APPENDIX FOR
A MIGRATION BASED THEORY OF THE EXCHANGE RATE AND THE NEW ZEALAND'S EXPERIENCE[1]

Debasis Bandyopadhyay
Department of Economics
University of Auckland, Auckland
New Zealand

Andrew Binning
Norges Bank, Oslo
Norway

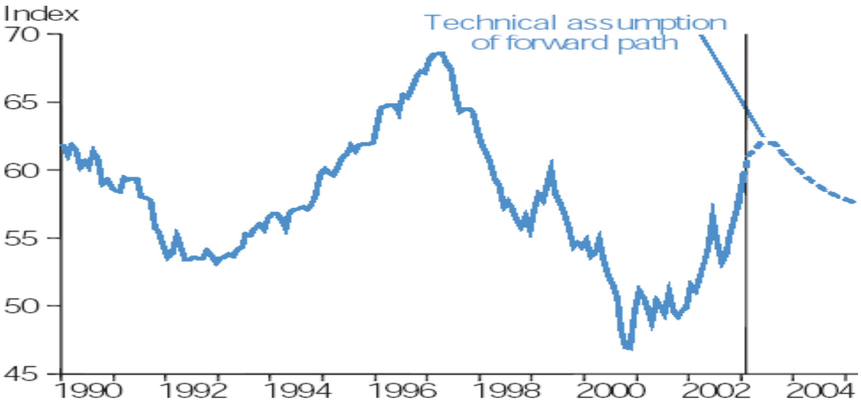
January 2016

[1] This paper originates from the B.Com (Hons) dissertation which Andrew Binning wrote under the supervision of Debasis Bandyopadhyay at the University of Auckland.

1. Figures 3 – 22:

Figure 1

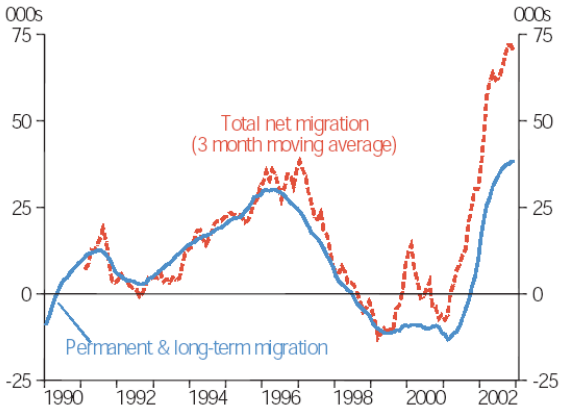
Nominal TWI exchange rate



Source: RBNZ.

Figure 2

Net migration
(annual total)



Source: Statistics New Zealand.

Figure 3.

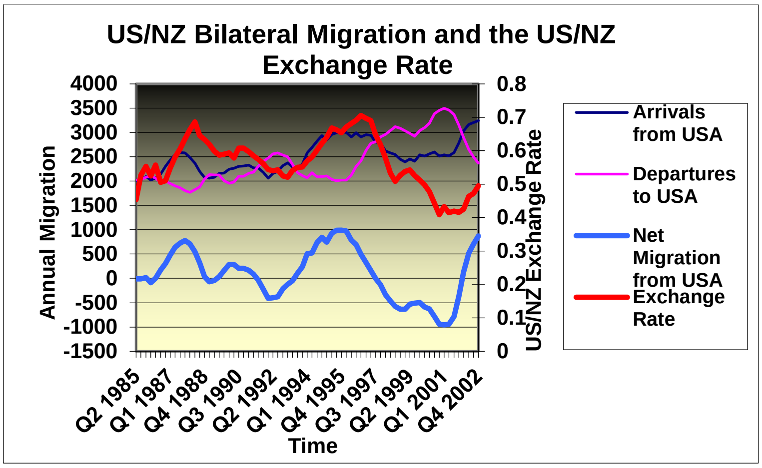
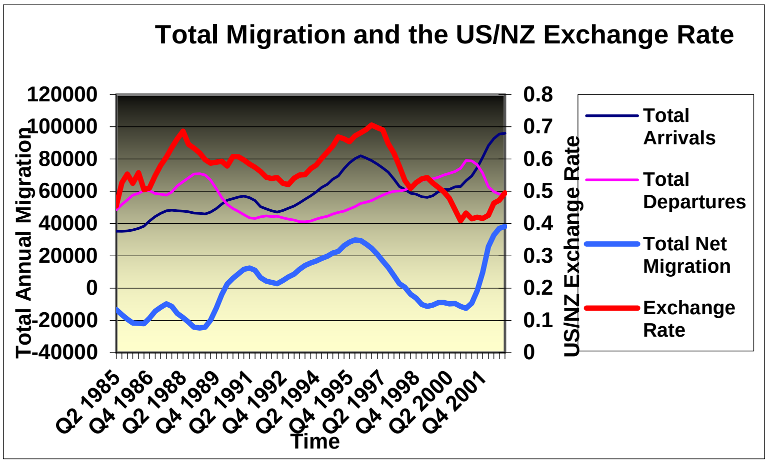


Figure 4.



Note: the exchange rate = $1/S_t$, i.e. the exchange rate (as defined here) is how much foreign currency is required to buy one unit of domestic currency.
All migration data is annual, measured quarterly.

Figure 5.

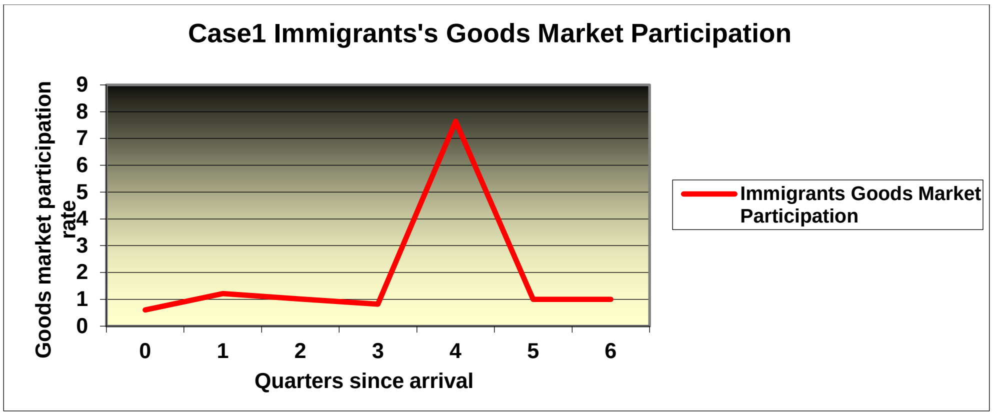


Figure 6.

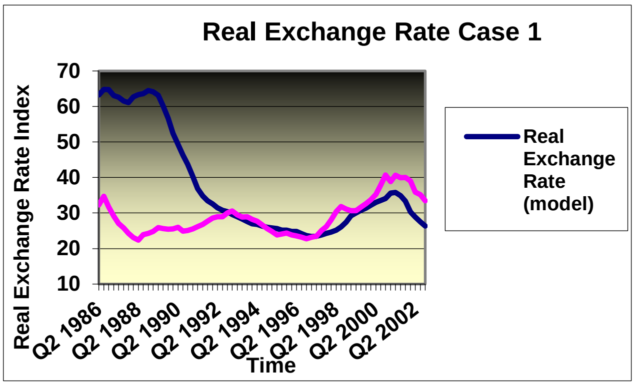


Figure 7.

