Gas Bubbles Motion in an Oscillating Fluid

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Abstract Motion of a gas bubble in an oscillating fluid is of fundamental interest for the theory of various widely used technological processes, in particular, for the flotation process. Therefore, many studies have been concerned with this problem, some of those being undertaken by eminent scientists. The main remarkable effects are gas bubbles sinking in vibrating fluid's volume and asynchronous self-induced vibration of emerging air cushion. In the authors' recent papers, the problem has been solved by means of the concept of vibrational mechanics and the method of direct separation of motions; experimental studies have been also conducted. The present paper generalizes those studies. It is shown that the condition of gas bubbles sinking is strongly dependent on its own compressibility as well as on the compressibility of the surrounding medium. A formula for the average velocity of gas bubble motion, which significantly depends on the depth of its submergence and on vibration parameters, is derived. A simple physical explanation of the experimentally observed and analytically studied effects is given.

Key words: Oscillating fluid volume, compressibility effects, method of direct separation of motions, sinking condition

1 Introduction

In the literature, the effect of gas bubbles sinking in a vertically oscillating fluid-filled volume has been attributed to two different mechanisms, which may be called a “wave-induced motion” [1, 2, 3] and a “vibration-induced motion” [4, 5]. The key feature of the first one is the gradient of the wave amplitude, while the key feature of the second mechanism is the compressibility of the bubble. In the present paper, these mechanisms are considered as being coupled. A gas bubble motion in vibrating fluid-filled volume is studied accounting for the compressibility of both the bubble and the gas-saturated layer, which is generated due to the turbulent fluid motion near its free surface [6]. In paper [1], to simplify the analysis, bubble's velocity relative to the fluid was considered to be equal to its velocity relative to the volume, whereas in the present study this assumption is omitted. A nonlinear differential equation, which describes bubble's motion relative to the volume, is derived, and, for the solution of this equation, the method of direct separation of motions is applied. The conditions of bubble sinking in gas-saturated fluid layer are determined. An expression for the critical thickness of this layer, starting from which it moves into the fluid volume, is derived, i.e. the condition of vibrational instability of the separate state of the gas-fluid system is obtained.

An approximate expression for the average velocity of bubble's motion in gas saturated layer, which strongly depends on the depth of its location and on vibration parameters, and an expression for the average velocity of penetration of this layer into the volume are derived.

Based on the obtained theoretical results, a simple physical explanation of the effects, experimentally observed in the fluid under the action of vibration, is provided. The results of recently conducted dedicated experiments are reported.

2 Governing equations

Motion of a bubble in vertically oscillating with amplitude A and frequency ω in harmonic law $\alpha = A\sin\omega t$ fluid-filled cylindrical volume is analyzed (Fig.1.).

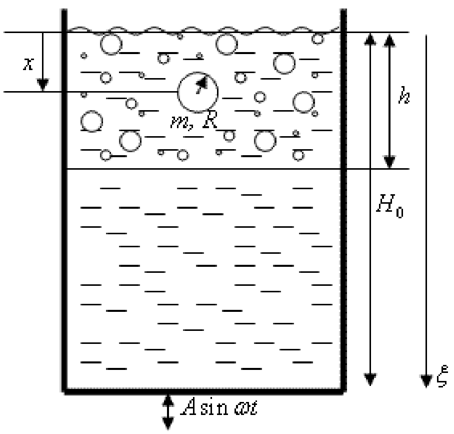

Fig. 1. Model of a bubble in gas saturated fluid.

It is assumed that the fluid is saturated with gas till the certain depth $h \le H_0$ (H_0 - the height of the fluid column) and that this fluid column may be considered as an elastic rod. The rest of the fluid is assumed to follow the oscillations of the vessel. The absolute displacement of the rod's cross-section, which in undisturbed state is situated at distance x from its free (upper) edge, is designated as $\xi(x,t)$. With the boundary conditions set as $\xi'|_{x=0} = 0$, $\xi|_{x=h} = A\sin\omega t$, this displacement is determined by the expression

$$\xi = \frac{g}{2c^2}(h^2 - x^2) + A\left(\cos\frac{\omega x}{c} \Big/ \cos\frac{\omega h}{c}\right)\sin\omega t \qquad (1)$$

here c - sound speed in a gas-saturated fluid.

