer (1986) and Lucas (1988), with empirical support from (among others), Benhabib and Spiegel (1994), Moretti (2004) and Klenow and Rodriguez-Clare (2005), and Gennaioli *et al* (*QJE* 2012). Policy also plays an important role in these models, in at least two ways. First, distortionary taxation can be used (in principle) to offset the externalities associated with the spillovers. Second, the size of the spillovers themselves can be influenced by the dispersion of skills which, in turn, reflect

[4] For studies that empahasize factor accumulation, see, for example, Young (1995), Nelson and Pack (1999), Hsieh (2002) , Caselli (2005), Baier, Dwyer and Tamura (2006), and Todaro and Smith (2006). Studies that emphasize the role of TFP include Hall and Jones (1996, 1999), Klenow and Rodriguez-Clare (1997), Prescott (1998, 2002), Acemoglu and Zilibotti (2001), Aiyar and Feyrer (2001), Easterly and Levine (2001), Hayashi and Prescott (2002), Kehoe and Ruhl (2003), Abrahamsen et al (2003) Aiyar and Dalgaard (2005), Koch (2006), Jerzmanowski (2007), and Sharma, Sylwester and Margono (2007).

[5] See Syverson (2011) for a survey.

the policy environment (Jovanovic and Rob (1989)).

Analysis of the effects of policy on both technology and efficiency, simultaneously, is largely unexplored ground. The aim of this paper is to provide a coherent theory of TFP that includes both components (technology and efficiency) in a consistent way, with misallocation, human capital and spillovers all playing major roles, and which is amenable to clear policy analysis. To this end, we present a model based on the framework developed in Benabou (2002), but with some significant modifications. As with the models of efficiency, discussed above, the fundamental source of inefficiency in Benabou's framework is capital market imperfections. However, unlike those models, in his model the imperfections are not induced by policy but by inherent informational asymmetries. Capital markets are absent, in fact, in this setting. Human capital plays two important roles here. First, as a productive asset (with spillovers); second, as a source of inequality – human capital is passed from generation to generation, in each individual family, through a stochastic technology. These two features lead to persistent dispersion of assets over time. One advantage of Benabou's framework is that it generates tractable measures (in fact, closed-form solutions) of this dispersion – the variance of the state variable (human capital) in equilibrium.We extend Benabou's model to allow for both physical and human capital, but maintain the absence of capital markets. Physical capital is productive, and is transferred across generations solely by bequests – chosen by families with a dynastic structure. We use the model to provide explicit formulas for the equilibrium values of efficiency, technology, and TFP.

In this setting, all of these variables are functions of the variances and covariance of the assets in equilibrium. As in Benabou's original framework, closed-form solutions for these dispersion measures, in equilibrium, are available, where dispersion of depends on the underlying technological, institutional, and policy parameters. The policy parameter that we mainly focus on is redistributive: the average marginal tax rate (τ) with a breakeven level of income. This allows us to examine the implications of redistributive policies that affect asset dispersion and, hence, efficiency, technology, and TFP. We also consider the effects on TFP and its components of changes in other key parameters in the model. Although closed form solutions for the endogenous variables are available, these functions are nonlinear in the parameters of interest, and most analytical comparative statics are ambiguous in sign. We therefore calibrate the model to the US economy and assess the comparative statics quantitatively through simulations.

We find that, in this framework, redistributive policies have unambiguously positive effects on efficiency, but their effects on technology and overall TFP depend on parameter values. That is, efficiency is strictly increasing in the value of τ across the entire range of parameters that we consider, but the effect of τ on technology, depends on the values of particular parameters and on the extent to which the distortionary impact of τ on investment decisions is neutralized through subsidies. If the distortionary impact is entirely neutralized, efficiency, technology, and TFP are strictly increasing in τ; otherwise, although the effect of τ will always be positive on efficiency in any economy, its effect on technology and overall TFP will depend on the specific parameter values of the particular economies being considered.

The first result stands in stark contrast with the results in, for example, Restuccia and Rogerson (2008). Here, as mentioned above, the root cause of inefficiency is not policy but a missing market: the market for capital. This generates misallocation and inefficiency because, in the absence of any redistribution, some families are unable to educate their young to the efficient level. (This problem is particularly acute for poor families with smart children.) Here, redistribution acts as a partial substitute for the missing market. The effect of redistribution on technology, in this framework, works through two different channels: the mean and the variance of human capital. Both of these channels affect technology through their impacts on spillovers. (In the absence of any spillovers, redistribution has no effect at all on technology and so its overall impact on TFP, through efficiency alone, is positive.)[6]

We also assess the optimal value of τ using the utilitarian welfare criterion used in Benabou (2002), with the economy calibrated to the US. We find that, despite its positive effect on TFP at low values, the optimal value of τ is zero for the US. This contrasts with the positive optimal values of τ found in both Benabou (2002) and Tang and King (2005), using Benabou's original model. The reason for this difference is the inclusion of physical capital in this case. The equilibrium savings rates for physical capital is distorted by τ here, and this is enough to overturn the positive effects found in the earlier studies without physical capital.

Overall we argue that, when capital market imperfections are driven mainly by informational asymmetries, redistributive tax policies can increase TFP. This effect occurs mainly (though not exclusively) through their influence on efficiency: by acting

[6] The positive effect of dispersion on spillovers is reminiscent of the effect discussed in Jovanovic and Rob (1989), but for different reasons.

as a needed substitute for missing capital markets (transferring some resources to where marginal products are high), they allow for more efficient allocations of human capital – reducing its variance and misallocation. These policies also influence the technology component of TFP, and can do so in a way that can increase or decrease TFP overall. When calibrated to the US economy, redistributive policies increase TFP, up to a point, but reduce welfare. However, in economies where the technological frontier is independent of the domestic economy (as in most developed countries), redistributive policies unambigously increase efficiency, TFP, and welfare.

The remainder of the paper is structured as follows. The model is presented in Section 2, its equilibrium dynamics are characterized in Section 3, and its steady state is analysed in Section 4. In Section 5 we present the quantitative analysis, and Section 6 contains some concluding remarks. Appendix A provides proofs of all the key lemmas and propositions in the paper, and Appendix B provides details of the calibrations.

2 The Model

2.1 *Model Environment*

The economy is populated by a continuum of infinitely lived dynasties, indexed by $i \in [0,1]$. Each dynasty is made up of a sequence of families consisting of individuals who live for two periods: first as a child and then as an adult. In each period t, the dynasty is represented by a family of one adult and one child. The adult, in period t, makes all decisions for that period subject to the constraint that she cannot pass on debt to her child. We call the adult of dynasty i in period t the "dynastical agent i "or simply "agent i". The preferences of agent i in period t are given by:

$$\ln U_t^i = E_t \left[\sum_{n=0}^{\infty} \rho^n \left(\ln c_{t+n}^i - \left(l_{t+n}^i \right)^{\eta} \right) \right] \tag{1}$$

where $c_t^i \geq 0$ and $l_t^i \in [0,\, 1]$ denote, respectively, consumption and labor supply by the adult of the dynasty i in period t; $\rho \in (0,1)$ is the discount factor, and $\eta > 1$.

We assume that each agent can operate the following production technology[7], which

[7]Notice that there are no firms in this model, *per se.* Each agent operates a production technology directly. The output from this process can be used for consumption, bequests, education expenditure, or taxes – as outlined below.

converts labor l_t^i , physical capital k_t^i, and human capital h_t^i into output y_t^i:

$$y_t^i = A_t \left(k_t^i\right)^{\lambda} \left(h_t^i\right)^{\mu} \left(l_t^i\right)^{1-\lambda-\mu}. \tag{2}$$

where $\lambda, \mu \in (0,1)$, and $\lambda + \mu < 1$. In the spirit of Lucas' (1988) model and, as modelled by Benabou (2002), we also allow for knowledge spillovers, of the following form: $A_t \equiv \bar{A} \left(\int_0^1 \left(h_t^i\right)^{\omega} di\right)^{\delta/\omega}$, $\bar{A} > 0$, $\omega \in (0,1)$ and where the parameter $\delta \geq 0$ captures the extent of the spillover.

As in Benabou (2002), the government has a scheme of progressive income taxation and transfers such that the disposable income of a typical agent at a date t, denoted $\hat{y}_t^i$, satisfies:

$$\hat{y}_t^i \equiv \left(y_t^i\right)^{1-\tau_t} \left(\tilde{y}_t\right)^{\tau_t}, \tag{3}$$

Here, $\tau_t \in (0,1)$ measures the average marginal tax rate and is identified as the degree of redistribution or *progressivity* in fiscal policy. In this scheme, those with income higher than $\tilde{y}_t$ pay net tax while those with income below $\tilde{y}_t$ receive net transfers. Thus, $\tilde{y}_t$ represents the break-even level of income with respect to the tax scheme. The government's balanced-budget constraint is:

$$\int_0^1 \left(y_t^i\right)^{1-\tau_t} \left(\tilde{y}_t\right)^{\tau_t} di = y_t, \tag{4}$$

where $y_t \equiv \int_0^1 y_t^i di$ denotes per-capita income.

At each date t, the disposable income of agent i must equal the total expenditure on consumption c_t^i, private education expenditure e_t^i, and the bequest b_t^i:

$$\hat{y}_t^i = c_t^i + e_t^i + b_t^i. \tag{5}$$

The human capital of the child, in dynasty i at time t, is given by h_{t+1}^i and is a function of her innate ability ξ_{t+1}^i, parental human capital h_t^i, and private investment on her education e_t^i, according to the stochastic technology:

$$h_{t+1}^i = \kappa \xi_{t+1}^i \left(h_t^i\right)^{\alpha} \left(e_t^i\right)^{\beta}. \tag{6}$$

Here, $\kappa > 0$, $\alpha \in (0,1)$, and $\beta \in (0,1)$ are parameters, and the term ξ_{t+1}^i represents *i.i.d.* idiosyncratic shocks with $\ln\, \xi_t^i \sim N\left(\varphi, \sigma^2\right)$, and constants $\varphi > 0$ and $\sigma > 0$.

Conceptually, the shocks arise from heterogeneity in innate ability or in the efficiency of human capital usage across different dynasties at time t, and over time within a dynasty. The parameter α measures the child's human capital elasticity with respect to its parent's human capital.[8] The parameter β measures the corresponding elasticity of human captial with regard to education expenditure (which, arguably, is primarily determined by the quality of the education system).

Capital goods are complementary to human capital and become obsolete at the end of each generation. (Intuitively, a tool loses value when its user dies. Parents buy new tools for their children and leave them as bequests.) To capture this feature we assume that they depreciate completely in the production process. Consequently, in $t+1$, agent i's physical capital k_{t+1}^i consists only of her parent's bequest b_t^i:

$$k_{t+1}^i = b_t^i. \tag{7}$$

Initial endowments of physical and human capital (k_0^i and h_0^i) are jointly lognormally distributed and the adult receives one unit of labor endowment in each period.

2.2 *Individual Optimization*

We focus on the stationary policy sequence, $\tau_t = \tau$. At each date t, let m_{ht}, m_{kt} denote the means and Δ_{ht}^2, Δ_{kt}^2 denote the variances of $\ln h_t^i$ and $\ln k_t^i$, respectively, and let cov_t denote the covariance between $\ln h_t^i$ and $\ln k_t^i$. Let $M_t \equiv (m_{ht}, m_{kt}, \Delta_{ht}^2, \Delta_{kt}^2, cov_t)$. Then for the agent's dynamic optimization problem, the state variables are $(h_t^i, k_t^i, M_t; \tau)$, the control variables are $(l_t^i,\ e_t^i,\ b_t^i)$ and the Bellman equation is:

$$\ln U\left(h_t^i, k_t^i, M_t; \tau\right) = \max_{l_t^i, e_t^i, b_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln c_t^i - (l_t^i)^\eta\right] \\ +\rho E_t\left[\ln U(h_{t+1}^i, k_{t+1}^i, M_{t+1}; \tau)\right] \end{array} \right\}, \tag{8}$$

subject to (2), (3), (5), (6) and (7).

As is common in this type of model, the first order conditions associated with the Bellman equation described by (8) yield simple closed-form solutions for each agent's optimal labor supply, education expenditure rates, and bequest rates. We present each of these in the following two lemmas, starting with labor supply.

[8] In the context of human capital inequality, Benabou (1996) called this a "neighborhood externality", and we will follow this convention here.

Lemma 1 *The optimal labor supply is invariant to time and the individual and aggregate state variables* (h_t^i, k_t^i, M_t) *and decreases with the average marginal income tax rate* τ *such that:*

$$l_t^i = l \equiv \left(\frac{((1-\lambda-\mu)/\eta)(1-\rho\alpha)(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)} \right)^{1/\eta}. \tag{9}$$

Next we consider investment rates for the two forms of capital. We denote by s_{jt}^i, $j = 1, 2$, respectively the *fraction* of disposable income that agent i invests in her children's education and for her bequest such that $s_{1t}^i \equiv e_t^i / \hat{y}_t^i$, $s_{2t}^i \equiv b_t^i / \hat{y}_t^i$.