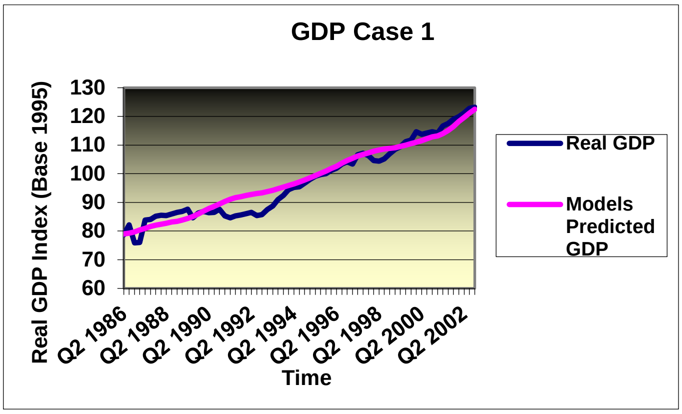


Figure 8.

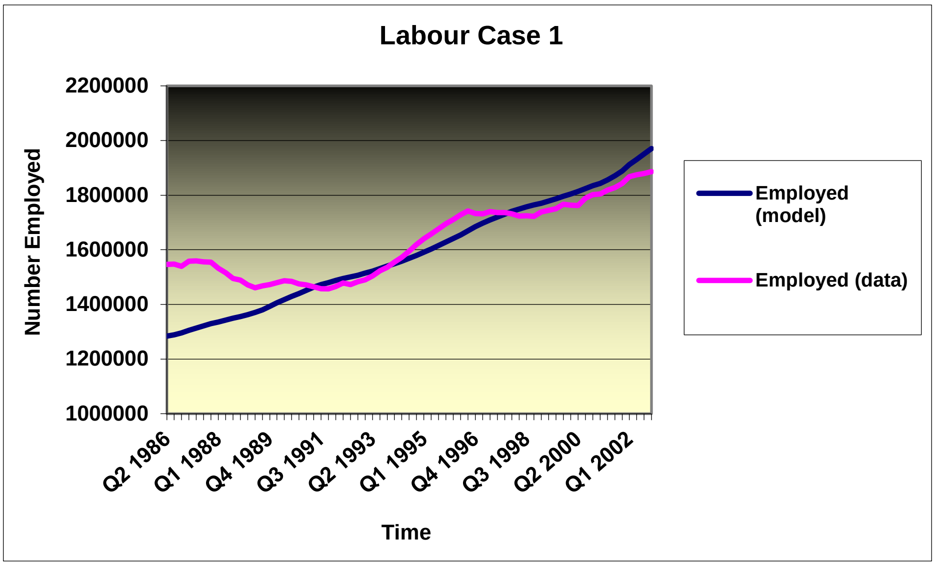


Figure 9.

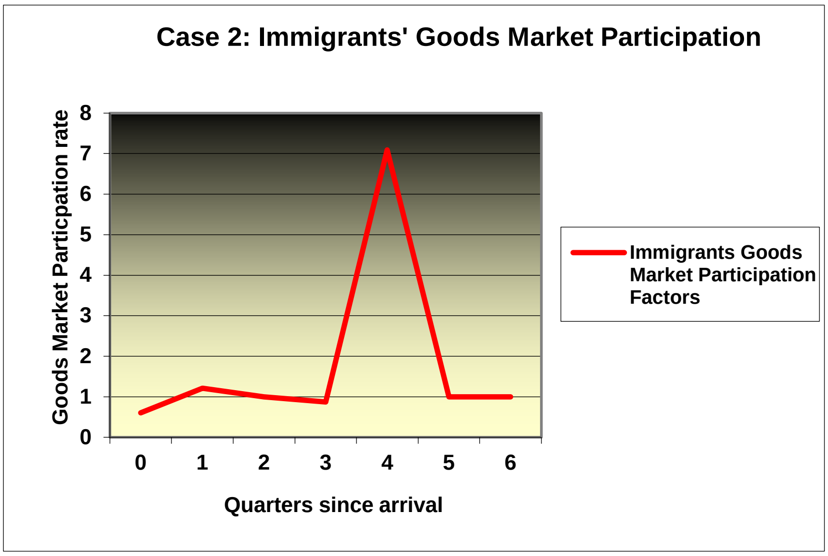


Figure 10.

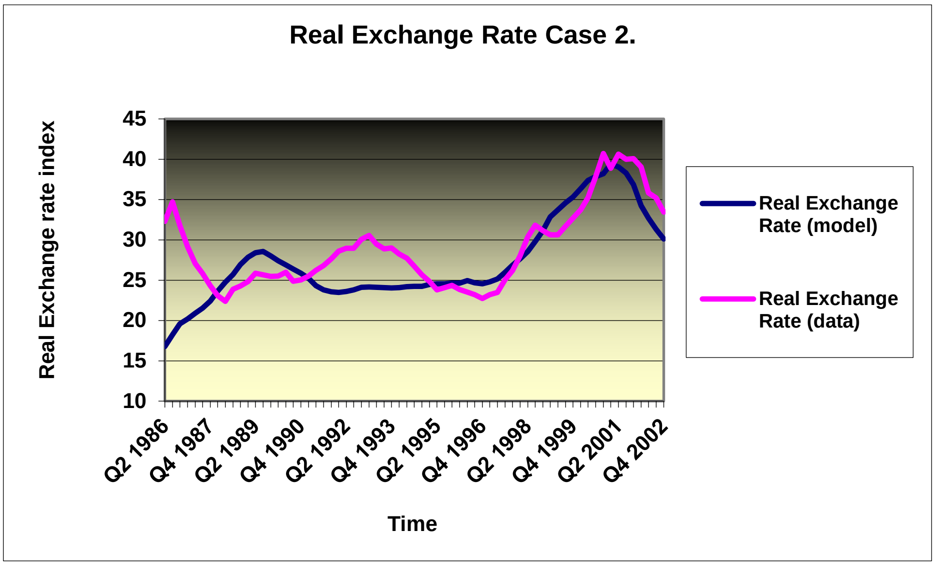


Figure 11.

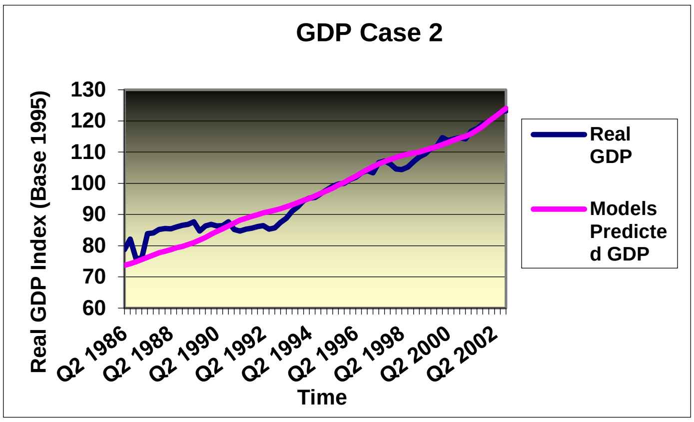


Figure 12.