A pressure in an oscillating fluid at the distance x ($x \le h$) from its free surface is defined by the formula [7]

$$P = P_e + \rho x g - \rho\int_0^x \frac{\partial^2\xi}{\partial t^2}(x_1,t)dx_1, \qquad (2)$$

here P_e is an external (say, atmospheric) pressure, ρ is the density of gas-saturated fluid, g - gravity acceleration.

Assuming the bubble to be compact (the condition $\frac{\omega R}{c} << 1$ is held, R is the radius of the bubble) and taking into account the expression (2), the equation of its motion in gas-saturated fluid can be written as

$$m\ddot{x} + \frac{d\left(m_0\dot{x}\right)}{dt} = -F(v_{rel}) + \left(m - \rho V_b\right)g + \rho V_b\frac{\partial^2\xi}{\partial t^2}(x,t) - m\ddot{\alpha} - \frac{d\left(m_0\left(\dot{\alpha} - \frac{\partial\xi}{\partial t}(x,t)\right)\right)}{dt} \qquad (3)$$

Here $v_{rel} = \dot{x} + \dot{\alpha} - \frac{\partial\xi}{\partial t}(x,t)$ is bubble's velocity relative to the surrounding medium (gas saturated fluid), m is the mass of the bubble (it accounts for the mass of a particle attached to the bubble), m_0 is the fluid added mass, defined by the formula $m_0 = \chi V_b \rho$, with χ being an added mass coefficient, V_b - instantaneous volume of the bubble (dot designates the full time derivative). $F(v_{rel}) = k_1 R^2 v_{rel}$ is the resistance force to gas bubble motion.

Assuming that gas bubble volume pulsations are small and isothermal, and taking into account expression (2), the following equation is used to describe the evolution of the volume of the bubble

$$\rho R_0\frac{d^2\Delta R}{dt^2} + 3\frac{\Delta R}{R_0}P_e = -\rho x g - \rho A\omega^2 f(x)\sin\omega t \qquad (4)$$

where $f(x)=\frac{c}{\omega}\left(\sin\frac{\omega x}{c}\Big/\cos\frac{\omega h}{c}\right)$, R_0 is the radius of the bubble near free surface of the fluid, $\Delta R = R - R_0$ (the surface tension is neglected, since it is assumed that the bubble has the radius, larger than 2 micrometer [1]).

3 Solution by the method of direct separation of motions

Inasmuch as the condition $\frac{c}{A\omega} >> 1$ is held, the variable x can be considered as parameter while solving the equation (4). Thus, we obtain the following expression for the instantaneous volume of the bubble

$$V_b = V_{b0}\left(1-\frac{\rho}{P_e}\left(xg + A\omega^2 f(x)\frac{\lambda^2}{\lambda^2-\omega^2}\sin\omega t\right)\right) \tag{5}$$

where $\lambda = \frac{1}{R_0}\sqrt{\frac{3P_e}{\rho}}$ is the eigenfrequency of bubble's radial oscillations.

For the solution of the problem we use the concept of vibrational mechanics and the method of direct separation of motions [8]. The solutions to equation (3) are sought in the form

$$x = X(t) + \psi(t,\tau) \tag{6}$$

where X - "slow", and ψ - "fast", 2π- periodic in dimensionless ("fast") time $\tau = \omega t$ variable, with the period-τ average being equal zero.

