UNIVERSITY OF AUCKLAND

Essays on Earnings Instability: The Impact of Search Effectiveness, Labour Market Policy Instruments, and Globalisation

A DISSERTATION

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By

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Contents

ABSTRACT

Chapter 1: We study the link between market thickness, labour market flexibility and wage dynamics. We consider an economy with two sectors; a risk-free sector that employs workers only, and a risky sector with matching frictions that employs both workers and employers. Workers are risk-averse, whereas employers are risk-neutral. In the risky sector, complete contracts are unavailable due to informational reasons; hence flexible self-enforcing contracts are the only means to share risk. We show that shifts out of stable employment into flexible employment engendered by improvements in search effectiveness increases the average real wage and wage volatility in the risky sector while raising the (expected) real wages and worker welfare in the whole economy. Furthermore, depending on parameter values, it may also increase economy-wide real wage volatility. Therefore, our model can explain the transitory variation in workers' earnings observed after the 1970s, even for job stayers.

Chapter 2: We study the relationship between unemployment insurance, employment creation and protection, and wage dynamics. We consider a labour market in which risk-averse workers and risk-neutral employers must match for production to occur. Contracts are incomplete; therefore self-enforcing contracts are the only means to share risk. We jointly analyse the impact of financing and spending aspects of labour market policies on the welfare of the parties and wage volatility. On the financing side, we show that either firing tax or hiring tax strictly dominate payroll tax in terms of efficiency gains and

may increase the employers' welfare. On the spending side, even though unemployment payment increases workers' welfare, it does so by increasing wage volatility and decreasing employers' welfare. On the other hand, hiring payment may increase the employers' welfare without causing any change in workers' welfare. While unemployment payment redistributes income between unemployed and employed workers, hiring payment produces inter-temporal redistribution during employment. The joint usage of hiring tax and hiring payment may function as minimum wage and increase the employers' welfare by smoothing the employees' income in the relationship.

Chapter 3: We employ a stylized model to analyse the relationship between factor market integration (i.e., off-shoring and immigration), unemployment, and earnings instability due to employment variation. We show that factor market integration can indeed increase both frictional and long-term unemployment. Moreover, it will raise earnings instability by increasing unemployment within a parameter space.

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1st Chapter

Nature of contribution by PhD candidate	Literature Review, Propositions and Proofs, Conclusion
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Introduction

This dissertation consists of three essays on the earnings instability and employer-employee relationship exploring three different set of circumstances. The main theme of the dissertation is how the strength of employer-employee relationship is affected by various factors, which are: (i) improvements in search effectiveness (first essay joint with Bilgehan Karabay), (ii) labour market policy instruments (second essay), (iii) factor market integration (third essay). The weakened (strengthened) employer-employee relationship in each case translates into increasing (decreasing) earnings instability. The basic model set-up is based on Karabay and McLaren (2010, 2011). First, we will give a summary of the common themes in this dissertation. Then, to help the reader navigate easily through each essay, we will list the common and distinguishing ways in which the basic model is applied to each essay.

The US Congressional Budget Office (2016) distinguishes income based on its source: market income and transfer income. Market income consists of labour income, business income, capital gains, and capital income. It is measured before government transfers and taxes. Labour income component includes cash wages and salaries. Transfer income includes cash payments from social security, unemployment insurance, etc. In general, market income and transfer income are denoted as before-tax income.[1] The US Congressional Budget Office (2011) reports an increase in concentration of income (even after

[1]In the second essay, we explore the impact of transfer income and taxes that support labour income.

taxes) from 1979 to 2007 and much of the increase is due to an increase in concentration of all of those sources of market income.[2] The labour income which is more evenly distributed than other income types (and probably for this reason) takes a lot of attention. A study of the labour income inequality by Autor *et al.* (2008) affirms that earnings differentials by education, occupation, and age and experience group have been growing rapidly in the US since the late 1970s. A parallel increase occurred in longitudinal variability in income according to Gottschalk and Moffitt (1994).[3] The longitudinal variability in income, also known as "earnings instability", is indicative of economic insecurity and risk and measures of transitory variation and volatility of income is used to summarise it.[4]

Even though the studies in the literature (i.e., the ones stated in footnote 3; Dynan *et al.*, 2012; Cameron and Tracy, 1998) disagree regarding the extent and the specific timing of the movements in earnings instability, they all agree that wages are much more volatile today than they were 40 years ago. However, relatively few studies actually investigate the sources of factors and identify the mechanisms through which earnings instability may have increased. Huff Stevens (2001), Violante (2002), and Farber (2007) explore the

[2]Also the composition of those sources has changed over those years and has contributed to the increased concentration of market income.

[3]In a follow-up study, Gottschalk and Moffitt (2009), by using 1970-2004 data from the Panel Study of Income Dynamics (PSID), find that even if increase in transitory variance is subsided in the 1990s, evidence on whether it increased in the 2000s is mixed. Celik *et al.* (2009) employ longitudinally matched Current Population Survey (matched CPS), the Survey of Income and Program Participation (SIPP), and the Longitudinal Employment and Household Dynamics (LEHD) data to study earnings instability. The CPS and SIPP data do not show increased earnings instability after 2000 while the LEHD data do. Shin and Solon (2011)'s analysis of the PSID and the National Longitudinal Survey of Youth (NLSY) shows increased income volatility in the 2000s.

[4]While change in wage differentials should be interpreted as the change in permanent incomes (the longer-term average), change in transitory income (the period-specific deviations from the average) reflect change in income risk. Income volatility is measured by the deviation of the distribution of individual incomes between consecutive periods. For a review of transitory variation and income volatility, see Jenkins, 2011, chapter 6.

contribution of job loss to increasing earnings instability and conclude that job loss is one factor that explains the increase in earnings instability during the 1970s and 1980s. Comin *et al.* (2009) suggest that the joint increase in firm-level instability and earnings instability after the 1980s may be the result of firms' policy of linking wages with the firms' output.[5] Bertrand (2004) shows that wage volatility in an industry is an increasing function of import penetration in that industry and that the relationship is amplified as the industry becomes more liquidity-constrained. While Karabay and McLaren (2010) argue that globalisation can be the source of the increase in earnings instability,[6] Gottschalk and Moffitt (1994) suggest the decline in regulation and unionisation can be the culprit behind the increase in earnings instability.

We believe one of the sources of the changes in the earnings instability lies in the changing nature of the employer-employee relationship. According to Okun (1981), it is the invisible handshake that governs (as opposed to the invisible hand that clears) the labour market. Invisible handshake is a term Okun coined to refer to the long-term relationship and a risk-sharing institution between workers and employers. There is some evidence that employment relationships have become more flexible. Prior to the 1970s, a long-term steady employment relationship was the norm. In the early 1970s, with the decline of routine mass-production jobs, the nature of employment relationships was altered to accommodate high levels of turnover, shorter periods of employment and the pervasive use of contingent employment contracts. The US labour market has experienced greater

[5]The reason for such a change in compensation policy, they and Gottschalk and Moffitt (2009) argue, may be because larger the firm gets more complex the tasks become and due to ineffective monitoring of the output of individual workers by the large firms, firms tie wages to relatively volatile firm-level performance indicators such as growth in sales.
[6]In particular, they find that the wage instability increases as import penetration increases.

instability and weaker ties between workers and employers — greater flexibility. Segal and Sullivan (1997) and Autor *et al.* (1999) document that temporary help services industry is growing 11% a year since the 1970s in the US. Cohany (1996) reports that in the US, 10% of labour force are in "alternative" arrangements: they work as independent contractors, temporary help agency workers, contract company workers, without an expectation of ongoing employment. Carnoy and Castells (1997) find that such changes are not specific to the US market. Accordingly, in G-7 countries, 30 to 45 percent of all workers have some form of flexible employment and this ratio is increasing. Thesmar and Thoenig (2002) document that, between 1982 and 1999 in France, the share of workers employed under fixed term contracts (FTC) tripled (from 2% to 6% of the total employment) and among workers with less than 6 months seniority, the share of the FTCs has increased from 17% to 28%.

Three changes that may increase the earnings instability are changes in the composition of industries which is the subject of the first essay (i.e., increase in temporary help services industry size that is associated with high income volatility), changes in income volatility in a given job which is the subject of the first and the second essays, and changes in employment stability in a given industry which is the subject of third essay (i.e. increase in job turnover may increase the earnings instability due to job change or unemployment). The main contribution of this dissertation, as in Bertrand (2004) and Karabay and McLaren (2010, 2011), is to connect the literature on the changing long-term employment relationship between workers and employers with the literature on earnings stability.

As we said before, three essays share a common model framework. We present the model in each essay for the reader's convenience even if there is some repetition. In the following section, we summarise the overlapping and exclusive features of model in each essay.

Common and Distinguishing Features of Model in Each Essay

The labour market outcomes depend on the nature of the employer-employee relationship that cannot be observed by outside parties as stated by Okun. So, we need a model that outside parties (i.e., governments) can influence outcomes only indirectly and macroeconomic measures (i.e., output, unemployment) emerge from the interaction of these employment relationship, government policies, and the exogenous parameters. That is why we have employed a common wage contract model that is borrowed from Karabay and McLaren (2010, 2011). In this model, employers and workers are infinitely lived and discount the future by a common factor. One employer and one worker team up to form a 'firm' where outsiders cannot observe the actions of the agents. Due to this informational condition, complete contracts cannot be written, but parties agree on an 'implicit' contract that is self-enforcing through threat of ending the partnership. Neither agent in a self-enforcing contract will have an incentive to shirk and thus such an agreement will be a subgame-perfect equilibrium of the game that they play together. In these contracts, employers promise to smooth out shocks to wages in return for long-run commitment by workers.

Within this common self-enforcing wage contract model; in the first essay, we analyse the relationship between market thickness, labour market flexibility and wage dynamics.

Applying the model in a setting characterized by two goods, two sectors (one sector is risky, other sector is risk-free), we show that a rise in market thickness caused by advances in search technology can generate on-the-job wage volatility[7] by increasing labour market flexibility. Our findings indicate that improvements in search effectiveness increases the average real wage and (potentially) on-the-job wage volatility in the risky sector while raising the expected real wages and worker welfare in the whole economy. Moreover, it may also raise economy-wide real wage volatility. As a result, within this simple framework we manage to explain the variation in the transitory component of workers' earnings observed after the 1970s.

In the second essay, we introduce unemployment and analyse the impact of a set of labour market institutions, which are (i) transfer payments (unemployment payment, hiring payment) and (ii) tax (payroll tax, firing tax, hiring tax)[8] that is used to finance those transfers, on the welfare of employers and workers and earnings instability of on-the-job workers. In particular, we have shown that workers' welfare is not affected under any type of financing, while the type of financing affects employers deeply. As payroll tax increases, it increases on-the-job wage volatility, thus weakens the employer/worker relationship. As firing tax increases, it decreases on-the-job wage volatility and may make employers better off. Hiring tax has the same effect albeit a smaller one. On the other hand, the type of spending has distributional effects across employers and workers. An

[7] If volatility is high enough employer and worker may not form a relationship at all. Thus, we take the level of wage volatility as a proxy for the strength of the employer-employee relationship in the first and second essays.

[8] Including all labour market policies within a single paper is a daunting task. Therefore, we constrain ourselves to these ones. In a companion paper, Avcioglu and Karabay (2017), consider minimum wage and severance payment and show that isolating the effect of a single labour market policy can be misleading to judge its real effects.

increase in unemployment payment increases workers' welfare at the expense of increased on-the-job wage volatility. An increase in the hiring payment will not have any effect on workers' welfare because higher first period expected utility is perfectly compensated by the lower expected utility in the second period, thus enabling a consumption smoothing within the employment relationship. Employers will be worse off as the unemployment payment increases, whereas they may be better off if on-the-job wage volatility is low enough as a result of increase in hiring payment. As a result, in Lazear (1990)'s terminology, hiring payment is neutral from the perspective of workers but not that of employers. Furthermore, financing hiring payment by hiring tax may function as minimum wage policy and increases the welfare of employers if the amount of the is low enough. As a conclusion, firing tax and hiring tax strictly dominate payroll tax in terms of efficiency gains. While unemployment payment redistributes income between unemployed and employed workers; hiring payment redistributes income across the initial phase and the later phase of the employment. Thus, even though unemployment payment increases workers' welfare it does so at the expense of higher on-the-job wage volatility and lower employers' welfare. Hiring payment leaves workers indifferent and may increase employers' welfare.

In the third essay, by making workers heterogeneous in terms effort cost they incur[9] and adding an international dimension, we analyse the relationship between factor market integration (i.e., off-shoring and immigration), labour market flexibility, unemployment, and earnings instability. We show that factor market integration can affect the nature

[9]When an employer matches with a worker who has an effort cost below the threshold level, that match can result in a productive relationship. Within the productive relationship, workers' wages will be a function of their effort costs.

of long-term employment relationships by weakening them.[10] Moreover, it will also raise earnings instability by increasing unemployment within a certain parameter space. As a result, within this simple framework we are able to provide an explanation for the increase in unemployment (and incidence of long-term unemployment) as well as the transitory variation in workers' earnings even when productivity has improved since the 1970s.

Table 1 in the Appendix lists the distinguishing features of the model variants in each essay.

[10]The proxy for the strength of employer-employee relationship is the threshold level of effort cost and an increase in the threshold level of effort cost is interpreted as the weakening of employer-employee relationship.

CHAPTER 1

Market Thickness, Labour Market Flexibility and Wage Dynamics

Co-authored *with Bilgehan Karabay*

1.1. Introduction

> *"People need to look at themselves as self-employed, as vendors who come to this company to sell their skills. In* AT&T*, we have to promote the whole concept of work force being contingent, though most of the contingent workers are inside our walls. Jobs are being replaced by projects and fields of work, giving rise to a society that is increasingly jobless but not workless."* – James Meadows, AT&T Vice President for Human Resources [excerpt taken from the article written by Edmund L. Andrews in New York Times, on February 13$^{\text{th}}$, 1996].

In the second half of the twentieth century, with recent advances made in information technology, the world has become much more interconnected than before. The rapid diffusion of information accompanied by ever decreasing communication costs accelerated the process of globalisation. This greater openness has brought many benefits, yet it may have important social costs. One such cost that is put forth in popular accounts is the rising insecurity and risk for workers.[1] This is what we explore in this paper.

During the same time period when globalisation has gained momentum, the U.S. labour market has undergone some discernible changes. First, it has become much more flexible. Prior to the 1970s, the long-term steady employment relationship was the norm. The stable, ordinary assembly jobs with high degrees of job security dominated the economy. However, a major turn of events took place in the early 1970s with the decline of routine mass-production jobs. The nature of employment relationship was altered to accommodate high levels of turnover, shorter periods of employment and the profound use of contingent employment contracts. The growth of temporary help services industry in the U.S. documented in the literature[2] is one example of such flexible employment arrangements. The upshot is that the U.S. labour market has experienced greater instability[3] and weaker ties between workers and employers — greater flexibility.[4,5]

Second, as a result of advancements in information technology, the functioning of labour markets has become much more efficient than before. This is manifested by the rising role played by labour market intermediaries in response to drastic changes observed

[1]See, for example, Gosselin (2008) and Hacker and Jacobs (2008).

[2]Temporary help services industry consists of agencies that find workers for client firms to do temporary jobs. Since 1972, employment in temporary help services has grown at 11 percent per year. More on this can be found, among others, in Segal and Sullivan (1997) and Autor, Levy and Murnane (1999).

[3]See Farber (2009).

[4]See Benner (2002).

[5]This observed trend is not particular to the U.S. In G-7 countries, 30 to 45 percent of all workers have some form of flexible employment and this ratio is increasing. See Carnoy and Castells (1997).

in employment conditions.[6] The increase in the number of intermediaries such as temporary help firms and head-hunters[7] can in fact raise the probability of finding a job for a worker or of filling a vacancy for an employer in a given time period without any change in the number of participants or their attributes.[8] Therefore, the improvements in search (matching) efficiency can make labour markets thicker.[9] Katz and Krueger (1999) and Autor (2001a) provide evidence that the surge of temporary help agencies since the 1970s increased matching efficiency. A similar efficiency in search can also be due to the emergence of head-hunters as documented by Finlay and Coverdill (2002). They state that head-hunters' knowledge of labour market allows them to connect employers with workers, as a result generating matches that would not have occurred otherwise. This efficiency effect is more pronounced especially after the 1970s when the head-hunting industry experienced major changes.[10]

Third, there has been a rise in the volatility of wages in the US after the 1970s as documented by Gottschalk, Moffitt, Katz and Dickens (1994).[11] Their analysis yields that

[6] Labour market intermediaries are those institutions that mediate work practices and provide matching activities for employers and workers. See Kazis (1998) for the extended role played by labour market intermediaries.

[7] Head-hunters are third-party agents who find job candidates for employers for a fee.

[8] More recently, the arrival of internet websites such as Monster.com, LinkedIn and Facebook has remarkably altered how employers and workers search for each other, see Autor (2001b) for examples.

[9] McLaren (2003) identifies three different ways that market thickness can occur: (a) rise in the number of market participants, (b) increased versatility of participants, and (c) improvements in search efficiency. Our focus in this paper is on the last one.

[10] Finlay and Coverdill (2002) identify two major changes. First, the payment of fees for head-hunters shifted from job seekers to employers. Second, head-hunters started to generate candidates for positions. This is quite different than what traditional employment agencies do, which is finding jobs for people, rather than finding people for jobs.

[11] A rise in earnings volatility during the same period is also documented for Canada (Baker and Solon, 2003) and Great Britain (Dickens, 2000).

much of the increase in the earnings volatility has arisen within jobs, and earnings instability has also increased even for job stayers. After their seminal work, several subsequent studies attempt to assess the recent trends in earnings volatility: Dynarski and Gruber (1997), Cameron and Tracy (1998), Haider (2001), Hacker (2008), Kambourov and Manovskii (2009), Shin and Solon (2011), Celik, Juhn, McCue and Thompson (2012), Dynan, Elmendorf, and Sichel (2012), Moffitt and Gottschalk (2012), Shin (2012) and Champagne and Kurmann (2013). Even though these studies disagree regarding the extent and the specific timing of the movements in earnings volatility, they all agree that wages are much more volatile today than they were 40 years ago.

In this paper, we ask whether there is any connection between these developments. In other words, whether the rise in market thickness brought by improvements in search effectiveness can cause an increase in short-term earnings variance by increasing labour market flexibility. We have a stylized model that connects these pieces together. We consider a risk-bearing employment relationship between risk-averse workers and risk-neutral employers in a labour market with search frictions. The environment is risky and complete contracts are unavailable due to informational reasons. Thus, the only way for an employer to share risk with a worker is to develop long-term employment agreement, in which employers promise wage insurance while purchasing labour. These implicit contracts,[12] also known as 'invisible handshake', are enforceable only through the threat that if any agent reneges, the relationship is severely damaged such that it is dissolved and parties to the contract must search for new partners. An increase in market thickness due to an improvement in search technology makes it easier to find a

[12]See Beaudry and DiNardo (1991) and McDonald and Worswick (1999) for empirical relevance of implicit contracts. Malcomson (1999, section 3) provides a survey.

new partner (either worker or employer) to work with and makes the termination of a given relationship less intimidating. This in turn weakens risk-sharing, makes existing relationships less stable and increases the volatility of wages.

There are other papers in the literature that analyse the relationship between market thickness and self-enforcing relationships. Kranton (1996) identifies that larger markets can destroy long-run relationships whereas increase in market search costs can facilitate them. Ramey and Watson (2001) study the effect of matching frictions on investment incentives of agents in a bilateral self-enforcing trading relationships. By confining their attention to stationary risk-sharing relationships, McLaren and Newman (2004) show that reductions in market frictions can potentially weaken cooperation and reduce welfare by increasing agents' outside options. When there is information asymmetry, Matouschek and Ramezzana (2007) show that an improvement in search frictions can make bilateral exchange more difficult. However, none of these papers are particular to labour market and wage volatility. Instead, our paper is closely related to Thomas and Worrall (1988) and Karabay and McLaren (2010). Thomas and Worrall examine long-run relationships between a risk-averse worker and a risk-neutral employer when each can alternatively participate an exogenous and randomly fluctuating labour spot market. The wage agreement within a given relationship is generally tethered by the ongoing wage in the spot market. Karabay and McLaren (2010) extend the work of Thomas and Worrall (1988) by endogenizing the spot market and adding moral hazard. They analyse the effects of free trade and offshoring on wage volatility and worker welfare. Unlike these papers, our focus is on matching efficiency. Therefore, we expand Karabay and McLaren (2010) framework by introducing search effectiveness and examine how changes in search effectiveness affect wage volatility and worker welfare. In our analysis, we look at not only the wage volatility in a particular sector as in Karabay and McLaren (2010) but also economy-wide wage volatility.

In our stylized model, we have two sectors, a 'careers sector' in which production is risky and requires unobservable effort by a worker and by an employer, and a 'spot market sector' with risk-free Ricardian technology. Here, the 'careers sector' represents a sector that is heavily characterized by flexible employment relationship such as services, and the 'spot market sector' represents a sector like manufacturing that involves routine production jobs.[13] We have three essential elements that shape our approach. The first element is the existence of non-diversifiable firm-specific risk. In the 'careers sector', firms are assumed to be hit by idiosyncratic shocks that are unobservable to people outside the firm. As a result, written contracts are not enforceable. Second, in the 'careers sector', there is no commitment. Since employers are risk-neutral, whereas workers are risk-averse, employers would like to commit credibly to a full wage insurance (constant wage), in effect reducing their expected wage payments to workers; but without enforceable contracts, they can only do so by reputational means (self-enforcing contracts). Consequently, in providing wage insurance, employers are constrained by their incentive compatibility constraints. The last element to our model is moral hazard. For production to occur in the 'careers sector', workers need to exert effort that is costly, unobservable and thus non-contractable. Therefore, in equilibrium, the wage payments

[13]Therefore, any move from spot market sector to careers sector (i.e., from manufacturing to services) represents a shift from more stable jobs to less stable ones. Schettkat and Yocarini (2006) reviews the literature analysing the shift to services.

are back-loaded in this sector.[14] The intuition is that when a worker needs to provide a non-contractable effort, it is generally optimal to promise wages that increase over time, so that the fear of losing high future wages deters shirking. Thus in the risky sector, new workers are always cheaper than incumbent workers. This is the essence of the employer's problem: if it is easy to find a replacement, the employer has a temptation to ditch the current senior worker for a new cheaper worker and this temptation is strongest if the firm is in difficulty. When this is the case, workers will know not to trust the employer's full wage insurance, and expecting a low wage in bad times, they will demand a high wage in good times. Therefore, if it is easy to find a new worker, an employer that makes only credible promises will promise a low wage in bad states and a high wage in good states, causing wage volatility in equilibrium. An increase in market thickness brought about by an advancement in search technology makes it easier for firms to hire workers, which in turn reduces the amount of wage insurance promised, raising the variance of wages in the 'careers sector'.

The rest of the paper is organized as follows. The next section lays out our model and characterize optimal wage contracts. Sections 1.3 to 1.4 derive the conditions under which those contracts will exhibit constant wages and volatile wages, respectively. Comparative statics of wage dynamics is analysed in section 1.5. In section 1.6, we turn to general equilibrium analysis. The last section concludes. All proofs are relegated to the appendix.

[14]This result is in the same spirit as Lazear (1979), Harris and Holmström (1982), Holmström (1983) and Shleifer and Summers (1988).

1.2. The Model

In this section, we describe the main features of our model. The set-up is an extended version of Karabay and McLaren (2010). Our focus is the effect of an improvement in search technology on wage dynamics. We consider a two-good, two factor model within a single country. First, we review the main model characteristics and then analyse how they change as labour market becomes thicker through an improvement in search effectiveness.

Production. There are two sectors; Y and X, and two factors of production: a measure L of workers and a measure E of employers. In the risk-free sector (spot-market sector), Y, there is a linear production technology such that one unit of worker can produce one unit of output per period. Let $p^y > 0$ represent the price of Y-sector output. Since we have constant returns to scale technology with only one factor, the wage workers earn in this sector must be $\omega^y = p^y$.

In the risky sector (careers sector), X, for production to occur one worker must team up with one employer and they must each put in one unit of non-contractible effort. We will call a given such partnership as a 'firm'. Workers suffer a disutility from effort equal to $k > 0$, while employers suffer no such disutility.[15] Within a given employment relationship, denote the effort put in by agent i by $e^i \in \{0, 1\}$, where $i = W$ indicates the worker and $i = E$ indicates the employer. Let sector X be the numeraire sector, i.e., $p^x \equiv 1$. The output and revenue generated in that period is then equal to $x_\epsilon e^W e^E$, where, ϵ is an idiosyncratic i.i.d. random variable that takes its value $\epsilon = G$ or B with respective

[15] Adding a disutility for employers would not change the results other than contracting the portion of the parameter space where it is possible to have efficient and self-enforcing contracts.

probabilities π_G and π_B, where $\pi_G + \pi_B = 1$ and $x_G > x_B > 0$. The random variable ϵ indicates whether the current period is one with a good state or a bad state for the firm's profitability. The average revenue is denoted by $\overline{x} \equiv \pi_G x_G + \pi_B x_B$. Employers without a worker are 'with vacancy' and do not produce anything.

Preferences. There is no storage, saving or borrowing. An agent's income in a given period is equal to that agent's consumption in that period.

Employers. Employers have the same linearly homogeneous and strictly quasi-concave per-period utility function $U(c^X, c^Y)$, defined over consumption of goods X and Y, (c^X and c^Y), respectively. Since employers are risk-neutral, their indirect utility is a linear function of income and is given by $v(p^x, p^y, I) = \dfrac{I}{\Gamma(p^x, p^y)}$, where I is income and $\Gamma(p^x, p^y)$ is a linear homogeneous consumer price index that represents the minimum expenditure required to obtain unit utility. Given that $p^x \equiv 1$, $\Gamma(1, p^y)$ can be written as $P(p^y)$. Notice that the elasticity of $P(p^y)$ with respect to p^y, $\frac{p^y P'}{P}$, represents the share of good Y in total consumption, α^y, due to Roy's identity and therefore we have $0 < \alpha^y = \frac{p^y P'}{P} < 1$.

Workers. Workers have the same per-period utility function $\mu(U(c^X, c^Y))$ over consumption of goods X and Y. The function μ is an increasing, differentiable and strictly concave von-Neumann-Morgenstern utility function. Worker's indirect utility is given by $\mu(\dfrac{I}{P(p^y)})$, where the properties of the function μ guarantees that workers are risk-averse.

In short, risk-neutral employers maximize their expected discounted lifetime profits, whereas risk-averse workers maximize their expected discounted lifetime utility and everyone discounts the future at a constant rate $\beta \in (0, 1)$.

Search. Those workers and employers seeking a partner, search until they have one. Search follows a specification of a type used extensively by Pissarides (2000). Let there be a measure n of workers and a measure m of employers searching in a given period, then $\Phi(n, m, \phi)$ matches occur. The function Φ is concave and increasing in all arguments and linear homogeneous in its first two arguments with $\Phi_{nm} = \Phi_{mn} > 0$, $\forall n, m, \phi$ and has an upper bound equal to $\min(n, m)$. ϕ is a measure of the effectiveness of search technology. It is the main parameter of interest in this paper and we will analyse how an increase in search effectiveness, ϕ, will affect wage volatility and workers' welfare. We denote by Q^E the steady-state probability that an employer will match with a worker in any given period, or in other words, $Q^E = \dfrac{\Phi(n, m, \phi)}{m}$, where n and m are set at their steady-state values. Similarly, we denote by $Q^W = \dfrac{\Phi(n, m, \phi)}{n}$ the steady-state probability that a worker will find a job in the X sector any given period. Search has no direct cost, but it does have an opportunity cost: if an agent is searching for a new partner, then she is unable to put in effort for production with her existing partner if she has one.

There is also a possibility in each period that a worker and employer who have been matched in that period or in the past will be exogenously separated from each other. This probability is given by a constant $(1 - \rho) \in (0, 1)$.

Goods market clearing. Total production of each good must be equal to the total consumption of each good. For a given relative price p^y, each worker and employer will consume each good in the same proportions, which corresponds to the condition that $p^y = \frac{U_2(1,r)}{U_1(1,r)}$, where the subscripts denote partial derivatives and r denotes the ratio of Y production to X production. In other words, the relative price must be equal to the marginal rate of substitution between the two goods determined by the production ratio. Given that U is strictly quasi-concave, the marginal rate of substitution is strictly decreasing in r, which in turn implies that p^y is strictly decreasing in r. We assume that $U_2(1,r) \longrightarrow \infty$ as $r \longrightarrow 0$, and $U_1(1,r) \longrightarrow \infty$ as $r \longrightarrow \infty$ so that for any $r \in (0,\infty)$, there is a unique, market clearing value of $p^y \in (0,\infty)$.

Timing of the game. The timing of events within a given period is as follows.

(1) Any readily matched employer and worker learn whether they will be exogenously separated this period.

(2) For each firm in the X sector, the idiosyncratic output shock ϵ is realized. This is common knowledge within the firm but unknown to agents outside the firm.

(3) The wage, if any, is paid (a claim on the firm's output at the end of the period).

(4) The employer and worker simultaneously choose their effort levels e^i. At the same time, the search mechanism operates. Within a firm, if $e^i = 0$, then agent i can participate in search and exert no effort. Workers in the Y sector and employers with vacancy always search and they do not incur any search cost.

(5) Each firm's revenue, $R = x_\epsilon e^W e^E$, as well as profits and consumption are realized.[16]

(6) For those agents who have found a new potential partner in this period's search, new partnerships with a new self-enforcing agreement are formed.[17] This is

[16]There is the possibility, off of the equilibrium path, that the firm's output will be zero because either agent has shirked. In such a case, we assume that the employer has deep pockets so that the wage claim promised to the worker can still be redeemed.
[17]Notice that for any agents matched in period t, the self-enforcing agreement is in effect in period $t+1$ after they realized there has been no exogenous separation but before the output shock is revealed.

achieved by a take-it-or-leave-it offer made by the employer to the worker. Therefore, we assume that employers have the full bargaining power.[18]

Our focus will be on steady-state equilibria. Let V^{ES} denote the expected lifetime discounted profit of an employer with vacancy and V^{WS} denote the expected lifetime discounted utility of a searching worker without an employer (i.e., Y-sector worker), where the superscript 'S' indicates the state of searching. Similarly, let V^{ER} and V^{WR} denote the lifetime pay-offs to employers and workers, respectively evaluated at the beginning of a cooperative X-sector relationship. Naturally, we must have $V^{WR} \geq V^{WS}$ in equilibrium, or no worker will accept an X-sector job. The values V^{ij} are endogenously determined as they depend on the endogenous probability of finding a match in any given period and the endogenous value of entering a relationship once a match occurs. When designing the wage contract, any employer will take them as given. We can write

$$V^{WS} = \mu(\frac{\omega^y}{P}) + Q^W \rho\beta V^{WR} + Q^W(1-\rho)\beta V^{WS} + (1-Q^W)\beta V^{WS} \tag{1.1}$$

$$V^{ES} = Q^E \rho\beta V^{ER} + Q^E(1-\rho)\beta V^{ES} + (1-Q^E)\beta V^{ES} \tag{1.2}$$

Y-sector worker's pay-off from search is the current Y-sector real wage plus the continuation values if the worker finds an X-sector job and is not immediately separated, finds an X-sector job but immediately separated, or fails to find an X-sector job. The pay-off from search for an X-sector employer with vacancy is given by the continuation values if the employer finds a worker and is not immediately separated, finds a worker

[18]Assigning full bargaining power to employers simplifies the model. Our results carry over even if we allow workers to capture some portion of X-sector rents. We will comment more on this in footnote 22.

but immediately separated, or fails to find a worker. If an X-sector worker, or an X-sector employer who already has a worker, chooses to search, the pay-off will be the same as in equations (1.1) and (1.2), respectively except for a straightforward change in the first-period pay-off.

A self-enforcing agreement between a worker and an employer is simply a subgame-perfect equilibrium of the game that they play together. Given that the employer has all of the bargaining power, the optimal agreement is the one that gives the highest expected discounted profit to the employer, subject to incentive constraints. If either agent reneges on the agreement, the relationship is severed and both agents must search for new partners. In other words, we will restrict our attention to 'grim punishment' strategies. Thus, the pay-off following a deviation would be V^{ES} for an employer and V^{WS} for a worker.

To sum up, risk-neutral employers search for risk-averse workers, and when they find each other, the employer offers the worker the profit-maximizing self-enforcing wage contract, which then remains in force until either party reneges or the two are exogenously separated. This pattern provides a steady flow of workers and employers into the search pool, where they receive endogenous pay-offs V^{WS} and V^{ES}. These values then act as parameters that constrain the optimal wage contract.

We now turn to the form of optimal contracts in the X sector. In our model the optimal employment contracts take one of two very simple forms, which we will call 'wage smoothing' and 'fluctuating wage.' This is what we will derive in this section.

The equilibrium can be characterized as the solution to a recursive optimization problem. Let $\Omega(W)$ be the highest possible expected present discounted profit the employer can receive in a subgame-perfect equilibrium, conditional on the worker receiving an expected present discounted pay-off of at least W. Arguments parallel to those in Lemma 1 of Thomas and Worrall (1988) can be used to show that the function Ω is defined on a compact interval $[W_{\min}, W_{\max}]$, decreasing, strictly concave and continuously differentiable, where $W_{\min}$ and $W_{\max}$ are respectively the lowest and highest worker pay-offs consistent with a subgame-perfect equilibrium of the game. This function must satisfy the following functional equation

$$\Omega(W) = \max_{\left\{\omega_\epsilon, \widetilde{W}_\epsilon\right\}} \sum_{\epsilon=G,B} \pi_\epsilon \left[x_\epsilon - \omega_\epsilon + \rho\beta\Omega(\widetilde{W}_\epsilon) + (1-\rho)\beta V^{ES} \right] \tag{1.3}$$

subject to

$$x_\epsilon - \omega_\epsilon + \rho\beta\Omega(\widetilde{W}_\epsilon) + (1-\rho)\beta V^{ES} \geq V^{ES} \tag{1.4}$$

$$\mu(\frac{\omega_\epsilon}{P}) - k + \rho\beta\widetilde{W}_\epsilon + (1-\rho)\beta V^{WS} \geq V^{WS} - \mu(\frac{\omega^y}{P}) + \mu(\frac{\omega_\epsilon}{P}) \tag{1.5}$$

$$\sum_{\epsilon=G,B} \pi_\epsilon \left[\mu(\frac{\omega_\epsilon}{P}) - k + \rho\beta\widetilde{W}_\epsilon + (1-\rho)\beta V^{WS} \right] \geq W \tag{1.6}$$

$$\omega_\epsilon \geq 0 \tag{1.7}$$

$$\widetilde{W}_\epsilon \geq 0 \tag{1.8}$$

The employer's problem stated in equation (1.3) is to choose the worker's current period wage ω_ϵ and continuation utility $\widetilde{W}_\epsilon$ at each state such that the employer's expected present discounted lifetime profit is maximized given that the worker's expected present discounted utility is at least equal to W. Constraint (1.4) is the employer's incentive compatibility constraint. If this is not satisfied in state ϵ, then the employer will in that state prefer to renege on the promised wage, understanding that this will cause the worker to lose faith in the relationship and sending both parties into the search pool. Constraint (1.5) is the worker's incentive compatibility constraint. The left-hand side is the worker's pay-off from putting in effort in the current period, collecting the wage, and continuing the relationship.[19] The right-hand side is the pay-off from shirking and searching, in which case the worker's pay-off is the same as it would be if she were in the Y sector except that in the current period her income is ω_ϵ instead of ω^y. If this constraint is not satisfied, the worker will prefer to shirk by searching instead of working. Constraint (1.6) is the target-utility constraint. In the first period of an employment relationship, since the employer has all the bargaining power, she must promise at least as much of a pay-off to the worker as remaining in the search pool would provide. Thus, in that case, denoting the target utility at the beginning of the relationship by W_0, we have $W = W_0 = V^{WS}$ (and so $V^{ER} = \Omega(V^{WS})$). Thereafter, the employer will in general be bound by promises of pay-offs she had made to the worker in the past. Finally, constraints (1.7) and (1.8) are natural bounds on the choice variables.

Constraint (1.5) can be replaced by the more convenient form

$$(1.5)' \qquad \widetilde{W}_\epsilon \geqslant \widetilde{W}^*, \text{ where } \widetilde{W}^* \equiv \frac{[1-(1-\rho)\,\beta]\,V^{WS} - \mu(\frac{\omega^y}{P}) + k}{\rho\beta}.$$

The value $\widetilde{W}^*$ is the minimum future utility stream that must be promised to the worker in order to convince the worker to incur effort and forgo search.

[19]Note that we are assuming that a worker cannot receive a Y-sector wage while searching if that worker is shirking on an X-sector job. This makes sense if, for example, *effort* is not observable and third-party verifiable but *physical presence* on the job site is, and a worker can search while physically at the X-sector job site but cannot produce Y-sector output while there. Thus, an X-sector employer would be able to sue to recover the wage just paid if the worker was absent, working another job, instead of on site at the location of the X firm.

The following lemma allows us to ignore constraint (1.8) in the employer's maximization problem.

Lemma 1. $\widetilde{W}_\epsilon \geqslant \widetilde{W}^* > V^{WS} > 0$. **Proof.** See Appendix B, page 128.

Given that $W_0 = V^{WS}$ and $\widetilde{W}^* > V^{WS}$, we have $W_{\min} = V^{WS}$ in the first period. Let the Kuhn-Tucker multiplier for (1.4) be denoted by ψ_ϵ, the multiplier for constraint (1.5)′ by υ_ϵ, the multiplier for constraint (1.6) by λ, and the multiplier for constraint (1.7) by χ_ϵ. The first-order conditions with respect to ω_ϵ and $\widetilde{W}_\epsilon$ respectively are

$$-\pi_\epsilon - \psi_\epsilon + \pi_\epsilon \lambda \frac{\mu'(\frac{\omega_\epsilon}{P})}{P} + \chi_\epsilon = 0, \tag{1.9}$$

$$\rho\beta\pi_\epsilon \Omega'(\widetilde{W}_\epsilon) + \rho\beta\psi_\epsilon \Omega'(\widetilde{W}_\epsilon) + \upsilon_\epsilon + \rho\beta\pi_\epsilon\lambda = 0, \tag{1.10}$$

and in addition there is an envelope condition

$$\Omega'(W) + \lambda = 0. \tag{1.11}$$

Since $\Omega'(W) < 0$, for equation (1.11) to hold, we must have $\lambda > 0$, hence the target utility constraint always bind. Therefore, at the beginning of the employment relationship it is feasible for the employer to push the worker's pay-off down to the opportunity pay-off. Since it is in the interest of the employer to do so, it is clear that workers joining X-sector employment receive the same pay-off that they would receive in the Y sector,

$V^{WR} = V^{WS}$. From equation (1.1), this immediately tells us[20]

$$V^{WS} = \frac{\mu(\frac{\omega^y}{P})}{1-\beta}, \tag{1.12}$$

and condition $(1.5)'$ can be rewritten as

$$\widetilde{W}_\epsilon \geqslant \widetilde{W}^*, \text{ where } \widetilde{W}^* \equiv \frac{\mu(\frac{\omega^y}{P})}{1-\beta} + \frac{k}{\rho\beta}. \tag{1.5$''$}$$

To summarise, the employer maximizes in each period equation (1.3), subject to constraints (1.4), $(1.5)''$, (1.6), and (1.7). In the first period of the relationship, the worker's target utility $W = W_0$ is given by V^{WS}, but in the second period it is determined by the values of $\widetilde{W}_\epsilon$ chosen in the first period and by the first-period state, and similarly in later periods it is determined by choices made at earlier dates. We impose an assumption.

Assumption 1. In the first period of an employment relationship, the employer's incentive-compatibility constraint (1.4) does not bind in either state.

We will discuss sufficient conditions for this later (see Lemma 2 in Section 1.4). We are now ready to describe the equilibrium.

[20] Of course, this implies that, in equilibrium, Y-sector workers are indifferent between searching and not searching, so if a small search cost were imposed, there would be no search (this is a version of the Diamond search paradox). However, this feature would disappear if any avenue were opened up to allow workers to capture some portion of X-sector rents. For example, for simplicity, we have assumed that employers have all of the bargaining power, but this could be relaxed. In addition, we have assumed that k is common knowledge, but it would be reasonable to assume that different workers have different values of k, and while employers know the distribution of this parameter, they do not know any given worker's value of it. Either of these modifications would very substantially increase the complexity of the model, but would give X-sector workers some portion of the rents and thus avoid the Diamond paradox.

Proposition 1. In the first period of an equilibrium employment relationship, the wage is set equal to ω^y in each state and the continuation pay-off for the worker in each state is set equal to $\widetilde{W}^*$. In the second period and all subsequent periods of the employment relationship, there is a pair of values ω^*_ϵ for $\epsilon = G, B$ such that regardless of history (provided neither partner has shirked), the wage is equal to ω^*_ϵ in state ϵ. In addition, the worker's continuation pay-off is always equal to $\widetilde{W}^*$. Furthermore, after the first period there are three possible cases:

(i) The employer's incentive compatibility constraint (1.4) never binds, and $\omega^*_G = \omega^*_B$.

(ii) The employer's incentive compatibility constraint (1.4) binds in the bad states but not in the good states, and $\omega^*_G > \omega^*_B$.

(iii) The employer's incentive compatibility constraint (1.4) always binds, and $x_G - \omega^*_G = x_B - \omega^*_B$.

Proof. See Appendix B, page 128.

Two types of wage agreements are possible in equilibrium. In each type, (under Assumption 1) the worker goes through an 'apprenticeship period' at the beginning of the relationship in which Y-sector wage, ω^y, is paid. Thereafter, if the employer's incentive constraint does not bind, the worker receives a constant wage $\omega^*_G = \omega^*_B$. We will call this type as *wage-smoothing* agreement. On the other hand, if the employer's constraint ever binds, then it binds only (and always) in the bad state, resulting in state-dependent wages with $\omega^*_G > \omega^*_B$.[21] We will call this type as *fluctuating-wage* agreement. The key idea is that

[21]The case in which the employer's incentive constraint binds in both states occurs in a zero-measure portion of the parameter space, hence will be ignored.

it is never optimal to promise more future utility than is required to satisfy the worker's incentive constraint (1.5)$''$, so after the first period of the relationship, the worker's target utility is always equal to $\widetilde{W}^*$ (Thus, in the first period we have $W_{\min} = V^{WS}$, whereas in any subsequent period we have $W_{\min} = \widetilde{W}^*$). This means that after the first period, the optimal wage settings by the employer are stationary. We will analyse each type of agreements in turn.[22]

1.3. Wage-Smoothing Agreement

In any type of equilibria (either wage smoothing or fluctuating wage), the worker's incentive compatibility constraint and the target utility constraint always bind (see the proof of Proposition 1 in the appendix). The former implies that $\widetilde{W}_\epsilon = \widetilde{W}^*$ for any ϵ, and after the first period, $W = \widetilde{W}^*$ and the latter implies that constraint (1.6) holds with equality. Moreover, under wage-smoothing equilibrium, the employer's incentive compatibility constraint is slack in both states, therefore paying a constant wage in both states is feasible. We can calculate this constant wage by substituting equations (1.12) and (1.5)$''$ into constraint (1.6)

$$\mu(\frac{\omega^*}{P}) = \mu(\frac{\omega^y}{P}) + \frac{k}{\rho\beta}, \tag{1.13}$$

[22]Note that in the current set-up, all bargaining power is allocated to the employer, therefore we have $V^{WR} = V^{WS}$. Giving some bargaining power to the worker makes the employer's incentive compatibility constraint tighter and expands the portion of the parameter space in which wage volatility occurs. However, as long as the worker's bargaining power is not too large, our main insights continue to hold, but at the cost of greater complexity. More specifically, for any value of V^{WR} with $V^{WS} \leq V^{WR} < \widetilde{W}^*$, the expected wage in the first period will be lower than the expected wage in the second period and it is possible to find a region in the parameter space where the same types of wage contracts as in the current set-up are offered in equilibrium.

where we denote the constant wage paid under wage-smoothing case with ω^*. We will name this as the 'efficiency wage'. It represents the lowest constant wage that can be given to the worker in a self-enforcing agreement. Given that employers are risk-neutral whereas workers are risk averse, employers always prefer wage smoothing, since it delivers the lowest expected wage payment to workers. However, wage-smoothing is not always possible since the employer's constraint may bind. If it binds, it does so only (and always) in the bad state. Hence, by computing the values of V^{ES} and $\Omega(\widetilde{W}^*)$ under wage-smoothing equilibrium, we can determine the conditions under which the employer's bad state incentive compatibility constraint (given in constraint (1.4) for $\epsilon = B$) is satisfied.

Now we are ready to find V^{ES}

$$V^{ES} = Q^E \rho\beta \left(\Omega(\widetilde{W}^*) + \omega^* - \omega^y\right) + Q^E(1-\rho)\beta V^{ES} + (1-Q^E)\beta V^{ES}. \tag{1.14}$$

In addition, $\Omega(\widetilde{W}^*)$ is given by

$$\Omega(\widetilde{W}^*) = \frac{\overline{x} - \omega^* + (1-\rho)\beta V^{ES}}{1-\rho\beta}. \tag{1.15}$$

If we substitute equation (1.15) into equation (1.14) and rearrange, we obtain

$$V^{ES} = \frac{Q^E \rho\beta \left[\overline{x} - \rho\beta\omega^* - (1-\rho\beta)\omega^y\right]}{(1-\beta)\left[1 - \left(1-Q^E\right)\rho\beta\right]}. \tag{1.16}$$

Notice that V^{ES} is increasing in Q^E ($\frac{\partial V^{ES}}{\partial Q^E} > 0$) and decreasing in $\omega^y = p^y$ ($\frac{\partial V^{ES}}{\partial \omega^y} < 0$).

The employer's bad-state incentive compatibility constraint is given by

$$x_B - \omega^* + \rho\beta\Omega(\widetilde{W}^*) + (1-\rho)\beta V^{ES} \geqslant V^{ES}. \tag{1.17}$$

Using equation (1.15), this becomes

$$x_B - \omega^* + \rho\beta\pi_G(x_G - x_B) \geqslant (1-\beta)\, V^{ES}. \tag{1.18}$$

We can interpret inequality (1.18) as follows. Suppose that the employer's incentive constraint just binds in the bad state so that the employer's pay-off is equal to V^{ES} in that state. The employer's average pay-off is then equal to $\pi_G\left(V^{ES} + x_G - x_B\right) + (1-\pi_G)\, V^{ES} = V^{ES} + \pi_G(x_G - x_B)$. From inequality (1.17), the employer's pay-off in the bad state if she does not renege is $x_B - \omega^* + \rho\beta\left[V^{ES} + \pi_G(x_G - x_B)\right] + (1-\rho)\beta V^{ES}$ and if she reneges is V^{ES}. Equating these two gives inequality (1.18) as an equality.

Next, substituting equation (1.16) into inequality (1.18) and rearranging, we obtain

$$\omega^* \leq \frac{Q^E\rho\beta\omega^y + x_B + (1-Q^E)\rho\beta\pi_G(x_G - x_B)}{1 + Q^E\rho\beta}. \tag{1.19}$$

At this stage, it will be instructive to look at the limiting cases. As $Q^E \to 1$, wage smoothing is sustainable if and only if $x_B \geqslant \omega^* + \rho\beta\,(\omega^* - \omega^y)$. If an employer can immediately find a new worker, reneging in the bad state involves paying no wage and receiving no output now, and starting a new relationship with a new worker next period. The loss from doing so is the current output, x_B. The benefit is the current wage that is not paid to the worker, plus the gain from paying a lower wage next period because the new worker will be in her apprenticeship period. Notice that since new workers are cheaper than old ones, the employer still has a temptation to renege even if the worker's productivity in the bad state exceeds her wage, e.g., $x_B \geqslant \omega^*$ but $x_B < \omega^* + \rho\beta\,(\omega^* - \omega^y)$.

In the other limiting case where $Q^E \to 0$, wage smoothing is sustainable if and only if $x_B + \rho\beta\pi_G(x_G - x_B) \geqslant \omega^*$. Given that the employer cannot find another worker at all, the wage-smoothing equilibrium can be sustained even if the employer makes losses in the bad state, e.g., $x_B < \omega^*$ but $x_B + \rho\beta\pi_G(x_G - x_B) \geqslant \omega^*$. Recalling that ω^* is determined by parameters through equation (1.13), we assume the following.

Assumption 2. The bad-state output satisfies the following condition

$$\omega^* < x_B < \omega^* + \rho\beta\left(\omega^* - \omega^y\right).$$

This assumption ensures that it is socially optimal to produce in both good and bad states. It also guarantees that for a given value of $Q^E \in (0, 1)$, the parameter space is partitioned into two regions where the wage-smoothing and the fluctuating-wage equilibria take place. Later on, the expression in (1.19) will be useful to do this partition.

Next, we turn to those fluctuating-wage equilibria.

1.4. Fluctuating-Wage Agreement

To reiterate, in any type of equilibria, the worker's incentive compatibility constraint and the target utility constraint always binds. However, in a fluctuating-wage equilibrium, unlike in a wage-smoothing equilibrium, the employer's bad-state incentive constraint binds, implying that in the bad state, the employer cannot afford to pay the same high wage she pays in the good state. Accordingly, we can follow the same steps as before in deriving (1.13) by substituting equations (1.12) and (1.5)$''$ into constraint (1.6) to obtain

$$E_\epsilon \mu(\frac{\omega_\epsilon^*}{P}) = \mu(\frac{\omega^y}{P}) + \frac{k}{\rho\beta}. \tag{1.20}$$

Equation (1.20) states that in any period after the first, the expected utility promised to an X-sector worker must be enough to compensate that worker next period, in expected value, for the current disutility of effort. This equation is represented in Figure 1.1 by the downward-sloping curve WW, which is strictly convex due to the worker's risk aversion.

Under the fluctuating-wage equilibrium, the employer's binding bad-state incentive compatibility constraint is given by

$$x_B - \omega_B^* + \rho\beta\Omega(\widetilde{W}^*) + (1-\rho)\beta V^{ES} = V^{ES}. \tag{1.21}$$

Developing expressions for $\Omega(\widetilde{W}^*)$ and V^{ES} by changing ω^* with $E_\epsilon\omega_\epsilon^*$ in equations (1.15) and (1.16), respectively and substituting them into equation (1.21) yields

$$\omega_B = \frac{-\rho\beta\pi_G\omega_G + Q^E\rho\beta\omega^y + x_B + (1-Q^E)\rho\beta\pi_G(x_G - x_B)}{1 - \rho\beta(\pi_G - Q^E)}, \tag{1.22}$$

which is the straight downward-sloping line EE in Figure 1.1.

