Endogenous Growth without Patents[1]

Debasis Bandyopadhyay

Department of Economics

University of Auckland, Auckland

New Zealand

Andrew Binning

Norges Bank, Oslo

Norway

February 2016

Abstract

This paper constructs an analytically tractable model of endogenous innovation with a special focus on the effects of barriers to entry, namely patents. Conventional models of endogenous growth rely on the existence and enforcement of intellectual property rights with patents. Those legal rights are seen as necessary evils, required to encourage innovation by ensuring successful innovators are rewarded with monopoly rents. This paper takes a different approach. By integrating Aghion, Harris and Vickers (1997) and the Boldrin and Levine (2003) framework into a conventional vintage capital growth model such as Greenwood, et al (1997), the paper characterises economic conditions under which patents may decrease the growth rate. The model is calibrated to the US economy to match its long run time series of GDP per hour. Simulations of that calibrated model provide important insights regarding growth promoting policies. The model also provides an explicit numerical algorithm to measure cross-country differences in total factor productivity due to barriers to technology adoption.

Key Words: Embodied Technology, Monopoly Rents from Innovation, Cost of Copying and Cost of Patents

JEL Codes: E23, E69, O34, L16, L51

---

[1] This paper originates from the M.Com dissertation which Andrew Binning wrote under the supervision of Debasis Bandyopadhyay at the University of Auckland.

1. Introduction

Sustained growth of GDP per hour has been the focal point in Macroeconomics for some time now. Economists have known for nearly fifty years that growth in technology is the primary determinant of GDP growth. Despite the importance technology growth, it has been only recently that attention has focussed on its determination within a macroeconomic framework. This paper is concerned with the determination of productivity growth through innovation and, in particular, the role that intellectual property rights play in this relationship. For a long time it has been a commonly held belief that growth through innovation requires the existence and enforcement of stringent intellectual property rights. Recently a second school of thought has emerged that believes growth can and does take place with out intellectual property rights. The contribution this paper makes is to set up a general equilibrium model capable of analysing these competing schools of thought and their implications for growth. The question this paper asks is can there be growth with out patents? To answer this question the paper offers a new approach to modelling endogenous innovation that allows for a more complete investigation of the relationship between market structure, the incentives to innovate in presence of patents and the growth rate of GDP. Given that the market structure can be influenced by the degree of intellectual property rights, there may be a role for policy to enhance the growth rate of GDP with a model such as this for providing useful insight into effective policy design.

It has been well documented that technological change is the dominant force behind GDP growth. Solow (1957) finds that Total Factor Productivity growth accounted for nearly 90% of GDP growth in the United States between 1908 and 1949, others have found similar results in different countries and over different time periods. Until quite recently, the large majority of growth models were capable of explaining many attributes of an economy, but left one feature, if not the most important feature unexplained, the growth rate of technology. Prescott (1998) investigates the importance of technology in explaining cross-country output differentials leading him to call for a theory of its determination. On a similar note Quah (2002a) has called for the explicit modelling of productivity as opposed to its treatment as a residual to be minimised. It has become clear that any serious theory of productivity must take into account it's determination within a model, hence its endogenous determination.

Romer (1990) was the first to recognise technology's determination by the decisions of individuals. He states three premises a theory of technology should satisfy. First, growth in technology is the source of GDP growth. Second, growth in technology comes about through the actions of rational maximising individuals responding to incentives and third, technology is non-rival, in relation to the cost of its discovery. It can be consumed by others at little additional cost. Romer (1990) mentions that models of exogenous growth are capable of accounting for the first and third premise, but the second requires something extra, something only a theory of endogenous productivity growth can account for, an incentive to innovate. The problem now becomes the creation and protection of incentives to innovate; this requires an understanding of the relevant properties of technology. The main body of literature has adopted the notion that technology is non-rival and partially excludable and through this characterisation has managed to justify the existence and enforcement of stringent intellectual property rights as a necessary condition for growth. A second school of thought has recently emerged, that has challenged this assertion. Boldrin and Levine (2003) have shown, based on technology's embodiment that innovation can and does take place without patents.

As mentioned, those that follow conventional wisdom have classified technology as both non-rival and partially excludable, using this near public good characterization to justify stringent patent laws.[2] Technology is considered non-rival, since the cost of reproducing or copying it is sufficiently small if nonexistent in relation to the cost of its discovery. Technology is partially excludable in the sense that the technology specific to a good can be patented, preventing its replication by competitors. However this does not prevent competitors from studying a specific technology and using it to invent new, separate and sufficiently distinct technologies. Intellectual property rights are seen as a necessary evil, while excluding many from the benefits of a new technology, they provide the required incentives for individuals to undertake innovation; that is a guarantee of monopoly rents, a reward for the risk and the costs associated with the discovery of a new technology. Models that rely on patents to create incentives for innovation, implicitly assume their perpetual duration. They make the additional assumptions that production occurs at full capacity upon the products release and that the adoption and diffusion of technology is instant. With out patents Bertrand competition ensues, profits are driven to zero instantaneously and the incentive to innovate disappears. By construction these models lead to the conclusion that innovation can not and does not take place without patents.

In recent years a second less prominent school of thought has emerged that believes innovation can and does take place without intellectual property rights.[3] Their argument relies upon the assumption that all technology is embodied, either in people or in goods. That is technology only has economic value through its embodiment, technology not embodied is irrelevant. Technology's embodiment means that it is both rival and excludable. It is rival because is it is costly to copy, it takes time for some one to learn a new idea, possibly requiring them to read a book, or to take apart the good in question so that it can be reverse engineered. Embodied technology is considered excludable because everyone other than the owners can be prevented from using it. Human capital is a special form of embodiment of technology. If an idea is embodied within a person its use can be prevented by not disclosing it. Similarly, quality of machines reflects another kind of embodiment. Technology's embodiment means that it is finitely expansible, that is it is capacity constrained, not enough can be produced in the initial periods to meet demand. Technology takes time to diffuse; firms must adopt the new technology before its use becomes widespread. All these properties of technology ensure that a successful innovator has a first mover advantage; that is they are able to make monopoly rents until competitors enter, which may take some time, so that the innovator does not need a patent. Innovation takes place so long as the revenues generated by an innovation cover

[2] For a good description of the conventional approach to technology see Romer (1990)
[3] For a good description of this alternative view of technology see Boldrin and Levine (2003) and (2004).

the cost of its development. Given that it takes time for competitors to enter, the successful innovator should be able recoup the cost of the innovation.

It is Quah's (2002b) rejection of the Boldrine and Levine (2003) conjecture when the copying process is extremely efficient that provides the motivation for this paper. While Quah is not an advocate for tough patent laws, his paper provides some justification for conventional wisdom under certain circumstances, recognition of the merits of both schools of thought. The aim of this paper is to set up an empirically tractable general equilibrium model that is capable of analyzing these competing views on technology and intellectual property rights and subsequently their effect on growth. In particular this paper aims to answer the question, can there be growth without patents? In a similar fashion to Boldrin and Levine (2003) a parameter is included in this model to capture the impact of barriers to entry on the profits of the innovating firm. These barriers to entry may include the intellectual property rights regime and the natural complexity of technology among other things.[4] The contribution this paper makes is the adaptation of the intuition behind the Aghion, Harris and Vickers (1997) "step by step" method of endogenous innovation to a more complete general equilibrium model, and subsequently using this model to examine the effect barriers to entry have on the growth rate of technology.[5]

The degree and scope of intellectual property rights could play an important role in the determination of the growth rate of technology and hence GDP growth. The model developed in this paper could have interesting and important implications for policy analysis. Given intellectual property rights can determine the market structure of an industry the model formulated in this paper could prove useful in showing how barriers to entry could affect an economy and the growth rate of technology. It may also prove useful in explaining cross country differences in income levels and growth rates especially where there is a known divergence in policy that may affect barriers to entry.

This paper proceeds as follows; the second section reviews relevant papers in the literature and includes a review of the conventional approaches taken to endogenising the growth rate of technology and the method that inspired the approach taken in this paper. The third section develops the model used in this paper, while the fourth section discusses the quantitative analysis. The fifth section performs some simulations to determine the properties of this model and the final section concludes. Two additional sections are included after the conclusion to look at extensions of the model which include using human capital in research and development and replacing the fixed saving rate assumption with intertemporal utility maximisation.

2. Literature Review

This section investigates the different approaches used in the literature to model the relationship between innovation and growth in technology. It begins by examining the

[4] Additional barriers to entry could include imperfect capital markets.

[5] Boldrin and Levine have yet to specify a fully endogenous framework for modelling the innovation process.

seminal works of Romer (1990) and the Schumpeterian "leapfrogging" approach to endogenising technology. Special attention is given to their set up and the mechanism used to endogenise the growth rate of technology. The alternative views and methodologies of Boldrin and Levine (2003) are investigated with in one of their models. The paper of Quah (2002b) is discussed briefly because it brings into question some of the arguments made by Boldrin and Levine and provides some justification for the existence of both schools of thought. The step by step approach developed by Aghion, Harris and Vickers (1997) is investigated because it is more conducive to modelling competition's effects on the growth rate, given the degree and scope of intellectual property rights will affect the market structure and the level of competition. This section concludes by looking at the vintage capital model, the framework chosen to model endogenous innovation in this paper, due to Boldrin and Levine's (2003) characterisation of technology.

Romer (1990) was among the first few to endogenise the rate of innovation in a model of monopolistic competition. The model is sectoral by design, consisting of three sectors, a perfectly competitive final goods producing sector, a monopolistic capital goods producing sector and a perfectly competitive research and development sector. Technology is measured by the number of designs in the economy where innovations are horizontal, increasing the variety of capital goods. Designs are produced by firms in the research and development sector and sold to firms in the capital goods producing sector. Upon purchasing the design, the capital goods producer becomes a monopolist, guaranteed the sole right to produce capital goods of that variety, retaining this right for eternity. Human capital is an input in both the production of final output and the discovery of new designs. The allocation of resources, namely human capital to research and development is the key determinant of growth in technology. This allocation is determined through an arbitrage condition that ensures in equilibrium human capital is paid the same wage regardless of their sector of employment.[6] Human capital in the final goods producing sector is paid their marginal product while human capital in the research and development sector is paid the expected discounted future stream of earnings an innovation generates. Clearly anything that affects the wage for human capital in the final goods producing sector and the future stream of earnings an innovation generates is going to affect the allocation of human capital to research and development and hence the growth rate of technology. The monopoly rents that accrue to the capital goods producer from owning the rights to produce a particular design and hence the wages paid to human capital engaged in R & D are guaranteed by the implicit assumption that patents last for ever. Romer's use of a static set up in the capital goods producing sector, assumes the instant adoption and diffusion of technology, and production at full capacity upon the products release. Without patents the capital goods producing sector reverts to Bertrand competition, firms earn zero profits and all human capital is allocated to production in the final goods producing sector so that no innovation takes place.

In recent years the Schumpeterian creative destruction or "leapfrogging" approach has emerged as the dominant method for endogenising the dynamic innovation process.

[6] The model implies that human capital used in research and development is a perfect substitute for human capital used in the production of final output.

Three prominent models that have used this mechanism include Grossman and Helpman (1991), Aghion and Howitt (1992) and Aghion and Howitt (1998b). The general set up of these models is similar to Romer (1990) with the economy divided into sectors contingent upon their function. However these models differ in their description and determination of technology. Their use of "quality ladders" means that innovations are vertical and sequential, raising the quality of technology in multiplicative increments. As in Romer (1990) only firms in the R and D sector undertake research. The capital goods producing sector is serviced by a monopolist that does not undertake research because it is unprofitable for them to do so, that there they have no incentive to innovate.[7] The innovation process is dynamic; it takes place in a "leap frogging" fashion. A successful innovator discovers a new higher quality of technology unseating the incumbent capital goods producer taking their position. The new capital goods producer produces capital goods while they hold the quality lead and are over taken when the next quality of capital is discovered. As in Romer (1990) the amount of research and development undertaken and hence the flow rate of innovation is determined by the allocation of resources to research and development through an arbitrage condition.[8] The reward a successful innovation generates determines the allocation of resources to research and development.[9] Again the existence and enforcement of patents is a necessary condition for these rewards, the incentive to innovate and hence the growth of technology. By construction these models lead to the conclusion that innovation can only take place when stringent intellectual property rights exist.

Boldrin and Levine (2003) set up their own model demonstrating that innovation can and does take place with out patents. They begin by assuming all technologies are embodied in goods. The goods can either be consumed or used in the production of more goods, introducing a degree of rivalry. Boldrin and Levine show that the price of a good when first released is positive and that innovation always takes place so long as the initial cost of development is covered by the sale of the first few units. This is satisfied in their model by the use of log utility which ensures price is bounded away from zero. The combined result is the innovator is guaranteed a degree of market power, the result of a

[7] This is shown by Grossman and Helpman (1991).

[8] Again, the resources allocated to the production of final output and research and development are perfect substitutes.

[9] The term "resources" devoted to research and development has been used in place of "human capital". In recent years the literature has moved away from using human capital and labour in the determination of research and development and instead has adopted the use of a fixed proportion of final output as the only input in research and development. This change is due to Aghion and Howitt (1998a) showing that the use of a Romer (1990) style human capital arbitrage condition leads to the conclusion that an increase in physical capital accumulation results in a decline in the growth rate of technology. This some what counter intuitive result is due to the marginal product of human capital in the final goods producing sector being an increasing function of physical capital. An increase in physical capital accumulation, all else constant, would result in an increase in the marginal product of human capital and hence the wage paid to human capital in equilibrium. An increase in the wage of human capital in the final goods producing sector results in the flow of human capital away from innovation and into production. The decline in human capital devoted to research and development causes a decline in the growth rate of technology and hence the some what perverse result that physical capital accumulation causes a decline in the growth rate of technology. The use of a fixed fraction of final output as the only input in research and development over comes this problem by ensuring both physical capital and human capital are used indirectly in research and development in the same proportions used in the final goods producing sector.

first mover advantage, consumers' impatience and a small degree of rivalry involved in the copying process. Their initial market power which eventually deteriorates enables them to at least cover the costs of undertaking research and development so that innovation takes place. Boldrin and Levine have yet to explain the endogenous determination of the innovation process and instead use models where the growth rate of technology is only partially endogenous, a function of consumers' preferences.

Although not specified in their paper, Boldrin and Levine's (2003) conclusions depend on their use of discrete time. Quah (2002b) repeats their analysis in continuous time.[10] He finds the conclusions change under continuous time. Using discrete time the price is bounded away from zero in the first period after the products release even with efficient copying processes. This is usually long enough to recoup the cost of an innovation. Likewise under continuous time, prices are bounded away from zero in the first period but time lengths are no longer fixed. An increase in the copying efficiency sees the length of time that prices remain above zero decrease and as the copying process gets extremely efficient, the amount of time the price remains strictly positive goes to zero in the limit. Boldrin and Levine's use of discrete time implies that demand and hence prices for each time period are fixed but can change at the start of each period. It is their use of discrete time that imposes an inefficiency on the copying process ensuring profits are positive for long enough to cover the cost of innovation. One of the main points that Quah highlights is that innovation will not take place in continuous time when the copying process is extremely efficient and the fixed sunk cost of an innovation is sufficiently large. Boldrin and Levine refer to this as the indivisibility of an idea. While the work of Quah (2002b) and Boldrin and Levine (2003) has redefined the way technology can be thought of, Quah and Boldrin and Levine have yet to specify how the innovation process could and should be endogenised in a growth model. The endogenising of innovation in a manner that is consistent with the Boldrin and Levine conjecture requires looking in a different direction.