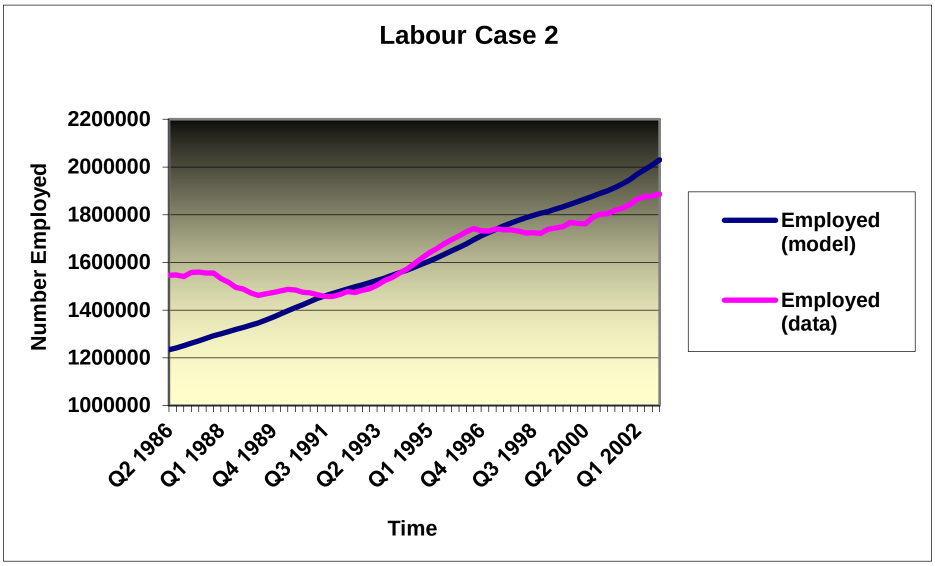


Figure 13.

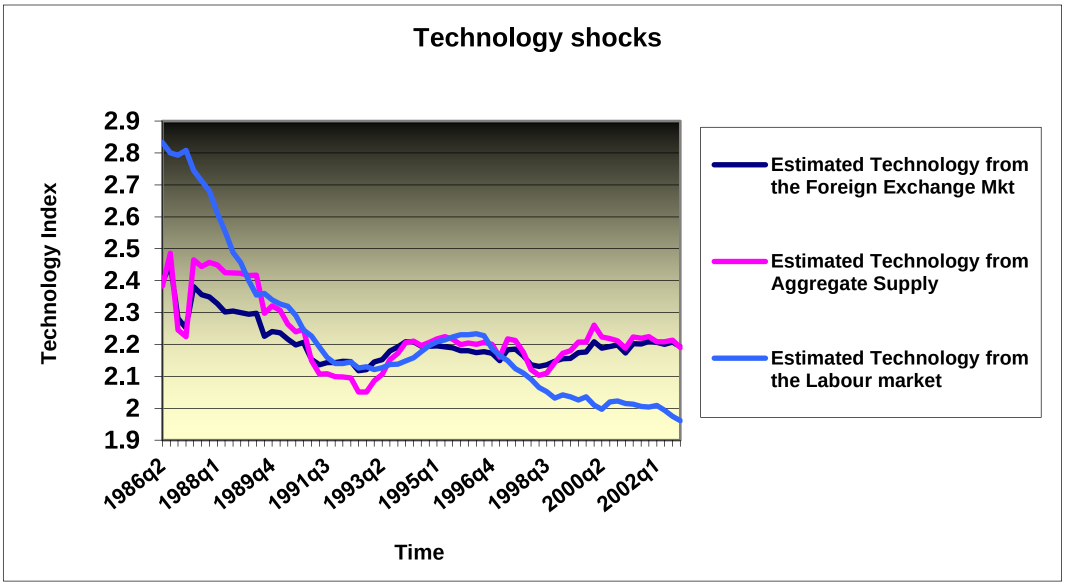


Figure 14.

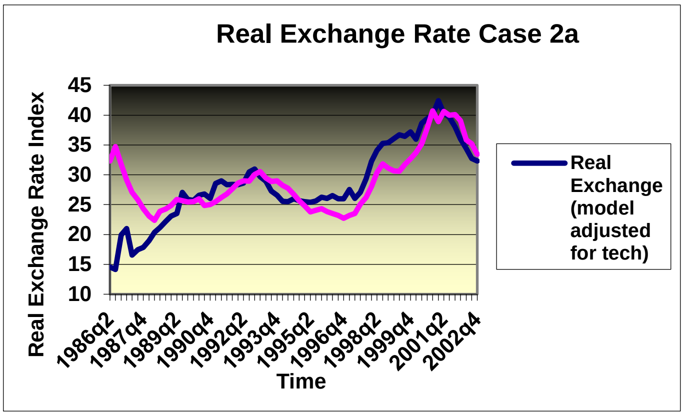


Figure 15.

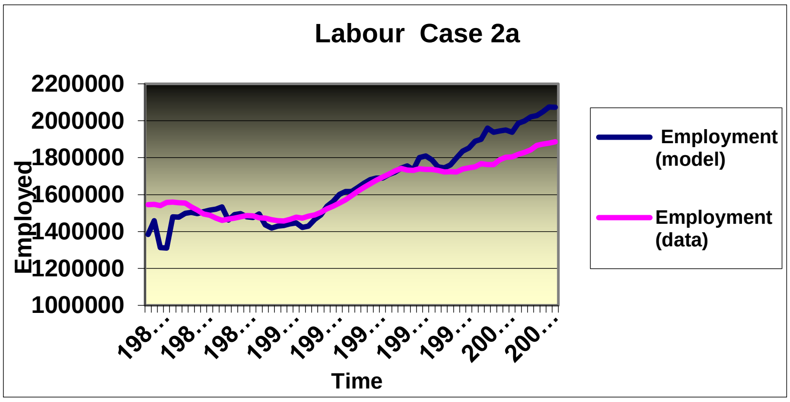


Figure 16.

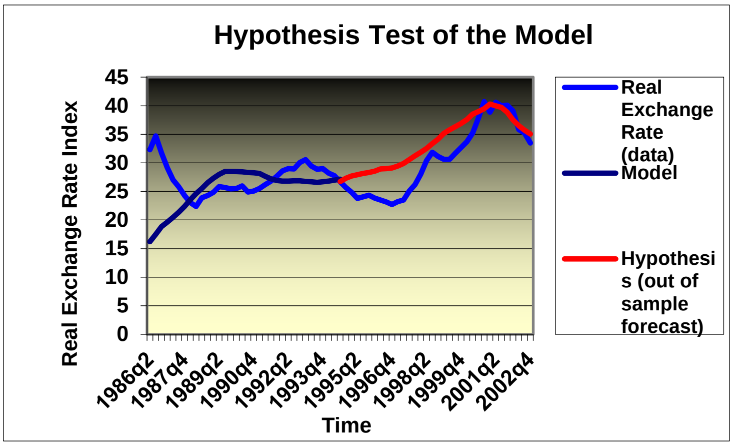
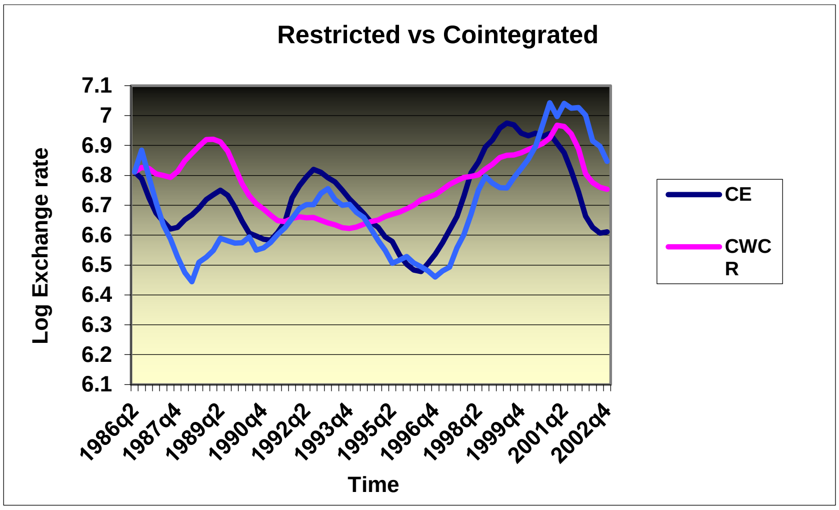
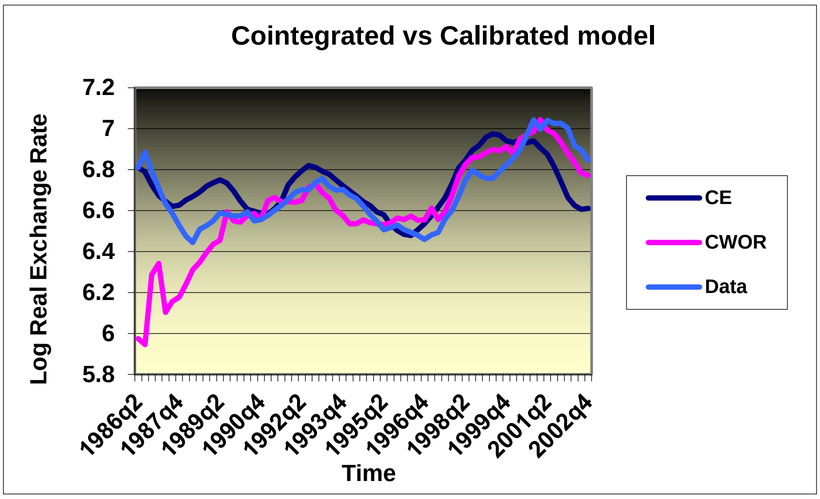