We can now summarize the equilibrium with the help of Figure 1.1. On the vertical axis we have the bad-state wage and on the horizontal axis we have the good-state wage. The EE line represents the employer's bad-state incentive compatibility constraint and the employer would not offer any wage combination above this line. The WW curve represents a combination of the worker's incentive compatibility constraint and the target utility constraint and the worker would not accept any wage combination below this curve. The efficiency wage, ω^*, is given by the intersection of WW with the 45°-line. Any movement along the WW curve toward that point increases the employer's profit.

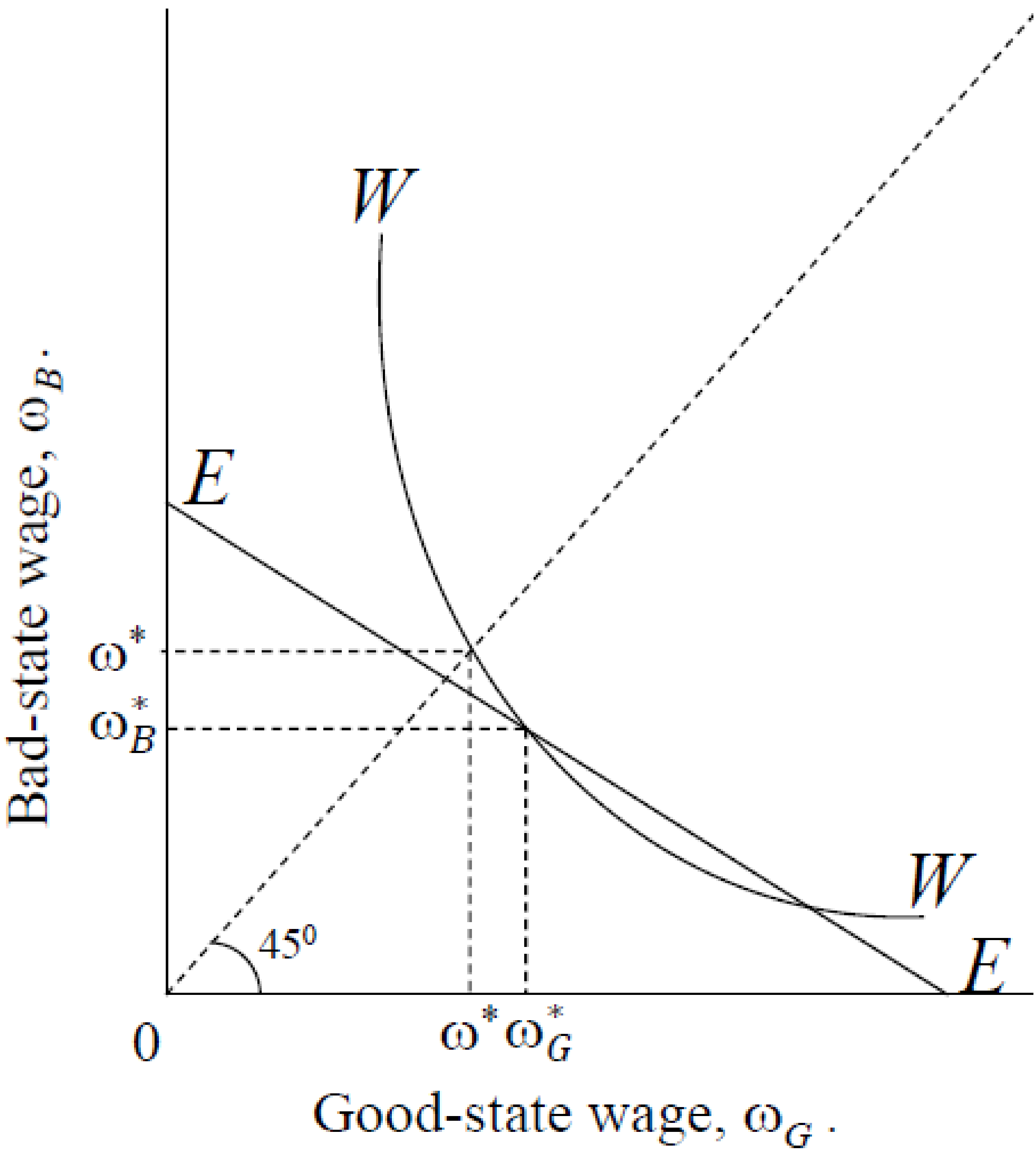

Figure 1.1. Fluctuating-wage equilibrium.

In equilibrium, the employer will choose the wage combination that minimizes expected wages, subject to the two constraints and this amounts to choosing ω^* if it is on or below EE, and choosing the intersection of EE and WW closest to the 45°-line otherwise. In this figure, by assumption, we focus on the fluctuating-wage equilibrium, so efficiency wage is unattainable as it is above the EE line. Therefore, we know that the intersection of EE with the 45°-line occurs below the intersection of WW with the 45°-line. Furthermore, since we have shown that in equilibrium the good-state wage is never below the bad-state wage, the WW curve and EE line must intersect *below* the 45°-line. Given the concavity of WW and the linearity of EE, there will clearly be two such intersections, but the one that will be chosen by the employer is the one closest to the 45°-line, as shown, because it will offer the lowest expected wage consistent with the constraints. This means that at the point of intersection that determines ω_B and ω_G, EE is flatter than WW. As a result, it is clear that anything that shifts the EE line down without shifting WW will raise ω_G and lower ω_B. In addition, it is useful to note that, since the WW curve is a worker indifference curve, holding k constant, anything that shifts up the WW line (whether or not it shifts the EE line) raises worker welfare.

We are ready now to state the sufficient condition for Assumption 1 to hold.

Lemma 2. A sufficient condition for Assumption 1 to hold is $x_B \geqslant \omega^y + \rho\beta(E_\epsilon \omega_\epsilon^* - \omega^y)$.[23]

Proof. See Appendix B, page 132.

[23]Note that there exists a portion of the parameter space where Assumption 1 and Assumption 2 are both satisfied. We will comment further on this in footnote 24 of Section 1.6.

In what follows we will do comparative statics on X-sector wages by changing Q^E and p^y.

1.5. Comparative Static Analysis

We will start our comparative statics by analyzing the effect of a rise in Q^E on X-sector wages and worker welfare, while keeping p^y constant. We will show that as it becomes easier for the employer to fill a vacancy, her temptation to cheat within a given relationship increases, which shifts the EE line down without shifting $WW.$ Whether this shift has any effect on wages depend on what type of equilibria prevails, i.e., wage smoothing versus fluctuating wage. We can summarize our findings in the following proposition.

Proposition 2. An increase Q^E holding $\omega^y = p^y$ constant will have no effect on worker welfare. Under fluctuating-wage equilibrium, it will raise ω_G and lower ω_B, in the process raising average X-sector wages. Under wage-smoothing equilibrium, it will have no effect on wages if wage smoothing is still possible; otherwise, the fluctuating-wage equilibrium with wage volatility and rising expected wage payment results.

Proof. See Appendix B, page 133.

An increase in Q^E makes it is easier for the employer to find a new worker. This in turn aggravates the employer's temptation to revoke on wage promises made to the seasoned worker, especially when profits are low. The increased temptation makes the employer's incentive constraint tighter, implying that a rise in Q^E will shift the EE down, without having any effect on WW (neither equation (1.13) in case of wage-smoothing equilibrium nor equation (1.20) in case of fluctuating-wage equilibrium will change). Since WW will not shift, worker welfare stays intact. Under the fluctuating-wage equilibrium, the downward shift of EE lowers the bad-state wage and to compensate the worker, raises the good-state wage. This raises the expected wage payment in the X sector due to worker's risk aversion. On the other hand, under wage smoothing, as long as the employer's bad-state incentive constraint does not bind, there will be no effect on wages. However, if it binds with an increase in Q^E, then it is not possible to sustain a constant wage, hence we move into a fluctuating-wage equilibrium with wage volatility and rising expected wage payment.

Consequently, Q^E affects the employer's well being in two ways. It has a positive direct effect given that it is easier for the employer to fill a vacancy and be productive. However, it may have a negative indirect effect such that in a given relationship with fluctuating wages, the employer's surplus is lower since the expected wage payment to the worker increases due to the increase in wage volatility.

Next, we analyse the effect of a change in p^y on X-sector wages and worker welfare.

Proposition 3. An increase in p^y always raises welfare of workers. Furthermore, under fluctuating-wage equilibrium, an increase in p^y will raise ω_G and lower ω_B, in the process raising average X-sector real wages. Instead, under wage-smoothing equilibrium, an increase in p^y will raise the efficiency wage in real terms if wage smoothing is still possible; otherwise, the fluctuating-wage equilibrium with wage volatility and rising expected real wage payment results.

Proof. See Appendix B, page 134.

In particular, a rise in p^y will shift both curves upward. The WW curve shifts up because the worker's opportunity cost has risen. The EE curve shifts up because, for given wages (either ω_G and ω_B in a fluctuating-wage equilibrium or ω^* in wage-smoothing equilibrium) the rise in the worker's opportunity cost lowers the degree to which new workers are cheaper than incumbents (recall that a new worker is paid her opportunity wage ω^y in the first period of employment). The former tends to increase the wage volatility while the latter tends to decrease it. Overall, the former effect dominates since the higher opportunity cost of X-sector workers lowers the joint surplus available to worker-employer pair in the X sector and also lowers the share of the surplus that can be captured by the employer. This sharpens the employer's incentive-compatibility constraint. In other words, the rise in the worker's opportunity cost makes the employer more prone to cheat in the bad-profitability state.

As a result, if fluctuating-wage equilibrium prevails, X-sector wages become more volatile as p^y rises. This also raises expected wage payment in the X sector. On the other hand, under wage smoothing, as long as the employer's bad-state incentive constraint does not bind, an increase in p^y will raise the efficiency wage without causing any wage volatility. Nevertheless, if it binds with an increase in p^y, then it is not possible to sustain a constant wage, hence we move into a fluctuating-wage equilibrium with wage volatility and rising expected wage payment.

Note that when p^y rises, the real wage in the Y sector, $\frac{\omega^y}{P(p^y)}$, goes up (so does $\mu(\frac{\omega^y}{P(p^y)})$) since the elasticity of $P(p^y)$ with respect to p^y is less than 1. To satisfy either equation

(1.20) or equation (1.13), the expected wage (in case of fluctuating-wage equilibrium) or the efficiency wage (in case of wage-smoothing equilibrium) must rise more than p^y. This implies that the expected wage or the efficiency wage rises not only in nominal terms but also in real terms. In turn, the rise in real wage causes the WW curve to shift upwards and increases worker welfare.

Propositions 2 and 3 imply the following.

Corollary. For a given p^y , there is a value $Q^E_{VV}(p^y) \in [0, 1]$, such that if $Q^E < Q^E_{VV}(p^y)$ a wage-smoothing equilibrium can be sustained, while if $Q^E > Q^E_{VV}(p^y)$ it cannot. Furthermore, $Q^E_{VV}(p^y)$ is decreasing in p^y.

Proof. See Appendix B, page 137.

In Figure 1.2, the downward-sloping VV curve represents the function $Q^E_{VV}(p^y)$. On this curve, (Q^E, p^y) combinations are such that the employer's incentive compatibility constraint holds with equality, and thus forms a border between the wage-smoothing and the fluctuating-wage equilibria. To the left of VV curve, we have wage smoothing and to the right, we have fluctuating wages since it is not possible to sustain efficiency wage. Consequently, we show that for given parameters, wage smoothing is possible if it is sufficiently difficult for an employer to find a new worker (i.e., for a given p^y, when Q^E *is* low) or if Y-sector output is sufficiently cheap (i.e., for a given Q^E, when p^y is low).

To see the effect of a change in Q^E and p^y, consider the part of the parameter space in Figure 1.2 where we have fluctuating-wage equilibrium. Any movement up and to the

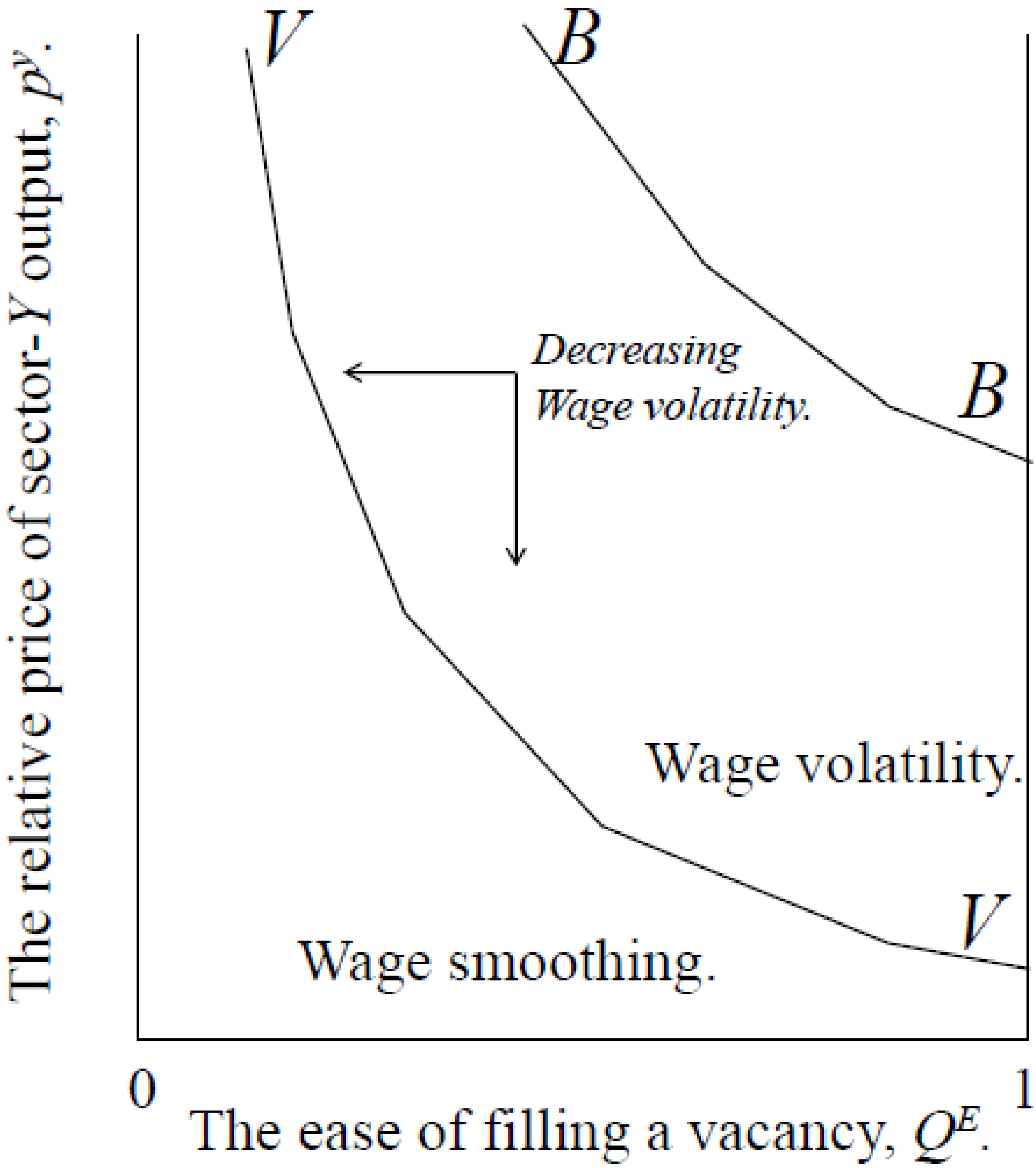

Figure 1.2. Type of Wage Contract and Comparative Statics.

right from a point on or above the VV curve results in a rise in wage volatility. Moreover, while any upward movement improves workers' welfare in both sectors, any horizontal movement has no such effect.

Additionally, when $(Q^E,\ p^y)$ combination is close to the VV curve, we are close to the efficiency wage region, thus ω_G^* is close to ω_B^*, so $x_G - \omega_G^* > x_B - \omega_B^*$. Increasing Q^E while holding p^y constant increases wage volatility by increasing ω_G^* and decreasing ω_B^*. As we continue increasing Q^E either in the limit we reach $Q^E = 1$ with the inequality $x_G - \omega_G^* > x_B - \omega_B^*$ still holds, or there exists a value of $Q_{BB}^E(p^y)$ such that the employer's incentive compatibility constraint binds in both states, so $x_G - \omega_G^* = x_B - \omega_B^*$ and for any value of $Q^E > Q_{BB}^E(p^y)$ we will have $x_G - \omega_G^* < x_B - \omega_B^*$, where no equilibrium exists as shown in Proposition 1. The function $Q_{BB}^E(p^y)$ is demonstrated by the downward-sloping BB curve in Figure 1.2.

In the following section, we will analyse the general equilibrium where Q^E and p^y are endogenously determined.

1.6. General Equilibrium

We consider the steady state equilibrium where the amount of each good produced is equal to the amount of each good consumed. We first determine the equilibrium value of Q^E. We know that in any period, the total number of matches in the X sector, $\Phi(n, m, \phi)$, is a function of number of workers searching (n), number of employers searching (m) and the effectiveness of search technology (ϕ). Therefore, at the steady state, the probability of a searching employer to find a partner is given by $Q^E = \frac{\Phi(n,m,\phi)}{m} = \Phi(\frac{n}{m}, 1, \phi)$, an increasing function of $\frac{n}{m}$ and ϕ. Thus the steady state level of m must satisfy

$$m = \Big(1 - \Phi(\frac{n}{m}, 1, \phi)\Big)\, m + (1-\rho)(E-m) + (1-\rho)\Phi(\frac{n}{m}, 1, \phi)m.$$

On the right-hand side, the first term represents the number of employers with no match; the second term represents the number of previously-matched employers that are exogenously separated; and the last term represents the number of newly-matched employers that are immediately exogenously separated. A straightforward simplification of the above equation yields

$$m = E - \frac{\rho}{1-\rho}\Phi(\frac{n}{m}, 1, \phi)m. \tag{1.23}$$

Derivation of the similar expression for the steady state number of workers results

$$n = L - \frac{\rho}{1-\rho}\Phi(\frac{n}{m}, 1, \phi)m. \tag{1.24}$$

Using these two equations, we have the following proposition.

Proposition 4. The steady-state value of $\frac{n}{m}$ and Q^E is uniquely determined for a given value of ϕ. Therefore, we can write $Q^E(\phi)$. Moreover, $Q^E(\phi)$ is strictly increasing.

Proof. See Appendix B, page 137.

An advancement in search technology affects the employer's steady state probability of finding a match both directly and indirectly. The direct effect, $\frac{\partial Q^E}{\partial \phi} = \frac{\partial \Phi(\frac{n}{m},1,\phi)}{\partial \phi} > 0$, increases the total number of matches in the X sector. The indirect effect, $\frac{\partial Q^E}{\partial\left(\frac{n}{m}\right)}\frac{d(\frac{n}{m})}{d\phi} =$

$\frac{\partial\Phi(\frac{n}{m},1,\phi)}{\partial\left(\frac{n}{m}\right)}\frac{d(\frac{n}{m})}{d\phi}$, can be positive or negative depending on whether $E < L$ or $E > L$, respectively. However, as we show in the appendix, even if the indirect effect is negative, it is always the case that the direct effect dominates, and thus an improvement in the effectiveness of search technology makes it easier for an employer to fill a vacancy.

Next, we will determine the relative price of good Y, p^y. To that purpose, since consumers have identical and homothetic demands, it is sufficient to pin down the relative supply of good Y, r.

Proposition 5. The steady-state supply of X-sector output is an increasing function of ϕ, while the steady-state supply of Y-sector output is a decreasing function of ϕ. Therefore, the relative supply of Y-sector output, r, is a decreasing function ϕ, and the relative price of Y-sector output, p^y, is an increasing function of ϕ.

Proof. See Appendix B, page 139.

We can illustrate Propositions 4 and 5 with the help of Figure 1.3. It is the same as Figure 1.2 with the addition of the upward-sloping curve *PP*, which is the locus of market-clearing values that complete the general equilibrium. It gives the combinations of Q^E and p^y obtained by varying ϕ over the positive real line. More precisely, for a given value of ϕ, we can find the steady-state value of Q^E (as in Proposition 4) and the

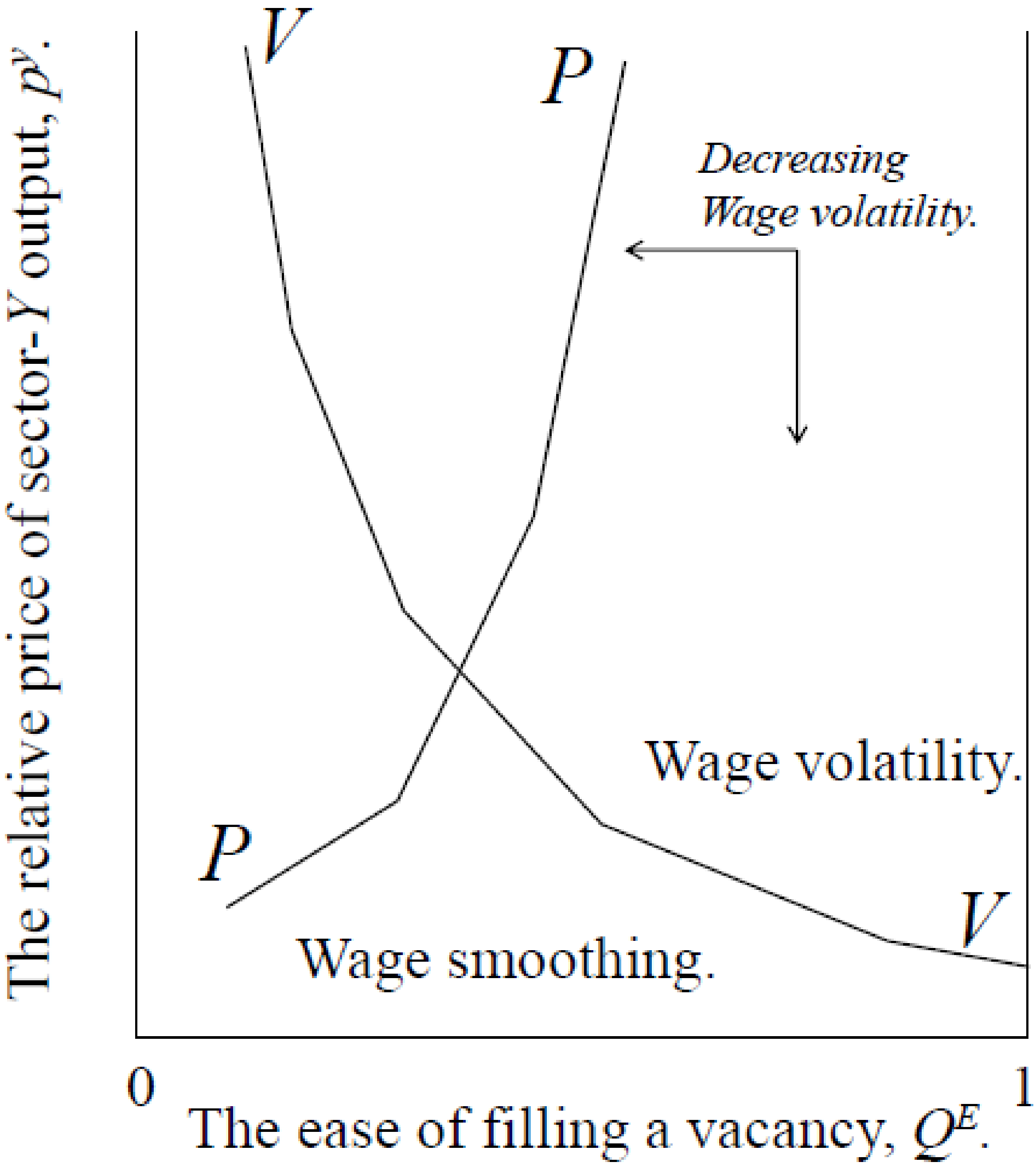

Figure 1.3. The effects of an improvement in search effectiveness.

steady-state value of the equilibrium relative price p^y (as in Proposition 5). Accordingly, as ϕ increases, the steady-state values of Q^E and p^y both go up.[24]

Note that the steepness of PP curve depends on elasticity of substitution implied by the utility function U between goods X and Y. Specifically, if the elasticity of substitution is high, then a given rise in ϕ and consequent drop in r will require only a small change in the relative price p^y to restore market clearing. Conversely, a low elasticity of substitution will require a large movement in p^y. For this reason, if they are very close substitutes, PP is arbitrarily flat, while if they are close to the case of perfect complementarity, it is arbitrarily steep.

Finally, we have all the tools to analyse the overall effect of a change in the effectiveness of search technology on equilibrium.

Theorem. An improvement in the effectiveness of search technology, i.e., a rise in ϕ, will raise the (average) real wage in both sectors, while raising the (expected) real wages and worker welfare in the whole economy. It also raises the wage volatility in the X sector if the new equilibrium has fluctuating wages. Besides, depending on the parameter values, it may also increase economy-wide wage volatility.

Proof. See Appendix B, page 140.

[24]We can now specify the portion of the parameter space where both Assumptions 1 and 2 are satisfied. First note that for the wage-smoothing agreement, the wage-smoothing condition is strictly stronger than Assumption 1 since $\omega^* > \omega^y$. Hence, for the whole length of PP curve to the left of VV and for at least a segment of positive length to the right of VV, Assumption 1 will be satisfied. If it is also true that Assumption 2 holds at the intersection of PP and VV, then there is a segment of PP including its intersection with VV plus some distance on both sides in which Assumptions 1 and 2 are both satisfied. We assume this and restrict our attention to that segment.

The rise in search effectiveness, ϕ, has two effects on the steady-state equilibrium. First, it increases the number of productive employers by increasing their probability of finding a worker (Proposition 4). This aggravates the employer's temptation to shirk on wage promise and increases (or does not affect if efficiency wage is still sustainable) the expected wage and X-sector wage volatility (Proposition 2). This is the direct effect. In addition, there is also an indirect effect such that as more matched pairs occur and increases the amount of good X, less workers will be left to be employed in the Y sector, causing Y-sector good production to go down while raising its relative price, p^y (Proposition 5). In turn, from Proposition 3, a rise in p^y also aggravates the employer's temptation to shirk and increases the expected wage (or the efficiency wage if it is still attainable) in both real and nominal terms and increases X-sector wage volatility (or does not affect it if efficiency wage is still sustainable). Overall, we can conclude that as search effectiveness improves, in the X sector, expected wage (or efficiency wage) increases both in nominal and real terms whereas wage volatility increases only if fluctuating wage prevails in the new equilibrium.

We can also see what happens to economy-wide wage dynamics. First, we focus on expected real wages. As ϕ rises, we know that $p^y = \omega^y$ increases. We also know that elasticity of $P(p^y)$ with respect to p^y is less than 1, since this elasticity measure also represents consumption share of good Y. These two observations imply that workers that are employed in the Y sector and those that are in the apprenticeship period of their X-sector employment will experience an increase in their real wage. Furthermore, for those incumbent workers in the X sector, a rise in ϕ increases the expected wage (or the efficiency wage in case of wage smoothing) more than the increase in p^y as can be seen from equation (1.20) (or from equation (1.13) in case of wage smoothing). Hence, their (expected) real wage increases even more than the former group of workers. Since any worker is employed either in the X sector or Y sector, this automatically implies economy-wide increase in (expected) real wages and worker welfare. Next, we turn our attention to economy-wide real wage volatility.[25] Once ϕ rises, there are two types of effects on real wage volatility which we call with a slight abuse of terminology as 'price' effect and 'compositional' effect.[26] The 'price' effect measures the change in wage volatility while holding industrial composition intact, i.e., keeping the number of workers in each sector constant. Instead, the 'compositional' effect measures the change in wage volatility in response to changes in sectoral compositions only, i.e., the movement of workers between sectors with different degrees of wage volatility. We show that the former effect is always positive (i.e., increases wage volatility), whereas the latter can be either positive or negative. Therefore, if the 'compositional' effect is positive, we can conclude that economy-wide real wage volatility increases; otherwise, it depends on how strong is the 'price' effect vis-à-vis the 'compositional' effect.

1.7. Conclusion

In this paper, we employ a stylized model to analyse the relationship between market thickness, labour market flexibility and wage dynamics. We have shown that a rise in market thickness caused by advances in search technology can generate wage volatility

[25]Our discussion is on real wage volatility, however nominal wage volatility can be directly inferred from it.

[26]See Autor, Katz and Kearney (2008). There are opposing views in the literature regarding the relative importance of these two effects. Lemieux (2006) finds that most of the residual (within group) wage dispersion observed from 1973 to 2003 is due to compositional effect, whereas Author, Katz and Kearney (2008) find that the price effect remains a key force in explaining residual wage inequality between 1989 to 2005.

by increasing labour market flexibility. Our findings indicate that improvements in search efficiency increases the average real wage and (potentially) wage volatility in the risky sector while raising the expected real wages and worker welfare in the whole economy. Moreover, it may also raise economy-wide real wage volatility. As a result, within this simple framework we manage to explain the transitory variation in workers' earnings observed after the 1970s.

Our results have also some implications regarding welfare of workers. At the outset, it may seem that the greater instability of earnings caused by transitory shocks would decrease the welfare of risk-averse agents. However, this conclusion is premature and in order to reach a convincing verdict we need to determine how insurable these shocks are. When it is possible to insure against earnings volatility via risk-sharing arrangements, this negative welfare effect may not be a concern. This is what we have found in this paper; following an advancement in search technology, even if wage volatility rises, worker welfare does not decrease but rather rises due to relative-price effects. This result is in harmony with the statement made by Edmund Phelps while discussing the work of Gottschalk, Moffitt, Katz and Dickens (1994):

> *"Insofar as increased transitory variance reflects wage flexibility, it means that labour markets are working more efficiently, which should be as welcome as increased price flexibility. Furthermore, individuals can take measures to soften the impact of transitory losses, and the welfare state offers additional insurance. In my view, efforts to make incomes more secure and insulate individuals from market signals would be the wrong*

response.” – Edmund Phelps, a general discussion on Gottschalk and Moffitt, Katz and Dickens (1994).

CHAPTER 2

Joint Design of Unemployment Insurance, Employment Creation and Protection: A Dynamic Model

2.1. Introduction

This paper explores three functions of taxation (where taxation refers to taxes and transfers); raising revenue, redistributing income, and correcting market failures[1] in the context of labour market policies. In particular, we are interested in how different types of taxes and redistribution policies jointly may have an impact on wage volatility and employer's and worker's welfare while two classes of market failure, a moral hazard problem and incomplete credit market, exists. We consider a labour market in which risk-averse workers and risk-neutral employers must match for production to occur. Contracts are incomplete; therefore self-enforcing contracts are the only means to share risk.

Even if, in principle, taxes and transfers are accepted as part of the optimal tax problem early on in the literature,[2] in practice with some notable exceptions (which will be noted later), joint investigation of the policies has not been usually followed. In this respect, labour market institutions are particularly important, since these policies are

[1]Stabilizing the economy is another use of taxation. However, this research will adopt a microeconomic viewpoint and exclude this role of taxation.
[2]See Diamond (1968) and Mirless (1971).

necessarily a bundle composed of tax and accompanying transfer. For instance, when unemployment payment[3] is financed by the payroll tax it is called unemployment insurance;[4] when firing payment is financed by the firing tax, it is called severance payment. So we need to evaluate the different parts of the policy as well as the policy itself as a whole.[5]

Existing labour markets institutions in most countries, such as unemployment insurance or Employment Protection Legislations (EPL)[6] create rigidities and distortions by preventing the setting of prices or quantities freely by private parties. In addition to existing policies, proposed institutions, such as hiring tax and hiring payment, potentially will have similar effects. Backed by the empirical evidence, presuming those institutions cause a sub-optimal equilibrium, reforms are on the policy agendas of many countries as well as discussions in academia. On the other hand, a subset of proposals argue that the government intervention on labour market may be Pareto improving.

For instance, unemployment payment is commonly believed to create distortions that adversely affect job search behaviour and lengthen unemployment spells (see Decker, 1997 for a summary), reduce precautionary savings (Engen and Gruber, 2001), and increase the incidence of temporary layoffs (Feldstein, 1978). With these issues in mind, Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) devise schemes optimizing the amount

[3]In the literature insurance payments and insurance benefits have different connotations (See Ozkan, 2014), but we do not distinguish them. Any regular payment for the unemployed is given the name, unemployment payment.

[4]According to Holmlund (1998), usually employers or workers, and government contributions, finance unemployment payment. We assume employers pay a tax while employer/worker relationship is intact and call it a payroll tax.

[5]Another important labour market policy, minimum wage, can be interpreted as a policy with tax and transfer. The additional wage employers pay to workers in excess of the amount they would pay if minimum wage had not been implemented can be thought of as a tax the employers pay and excess payment to workers as a redistribution of that tax, *ceteris paribus*.

[6]Severance payment is a component of EPL. In this research, we focus only on the financing side of it. See footnote 20 for an additional comment.

and timing of the unemployment payment. Also, Riboud *et al.* (2002) discuss institutional reforms in transition economies of Central and Eastern Europe and suggests high payroll taxes used to finance unemployment payment may be partly responsible for the unemployment. In his seminal paper, Feldstein (1976) asserts that payroll tax does not internalize the social cost of unemployment and increases the cost of workers, so that employers layoff workers excessively, above the socially optimal level of layoff. On the other hand, it is no secret that the main motivation of the introduction of the unemployment payment is to decrease income (and thus consumption) volatility.[7] Unemployed workers have to decrease their consumption drastically unless they have enough savings to use as a cushion or they have access to capital markets. Thus, being unemployed has big economic, psychological, and social costs. So, most studies focus on unemployment payments' consumption-smoothing and risk-sharing function (Gruber, 1997). Acemoglu and Shimer (2000) argue that unemployment payment motivates workers to seek high paying jobs by increasing labour productivity.

Another safety net that appears in most countries is EPL. It is intended to protect workers against unfair actions of employers, but may increase the cost of employment and thus lower employment levels. Given that most OECD countries have some form of compensation[8] for employer-initiated separations mandated by the law,[9] an OECD (2004) report brings attention to the debate on the benefits and costs of European EPL.[10] In

[7]Krueger and Perri (2005) and Heathcote *et al.* (2004) have shown that the wage volatility has direct effects on consumption volatility and large effects on welfare.

[8]Privately provided or legally mandated insurance benefits after a dismissal incident of a worker are usually called severance pay or dismissal pay (see Burguet and Caminal, 2008). In such schemes, the amount the employer pays is equal to the amount the employee receives after a dismissal.

[9]The notable exception is the U.S. See OECD (1999), Table 2.A.3.

[10]Heckman and Páges (2000) show how job security policies may increase unemployment and increase inequality in Latin America.

particular, OECD members and European policy-makers endorse *flexicurity* by relaxing the restrictions of EPL (i.e., reductions in firing and hiring restrictions) given that they blame EPL for the chronic high unemployment rate, and by implementing Active Labour Market Policies (ALMPs). On the other hand, Feldstein (1976) advocates that mandating firms to pay a firing tax is a way of internalizing the social cost of firing. There are other studies that extend the work of Feldstein (1976) and examine the effects of experience rating (as represented by firing taxes), such as Cahuc and Malherbet (2004), Cahuc and Zylberberg (2008), and Baumann and Stähler (2006). They all propose a US style experience rating system for the Continental European countries. Feldstein and Altman (1998) propose unemployment insurance savings accounts (UISAs) that reduce unemployment payment's adverse effects on incentives without decreasing protection for the unemployed. Robalino *et al.* (2009) argues that developing countries may benefit most from the self-policing nature of the UISAs system due to their weak institutions and large informal sectors.[11]

One of the categories of ALMPs involves subsidies to employers for creating jobs and/or paying workers for a certain period of time until they find a job.[12] Like the other policies, ALMPs have the potential to distort the labour market and create inefficiencies.

[11]Under UISAs system individuals have to save a certain percentage of their wages and use it when they are unemployed unless the account balance is negative. The amount that is left in the account is used for pension after the individual reaches the retirement age. So, in addition to financing the unemployment payments, the UISAs system finance the pension as well. The fund we are proposing is i) a pool of contributions (and not individual accounts) and ii) strictly reserved for funding the unemployment and hiring payments. This two characteristics makes our fund less distortionary (requires less fund) compared to the UISAs system.

[12]There are three main categories of Active Labour Market Policy: Public employment services; Training schemes; Employment subsidies. See OECD (2005) and European Commission (2010) for flexicurity discussion and Martin (2014) for macroeconomic evidence on the impact of Active Labour Market Policies on employment and unemployment rates.

Supported by the prediction of competitive labour market theory, the minimum wage has been identified as a cause of unemployment since the 1980s. For instance, the empirical study by Abowd *et al.* (2000), covering France and US, suggests low-skilled or young workers should be excluded from the mandatory minimum wage.[13] Several studies[14] criticise the minimum wage for having a role in increasing wage inequality. However, recent empirical research does not support this finding (See Holmlund, 2014). Cahuc and Michel (1996) argue that an increase in unemployment in unskilled labour due to higher minimum wage may not be a negative result *per se*, since higher relative demand for skilled labour may induce workers to improve their level of skills, to accumulate human capital and to improve economic efficiency. They build this intuition on the lack of evidence of differences in efficiencies across countries with different minimum wage regimes. Given that the employment effects of minimum wage appear to be small. Besides, Card and Krueger (1994, 1995) and Machin and Manning (1997) assert that the minimum wage can be an useful tool to alleviate poverty with its redistributive effect.[15]

The research that is being conducted usually examines one labour market policy at a time. One prominent research that investigates different labour market policies concurrently is Blanchard and Tirole (2008) where they show that unemployment insurance and employment protection policies (as represented by firing taxes) are tightly linked.[16] In a

[13]Survey of Neumark and Wascher (2008) on minimum wage literature report that a majority of the studies, some are statistically significant, indicate a consistent negative employment effects of minimum wage.

[14]See Autor *et al.* (2008), Card and DiNardo (2002), Lee (1999), and Lemieux (2006) .

[15]Even if we are not going to directly investigate minimum wage, under certain conditions hiring tax and hiring payment may jointly behave like minimum wage.

[16]Optimum has two characteristics in their benchmark case: (i) firing tax is equal to unemployment payment and (ii) there is no role for the government. Different extensions (limits to full insurance, heterogeneity of firms and workers, etc.) of the benchmark supports a shift from payroll tax to firing

static model similar to Blanchard and Tirole (2008), Mongrain and Roberts (2005) also study these two aspects of the system together.[17] They find that under moral hazard, a full-experience rating system, where the cost of unemployment is covered by firing tax, maximizes workers' expected utility in equilibrium. L'Haridon and Malherbet (2009) evaluate the combination of firing tax and payroll subsidy.[18] They show that such a reform, among other benefits, improves the overall labour-market performance and decreases the size of the unemployment insurance budget. Another study that examines the effects of firing tax in a matching model with a balanced-budget constraint is Gavrel and Lebon (2008).[19] They find that firing taxes lowers unemployment by making firms internalize the unemployment costs of their firing behaviour.

Our research contributes to the literature in three ways. First, we expand the question Blanchard and Tirole (2008) ask and build a dynamic framework. We provide answers as to when different type of taxes and transfer schemes may have welfare enhancing effect for workers and employers. We explicitly evaluate the financing and spending side of the labour market policies. On the financing side we investigate payroll tax, firing tax, and hiring tax; and on the spending side we investigate unemployment payment and hiring payment.[20]

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tax will improve the results and the government may have a role if unemployment payment is not fully financed by firing tax.

[17]Other prominent papers that incorporate search friction and risk-averse workers into their dynamic models are Alvarez and Veracierto (2001) and Bertola (2004). They demonstrate that mandatory severance payments may improve welfare and efficiency.

[18]They model firing tax and payroll subsidy in a way that mimics the experience rating system in the US.

[19]Our model enables us to examine the impact of policies both with and without a balanced-budget constraint.

[20]The natural counterpart of firing tax would be firing payment. However, firing payment is not going to be qualitatively different from unemployment payment. Both payments would increase the opportunity pay-off of the worker albeit the impact of unemployment payment will be higher due to worker's risk

Second, it is well-documented[21] that there is a seniority effect, that is, workers are paid more as they stay longer in a job. Medoff and Abraham (1980) show that most of the increase in earnings can be explained by the experience (seniority), not by the performance. Lazear (1979) points out that it is generally optimal to "pay workers less than their marginal productivity when they are young and pay more than their marginal productivity when they are old" and calls this deferred compensation 'back-loading' of wages. In this payment scheme, employers are left with production surplus at initial stages of employment in order to incentivize workers. Taxing employers at the initial stage of the employment, thus, can be a source of income for financing payments and will not distort workers' incentives, since it does not involve any direct payment to workers. On the other hand, Mortenson (1994) finds that hiring subsidies, one component of ALMPs, may have welfare-improving effects.[22] One interesting result of studying hiring tax and hiring payment jointly will be that, under certain conditions, they will act like a minimum wage policy. We assess the viability of hiring tax as a source of finance and of hiring payment as a cushion to a worker at his/her initial stage of employment. Hence, the inclusion of hiring tax/payment will deliver insights.

Lastly, we add another component/layer to the behavioural distortion. Specifically, induced behavioural responses – the 'moral hazard' problem – is a concern for any insurance scheme.[23] In our case, the type of financing and spending may strengthen or weaken

aversion (See Boeri *et al.*, 2016). Thus, to minimize the number of cases, we assume during unemployment spells only type of compensation is unemployment payments.

[21]See Flabbi and Ichino (2001) and Kotlikoff and Gokhale (1992) for empirical evidence.

[22]In Mortensen (1994), source of the welfare (aggregate income) increase is through the decrease in unemployment rate.

[23]In particular, we focus our attention to the agency problem in the relationship. Another moral hazard issue that may arise is that level and profile of unemployment payment may have an impact on job finding probability. In turn, it may affect the likelihood of forming a relationship. We eliminate this type of

the employer/worker relationship. So even if the duration of unemployment and the probability of lay-off stay the same, financing and spending scheme may have volatility[24] increasing effect for workers who are currently employed.

In section 2.2, we give a brief summary of the model and the results; in section 2.3 and section 2.4, we lay out the model, introduce the budget constraint, transition equations, and objective function. In section 2.5, we examine the effects of particular financing and spending choices of funds on wage dynamics and the welfare of parties. First, we study the impact of three financing methods on the welfare: payroll tax, firing tax, hiring tax. By changing the composition of financing in a particular way and keeping the spending constant, we seek to find whether the employer/worker relationship becomes stronger or weaker. Second, we examine the effects of particular spending choices of funds while keeping the financing constant on wage dynamics and welfare of parties. The policy-maker chooses among two types of spending: unemployment payment and hiring payment. The objective in doing this exercise is to figure out whether increasing unemployment payment and hiring payment strengthen or weaken the employer/worker relationship. Finally, we examine two joint labour market policies: financing unemployment payment with payroll tax and financing hiring payment with hiring tax.

moral hazard problem by assuming job search effort is effectively monitored by the administration so that constant unemployment payment is optimal (See Holmlund, 1998).

[24]If volatility is high enough employer and worker may not form a relationship at all. Thus, we take level of wage volatility as a proxy for the strength of the employer/worker relationship. If the volatility exceeds a certain threshold no relationship may be formed.

2.2. Non-technical Summary

In our model, firms are hit by firm-specific shocks (or idiosyncratic shocks) that are unobservable to people outside the firm. As a result, written contracts are not enforceable. Only reputation within long-run employment relationship can work as a risk-sharing mechanism. In this risky environment, since employers are risk-neutral whereas workers are risk-averse, employers would like to provide full wage insurance or in other words, they would like to commit credibly to a constant wage (in effect reducing their expected wage payments to workers), but without enforceable contracts they can do so only by reputational means and so are constrained by their incentive compatibility constraints (ICCs). For production to occur, workers need to put costly noncontractible effort into the production process. As a result, in equilibrium, wage payments are back-loaded. The intuition is that when a non-contractible effort must be elicited from a worker, it is generally optimal to promise wages that increase over time so that the fear of losing high future wages deters shirking. Because of this back-loading of wages, new workers are cheaper than incumbent ones. This is the source of the firm's problem: Since the worker or employer has a temptation to shirk if the pay-off from shirking is high and because of back-loading wages are higher in later periods, employer's and worker's pay-off from shirking is high. Of course, an increase in the pay-off from shirking will tighten the ICCs so that employers will have to pay a lower wage in bad times and to compensate for that they will have to pay a high wage in good times.

Given this set-up, we study how financing and spending of funds used for labour market policies affect the wage volatility, employers' and workers' welfare. We find that the choice of financing and spending mechanism *per se* may be a source of volatility for the employed workers, and can decrease the strength of the employer/worker relationship.

In particular, we find that firing tax strictly dominates payroll tax and hiring tax in terms of efficiency gains. While the firing tax and hiring tax decreases wage volatility and expected wages through decreasing employers' pay-off of shirking, payroll tax increases it. If the decrease in wage volatility is large enough, an increase in firing tax and hiring tax has the potential to improve the employer's welfare. Since, unemployment payment aims to cure consumption volatility,[25] even if on-the-job wage volatility increases and employers are worse off as unemployment payment increases, one can defend it on the basis that it decreases the inequality between the unemployed and the employed. On the other hand, hiring payment increases the worker's income (wages and transfers) in the early phase of employment while decreasing it in later stages (thus smoothing out consumption during their employment) without having any affect on unemployed workers. When hiring tax is used to finance hiring payment, this joint policy may work as minimum wage under certain conditions. Unless wage volatility is high in the later phase of unemployment, this joint policy has the potential to decrease wage volatility and to increase the welfare of employers.

[25]Nicholson (2008) lists several causes of wage volatility such as job loss, changes in hours of work or wages on a given job or as a result of a job change, temporary absences from work, work-related disabilities or injuries, time out of employment for training needs and declining worker productivity because of age or other reasons.

2.3. The Model

In this section, we describe the main features of our model. The set-up is an extended version of Karabay and McLaren (2011).

Production. There are two factors of production: a measure E of Employers and a measure L of Workers. A worker can be unemployed or can team up with an employer and form a partnership for production to occur. We will call this partnership a 'firm'. In each period, a worker and employer must both put in one unit of non-contractible effort. Workers suffer a disutility from effort equal to $k > 0$, while employers suffer no such disutility. Within a given employment relationship, denote the effort put in by agent i by $e^i \in \{0, 1\}$, where $i = W$ indicates the worker and $i = E$ indicates the employer. The output and revenue generated in that period is then equal to $x_\epsilon e^W e^E$ where price of output is normalized to 1 and ϵ is an idiosyncratic independent and identically distributed random variable that takes a value $\epsilon = G$ or B with respective probabilities π_G and π_B, where $\pi_G + \pi_B = 1$ and $x_G > x_B > 0$. The random variable ϵ indicates whether the current period is one with a good state or a bad state for the firm's profitability. The average revenue is denoted by $\overline{x} \equiv \pi_G x_G + \pi_B x_B$. Employers without a worker are 'with vacancy' and do not produce anything.

Search. Workers and employers search for a partner until they have one. Search follows a specification of a type used extensively by Pissarides (2000). Let there be a measure n workers and a measure m employers looking for a match in a given period, then $\Phi(n, m)$ matches occur. Φ is concave, increasing in all arguments, and linearly homogeneous of degree 1 with $\Phi(n, m) < \min(n, m)$ and $\Phi_{nm} = \Phi_{mn}$ $\forall n, m$. Q^E is the steady-state probability that an employer will match with a worker in any given period, or in other words, $Q^E = \dfrac{\Phi(n,m)}{m}$, where n and m are set at their steady-state values. Similarly, $Q^W = \dfrac{\Phi(n,m)}{n}$ is the steady-state probability that the worker will find a job any given period. Even if search has no direct cost, it has an indirect cost such that no production occurs if parties are searching.

There is also a possibility in each period that a worker and employer who have been matched in that period or in the past will be exogenously separated from each other. This probability is given by a constant $(1-\rho) \in (0,1)$.

Preferences. There is no storage, saving or borrowing (incomplete credit market).[26] An agent's income in a given period is equal to that agent's consumption in that period. All agents discount the future at a constant rate $\beta \in (0,1)$.

The workers are risk-averse, with increasing, differentiable and strictly concave utility function μ, while the employers are risk-neutral. There is a finite lower bound, $\mu(0)$, to the worker's utility (or, equivalently, there is some exogenous source of consumption on which workers can rely on even if they are unemployed). Workers maximize expected discounted lifetime utility, and employers maximize expected discounted lifetime profits.

Due to informational constraints, parties outside the firm do not know the reason for the termination of a partnership. It can be due to either an exogenous separation from the previous period or shirking.

In any period, the government can deduce whether agents are in a productive relationship by checking if any output is produced in that period without observing the exact

[26]If saving or borrowing is allowed, given workers are foresighted, unemployment payment by the government would not be needed. However, a drastic decrease in consumption during unemployment unless there exists a safety net suggests that capital markets are incomplete. Our approach is also consistent with the existing literature that study unemployment payment. For instance, Shave and Weiss (1979) and Hopenhayn and Nicolini (1995) consider cases where consumers have no ability to save and a limited ability to self-insure, respectively.

amount of that output, i.e., x_G or x_B. In other words, the government learns both the beginning and the end of an employer-worker relationship by detecting output production. If there is a match and immediate separation without any output produced, then the government cannot distinguish this case from no-match case.

Timing of the game. We analyse steady-state equilibrium and the sequence of events within each period is as follows.

(1) Any readily matched employer and worker learn whether or not they will be exogenously separated in this period.

(2) The idiosyncratic output shock ϵ, for each firm, is realized. Within a given employment relationship, this is immediately common knowledge. The value of ϵ is not available to any agent outside of the firm, however.

(3) The wage, if any, is paid (a claim on the firm's output at the end of the period).

(4) The employer and worker simultaneously choose their effort levels e^j, $j \in \{E, W\}$. At the same time, the search mechanism operates. Within a firm, if $e^j = 0$, then agent j can participate in search and put in no effort. Unemployed worker and employer with vacancy always search and do it without any cost.

(5) Each firm's output/revenue, $x_\epsilon e^W e^E$ and profit is realized, and then consumption is realized.

(6) For those agents who have found a new potential partner in this period's search, new partnerships with a new self-enforcing agreement are formed. This is achieved by a take-it-or-leave-it offer made by the employer to the worker. Therefore, it is assumed that employers have the full bargaining power.