Recently some theoretical papers have modelled the relationship between competition and innovation. Aghion, Harris and Vickers (1997), troubled by empirical evidence supporting a relationship between competition and innovation, abandoned the leapfrogging approach in favour of the "step by step" approach to innovation. This method is more conducive to endogenising the relationship between innovation and product market competition, the likely outcome of relaxing patent laws. They set up a model of static duopoly, where both firms are assumed symmetric. The product market can be in one of two states, either one firm holds an out right quality lead in which case they make monopoly profits, while the other firm, one step behind in terms of their product quality, makes zero profits. Alternatively if both firms jointly hold the quality lead they make duopoly profits which are larger than zero but less than monopoly profits. Firms are required to be at the technological frontier if they are to innovate. If a single firm holds the quality lead, they enjoy monopoly profits with no incentive to innovate. If a firm is a laggard they have an incentive to catch up and make monopoly profits. If both firms are at the frontier they have an incentive to innovate, that is the monopoly profits from holding the quality lead outright. While their model is analytically complicated and

[10] Quah (2002) uses Boldrin and Levine's (2003) model from an earlier draft.

somewhat incomplete,[11] it is the intuition they develop that is of importance. Firms innovate to escape competition, so that increasing competition increases the rate of innovation. However, too much competition leads to too much innovation and a shorter duration of monopoly profits, destroying the incentive to innovate.[12] Aghion, Harris and Vickers (1997) find an inverted "U" shape relationship between the growth rate of technology and degree of product market competition.[13]

The model of Aghion, Harris and Vickers (1997) and its subsequent extensions provide the intuition required to endogenise innovation when competition is present.[14] The problem however is the complicated nature of their model.[15] To gain a better understanding of how intellectual property rights affect the growth rate, a more complete framework is required. The Boldrin and Levine conjecture requires that technology be embodied in either goods or in people. Vintage capital models are built on the assumption that some if not all technology is embodied in capital goods. Restricting attention to technology embodied in capital goods allows the use of the vintage capital model to examine the Boldrin and Levine conjecture.

The earliest and simplest vintage capital model is due to Solow (1960). Based upon his classic 1956 model, Solow extends the framework to cover the case where improvements in capital augmenting technology only take place in new capital goods. More recently Hulten (1992) and Greenwood, Hercowitz and Krusell (1997) have extended this model further to include both neutral and investment specific technologies. The models of Krusell (1998), Boucekkine, del Rio and Licandro (2000), (2004) Boucekkine and delaCroix (2000) and Hsieh (2001) have all, based on the Schumpeterian "leapfrogging" approach, attempted to endogenise the growth rate of technology in a vintage capital model using the Greenwood, Hercowitz and Krusell (1997) framework.

Vintage capital models are characterised by their heterogeneous capital stocks, differentiated by quality. Standard growth models that incorporate Hicks, Harrod and Solow neutral technology exhibit homogeneous capital stocks, so that capital is indistinguishable by date of construction and participates equally in the production of final output. Solow's vintage capital model displays a special case of capital augmenting technology. The usual form of capital augmenting technology implies improvements in technology are capital saving and affect all capital goods in the same way. The vintage

---

[11] Their model is a general equilibrium model, but it lacks a capital market, the final goods producing sector is imperfectly competitive and the interest rate and wage are exogenous. A perfectly competitive final goods producing sector is consistent with the National Accounting framework.

[12] This is the well known Schumpeterian creative destruction effect in action, the rewards an innovating firm can expect to receive decline in the amount of innovation. Too much innovation sees the time that a successful innovator remains in the lead decline so that the reward from innovating also declines.

[13] Aghion, Harris and Vickers (1997) measure product market competition using the Lerner index, a measure of market power.

[14] When patent laws are relaxed it is likely that competition will increase since it will become more difficult to exclude other firms from producing in the capital goods producing sector. Hence any model that is to deal with the effects of reducing intellectual property rights must also deal with innovation under some competition.

[15] It is more reminiscent of the "patent race" models of Industrial Organisation than of a growth model, with the interest rate and wage both exogenous.

capital model is separated as a special case because improvements in technology only affect the leading vintage of capital, making it more productive than previous vintages. This is due to each vintage of capital embodying the leading technology at its date of construction, so that capital goods can be indexed by time. The technology embodied in the leading vintage grows over time, making new capital more valuable than old with no further improvements taking place in the quality of capital already in existence. Each vintage of capital depreciates at the standard rate so that the quantity of a particular vintage of capital used in production declines with time. This suggests that newer vintages of capital have a higher participation in the production of final output than older vintages. The aggregate capital stock is constructed by summing over the surviving quantities of the different vintages of capital in existence at a given time and can be thought of as an effective capital stock because of the embodied technology.

Jorgenson (1966) raises the problem of identifying embodied technology in vintage capital models. Identification is a common problem in both econometrics and calibration and results from different sets of structural parameters being compatible with the same set of data. The problem arises in vintage capital models because it is difficult to identify which part of technology growth is due to embodied technology and which part is due to disembodied technology. Given the long run growth rate of GDP per hour there are infinitely many combinations of embodied and disembodied technology growth that would be consistent with the data. Jorgenson (1966) shows using a variant of Solow's vintage capital model that the inverse of quality adjusted capital prices provide an index of embodied technology. Greenwood, Hercowitz and Krusell (1997) develop a quality adjusted capital price series based on Gordon's (1990) data. Using this index they are able to pin down the growth rate of embodied technology allowing them to establish the growth rate of disembodied technology that is consistent with the data. The method of calibration is used to match their model to certain features of the United States economy. They find that nearly 60% of all technology in the United States is investment specific, significantly larger than Hulten's (1992) finding of 20%.

3. The Model Economy

This paper presents a vintage capital model of endogenous embodied technology growth. The model has been developed to investigate the effects of barriers to entry namely intellectual property rights on the growth rate of technology. The model's novelty is in its treatment of innovation without intellectual property rights. Insight from Boldrin and Levine (2003) is combined with intuition from Aghion, Vickers and Harris (1997) and (2001) to model the innovation process in an original manner. The model's basic vintage capital structure is similar to Boucekkine and DelaCroix (2003), Boucekkine, del Rio and Licandro (2000), Solow (1960) and Greenwood, Hercowitz and Krusell (1997).[16] The addition of human capital as a factor of production in the manufacture of final goods means this model can be thought of as a vintage capital version of Mankiw, Romer and Weil (1992).

[16] The model developed in this dissertation is able to replicate many of the results in Greenwood, Hercowitz and Krusell (1997) through certain parameter restrictions, see Appendix II.

The model economy is sectoral consisting of a representative household and two production sectors, a representative final goods producing sector and a representative capital goods producing sector. The final goods producing sector is perfectly competitive, there are infinitely many firms that produce a single homogeneous product, using quality capital, labour and human capital as inputs in production. These firms demand quality capital goods from the capital goods producing sector taking their price as given. Firms buy machinery paying for the quantity and receiving the embodied quality for free. In addition they take the factor prices of labour and human capital as given. Labour and human capital are supplied by households to the final goods producing sector and raw physical capital is supplied by households to the capital goods producing sector. Firms in the final goods producing sector choose allocations of these factors to maximise profits.

The use of vintage capital in this model economy means both capital goods and the capital stock are heterogeneous. A new variety of capital good is produced each year which embodies the leading quality of technology at its date of construction. Capital goods exist in quantities proportional to their age and differ in productivity depending on the leading technology available at their date of construction. Improvements in embodied technology only take place in new machinery in contrast with models that use the more general specification for capital augmenting technology.[17] Capital goods produced today have a higher participation in the production of final output in comparison with those produced in previous periods.

The growth rate of embodied technology is determined in the capital goods producing sector. The capital goods producing sector consists of two types of firms existing in various states of oligopoly at any given time. Both types of firm produce quality capital goods by costlessly adding technology to raw capital supplied by households. The first type of firm, of which there is only one in this representative sector, is known as an innovator with the remaining firms known as copying firms. The innovator is the only firm able to conduct research and development to discover new qualities of technology, the remaining firms copy the innovator's technology. The amount of research and development carried out by the innovating firm and hence the growth rate of embodied technology depends on the incentives to innovate. When the innovating firm decides to release a new quality of capital they incur a fixed sunk cost associated with the discovery of this technology and they become by default a monopolist. They retain this first mover advantage until the copying firms learn to copy the technology and enter the market so that the industry becomes oligopolistic. Consequently it is the erosion of the innovator's market power and hence profit or more specifically the speed of this erosion that plays a key role in determining the innovation rate.

The technology discovered by the innovating firm may be inherently difficult to copy. Frictions in the copying process may result from protection by intellectual property rights or the natural complexity of a technology. Copying firms observe the new technology, imitating it at some cost which may be a realised fixed sunk cost or the opportunity cost

[17] This is true for models of labour augmenting and neutral technological change where improvements in technology affect all machinery in the same way.

of the profits foregone when learning to copy the technology.[18] This cost means firms take some time to copy the technology and enter the market. This will be referred to as "barriers to entry" and is a function of both the intellectual property rights regime as well as the natural complexity of the technology. The innovating firm chooses the duration of an innovation to maximise the discounted stream of future profits. A reduction in the barriers to entry allows firms to enter the capital goods producing sector at a faster rate, hence speeding up the decline in an innovator's profits. The innovating firm earns larger gross profits per unit of time in the early stages of an innovation's life while they retain market power, in the later stages of an innovation's life the gross profit per unit of time declines so that they must innovate more frequently to retain market power and remain in the earlier phase of the innovation when barriers to entry are relaxed. The feature that prevents firms from innovating too much is the fixed sunk cost of an innovation; the firm must weigh up the gain in extra profits from a new innovation with the cost of discovering that innovation. When barriers to entry are sufficiently low, the erosion of profits will be too swift so that the fixed cost of the innovation will dominate the decision to innovate resulting in less innovation taking place.

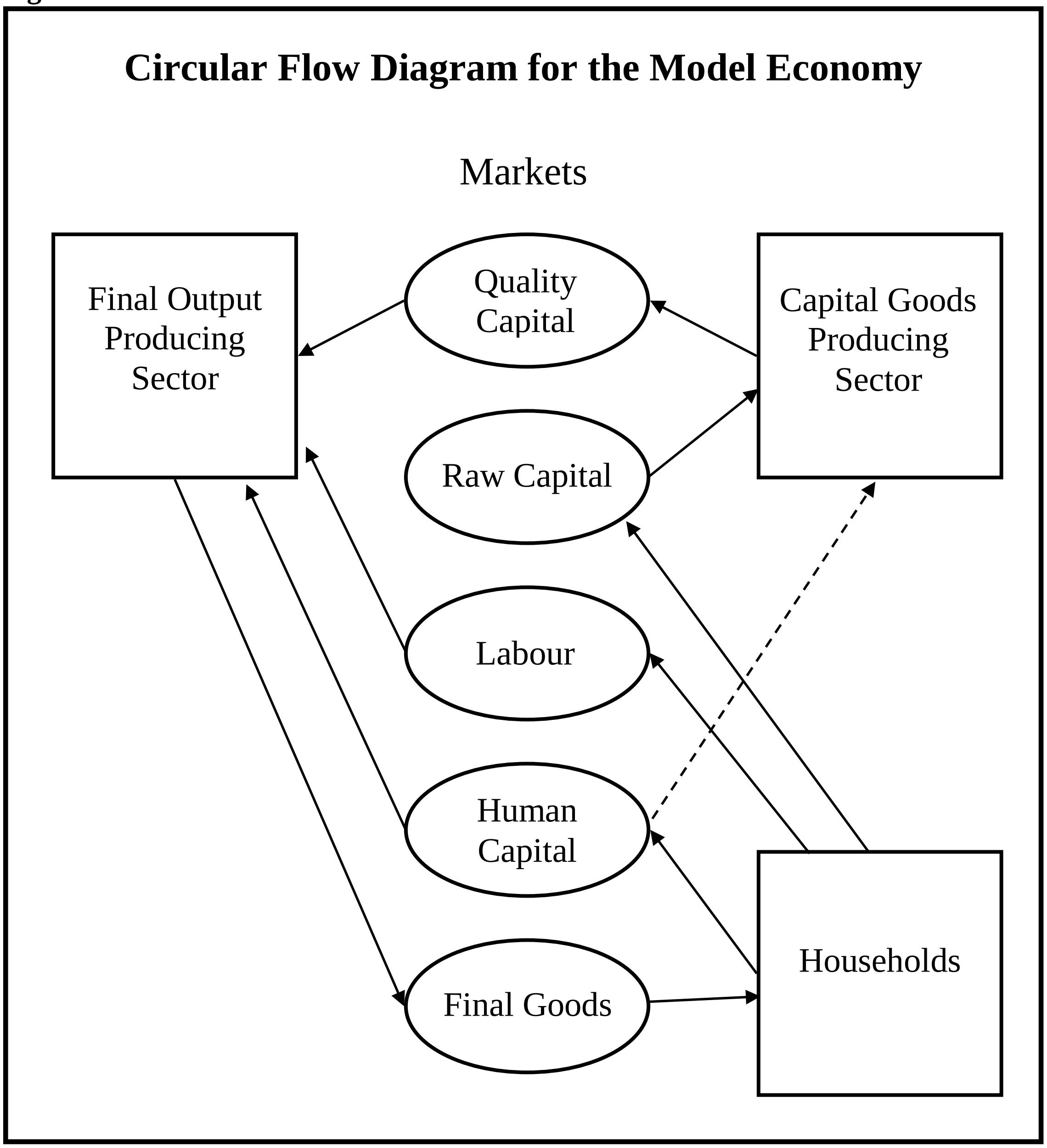
The structure of the capital goods producing sector and the mechanism used to endogenise technology rely on the key assumption that the innovator is the only firm able to release innovations. This is quite a strong assumption and is used to simplify the modelling of the innovation process in a manner that is consistent with the Boldrin and Levine (2004) and Aghion, Harris and Vickers (1997) frameworks. The addition of some kind of "leapfrogging" or overtaking mechanism due to more than one firm being able to innovate would complicate things unnecessarily at this stage. One could think of the innovator as being the only firm able to innovate because they have held the technology lead for the longest, allowing them to release innovations before they are overtaken due to better knowledge and understanding of the technology than their competitors. While this assumption may not be wholly realistic and its relaxation could see the results generated by the model change, it does however fit the purposes of this paper.

Figure 1 below gives a diagrammatic representation of this model economy. There are five markets where interaction between households, the capital goods producing sector and the final output producing sector take place. The first four markets from the top are for the factors of production. These markets include the market for quality capital goods, the market for raw capital, the labour market and the market for human capital. The last market at the bottom of figure 1 is the final goods market. Households supply human capital and labour which are demanded by firms in the final goods producing sector. Households supply raw capital to firms in the capital goods producing sector. Firms in the capital goods producing sector costlessly add technology to raw capital to produce quality capital which is then supplied to the final goods producing sector. Firms in the final goods producing sector own their capital, so that they pay themselves an imputed rental. A dashed line has been used to link the market for human capital with the capital goods producing sector. This is because the model is extended to relate the fixed sunk

[18] That is the copying firms were learning to copy the technology, so that they were unable to produce anything during this period.

cost of an innovation to the fixed amount of human capital required to produce an innovation and the rental rate of human capital in the final goods producing sector.

Figure 1.



Given this is a model of endogenous productivity growth, the primary objective of the subsequent sections is to derive the growth rate of technology, the following describes the steps in which this is done. The derivation of the growth rate of technology requires a closer inspection of the capital goods producing sector. It is the profit maximising actions of the innovating firm that determine the growth rate of technology in this model. To determine their actions requires the modelling of the innovator's profits, a function of the demand for new capital goods. The description of this model begins in the final goods producing sector where the demand for embodied capital goods is derived as part of the first step. The second step uses this demand to determine the gross profits of a capital goods producing firm. The third step involves working out the period profits of an innovating firm and the fourth and final step sees the innovator choose the optimal length of an innovation to maximise their period profits. The length of an innovation they choose is used to determine the rate of innovation and the growth rate of technology.

3.1.1 Final Goods Producing Sector

This section characterises the final goods producing sector by describing the supply of final output and factors used in its production.

Final output Y_t is produced using labour L_t, human capital H_t and physical capital K_t and supplied according to the following Cobb Douglas production function where B is a scaling parameter.[19] [20]

$$Y_t = BK_t^{\alpha} H_t^{\beta} L_t^{1-\alpha-\beta} \tag{1}$$

This production function resembles Mankiw, Romer and Weil (1992). It is also similar to Romer (1990) except embodied capital is used in place of the composite intermediate good. [21]

Labour L_t is supplied inelasticically and is assumed to grow at the constant rate n where L_0 denotes the initial endowment of labour at date 0, so that

$$L_t = (1+n)^t L_0 \tag{2}$$

Let I_t^H denote investment in human capital and δ the depreciation rate of both physical and human capital

Human capital H_t is supplied by households and grows according to the law of motion

$$H_t = I_{t-1}^H + (1-\delta) H_{t-1} \tag{3}$$

The capital goods producing firms costlessly add q_t units of quality to each unit of raw capital where q_t is an index representing the level of embodied technology at date t. The

[19] See page 131 for a complete list and description of the parameters and variables used in this model.