Lemma 2 *The optimal education investment rate* s_{1t}^i *and the bequest rate* s_{2t}^i *are invariant to time and the individual and aggregate state variables* (h_t^i, k_t^i, M_t) *and decrease with the average marginal income tax rate* τ *such that:*

$$s_{1t}^i = s_1 \equiv \frac{\rho\beta\mu(1-\tau)}{1-\rho\alpha}, \tag{10}$$

$$s_{2t}^i = s_2 \equiv \rho\lambda(1-\tau). \tag{11}$$

Lemmas 1 and 2 make clear the *negative* effects of redistribution on the incentives to supply labor and capital inputs. Intuitively, τ is a distortionary tax, which reduces the return, from each individual's point of view, from both labor supply and investment. However, as we will see below, and as is standard in this type of model, τ can have an offsetting positive effect through the equilibrium dynamics. We now turn to analyse these dynamics.

3 The Equilibrium Dynamics

The optimization problem (8) yields (9)–(11) and the following decision rules:

$$\ln c_t^i = \ln(1-s_1-s_2) + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t, \tag{12}$$

$$\ln e_t^i = \ln s_1 + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t, \tag{13}$$

$$\ln b_t^i = \ln s_2 + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t. \tag{14}$$

Together with the government's budget constraint (4), the above decision rules imply a unique sequence of aggregate state variables $\{M_t\}$ that coincides with what the agent i takes as given in (8) such that, at each date $t = 0, 1, 2, ..,$ the following aggregate consistency condition holds:

$$\int_0^1 y_t^i di = \int_0^1 (c_t^i + e_t^i + b_t^i) di \tag{15}$$

3.1 *The Dynamic Paths of Physical Capital, Human Capital and Income*

The dynamic path of physical capital for dynasty i can be found by combining the logarithm of (7) with (2) and (14):

$$\begin{aligned} \ln k_{t+1}^i = {} & \ln s_2 + (1-\tau) \ln A_t + (1-\lambda-\mu)(1-\tau) \ln l + \lambda (1-\tau) \ln k_t^i \\ & + \mu (1-\tau) \ln h_t^i + \tau \ln \tilde{y}_t. \end{aligned} \tag{16}$$

Similarly, the dynamic path of human capital can be found by combining the logarithm of (6) with (2) and (13):

$$\begin{aligned} \ln h_{t+1}^i = {} & \ln \kappa + \beta \ln s_1 + \beta (1-\tau) \ln A_t + \beta (1-\lambda-\mu)(1-\tau) \ln l + \ln \xi_{t+1}^i \\ & + \beta\lambda (1-\tau) \ln k_t^i + (\alpha + \beta\mu (1-\tau)) \ln h_t^i + \beta\tau \ln \tilde{y}_t. \end{aligned} \tag{17}$$

Also, substitution of (16) and (17) into (2) yields the equilibrium path of income for agent i:

$$\begin{aligned} \ln y_{t+1}^i = {} & \psi + \ln A_{t+1} - \alpha \ln A_t + (1-\alpha)(1-\lambda-\mu) \ln l \\ & + \mu \ln \xi_{t+1}^i + (\lambda + \beta\mu) \tau \ln \tilde{y}_t - \alpha\lambda\tau \ln \tilde{y}_{t-1} \\ & + (\alpha + (\lambda+\beta\mu)(1-\tau)) \ln y_t^i - \alpha\lambda (1-\tau) \ln y_{t-1}^i, \end{aligned} \tag{18}$$

where $\psi \equiv \mu \ln \kappa + \mu\beta \ln s_1 + \lambda (1-\alpha) \ln s_2$ and where s_1 and s_2 are given by (10) and (11).

Notice that the intergenerational persistence of human capital $p^h \equiv \alpha + \beta\mu (1-\tau)$ and physical capital $p^k \equiv \lambda (1-\tau)$ together imply intergenerational persistence of income $p \equiv \alpha + (\lambda + \beta\mu)(1-\tau)$ between parents and children. This has a structural component α reflecting the neighborhood effect, which cannot be lowered with redistribution alone. The other component of intergenerational persistence decreases with τ, the degree of redistribution, and through this channel a policy of redistribution enhances intergenerational income mobility. Next, we characterize the dynamic paths of the aggregate state variables.

3.2 *Dynamics of the Economy-wide State Variables*

Given the initial lognormal distribution, by (16) and (17), physical and human capital and income are also lognormally distributed over time. Thus, at each date t, M_t satisfies:

$$\begin{aligned} m_{kt+1} = {} & \ln s_2 + \ln \bar{A} + \delta\omega\Delta_{ht}^2/2 + (1-\lambda-\mu)\ln l + \lambda m_{kt} + (\delta+\mu)\, m_{ht} \\ & + \tau\,(2-\tau)\left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2, \end{aligned} \tag{19}$$

$$\Delta_{kt+1}^2 = (1-\tau)^2\left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right), \tag{20}$$

$$\begin{aligned} m_{ht+1} = {} & \ln\kappa + \varphi + \beta\ln\bar{A} + \beta\ln s_1 + \beta(1-\lambda-\mu)\ln l \\ & + \beta\lambda m_{kt} + (\alpha+\beta\mu+\beta\delta)\, m_{ht} \\ & + \beta\tau\,(2-\tau)\left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2 + \beta\delta\omega\Delta_{ht}^2/2, \end{aligned} \tag{21}$$

$$\begin{aligned} \Delta_{ht+1}^2 = {} & \sigma^2 + \beta^2\lambda^2\,(1-\tau)^2\,\Delta_{kt}^2 + (\alpha+\beta\mu\,(1-\tau))^2\,\Delta_{ht}^2 \\ & + 2\beta\lambda\,(1-\tau)\,(\alpha+\beta\mu\,(1-\tau))\, cov_t, \end{aligned} \tag{22}$$

$$\begin{aligned} cov_{t+1} = {} & \beta\lambda^2\,(1-\tau)^2\,\Delta_{kt}^2 + \mu\,(1-\tau)\,(\alpha+\beta\mu\,(1-\tau))\,\Delta_{ht}^2 \\ & + \lambda\,(1-\tau)\,(\alpha+2\beta\mu\,(1-\tau))\, cov_t. \end{aligned} \tag{23}$$

We define, for each date t, an index of inequality Λ_t as the logarithm of the ratio of mean to median income.[9] The following Lemma describes the equilibrium dynamics of per capita income and inequality jointly.

[9] This is a particularly convenient index of inequality for this class of models. See Benabou (2002) for a discussion.

Lemma 3 *At each date t, the inequality index Λ_t equals the variance of logarithmic earnings of agents such that*

$$\Lambda_t = \left(\lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2, \tag{24}$$

and the evolution of earnings of adults is governed by a lognormal distribution such that

$\ln y_t^i \sim N\left(\ln A_t + \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l,\ 2\Lambda_t\right)$.

Also, the break-even level of income $\tilde{y}_t$ satisfies:

$$\begin{aligned} \ln \tilde{y}_t &= \ln y_t + (1-\tau)\Lambda_t, \\ &= \ln A_t + \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l + (2-\tau)\Lambda_t. \end{aligned} \tag{25}$$

Following Solow (1957), TFP is defined to be the ratio of average output to the weighted average of inputs. We use a Cobb-Douglas production technology similar to those assigned to each individual to compute the TFP for the economy as follows

$$TFP \equiv \frac{\int_0^1 y^i di}{\left(\int_0^1 k^i di\right)^{\lambda} \left(\int_0^1 h^i di\right)^{\mu} \left(\int_0^1 l^i di\right)^{1-\lambda-\mu}}. \tag{26}$$

Lemma 4 *In equilibrium, TFP satisfies*

$$TFP_t = E_t * A_t, \tag{27}$$

where $E_t \equiv \exp\left(\left((\lambda-1)\lambda\Delta_{kt}^2 + (\mu-1)\mu\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2\right)$ denotes "efficiency", and $A_t = \bar{A}\exp\left(\delta\left(m_{ht} + \frac{1}{2}\omega\Delta_{ht}^2\right)\right)$ denotes "technology".

Notice that efficiency, here, is a function of the *dispersion* of productive assets – as in Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and others. Technology is a function of the mean of human capital (as in Lucas (1988)) and the dispersion of human capital (as in Jovanovic and Rob (1989)).

4 The Steady State

The following proposition identifies a necessary and sufficient condition for the existence of a unique, stable, steady state equilibrium.

Proposition 5 *The equilibrium sequence M_t monotonically converges to a unique steady state, as a function of τ, if and only if $(1-\alpha)(1-\lambda) > \beta(\mu+\delta)$.*

Intuitively, the condition $(1-\alpha)(1-\lambda) > \beta(\mu+\delta)$ implies economy-wide diminishing returns to input accumulation. In other words, that the degree δ of the external spillover of knowledge is not strong enough to offset the diminishing returns to composite capital present in the production technology.[10]

In this steady state, our index of inequality is time invariant: $\Lambda_t = \Lambda$.Using (20), (22) and (23) in (24), we obtain:

$$\Lambda \equiv \frac{\mu^2 (1+\lambda\alpha(1-\tau))}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+\beta\mu)(1-\tau)+\alpha)^2\right)}\frac{\sigma^2}{2}. \tag{28}$$

By (28), $\partial\Lambda/\partial\tau < 0$, that is, an increase in τ decreases the steady state inequality Λ (as one should expect).

Closed form solutions for the key variables in the steady state equilibrium are provided in the following proposition.

Proposition 6 *The steady state equilbrium is characterized by the following equations:*

$$y = TFP * k^{\lambda} h^{\mu} l^{1-\lambda-\mu}. \tag{29}$$

$$TFP = E * A, \tag{30}$$

$$\ln k = m_k + \Delta_k^2/2, \tag{31}$$

$$\ln h = m_h + \Delta_h^2/2, \tag{32}$$

$$E \equiv \exp\left(\left((\lambda-1)\lambda\Delta_k^2 + (\mu-1)\mu\Delta_h^2 + 2\lambda\mu cov\right)/2\right), \tag{33}$$

$$A = \bar{A}\exp\left(\delta\left(m_h + \frac{1}{2}\omega\Delta_h^2\right)\right), \tag{34}$$

[10] In the limiting case, when $(1-\alpha)(1-\lambda) = \beta(\mu+\delta)$, the economy exhibits balanced endogenous growth. In this paper we focus on income levels and, so, set aside issues of endogenous growth.

$$m_h = \frac{\left(\begin{array}{c} (1-\lambda)(\ln \kappa + \varphi + \beta \ln s_1) + \beta\lambda \ln s_2 + \beta \ln \bar{A} \\ +\beta(1-\lambda-\mu)\ln l + \beta\tau(2-\tau)\Lambda + \beta\delta\omega\Delta_h^2/2 \end{array} \right)}{(1-\alpha)(1-\lambda) - \beta(\mu+\delta)}, \tag{35}$$

$$m_k = \frac{\left(\begin{array}{c} (\delta+\mu)(1-\lambda)(\ln \kappa + \varphi + \beta \ln s_1) + (1-\lambda)(1-\alpha-\beta(\mu+\delta))\ln s_2 \\ +(1-\lambda)(1-\alpha)\left((1-\lambda-\mu)\ln l + \tau(2-\tau)\Lambda + \delta\omega\Delta_h^2/2 + \ln \bar{A}\right) \end{array} \right)}{(1-\lambda)((1-\alpha)(1-\lambda) - \beta(\mu+\delta))}, \tag{36}$$

$$\Delta_k^2 = \frac{\mu^2(1-\tau)^2(1+\lambda\alpha(1-\tau))}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+\beta\mu)(1-\tau)+\alpha)^2\right)}\sigma^2, \tag{37}$$

$$\Delta_h^2 = \frac{(1-\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau)))\left(1-\lambda^2(1-\tau)^2\right) - 2\mu\beta\lambda^3(1-\tau)^4}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+\beta\mu)(1-\tau)+\alpha)^2\right)}\sigma^2, \tag{38}$$

$$cov = \frac{\beta\lambda^2\mu^2(1-\tau)^4 + \mu(1-\tau)(\alpha+\beta\mu(1-\tau))\left(1-\lambda^2(1-\tau)^2\right)}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+\beta\mu)(1-\tau)+\alpha)^2\right)}\sigma^2, \tag{39}$$

where l, s_1, s_2 and Λ are given by (9), (10), (11) and (28).

Thus, total factor productivity, in the steady state equilibrium, is broken down into both its "efficiency" and "technology" components, and closed-form solutions for these components are presented in Proposition 2. Efficiency measures the distance between actual production and the production possibility frontier, whereas technology measures the influence of spillovers on production. Moreover, by (38) and (35), we can see that technology is affected not only by institutional and technological parameters, such as the neighborhood externality α, the quality of education system β, the shares of physical λ and human μ capital, the demographic parameter σ^2 and fiscal policy parameter τ, but also affected by labour supply and saving rates, human capital index κ and the mean of logarithm of innate ability φ. This implies that, in the economy with externality in production technology, TFP is influenced by all of the parameters of the economy.

5 Quantitative Analysis

In this section, we first estimate key parameter values for the U.S., Japan and Australian economies. Then, we discuss the property of TFP and see how TFP changes with institutional factors, technological parameters and fiscal policy. Finally, we discuss how much differences of per capita income could be explained by TFP differences across U.S., Japan and Australia.

5.1 Parameter Values

The following two subsections in two parts describe what we do to complete the task of calibration. Part 1 lists 10 parameters of our paper that we need to calibrate along with their plausable values from others work on comparable models, followed by Part 2 where we list 10 equations for solving the values of those parameters using equations generated solely in our model.[11]

5.1.1 Part 1 - Information from other papers regarding plausible values for the parameters of our model.

The output elasticity of physical and human capital λ and μ, and elasticity of raw labor $\varepsilon \equiv 1-\lambda-\mu$:

1. λ - Gollin (2002) provides an estimate = 0.33.

