PUBLIC DEBT WITH VALUED FIAT CURRENCY IN A MODEL OF ECONOMIC GROWTH

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ABSTRACT
The paper incorporates Tobin's portfolio balance theory into an overlapping generations model of growth with endogenously valued money in which fiscal policy and/or monetary policy can change the steady state level of the capital stock. The optimal inflation rate that maximises the steady state capital stock is a function of the nominal interest rate and the income tax rate. For example when the nominal interest rate equals 6%, government balances its budget and sets the average income tax rate to be 20%, then the optimal inflation rate for the model economy is about 3.39%. The model can be used to demonstrate how open market intervention could hinder economic growth when the targeted inflation rate is not equal to the optimal inflation rate. In the model money is neutral but not super neutral. In contrast with most models that explain real effects of inflation, anticipated changes in the inflation rate have a real effect in this model. This occurs because money and nominal government debt enters the economy not as a helicopter drop but as seignorage and the Ricardian Equivalence does not hold.

Key Words: Inflation-Tax, Overlapping Generations, Asset Demand for Money, Capital Accumulation, Public Debt

JEL Codes: E21, E41, E62, O42

[1] I acknowledge research assistance from Gaylene Hunter who wrote her master's thesis on a related topic under my supervision.

1. Introduction

Although the idea that tight monetary policy could be harmful to growth has been around for some time, it is generally not brought up in polite conversation. Conventional wisdom typically relies on empirical studies on cross-country data that report (see Chari, 1995) a negative relationship between inflation and growth of per capita output of country. A casual inspection of time series of developing economies such as China, India and the East Asian Countries would suggest a different scenario. Growth in those countries thrives with moderate inflation. Developing a model to provide a theoretical framework for an empirical observation is a respected method of analysis. However, doing so involves a certain amount of faith that the relationship that you observe in the data is irrefutable. A model that could rationalise both observations should enable us to identify conditions when inflation helps the economy to grow and conditions when inflation is harmful for growth. In particular it should provide an algorithm to determine the optimal inflation rate conditional on relevant economic factors, and tell us if and how exactly economic growth could be hampered if for whatever reasons an economy does not achieve that rate. This paper provides a model that does just that.

We combine Diamond (1965) and Wallace (1980) to develop an over-lapping generations model of economic growth with valued money. The model is empirically tractable. It provides a numerical algorithm to determine the value of government spending that is fiscally responsible (i.e., zero deficit) and to forecast economic growth as a function of the inflation rate, given specified rates of income tax and a nominal interest. The model accomplishes its goal essentially by characterising a micro-foundation of money demand that is sensitive to changes in the country's fiscal policies. Conventional models for analysing effects of monetary policies typically ignore specification of a rational money demand function. This model fills that gap to demonstrate that an optimal monetary policy is not independent of the fiscal policy arrangement of a country. It argues that monetary policies such as interest targeting and various activities for implementing that target without any respect for the changes in the fiscal policy conditions may be harmful for growth.

Money Demand

One common way of explaining the existence of money is to start with an equation of exchange, say MV = PY. Then the real balances in the economy, M/P are equal to Y/V. Thus the quantity theory asserts that the demand for money, is equal to the real balances in the economy, which in turn is proportional to the real amount of goods in the economy. This is effectively the same thing as the 'cash in advance' condition i.e. something along the lines of $M/P \geq \gamma Y$, where we say that agents in the economy are subject to a transaction constraint. We can think of M/P as being the 'value of money', where the price of money is the inverse of the general price level P, and M is the total stock of fiat currency. A disadvantage to using a cash in advance constraint is that this method generally requires the modeller to impose the value of money in the model by exogenous choice of γ. Or, if you had a money growth rule, the choice of γ determines the price level. A model with overlapping generations does not require such an explanation for holding money. In such a model, fiat currency can be used as a means of transferring wealth from one time period to the next. In this sense, money in the overlapping generations model gains an inherent value by eliminating the problem of a missing market across generations. This model, however, allows for both types of demand for money: a transaction demand due to the presence of cash goods that must be bought by cash only and a speculative demand or asset demand due to the portfolio choice option between cash and capital as discussed by Tobin (1965). The model also considers extending the cash-in-advance constrained transaction demand to a more general case that Baumol (1952) considered where the transaction demand depends on the nominal interest rate.