Figure 17.



CE = Cointegrating equation (5*) from Section 4.
CWCR = Calibrated equation with cointegration style restrictions
Data = real exchange rate data

Figure 18.



CE = Cointegrating equation (5*) taken from Section 4.
CWOR = calibrated equation without cointegrating restrictions, from Case 2a
Data = the real exchange rate data

Figure 19.

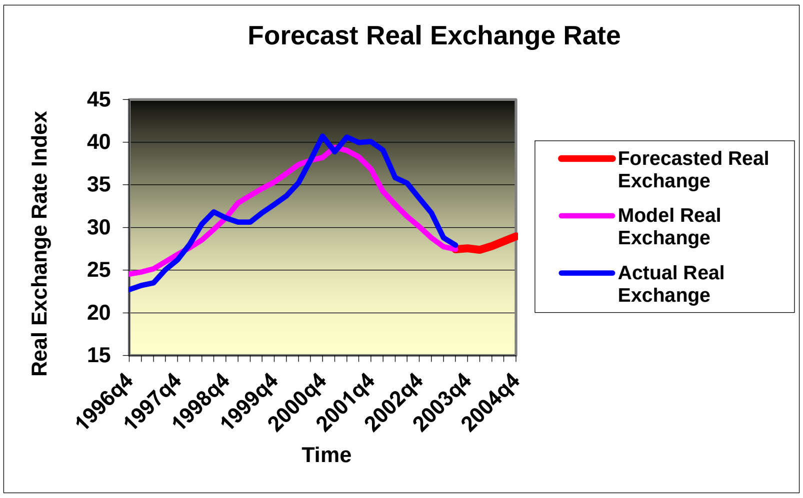
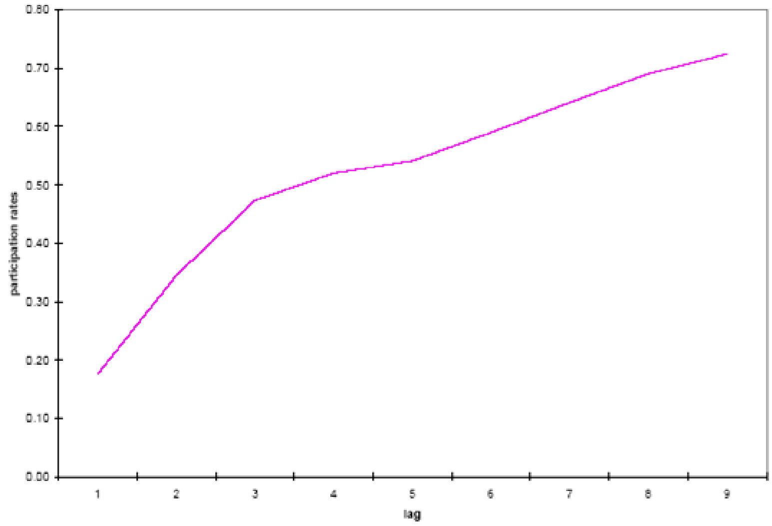


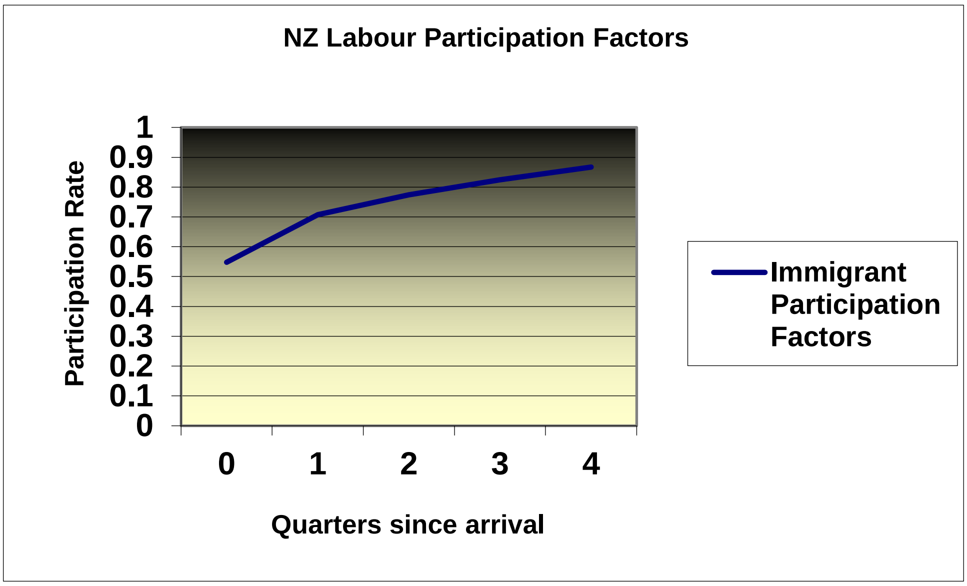
Figure 20.

Israeli Labour Participation Factors



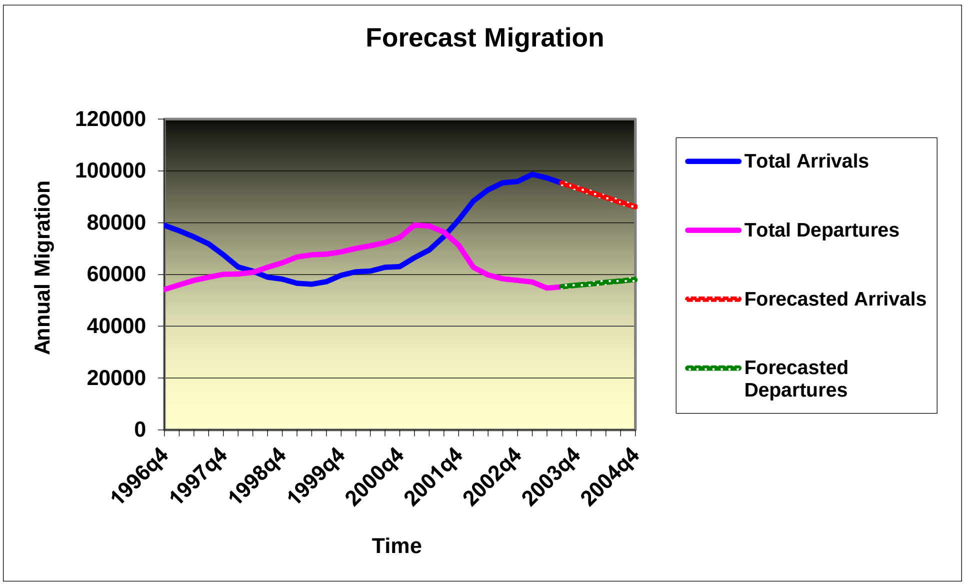
Source: Hercowitz and Yashiv (2002)

Figure 21.



