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ENDOGENOUS EMIGRATION OF SKILLED LABOUR

Debasis Bandyopadhyay,
Matthew Radich and Graham F Rhodes

Economics Department
University of Auckland

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ABSTRACT

There is an increasing trend in the pattern of world migration of educated and skilled individuals leaving developing countries for developed ones. This has created much concern about a so-called "brain drain". As the name suggests, the brain drain was believed by early commentators to be unequivocally harmful to those countries which suffered from it. More recent literature does not directly refute this belief but puts it under closer scrutiny, and finds the possibility of many different mechanisms through which the negative effects of the brain drain may be mitigated. This paper aims to illuminate by surveying the literature on the brain drain. It does so by exploring the brain drain in the current context of globalisation and by making comparisons to previous periods of globalisation, by examining the theoretical analyses of migration and the brain drain, by exploring an economic model of the brain drain, and by analysing the particular case of New Zealand, a developed country with a high rate of skilled and educated emigration.

Key Words: Education, Persistent Wage-Differential, Human Capital, Growth with Increasing Returns

JEL Codes: I25, J24, J61, O15

Introduction

We model Brain Drain endogenously and examine its relationship with economic growth. We address the issue in an environment when emigrants can not be taxed, foreign countries have maximum quotas and restrictions on the number and the type of emigrants and the government keeps a balanced budget. Our paper develops the model of Wong-Yip (1999) by introducing physical capital and by considering alternative specification of how teacher to student ratio affects the quality of education. Wong-Yip's model shows that brain-drain hurts growth and generally hurts the non-immigrants by adversely affecting human capital accumulation in the home country. With minor modification as suggested above we make brain drain an endogenous function of fiscal policy and show that brain-drain could be good some time and there is an optimal profile of brain drain that can be manipulated by the government. Then by solving the Ramsey (1928) problem, we explicitly derive the fiscal policy that ensures the optimal profile of brain drain.

An outline of the optimal policy follows:

Three occupational choices of the skilled workforce:

- Educators (E_t)
- Entrepreneurs (L_{St})
- Emigrant (π_t)

Given an objective function involving a sequence of utility or growth rate we develop a concept of the socially efficient proportion of skilled workers who should be employed as educators (γ_t).

Two main trade-offs:

1. In a closed economy, public policy determines the proportion of the skilled labour force who work as entrepreneurs. A greater proportion of entrepreneurs means a higher national output (Y_t), but lower productivity due to a decrease in the number of educators (E_t). This could possibly be developed into a Business Cycle Model of output and productivity.

2. In an open economy, the foreign labour market relative to the domestic labour market not only alters the size of the skilled labour force who remain in

the country, but also the proportion of the skilled workers who choose to be entrepreneurs (i.e. effects l_t, n_t and γ_t).

The Model

Our model abstracts a Diamond (1965) type overlapping-generations model of a small, open economy with commodity and education production sectors similar to Wong-Yip model. A single good is produced by perfectly competitive firms and for simplicity, consumed domestically. Agents live for two periods. In the first period of their life, they devote a certain proportion of their time to education and the remainder is spent working as unskilled labour. In their second period of life, having received education, agents now contribute to the pool of skilled labour. Of these skilled workers, some proportion work as educators and the rest input their labour in commodity production, and can be thought of as entrepreneurs. Government uses a proportional income tax on the skilled workforce in both the production and education sectors to fully fund the education system. Wong and Yip motivate the Brain Drain through a wage differential in returns to skilled work at home and abroad. After the Brain Drain is permitted, at the beginning of each period a proportion of the skilled labour force leaves the source country permanently. Following is a detailed description of the model.

1.1 Technology

Competitive firms produce the homogeneous good with a mixture of two types of labour, skilled and unskilled. Departing from Wong and Yip's formulation, physical capital is also considered. In period *t*, $t = 0,1,...,\infty,$ production is represented by:

$$Y_t = F(K_t, L_{Ut}, L_{St}) \tag{1.1}$$

which, in this case, is assumed to take the following functional form:

$$Y_t = K_t^{\alpha} L_{St}^{\beta} L_{Ut}^{1-\alpha-\beta} \tag{1.2}$$

where, Y_t is the output, K_t is the total stock of physical capital, L_{St} the input of skilled labour measured in efficiency units and L_{Ut} is the input of unskilled labour measured in efficiency units. Each type of worker is paid their marginal products per efficiency unit of labour, w_{Ut} and w_{St} for unskilled and skilled workers respectively, such that:

$$w_{Ut} = (1-\alpha-\beta)K_t^{\alpha} {L_{St}}^{\beta} {L_{Ut}}^{-(\alpha+\beta)} \tag{1.3}$$

$$w_{St} = \beta K_t^{\alpha} L_{St}^{\beta-1} L_{Ut}^{1-\alpha-\beta} \tag{1.4}$$

Each skilled worker can be thought of in this economy as an entrepreneur. Each entrepreneur uses the pool of unskilled labour to produce the nation's homogenous good.