[20] The parameter B could be given a time subscript and interpreted as neutral technology.

[21] Romer's (1990) production function is given by $Y_t = BH_{Y,t}^{\beta} L_t^{1-\alpha-\beta} \int_0^{\infty} x_i^{\alpha} di$, where physical capital is disaggregated into durable intermediate goods and $\int_0^{\infty} x_i^{\alpha} di$ is the sum of all intermediate durable goods in use.

final goods producing firms demand X_t units of new embodied machinery from the capital goods producing sector at date *t*.

Physical capital K_t grows according to the law of motion

$$K_t = q_t X_t + (1-\delta)K_{t-1} \tag{4}$$

3.1.2 The Firm's Problem

This section characterises the perfectly competitive final goods producing sector by deriving the demand for embodied capital, labour and human capital, where the demand for new capital goods will be used to determine both the profits of capital goods producers and the growth rate of embodied technology. At each date *t*, the representative firm for the model economy takes implicit rental prices P_t^K for all vintages of capital goods, the wage rate w_t and the rental price of human capital r_t^H as given and chooses human capital, labour and physical capital so as to solve the following maximisation problem.

$$\Pi_t = BK_t^{\alpha} H_t^{\beta} L_t^{1-\alpha-\beta} - P_t^K K_t - r_t^H H_t - w_t L_t \tag{5}$$

The First Order Conditions are given by

$$\frac{\partial \Pi_t}{\partial K_t} = \alpha B K_t^{\alpha-1} H_t^{\beta} L_t^{1-\alpha-\beta} - P_t^K = 0, \tag{6}$$

$$\frac{\partial \Pi_t}{\partial H_t} = \beta B K_t^{\alpha} H_t^{\beta-1} L_t^{1-\alpha-\beta} - r_t^H = 0, \tag{7}$$

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha-\beta) B K_t^{\alpha} H_t^{\beta} L_t^{-\alpha-\beta} - w_t = 0, \tag{8}$$

From (6) the price of capital services is given by;

$$P_t^K = \alpha B K_t^{\alpha-1} H_t^{\beta} L_t^{1-\alpha-\beta} \tag{9}$$

This is related to Jorgenson's (1963) neoclassical theory of investment which will be discussed below. Because firms own their machinery the price of capital services is an imputed rental firms pay to themselves.

From (7) the inverse demand for human capital is given by;

$$r_t^H = \beta B K_t^{\alpha} H_t^{\beta-1} L_t^{1-\alpha-\beta} \tag{10}$$

From (8) the inverse demand for labour is given by;

$$w_t = (1-\alpha-\beta) B K_t^{\alpha} H_t^{\beta} L_t^{-\alpha-\beta} \tag{11}$$

Rearranging equation (9) gives the demand for physical capital

$$K_t = \left[\frac{\alpha B H_t^{\beta} L_t^{1-\alpha-\beta}}{P_t^K} \right]^{\frac{1}{1-\alpha}} \tag{12}$$

Rearranging equation (10) gives the demand for human capital

$$H_t = \left[\frac{\beta B K_t^{\alpha} L_t^{1-\alpha-\beta}}{r_t^H} \right]^{\frac{1}{1-\beta}} \tag{13}$$

Rearranging equation (11) gives the demand for labour

$$L_t = \left[\frac{(1-\alpha-\beta) B K_t^{\alpha} H_t^{\beta}}{w_t} \right]^{\frac{1}{\alpha+\beta}} \tag{14}$$

Letting C_t denote aggregate consumption, for completeness aggregate demand is given by

$$Y_t = C_t + I_t^H + X_t \tag{15}$$

All markets clear so that (12) equals (4), (13) equals (3), (14) equals (2) and (15) equals (1).

Households supply raw physical capital to the capital goods producing sector which is used to produce embodied capital. Households also save a fraction of the final good to be invested in human capital. The following additional clearing conditions are assumed to hold.

Let s_K denote the saving rate for physical capital so that investment in physical capital is given by

$$X_t = s_K Y_{t-1} \tag{16}$$

The Solow (1956) fixed saving rate assumption has been used for simplicity.[22] Capital goods producing firms demand X_t units of raw capital while households supply $s_K Y_{t-1}$ units of investment. From (16) the household's supply of raw capital equals the demand for raw capital via the flexible adjustment of the market interest rate.

Let s_H denote the saving rate for human capital so that investment in human capital is given by

$$I_t^H = s_H Y_t \tag{17}$$

This is the standard fixed saving rate assumption as used in Mankiw, Romer and Weil (1992). This has been used for simplicity and like the equilibrium condition for investment in physical capital holds through the flexible adjusted of the market interest rate.

[22] When introducing their new generation of endogenous growth models in the Solow framework, Howitt (1998) and Aghion and Howitt (1998b) make use of the Solow fixed saving rate assumption for simplicity. It allows their model to be compared to Solow (1956) and assists readers understanding of the new ideas and concepts they develop. They make a trade off between including additional complexities in the neoclassical framework by endogenising the growth rate and simplifying the representative household's problem. In later papers, once their framework has been recognised and understood they include intertemporal utility maximisation for the representative household. In a similar vein, the Solow fixed saving rate assumption has been used in this dissertation for simplicity. A trade off is made to aid readers comprehension, a complex and new method of endogenising technology is developed while the consumers problem is reduced to the constant saving rate assumption. At a later stage intertemporal utility maximisation for the representative household could be included in the model developed in this dissertation where log utility would result in the same outcome as the constant saving rate assumption. The foundations for this are developed in an extension in section 8 in this dissertation.

As mentioned there are as many qualities or vintages of physical capital in use by firms in the final goods producing sector as there have been time periods. This is a result of a new vintage of capital good being produced at each date with the surviving stock of each vintage depreciating at the standard rate δ. Consequently the representative firm chooses allocations of human capital and labour to be used with each vintage of machinery such that the marginal products of all human capital and labour are equated regardless of the vintage of machinery they are used with.[23] Any differential would be eliminated through arbitrage.

More conventional growth models leave out the dynamic considerations of purchasing a new machine which yield benefits long after its acquisition. The demand for a new machine is derived following Jorgensen's (1963) neoclassical theory of investment.[24] The firm decides to add to its inventory new machinery by comparing the marginal cost and the present discounted value of all future benefits from that machine by solving the following problem.

Define P_t as the discounted sum of the future rentals of physical capital P_t^K adjusted for the surviving quantity of date t vintage capital so that the inverse demand for vintage t capital is given by

$$P_t = \alpha B q_t^{\alpha} X_t^{\alpha-1} \sum_{u=t}^{\infty} \left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u} \right] H_{t,u}^{\beta} L_{t,u}^{1-\alpha-\beta} \equiv P_t(X_t), \tag{18}$$

where $R_t^t = 1$ and $R_t^u = \prod_{\tau=t+1}^{u} (1+r_\tau)$ and a subscript on human capital and labour denotes their allocation to vintage t capital at some future date u where $u \geq t$. A full derivation of equation (18) is given in Appendix I and the interest rate r_t in Appendix II.

Equation (18) represents the inverse demand for vintage t capital goods at their date of construction. It is the industry demand for new capital goods that firms in the capital goods producing sector face. The demand for vintage t capital goods at date t reflects the discounted future stream of earnings a unit of vintage capital generates over its useful life in combination with its corresponding equilibrium allocations of both human capital and labour. The inverse demand for vintage t capital at date t is a function of date t embodied technology, the depreciation rate, the real interest rate and the future allocations of human capital and labour used with vintage t capital in the production of final output. If the real interest rate increases demand declines because the future income streams generated by vintage t capital are discounted by a larger amount. If the rate of depreciation increases, machines wear out at a faster rate so that the future stock of vintage t capital and hence the decline in earnings generated is quickened resulting in a lower demand for vintage t capital at date t. If future allocations of either human capital or labour used in

[23] See Appendix I for a full derivation.
[24] See Appendix I for a full derivation.

conjunction with vintage t capital increase then demand for vintage t capital also increases.

Now given this derived demand function the focus will shift to the capital goods producing sector where the profits of the capital goods producing firms will be determined.

3.2 The Capital Goods Producing Sector

This section describes the production of machinery in the capital goods producing sector which is supplied to firms in the final goods producing sector. The equilibrium quantity of new capital goods produced X_t and their corresponding price P_t will be determined. The capital goods producing sector exists in various stages of oligopoly at any given time depending on barriers to entry and the time since an innovation's release among other things. As mentioned, machinery is produced by two types of firms, both solve the same basic gross profit maximisation problem choosing the quantity of capital to produce, this is described below. [25]

The capital goods producing sector is oligopolistic consisting of J symmetric[26] capital goods producing firms at any point in time. When J equals 1 the capital goods producing sector is serviced by a monopoly, as J gets larger this sector begins to approximate perfect competition. Each firm produces an identical capital good that embodies the leading technology available at its date of construction.[27] Raw capital is demanded from households and technology added to create quality capital goods which are sold to firms in the final goods producing sector. Firms in the final goods producing sector demand capital goods paying for the quantity and receiving technology free. Firms in the capital goods producing sector face the industry demand for this new capital good. The equilibrium price and quantity of capital goods produced are determined by solving the typical jth firm's problem under Cournot competition.

The total revenue of firm j is equal to the inverse market demand multiplied by the quantity of capital goods firm j produces denoted by X_{jt}. As is standard in the Cournot methodology the jth firm does not face the entire market demand unless the capital goods producing sector is monopolistic. Instead the jth firm faces their residual demand, that is they face their share of demand not served by the other J -1 firms.

Let $\Pi_{j,t}$ denote the gross profit of the jth firm in the capital goods producing sector at date t, $P(X_t)$ is the inverse market demand for capital goods, X_{jt} denotes the residual demand

[25] Only one firm belongs to the first type, this firm is called an innovator and carries out research and development to discover new higher qualities of embodied technology as well as producing capital goods. The remaining firms, called copiers, imitate the technology first discovered and produced by the innovator.

[26] Firms are symmetric in gross profits and hence their gross profit maximisation problem.

[27] A new vintage of capital is produced in each period so that capital goods are indexed by date of construction.

firm j faces for capital goods at date t, X_t denotes the quantity of capital goods produced by the entire market at date t and C denotes the constant marginal cost of producing quality capital goods.

The jth firm maximises gross profits by choosing the quantity of capital goods to produce.[28] The gross profits of the jth firm at date t are given by

$$\Pi_{j,t} = P(X_t)X_{j,t} - CX_{j,t} \tag{19}$$

Substituting equation (18) into (19) gives

$$\Pi_{j,t} = X_{j,t}\alpha Bq_t^{\alpha} X_t^{\alpha-1}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta} - CX_{j,t}$$

Firm j has some influence determining prices in the capital goods producing sector when J, the number of firms is small. They choose the amount of capital goods to produce, taking the effect on price into account. However firm j is unable to control directly what the other J - 1 firms do and instead responds optimally to their output decisions.[29]

The firm's First Order Condition is given by

$$\frac{\partial \Pi_{j,t}}{\partial X_{j,t}} = \alpha Bq_t^{\alpha} X_t^{\alpha-1}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta} +$$
$$X_{j,t}(\alpha-1)\alpha Bq_t^{\alpha} X_t^{\alpha-2}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta} - C = 0$$

Setting marginal revenue equal to marginal cost gives

$$\alpha Bq_t^{\alpha} X_t^{\alpha-2}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta}\left[X_t + (\alpha-1)X_{j,t}\right] = C$$

[28] Note that the jth firm is maximising gross profits. This is because all firms regardless of type have the same gross profit maximisation problem. The innovating firm will have a different fixed cost associated with the discovery of a new technology compared with the copying firms which will have some cost associated with learning to copy the technology.
[29] That is firms are assumed to behave non-strategically for expositional purposes.

$$X_t^{\alpha-2}\left[X_t-(1-\alpha)X_{j,t}\right]=\frac{C}{\alpha Bq_t^{\alpha}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta}}$$

Since all firms in the capital goods producing sector are symmetric,[30] they produce the same quantity in equilibrium. Define $\overline{X}_t$ to be the typical quantity of capital goods produced by a capital goods producer when there are *J* firms in this sector. As a result the following holds, $X_{j,t}=\overline{X}_t$ and $J\overline{X}_t=X_t$. That is each firm produces their best response output given what their competitors produce. Substituting these in gives

$$(J\overline{X}_t)^{\alpha-2}(\alpha+J-1)\overline{X}_t=\frac{C}{\alpha Bq_t^{\alpha}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta}}$$

So that any given capital goods producing firm produces the following in equilibrium

$$\overline{X}_t=\left[\frac{\alpha Bq_t^{\alpha}\sum_{u=t}^{\infty}\left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u}\right]H_{t,u}^{\beta}L_{t,u}^{1-\alpha-\beta}\left[J^{\alpha-1}-(1-\alpha)J^{\alpha-2}\right]}{C}\right]^{\frac{1}{1-\alpha}}$$

This is the firms Nash Equilibrium quantity of new capital goods.

Multiplying by *J*, the number of firms in the market at present gives equilibrium industry output. [31]

---

[30] All firms are symmetric in the gross profits they make at a given time, but there net profits and period profits will not be the same due to different fixed costs and entry times.

[31] Note that the equilibrium quantity produced is a function of the market interest rate. Flexible adjusted on the interest rate ensures that the equilibrium quantity produced equals the supply of raw capital from households.

$$X_t = \left[\frac{\alpha B q_t^{\alpha} \sum_{u=t}^{\infty} \left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u} \right] H_{t,u}^{\beta} L_{t,u}^{1-\alpha-\beta} \left[1 - \frac{(1-\alpha)}{J} \right]}{C} \right]^{\frac{1}{1-\alpha}} \tag{20}$$

Recall the inverse demand for new capital goods from (18) is given by

$$P(X_t) = \alpha B q_t^{\alpha} X_t^{\alpha-1} \sum_{u=t}^{\infty} \left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u} \right] H_{t,u}^{\beta} L_{t,u}^{1-\alpha-\beta}$$

Substituting (20) into (18) gives

$$P_t = \alpha B q_t^{\alpha} \left[\frac{C}{\alpha B q_t^{\alpha} \sum_{u=t}^{\infty} \left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u} \right] H_{t,u}^{\beta} L_{t,u}^{1-\alpha-\beta} \left[1 - \frac{(1-\alpha)}{J} \right]} \right] \times \sum_{u=t}^{\infty} \left[\frac{((1-\delta)^{u-t})^{\alpha}}{R_t^u} \right] H_{t,u}^{\beta} L_{t,u}^{1-\alpha-\beta}$$

The equilibrium price for new capital goods in the capital goods producing sector is given by

$$P = \left[\frac{C}{1 - \frac{(1-\alpha)}{J}} \right] \tag{21}$$

The equilibrium price for a new capital good depends only on the market structure and the cost of raw materials and does not depend on q_t, X_t, H_t, L_t, or δ. The price is a mark up on marginal cost, where the mark up depends on the number of firms in the market or the market power of an individual firm. This is the standard Cournot pricing rule where price is a constant mark-up over marginal cost. The general Cournot formula is given below where ε is the price elasticity of demand

$$P\left[1+\frac{1}{J\varepsilon}\right]=C, \tag{22}$$

$$P=\frac{C}{1+\frac{1}{J\varepsilon}}. \tag{23}$$

In the specific example outlined in this paper the price elasticity of demand is given by

$$\varepsilon=\frac{-1}{1-\alpha} \tag{24}$$

Increasing capital's share of income α decreases the price elasticity of demand, so that a lower share of capital implies more market power for capital goods producing firms.

Equation (21) implies the equilibrium price for new capital goods is constant over time. The general result is an artefact of constant returns to scale in the production technology and the proportionality of marginal revenue to inverse demand.