The participation rate = the percentage of employed immigrants that arrived q quarters ago out of all immigrants of working age that arrived q quarters ago.

Figure 22.



Policy Experiment: Forecasting Migration

The forecasted migration is calculated using the following arbitrary procedure. It is assumed that immigration starts to decline slowly and that emigration increases slowly. An autoregressive procedure is applied and is presented below.

$$\Delta P_{t+1}^{E} = (1.01) * \Delta P_{t}^{E}$$

$$\Delta P_{t+1}^{I} = (0.98) * \Delta P_{t}^{I}$$

The coefficient on emigration is slightly larger than unity, implying that emigration will gradually increase. The coefficient on immigration is slightly less than unity implying that immigration will gradually decrease. This is a reasonably fair way of calculating future arrivals and departures since it is known that Total Arrivals and Total Departures follow a random walk from the Diagnostic Statistics section. Therefore imposing coefficients of slightly less or slightly more than unity will give near random walk processes. The forecasted migration is displayed in figure 22.

The forecasted level of domestic capital and the native born population are determined by using the preceding period's growth rate and extending the data.

A Technical Note Total Factor Productivity

In this section Z_{1t}, Z_{2t}, Z_{3t} and Z_{4t} will be solved for as functions of technology A_t, the aggregate demand shocks ξ_t, the parameters α, β, ε and λ. The exact functional forms of Z_{1t}, Z_{3t} and Z_{4t} must be known in order to estimate technology and aggregate demand shocks in the labour, the goods and the foreign exchange markets and to estimate the employed, GDP and the real exchange rates as functions of technology and aggregate demand shocks. Z_{1t}, Z_{2t}, Z_{3t} and Z_{4t} have all been allowed to vary with A_t and ξ_t over time.

From equation (1) in section 5.

$$Y_t = A_t L_t^{\alpha} Km_t^{\beta} K_t^{1-\alpha-\beta} \tag{A.1}$$

where A_t is an index of technology that varies with time

Differentiating the profit function with respect to labour and imported capital gives the following first order conditions.

$$\alpha A_t L_t^{\alpha-1} Km_t^{\beta} K_1^{1-\alpha-\beta} = w_t \tag{A.2}$$

$$\beta A_t L_t^{\alpha} Km_t^{\beta-1} K_t^{1-\alpha-\beta} = S_{Rt} \tag{A.3}$$

From equations (5) and (6) in section 5, the labour supply is obtained

$$L_t = w_t^{\lambda} \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right)$$

Setting labour demand equal to labour supply and solving for L_t

$$\alpha A_t L_t^{\alpha-1} Km_t^{\beta} K_t^{1-\alpha-\beta} = \left(\frac{L_t}{P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I} \right)^{\frac{1}{\lambda}}$$

$$\alpha^{\lambda} A_t^{\lambda} L_t^{\alpha\lambda-\lambda} Km_t^{\beta\lambda} K_t^{\lambda(1-\alpha-\beta)} = \frac{L_t}{P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I}$$

$$\alpha^{\lambda} A_t^{\lambda} L_t^{\alpha\lambda-\lambda} Km_t^{\beta\lambda} K_t^{\lambda(1-\alpha-\beta)} \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right) = L_t$$

$$\alpha^{\lambda} A_t^{\lambda} Km_t^{\beta\lambda} K_t^{\lambda(1-\alpha-\beta)} \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right) = L_t^{1-\alpha\lambda+\lambda} \qquad (A.4)$$

Making Km_t the subject of $(A.3)$ gives

$$Km_t^{\beta-1} = S_{Rt} \beta^{-1} A_t^{-1} L_t^{-\alpha} K_t^{-(1-\alpha-\beta)}$$

$$Km_t = S_{Rt}^{\frac{1}{\beta-1}} \beta^{\frac{1}{1-\beta}} A_t^{\frac{1}{1-\beta}} L_t^{\frac{\alpha}{1-\beta}} K_t^{\frac{(1-\alpha-\beta)}{1-\beta}} \qquad (A.5)$$

Substituting (A.4) into (A.5) gives

$$\alpha^{\lambda} A_t^{\frac{\lambda}{1-\beta}} S_{Rt}^{\frac{\beta\lambda}{\beta-1}} \beta^{\frac{\beta\lambda}{1-\beta}} L_t^{\frac{\alpha}{1-\beta}} K_t^{\frac{\beta\lambda(1-\alpha-\beta)}{1-\beta}} K_t^{\lambda(1-\alpha-\beta)} \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right) = L_t^{1-\alpha\lambda+\lambda}$$

$$\alpha^{\lambda} A_t^{\frac{\lambda}{1-\beta}} S_{Rt}^{\frac{-\beta\lambda}{1-\beta}} \beta^{\frac{\beta\lambda}{1-\beta}} K_t^{\frac{\lambda(1-\alpha-\beta)}{1-\beta}} \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right) = L_t^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{1-\beta}}$$

$$\left(A_t^{\frac{\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\alpha^{\frac{\lambda(1-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\beta^{\frac{\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(S_{Rt}^{\frac{-\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(K_t^{\frac{\lambda(1-\alpha-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}} \right)$$

$$\times \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right)^{\frac{1-\beta}{\lambda(1-\alpha-\beta)+1-\beta}} = L_t \qquad (A.6)$$

Comparing (7) and (A.6) denotes

$$Z_{1t} \equiv A_t^{\frac{\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \left(\alpha^{\frac{\lambda(1-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\beta^{\frac{\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \qquad (A.7)$$

Z_{1t} varies with time because A_t varies with time.

From (A.5)

$$Km_t = S_{Rt}^{\frac{1}{\beta-1}} A_t^{\frac{1}{1-\beta}} \beta^{\frac{1}{1-\beta}} L_t^{\frac{\alpha}{1-\beta}} K_t^{\frac{1-\alpha-\beta}{1-\beta}}$$

Substituting $(A.6)$ into $(A.5)$ gives

$$Km_t = \left(A_t^{\frac{\lambda+1}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\alpha^{\frac{\alpha\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\beta^{\frac{\lambda(1-\alpha-\beta)+1-\beta+\beta\lambda\alpha}{(1-\beta)[\lambda(1-\alpha-\beta)+1-\beta]}} \right) \left(S_{Rt}^{\frac{-\lambda(1-\alpha-\beta)+1-\beta+\lambda\alpha\beta}{(1-\beta)[\lambda(1-\alpha-\beta)+1-\beta)}} \right)$$
$$\times \left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right)^{\frac{\alpha}{\lambda(1-\alpha-\beta)+1-\beta}}$$

Comparing the above equation with (8) denotes

$$Z_{2t} \equiv \left(A_t^{\frac{\lambda+1}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\alpha^{\frac{\alpha\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\beta^{\frac{\lambda(1-\alpha-\beta)+1-\beta+\beta\lambda\alpha}{(1-\beta)[\lambda(1-\alpha-\beta)+1-\beta]}} \right) \qquad (A.8)$$

Z_{2t} varies with time because A_t varies with time.

Substituting $(A.6)$ and $(A.8)$ into $(A.1)$ denotes

$$Z_{3t} \equiv Z_{1t}^{\alpha} Z_{2t}^{\beta} A_t$$

Z_{3t} varies with time because Z_{1t}, Z_{2t} and A_t vary with time.