As a result, considering the mass of the bubble to be negligibly small in comparison with the mass of the medium in its volume $m << \rho V_{b0}$, we obtain the following equation of its "slow" motions:

$$\ddot{X} + \eta\dot{X} = \frac{A^2\omega^4}{2\chi}\left[\frac{1}{c^2}\left(\frac{\omega^2}{\eta^2+\omega^2}(1+\frac{1}{\chi})f'(X)-1\right)+\frac{\rho}{P_e}\frac{\lambda^2}{\lambda^2-\omega^2}\frac{\omega^2+\frac{1}{3}\eta^2}{\eta^2+\omega^2}f'(X)\right]f(X)-\frac{1}{\chi}g \tag{7}$$

Here $\eta = \frac{k_1 R_0^2}{\chi\rho V_{b0}}$ (V_{b0} is the volume of the bubble near free surface of the fluid), prime designates a spatial derivative.

Vibrational force, induced on the bubble in gas-saturated fluid layer, has two components. The first one is controlled by the bubble's compressibility, while the second one is controlled by the compressibility of the surrounding medium. The ranges of the parameters, when one of these force components dominates the other, are determined.

The conditions of gas bubble sinking (or rising) in gas-saturated fluid layer, can be obtained from the equation (7). If the expression written in the right hand side of this equation is positive, then bubble sinks, and if it is negative, then bubble rises. Thus, the condition of bubble sinking in gas saturated fluid layer has the form

$$\frac{A^2\omega^4}{2g}\left[\frac{1}{c^2}\left(\frac{\omega^2}{\eta^2+\omega^2}(1+\frac{1}{\chi})f'(X)-1\right)+\frac{\rho}{P_e}\frac{\lambda^2}{\lambda^2-\omega^2}\frac{\omega^2+\frac{1}{3}\eta^2}{\eta^2+\omega^2}f'(X)\right]f(X)>1 \quad (8)$$

If the inequality (8) is held at the front of the expanding area $X=h$ of the fluid-gas phase, then bubbles, generated in the layer with thickness h, will move deeper into the fluid, and the gas-saturated fluid layer will continue to expand. Thereby, by employing the obtained condition of bubble sinking (8), the condition of vibrational instability of the separate state of the gas-fluid system is determined.

An approximate expression for the velocity of bubble's "slow" motion in gas saturated fluid layer is defined from the equation (7) with its "slow" acceleration $\ddot{X}$ being considered as small. The velocity of this layer penetration into the volume is equal to the sinking velocity of the bubble, situated at its boundary $X=h$.

To verify analytically obtained results, a numerical experiment was conducted. Original equations of the gas bubble motion were integrated directly by means of Mathematica 7, and the obtained results are in good agreement with analytical solution.

Series of experiments has been conducted on the multi-functional vibration test rig of the Joint laboratory of Vibrational Mechanics IPME RAS and REC "Mekhanobr-Tekhnika". As a result, the assumption that the vibrational force, induced on the bubble in gas-saturated fluid, has the component controlled by the bubble's compressibility and the component controlled by the compressibility of the surrounding medium, has been entirely confirmed. It has been shown that, at some ranges of external excitation parameters, either of those can lead to gas bubble sinking.

4 Conclusions

Motion of a bubble in vibrating volume of a fluid, saturated with gas on a certain depth, is studied, with compressibility of both the bubble and the gas saturated layer being taken into account. The conditions of gas bubble sinking in the layer are determined. The expression for the critical thickness of gas-saturated fluid layer, starting from which it penetrates into the volume, is derived, i.e. the condition of vibrational instability of the separate state of the gas-fluid system is obtained. An approximate expression for the average velocity of bubble's motion in gas saturated layer, which strongly depends on the depth of its location and on vibration parameters, and an expression for the average velocity of penetration of this layer into the volume are derived. The reported results are applicable for control and optimization of relevant technological processes.

It is shown that vibrational force, induced on the bubble in oscillating gas saturated fluid, has two components controlled by the bubble's compressibility and by the compressibility of the surrounding medium, respectively.

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