The Financing and Spending of the Fund. Government can collect taxes from employers as well as contribute itself to the funds to finance the spending. There is no borrowing or roll-over of funds between periods. Thus the budget of the fund clears in every period as below.

$$F + N_u u + N_c c + N_h h = N_{\omega^u}\omega^u + N_{\omega^h}\omega^h. \tag{2.1}$$

The left-hand side shows how the fund is financed. F represents the total amount of (windfall) transfer to the fund by the government. F is like a lump-sum tax, and is non-distortionary. When $F = 0$, the fund is balanced and when $F > 0$, the fund is in deficit. The main difference between lump-sum tax and windfall is that changing the lump-sum tax *per se* will ultimately will have welfare effect on employers and workers. u is payroll tax paid by the employer at each period while parties are in a relationship. In other words, the government will collect payroll tax only when there is production. c is firing tax paid by the employer as a result of separation and h denotes hiring tax paid by the employer as a result of hiring.

The right-hand side shows how the fund is spent. Suppose the government aims to support unemployed workers and support workers during the initial phase of their employment. ω^u is unemployment payment to workers who are searching for two or more periods. ω^h is hiring payment made to each hired worker. Let α denote the type of tax employers pay and the type of transfer workers receive. We have $\alpha \in \{u, c, h, \omega^u, \omega^h\}$. Therefore N_α represents the measure of employers who pay α or the measure of workers who receive α.

Total number of employers/workers in relationship period i is given

$$E - m_i = \rho \left(E - m_{i-1}\right) + \rho \Phi \left(n_{i-1}, m_{i-1}\right)$$

We analyse steady-state equilibrium, not how variable of interest changes during the transitory periods. At the steady state we must have:

$$E - m = \frac{\rho}{1-\rho} \Phi \left(n, m\right)$$

So, financing by payroll tax amounts to $N_u u = \frac{\rho}{1-\rho} \Phi \left(n, m\right) u$.

In the steady state, the number of separations will be equal to the number of matches, $\left(1 - \rho\right) \left(E - m\right) = \rho \Phi \left(n, m\right)$. So the financing by firing tax amounts to $N_c c = \rho \Phi \left(n, m\right) c$.[27]

The total number of matches in the steady state is equal to $\Phi \left(n, m\right)$. Among the newly-matched employers, $\rho \Phi \left(n, m\right)$ of them do not separate immediately in the first period of their employment relationship, whereas $\left(1 - \rho\right) \Phi \left(n, m\right)$ of them are matched and separated immediately. So the financing by hiring tax amounts to $N_h h = \rho \Phi \left(n, m\right) h$.

Total amount of unemployment payment is equal to $N_{\omega^u} \omega^u = \left(n - \rho \Phi \left(n, m\right)\right) \omega^u$, where $\rho \Phi \left(n, m\right)$ refers to the number of workers in the first period of the unemployment. Total amount of hiring payment is equal to $N_{\omega^h} \omega^h = \rho \Phi \left(n, m\right) \omega^h$, where $\rho \Phi \left(n, m\right)$ refers to the number of matches that are not immediately separated (first period of the employment).

Timing of Taxes and Payments. The timing of taxes and payments is as follows.

[27]We assume the employer does not have to pay firing tax if there is matching and immediate separation or no production in the first period.

(i) payroll tax is charged in every period when production takes place. Firing tax is charged in the second period after the separation for analytical simplicity. If shirking occurs, that period is accepted as the first period of the separation. Hiring tax is charged in the first period in which production takes place.

(ii) unemployment payment is made while worker is searching except in the first period of search.[28] Thus, if either party shirks, worker would not get paid during that period. Hiring payment is made in the first period when production takes place.

Transition Equations. We will focus on steady-state equilibria. To avoid excessive notation, we will express all value functions in terms of the value function of agents that are searching for two or more periods.

The expected lifetime discounted profit of an employer with vacancy (thus searching) is denoted by V^{ES}. The superscript S indicates the state of searching. Here, V^{ER} denotes the *ex-ante* expected lifetime discounted profit of an employer evaluated at the *beginning of a cooperative relationship* (both agents incur effort).

The expected lifetime discounted utility of an unemployed (thus searching) worker is denoted by V^{WS}. The value functions of employers and workers after separation as well

[28]Even if our primary reason for a deferred unemployment payment is to avoid dual income source (the wage payment by the employer and unemployment payment by the fund) when worker shirks, it may also be optimal to hold unemployment payment initially to discourage the use of temporary layoffs subsidized by unemployment payment (See Frederiksson and Holmlund, 2006).

as the payment made to workers following a separation are given below.

(After Separation)	Employer's Value Function	Government's Payment to the Worker	Worker's Value Function
1^{st} period	$V^{ES}-\beta c$	0	$V^{WS}-(\mu(\omega^u)-\mu(0))$
2^{nd} period	$V^{ES}-c$	ω^u	V^{WS}
3^{rd} period	V^{ES}	ω^u	V^{WS}

We can write the transition equations as:

$$V^{ES} = Q^E\rho\beta V^{ER}+Q^E(1-\rho)\beta V^{ES}+\left(1-Q^E\right)\beta V^{ES} \tag{2.2}$$

$$V^{WS} = \mu(\omega^u)+Q^W\rho\beta V^{WR}+Q^W(1-\rho)\beta V^{WS}+\left(1-Q^W\right)\beta V^{WS} \tag{2.3}$$

The expected lifetime discounted profit of an employer with vacancy (for more than two periods), denoted by V^{ES}, is equal to the continuation pay-off of the employer if the employer finds a worker who is not immediately exogenously separated, finds a worker who is immediately exogenously separated, or fails to find a worker. The expected lifetime discounted utility of an unemployed worker (for more than two periods), denoted by V^{WS}, is equal to the $\mu(\omega^u)$ plus the continuation utility value of the worker if the worker finds work and is not immediately exogenously separated, finds work and is immediately exogenously separated, or fails to find work.

Although employers and workers will take V^{ES} and V^{WS} as given, these values depend on the endogenous probability of matching $\left(Q^W, Q^E\right)$.

Given those values, and the assumption that the employer holds all of the bargaining power, a self-enforcing agreement between a worker and an employer is a subgame-perfect equilibrium of the game that they play together. In this game, the employer chooses a wage scheme that will give him/her the highest expected discounted lifetime pay-off subject to constraints. If either agent defects (i.e., shirking, putting no effort, or not paying wages) from the agreement at any time, the existing agreement becomes void, and both agents must search for new partners immediately. This assumption[29] is sufficient to ensure the credibility of promises.[30]

As noted earlier, all three essays share a common framework with some adaptations. Here, we will summarize the key differences between the first essay's and the current essay's model. In this essay, there is one (risky) sector and that sector corresponds to the careers sector in the first essay. In this essay, $\Phi(n, m)$ matches occur, thus there is no search effectiveness parameter, φ, as in the first essay. In this essay, since there is only one good produced in the economy, utility function is much simpler and workers have the same per-period utility function μ over the consumption of the good. In this essay, when workers' relationship dissolves, they are unemployed, but in the first essay they work in the risk-free sector in such a case. In both models, workers who are not in a relationship always search. Table 1 in the Appendix lists the distinguishing features of the model variants in each essay.

We will now characterize the optimal labour contracts below for the employer and the worker who paired after searching more than one period.

[29] Thomas and Worall (1989) use a much stronger assumption. In their paper, defecting party is banished from entering any relationship forever.

[30] See MacLeod and Malcomson (1989) for a similar study with moral hazard problem. Inability to verify information that limits explicit contracts is the main motivation for implicit contracts that supports incentive compatibility.

2.4. Optimal Contracts

The equilibrium can be characterized as the solution to a recursive optimization problem. $\Omega(W)$ is the highest possible expected pay-off the employer can receive, given the worker will receive W. Ω is decreasing, strictly concave, and a differentiable function as in Benvenista and Scheinkman (1979).

$$\Omega(W) = \max_{\{\omega_\epsilon, \widetilde{W}_\epsilon\}} \sum_{\epsilon=G,B} \pi_\epsilon \left[x_\epsilon - \omega_\epsilon - t + \rho\beta\Omega(\widetilde{W}_\epsilon) + (1-\rho)\beta\left(V^{ES} - \beta c\right)\right] \tag{2.4}$$

subject to

$$x_\epsilon - \omega_\epsilon - t + \rho\beta\Omega(\widetilde{W}_\epsilon) + (1-\rho)\beta\left(V^{ES} - \beta c\right) \geq V^{ESHIRK} \tag{2.5}$$

$$\mu(p_\epsilon) - k + \rho\beta\widetilde{W}_\epsilon + (1-\rho)\beta\left(V^{WS} - (\mu(\omega^u) - \mu(0))\right) \geq V^{WSHIRK} \tag{2.6}$$

$$\sum_{\epsilon=G,B} \pi_\epsilon \left[\mu(p_\epsilon) - k + \rho\beta\widetilde{W}_\epsilon + (1-\rho)\beta\left(V^{WS} - (\mu(\omega^u) - \mu(0))\right)\right] \geq W \tag{2.7}$$

$$\omega_\epsilon \geq 0 \tag{2.8}$$

$$\widetilde{W}_\epsilon \geq 0 \tag{2.9}$$

The employer's problem (2.4) is to choose the current period wage and the worker's continuation pay-off (ω_ϵ, $\widetilde{W}_\epsilon$) at each state such that the employer's expected discounted lifetime profit is maximized given the worker's expected present discounted pay-off is equal to at least W. (2.5) is the employer's incentive compatibility constraint (ICC). In equilibrium, (ω_ϵ, $\widetilde{W}_\epsilon$) will be such that it will always be in the employer's best interest to commit to a cooperative relationship at each state. (2.6) is the ICC for the worker. We articulate the pay-off from shirking by the employer, V^{ESHIRK}, and by worker, V^{WSHIRK}, after summarising below how much tax the employer is going to pay and the total payment the worker is going to receive in each period *during the relationship*.

(In the relationship)	Tax paid by employer, t	Payment received by worker, p_ϵ
1^{st} period	$u+h$	$\omega_\epsilon + \omega^h$
2^{nd} period	u	ω_ϵ
3^{rd} period	u	ω_ϵ

Given that no party shirks; after the production, the employer will pay $t = u + h$ (payroll and hiring tax) in the first period,; then will pay $t = u$, payroll tax, in later periods. The worker, on the other hand will get paid $p_\epsilon = \omega_\epsilon + \omega^h$, the labour wage plus hiring payment in the first period; then will get paid $p_\epsilon = \omega_\epsilon$, the labour wage, in later periods. Now we will write the value function of shirking employers and workers below.

In the relationship	Employer's Value Function if employer shirks, V^{ESHIRK}	Worker's Value Function if worker shirks, V^{WSHIRK}	W
1^{st} period	V^{ES}	$V^{WS} - (\mu(\omega^u) - \mu(\omega_\epsilon + \omega^u))$	W_0
2^{nd} period	$V^{ES} - \beta c$	$V^{WS} - (\mu(\omega^u) - \mu(\omega_\epsilon))$	W_1
3^{rd} period	$V^{ES} - \beta c$	$V^{WS} - (\mu(\omega^u) - \mu(\omega_\epsilon))$	W_2

In case of shirking, the value function of either the employer or the worker will change depending on the period when shirking occurs. In the first period of the employment, if any party shirks (no production), the government will not able to distinguish it from a matching and immediate separation case and will consider the situation as if agents stayed vacant/unemployed. In such a case, the employer will not pay any tax and the worker will not get any hiring payment. If the worker shirks, the worker's first period income will be equal to unemployment payment plus the first period wage; if the employer decides to shirk, the employer will not pay any wage to the worker. On the other hand, in later periods, if the worker shirks, the worker's period income will only consist of wages; if the employer shirks, the employer will pay firing tax one period after the shirking.

We will denote inequality (2.7) as the target utility constraint. The employer has a target expected present discounted pay-off intended for the worker which is equal to at least W. The last constraint is the positive current wage constraint. Next we will have an assumption that will enable us to focus on a particular parameter space.

Assumption 1. $\mu(0) \leq \mu(\omega^u) < \mu(\omega^h) \leq \mu(\omega^u) + k$.

This assumption has two requirements. First, the hiring payment is larger than the unemployment payment. This requirement is necessary to make the solution history-independent.[31] Second, the hiring payment must have an upper bound.

In the first period of the employment relationship, the employer must pay the worker at least as much as the worker's outside option, otherwise she will never accept employment, thus, $V^{WR} \geq V^{WS}$. If we substitute V^{WS} for V^{WR} in equation (2.3), we obtain:

$$V^{WS} \geq \frac{\mu(\omega^u)}{1-\beta} > 0 \tag{2.10}$$

[31]Equilibrium wage when $\omega^h < \omega^u$ will be both state-dependent and history-dependent. Thus, such a case will increase the complexity of the analysis. For an equilibrium wage that is both state-dependent and history-dependent, reader may refer to Thomas and Worrall (1988). Solving for $\omega^h = \omega^u$ is a special case and will not provide extra benefit.

Suppose the first period and the second and subsequent periods continuation pay-off, $\widetilde{W}_\epsilon$, is denoted as $\widetilde{W}_\epsilon^{R1}$ and $\widetilde{W}_\epsilon^{R2}$, respectively. By assumption 1 and inequality (2.10), we can write the worker's ICC (2.6) as:[32]

$$\widetilde{W}_\epsilon^{R1} \geq \widetilde{W}^* - \frac{\mu\left(\omega_\epsilon^{R1} + \omega^h\right) - \mu\left(\omega_\epsilon^{R1} + \omega^u\right)}{\rho\beta} \tag{2.6a}$$

$$\widetilde{W}_\epsilon^{R2} \geq \widetilde{W}^* \tag{2.6b}$$

$$\text{where, } \widetilde{W}^* \equiv \frac{(1-(1-\rho)\beta)\left(V^{WS} - (\mu(\omega^u) - \mu(0))\right) - \mu(0)}{\rho\beta} + \frac{k}{\rho\beta} \tag{2.11}$$

As can be seen from the above the inequalities, (2.6a) and (2.6b), continuation pay-offs, $\widetilde{W}_\epsilon^{R1}$ and $\widetilde{W}_\epsilon^{R2}$, are positive, so we do not need constraint (2.9) in the employer's maximization problem. The value $\widetilde{W}^*$ is the minimum future utility stream promise in the second period to the worker in order to convince the worker to exert effort and forgo search. The first period continuation pay-off, $\widetilde{W}_\epsilon^{R1}$, is lower than the second and subsequent period continuation pay-off, $\widetilde{W}_\epsilon^{R2}$, due to the lower opportunity pay-off of the worker in the first period. By the following assumption, we will able to provide a feasible solution set to the employer's maximization problem.

Assumption 2. In the first period of an employment relationship, the employer's incentive-compatibility constraint (2.5) does not bind in either state.

[32] By Assumption 1 and the concavity of the utility function we obtain

$$-\left(\mu\left(\omega_\epsilon + \omega^h\right) - \mu\left(\omega_\epsilon + \omega^u\right)\right) \geq -\left(\mu\left(\omega^h\right) - \mu\left(\omega^u\right)\right) \geq -k$$

We will discuss sufficient conditions for this assumption later (Lemma 4 in Section 4). We can now prove that under Assumption 2, the equilibrium always takes the same form through the following proposition.

Proposition 1. (i) In the first period of an equilibrium employment relationship, the wage is set equal to zero in each state. (ii) In the second period of the employment relationship, there is a pair of values ω_ϵ^{*R2} for $\epsilon = G, B$ such that regardless of the history (provided neither partner has shirked), the wage is equal to ω_ϵ^{*R2} in state ϵ. (iii) In the third period and all subsequent periods of the employment relationship, there is a pair of values ω_ϵ^{*R3} for $\epsilon = G, B$ such that regardless of the history (provided neither partner has shirked), the wage is equal to ω_ϵ^{*R3} in state ϵ. In addition, the worker's continuation pay-off in the second and subsequent periods is always equal to $\widetilde{W}^*$. Furthermore, after the first period there are the following agreement types:

(1) Wage Smoothing Agreement when the employer's ICC (2.5) never binds.

(2) Partially Wage Fluctuating Agreement when the third and subsequent period employer's ICC (2.5) binds in the bad state.

(3) Fully Wage Fluctuating Agreement when the second and subsequent period employer's ICC (2.5)to avoid excessive notation.

Proof. See Appendix C, page 151.

In the first period, the employer needs to provide the worker with a target utility, W_0, of at least V^{WS} and a future utility stream, $\widetilde{W}_\epsilon^{R1}$, of at least $\widetilde{W}^* - \frac{\mu\left(\omega_\epsilon+\omega^h\right)-\mu(\omega_\epsilon+\omega^u)}{\rho\beta}$. We know that the employer has all the bargaining power and there is no need to give the worker more than what is required. As a result, the promise of the current wage and the future utility stream is such that the first period worker's ICC (2.6a) and target utility constraint (2.7) just bind.[33] Setting $\widetilde{W}_{\epsilon}^{R1} = \widetilde{W}^{*} - \frac{\mu\left(\omega_{\epsilon,1}+\omega^{h}\right)-\mu(\omega_{\epsilon,1}+\omega^{u})}{\rho\beta}$, where $\widetilde{W}^{*}$ is given in (2.11), and plugging this value into the binding target utility constraint gives us zero wage in each state. Thus, the worker goes through an 'apprenticeship period' at the beginning of the relationship in which zero wage is paid. Binding target utility constraint means, at the beginning of the employment relationship it is feasible for the employer to push the worker's pay-off down to the opportunity pay-off. Since it is in the interest of the employer to do so, it is clear that workers starting the employment relationship receive the same pay-off that they would receive while they are unemployed, $V^{WR} = V^{WS}$. From equation (2.3), this immediately tells us

$$V^{WS} = \frac{\mu(\omega^{u})}{1-\beta} \tag{2.12}$$

and $\widetilde{W}^{*}$ in (2.11) can be rewritten as

$$\widetilde{W}^{*} \equiv \frac{\mu(\omega^{u})}{1-\beta} + \frac{(1-\rho)\,\beta\,(\mu\,(\omega^{u}) - \mu\,(0))}{\rho\beta} + \frac{k}{\rho\beta} \tag{2.13}$$

If the employer's ICC (2.5) in the bad state does not bind in any period; in this equilibrium, the worker will receive a constant wage $\omega_{G}^{*R2} = \omega_{B}^{*R2} = \omega^{*R2}$ in the second period and $\omega_{G}^{*R3} = \omega_{B}^{*R3} = \omega^{*R3}$ in the third period where $\omega^{*R3} > \omega^{*R2}$. We will call this type of agreement, a *Wage Smoothing* agreement. If the employer's ICC (2.5) in the bad state binds in the third and subsequent periods and not in the second period; in this equilibrium, worker will receive fluctuating wage in the third and subsequent periods and constant wage in the second period, $\omega_{G}^{*R3} > \omega_{B}^{*R3} > \omega_{G}^{*R2} = \omega_{B}^{*R2} = \omega^{*R2}$. We will call this

[33]Since $\Omega'(W) < 0$, for envelope condition to hold, we must have λ^{R1}, λ^{R2}, $\lambda^{R3} > 0$, hence the target utility constraint always bind.

type, *Partially Fluctuating Wage* agreement. Lastly, if the employer's ICC (2.5) at bad state binds in the second, third and subsequent periods; in this equilibrium, the worker will receive fluctuating wage, $\omega_G^{*R3} > \omega_G^{*R2} > \omega_B^{*R2} = \omega_B^{*R3}$. We will call this type, *Fully Fluctuating Wage* agreement.

The key idea is that it is never optimal to promise more future utility than is required to satisfy the worker's ICC, (2.6a) and (2.6b), so after the second period of the relationship, the worker's target utility is always equal to $\widetilde{W}^*$ (Thus, in the first period we have $W_0 = V^{WS}$, in the second period $W_1 = \widetilde{W}^* - \frac{\mu(\omega^h) - \mu(\omega^u)}{\rho\beta}$, whereas in any subsequent period we have $W_2 = \widetilde{W}^*$). This means that after the second period, the optimal wage settings by the employer are stationary.

While solving the dynamic optimization problem (see the proof of Proposition 1), we've stated that the Kuhn-Tucker multiplier for (2.5) be denoted by ψ_ϵ, the multiplier for (2.6a) and (2.6b) by υ_ϵ, multiplier for (2.7) by λ, multiplier for (2.8) by χ_ϵ. The following Lemma provides an interpretation for the multiplier of employer's ICC (2.5),ψ_ϵ, which we will use extensively.

Lemma 1. The value of the Kuhn-Tucker multiplier on the employer's ICC at bad-state incentive, ψ_B^{R2} and ψ_B^{R3} satisfies:

$$\psi_B^{R2} = (1 - \pi_G)\left[\frac{\mu'\left(\omega_B^{R2}\right)}{\mu'\left(\omega_G^{R2}\right)} - 1\right] \tag{2.14}$$

$$\psi_B^{R3} = (1 - \pi_G)\left[\frac{\mu'\left(\omega_B^{R3}\right)}{\mu'\left(\omega_G^{R3}\right)} - 1\right] \tag{2.15}$$

Proof. See Appendix C, page 157.

We will use ψ_B^{R2} and ψ_B^{R3} as a measure of wage volatility. The following Lemma will enable us to rewrite the employer's ICC (2.5) in terms of the second period, ω_ϵ^{*R2}, and third and subsequent period wages, ω_ϵ^{*R3}.

Lemma 2. Values of Ω in the first, second, and third and subsequent periods are

$$\Omega\left(V^{WS}\right) = V^{ER} = \frac{\left(1-\beta+Q^E\rho\beta\right)}{\left(1-\beta\right)\left(1-\left(1-Q^E\right)\rho\beta\right)}X \tag{2.16}$$

$$\Omega\left(\widetilde{W}_\epsilon^{R1}\right) = V^{ER} - \left(\left(1-\rho\beta\right)E_\epsilon\omega_\epsilon^{*R2} + \rho\beta E_\epsilon\omega_\epsilon^{*R3}\right) + h \tag{2.17}$$

$$\Omega\left(\widetilde{W}_\epsilon^{R2}\right) = V^{ER} + \left(\rho\beta E_\epsilon\omega_\epsilon^{*R2} - \left(1+\rho\beta\right)E_\epsilon\omega_\epsilon^{*R3}\right) + h \tag{2.18}$$

$$X = \begin{bmatrix} E_\epsilon x_\epsilon - \rho\beta\left(\left(1-\rho\beta\right)E_\epsilon\omega_\epsilon^{*R2} + \rho\beta E_\epsilon\omega_\epsilon^{*R3}\right) \\ -u - \beta\left(1-\rho\right)\beta c - \left(1-\rho\beta\right)h \end{bmatrix} \tag{2.19}$$

Proof. See Appendix C, page 158.

Here, X can be interpreted as the employer's annuity. So, at the beginning of the relationship, the employer is indifferent between receiving X each period during the relationship and solving the maximization problem we have so far described. As can be seen from X in equation (2.19), what matters for the employer is the amount of the weighted average of second and subsequent period expected wages $\left(\left(1-\rho\beta\right)E_\epsilon\omega_\epsilon^{*R2} + \rho\beta E_\epsilon\omega_\epsilon^{*R3}\right)$ paid to the worker and the amount of average tax $\left(u+\beta\left(1-\rho\right)\beta c + \left(1-\rho\beta\right)h\right)$ paid to the fund.

We can rearrange the equation (2.2) and obtain

$$V^{ER} = \frac{\left(1-\beta+Q^E\rho\beta\right)}{Q^E\rho\beta}V^{ES} \tag{2.20}$$

Using equations (2.17), (2.18), (2.19), and (2.20), the employer's ICC (2.5) for the second and third and subsequent periods of the relationship can be written respectively as follows

$$(2.5a)\qquad (1-\rho\beta)\left(x_\epsilon-\omega_\epsilon^{R2}-u\right)+\rho\beta\left(E_\epsilon x_\epsilon-E_\epsilon\omega_\epsilon^{*R3}-u\right)\geq(1-\beta)\left(V^{ES}-\beta c\right)$$

$$(2.5b)\qquad (1-\rho\beta)\left(x_\epsilon-\omega_\epsilon^{R3}-u\right)+\rho\beta\left(E_\epsilon x_\epsilon-E_\epsilon\omega_\epsilon^{*R3}-u\right)\geq(1-\beta)\left(V^{ES}-\beta c\right)$$

We can express V^{ES} in terms of X by substituting V^{ER} from equation (2.16) into (2.20) and obtain

$$(2.21)\qquad V^{ES}=\frac{Q^E\rho\beta}{(1-\beta)\left(1-\left(1-Q^E\right)\rho\beta\right)}X$$

Since the target utility constraint (2.7) always binds, second and third and subsequent period target utility constraints can be rewritten by substituting equations (2.12), (2.6a), and (2.6b), into the constraint given in (2.7), and we obtain target utility constraints for second period and third and subsequent periods as:

$$(2.7a)\qquad \sum_{\epsilon=G,B}\pi_\epsilon\mu\left(\omega_\epsilon^{R2}\right)\geq\mu(0)+\frac{k}{\rho\beta}+\beta\frac{\mu(\omega^u)-\mu(0)}{\rho\beta}-\frac{\mu\left(\omega^h\right)-\mu(\omega^u)}{\rho\beta}$$

$$(2.7b)\qquad \sum_{\epsilon=G,B}\pi_\epsilon\mu\left(\omega_\epsilon^{R3}\right)\geq\mu(0)+\frac{k}{\rho\beta}+\beta\frac{\mu(\omega^u)-\mu(0)}{\rho\beta}$$

To summarise, the employer maximizes in each period equation (2.4), subject to employer's ICCs ((2.5a) and (2.5b)), worker's ICCs ((2.6a) and (2.6b)), target utility constraints ((2.7a) and (2.7b)), and (2.8). We will analyse three types of agreements in turn.

2.4.1. Wage Smoothing agreement

In any type of equilibria (either Wage Smoothing or Wage Fluctuating), the worker's ICCs ((2.6a) and (2.6b)) and the target utility constraints ((2.7a) and (2.7b)) always bind (see the proof of Proposition 1 in the Appendix). Moreover, under a Wage Smoothing agreement, the employer's ICCs ((2.5a) and (2.5b)) are slack in both states, therefore paying a constant wage in both states is feasible. We denote the constant wage paid under the Wage Smoothing case with $\omega_G^{R2} = \omega_B^{R2} = \omega^{*R2}$ and $\omega_G^{R3} = \omega_B^{R3} = \omega^{*R3}$. We will name both of these as the 'efficiency wage.' It represents the lowest constant wages that can be paid in a given period to the worker in a self-enforcing agreement. Given that employers are risk-neutral whereas workers are risk averse, employers always prefer Wage Smoothing, since it delivers the lowest expected wage payment to workers. The employer's ICCs ((2.5a) and (2.5b)) will be in strict inequality and also $\omega^{*R2} < \omega^{*R3}$ due to target utility constraints ((2.7a) and (2.7b)). However, Wage Smoothing is not always possible since the employer's ICCs ((2.5a) or (2.5b)) may bind. Next, we turn to those fluctuating wage equilibria.

2.4.2. Partially and Fully Wage Fluctuating agreement

To reiterate, in any type of equilibria, the worker's ICCs ((2.6a) and (2.6b)) and the target utility constraints ((2.7a) and (2.7b)) always bind. However, under *Partially Wage Fluctuating* agreement, only the third period employer's ICCs (2.5b) in the bad state binds, implying that in the bad state in the third period, the employer cannot afford to pay the same high wage she pays in the good state. We denote the wage paid under *Partially Wage Fluctuating* case by $\omega_G^{R3} > \omega_B^{R3} > \omega_G^{R2} = \omega_B^{R2} = \omega^{*R2}$. Similarly, under *Fully Wage Fluctuating* agreement, the second and subsequent period employer's ICCs ((2.5a) and (2.5b)) in the bad state bind. We denote the wage paid under *Fully Wage Fluctuating* case by $\omega_G^{R3} > \omega_G^{R2} > \omega_B^{R2} = \omega_B^{R3}$.

The following Lemma provides an upper bound for ψ_B^{R3}, which will be helpful when we perform comparative static analysis.

Lemma 3. Upper bound for ψ_B^{R3} when we have partially and fully Wage Fluctuating agreement, respectively, is

$$\text{(2.22)}\quad \psi_B^{R3} < \left[\begin{array}{l} \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}, \text{ while in partially Wage Fluctuating agreement} \\ \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta}, \text{ while in fully Wage Fluctuating agreement} \end{array}\right.$$

Proof. See Appendix C, page 160.

Lemma 4. The first period employer's ICC will not bind iff:

$$\begin{array}{rcl} \pi_G\left(1-Q^E\right)\rho\beta x_G & & \rho\beta\left(\left(1-\rho\beta\right)E_\epsilon\omega_\epsilon^{*R2}+\rho\beta E_\epsilon\omega_\epsilon^{*R3}\right) \\ & > & \\ +\left(1-\pi_G\left(1-Q^E\right)\rho\beta\right)x_B & & +\left(u+\beta\left(1-\rho\right)\beta c+\left(1-\rho\beta\right)h\right) \end{array}$$

Proof. See Appendix C, page 163.

In what follows we will perform comparative static on wages and the welfare of workers and employers by changing taxes and transfer payments.

2.5. Comparative Statics Analysis

2.5.1. Effect of a Change in Financing

In this part we focus our attention to the financing method, so we assume there is no change in the spending pattern: $d\omega^u = d\omega^h = 0$. Then, the total derivative of accounting identity (2.1) becomes

$$dF + \frac{\rho}{1-\rho}\Phi du + \rho\Phi dc + \rho\Phi dh = 0$$

In the following proposition, we summarize how payroll tax, firing tax, and hiring tax have an effect on wage volatility, expected wages, and employers' and workers' welfare in different agreement types.

Proposition 2.

Volatility: Under Wage Smoothing agreement, there will be no change in volatility. Under partially/fully Wage Fluctuating agreements, an increase in payroll tax *will* increase the wage volatility. On the other hand, an increase in firing tax or hiring tax *will* decrease the wage volatility.

Wages: Under Wage Smoothing agreement, increase in payroll tax, firing tax, and hiring tax will not change the wage employer pays to the worker in the second and subsequent periods. In contrast, under partially/fully Wage Fluctuating agreements, an increase in payroll tax *will* increase the weighted average of expected wages, $(1-\rho\beta)\, E_\epsilon \omega_\epsilon^{*R2} +$

$\rho\beta E_\epsilon \omega_\epsilon^{*R3}$. On the other hand, an increase in firing tax or hiring tax *will* decrease the weighted average of expected wages.

Welfare: Worker's welfare does not change with respect to any type of tax in any type of agreement.[34] Under Wage Smoothing agreement, an increase in payroll tax, firing tax, and hiring tax will be absorbed by the employer, who will thus be worse off. Under partially/fully Wage Fluctuating agreements, an increase in payroll tax *will* decrease the employer's welfare. On the other hand, an increase in firing tax or hiring tax *may* increase the employer's welfare if volatility is high enough.

Proof. See Appendix C, page 163.

The main idea behind Proposition 2 is quite simple. As can be seen from X in equation (2.19), any increase in any type of tax definitely has a *direct* tax effect which will be negative on the welfare of the employer regardless of whether she is in a relationship (2.16) or has a vacant position (2.21). If an increase in taxes decreases the weighted average of expected wages, $((1-\rho\beta)\, E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3})$, it will have an *indirect* tax effect (or volatility effect due to an increase in tax) which will be positive on the employer's welfare. If that impact is strong enough, then an increase in tax may have a *final* positive effect on the welfare of the employer. This *indirect* tax effect decreases the weighted average of expected wages only when wage volatility decreases. We also know that wage volatility decreases when a particular type of tax employed relaxes the employer's ICCs ((2.5a) and (2.5b)).

[34]When workers have some bargaining power, workers' wage will decrease in a Wage Smoothing agreement, as payroll tax, firing tax, and hiring tax increase. In that case, workers will be worse off, too.

The worker's welfare depends on the outside option, and since the worker's pay-off does not change when taxes change, the right-hand side of target utility constraints ((2.7a) and (2.7b)), will remain the same and workers will be indifferent. In a Wage Smoothing agreement, constant expected utility will translate into constant wages; the *indirect* tax effect will be zero, and employers will be worse off. In a partially/fully Wage Fluctuating agreement, the *indirect* tax effect works through the employer's ICCs ((2.5a) and (2.5b)). If a tax relaxes those constraints, the employer will able to give smoother wages and given that worker will be indifferent to tax changes, the weighted average of expected wages will decrease. The employer's ICCs ((2.5a) and (2.5b)) may relax if the change in weighted average of expected net output in the bad state (left-hand side, $(1-\rho\beta)(x_\epsilon - u) + \rho\beta(E_\epsilon x_\epsilon - u)$) outweighs employer's opportunity pay-off of shirking (right-hand side, $V^{ES} - \beta c$). Now we will assess the impact of payroll tax, firing tax, and hiring tax on wage volatility, expected wages, and the welfare of the employers.

In a Wage Smoothing agreement, the employer's ICCs ((2.5a) and (2.5b)) do not bind, so the employer will not change the wage payments. In a partially/fully Wage Fluctuating agreement, employers are worse off (compared to Wage Smoothing agreement) due to the increase in weighted average of expected wages. An increase in payroll tax, u, will have the same effect as a decrease in output. Since the employers are left with less output after taxes while keeping the output spread, $x_G - x_B$, constant, it decreases the net benefit of staying in the relationship. So, the left-hand side of the employer's ICCs ((2.5a) and (2.5b)) will decrease more than the right-hand side and as a result the employer has to offer lower wages in bad times. So, this *indirect* tax effect will increase the wage volatility and the weighted average of expected wages.

Unlike the increase in payroll tax, increases in firing tax, c, or hiring tax, h, decrease right-hand side of the employer's ICCs ((2.5a) and (2.5b)) by making employer's shirking less attractive. Since no firing tax or hiring tax is paid in the second and subsequent periods of the relationship (no change in left-hand side), employers will able to pay a less volatile (and lower expected) wage and keep workers indifferent. If this *indirect* tax effect is strong enough, decrease in the weighted average of expected wages may outweigh the *direct* tax effect and have a positive *final* tax effect on the welfare of employers, V^{ER}. This positive *indirect* tax effect is strongest when volatility is high, so giving more room for a decrease in the weighted average of expected wages.[35]

An interesting question would be how to rank taxes in terms of the welfare effects they create. Both the *direct* and *indirect* tax effects play a role in ranking the welfare effects of taxes. First, frequency and timing of the taxes determines the magnitude of the *direct* tax effect: While payroll tax is paid every period of the relationship and the employer pays less tax per period for each additional spending (for each unit increase in spending, payroll tax , u, increases by $\frac{1-\rho}{\rho\Phi}$), hiring tax is paid in the first period of the relationship, and firing tax is paid after the relationship dissolves (for each unit increase in spending, hiring tax or firing tax increase by $\frac{1}{\rho\Phi}$).[36] Both firing tax and hiring tax are paid once. Because of this frequency and timing of the taxes, hiring tax will be discounted the least and firing tax will be discounted the most. Thus, the increase in hiring tax will decrease the employer's welfare the most and the increase in firing tax will decrease the employer's welfare the least. Second, the impact of wages on wage volatility determines

[35]Although in our model matching and separation rates are exogenous, we recognize that similar to firing tax, hiring tax may also decrease the rate of hiring. Nevertheless, we conjecture that the marginal benefit of such hiring tax may outweigh its marginal cost, at least for low levels of hiring tax.
[36]See the accounting identity (2.1).

the magnitude of the *indirect* tax effect: While payroll tax increases the wage volatility and the weighted average of expected wages, hiring tax and firing tax decrease the wage volatility and the weighted average of expected wages with the the *indirect* tax effect being strongest with a firing tax.

When we have Wage Smoothing agreement, (only *direct* tax effect exists) firing tax dominates the other two tax types and hiring tax is dominated by all with respect to Welfare gain. When we have partially/fully Wage agreement, firing tax will keep dominating the other two tax types. However, if third period wage volatility is high enough ($\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$) hiring tax may dominate payroll tax.

2.5.2. Effect of a Change in Spending

In this part we focus our attention to the spending method, so we assume there is no change in the financing pattern: $du = dh = dc = 0$. Then, the total derivative of the accounting identity (2.1) becomes

$$dF = (n - \rho\Phi(n,m))\, d\omega^u + \rho\Phi(n,m)\, d\omega^h$$

Here we will summarize how unemployment payment and hiring payment have an effect on wage volatility, expected wages, and the welfare of parties.

Proposition 3.

Volatility: Under Wage Smoothing agreement, there will be no change in wage volatility when transfer payments change. Under partially/fully Wage Fluctuating agreements, an increase in unemployment payment *will* increase the wage volatility. Increase in hiring payment *may* decrease the second period wage volatility (see footnote 38) and *will* increase the third period wage volatility.

Wages: Under Wage Smoothing agreement, the weighted average of expected wages *will* increases when unemployment payment increases and *will* decrease when hiring payment increases. Under partially/fully Wage Fluctuating agreements, an increase in unemployment payment *will* increase the weighted average of expected wages. Increase in hiring payment *may* decrease the weighted average of expected wages if the third period wage volatility is below a certain level, $\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$.

Welfare: An increase in unemployment payment *will* make employers worse off and workers better off in any type of agreement. Under Wage Smoothing agreement, an increase in hiring payment *will* make employers better off and workers indifferent. In a partially/fully Wage Fluctuating agreements, an increase in hiring payment *may* make employers better off if the third period wage volatility is below a certain level, $\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$. Hiring payment *will* leave the workers indifferent.

Proof. See Appendix C, page 163.

Just like the impact of tax, payments have a direct and indirect effect on employers' welfare. We interpret the direct effect as the effect of workers' welfare and indirect effect as the effect of resulting wage volatility.

When unemployment payment to the worker increases, since worker's opportunity pay-off increases, the worker demands higher wages during their employment. Thus, increase in the weighted average of expected wages will make the worker better off in all types of agreement. The impact of unemployment payment to the employer will be from two sources: First, due to higher wages demanded by the worker as a result of the *direct* payment effect, the employer will be worse off. Second, depending on the agreement type, the employer may pay an additional amount due to the *indirect* payment effect.

When we have Wage Smoothing agreement, the employer will face only the *direct* payment effect, which is an increase in the weighted average of expected wages due to higher opportunity pay-off. When we have partially/fully Wage Fluctuating agreements, the weighted average of expected wages will increase more due to the tightening of the employer's ICCs ((2.5a) and (2.5b)), so that the employer will be even more worse off. To see exactly how it happens, we need to look at the employer's ICCs ((2.5a) and (2.5b)), to find out the impact of wage volatility on the weighted average of expected wages. On the one hand, it makes shirking by the employer less attractive and decreases the term in right-hand side of the employer's ICCs ((2.5a) and (2.5b)), $V^{ES} - \beta c$; on the other hand, higher wages that are going to be paid while in the relationship ($(1-\rho\beta)\,\omega_B^{R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}$ in the second period and $(1-\rho\beta)\,\omega_B^{R3} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}$ in the third and subsequent periods) makes staying in the relationship less attractive and decreases the left-hand side of the employer's ICCs ((2.5a) and (2.5b)). So, the left-hand side of the employer's ICCs ((2.5a) and (2.5b)) will decrease more than the right-hand side and as a result the employer has to offer lower wages in bad times and higher wages in good times. So, this *indirect* payment effect will increase the wage volatility and the weighted average of expected wages.

When hiring payment to the worker increases, since the worker's opportunity pay-off stays the same, the worker will be indifferent. However, an increase in hiring payment will enable the employer to implement inter-temporal substitution on the worker's wage. Knowing that the worker will get a higher income (wages and transfers) in the first period, the employer can decrease the second period expected wage payment while keeping the worker's welfare constant. This has three consequences. First, the worker's inter-temporal income (wage plus transfers) will be smoother. Second, the employer will pay less in the second period and thus will have the potential to be better off. Third, the second period wage volatility may decrease because of the relaxation of the second period employer's ICC (2.5a).

In particular, when we have a Wage Smoothing agreement, an increase in hiring payment decreases the weighted average of expected wages[37] and thus, will make the employer better off due to the *direct* payment effect. When we have partially/fully Wage Fluctuating agreements, the *indirect* payment effect will work towards increasing the weighted average of expected wages. [38] The welfare of the employer will increase (the *direct* payment effect will dominate) as long as the third period wage volatility is low enough, $\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$.

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[37]The second period expected utility will decrease while the expected utility in later periods will stay the same.

[38]The *indirect* payment effect is the weighted average of the second period *indirect* payment effect and the third period *indirect* payment effect. Below we show the necessary and sufficient condition for the decreasing second period wage volatility with respect to ω^h. The third period wage volatility will increase at all times. Change in wage volatility (ψ_B) with respect to r, $\frac{d\psi_B}{dr}$, can be written as

$$\frac{d\psi_B}{dr} = \left[\psi_B + (1-\pi_G)\right] A(\omega_G) \left(-\frac{d\omega_B}{dr}\right) \left(\frac{\left(\frac{d\omega_G}{dr}\right) + \left(-\frac{d\omega_B}{dr}\right)}{\left(-\frac{d\omega_B}{dr}\right)} + \frac{A(\omega_B) - A(\omega_G)}{A(\omega_G)} \right)$$

where, $A(\omega_\epsilon) = -\frac{\mu''(\omega_\epsilon)}{\mu'(\omega_\epsilon)}$, $\epsilon = G, B$ is the absolute risk aversion parameter. Given that we have fully Wage Fluctuating agreement and $r = \omega^h$, the necessary and sufficient condition for a decreasing second period wage volatility (ψ_B^{R2}) will be $\frac{\left(\frac{d\omega_G^{R2}}{d\omega^h}\right) + \left(-\frac{d\omega_B^{R2}}{d\omega^h}\right)}{\left(-\frac{d\omega_B^{R2}}{d\omega^h}\right)} < -\frac{A(\omega_B^{R2}) - A(\omega_G^{R2})}{A(\omega_G^{R2})}$. After we plug the values for $\frac{d\omega_G^{R2}}{d\omega^h}$ and $\frac{d\omega_B^{R2}}{d\omega^h}$, we obtain the necessary and sufficient condition for decreasing ψ_B^{R2} with respect to ω^h as

$$\psi_B^{R3} < \frac{(1-\rho\beta)}{\rho\beta} - \frac{\pi_G Q^E \rho\beta}{1 + Q^E \rho\beta} \frac{A(\omega_B^{R2}) - A(\omega_G^{R2})}{A(\omega_G^{R2})}.$$

2.5.3. Joint Design of Labour Market Policies

In this section, we consider the joint design of policies, in which the combined effect of policies in both sides of the fund (financing and spending) are taken into account. We analyse two scenarios: (i) Financing unemployment payment with payroll tax, (ii) Financing hiring payment with hiring tax.

2.5.3.1. Financing unemployment payment with payroll tax. To analyse this scenario, we assume $dF = dc = dh = d\omega^h = 0$. Then, (2.1) becomes

$$\frac{\rho}{1-\rho}\Phi\left(n,m\right)du = \left(n - \rho\Phi\left(n,m\right)\right)d\omega^u$$

The following proposition summarizes how an increase in unemployment payment financed by an increase in payroll tax affects the welfare of parties evaluated at the beginning of a productive relationship (positive output).

Proposition 4. An increase in unemployment payment ($d\omega^u > 0$) that is financed by payroll tax ($du > 0$) *will* increase the welfare of workers and decrease the welfare of employers.

Proof. See Appendix C, page 189.

In any type of agreement (Wage Smoothing, partially/fully Wage Fluctuating), an increase in unemployment insurance will increase the worker's welfare (due to an increase in unemployment payment) and will decrease the employer's welfare (due to both payroll tax and unemployment payment). In partially/fully Wage Fluctuating agreements, there will also be an efficiency loss due to increased wage volatility (due to both payroll tax and unemployment payment), while in Wage Smoothing agreement no such loss exists.

2.5.3.2. Financing hiring payment with hiring tax. For this scenario, we assume $dF = dc = du = d\omega^u = 0$. Then, (2.1) becomes

$$\rho\Phi(n,m)\,dh = \rho\Phi(n,m)\,d\omega^h.$$

The following proposition summarizes how an increase in hiring payment financed by an increase in hiring tax affects the welfare of parties evaluated at the beginning of a productive relationship (V^{ER} and V^{WR}).

Proposition 5. An increase in hiring payment ($d\omega^h > 0$) that is financed by hiring tax ($dh > 0$) does not affect the welfare of workers. For employers, there is a critical value of ω^h, ω_G^{R2}, such that if $\omega^h < \omega_G^{R2}$ ($\omega^h > \omega_G^{R2}$), it increases the welfare of employers if wage volatility is low (high) enough, i.e., $\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$ ($\psi_B^{R3} > \frac{1-\rho\beta}{\rho\beta}$).
Proof. See Appendix (page 189).

We know from Propositions 2 and 3 that neither an increase in hiring tax nor an increase in hiring payment has any effect on workers. The effect on employers is more subtle. Four counter effects compete in determining the employer's welfare. On the one hand, an increase in hiring tax paid by the employer (the *direct* tax effect) decreases the employer's welfare. On the other hand, higher hiring payment paid to the worker (the *direct* payment effect) decreases the second period wages and thus increases the employer's welfare. Hiring tax and hiring payment has an *indirect* (tax and payment) effect which arises due to volatility. While the *indirect* tax effect increases the employer's welfare, the *indirect* payment effect decreases it. Which effect dominates depends on the relative size of the initial hiring payment and the resulting change in wage volatility.

If the hiring payment is not too high ($\omega^h < \omega_G^{R2}$), in a Wage Smoothing agreement, (since there is only the *direct* (tax or payment) effect), an increase in hiring payment will decrease the second period wages considerably. The *direct* payment effect will dominate the *direct* tax effect and will increase the employer's welfare. In partially/fully Wage Fluctuating agreements, as long as the third period wage volatility is below a certain level, $\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$, the *direct* payment effect will keep dominating and the employer will be better off.

If hiring payment is high enough ($\omega^h > \omega_G^{R2}$), the *direct* tax effect will start to dominate; in a Wage Smoothing agreement, the employer will be worse off,[39] and only when wage volatility is above a certain level, $\psi_B^{R3} > \frac{1-\rho\beta}{\rho\beta}$, the employer will be better off.

2.5.3.3. When hiring payment with hiring tax acts as a minimum wage. Suppose we have a minimum wage ($\omega > \omega^{MW}$) policy where ω^{MW} is the minimum wage the employer has to pay to the worker. If the minimum wage has an upper bound ($\omega^{MW} < \omega_B^{R2}$), then in the first period of the relationship, minimum wage constraint ($\omega > \omega^{MW}$) will bind and in the later periods minimum wage constraint will not bind. Then, as long as $\omega^h < \omega_B^{R2}$, the joint policy of financing hiring payment with hiring tax will function as the way minimum wage policy does. This also implies, as long as minimum wage is below a critical value, minimum wage legislation has the potential to increase the

[39]Assumption 1, $\mu\left(\omega^h\right) \leq \mu\left(\omega^u\right) + k$, should still hold.

employer's welfare if wage volatility is low enough (i.e., $\psi_B^{R3} < \frac{1-\rho\beta}{\rho\beta}$). This inter-temporal substitution of wages will not the change the worker's welfare, but provided that the wage volatility is low enough, the employer can benefit from such a policy. The inter-temporal substitution of wages can be an alternative explanation for a phenomenon noted in introduction, namely the lack of evidence for differences in efficiencies across countries with different minimum wages and small employment effects of minimum wages.

2.6. Conclusion

We have shown that the worker's welfare is not affected under any type of financing, while the type of financing affects the employer deeply. An increase in payroll tax will increase the wage volatility, thus weakening the employer/worker relationship. An increase in firing tax will decrease the wage volatility and may make the employer better off. Hiring tax has the same effect albeit a smaller one.

On the other hand, the type of spending has distributional effects across employers and workers. An increase in unemployment payment increases the worker's welfare at the expense of an increase in the wage volatility. An increase in hiring payment will not have any effect on the worker's welfare because of perfect compensation of higher first period expected utility by the lower expected second period utility, thus enabling a consumption smoothing within the employment relationship. If contracts were complete, hiring payment would be neutral.[40] In our model, contracts are not complete, but nevertheless still hiring payment made in the first period were compensated with the lower expected wages in the second period so that the first period value function of the worker has not changed.

The employer will be worse off as unemployment payment increases, and may be better off as hiring payment increases if wage volatility is low enough. The joint policy, financing hiring payment with hiring tax, may function as a minimum wage policy and increases the welfare of the employer if hiring payment and wage volatility is low enough.

As a conclusion, firing tax strictly dominates payroll tax and hiring tax in terms of efficiency gains. Depending on the volatility, hiring tax may be preferable to payroll tax. While unemployment payment redistributes income between unemployed and employed

[40] See Lazear (1990).

workers; hiring payment redistributes income across the initial phase and the later phase of the employment. Thus, even though unemployment payment increases the worker's welfare it does so at the expense of higher volatility and lower employer's welfare. Hiring payment leaves the worker indifferent and may increase the employer's welfare.

CHAPTER 3

Factor Market Integration, Unemployment, and Earnings Instability under Invisible Handshake with Heterogeneous Workers

3.1. Introduction

> *"...the structural evolution of the global economy today and its effects on the US economy mean that, for the first time, growth and employment in the US are starting to diverge."* [excerpt taken from the article written by Michael Spence in Foreign Affairs, on July/August 2011].

We analyse the effect of increased factor market integration, in the form of cost reducing off-shoring and immigration, on earnings instability. In particular, we adapt an implicit contracting model by embedding it into a dynamic general equilibrium model to study the resulting frictional unemployment, long-term unemployment, and how earnings instability arise from the increases in unemployment. The model results, based on differences in factor endowments across countries, suggest that this factor market integration that can be viewed as one manifestation of globalisation can indeed have negative effects on unemployment and consequently earnings instability.

We conceptualize factor market integration, arising either via the workers' decision on where to work or the firms decision on where to invest. Thus low barriers on immigration and investment will both work towards the integration of factor markets. While the movement of workers may be formally due to the deregulation of worker movements (as in UK with East European workers), the regulation of intake of workers (as in Germany with Turkish workers), the influx of refugees (as in Europe recently with Syrian workers), or unauthorized immigrants (as in US with illegal Mexican of the workers), the end result is the same; foreign workers legally or illegally seek employment in countries other than where they reside to improve their welfare. Similarly, employers may choose to (partially or completely) relocate their operations in a foreign country (i.e., off-shoring) if they find it advantageous to do so.