2. μ - Mankiw, Romer and Weil (1992), $\mu = (1-ratio)*labour\ share = (1-0.26)*0.67 = 0.5$, where $ratio =$ the ratio of minimum wage to average wage = 0.26.

Following Barro, Mankiw, and Sala-i-Martin (1995), we choose $\lambda = 0.3$, $\mu = 0.5$.

3. ρ - Each generation lasts 25 years, and an annual time discount rate is around 0.96. Then this implies a discount factor of 0.36 for each generation, i.e., $\rho = 0.36$.

4. η : intertemporal elasticity of labor supply $\epsilon = \frac{1}{\eta-1} = 0.25$ - Chetty, Guren, Manoli and Weber (2011); Blundell, Bozio and Laroque (2011) find the elasticity 0.34 for women, 0.25 for men. It implies $\eta = 5$.

5. $\alpha = 0.2$,de la Croix and Doepke (2003)
   $= 0.24$, Rosenweig and Wolpin (1994)
   $= 0.3$, Erosa and Koreshkova (2007) - JME paper

6. $\beta = 0.6$ - de la Croix and Doepke (2003)
   $= 0.45$ - Erosa and Koreshkova (2007);
   $= 0.4$ - Benabou (2002).

[11] We show the calibration for U.S. here and leave the calibration for Australia and Japan in Appendix B.

7. σ^2: This parameter can be determined by inocme inequality $\Lambda \in [0.17, 0.29]$ which is available from U.S. Census.

8. $\delta = 0.29$ - Choi (2010)

$= 0.28$ - Iacopetta (2011)

$= 0.1$ - de la Croix and Doepke (2003)

9. κ can be calibrated to match the average years of schooling, 12.6 in 1995, which is available in Barro-Lee dataset 2010.

10. $\bar{A}$ is can be calibrated to match the per capita income, \$43,842 in 1995, estimated in Jerzmanowski (2007).

5.1.2 Part 2 - Keeping in mind the above information, guess and calibrate the above 10 parameters to match the following targets

(i) Following Barro, Mankiw, and Sala-i-Martin (1995), we use shares of 0.3 for physical capital, 0.5 for human capital, and 0.2 for labor, i.e., $\lambda = 0.3$, $\mu = 0.5$.

(ii) The intergenerational persistence of income between parents and children is as follows

$$p(\tau) \equiv \alpha + (\lambda + \beta\mu)(1 - \tau). \tag{40}$$

Setting $\alpha = 0.2$ and $\beta = 0.5$ allows $p(\tau)$ to range from 0.2 to 0.75. This range is consistent with Solon's (1992) and Mulligan's (1997) estimation for U.S. intergenerational persistence.

(iii) In Benabou (2002), he measures the family income inequality by using the logarithm of the ratio of mean to median income. Moreover, by approximating the US distribution of family incomes as a lognormal, Benabou (2002) gives the standard deviation of log-income 0.61 in 1990 Census and 0.69 in that of 1995. It implies that the income inequality which equals half of variance of log-income ranges from 0.19 to 0.24. We set $\sigma^2 = 1.15$,[12] so that the feasible range is [0.15, 0.29]. The mean of the logarithm of innate ability φ is set to -0.575 so that $E[\xi] = 1$.[13]

[12] In Benabou (2002), $\sigma^2 = 1$. Introducing physical capital leads to higher income inequality so that $\sigma^2 = 1.15$ in this paper.

[13] Note that here we follow Benabou (2002), and φ could be a free parameter without such restriction.

(iv) A is chosen to 1000. Any A between 400~1000 satisfy delta between 0.1 and 0.5

(v) the degree of externality δ and human capital index κ are calibrated to minimized the error

$$error = \left(\left(h - 12.6 \right) / 12.6 \right)^2 + \left(\left(y - 43842 \right) / 43842 \right)^2 .$$

Thus we choose $\delta = 0.136$ and $\kappa = 0.045$.

The following Table summarizes the benchmark parameter values.[14]

Table 1: Calibrated model parameters for the U.S. benchmark economy

Targets	Parameter	Benchmark
share of physical capital, (Barro, Mankiw, and Sala-i-Martin, 1995)	λ	0.33
share of human capital, (Barro, Mankiw, and Sala-i-Martin, 1995)	μ	0.4757
time discount	ρ	0.37
(annual time discount rate is 0.96, so that each generation is $0.96^{25} = 0.36$)		
intertemporal elasticity of labor supply,	η	4
(Chetty, Guren, Manoli and Weber, 2011 and Blundell, Bozio and Laroque, 2011)		
human capital externality, (Acemoglu and Angrist 2001)	δ	0.101
the elasticity of human capital with respect to education,	β	0.219
(de la Croix and Doepke, 2003 and Benabou, 2002)		
intergenerational persistence of income,	α	0.301
(Lee and Solon, 2009 and Murtazashvili, 2011)		
Income Inequality, (World Income Inequality Database)	σ^2	1.29
average years of schooling, 1995, (Jerzmanowski, 2007)	κ	1.43
per capita income, 1995, (Jerzmanowski, 2007)	$\bar{A}$	948.6
	τ	33%

[14] According to OECD Table I.4, $\tau_{U.S.} = 21.7\%$, $\tau_{Japan} = 13.6\%$ and $\tau_{Australia} = 31.5\%$ in 2008.

Table : Calibrated model parameters for the Australian economy

Targets	Parameter
share of capital in national income	share of physical capital
	share of human capital
interest rate	time discount
intertemporal elasticity of labor supply,	
	human capital externality
the return to investment in education	the elasticity of children's human capital with respect to education
intergenerational persistence of income,	the elasticity of children's human capital with respect to parents' human c
Income Inequality	varaince of innate ability
average years of schooling, 1995	human capital index
per capita income, 1995	technology index

Table : Calibrated model parameters for the Japan economy

Targets	Parameter
share of capital in national income	share of physical capital
	share of human capital
interest rate	time discount
intertemporal elasticity of labor supply,	
	human capital externality
the return to investment in education	the elasticity of children's human capital with respect to education
intergenerational persistence of income,	the elasticity of children's human capital with respect to parents' human c
Income Inequality	varaince of innate ability
average years of schooling, 1995	human capital index
per capita income, 1995	technology index

5.2 Properties of TFP

In this section, to explore the property of TFP, we plot graphs of TFP against parameters α, β, λ, μ, τ and δ while keep other parameters at the baseline values.

Property 1: *A more segregated society would have a lower level of TFP .*

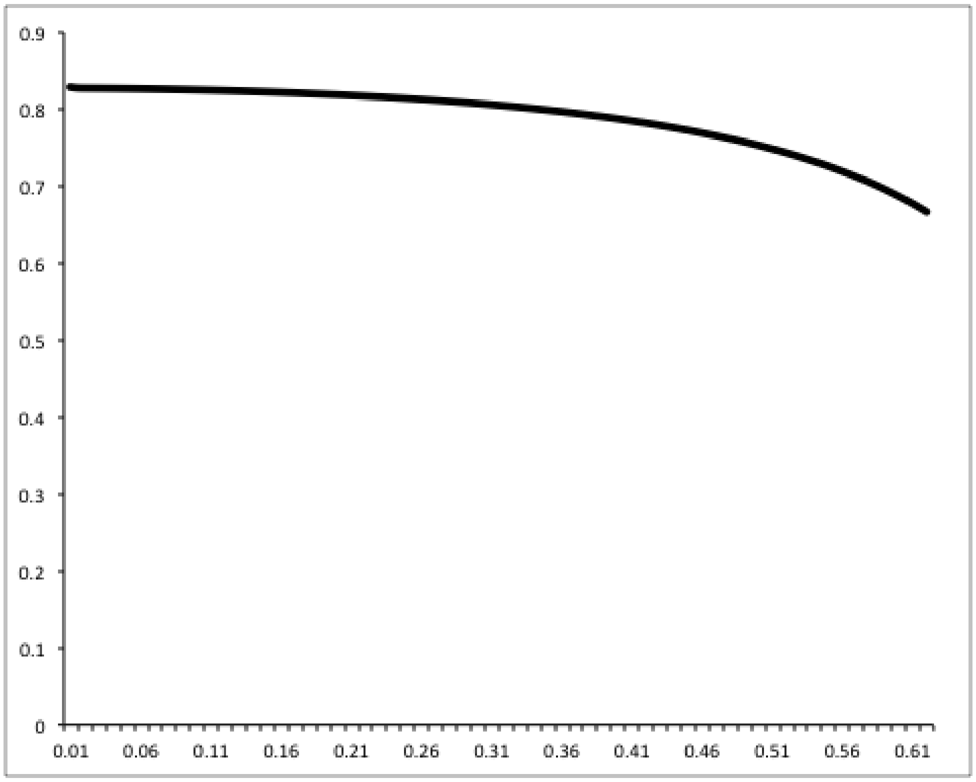


Figure 1a—EFF vs Neighbourhood Externality, α.

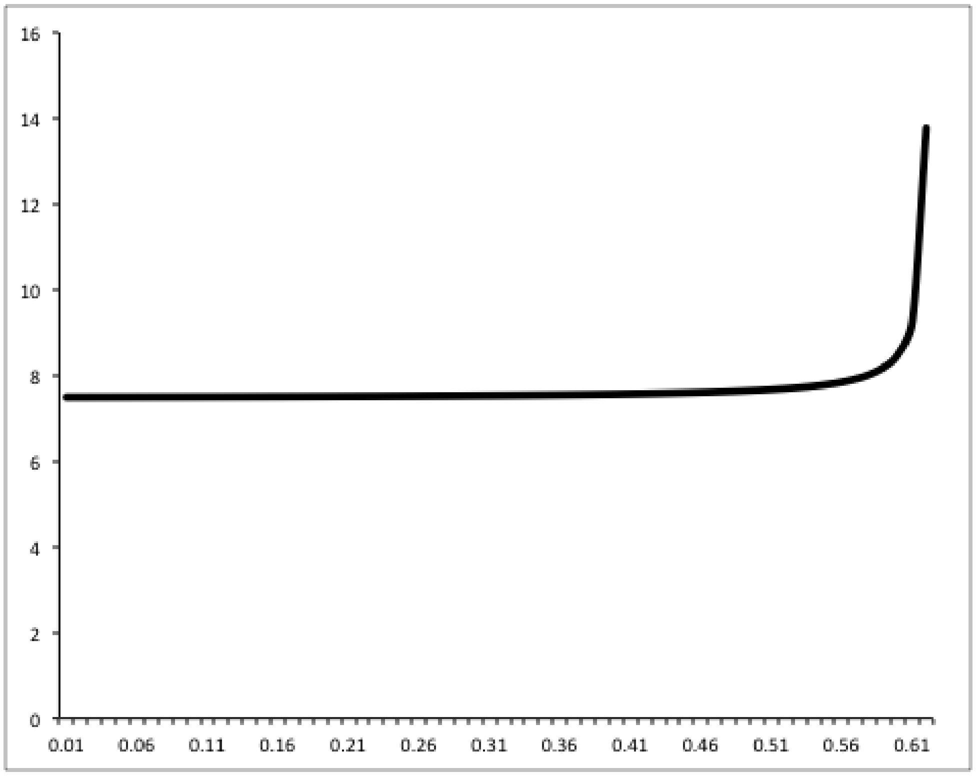


Figure 1b—lnTECH vs Neighbourhood Externality, α.

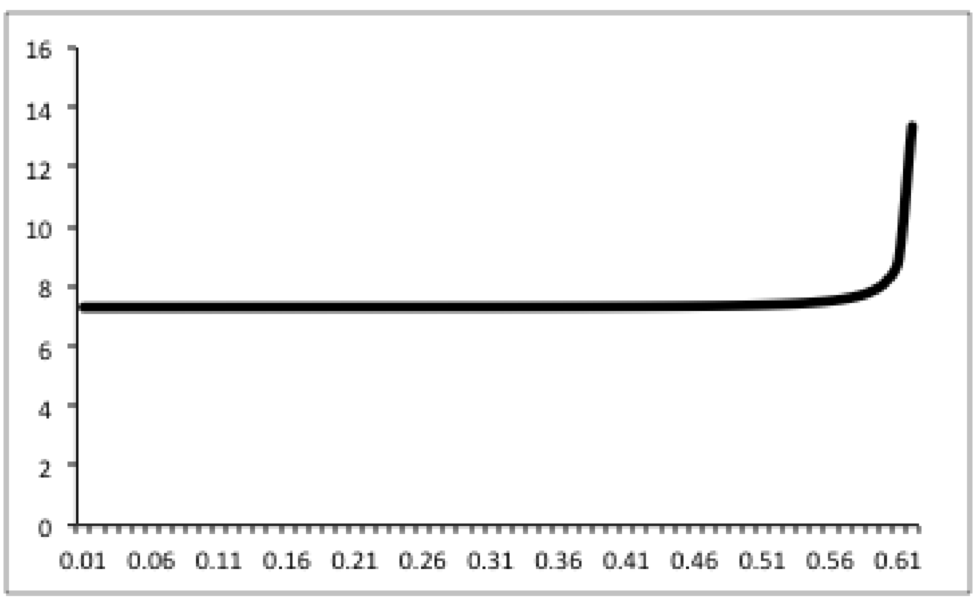


Figure 1c—lnTFP vs Neighbourhood Externality, α.