The model can be reparameterised with ϕ used to denote the inverse of the number of firms in the capital goods producing sector *J*. The parameter restriction $0 \le \phi \le 1$ is applied, with $\phi = 1$ corresponding to a monopoly market structure and $\phi = 0$ to perfect competition.
Rewriting the equilibrium price in terms of ϕ gives

$$P=\left[\frac{C}{1-\phi[1-\alpha]}\right] \tag{25}$$

If ϕ equals 1 then the standard monopoly result is recovered

$$P=\left[\frac{C}{\alpha}\right] \tag{26}$$

This is the markup over marginal cost used in Romer (1990), Aghion and Howitt (1992) and by others.

If ϕ equals 0 then firms use marginal cost pricing

$$P=C \tag{27}$$

This is the standard result with perfect competition.

Normalising the marginal cost to 1 and solving for the differential between price and marginal cost gives[32]

$$P - C = \left[\frac{\phi(1-\alpha)}{1-\phi\left[1-\alpha\right]} \right] \tag{28}$$

This step concludes the derivation of the optimal quantity of raw capital the capital goods producing sector purchases from households and the price at which they sell their capital goods to the final goods sector.

3.3 Embodied Technology

This section discusses the nature of embodied technology used in this model. Embodied technology can be thought of as a measure of the quality of capital. It is a special case of capital augmenting technology where improvements in the quality of capital only take place in new capital machinery. Technology is modelled as "quality ladders" in the tradition of Aghion and Howitt (1992) and Grossman and Helpman (1991). Following the standard characterisation of quality ladders innovations are vertical with successive innovations raising the quality of capital, they are sequential, occurring one after another and they drastic so that firms can only make non-zero profits by producing capital goods that embody the leading technology. Firms cease production of capital goods that embody previous technologies with firms unable to sell new capital machinery that does not embody the leading technology. However capital goods that do not embody the leading technology continue to be used by firms in the final goods producing sector until they wear out.

Following the quality ladders literature, it is assumed that the size of an innovation, that is the incremental increase in the quality of capital resulting from a single innovation, is determined exogenously. Keeping with tradition the flow rate of innovations or the expected number of innovations with in a fixed period of time is determined endogenously. The determination of the innovation rate, a precursor to the growth rate of technology, is left for a subsequent section once the basic mechanics of the model have been developed.

---

[32] The differential between the price and the marginal cost of a capital good is given by:

$$P - C = \left[\frac{C}{1-\phi\left[1-\alpha\right]} \right] - C = \left[\frac{C - C + C\phi(1-\alpha)}{1-\phi\left[1-\alpha\right]} \right] = C\left[\frac{\phi(1-\alpha)}{1-\phi\left[1-\alpha\right]} \right]$$

The differential between price and marginal cost is just a mark up over marginal cost. Changing *C* results in a proportional increase in *P* and *P* – *C*, so that normalising *C* to 1 will not change the results.

Let q_t denote an index of embodied technology at date t, γ the size of an innovation where the restriction $\gamma > 1$ is imposed and Z_t the number of innovations that have occurred up to date t so that the measure of embodied technology used in this paper is defined as

$$q_t = \gamma^{Zt} \tag{29}$$

Equation (29) is consistent with the standard specification of technology in models of quality ladders.[33]

Let η_i denote the flow rate of innovations at time i so that the cumulative number of innovations that have taken place up to time t, denoted by Z_t is given by

$$Z_t = \sum_{i=-\infty}^{t} \eta_i \tag{30}$$

From equations (30) and (29) embodied technology can be rewritten in terms of the previous period's level of technology, the number of innovations that have taken place in the last time period and the incremental increase in technology brought about by a single innovation so that

$$q_t = \gamma^{\eta_t} q_{t-1} \tag{31}$$

Taking logs and rearranging gives the growth rate of embodied technology at time t as

[33] Solow (2000) questions the use of a multiplicative technology function in place of an additive function when reviewing the model of Aghion and Howitt (1992). An additive function where technology increases with a fixed incremental step each period would be unable to replicate the exponential GDP profiles observed in the data. Solow (2000) concludes that Aghion and Howitt's (1992) use of a multiplicative function is primarily to simplify modelling in a manner consistent with the data as opposed to conforming with any specific micro-foundation. A suitable micro-foundation for this multiplicative specification of technology can be obtained from an appropriate interpretation of Moore's Law. Moore's Law was an empirical observation made by Intel cofounder Gordon Moore in 1965 about the rate of growth in the number of transistors in an integrated circuit, in particular their number doubled every two years (Intel Corporation). The number of transistors on an integrated circuit could be used to proxy embodied technology. An increase in the number of transistors improves both the performance and the productivity of the computers they exist within. Figure 10 in Appendix IV gives a graphical representation of Moore's Law which has continued to hold long after Moore's initial observation. It can be seen that the number of transistors has grown exponentially and yet the release rate for each new generation of computer chip has remained reasonably constant at about one every couple of years. This is consistent with the multiplicative specification of technology used in this dissertation and by others that have taken the quality ladders approach. Moore's law has also found to hold with other technologies like hard disk space per dollar cost and RAM storage capacity (Wikipeadia). It is conceivable that similar relationships existed with technologies prior to the computer age.

$$g_{qt} = \eta_t \ln \gamma \tag{32}$$

Each period innovations come in sequence and each innovation lasts for a length of time denoted by χ_t. Consequently, η_t the number of innovations at each date *t* is given by

$$\eta_t \equiv \frac{1}{\chi_t} \tag{33}$$

Combining (32) and (33) gives

$$g_{qt} = \frac{\ln \gamma}{\chi_t} \tag{34}$$

Given that γ is exogenous, it is the endogenous determination of χ_t the length of an innovation that determines the growth rate of embodied technology. The next section describes the profit maximising exercise of the innovating firm and the χ_t they choose to solve this problem.

3.4 Derivation of the Equilibrium Growth Rate

The growth rate of embodied technology is determined by the profit maximising actions of the innovating firm. More specifically the innovating firm chooses the length or duration of an innovation to maximise the profits they make in a single time period referred to as period profits. The duration of an innovation that maximises their profits can be combined with equation (34) to determine the growth rate of embodied technology. The derivation of the period profits the innovator makes and the duration of an innovation are described in the following four steps.

The first step requires deriving the gross profit a capital goods producing firm makes at a particular stage in the life of the innovation that is "*Gross Profits as a Function of the Age of an Innovation*". Once this has been done the "*The Value of an Innovation at the Date of its Release*" to the innovating firm can be determined by integrating over the discounted gross profits made at all stages of the innovations life and subtracting the fixed sunk cost of the innovation. The next step is to calculate the "*Innovator's Period Profits*", this is done by integrating over the discounted value of an innovation. The final step involves the innovating firm choosing "*The Optimum Length of an Innovation*", that is the length of an innovation that maximises their period profits.

To simplify the analysis, those variables that grow are transformed by scaling them. A suitable choice of units is used to ensure the transformed variables are stationary on a balanced growth path. The transformed variables enable the characterisation of the equilibrium growth rate. The scaling unit, called effective units of labour turns out to be

$\frac{1}{B^{\frac{1}{1-\alpha-\beta}} q_t^{\frac{\alpha}{1-\alpha-\beta}} L_t}$, the details left out are reproduced in Appendix V. In units of the effective labour defined above the quantity of capital can be rewritten as

$$x_t = \frac{X_t}{B^{\frac{1}{1-\alpha-\beta}} q_t^{\frac{\alpha}{1-\alpha-\beta}} L_t}$$

Gross profits per effective worker in the capital goods producing sector are defined as

$$\pi_t = \frac{\Pi_{j,t}}{B^{\frac{1}{1-\alpha-\beta}} q_t^{\frac{\alpha}{1-\alpha-\beta}} L_t}$$

The gross profit per effective labour made by a capital goods producer is equal to the mark up over marginal cost multiplied by their share of equilibrium investment per effective labour. Given that all firms gross profits are symmetric a firms share of investment is equal to the market equilibrium investment divided by the number of firms *J* in the industry. The maximal profit per effective labour in the capital goods producing sector is given by

$$\pi_t = (P-C)\left[\frac{x_t}{J}\right] = (P-C)\phi x_t \tag{35}$$

Substituting equation (28) for $P - C$ in (35) gives

$$\pi_t = \left[\frac{\phi^2(1-\alpha)}{1-\phi(1-\alpha)}\right] x_t \tag{36}$$

Equation (36) gives gross profits per effective labour for a capital goods producing firm. Gross profits per effective labour increase with the equilibrium quantity of capital goods produced a result of the mark-up being independent of the quantity produced. In addition profits decrease with capital's share of income because a larger share of income means demand for capital goods is more elastic.[34] Firms prefer less elastic demand because they can charge higher prices with a lower trade-off in the quantity demanded. The firm's

[34] See equation (24).

mark-up and hence market power decreases in the number of firms in the capital goods producing sector. A lower mark-up results in a lower profit.

***Gross Profits as a Function of the Age of an Innovation*:** Let the subscript υ denote the time since an innovation was released or the age of an innovation, where $\upsilon \leq \chi_t$. The number of firms who can copy the original innovation and supply additional capital goods of the same quality are assumed to increase with the innovation's age so that gross profits at time *t* from an innovation of age υ are given by[35]

$$\pi_{\upsilon,t} = \left[\frac{\phi_\upsilon^2 (1-\alpha)}{1-\phi_\upsilon (1-\alpha)} \right] x_t , \tag{37}$$

$$\text{where } \frac{\phi_\upsilon^2 \alpha}{1-\phi_\upsilon (1-\alpha)} = \theta^\upsilon , \quad 0 \leq \theta \leq 1,$$

$$\text{and } \phi_\upsilon = \frac{-\theta^\upsilon (1-\alpha) + \sqrt{\left[\theta^\upsilon (1-\alpha)\right]^2 + 4\alpha\theta^\upsilon}}{2\alpha}. \tag{38}$$

The profits that a particular firm in the capital goods producing sector earn υ after an innovation has been released are given by equation (37). It is assumed that when an innovation is released the innovator is the only firm able to produce capital goods embodying the new technology. As the age of the innovation increases, copying firms learn to copy the technology and enter the market. Note the reparameterisation implies that $\left(\frac{1-\alpha}{\alpha}\right)\theta^\upsilon = \frac{P_\upsilon - C}{J_{\upsilon,t}}$, that is the average mark-up in the industry declines as the number of firms increase or equivalently as the age of the innovation increases. If $\theta = 1$ the average mark-up does not decline while if it is zero there is no mark-up. Consequently, the parameter θ can be interpreted as "barriers to entry" in the industry,[36]

[35] Firms could enter the capital goods producing sector at any rate. The standard assumption used in the literature would be firms entering according to a logistic profile, i.e. there is some diffusion or adoption lag (Griliches 1957) or the industry follows some sort of product life cycle, with low entry initially followed by faster rates of entry which then taper off. While this is a more realistic assumption, the diffusion or adoption lag will complicate things unnecessarily, a simpler assumption is that firms enter at such a rate that the monopoly mark up over marginal cost declines at a constant rate. This fits closely with the logistic profile except the tapering off phase is not observed. One can think of the innovator innovating before this phase is reached.

[36] The parameter θ has a similar interpretation to the "photocopying" parameter denoted "β" in Boldrin and Levine (2003). In their paper this parameter measures the multiplicative increase in the good produced each period. At the same time the price of the good falls at the inverse rate of this parameter. In this model θ measures the increase in competition each period which results in a decline in the monopoly mark-up.

where $\theta = 1$ represents strong barriers to entry resulting in monopoly and $\theta = 0$ implies that there are no barriers to entry so that all firms enter instantaneously and the industry is perfectly competitive at the date of an innovations release. If $0 < \theta < 1$, gross profits decay with the age of the innovation. Barriers to entry may include patents and the degree of ease with which a technology can be copied. Since government can control the scope of intellectual property rights, θ could be influenced by government policy.[37]

***The Value of an Innovation at the Date of its Release*:** Following Boldrin and Levine (2004), an innovation is associated with a fixed sunk cost, that is the discovery of an innovation requires some fixed amount of labour or human capital.[38] The value of an innovation to an innovating firm at the date of its release is equal to the present discounted stream of gross profits it generates less this fixed sunk cost.

Let $V_{0,t}$ denote the present discounted value of an innovation at the date of its release and F the fixed sunk cost associated with the discovery of a new quality of capital. An innovation of length χ_t has the following value to the innovating firm at the date of its release

$$V_{0,t} = \int_0^{\chi_t} \pi_{\upsilon,t} e^{-r_t \upsilon} d\upsilon - F \,, \tag{39}$$

where

[37] Alternatively if technology is extremely difficult to copy government could make firms disclose their trade secrets.

[38] One can think of this assumption in terms of an innovation requiring a fixed quantity of man hours to take place or something to that effect. This is a departure from the more conventional literature. The standard approach involves choosing the allocation of resources used in R & D to increase the growth rate of technology, equating the derived demand for the resource with its supply. Here the innovating firm chooses the length of an innovation with the amount of resources required for one innovation fixed. Choosing the length of an innovation amounts to choosing the number of innovations that occur within a fixed time period. The innovating firm weighs up the cost against the gain in revenues they receive from cutting the length of an innovation and releasing an additional innovation. As well as being consistent with Boldrin and Levine (2004), the approach developed in this dissertation gets around the well known scale effect that has plagued this literature. The scale effect is associated with the research production function in more conventional models of endogenous innovation. Most specifications have the number of innovations or designs determined directly by the amount of human capital allocated to research and development. This leads to the problem of human capital growth causing an increase in the growth rate of technology. This specification and the results obtained are inconsistent with stylised facts. Over the last century human capital has grown while the growth rate of technology has remained constant. Aghion and Howitt (1998b) and Howitt (1998) deal with the scale effect by introducing proliferation, that is the variety of products increases horizontally at the rate of labour growth (or human capital growth) so that some resources are devoted to unproductive innovation. This results in the growth rate of technology being unrelated to labour growth. The alternative specification developed in this dissertation is able to treat the scale effect and is consistent with stylised facts. Using a fixed cost for an innovation ensures that the resources devoted to research and development do not grow in the steady state so that the growth rate of embodied technology is constant on a balanced growth path

$$\pi_{\upsilon,t} = \left(\frac{1-\alpha}{\alpha}\right)\theta^{\upsilon} x_t .$$

The innovating firm is the only firm to earn gross profits for the full duration of the innovation that is from when the industry is ruled by a monopoly through various degrees of oligopoly until the next innovation is released. Copying firms enter at various stages through the life span of the innovation.[39]

The cost of copying is implicitly determined by the value of θ, a measure of the barriers to entry. In fact it increases with θ. Note that given F there is a critical threshold of θ such that maximal profit of the innovator and the profit of the first copier are equal. If θ is less than this critical threshold then there will be no innovation. So there is some participation constraint that must be satisfied for the innovator to want to participate as an innovator. Note also, however, the critical threshold for θ falls as the fixed cost F falls.[40]

***The Innovator's Period Profits*:** The number of innovations that take place within a fixed time period[41] could range from none taking place, in which case there is no growth, through to infinitely many taking place.[42] This means that the duration of an innovation could cover a fraction of a discrete time period or that a discrete time period could cover a fraction of the duration of an innovation.

Let $\Phi_{0,t}$ denote the stream of profits the innovating firm earns during a fixed time period discounted to the start of this period, called period profits so that

$$\Phi_{0,t} = V_{0,t}\int_0^{\frac{1}{\chi_t}} e^{-r_t \chi_t \tau} d\tau \tag{40}$$

The key feature to note about equation (40) is the use of the integral to approximate a summation. The integral gives a reasonable approximation of the summation of the discounted stream of earnings the period's innovations generate. As determined by equation (33) the number of innovations that occur during a fixed period of time is given by $\eta_t \equiv 1/\chi_t$. From equation (39) the value of an innovation $V_{0,t}$ is discounted back to the date of its release, however many innovations may occur during a fixed period of time. Future innovations that occur with in this period need to be discounted back to the start of the time period. The additional advantage of the integral is that it takes into account fractions of innovations.