Define:

$$l_t \equiv \frac{L_{Ut}}{L_{St}} \tag{1.5}$$

as the unskilled to skilled labour input ratio. Thus, each skilled entrepreneur has l_t units of unskilled labour at their disposal for commodity production. As a result, using 1.5 in 1.2, the production function can be written in intensive form as:

$$\begin{aligned} y_t &= f(l_t, k_t) \\ &= k_t^{\alpha} l_t^{1-\alpha-\beta} \end{aligned} \tag{1.6}$$

where, $y_t \equiv \frac{Y_t}{L_{St}}$ and $k_t \equiv \frac{K_t}{L_{St}}$. Similarly, the marginal products of skilled and unskilled labour and physical capital can also be described in terms of l_t and k_t:

$$\begin{aligned} w_{Ut} &= f_1(k_t, l_t) \\ &= (1-\alpha-\beta)k_t^{\alpha} l_t^{-(\alpha+\beta)} \end{aligned} \tag{1.7}$$

$$r_t = f_2(k_t, l_t) \\ = \alpha k_t^{\alpha-1} l_t^{1-\alpha-\beta} \quad (1.8)$$

$$w_{St} = y_t - w_{Ut} l_t - r_t k_t \\ = f(k_t, l_t) - f_1(k_t, l_t) l_t - f_2(k_t, l_t) k_t \\ = k_t^{\alpha} l_t^{1-\alpha-\beta} - (1-\alpha-\beta) k_t^{\alpha} l_t^{-(\alpha+\beta)} l_t - \alpha k_t^{\alpha-1} l_t^{1-\alpha-\beta} k_t \\ = \beta y_t \quad (1.9)$$

Using 1.5, 1.7, 1.8 and 1.9 we have:

$$w_{Ut} L_{Ut} = (1-\alpha-\beta) Y_t \\ w_{St} L_{St} = \beta Y_t \\ r_t K_t = \alpha Y_t \quad (1.10)$$

which ensure that the zero profit condition for perfect competition in commodity is met as follows:

$$\Pi_t = Y_t - w_{St} L_{St} - w_{Ut} L_{Ut} - r_t K_t \\ = Y_t - \beta Y - (1-\alpha-\beta) Y_t - \alpha Y_t \\ = 0 \quad (1.11)$$

1.2 Workers

Each worker lives for two periods, labelled 1 and 2. In period t, when the individual is born, they have human capital, h_{t-1} inherited from the generation of people old in the previous period, which is the labour efficiency they supply when working as unskilled labour. Thus h_t is the human capital of the old in period *t* and the young in period $t+1$. When young, agents spend a proportion of their time getting education and the remainder working as unskilled labour. Assuming that each individual is endowed with one unit of non-leisure time for each period, they can choose to devote a proportion of their time, $e_t \in (0,1)$, to education in the first period of their life to give them a particular level of human capital, h_{t+1}, in their second period of life. Thus the amount of time, when each worker is young, they can work as an unskilled worker is $(1-e_t)$, giving them income of $(1-e_t)h_{t-1}w_{Ut}$.

Education is provided free by the government, and funded through an ad valorem tax at rate τ_t on skilled workers. After choosing to get education when young, an individual will attain a higher level of human capital in the next period, h_{t+1} and supply the same in efficiency units of labour. Working as skilled labour will provide an after tax income of $(1-\tau_{t+1})h_{t+1}w_{St+1}$. Section 1.4 gives a complete description of the education process and the accumulation of human capital.