Money Supply: Seignorage versus Helicopter Drop

The model explicitly derives a money supply function that depends on the fiscal policy. If the long run level of government spending exceeds the long run income tax revenue, then the government would need to increase the growth rate of the money supply to make up for the shortfall. Therefore the supply of money is a function of the fiscal balance. Money enters the economy as seignorage and not by as lump-sum transfer or as the so-called helicopter-drop. The economic history motivates the former channel despite the academic tradition (see, e.g. Champ and Freeman (1990) that conveniently relies on the latter channel. This distinction turns out to make an important contribution in the policy debate. The model argues that if money enters the economy through seignorage, then even an anticipated change in the inflation rate would have real effects. Suppose that the goal of the central bank is to indirectly act to raise capital stock by choosing a particular inflation target, it is then also necessary that the inflation target is consistent with fiscal policy. That is, consistent in the sense that fiscal policy doesn't act to raise money demand (by increasing the fiscal deficit) at a time when monetary policy is attempting to lower it (by reducing inflation). The real effect arises when a higher inflation rate decreases demand for money by making money less attractive as an asset. Following an argument similar to Tobin's portfolio balance theory the model demonstrates how inflation can discourage people from holding (hoarding) cash, and encourage holding capital. Furthermore, inflation can have a positive effect on growth by acting as a replacement to government revenue for income tax.[2]

The Phillips Curve Debate

The Phillips curve debate has dominated the New Zealand media and has also received a place in the latest RBNZ model for forecasting. Despite what monetarists claimed regarding the neutrality of money, a positive relationship between inflation and growth is generally accepted to exist in the short run. However, in absence of an inertia in updating expectation and more directly in presence of the assumption of rational expectation even a downward sloping Phillips curve would preclude a inflation output trade-off that a government can exploit. In those models, only unanticipated shocks to the money base are capable of producing real effects. (see, Lucas,1972; Freeman and Champ,1990). This paper envisages a perfectly flexible neoclassical model without any expectation inertia in which it is possible to have a positive exploitable trade-off between inflation and output. It is also possible in such a model for an anticipated change in the inflation target to have real effects. The

[2] Note that the income tax take is measured in real goods units. As seignorage increases inflation and the price level, it also increases the nominal value of the income tax. There is no effect on the real value of the income tax.

model derives an upper bound for steady state inflation, beyond which money loses its value as an asset. Inflation above this rate is harmful for growth. The upper bound on inflation depends on the fundamentals of the economy, including the depreciation, savings, and population growth rates, and the proportion of income that goes to the owners of capital. However, the upper bound is also related to the income tax rate.

Section 2 describes the model, Section 3 characterises the optimal inflation rate and Section 4 includes a few numerical results to illustrate the idea of optimal inflation rate. There are two other sections to be included in the paper later. They are regarding the idea of equilibrium inflation rate given a specific fiscal policy set-up and what happens when monetary authority targets a inflation interest rate pair that is not optimal. Given the value of government spending per capita and the income tax rate, the model can determine an equilibrium inflation rate as a function of the nominal interest rate. Some discussion of an equilibrium inflation rate is given in Note 2 in the Appendix.

2. The Model

The Environment

We proceed along the same lines as Freeman and Champ (1990).[3]The model is an adapted version of Diamond's (1965) overlapping generations model, with money included in the model following McCandless and Wallace (1980). In each period $t \geq 1$, N_t agents are born. The population grows at the gross rate n. An agent born at t maximises the present value of their utility, which depends on their consumption when young, c_{1t} and consumption when old, c_{2t+1}. For simplicity utility is represented as logarithmic, (1). The function $U(\cdot)$ is strictly concave and strictly increasing.

(1) $U = lnc_{1t} + \beta lnc_{2t+1}$,

Each young agent is endowed with one unit of labour which is supplied to the labour market. The initial old consume out of an endowment of fiat currency. Everything (including money) is measured in goods units. The young get a wage which they can consume, keep as capital, use to purchase private or government bonds, or use to purchase money. A Cobb- Douglas production function is used, and is assumed to have constant returns to scale. Therefore, per capita output, y_t is Ak_t^{α}.

In each time period t, government creates money at the rate μ_t. The price level grows at a rate π_t. Later we consider the set of equilibria such that the real money balance per capita remains constant, i.e. $M_t/L_tP_t = M_{t+1}/P_{t+1}L_{t+1} = m$. We call these equilibria steady states.

(2) $M_t = (1 + \mu_t) M_{t-1}$

Money enters the economy only when the government uses it to partly finance its expenditure. Per capital real money balances are denoted m_t. The rest of government

[3] Differences include: the use of seignorage to make government purchases; income tax instead of lump sum tax; tax on return to capital; and a cash in advance constraint that does not necessarily bind with equality.

expenditure is financed either by income tax, or bonds, B_t^g. All government bonds have one period maturity and earn the nominal interest rate R_t. There are no transfers in this model, and government expenditure is assumed to have no direct effect on utility or production. There is a flat income tax rate, τ. The aggregate government budget constraint is:

$$(3) \qquad G_t + \frac{R_t B_{t-1}^g}{P_t} = \tau_t Y_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t^g - B_{t-1}^g}{P_t}$$

Dividing both sides by the population we get the government's budget in per capita terms as follows[4]:

$$(4) \qquad g_t + \frac{R_t b_{t-1}^g}{P_t} = \tau_t A k_t^\alpha + \left(\frac{\mu_t}{1+\mu_t}\right) m_t + \frac{b_t^g - b_{t-1}^g}{P_t},$$

where the real value of money $m_t = M_t/L_tP_t$ and the growth rate of m_t is given by the difference between the growth rate of money μ_t and $(\pi_t + n)$.