In our model, we interpret α in the human capital accumulation equation (6) as the parameter that captures the economy's degree of segregation generating a "neighborhood-externality". Segregation of different kinds prevents interaction across communities and enhances the value of family connection in building one's human capital. Englander and Mittelstadt (1988) find a similar conclusion. Segregation prohibits spillover of knowledge across families and communities and then hinders the improvement of overall productivity. The examples of segregation are like interpersonal differences in productivity due to barriers to education, communication problems due to language difficulties, or discrimination because of race, religions, gender, national identity, and age. Because such segregation helps to confine knowledge within families and creates knowledge-gaps across communities, they cause a persistent difference in marginal productivity across individuals. These interpersonal differences in productivity due to barriers to knowledge spillover across families or communities imply a lower level of TFP.

Property 2: *A better quality of educational system implies a lower level of TFP.*

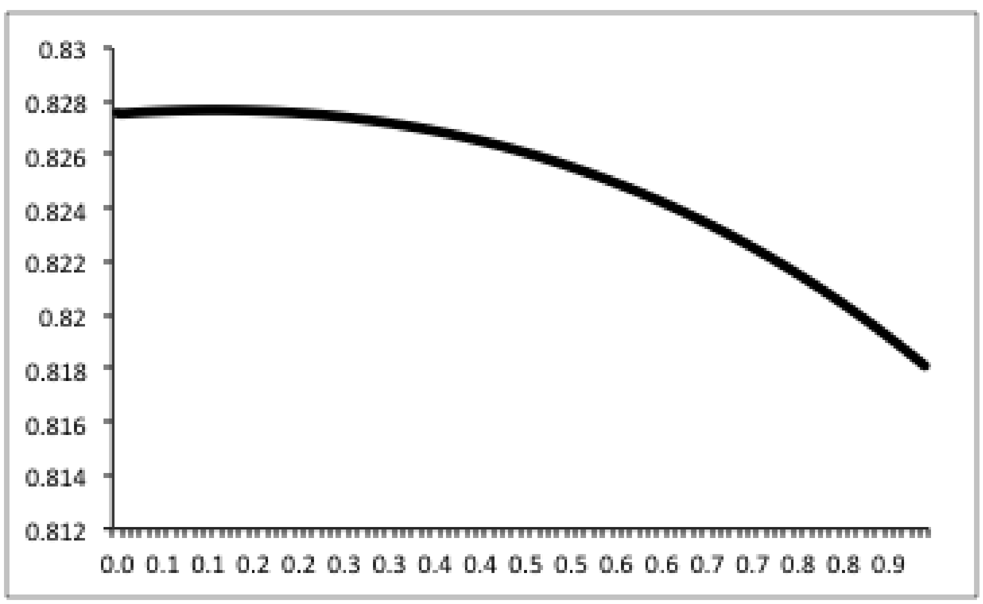


Figure 2a—Efficiency vs Quality of Education System, β.

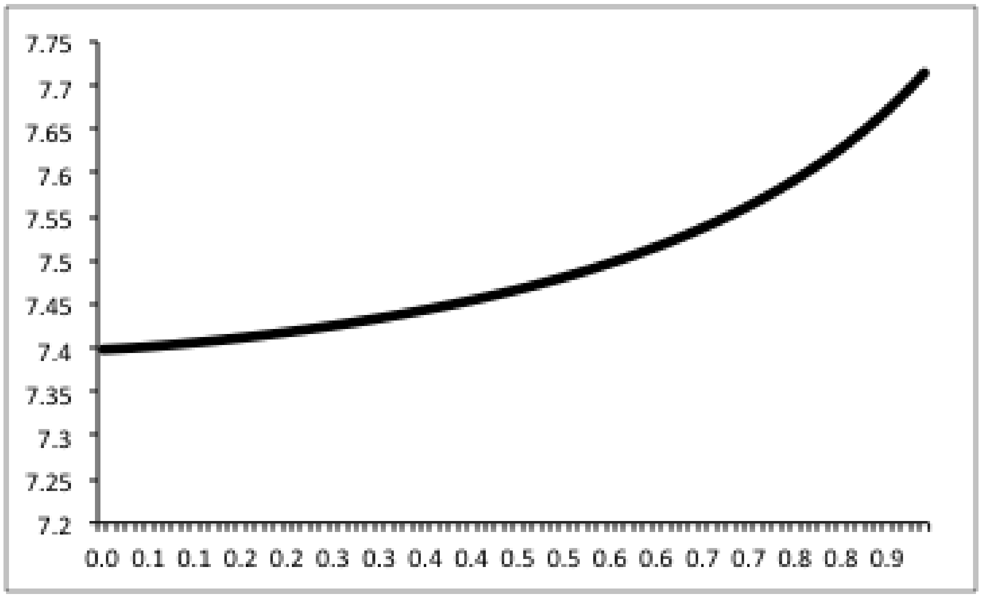


Figure 2b—lnTECH vs Quality of Education System, β.

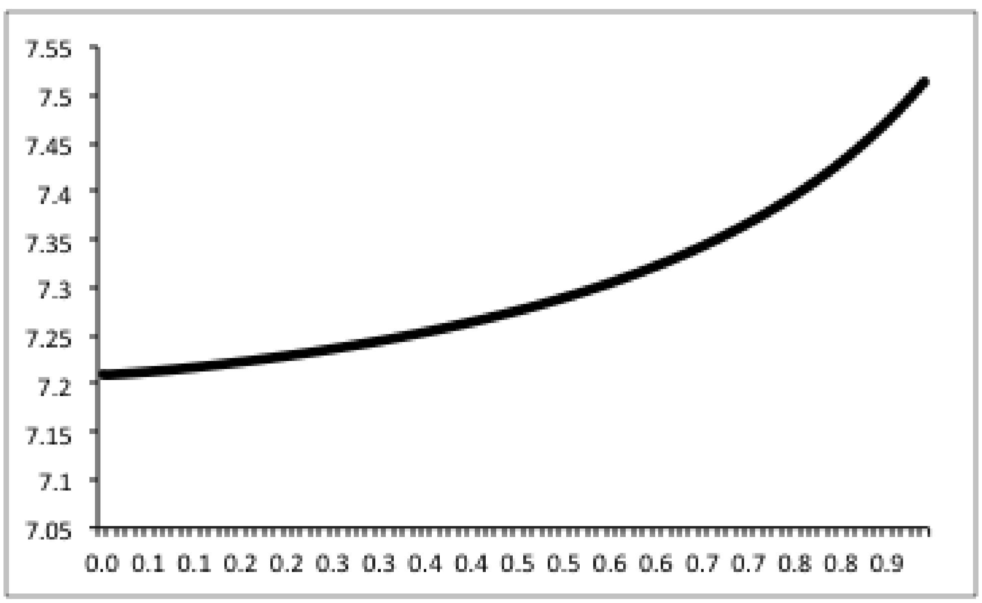


Figure 2c—lnTFP vs Quality of Education System, β.

Figure 2 shows that TFP decreases with the education quality parameter β. By (10), the optimal proportion of income that agents invest in schooling for their children increases with this parameter and that leads to a higher degree of inequality. A greater inequality, by Lemma 5, leads to a lower level of efficiency. Moreover, m_h decreases with β. This is because by (35) we can see that higher β may lower m_h when the effects of the negative terms, $\ln s_1$, $\ln s_2$ and $\ln l$, dominate. Therefore, TFP decreases with β.

In addition, Figure 2 shows that TFP could be greater than one when β is small. This is because by following Lucas (1988) to include externality in production, TFP measures both efficiency and externality. Externality could be very high such that TFP could be greater than one. This is because by (6), a very low β could reduce the negative effect of credit constraint on education expenditure and then leads to a high level of aggregate human capital.

The parameter δ measures the degree of knowledge spillover. The following graph shows how TFP changes with δ.

Property 3: *A higher degree of knowledge spillover leads to a lower level of* TFP .

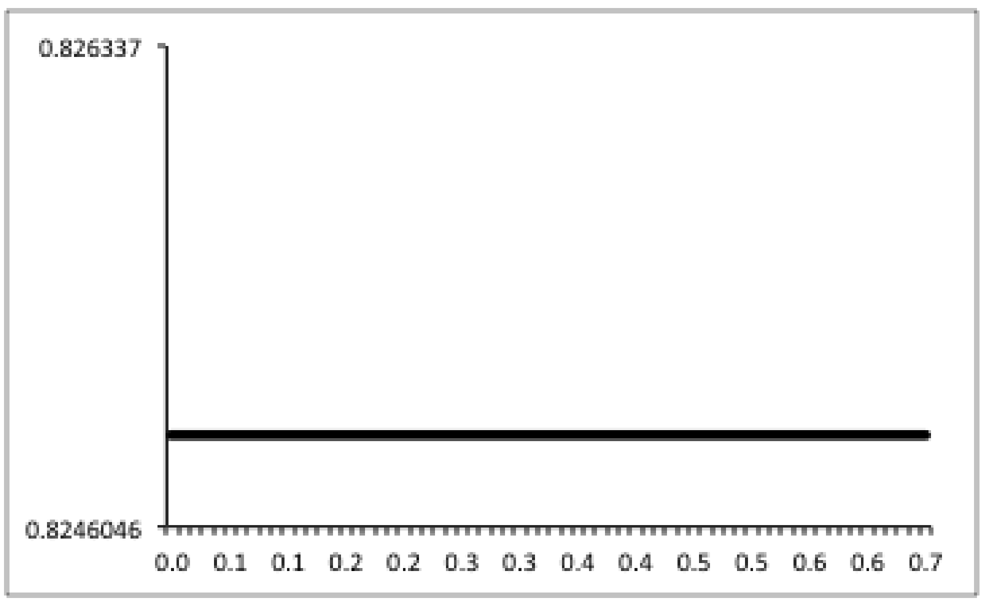


Figure 3a—Efficiency vs Degree of Knowledge Spillover, δ.

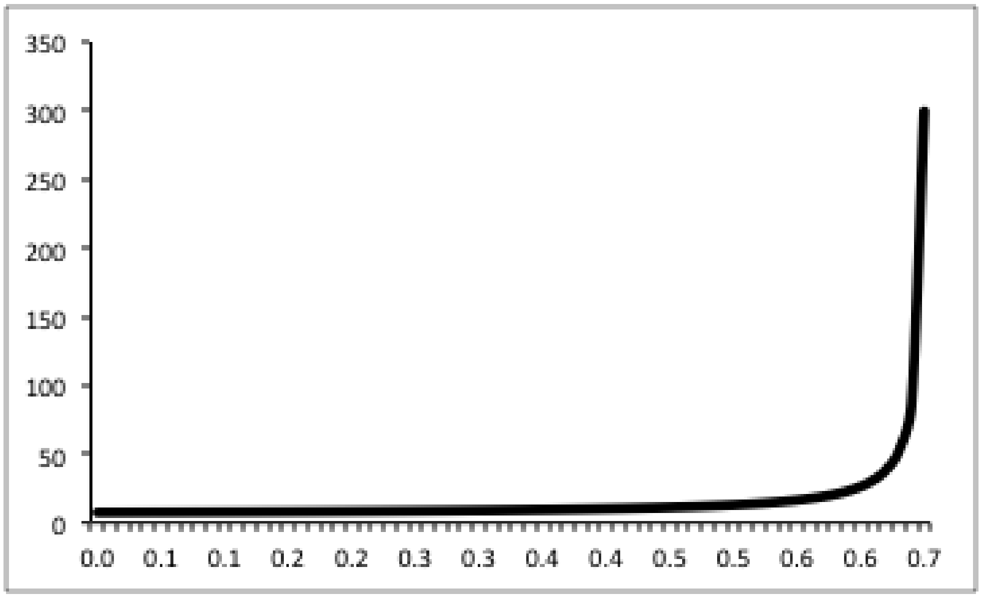


Figure 3b—lnTECH vs Degree of Knowledge Spillover, δ.

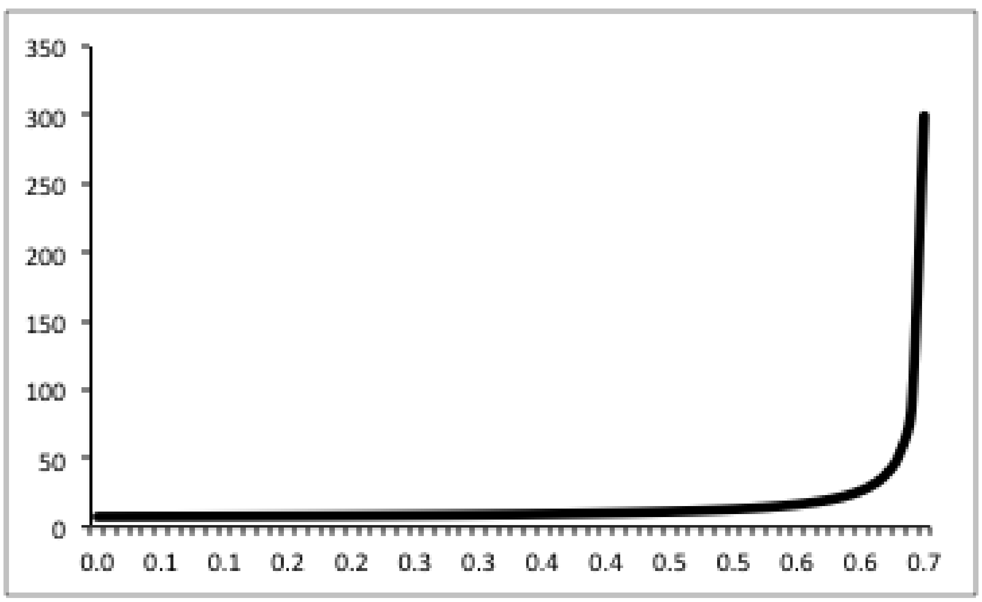


Figure 3c—lnTFP vs Degree of Knowledge Spillover, δ.