Just as increased goods market integration (i.e., trade) did before the 1990s, during the last two decades factor market integration is evoking suspicion and insecurity among general public in developed countries. Even if it can be demonstrated that factor market integration will increase the total value of production and that countries involved in those developments will be better off overall, there is not much consensus on subgroups that may be adversely affected during the transition and afterwards once market conditions stabilize (See Bhagwati and Blinder, 2009 for a debate in the context of services off-shoring). As a result of public ambivalence towards globalisation, an increasingly dominant characteristic of political debate since the new millennium – in the US and other developed countries – is anxiety over factor market integration. The common resentment that is the fuel of those campaigns against globalisation is the expectation of severe deterioration in the economic fortunes of the home country workers. Consistent with high unemployment rates could be seen as primarily a European problem from the 1970s until the recession in late 2007. Katz (2010) implies that the recession from late 2007 ended this dichotomy and merely exposed the deferred impact of globalisation[1] on unemployment in the US.[2] In fact, Daly *et al.* (2012)'s estimate that the natural rate of unemployment in the US has risen from a pre-recession level of 5 percent to 6 percent and that long-term unemployment has almost tripled since the 1990s is consistent with the jobless recovery argument of Katz. In particular, on both side of the Atlantic Ocean, incidence of long-term unemployment[3] has been increasing; in the US from 6.3 percent in 1991 to 18.7 percent in 2015 percent, and in the EU from 43.4 percent in 1991 to 48.3 percent in 2015.

Immigration and off-shoring are widely viewed as the "usual suspects" for the worsening of labour market conditions. During the 2004 US election campaign, there were calls to tighten visa restrictions on foreign software engineers to slow down the off-shoring of Information and Communication Technologies (ICT) jobs.[4] Trade statistics suggest off-shoring is not limited to the ICT industry and it is an inevitable trend in the whole economy. For example, the index of off-shoring of goods and services production[5] rose from 7% to 10% in the US and 12% to 16% in OECD member countries between 1995 and 2005 (OECD, 2010).

Recently, relevant political discourse has mostly focused on immigration. During the 2016 US presidential election, the GOP nominee consistently stated that he wants to build a wall between the US and Mexico to deter illegal immigrants (BallotPedia, 2016).

[1]Other plausible explanation for the long-term trend in labour market is the rapid skill-biased technological change.

[2]Katz (2010) lists other long-term trends that were masked during the boom period as longer-term job polarization and rising wage inequality.

[3]Incidence of long-term unemployment is defined as the number of long-term unemployed (12 months or more) as a percentage of total unemployment. For reference, see OECD (2016).

[4]Foreign companies use these visa programs (known as L-1 and H-1B) to bring employees into the US to train them for the jobs they will do before returning home. See Koch (2003) and Reuters (2004) for a detailed discussion and impact of already tightened restrictions.

[5]The index of off-shoring is based on the indicator proposed by Feenstra and Hanson (1996, 1999).

Also, migration from EU member countries has been a central issue in the UK referendum campaign, famously called BREXIT, over whether to stay in EU. The data also supports the increase in immigration. After the Immigration and Nationality Act of 1965[6] passed in the US, the number of immigrants entering the country did increase. But the more significant increase in the presence of foreign-born workers in the United States occurred during the 1990s and 2000s. For example, the number of people obtaining lawful permanent resident (LPR) status (those entering with permanent resident visas) annually was 0.178 million between 1925 to 1965, 0.479 million between 1966 to 1988, and 1,012 million between 1989 to 2014 (See Office of Immigration Statistics, 2016). The stock of LPRs was estimated in 2013 at 13.1 million, with an additional 8.8 million eligible to naturalize, and 63 percent of LPRs obtained that status in 2000 or later (See Baker and Rytina, 2013). An unintended consequence of the Immigration and Nationality Act of 1965 was the increase in unauthorized immigration to the US due to the termination of guest worker program with Mexico, also known as the Bracero Program. As of 2012, the unauthorized immigrant population residing in the US was estimated to be 11.4 million, 42 percent of whom entered in 2000 or later, and 59 percent of whom were from Mexico (See Baker and Rytina, 2014). Recent increases in immigration are not limited to the US. Between 1995 and 2015 in the UK, among working age adults the share of immigrants more than doubled – from 8.2 percent to 16.6 percent (from 3 million to 6.7 million).[7]

[6]The earlier Immigration Act of 1924 was enacted to preserve the ideal of American homogeneity. More specifically Africans and Asians was prohibited and Jews from the Russian Pale and Catholics from Poland and Italy were deemed unassimilable and accordingly subject to restrictive quotas on their immigration. The Immigration and Nationality Act of 1965 was intended to replace these discriminatory quotas with a new policy (See Massey and Pren, 2012).

[7]See Wadsworth (2015) and Wadsworth et al. (2016).

Another observed development accompanying factor market integration is the increase in earnings instability since 1970. Earnings instability is indicative of economic insecurity and risk and measures of transitory variation and volatility of income is used to summarise it. In the previous essays, wage volatility was the the main indicator of earnings instability. In this essay, we will use transitory earnings variation as measure of income risk. Typically, in a given period, income variation can be decomposed into 'permanent' and 'transitory' variance components. While permanent income variance is a subject of income inequality, transitory component fluctuates around the permanent income and thus it is related to earnings instability. Gottschalk and Moffitt (1994) find that increased transitory variation explains one-third of the expansion in overall cross-sectional earnings inequality in the 1980s. When transitory earnings variance is decomposed according to its source, i.e., wage variation versus employment variation,[8] it turns out that 40 percent of the increase in the transitory earnings variance is due to employment variation. Gottshalk and Moffitt (2009), in a follow-up research, note that the transitory earnings variance has risen through the late 1980s and has not fallen back since then.

There are numerous studies that investigate the impact of immigration and off-shoring on unemployment in isolation. For instance, Brucker (2011) studies the effect of immigration on unemployment in the US and Europe. He finds that unemployment increase by 0.3 percent when immigration increase by 1 percent of the population.[9] Crinò (2010) focuses on services off-shoring in the US and finds positive employment effects of off-shoring for

[8]See footnote 25 (Chapter 2, page 57) for a list of other causes of earnings instability. These causes can be grouped into transitory earnings variance due to wage variation and employment variation.
[9]Longhi *et al.* (2008)'s meta-analysis conclude that the immigration's impact is quantitatively small on home country workers and majority of the research find estimated coefficients are statistically insignificant.

relatively skilled home country workers while less skilled home country workers may have to be displaced as expected of Feenstra and Hanson (1996).[10]

In our view, off-shoring and immigration are closely related, so that studying them separately might potentially underestimate the impact of factor market integration. One study that investigates off-shoring and immigration as a joint phenomena is Ottaviano *et al.* (2013). In their model, immigrant workers and offshore workers compete with each other, and exhibit stronger substitutability between them than occurs between immigrant workers and home country workers. Their finding is based on the US data and supports credibility to our model, in which no distinction is made between immigrant workers and offshore workers, but rather developments in immigration and off-shoring since the 1990s are seen as comprising an overall integration of factor markets. Another study that analyses the impact of globalisation using tools from contract theory and international trade theory is Karabay and McLaren (2010). While their primary concern is to study the impact of globalisation on earnings instability due to wage rate variation, we focus our attention on earnings instability due to employment variation, in particular increasing earnings instability due to an increase in frictional unemployment and long-term unemployment.[11]

For this purpose, we have a stylized open-economy general equilibrium model in which implicit contracts are embedded. In our model, all agents are risk-neutral. There is one sector in which production requires unobservable effort by a worker and by an employer.

[10]Hummels *et al.* (2010) focus on Denmark and find similar results. Wright (2014) finds off-shoring increases productivity of home country firms, but decreases the employment in low-skill labour in home country.

[11]According to OECD (2016), “long-term unemployment refers to people who have been unemployed for 12 months or more.” In our stylized model, workers are deemed long-term unemployed if employers cannot form a productive relationship with them.

Workers suffer a disutility from an effort and they are heterogeneous in the disutility. Employers suffer no such disutility. Complete contracts are not feasible due to verification problem by third parties so employers are constrained by their incentive-compatibility constraints. Such a contract is enforceable by the threat of ending the relationship if the other party deviates from the implicit contract. In our model, firms promise workers wages that increase over time so that workers exert effort out of fear of losing high future wages. This "back-loaded" wage compensation in equilibrium enables firms to pay lower wages to new workers than the incumbent ones. Unemployed workers seeking jobs and employers with vacancies search until they have a match.[12]

There are two countries, which differ only in their ratios of workers to employers. Globalisation take the form of integration of factor markets, either by off-shoring or immigration. As a result of this integration, employers can choose to employ workers in either country, and workers can choose which of the countries they wish to work. By making it easier for a firm in a labour-scarce economy to hire workers, integration creates efficiencies in matching workers to employers, but also gives rise to an externality, in general equilibrium, in the form of adverse affects on home country workers, raising unemployment by increasing both the length of unemployment spells and long-term unemployment.

The model can shed light on a number of empirical findings. First, we can rationalize a rise in earnings instability[13] as a result of factor market integration, as documented by Gottschalk and Moffitt (2009). Second, as mentioned above, OECD data shows that US

[12]Matouschek *et al.* (2008) also consider a standard search model of the labour market, but with wage bargaining under private information. They show that a fall in labour market frictions will lead to an increase in unemployment. They establish their result on "recent economic and technological changes", but no further detail is given.

[13]Again, we will use transitory earnings variation measure as an indication of earnings instability.

and European workers saw a rise in incidence of long-term unemployment. Our model produces results that are consistent with that observation.

We present the formal model in the next section. In section 3.3 we characterize optimal wage contracts. In section 3.4, we show how the general equilibrium is changed by international integration of factor markets.

3.2. The Model

Production. There are two factors of production: A measure E of employers and a measure L of workers. A worker can be unemployed or can team up with an employer and form a partnership in order for production to occur. We will call this partnership a 'firm'. In each period, a worker and an employer must both put in one unit of non-contractible effort. Worker i suffers a disutility from effort equal to $k_i > 0$, where k_i is independent identically distributed with cumulative distribution function $G(k_i)$ and has support of $[0, \infty)$, while employers suffer no such disutility. Within a given employment relationship, denote the effort put in by agent j by $e^j \in \{0, 1\}$, where $j = W$ indicates the worker and $j = E$ indicates the employer. The output and revenue generated in that period is then equal to $xe^W e^E$, where the price of output is normalized to 1. Employers without a worker are 'with vacancy' and do not produce anything.

Preferences. There is no storage, saving or borrowing. An agent's income in a given period is equal to that agent's consumption in that period. All agents discount the future at a constant rate $\beta \in (0, 1)$.

All agents are risk-neutral. There is a finite lower bound, α, to workers' utility (or, equivalently, there is some exogenous source of consumption on which workers can rely even if they are unemployed) and workers' utility function is in the form of $\alpha + \omega$.

Search. Workers and employers, if they are seeking for a match, search until they have one. Search follows a specification of a type used extensively by Pissarides (2000). Let there be a measure n workers and a measure m employers looking for a match in a given period, then $H\left(\overline{k}\right) \Phi(n, m)$ matches occur. Here $\Phi(n, m)$ is the number of *blind* matches and $H\left(\overline{k}\right)$ is the probability that those *blind* matches are suitable for production with $\overline{k}$

representing the threshold disutility from effort. Namely, anyone with $k_i > \overline{k}$ will never stay in a productive relationship. Φ is a concave function increasing in all arguments, linearly homogeneous in its arguments, with an upper bound equal to $\min(n,m)$, and has equal mixed partials ($\frac{\partial^2 \Phi(n,m)}{\partial n \partial m} = \frac{\partial^2 \Phi(n,m)}{\partial m \partial n}$) $\forall n, m$. Later we will show that such a $\overline{k}$ exists and is unique for a given parameter space. Q^E is the steady-state probability that an employer will match with a worker with $k_i \leq \overline{k}$ in any given period, or in other words, $Q^E = \frac{H(\overline{k})\Phi(n,m)}{m}$, where n, m and $\overline{k}$ are set at their steady-state values. Similarly, $Q^W = \frac{H(\overline{k})\Phi(n,m)}{n}$ is the steady-state probability that a worker with $k_i \leq \overline{k}$ will find a job any given period.[14] Search has no direct cost, but has an indirect cost - parties can not exert effort for production while searching. There is also a possibility in each period that a worker and an employer who have been matched in that period or in the past will be exogenously separated from each other. This probability is given by a constant $(1-\rho) \in (0,1)$.

Timing of the game. The sequence of events within each period is as follows.

(1) Any readily matched employer and worker learn whether or not they will be exogenously separated in this period.[15]

(2) The wage, if any, is paid (a claim on the firm's output at the end of the period).

(3) The employer and worker simultaneously choose their effort levels $e^j \in \{0,1\}$, where $j = W$ indicates the worker and $j = E$ indicates the employer.. At the same time, the search mechanism operates. Within a firm, if $e^j = 0$, then agent j can participate in search and put no effort. Unemployed worker and employer with vacancy always search and do it without any cost.

[14]If worker suffer a disutility from effort equal to $k_i > \overline{k}$, $Q^W = 0$.
[15]In this model, there is no idiosyncratic output shock.

(4) Each firm's output/revenue, $xe^W e^E$ and profit is realized, and then consumption is realized.

(5) For those agents who have found a new potential partner in this period's search, new partnerships with a new self-enforcing agreement are formed. This is achieved by a take-it-or-leave-it offer made by the employer to the worker. The employer has the full bargaining power.

Total number of employers/workers in a relationship in period t is given

$$E - m_t = \rho \left(E - m_{t-1}\right) + \rho H \left(\overline{k}_{t-1}\right) \Phi \left(n_{t-1}, m_{t-1}\right)$$

At the steady state we must have:

$$E - m = \frac{\rho}{1-\rho} H \left(\overline{k}\right) \Phi \left(n, m\right)$$

Given that $Q^E = H \left(\overline{k}\right) \frac{\Phi(n,m)}{m}$, we can write the above equation as

$$\frac{E}{m} = 1 + \frac{\rho}{1-\rho} Q^E \tag{3.1}$$

Transition Equations. We will focus on steady-state equilibria. The superscripts 'E' and 'W' refer to employers and workers, while the superscript 'S' and 'R' indicate the state of searching and being in a cooperative relationship, respectively. The expected lifetime discounted profit of an employer with vacancy (thus searching) is denoted by V^{ES}. The expected lifetime discounted utility of a worker with effort cost equal to k_i without a job (thus searching) is denoted by $V_{k_i}^{WS}$. Similarly, let V^{ER} and $V_{k_i}^{WR}$ denote the *ex-ante* expected lifetime discounted profit of an employer and the expected lifetime utility of a worker evaluated at the *beginning of a cooperative relationship* (both agents put in effort), respectively.

We can write the transition equations as

$$V^{ES} = Q^E \rho\beta V^{ER} + Q^E (1-\rho)\beta V^{ES} + \left(1-Q^E\right)\beta V^{ES} \tag{3.2}$$

$$V_{k_i}^{WS} = \alpha + Q^W \rho\beta V_{k_i}^{WR} + Q^W (1-\rho)\beta V_{k_i}^{WS} + \left(1-Q^W\right)\beta V_{k_i}^{WS} \tag{3.3}$$

The expected lifetime discounted profit of an employer with vacancy, V^{ES}, is equal to the continuation pay-off of the employer if the employer finds a worker and is not immediately exogenously separated, finds a worker and is immediately exogenously separated, or fails to find a worker. The expected lifetime discounted utility of an unemployed worker, $V_{k_i}^{WS}$, is equal to the exogenous source of income, α, plus the continuation utility value of the worker if the worker finds work and is not immediately exogenously separated, finds work and is immediately exogenously separated, or fails to find work.

Although employer and worker will take V^{ab} as given, where, $a \in \{E, W\}$ and $b \in \{S, R\}$, they depend on the endogenous probability of matching (Q^W, Q^E). Given those values, a self-enforcing agreement between a worker and an employer is a subgame-perfect equilibrium of the game that they play together. Since the employer has the full bargaining power, the employer chooses a wage scheme that will give him/her the highest expected discounted lifetime pay-off subject to constraints. If either agent defects (putting no effort or not paying wages) from the agreement at any time, the existing agreement becomes void, and both agents must search for new partners immediately. This assumption is sufficient to ensure the credibility of promises.[16]

As stated before, all three essays share a common framework with some adaptations. Here, we will summarize the key differences between the first essay's and current essay's model. In this essay, there is no shock to the economy, there is one (non-risky) sector and that sector corresponds to the careers sector in first essay. In the first essay, workers are identical and they all suffer a disutility from effort equal to $k > 0$. In this essay, worker i suffers a disutility from effort equal to $k_i > 0$, where k_i is independent and identically distributed with cumulative distribution function $G(k_i)$. In the first essay, $\Phi(n, m)$ matches occur and in this essay, $H\left(\overline{k}\right)\Phi(n, m)$ matches occur. $\Phi(n, m)$ has a slightly different meaning than first essay. Here $\Phi(n, m)$ is the number of blind matches and $H\left(\overline{k}\right)$ is the probability that those blind matches are suitable for production with $\overline{k}$ representing the threshold disutility from effort. Also, there is no search effectiveness parameter in this essay, φ, as in the first essay. In this essay, since there is only one good produced in the economy and workers are risk-neutral, the utility function is much simpler and workers have the same per-period utility function. Workers utility function is in the form of $\alpha + \omega$ (which is equal to the consumption), where α is a finite lower bound to workers' utility and ω is the wage earned. In this essay, when workers' relationship dissolves, they are unemployed (in the first essay they change sectors), workers who are not in relationship always search (in the first essay workers search, too). Table 1 in the Appendix lists the distinguishing features of the model variants in each essay.

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[16] Thomas and Worall (1988) uses a stronger assumption. In their paper, a defecting party is banished from entering any relationship forever. See also MacLeod and Malcomson (1989) for a similar study with moral hazard problem. Inability to verify information that limits explicit contracts was the main motivation for implicit contracts that support incentive compatibility.

We will now characterize the optimal labour contracts below.

3.3. Optimal Contracts

The equilibrium can be characterized as the solution to a recursive optimization problem. $\Omega(W_{k_i})$ is the highest possible expected pay-off the employer (who is in a relationship with a worker with $k = k_i$) can receive, given the worker will receive W_{k_i}. Ω is decreasing, concave and a differentiable function as in Benvenista and Scheinkman (1979).

$$\Omega(W_{k_i}) = \max_{\{\omega_{k_i}, \widetilde{W}_{k_i}\}} x - \omega_{k_i} + \rho\beta\Omega(\widetilde{W}_{k_i}) + (1-\rho)\,\beta V^{ES} \tag{3.4}$$

subject to

$$x - \omega_{k_i} + \rho\beta\Omega(\widetilde{W}_{k_i}) + (1-\rho)\,\beta V^{ES} \geq V^{ES} \tag{3.5}$$

$$(\alpha + \omega_{k_i}) - k_i + \rho\beta\widetilde{W}_{k_i} + (1-\rho)\,\beta V_{k_i}^{WS} \geq V_{k_i}^{WS} - \alpha + (\alpha + \omega_{k_i}) \tag{3.6}$$

$$(\alpha + \omega_{k_i}) - k_i + \rho\beta\widetilde{W}_{k_i} + (1-\rho)\,\beta V_{k_i}^{WS} \geq W_{k_i} \tag{3.7}$$

$$\omega_{k_i} \geq 0 \tag{3.8}$$

$$\widetilde{W}_{k_i} \geq 0 \tag{3.9}$$

The employer's problem given in (3.4) is to choose current period wage and worker's continuation pay-off $(\omega_{k_i}, \widetilde{W}_{k_i})$ such that the employer's discounted lifetime profit is maximized given the worker's present discounted pay-off is equal to at least W_{k_i}. Constraint (3.5) is the employer's incentive compatibility constraint. In equilibrium, $(\omega_{k_i}, \widetilde{W}_{k_i})$ will be such that it will always be in employer's best interest to commit to a cooperative relationship. Constraint (3.6) is the worker's incentive compatibility constraint. We should emphasize that "α" is obtained whether the worker is employed or not. We will call constraint(3.7) the target utility constraint. The employer has a target present discounted pay-off intended for the worker which is equal to at least W_{k_i}. The last two constraints are the natural bounds on choice variables, namely, the current wage and the future utility stream to be promised to the worker.

At the beginning of the first period of an employment relationship, since the employer has all the bargaining power, she must promise at least as much of a pay-off to the worker as remaining in the search pool would provide. Thus, in that case, denoting the target utility at the beginning of the relationship by W_{0,k_i}, we have $W_{0,k_i} = V_{k_i}^{WS}$ (and so $V^{ER} = E_{k_i}\Omega(V_{k_i}^{WS})$). Thereafter, the employer will in general be bound by promises of pay-offs she made to the worker in the past. The following two lemmas will help us to simplify the problem.

Lemma 1. The target utility constraint always binds.

Proof. Let λ_{k_i} represent the multiplier for target utility constraint (3.7). From the envelope condition, we have $\lambda_{k_i} + \Omega'(W_{k_i}) = 0$. Since $\Omega'(W) < 0$, we must have $\lambda_{k_i} > 0$, which means target utility constraint always binds.

The next lemma will allow us to ignore constraint (3.9) in the employer's problem.

Lemma 2. $\widetilde{W}_{k_i} \geq \widetilde{W}^*_{k_i} > V^{WS}_{k_i}$, where $V^{WS}_{k_i} = \frac{\alpha}{1-\beta}$ and $\widetilde{W}^*_{k_i} \equiv \frac{\alpha}{1-\beta} + \frac{k_i}{\rho\beta}$.

Proof. We can write the worker's ICC (3.6) as[17]

$$\widetilde{W}_{k_i} \geq \frac{(1-(1-\rho)\beta)\, V^{WS}_{k_i} - \alpha}{\rho\beta} + \frac{k_i}{\rho\beta}. \tag{3.6$'$}$$

At the beginning of the first period of an employment relationship, since the employer has all the bargaining power, she must promise at least as much of a pay-off to the worker as remaining in the search pool would provide. In addition, Lemma 1 shows us that the employer does not promise a pay-off more than the worker's outside option. Thus, denoting the target utility at the beginning of the relationship by W_{0,k_i}, we must have $W_{0,k_i} \equiv V^{WR}_{k_i} = V^{WS}_{k_i}$ (and so $V^{ER} = E_{k_i}\Omega(V^{WS}_{k_i})$). Then, the equation (3.3) becomes

$$V^{WS}_{k_i} = \frac{\alpha}{1-\beta}. \tag{3.10}$$

So, the value of expected lifetime discounted utility of an unemployed worker, $V^{WS}_{k_i}$, does not depend on k_i. Hereafter, we will refer it as V^{WS}. If we plug (3.10) into the right-hand side of the inequality given in (3.6)$'$, we obtain $\widetilde{W}_{k_i} \geq \frac{\alpha}{1-\beta} + \frac{k_i}{\rho\beta}$. Let $\widetilde{W}^*_{k_i} \equiv \frac{\alpha}{1-\beta} + \frac{k_i}{\rho\beta}$. Then, we have

$$\widetilde{W}_{k_i} \geq \widetilde{W}^*_{k_i} > V^{WS} > 0$$

The value $\widetilde{W}^*_{k_i}$ represents the minimum future utility stream that can be promised to the worker in order to convince the worker to exert effort and forgo search. We can now

[17] See Lemma 1 in Chapter 1, page 24, for a similar proof.

replace the constraint (3.6) by the more convenient form

$$\widetilde{W}_{k_i} \geq \widetilde{W}^*_{k_i} \tag{3.6$''$}$$

where

$$\widetilde{W}^*_{k_i} \equiv \frac{\alpha}{1-\beta} + \frac{k_i}{\rho\beta}$$

We are now ready to describe the equilibrium.

Proposition 1. In the first period of an equilibrium employment relationship, the wage is set equal to zero and continuation pay-off for the worker with k_i is set equal to $\widetilde{W}^*_{k_i}$. In the second and subsequent periods of the employment relationship, regardless of history (provided neither partner has shirked), the wage is equal to $\omega^*_{k_i} = \frac{k_i}{\rho\beta}$.
Proof. See Appendix D, page 192.

The worker goes through an 'apprenticeship period' at the beginning of the relationship in which zero wage is paid. The key idea is that it is never optimal to promise more future utility than required to satisfy the worker's incentive constraint (3.6)$''$, so in the first period we have $W_{k_i} = V^{WS}$, whereas after the first period $W_{k_i} = \widetilde{W}^*_{k_i}$. This means that after the first period, the optimal wage setting by the employer is stationary.

Since the target utility constraint (3.7) always binds, by substituting equation (3.10) and constraint (3.6)$''$ into constraint (3.7), second and subsequent period target utility constraint can be written as,

$$\omega_{k_i} \geq \frac{k_i}{\rho\beta}. \tag{3.7$'$}$$

It is obvious that right-hand side of the above inequality is an increasing and linear function of k_i. During the vacancy period, the employer has an expectation of the level of wage she is going to pay to the worker and will base her decision on that expectation. The following Lemma gives the formula for it.

Lemma 3. There exists a parameter space for a unique $\overline{k}$ where for $k_i < \overline{k}$, employer/worker relationship is feasible and employer's ICC (3.5) does not bind, for $k_i = \overline{k}$, employer/worker relationship is feasible and employer's ICC (3.5) does bind, and for $k_i > \overline{k}$, relationship is not feasible. The expected wage the employer is going to pay is denoted by $\widetilde{\omega}$ and it is given by

$$\widetilde{\omega} = \frac{1}{G\left(\overline{k}\right)} \int_0^{\overline{k}} \omega_{k_i} g\left(k_i\right) dk_i.$$

Moreover, there is a critical value of $\frac{E}{L}$, $\left(\frac{E}{L}\right)^{crit} < 1$, such that for $\overline{k}$ to be unique, it is sufficient to have $\frac{E}{L} > \left(\frac{E}{L}\right)^{crit}$.

Proof. See Appendix D, page 194.

As stated in Proposition 1, when the employer matches with a worker with $k_i \leq \overline{k}$, the current wage that has to be paid after the first period is equal to $\frac{k_i}{\rho\beta}$, increasing with k_i. On the other hand, for the employer's ICC to be satisfied, the wage cannot exceed a certain level, which is given by

$$\omega_{k_i} \leq x - (1-\beta) V^{ES}. \qquad (3.5)'$$

Here V^{ES} is affected by two competing forces. On the one hand, an increase in $\overline{k}$ increases the probability of finding a suitable match, Q^E, and in return increases V^{ES}. On the other hand, an increase in $\overline{k}$ increases the expected wage, $\widetilde{\omega}$, that is going to be paid to workers. Higher $\widetilde{\omega}$ will leave less surplus to the employer, thus decreasing V^{ES}. Lemma 3 shows that it is sufficient to have $\frac{E}{L} > \left(\frac{E}{L}\right)^{crit}$ for V^{ES} to be increasing in $\overline{k}$, implying that the former effect dominates. Therefore, there exists a unique $\overline{k}$ where right-hand side of $(3.5)'$ intersects with $(3.7)'$ only once. When the employer matches with a worker with $k_i \leq \overline{k}$, relationship is feasible and with $k_i > \overline{k}$, relationship is not feasible. When the employer matches with a worker with $k_i = \overline{k}$, the employer's ICC $(3.5)'$ and the target utility constraint $(3.7)'$ does bind.[18]

In Figure 3.1 below, k is on the x-axis and ω is on the y-axis and we have the graphical representation of the target utility constraint $(3.7)'$ and the employer's ICC $(3.5)'$. We will use the derivation in (D.30, Appendix D, page 201) to establish the existence and uniqueness of $\overline{k}$, where $x - (1-\beta)\, V^{ES} < \frac{x}{1+Q^E\rho\beta}$. Here, $\frac{x}{1+Q^E\rho\beta}$ provides an upper-bound and is decreasing in $\overline{k}$. Then, $\omega_{k_i} = \frac{k_i}{\rho\beta}$ and $\omega_{\overline{k}} = \frac{x}{1+Q^E\rho\beta}$ will intersect only once for $k_i = \overline{k}$. Thus, such a $\overline{k}$ exists. Furthermore, if $\frac{E}{L} > \left(\frac{E}{L}\right)^{crit}$ (by Lemma 3), then right-hand side of the employer's ICC $(3.5)'$, $x - (1-\beta)\, V^{ES}$, will be a negatively sloped and then there will be a unique $\overline{k}$.

[18]Another sufficient condition for the uniqueness of $\overline{k}$ would be to focus on the range of $\frac{E}{L}$ that satisfies the condition that slope of the employer's ICC $(3.5)'$ is less than the slope of the target utility constraint.

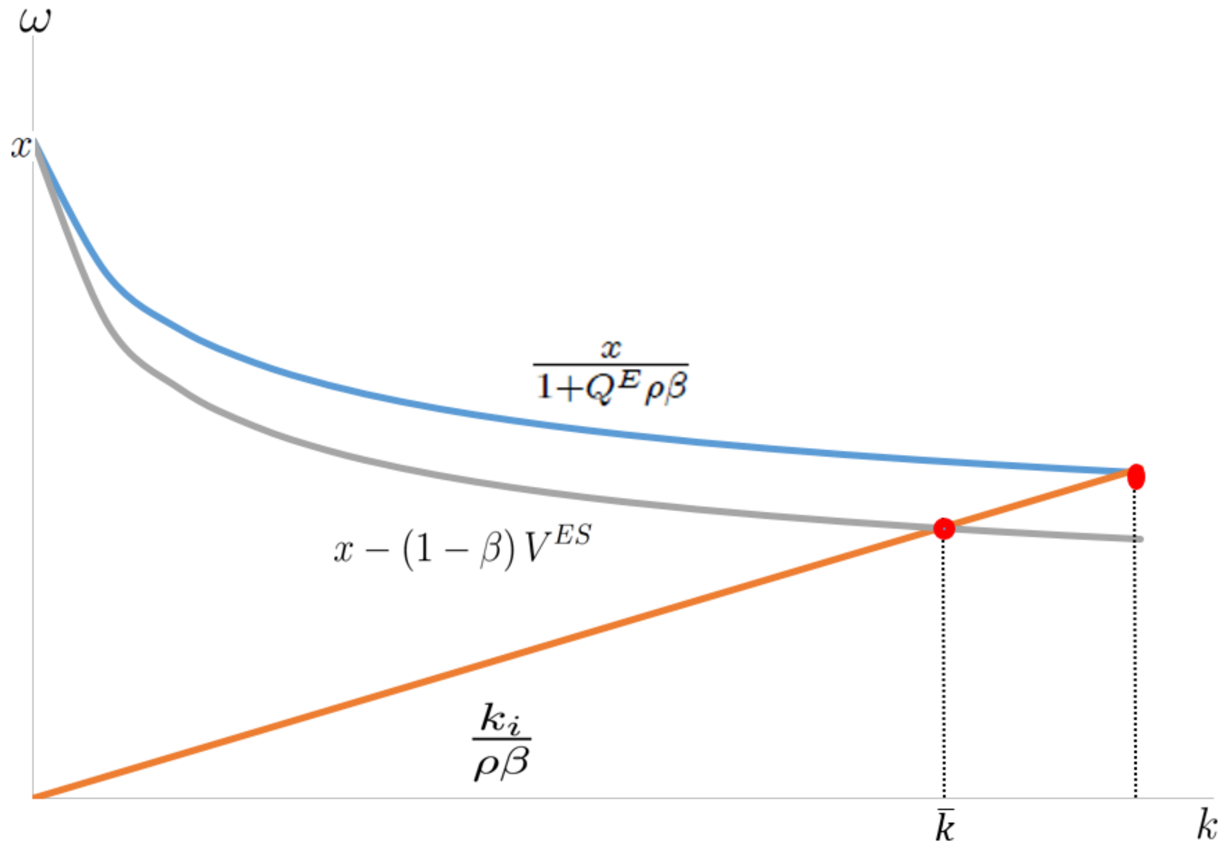

Figure 3.1. Endogenous Threshold Effort Level

3.4. Comparative Statics Analysis

The following lemma provides answer for the change in equilibrium threshold k when a exogenous change occurs in relative size of employers in a closed economy.

Lemma 4. An increase in $\frac{E}{L}$ increases the equilibrium $\overline{k}$.

Proof. See Appendix D, page 208.

When employers become more abundant relative to workers, it is harder for employers to find a worker, thus to counterbalance this, they are now willing to stay in the relationship with workers that have higher k than before. As a result, there will be more successful matches resulting production, the unemployment rate of the workers, $\frac{n}{L}$, will decrease and it will drive up the equilibrium $\overline{k}$. In other words, increase in $\frac{E}{L}$ will shift up the right-hand side of the employer's ICC $(3.5)'$ by decreasing V^{ES} represented in Figure 3.1.

3.4.1. The Effect of Factor Market Integration

Given countries have the same distribution of k; before off-shoring, the country that has a higher $\frac{E}{L}$ ratio will have a lower unemployment rate due to a higher probability of finding a suitable match and a higher equilibrium $\overline{k}$ (by Lemma 4). The following proposition state the impact of factor market integration on each country.

Proposition 2. When factor market integration is possible for two countries with different $\frac{E}{L}$ ratios, the (frictional and long-term) unemployment rate, total production, and the employers' pay-off increase and average wage decreases for the country that has a higher $\frac{E}{L}$ ratio and the opposite is true for the other country. After factor market integration, earnings instability of the worker with $k_i \leq \overline{k}$ due to employment variation may increase.

We consider a stylized version of factor market integration such that we combine two economies to reach one big world economy. As a result, from the high $\frac{E}{L}$ country's perspective, the equilibrium $\overline{k}$ will decrease (by Lemma 4). The probability of finding a suitable match for the employer in the high $\frac{E}{L}$ country, Q^E, will increase. At the steady state, since $E = m\left[1 + \frac{\rho}{1-\rho}Q^E\right]$ by equation (3.1), the number of vacancies, m, will decrease and total production by home country employers, $E - m$, will increase. Since the economy is operating at a lower equilibrium $\overline{k}$, the measure of home country workers employed will decrease after the integration. As a result unemployment, $\frac{n}{L}$, will increase. Since the average wage $\widetilde{\omega}$ decreases due to lower equilibrium $\overline{k}$ and Q^E increases, the value function of the employer with a vacancy (V^{ES}) and ex-ante value function of the employer in relationship (V^{ER}) increase (by Equations D.8 and D.9 at Appendix D, page 196).

A worker whose effort cost is considered to be low to form a relationship before the factor market integration, but considered to be high (i.e., higher than than the threshold effort level) after the integration, will have no transitory earnings variation due to employment variation but will experience long-term unemployment. A worker whose effort cost is low enough before and after integration will face higher frictional unemployment, implying longer unemployment spell. Such a worker will also have higher transitory earnings variation due to employment variation if unemployment rate (for workers whose $k_i \leq \overline{k}$) is less than $\frac{2\rho-1}{2\rho}$ (see Appendix D, page 211). Economy-wide income inequality will decrease because average wages ($\widetilde{\omega}$) and variance of workers' wage after second and subsequent periods decrease. The increase in unemployment (for $k_i \leq \overline{k}$) will work towards increasing the income inequality, but will not be sizable enough to dominate. So, even if income inequality decreases due to integration, earnings instability of workers with $k_i \leq \overline{k}$ will increase if unemployment rate (for workers whose $k_i \leq \overline{k}$) is less than a certain threshold level.

3.5. Conclusion

In this essay, we employ a stylized model to analyse the relationship between factor market integration (i.e., off-shoring and immigration), labour market flexibility, unemployment, and earnings instability. We have shown that factor market integration can indeed affect the nature of long-term employment relationships by weakening them. Moreover, it may raise earnings instability by increasing unemployment.

Furthermore, it will be interesting to note the interaction of second and third essay. In the second essay, we investigate whether labour market policy instruments change the on-the-job volatility. The impact of these policy instruments work through the redistribution of welfare between employers and workers without having any impact on unemployment and total production. Third essay complements the second essay by studying the impact of unemployment on workers' earnings instability. We conjecture that, if worker's in the second essay are taken as heterogeneous, any labour market policy instrument in the second essay that tighten the employer's incentive compatibility constraint, will have a similar effect as factor market integration has on home country workers.

To sum up, within this simple framework we manage to explain the increase in unemployment (and incidence of long-term unemployment) and transitory variation in workers' earnings even when productivity was improved as observed after the 1970s.

Conclusion

The first essay discusses whether an increase in search effectiveness, by weakening the long-term employment relationship and undermining this risk-sharing institution, may have a role in increasing earnings instability of on-the-job workers and in the overall economy by changing the composition of industries. Also, it states that for a parameter space identified in the analysis earnings instability increases in the overall economy.

The second essay asks whether labour market institutions may have a role in the earnings instability of on-the-job workers. In particular, the impact of two types of transfer income (unemployment payment, hiring payment) and three types of financing those transfer incomes (payroll tax, firing tax, hiring tax) on earnings instability is investigated. While unemployment payment and payroll tax increase earnings instability of the on-the-job workers, hiring payment, firing tax, and hiring tax decrease it.

The third essay explores the impact of international factor market integration on the long-term and frictional unemployment and earnings instability. The third essay not only complements the second essay and Karabay and McLaren (2010) by incorporating the role of unemployment in explaining the increase in earnings instability, but also highlights a recent phenomena of increase in the long-term unemployment.

References

[1] Acemoglu, D., Shimer, R., 2000. Productivity gains from unemployment insurance. *European Economic Review* 44, pp. 1195–1224.

[2] Alvarez, F. and M. Veracierto, 2001. Severance payments in an economy with frictions. *Journal of Monetary Economics*, 47, pp. 477–498.

[3] Andrews, E.L., 1996. AT&T's plan for cuts: be gentle but stay firm. *The New York Times*, February 13.

[4] Autor, D.H., F. Levy and R. Murnane, 1999. Skills training in the temporary help sector: employer motivations and worker impacts. A report to the Department of Labor Education and Training Administration, Washington, DC.

[5] Autor, D.H., 2001a. Why do temporary help firms provide free general skills training? *Quarterly Journal of Economics* 116, pp. 1409-1448.

[6] Autor, D.H., 2001b. Wiring the labor market. *Journal of Economic Perspectives* 15, pp. 25-40.

[7] Autor, D.H., Katz, L.F., and Kearney, M.S., 2008. Trends in U.S. wage inequality revising the revisionists. *The Review of Economics and Statistics* 90, pp 300–323.

[8] Avcioglu, S. and Karabay, B., 2017. It takes two to tango: the interplay of severance payment and minimum wage under invisible handshake. Working paper, RMIT University.

[9] Baker, B. and Rytina, N., 2013. Estimates of the Unauthorized Immigrant Population Residing in the United States: January 2012. Washington, DC: Department of Homeland Security.

[10] Baker, B. and Rytina, N., 2014. Estimates of the Lawful Permanent Resident Population in the United States: January 2013. Washington, DC: Department of Homeland Security.

[11] Baker, M. and G. Solon, 2003. Earnings dynamics and inequality among Canadian men, 1967-1992: evidence from longitudinal income tax records. *Journal of Labor Economics* 21, pp. 289–322.

[12] BallotPedia, 2016. 2016 Presidential Candidates on Immigration. Available at https://ballotpedia.org/2016_presidential_candidates_on_immigration. Accessed September 04, 2016.

[13] Baumann, F. and N. Stähler, 2006. Financing unemployment benefits: dismissal versus employment taxes. *Review of Labour Economics and Industrial Relations* 20, pp. 433–451.

[14] Bhagwati, J., 1998. A Stream of Windows: Unsettling Reflections on Trade, Immigration, and Democracy. Cambridge, MA: The MIT Press.

[15] Blinder, A.S., 2009. How many US jobs might be offshorable? *World Economics* 10(2), pp. 41–78.

[16] Beaudry, P. and J. DiNardo, 1991. The effects of implicit contracts on the behavior of wages over the business cycle. *Journal of Political Economy* 99, pp. 665–688.

[17] Benner, C., 2002. *Work in the New Economy: Flexible Labor Markets in Silicon Valley*. Oxford: Blackwell Publishers.

[18] Benveniste, L.M. and J.A. Scheinkman, 1979. On the differentiability of the value function in dynamic models of economics, *Econometrica* 47, pp. 727–732.

[19] Bertola, G., 2004. A pure theory of job security and labor income risk. *Review of Economic Studies* 71, pp. 43–61.

[20] Bertrand, M. 2004. From the Invisible Handshake to the Invisible Hand? How Import Competition Changes the Employment Relationship. *Journal of Labor Economics* 22(4), pp. 723–765.

[21] Blanchard, O.J., 2004. Explaining European unemployment. NBER Reporter, June.

[22] Blanchard, O.J. and J. Tirole, 2008. The joint design of unemployment insurance and employment protection: a first pass. *Journal of the European Economic Association* 6, pp. 45–77.

[23] Boeri, T., P. Garibaldi and E. Moen, 2016. Inside Severance Payment. LIEPP Working Paper #47.

[24] Brucker, H., 2011. The Labor Market Challenge: Does International Migration Challenge Labor Markets in Host Countries? A Critical Review of the Recent and Traditional Literature. EU-US Immigration Systems 2011/21. European University Institute.

[25] Burguet, R. and R. Caminal, 2008. Does the market provide sufficient employment protection? *Labour Economics*, 15, pp. 406–422.

[26] Cameron, S. and J. Tracy, 1998. Earnings variability in the United States: an examination using matched-CPS data. Mimeo: Federal Reserve Bank of New York.

[27] Cahuc, P. and F. Malherbet, 2004. Unemployment compensation finance and labor market rigidity. *Journal of Public Economics,* 88, pp. 481–501.

[28] Cahuc, P. and A. Zylberberg, 2008. Optimum income taxation and layoff taxes. *Journal of Public Economics*, 92, pp. 2003–2019.

[29] Card, D. and DiNardo, J.E., 2002. Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles. *Journal of Labor Economics* 20, pp. 733–783.

[30] Card, D., Krueger, A., 1994. Minimum wages and employment: A case study of the fast food industry in New Jersey and Pennsylvania. *American Economic Review* 84, pp. 772–793.

[31] Card, D., Krueger, A., 1995. Myth and measurement: The new evidence of the minimum wage. Princeton University Press, Princeton.

[32] Carnoy, M. and Castells, M., 1997. Sustainable flexibility: a prospective study on work, family and society in the information age. OECD Working Papers 5, No. 29.

[33] Celik, S., Juhn, C., McCue., K., and Thompson, J., 2012. Recent trends in earnings volatility: evidence from survey and administrative data. *B.E. Journal of Economic Analysis & Policy (Contributions)* 12(2), Article 1.

[34] Champagne, J. and A. Kurmann, 2013. The great increase in relative volatility of real wages in the United States. *Journal of Monetary Economics* 60, pp. 166–183.

[35] Comin, D., E.L. Groshen, and B. Rabin, 2009. Turbulent Firms, Turbulent-Wages? *Journal of Monetary Economics* 56, pp. 109–133.

[36] Cohany, S., 1996. Workers in Alternative Employment Arrangements. *Monthly Labor Review*, 119, pp. 31–45.

[37] Crinò, R., 2010. Service Offshoring and White-Collar Employment. *Review of Economic Studies* 77(2), pp. 595–632.

[38] Daly, M.C., Hobijn, B., Şahin, A. and Valletta, R.G., 2012. A search and matching approach to labor markets: Did the natural rate of unemployment rise?. *The Journal of Economic Perspectives* 26(3), pp.3–26.

[39] Dickens, R., 2000. The evolution of individual male earnings in Great Britain: 1975-1995. *Economic Journal* 110, pp. 27-49.

[40] Dynan, K., Elmendorf, D., and Sichel, D.E., 2012. The evolution of household income volatility. *B.E. Journal of Economic Analysis & Policy (Advances)* 12(2), Article 3.

[41] Dynarski, S. and J. Gruber, 1997. Can families smooth variable earnings? *Brookings Papers on Economic Activity* 1, pp. 229-284.

[42] Engen, E.M. and J. Gruber, 2001. Unemployment insurance and precautionary saving. *Journal of Monetary Economics* 47, pp. 545–579.

[43] European Commission, 2010. Europe 2020: A European strategy for smart, sustainable and inclusive growth, March.

[44] Farber, H.S., 2007. Is the Company Man an Anachronism? Trends in Long Term Employment in the U.S., 1973–2006. Working Paper No. 518, Industrial Relations Section. Princeton University.

[45] Farber, H., 2009. Job loss and the decline in job security in the United States. Princeton University Industrial Relations Section Working Paper No. 520.

[46] Feenstra, R.C. and Hanson, G.H., 1996. Globalisation, Outsourcing and Wage Inequality. *American Economic Review* 86(2), pp. 240–245.

[47] Feenstra, R.C. and Hanson, G.H., 1999. The Impact of Outsourcing and High-Technology Capital on Wages: Estimates for the United States, 1979-1990. *Quarterly Journal of Economics* 114(3), pp. 907–940.

[48] Feldstein, M., 1976. Temporary layoffs in the theory of unemployment. *Journal of Political Economy* 84, pp. 937–957.

[49] Feldstein, M., 1978. The effect of unemployment insurance on temporary layoff unemployment. *American Economic Review* 68, pp. 834–846.

[50] Feldstein, M. and D. Altman, 1998. Unemployment insurance savings accounts. NBER Working Paper #6860.

[51] Finlay, W. and J.E. Coverdill, 2002. *Headhunters: Matchmaking in the Labor Market*. Ithaca, NY: Cornell University Press.

[52] Flabbi, L. and A. Ichino, 2001. Productivity, seniority and wages: new evidence from personnel data, *Labour Economics* 8, pp 359–387.

[53] Fredriksson, P. and Holmlund, B., 2006. Improving Incentives in Unemployment Insurance: A Review of Recent Research. *Journal of Economic Surveys* 20: pp. 357–386.

[54] Gavrel, F. and and I. Lebon, 2008. Firing costs, payroll taxes and unemployment. *Annals of Economics and Statistics* 89, pp. 121–129.

[55] Gosselin, P., 2008. *High Wire: The Precarious Financial Lives of American Families*. Basic Books.

[56] Gottschalk, P., Moffitt, R.A., Katz, L.F., and Dickens, W.T., 1994. The growth of earnings instability in the U.S. labor market. *Brookings Papers on Economic Activity* 2, pp. 217–272.

[57] Gottschalk, P. and R. Moffitt, 2009. The Rising Instability of U.S. Earnings. *Journal of Economic Perspectives* 23, pp. 3–24.

[58] Hacker, J.S. and E. Jacobs, 2008. The rising instability of American family incomes, 1969-2004: evidence from the panel study of income dynamics. EPI Briefing Paper 213 (May), Economic Policy Institute.

[59] Hacker, J.S., 2008. *The Great Risk Shift: The New Economic Insecurity and the Decline of the American Dream*. Revised and Expanded Edition. Oxford, UK: Oxford University Press.

[60] Haider, S.J., 2001. Earnings instability and earnings inequality in the United States, 1967–1991. *Journal of Labor Economics* 19, pp. 799–836.

[61] Harris, M. and B. Holmström, 1982. A theory of wage dynamics. *Review of Economic Studies* 49, pp. 315–333.

[62] Heathcote, J., Storesletten, K., and Violante, G.L., 2004. The Cross-Sectional Implications of Rising Wage Inequality in the United States. CEPR Discussion Paper #4296.

[63] Heckman J. and C. Páges, 2000. The cost of job security regulation: evidence from the Latin American labor market. NBER Working Paper #7773.

[64] Holmlund, B., 1998. Unemployment insurance in theory and practice. *Scandinavian Journal of Economics* 100, pp. 113–141.

[65] Holmlund, B., 2014. What do labor market institutions do? *Labor Economics* 30, pp. 62–69.

[66] Holmström, B., 1983. Equilibrium long-term labour contracts. *Quarterly Journal of Economics* (Supplement) 98, pp. 23–54.

[67] Hopenhayn, H., Nicolini, J.P., 1997. Optimal unemployment insurance. *Journal of Political Economy* 105, pp. 412–438.

[68] Huff Stevens, A., 2001. Changes in Earnings Instability and Job Loss. *Industrial and Labor Relations Review* 55, pp.60–78.

[69] Hummels, D., Jorgenson, R., Munch, J., and Xiang, C., 2011. Wage and Employment Effects of Outsourcing: Evidence from Danish Matched Worker-Firm Data. NBER #17496.

[70] Jenkins, S.P., 2011. Changing Fortunes: Income Mobility and Poverty Dynamics in Britain. Oxford University Press.

[71] Kambourov, G. and I. Manovskii, 2009. Occupational mobility and wage inequality. *Review of Economic Studies* 76, pp. 731–759.

[72] Karabay, B., McLaren, J., 2010. Trade, offshoring, and the invisible handshake. J*ournal of International Economics* 82(1), pp. 26–34.

[73] Karabay, B. and McLaren, J., 2011. Pareto-Improving Firing Costs? *European Economic Review* 55, pp. 1083–1093.

[74] Katz, F.L., 2010. Long-Term Unemployment in the Great Recession. Testimony to the Joint Economic Committee, U.S. Congress, April 29.

[75] Katz, L.F. and A.B. Krueger, 1999. The high-pressure U.S. labor market of the 1990s. *Brookings Papers on Economic Activity* 1, pp. 1–65.

[76] Kazis, R., 1998. New Labor Market Intermediaries: What's driving them? Where are they headed? Task Force Working Paper No. 03, MIT Sloan School of Management, Cambridge, MA.

[77] Koch, C., 2003. Temporary Visas Give Offshoring a Boost. Available at http://www.cio.com/article/2442092/offshoring/temporary-visas-give -offshoring-a-boost.html#tk.drr_mlt. Accessed September 04, 2016.

[78] Kotlikoff, L. and J. Gokhale, 1992. Estimating a firm's age-productivity profile using the present value of workers' earnings. *Quarterly Journal of Economics* 107, pp. 1215–1242.

[79] Kranton, R.E., 1996. Reciprocal exchange: a self-sustaining system. *American Economic Review* 86, pp. 830–851.

[80] Krueger, D. and Perri, F., 2005. Does income inequality lead to consumption equality? evidence and theory. Staff Report #363, Federal Reserve Bank of Minneapolis.

[81] Longhi, S., Nijkamp, P., and Poot, J., 2008. Meta-Analysis of Empirical Evidence on the Labour Market Impacts of Immigration. The Institute for the Study of Labor (IZA) Discussian Paper #3418.

[82] Lazear, E.P., 1979. Why is there mandatory retirement? *Journal of Political Economy* 87, pp. 1261–1284.

[83] Lazear, E.P., 1990. Job security provisions and employment. Qu*arterly Journal of Economics* 105, pp. 699–726.