A further extension beyond Wong and Yip's model allows for individuals to save, at a rate s_t, in their first period of life as a young, unskilled student. Consider a representative consumer born at the beginning of period t. Let inter-temporal utility be:

$$u_t = \ln c_{1t} + \rho \ln c_{2t+1} \qquad (1.12)$$

where $\rho \in (0,1)$ is the discount factor, consumption in the first period when they are young and working as an unskilled worker is:

$$c_{1t} = (1-e_t)w_{ut}h_{t-1} - s_t \qquad (1.13)$$

and consumption in the second period of their life as a skilled worker is:

$$c_{2t+1} = (1-\tau_{t+1})w_{st}h_{t+1} + r_w s_t + (1-\delta)s_t \qquad (1.14)$$

where δ is the rate of depreciation and r_w is the world rate of interest which this small, open economy takes as given.

1.3 Population and Brain Drain

Generation numbers of young and old in each period are denoted as N_t^Y and N_t^O respectively. Whereas Wong and Yip assume each individual on average has one child; consequently there is no population growth within the economy, our model allows for natural population increase. Each person has on average $(1+n+\varepsilon_t)$ children at the beginning of their second period of life who constitute the generation of young in that period, where n is the average rate of natural population increase and ε_t is a stochastic population shock and $\varepsilon_t \sim iidN(0,\sigma^2)$. Thus we have the number of young people born at the beginning of each period is:

$$N_t^Y = (1+n+\varepsilon_t)N_t^O \quad \text{where, } N_t^O = N_{t-1}^Y \tag{1.15}$$

Before a Brain Drain is introduced, each generation is normalised. Thus in the simulation, in period zero and autarky, the number of both young and old is one ($N^* = N_0^Y = N_0^O = 1$) and the total population is two.

In the simulation, a Brain Drain is introduced at time $t=1$ between the home economy and a foreign one. For simplicity, it is assumed that the Brain Drain has the following characteristics:

- The home country is small compared to the foreign country. Prices and policies abroad are not affected by labour movements and the home country takes conditions abroad as given.
- Before the Brain Drain the home country is assumed to be in autarky.
- Migration is riskless, costless and instantaneous.
- The foreign country will only accept skilled immigrants, and as such a Brain Drain is the only type of migration considered.
- In order to simplify the analysis, there is neither return migration of workers from abroad and nor any immigration into the home country of foreign workers. This allows the investigation to focus purely on the effects of the Brain Drain.

Furthermore, to motivate migration in Wong and Yip's framework, it is assumed that after tax wage levels for skilled workers abroad ($F*$) are strictly greater than autarkic after tax income at home:

$$(1-\tau_t)w_S^{autarky} < F* \qquad (1.16)$$

It is assumed that the foreign economy is on a balanced growth path. As a result $F*$, the trend adjusted after tax foreign wage for skilled workers, is constant. The condition 1.16 ensures that when migration is allowed and a Brain Drain takes place for the host country. Over time as the skilled labour supply contracts due to Brain Drain, by 1.4, skilled wages at home will increase bringing 4.8 into equilibrium. At this stage the Brain Drain will cease.

When emigration is allowed (when 1.16 holds) a proportion, π_t, of the skilled workers leave for greener pastures abroad, and as a result, the domestic population contracts. It is further assumed that migration frictions, such as heterogeneity of risk attitudes, foreign countries' immigration policies and the inevitable uncertainty of finding employment abroad, prevent all the skilled workers leaving at once.

Population dynamics under Brain Drain can be described as follows. The numbers of young and old in period t are given by 1.15. During a Brain Drain when π_t proportion of the young of the previous period leave for more appealing climes the number of old, skilled workers remaining is:

$$N_t^O = (1-\pi_t)N_{t-1}^Y \qquad (1.17)$$

Thus the Brain Drain shrinks each old generation at the rate $\frac{1}{1-\pi_t}$, in contrast to natural increase which swells each successive young generation at the rate $(1+n+\varepsilon_t)$. However, it must be noted, that in Wong and Yip's and our formulation π_t cannot be constant under a balanced growth path. In both models, π_t is endogenous and a function of the wage gap, $F^* - w_{St}$, and as such:

$$\pi_t \to 0, as\ w_{St} \to F^* \tag{1.18}$$

Eventually a marginal skilled worker will receive the same level of welfare whether they migrate or not and the Brain Drain will cease. Namely when:

$$(1-\tau_t)w_{St} = F^* \tag{1.19}$$

1.4 Education Production

In the model, education enables human capital accumulation. The growth of knowledge depends on three factors: current level of human capital, h_t, which is also the labour efficiency supplied when working; time spent in education, e_t; the quality of education provided by the government, γ_t. With this final parameter, γ_t, we again diverge slightly from the course chosen by Wong and Yip, who rather than quality of education, use the number of teachers, E_t Education production is assumed to take the form:

$$\begin{aligned} h_{t+1} &= h_t f(e_t) g(\gamma_t) \\ h_{t+1} &= h_t (1+e_t)^{\theta} (1+\gamma_t)^{\sigma} \end{aligned} \tag{1.20}$$

where the following conditions are satisfied:

$$\begin{aligned} &f(e_t) > 1 \text{ for } e_t > 0 \\ &f(0) = 1 \\ &f'(e_t) > 0 \\ &f''(e_t) < 0 \text{ for } \mathrm{e_t} \geq 0 \\ &g(0) = 1 \\ &g'(\gamma_t) > 0 \end{aligned}$$

The parameter θ can be interpreted as the ability of students. Similarly σ can be interpreted as the degree of ineffectiveness of teachers, and it is assumed that σ and θ are non-negative.

Moreover, the condition $g(0)=1$ captures the necessity of quality of teaching. Without a lower quality education system, agents in the economy would be hindered in their learning, as any proportion of time in education would be less effective and growth in human capital would come solely from the students' own level of ability. Furthermore, from 1.21, education is a function of the teachers' human capital in period t, which is h_t, not of students' human capital, h_{t-1}. Thus through a combination of attending school and effective teaching, knowledge grows in this economy.

Thus in period t the unskilled young have human capital h_{t-1}, which they inherit from the previous period's, $t-1$, generation of old. The old, skilled generation of period t have human capital h_t, which some of the skilled workforce employed as Educators use to imbue the young of that generation with human capital h_{t+1}. The old of period t pass on their inherent knowledge, h_t, to the next generation, the young of period $t+1$. When the young of period t graduate to period $t+1$ and become members of the old, skilled generation, their time in education in period t enables them to provide h_{t+1} efficiency units of labour when working as skilled labour in commodity production or as Educators.

Educators are themselves skilled workers, and as such come from the pool of skilled labour, thus reducing the pool of skilled workers available for commodity production. It is necessary to make the following clarification of young and unskilled and old and skilled. The number of unskilled workers in the economy is the same as the number of young people:

$$N_{Ut} = N_t^Y \tag{1.21}$$

However, the number of skilled workers for *commodity* production is:

$$N_{St} = N_t^O - E_t \tag{1.22}$$

Thus, although the numbers of young and unskilled are interchangeable, the numbers of old and skilled workers available for *commodity* production are not.

In period t, the government hires a proportion of the skilled workers to act as educators, E_t. The ratio of teachers to students is $\gamma_t \in (0,1)$, and can be interpreted as the quality and level of government subsidisation of education: a proportion of national output invested in funding human capital accumulation. If all young people spend some time in education, then the number of young people is the same as the number of students, and thus from equation 4.15:

$$E_t = \gamma_t N_t^Y \tag{1.23}$$

While not in school getting education, unskilled workers contribute to commodity production, as do skilled workers less educators. Thus the Labour Market clearing conditions are as follows:

$$L_{Ut} = (1 - e_t) h_{t-1} N_{Ut} \tag{1.24}$$

and,

$$L_{St} = h_t N_{St} \tag{1.25}$$

1.5 Labour Markets and Brain Drain

We define a new variable:

$$n_t \equiv \frac{(1 - e_t) N_{Ut}}{N_{St}} \tag{1.26}$$

called the unskilled to skilled worker ratio, which is the supply of unskilled labour per entrepreneur. The labour market clearing condition under balanced growth is (by 1.5, 1.24, 1.25 and 1.39):

$$n_t = l_t(1+g_{t-1}) \tag{1.27}$$

which relates demand and supply for both types of labour.