The above graph shows that as the degree of knowledge spillover δ increases, total factor productivity decreases due to the negative effect of m_h on the technology such that in (**??**), the term $m_h + \frac{1}{2}\omega\Delta_h^2$ is negative. Then the increase of δ multiplied by

$m_h + \frac{1}{2}\omega\Delta_h^2$ drags down the level of TFP[15] while aggregate efficiency keeps unchanged. The economic intuition is that in an economy with low level of technology, higher degree of knowledge/technology spillover may not be good for improving TFP because individuals can not benefit from such low technology. In contrast, from the point of microeconomic angle, decreasing the barriers to knowledge spillover across families or communities helps increase TFP.

By increasing λ and μ proportionately while keeping the ratio of the two capital shares constant, $\lambda/\mu = 3/5$, as implied by Barro, Mankiw and Sala-i-Martin (1995), the following graph shows how TFP changes with λ.

Property 4: *In the presence of capital-skill complementarity, TFP has an inverted U shape with respect to an increase of capital intensity.*

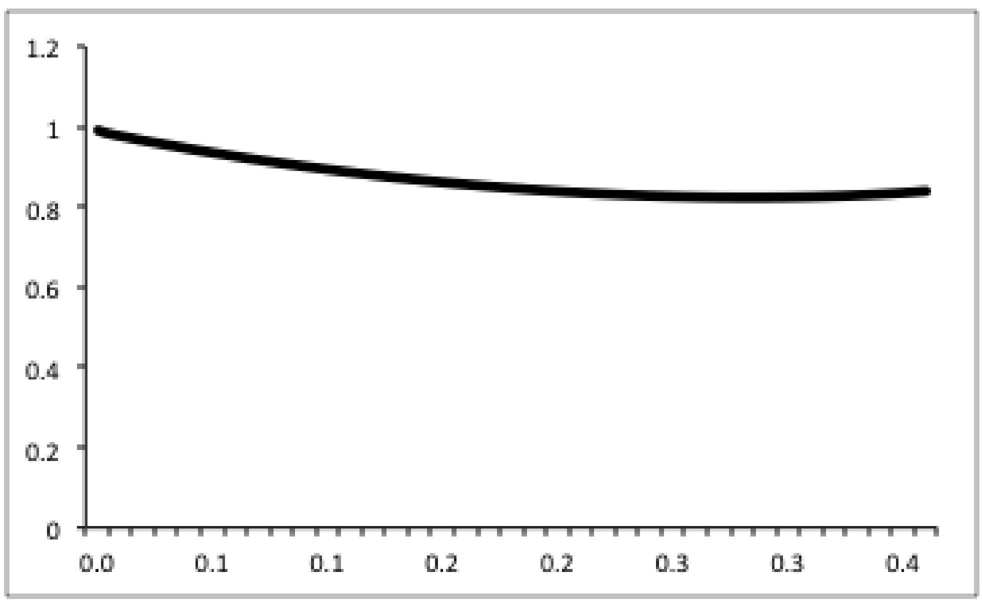


Figure 4a—Efficiency vs Capital Intensity with Capital-Skill Complementarity, $\lambda/\mu = 3/5$.

Figure 4b—lnTECH vs Capital Intensity with Capital-Skill Complementarity, $\lambda/\mu = 3/5$.

[15]If however, an economy has a very high κ, for example, $\kappa = 8.1$, so that $m_h + \frac{1}{2}\omega\Delta_h^2$ is positive, the technology and then TFP could increase with δ.

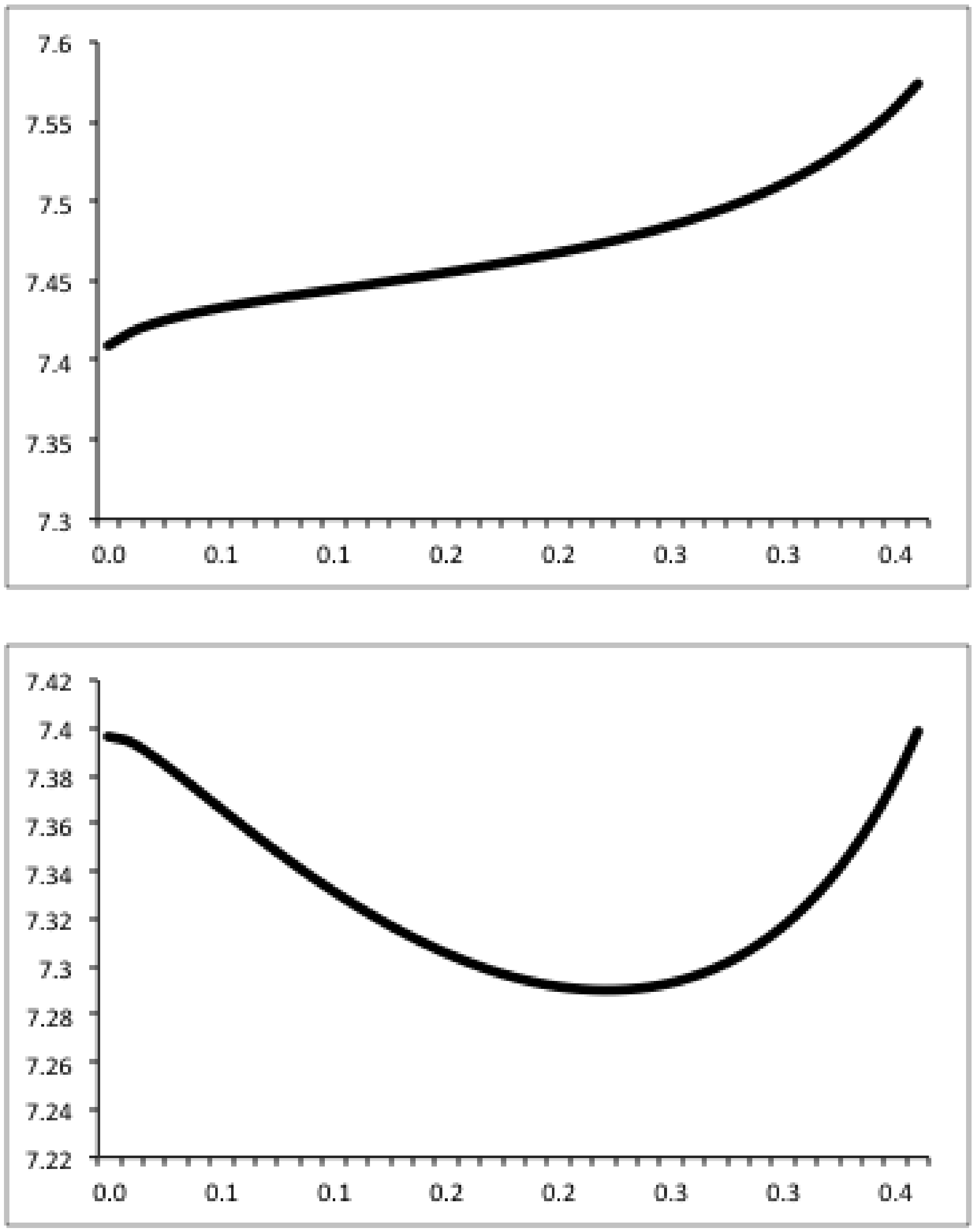
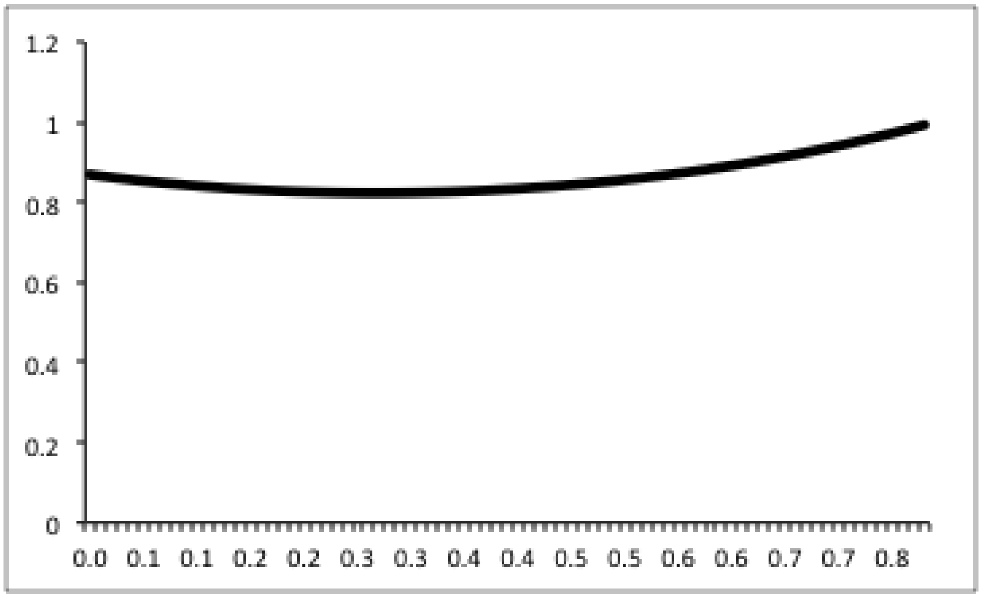


Figure 4c—lnTFP vs Capital Intensity with Capital-Skill Complementarity, $\lambda/\mu = 3/5$.

The above Figure shows that TFP increases first and then decreases as λ surpasses beyond about 0.1. This is because an increasing proportion of skilled labour and capital could lead to higher levels of technology. Therefore, TFP increases through benefiting from the high technology. But as the proportion of skilled labour and capital exceeds a critical point, labor supply which has no effect on efficiency decreases significantly.



The decrease of labor supply combined with high levels of income inequality in the absence of credit market drags down TFP as λ/μ surpasses a critical point.

The following graph shows how TFP changes with λ such that $\lambda + \mu = 0.8$.

Property 5: *TFP decreases as the share of physical capital λ increases and the share of human capital μ decreases such that $\lambda + \mu$ is constant.*

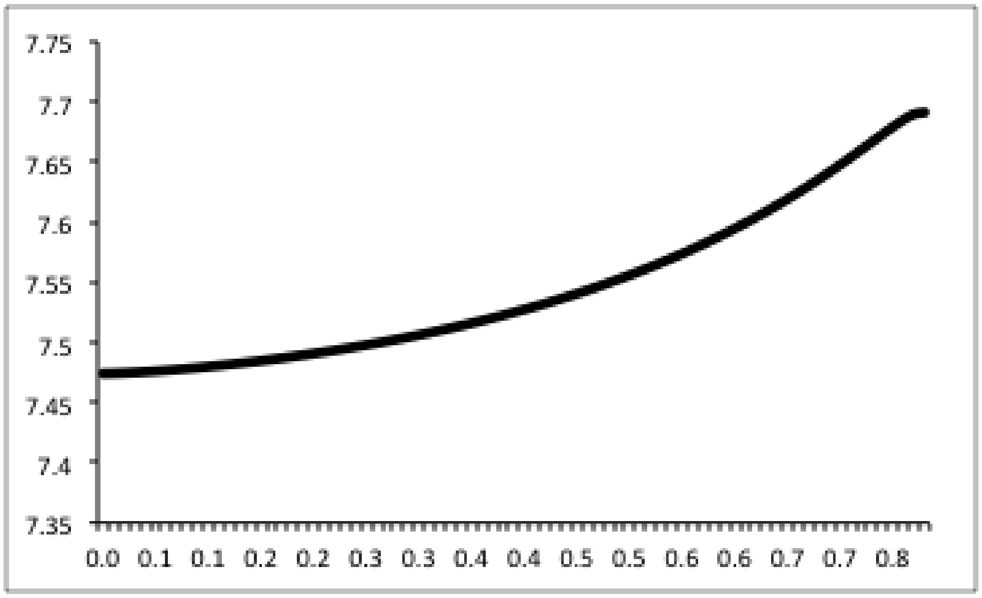
Figure 5a—Efficiency vs Capital Intensity with Capital-Skill Complementarity, $\lambda + \mu = 0.8$.

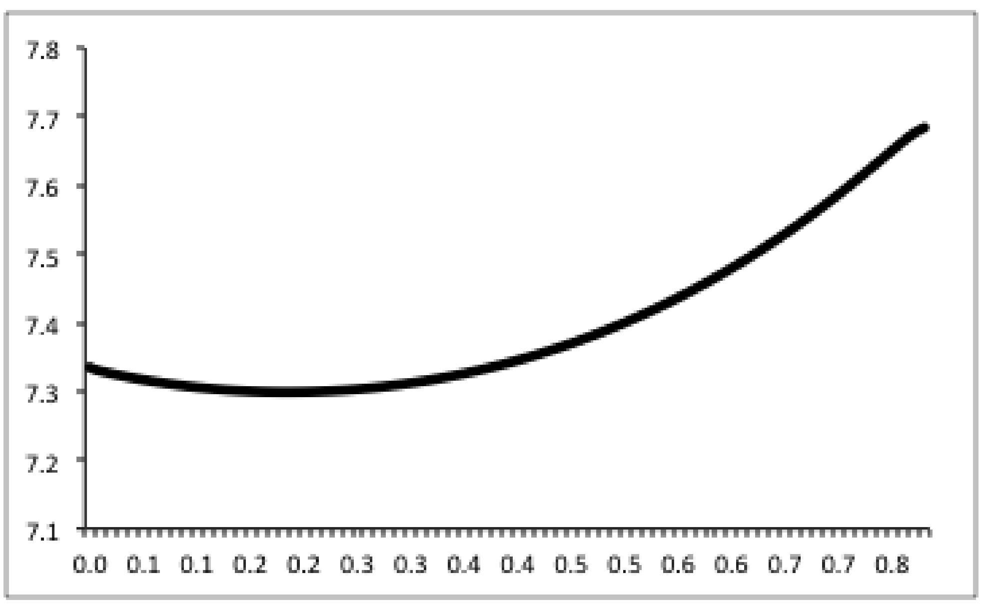
Figure 5b—lnTECH vs Capital Intensity with Capital-Skill Complementarity, $\lambda + \mu = 0.8$.