[84] Lemieux, T., 2006. Increased Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill. *American Economic Review* 96, pp. 461–498.

[85] Lee, D.S., 1999. Wage Inequality in the U.S. during the 1980s: Rising Dispersion or Falling Minimum Wage? *Quarterly Journal of Economics* 114, pp. 977–1023.

[86] L'Haridon, O. and F. Malherbet, 2009. Employment protection reform in search economies. *European Economic Review* 53, pp. 255–273.

[87] Machin, S. and Manning, A., 1997. Minimum wages and economic outcomes. *Europe European Economic Review* 41, pp. 733-742.

[88] MacLeod, W.B. and J.M. Malcomson, 1989. Implicit contracts, incentive compatibility, and involuntary unemployment. *Econometrica* 57, pp. 447–480.

[89] Malcomson, J.M., 1999. Individual employment contracts. Chapter 35 of O. Ashenfelter and D. Card, eds., *The Handbook of Labor Economics*, Vol. 3, Amsterdam: Elsevier, pp. 2291–2372.

[90] Mankiw, N. G., 2016. Why Voters Don't Buy It When Economists Say Global Trade Is Good. 29 July, The New York Times online.

[91] Mansfield, E.D. and Mutz, D.C., 2013. US versus Them: Mass Attitudes toward Offshore Outsourcing. *World Politics* 65, pp. 571–608.

[92] Martin, J.P., 2014. Activation and Active Labour Market Policies in OECD Countries: Stylized Facts and Evidence on their Effectiveness. IZA Policy Paper #84.

[93] Massey, D.S. and Pren, K.A., 2012. Unintended Consequences of US Immigration Policy: Explaining the Post-1965 Surge from Latin America. *Population and Development Review* 38(1), pp. 1–29.

[94] Matouschek, N. and P. Ramezzana, 2007. The role of exclusive contracts in facilitating market transactions. *Journal of Industrial Economics* 55, pp. 197–222.

[95] Matouschek, N., Ramezzana, P., and Robert-Nicoud, F., 2008. Labor market reforms, job instability, and the flexibility of the employment relationship. *European Economic Review* 53(1), pp. 19–36.

[96] McDonald, J.T. and C. Worswick, 1999. Wages, implicit contracts and the business cycle: evidence from Canadian micro data. *Journal of Political Economy* 107, pp. 884–892.

[97] McLaren, J., 2003. Trade and market thickness: effects on organizations. *Journal of the European Economic Association* 1, pp. 328–336.

[98] McLaren, J. and A. Newman, 2004. Globalisation and insecurity. Mimeo: University of Virginia.

[99] Medoff, J.L. and K.G. Abraham, 1980. Experience, performance, and earnings. *Quarterly Journal of Economics* 95, pp. 703–736.

[100] Mirrlees, J.A., 1971. An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies* 38, pp. 175–208.

[101] Moffitt, R.A. and P. Gottschalk, 2012. Trends in the transitory variance of male earnings: methods and evidence. *Journal of Human Resources* 47, pp. 204-236.

[102] Mongrain, S. and J. Roberts, 2005. Unemployment insurance and experience rating: insurance versus efficiency. *International Economic Review* 46, pp. 1303–1319.

[103] Mortenson, D., 1994. Reducing supply-side disincentives to job creation. In *Reducing Unemployment: Current Issues and Policy Options*. Kansas City: Federal Reserve Bank.

[104] Nicholson, W., 2008. The Evolution of Unemployment Insurance in the United States. *Comparative Labor Law and Policy Journal* 30, pp. 101–113.

[105] OECD, 1999. Employment Outlook, June.

[106] OECD, 2004. Employment Outlook, June.

[107] OECD, 2005. Employment Outlook, June.

[108] OECD, 2010. Measuring Globalisation OECD Economic Globalisation Indicators 2010. OECD Publishing, pp. 218–221.

[109] OECD, 2016. Long-term unemployment rate (indicator). doi: 10.1787/76471ad5-en. Available at https://data.oecd.org/unemp/long-term-unemployment-rate.htm. Accessed September 04, 2016.

[110] Office of Immigration Statistics, 2016. 2014 Yearbook of Immigration Statistics, Table 1. Washington, DC: Department of Homeland Security.

[111] Okun, A., 1981. Prices and quantities. Washington, DC: Brookings Institution.

[112] Ottaviano, G.I.P., Peri, G., and Wright, G.C., 2013. Immigration, Offshoring, and American Jobs. *American Economic Review* 103(5), pp. 1925–59.

[113] Ozkan, U.R., 2014. Comparing formal unemployment compensation systems in 15 OECD countries. *Social Policy and Administration* 48, pp. 44–66.

[114] Pissarides, C., 2000. *Equilibrium Unemployment Theory* (2nd edition). Cambridge, MA: MIT Press.

[115] Portes, J., 2016. Immigration, Free Movement and the UK Referendum. *National Institute Economic Review* 236, pp. 14–22.

[116] Ramey, G. and J. Watson, 2001. Bilateral trade and opportunism in a matching market. *B.E. Journal of Theoretical Economics* (*Contributions*) 1(1), Article 3.

[117] Ranjan, P., 2013 Offshoring, unemployment, and wages The role of labor market institutions. *Journal of International Economics* 89, pp. 172–186.

[118] Reuters, 2004. Tight visa rules hurt U.S. business, companies say. Available at http://www.siliconinvestor.com/readmsgs.aspx?subjectid=53920&msgnum=88649&batchsize=10&batchtype=Previous. Accessed September 04, 2016.

[119] Riboud, M., Sánchez-Páramo, C. and Silva-Jáuregui, C., 2002. Does Eurosclerosis matter? Institutional reform and labour market performance in Central and Eastern European countries. *Labour, Employment and Social Policies in the EU Enlargement Process.* Washington DC: World Bank, pp.243–311.

[120] Robalino, D.A., M. Vodopivec and A. Bodor, 2009. Savings for unemployment in good or bad times: options for developing countries. IZA Discussion Paper #4516.

[121] Schettkat, R. and L. Yocarini. The shift to services employment: a review of the literature. *Structural Change and Economic Dynamics* 17, pp. 127-147.

[122] Segal, L. and D. Sullivan, 1997. Temporary services employment durations: evidence from state UI data. Working Paper Series, Macroeconomic Issues WP-97-23, Federal Reserve Bank of Chicago.

[123] Shavell, S., Weiss, L., 1979. The optimal payment of unemployment benefits over time. *Journal of Political Economy* 87, pp. 1347–1362.

[124] Shin, D. and Solon, G., 2011. Trends in men's earnings volatility: What does the Panel Study of Income Dynamics show? *Journal of Public Economics* 95, pp.973–982.

[125] Shin, D., 2012. Recent trends in men's earnings volatility: evidence from the National Longitudinal Survey of Youth, 1985-2009. *B.E. Journal of Economic Analysis & Policy (Topics)* 12(2), Article 2.

[126] Shleifer, A. and L.H. Summers, 1988. Breach of trust in hostile takeovers. In Alan J. Auerbach, ed., *Corporate Takeovers: Causes and Consequences*, Chicago, IL: The University of Chicago Press, pp. 33-56.

[127] Spence, M., 2011. The Impact of Globalisation on Income and Employment, *Foreign Affairs* 90(4), pp. 28–41.

[128] Sundaram, R., Hoerning, U., Falcão, N., Millán, N., Tokman, C., and Zini, M. 2015. Portraits of Labor Market Exclusion. Directions in Development. Washington, DC: World Bank. doi:10.1596/978-1-4648-0539-4.

[129] Thesmar, D. and Thoenig, M., 2002. Why is a Flexible World More Insecure? The Way Outsourcing Amplifies Uncertainty. manuscript, INSEE and CERAS, Paris.

[130] Thomas, J. and T. Worrall, 1988. Self-enforcing wage contracts. *Review of Economic Studies* 55, pp. 541–554.

[131] U.S. Congressional Budget Office, 2011. Trends in the Distribution of Household Income Between 1979 and 2007.Washington, D.C.

[132] U.S. Congressional Budget Office, 2016. The Distribution of Household Income and Federal Taxes, 2013. Washington, D.C.

[133] Violante, G.L., 2002. Technological Acceleration, Skill Transferability and the Rise in Residual Inequality. *Quarterly Journal of Economics* 117, pp. 297–338.

[134] Wadsworth, J., 2015. Immigration and the UK Labour Market. Brexit and the Impact of Immigration on the UK. CEP 2015 Election Analysis Series #1.

[135] Wadsworth, J., Dhingra, S., Ottaviano, G., and Van Reenen, J., 2016. Brexit and the Impact of Immigration on the UK. CEP Brexit Analysis #5.

[136] Wright, G.C., 2014. Revisiting the employment impact of offshoring. *European Economic Review* 66, pp. 63–83.

[137] World Trade Organization, 2001. Guidelines for the Scheduling of Specific Commitments under the General Agreement On Trade in Services (GATS). Available at https://www.wto.org/english/tratop_e/serv_e/sl92.doc. Accessed September 07, 2016.

APPENDIX A

Distinguishing Features of Each Model

	First Chapter	**Second Chapter**	**Third Chapter**
Production	There are two sectors; Y and X. In the risk-free sector (spot-market sector), Y, there is a linear production technology and positive price of Y-sector output, p^y. In the risky sector (careers sector), X, (a numeraire sector, $p^x \equiv 1$.) for production to occur one worker must team up with one employer and they must each put in one unit of non-contractible effort. Only workers suffer a disutility from effort equal to k>0. The output and revenue generated in that period is then equal to $x_\varepsilon e^W e^E$ where the effort put in by agent j is denoted by $e^j \in \{0,1\}$, where j=W indicates the worker and j=E indicates the employer. ε is an idiosyncratic i.i.d. random variable that takes a value ε=G or B with respective probabilities π_G and π_B, where $\pi_G+\pi_B=1$ and $\pi_G>\pi_B=>0$.	There are one (risky) sector where output price, $p \equiv 1$. It corresponds to the careers sector in First Chapter. For production to occur one worker must team up with one employer and they must each put in one unit of non-contractible effort. Only workers suffer a disutility from effort equal to k>0. The output formulation is same as the First Chapter.	There are one (non-risky) sector where output price, $p \equiv 1$. It corresponds to the careers sector in First Chapter. For production to occur one worker must team up with one employer and they must each put in one unit of non-contractible effort. Worker *i* suffer a disutility from effort equal to $k_i > 0$, where k_i is i.i.d. with cdf $G(k_i)$. The output and revenue generated in that period is then equal to $x e^W e^E$ where the effort put in by agent j is denoted by $e^j \in \{0,1\}$, where j=W indicates the worker and j=E indicates the employer.
Search	$\Phi(n,m,\varphi)$ matches occur where a measure n of workers, a measure m of employers searching in a given period and φ is a measure of the effectiveness of search technology.	$\Phi(n,m)$ matches occur.	$H(\bar{k})\Phi(n,m)$ matches occur. Here $\Phi(n,m)$ is the number of blind matches and $H(\bar{k})$ is the probability that those blind matches are suitable for production with $\bar{k}$ representing the threshold disutility from effort.
Preferences	The workers are risk-averse and the employers are risk-neutral. Employers have the same linearly homogeneous and quasi-concave per-period utility function $U(c^X,c^Y)$, defined over consumption of goods X and Y, (c^X and c^Y), respectively. Workers have the same per-period utility function $\mu(U(c^X,c^Y))$ over consumption of goods X and Y.	The workers are risk-averse and the employers are risk-neutral. Workers have the same per-period utility function μ over consumption of the good. There is a finite lower bound, $\mu(0)$, to workers' utility.	The workers and the employers are risk-neutral. Workers utility function is in the form of $\alpha+\omega$, where α is a finite lower bound to workers' utility and ω is the wage earned (which is equal to the consumption).
Timing of the Game	1. Any readily matched employer and worker learn whether they will be exogenously separated this period. 2. For each firm in the risky (X) sector, the idiosyncratic output shock ε is realized. This is common knowledge within the firm but unknown to agents outside the firm. 3. The wage, if any, is paid (a claim on the firm's output at the end of the period). 4. The employer and worker simultaneously choose their effort levels e^j. At the same time, the search mechanism operates. Within a firm, if $e^j=0$, then agent j can participate in search and exert no effort. Workers in the non-risky (Y) sector and employers with vacancy always search and they do not incur any search cost. 5. Each firm's revenue, $R = x_\varepsilon e^W e^E$, as well as profits and consumption are realized. 6. For those agents who have found a new potential partner in this period's search, new partnerships with a new self-enforcing agreement are formed. This is achieved by a take-it-or-leave-it offer made by the employer to the worker. Therefore, we assume that employers have the full bargaining power.	Same as the Timing of The Game of the First Chapter except the underlined sentence in bullet no 4. … 4. … Unemployed workers and employers with vacancy always search and do it without any cost. …	Same as the Timing of The Game of the First Chapter except the bullet no 2 and the underlined sentence in bullet no 4. … 2. There is no shock in the economy. … 4. … Unemployed workers and employers with vacancy always search and do it without any cost. …

Figure A.1. Distinguishing Features of Each Model.

APPENDIX B

Appendix to Chapter 1

Proof of Lemma 1. In the first-period of the employment relationship, the employer must give the worker at least as much as the worker's outside option, otherwise she will never accept employment, thus, $V^{WR} \geq V^{WS}$. If we substitute V^{WS} for V^{WR} in equation (1.1), we obtain

$$V^{WS} \geq \mu(\frac{\omega^y}{P}) + Q^W \rho\beta V^{WS} + Q^W(1-\rho)\beta V^{WS} + (1-Q^W)\beta V^{WS}, \text{ or}$$

$$V^{WS} \geq \frac{\mu(\dfrac{\omega^y}{P})}{1-\beta} > 0. \tag{B.1}$$

Next, using equations $(1.5)'$ and (B.1), we have

$$\widetilde{W}_\epsilon \geqslant \widetilde{W}^* = V^{WS} + \frac{(1-\beta)\,V^{WS} - \mu(\dfrac{\omega^y}{P}) + k}{\rho\beta} > V^{WS} > 0.\blacksquare$$

Proof of Proposition 1. In accordance with Assumption 1, when solving the first period problem, we will assume that the employer's incentive compatibility constraint does not bind. A sufficient condition for this is provided in Lemma 2 in the main text.

Consider the first period problem. From the envelope condition given in equation (1.11), we know that the target utility constraint binds, so $\lambda > 0$ and $V^{WR} = V^{WS}$.

Suppose that the worker's incentive-compatibility constraint does not bind in state ϵ in the first period. Then $\upsilon_\epsilon = 0$, and since $\psi_\epsilon = 0$ because of Assumption 1, equation (1.10) becomes

$$\Omega'(\widetilde{W}_\epsilon) + \lambda = 0.$$

The envelope condition given in equation (1.11) implies that in the first period we have $\Omega'(W_0) + \lambda = 0$. This means $\widetilde{W}_\epsilon = W_0 = V^{WS}$. But since $V^{WS} < \widetilde{W}^*$ as shown in Lemma 1, this implies that the worker's incentive compatibility constraint $(1.5)''$ will be violated, a contradiction. Therefore, the worker's incentive compatibility constraint must bind in each state, ensuring that $\widetilde{W}_\epsilon = \widetilde{W}^*$. Given that $\widetilde{W}_\epsilon = \widetilde{W}^*$ and $W_0 = V^{WS}$, the target utility constraint (1.6) in the first period becomes

$$\sum_{\epsilon=G,B} \pi_\epsilon \mu(\frac{\omega_\epsilon}{P}) = \mu(\frac{\omega^y}{P}). \tag{B.2}$$

In addition, condition (1.9), with $\psi_\epsilon = 0$, becomes

$$-\pi_\epsilon + \pi_\epsilon \lambda \frac{\mu'(\frac{\omega_\epsilon}{P})}{P} + \chi_\epsilon = 0.$$

If $\chi_\epsilon > 0$ for some ϵ, then $\omega_\epsilon = 0$. This clearly cannot be true for both values of ϵ, because that would imply a permanent zero wage, and it would not be possible to satisfy equation (B.2). Therefore, for at most one state, say ϵ', $\chi_{\epsilon'} > 0$. Denote by ϵ'' the state with $\chi_{\epsilon''} = 0$. Then $\frac{\mu'(0)}{P} = \frac{1}{\lambda}\left(1 - \frac{\chi_{\epsilon'}}{\pi_{\epsilon'}}\right) < \frac{1}{\lambda} = \frac{\mu'(\frac{\omega_{\epsilon''}}{P})}{P}$. However, given that $\omega_{\epsilon''}$ is non-negative and μ is strictly concave, this is impossible. We conclude that $\chi_\epsilon = 0$ in both states, and therefore $\omega_G = \omega_B = \omega^y$. Therefore, ω^y is the minimum first period wage required to make the worker willing to accept the job.

Consider now the second-period problem. As before due to the envelope condition, the target utility constraint binds. We know that the target continuation pay-off for the worker is $\widetilde{W}^*$. We claim that the choice of next-period continuation pay-off $\widetilde{W}_\epsilon$ will be equal to $\widetilde{W}^*$ for $\epsilon = G, B$. If $\upsilon_\epsilon > 0$, then complementary slackness implies that $\widetilde{W}_\epsilon = \widetilde{W}^*$. Therefore, suppose that $\upsilon_\epsilon = 0$. This implies that condition (1.10) becomes

$$\Omega'(\widetilde{W}_\epsilon) = -\lambda \frac{\pi_\epsilon}{\pi_\epsilon + \psi_\epsilon}.$$

Since, by the envelope condition, $-\lambda = \Omega'(W)$, and as we recall for the second-period problem the worker's target utility $W = \widetilde{W}^*$, this becomes

$$\Omega'(\widetilde{W}_\epsilon) = \Omega'(\widetilde{W}^*) \frac{\pi_\epsilon}{\pi_\epsilon + \psi_\epsilon}. \tag{B.3}$$

If $\psi_\epsilon = 0$, this implies through the strict concavity of Ω that $W = \widetilde{W}^*$, and we are done. On the other hand, if $\psi_\epsilon > 0$, equation (B.3) then implies that $0 > \Omega'(\widetilde{W}_\epsilon) > \Omega'(\widetilde{W}^*)$, implying that $\widetilde{W}_\epsilon < \widetilde{W}^*$. However, this violates constraint $(1.5)''$. Therefore, all possibilities either imply that $W = \widetilde{W}^*$ or lead to a contradiction, and the claim is proven.

Since $W = \widetilde{W}^*$, the optimization problem in the third period of the relationship is identical to that of the second period. By induction, the target utility for the worker in every period after the first, regardless of history, is equal to $\widetilde{W}^*$, and so the wage chosen for each state in every period after the first, regardless of history, is the same.

Now, to establish the three possible outcomes, we consider each possible case in turn. Consider the optimization problem (1.3) at any date after the first period of relationship. First, suppose that the employer's constraint does not bind in either state. In this case,

$\psi_\epsilon = 0$ for $\epsilon = G, B$. Condition (1.9) now becomes

$$-\pi_\epsilon + \pi_\epsilon \lambda \frac{\mu'(\frac{\omega_\epsilon}{P})}{P} + \chi_\epsilon = 0.$$

If $\chi_\epsilon > 0$ for some ϵ, then $\omega_\epsilon = 0$. This clearly cannot be true for both values of ϵ, because that would imply a permanent zero wage, and it would not be possible to satisfy constraint (1.6). (To see this, formally, substitute $W = \widetilde{W}_\epsilon = \widetilde{W}^*$, the expression for V^{WS}, and $\omega_G = \omega_B = 0$ into constraint (1.6), and note that the constraint is violated.) Therefore, for at most one state, say ϵ', $\chi_{\epsilon'} > 0$. Denote by ϵ'' the state with $\chi_{\epsilon''} = 0$. Then $\frac{\mu'(0)}{P} = \frac{1}{\lambda}\left(1 - \frac{\chi_{\epsilon'}}{\pi_{\epsilon'}}\right) < \frac{1}{\lambda} = \frac{\mu'(\frac{\omega_{\epsilon''}}{P})}{P}$. However, given that $\omega_{\epsilon''}$ is non-negative and μ is strictly concave, this is impossible. We conclude that $\chi_\epsilon = 0$ in both states, and therefore $\omega_G = \omega_B$.

Next, suppose that we have $\psi_G > 0$ and $\psi_B = 0$, so that the employer's constraint binds only in the good state. We will show that this leads to a contradiction. Recall from the previous discussion that $\widetilde{W}_\epsilon = \widetilde{W}^*$ for both states, and note that, by assumption, constraint (1.4) is satisfied with equality for $\epsilon = G$. Since $x_B < x_G$, we now see that constraint (1.4) must be violated for $\epsilon = B$ if $\omega_G \leq \omega_B$. Therefore, $\omega_G > \omega_B \geq 0$. This implies that $\chi_G = 0$. Applying condition (1.9), then, we have

$$\frac{\mu'(\frac{\omega_G}{P})}{P} = \frac{1}{\lambda}\left(1 + \frac{\psi_G}{\pi_G}\right) > \frac{1}{\lambda}\left(1 - \frac{\chi_B}{\pi_B}\right) = \frac{\mu'(\frac{\omega_B}{P})}{P},$$

which contradicts the requirement that $\omega_G > \omega_B$. This shows that it is not possible for the employer's constraint to bind only in the good state.

Now suppose that we have $\psi_G = 0$ and $\psi_B > 0$, so that the employer's constraint binds only in the bad state. We now wish to prove that in this case $\omega_G > \omega_B$. Suppose to the contrary that $\omega_G \leq \omega_B$. This implies that $\omega_B > 0$ (since, as shown earlier, it is not possible to have zero wage in both states), so that $\chi_B = 0$. Then, from condition (1.9)

$$\frac{\mu'(\frac{\omega_B}{P})}{P} = \frac{1}{\lambda}\left(1 + \frac{\psi_B}{\pi_B}\right) > \frac{1}{\lambda}\left(1 - \frac{\chi_G}{\pi_G}\right) = \frac{\mu'(\frac{\omega_G}{P})}{P},$$

which implies that $\omega_G > \omega_B$. Therefore, we have a contradiction, and we conclude that $\omega_G > \omega_B$.

Finally, suppose that the employer's constraint binds in both states. Given that $\widetilde{W}_\epsilon = \widetilde{W}^*$ in both states, equality in both states for constraint (1.4) requires that short-term profits $x_\epsilon - \omega_\epsilon^*$ are equal in the two states.

We have thus eliminated all possibilities aside from those listed in the statement of the proposition.■

Proof of Lemma 2. First note that under wage-smoothing agreement since the employer's incentive compatibility does not bind in the second period and thereafter, it will not bind in the first period as well since $\omega^* > \omega^y$. Therefore, we need to focus on the fluctuating-wage agreement where the employer's bad-state incentive compatibility constraint binds in the second and subsequent periods. In the first period of a fluctuating-wage agreement, for the employer's bad-state incentive compatibility constraint to be slack, we need

$$x_B - \omega^y + \rho\beta\Omega(\widetilde{W}^*) + (1-\rho)\beta V^{ES} \geqslant V^{ES}.$$

Using the expressions for $\Omega(\widetilde{W}^*)$ and V^{ES} by changing ω^* with $E_\epsilon\omega^*_\epsilon$ in equations (1.15) and (1.16), respectively and substituting them into above equation, we obtain

$$x_B - [\omega^y + \rho\beta(E_\epsilon\omega^*_\epsilon - \omega^y)] + (1 - Q^E)\rho\beta\pi_G(x_G - x_B) \geqslant 0. \tag{B.4}$$

The last term is positive. Therefore, for condition (B.4) to hold, it is sufficient to have $x_B \geqslant \omega^y + \rho\beta(E_\epsilon\omega^*_\epsilon - \omega^y)$.■

Proof of Proposition 2. Consider the fluctuating-wage equilibrium. Differentiating equation (1.22) with respect Q^E while holding $\omega^y = p^y$ constant, we obtain

$$\frac{d\omega_B}{dQ^E}_{|dp^y=0} = -\rho\beta\frac{x_B - [\rho\beta\pi_G\omega_G + (1-\rho\beta\pi_G)\,\omega^y] + [1 + \rho\beta\,(1-\pi_G)]\,\pi_G(x_G - x_B)}{[1 - \rho\beta(\pi_G - Q^E)]^2}.$$

Therefore,

(B.5)
$$\frac{d\omega_B}{dQ^E}_{|dp^y=0} < 0 \Leftrightarrow x_B - [\rho\beta\pi_G\omega_G + (1-\rho\beta\pi_G)\,\omega^y] + [1 + \rho\beta\,(1-\pi_G)]\,\pi_G(x_G - x_B) > 0.$$

We can rewrite the condition in (B.5) as

$$\left.\begin{array}{l} \left[x_B - [\omega^y + \rho\beta(E_\epsilon\omega^*_\epsilon - \omega^y)] + (1 - Q^E)\rho\beta\pi_G(x_G - x_B)\right] \\ \\ +[1 - \rho\beta(\pi_G - Q^E)]\pi_G(x_G - x_B) + \rho\beta\pi_B(\omega_B - \omega^y) \end{array}\right\} > 0.$$

Notice that the expression in the first line is the same expression given in condition (B.4) (which is necessary and sufficient for Assumption 1 to hold) and it is non-negative. The expression in the second line needs to be positive since the first term is positive and the second term is non-negative since $\omega_B \geqslant \omega^y$. The latter condition needs to be true otherwise it would not be possible to pay ω^y in the first period. Therefore, the above equation must be positive and this proves that $\frac{d\omega_B}{dQ^E}_{|dp^y=0} < 0$.

Now, consider equation (1.20). Using this equation, we can find

$$\frac{d\omega_G}{d\omega_B} = -\frac{1-\pi_G}{\pi_G}\frac{\mu'(\frac{\omega_B}{P})}{\mu'(\frac{\omega_G}{P})} < 0.$$

Since $\frac{d\omega_B}{dQ^E}_{|dp^y=0} < 0$ and $\frac{d\omega_G}{d\omega_B} < 0$, we must have $\frac{d\omega_G}{dQ^E}_{|dp^y=0} > 0$. Given the strict concavity of μ, expected wage payment in the X sector must rise, i.e., $\frac{dE_\epsilon \omega_\epsilon}{dQ^E}_{|dp^y=0} > 0$, for equation (1.20) to hold.

In the case of wage smoothing, differentiating the right-hand side (RHS) of expression (1.19) with respect Q^E while holding $\omega^y = p^y$ constant, we obtain

$$\frac{dRHS}{dQ^E}_{|dp^y=0} = -\rho\beta\frac{x_B - \omega^y + (1+\rho\beta)\,\pi_G(x_G - x_B)}{(1+Q^E\rho\beta)^2} < 0.$$

The inequality that $x_B - \omega^y + (1+\rho\beta)\,\pi_G(x_G - x_B) > 0$ follows since condition (B.4) must be true for Assumption 1 to hold. This means that as Q^E increases the employer's bad state incentive constraint becomes tighter, but it will have no effect on the efficiency wage, ω^*, as long as the constraint does not bind. On the other hand, if the constraint starts to bind once Q^E rises, then we are in the fluctuating-wage zone, so wage volatility as well as expected wage payment increases in the X sector.■

Proof of Proposition 3. First consider the fluctuating-wage equilibrium. Totally differentiating the WW curve given in equation (1.20) with respect to p^y and recalling that

$\omega^y = p^y$, we obtain

(B.6)
$$\frac{\pi_G \mu'(\frac{\omega_G}{P})\frac{d\omega_G}{dp^y} + \pi_B \mu'(\frac{\omega_B}{P})\frac{d\omega_B}{dp^y}}{P} - \frac{\left[\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P})\right] P'}{P^2} = \frac{(P - p^y P')\mu'(\frac{p^y}{P})}{P^2}.$$

Now, using Roy's identity, for any consumer (either employer or worker) we can get the Marshallian demand for good Y as $c^Y(p^y, I) = \frac{P'I}{P}$. Therefore, good Y's share in total consumer expenditure is $\frac{p^y c^Y}{I} = \frac{p^y P'}{P} = \alpha^y$. Hence, equation (B.6) can be rewritten as

$$\pi_G \mu'(\frac{\omega_G}{P})\frac{d\omega_G}{dp^y} + (1-\pi_G)\,\mu'(\frac{\omega_B}{P})\frac{d\omega_B}{dp^y} = \frac{\alpha^y}{p^y}\left[\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P})\right] + (1-\alpha^y)\mu'(\frac{p^y}{P}).$$

Similarly, totally differentiating the EE line given in equation (1.22) with respect to p^y and recalling that $\omega^y = p^y$, we obtain

$$\rho\beta\pi_G \frac{d\omega_G}{dp^y} + [1 - \rho\beta(\pi_G - Q^E)]\frac{d\omega_B}{dp^y} = Q^E \rho\beta. \tag{B.7}$$

Equations (B.6) and (B.7) are then a system of two linear equations in two unknowns, $\frac{d\omega_G}{dp^y}$ and $\frac{d\omega_B}{dp^y}$. Solving for $\frac{d\omega_B}{dp^y}$, we obtain

$$\frac{d\omega_B}{dp^y} = -\rho\beta\pi_G \frac{\alpha^y \frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P})}{\omega^y} + (1-\alpha^y)\mu'(\frac{p^y}{P}) - Q^E \mu'(\frac{\omega_G}{P})}{D},$$

where $D \equiv (1-\pi_G)\,\mu'(\frac{\omega_B}{P})[1 - \rho\beta(\pi_G - Q^E)]\left[\frac{\pi_G}{1-\pi_G}\frac{\mu'(\frac{\omega_G}{P})}{\mu'(\frac{\omega_B}{P})} - \frac{\rho\beta\pi_G}{1-\rho\beta(\pi_G - Q^E)}\right]$ is the determinant of the system. The term in the last bracket represents the difference (in absolute value) between the slope of WW curve and EE line. Since at the equilibrium the WW curve is steeper than the EE line, it is positive. Hence, $D > 0$. Note that

$$\frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + (1-\pi_G)\,\omega_B \mu'(\frac{\omega_B}{P})}{\omega^y} > \frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + (1-\pi_G)\,\omega_B \mu'(\frac{\omega_B}{P})}{\pi_G \omega_G + (1-\pi_G)\,\omega_B} > \mu'(\frac{\omega_G}{P}).$$

The first inequality holds because the condition defining the WW curve implies that $\omega^y < \pi_G\omega_G + (1-\pi_G)\omega_B$, and the second holds because the middle expression is a weighted average of $\mu'(\frac{\omega_G}{P})$ and $\mu'(\frac{\omega_B}{P})$, of which the former is smaller. This implies that

$$\alpha^y \frac{\pi_G\omega_G\mu'(\frac{\omega_G}{P}) + \pi_B\omega_B\mu'(\frac{\omega_B}{P})}{\omega^y} + (1-\alpha^y)\mu'(\frac{p^y}{P}) \;>\; \mu'(\frac{\omega_G}{P}) > Q^E\mu'(\frac{\omega_G}{P}), \text{ so}$$

$$\frac{d\omega_B}{dp^y} \;<\; 0.$$

Since $\frac{d\omega_B}{dp^y} < 0$, equation (B.7) requires that $\frac{d\omega_G}{dp^y} > 0$, and therefore $\frac{d(\omega_G-\omega_B)}{dp^y} > 0$. Moreover, it is easy to see that $\frac{d\left(\frac{p^y}{P(p^y)}\right)}{dp^y} = \frac{1-\alpha^y}{P(p^y)} > 0$. Therefore, given the strict concavity of μ, we must have $\frac{dE_\epsilon\omega_\epsilon}{dp^y} > 1$, for equation (1.20) to hold.

Now consider the wage-smoothing equilibrium. If after a rise in p^y, it is not possible to sustain wage smoothing anymore, then all the results derived above for the fluctuating-wage equilibrium hold. Conversely, if it is still possible to have wage smoothing after a rise in p^y, by totally differentiating equation (1.13) with respect to p^y and recalling that $\omega^y = p^y$, we obtain

$$\frac{\mu'(\frac{\omega^*}{P})\frac{d\omega^*}{dp^y}}{P} - \frac{\omega^*P'\mu'(\frac{\omega^*}{P})}{P^2} = \frac{(P - p^yP')\mu'(\frac{\omega^y}{P})}{P^2}$$

Hence, we have

$$\text{(B.8)} \qquad \frac{d\omega^*}{dp^y} = \alpha^y\frac{\omega^*}{\omega^y} + (1-\alpha^y)\frac{\mu'(\frac{\omega^y}{P})}{\mu'(\frac{\omega^*}{P})} > 1,$$

since μ is strictly concave and $\omega^* > \omega^y$.■

Proof of Corollary. For a given p^y, $Q^E_{VV}(p^y)$ can be defined as the value of Q^E that makes inequality (1.19) hold with equality,

$$Q^E_{VV}(p^y) = \frac{x_B - \omega^* + \rho\beta\pi_G(x_G - x_B)}{\rho\beta\left[\pi_G(x_G - x_B) + \omega^* - p^y\right]}, \tag{B.9}$$

where we use $\omega^y = p^y$. Furthermore, under Assumption 2, we have $0 < Q^E_{VV}(p^y) < 1$.

Given equation (B.8), it is straightforward to verify that $\frac{dQ^E_{VV}(p^y)}{dp^y} < 0$.■

Proof of Proposition 4. The number of matched employers is equal to $E - m$, and the number of matched workers is equal to $L - n$. These must always be equal, so

$$E - L = m - n. \tag{B.10}$$

Assume initially that $E > L$. Dividing equation (1.23) by equation (1.24) and rearranging, we obtain

$$\frac{E}{L} = 1 + \frac{m-n}{\frac{\rho}{1-\rho}\Phi(\frac{n}{m}, 1, \phi)m + n}, \text{ or}$$

$$\frac{E}{L} = 1 + \frac{1 - \frac{n}{m}}{\frac{\rho}{1-\rho}\Phi(\frac{n}{m}, 1, \phi) + \frac{n}{m}}. \tag{B.11}$$

Since $E > L$, $\frac{n}{m} < 1$ must hold for the right-hand side of equation (B.11) to be greater than unity. Therefore, at an equilibrium, the right-hand side of equation (B.11) is strictly decreasing in $\frac{n}{m}$, so the equilibrium level of $\frac{n}{m}$ is uniquely determined for given values of $\frac{E}{L}$, ϕ and ρ.

Now consider the case where $E < L$. We can instead rewrite equation (B.11) as

$$\text{(B.12)} \qquad \frac{L}{E} = 1 + \frac{1 - \frac{m}{n}}{\frac{\rho}{1-\rho}\Phi(1, \frac{m}{n}, \phi) + \frac{m}{n}}.$$

Since $E < L$, for the right-hand side of equation (B.12) to be greater than unity, $\frac{m}{n} < 1$ (or alternatively, $\frac{n}{m} > 1$) must hold. Therefore, at an equilibrium, the right-hand side of equation (B.12) is strictly increasing in $\frac{n}{m}$, so the equilibrium level of $\frac{n}{m}$ is uniquely determined for given values of other parameters.

Two observations are in order. First, from equation (B.10), we have $dm = dn$, for given values of E and L. Second, for given values of E, L and ρ, if $E > L$, then $\frac{n}{m}$ is a locally decreasing function of ϕ with $\frac{n}{m} < 1$, whereas if $E < L$, then $\frac{n}{m}$ is a locally increasing function of ϕ with $\frac{n}{m} > 1$. Then, these two observations imply that $\frac{dm}{d\phi} = \frac{dn}{d\phi} < 0$.

Now, we are ready to show that $Q^E(\phi)$ is strictly increasing. Using $Q^E(\phi) = \frac{\Phi(n(\phi), m(\phi), \phi)}{m(\phi)} = \Phi(\frac{n}{m}(\phi), 1, \phi)$, we can rewrite equation (1.23) as

$$\text{(B.13)} \qquad E = m(\phi)\left[1 + \frac{\rho}{1-\rho}Q^E(\phi)\right].$$

Totally differentiating equation (B.13) with respect to ϕ yields

$$\begin{aligned}
\frac{dQ^E}{d\phi} &= -\frac{1-\rho}{\rho}\frac{m'(\phi)}{m(\phi)}\left[1 + \frac{\rho}{1-\rho}Q^E(\phi)\right], \text{ or} \\
&= -\frac{1-\rho}{\rho}\frac{m'(\phi)}{[m(\phi)]^2}E > 0, \text{ since } m'(\phi) < 0. \blacksquare
\end{aligned}$$

Proof of Proposition 5. The number of employers producing X-sector output at time t is given by

$$E - m_t = \rho \left[E - m_{t-1} + \Phi \left(n_{t-1}, m_{t-1}, \phi \right) \right], \tag{B.14}$$

where on the right-hand side, the first term represents the number of previously-matched employers that are not exogenously separated at time t, and the second term represents the number of newly-matched employers that are not immediately exogenously separated.

Denote the aggregate X-sector output produced in period t by x_t. Since the average X-sector output of an operating firm in any period is given by $\overline{x}$, the number of employers producing X-sector output at time t must be also equal to $\dfrac{x_t}{\overline{x}}$. Thus, equation (B.14) becomes

$$\frac{x_t}{\overline{x}} = \rho \left[\frac{x_{t-1}}{\overline{x}} + \Phi \left(n_{t-1}, m_{t-1}, \phi \right) \right].$$

Furthermore, evaluating the above equation at the steady state, where $x_t = x_{ss}$, $n_t = n$, $m_t = m$, we obtain

$$\begin{aligned} \frac{x_{ss}}{\overline{x}} &= \frac{\rho}{1-\rho} \Phi \left(n, m, \phi \right), \text{ or} \\ &= \frac{\rho}{1-\rho} \Phi \left(L - \frac{x_{ss}}{\overline{x}}, E - \frac{x_{ss}}{\overline{x}}, \phi \right). \end{aligned}$$

Since Φ is a linear homogeneous in n and m, dividing each side by $\dfrac{x_{ss}}{\overline{x}}$ yields

$$\Phi \left(\frac{\overline{x}}{x_{ss}} L - 1, \frac{\overline{x}}{x_{ss}} E - 1, \phi \right) = \frac{1-\rho}{\rho}.$$

From the above equation, it is easy to see that for given values of E, L, and ρ, $x_{ss}(\phi)$ is an increasing function of ϕ.

Now we return to Y-sector output. In period t, it is given by

$$\begin{aligned} y_t &= L - \frac{x_t}{\overline{x}}, \\ &= n_t. \end{aligned}$$

In the steady state, this becomes

$$\begin{aligned} y_{ss}(\phi) &= L - \frac{x_{ss}(\phi)}{\overline{x}}, \text{ or} \\ &= n(\phi). \end{aligned}$$

Since $x'_{ss}(\phi) > 0$ (alternatively, $n'(\phi) < 0$), $y_{ss}(\phi)$ is an decreasing function of ϕ. Hence, the ratio of Y-sector production to X-sector production, $r(\phi) = \dfrac{y_{ss}(\phi)}{x_{ss}(\phi)}$, is strictly decreasing function of ϕ.

Goods market clearing together with utility maximization condition implies $p^y(\phi) = \frac{U_2(1,r(\phi))}{U_1(1,r(\phi))}$. Since p^y is strictly decreasing in r and r is strictly decreasing in ϕ, p^y must be strictly increasing in ϕ.■

Proof of Theorem. The first part of the theorem follows from Propositions 2, 3, 4, and 5. Therefore, consider the second part regarding the economy-wide wage volatility. Before analyzing the economy-wide real wage volatility, it will be helpful to derive the following.

$$\begin{aligned} \frac{d\left[\frac{\omega^y}{P}(\phi)\right]}{d\phi} &= \frac{\partial(\omega^y/P)}{\partial\omega^y}\frac{\partial\omega^y}{\partial\phi} \\ &= \frac{(1-\alpha^y)}{P}\frac{\partial\omega^y}{\partial\phi} > 0, \text{ since } \frac{\partial\omega^y}{\partial\phi} > 0. \end{aligned}$$

Next, under wage-smoothing equilibrium, we can use equation (1.13) to obtain

$$\begin{aligned}
\frac{d\left[\frac{\omega^*}{P}(\phi)\right]}{d\phi} &= \frac{\partial(\omega^*/P)}{\partial\omega^y}\frac{\partial\omega^y}{\partial\phi} \\
&= \frac{\mu'(\omega^y/P)}{\mu'(\omega^*/P)}\frac{\partial(\omega^y/P)}{\partial\omega^y}\frac{\partial\omega^y}{\partial\phi} \\
&= \frac{\mu'(\omega^y/P)}{\mu'(\omega^*/P)}\frac{(1-\alpha^y)}{P}\frac{\partial\omega^y}{\partial\phi} > 0, \text{ since } \frac{\partial\omega^y}{\partial\phi} > 0.
\end{aligned}$$

Similarly, under fluctuating-wage equilibrium, we can use equation (1.20) to get

$$\begin{aligned}
\frac{d\left[\frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right]}{d\phi} &= \frac{\partial(E_\epsilon\omega_\epsilon/P)}{\partial\omega^y}\frac{\partial\omega^y}{\partial\phi} + \frac{\partial(E_\epsilon\omega_\epsilon/P)}{\partial Q^E}\frac{\partial Q^E(\phi)}{\partial\phi} \\
&= \frac{\mu'(\omega^y/P)}{E_\epsilon\mu'(\omega_\epsilon/P)}\frac{\partial(\omega^y/P)}{\partial\omega^y}\frac{\partial\omega^y}{\partial\phi} + \frac{\partial(E_\epsilon\omega_\epsilon/P)}{\partial Q^E}\frac{\partial Q^E(\phi)}{\partial\phi} \\
&= \frac{\mu'(\omega^y/P)}{E_\epsilon\mu'(\omega_\epsilon/P)}\frac{(1-\alpha^y)}{P}\frac{\partial\omega^y}{\partial\phi} + \frac{\partial(E_\epsilon\omega_\epsilon/P)}{\partial Q^E}\frac{\partial Q^E(\phi)}{\partial\phi} \\
&> 0, \text{ since } \frac{\partial\omega^y}{\partial\phi} > 0, \frac{\partial(E_\epsilon\omega_\epsilon/P)}{\partial Q^E} > 0 \text{ and } \frac{\partial Q^E(\phi)}{\partial\phi} > 0.
\end{aligned}$$

Furthermore, given that $\mu(.)$ is strictly concave, $\frac{E_\epsilon\omega_\epsilon}{P}(\phi) > \frac{\omega^y}{P}(\phi)$ and $\frac{\omega^*}{P}(\phi) > \frac{\omega^y}{P}(\phi)$, we can conclude that

$$\frac{d\left[\frac{\omega_G}{P}(\phi)\right]}{d\phi} > \frac{d\left[\frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right]}{d\phi} > \frac{d\left[\frac{\omega^y}{P}(\phi)\right]}{d\phi} > 0 > \frac{d\left[\frac{\omega_B}{P}(\phi)\right]}{d\phi}, \text{ and}$$

(B.15)

$$\frac{d\left[\frac{\omega^*}{P}(\phi)\right]}{d\phi} > \frac{d\left[\frac{\omega^y}{P}(\phi)\right]}{d\phi} > 0.$$

Now, we are ready to look at the economy-wide real wage volatility under each type of equilibria. First, consider the fluctuating-wage equilibrium. There are three types of wages at the steady state: for those that are in the second or later periods of their X-sector employment, we have the good-state wage, ω_G and the bad-state wage, ω_B, and for those that are either employed in the Y sector or in the first period of their X-sector employment, we have Y-sector wage, ω^y. The proportion of workers that receive a particular type of wage is given below.

Sector of Employment	Employment Period	Wage	Proportion of Workers
X	2^{nd} or later	$\omega_G(\phi)$	$\rho\pi_G\left(\frac{L-n(\phi)}{L}\right)$
X	2^{nd} or later	$\omega_B(\phi)$	$\rho(1-\pi_G)\left(\frac{L-n(\phi)}{L}\right)$
X	1^{st}	$\omega^y(\phi)$	$(1-\rho)\left(\frac{L-n(\phi)}{L}\right)$
Y	Any	$\omega^y(\phi)$	$\frac{n(\phi)}{L}$

Let Pr_G, Pr_B, and Pr_Y represent the steady-state probability that a worker receives the good-state wage, the bad state wage and the Y-sector wage, respectively. Using the table above, we have

$$\begin{aligned}
Pr_G(\phi) &= \rho\pi_G\left(\frac{L-n(\phi)}{L}\right) \\
Pr_B(\phi) &= \rho(1-\pi_G)\left(\frac{L-n(\phi)}{L}\right) \\
Pr_Y(\phi) &= (1-\rho)\left(\frac{L-n(\phi)}{L}\right)+\frac{n(\phi)}{L} \\
&= 1-\rho\left(\frac{L-n(\phi)}{L}\right).
\end{aligned}$$

Denote $\overline{\omega}(\phi)$ as the expected nominal wage of a worker who might be working either in the X or Y sector. Then, the expected real wage of a worker is given by

$$\begin{aligned} \frac{\overline{\omega}(\phi)}{P(\phi)} &= Pr_G(\phi)\frac{\omega_G(\phi)}{P(\phi)} + Pr_B(\phi)\frac{\omega_B(\phi)}{P(\phi)} + Pr_Y(\phi)\frac{\omega^y(\phi)}{P(\phi)}, \text{ which is} \\ &= (1 - Pr_Y(\phi))\,\frac{E_\epsilon\omega_\epsilon(\phi)}{P(\phi)} + Pr_Y(\phi)\frac{\omega^y(\phi)}{P(\phi)}. \qquad \text{(B.16)} \end{aligned}$$

Denote $\sigma^2_{\frac{\omega}{P}}(\phi)$ the variance of the real wage. It can be easily calculated as

$$\begin{aligned} \sigma^2_{\frac{\omega}{P}}(\phi) &= Pr_G(\phi)\left(\frac{\omega_G(\phi) - \overline{\omega}(\phi)}{P(\phi)}\right)^2 + Pr_B(\phi)\left(\frac{\omega_B(\phi) - \overline{\omega}(\phi)}{P(\phi)}\right)^2 \\ &\quad + Pr_Y(\phi)\left(\frac{\omega^y(\phi) - \overline{\omega}(\phi)}{P(\phi)}\right)^2, \text{ or} \end{aligned}$$

$$\begin{aligned} \sigma^2_{\frac{\omega}{P}}(\phi) &= Pr_G(\phi)\left(\frac{[\omega_G(\phi) - E_\epsilon\omega_\epsilon(\phi)] + [E_\epsilon\omega_\epsilon(\phi) - \overline{\omega}(\phi)]}{P(\phi)}\right)^2 \\ &\quad + Pr_B(\phi)\left(\frac{[\omega_B(\phi) - E_\epsilon\omega_\epsilon(\phi)] + [E_\epsilon\omega_\epsilon(\phi) - \overline{\omega}(\phi)]}{P(\phi)}\right)^2 \\ &\quad + Pr_Y(\phi)\left(\frac{\omega^y(\phi) - \overline{\omega}(\phi)}{P(\phi)}\right)^2. \end{aligned}$$

Using equation (B.16), we can replace $[\omega^y(\phi) - \overline{\omega}(\phi)]$ with $(1 - Pr_Y(\phi))\,[\omega^y(\phi) - E_\epsilon\omega_\epsilon(\phi)]$ and also $[E_\epsilon\omega_\epsilon(\phi) - \overline{\omega}(\phi)]$ with $Pr_Y(\phi)\,[E_\epsilon\omega_\epsilon(\phi) - \omega^y(\phi)]$ to obtain

$$\begin{aligned}
\sigma^2_{\frac{\omega}{P}}(\phi) \;&=\; Pr_G(\phi)\left[\begin{array}{c} \left(\frac{\omega_G(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right)^2 + [Pr_Y(\phi)]^2\left(\frac{E_\epsilon\omega_\epsilon(\phi)-\omega^y(\phi)}{P(\phi)}\right)^2 \\ +2Pr_Y(\phi)\left(\frac{E_\epsilon\omega_\epsilon(\phi)-\omega^y(\phi)}{P(\phi)}\right)\left(\frac{\omega_G(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right) \end{array}\right] \\
&\quad +Pr_B(\phi)\left[\begin{array}{c} \left(\frac{\omega_B(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right)^2 + [Pr_Y(\phi)]^2\left(\frac{E_\epsilon\omega_\epsilon(\phi)-\omega^y(\phi)}{P(\phi)}\right)^2 \\ +2Pr_Y(\phi)\left(\frac{E_\epsilon\omega_\epsilon(\phi)-\omega^y(\phi)}{P(\phi)}\right)\left(\frac{\omega_B(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right) \end{array}\right] \\
&\quad +Pr_Y(\phi)\,[1-Pr_Y(\phi)]^2\left(\frac{\omega^y(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right)^2.
\end{aligned}$$

The above equation can be rearranged as

$$\begin{aligned}
\sigma^2_{\frac{\omega}{P}}(\phi) \;&=\; Pr_G(\phi)\left(\frac{\omega_G(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right)^2 + Pr_B(\phi)\left(\frac{\omega_B(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right)^2 \\
&\quad + [Pr_G(\phi)+Pr_B(\phi)]\,[Pr_Y(\phi)]^2\left(\frac{E_\epsilon\omega_\epsilon(\phi)-\omega^y(\phi)}{P(\phi)}\right)^2 \\
&\quad +2Pr_Y(\phi)\left(\frac{E_\epsilon\omega_\epsilon(\phi)-\omega^y(\phi)}{P(\phi)}\right)\left[\begin{array}{c} Pr_G(\phi)\left(\frac{\omega_G(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right) \\ +Pr_B(\phi)\left(\frac{\omega_B(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right) \end{array}\right] \\
&\quad +Pr_Y(\phi)\,[1-Pr_Y(\phi)]^2\left(\frac{\omega^y(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right)^2.
\end{aligned}$$

Notice that

$$Pr_G(\phi)\left(\frac{\omega_G(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right) + Pr_B(\phi)\left(\frac{\omega_B(\phi)-E_\epsilon\omega_\epsilon(\phi)}{P(\phi)}\right) = 0, \text{ and}$$

$$Pr_G(\phi) + Pr_B(\phi) = 1 - Pr_Y(\phi).$$

Using these two, we obtain

$$
\begin{aligned}
\sigma^2_{\frac{\omega}{P}}(\phi) &= [1 - Pr_Y(\phi)] \left[\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2 + (1-\pi_G)\left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2\right] \\
&\quad + Pr_Y(\phi)\left[1 - Pr_Y(\phi)\right]\left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2, \qquad \text{(B.17)}
\end{aligned}
$$

where on the right-hand side, the first line represents the within-group variance of those earning fluctuating wage in the X sector weighted by its steady-state probability and the second line represents between-group wage variance. Therefore, the variance consists of sum of within group variance and between group variance.