Equilibrium Conditions

Agents are assumed to have rational expectations, and the market is perfectly competitive. Therefore, both capital and labour earn their marginal product. The rate of return on capital net of tax and depreciation is given in equations (5), and (6) is the wage rate.

$$(5) \qquad r_t^K = \alpha A k_t^{\alpha-1} - \delta$$

where δ is the rate of depreciation, and takes a value between zero and one; and

$$(6) \qquad w_t = (1-\alpha) A k_t^\alpha$$

The optimisation problem of an individual born at t can now be expressed as a choice of personal holdings of capital, (k_t), government bonds ($b^g{}_t$), private bonds, (l_t), and real balance, (m_t) to maximise utility (1) subject to the following budget constraints:

$$(7.1) \qquad (1-\gamma)c_{1t} = (1-\tau_t)w_t - k_{t+1} - \frac{l_t}{P_t} - \frac{b_{t+1}^g}{P_t} - m_t, \; 0<\gamma<1.$$

(7.2a(i)) $\quad m_t \geq \gamma c_{1t}$ (Cash in advance constraint: CIA)

(7.2a(ii.)) (or, $m_t \geq \hat{\gamma} A k_t^\alpha$ where $\hat{\gamma}$ becomes a function of τ.)

Note that $\quad m_t - \gamma c_{1t} = m_t^A$

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[4] Since $\dfrac{M_t - M_{t-1}}{P_t} = \dfrac{M_t - M_{t-1}}{M_{t-1}} * \dfrac{M_{t-1}}{M_t} * \dfrac{M_t}{P_t} = \dfrac{\mu_t}{1+\mu_t} * \dfrac{M_t}{P_t}$ and $m_t = M_t/P_tL_t$

where m_t^A is the real value of cash balances held due to asset demand.

(7.2b) $$m_t = \sqrt{\frac{c_{1t}(\phi / P_t)}{2(1+\tilde{R}_t)}}\text{, or } Ln\, m_t = \gamma_0 + \gamma_1 Lnc_{1t} - \gamma_2 \tilde{R}_t$$

Note that combining (7.1) and (7.2a) gives

(7.3) $$c_{1t} = (1-\tau_t)w_t - k_{t+1} - \frac{l_t}{P_t} - \frac{b_{t+1}^g}{P_t} - m_t^A$$

We allow for two alternative methods of modelling transaction demand for money. The first is a simple cash in advance constraint (CIA), (7.2a). The government requires the young to carry out a certain fraction of their purchases of the consumption good in fiat currency. The cash in advance constraint is not applicable to the old, since they are assumed to consume goods at least to the value of, and probably more than, their cash balances. If the CIA constraint binds with equality, it can be inferred that the nominal interest rate is greater than zero. If it does not bind with equality, then the nominal interest rate must be zero. Whenever the constraint binds with equality, the nominal interest rate is indeterminate.

The second modelling technique is the Baumol's inventory model of money demand (7.2b). Here, the real money balance is a function of the real cost per transaction, ϕ/P_t, and the nominal interest rate. Unlike the CIA constraint, the nominal interest rate can be determined using Baumol money demand. However, since this means that the nominal interest rate is a function of ϕ, instead of estimating ϕ by fitting it to some interest rate data, I have simply taken R from the data.

The budget constraint for the old allows consumption of all forms of saving. However, the old must pay tax on their return from capital and bonds. If the price level has increased from t to $t+1$ then they will additionally pay an inflation tax. Note that if the old have any money then they must have kept it for its value as an asset. The old are not subject to the cash in advance constraint.

(8) $$c_{2t+1} = [1+(1-\tau_t)r_{t+1}^K]k_{t+1} + \frac{[1+(1-\tau_t)R_{t+1}]l_t}{P_{t+1}} + \frac{P_t m_t^A}{P_{t+1}} + \frac{[1+(1-\tau_t)R_{t+1}]b_{t+1}^g}{P_{t+1}}$$

Combining (7.3) and (8) we get

(9)
$$c_{1t} + \frac{(1+\pi_t)c_{2t+1}}{(1+\tilde{R}_{t+1})} = (1-\tau_t)w_t + \left(\frac{(1+\pi_t)(1+\tilde{r}_{t+1}^K)}{1+\tilde{R}_{t+1}} - 1\right)k_{t+1} + P_t m_t^A\left(\frac{1}{P_t(1+\tilde{R}_{t+1})} - \frac{1}{P_t}\right)$$

where $\tilde{r}_{t+1}^K$ is after tax return to capital at time $t+1$, $(1-\tau)r_{t+1}^K$, and $\tilde{R}_{t+1}$ is the after-tax nominal interest rate at time $t+1$, $(1-\tau)R_{t+1}$.

To ensure that both capital and bonds are held, we equate their returns. By doing so we derive the Fisher equation for the model, (11). We will observe situations where this equation does not bind, due to inflation or interest rate targeting. In these situations there will be some portfolio adjustment of holdings of capital and bonds in order to return to steady state.