Agents observe *last* period's difference in domestic and foreign skilled wages and emigrate. As a result, there is a one period lag between wage differentials and their effect on the rates of emigration:

$$\pi_t = f(n_{t-1} - n_{t-1}^{FF}) \tag{1.28}$$

1.6 Government

Public education is financed through income tax on all skilled workers (including Educators). Bond markets are allowed for, thus a balanced government budget implies:

$$r_t B_t^g + h_t E_t w_{St} = \tau_t h_t [E_t + N_{St}] w_{St} + B_{t+1}^g \tag{1.29}$$

So, by 1.21, 1.22 and 1.23, the tax rate that balances the government budget is:

$$\tau_t = \frac{\gamma_t N_{Ut}}{N_{St} + \gamma_t N_{Ut}} - \frac{NFD_t}{h_t w_{St}\left[N_{St} + \gamma_t N_{Ut}\right]} \tag{1.30}$$

where, $NFD_t = B_{t+1}^g - r_t B_t^g$

1.7 Individuals' Optimisation

The individual's problem is to choose e_t such that:

$$\max_{\{e_t,s_t\}} u_t = \ln\left[(1-e_t)w_{Ut}h_t - s_t\right] + \rho\ln\left[(1-\tau_{t+1})w_{St+1}h_{t+1} + (1+r_w-\delta)s_t\right] \quad (1.31)$$

subject to:

$$\begin{aligned} h_{t+1} &= h_t(1+e_t)^{\theta}\left(1+\gamma_t\right)^{\sigma} \\ c_t^y + s_t &= y_t^y \\ c_{t+1}^o &= y_{t+1}^o + r_t s_t \end{aligned} \quad (1.32)$$

where s_t is savings, $y_t^y = (1-e_t)w_{Ut}h_{t-1}$ is the wage income when young, $y_{t+1}^o = (1-\tau_{t+1})w_{St+1}h_{t+1}$ is the wage income when old and r_t is the interest rate. The economy is assumed to small such that domestic interest rates are given by world interest rates, so that $r_t = r_w$, where r_w is the world interest rate. Agents save depending on the magnitude of returns to education (r^E) relative to returns to physical capital (r_w). If the returns to physical capital are greater than those to education, individuals will save rather than go to school, and when returns to education outweigh the returns from physical capital, individuals will attain education rather than save, to improve their future income when old.

Returns to education are given by:

$$1+r^E = \frac{(1-\tau_{t+1})w_{St+1}\,{}^{\delta h_{t+1}}\!/_{\delta e}}{w_{Ut}h_{t-1}} \quad (1.33)$$

thus, by 1.20:

$$r^E = \frac{\theta(1-\tau_{t+1})w_{St+1}h_{t+1}}{w_{Ut}h_{t-1}(1+e)} - 1 \quad (1.34)$$

In the event that $r^E < r_w$, savings yields a higher income when old than would education. Under these conditions, education is substituted by savings when agents are young. Solving the first order conditions for 1.31 subject to 1.32 and choosing s_t to maximise utility when $r^E < r_w$ implies:

$$s_t = sh_{t-1}w_{Ut} - s(r_w)(1-\tau_t)h_{t+1}w_{St+1}$$
$$\text{where, } s = \frac{\rho}{1+\rho} \text{ and } s(r_w) = \frac{1}{(1+\rho)(1+r_w-\delta)} \tag{1.35}$$

Solving the first order conditions for 1.31 subject to 1.32 and choosing e_t to maximise utility when $r^E > r_w$ implies:

$$e = \frac{\rho\theta - 1}{\rho\theta + 1} \tag{1.36}$$

From equation 1.36, the optimal amount of time spent in education when young by the individual is a function of the parameters, ρ (the discount rate) and θ (the level of ability), is constant over time and independent of any Brain Drain effect. Furthermore, since agents are homogenous and the choice of time at school constant, e, can be interpreted as the level of compulsory education in the economy.

1.8 Growth under Brain Drain

The growth of output is given by:

$$1 + g_t^Y = \frac{Y_{t+1}}{Y_t} \tag{1.37}$$

which, by 1.6, is:

$$\frac{Y_{t+1}}{Y_t} = \frac{L_{St+1}k_{t+1}^{\alpha}l_{t+1}^{1-\alpha-\beta}}{L_{St}k_t^{\alpha}l_t^{1-\alpha-\beta}} \tag{1.38}$$

However, on a balanced growth path l_t is constant, and as a result k_t is also constant. Consequently,

$$\begin{aligned}\frac{Y_{t+1}}{Y_t} &= \frac{L_{St+1}}{L_{St}} \\ &= \frac{h_{t+1}N_{St+1}}{h_t N_{St}}\end{aligned} \tag{1.39}$$

Thus, on a balanced growth path the growth rate of output must be the same as the growth rate of human capital such that:

$$\frac{Y_{t+1}}{Y_t} = \frac{h_{t+1}}{h_t} \tag{1.40}$$

and from equations 1.20 and 1.23 we get:

$$1+g_t = \frac{h_{t+1}}{h_t} = (1+e)^{\theta}(1+\gamma_t)^{\sigma}, \quad g \geq 0 \tag{1.41}$$

where g_t is the autarkic growth rate. When the government maintains a constant education policy of $\gamma_t = \gamma *$ growth will remain constant. However, a fixed proportional tax scheme results in fluctuations in the level of government subsidisation of education which in turn affects the growth rate of the economy.