Differentiating equation (B.17) with respect to ϕ, we obtain

$$
\begin{aligned}
d\sigma^2_{\frac{\omega}{P}}(\phi) &= \left\{ \begin{array}{l} [1 - Pr_Y(\phi)]\, d\left[\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2 + (1-\pi_G)\left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2\right] \\ + Pr_Y(\phi)\left[1 - Pr_Y(\phi)\right] d\left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2 \end{array} \right. \\
&\quad \left\{ \begin{array}{l} + \left[\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2 + (1-\pi_G)\left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2\right] d\left[1 - Pr_Y(\phi)\right] \\ + \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)}\right)^2 dPr_Y(\phi)\left[1 - Pr_Y(\phi)\right] \end{array} \right. \qquad \text{(B.18)}
\end{aligned}
$$

where the terms in the first brace represent the 'price' effect, which measures the change in wage volatility while holding industrial composition intact, i.e., keeping the proportion of workers in each sector constant. The terms in the second brace represent the 'compositional' effect, which measures the change in wage volatility in response to changes in sectoral compositions only, i.e., the movement of workers between sectors with different degrees of wage volatility. We can rewrite the 'price' effect as

$$= 2\left[1 - Pr_Y(\phi)\right] \left\{ \begin{array}{l} \pi_G \left(\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 \left[\frac{d\left(\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)}{\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)}\right] \\ +(1-\pi_G)\left(\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 \left[\frac{d\left(\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)}{\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)}\right] \\ +Pr_Y(\phi)\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 \left[\frac{d\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)}{\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)}\right] \end{array} \right\} > 0 \text{ for } d\phi > 0.$$

Using conditions given in (B.15), we can see that the 'price' effect is always positive. We can also rewrite the 'compositional' effect as

$$= \rho \frac{n}{L}\left[-\frac{dn(\phi)}{n}\right]\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 \left\{ \begin{array}{l} \frac{\pi_G\left(\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 + (1-\pi_G)\left(\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2}{\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2} \\ +1 - 2\rho\left(\frac{L-n(\phi)}{L}\right) \end{array} \right\}.$$

Notice that we have $1 - 2\rho\left(\frac{L-n(\phi)}{L}\right) > -1$, since $\rho\left(\frac{L-n(\phi)}{L}\right) = [1 - Pr_Y(\phi)] \in (0,1)$. In addition, we know that $dn(\phi) < 0$ for $d\phi > 0$. Therefore, a sufficient condition for the compositional effect to be non-negative is

$$\frac{\pi_G\left(\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 + (1-\pi_G)\left(\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2}{\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2} \geqslant 1, \text{ or}$$

$$\pi_G(1-\pi_G)\left(\frac{\frac{\omega_G}{P}(\phi) - \frac{\omega_B}{P}(\phi)}{\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)}\right)^2 \geqslant 1.$$

If this condition is satisfied, then the compositional effect is non-negative and economy-wide wage volatility increases as a result of an improvement in search efficiency.

After some algebra, we can rewrite equation (B.18) as

$$d\sigma^2_{\frac{\omega}{P}}(\phi) = \begin{cases} 2Pr_Y(\phi)\left[1 - Pr_Y(\phi)\right]\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2 \left[-\frac{dn(\phi)}{n}\right] \\ * \left[\begin{array}{l} \gamma_G \varepsilon_{(\omega_G - E_\epsilon \omega_\epsilon),n} + \gamma_B \varepsilon_{(\omega_B - E_\epsilon \omega_\epsilon),n} + \varepsilon_{(\omega^y - E_\epsilon \omega_\epsilon),n} \\ + \frac{1}{2}\frac{\frac{n(\phi)}{L}}{1-\frac{n(\phi)}{L}}\left(\gamma_G + \gamma_B + 1 - \frac{1-Pr_Y(\phi)}{Pr_Y(\phi)}\right) \end{array}\right] \end{cases} \text{, where} \tag{B.19}$$

$$\varepsilon_{(\omega_\epsilon - E_\epsilon \omega_\epsilon),n} = \frac{\left[\frac{d\left(\frac{\omega_\epsilon}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)}{\frac{\omega_\epsilon}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)}\right]}{\left[-\frac{dn(\phi)}{n}\right]} > 0,\ \varepsilon_{(\omega^y - E_\epsilon \omega_\epsilon),n} = \frac{\left[\frac{d\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)}{\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)}\right]}{\left[-\frac{dn(\phi)}{n}\right]} > 0,$$

$$\gamma_G = \frac{\pi_G}{Pr_Y(\phi)}\frac{\left(\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2}{\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2} > 0,\ \gamma_B = \frac{(1-\pi_G)}{Pr_Y(\phi)}\frac{\left(\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2}{\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon \omega_\epsilon}{P}(\phi)\right)^2} > 0.$$

Here, ε's represent employment elasticity of wage differences, $(\gamma_G + \gamma_B)$ is the ratio of within group variance (weighted by its steady-state probability) to between group variance, and γ_G and γ_B are the contribution of within group good-state and bad-state wage variance to this ratio. From equation (B.19), we can see that the economy-wide real wage volatility increases as a result of an improvement in search effectiveness ($d\phi > 0$) *iff*

$$\gamma_G \varepsilon_{(\omega_G - E_\epsilon \omega_\epsilon),n} - \gamma_B \varepsilon_{(\omega_B - E_\epsilon \omega_\epsilon),n} + \varepsilon_{(\omega^y - E_\epsilon \omega_\epsilon),n} > -\frac{1}{2}\frac{\frac{n(\phi)}{L}}{1-\frac{n(\phi)}{L}}\left(\gamma_G + \gamma_B + 1 - \frac{1-Pr_Y(\phi)}{Pr_Y(\phi)}\right).$$

Next, consider the wage-smoothing equilibrium. There are two types of wages at the steady state: for those that are in the second or later periods of their X-sector employment, we have the efficiency wage, ω^*, and for those that are either employed in the Y sector or in the first period of their X-sector employment, we have Y-sector wage, ω^y. The proportion of workers that receive a particular type of wage is given below.

Sector of Employment	Employment Period	Wage	Proportion of Workers
X	2^{nd} or later	$\omega^*(\phi)$	$\rho\left(\frac{L-n(\phi)}{L}\right)$
X	1^{st}	$\omega^y(\phi)$	$(1-\rho)\left(\frac{L-n(\phi)}{L}\right)$
Y	Any	$\omega^y(\phi)$	$\frac{n(\phi)}{L}$

Using the table above, we have $Pr_Y(\phi) = 1 - \rho\left(\frac{L-n(\phi)}{L}\right)$ as before and the expected real wage of a worker who might be working either in the X or Y sector is given by

$$\frac{\overline{\omega}(\phi)}{P} = (1 - Pr_Y(\phi))\,\frac{\omega^*(\phi)}{P(\phi)} + Pr_Y(\phi)\frac{\omega^y(\phi)}{P(\phi)}. \tag{B.20}$$

The variance of the real wage can be easily calculated as

$$\sigma^2_{\frac{\omega}{P}}(\phi) = [1 - Pr_Y(\phi)]\left(\frac{\omega^*(\phi) - \overline{\omega}(\phi)}{P(\phi)}\right)^2 + Pr_Y(\phi)\left(\frac{\omega^y(\phi) - \overline{\omega}(\phi)}{P(\phi)}\right)^2.$$

Using equation (B.20), we can replace $[\omega^y(\phi) - \overline{\omega}(\phi)]$ with $(1 - Pr_Y(\phi))\,[\omega^y(\phi) - \omega^*(\phi)]$ and also $[\omega^*(\phi) - \overline{\omega}(\phi)]$ with $Pr_Y(\phi)\,[\omega^*(\phi) - \omega^y(\phi)]$ to obtain

$$\sigma^2_{\frac{\omega}{P}}(\phi) = Pr_Y(\phi)\,[1 - Pr_Y(\phi)]\left(\frac{\omega^*(\phi) - \omega^y(\phi)}{P(\phi)}\right)^2. \tag{B.21}$$

Differentiating equation (B.21) with respect to ϕ, we obtain

$$\begin{aligned} d\sigma^2_{\frac{\omega}{P}}(\phi) \;=\; & \Bigg\{ Pr_Y(\phi)\,[1 - Pr_Y(\phi)]\,d\left(\frac{\omega^*(\phi) - \omega^y(\phi)}{P(\phi)}\right)^2 \\ & \Bigg\{ + \left(\frac{\omega^*(\phi) - \omega^y(\phi)}{P(\phi)}\right)^2 dPr_Y(\phi)\,[1 - Pr_Y(\phi)] \end{aligned} \tag{B.22}$$

As before, the terms in the first brace represent the 'price' effect and the terms in the second brace represent the 'compositional' effect. We can rewrite the 'price' effect as

$$= 2Pr_Y(\phi)\left[1 - Pr_Y(\phi)\right]\left(\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi)\right)^2 \left[\frac{d\left(\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi)\right)}{\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi)}\right] > 0, \text{ for } d\phi > 0.$$

Again using condition given in (B.15), we can see that the 'price' effect is always positive. We can also rewrite the 'compositional' effect as

$$= \rho\frac{n}{L}\left[-\frac{dn(\phi)}{n}\right]\left(\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi)\right)^2 \left[1 - 2\rho\left(\frac{L - n(\phi)}{L}\right)\right].$$

Notice that we have $1 - 2\rho\left(\frac{L-n(\phi)}{L}\right) = 1 - 2\left[1 - Pr_Y(\phi)\right]$. As before, we also know that $dn(\phi) < 0$, for $d\phi > 0$. Thus, a sufficient condition for the compositional effect to be non-negative is

$$\begin{aligned} 1 - 2\left[1 - Pr_Y(\phi)\right] &\geqslant 0, \text{ or} \\ Pr_Y(\phi) &\geqslant \frac{1}{2}. \end{aligned}$$

If this condition is satisfied, then the compositional effect is non-negative and economy-wide real wage volatility increases as a result of an improvement in search efficiency.

After some algebra, we can rewrite equation (B.22) as

$$\text{(B.23)} \qquad d\sigma^2_{\frac{\omega}{P}}(\phi) = \begin{cases} 2Pr_Y(\phi)\left[1 - Pr_Y(\phi)\right]\left(\frac{\omega^y}{P}(\phi) - \frac{\omega^*}{P}(\phi)\right)^2\left[-\frac{dn(\phi)}{n}\right] \\ \\ * \left[\varepsilon_{(\omega^y - \omega^*),n} + \frac{1}{2}\frac{\frac{n(\phi)}{L}}{1 - \frac{n(\phi)}{L}}\left(1 - \frac{1 - Pr_Y(\phi)}{Pr_Y(\phi)}\right)\right], \end{cases}$$

where $\varepsilon_{(\omega^y-\omega^*),n} = \frac{\left[\frac{d\left(\frac{\omega^y}{P}(\phi)-\frac{\omega^*}{P}(\phi)\right)}{\frac{\omega^y}{P}(\phi)-\frac{\omega^*}{P}(\phi)}\right]}{\left[-\frac{dn}{n}(\phi)\right]} > 0$. From equation (B.23), we can see that the economy-wide real wage volatility increases as a result of an improvement in search efficiency ($d\phi > 0$) *iff*

$$\varepsilon_{(\omega^y-\omega^*),n} > -\frac{1}{2}\frac{\frac{n(\phi)}{L}}{1-\frac{n(\phi)}{L}}\left(1-\frac{1-Pr_Y(\phi)}{Pr_Y(\phi)}\right). \blacksquare$$

APPENDIX C

Appendix to Chapter 2

Proof of Proposition 1.

The Kuhn-Tucker multiplier for (2.5) be denoted by ψ_ϵ^{Ri}, the multiplier for (2.6) by υ_ϵ^{Ri}, multiplier for (2.7) by λ, multiplier for (2.8) by χ_ϵ^{Ri}. The superscript, i, refers to the period of the relationship. The first order conditions in the **first period** with respect to ω_ϵ^{R1} and $\widetilde{W}_\epsilon^{R1}$ are

$$\pi_\epsilon - \psi_\epsilon^{R1} + \upsilon_\epsilon^{R1} \frac{\mu'\left(\omega_\epsilon^{R1} + \omega^h\right) - \mu'\left(\omega_\epsilon^{R1} + \omega^u\right)}{\rho\beta} + \lambda^{R1}\pi_\epsilon\mu'\left(\omega_\epsilon^{R1} + \omega^h\right) + \chi_\epsilon^{R1} = 0 \tag{C.1}$$

$$\rho\beta\pi_\epsilon\Omega'\left(\widetilde{W}_\epsilon^{R1}\right) + \rho\beta\psi_\epsilon^{R1}\Omega'\left(\widetilde{W}_\epsilon^{R1}\right) + \upsilon_\epsilon^{R1} + \rho\beta\lambda^{R1}\pi_\epsilon = 0 \tag{C.2}$$

The first order conditions in the **second period** with respect to ω_ϵ^{R2} and $\widetilde{W}_\epsilon^{R2}$ are

$$-\pi_\epsilon - \psi_\epsilon^{R2} + \lambda^{R2}\pi_\epsilon\mu'\left(\omega_\epsilon^{R2}\right) + \chi_\epsilon^{R2} = 0 \tag{C.3}$$

$$\rho\beta\pi_\epsilon\Omega'\left(\widetilde{W}_\epsilon^{R2}\right) + \rho\beta\psi_\epsilon^{R2}\Omega'\left(\widetilde{W}_\epsilon^{R2}\right) + \upsilon_\epsilon^{R2} + \rho\beta\lambda^{R2}\pi_\epsilon = 0 \tag{C.4}$$

The first order conditions in the **third and subsequent periods** with respect to ω_ϵ^{R3} and $\widetilde{W}_\epsilon^{R3}$ are

$$-\pi_\epsilon - \psi_\epsilon^{R3} + \lambda^{R3}\pi_\epsilon\mu'\left(\omega_\epsilon^{R3}\right) + \chi_\epsilon^{R3} = 0 \tag{C.5}$$

$$\rho\beta\pi_\epsilon\Omega'\left(\widetilde{W}_\epsilon^{R3}\right) + \rho\beta\psi_\epsilon^{R3}\Omega'\left(\widetilde{W}_\epsilon^{R3}\right) + \upsilon_\epsilon^{R3} + \rho\beta\lambda^{R3}\pi_\epsilon = 0 \tag{C.6}$$

In addition, there is an envelope condition for first, second, third and subsequent periods

$$\lambda^{R1} + \Omega'(W_0) = 0 \tag{C.7}$$

$$\lambda^{R2} + \Omega'(W_1) = 0 \tag{C.8}$$

$$\lambda^{R3} + \Omega'(W_2) = 0 \tag{C.9}$$

where

$$\begin{aligned} W_0 &= V^{WS} - \mu(\omega^u) + E_\epsilon \mu\left(\omega_\epsilon^{R1} + \omega^u\right) \\ W_1 &= \widetilde{W}_\epsilon^{R1} \\ W_2 &= \widetilde{W}_\epsilon^{R2} \end{aligned}$$

Since $\Omega'(W) < 0$, for envelope condition to hold, we must have λ^{R1}, λ^{R2}, $\lambda^{R3} > 0$, hence the target utility constraint (2.7) always binds.

Employer will choose a target utility, W_0 at least as much as $V^{WSHIRK} = V^{WS} - \mu(\omega^u) + E_\epsilon \mu\left(\omega_\epsilon^{R1} + \omega^u\right)$. Employer will choose a ω_ϵ^{R1} that will minimize $V^{WS} - \mu(\omega^u) + E_\epsilon \mu\left(\omega_\epsilon^{R1} + \omega^u\right)$. So, $\omega_G^{*R1} = \omega_B^{*R1} = 0$ and first period $V^{WSHIRK} = V^{WS}$. Now the first period worker's ICC (2.6a) is $\widetilde{W}_\epsilon^{R1} \geq \widetilde{W}^* - \frac{\mu(\omega^h) - \mu(\omega^u)}{\rho\beta}$.

Here, we will show that first period worker's ICC (2.6a) always binds, $\upsilon_\epsilon^{R1} > 0$. Now, we return to the first order conditions. By the Assumption 2, $\psi_\epsilon^{R1} = 0$. Suppose that the worker's ICC does not bind in state ϵ ($\upsilon_\epsilon^{R1} = 0$) in the first period. Then, equation (C.2) becomes $\Omega'\left(\widetilde{W}_\epsilon^{R1}\right) + \lambda^{R1} = 0$. The envelope condition given in equation (C.7) implies that in the first period we have $\Omega'\left(V^{WS}\right) + \lambda^{R1} = 0$ given that $W_0 = V^{WS}$. This means

$\widetilde{W}_\epsilon^{R1} = V^{WS}$. But since $\widetilde{W}_\epsilon^{R1} > \widetilde{W}^* - \frac{k}{\rho\beta} > V^{WS}$ by the worker's ICC (2.6a), this implies that the worker's ICC (2.6a) will be violated, a contradiction. Therefore, the worker's ICC (2.6a) must bind in each state, ensuring that $\widetilde{W}_\epsilon^{R1} = \widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}$.

Consider now the second-period problem. As before due to the envelope condition, the target utility constraint (2.7) binds. We know that the second period target utility, W_1, is equal to the first period continuation pay-off for the worker, $\widetilde{W}_\epsilon^{R1} = \widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}$. We claim that the choice of third period target utility, $W_2 = \widetilde{W}_\epsilon^{R2}$ will be equal to $\widetilde{W}^*$ for $\epsilon = G, B$. If $\upsilon_\epsilon^{R2} > 0$, then complementary slackness implies that $\widetilde{W}_\epsilon^{R2} = \widetilde{W}^*$. Therefore, suppose that $\upsilon_\epsilon^{R2} = 0$. This implies that condition (C.4) becomes $\Omega'\left(\widetilde{W}_\epsilon^{R2}\right) = -\lambda^{R2}\frac{\pi_\epsilon}{\pi_\epsilon+\psi_\epsilon^{R2}}$. Since, by the envelope condition, $\lambda^{R2} = -\Omega'\left(\widetilde{W}_\epsilon^{R1}\right)$, and as we recall the binding first-period worker's ICC (2.6a), $\widetilde{W}_\epsilon^{R1} = \widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}$, this becomes

$$\text{(C.10)} \qquad \Omega'\left(\widetilde{W}_\epsilon^{R2}\right) = \Omega'\left(\widetilde{W}^* - \frac{\mu\left(\omega^h\right)-\mu\left(\omega^u\right)}{\rho\beta}\right)\frac{\pi_\epsilon}{\pi_\epsilon+\psi_\epsilon^{R2}}.$$

If $\psi_\epsilon^{R2} = 0$, equation (C.10) then implies that $\widetilde{W}_\epsilon^{R2} = \widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}$, implying $\widetilde{W}_\epsilon^{R2} < \widetilde{W}^*$. However, this violates second and subsequent period worker's ICC (2.6b). Similarly, if $\psi_\epsilon^{R2} > 0$, equation (C.10) then implies that $0 > \Omega'\left(\widetilde{W}_\epsilon^{R2}\right) > \Omega'\left(\widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}\right)$, implying $\widetilde{W}_\epsilon^{R2} < \widetilde{W}^*$. However, this violates second and subsequent period worker's ICC (2.6b). Therefore, all possibilities either imply that $\widetilde{W}_\epsilon^{R2} = \widetilde{W}^*$ and $\upsilon_\epsilon^{R2} > 0$ or lead to a contradiction.

Since $\widetilde{W}_\epsilon^{R2} = \widetilde{W}_\epsilon^{R3} = \widetilde{W}^*$, the optimization problem in the fourth and subsequent periods of the relationship is identical to that of the third period. By induction, the target utility for the worker in every period after the second, regardless of history, is equal to $\widetilde{W}^*$, and so the wage chosen for each state in every period after the second, regardless of history, is the same. Thus third and subsequent period Kuhn-Tucker multipliers are equal. Target utility in the second and third and subsequent periods are

$$\begin{aligned} W_1 &= \widetilde{W}_\epsilon^{R1} = \widetilde{W}^* - \frac{\mu\left(\omega^h\right) - \mu\left(\omega^u\right)}{\rho\beta} \\ W_2 &= \widetilde{W}_\epsilon^{R2} = \widetilde{W}^* \end{aligned}$$

We can rewrite the Kuhn-Tucker conditions given in (C.1)-(C.6) below by using the envelope conditions $\lambda^{R2} = -\Omega'\left(\widetilde{W}_\epsilon^{R1}\right)$ and $\lambda^{R3} = -\Omega'\left(\widetilde{W}_\epsilon^{R2}\right)$. Also we use the fact that after the first period continuum utility, $\widetilde{W}_\epsilon$, is stationary, thus $\lambda^{R3} = \lambda^{R4}$.

After rewriting the first order conditions in the **first period** with respect to ω_ϵ^{R1} and $\widetilde{W}_\epsilon^{R1}$ we obtain

$$-\pi_\epsilon + \upsilon_\epsilon^{R1}\frac{\mu'\left(\omega^h\right) - \mu'\left(\omega^u\right)}{\rho\beta} + \lambda^{R1}\pi_\epsilon\mu'\left(\omega^h\right) + \chi_\epsilon^{R1} = 0 \tag{C.11}$$

$$\rho\beta\pi_\epsilon\left(\lambda^{R2} - \lambda^{R1}\right) = \upsilon_\epsilon^{R1} \tag{C.12}$$

After rewriting the first order conditions in the **second period** with respect to ω_ϵ^{R2} and $\widetilde{W}_\epsilon^{R2}$ we obtain

$$-\pi_\epsilon - \psi_\epsilon^{R2} + \lambda^{R2}\pi_\epsilon\mu'\left(\omega_\epsilon^{R2}\right) + \chi_\epsilon^{R2} = 0 \tag{C.13}$$

$$\rho\beta\pi_\epsilon\left(\lambda^{R3} - \lambda^{R2}\right) + \rho\beta\psi_\epsilon^{R2}\lambda^{R3} = \upsilon_\epsilon^{R2} \tag{C.14}$$

After rewriting the first order conditions in the **third period** with respect to ω_ϵ^{R2} and $\widetilde{W}_\epsilon^{R2}$ we obtain

$$-\pi_\epsilon - \psi_\epsilon^{R3} + \lambda^{R3}\pi_\epsilon\mu'\left(\omega_\epsilon^{R3}\right) + \chi_\epsilon^{R3} = 0 \tag{C.15}$$

$$\rho\beta\psi_\epsilon^{R3}\lambda^{R3} = \upsilon_\epsilon^{R3} \text{ since } \lambda^{R3} = \lambda^{R4} \tag{C.16}$$

Now, to establish the three possible outcomes stated in Proposition 1, we consider each possible case in turn. Consider the optimization problem (2.4) at any date after the first period of relationship. First, suppose that the employer's ICC (2.5) does not bind in either state. In this case, ψ_ϵ^{R2}, $\psi_\epsilon^{R3} = 0$ for $\epsilon = G, B$. First order conditions in the **second period** and the **third period** with respect to ω_ϵ^{R2}, (C.13) and (C.15), now become

$$-\pi_\epsilon + \lambda^{R2}\pi_\epsilon\mu'\left(\omega_\epsilon^{R2}\right) + \chi_\epsilon^{R2} = 0 \tag{C.17}$$

$$-\pi_\epsilon + \lambda^{R3}\pi_\epsilon\mu'\left(\omega_\epsilon^{R3}\right) + \chi_\epsilon^{R3} = 0 \tag{C.18}$$

If $\chi_\epsilon^{R2} > 0$ for some ϵ, then $\omega_\epsilon^{R2} = 0$. This clearly cannot be true for both values of ϵ, because it would not be possible to satisfy target utility constraint (2.7). (To see this, formally, substitute $W_1 = \widetilde{W}_\epsilon^{R1} = \widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}$, $\widetilde{W}_\epsilon^{R2} = \widetilde{W}^*$, the expression for V^{WS}, and $\omega_G^{R2} = \omega_B^{R2} = 0$ into target utility constraint (2.7), and note that the constraint is violated.) Therefore, for at most one state, say ϵ', $\chi_{\epsilon'}^{R2} > 0$. Denote by ϵ'' the state with $\chi_{\epsilon''}^{R2} = 0$. Then $\mu'(0) = \frac{1}{\lambda}\left(1 - \frac{\chi_{\epsilon'}}{\pi_{\epsilon'}}\right) < \frac{1}{\lambda} = \mu'(\omega_{\epsilon''})$ by equation (C.17). However, given that $\omega_{\epsilon''}$ is non-negative and μ is strictly concave, this is impossible. We conclude that $\chi_\epsilon^{R2} = 0$ in both states, and therefore $\omega_G^{R2} = \omega_B^{R2}$. By induction, the same argument is true for the third and subsequent periods, as well. Given that $\lambda^{R2} = -\Omega'\left(\widetilde{W}^* - \frac{\mu(\omega^h)-\mu(\omega^u)}{\rho\beta}\right)$

and $\lambda^{R3} = -\Omega'\left(\widetilde{W}^*\right)$, $\lambda^{R3} > \lambda^{R2}$. We plug the χ_ϵ^{R2}, $\chi_\epsilon^{R3} = 0$ into the above equations (C.17) and (C.18), obtain $\lambda^{R2} = \frac{1}{\mu'(\omega_\epsilon^{R2})}$ and $\lambda^{R3} = \frac{1}{\mu'(\omega_\epsilon^{R3})}$ for $\epsilon = G, B$. Then we will have $\omega_B^{R3} = \omega_G^{R3} > \omega_G^{R2} = \omega_B^{R2}$ while ψ_ϵ^{R2}, $\psi_\epsilon^{R3} = 0$ for $\epsilon = G, B$.

Next, suppose that we have $\psi_G^{R2} > 0$ and $\psi_B^{R2} = 0$, so that the second period employer's constraint binds only in the good state. We will show that this leads to a contradiction. Recall from the previous discussion that $\widetilde{W}_\epsilon^{R2} = \widetilde{W}^*$ for both states, and note that, by assumption, constraint (2.5) is satisfied with equality for $\epsilon = G$. Since $x_B < x_G$, we now see that constraint (2.5) must be violated for $\epsilon = B$ if $\omega_G^{R2} \leq \omega_B^{R2}$. Therefore, $\omega_G^{R2} > \omega_B^{R2} \geq 0$. This implies that $\chi_G^{R2} = 0$. Applying condition (C.13), then, we have

$$\mu'(\omega_G^{R2}) = \frac{1}{\lambda^{R2}}\left(1 + \frac{\psi_G^{R2}}{\pi_G}\right) > \frac{1}{\lambda^{R2}}\left(1 - \frac{\chi_B^{R2}}{\pi_B}\right) = \mu'(\omega_B^{R2}),$$

which contradicts the requirement that $\omega_G^{R2} > \omega_B^{R2}$. This shows that it is not possible for the employer's constraint to bind only in the good state. The same argument is true for the third period, as well.

Now suppose that we have $\psi_G^{R2} = 0$ and $\psi_B^{R2} > 0$, so that the employer's constraint binds only in the bad state. We now wish to prove that in this case $\omega_G^{R2} > \omega_B^{R2}$. Suppose to the contrary that $\omega_G^{R2} \leq \omega_B^{R2}$. This implies that $\omega_B^{R2} > 0$ (since, as shown earlier, it is not possible to have zero wage in both states), so that $\chi_B^{R2} = 0$. Then, from condition (C.13)

$$\mu'(\omega_B^{R2}) = \frac{1}{\lambda^{R2}}\left(1 + \frac{\psi_B^{R2}}{\pi_B}\right) > \frac{1}{\lambda^{R2}}\left(1 - \frac{\chi_G^{R2}}{\pi_G}\right) = \mu'(\omega_G^{R2}),$$

which implies that $\omega_G^{R2} > \omega_B^{R2}$. Therefore, we have a contradiction, and we conclude that $\omega_G^{R2} > \omega_B^{R2}$. The same argument is true for the third period, as well ($\omega_G^{R3} > \omega_B^{R3}$). If

ψ_B^{R2}, $\psi_B^{R3} > 0$ and ψ_G^{R2}, $\psi_G^{R3} = 0$ then (2.5) is going to bind in the bad state in second and third period which implies $\omega_B^{R2} = \omega_B^{R3}$ and obtain $\lambda^{R2} = \frac{1}{\mu'\left(\omega_G^{R2}\right)}$ and $\lambda^{R3} = \frac{1}{\mu'\left(\omega_G^{R3}\right)}$. Given that $\lambda^{R2} = -\Omega'\left(\widetilde{W}^* - \frac{\mu\left(\omega^h\right)-\mu(\omega^u)}{\rho\beta}\right)$ and $\lambda^{R3} = -\Omega'\left(\widetilde{W}^*\right)$, $\lambda^{R3} > \lambda^{R2}$. So, $\omega_G^{R3} > \omega_G^{R2} > \omega_B^{R2} = \omega_B^{R3}$ if ψ_B^{R2}, $\psi_B^{R3} > 0$ and ψ_G^{R2}, $\psi_G^{R3} = 0$.

Suppose that $\psi_B^{R2} > 0$, $\psi_G^{R2} = 0$ and $\psi_\epsilon^{R3} = 0$ for $\epsilon = G$. This implies $\omega_B^{R3} = \omega_G^{R3} > \omega_G^{R2} > \omega_B^{R2}$ (by using the $\lambda^{R3} > \lambda^{R2}$ by the envelope theorem). Then since employer's ICC in the bad state (2.5) is binding ($\psi_B^{R2} > 0$) in the second period, third period employer's ICC in the bad state will be violated. Thus, $\psi_B^{R2} > 0$ and $\psi_B^{R3} = 0$ is not feasible and when $\psi_B^{R2} > 0$, we have to have $\psi_B^{R3} > 0$ or when $\psi_B^{R3} = 0$, we have to have $\psi_B^{R2} = 0$.

Also, we will always have $\psi_G^{R2} = 0$. Otherwise, employer's ICC (2.5) at good state in the third period will be violated. Lastly, employer's ICC (2.5) may bind at both states in the third period. Equality at both states for employer's ICC (2.5) requires that short-term profits $x_\epsilon - \omega_\epsilon^{R3*}$ are equal at two states.

We have thus eliminated all possibilities aside from those listed in the statement of the proposition.

Proof of Lemma 1.

We conclude that $\chi_\epsilon^{R2} = 0$ in both states and $\psi_G^{R2} = 0$ (from Proposition 1). So, the first order condition in the **second period** with respect to ω_ϵ^{R2} (C.13) when $\epsilon = G$ becomes $\lambda^{R2} = \frac{1}{\mu'\left(\omega_G^{R2}\right)}$. Then, we plug $\lambda^{R2} = \frac{1}{\mu'\left(\omega_G^{R2}\right)}$ and $\chi_B^{R2} = 0$ into equation (C.17) when $\epsilon = B$, and obtain

$$\psi_B^{R2} = (1 - \pi_G)\left[\frac{\mu'\left(\omega_B^{R2}\right)}{\mu'\left(\omega_G^{R2}\right)} - 1\right]$$

Similarly, the first order condition in the **third and subsequent periods** with respect to ω_ϵ^{R3} equation (C.18) when $\epsilon = B$ becomes

$$\psi_B^{R3} = (1-\pi_G)\left[\frac{\mu'\left(\omega_B^{R3}\right)}{\mu'\left(\omega_G^{R3}\right)} - 1\right]$$

Proof of Lemma 2.

We will find V^{ER}, recursively. We write the Value Function of an employer in a relationship in the first, second, third and subsequent period, respectively as

$$V^{ER} = \begin{cases} E_\epsilon x_\epsilon - E_\epsilon \omega_\epsilon^{*R1} - (u+h) \\ +\rho\beta E_\epsilon \Omega\left(\widetilde{W}_\epsilon^{*R1}\right) + (1-\rho)\beta\left(V^{ES} - \beta c\right) \end{cases} \tag{C.19}$$

$$\Omega\left(\widetilde{W}_\epsilon^{*R1}\right) = \begin{cases} E_\epsilon x_\epsilon - E_\epsilon \omega_\epsilon^{*R2} - u \\ +\rho\beta E_\epsilon \Omega\left(\widetilde{W}_\epsilon^{*R2}\right) + (1-\rho)\beta\left(V^{ES} - \beta c\right) \end{cases} \tag{C.20}$$

$$\Omega\left(\widetilde{W}_\epsilon^{*R2}\right) = \begin{cases} E_\epsilon x_\epsilon - E_\epsilon \omega_\epsilon^{*R3} - u \\ +\rho\beta E_\epsilon \Omega\left(\widetilde{W}_\epsilon^{*R3}\right) + (1-\rho)\beta\left(V^{ES} - \beta c\right) \end{cases} \tag{C.21}$$

We use the information, $\Omega\left(\widetilde{W}_\epsilon^{*R2}\right) = \Omega\left(\widetilde{W}_\epsilon^{*R3}\right)$ (from Proposition 1), to isolate $\Omega\left(\widetilde{W}_\epsilon^{*R2}\right)$ in equation (C.21) and obtain

$$\Omega\left(\widetilde{W}_\epsilon^{*R2}\right) = \begin{cases} \frac{E_\epsilon x_\epsilon - E_\epsilon \omega_\epsilon^{*R3} - u - \beta(1-\rho)\beta c}{(1-\rho\beta)} \\ +\frac{(1-\rho)\beta}{(1-\rho\beta)} V^{ES} \end{cases} \tag{C.22}$$

We substitute (C.22) into the equation (C.20) , rearrange the terms, and obtain

$$\Omega\left(\widetilde{W}_\epsilon^{*R1}\right) = \begin{cases} \frac{E_\epsilon x_\epsilon - \left((1-\rho\beta)E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) - u - \beta(1-\rho)\beta c}{(1-\rho\beta)} \\ +\frac{(1-\rho)\beta}{(1-\rho\beta)} V^{ES} \end{cases} \tag{C.23}$$

We now substitute equation (C.23) into the equation (C.19) and obtain

$$V^{ER} = \begin{cases} E_\epsilon x_\epsilon - E_\epsilon \omega_\epsilon^{*R1} - (u+h) - \beta\,(1-\rho)\,\beta c \\ +\rho\beta \frac{E_\epsilon x_\epsilon - \left((1-\rho\beta)E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) - u - \beta(1-\rho)\beta c}{(1-\rho\beta)} \\ +\frac{(1-\rho)\beta}{(1-\rho\beta)} V^{ES} \end{cases}$$

We multiply and divide the $\left(E_\epsilon x_\epsilon - E_\epsilon \omega_\epsilon^{*R1} - (u+h) - \beta\,(1-\rho)\,\beta c\right)$ term with $(1-\rho\beta)$ and rearrange them. Then, we obtain

$$V^{ER} = \frac{\begin{cases} E_\epsilon x_\epsilon - (1-\rho\beta)\,E_\epsilon \omega_\epsilon^{*R1} - u - (1-\rho\beta)\,h - \beta\,(1-\rho)\,\beta c \\ -\rho\beta\left((1-\rho\beta)\,E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) \\ +\,(1-\rho)\,\beta V^{ES} \end{cases}}{(1-\rho\beta)} \tag{C.24}$$

We know $E_\epsilon \omega_\epsilon^{*R1} = 0$, so we simplify the equation (C.24) further and obtain

$$V^{ER} = \frac{\begin{cases} E_\epsilon x_\epsilon - u - (1-\rho\beta)\,h - \beta\,(1-\rho)\,\beta c \\ -\rho\beta\left((1-\rho\beta)\,E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) \\ +\,(1-\rho)\,\beta V^{ES} \end{cases}}{(1-\rho\beta)} \tag{C.25}$$

We express the equation (C.25) as

$$V^{ER} = \frac{1}{(1-\rho\beta)} X + \frac{(1-\rho)\beta}{(1-\rho\beta)} V^{ES} \tag{C.26}$$

where

$$X = E_\epsilon x_\epsilon - \rho\beta\left((1-\rho\beta)\,E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) - u - \beta\,(1-\rho)\,\beta c - (1-\rho\beta)\,h \tag{C.27}$$

We substitute equation (C.26) into the equation (2.2) and obtain

$$V^{ES} = \frac{Q^E \rho\beta}{(1-\beta)\left(1-\left(1-Q^E\right)\rho\beta\right)} X \tag{C.28}$$

We now substitute equation (C.28) into the equation (C.26) and obtain

$$V^{ER} = \frac{\left(1-\left(1-Q^E\rho\right)\beta\right)}{(1-\beta)\left(1-\left(1-Q^E\right)\rho\beta\right)} X \tag{C.29}$$

where, X is as in equation (C.27).

Value Function of an employer in a relationship in the second, third and subsequent periods (C.20 and C.21) can be written by using equations (C.22), (C.23), and (C.25)

$$\begin{aligned}
\Omega\left(\widetilde{W}_\epsilon^{*R1}\right) &= V^{ER} - \left((1-\rho\beta) E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) + h \\
\Omega\left(\widetilde{W}_\epsilon^{*R2}\right) &= V^{ER} + \left(\rho\beta E_\epsilon \omega_\epsilon^{*R2} - (1+\rho\beta) E_\epsilon \omega_\epsilon^{*R3}\right) + h
\end{aligned}$$

Proof of Lemma 3.

We can further refine the employer's ICCs ((2.5a) and (2.5b)) by substituting equation (2.21) into inequalities (2.5a) and (2.5b) to obtain

$$\omega_B^{R2}, \omega_B^{R3} \leq \frac{K}{\left(1-\left(1-Q^E\right)\rho\beta\right)} \tag{C.30}$$

where

$$K = \begin{bmatrix} x_B + \pi_G \rho\beta\left(1-Q^E\right)(x_G - x_B) \\ +\rho\beta Q^E \rho\beta E_\epsilon \omega_\epsilon^{*R2} - \rho\beta\left(1+Q^E\rho\beta\right) E_\epsilon \omega_\epsilon^{*R3} \\ -u + Q^E\rho\beta h + \left((1-\beta)+Q^E\rho\beta\right)\beta c \end{bmatrix}$$

The determination of the third period equilibrium wages can be summarized in Figure 1.1. Slightly adapted to this essay, the horizontal axis shows the good-state wage, ω_G^{R3}, and the vertical axis shows the bad-state wage, ω_B^{R3}. The second equation in target utility constraint (2.7b) states that in any period after the second, the expected utility promised to a worker must be enough to compensate that worker, in expected value, for the current disutility of effort. This equation is represented in Figure 1.1 by the downward-sloping curve WW, which is strictly convex due to the worker's risk aversion. Employer's ICC at bad state constraint C.30 is the downward sloping line EE in the figure. Notice that EE line will shift up by an increase in second period good state wage, ω_G^{R2}, in partially/fully Wage Fluctuating agreement.

In particular, when we have fully Wage Fluctuating agreement, then $\omega_B^{R2} = \omega_B^{R3} < \omega_G^{R2} < \omega_G^{R3}$ employer's ICC in the bad state, (2.5a) and (2.5b), can be written as

$$\omega_B^{R2} = \omega_B^{R3} \leq \frac{K}{\left(1 - \left(1 - Q^E\right)\rho\beta\right)}$$

where,

$$K = -\left(1 - \pi_G\right)\rho\beta\omega_B^{R3} + \left[\begin{array}{c} x_B + \pi_G\rho\beta\left(1 - Q^E\right)\left(x_G - x_B\right) \\ +\pi_G\rho\beta\left(Q^E\rho\beta\omega_G^{R2} - \left(1 + Q^E\rho\beta\right)\omega_G^{R3}\right) \\ -u + Q^E\rho\beta h + \left(\left(1-\beta\right) + Q^E\rho\beta\right)\beta c \end{array}\right]$$

The slope of the EE line and WW curve in Figure 1.1 will be equal to

$$\begin{aligned} \frac{d\omega_B^{R3}}{d\omega_G^{R3}} &= -\frac{\pi_G\left(1 + Q^E\rho\beta\right)\rho\beta}{\left(1 - \left(1 - Q^E\right)\rho\beta\right) + \left(1 - \pi_G\right)\rho\beta} \\ \frac{d\omega_B^{R3}}{d\omega_G^{R3}} &= -\frac{\pi_G\mu^{/}\left(\omega_G^{R3}\right)}{\left(1 - \pi_G\right)\mu^{/}\left(\omega_B^{R3}\right)} \end{aligned}$$

Also, we can rewrite the slope of WW curve as $\frac{d\omega_B^{R3}}{d\omega_G^{R3}} = -\frac{\pi_G}{\psi_B^{R3}+(1-\pi_G)}$ by using the equation (2.15). Since we have to have $-\frac{\pi_G}{\psi_B^{R3}+(1-\pi_G)} < -\frac{\pi_G\left(1+Q^E\rho\beta\right)\rho\beta}{(1-(1-Q^E)\rho\beta)+(1-\pi_G)\rho\beta}$ for a feasible solution, then $\psi_B^{R3} < \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta}$, while in fully Wage Fluctuating agreement. If we have partially Wage Fluctuating agreement ($\omega_G^{R2} = \omega_B^{R2} < \omega_B^{R3}$), then only third period employer's ICC (2.5b) will be relevant and

$$\omega_B^{R3} \leq \frac{K}{(1-(1-Q^E)\,\rho\beta)}$$

where,

$$K = -(1-\pi_G)\,\rho\beta\left(1+Q^E\rho\beta\right)\omega_B^{R3} + \begin{bmatrix} x_B + \pi_G\rho\beta\left(1-Q^E\right)(x_G - x_B) \\ +\rho\beta Q^E\rho\beta E_\epsilon\omega_\epsilon^{*R2} - \pi_G\rho\beta\left(1+Q^E\rho\beta\right)\omega_G^{R3} \\ -u + Q^E\rho\beta h + \left((1-\beta)+Q^E\rho\beta\right)\beta c \end{bmatrix}$$

Also, the slope of the EE line and WW curve in Figure 1.1 will be equal to

$$\begin{aligned} \frac{d\omega_B^{R3}}{d\omega_G^{R3}} &= -\frac{\pi_G\left(1+Q^E\rho\beta\right)\rho\beta}{(1-(1-Q^E)\,\rho\beta)+(1-\pi_G)\,\rho\beta\,(1+Q^E\rho\beta)} \\ \frac{d\omega_B^{R3}}{d\omega_G^{R3}} &= -\frac{\pi_G}{\psi_B^{R3}+(1-\pi_G)} \end{aligned}$$

Since we have to have $-\frac{\pi_G}{\psi_B^{R3}+(1-\pi_G)} < -\frac{\pi_G\left(1+Q^E\rho\beta\right)\rho\beta}{(1-(1-Q^E)\rho\beta)+(1-\pi_G)\rho\beta(1+Q^E\rho\beta)}$ for a feasible solution, then $\psi_B^{R3} < \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}$, while in partial Wage Fluctuating agreement. It is easy to see the upper bound of ψ_B^{R3} while we have fully Wage Fluctuating agreement is lower than the upper bound of ψ_B^{R3} when we have partially Wage Fluctuating agreement.

$$\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)\,Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\,\rho\beta} < \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\,\rho\beta}$$

Proof of Lemma 4.

First period employer's ICC (2.5) at bad state is

$$x_B - \omega_B^{R1} - u - h + \rho\beta\Omega(\widetilde{W}_\epsilon^{R1}) + (1-\rho)\,\beta\left(V^{ES} - \beta c\right) \geq V^{ES}$$

After we rewrite the first two terms as $x_B = E_\epsilon x_\epsilon - \pi_G\,(x_G - x_B)$ and $\omega_B^{R1} = E_\epsilon \omega_\epsilon^{R1} - \pi_G\left(\omega_G^{R1} - \omega_B^{R1}\right)$, we obtain

$$\left\{\begin{array}{c} E_\epsilon x_\epsilon - \pi_G\,(x_G - x_B) - \left(E_\epsilon \omega_\epsilon^{R1} - \pi_G\left(\omega_G^{R1} - \omega_B^{R1}\right)\right) \\ -u - h + \rho\beta\Omega(\widetilde{W}_\epsilon^{R1}) + (1-\rho)\,\beta\left(V^{ES} - \beta c\right) \end{array}\right. \geq V^{ES}$$

We use equation (C.19) to simplify the above equation, then obtain

$$V^{ER} - V^{ES} - \pi_G\,(x_G - x_B) + \pi_G\left(\omega_G^{R1} - \omega_B^{R1}\right) \geq 0$$

We use equations (C.28), (C.29), and the information that $\omega_\epsilon^{R1} = 0$ to we obtain

$$\frac{1}{\left(1 - \left(1 - Q^E\right)\rho\beta\right)} X \geq \pi_G\,(x_G - x_B)$$

where, X is in equation (C.27). We rearrange the terms and find the first period employer's ICC will not bind iff

$$\begin{array}{c} \pi_G\left(1 - Q^E\right)\rho\beta x_G \\ +\left(1 - \pi_G\left(1 - Q^E\right)\rho\beta\right)x_B \end{array} > \begin{array}{c} \rho\beta\left(\left(1-\rho\beta\right)E_\epsilon \omega_\epsilon^{*R2} + \rho\beta E_\epsilon \omega_\epsilon^{*R3}\right) \\ +\left(u + \beta\,(1-\rho)\,\beta c + (1-\rho\beta)\,h\right) \end{array}$$

Proof of Proposition 2 and 3.

We take the derivative of equations C.30 with respect to $r \in \{u, h, c, \omega^u, \omega^h\}$.

$$\begin{aligned}
\left(1-\left(1-Q^E\right)\rho\beta\right)\frac{d\omega_B^{R2}}{dr} &= \frac{dK}{dr} \\
\left(1-\left(1-Q^E\right)\rho\beta\right)\frac{d\omega_B^{R3}}{dr} &= \frac{dK}{dr}
\end{aligned}$$

where,

$$\frac{dK}{dr} = Q^E\rho\beta\rho\beta\frac{dE_\epsilon\omega_\epsilon^{R2}}{dr} - \left(1+Q^E\rho\beta\right)\rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{dr} + \vartheta^r \tag{C.31}$$

and

$$\vartheta^r = \left(\begin{array}{rl}
-1 & \text{if } r = u \\
Q^E\rho\beta & \text{if } r = h \\
\left(1-\left(1-Q^E\rho\right)\beta\right)\beta & \text{if } r = c \\
0 & \text{if } r = \omega^u \\
0 & \text{if } r = \omega^h
\end{array}\right)$$

ϑ^r is the derivative of the term, $(-u + Q^E\rho\beta h + \left((1-\beta)+Q^E\rho\beta\right)\beta c)$, in the right-hand side of employer's ICC C.30 with respect to r. Notice derivative of equation (C.31) is only relevant if employer's ICC C.30 is binding. Also, it is going to be important to remember when we have fully Wage Fluctuating agreement, $\omega_B^{R2} = \omega_B^{R3}$ and $\frac{d\omega_B^{R2}}{dr} = \frac{d\omega_B^{R3}}{dr}$ and when we have partially Wage Fluctuating agreement, $\omega_B^{R2} = \omega_G^{R2}$ and $\frac{d\omega_B^{R2}}{dr} = \frac{d\omega_G^{R2}}{dr}$. We now take the derivative of target utility constraint ((2.7a) and (2.7b)), with respect to $r \in \{u, h, c, \omega^u, \omega^h\}$.

$$\text{(C.32)} \qquad \begin{aligned} \mu^{/}\left(\omega_G^{R2}\right)\left(\pi_G \frac{d\omega_G^{R2}}{dr} + (1-\pi_G)\frac{\mu^{/}\left(\omega_B^{R2}\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\frac{d\omega_B^{R2}}{dr}\right) &= \Delta^r + d_3^r \\ \mu^{/}\left(\omega_G^{R3}\right)\left(\pi_G \frac{d\omega_G^{R3}}{dr} + (1-\pi_G)\frac{\mu^{/}\left(\omega_B^{R3}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\frac{d\omega_B^{R3}}{dr}\right) &= d_3^r \end{aligned}$$

where,

$$\Delta^r = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{1}{\rho\beta}\mu^{/}\left(\omega^u\right) & \text{if } r = \omega^u \\ -\frac{1}{\rho\beta}\mu^{/}\left(\omega^h\right) & \text{if } r = \omega^h \end{pmatrix}, \; d_3^r = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{\beta}{\rho\beta}\mu^{/}\left(\omega^u\right) & \text{if } r = \omega^u \\ 0 & \text{if } r = \omega^h \end{pmatrix}$$

Here, $\Delta^r + d_3^r$ and d_3^r are the derivative of right-hand side of second and third and subsequent periods target utility constraint ((2.7a) and (2.7b)), with respect to r, respectively. We add and subtract $(1-\pi_G)\frac{d\omega_B^{R2}}{dr}$ and $(1-\pi_G)\frac{d\omega_B^{R3}}{dr}$ in the equation system C.32 and obtain

$$\begin{aligned} \pi_G \frac{d\omega_G^{R2}}{dr} + (1-\pi_G)\frac{d\omega_B^{R2}}{dr} + (1-\pi_G)\frac{\mu^{/}\left(\omega_B^{R2}\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\frac{d\omega_B^{R2}}{dr} - (1-\pi_G)\frac{d\omega_B^{R2}}{dr} &= \frac{\Delta^r + d_3^r}{\mu^{/}\left(\omega_G^{R2}\right)} \\ \pi_G \frac{d\omega_G^{R3}}{dr} + (1-\pi_G)\frac{d\omega_B^{R3}}{dr} + (1-\pi_G)\frac{\mu^{/}\left(\omega_B^{R3}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\frac{d\omega_B^{R3}}{dr} - (1-\pi_G)\frac{d\omega_B^{R3}}{dr} &= \frac{d_3^r}{\mu^{/}\left(\omega_G^{R3}\right)} \end{aligned}$$

In both equations above, first two terms give the expected derivative of expected wages with respect to r. We take take common paranthesis of the derivative of bad state wage with respect to r. We use (2.14) and (2.15) from Lemma 1 to obtain the coefficients ψ_B^{R2} and ψ_B^{R3}, respectively. When we isolate the derivative of expected wages with respect to

r, we obtain

$$\text{(C.33)} \qquad \frac{dE_\epsilon\omega_\epsilon^{R2}}{dr} = \frac{\Delta^r + d_3^r}{\mu^/\left(\omega_G^{R2}\right)} - \psi_B^{R2}\frac{d\omega_B^{R2}}{dr}$$

$$\text{(C.34)} \qquad \frac{dE_\epsilon\omega_\epsilon^{R3}}{dr} = \frac{d_3^r}{\mu^/\left(\omega_G^{R3}\right)} - \psi_B^{R3}\frac{d\omega_B^{R3}}{dr}$$

We have three equations, (C.31), (C.33), and (C.34), three unknowns ($\frac{d\omega_B^{R3}}{dr}$, $\frac{dE_\epsilon\omega_\epsilon^{R2}}{dr}$, $\frac{dE_\epsilon\omega_\epsilon^{R3}}{dr}$).