Substituting in Z_{1t} and Z_{2t}, Z_{3t} can be written in terms of A_t and parameters giving

$$Z_{3t} = \left(A_t^{\frac{\lambda+1}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\alpha^{\frac{\alpha\lambda}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(\beta^{\frac{\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \qquad (A.9)$$

Aggregate supply is taken from equation (9)

$$Y_t^S = Z_{3t} \left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}} \right) \left(P_t^N - \Delta P_t^E \theta_{l_t}^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right)^{\frac{\alpha}{\lambda(1-\alpha-\beta)+1-\beta}} S_{Rt}^{\left(\frac{-\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta} \right)} \qquad (A.10)$$

Aggregate demand is taken from equation (10)

$$Y_t^D = \xi_t S_{Rt}^{\varepsilon} \left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{y_q}^I \Delta P_{t-q}^I \right) \quad , \varepsilon > 0 \qquad (A.11)$$

ξ_t has been allowed to vary with time and can be thought of as measuring aggregate demand shocks, the models deviations from true GDP. Setting equation (A.8) equal to equation (A.9) and solving for S_{Rt} gives

$$Z_{3t}K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\left(P_t^N-\Delta P_t^E\theta_{l_t}^E+\sum_{q=0}^{t}\theta_{l_q}^I\Delta P_{t-q}^I\right)^{\frac{\alpha}{\lambda(1-\alpha-\beta)+1-\beta}}S_{Rt}^{\left(\frac{-\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}\right)}=$$

$$\xi_t S_{Rt}^{\varepsilon}\left(P_t^N-\theta_y^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{y_q}^I\Delta P_{t-q}^I\right)$$

$$S_{Rt}=\left(\frac{Z_{3t}}{\xi_t}\right)^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\mu}}\left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\mu}}\right)$$
$$\times\left(P_t^N-\theta_y^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{y_q}^I\Delta P_{t-q}^I\right)^{\frac{-[\lambda(1-\alpha-\beta)+(1-\beta)]}{\mu}}$$
$$\times\left(P_t^N-\theta_l^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{l_q}^I\Delta P_{t-q}^I\right)^{\frac{\alpha}{\mu}} \qquad (A12)$$

where $\mu\equiv\beta(1+\lambda)+\varepsilon\lambda(1-\alpha-\beta)+\varepsilon(1-\beta)\ >0$

comparing (A12) with (11) denotes

$$Z_{4t}\equiv\left(\frac{Z_{3t}}{\xi_t}\right)^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\beta(1+\lambda)+\varepsilon\lambda(1-\alpha-\beta)+\varepsilon(1-\beta)}}$$

Z_{4t} varies with time because Z_{3t} varies with time. Writing Z_{4t} in terms of A_t, ξ_t and parameters gives

$$Z_{4t}=\left(\frac{\left(A_t^{\frac{\lambda+1}{\mu}}\right)\left(\alpha^{\frac{\alpha\lambda^2(1-\alpha-\beta)+\alpha\lambda-\alpha\lambda\beta}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}}\right)\left(\beta^{\frac{\beta(1+\lambda)\lambda(1-\alpha-\beta)+\beta(1+\lambda)-\beta^2(1+\lambda)}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}}\right)}{\xi_t^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\mu}}}\right) \qquad (A.13)$$

Solving for Technology and Aggregate Demand Shocks

Technology and aggregate demand shocks are solved for simultaneously by setting the models prediction of the real exchange rate, the employed and GDP equal to their data equivalents, in a spread sheet model. The formulas used are derived below.

Aggregate demand shocks:

From (A11) aggregate demand is given by,

$$Y_t^D = \xi_t S_{Rt}^{\varepsilon}\left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{y_q}^I \Delta P_{t-q}^I \right) \quad , \varepsilon > 0$$

Setting aggregate demand equal to GDP

$$Y_t^{Data} = Y_t^D$$

$$Y_t^{Data} = \xi_t \left(S_{Rt}^{Data} \right)^{\varepsilon} \left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{y_q}^I \Delta P_{t-q}^I \right)$$

Where S_{Rt}^{Data} is the real exchange rate data, used because it assumed to contain the true aggregate demand shocks, and will not change with the estimate of ξ_t.

Solving for the aggregate demand shocks

$$\xi_t = \frac{Y_t^{Data}}{\left(S_{Rt}^{Data} \right)^{\varepsilon} \left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{y_q}^I \Delta P_{t-q}^I \right)} \qquad (A.14)$$

Technology from the goods market:

From (A10) and (A9) aggregate supply is given by,

$$Y_t^S = \left(A_t^{\frac{\lambda+1}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\alpha^{\frac{\alpha\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\beta^{\frac{\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)$$

$$\times\left(P_t^N - \Delta P_t^E \theta_{l_t}^E + \sum_{q=0}^{t}\theta_{l_q}^I \Delta P_{t-q}^I\right)^{\frac{\alpha}{\lambda(1-\alpha-\beta)+1-\beta}} S_{Rt}^{\left(\frac{-\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}\right)}$$

Setting aggregate supply equal to GDP gives,

$$Y_t^{Data} = Y_t^S$$

$$Y_t^{Data} = \left(A_t^{\frac{\lambda+1}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\alpha^{\frac{\alpha\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\beta^{\frac{\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)$$

$$\times\left(P_t^N - \Delta P_t^E \theta_{l_t}^E + \sum_{q=0}^{t}\theta_{l_q}^I \Delta P_{t-q}^I\right)^{\frac{\alpha}{\lambda(1-\alpha-\beta)+1-\beta}} \left(S_{Rt}^{Data}\right)^{\left(\frac{-\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}\right)}$$

Where S_{Rt}^{Data} is the real exchange rate data and has been used because it is assumed that it contains the true value of technology shocks and will not change with the estimate of A_t.

Solving for technology,

$$A_t^{\frac{\lambda+1}{\lambda(1-\alpha-\beta)+1-\beta}} = \frac{Y_t^{Data}}{\left(\begin{array}{l}\left(\alpha^{\frac{\alpha\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\beta^{\frac{\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\right) \\ \times\left(P_t^N - \Delta P_t^E \theta_{l_t}^E + \sum_{q=0}^{t}\theta_{l_q}^I \Delta P_{t-q}^I\right)^{\frac{\alpha}{\lambda(1-\alpha-\beta)+1-\beta}} \left(S_{Rt}^{Data}\right)^{\frac{-\beta(1+\lambda)}{\lambda(1-\alpha-\beta)+1-\beta}}\end{array}\right)}$$

$$A_t = \frac{\left(Y_t^{Data}\right)^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\lambda+1}}}{\left(\left(\alpha^{\frac{\alpha\lambda}{\lambda+1}}\right)\left(\beta^{\beta}\right)K_t^{(1-\alpha-\beta)}\left(P_t^N - \Delta P_t^E \theta_{l_t}^E + \sum_{q=0}^{t}\theta_{l_q}^I \Delta P_{t-q}^I\right)^{\left(\frac{\alpha}{\lambda+1}\right)}\left(S_{Rt}^{Data}\right)^{-\beta}\right)} \qquad (A.15)$$

Technology from the labour market:

From (A6) and (A7), employment is given by,

$$\left(A_t^{\frac{\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\alpha^{\frac{\lambda(1-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\beta^{\frac{\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(S_{Rt}^{\frac{-\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(K_t^{\frac{\lambda(1-\alpha-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)$$

$$\times\left(P_t^N-\theta_l^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{l_q}^I\Delta P_{t-q}^I\right)^{\frac{1-\beta}{\lambda(1-\alpha-\beta)+1-\beta}}=L_t^{Model}$$