The intertemporal budget constraint also implies that R_t must be greater than or equal to zero, since negative R_t would mean that people would switch from bonds to money, and try to hold as much money as possible (infinite demand for money).

R_t equal to zero would equate the return on money and bonds. Zero nominal interest rates can occur with a combination of either deflation and positive return to capital, or inflation and negative return to capital. This case can create an asset demand for money, depending on what the return to capital is.

(10) $$(1+\pi_t)(1+\tilde{r}^K_{t+1}) = 1+\tilde{R}_{t+1}$$

Using the intertemporal budget constraint and the utility function, the (per capita) saving function is derived to be:[5]

(11) $$s_t = \left(\frac{\beta}{1+\beta}\right)(1-\tau_t)w_t$$

Let I_t denote aggregate investment in (purchases of) new capital goods at time t. Then the quantity of goods available for investment is equal to the amount of goods that the young did not consume, S_t, less the amount of goods used by the government, less the goods used to purchase cash that has an asset demand. Balances purchased from an asset demand are the balances that exist at t which can be purchased from the current old and balances obtained from the government in the current period in exchange for goods. Therefore the market clearing condition is:

(12) $$I_t = S_t - G_t - \frac{M_{t-1}}{P_t}$$

(Since we have a closed economy, aggregate private bond holding must be zero, i.e. $\Sigma l_t = 0$.)

Capital accumulates over time according to the following law of motion (Diamond, 1965; Solow, 1956):

(13) $$K_{t+1} = (1-\delta)K_t + I_t$$

(14) $$K_{t+1} = (1-\delta)K_t + S_t - G_t - \frac{M_{t-1}}{P_t}$$

This gives the capital accumulation path per capita as[6]:

[5] For the derivation see Note 1 in the Appendix

[6] At steady state $\frac{M_{t-1}}{P_t L_{t+1}} = \frac{M_t}{P_t L_t (1+\mu_t)} * \frac{L_t}{L_{t+1}} = \frac{m}{(1+\pi+n)(1+n)}$.

(15) $$k_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}(1-\tau_t)Ak_t^{\alpha} - \frac{m_t}{(1+\mu_t)(1+n)} + \frac{(1-\delta)k_t}{1+n} - \frac{g_t}{1+n}$$

The steady state version of (15) is

(16)
$$k = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}(1-\tau)Ak^{\alpha} - \frac{m}{(1+\pi+n)(1+n)} + \frac{(1-\delta)k}{1+n} - \frac{g}{1+n}$$

The model can be used to simulate the evolution of capital and the value of money[7] over time. **It can be shown that the equilibrium always converges to a unique steady state, where k_t = k, m_t = m, μ_t = μ, and π_t = π.** The equilibrium can be characterised by the government budget constraint (5), the Fisher equation, (either (11) or (12) depending on whether the nominal interest rate is zero or positive), and the capital accumulation path (18).

3. Characterisation of an optimal inflation rate

An optimal nominal interest rate - inflation target combination:
Given the existence of a maximum steady state capital stock and a positive nominal interest rate, there exists an associated inflation rate that will satisfy the Fisher equation. At this point, the returns on capital and government bonds are equal.[8] *If the government wishes to maximise output, it should not sell bonds.*

The objective is to find the inflation target that achieves the maximum possible steady state capital stock, k^S_{max}. Suppose that in addition to targeting the inflation rate, government also sets the income tax rate and can issue bonds. Then, in order to keep to a balanced budget, government spending is limited by the amount of revenue generated by income and inflation tax, and the issue of new bonds.

To determine k^S_{max} we analyse the steady state version of the capital accumulation path, which can be rearranged as in (19)[9]: We replace the bond terms by b, which will be constant in the steady state. Then we make use of the assumption that in the steady state m is constant, and is made up of a transaction component that satisfies the CIA constraint, $m^T = \hat{\gamma}Ak^{\alpha}$, and under certain circumstances may have an asset demand component, m^A.

(17) $$\frac{\beta(1-\alpha)}{(1+\beta)}(1-\tau)Ak^{\alpha} - (\delta+n)k = (\tau+\hat{\gamma})Ak^{\alpha} + m^A + b$$

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7 The value of money is determined by its price in goods units $p^m = 1/P_t$.

8 The representative agent is assumed to hold government bonds, but not private bonds.

9 $$g_t + \frac{m}{(1+\pi+n)} = \tau Ak^{\alpha} + \left(\frac{\pi+n}{1+\pi+n}\right)m + b + \frac{m}{(1+\pi_t+n)}$$

$$g_t + \frac{m}{(1+\pi+n)} = \tau Ak^{\alpha} + m + b$$

where $b = \frac{\Delta b_t^g}{P_t} - \frac{R_t b_{t-1}^g}{P_t}$ and is equal to the real fiscal deficit net of interest payments[10].

The left-hand side of (17) represents savings minus the minimum required investment to keep per capita capital stock at its existing level. The right hand side represents bonds and asset demand money plus the tax burden and requirements for transaction cash. If we subtract tax and transaction demand money from the left-hand side we get the steady state real balance of asset demand money per capita, plus the change in bond holdings and the interest payment on bonds. This is depicted in Fig.1, where [1] - [2] = $m^A + b$.