First, we will solve for the case when we have Wage Smoothing agreement, then when we have partially Wage Fluctuating agreement, and when we have fully Wage Fluctuating agreement.

Wage Smoothing agreement: Employer's ICC does not bind, so equation (C.31) is irrelevant. Since $\psi_B^{R2} = 0$ and $\psi_B^{R3} = 0$, we will have ($\omega_B^{R2} = \omega_G^{R2} < \omega_B^{R3} = \omega_G^{R3}$) and

$$\frac{d\omega_G^{R2}}{dr} = \frac{d\omega_B^{R2}}{dr} = \frac{\Delta^r + d_3^r}{\mu^/\left(\omega_G^{R2}\right)}$$

$$\frac{d\omega_G^{R3}}{dr} = \frac{d\omega_B^{R3}}{dr} = \frac{d_3^r}{\mu^/\left(\omega_G^{R3}\right)}$$

We plug Δ^r and d_3^r, we obtain

$$\frac{d\omega_G^{R2}}{dr} = \frac{d\omega_B^{R2}}{dr} = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{\frac{1+\beta}{\rho\beta}\mu^/(\omega^u)}{\mu^/\left(\omega_G^{R2}\right)} & \text{if } r = \omega^u \\ -\frac{\frac{1}{\rho\beta}\mu^/\left(\omega^h\right)}{\mu^/\left(\omega_G^{R2}\right)} & \text{if } r = \omega^h \end{pmatrix}$$

and

$$\frac{d\omega_G^{R3}}{dr} = \frac{d\omega_B^{R3}}{dr} = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{\frac{\beta}{\rho\beta}\mu^{/}(\omega^u)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r = \omega^u \\ 0 & \text{if } r = \omega^h \end{pmatrix}$$

Partially Wage Fluctuating agreement: Since $\psi_B^{R2} = 0$ and $\psi_B^{R3} > 0$, we will have ($\omega_B^{R2} = \omega_G^{R2} < \omega_B^{R3} < \omega_G^{R3}$) and by equation (C.38), we obtain

$$\text{(C.35)} \qquad \frac{dE_\epsilon \omega_\epsilon^{R2}}{dr} = \frac{d\omega_G^{R2}}{dr} = \frac{d\omega_B^{R2}}{dr} = \frac{\Delta^r + d_3^r}{\mu^{/}\left(\omega_G^{R2}\right)}$$

We plug the $\frac{dE_\epsilon \omega_\epsilon^{R2}}{dr}$ and $\frac{dE_\epsilon \omega_\epsilon^{R3}}{dr}$ in (C.35) and (C.34) into (C.31) and solve for $\frac{d\omega_B^{R2}}{dr}$ and obtain

$$\text{(C.36)} \qquad \frac{d\omega_B^{R3}}{dr} = \frac{Q^E\rho\beta\rho\beta\frac{\Delta^r + d_3^r}{\mu^{/}\left(\omega_G^{R2}\right)} - \left(1 + Q^E\rho\beta\right)\rho\beta\frac{d_3^r}{\mu^{/}\left(\omega_G^{R3}\right)} + \vartheta^r}{\left(\left(1 - \left(1 - Q^E\right)\rho\beta\right) - \left(1 + Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)}$$

where,

$$\text{(C.37)} \qquad \vartheta^r = \begin{pmatrix} -1 & \text{if } r = u \\ Q^E\rho\beta & \text{if } r = h \\ \left(1-\left(1-Q^E\rho\right)\beta\right)\beta & \text{if } r = c \\ 0 & \text{if } r = \omega^u \\ 0 & \text{if } r = \omega^h \end{pmatrix}$$

$$\Delta^r = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{1}{\rho\beta}\mu^{/}\left(\omega^u\right) & \text{if } r = \omega^u \\ -\frac{1}{\rho\beta}\mu^{/}\left(\omega^h\right) & \text{if } r = \omega^h \end{pmatrix}, \; d_3^r = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{\beta}{\rho\beta}\mu^{/}\left(\omega^u\right) & \text{if } r = \omega^u \\ 0 & \text{if } r = \omega^h \end{pmatrix}$$

We plug Δ^r and d_3^r, we obtain

$$\frac{d\omega_G^{R2}}{dr} = \frac{d\omega_B^{R2}}{dr} = \begin{pmatrix} 0 & \text{if } r = u \\ 0 & \text{if } r = h \\ 0 & \text{if } r = c \\ \frac{1+\beta}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r = \omega^u \\ -\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r = \omega^h \end{pmatrix}$$

and

$$\frac{d\omega_B^{R3}}{dr} = \begin{pmatrix} \frac{-1}{\left((1-(1-Q^E)\rho\beta)-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = u \\ \frac{Q^E\rho\beta}{\left((1-(1-Q^E)\rho\beta)-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = h \\ \frac{\left(1-\left(1-Q^E\rho\right)\beta\right)\beta}{\left((1-(1-Q^E)\rho\beta)-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = c \\ \frac{\rho\beta\left(Q^E\rho\beta\frac{1+\beta}{\mu^{/}\left(\omega_G^{R2}\right)}-\left(1+Q^E\rho\beta\right)\frac{\beta}{\mu^{/}\left(\omega_G^{R3}\right)}\right)\frac{1}{\rho\beta}\mu^{/}(\omega^u)}{\left((1-(1-Q^E)\rho\beta)-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = \omega^u \\ \frac{-Q^E\rho\beta\rho\beta\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}}{\left((1-(1-Q^E)\rho\beta)-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = \omega^h \end{pmatrix}$$

Fully Wage Fluctuating agreement: We plug the $\frac{dE_\epsilon\omega_\epsilon^{R2}}{dr}$ and $\frac{dE_\epsilon\omega_\epsilon^{R3}}{dr}$ in (C.33) and (C.34) into (C.31) and solve for $\frac{d\omega_B^{R2}}{dr}$ and obtain

$$\text{(C.38)} \qquad \frac{d\omega_B^{R2}}{dr} = \frac{d\omega_B^{R3}}{dr} = \frac{Q^E\rho\beta\rho\beta\frac{\Delta^r+d_3^r}{\mu^{/}\left(\omega_G^{R2}\right)} - \left(1+Q^E\rho\beta\right)\rho\beta\frac{d_3^r}{\mu^{/}\left(\omega_G^{R3}\right)} + \vartheta^r}{\left((1-(1-Q^E)\,\rho\beta) + Q^E\rho\beta\rho\beta\psi_B^{R2} - (1+Q^E\rho\beta)\,\rho\beta\psi_B^{R3}\right)}$$

where, ϑ^r, Δ^r, and d_3^r are as in (C.37). We plug ϑ^r, Δ^r, and d_3^r in (C.37) and obtain

$$\text{(C.39)} \qquad \frac{d\omega_B^{R3}}{dr} = \begin{pmatrix} \frac{-1}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = u \\ \frac{Q^E\rho\beta}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = h \\ \frac{\left(1-\left(1-Q^E\rho\right)\beta\right)\beta}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = c \\ \frac{\rho\beta\left(Q^E\rho\beta\frac{1+\beta}{\mu^{/}\left(\omega_G^{R2}\right)}-\left(1+Q^E\rho\beta\right)\frac{\beta}{\mu^{/}\left(\omega_G^{R3}\right)}\right)\frac{1}{\rho\beta}\mu^{/}(\omega^u)}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = \omega^u \\ \frac{-Q^E\rho\beta\rho\beta\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} & \text{if } r = \omega^h \end{pmatrix}$$

First, we will use Lemma 3 to show that the

$$Denominator = \left(\left(1-\left(1-Q^E\right)\rho\beta\right) + Q^E\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)$$

in equation (C.39) is positive. Remember the third and subsequent periods Employer's ICC Kuhn-Tucker Multiplier, ψ_B^{R3}, is used as a measure of wage volatility and has an upper bound as in inequality (2.22) from Lemma 3:

$$\psi_B^{R3} < \left[\begin{array}{l} \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}, \text{ while in partially Wage Fluctuating agreement} \\ \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta}, \text{ while in fully Wage Fluctuating agreement} \end{array} \right.$$

When we plug the upper bound of ψ_B^{R3} for each agreement type into the denominator,[1] we will have a lower bound for the denominator term: *Denominator* will be positive while in partially Wage Fluctuating agreement and will be bigger than $Q^E\rho\beta\rho\beta\left(\psi_B^{R2}\left(1-\pi_G\right)\right)$ while in fully Wage Fluctuating agreement. So, the denominator term is positive for all $r \in \{u, h, c, \omega^u, \omega^h\}$. Next, we will investigate the sign of the numerator of $\frac{d\omega_B^{R3}}{dr}$. The sign of the numerator of $\frac{d\omega_B^{R3}}{dr}$ is obvious for all $r \in \{u, h, c, \omega^u, \omega^h\}$, but for $r = \omega^u$. So, we will focus on the sign of the numerator of $\frac{d\omega_B^{R3}}{d\omega^u}$. From equation (C.39), we know, *iff* $(Q^E\rho\beta\frac{1+\beta}{\mu^/\left(\omega_G^{R2}\right)} - \left(1+Q^E\rho\beta\right)\frac{\beta}{\mu^/\left(\omega_G^{R3}\right)})$ is negative, *then* numerator of $\frac{d\omega_B^{R3}}{d\omega^u}$ is negative. When we rearrange the terms, then the condition becomes *iff* $(\frac{Q^E\rho\beta+\beta Q^E\rho\beta}{\beta+\beta Q^E\rho\beta} < \frac{\mu^/\left(\omega_G^{R2}\right)}{\mu^/\left(\omega_G^{R3}\right)})$, *then* numerator of $\frac{d\omega_B^{R3}}{d\omega^u}$ is negative. Since $\frac{Q^E\rho\beta+\beta Q^E\rho\beta}{\beta+\beta Q^E\rho\beta} < 1 < \frac{\mu^/\left(\omega_G^{R2}\right)}{\mu^/\left(\omega_G^{R3}\right)}$ (from $\omega_G^{R2} < \omega_G^{R3}$ and strictly concave and increasing function, μ), then numerator of $\frac{d\omega_B^{R3}}{d\omega^u}$ is negative.

So, $\frac{d\omega_B^{R3}}{du} < 0$,$\frac{d\omega_B^{R3}}{dh} > 0$,$\frac{d\omega_B^{R3}}{dc} > 0$,$\frac{d\omega_B^{R3}}{d\omega^u} < 0$,$\frac{d\omega_B^{R3}}{d\omega^h} < 0$ and $\frac{d\omega_B^{R2}}{dr} = \frac{d\omega_B^{R3}}{dr}$ for all $r \in \{u, h, c, \omega^u, \omega^h\}$ when we have full Wage Fluctuating agreement.

To sum up, when we have Wage Smoothing agreement (first row), partial Wage Fluctuating agreement (second row), and fully Wage Fluctuating agreement (third row); $\frac{d\omega_B^{R2}}{dr}$

[1]Remember when we have partially Wage Fluctuating agreement, $\psi_B^{R2} = 0$.

for $r \in \{u, h, c, \omega^u, \omega^h\}$ has the following signs.

$$\left(\begin{array}{l} \frac{d\omega_B^{R2}}{du} = 0,\ \frac{d\omega_B^{R2}}{dh} = 0,\ \frac{d\omega_B^{R2}}{dc} = 0,\ \frac{d\omega_B^{R2}}{d\omega^u} > 0,\ \frac{d\omega_B^{R2}}{d\omega^h} < 0 \\ \frac{d\omega_B^{R2}}{du} = 0,\ \frac{d\omega_B^{R2}}{dh} = 0,\ \frac{d\omega_B^{R2}}{dc} = 0,\ \frac{d\omega_B^{R2}}{d\omega^u} > 0,\ \frac{d\omega_B^{R2}}{d\omega^h} < 0 \\ \frac{d\omega_B^{R2}}{du} < 0,\ \frac{d\omega_B^{R2}}{dh} > 0,\ \frac{d\omega_B^{R2}}{dc} > 0,\ \frac{d\omega_B^{R2}}{d\omega^u} < 0,\ \frac{d\omega_B^{R2}}{d\omega^h} < 0 \end{array} \right)$$

$\frac{d\omega_B^{R3}}{dr}$ for $r \in \{u, h, c, \omega^u, \omega^h\}$ has the following signs.

$$\left(\begin{array}{l} \frac{d\omega_B^{R3}}{du} = 0,\ \frac{d\omega_B^{R3}}{dh} = 0,\ \frac{d\omega_B^{R3}}{dc} = 0,\ \frac{d\omega_B^{R3}}{d\omega^u} > 0,\ \frac{d\omega_B^{R3}}{d\omega^h} = 0 \\ \frac{d\omega_B^{R3}}{du} < 0,\ \frac{d\omega_B^{R3}}{dh} > 0,\ \frac{d\omega_B^{R3}}{dc} > 0,\ \frac{d\omega_B^{R3}}{d\omega^u} < 0,\ \frac{d\omega_B^{R3}}{d\omega^h} < 0 \\ \frac{d\omega_B^{R2}}{dr} = \frac{d\omega_B^{R3}}{dr} \text{ for all } r \in \{u, h, c, \omega^u, \omega^h\} \end{array} \right)$$

We want to know how the employer's and worker's welfare, V^{ER} and V^{ES}, change when a policy variable, $r \in \{u, h, c, \omega^u, \omega^h\}$, changes. By equations (2.16) and (C.28) , change in V^{ER} and V^{ES}, $\frac{dV^{ER}}{dr}$ and $\frac{dV^{ES}}{dr}$ depend on change in X, $\frac{dX}{dr}$. By equation (2.19),

$$\frac{dX}{dr} = -\left(\xi^r + \rho\beta \left((1-\rho\beta) \frac{dE_\epsilon \omega_\epsilon^{R2}}{dr} + \rho\beta \frac{dE_\epsilon \omega_\epsilon^{R3}}{dr} \right) \right) \tag{C.40}$$

where,

$$\xi^r = \left(\begin{array}{rl} 1 & \text{if } r = u \\ (1-\rho\beta) & \text{if } r = h \\ \beta(1-\rho)\beta & \text{if } r = c \\ 0 & \text{if } r = \omega^u \\ 0 & \text{if } r = \omega^h \end{array} \right) \tag{C.41}$$

$\frac{dX}{dr}$ is a function of change in weighted average of expected wages, $\left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dr}+\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dr}\right)$, and $\left(\xi^{r}=\frac{d(u-\beta(1-\rho)\beta c-(1-\rho\beta)h)}{dr}\right)$ for $r\in\{u,h,c,\omega^{u},\omega^{h}\}$.

We will use the equation system (C.33) and (C.34) that relates the change in second period and third and subsequent periods expected wages, $\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dr}$ and $\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dr}$, to the wage volatility measures (ψ_{B}^{R2} and ψ_{B}^{R3}) and change in the bad state wages, $\frac{d\omega_{B}^{R2}}{dr}$ and $\frac{d\omega_{B}^{R3}}{dr}$ for $r\in\{u,h,c,\omega^{u},\omega^{h}\}$ to derive change in weighted average of expected wages, $\left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dr}+\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dr}\right)$.

(C.42)

$$\begin{pmatrix}(1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dr}\\ +\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dr}\end{pmatrix}=(1-\rho\beta)\left(\frac{\Delta^{r}+d_{3}^{r}}{\mu^{/}\left(\omega_{G}^{R2}\right)}-\psi_{B}^{R2}\frac{d\omega_{B}^{R2}}{dr}\right)+\rho\beta\left(\frac{d_{3}^{r}}{\mu^{/}\left(\omega_{G}^{R3}\right)}-\psi_{B}^{R3}\frac{d\omega_{B}^{R3}}{dr}\right)$$

When we have fully Wage Fluctuating agreement, we know $\frac{d\omega_{B}^{R2}}{dr}=\frac{d\omega_{B}^{R3}}{dr}$ and $\psi_{B}^{R3}>\psi_{B}^{R2}>0$. We plug the $\frac{d\omega_{B}^{R2}}{dr}$ in equation (C.38) into equation (C.42) and obtain

(C.43)

$$\begin{pmatrix}(1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dr}\\ +\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dr}\end{pmatrix}=\frac{\begin{pmatrix}\left(1-\left(1-Q^{E}\right)\rho\beta\right)\begin{pmatrix}\left((1-\rho\beta)\frac{\Delta^{r}+d_{3}^{r}}{\mu^{/}\left(\omega_{G}^{R2}\right)}+\rho\beta\frac{d_{3}^{r}}{\mu^{/}\left(\omega_{G}^{R3}\right)}\right)\\ +\rho\beta\left(\frac{d_{3}^{r}}{\mu^{/}\left(\omega_{G}^{R3}\right)}\psi_{B}^{R2}-\frac{\Delta^{r}+d_{3}^{r}}{\mu^{/}\left(\omega_{G}^{R2}\right)}\psi_{B}^{R3}\right)\end{pmatrix}\\ -\left((1-\rho\beta)\psi_{B}^{R2}+\rho\beta\psi_{B}^{R3}\right)\vartheta^{r}\end{pmatrix}}{\left(\left(1-\left(1-Q^{E}\right)\rho\beta\right)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-\left(1+Q^{E}\rho\beta\right)\rho\beta\psi_{B}^{R3}\right)}$$

where, ϑ^{r}, Δ^{r}, and d_{3}^{r} are as in (C.37). We plug ϑ^{r}, Δ^{r}, and d_{3}^{r} in (C.37) and obtain

(C.44)

$$\begin{pmatrix}(1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dr}\\ +\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dr}\end{pmatrix}=\frac{\left(1-\left(1-Q^{E}\right)\rho\beta\right)\rho\beta}{\left(\left(1-\left(1-Q^{E}\right)\rho\beta\right)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-\left(1+Q^{E}\rho\beta\right)\rho\beta\psi_{B}^{R3}\right)}\kappa^{r}$$

where,

$$\text{(C.45)}\quad \kappa^r = \left(\begin{array}{rl} \frac{\left((1-\rho\beta)\psi_B^{R2}+\rho\beta\psi_B^{R3}\right)}{(1-(1-Q^E)\rho\beta)\rho\beta} & \text{if } r=u \\ -\frac{\left((1-\rho\beta)\psi_B^{R2}+\rho\beta\psi_B^{R3}\right)Q^E\rho\beta}{(1-(1-Q^E)\rho\beta)\rho\beta} & \text{if } r=h \\ -\frac{\left((1-\rho\beta)\psi_B^{R2}+\rho\beta\psi_B^{R3}\right)\left(1-\left(1-Q^E\rho\right)\beta\right)\beta}{(1-(1-Q^E)\rho\beta)\rho\beta} & \text{if } r=c \\ \left(\beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\left(\psi_B^{R2}+1\right)-(1+\beta)\left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{1}{\rho\beta}\frac{\mu^{/}(\omega^u)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r=\omega^u \\ \left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r=\omega^h \end{array} \right)$$

Given that we already determined the denominator is positive, sign of the right-hand side will depend on the sign of the numerator. It easy to see increase in u will increase the weighted average of expected wages and increase in h and c will decrease the weighted average of expected wages, $((1-\rho\beta)\,E_\epsilon\omega_\epsilon^{R2}+\rho\beta E_\epsilon\omega_\epsilon^{R3})$. Increase in ω^u will increase the weighted average of expected wages (where we show it below). Increase in ω^h will decrease weighted average of expected wages iff $\psi_B^{R3}<\frac{(1-\rho\beta)}{\rho\beta}$. Here, we will show an increase in ω^u will increase the weighted average of expected wages for all parameter space and for all agreement types. From equation (C.44), we observe that increase in ω^u will increase the weighted average of expected wages iff

$$\beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\left(\psi_B^{R2}+1\right)-(1+\beta)\left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)>0$$

Wage Smoothing agreement: Since $\psi_B^{R2}=0$ and $\psi_B^{R3}=0$,

$$\beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}+\frac{(1+\beta)\,(1-\rho\beta)}{\rho\beta}>0$$

increase in ω^u increases weighted average of expected wages.

Partially Wage Fluctuating agreement: Since $\psi_B^{R2} = 0$,

$$\kappa^{\omega^u} = \left(\beta \frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\left(\psi_B^{R2} + 1\right) - (1+\beta)\left(\psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)}$$

Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{\left(1+Q^E\rho\beta\right)\rho\beta}$, so we have to have

$$\kappa^{\omega^u} > \left(\beta \frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)} - (1+\beta)\left(\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{\left(1+Q^E\rho\beta\right)\rho\beta} - \frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)}$$

Since $\mu^{/}\left(\omega_G^{R3}\right) < \mu^{/}\left(\omega_B^{R3}\right) < \mu^{/}\left(\omega_G^{R2}\right) = \mu^{/}\left(\omega_B^{R2}\right)$, we can write an inequality as:

$$\kappa^{\omega^u} > \left(\beta - (1+\beta)\left(\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{\left(1+Q^E\rho\beta\right)\rho\beta} - \frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)}$$

After we arrange the right-hand side we obtain

$$\kappa^{\omega^u} > \frac{\beta - Q^E\rho\beta}{\left(1+Q^E\rho\beta\right)}\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)}$$

Right-hand side is always positive, so increase in ω^u increases weighted average of expected wages.

Fully Wage Fluctuating agreement:

Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-\left(1-\pi_G\right)Q^E\rho\beta\rho\beta}{\left(1+Q^E\rho\beta\right)\rho\beta}$, so we have to have

$$\kappa^{\omega^u} > \left(\begin{array}{c} \beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\left(1+\psi_B^{R2}\right) \\ -(1+\beta)\left(\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-\left(1-\pi_G\right)Q^E\rho\beta\rho\beta}{\left(1+Q^E\rho\beta\right)\rho\beta} - \frac{(1-\rho\beta)}{\rho\beta}\right)\end{array}\right)\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)}$$

After we arrange the right-hand side we obtain

$$\kappa^{\omega^u} > \left(\beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\psi_B^{R2} + (1+\beta)\frac{(1-\pi_G)\,Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\,\rho\beta} + \frac{\beta - Q^E\rho\beta}{(1+Q^E\rho\beta)}\right)\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^u\right)}{\mu^{/}\left(\omega_G^{R2}\right)}$$

Right-hand side is always positive, so increase in ω^u increases weighted average of expected wages.

So, increase in ω^u will increase the weighted average of expected wages in all agreement types.

Summary 1. *To sum up,*

$$\left((1-\rho\beta)\frac{dE_\epsilon\omega_\epsilon^{R2}}{dr} + \rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{dr}\right) = \begin{pmatrix} > 0 & \textit{if } r = u \\ < 0 & \textit{if } r = h \\ < 0 & \textit{if } r = c \\ > 0 & \textit{if } r = \omega^u \\ < 0 \ \textit{iff } \psi_B^{R3} < \frac{(1-\rho\beta)}{\rho\beta} & \textit{if } r = \omega^h \end{pmatrix}$$

We will now derive $\frac{dX}{dr}$. From equation (C.40), we already derived how weighted average of expected wages change when when a policy variable, $r \in \{u, h, c, \omega^u, \omega^h\}$, changes in equation (C.44). Here, we will arrange the terms and find

$$\frac{dX}{dr} = -\left(\xi^r + \rho\beta\left((1-\rho\beta)\frac{dE_\epsilon\omega_\epsilon^{R2}}{dr} + \rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{dr}\right)\right)$$

where, ξ^r is as in (C.41). We can interpret ξ^r as the *direct* tax effect on X. While taxes (i.e., u, h, c) have a *direct* tax effect, spending policies (i.e., ω^u, ω^h) do not have. So it is easy to see

$$\frac{dX}{dr} = -\rho\beta \left((1-\rho\beta) \frac{dE_\epsilon \omega_\epsilon^{R2}}{dr} + \rho\beta \frac{dE_\epsilon \omega_\epsilon^{R3}}{dr} \right), \text{ for } r \in \{\omega^u, \omega^h\} \tag{C.46}$$

Here, we will find the $\frac{dX}{dr}$ for $r \in \{u,h,c\}$ and then assemble the results at the end. We can rewrite ξ^r to get a common denominator with the change in weighted average of expected wages term. Then, using equations (C.40), (C.44), and (C.41), and after we arrange the terms, for $r \in \{u,h,c\}$, we obtain

$$\frac{dX}{dr} = \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta}{\left(\left(1-\left(1-Q^E\right)\rho\beta\right) + Q^E\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)} interim\varrho^r$$

where,

$$interim\varrho^r = \left(\begin{array}{rl} -\left(\psi_B^{R2} - \psi_B^{R3}\right) - \frac{1}{\rho\beta} & \text{if } r = u \\ \psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta} & \text{if } r = h \\ \left(\left(1-\beta\right)\psi_B^{R2} + \beta\psi_B^{R3}\right) - \frac{(\beta-\rho\beta)}{\rho\beta} & \text{if } r = c \end{array} \right) \tag{C.47}$$

After we arrange the terms, for $r \in \{\omega^u, \omega^h\}$, we obtain

$$\frac{dX}{dr} = \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta}{\left(\left(1-\left(1-Q^E\right)\rho\beta\right) + Q^E\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)} \left(-\rho\beta\kappa^r\right) \tag{C.48}$$

where, κ^r is as in (C.45) for $r \in \{\omega^u, \omega^h\}$. Now we will express the $\frac{dX}{dr}$ for $r \in \{u,h,c\}$, and $\frac{dX}{dr}$ for $r \in \{\omega^u, \omega^h\}$, in one equation.

$$\frac{dX}{dr} = \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta}{\left(\left(1-\left(1-Q^E\right)\rho\beta\right) + Q^E\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)} \varrho^r \tag{C.49}$$

where,

$$
\text{(C.50)}\quad \varrho^{r}=\left(\begin{array}{rl}
-\left(\psi_B^{R2}-\psi_B^{R3}\right)-\frac{1}{\rho\beta} & \text{if } r=u\\
\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta} & \text{if } r=h\\
\beta\left(\left(1-\beta\right)\psi_B^{R2}+\beta\psi_B^{R3}\right)-\frac{\beta(\beta-\rho\beta)}{\rho\beta} & \text{if } r=c\\
-\left(\beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\left(1+\psi_B^{R2}\right)-\left(1+\beta\right)\left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{\mu^{/}\left(\omega^{u}\right)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r=\omega^{u}\\
-\left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)\frac{\mu^{/}\left(\omega^{h}\right)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r=\omega^{h}
\end{array}\right)
$$

Increase in u decreases the X for all agreement types (see below for exposition), increase in h increases the X iff $\psi_B^{R3} > \frac{(1-\rho\beta)}{\rho\beta}$, and increase in c increases the X iff $\left((1-\beta)\,\psi_B^{R2}+\beta\psi_B^{R3}\right) > \frac{(\beta-\rho\beta)}{\rho\beta}$. Increase in ω^u decreases the X (see page 173). Increase in ω^h increases the X iff $\psi_B^{R3} < \frac{(1-\rho\beta)}{\rho\beta}$.[2] Here, we will show an increase in u decreases the X in all agreement types.

Wage Smoothing agreement: Since $\psi_B^{R2}=0$ and $\psi_B^{R3}=0$, $\varrho^u=-\frac{1}{\rho\beta}<0$, increase in u decreases the X.

Partially Wage Fluctuating agreement: Since $\psi_B^{R2}=0$, $\varrho^u=\psi_B^{R3}-\frac{1}{\rho\beta}$. Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}$, so we have to have $\varrho^u<\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}-\frac{1}{\rho\beta}$. After we arrange the right-hand side we obtain $\varrho^u<-\frac{1}{(1+Q^E\rho\beta)}$. Right-hand side is always negative, so $\varrho^u<0$, increase in u decreases the X.

Fully Wage Fluctuating agreement: Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta}$, so we have to have

$$
\varrho^u < -\psi_B^{R2}+\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-\left(1-\pi_G\right)Q^E\rho\beta\rho\beta}{\left(1+Q^E\rho\beta\right)\rho\beta}-\frac{1}{\rho\beta}
$$

[2]We have already shown X is negatively related with the weighted average of expected wages for $r\in\{\omega^u,\omega^h\}$. So, we are not going to repeat it here.

After we arrange the right-hand side we obtain $\varrho^u < -\left(\psi_B^{R2} + \frac{1+(1-\pi_G)Q^E\rho\beta}{(1+Q^E\rho\beta)}\right)$. Right-hand side is always negative, so $\varrho^u < 0$, increase in u decreases the X. As a result, increase in u decreases the X in all agreement types.

Summary 2. *To sum up,*

$$\frac{dX}{dr} = \begin{pmatrix} < 0 & \textit{if } r = u \\ > 0 \textit{ iff } \psi_B^{R3} > \frac{(1-\rho\beta)}{\rho\beta} & \textit{if } r = h \\ > 0 \textit{ iff } \left((1-\beta)\,\psi_B^{R2} + \beta\psi_B^{R3}\right) > \frac{\beta-\rho\beta}{\rho\beta} & \textit{if } r = c \\ < 0 & \textit{if } r = \omega^u \\ > 0 \textit{ iff } \psi_B^{R3} < \frac{(1-\rho\beta)}{\rho\beta} & \textit{if } r = \omega^h \end{pmatrix}$$

By equations (2.16) and (2.2) , change in V^{ER} and V^{ES}, $\frac{dV^{ER}}{dr}$ and $\frac{dV^{ES}}{dr}$ depend on change in X, $\frac{dX}{dr}$.

$$\begin{aligned} \frac{dV^{ES}}{dr} &= \frac{Q^E\rho\beta\rho\beta}{(1-\beta)\left((1-(1-Q^E)\,\rho\beta) + Q^E\rho\beta\rho\beta\psi_B^{R2} - (1+Q^E\rho\beta)\,\rho\beta\psi_B^{R3}\right)}\varrho^r \\ \frac{dV^{ER}}{dr} &= \frac{\left(1-\beta+Q^E\rho\beta\right)\rho\beta}{(1-\beta)\left((1-(1-Q^E)\,\rho\beta) + Q^E\rho\beta\rho\beta\psi_B^{R2} - (1+Q^E\rho\beta)\,\rho\beta\psi_B^{R3}\right)}\varrho^r \end{aligned}$$

where ϱ^r is as in (C.50). So, sign of $\frac{dX}{dr}$ will be same as the sign of $\frac{dV^{ES}}{dr}$ and $\frac{dV^{ER}}{dr}$ for $r \in \{u, h, c, \omega^u, \omega^h\}$.

Comparison of Taxes

So far, we have derived the impact of only one labour market policy on the employer's welfare. In fact what we really want is to compare the policies. While doing that we will compare the κ^r (indicator of how the weighted average of expected wages changes) and ϱ^r (indicator of how the employer's welfare changes) of different policies.

How increasing c and decreasing u such that total tax is constant effects equilibrium

First, we will answer the question of how increasing c and decreasing u such that total tax is constant ($\frac{\rho}{1-\rho}\Phi du + \rho\Phi dc = 0$) will affect the weighted average of expected wages and the welfare of employer.

$$\left(\begin{array}{c} \left((1-\rho\beta)\frac{dE_\epsilon\omega_\epsilon^{R2}}{dc} + \rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{dc}\right)|_{du=-(1-\rho)dc} \\ = \\ \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta(\kappa^c-(1-\rho)\kappa^u)}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} \end{array}\right)$$

where κ^c and κ^u is as in equation (C.45). We rearrange the terms and obtain

$$(\kappa^c - (1-\rho)\kappa^u) = -\frac{\left((1-\rho\beta)\psi_B^{R2} + \rho\beta\psi_B^{R3}\right)\left(\left(1-\left(1-Q^E\right)\rho\beta\right)\beta + (1-\rho)\right)}{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta}$$

Weighted average of expected wage decreases (since $(\kappa^c - (1-\rho)\kappa^u) < 0$) when transfer payments are financed more by firing tax and less by payroll tax. Change in employer's Value function in the first period when we increase c and decrease u such that total tax is constant will be:

$$\frac{dV^{ER}}{dc}|_{du=-(1-\rho)dc} = \frac{\left(1-\beta+Q^E\rho\beta\right)\rho\beta\left(\varrho^c-(1-\rho)\varrho^u\right)}{(1-\beta)\left((1-\left(1-Q^E\right)\rho\beta) + Q^E\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)}$$

where, ϱ^c and ϱ^u is as in equation (C.50). We rearrange the terms and obtain

$$(\varrho^c - (1-\rho)\varrho^u) = (\beta - (\beta\beta - (1-\rho)))\psi_B^{R2} + (\beta\beta - (1-\rho))\psi_B^{R3} + \frac{(1-\rho)(1-\beta\beta)}{\rho\beta}$$

Employer's Value Function increases with the changing composition of financing iff the term above is positive, $(\varrho^c - (1-\rho)\,\varrho^u) > 0$.

Wage Smoothing agreement: Since $\psi_B^{R2} = 0$ and $\psi_B^{R3} = 0$, $(\varrho^c - (1-\rho)\,\varrho^u) = \frac{(1-\rho)(1-\beta\beta)}{\rho\beta} > 0$ and $\frac{dV^{ER}}{dc}|_{du=-(1-\rho)dc} > 0$.

Partially Wage Fluctuating agreement: Since $\psi_B^{R2} = 0$, for $\beta\beta > (1-\rho)$, it is obvious that $(\varrho^c - (1-\rho)\,\varrho^u) > 0$. So, we will look at $(\varrho^c - (1-\rho)\,\varrho^u)$ for $\beta\beta < (1-\rho)$. Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}$, so we have to have

$$(\varrho^c - (1-\rho)\,\varrho^u) > -((1-\rho)-\beta\beta)\,\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{\left(1+Q^E\rho\beta\right)\rho\beta} + \frac{(1-\rho)\,(1-\beta\beta)}{\rho\beta}$$

After we arrange the right-hand side we obtain

$$(\varrho^c - (1-\rho)\,\varrho^u) > \frac{(1-\rho)+\beta\left(1-\beta+Q^E\rho\beta\right)}{\left(1+Q^E\rho\beta\right)}$$

Right-hand side is always positive, so $\frac{dV^{ER}}{dc}|_{du=-(1-\rho)dc} > 0$.

Fully Wage Fluctuating agreement: For $\beta\beta > (1-\rho)$ it is obvious that right-hand side of the above inequality is positive, so $(\varrho^c - (1-\rho)\,\varrho^u) > 0$. Now, we will look at whether $(\varrho^c - (1-\rho)\,\varrho^u) > 0$ for $\beta\beta < (1-\rho)$.

Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta}$, so we have to have

$$(\varrho^c - (1-\rho)\,\varrho^u) > \left(\begin{array}{c} (\beta-(\beta\beta-(1-\rho)))\,\psi_B^{R2} \\ +\,(\beta\beta-(1-\rho))\,\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta} + \frac{(1-\rho)(1-\beta\beta)}{\rho\beta} \end{array}\right)$$

After we arrange the right-hand side we obtain

$$\left(\varrho^{c}-\left(1-\rho\right)\varrho^{u}\right)>\left(\begin{array}{c}\frac{\left(1-\rho\right)+\beta\left(1-\beta+Q^{E}\rho\beta\right)}{\left(1+Q^{E}\rho\beta\right)}\\ +\left(\left(1-\rho\right)+\beta\left(1-\beta\right)\right)\psi_{B}^{R2}+\left(\left(1-\rho\right)-\beta\beta\right)\frac{\left(1-\pi_{G}\right)Q^{E}\rho\beta}{\left(1+Q^{E}\rho\beta\right)}\end{array}\right)$$

Right-hand side is always positive, so $\frac{dV^{ER}}{dc}|_{du=-(1-\rho)dc}$. As a result, $\frac{dV^{ER}}{dc}|_{du=-(1-\rho)dc}>0$ in all agreement types.

How increasing h and decreasing u such that total tax is constant effects equilibrium

We will answer the question of how increasing h and decreasing u such that total tax is constant ($\frac{\rho}{1-\rho}\Phi du + \rho\Phi dh = 0$) will affect the weighted average of expected wages and the welfare of employer.

$$\left(\begin{array}{c} \left((1-\rho\beta)\frac{dE_\epsilon\omega_\epsilon^{R2}}{dh} + \rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{dh}\right)|_{du=-(1-\rho)dh} \\ = \\ \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta\left(\kappa^h-(1-\rho)\kappa^u\right)}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} \end{array}\right)$$

where κ^h and κ^u is as in equation (C.45). After we rearrange the terms, we obtain

$$\left(\kappa^h - (1-\rho)\,\kappa^u\right) = -\frac{\left(Q^E\rho\beta + (1-\rho)\right)\left((1-\rho\beta)\,\psi_B^{R2} + \rho\beta\psi_B^{R3}\right)}{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta}$$

Weighted average of expected wage decreases (since $\left(\kappa^h - (1-\rho)\,\kappa^u\right) < 0$) when transfer payments are financed more by hiring tax and less by payroll tax. Change in employer's Value function when we increase h and decrease u such that total tax is constant will be:

$$\frac{dV^{ER}}{dh}|_{du=-(1-\rho)dh} = \frac{\left(1-\beta+Q^E\rho\beta\right)\rho\beta\left(\varrho^h - (1-\rho)\,\varrho^u\right)}{(1-\beta)\left(\left(1-\left(1-Q^E\right)\rho\beta\right) + Q^E\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)}$$

where, ϱ^h and ϱ^u is as in equation (C.50). We rearrange the terms and obtain

$$\left(\varrho^h - (1-\rho)\,\varrho^u\right) = \left((1-\rho)\,\psi_B^{R2} + \rho\psi_B^{R3} - \frac{1-\beta}{\beta}\right)$$

Employer's Value Function increases with the changing composition of financing only when right-hand side of the above equation is positive.

Wage Smoothing agreement: Since $\psi_B^{R2} = 0$ and $\psi_B^{R3} = 0$, $\left(\varrho^h - (1-\rho)\,\varrho^u\right) = -\frac{1-\beta}{\beta} < 0$ and $\frac{dV^{ER}}{dh}|_{du=-(1-\rho)dh} < 0$.

Partially Wage Fluctuating agreement: Since $\psi_B^{R2} = 0$, $\frac{dV^{ER}}{dh}|_{du=-(1-\rho)dh} > 0$ iff $\psi_B^{R3} > \frac{1-\beta}{\rho\beta}$.

Fully Wage Fluctuating agreement: $\frac{dV^{ER}}{dh}|_{du=-(1-\rho)dh} > 0$ iff $\left((1-\rho)\,\psi_B^{R2} + \rho\psi_B^{R3}\right) > \frac{1-\beta}{\beta}$.

How increasing c and decreasing h such that total tax is constant effects equilibrium

First, we will answer the question of how increasing c and decreasing h such that total tax is constant ($dh + dc = 0$) will affect the weighted average of expected wages and the welfare of employer.

$$\left(\begin{array}{c} \left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{dc} + \rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{dc}\right)|_{dh=-dc} \\ = \\ \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta\left(\kappa^c-\kappa^h\right)}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2}-(1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} \end{array}\right)$$

where κ^c and κ^h is as in equation (C.45). After we rearrange the terms, we obtain

$$\kappa^c - \kappa^h = -\frac{(1-\beta)\left(\beta - Q^E\rho\beta\right)\left((1-\rho\beta)\psi_B^{R2} + \rho\beta\psi_B^{R3}\right)}{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta}$$

Weighted average of expected wage decreases (since $\left(\kappa^c - \kappa^h\right) < 0$) when transfer payments are financed more by firing tax and less by hiring tax. Change in employer's Value function when we increase c and decrease h such that total tax is constant will be:

$$\frac{dV^{ER}}{dc}|_{dh=-dc} = \frac{\left(1-\beta+Q^E\rho\beta\right)\rho\beta\left(\varrho^c-\varrho^h\right)}{(1-\beta)\left(\left(1-\left(1-Q^E\right)\rho\beta\right)+Q^E\rho\beta\rho\beta\psi_B^{R2}-\left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)}$$

where, ϱ^c and ϱ^h is as in equation (C.50). After we rearrange the terms, we obtain

$$\left(\varrho^c - \varrho^h\right) = -(1-\beta\beta)\left(\psi_B^{R3} - \frac{\frac{1+\beta-\rho\beta}{\rho\beta} + \beta\psi_B^{R2}}{1+\beta}\right)$$

Employer's Value Function increases, $\frac{dV^{ER}}{dc}|_{dh=-dc} > 0$, with the changing composition of financing iff $\psi_B^{R3} < \frac{\frac{1+\beta-\rho\beta}{\rho\beta}+\beta\psi_B^{R2}}{1+\beta}$.

Wage Smoothing agreement: Since $\psi_B^{R2} = 0$ and $\psi_B^{R3} = 0$, $\left(\varrho^c - \varrho^h\right) = \frac{1+\beta-\rho\beta}{(1-\beta)\rho\beta} > 0$ and $\frac{dV^{ER}}{dc}|_{dh=-dc} > 0$.

Partially Wage Fluctuating agreement: Since $\psi_B^{R2} = 0$, right-hand side of $\left(\varrho^c - \varrho^h\right)$ is equal to $-(1-\beta\beta)\left(\psi_B^{R3} - \frac{\frac{1+\beta-\rho\beta}{\rho\beta}}{1+\beta}\right)$. Upper bound for ψ_B^{R3} is $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{(1+Q^E\rho\beta)\rho\beta}$, so we have to have

$$\left(\varrho^c - \varrho^h\right) > -(1-\beta\beta)\left(\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)}{\left(1+Q^E\rho\beta\right)\rho\beta} - \frac{\frac{1+\beta-\rho\beta}{\rho\beta}}{1+\beta}\right)$$

After we arrange the right-hand side we obtain

$$\left(\varrho^c - \varrho^h\right) > (1-\beta)\left(\frac{\beta - Q^E\rho\beta}{(1+Q^E\rho\beta)}\right)$$

Right-hand side is always positive, so $\frac{dV^{ER}}{dc}|_{dh=-dc} > 0$.

Fully Wage Fluctuating agreement: Since the upper bound for ψ_B^{R3} is equal to $\frac{\left(1-\left(1-Q^E\right)\rho\beta\right)-(1-\pi_G)Q^E\rho\beta\rho\beta}{(1+Q^E\rho\beta)\rho\beta}$, we plug it into the right-hand side of $\left(\varrho^c - \varrho^h\right)$, we arrange the terms and obtain

$$\left(\varrho^c - \varrho^h\right) > (1-\beta)\left(\frac{\beta - Q^E\rho\beta}{(1+Q^E\rho\beta)} + \frac{(1+\beta)(1-\pi_G)Q^E\rho\beta}{(1+Q^E\rho\beta)} + \beta\psi_B^{R2}\right)$$

Right-hand side is always positive, so $\frac{dV^{ER}}{dc}|_{dh=-dc} > 0$. As a result, $\frac{dV^{ER}}{dc}|_{dh=-dc} > 0$ in all agreement types.

How increasing ω^h and decreasing ω^u such that total payment is constant effects equilibrium

Now, we will investigate the parameter space in which employer prefers the hiring payment to the unemployment payment. We want to keep total spending constant so the following equation should hold.

$$(n - \rho\Phi(n,m))\, d\omega^u + \rho\Phi(n,m)\, d\omega^h = 0$$

An unit increase in hiring payment should accompany $\frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}$ decrease in unemployment payment. So,

$$d\omega^u = -\frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))} d\omega^h$$

$$\left(\begin{array}{c} \left((1-\rho\beta)\frac{dE_\epsilon \omega_\epsilon^{R2}}{d\omega^h} + \rho\beta \frac{dE_\epsilon \omega_\epsilon^{R3}}{d\omega^h}\right)|_{d\omega^u = -\frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))} d\omega^h} \\ = \\ \frac{\left(1-\left(1-Q^E\right)\rho\beta\right)\rho\beta\left(\kappa^{\omega^h} - \frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\kappa^{\omega^u}\right)}{\left((1-(1-Q^E)\rho\beta)+Q^E\rho\beta\rho\beta\psi_B^{R2} - (1+Q^E\rho\beta)\rho\beta\psi_B^{R3}\right)} \end{array} \right)$$

where κ^{ω^h} and κ^{ω^u} is as in equation (C.45). Also remember, $-\rho\beta\kappa^{\omega^h} = \varrho^{\omega^h}$ and $-\rho\beta\kappa^{\omega^u} = \varrho^{\omega^u}$. Then,

$$\begin{array}{c} \left(\kappa^{\omega^h} - \frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\kappa^{\omega^u}\right) \\ = \end{array}$$

$$-\frac{1}{\rho\beta}\frac{\mu^/\left(\omega^h\right)}{\mu^/\left(\omega_G^{R2}\right)} \left(\begin{array}{c} -\left(\psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta}\right) \\ +\frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\left(\beta\frac{\mu^/\left(\omega_G^{R2}\right)}{\mu^/\left(\omega_G^{R3}\right)}\left(1+\psi_B^{R2}\right) - (1+\beta)\left(\psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{\mu^/(\omega^u)}{\mu^/\left(\omega^h\right)} \end{array} \right)$$

Since $\frac{\mu^{/}(\omega^u)}{\mu^{/}\left(\omega^h\right)} > 1$ and $\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)} > 1$,

$$\left(\begin{array}{c} \left(\kappa^{\omega^h} - \frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\kappa^{\omega^u}\right) \\ < \\ -\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\left(\begin{array}{c} -\left(\psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta}\right) \\ +\frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\left(\beta\left(1+\psi_B^{R2}\right) - (1+\beta)\left(\psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta}\right)\right) \end{array}\right) \end{array}\right)$$

We rearrange the terms and obtain:

$$\left(\begin{array}{c} \left(\kappa^{\omega^h} - \frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\kappa^{\omega^u}\right) \\ < \\ -\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\left(\frac{\frac{(1-\rho\beta)}{\rho\beta}n+\Phi(n,m)}{(n-\rho\Phi(n,m))} + \frac{\rho\beta\Phi(n,m)}{(n-\rho\Phi(n,m))}\psi_B^{R2} - \frac{n+\rho\beta\Phi(n,m)}{(n-\rho\Phi(n,m))}\psi_B^{R3}\right) \end{array}\right)$$

We multiply and divide right-hand side with n (since $Q^W = \frac{\Phi(n,m)}{n}$) and obtain

$$\left(\begin{array}{c} \left(\kappa^{\omega^h} - \frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\kappa^{\omega^u}\right) \\ < \\ -\frac{1}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\left(\frac{1}{\rho\beta}\frac{\left(1-\left(1-Q^W\right)\rho\beta\right)}{1-Q^W\rho} + \frac{Q^W\rho\beta}{1-Q^W\rho}\psi_B^{R2} - \frac{1+Q^W\rho\beta}{1-Q^W\rho}\psi_B^{R3}\right) \end{array}\right)$$

$$\left(\begin{array}{c} \left(\kappa^{\omega^h} - \frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}\kappa^{\omega^u}\right) \\ < \\ -\frac{1}{\rho\beta\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\frac{\left(1-\left(1-Q^W\right)\rho\beta\right)+Q^W\rho\beta\rho\beta\psi_B^{R2}-\left(1+Q^W\rho\beta\right)\rho\beta\psi_B^{R3}}{1-Q^W\rho} \end{array}\right)$$

If $\left(1-\left(1-Q^W\right)\rho\beta\right) + Q^W\rho\beta\rho\beta\psi_B^{R2} - \left(1+Q^W\rho\beta\right)\rho\beta\psi_B^{R3} > 0$, then

$$\left((1-\rho\beta)\frac{dE_\epsilon\omega_\epsilon^{R2}}{d\omega^h} + \rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{d\omega^h}\right)|_{d\omega^u=-\frac{\rho Q^W}{1-\rho Q^W}d\omega^h} < 0 \text{ and } \frac{dV^{ER}}{d\omega^h}|_{d\omega^u=-\frac{\rho Q^W}{1-\rho Q^W}d\omega^h} > 0$$

Remember the

$$(1-\beta)\frac{dV^{ER}}{dr} = \frac{\left(1-\beta+Q^{E}\rho\beta\right)\rho\beta}{\left((1-(1-Q^{E})\rho\beta)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-(1+Q^{E}\rho\beta)\rho\beta\psi_{B}^{R3}\right)}\varrho^{r}$$

We know denominator, $\left(1-\left(1-Q^{E}\right)\rho\beta\right)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-\left(1+Q^{E}\rho\beta\right)\rho\beta\psi_{B}^{R3}$, is positive.

If

$$\left[\begin{array}{c} \left(1-\left(1-Q^{W}\right)\rho\beta\right)+Q^{W}\rho\beta\rho\beta\psi_{B}^{R2}-\left(1+Q^{W}\rho\beta\right)\rho\beta\psi_{B}^{R3} \\ > \\ \left(1-\left(1-Q^{E}\right)\rho\beta\right)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-\left(1+Q^{E}\rho\beta\right)\rho\beta\psi_{B}^{R3} \end{array}\right]$$

then

$$\left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{d\omega^{h}}+\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{d\omega^{h}}\right)|_{d\omega^{u}=-\frac{\rho Q^{W}}{1-\rho Q^{W}}d\omega^{h}}<0 \text{ and } \frac{dV^{ER}}{d\omega^{h}}|_{d\omega^{u}=-\frac{\rho Q^{W}}{1-\rho Q^{W}}d\omega^{h}}>0$$

We rearrange the terms. If

$$\left(Q^{W}-Q^{E}\right)\rho\beta\left(\frac{1}{\rho\beta}+\left(\psi_{B}^{R2}-\psi_{B}^{R3}\right)\right)>0$$

then

$$\left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{d\omega^{h}}+\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{d\omega^{h}}\right)|_{d\omega^{u}=-\frac{\rho Q^{W}}{1-\rho Q^{W}}d\omega^{h}}<0 \text{ and } \frac{dV^{ER}}{d\omega^{h}}|_{d\omega^{u}=-\frac{\rho Q^{W}}{1-\rho Q^{W}}d\omega^{h}}>0$$

Since $\frac{1}{\rho\beta}+\left(\psi_{B}^{R2}-\psi_{B}^{R3}\right)>0$, (check the case at page 177) if

$$Q^{W}>Q^{E}$$

then

$$\left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{d\omega^{h}}+\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{d\omega^{h}}\right)|_{d\omega^{u}=-\frac{\rho Q^{W}}{1-\rho Q^{W}}d\omega^{h}}<0 \text{ and } \frac{dV^{ER}}{d\omega^{h}}|_{d\omega^{u}=-\frac{\rho\Phi(n,m)}{(n-\rho\Phi(n,m))}d\omega^{h}}>0$$

Proof of Proposition 4.