A greater growth rate of human capital increases the quality of the skilled entrepreneurs, leading to a greater demand for unskilled labour. However, since the supply of unskilled labour per entrepreneur is constant (by equation 24), the unskilled wage increases and by substitution the quantity of unskilled labour employed falls.

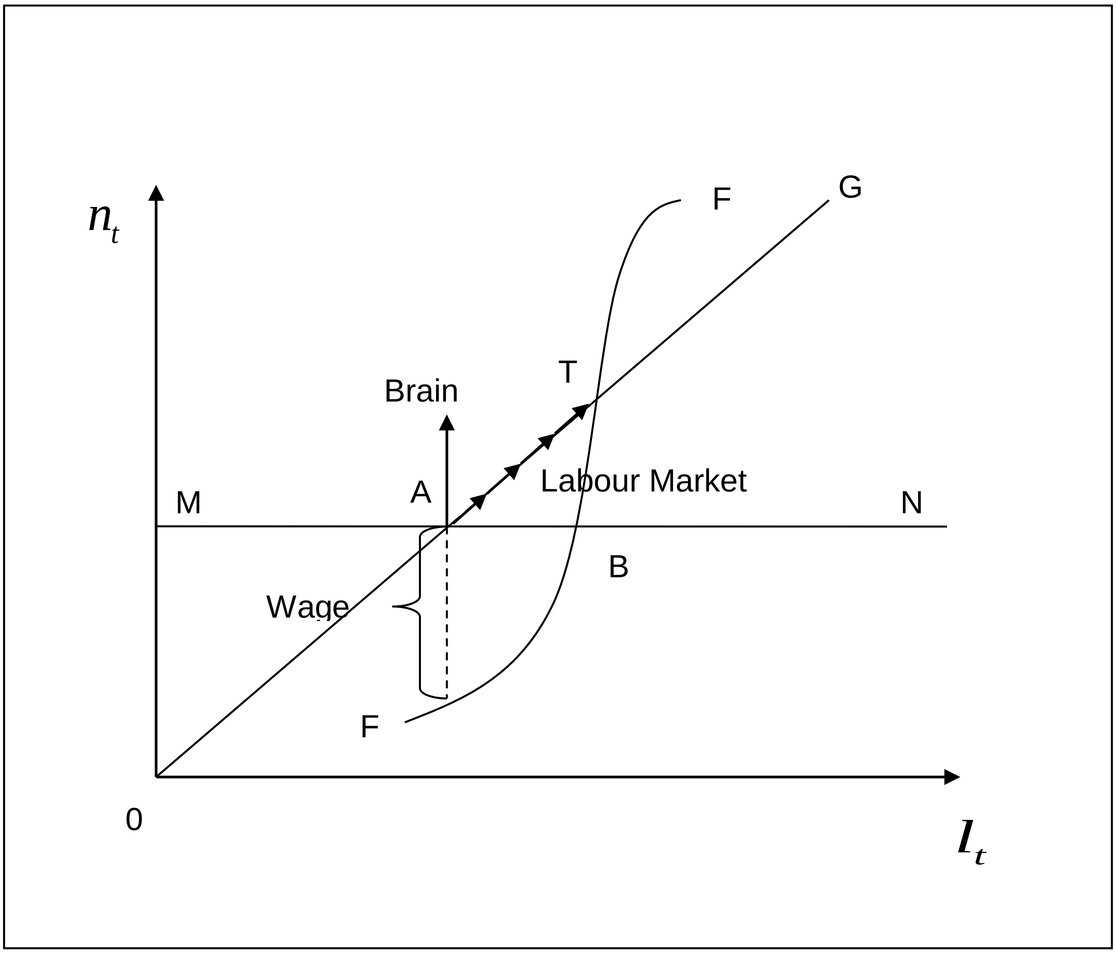
In response to the growth of the young population, if the government employs a greater number of teachers to maintain a constant teachers to students ratio ( ), then the number of skilled entrepreneurs in the economy will fall. Demand for unskilled labour decreases to clear the labour market and the new value of will be lower. Also note that will be higher, which means an increase in unskilled workers per entrepreneur.