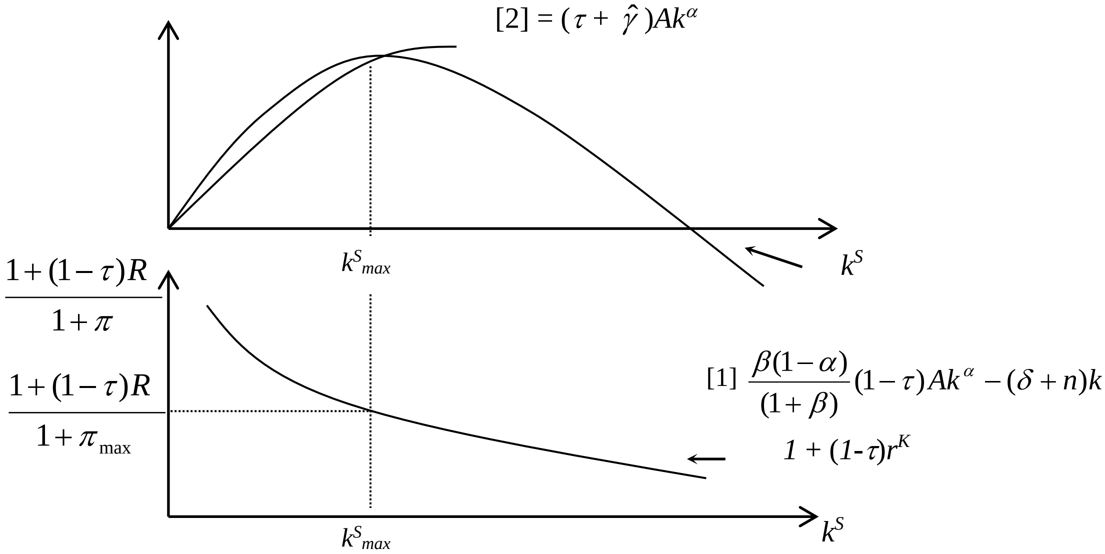


Fig.1 Government sets the rates of inflation and income tax

The maximum possible steady state capital stock is where $m^A + b$ is zero. That is, the maximum possible capital stock is obtained where aggregate saving is only in the form of capital. Agents have zero asset demand for money, and zero bonds for all t. To achieve this outcome we require the return on capital to be greater than the maximum return on either money or bonds. Therefore we require the following condition to hold:

(18) $$\frac{1+\tilde{\tilde{R}}}{1+\pi} \leq 1 + \tilde{r}^K$$

Points to note:

(a) This constraint (18) implies that the given the appropriate inflation rate, any nominal interest rate is consistent with the maximum steady state capital stock.
(b) Additionally, given a steady state constant level of bond holding, the inflation rate does not enter into (19). Consequently, the inflation target rate has no effect on the determination of the maximum possible steady state stock of

[10] See Note 3 in the Appendix

capital. The role of the inflation target in this setting is to determine whether the maximum possible steady state is achieved or not.

(c) The imposition of the CIA constraint lowers the steady state capital stock level.

(d) By reducing the tax rate, we can lower the total tax liability of the private sector [2], and increase the amount of private savings, thereby increasing the maximum possible steady state capital stock.

(e) k^S above k^S_{max} are not feasible since they do not yield enough saving in the economy to be able to pay the tax and purchase sufficient real balances to satisfy the CIA constraint. After deductions for capital depreciation, expansion of the capital stock for new population, and taxes, there is too little savings left to purchase sufficient real balances.

The maximum steady state capital stock, k^S_{max} is such that the following condition is satisfied:

$$(19) \qquad \frac{\beta(1-\alpha)}{(1+\beta)}(1-\tau)Ak^{\alpha} - (\delta+n)k - (\tau+\hat{\gamma})Ak^{\alpha} = 0$$

Solving for k^S_{max} gives:

$$(20) \qquad k^S_{\max} = \left[\frac{A[D(1-\tau)-\tau-\hat{\gamma}]}{\delta+n}\right]^{\frac{1}{1-\alpha}}$$

We can find the associated inflation rate π^S_{max} by substituting this k^S_{max} into the Fisher equation (12). That is, this inflation rate solves (23) given capital equal to k^S_{max}.

$$(21) \qquad \pi^S_{\max} = \frac{\tilde{R} - \tilde{r}^{SK}_{\max}}{1+\tilde{r}^{K}}$$

Note that this implies that π^S_{max} will be positive if $\tilde{R} > \tilde{r}^{SK}_{\max}$, and zero only when $\tilde{r}^{SK}_{\max}$ is equal to $\tilde{R}$.