Setting the models prediction of employment equal to the employment data gives,

$$L_t^{Data}=L_t^{Model}$$

$$\left(A_t^{\frac{\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\alpha^{\frac{\lambda(1-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\beta^{\frac{\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\left(S_{Rt}^{Data}\right)^{\frac{-\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(K_t^{\frac{\lambda(1-\alpha-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)$$

$$\times\left(P_t^N-\theta_l^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{l_q}^I\Delta P_{t-q}^I\right)^{\frac{1-\beta}{\lambda(1-\alpha-\beta)+1-\beta}}=L_t^{Data}$$

Solving for technology,

$$A_t^{\frac{\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}=\frac{L_t^{Data}}{\left(\left(\alpha^{\frac{\lambda(1-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\beta^{\frac{\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(\left(S_{Rt}^{Data}\right)^{\frac{-\beta\lambda}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\left(K_t^{\frac{\lambda(1-\alpha-\beta)}{\lambda(1-\alpha-\beta)+1-\beta}}\right)\times\left(P_t^N-\theta_l^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{l_q}^I\Delta P_{t-q}^I\right)^{\frac{1-\beta}{\lambda(1-\alpha-\beta)+1-\beta}}\right)}$$

$$A_t=\frac{\left(L_t^{Data}\right)^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\lambda}}}{\left(\left(\alpha^{1-\beta}\right)\left(\beta^{\beta}\right)\left(\left(S_{Rt}^{Data}\right)^{-\beta}\right)\left(K_t^{(1-\alpha-\beta)}\right)\left(P_t^N-\theta_l^E\Delta P_t^E+\sum_{q=0}^{t}\theta_{l_q}^I\Delta P_{t-q}^I\right)^{\frac{1-\beta}{\lambda}}\right)}$$

$$(A.16)$$

Technology from the foreign exchange market:

From (A12) and (A13)

$$S_{Rt}^{Model} = \left(\frac{\left(A_t^{\frac{\lambda+1}{\mu}} \right) \left(\alpha^{\frac{\alpha\lambda^2(1-\alpha-\beta)+\alpha\lambda-\alpha\lambda\beta}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}} \right) \left(\beta^{\frac{\beta(1+\lambda)\lambda(1-\alpha-\beta)+\beta(1+\lambda)-\beta^2(1+\lambda)}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}} \right)}{\xi_t^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\mu}}} \right) \left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\mu}} \right)$$

$$\times \left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{y_q}^I \Delta P_{t-q}^I \right)^{\frac{-[\lambda(1-\alpha-\beta)+(1-\beta)]}{\mu}}$$

$$\times \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right)^{\frac{\alpha}{\mu}}$$

where ξ_t, the aggregate demand shock, is determined in the goods market using (A.14).

Setting the models prediction of the real exchange rate equal to the real exchange rate data gives,

$$S_{Rt}^{Data} = S_{Rt}^{Model}$$

$$S_{Rt}^{Data} = \left(\frac{\left(A_t^{\frac{\lambda+1}{\mu}} \right) \left(\alpha^{\frac{\alpha\lambda^2(1-\alpha-\beta)+\alpha\lambda-\alpha\lambda\beta}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}} \right) \left(\beta^{\frac{\beta(1+\lambda)\lambda(1-\alpha-\beta)+\beta(1+\lambda)-\beta^2(1+\lambda)}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}} \right)}{\xi_t^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\mu}}} \right) \left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\mu}} \right)$$

$$\times \left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{y_q}^I \Delta P_{t-q}^I \right)^{\frac{-[\lambda(1-\alpha-\beta)+(1-\beta)]}{\mu}}$$

$$\times \left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t} \theta_{l_q}^I \Delta P_{t-q}^I \right)^{\frac{\alpha}{\mu}}$$

Solving for technology,

$$A_t^{\frac{\lambda+1}{\mu}} = \frac{S_{Rt}^{Data}}{\left(\left(\frac{\left(\alpha^{\frac{\alpha\lambda^2(1-\alpha-\beta)+\alpha\lambda-\alpha\lambda\beta}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}}\right)\left(\beta^{\frac{\beta(1+\lambda)\lambda(1-\alpha-\beta)+\beta(1+\lambda)-\beta^2(1+\lambda)}{[\lambda(1-\alpha-\beta)+1-\beta]\mu}}\right)}{\xi_t^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{\mu}}}\right)\left(K_t^{\frac{(1-\alpha-\beta)(1+\lambda)}{\mu}}\right) \times\left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t}\theta_{y_q}^I \Delta P_{t-q}^I\right)^{\frac{-[\lambda(1-\alpha-\beta)+(1-\beta)]}{\mu}} \times\left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t}\theta_{l_q}^I \Delta P_{t-q}^I\right)^{\frac{\alpha}{\mu}}\right)}$$

$$A_t = \frac{\left(S_{Rt}^{Data}\right)^{\frac{\mu}{\lambda+1}}}{\left(\left(\frac{\left(\alpha^{\frac{\alpha\lambda^2(1-\alpha-\beta)+\alpha\lambda-\alpha\lambda\beta}{[\lambda(1-\alpha-\beta)+1-\beta](\lambda+1)}}\right)\left(\beta^{\frac{\beta\lambda(1-\alpha-\beta)+\beta-\beta^2}{[\lambda(1-\alpha-\beta)+1-\beta]}}\right)}{\xi_t^{\frac{\lambda(1-\alpha-\beta)+1-\beta}{(1+\lambda)}}}\right)\left(K_t^{(1-\alpha-\beta)}\right) \times\left(P_t^N - \theta_y^E \Delta P_t^E + \sum_{q=0}^{t}\theta_{y_q}^I \Delta P_{t-q}^I\right)^{\frac{-[\lambda(1-\alpha-\beta)+(1-\beta)]}{(1+\lambda)}} \times\left(P_t^N - \theta_l^E \Delta P_t^E + \sum_{q=0}^{t}\theta_{l_q}^I \Delta P_{t-q}^I\right)^{\frac{\alpha}{(1+\lambda)}}\right)} \qquad (A.17)$$

The estimated technology from the labour, the goods and the foreign exchange markets is reported in figure 13.

The models prediction of the real exchange rate and the employed can be estimated by using the aggregate demand and technology shocks generated in the goods market. (A.14) and

(A.15) can be substituted into (A.13) which can in turn be substituted in equation (11), to get the real exchange rate as a function of technology and aggregate demand shocks generated in the goods market. Likewise, (A.15) can be substituted into (A.7) which can in turn be substituted in equation (7), to get the models prediction of the employed as a function of technology generated in the goods market. The results for these are reported in figures 14 and 15 respectively.

Findings from Econometric Tests:

(E.1) Table 6: Unit root test results

Variable	**Phillips-Perron Probability values**	**Unit Root test Specification**
s_t	0.2279	Intercept
s_{Rt}	0.0977	Intercept
$\ln(Tot\ Arr) - \ln(Tot\ Dep) = Totnet$	0.1599	None
$\ln(US\ Arr) - \ln(US\ Dep) = USnet$	0.0629	None
$\ln(Tot\ Arr)$	0.5052	Trend & Intercept
$\ln(Tot\ Dep)$	0.7306	Trend & Intercept
$\ln(USArr)$	0.5514	Trend & Intercept
$\ln(USDep)$	0.6748	Trend & Intercept

Note: none refers to any intercept or trend used in the Unit root test.

$s_t = \ln S_t$, the log of the nominal exchange rate, where the nominal exchange rate is the how much domestic currency it takes to buy one unit of foreign currency.