Population Change: a) Natural Increase

Consider, unlike physical capital and output, the case when the number of unskilled workers increases through population growth, but supply of skilled workers remains constant. In this instance, the supply of unskilled labour per entrepreneur will increase, given by 24' on page 6a, and the effect in this case is not clear.

Population Change: b) Emigration

If skilled can emigrate, while unskilled cannot, we will get a similar effect on n_t, the ratio of the supply of unskilled to skilled labour, but OG in figure 1 does not shift, so l_t goes up, resulting in a lower unskilled wage and a higher skilled



$$F^* = \frac{(1-e_t)}{\gamma_t n_t + (1-e_t)} w_{St}$$

FF describes the locus of n_t and l_t such that the after tax skilled wage at home is equated with foreign after tax earnings for the skilled, namely when 1.16 holds with equality. At point A, n_t is greater than what is required by FF for a given l_t. it follows that the domestic after tax wage of the skilled is less than the foreign, which leads to emigration. As people emigrate, n_t rises because the supply of unskilled labour per entrepreneur increases.

In Wong and Yip's paper, "Brian Drain is equivalent to a fall in population," because they assume skilled workers emigrate with children. This results in a fall in the total number of educators for a given teacher to student ratio.

If we adopt a new human capital production function such that quality of education is proxied by the teacher-student ratio (γ_t), and not the total number of teachers, then to ensure balanced growth, government must maintain a constant teacher-student ratio when optimal time spent in education when young (e_t) is constant.

Interestingly, even without assuming a constant γ_t, although maintaining a constant E_t, we can generate a growth path converging to a steady growth path.

<u>Transitional Dynamics:</u>

Start at point A with a wage gap. This causes Brian Drain which leads to a domestic labour market adjustment resulting in an increase in l_t. Assuming the autarkic growth rate continues temporarily, the economy moves from point A to point T (assuming γ_t is constant, equilibrium is possible).

At $t = 0$, some skilled workers (not unskilled) leave, resulting in a fall in the supply of skilled labour. Consequently, l_t falls and n_t falls. The marginal product of skilled labour increases at home by diminishing returns and thus there will be more demand for unskilled labour which will increase l_0.

Immediately following the Brain Drain, n_0 increases or equivalently, the supply of unskilled relative to skilled labour increases. With the growth rate of human capital remaining constant, demand for unskilled labour per entrepreneur remains unchanged. The excess supply of unskilled labour leads to a fall in the unskilled wage and that in turn encourages each existing entrepreneur to employ more skilled labour.

1.9 Simulation

Solving the model gives the following system of equations used to simulate the economy before, during and after a Brain Drain in terms of parameters and constants, or preceding variables.

For simulation purposes, as described previously in section 1.5, π_t is a function of the wage differentials of the *previous* period which, can be described as a function of the difference between the unskilled-skilled worker ratios at home and abroad. Namely:

$$\pi_t = f\left(\frac{n_{t-1} - n_{t-1}^{FF}}{n_{t-1} + n_{t-1}^{FF}}\right) \tag{1.42}$$

The teacher-student ratio for simulation purposes was either fixed at a constant rate $\gamma_t = \gamma *$, or when the government chooses to maintain a constant fiscal structure is given by the following:

$$\gamma_t = \frac{\tau}{1 + n + \varepsilon_t} \tag{1.43}$$

Population dynamics during the Brain Drain are given by:

$$N_t^O = (1-\pi_t)N_{t-1}^Y$$
$$\text{and, } N_t^Y = (1+n+\varepsilon_t)N_t^O \tag{1.44}$$

In the economy in question, the number of unskilled workers is the same as the number of young, who are also students:

$$N_{Ut} = N_t^Y \tag{1.45}$$

The number of Educators in the economy is dictated by government education policy and is proportional to the number of students:

$$E_t = \gamma_t N_t^Y \tag{1.46}$$

\
The growth of the economy is:

$$g_t^{BD} = (1+e)^{\theta}(1+\gamma_t)^{\sigma} - 1 \tag{1.47}$$

When the government maintains a constant education policy, namely when $\gamma_t = \gamma *$, in the presence of a Brain Drain, growth in the economy remains constant. Under a constant proportional tax regime however, growth rates are affected by the emigration of skilled workers.