From (20) an upper bound can be derived for τ, such that $k^S_{\max} > 0$.

$$(22) \qquad \tau < \frac{(1-\alpha)(\beta-\gamma)}{(1-\alpha)(\beta-\gamma)+(1+\beta)} = \tau_{\max}$$

Government has set the income tax rate, therefore we know that if the Fisher equation holds, then at the steady state inflation and the nominal interest rate have to be such that the ratio $[1+(1-\tau)R]/(1+\pi)$ is equal to $[1+(1-\tau)r^{KS}_{max}]$, by (11). However, we have no means of separately determining π and R.

However, *given* a nominal interest rate target, we can determine the inflation rate. Let $\theta(\tau)_{max}$ be equal to $[1+(1-\tau)r^{KS}_{max}]$. Then there is a direct correspondence between the targeted nominal interest rate and the target inflation rate.

$$(23) \qquad \frac{1+(1-\tau)R}{1+\pi} = 1+(1-\tau)r^{KS}_{\max} = \theta(\tau)_{\max} \Leftrightarrow \frac{1+(1-\tau)R^T}{\theta(\tau)_{\max}} = 1+\pi^T$$

To summarize, the choice of income tax rate will determine the maximum possible steady state level of capital. At this steady state the nominal interest rate is indeterminate. However, we are able to target inflation if we select a nominal interest rate target in accordance with (23).

We can re-write (23) as (24).

$$(24) \qquad R^T = r_{\max}^{KS} + \left(\frac{\theta(\tau)_{\max}}{1-\tau} \right) \pi^T$$

The most striking feature of (24) is that it shows that the relationship between the nominal interest target and the inflation target is dependent on the income tax rate. That is, the effectiveness of nominal interest rate targeting to attain the target inflation rate, depends on fiscal policy. If fiscal policy changes $\theta(\tau)_{max}$, then the relationship between π and R changes.

So far we have determined the inflation rate - nominal interest rate combination in order to achieve k^S_{max}. At all levels of capital below k^S_{max} the return on capital has outweighed the return on bonds and capital. However, at k^S_{max} the Fisher equation (11) holds. That is, capital stock is sufficiently large to equate the returns on bonds and money. Therefore, if the government wants the economy to be at the maximum steady state, then it should not sell bonds.

3. Estimation

In Table 1 I have estimated the optimal inflation rates from a given nominal interest rate and for different levels of income tax rate. In Table 2, given an inflation rate target of zero or 3%, the associated nominal interest rates that will ensure maximum steady state capital are calculated. For estimation of values in Tables 1 and 2 I have used equations (22),(24) and (26). The parameter settings were $\alpha = 0.3$, $\beta = 0.95$, $\gamma = 0.014$, $\delta = 0$.

Calibration of parameter values

The output elasticity of capital, or α can be estimated from NZ data by the ratio of output that goes to the owners of capital. Values of α range from about 0.2 to 0.4 and appear to have an upward trend. Therefore we have decided to settle on 0.3. This value is in line with that used by the RBNZ's FPS model, i.e. 0.35. It is also comparable to Mendoza's (1991) estimate of α for the Canadian economy, of 0.32.

The discount factor, β is also selected in line with the literature. For example Chari, Christiano and Kehoe's (1994) choice of $\beta = 0.98$.

The proportion of consumption goods that the young must by with cash, γ, is selected from NZ data. It is estimated as the ratio of currency to GDP.

- δ is set at 0 in order to determine the minimum possible value of the inflation rate. Note that a higher value of δ raises the optimal inflation rate.

In Table 1 the nominal interest rate is assumed to be beyond the control of the central bank. This could be justified by a small open economy interest rate parity assumption, where the nominal interest rate is equal to the world interest rate when there is no expected change in the exchange rate. When the central bank is able to set the nominal interest rate, then Table 2 gives the nominal interest targets that are consistent with 0% inflation, and 3% inflation, so that the inflation-nominal interest rate combination will yield the steady state capital stock.

Algorithm for estimation of Tables 1&2

The first step was to select a nominal interest rate. For illustration, I selected 6% as a value that corresponds to current nominal interest rates. The income tax rate is varied, and for each income tax - nominal interest rate combination an optimal inflation rate can be found using (26). This is the "optimal" inflation rate in the sense that it is the inflation rate, for the given nominal interest rate, that will set the return on bonds equal to the return on capital at the maximum possible steady state capital stock. Once the maximum steady state capital stock is known, using (22), the per capita output of the economy and the per capita level of government spending can be ascertained with equations (5) and the specification of the production function, $y = Ak^{\alpha}$. For simplicity I have set A equal to 100, however it would be possible to calibrate the output series more closely to the actual NZ real GDP series by adjusting A. Note that in the maximum possible steady state the level of government bonds issued is zero. If the aim of the government is to maximise output, then government should not issue any bonds.

In Table 2 the income tax rate is varied and the nominal interest rate that would be consistent with the current inflation targets of 0% and 3% are calculated.