$s_{Rt} = \ln S_{Rt}$, the log of the real exchange rate, where the real exchange rate is relative price of a basket of domestic goods compared to a basket of foreign goods.

(E.2) Table 7: Cointegrating Results

Variables in the Cointegrating Equation	**Phillips-Perron significance level**
(1) $s_R, Totnet$	5%
(2) $s, Totnet$	12.5%
(3) $s_R, USnet$	
(4) $s, USnet$	
(5) $s_R, \ln(TotArr), \ln(TotDep)$	15%
(6) $s, \ln(TotArr), \ln(TotDep)$	
(7) $s_R, \ln(USArr), \ln(USDep)$	
(8) $s, \ln(USArr), \ln(USDep)$	
(9) $s_R, \ln(USArr)$	10%
(10) $s, \ln(USArr)$	
(11) $s_R, \ln(TotArr)$	10%
(12) $s, \ln(TotArr)$	
(13) $s_R, \ln(USDep)$	12.5%
(14) $s, \ln(USDep)$	
(15) $s_R, \ln(TotDep)$	5%
(16) $s, \ln(TotDep)$	12.5%

Note: The number in brackets in the first column refers to the cointegrating equation tested and all unit root tests performed on the residuals do not include trend or intercept.

(E.3) Error Correction Models and Granger Causality Tests (standard errors in brackets)

(1**) $$\Delta(s_R)_t = \underset{(0.0044)}{0.0028} - \underset{(0.0381)}{0.1077}(\varepsilon_{1*})_{t-1} - \underset{(0.0770)}{0.1458}\Delta Totnet_{t-1} - \underset{(0.0980)}{0.2166}\Delta(s_R)_{t-1}$$

$$\Delta Totnet_t = \underset{(0.0043)}{0.0032} - \underset{(0.0372)}{0.0335}(\varepsilon_{1*})_{t-1} + \underset{(0.0751)}{0.8009}\Delta Totnet_{t-1} + \underset{(0.0956)}{0.0856}\Delta(s_R)_{t-1}$$

$$(2^{**})\ \Delta s_t = \underset{(0.0043)}{0.0041} - \underset{(0.0480)}{0.1061}(\varepsilon_{2*})_{t-1} - \underset{(0.0829)}{0.1301}\Delta Totnet_{t-1} + \underset{(0.0993)}{0.2269}\Delta s_{t-1}$$

$$\Delta Totnet_t = \underset{(0.0043)}{0.0030} - \underset{(0.0475)}{0.0439}(\varepsilon_{2*})_{t-1} + \underset{(0.0820)}{0.7886}\Delta Totnet_{t-1} + \underset{(0.0982)}{0.1185}\Delta s_{t-1}$$

$$(5^{**})\ \Delta(s_R)_t = \underset{(0.0046)}{0.0073} - \underset{(0.0452)}{0.1497}(\varepsilon_{5*})_{t-1} - \underset{(0.1556)}{0.3108}\Delta\ln(Tot\ Arr)_{t-1} - \underset{(0.1237)}{0.0969}\Delta\ln(Tot\ Dep)_{t-1}$$

$$+ \underset{(0.1012)}{0.1012}\Delta(s_R)_{t-1}$$

$$\Delta\ln(TotArr)_t = \underset{(0.0028)}{0.0016} - \underset{(0.0270)}{0.0031}(\varepsilon_{5*})_{t-1} - \underset{(0.0930)}{0.8978}\Delta\ln(TotArr)_{t-1} + \underset{(0.0739)}{0.0738}\Delta\ln(TotDep)_{t-1}$$

$$+ \underset{(0.0605)}{0.0657}\Delta(s_R)_{t-1}$$

$$\Delta\ln(Tot\ Dep)_t = \underset{(0.0030)}{0.0012} + \underset{(0.0296)}{0.0096}(\varepsilon_{5*})_{t-1} - \underset{(0.1020)}{0.1364}\Delta\ln(TotArr)_{t-1} + \underset{(0.0810)}{0.7452}\Delta\ln(TotDep)_{t-1}$$

$$- \underset{(0.0663)}{0.0527}\Delta\ln(s_R)_{t-1}$$

$$(9^{**})\ \Delta(s_R)_t = \underset{(0.0041)}{0.0013} - \underset{(0.0426)}{0.1432}(\varepsilon_{9*})_{t-1} - \underset{(0.1319)}{0.0101}\Delta\ln(USArr)_{t-1} - \underset{(0.1393)}{0.2476}\Delta\ln(USArr)_{t-2}$$

$$+ \underset{(0.1390)}{0.1306}\Delta\ln(US\ Arr)_{t-3} + \underset{(0.1172)}{0.0800}\Delta(s_R)_{t-1} + \underset{(0.1080)}{0.1870}\Delta(s_R)_{t-2} + \underset{(0.0981)}{0.0283}\Delta(s_R)_{t-3}$$

$$\Delta\ln(USArr)_t = \underset{(0.0041)}{0.0039} + \underset{(0.0359)}{0.0212}(\varepsilon_{9*})_{t-1} + \underset{(0.1302)}{0.3883}\Delta\ln(USArr)_{t-1} + \underset{(0.1010)}{0.0137}\Delta(s_R)_{t-1}$$

$$(11^{**})\ \Delta(s_R)_t = \underset{(0.0045)}{0.0072} - \underset{(0.0448)}{0.1346}(\varepsilon_{11*})_{t-1} - \underset{(0.1486)}{0.2784}\Delta\ln(USArr)_{t-1} + \underset{(0.0983)}{0.1334}\Delta(s_R)_{t-1}$$

$$\Delta\ln(US\ Arr)_t = 0.0023 - 0.0046(\varepsilon_{11*})_{t-1} + 0.8625\Delta\ln(US\ Arr)_{t-1} + 0.0524\Delta(s_R)_{t-1}$$

(0.0027) (0.0266) (0.0882) (0.0583)

$$(13^{**})\quad \Delta(s_R)_t = -0.0014 - 0.1844(\varepsilon_{13^*})_{t-1} + 0.0214\Delta(USDep)_{t-1} + 0.0214\Delta(s_R)_{t-1}$$

(0.0041) (0.0414) (0.1329) (0.0985)

$$\Delta(US\ Dep) = 0.0006 + 0.0100(\varepsilon_{13^*})_{t-1} + 0.6344\Delta(USDep)_{t-1} + 0.1093\Delta(s_R)_{t-1}$$

(0.0037) (0.0379) (0.1215) (0.0900)

$$(15^{**})\quad \Delta(s_R)_t = -0.0003 - 0.0796(\varepsilon_{15^*})_{t-1} + 0.1381\Delta\ln(Tot\ Dep)_{t-1} + 0.2984\Delta(s_R)_{t-1}$$

(0.0046) (0.0325) (0.1232) (0.1008)

$$\Delta\ln(US\ Dep)_t = -0.0005 - 0.0121(\varepsilon_{15^*})_{t-1} + 0.7839\Delta\ln(US\ Dep)_{t-1} - 0.0280\Delta(s_R)_{t-1}$$

(0.00027) (0.0190) (0.0723) (0.0591)

$$(16^{**})\quad \Delta s_t = 0.0007 - 0.0777(\varepsilon_{16^*})_{t-1} + 0.1383\Delta\ln(TotDep)_{t-1} + 0.2871\Delta s_{t-1}$$

(0.0045) (0.0396) (0.1248) (0.1032)

$$\Delta\ln(US\ Dep)_t = -0.0005 - 0.0201(\varepsilon_{16^*}) + 0.7700\Delta\ln(TotDep)_{t-1} - 0.0499\Delta s_{t-1}$$

(0.0027) (0.0237) (0.0747) (0.0618)