In this part, we will find how joint policies effect the welfare of employer. When we finance the unemployment payment with the payroll tax ($d\omega^{u}=\frac{\frac{\rho}{1-\rho}Q^{W}}{1-\rho Q^{W}}du$):

$$\left(\begin{array}{c}\left((1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{du}+\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{du}\right)|_{d\omega^{u}=\frac{\frac{\rho}{1-\rho}Q^{W}}{1-\rho Q^{W}}du}\\ =\\ \frac{\left(1-\left(1-Q^{E}\right)\rho\beta\right)\rho\beta\left(\kappa^{u}+\frac{\rho}{1-\rho}\frac{\Phi(n,m)}{n-\rho\Phi(n,m)}\kappa^{\omega^{u}}\right)}{\left((1-(1-Q^{E})\rho\beta)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-(1+Q^{E}\rho\beta)\rho\beta\psi_{B}^{R3}\right)}\end{array}\right)$$

Similarly,

$$\frac{dV^{ER}}{du}|_{d\omega^{u}=\frac{\frac{\rho}{1-\rho}Q^{W}}{1-\rho Q^{W}}du}=\frac{\left(1-\beta+Q^{E}\rho\beta\right)\rho\beta\left(\varrho^{u}+\frac{\rho}{1-\rho}\frac{\Phi(n,m)}{n-\rho\Phi(n,m)}\varrho^{\omega^{u}}\right)}{(1-\beta)\left((1-(1-Q^{E})\rho\beta)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-(1+Q^{E}\rho\beta)\rho\beta\psi_{B}^{R3}\right)}$$

We've shown $\kappa^{u}>0$, $\kappa^{\omega^{u}}>0$, $\varrho^{u}<0$, and $\varrho^{\omega^{u}}<0$. So, when we finance unemployment payment with the payroll tax, weighted average of expected wages increase and employer's welfare decrease.

Proof of Proposition 5.

We will answer the question of how financing the hiring payment with the hiring tax ($d\omega^{h}=dh$) will affect the weighted average of expected wages and the welfare of employer.

$$\left(\begin{array}{c}(1-\rho\beta)\frac{dE_{\epsilon}\omega_{\epsilon}^{R2}}{d\omega^{h}}\\ +\rho\beta\frac{dE_{\epsilon}\omega_{\epsilon}^{R3}}{d\omega^{h}}\end{array}\right)|_{dh=d\omega^{h}}=\frac{\left(1-\left(1-Q^{E}\right)\rho\beta\right)\rho\beta\left(\kappa^{h}+\kappa^{\omega^{h}}\right)}{\left((1-(1-Q^{E})\rho\beta)+Q^{E}\rho\beta\rho\beta\psi_{B}^{R2}-(1+Q^{E}\rho\beta)\rho\beta\psi_{B}^{R3}\right)}$$

where,

$$\kappa^h + \kappa^{\omega^h} = \frac{1}{\rho\beta}\left(-\frac{\left(\left(1-\rho\beta\right)\psi_B^{R2} + \rho\beta\psi_B^{R3}\right)Q^E\rho\beta}{\left(1-\left(1-Q^E\right)\rho\beta\right)} + \left(\psi_B^{R3} - \frac{\left(1-\rho\beta\right)}{\rho\beta}\right)\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\right)$$

We rearrange the numerator in the left-hand side and and obtain

$$\left(\begin{array}{c} \left(\left(1-\rho\beta\right)\frac{dE_\epsilon\omega_\epsilon^{R2}}{d\omega^h} + \rho\beta\frac{dE_\epsilon\omega_\epsilon^{R3}}{d\omega^h}\right)|_{dh=d\omega^h} < 0 \\ \text{iff} \\ \psi_B^{R3} < \frac{(1-\rho\beta)}{\rho\beta}\left(1 + \frac{\psi_B^{R2}+1}{\frac{(1-(1-Q^E)\rho\beta)}{\rho\beta Q^E\rho\beta}\left(\frac{\mu^{/}(\omega^h)}{\mu^{/}\left(\omega_G^{R2}\right)}-1\right)+\frac{(1-\rho\beta)(1+Q^E\rho\beta)}{\rho\beta Q^E\rho\beta}}\right) \end{array}\right)$$

Similarly,

$$\frac{dV^{ER}}{d\omega^h}|_{dh=d\omega^h} = \frac{\left(1-\beta+Q^E\rho\beta\right)\rho\beta\left(\varrho^h+\varrho^{\omega^h}\right)}{\left(1-\beta\right)\left(\left(1-\left(1-Q^E\right)\rho\beta\right)+Q^E\rho\beta\rho\beta\psi_B^{R2}-\left(1+Q^E\rho\beta\right)\rho\beta\psi_B^{R3}\right)}$$

$$\varrho^r = \left(\begin{array}{rl} -\left(\psi_B^{R2}-\psi_B^{R3}\right) - \frac{1}{\rho\beta} & \text{if } r=u \\ \psi_B^{R3} - \frac{(1-\rho\beta)}{\rho\beta} & \text{if } r=h \\ \beta\left(\left(1-\beta\right)\psi_B^{R2}+\beta\psi_B^{R3}\right) - \frac{\beta(\beta-\rho\beta)}{\rho\beta} & \text{if } r=c \\ -\left(\beta\frac{\mu^{/}\left(\omega_G^{R2}\right)}{\mu^{/}\left(\omega_G^{R3}\right)}\left(1+\psi_B^{R2}\right)-\left(1+\beta\right)\left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)\right)\frac{\mu^{/}(\omega^u)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r=\omega^u \\ -\left(\psi_B^{R3}-\frac{(1-\rho\beta)}{\rho\beta}\right)\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)} & \text{if } r=\omega^h \end{array}\right)$$

Thus,

$$\left(\varrho^h+\varrho^{\omega^h}\right) = -\left(\psi_B^{R3}-\frac{\left(1-\rho\beta\right)}{\rho\beta}\right)\rho\beta\left(\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}-1\right)$$

$$-\frac{(1-\rho\beta)}{\rho\beta}+\frac{(1-\rho\beta)}{\rho\beta}\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}+\psi_B^{R3}-\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}\psi_B^{R3} = \frac{(1-\rho\beta)}{\rho\beta}\left(\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}-1\right)-\left(\frac{\mu^{/}\left(\omega^h\right)}{\mu^{/}\left(\omega_G^{R2}\right)}-1\right)\psi_B^{R3}$$

When we finance the hiring payment with the hiring tax ($d\omega^h = dh$), employer's welfare increases iff

$$\left(\left(\psi_B^{R3} < \tfrac{(1-\rho\beta)}{\rho\beta}\right) \text{ and } \left(\frac{\mu^{/}\left(\omega^{h}\right)}{\mu^{/}\left(\omega_G^{R2}\right)} > 1\right)\right) \text{ or } \left(\left(\psi_B^{R3} > \tfrac{(1-\rho\beta)}{\rho\beta}\right) \text{ and } \left(\frac{\mu^{/}\left(\omega^{h}\right)}{\mu^{/}\left(\omega_G^{R2}\right)} < 1\right)\right).$$

APPENDIX D

Appendix to Chapter 3

Proof of Proposition 1.

Consider the first period problem. Since the left hand side of worker's ICC (3.6) and target utility constraint (3.7) are identical and the constraint (3.7) binds as shown in Lemma 2, it is possible to rewrite the worker's ICC (3.6) in the first period as

$$W_0 \geq V^{WS} + \omega_{k_i}^*,$$

where W_0 is the target utility promised to worker at the beginning of the relationship. Since $\omega_{k_i} \geq 0$ and also $W_0 = V^{WS}$, we must have $\omega_{k_i}^* = 0$ in the first period. Plugging $\omega_{k_i}^* = 0$ and (3.10) into (3.7) yields $\widetilde{W}_{k_i} = \frac{\alpha}{1-\beta} + \frac{k_i}{\rho\beta}$, which, using (3.6)$''$, implies $\widetilde{W}_{k_i} = \widetilde{W}_{k_i}^*$. Hence, the worker's incentive compatibility constraint binds in the first period and the future utility stream that is promised to the worker is equal to $\widetilde{W}_{k_i}^*$. This also represents the target utility for the worker in the second period problem.

We are now ready to analyse the subsequent periods. Let the Kuhn-Tucker multiplier for (3.5) be denoted by ψ_{k_i}, the multiplier for (3.6) be by υ_{k_i}, the multiplier for (3.7) by λ_{k_i} and the multiplier for (3.8) be by χ_{k_i}. The first order conditions with respect to $\omega_{k_i}^*$

and $\widetilde{W}_{k_i}$ are

$$\text{(D.1)} \qquad -1 - \psi_{k_i} + \lambda_{k_i} + \chi_{k_i} = 0$$

$$\text{(D.2)} \qquad \rho\beta\Omega'\left(\widetilde{W}_{k_i}\right) + \rho\beta\psi_{k_i}\Omega'\left(\widetilde{W}_{k_i}\right) + \upsilon_{k_i} + \rho\beta\lambda_{k_i} = 0$$

Similar to the first period problem, we can rewrite the worker's incentive compatibility constraint (ICC) in the second period as

$$\widetilde{W}^*_{k_i} \geq V^{WS} + \omega^*_{k_i}.$$

Here, $\omega^*_{k_i}$ denotes the wage in the second period and $\widetilde{W}^*_{k_i}$ is the promise made to the worker in the first period. Notice that if $k_i = 0$, then we have $\widetilde{W}^*_{k_i} = W_0$, and the second period problem becomes identical to the first period problem, indicating zero wage forever. On the other hand, if $k_i > 0$, we have $\widetilde{W}^*_{k_i} > W_0 = V^{WS}$. In this case, if $\upsilon^{R2}_{k_i} > 0$, by complementary slackness, we have $\widetilde{W}_{k_i} = \widetilde{W}^*_{k_i}$, where $\widetilde{W}_{k_i}$ denotes the future utility stream that is promised to the worker in the second period. On the other hand, if $\upsilon^{R2}_{k_i} = 0$, then (D.2) implies that $\lambda_{k_i} = -\left(1 + \psi_{k_i}\right)\Omega'\left(\widetilde{W}_{k_i}\right)$. Since $\lambda_{k_i} = -\Omega'\left(\widetilde{W}^*_{k_i}\right)$ from the envelope condition, $\Omega'(W) < 0$ and $\psi_{k_i} \geqslant 0$, we must have $\widetilde{W}^*_{k_i} \geq \widetilde{W}_{k_i}$. At the same time by (3.6)$''$, we must have $\widetilde{W}_{k_i} \geq \widetilde{W}^*_{k_i}$. These two conditions imply that $\widetilde{W}_{k_i} = \widetilde{W}^*_{k_i}$. As a result, the worker's incentive compatibility constraint binds and for all period we have $\widetilde{W}_{k_i} = \widetilde{W}^*_{k_i}$. Now consider the binding target utility constraint given in (3.7). We know that on the left hand side, $\widetilde{W}_{k_i} = \widetilde{W}^*_{k_i}$, and on the right hand side, $W_{k_i} = \widetilde{W}^*_{k_i}$. Plugging these values gives us $\omega^*_{k_i} = \frac{k_i}{\rho\beta}$ in all periods other than the first period.

Proof of Lemma 3. Here, we will first assume that such a unique $\overline{k}$ exists where workers with $k_i \leq \overline{k}$ able to form a productive relationship and workers with $k_i > \overline{k}$ stays unemployed permanently. Then, we show such a $\overline{k}$ exists and unique for a parameter space.

First, we will simplify the employer's ICC (3.5). The Value Function of an employer in a relationship with a worker with $k = k_i$ (where $k_i \leq \overline{k}$) after the first period is

$$\Omega\left(\widetilde{W}_{k_i}^*\right) = x - \omega_{k_i}^* + \rho\beta\Omega\left(\widetilde{W}_{k_i}^*\right) + (1-\rho)\,\beta V^{ES}$$

We isolate the $\Omega\left(\widetilde{W}_{k_i}^*\right)$ in the equation and obtain

$$\Omega\left(\widetilde{W}_{k_i}^*\right) = \frac{1}{(1-\rho\beta)}\left(x - \omega_{k_i}^*\right) + \frac{(1-\rho)\,\beta}{(1-\rho\beta)} V^{ES} \tag{D.3}$$

Now we will rewrite employer's ICC (3.5) by plugging $\Omega\left(\widetilde{W}_{k_i}^*\right)$ in equation (D.3) and obtain

$$\omega_{k_i} \leq x - (1-\beta)\,V^{ES} \tag{D.4}$$

Also we import the target utility constraint ((3.7)′) here.

$$\omega_{k_i} \geq \frac{k_i}{\rho\beta} \tag{D.5}$$

Assuming a unique $\overline{k}$ exists, workers with $k_i < \overline{k}$ will get paid $\frac{k_i}{\rho\beta}$ and employer's ICC (D.4) will not bind. If the worker with $k_i = \overline{k}$ form a relationship, that worker will get paid $\frac{\overline{k}}{\rho\beta}$ and employer's ICC (D.4) will bind.

If we show right-hand side of the employer's ICC (D.4), $x-(1-\beta)\,V^{ES}$, is decreasing in $\overline{k}$, it is sufficient to conclude there exists a unique $\overline{k}$. Since $G\left(\overline{k}\right)$ is an increasing function of $\overline{k}$, if right-hand side of the employer's ICC (D.4), $x-(1-\beta)\,V^{ES}$, is decreasing in $G\left(\overline{k}\right)$, there exists a unique $\overline{k}$.

We will first take a total derivative of logarithm of right-hand side of the employer's ICC (D.4) and find

$$d\ln\left(x-(1-\beta)\,V^{ES}\right)=-\frac{(1-\beta)\,V^{ES}}{x-(1-\beta)\,V^{ES}}d\ln V^{ES} \tag{D.6}$$

Next, we will derive the V^{ES}. To do that, first we simplify the V^{ES} in equation (3.2) and obtain

$$V^{ES}=\frac{Q^E\rho\beta}{1-(1-Q^E\rho)\,\beta}V^{ER} \tag{D.7}$$

First period Value Function of the employer that forms a relationship with a worker with $k=k_i$ (where $k_i\leq\overline{k}$) is

$$V^{ER}_{k_i}=x+\rho\beta\Omega\left(\widetilde{W}^*_{k_i}\right)+(1-\rho)\beta V^{ES}$$

We plug $\Omega\left(\widetilde{W}^*_{k_i}\right)$ in equation (D.3) into the above equation and obtain,

$$V^{ER}_{k_i}=\frac{1}{(1-\rho\beta)}\left(x-\rho\beta\omega^*_{k_i}\right)+\frac{(1-\rho)\beta}{(1-\rho\beta)}V^{ES}$$

$V^{ER}_{k_i}$ is the value function of the employer who has formed a relationship with a worker with $k=k_i$. Before k_i is known by the employer, she can only form an expectation on the wage she is going to pay. Suppose we denote such expected wage as $\widetilde{\omega}$. Also, expected effort level employer will face is $\widetilde{\omega} = \frac{\widetilde{k}}{\rho\beta}$ by the binding target utility constraint ((3.7)′).

Then the ex-ante value function of the employer, V^{ER}, will be

$$V^{ER} = \frac{1}{(1-\rho\beta)}\left(x - \rho\beta\widetilde{\omega}\right) + \frac{(1-\rho)\beta}{(1-\rho\beta)} V^{ES}$$

Then we solve for V^{ES} and V^{ER} using equation (D.7) and above equation and obtain,

$$\text{(D.8)} \qquad (1-\beta)\, V^{ES} = \frac{Q^E \rho\beta}{1-(1-Q^E)\,\rho\beta}\left(x - \rho\beta\widetilde{\omega}\right)$$

$$\text{(D.9)} \qquad (1-\beta)\, V^{ER} = \frac{1-\left(1-Q^E\rho\right)\beta}{1-(1-Q^E)\,\rho\beta}\left(x - \rho\beta\widetilde{\omega}\right)$$

We take the total derivative of logarithm of V^{ES} in (D.8) and use $\widetilde{\omega} = \frac{\widetilde{k}}{\rho\beta}$ to obtain

$$\text{(D.10)} \qquad d\ln V^{ES} = \frac{1-\rho\beta}{1-(1-Q^E)\,\rho\beta} d\ln Q^E - \frac{\widetilde{k}}{x-\widetilde{k}} d\ln \widetilde{k}$$

After we substitute $d\ln V^{ES}$ in (D.10) into equation (D.6), we obtain

$$\text{(D.11)} \quad d\ln\left(x - (1-\beta)\, V^{ES}\right) = -\frac{(1-\beta)\, V^{ES}\left(\frac{1-\rho\beta}{1-(1-Q^E)\rho\beta} d\ln Q^E - \frac{\widetilde{k}}{x-\widetilde{k}} d\ln \widetilde{k}\right)}{x-(1-\beta)\, V^{ES}}$$

Here, right-hand side of employer's ICC (D.4), $x-(1-\beta)\, V^{ES}$, is affected by two competing forces. On the one hand, an increase in $\overline{k}$ changes the probability of finding a suitable match, Q^E, and in return changes V^{ES}. On the other hand, an increase in $\overline{k}$ changes the expected wage, $\widetilde{\omega}$, that is going to be paid to workers. Thus surplus left to the employer will change, as well. Next we will determine $d\ln Q^E$ and $d\ln \widetilde{k}$.

We first determine the $d\ln Q^E$. The steady state level of m must satisfy

$$m = \left[1 - H\left(\overline{k}\right)\Phi\left(\frac{n}{m},1\right)\right]m + (1-\rho)\left(E-m\right) + (1-\rho)\,H\left(\overline{k}\right)\Phi\left(\frac{n}{m},1\right)m.$$

On the right-hand side, the first term represents the number of employers with no match; the second term represents the number of previously-matched employers that are exogenously separated; and the last term represents the number of newly-matched employers that are immediately exogenously separated. A straightforward simplification of the above equation yields

$$m = E - \frac{\rho}{1-\rho}H\left(\overline{k}\right)\Phi\left(\frac{n}{m},1\right)m. \tag{D.12}$$

We can rewrite equation (D.12) as

$$\frac{E}{m} = \left[1 + \frac{\rho}{1-\rho}Q^E\right]. \tag{D.13}$$

We take the total derivative of logarithm of each side and find

$$d\ln Q^E = \frac{E}{E-m}d\ln\left(\frac{E}{m}\right) \tag{D.14}$$

Later, we will use the following expression several times.

$$d\ln\left(\frac{n}{L}\right) = -\frac{L-n}{n}d\ln\left(1-\frac{n}{L}\right) \tag{D.15}$$

We would like to express the change in Q^E in terms of change in ratio of workers who are in the relationship, $(1-\frac{n}{L})$, and the change in employer to labour ratio, $\frac{E}{L}$. We will rewrite $\frac{E}{m}$ as $\frac{1}{\frac{E}{L}-\left(1-\frac{n}{L}\right)}\frac{E}{L}$ in equation (D.14) using the identity, $E-m \equiv L-n$. Then the equation (D.14) becomes

$$d\ln Q^E = -\frac{E}{E-m}d\ln\left(\frac{E}{L}-\left(1-\frac{n}{L}\right)\right)+\frac{E}{E-m}d\ln\left(\frac{E}{L}\right) \tag{D.16}$$

We write the first term as

$$d\ln\left(\frac{E}{L}-\left(1-\frac{n}{L}\right)\right) = -\frac{E-m}{m}d\ln\left(1-\frac{n}{L}\right)+\frac{E}{m}d\ln\left(\frac{E}{L}\right) \tag{D.17}$$

We plug relevant $d\ln\left(\frac{E}{L}-\left(1-\frac{n}{L}\right)\right)$ in the equation (D.17) into equation (D.16) and obtain

$$d\ln Q^E = \frac{E}{m}d\ln\left(1-\frac{n}{L}\right)-\frac{E}{m}d\ln\left(\frac{E}{L}\right) \tag{D.18}$$

Now we will determine $d\ln\widetilde{\omega}$. Employer will able to form a relationship with a worker that has $k_i \leq \overline{k}$. We know $G\left(\overline{k}\right)$ is the population cumulative distribution function of workers and gives the ratio of workers with $k_i \leq \overline{k}$ in the labour population. For those workers with $k_i > \overline{k}$, will always stay in the search pool and never able to have a successful match. So their number, $L\left[1 - G\left(\overline{k}\right)\right]$, is going to be constant in the search pool . If $H\left(\overline{k}\right)$ is the probability of the unemployed workers in the search pool with $k_i \leq \overline{k}$, then we will have an identity as below.

(D.19) $$n\left[1 - H\left(\overline{k}\right)\right] \equiv L\left[1 - G\left(\overline{k}\right)\right]$$

We isolate $H\left(\overline{k}\right)$ from the equation above and obtain

(D.20) $$H\left(\overline{k}\right) = \frac{G\left(\overline{k}\right) - \left(1 - \frac{n}{L}\right)}{1 - \left(1 - \frac{n}{L}\right)}$$

Notice, $H\left(\overline{k}\right) < G\left(\overline{k}\right)$, because a portion of the workers who has $k_i \leq \overline{k}$ are in a relationship and are not searching. The formula for the expected wage employer is going pay is

$$\widetilde{\omega} = \frac{1}{G\left(\overline{k}\right)} \int_0^{\overline{k}} \omega_{k_i} g\left(k_i\right) dk_i$$

Similarly, the expected effort level the employer is going to face, $\widetilde{k}$, as

(D.21) $$\widetilde{k} = \frac{1}{G\left(\overline{k}\right)} \int_0^{\overline{k}} k_i g\left(k_i\right) dk_i$$

We take the total derivative of $\widetilde{\omega} = \frac{\widetilde{k}}{\rho\beta}$ and find $d\ln\widetilde{\omega} = d\ln\widetilde{k}$. Then, using equation (D.21), we obtain

$$d\ln\widetilde{\omega} = d\ln\int_0^{\overline{k}} k_i g\left(k_i\right) dk_i - d\ln G\left(\overline{k}\right) \tag{D.22}$$

The first term, $d\ln\int_0^{\overline{k}} k_i g\left(k_i\right) dk_i$, can be written as

$$d\ln\int_0^{\overline{k}} k_i g\left(k_i\right) dk_i = \frac{d\int_0^{\overline{k}} k_i g\left(k_i\right) dk_i}{\int_0^{\overline{k}} k_i g\left(k_i\right) dk_i}$$

The numerator, $d\int_0^{\overline{k}} k_i g\left(k_i\right) dk_i$, is equal to $\overline{k}g\left(\overline{k}\right)$ and by the equation (D.21) denominator is equal to $\widetilde{k}G\left(\overline{k}\right)$. Then, equation (D.22) becomes

$$d\ln\widetilde{\omega} = \frac{\overline{k}-\widetilde{k}}{\widetilde{k}} d\ln G\left(\overline{k}\right) \tag{D.23}$$

Now, we can plug the equation (D.18) and (D.23) into (D.10) to obtain

$$d\ln V^{ES} = \frac{1-\rho\beta}{1-\left(1-Q^E\right)\rho\beta}\frac{E}{m}\left(d\ln\left(1-\frac{n}{L}\right) - d\ln\left(\frac{E}{L}\right)\right) - \frac{\overline{k}-\widetilde{k}}{x-\widetilde{k}} d\ln G\left(\overline{k}\right) \tag{D.24}$$

We will show that right-hand side of the employer's ICC (D.4) is decreasing with $\overline{k}$ when $\frac{E}{L}$ exceeds a certain threshold. Given $\frac{E}{L}$, $d\ln\left(\frac{E}{L}\right) = 0$, we take the derivative of (D.24) with respect to $d\ln G\left(\overline{k}\right)$ and obtain

$$\frac{d\ln V^{ES}}{d\ln G\left(\overline{k}\right)} = \frac{1-\rho\beta}{1-\left(1-Q^E\right)\rho\beta}\frac{E}{m}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)} - \frac{\overline{k}-\widetilde{k}}{x-\widetilde{k}} \tag{D.25}$$

Now, we will show the right-hand side of the employer's ICC (D.4) is bounded from above. After we plug equation (D.8) into the employer's ICC (D.4) we can rewrite the employer's ICC (D.4) as

$$\omega_{k_i} \leq \frac{(1-\rho\beta)\,x + Q^E\rho\beta\rho\beta\widetilde{\omega}}{1-(1-Q^E)\,\rho\beta} \tag{D.26}$$

We will use the information that the worker's wage with $k_i = \overline{k}$ will be higher than the average wage, $\widetilde{\omega}$, namely $\widetilde{\omega} < \omega_{\overline{k}}$. So,

$$\frac{(1-\rho\beta)\,x + Q^E\rho\beta\rho\beta\widetilde{\omega}}{1-(1-Q^E)\,\rho\beta} < \frac{(1-\rho\beta)\,x + Q^E\rho\beta\rho\beta\omega_{\overline{k}}}{1-(1-Q^E)\,\rho\beta} \tag{D.27}$$

Recognize that employer's ICC (D.4) will bind for the worker with $k_i = \overline{k}$. So, inequality (D.26) will be an equality for the worker with $k_i = \overline{k}$. Using the (D.26) and (D.27) we obtain

$$\omega_{\overline{k}} < \frac{(1-\rho\beta)\,x + Q^E\rho\beta\rho\beta\omega_{\overline{k}}}{1-(1-Q^E)\,\rho\beta} \tag{D.28}$$

Then, we isolate the $\omega_{\overline{k}}$ and obtain

$$\omega_{\overline{k}} < \frac{x}{1+Q^E\rho\beta} \tag{D.29}$$

So,

$$x - (1-\beta)\,V^{ES} < \frac{x}{1+Q^E\rho\beta} \tag{D.30}$$

It is easy to show $\frac{x}{1+Q^E\rho\beta} \leq x$. Existence of an upper bound to the right-hand side of employer's ICC (D.4) proves employer's ICC (D.4) and target utility constraint (D.5) intersects at least once. The upper bound for the right-hand of employer's ICC (D.4), $\frac{x}{1+Q^E\rho\beta}$, and the right-hand side of the target utility constraint (D.5), $\frac{\overline{k}}{\rho\beta}$, intersects at $\overline{k} = \frac{\rho\beta x}{1+Q^E\rho\beta}$. This implies, right-hand side of the employer's ICC (D.4) and right-hand of the target utility constraint (D.5) have to intersect at $\overline{k} < \frac{\rho\beta x}{1+Q^E\rho\beta}$ in equilibrium (see the Figure 3.1). Then, the second term in the equation (D.25) will be

$$\frac{\overline{k}-\widetilde{k}}{x-\widetilde{k}} < \frac{\rho\beta}{1+Q^E\rho\beta}\frac{x-\frac{1+Q^E\rho\beta}{\rho\beta}\widetilde{k}}{x-\widetilde{k}}$$

Since $\frac{x-\frac{1+Q^E\rho\beta}{\rho\beta}\widetilde{k}}{x-\widetilde{k}} < 1$, above inequality can be written as

$$\frac{\overline{k}-\widetilde{k}}{x-\widetilde{k}} < \frac{\rho\beta}{1+Q^E\rho\beta}$$

So, $\frac{d\ln V^{ES}}{d\ln G\left(\overline{k}\right)}$ in equation (D.25) will have an upper bound,

$$\frac{d\ln V^{ES}}{d\ln G\left(\overline{k}\right)} > \frac{1-\rho\beta}{1-\left(1-Q^E\right)\rho\beta}\frac{E}{m}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)} - \frac{\rho\beta}{1+Q^E\rho\beta}$$

If, $\frac{1-\rho\beta}{1-(1-Q^E)\rho\beta}\frac{E}{m}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)} - \frac{\rho\beta}{1+Q^E\rho\beta} > 0$ then $\frac{d\ln V^{ES}}{d\ln G\left(\overline{k}\right)} > 0$. Using $\frac{E}{m} = \left[1+\frac{\rho}{1-\rho}Q^E\right]$ in equation (D.13), it is sufficient to have $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)} > \frac{1+\frac{\rho\beta}{1-\rho\beta}Q^E}{1+\frac{\rho}{1-\rho}Q^E}\frac{\rho\beta}{1+Q^E\rho\beta}$ for a decreasing right-hand side of the employer's ICC (D.4). Since $\frac{1+\frac{\rho\beta}{1-\rho\beta}Q^E}{1+\frac{\rho}{1-\rho}Q^E}\frac{\rho\beta}{1+Q^E\rho\beta}$ is decreasing in Q^E, it is maximum at $Q^E = 0$. From it, we conclude,

(D.31) $$\frac{d\ln V^{ES}}{d\ln G\left(\overline{k}\right)} > 0, \text{ if } \frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)} > \rho\beta$$

Since we have established the sufficient condition for the uniqueness of $\overline{k}$, we will now find whether this sufficient condition is satisfied. To do that, we will find the $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)}$. We already have an equation (D.18) that relates $d\ln Q^E$ in terms of $d\ln\left(1-\frac{n}{L}\right)$. If we have another equation that shows the relationship between $d\ln Q^E$ and $d\ln\left(1-\frac{n}{L}\right)$, we will be able to find $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)}$.

We know $Q^E = \frac{H\left(\overline{k}\right)\Phi(n,m)}{m}$. We take the total derivative of logarithm of each side and find

$$d\ln Q^E = d\ln H\left(\overline{k}\right) + d\ln\Phi\left(\frac{n}{m},1\right) \tag{D.32}$$

First, we take take total derivative of logarithm of both sides of equation (D.20) to find $d\ln H\left(\overline{k}\right)$

$$d\ln H\left(\overline{k}\right) = d\ln(G\left(\overline{k}\right) - \left(1-\frac{n}{L}\right)) - d\ln(1-\left(1-\frac{n}{L}\right))$$

Then, using equation (D.15), we obtain

$$d\ln H\left(\overline{k}\right) = \frac{L}{n}\frac{G\left(\overline{k}\right)}{H\left(\overline{k}\right)}d\ln G\left(\overline{k}\right) - \frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{L-n}{n}d\ln\left(1-\frac{n}{L}\right) \tag{D.33}$$

Next, we can decompose the $d\ln\Phi\left(\frac{n}{m},1\right)$ by

$$d\ln\Phi\left(\frac{n}{m},1\right) = \frac{\partial\ln\Phi\left(\frac{n}{m},1\right)}{\partial\ln\left(\frac{n}{m}\right)}d\ln\left(\frac{n}{m}\right) \tag{D.34}$$

The first term, $\frac{\partial\ln\Phi\left(\frac{n}{m},1\right)}{\partial\ln\left(\frac{n}{m}\right)}$ is the elasticity of the match function, $\varepsilon_{\frac{n}{m},\Phi}$ and $\varepsilon_{\frac{n}{m},\Phi} < 1$. We can write the $d\ln\left(\frac{n}{m}\right)$ as

$$d\ln\left(\frac{n}{m}\right) = d\ln\left(\frac{n}{L}\right) + d\ln\left(\frac{E}{m}\right) - d\ln\left(\frac{E}{L}\right)$$

We plug the $d\ln\left(\frac{n}{L}\right)$ in equation (D.15) and $d\ln\left(\frac{E}{m}\right)$ using equations (D.14)and (D.18) into above equation and get

$$d\ln\left(\frac{n}{m}\right)=\frac{n-m}{m}\frac{L-n}{n}d\ln\left(1-\frac{n}{L}\right)-\frac{E}{m}d\ln\left(\frac{E}{L}\right)$$

Then, the equation (D.34) becomes

$$d\ln\Phi\left(\frac{n}{m},1\right)=\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\frac{L-n}{n}d\ln\left(1-\frac{n}{L}\right)-\frac{E}{m}d\ln\left(\frac{E}{L}\right)\right)\tag{D.35}$$

The total derivative of probability of finding a suitable match after we substitute equations (D.33) and (D.35) into equation (D.32) and rearrange will be

$$d\ln Q^E=\left[\begin{array}{r}\frac{L}{n}\frac{G\left(\overline{k}\right)}{H\left(\overline{k}\right)}d\ln G\left(\overline{k}\right)\\+\left(-\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\right)\left(\frac{L-n}{n}\right)\right)d\ln\left(1-\frac{n}{L}\right)\\-\varepsilon_{\frac{n}{m},\Phi}\frac{E}{m}d\ln\left(\frac{E}{L}\right)\end{array}\right.\tag{D.36}$$

Now, we are ready to show that right-hand side of the employer's ICC (D.4) is decreasing with $\overline{k}$ when $\frac{E}{L}$ exceeds a certain threshold. Given $\frac{E}{L}$, $d\ln\left(\frac{E}{L}\right)=0$, we take the derivative of (D.18) and (D.36) with respect to $d\ln G\left(\overline{k}\right)$ and obtain

$$\frac{d\ln Q^E}{d\ln G\left(\overline{k}\right)}=\frac{E}{m}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)}\tag{D.37}$$

$$\frac{d\ln Q^E}{d\ln G\left(\overline{k}\right)}=\left(\begin{array}{c}\frac{L}{n}\frac{G\left(\overline{k}\right)}{H\left(\overline{k}\right)}\\+\left(-\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\right)\left(\frac{L-n}{n}\right)\right)\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)}\end{array}\right)\tag{D.38}$$

Now have two equations two unknowns $\left(\frac{d\ln Q^E}{d\ln G\left(\overline{k}\right)},\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\overline{k}\right)}\right)$ and solve

$\frac{E}{m}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} = \frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} + \left(-\frac{1-H\left(\bar{k}\right)}{H\left(\bar{k}\right)}\frac{L-n}{n} + \varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\right)\frac{L-n}{n}\right)\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)}$ We isolate $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)}$ and obtain

$$\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} = \frac{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)}}{\frac{E}{m} + \frac{1-H\left(\bar{k}\right)}{H\left(\bar{k}\right)}\frac{L-n}{n} - \varepsilon_{\frac{n}{m},\Phi}\frac{n-m}{m}\frac{L-n}{n}}$$

We add and subtract $\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)}$ in the denominator and obtain

$$\text{(D.39)}\qquad \frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} = \frac{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)}}{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} + \frac{E}{m} - \left(\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} - \frac{1-H\left(\bar{k}\right)}{H\left(\bar{k}\right)}\frac{L-n}{n}\right) - \varepsilon_{\frac{n}{m},\Phi}\frac{n-m}{m}\frac{L-n}{n}}$$

We take the term $\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} - \frac{1-H\left(\bar{k}\right)}{H\left(\bar{k}\right)}\frac{L-n}{n}$, manipulate a bit, use equation (D.20), and obtain

$$\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} - \frac{1-H\left(\bar{k}\right)}{H\left(\bar{k}\right)}\frac{L-n}{n} = \frac{L}{n}$$

Now we plug the equivalent term into the equation (D.39) and obtain

$$\text{(D.40)}\qquad \frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} = \frac{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)}}{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} + \frac{E}{m} - \frac{L}{n} - \varepsilon_{\frac{n}{m},\Phi}\frac{n-m}{m}\frac{L-n}{n}}$$

We use identity $E - m \equiv L - n$, and obtain $\frac{E}{m} - \frac{L}{n} = \frac{n-m}{n}\frac{E-m}{m}$. Then we plug it into (D.40) and get

$$\text{(D.41)}\qquad \frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} = \frac{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)}}{\frac{L}{n}\frac{G\left(\bar{k}\right)}{H\left(\bar{k}\right)} + \frac{n-m}{n}\frac{E-m}{m} - \varepsilon_{\frac{n}{m},\Phi}\frac{n-m}{n}\frac{E-m}{m}}$$

We take the terms in the denominator the $\frac{n-m}{n}\frac{E-m}{m}$ common parenthesis and numerator and denominator with $\frac{L}{n}\frac{G(\bar{k})}{H(\bar{k})}$ and obtain

$$\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)}=\frac{1}{1+\frac{H(\bar{k})}{G(\bar{k})}\left(1-\varepsilon_{\frac{n}{m},\Phi}\right)\frac{n-m}{n}\frac{n}{L}\frac{E-m}{m}}$$

We use $\frac{E-m}{m}=\frac{\rho}{1-\rho}Q^{E}$ (D.13) and $\frac{n-m}{n}=\frac{1-\frac{E}{L}}{\frac{n}{L}}$ and obtain

$$\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)}=\frac{1}{1+\frac{H(\bar{k})}{G(\bar{k})}\left(1-\varepsilon_{\frac{n}{m},\Phi}\right)\left(1-\frac{E}{L}\right)\frac{\rho}{1-\rho}Q^{E}} \tag{D.42}$$

Since $\varepsilon_{\frac{n}{m},\Phi}<1$, if $\frac{E}{L}<1$, then $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G(\bar{k})}<1$ and if $\frac{E}{L}>1$, then $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G(\bar{k})}>1$.

$$\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)}=\begin{cases}<1 \text{ for} \frac{E}{L}<1\\ =1 \text{ for} \frac{E}{L}=1\\ >1 \text{ for} \frac{E}{L}>1\end{cases}$$

We know the $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)}$ from (D.42). Also we have found the sufficient condition for decreasing employer's ICC (D.31): $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} > \rho\beta$. We will now check parameter space for which right-hand side of employer's ICC (D.4) is decreasing (or equivalently V^{ES} is increasing).

Case 1

If $\frac{E}{L} = 1$, since $\left(1 - \varepsilon_{\frac{n}{m},Q^E}\right)\left(1 - \frac{E}{L}\right) = 0$ and $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} = 1$ where $1 > \rho\beta$.

So, right-hand side of the employer's ICC (D.4) is decreasing for $E = L$.

Case 2

If $\frac{E}{L} > 1$, then $\left(1 - \varepsilon_{\frac{n}{m},Q^E}\right)\left(1 - \frac{E}{L}\right) < 0$ and $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} > 1$

So, right-hand side of the employer's ICC (D.4) is decreasing for $\frac{E}{L} > 1$.

Case 3

If $\frac{E}{L} < 1$, then $\left(1 - \varepsilon_{\frac{n}{m},Q^E}\right)\left(1 - \frac{E}{L}\right) > 0$ and $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} < 1$

Right-hand side of the employer's ICC (D.4) is decreasing for $\frac{E}{L} > \left(\frac{E}{L}\right)^{crit}$ that makes the $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln G\left(\bar{k}\right)} > \rho\beta$.

Proof of Lemma 4.

We will show V^{ES} is decreasing with $\frac{E}{L}$ while $\overline{k}$ is constant. Given that $\overline{k}$ is constant, $(d\ln G\left(\overline{k}\right)=0)$, we take the derivative of (D.18) and (D.36) with respect to $d\ln\left(\frac{E}{L}\right)$ and obtain

$$\frac{d\ln Q^E}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} = -\frac{E}{m}+\frac{E}{m}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} \tag{D.43}$$

$$\frac{d\ln Q^E}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} = \begin{pmatrix} -\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{L-n}{n} \\ +\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\right)\left(\frac{L-n}{n}\right)\end{pmatrix}\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant}-\varepsilon_{\frac{n}{m},\Phi}\frac{E}{m} \tag{D.44}$$

Now we have an equation system with two equations (D.43), (D.44) and two unknowns $\left(\frac{d\ln Q^E}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant}, \frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant}\right)$.

$-\frac{E}{m}+\varepsilon_{\frac{n}{m},\Phi}\frac{E}{m}=\left(-\frac{E}{m}-\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\right)\left(\frac{L-n}{n}\right)\right)\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant}$

We multiply both sides with $\frac{m}{E}$ and obtain

$-1+\varepsilon_{\frac{n}{m},\Phi}=\left(-1-\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\right)\left(\frac{L-n}{n}\right)\frac{m}{E}\right)\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant}$

We add and subtract $\varepsilon_{\frac{n}{m},Q^E}$ from the left-hand side, arrange the terms, and obtain

$$\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant}=\frac{1-\varepsilon_{\frac{n}{m},\Phi}}{1-\varepsilon_{\frac{n}{m},\Phi}+\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}-\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\frac{L-n}{n}\frac{m}{E}-1\right)} \tag{D.45}$$

We will use $E-L=m-n$ to rewrite the term in the denominator as

$$\varepsilon_{\frac{n}{m},\Phi}\left(\frac{n-m}{m}\frac{L-n}{n}\frac{m}{E}-1\right)=-\varepsilon_{\frac{n}{m},\Phi}\frac{m}{E}\frac{L}{n} \tag{D.46}$$

We plug the above term into (D.45) and obtain

$$\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} = \frac{1-\varepsilon_{\frac{n}{m},\Phi}}{1-\varepsilon_{\frac{n}{m},\Phi}+\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\frac{m}{E}\frac{L}{n}} \tag{D.47}$$

Since denominator of (D.47) is bigger than numerator, $\frac{d\ln\left(1-\frac{n}{L}\right)}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} < 1$. Using equation (D.47), equations (D.43) and (D.44) becomes

$$\frac{d\ln Q^E}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} = -\frac{E}{m}\frac{\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\frac{m}{E}\frac{L}{n}}{1-\varepsilon_{\frac{n}{m},\Phi}+\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\frac{m}{E}\frac{L}{n}} \tag{D.48}$$

$$\frac{d\ln V^{ES}}{d\ln\left(\frac{E}{L}\right)}|_{\overline{k}=constant} = -\frac{1-\rho\beta}{1-\left(1-Q^E\right)\rho\beta}\frac{E}{m}\frac{\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\frac{m}{E}\frac{L}{n}}{1-\varepsilon_{\frac{n}{m},\Phi}+\frac{1-H\left(\overline{k}\right)}{H\left(\overline{k}\right)}\frac{m}{E}\frac{L-n}{n}+\varepsilon_{\frac{n}{m},\Phi}\frac{m}{E}\frac{L}{n}} \tag{D.49}$$

So, Q^E and V^{ES} will decrease as it can be seen from equation (D.48) and (D.49). For all E, L combinations, increase in $\frac{E}{L}$ will shift the V^{ES} and the employer's ICC curve (D.4) upwards . So, employer's ICC and target utility constraint will intersect at a higher $\overline{k}$.

Here, first we will find out the earnings instability of the workers with $k_i \leq \overline{k}$ and then find-out the economy-wide income variance. There are two types of wages at the steady state: for those that are in the second or later periods of their employment, we have the wage, ω_{k_i}, and for those that are either unemployed or in the first period of their employment, we have the wage, 0. The proportion of workers that receive a particular type of wage is given below.

Employment Status	Employment Period	Wage	Proportion of Home Country Workers
Employed	1^{st}	0	$(1-\rho)\left(1-\frac{n}{L}\right)$
Employed	2^{nd} or later	ω_{k_i}, where $k_i \leq \overline{k}$	$\rho\left(1-\frac{n}{L}\right)$
Unemployed $\left(k_i \leq \overline{k}\right)$	-	0	$\frac{n}{L}-\left(1-G\left(\overline{k}\right)\right)$
Unemployed $\left(k_i > \overline{k}\right)$	-	0	$1-G\left(\overline{k}\right)$

Denote $\overline{\omega}_{k_i}$ as the permanent income of a worker with $k_i \leq \overline{k}$. Then, $\overline{\omega}_{k_i}$ is given by

(D.50) $$\overline{\omega}_{k_i \leq \overline{k}} = Pr_{\omega_{k_i}>0}\,\omega_{k_i \leq \overline{k}}$$

where $Pr_{\omega_{k_i}>0}$ is the probability of a worker with $k_i \leq \overline{k}$ receiving positive wage (in the above table second row of last column divided by $G\left(\overline{k}\right)$). So,

$$\begin{aligned} Pr_{\omega_{k_i}>0} &= \frac{1}{G\left(\overline{k}\right)}\rho\left(1-\frac{n}{L}\right) \\ Pr_{\omega_{k_i}=0} &= 1-Pr_{\omega_{k_i}>0} \end{aligned}$$

Worker's transitory earnings variation, $\sigma^2_{\omega_{k_i \leq \bar{k}}}$, will be

$$\sigma^2_{\omega_{k_i \leq \bar{k}}} = Pr_{\omega_{k_i}>0}\left(\omega_{k_i \leq \bar{k}} - \overline{\omega}_{k_i \leq \bar{k}}\right)^2 + Pr_{\omega_{k_i}=0}\left(0 - \overline{\omega}_{k_i \leq \bar{k}}\right)^2 \tag{D.51}$$

After using the permanent income equation (D.50) in (D.51) we obtain,

$$\sigma^2_{\omega_{k_i \leq \bar{k}}} = \left[1 - Pr_{\omega_{k_i}>0}\right] Pr_{\omega_{k_i}>0}\omega^2_{k_i \leq \bar{k}} \tag{D.52}$$

We take the total derivative of (D.52) and obtain,

$$d\sigma^2_{\omega_{k_i \leq \bar{k}}} = \left[1 - 2Pr_{\omega_{k_i \leq \bar{k}}>0}\right] \omega^2_{k_i \leq \bar{k}} dPr_{\omega_{k_i}>0}$$

Factor market integration will decrease the probability of the home country worker being in the relationship, so $dPr_{\omega_{k_i}>0} < 0$. As a result, transitory earning variation will increase iff $Pr_{\omega_{k_i}>0} > \frac{1}{2}$ or equivalently $\frac{1}{G(\overline{k})}\rho\left(1 - \frac{n}{L}\right) > \frac{1}{2}$. Unemployment rate (for workers whose $k_i \leq \overline{k}$) is $\frac{\frac{n}{L} - \left(1 - G\left(\overline{k}\right)\right)}{G\left(\overline{k}\right)}$. After some manipulation, we find that transitory earnings variation will increase iff unemployment rate (for workers whose $k_i \leq \overline{k}$) is less than $\frac{2\rho - 1}{2\rho}$.

Now, we are ready to look at the economy-wide income variance. Let $Pr_{\omega>0}$ and $Pr_{\omega=0}$ represent the steady-state probability that a worker is in the second and subsequent periods of employment and a worker is unemployed or in the first periods of employment, respectively. Then, we have

$$\begin{aligned} Pr_{\omega>0} &= \rho\left(1-\frac{n}{L}\right) \\ Pr_{\omega=0} &= 1-\rho\left(1-\frac{n}{L}\right). \end{aligned}$$

Denote $E(\omega)$ as the expected wage of a worker who might be working or unemployed. Then, $E(\omega)$ is given by

$$E(\omega) = [1 - Pr_{\omega=0}]\,\widetilde{\omega}. \tag{D.53}$$

Denote σ_ω^2 the variance of the wage. It can be calculated as

$$\sigma_\omega^2 = [1 - Pr_{\omega=0}]\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left(\omega_{k_i} - E(\omega)\right)^2 g(k_i)\,dk_i + Pr_{\omega=0}\left(E(\omega)\right)^2$$

The first term, $\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left(\omega_{k_i} - E(\omega)\right)^2 g(k_i)\,dk_i$, measures the deviation of the income from average for those workers who earn positive income.

We add and subtract $\widetilde{\omega}$ from the first term and obtain

$$\sigma_\omega^2 = \left[\begin{array}{c} \left[1-Pr_{\omega=0}\right]\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left(\omega_{k_i}-\widetilde{\omega}+\widetilde{\omega}-E\left(\omega\right)\right)^2 g\left(k_i\right)dk_i \\ +Pr_{\omega=0}\left(E\left(\omega\right)\right)^2 \end{array}\right]$$

$$\sigma_\omega^2 = \left[\begin{array}{c} \left[1-Pr_{\omega=0}\right]\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left[\begin{array}{c} \left(\omega_{k_i}-\widetilde{\omega}\right)^2+\left(\widetilde{\omega}-E\left(\omega\right)\right)^2 \\ +2\left(\omega_{k_i}-\widetilde{\omega}\right)\left(\widetilde{\omega}-E\left(\omega\right)\right) \end{array}\right] g\left(k_i\right)dk_i \\ +Pr_{\omega=0}\left(E\left(\omega\right)\right)^2 \end{array}\right]$$

$$\sigma_\omega^2 = \left[\begin{array}{c} \left[1-Pr_{\omega=0}\right]\left(\widetilde{\omega}-E\left(\omega\right)\right)^2 \\ +Pr_{\omega=0}\left(E\left(\omega\right)\right)^2 \\ +\left[1-Pr_{\omega=0}\right]\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left(\omega_{k_i}-\widetilde{\omega}\right)^2 g\left(k_i\right)dk_i \end{array}\right]$$

$$\sigma_\omega^2 = \left[\begin{array}{c} \left[1-Pr_{\omega=0}\right]\left(\widetilde{\omega}-\left[1-Pr_{\omega=0}\right]\widetilde{\omega}\right)^2 \\ +Pr_{\omega=0}\left[1-Pr_{\omega=0}\right]^2\widetilde{\omega}^2 \\ +\left[1-Pr_{\omega=0}\right]\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left(\omega_{k_i}-\widetilde{\omega}\right)^2 g\left(k_i\right)dk_i \end{array}\right]$$

$$\sigma_\omega^2 = \left[\begin{array}{c} \left[1-Pr_{\omega=0}\right]Pr_{\omega=0}\widetilde{\omega}^2 \\ +\left[1-Pr_{\omega=0}\right]\frac{1}{G\left(\overline{k}\right)}\int_0^{\overline{k}}\left(\omega_{k_i}-\widetilde{\omega}\right)^2 g\left(k_i\right)dk_i \end{array}\right]$$

$$\sigma_\omega^2 = \left[1-Pr_{\omega=0}\right]Pr_{\omega=0}\widetilde{\omega}^2+\left[1-Pr_{\omega=0}\right]Var\left(\omega_{k_i}\right)$$

where $Var(\omega_{k_i})$ is the variance of wages for the workers in the second and subsequent period relationship.

$$\begin{aligned}
\sigma_\omega^2 &= [1 - Pr_{\omega=0}] \left[Pr_{\omega=0}\widetilde{\omega}^2 + Var(\omega_{k_i})\right] \\
\text{where, } Var(\omega_{k_i}) &= \frac{1}{G(\overline{k})} \int_0^{\overline{k}} (\omega_{k_i} - \widetilde{\omega})^2 g(k_i)\, dk_i
\end{aligned}$$

We take total derivative and obtain

(D.54)

$$d\sigma_\omega^2 = -\left[(-1 + 2Pr_{\omega=0})\widetilde{\omega}^2 + Var(\omega_{k_i})\right] dPr_{\omega=0} + 2[1 - Pr_{\omega=0}] Pr_{\omega=0}\widetilde{\omega} d\widetilde{\omega} + [1 - Pr_{\omega=0}] dVar(\omega_{k_i})$$

Factor market integration will decrease the probability of the home country worker being in the relationship, so $dPr_{\omega>0} < 0$, the average wages employer will pay, $\widetilde{\omega}$, and the variance of wages for the workers in the relationship, $Var(\omega_{k_i})$. As a result, economy-wide income inequality, σ_ω^2, will decrease, if $Pr_{\omega=0} > \frac{1}{2}$.