Table 1

income tax rate	*R* (Nominal Interest Rate)	optimal inflation rate	*y*	*g*
0.00%	6.00%	5.67%	5079.42	4.65
5.00%	6.00%	5.31%	4606.49	234.10
10.00%	6.00%	4.90%	4057.85	408.72
15.00%	6.00%	4.38%	3386.36	510.05
20.00%	6.00%	3.39%	2461.01	493.35
21.00%	6.00%	2.96%	2210.97	465.21
22.00%	6.00%	2.24%	1916.02	422.13
23.00%	6.00%	0.70%	1542.95	355.08
24.00%	6.00%	-6.00%	974.86	233.25

y is real per capita output
g is real per capita government spending

Table 2		
income tax rate	*R* given 0% inflation target	*R* given 3% inflation target
0.00%	0.31%	3.32%
5.00%	0.39%	3.56%
10.00%	0.53%	3.88%
15.00%	0.81%	4.36%
20.00%	1.70%	5.50%
21.00%	2.19%	6.05%
22.00%	3.05%	6.99%
23.00%	5.06%	9.11%
24.00%	14.78%	19.17%

Policy Implications

Table 1 has certain policy implications, depending on what the objective of government is. If the objective is to maximise government spending, then the optimal tax to GDP ratio is 15%, and the optimal inflation rate is 4.38%.

If the government wishes to maximise output, then the optimal income tax - inflation rate combination is 0% income tax, and 5.67% inflation. If the objective was to have zero inflation, then government should choose an income tax rate slightly below 23%.

4. Conclusions

Given that the income tax rate that is beyond the control of the central bank, the steady state capital stock is maximised when there is no incentive to hold nominal assets. In order to achieve this point, there is an "optimal" combination of inflation and interest rates that equates the returns on bonds and capital.

Therefore, if the central bank is committed to targeting a particular inflation rate, then to attain the maximum possible steady state capital stock, there is an implied "optimal" nominal interest rate that should be targeted. This nominal interest rate will change if the income tax rate changes. On the other hand, if the central bank wishes to target (through open market operations for example) a certain nominal interest rate, then there is an optimal inflation rate, that again, depends on the income tax rate. For example, if the ratio of income tax to GDP is 20%, then the nominal interest rate target should be between 1.7 and 5.5%. Or, for an income tax to GDP ratio of 20% the optimal inflation found by the model was 3.39%.

The implication is simply that the central bank should conduct monetary policy, in accordance with changes in fiscal policy.

APPENDIX

Note 1: Derivation of the savings function

$$L = \ln c_{1t} + \beta \ln c_{2t+1} - \lambda \left[c_{1t} + \frac{(1+\pi_t)c_{2t+1}}{(1+\tilde{R}_{t+1})} - (1-\tau_t)w_t - k_{t+1}(\cdot) - m_t(\cdot) \right]$$

first order conditions:

[1.1] $\lambda = 1/c_{1t}$

[1.2] $$\frac{\beta}{c_{2t+1}} = \frac{\lambda(1+\pi_t)}{(1+\tilde{R}_{t+1})}$$

therefore,

[1.3] $$\beta c_{1t} = \frac{c_{2t+1}(1+\pi_t)}{1+\tilde{R}_{t+1}}$$

Substituting [3] into the lifetime budget constraint, (10) gives:

[1.4] $$c_{1t} = \left(\frac{1}{1+\beta}\right)\left[(1-\tau_t)w_t + \left(\frac{(1+\pi_t)(1+\tilde{r}^K_{t+1})}{1+\tilde{R}_{t+1}} - 1\right)k_{t+1} + m^A_t\left(\frac{1}{P_t(1+\tilde{R}_{t+1})} - \frac{1}{P_t}\right)\right]$$

[1.5]

$$c_{1t} = \frac{(1-\tau_t)w_t}{1+\beta} + \left(\frac{k_{t+1}}{1+\beta}\right)\left(\frac{(1+\pi_t)(1+\tilde{r}^K_{t+1})}{1+\tilde{R}_{t+1}} - 1\right) + \left(\frac{m^A_t}{1+\beta}\right)\left(\frac{1}{P_t(1+\tilde{R}_{t+1})} - \frac{1}{P_t}\right)$$

The savings function for each individual is therefore:

[1.6] $s_t = (1-\tau_t)w_t - c_{1t}$

$$s_{1t} = \frac{\beta(1-\tau_t)w_t}{1+\beta} - \left(\frac{k_{t+1}}{1+\beta}\right)\left(\frac{(1+\pi_t)(1+\tilde{r}^K_{t+1})}{1+\tilde{R}_{t+1}} - 1\right) - \left(\frac{m^A_t}{1+\beta}\right)\left(\frac{1}{P_t(1+\tilde{R}_{t+1})} - \frac{1}{P_t}\right)$$

In a steady state the savings function is [7]. This is the steady state function no matter what the nominal after-tax nominal interest rate is. (Given that its not negative.) If $(1-\tau)R$ is zero, then the last term in [6] is zero. If $(1-\tau)R$ is positive, then m_t^A is zero, and again, the last term in [6] is zero.

[1.7]

$$s_{1t} = \frac{\beta(1-\tau_t)w_t}{1+\beta}$$

<u>Note 2 - an equilibrium inflation rate</u>: Given that the government selects the tax rate and the level of government spending, there exists an endogenously determined equilibrium inflation rate.