Figure 5c—lnTFP vs Capital Intensity with Capital-Skill Complementarity, $\lambda + \mu = 0.8$.

The above Figure shows that TFP always decreases with λ as the sum of λ and μ keeps constant. It implies that as the society relies more on physical capital and less on human capital, TFP always decreases when the society can benefit from externality. This is because the externality comes from aggregate human capital which decreases as μ decreases. The decrease of externality drags down TFP even though aggregate efficiency could increase.

Property 6: *TFP has an inverted U-shape with respect to an increase of τ.*


7.75
7.7
7.65
7.6
7.55
7.5
7.45
7.4
7.35
0.0 0.1 0.1 0.2 0.2 0.3 0.3 0.4 0.4 0.5 0.5 0.6 0.6 0.7 0.7 0.8


7.8
7.7
7.6
7.5
7.4
7.3
7.2
7.1
0.0 0.1 0.1 0.2 0.2 0.3 0.3 0.4 0.4 0.5 0.5 0.6 0.6 0.7 0.7 0.8

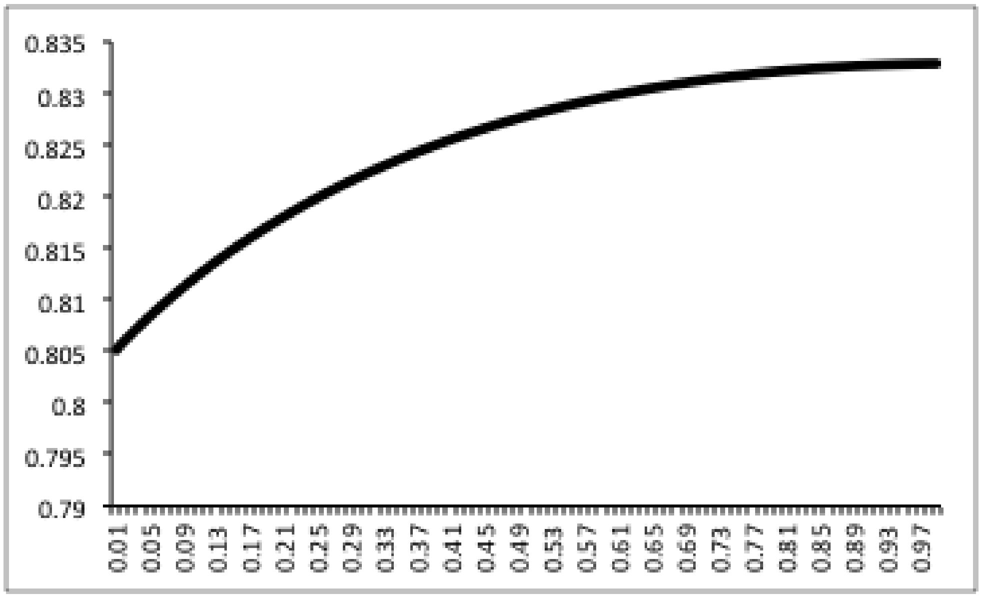


Figure 6a—Efficiency vs Redistribution, τ.

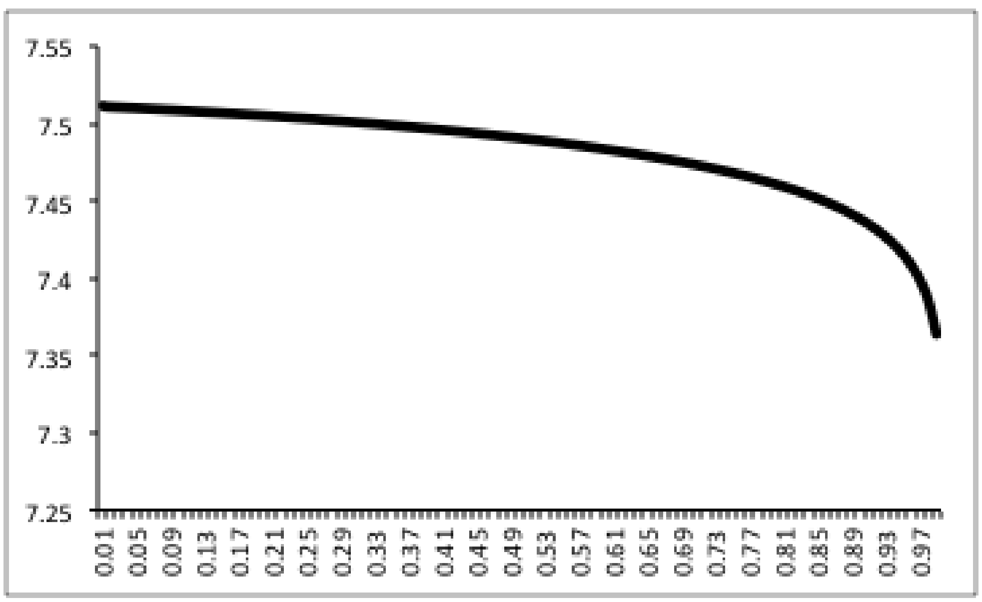


Figure 6b—lnTECH vs Redistribution, τ.

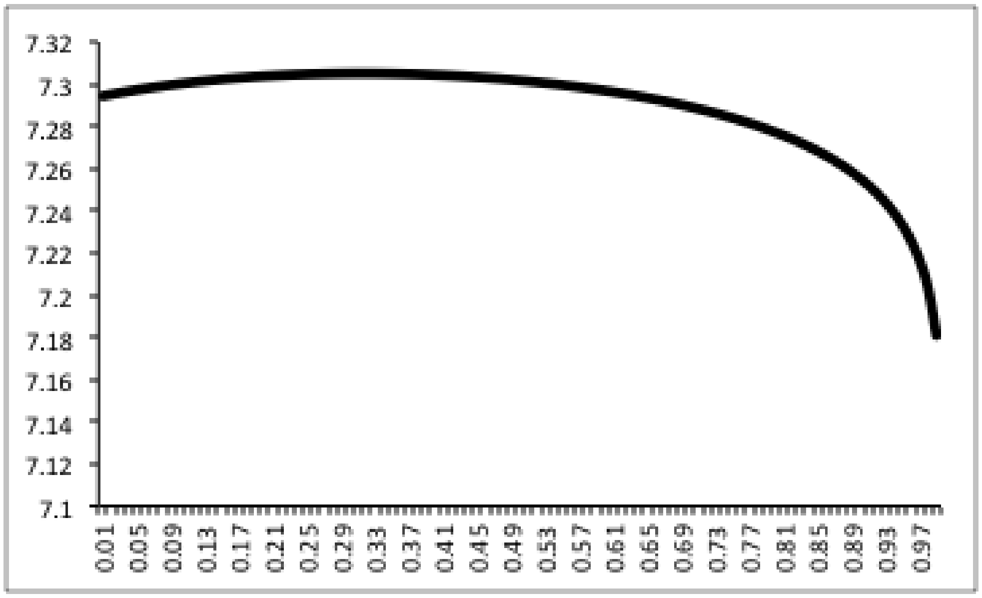


Figure 6c—lnTFP vs Redistribution, τ.

Figure 6 shows that TFP increases with τ as τ is less than 10.8% and then decreases. Redistribution does not only help to improve efficiency but also obtains some gains to m_h through reallocating education expenditure to reduce the imbalance of opportunities of taking education because of credit constraints. That's why TFP goes up when τ is low. But as τ is greater than 10.8%, TFP is dragged down by m_h due to the high distortions of labour supply and saving rate s_2.

The above discussion helps us to understand how and why TFP varies across countries. In the following section, we estimate how much differences of per capita income could be explained by TFP differences through changing institutional and policy parameters.

5.3 Effects of TFP Differences on the Per Capita Income Differences

In this subsection, we first present estimations of Efficiency, Technology and Per capita Income of Japan and Australia relative to the U.S. and then discuss how much the differences of per capita income could be explained by TFP across U.S., Japan and Australia.

The following Table shows the ratio of TFP, Efficiency, Technology and Per capita Income of Japan and Australia relative to the U.S.

Table 2

Ratio of TFP, Efficiency, Technology and Per capita Income relative to the U.S.

	Japan		
	Model	Jerzmanowski (2007), [HJ 1999]	
TFP	37%	68%	[66%]
Efficiency	93%	79%	
Technology	40%	86%	

	Australia	
	Model	Jerzmanowski (2007)
TFP	53%	76%
Efficiency	86%	80%
Technology	62%	93%

Note: HJ 1999 refers to Hall and Jones (1999)

From the above Table, it can be seen that levels of TFP, technology, efficiency and per capita income in the U.S. are higher than Japan and Australia. Prescott (2002, page1) claimed that "The United States is prosperous relative to Japan because production efficiency is higher in the United States...In the United States, total factor productivity is approximately 20 percent higher than in Japan". From the above Table, it can be seen that TFP is about 85% higher in the U.S. than in Japan. Efficiency levels are quite similar but levels of technology in U.S. are much higher than Japan. The above result is a little bit higher than Prescott's (2002) estimation but consistent with Hall and Jones (1999) and Jerzmanowski (2007). Hall and Jones (1999) found that productivity in Japan is around 66% of U.S.[16] Jerzmanowski (2007) decomposed TFP into technology and efficiency and found that technology in Japan is around 86% of U.S. which is a little bit higher than the above result, 60%, but efficiency in Japan is 79% of U.S. which is much lower than our estimation, 90%. Overall, Jerzmanowski

[16] Please see Hall and Jones (1999), Table 1.

(2007) find that TFP in Japan is around 68% of U.S. in 1995,[17] which is close to our estimation, 54%.

The above Table shows that TFP in Australia is around 79% of U.S. This is close to the result in Jerzmanowski (2007) where he found that TFP in Australia is around 76% of U.S.[18] Moreover, he found that technology and efficiency in Australia are around 93% and 80% of U.S. which are close to our estimations, 88% and 90%.

From the above analysis, we can say that our results are consistent with literature.

To answer how much the difference of per capita income across countries could be explained by differences in TFP, by (26) we get

$$\Delta \ln y = \Delta \ln TFP + \lambda \Delta \ln k + \mu \Delta \ln h + (1 - \lambda - \mu) \Delta \ln l. \tag{41}$$

From equation (41), we can see that the percentage of the differnce of per capita income explained by the difference of TFP is $\Delta \ln TFP / \Delta \ln y * 100\%$.

Using equation (41), we estimate how much of the difference of per capita income can be explained by differences in TFP across two countries due to different institutional, techology or policy parameters. The following Table shows the results when one parameter value changes by 1% at each point value while the other parameters are fixed at the U.S. benchmark values. For example, the first result in the third column shows that the TFP difference can explain 11.58% of per capita income difference when one country has $\alpha_1 = 0.101$ while the other has $\alpha_2 = 0.1$, i.e., $11.58\% = (\ln TFP(\alpha_1) - \ln TFP(\alpha_2)) / (\ln y(\alpha_1) - \ln y(\alpha_2))$.

[17] Please see Jerzmanowski (2007), Table 4.
[18] Please see Jerzmanowski (2007), Table 7.

Table 3

Percentage of The Difference of Per Capita Income Explained by The Difference of TFP

Across Countries Due to Different β, α, λ, μ and τ

$(\beta,\ \alpha,\ \lambda,\ \mu,\ \tau)$	β	α	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	δ	τ
0.1	11.94%	11.58%	9.98%	7.96%	52.10%	-0.35%
0.2	11.97%	12.77%	6.11%	5.77%	52.10%	2.22%
0.3	11.96%	13.50%	1.20%	3.66%	52.10%	3.81%
0.4	11.93%	13.88%	–	2.08%	52.10%	4.82%
0.5	11.88%	13.94%	–	1.20%	52.10%	5.46%
0.6	11.82%	13.67%	–	1.10%	52.10%	5.85%

The second through the last column of the above Table shows the percentage of the difference of per capita income explained by TFP across countries due to 1% difference at each point of β, α, λ, μ or τ.

The results in column 4 show that, by increasing λ and μ proportionately while keeping the ratio of the two capital shares constant, as implied by Barro, Mankiw and Sala-i-Martin (1995), how much the difference of per capita income can be explained by the difference in TFP.

The results in column 5 show that, as the share of physical capital λ increases and the share of human capital μ decreases such that $\lambda + \mu$ is constant, how much the difference of per capita income can be explained by the difference in TFP.

The following Table shows how much the variation of per capita income can be explained by the variation of TFP, i.e., $var\left(\ln TFP\right)/var\left(\ln y\right)$, when one parameter value changes at each point value with $\pm 10\%$ range while other parameters are fixed at the U.S. benchmark values. For example, the first result in the third column shows that the variation of TFP can explain 1.43% of the variation of per capita income as α increases from 9% to 11% with 100 steps and each step=0.0002.