[39] Given copying firms enter at different stages during the life of an innovation; the general form of profits for a copying firm will not be presented here.
[40] See Appendix VI for a derivation of the participation constraint.
[41] In this case one year.
[42] The number of innovations that take place will be a function of the parameter values in the model.

Substituting equation (39) into (40) gives

$$\Phi_{0,t} = \int_0^{\frac{1}{\chi_t}} e^{-r_t\chi_t\tau}\left[\int_0^{x_t}\left(\frac{1-\alpha}{\alpha}\right)\theta^{\upsilon} x_t e^{-r_t\upsilon} d\upsilon - F\right] d\tau \tag{41}$$

Let $k_{t\text{-}1}$ and $h_{t\text{-}1}$ denote date t - 1 physical and human capital per effective labour. Rewriting equation (16) in terms of effective labour and substituting into equation (41) gives

$$\Phi_{0,t} = \int_0^{\frac{1}{\chi_t}} e^{-r_t\chi_t\tau}\left[\int_0^{x_t}\left(\frac{1-\alpha}{\alpha}\right)\theta^{\upsilon} s_K B k_{t-1}^{\alpha} h_{t-1}^{\beta} e^{-r_t\upsilon} d\upsilon - F\right] d\tau \tag{42}$$

***The Optimum Length of an Innovation*: ** The innovating firm chooses the duration of an innovation that maximises their discounted flow of net profits in a discrete time period. The length of an innovation they choose is determined by differentiating equation (40) with respect to χ_t. This gives

$$\frac{\partial\Phi_{0,t}}{\partial\chi_t} = \left(\frac{e^{-r_t}-1}{\chi_t^2 r_t}\right)V_{0,t} + \left(\frac{1-e^{-r_t}}{\chi_t r_t}\right)\frac{\partial V_{0,t}}{\partial\chi_t} = 0 \tag{43}$$

The left side of (43) which is negative is the marginal loss from increasing the duration of an innovation and hence decreasing the number of innovations that occur within a fixed period of time holding the value of an innovation constant. The right side is the marginal gain from increasing the duration of the innovation that is the increase in the value of an innovation due to the innovation lasting longer while holding the number of innovations constant. The optimal length of an innovation is obtained by setting the marginal gain equal to the marginal loss from increasing the length of an innovation.

Equation (43) can be rewritten explicitly by substituting in the value of an innovation.

$$\begin{aligned}\frac{\partial\Phi_{0,t}}{\partial\chi_t} = \left(\frac{e^{-r_t}-1}{\chi_t^2 r_t}\right)\left[\left(\frac{1-\alpha}{\alpha}\right) s_K B k_{t-1}^{\alpha} h_{t-1}^{\beta}\left(\frac{1-\theta^{\chi_t}e^{-r_t\chi_t}}{r_t-\ln\theta}\right) - F\right] \\ + \left(\frac{1-e^{-r_t}}{r_t\chi_t}\right)\left(\frac{1-\alpha}{\alpha}\right) s_K B k_{t-1}^{\alpha} h_{t-1}^{\beta}\theta^{\chi_t} e^{-r_t\chi_t} = 0\end{aligned} \tag{44}$$

Rearranging equation (44) gives

$$\frac{F}{\left(\frac{1-\alpha}{\alpha}\right)Bk_{t-1}^{\alpha}h_{t-1}^{\beta}}=\left[\left(\frac{1-\theta^{\chi_t}e^{-\chi_t r_t}}{r_t-\ln\theta}\right)-\chi_t\theta^{\chi_t}e^{-\chi_t r_t}\right] \tag{45}$$

The χ_t that solves (45) is the duration of an innovation that maximises the innovator's period profits. Given the complexity of (45) χ_t can not be solved for directly and instead must be determined through numerical simulations. Plotting both sides of (45) on the same axis for a large range of χ values confirms there is a unique solution to condition (45).

Given the χ_t that solves (45), the equilibrium growth rate of embodied technology can be determined from equations (33) and (32). In addition the χ_t that solves (45) can be combined with (33), (30) and (29) to determine the index of embodied technology at date *t*. This concludes the determination of the equilibrium growth rate and the model.

4. Quantitative Analysis

In this section the model developed in this paper is used to explain features of embodied technology growth in the United States. In particular the model's predicted series for output per labour is matched to GDP per hour for the US. Consequently the parameterisation of the model allows the investigation of the models properties and their implications for the US economy and their growth rate.

Begin by defining the model's prediction of output per labour as $\tilde{y}_t$. The inclusion of neutral technology is important if the model is to be able to replicate features of the US economy. As Greenwood, Hercowitz and Krusell (1997) assert, technological change in the US is both neutral and investment specific. The scaling parameter B is replaced with B_t representing an index of disembodied neutral technology which grows at the exogenous rate g_B so that $B_t=B_0(1+g_B)^t$. Appendix V develops steady state specifications for the variables in this model. From equation (A.V.28) in Appendix V output per labour can be written as

$$\tilde{y}_t=B_t^{\frac{1}{1-\alpha-\beta}}q_t^{\frac{\alpha}{1-\alpha-\beta}}k_t^{\alpha}h_t^{\beta} \tag{46}$$

On a balanced growth path physical and human capital per effective labour are stationary so that the steady state values for k and h derived in Appendix V can be substituted into equation (46). This gives

$$\tilde{y}_t = \left(B_0(1+g_B)^t\right)^{\frac{1}{1-\alpha-\beta}}\left(\gamma^{\eta(t+1)}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left[\frac{s_K}{\left(\frac{1-\beta}{1-\alpha-\beta}\right)\eta\ln\gamma+n+\delta+\left(\frac{1}{1-\alpha-\beta}\right)g_B}\right]^{\frac{\alpha}{1-\alpha-\beta}} \times\left[\frac{s_H}{\left(\frac{\alpha}{1-\alpha-\beta}\right)\eta\ln\gamma+n+\delta+\left(\frac{1}{1-\alpha-\beta}\right)g_B}\right]^{\frac{\beta}{1-\alpha-\beta}} \tag{47}$$

Taking logs

$$\ln\tilde{y}_t = \ln B_0 + \left(\frac{1}{1-\alpha-\beta}\right)g_B t + \left(\frac{\alpha}{1-\alpha-\beta}\right)\eta(t+1)\ln\gamma + \left(\frac{\alpha}{1-\alpha-\beta}\right)\ln(s_K) - \left(\frac{\alpha}{1-\alpha-\beta}\right)\ln\left[\left(\frac{1-\beta}{1-\alpha-\beta}\right)\eta\ln\gamma+n+\delta+\left(\frac{1}{1-\alpha-\beta}\right)g_B\right] + \left(\frac{\beta}{1-\alpha-\beta}\right)\ln(s_H) - \left(\frac{\beta}{1-\alpha-\beta}\right)\ln\left[\left(\frac{\alpha}{1-\alpha-\beta}\right)\eta\ln\gamma+n+\delta+\left(\frac{1}{1-\alpha-\beta}\right)g_B\right] \tag{48}$$

Note the similarity between equation (48) and Mankiw, Romer and Weil (1992), the key difference being the inclusion of embodied technology.[43] Setting the frequency of innovations η equal to zero gives Mankiw, Romer and Weil's (1992) Equation (11).[44]

[43] This model makes an additional contribution to explaining cross country income disparity. The model of Mankiw, Romer and Weil (1992) made significant inroads in the debate that surrounds the issue of cross country income differentials. The inclusion of embodied technology adds an extra dimension to this puzzle with further benefit gained from its endogenous determination. Differences in structural parameters that affect the growth rate of embodied technology between countries could provide additional sources for the existence and persistence of cross country income differentials. The parameter of most interest that could differ between countries depending on government policy and intellectual property rights among other things is θ, a measure of the barriers to entry. Overly restrictive barriers to entry could impose a low "speed limit" on a countries growth potential. Like wise the lack of barriers to entry could destroy completely the incentive to innovate. These issues will be examined in some depth once the model has been parameterised.

[44] The coefficient in front of the growth rate of disembodied technology differs from Mankiw, Romer and Weil (1992) because disembodied technology is neutral in this dissertation where is it labour augmenting in Mankiw, Romer and Weil (1992).

$$\ln \tilde{y}_t = \ln B_0 + \left(\frac{1}{1-\alpha-\beta}\right) g_B t + \left(\frac{\alpha}{1-\alpha-\beta}\right) \ln(s_K)$$

$$- \left(\frac{\alpha+\beta}{1-\alpha-\beta}\right) \ln\left[n + \delta + \left(\frac{1}{1-\alpha-\beta}\right) g_B \right] + \left(\frac{\beta}{1-\alpha-\beta}\right) \ln(s_H) \tag{49}$$

One thing is noteworthy, despite the complicated model the end results develop analytically tractable and numerically explicit formulas that can be compared with their counterparts in the conventional Mankiw, Romer and Weil (1992) model. In particular output per labour given by equation (47) and the profit maximising condition for the innovating firm given by equation (45) constitute important parts for calibration. The following sections discuss the model, data, calibration and the solution method.

4.1 National Accounting

To be able to match a model with the data requires the model's compatibility with the national accounting framework. This is easily satisfied in models that comprise of heterogeneous capital stocks and use Cobb Douglas production technologies with constant returns to scale. Labour is paid its share of income known as compensation to employees with physical capital paid the residual. Using vintage capital changes things slightly, for one thing capital now includes embodied technology. Constant returns to scale in the factors of production and a Cobb Douglas production technology ensure that all factors are paid their respective shares of income. Some simple algebra confirms

$$Y_t = w_t L_t + r_t^H H_t + P_t^K K_t \tag{50}$$

This is consistent with the national accounting framework, except K_t is the stock of quality capital which includes embodied technology and P_t^K is the rental price of embodied capital services. Appendix II shows P_t^K declines with time at the inverse of the rate of technology growth.

The national accounting framework derived in this model can be compared with its equivalent in Mankiw, Romer and Weil (1992). Let $\tilde{K}_t$ denote the homogenous capital stock excluding embodied technology so that $\tilde{K}_t = I_{t-1} + (1-\delta)\tilde{K}_{t-1}$ where $I_{t-1} = s_K Y_{t-1}$, and r_t^K the rental rate on physical capital, from Mankiw, Romer and Weil (1992) all factors are paid their marginal products so that[45]

$$Y_t = w_t L_t + r_t^H H_t + r_t^K \tilde{K}_t \tag{51}$$

[45] The similarities between the two models mean the Mankiw, Romer and Weil (1992) parameter estimates can be used when calibrating the model in this dissertation.

Equation (50) and (51) imply

$$r_t^K \tilde{K}_t = P_t^K K_t \tag{52}$$

Equation (52) mirrors Hulten's (1992) result. Capital regardless of whether technology is embodied is paid its respective share of income which happens to be the same under investment specific and neutral technologies. This is an artefact of constant returns to scale and both labour and human capital's shares of income remaining the same regardless of the capital specification. Equations (50) and (51) imply that both sides of (52) must grow at the same rate in this case the rate Y_t grows. In Mankiw, Romer and Weil (1992), r_t^K is constant along a balanced growth path while $\tilde{K}_t$ grows at the rate $(1+g_Y)$. In this vintage capital model K_t grows at the rate $(1+g_q)(1+g_Y)$ and P_t^K grows at the rate $1/(1 + g_q)$ so that both sides of equation (52) grow at the rate $(1 + g_Y)$.[46]

4.2 Data

This section discusses the data used in this paper, their sources and the method of their construction where applicable. Data needs to be collected to match $\tilde{y}_t$ and q_t. Output per labour $\tilde{y}_t$ is matched with the annual NIPA Real GDP per hour series for the United States. Calibrating technology parameters proves more difficult because of the identification problem associated with vintage capital models. How should technology be partitioned into its investment specific and neutral components? Jorgenson (1966) provides an answer to this question, using a variant of Solow's (1960) vintage capital model he shows that a measure of embodied technology can be derived by inverting quality adjusted capital prices. Greenwood, Hercowitz and Krusell (1997) following Jorgenson's lead construct an annual series for quality adjusted capital prices using data from Gordon (1990) and the NIPA for the 1954-1990 period. Subsequently they use their measure of embodied technology to disaggregate total factor productivity and calibrate their vintage capital model. Using the Greenwood, Hercowitz and Krusell (1997) data series an index for quality of capital or embodied technology is constructed in this paper.[47]

4.3 Calibration

The parameters that need to be assigned values include the factor income shares α and β, the saving rates s_H and s_K, the rate of depreciation δ, the technology parameters γ and g_B, the growth rate of labour n, the fixed sunk cost of an innovation F and the barriers to entry index θ. The model in this paper is matched to the data using the method of

---

[46] See Appendix II for a full derivation.

[47] The Author would like to thank Jeremy Greenwood for promptly supplying the data when it was requested.

calibration described in Cooley and Prescott (1995). As many parameters as possible are fixed a priori, either set to their long run average values or taken from previous empirical studies. The remaining free parameters are chosen to fit the model by minimising the residual between the models predicted series for Gross Domestic Product per hour on a balanced growth path and its time series equivalent in the data. The model is calibrated for the 1954-1990 period given this is the coverage of the Greenwood, Hercowitz and Krusell (1997) capital price series.

The parameters to be fixed using a priori information include, α, β, s_K, s_H, δ, γ, g_B and n. Physical and human capital's shares of income are set to $\alpha = 1/3$ and $\beta = 1/3$ respectively, these are the parameter estimates generated in Mankiw, Romer and Weil (1992). The size of an innovation can be determined from the data if a Poisson arrival rate is assumed for innovations.[48] Imposing the Poisson restriction parameterises the statistical properties of the data where γ is determined by the Poisson mean and variance. The description of this method is left to Appendix VII. An estimate of $\gamma = 1.018$ is found, suggesting that each innovation raises the quality of capital by 1.8%. The rate of depreciation has been set to $\delta = 0.05$, the standard rate in the literature which implies that both physical and human capital have half lives of about 13.5 years. The saving rate for physical capital is derived by using the average percentage of Gross Fixed Capital Formation as a share of GDP which gives $s_K = 0.2$.[49] The saving rate for human capital is set to $s_H = 0.1$, the estimate used for the United States in Mankiw, Romer and Weil (1992). The growth rate of labour is set to $n = 0.01$, which is the average growth rate in the NIPA labour hours series.

The growth rate of disembodied technology is determined using GDP per hour data and the Greenwood, Hercowitz and Krusell (1997) series for quality adjusted capital prices. The capital price series is inverted to obtain an index of embodied technology from which the average growth rate of embodied technology is recovered. Combining this with the average growth rate of the GDP per hour series and the factor shares assigned previously the average growth rate of disembodied technology is obtained from equation (A.V.13) in Appendix V. This is reproduced below where $\bar{g}_Y^{DATA}$ is the average growth rate of GDP observed in the data and $\bar{g}_q^{DATA}$ is the average growth rate of embodied technology.

$$g_B = (1-\alpha-\beta)\left[\bar{g}_Y^{DATA} - n - \left(\frac{\alpha}{1-\alpha-\beta}\right)\bar{g}_q^{DATA}\right]$$

This procedure generates an estimate of $g_B = 0.003$. All parameters chosen a priori are listed in Table 1 below.

[48] Note, in this case innovations are assumed to be deterministic, other models in the literature have assumed stochastic innovations which follow a Poisson arrival rate, so given the size of γ can easily be determined by assuming a Poisson arrival rate, this seems like a reasonable approach to take. The full methodology is defined in Appendix VII.

[49] Gross Fixed Capital Formation and GDP data are sourced from the International Financial Statistics Database.

Table 1. Parameters Fixed A priori

Parameter Symbol	Parameter Value	Description
α	0.33	Physical Capitals Share of Income
β	0.33	Human Capitals share of income
γ	1.018	The size of an innovation
δ	0.05	The rate of depreciation
s_K	0.2	Saving Rate for Physical Capital
s_H	0.1	Saving Rate for Human Capital
n	0.01	Growth Rate of the labour force
g_B	0.003	The growth rate of neutral technology

The free parameters that have yet to be chosen include F and θ. The following describes the procedure used to determine the value of these parameters.