This section considers the determination of inflation rates within the model, when government does not sell bonds. For this we suppose that fiscal policy is exogenous and monetary is endogenous. That is, the policy instruments are government spending and the tax rate. The CAP is just a simplified version of (16):

[2.1] $$\frac{\beta(1-\alpha)}{(1+\beta)}(1-\tau)Ak^{\alpha}-(\delta+n)k = g+\frac{m}{(1+\pi+n)}$$

In contrast to the method used for finding the optimal inflation rate, in determining the endogenous inflation rate we will use capital stock (and not real balances) to equate the left and right hand sides of the CAP. To do this we need to be able to substitute out the last term involving m and π, since these are both unknowns. This can be done using the GBC and the Fisher equation. By the GBC we know that:

[2.2] $$\frac{m}{1+\pi+n}=\frac{g-\tau Ak^{\alpha}}{\pi+n}$$

Equation [2.2] states that the ratio of the money supply to the gross rate of money ($1+\mu_t$), is equal to the size of the fiscal deficit. This occurs because, in the absence of bond finance, if the government faces a deficit, it raises the needed revenue through seignorage. Since we implicitly assume that at all times markets clear, the demand for money is also dependent on the size of the fiscal deficit.

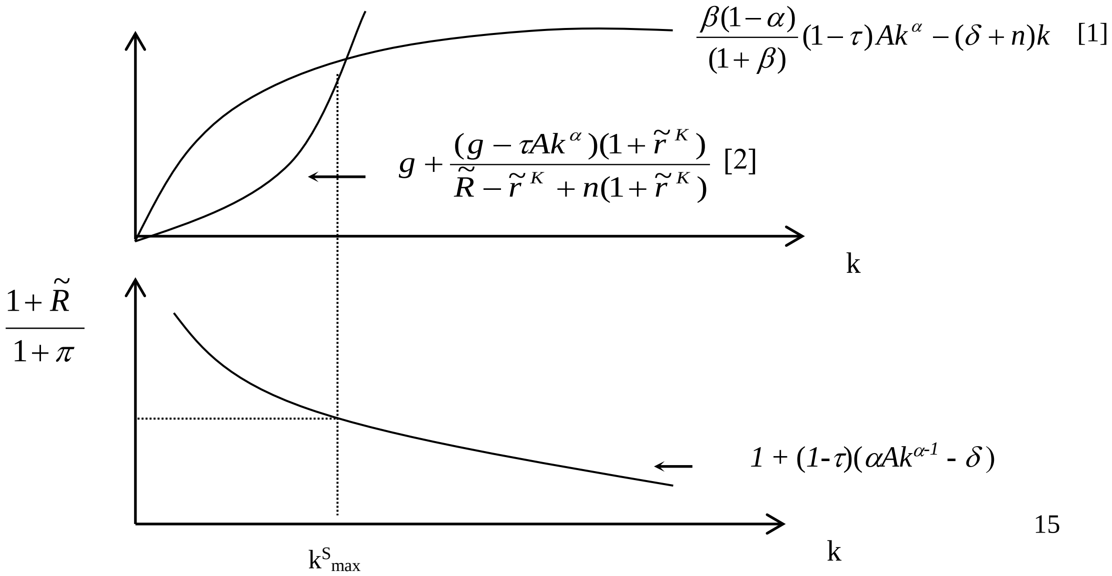
Next we have to substitute out π. To do this we can rearrange the Fisher equation as in (21). This is the endogenously determined inflation rate.

Therefore the CAP can now be expressed as a function of government spending and tax.

[2.3] $$\frac{\beta(1-\alpha)}{(1+\beta)}(1-\tau)Ak^{\alpha}-(\delta+n)k = g+\frac{(g-\tau Ak^{\alpha})(1+\tilde{r}^{K})}{\tilde{R}-\tilde{r}^{K}+n(1+\tilde{r}^{K})}$$

The steady state capital is the capital stock that equates the left and right sides of[2.3].

Fig.2 Government sets the level of government spending, and the tax rate



The government sets the level of government spending and the tax rate, which determine curves [1] and [2] in the top half of Fig.2. Then the equilibrium inflation rate is determined from the Fisher equation, depicted in the lower half of the diagram.

Note 3 : Steady state b

[3.1] $$b = \frac{b_t^g}{P_t} - \frac{b_{t-1}^g(1+R)}{P_t} = b_t^{g*} - \frac{b_{t-1}^{g*}(1+\tilde{R})}{1+\pi}$$ where $b_t^g/P_t = b_t^{g*}$

Let $b_t^{g*} = b_{t-1}^{g*} = b^{g*}$ = the amount of bonds issued each period, then

[3.2] $$b^{g*} = \frac{b}{-\tilde{r}^K}$$

Therefore, if $\tilde{r}^K$ and b^{g*} are positive, then b is negative. That is, in the steady state, if the real after tax return on capital is positive, and the government is issuing bonds, then by [3.2] the government must have a surplus.

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