Table 4

Variation of Per Capita Income Explained by the Variation of TFP

Across Countries Due to Different β, α, λ, μ and τ with $\pm 10\%$ range

$(\beta,\ \alpha,\ \lambda,\ \mu,\ \tau)$	β	α	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	δ	τ
0.1	1.43%	1.34%	1.00%	0.64%	27.14%	0.002%
0.2	1.43%	1.63%	0.38%	0.34%	27.14%	0.05%
0.3	1.43%	1.82%	0.02%	0.14%	27.14%	0.14%
0.4	1.42%	1.92%	0.07%	0.04%	27.14%	0.30%
0.5	1.41%	1.94%	–	0.01%	27.14%	0.34%
0.6	1.40%	1.8%	–	0.01%	27.14%	0.37%

It can be seen from the above Table that if there are 100 countries with all the same parameter values except one parameter from the set $(\beta,\ \alpha,\ \lambda,\ \mu,\ \tau)$, then the role of parental influence and education effect on human capital accumulation, measured by α and β, is more significant on the variation of per capita income explained by the variation of TFP than other parameters. In overall, Table 5 shows the similar results with the Table 4 but with smaller magnitude.

Table 5: Variation of Per Capita Income Explained by the Variation of TFP (%)

$$V\_TFP = (var(\ln TFP) + cov(\ln TFP, \ln Input))/var(\ln Y)$$

Across Countries Due to Different δ, α, β and τ

Sources of Variation	V_TFP
δ	52.10
δ, α	38.81
δ, α, β	21.49
δ, α, β^*	17.95
δ, α, β^{**}	18.26
δ, α, β, τ	15.87
δ, α, τ	35.05
δ, α, β, τ^*	16.25
δ, τ^*	58.76
α, β, τ^*	11.41
α, β^*, τ^*	11.90
κ^*	11.73
κ^*, δ	3.26
κ^*, δ, τ^*	2.70

Note: * means the rich and poor countries have different mean and variance of that parameter, and ** means the rich and poor countries have different mean but the same variance of that parameter.

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Table5-1 contains the average of (del TFP/ del Y)

(β, α, λ, μ, τ)	β	α	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	δ	τ
0.1	12.64%	29.35%	1.17%	1.16%	52.12%	17.91%
0.2	12.36%	23.26%	3.06%	3.06%	52.12%	16.91%
0.3	12.17%	19.41%	4.46%	4.46%	52.12%	15.96%
0.4	12.02%	16.89%	5.46%	5.65%	52.12%	15.00%
0.5	11.90%	15.13%	–	6.76%	52.12%	13.99%
0.6	–	13.81%	–	7.81%	52.12%	12.87%

Table5-2 contains the variance of (del TFP / del Y)

$(\beta,\ \alpha,\ \lambda,\ \mu,\ \tau)$	β	α	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	δ	τ
0.1	0.02%	10.08%	0.94%	0.95%	0.00%	0.17%
0.2	0.03%	13.77%	1.64%	1.64%	0.00%	0.64%
0.3	0.04%	14.77%	2.35%	2.32%	0.00%	1.30%
0.4	0.05%	11.98%	2.11%	3.79%	0.00%	2.59%
0.5	0.04%	9.37%	–	4.40%	–	4.77%
0.6	–	7.46%	–	6.01%	–	7.73%

Table5-3 contains the (variance of del TFP)/ (variance of del Y)

$(\beta,\ \alpha,\ \lambda,\ \mu,\ \tau)$	β	α	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	δ	τ
0.1	1.60%	8.61%	0.01%	0.01%	27.17%	3.21%
0.2	1.53%	5.40%	0.09%	0.09%	27.17%	2.86%
0.3	1.48%	3.76%	0.20%	0.20%	27.17%	2.55%
0.4	1.44%	2.84%	0.32%	0.32%	27.17%	2.25%
0.5	1.42%	2.27%	–	0.46%	27.17%	1.96%
0.6	–	1.89%	–	0.62%	27.17%	1.65%

6 Concluding Remarks

We present an analytical expression for TFP based on a dynamic general equilibrium theory to estimate quantitatively how much of the variations of per capita income across countries can be explained by variations in TFP which could be decomposed into technology and efficiency, as opposed to variations in inputs. Our theory of TFP indicates that measured TFP would be lower in a country with a greater degree of segregation that hinders knowledge spillover, and with a better quality of educational system. It would also be lower in a country which operates a technology with a smaller elasticity of unskilled labour, provided it is not too small, and with a relatively symmetric distribution of elasticity of physical and human capital. A country with an appropriate degree of redistribution would have a higher level of TFP.

In addition, in our paper, TFP could be decomposed into two parts, technology and efficiency. Through analysing across-country income differences, we can estimate the fraction of TFP differences due to technology differences or efficiency differences. It then gives policy makers an explicit implication that focusing on improving technology or increasing efficiency is more important.

The numerical results highlight that TFP differences play a significant role in explaining the cross-country differences in output per capita across U.S., Japan and Australia. More than 50% of income differences of income per capita between U.S. and Japan is explained by TFP differences. This finding supports Prescott's (2002) estimation that U.S. TFP is much higher than Japan. More than 100% of the per capita income differences explained by TFP differences implies that the most efficient way for Australia to catch U.S. is to improve TFP.

Young (1995) and Hsieh (2002), using empirical models, analyze the role of TFP across East Asian countries and find that rate of output growth was due primarily to factor accumulation. Future research may apply our general theory to calibrate countries other than the U.S., Japan and Australia to examine if the empirical estimation is consistent with our theory prediction.

Appendix

Appendix A: PROOFS OF LEMMAS AND PROPOSITIONS

PROOFS OF LEMMAS 1 AND 2:

By (5), we rewrite (8) as follows:

$$\ln U\left(h_t^i, k_t^i, M_t; T\right) = \max_{s_{1t}^i, s_{2t}^i, l_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln\left(1 - s_{1t}^i - s_{2t}^i\right) + \ln \hat{y}_t^i - \left(l_t^i\right)^\eta\right] \\ +\rho E_t\left[\ln U\left(h_{t+1}^i, k_{t+1}^i, M_{t+1}; T\right)\right] \end{array} \right\}. \tag{A.1}$$

Agent solves (A.1) subject to (2), (3) and

$$h_{t+1}^i = \kappa\left(s_{1t}^i\right)^\beta \xi_{t+1}^i \left(A_t\right)^{\beta(1-\tau)} \left(k_t^i\right)^{\beta\lambda(1-\tau)} \left(h_t^i\right)^{\alpha+\beta\mu(1-\tau)} \left(l_t^i\right)^{\beta(1-\lambda-\mu)(1-\tau)} \left(\tilde{y}_t\right)^{\beta\tau}, \text{ and} \tag{A.2}$$

$$k_{t+1}^i = s_{2t}^i \left(A_t\right)^{(1-\tau)} \left(k_t^i\right)^{\lambda(1-\tau)} \left(h_t^i\right)^{\mu(1-\tau)} \left(l_t^i\right)^{(1-\lambda-\mu)(1-\tau)} \left(\tilde{y}_t\right)^{\tau}. \tag{A.3}$$

We guess the value function as: $\ln U\left(h_t^i, k_t^i, M_t; T\right) = Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t$. Then by substituting this value function, (A.2) and (A.3) into (A.1), we get

$$
\begin{aligned}
Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = \; & (1-\rho)\begin{pmatrix} \ln(1-s_{1t}^i - s_{2t}^i) + (1-\tau)\ln A_t \\ +(1-\lambda-\mu)(1-\tau)\ln l_t^i + \tau \ln \tilde{y}_t - (l_t^i)^\eta \end{pmatrix} \\
& + (1-\rho+\rho\beta Z_1 + \rho Z_2)\,\lambda\,(1-\tau)\ln k_t^i \\
& + ((1-\rho+\rho\beta Z_1 + \rho Z_2)\,\mu\,(1-\tau) + \rho\alpha Z_1)\ln h_t^i \\
& +\rho\left(\begin{array}{l} Z_1 \begin{pmatrix} \ln\kappa + \beta\ln s_{1t}^i + \varphi + \beta(1-\tau)\ln A_t \\ +\beta(1-\lambda-\mu)(1-\tau)\ln l_t^i + \beta\tau\ln\tilde{y}_t \end{pmatrix} \\ +Z_2 \begin{pmatrix} \ln s_{2t}^i + (1-\tau)\ln A_t + \tau\ln\tilde{y}_t \\ +(1-\lambda-\mu)(1-\tau)\ln l_t^i \end{pmatrix} + B_{t+1} \end{array}\right).
\end{aligned} \tag{A.4}
$$

Taking partial differentials with respect to $\ln k_t^i$ and $\ln h_t^i$ yield

$$Z_1 = (1-\rho+\rho\beta Z_1 + \rho Z_2)\,\mu\,(1-\tau) + \rho\alpha Z_1, \tag{A.5}$$

$$Z_2 = (1-\rho+\rho\beta Z_1 + \rho Z_2)\,\lambda\,(1-\tau). \tag{A.6}$$

Rearranging (A.5) and (A.6), we verify the guess and confirm the existence of (A.4) and get

$$Z_1 = \frac{(1-\rho)\mu(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)}, \tag{A.7}$$

$$Z_2 = \frac{(1-\rho\alpha)(1-\rho)\lambda(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)}. \tag{A.8}$$

The values of human capital and physical capital as expressed by their utility elasticities are respectively given by Z_1 and Z_2. Note the tax rate τ can alter these values individually but does not alter the relative value of human to physical capital, $\frac{\mu}{\lambda(1-\rho\alpha)}$, which increases with output elasticity of human capital μ, neighborhood effect α and patience ρ but remains unaffected by the quality of education β.

The first-order conditions of (A.1) with respect to the saving rates and labour supply are

$$\frac{1-\rho}{1-s_{1t}^i - s_{2t}^i} = \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_{1t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial s_{1t}^i}\right), \tag{A.9}$$

$$\frac{1-\rho}{1-s_{1t}^i - s_{2t}^i} = \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_{2t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial s_{2t}^i}\right), \tag{A.10}$$

$$(1-\rho)\,\eta\left(l_t^i\right)^{\eta-1} = (1-\rho)(1-\lambda-\mu)(1-\tau)/l_t^i + \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial l_t^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial l_t^i}\right), \tag{A.11}$$

where $\partial \ln k_{t+1}^i/\partial s_{1t}^i = 0$, $\partial \ln k_{t+1}^i/\partial s_{2t}^i = 1/s_{2t}^i$, $\partial \ln h_{t+1}^i/\partial s_{1t}^i = \beta/s_{1t}^i$, $\partial \ln h_{t+1}^i/\partial s_{2t}^i = 0$, $\partial \ln k_{t+1}^i/\partial l_t^i = (1-\lambda-\mu)(1-\tau)/l_t^i$ and $\partial \ln h_{t+1}^i/\partial l_t^i = \beta(1-\lambda-\mu)(1-\tau)/l_t^i$.

The above optimization problem (A.4) is strictly concave. Consequently, (A.9)—(A.11) are sufficient for the optimization exercise and the Lemmas 1 and 2 follow immediately after we substitute (A.7) and (A.8) into (A.9)—(A.11). □

PROOF OF LEMMA 3: By assumption, at the initial date $t=0$, physical and human capitals are lognormally distributed. By (16) and (17), it follows that k_t^i and h_t^i remain lognormally distributed over time and hence by (2) y_t^i is lognormal and is given by,

$$\ln y_t^i = \ln A_t + \lambda \ln k_t^i + \mu \ln h_t^i + (1-\lambda-\mu)\ln l. \tag{A.12}$$

The mean of the lognormal distribution of y_t^i is given by,

$$\int_0^1 \ln y_t^i di = \ln A_t + \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l. \tag{A.13}$$

The variance of $\ln y_t^i$ is the sum of variances of $\ln k_t^i$, $\ln h_t^i$ plus the covariance of these two variables

$$\text{var}\left[\ln y_t^i\right] = \lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t. \tag{A.14}$$

The income per capita y_t is[19]

$$y_t = \int_0^1 y_t^i di = \exp\left(\int_0^1 \ln y_t^i di + \frac{1}{2}\text{var}\left[\ln y_t^i\right]\right). \tag{A.15}$$

The median income is

$$y_{t,median} = \exp\left(\int_0^1 \ln y_t^i di\right). \tag{A.16}$$

Therefore, inequality index is

$$\Lambda_t \equiv \log\left(\frac{y_t}{y_{t,median}}\right) = \frac{1}{2}\text{var}\left[\ln y_t^i\right] = \left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2. \tag{A.17}$$

[19] Please see Shimizu (1988) for detailed description about properties of moment generating function on lognormal distribution.