The growth of human capital is described by:

$$h_t = (1+g_t)^t h_0 \tag{1.48}$$

The number of unskilled, young people and skilled old people available for commodity production is:

$$N_{Ut} = N_t^Y \tag{1.49}$$

$$N_{St} = N_t^O - \gamma_t N_t^Y \tag{1.50}$$

where $N_t^O = (1-\pi_t)N_{t-1}^Y$.

The number of unskilled workers per entrepreneur, or unskilled to skilled worker ratio is:

$$n_t = \frac{(1-e)N_{Ut}}{N_{St}} \quad (1.51)$$

and the supplies of unskilled and skilled labour for commodity production are:

$$L_{Ut} = (1-e)h_{t-1}N_{Ut} \quad (1.52)$$

$$L_{St} = h_t N_{St} \quad (1.53)$$

and so consequently the unskilled-skilled labour ratio is:

$$l_t = \frac{L_{Ut}}{L_{St}} \quad (1.54)$$

Physical capital available to each entrepreneur is:

$$k_t = \left[\frac{\alpha l_t^{1-\alpha-\beta}}{r_w}\right]^{\frac{1}{1-\alpha}} \quad (1.55)$$

The economy's total capital stock is:

$$K_t = L_{St}k_t \quad (1.56)$$

National output is:

$$\begin{aligned} Y_t &= L_{St}y_t \\ &= L_{St}k_t^{\alpha}l_t^{1-\alpha-\beta} \end{aligned} \quad (1.57)$$

The wage rates of unskilled and skilled labour are:

$$w_{Ut} = (1-\alpha-\beta)k_t^{\alpha}l_t^{-(\alpha+\beta)} \quad (1.58)$$

$$w_{St} = \beta y \quad (1.59)$$

$$n_t^{FF} = \frac{1-e}{\gamma_t}\left[\frac{w_{St}}{F*} - 1\right] \quad (1.60)$$

Under Brain Drain, the government must choose either to maintain a constant tax rate $\tau_t = \tau *$, or adjust taxes to maintain the education system with a constant teacher to student ratio and a tax base affected by the Brain Drain, using the following rule:

$$\tau_t = \frac{\gamma_t n_t}{(1-e)+\gamma_t n_t} \tag{1.61}$$

The returns to education are given by:

$$r^E = \frac{\theta(1-\tau_{t+1})w_{St+1}h_{t+1}}{w_{Ut}h_{t-1}(1-e)} - 1 \tag{1.62}$$

As detailed in section 1.7, savings are dictated by the returns to education relative to the returns to physical capital such that:

$$s_t = \begin{cases} 0 & \text{if } r^E > r_w \\ sh_{t-1}w_{Ut} - s(r_w)(1-\tau_t)h_{t+1}w_{St+1} & \text{if } r^E < r_w \end{cases} \tag{1.63}$$

Consumption when young is:

$$c_t^y = \begin{cases} (1-e)h_{t-1}w_{Ut} & \text{if } r^E > r_w \Rightarrow s_t = 0 \\ h_{t-1}w_{Ut} - s_t & \text{if } r^E < r_w \Rightarrow e = 0 \end{cases} \tag{1.64}$$

and when old:

$$c_t^o = \begin{cases} (1-\tau_{t+1})h_{t+1}w_{St} & \text{if } r^E > r_w \Rightarrow s_t = 0 \\ (1-\tau_{t+1})h_{t+1}w_{St} + (1+r_w\text{-}\delta)s_t & \text{if } r^E < r_w \Rightarrow e = 0 \end{cases} \tag{1.65}$$

Thus utility is given as:

$$u_t = \ln c_t^y + \rho \ln c_{t+1}^o \tag{1.66}$$

By setting $\pi_t = 0$, the same system of equations models the economy in autarky.

Reference:

Diamond, P. A. 1965. “National Debt in a Neoclassical Growth Model,” *American Economic Review*, 55, 1126-1150

Ramsey, F. P. 1928. “A Mathematical Theory of Saving,” *Economic Journal*, 38, 543-559.

Wong, K. and Yip, C.K. 1999. “Education, Economic Growth and Brain Drain,” *Journal of Economic Dynamics and Control*, Vol. 23, 699-726.