Solution Method: Calculating Barriers to Entry for a Country

This section describes a new method for determining barriers to entry or at least giving some indication of their magnitude. Given the parameters fixed A priori, the remaining free parameters which include the fixed sunk cost of an innovation F and barriers to entry θ are chosen to match the model's predicted growth rate of embodied technology with its equivalent in the data. These parameters are chosen by setting the model's predicted growth rate of embodied technology equal to the long run average growth rate of embodied technology in the data and then working backwards to find combinations of these parameters that give such a result. With two free parameters remaining there is only one degree of freedom when fitting this model, that is once one of these parameters has been chosen the other is automatically pinned down to ensure the exact match of the model's predicted growth rate of technology with its counterpart in the data. The question is which of these parameters should be chosen?

Obviously the economic interpretation and the availability of relevant and comparable data will play a key part in answering this question. The parameter θ has an abstract interpretation implying it would be difficult to pin down outside of this model. The parameter F has a more natural interpretation so that there are some candidate data series that could be used to guide its estimation within the context of this model. F could be chosen so that the period costs of an innovation as a percentage of GDP match the average share of GDP spent on research and development observed in the data. The period expenditure on innovation is the total discounted sum of expenditure on the fixed sunk cost of an innovation with in the time period. With F pinned down the corresponding θ will reveal itself.

Putting this method into practice requires an estimate for the long run average growth rate of embodied technology, the long run average share of R & D spending as a percentage of GDP and the construction of a measure of period expenditure on innovation. From the data the average growth rate of embodied technology is 1.3%[50] and the average R & D share of GDP is 0.025.[51]

Let $\bar{\chi}$ denote the length of an innovation implied by the data, this can be obtained by rearranging (33) which gives

$$\bar{\chi} = \frac{\ln \gamma}{\bar{g}_q^{DATA}} \tag{53}$$

Rearranging equation (45) by making F the subject and substituting in $\bar{\chi}$ from equation (53) gives

$$F = \left[\left(\frac{1-\theta^{\bar{\chi}}e^{-\bar{\chi}r}}{r-\ln\theta}\right) - \bar{\chi}\theta^{\bar{\chi}}e^{-\bar{\chi}r}\right]\left(\frac{1-\alpha}{\alpha}\right)Bk^{\alpha}h^{\beta} \tag{54}$$

Note with the growth rate of embodied technology set to its long run average and all the fixed parameters set to their levels in Table 1, only F and θ will be permitted to vary. Equation (54) enables the fixed sunk cost of an innovation to be plotted against the barriers to entry.

Using equation (41) the discounted expenditure on innovations for one time period denoted by ζ_t is given by

$$\zeta_t = F\int_0^{\frac{1}{\chi_t}} e^{-r_t\chi_t\tau}d\tau = F\left(\frac{1-e^{-r_t}}{\chi_t r_t}\right) \tag{55}$$

All that remains is to find the F that matches the period expenditure as a percentage of GDP to the observed share of GDP spent on R & D. Doing this will determine the value of θ that fits the model.

Equation (55) is used in conjunction with equation (A.V.22) from Appendix V to obtain the period expenditure on innovation as a fraction of GDP. F is chosen such that the period expenditure on innovations equals 0.025, that is

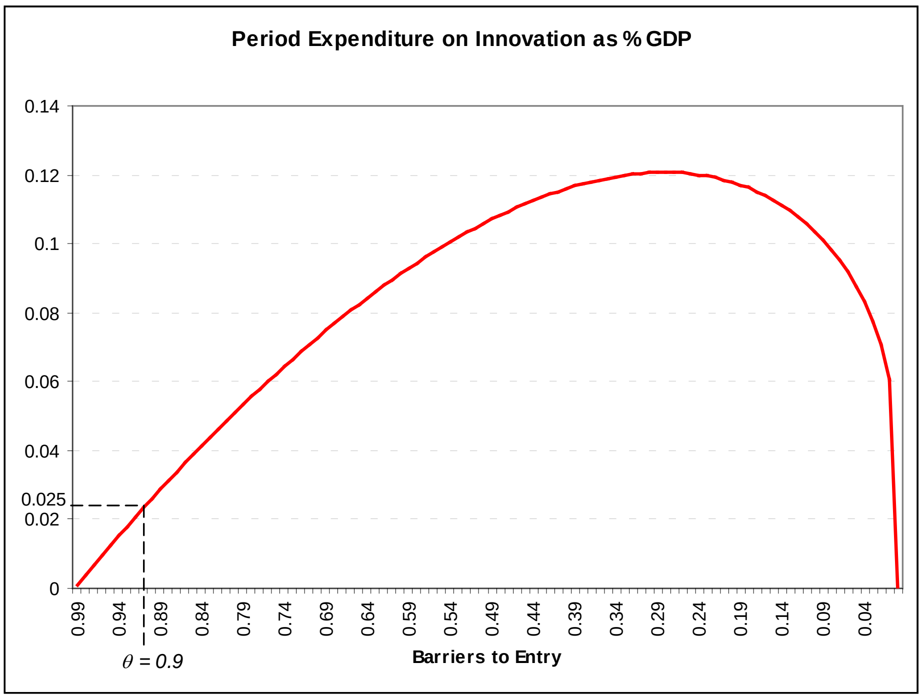
$$\frac{R\&D}{GDP} = 0.025 = \frac{\zeta}{y} = \left(\frac{F}{y}\right)\left(\frac{1-e^{-r}}{\bar{\chi}r}\right) \tag{56}$$

[50] This number is slightly different to Greenwood, Hercowitz and Krusell (1997) because the parameter weights are slightly different due to their disaggregation of capital into structures and equipment.
[51] The data for Research and Development spending come from the National Science Foundation.

Note the subscripts have been dropped because these are steady state values so they are time invariant on a balance growth path. With the F that matches the period expenditure on innovation with research and development expenditure fixed the degree of freedom is eliminated and the value of θ required to fit the model is pinned down.[52]

Figure 2 below gives a graphical representation of the process used to choose F and θ.

Figure 2.



The period's cost of innovation is plotted against the θ implied by equation (54) when all the fixed parameters are set to their long run averages and the growth rate of embodied technology is fixed to its long run average. There are two points in this graph that correspond with the observed R & D spending as a percentage of GDP. Given the United States has quite strong intellectual property rights the more pessimistic estimate of barriers to entry, $\theta = 0.9$ is used. The parameters chosen to fit the model to the data are presented in table 2 below.

52 This procedure generates estimates of χ and hence η which could prove useful if a regression were performed using equation (50). With suitable estimates of variables the other parameters could easily be estimated by equation (50).

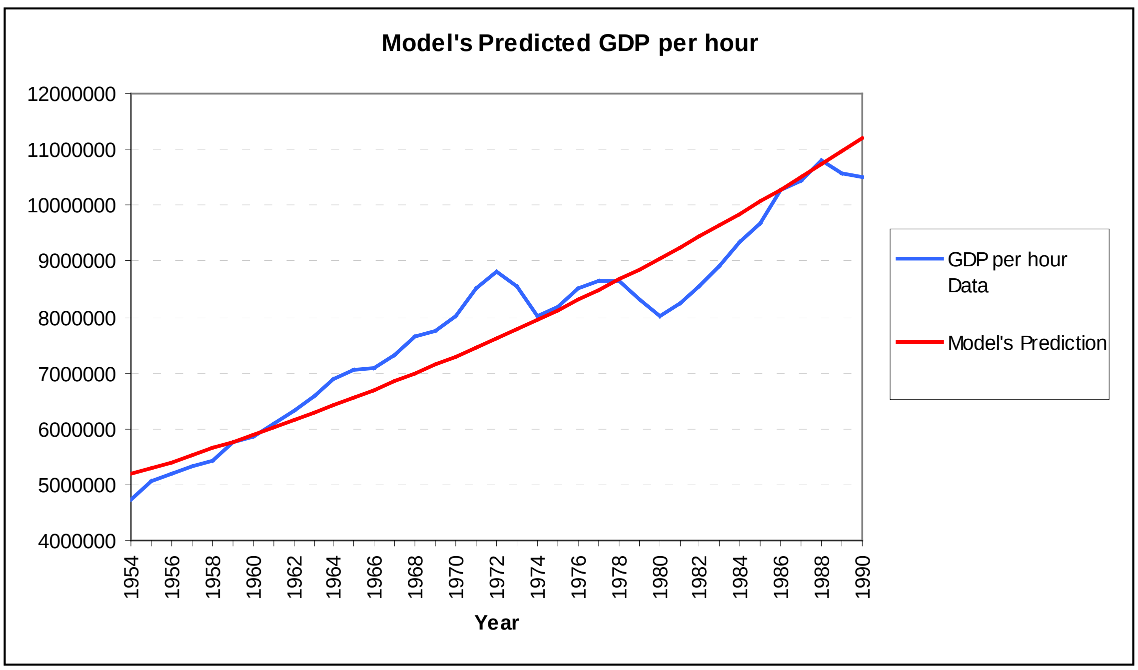
Table 2. Parameters Chosen to Fit the Model A Posteriori

Parameter Symbol	Estimate	Description
F	0.09	The Fixed cost of an innovation
θ	0.9	A measure of the barriers to entry

By itself the fitted parameter value for F has little economic interpretation given this estimate will vary with the scaling parameter B_0. However the relative size of F may have some interpretation when comparing between countries where it may prove a source of growth and income differentials. It may also have an interpretation if a change in F explains the change in the growth rate. The parameter θ on the other hand has quite an important interpretation. This parameter measures the degree of barriers to entry, the rate at which other firms are able to enter this industry. The model implies using US parameters and US data that $\theta = 0.9$. This can be interpreted as the gross monopoly profits of the innovator having a half life of 6.6 years. However the parameter values used in this model imply that the duration of an innovation is only 1.39 years. Using these parameter values and equation (38) the innovator decides to release a new innovation when there are approximately 1.5 firms in the industry.

The model's predicted series for GDP per hour is plotted along side NIPA data for GDP per hour in Figure 3 below.

Figure 3.



The model's fitted prediction runs through the GDP data a property consistent with Kaldor's stylised fact that gross domestic product grows at a constant rate for long periods of time. The root mean square error as a percentage of the average GDP per hour is equal to 0.069.

Given the parameter values chosen a priori and those chosen to fit the model, the properties of this model and their suggestions for US growth potential can be investigated. The parameter of most interest is θ, since this is a function of the intellectual property rights regime and the complexity of technology and could potentially be influenced by government policy. The effects of changing θ are investigated in the next section.

5. Numerical Simulations

This section uses numerical simulations to evaluate the properties of this model with special attention given to the effects barriers to entry have on the growth rate of embodied technology. The complexity of the equilibrium growth condition (45) means the optimum length of an innovation can not be solved for directly and instead is solved by performing iterations. All numerical simulations are performed with the parameter values specified in the calibration section except where noted otherwise. All simulations are performed on the balanced growth path for expositional purposes. Iterations are performed using Excel's Solver algorithm which uses Newton's method.

To find out the effect barriers to entry have on this model requires the determination of the optimum length of an innovation. This is the key determinant of the growth rate of embodied technology and determines the variables in the model. The simulation process begins by substituting the steady state values of the variables k, h and r determined in Appendix V into equation (45). All parameters are fixed to the values chosen in the calibration section except for θ which is allowed to vary. Starting with θ set to 1 the unique solution to (45) is determined by performing iterations using Excel's solver algorithm. When a solution is found it is recorded before θ is decreased by an increment of 0.01 to 0.99. The iteration process is repeated again until all values of θ between 0 and 1 in steps of 0.01 have been covered. This generates a series of χ where each corresponds to a different θ. This series can be used to determine how the model responds to changes in the barriers to entry on a balanced growth path.

The series for χ is plotted against θ below.

Figure 4.

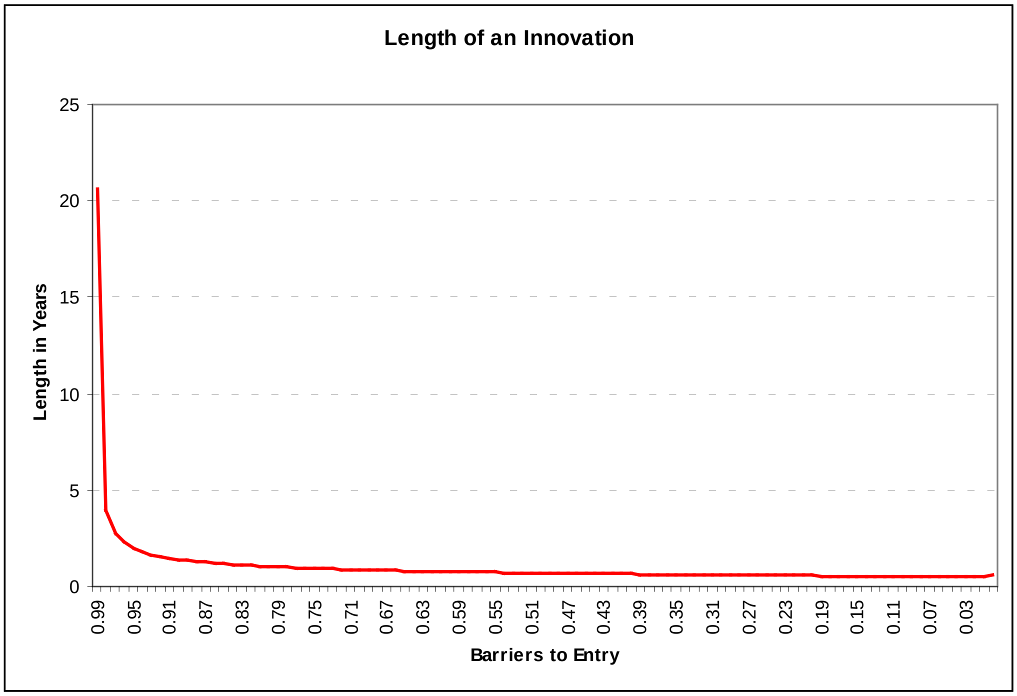


Figure 4 displays the profit maximising duration of an innovation in years for the innovating firm. When barriers to entry are strong, that is θ tends to 1, the optimal duration of an innovation approaches infinity. As the barriers to entry are relaxed the duration falls rapidly before levelling off. When the barriers to entry are very weak, that is θ tends to 0, the optimal length of an innovation begins to increase again.

The length of time it takes the innovating firm to pay off the fixed sunk cost of an innovation will depend on the current level of barriers to entry. Given the optimum length of an innovation that corresponds to each θ, the following describes how the length of time taken to pay off the fixed sunk cost of an innovation is determined.
Let κ denote the length of time it takes to pay off the fixed sunk cost associated with an innovation so that

$$\int_0^{\kappa}\left(\frac{1-\alpha}{\alpha}\right)\theta^{\upsilon}xe^{-r\upsilon}d\upsilon = F$$

Solving for κ as a function of θ gives

$$\kappa(\theta) = \frac{\ln\left(1-F\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{r-\ln\theta}{x}\right)\right)}{\ln\theta - r} \tag{57}$$

Equation (57) is solved for $\kappa(\theta)$ for each value of θ between 0 and 1 moving in steps of 0.01. The length of an innovation that corresponds to each θ is used to determine the steady state values of x and r. The series for $\kappa(\theta)$ is plotted against θ in figure 5 below.

Figure 5.