To derive the expression for the break-even point defined by (4), we note the mean of y_t^i in logarithm, by (A.15), satisfies

$$\ln y_t = \ln A_t + \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l + \Lambda_t, \tag{A.18}$$

and the mean of $\left(y_t^i\right)^{1-\tau}$ in logarithm is

$$\ln \int_0^1 \left(y_t^i\right)^{1-\tau} di = (1-\tau)\left(\ln A_t + \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l\right) + (1-\tau)^2 \Lambda_t. \tag{A.19}$$

Taking the difference between before and after tax income yields

$$\ln y_t - \ln \int_0^1 \left(y_t^i\right)^{1-\tau} di = \tau \ln A_t + \lambda\tau m_{kt} + \mu\tau m_{ht} + (1-\lambda-\mu)\tau\ln l + \tau(2-\tau)\Lambda_t. \tag{A.20}$$

It means that $\tau \ln \tilde{y}_t = \tau \ln A_t + \lambda\tau m_{kt} + \mu\tau m_{ht} + (1-\lambda-\mu)\tau\ln l + \tau(2-\tau)\Lambda_t$, then we can get (25). □

PROOF OF LEMMA 4: By (16) and (17), we know that physical and human capital distribute lognormally. Then from the property of the moment generating function for lognormal distribution, we get (**??**) and (**??**). By the definition of A_t, we get

$$A_t = \bar{A}\exp\left(\delta\left(m_{ht} + \frac{1}{2}\omega\Delta_{ht}^2\right)\right). \tag{A.23}$$

Substituting (**??**) and (**??**) into (A.18) yields (**??**). Then, by (26), (**??**) and (A.23), we can get (27). □

PROOF OF PROPOSITION 1: Writing the system of linear equations (20), (22) and (23) in a matrix form, we get

$$M_{t+1} = A_0 + A_1 * M_t, \tag{A.25}$$

where

$$M_{t+1} \equiv \begin{bmatrix} \Delta_{kt+1}^2 \\ \Delta_{ht+1}^2 \\ cov_{t+1} \end{bmatrix},\ A_0 \equiv \begin{bmatrix} 0 \\ \sigma^2 \\ 0 \end{bmatrix},$$

$$A_1 \equiv \begin{bmatrix} (1-\tau)^2 \lambda^2 & (1-\tau)^2 \mu^2 & 2\lambda\mu (1-\tau)^2 \\ (1-\tau)^2 \beta^2\lambda^2 & (\alpha+\beta\mu(1-\tau))^2 & 2\beta\lambda(1-\tau)(\alpha+\beta\mu(1-\tau)) \\ (1-\tau)^2 \beta\lambda^2 & \mu(1-\tau)(\alpha+\beta\mu(1-\tau)) & \lambda(1-\tau)(\alpha+2\beta\mu(1-\tau)) \end{bmatrix}.$$

The sequence M_t converges to a steady state if all eigenvalues of A_1, denoted by $S(A_1) \equiv (E_j)$, $j = 1, 2, 3$, are less than one[20]. By setting $\tau = 0$ to avoid unnecessary details, we solve $\det |A_1 - S(A_1) I| = 0$, where I is identity matrix, to get

$$E_1 \equiv \lambda\alpha < 1, \tag{A.26}$$

Eigenvalue E_1 is less than one since $\alpha, \lambda \in (0,1)$. Note that when $\tau = 0$, $|A_1| = \lambda^3\alpha^3$. It implies

$$E_2 * E_3 = \lambda^2\alpha^2, \tag{A.27}$$

since A_1 is symmetric. The symmetry of A_1 implies also that the $trace\{A_1\} = \sum_{j=1,\dots 3} E_j$. Or, equivalently,

$$E_2 + E_3 = \lambda(\lambda + 2\beta\mu) + (\alpha + \beta\mu)^2 . \tag{A.28}$$

By (A.27) and (A.28) and the assumption $(1-\alpha)(1-\lambda) - \beta(\mu+\delta) > 0 \Rightarrow (1-\alpha)(1-\lambda) - \beta\mu > 0$, it follows that both $E_2 < 1$ and $E_3 < 1$. Thus, $S(A_1) < 1$. Consequently, M_t converges to a unique steady state which we denote as M. By (A.25), M satisfies the following fixed point problem

$$M = A_0 + A_1 * M, \tag{A.29}$$

and has a unique solution, since $I - A_1$ is nonsingular. It follows, therefore, a unique steady state exists and the equilibrium sequence of M_t converges to it. Moreover, $0 < S(A_1) < 1$ implies that $\{M_t\}$ constitutes a monotone sequence[21].

Similarly, substituting (A.23) into equations (19) and (21) and then writing in a matrix form, we get

$$N_{t+1} = A_0 + A_1 * N_t, \tag{A.30}$$

[20] For detailed discussion about this property, please see Reich (1949), Lorenz (1993) and Young (2003).

[21] For details, see page 255, Lorenz (1993).

where[22]

$$N_{t+1} \equiv \begin{bmatrix} m_{kt+1} \\ m_{ht+1} \end{bmatrix},$$

$$A_0 \equiv \begin{bmatrix} \ln \bar{A} + \ln s_2 + \delta\omega\Delta_h^2/2 + (1-\lambda-\mu)\ln l + \tau(2-\tau)\Lambda, \\ \ln\kappa + \beta\ln\bar{A} + \varphi + \beta\ln s_1 + \beta(1-\lambda-\mu)\ln l + \beta\tau(2-\tau)\Lambda + \beta\delta\omega\Delta_h^2/2. \end{bmatrix}$$

$$A_1 \equiv \begin{bmatrix} \lambda & \delta+\mu \\ \beta\lambda & \alpha+\beta\mu+\beta\delta \end{bmatrix}.$$

The sequence N_t converges to a steady state if all eigenvalues of A_1, denoted by $S(A_1) \equiv (E_j)$, $j = 1, 2$, are less than one. We solve $\det |A_1 - S(A_1) I| = 0$, where I is identity matrix, to get

$$\begin{aligned} E_1 &\equiv \frac{1}{2}(\alpha+\lambda+\beta(\mu+\delta)) \\ &+ \frac{1}{2}\sqrt{(\alpha+\beta\mu)^2 + 2\alpha\beta\delta + 2\beta\lambda\delta + 2\beta^2\mu\delta + \beta^2\delta^2 + 2\beta\lambda\mu - 2\alpha\lambda + \lambda^2}, \\ E_2 &\equiv \frac{1}{2}(\alpha+\lambda+\beta(\mu+\delta)) \\ &- \frac{1}{2}\sqrt{(\alpha+\beta\mu)^2 + 2\alpha\beta\delta + 2\beta\lambda\delta + 2\beta^2\mu\delta + \beta^2\delta^2 + 2\beta\lambda\mu - 2\alpha\lambda + \lambda^2}. \end{aligned} \tag{A.31}$$

Note that $|A_1| = \alpha\lambda$. It implies that

$$E_1 * E_2 = \alpha\lambda, \tag{A.32}$$

since A_1 is symmetric. The symmetry of A_1 implies also that the $trace\{A_1\} = E_1 + E_2$. Or, equivalently,

$$E_1 + E_2 = \alpha + \lambda + \beta(\mu+\delta). \tag{A.33}$$

By (A.32) and (A.33) and the assumption $(1-\alpha)(1-\lambda) - \beta(\mu+\delta) > 0$ and $\alpha, \beta, \lambda, \mu \in (0,1)$, it follows that both $E_1 < 1$ and $E_2 < 1$. Then a unique steady state exists and the equilibrium sequence of N_t converges to it. The Proposition 1 is proved. $\square$

PROOF OF PROPOSITION 2: From Proposition 1, we know that $\Delta_{kt+1}^2 = \Delta_{kt}^2 = \Delta_k^2$, $\Delta_{ht+1}^2 = \Delta_{ht}^2 = \Delta_h^2$, $cov_{t+1} = cov_t = cov$, $m_{kt+1} = m_{kt} = m_k$ and $m_{ht+1} = m_{ht} = m_h$. By (20), (22) and (23), we can get (**??**), (38) and (**??**). And by (19) and

[22] Note that for simplicity, Δ_{kt}^2, Δ_{ht}^2 and cov_t are regarded as constant here since they converge to steady state in the long run.

(21), we can get (35) and (**??**). If there is no externality, i.e., $\delta = 0$, then TFP measures aggregate efficiency, denoted as E. It is given as follows

$$E = \exp\left(\left(\left(\lambda - 1\right)\lambda\Delta_k^2 + \left(\mu - 1\right)\mu\Delta_h^2 + 2\lambda\mu cov\right)/2\right). \tag{A.35}$$

Then the rest of TFP in (**??**) is technology. $\square$

Appendix B: Calibration of Parameter Values for Japan and Australia

Japan

1). The parameters we calibrate to match targeted values of variables are: the 'neighborhood externality' parameter α, the quality of education system β, the mean and variance of logarithm innate ability φ and σ^2.

Accumulation

The intergenerational persistence of income between parents and children is defined in (18). Lefranc, Ojima and Yoshida (2008) gives internenerational earnings elasticity of son's annual earnings to father's income, 0.25.[23] I set $\alpha = 0.14$, and $\beta = 0.45$, which allows $p(\tau)$ to range from 0.14 to 0.64 which is consistent with Lefranc, Ojima and Yoshida (2008).

Inequality

In Benabou (2002), he measures the family income inequality by using the logarithm of the ratio of mean to median income. In Japan, according to the data from Statistics Japan[24], the logarithm of the ratio of mean to median income increases from 0.24 to 0.25 from 2003 to 2008. By (28), I set $\sigma^2 = 1.5$, so that the feasible range is [0.21, 0.33]. The mean of log-innate ability φ is set to -0.75.

2). The parameters which are picked from literature are: the output elasticity of physical and human capital λ and μ, agent's attitude towards patience ρ, and in-

[23] Please see Lefranc, Ojima and Yoshida (2008), Table 1.
[24] Table 3 at http://www.stat.go.jp/english/data/sousetai/1.htm.

tertemporal elasticity of labor supply $\epsilon = \frac{1}{\eta - 1}$ which identifies the preference parameter η.

Production Parameters

Gollin (2002) gives the labour share of national income in Japan 0.73.[25] I use the measure which is introduced by Mankiw, Romer and Weil (1992) (MRW) to specify shares of human capital μ and raw labour $\varepsilon \equiv 1 - \lambda - \mu$. In MRW (1992), they regard the labour who earn minimum wage as having no human capital. The share of human capital is calculated by multiplying one minus the ratio of minimum wage to average wage by the labour share, i.e., $\mu = (1 - ratio) * labour\ share$. The ratio of minimum wage to average wage, according to OECD in 1996, is 0.29. Therefore, I get $\lambda = 0.27$, $\mu = 0.52$ and $\varepsilon = 0.21$.

Labor Supply and Discount Factor

Kuroda and Yamamoto (2007) estimate the Frisch labour supply elasticity in Japan is around $0.1 \sim 0.2$. Bessho and Hayashi (2006) estimate Marshallian elasticity find it around 0.178 with a linear labor supply function. We choose 0.2. So, it equivalently means that $\eta = 6$. Following Benabou (2002), we set $\rho = 0.4$.

3). The parameters which are chosen **arbitrarily** are: the degree of externality δ and the degree of efficiency unit of human capital ω, human capital index κ and the constant $\bar{A}$ in technology function.

The parameter values of δ, ω, μ and κ are kept the same as U.S. due to the data limitation. The value of $\bar{A}$ is calibrated to match the ratio of per capita income of Japan to the U.S. Since the estimated per capita income ratio is close to Jerzmanowski (2007) Table 7, $y_{Japan}/y_{U.S.} = 0.747$, we set $\bar{A} = 1$ for Japan.

Australia

1). The parameters we calibrate to match targeted values of variables are: the 'neighborhood externality' parameter α, the quality of education system β, the mean and variance of logarithm innate ability φ and σ^2.

Accumulation

[25] Please see Gollion (2002), Table 2, Adjustment 3.

The intergenerational persistence of income between parents and children is defined in (18). In Leigh (2007), he found that the correlation between the log earnings of fathers and sons in Australia is around 0.14 to 0.19. I set $\alpha = 0.1$, and $\beta = 0.3$, which allows $p(\tau)$ to range from 0.10 to 0.50.

Inequality

In Australia, according to the data from Australian Bureau of Statistics, the logarithm of the ratio of mean to median income decreases from 0.21 to 0.13 from 1995 to 2006. By (28), I set $\sigma^2 = 4.5$, so that the feasible range is [0.13, 0.17]. The mean of log-innate ability φ is set to -2.25.

2). The parameters which are picked from literature are: the output elasticity of physical and human capital λ and μ, agent's attitude towards patience ρ, and intertemporal elasticity of labor supply $\epsilon = \frac{1}{\eta - 1}$ which identifies the preference parameter η.

Production Parameters

Gollin (2002) estimates that the share of labour in the production in Australia is 0.68. Minimum and average wage rates are available in OECD. The ratio is 0.65. Therefore, following MRW (1992), the share of human capital equals 0.68 multiplied by one minus the ratio of minimum wage to average wage, $\mu = 0.68 * (1 - 0.65) = 0.24$ and $\lambda = 1 - 0.68 = 0.32$, $\varepsilon = 0.44$.

Labor Supply and Discount Factor

For Australia, Norris (1996) reports that wage elasticities for women range from 0.40 to 0.50. Dandie and Mercante (2007) did a comprehensive literature survey and summarized the published labour supply elasticity estimates for Australia. They found the range would be from 0 to 0.52 if estimating married men, married women, single men, single women and lone parents. In the thesis, I set $\eta = 4.3$ so that the average labour supply elasticity of Australia is 0.3. Following Benabou (2002), we set $\rho = 0.4$.

3). The parameters which are chosen **arbitrarily** are: the degree of externality δ and the degree of efficiency unit of human capital ω, human capital index κ and the constant $\bar{A}$ in technology function.

Due to the data availability, the parameter values of δ, ω, μ and κ are kept the same as the U.S. economy. From Jerzmanowski (2007) Table 7, we find $y_{Australia}/y_{U.S.} = 0.771$. Consequently, we set $\bar{A} = 0.808$ for Australia to match this ratio.

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