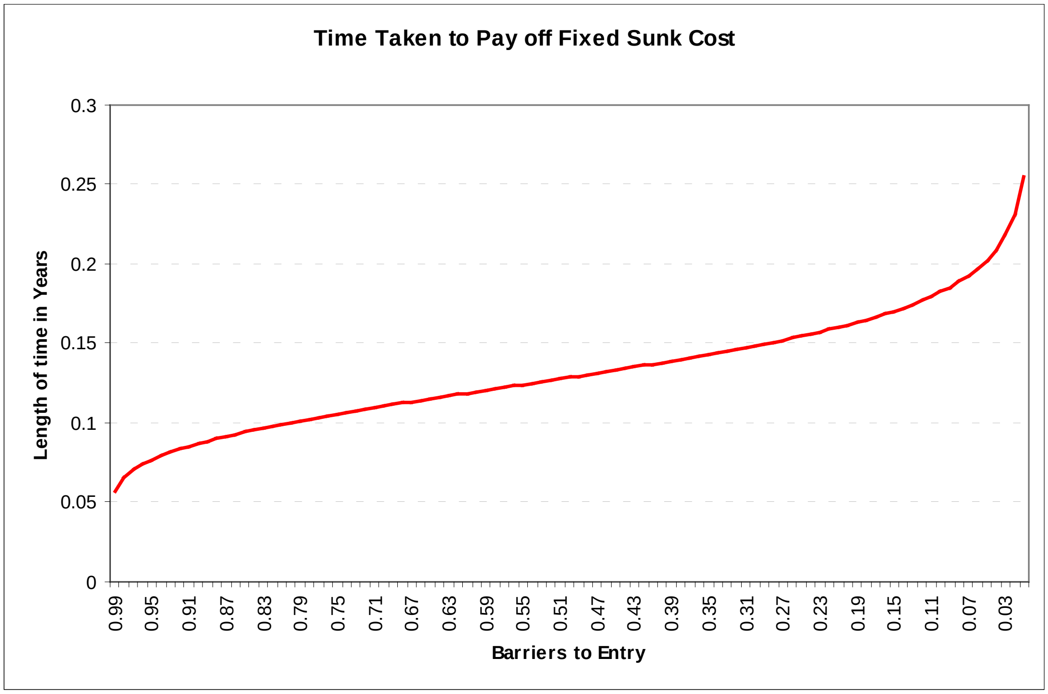
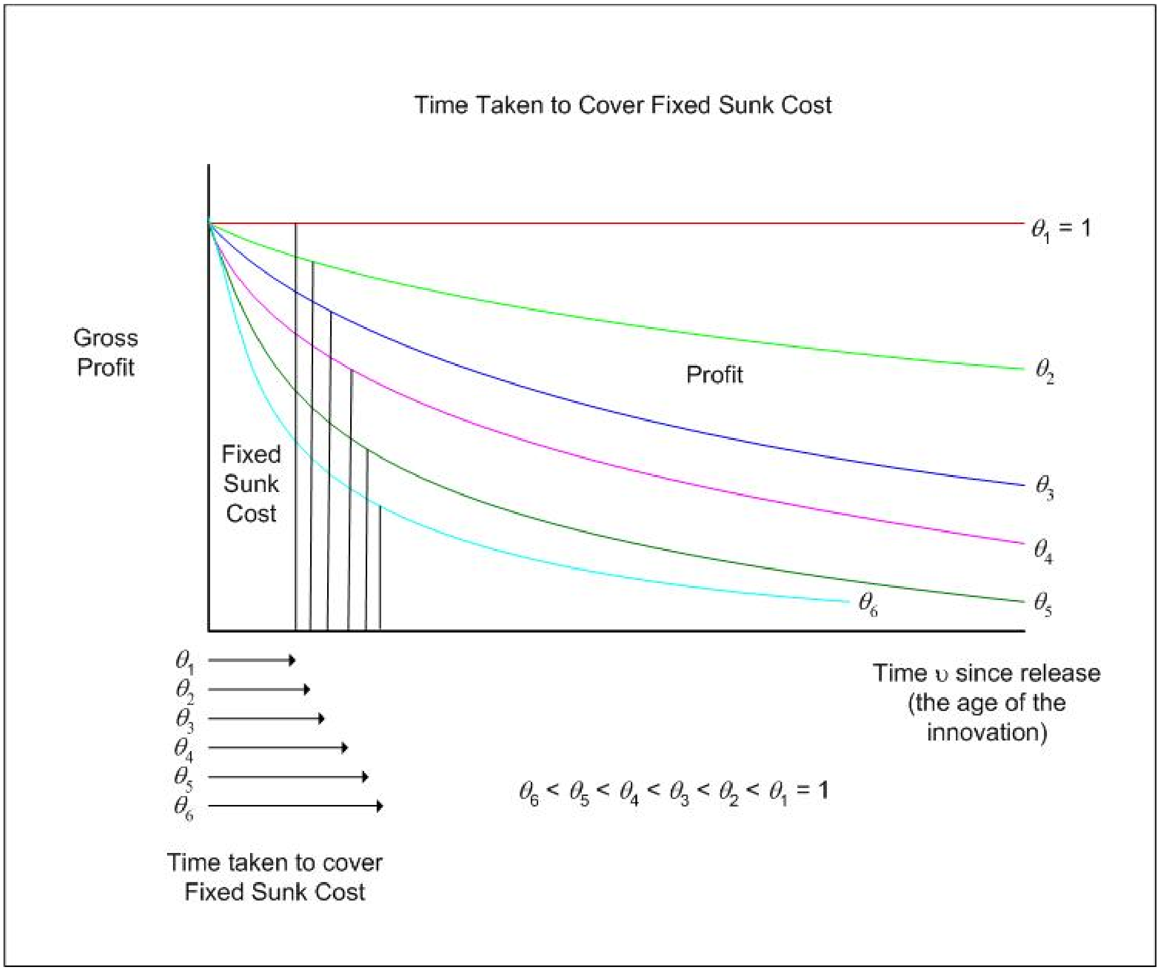


Figure 5 shows the length of time that it takes the innovating firm to generate enough revenue from an innovation to cover the cost of its discovery. When barriers to entry are very strong, that is as θ tends to 1, the length of time taken to cover the cost is the shortest. As barriers to entry are relaxed, the time taken to cover the cost of an innovation increases. When barriers to entry are extremely weak, that is as θ tends to 0, the time taken to cover the cost of an innovation gets extremely large tending to infinity. The result being the length of time it takes to pay off the fixed sunk cost associated with an innovation has a characteristic "S" shape in θ the barriers to entry.

Result 1: Decreasing barriers to entry θ increases the time it takes for the innovating firm to pay off the fixed sunk cost F associated with an innovation.

Decreasing the barriers to entry increases the entry rate of competitors. As a consequence the innovator's gross profits decline at a faster rate a result of more competition at an earlier stage. The total gross profit an innovator earns from a single innovation is therefore smaller. Given the fixed sunk cost is invariant to competition, lower gross profits mean it takes longer for the innovator to cover this cost. Figure 6 below gives a diagrammatic representation of this result and the profits earned from a single innovation. Note profits have not been discounted for expositional purposes.

Figure 6.



In figure 6 the gross profit profile associated with $\theta_1 = 1$ does not decline with time so that the innovating firm earns monopoly profits forever. If θ decreases, the gross profit profiles shift inward because profits are declining at a faster rate. A lower θ means that it is easier for competitors to enter, entering at a faster rate and hastening the erosion of gross profits. As a result the lower gross profits mean it will take longer for the firm to pay off the fixed cost associated with the discovery of an innovation.

The profit maximising length of an innovation forms a key component of the growth rate of embodied technology. Using the series of χ generated for the different values of θ the growth rate of embodied technology can be determined as a function of θ by using equation (34)

$$g_q = \frac{\ln \gamma}{\chi(\theta)}$$

The corresponding series for the growth rate of embodied technology is plotted in figure 7 below.

Figure 7.

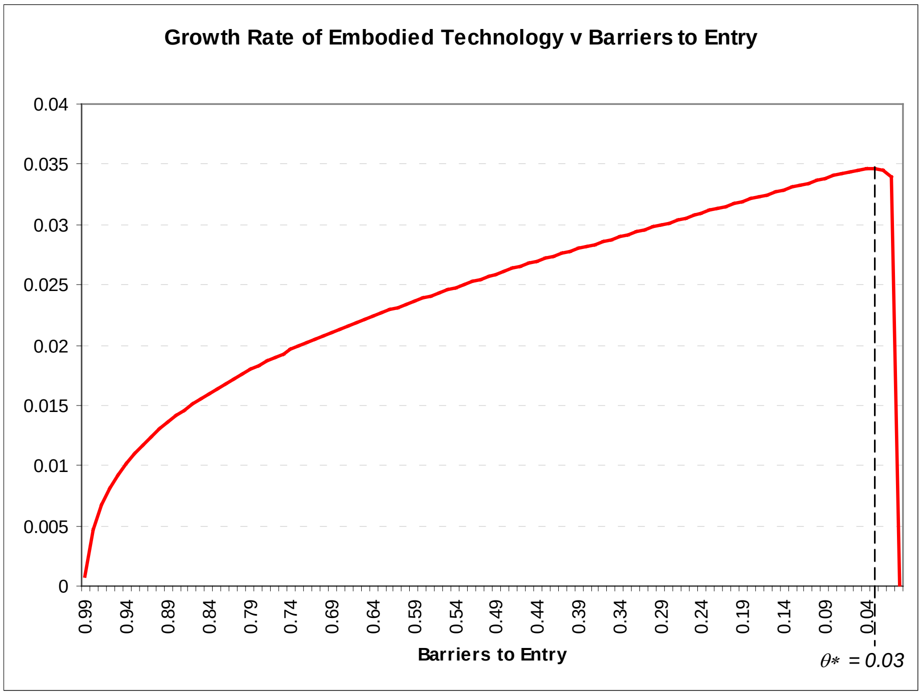


Figure 7 shows that as barriers to entry denoted by θ tend to one the growth rate of embodied technology approaches zero. As θ declines the growth rate of technology increases until it reaches a maximum (denoted by θ^*). As θ goes to zero the growth rate of embodied technology tends to zero. From the calibration section the estimate for barriers to entry in the United States is $\theta = 0.9$. The models "growth profile" plotted in figure 7 above implies, holding all other parameters constant, that lowering the barriers to entry θ from their current level will result in an increase in the growth rate of embodied technology. The model predicts the growth maximising level of barriers to entry θ^* to be 0.03 and at this level the ceiling on embodied technology growth is just under 3.5%, over 2% larger than the long run average observed in the data that correspond with $\theta = 0.9$.[53] Relaxing the current intellectual property rights regime in the United States may result in a decline in the parameter θ since less stringent patent rules mean that it will be easier to copy a technology, but it is unlikely that this alone will be enough to decrease the current θ to 0.03. There are likely to be other obstacles that affect barriers to entry like the natural complexity of a technology and imperfect capital markets. If this is the case the model implies that there will be growth with out patents and that growth will be higher without patents.

[53] From the calibration section the long run average growth rate of embodied technology is 1.3%.

The following results are stated based on the simulation plotted in figure 7 and some general intuition from the model.

Result 2: For barriers to entry θ = 1 the innovating firm chooses the length of an innovation to be infinite so that no innovation occurs. For barriers to entry θ = 0 the innovating firm releases no innovations. This means growth in embodied technology g_q is zero when barriers to entry are very strong or non existent.

When θ is set to 1 the innovating firm is a monopolist because the barriers to entry are so strong that all competitors are prevented from entering the capital goods producing sector. The innovating firm releases one innovation at time zero and as a monopolist earns monopoly rents on this one innovation for ever. Given that innovations are costly and the innovator does not face competition there is no incentive to release any subsequent innovations which means the length of this first innovation is infinite.[54]

A necessary condition for an innovation to take place is that the discounted revenue an innovation generates is able to cover the fixed sunk cost associated with discovering that innovation, that is [55]

$$\int_0^{\chi_t} \left(\frac{1-\alpha}{\alpha}\right)\theta^{\upsilon} x_t e^{-r_t \upsilon} d\upsilon \geq F \tag{58}$$

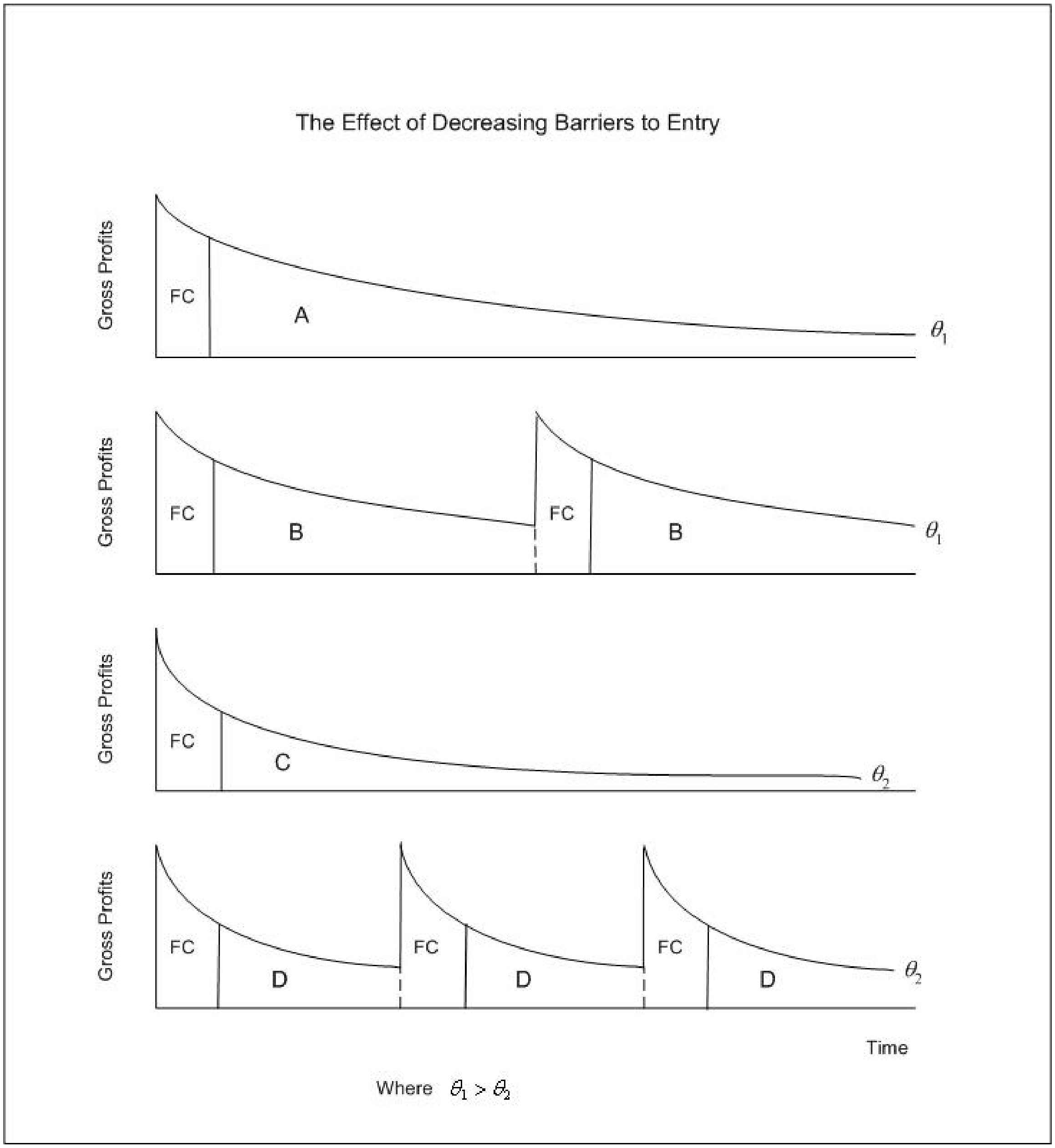
The value of an innovation to the innovator at the date of its release must be non negative. If θ = 0 then the left side of equation (58) is equal to zero, implying that $V_{0,t} < 0$, in which case no innovation takes place. If the innovating firm releases an innovation, infinitely many copying firms enter the market instantaneously producing identical capital goods so that the market is perfectly competitive. The innovating firm makes zero gross profits and negative net profits being unable to cover the fixed sunk cost of the innovation. Given the next best alternative for the innovating firm is not releasing an innovation and making zero net profits, they would choose this option over innovating. This is the same result Quah (2002b) obtains, if the copying process is extremely efficient and there are no patents, implying there are no barriers to entry, the innovator is unable to cover the cost of the innovation in which case no innovation takes place. Boldrine and Levine (2003) refer to this as an indivisibility of technology. This provides some justification for introducing patents when θ approaches zero.

Result 3: The growth rate of embodied technology is maximised for some level of barriers to entry $\theta = \theta^*$ where $0 < \theta^* < 1$. For barriers to entry θ such that $1 \geq \theta > \theta^*$ decreasing the barriers to entry results in the innovating firm decreasing the duration and increasing the frequency of their innovations which increases the growth rate of embodied technology.

[54] This result is similar to Grossman and Helpman (1991) who show that a monopolist has no incentive to undertake research and development.
[55] This is a necessary condition that Boldrin and Levine (2003) outline for innovation to take place.

When choosing the length of an innovation the innovating firm weighs up the additional profits gained by shortening the life of an innovation and fitting more innovations within a time period against the extra fixed sunk costs incurred with each innovation. As outlined in result 2, if θ is set to 1, the length of an innovation is infinite. If θ is lowered slightly the innovating firm experiences some competition in the future eroding the profits they make. Given this erosion is more severe in the later stages of an innovation's life, the innovator does better to release another innovation before they reach this later stage, thereby cutting the length of the current innovation. Decreasing the barriers to entry increases the entry rate of competitors and hastens the erosion of the innovator's profits; the innovating firm responds by choosing shorter durations enabling the release of more innovations with in that period. For some level of barriers to entry such that $\theta = \theta^*$ where $\theta^* < 1$ the firm chooses the shortest length of an innovation. For $\theta < \theta^*$ the effect of decreasing barriers to entry increases the length of an innovation because gross profits are sufficiently small that the size of the fixed sunk cost dominates. The innovating firm must increase the length of an innovation so that their gross profits are large enough to cover the fixed sunk cost of the innovation.
Figure 8 below gives a diagrammatic representation of the intuition behind this result.

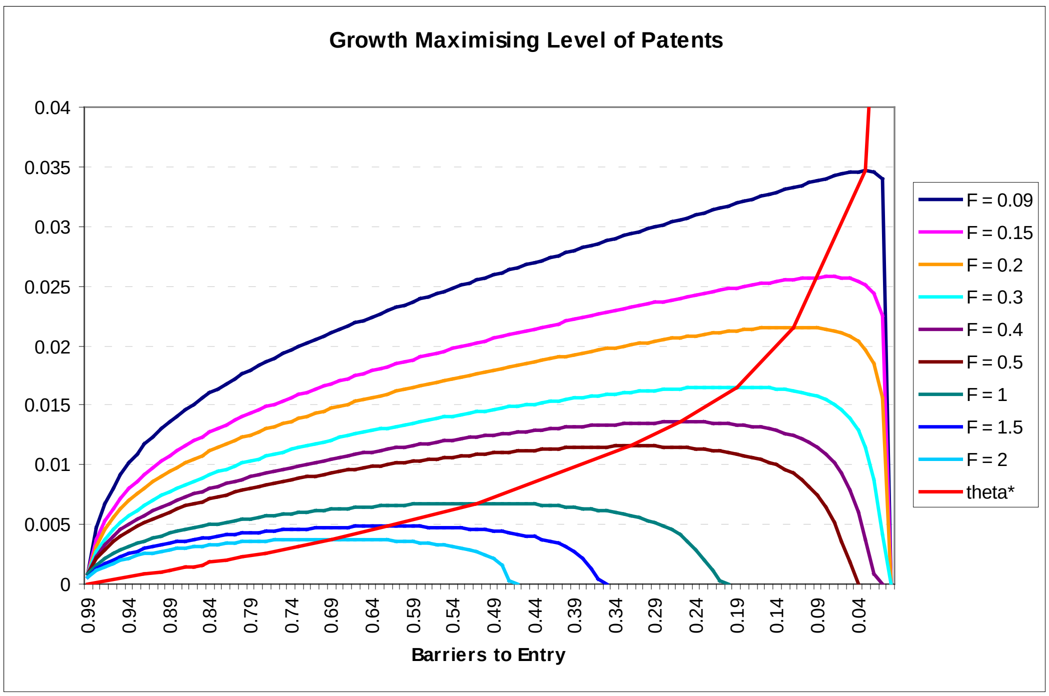
Figure 8.



The top 2 graphs are the gross profit profiles of the innovating firm for the same parameters and barriers to entry. The key thing to note is the innovator does better by releasing two innovations in this time period than only one innovation, that is 2B > A. The bottom two graphs are for the same scenario except the barriers to entry have declined. This shifts the gross profit profiles in ward. The innovator does better by releasing 3 innovations in this time period than only one innovation, that is 3D > C. The second and the fourth graph show for lower barriers to entry the innovating firm wants to release more innovations than if the barriers to entry are higher. The reason for this is the innovating firm earns more profits in the earlier stages of an innovation's life after they have paid off the fixed cost than in the later stages when they have less market power and face more competition. At some point in time they will do better to release another innovation than to continue with the current innovation. By decreasing barriers to entry more competitors enter at an earlier stage moving the gross profit profile inward. This shortens the length of time the innovating firm has market power and earns their highest profits. The innovating firm must release more innovations within this fixed period to maintain market power and earn the highest profits they can.[56]

To determine the affects the fixed sunk cost has on the growth rate, the iteration procedure outlined at the start of this section is repeated for many different values of F. Each value of F corresponds to a different series of χ where F is fixed for the whole series. The series for each F can be used to calculate a series of growth rates as a function of θ with one series for each F. These series can be plotted on the same axis with each series representing a different "growth profile". This is done in Figure 8 below. The additional step is taken to plot a curve through the maximal points on each of the profiles interpolating the points between the profiles.

Figure 9.[57]



The level of patenting that maximises growth varies depending upon the size of the fixed sunk cost associated with an innovation. Each profile in figure 9 plots the growth rate of

[56] This relationship does not hold indefinitely, if barriers to entry are lowered below some critical level the length of time that it takes to pay off an innovation begins to dominate. If barriers to entry are sufficiently low the total revenue earned by an innovation will be much lower so that the fixed sunk cost will have to be spread over a much larger period, this effect increases the profit maximising length of an innovation decreasing the growth rate of embodied technology.

[57] Although not plotted in this graph, other profiles were calculated for both higher and lower fixed sunk costs than those plotted. This aided the generation of the curve corresponding to the maximum growth rates of technology given the fixed sunk cost.

embodied technology against barriers to entry for a given fixed cost. Profiles that are closer to the x axis represent higher fixed sunk costs, while profiles that are further from the x axis represent lower fixed sunk costs. The red line denoted “theta*” traces the locus of the maximal growth rates as a function of the optimal level of patents. If the fixed sunk cost of an innovation is large the growth maximising level of patents would be large too.

Result 4: The growth maximising level of barriers to entry θ^* increase with the fixed sunk cost of an innovation.

The innovating firm requires higher barriers of entry to delay entry by competitors so that they are able to cover the higher cost of the innovation. Result 4 may have important policy implications, in industries with higher fixed sunk costs of innovation, more protection or barriers to entry may be required to maximise the growth rate of embodied technology. This could justify the use of more stringent intellectual property rights where competition is fierce and the cost of discovering an innovation is high. However the maximum growth rate of embodied technology declines when the fixed sunk cost of an innovation increases. Alternatively more effort could be put into minimising the fixed sunk cost associated with an innovation.

The results determined in this section all depend on the assumption that only one firm is able to innovate. This is a strong assumption used for modelling simplicity and may only be realistic for some cases. If this assumption were relaxed the results may change. If more than one firm were allowed to innovate and products were differentiated, or innovations were horizontal and there were 100% barriers to entry, the model may end up resembling Romer (1990). Alternatively if more than one firm were allowed to innovate and technology were still modelled using quality ladders, some kind of “leap frogging” or over taking may have to be introduced, which may also change the results.

This section has shown the model’s key prediction of higher growth when some competition is present. The model predicts no growth with monopoly or perfect competition. This is different to the conclusions reached using the Schumpeterian leapfrogging and Romer (1990) frame works where competition results in the erosion of the capital goods producer’s profits and the innovators’ incentive to undertake research and development. The reason for this difference lies in the models construction. The Romer style growth model and the Schumpeterian leapfrogging models use resource arbitrage conditions to determine the allocation of resources to the research and development and production sectors of the model. This means that resources allocated to research and development and to production are perfect substitutes and they seek out activities that give the highest return. As such, competition in the capital goods producing sector lowers the return from research and development resulting in less resources being devoted to this activity. Competition results in a pure substitution effect, resources are substituted into the final goods producing sector which is more profitable.

In this model increased competition results in an increase in the growth rate of embodied technology. This is because research and development is internalised by one of the capital goods producing firms. There is no arbitrage condition between resources devoted to research and development and the firm must release more innovations to increase their profits in the face of competition. That is competition results in a pure income effect, prevented from substituting resources into other more profitable areas the innovating firm has to release more innovations to maximise their profits when barriers to entry are relaxed.

6. Conclusion

This paper constructs an analytically tractable model of endogenous innovation with a special focus on the effects of barriers to entry, namely patents. More specifically it develops a theory of the determination of both embodied technology and its growth rate. As is the convention in this literature, the growth rate of technology is determined by the profit maximising actions of the innovators, however the model differs in the mechanism used to endogenise innovation. Based on Aghion, Harris and Vickers (1997) "step by step" approach, the innovating firm innovates to escape competition. Competition is modelled as barriers to entry, a measure of the speed at which competitors enter the market an idea that underlies the Boldrine and Levine school of thought. Barriers to entry could include intellectual property rights or the natural complexity of a technology. The model is calibrated to match features of the US economy. The estimated barriers to entry in the US are found to far exceed the growth maximising level of barriers to entry suggested by this model and the US by lowering their current barriers to entry could raise their growth rate of embodied technology by over 2%. The intellectual property rights regime in the US is likely to make up some of these barriers to entry, so that the model implies relaxing or even eliminating patents would increase the growth rate of embodied technology. However there are likely to be other barriers to entry like the natural complexity of a technology and imperfect capital markets that may have a larger impact.

The model's key contribution is the application of the "step by step" intuition into a more complete framework. Although somewhat complicated the model is able to generate explicit formulas that can easily be compared with Mankiw, Romer and Weil (1992) it's more conventional counterpart. The inclusion of a parameter to measure the degree of barriers to entry means this model has useful applications in determining the optimum degree of enforcement on intellectual property rights issues. A new calibration approach is formulated which allows for the estimation of the parameter that measures barriers to entry. The model may also prove useful in explaining some of the disparity that exists between countries in terms of both their income levels and their growth rates.

The model's key limitation is the assumption that the innovator is the only firm able to release innovations. This is quite a strong assumption used for modelling simplicity and to develop intuition for how competition could increase the growth rate of technology. Relaxing this assumption and allowing other firms to innovate may change the results.[58]

[58] If more than one firm were able to innovate and innovations were non drastic or horizontal then the model may end up giving similar results to Romer (1990) when $\theta = 1$.

Future research could expand upon some of the concepts dealt with in this paper. As mentioned conventional models in this literature find that increasing competition decreases the growth rate of technology. This is the result of using resource arbitrage conditions to explain the allocation of resources to production and research and development. That is competition leads to a substitution effect, resources are substituted into more profitable areas when research and development becomes more profitable and there is no income effect. The mechanism used in this paper uses an income effect. Resources are restricted to be used only in research and development, more competition means the innovator must release more innovations to maintain profits. Future research could look at designing a mechanism where both income and substitution effects come into play. A parameter could be included that takes into account the net effect of the income and substitution effect associated with increasing competition. A human capital or resource arbitrage condition could be included where human capital used in research and development is an imperfect substitute for human capital used in final goods production. Then the degree of substitutability will determine the affects that competition will have on the growth rate of embodied technology.

References

AGHION P, C. HARRIS AND J. VICKERS, "Competition and Growth with Step-by-Step innovation: An Example," *European Economic Review* 41, (1997), 771-782.

AGHION P, C. HARRIS AND J. VICKERS, "Competition, Imitation and Growth with Step-by-Step Innovations," *Review of Economic Studies* 68, (2001), 467-492.

AGHION P. AND P. HOWITT, "A Model of Growth through Creative Destruction," *Econometrica* 60, (1992), 323-351.

AGHION P. AND P. HOWITT, "Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth," *Journal of Economic Growth* 3, (1998a), 111-130.

AGHION P. AND P. HOWITT, "Endogenous Growth Theory," (Cambridge: MIT Press 1998b)

BOLDRIN M. AND D. LEVINE, "Perfectly Competitive Innovation," Staff Report 303, FRB Minneapolis Research Department, (2003), Minneapolis.

BOLDRIN M. AND D. LEVINE, “Against Intellectual Property,” (Unpublished book 2004) <http://levine.sscnet.ucla.edu/general/intellectual/against.htm>.

BOUCEKKINE R, AND D. DELACROIX, “Information technologies, embodiment and growth,” *Journal of Economic Dynamics and Control* 27, (2003), 2007-2034.

BOUCEKKINE R, F. DEL RIO AND O. LICANDRO, “Obsolescence Vs modernisation in a Shumpeterian vintage capital model,” Documento de Trabajo 2000-27, (2000).

BOUCEKKINE R, F. DEL RIO AND O. LICANDRO, “Obsolescence and Modernization in the Growth Process,” IRES Universite catholique de Louvain working paper, (2004).

COOLEY, T. AND E. PRESCOTT, “Economic Growth and Business Cycles”. Cooley, T. (Ed.). *Frontiers of Business Cycle Research* (Princeton: Princeton University Press 1995).

GORDON R., *The Measurement of Durable Goods Prices* (Chicago: University of Chicago Press 1995).

GREENWOOD J, Z. HERCOWITZ AND P. KRUSSEL, “The Long-run implications of Investment Specific Technological Change,” *The American Economic Review* 87, (1997), 342-362.

GREENWOOD J, AND B. JOVANOVIC, “Accounting for Growth,” in *New Developments in Productivity Analysis*, (eds) C. Hulten, E. Dean and M. Harper. (Chicago: University of Chicago Press 2001), 179-222.

GRILICHES Z., “Hybrid corn: an exploration in the economics of technological change,” *Econometrica* 4, (1957), 501-522.

GROSSMAN G. AND E. HELPMAN, “Quality Ladders in the Theory of Growth,” *The Review of Economic Studies* 58, (1991), 43-61.

HOWITT P., “Measurement Obsolescence, General Purpose Technologies,” Helpman E. (Ed) *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press 1998).

HSIEH C., “Endogenous growth and obsolescence,” *Journal of Development Economics* No.66, (2001), 153-171.

HULTEN C., “Growth Accounting When Technical Change is Embodied in Capital,” *American Economic Review* 82, (1992), 964-980.

INTEL CORPORATION, “Moore’s Law,” Research at Intel, <http://www.intel.com/research/silicon/mooreslaw.htm>

JORGENSON D., “Capital Theory and Investment Behaviour,” *The American Economic Review* 53, (1963), 247-259.

JORGENSON D., “The Embodiment Hypothesis,” *The Journal of Political Economy* 74, (1966), 1-17.

KRUSELL P., “Investment-Specific R&D and the Decline in the Relative Price of Capital,” *Journal of Economic Growth* 3, (1998), 131-141.

MANKIW N.G, D. ROMER and D.WEIL, “A Contribution to the Empirics of Economic Growth,” *The Quarterly Journal of Economics* 107, (1992), 407-437.

PRESCOTT E., “Lawrence R. Klein Lecture 1997: Needed: A Theory of Total Factor Productivity,” *International Economic Review* 39, (1998), 525-551.

QUAH D., “Economic Growth Measurement,” *International Encyclopedia of the Social and Behavioral Sciences* 3.4, (2002a).

QUAH D., “24-7 Competitive Innovation,” CEP Working Paper #1218, (2002b).

ROMER P., “Endogenous Technological Change,” *The Journal of Political Economy* 98, (1990), S71-S102.

SOLOW R., “A Contribution to the Theory of Economic Growth,” *Quarterly Journal of Economics* 70, (1956), 65-94.

SOLOW R., “Technical Change and the Aggregate Production Function” *Review of Economics and Statistics* 39, (1957), 312-320.

SOLOW R., “Investment and Technical Progress,” In Arrow, K., Karbin, S., & Suppes, P. (Eds.), *Mathematical Methods in the Social Sciences* (Stanford: Stanford University Press 1960).

SOLOW R., *Growth Theory: An exposition*, (New York, Oxford: Oxford University Press 2000)

WIKIPEDIA <http://en.wikipedia.org/wiki/Moore's_law> (2004)