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Modelling Contracts and Incentives in Agricultural Cooperatives with an Emphasis on Quality and Financial Decisions

By

Xiaoyan Qian

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Operations and Supply Chain Management, the University of Auckland, 2017.

The University of Auckland
2017
Abstract

An agricultural cooperative (co-op) is an organisation that is owned, financed, and controlled by a group of farmers who collaborate by pooling resources for their own mutual benefit. The success of a co-op requires a highly coordinated supply chain and a well-managed capital structure. However, the principles of user-owner, user-control, and user-benefits result in several problems inherent to co-ops that create disadvantages for co-op members. This thesis focuses on two specific issues in this class: the quality coordination problem and the financing problem. To investigate the relevant trade-offs, this thesis builds models and employs the approaches of game theory, dynamic programming, case study, and numerical experiments.

This thesis first considers the quality coordination problem, or more specifically, contractual coordination for a co-op that proposes quality provisions in its contracts with farmers. Included in the research is the practice of multi-period payment schemes and farmers’ time preferences. The thesis derives the optimal decisions for a quality standard, together with optimal payment schemes. This work provides guidelines for the implementation of quality provisions and incentive mechanisms for current co-ops. In addition, because, in co-ops, equity investment is linked to members’ economic participation, the thesis next considers financing problems. In particular, it considers the interaction of operational and financial decisions. The results provide insight into a co-op’s decision making, cash position, and risk management. To further investigate cooperative finance, this thesis conducts a case study of co-op capital structure optimisation in the New Zealand Dairy Industry using Fonterra Co-op Group Limited. A Markov decision process (MDP) model captures the relationships among the various financial instruments, and a numerical analysis by an Approximate Dynamic Programming (ADP) algorithm derives results and evaluates the financial performance under different scenarios. These results show the trade-offs between the equity holders’ returns and financial risks. Overall, this thesis provides implications for agricultural co-ops in achieving coordination on quality standards and alleviating financial challenges.
Dedication

This thesis is dedicated to my parents who love, educate, and support me.
Acknowledgements

I am a lucky person. In my life, I have met so many wonderful people who have always supported, encouraged, accompanied and even challenged me along the way, and I need to thank them all.

First and foremost, I would like to say Thank You to my supervisor Professor Tava Olsen. Tava is an excellent supervisor with both a high IQ and a high EQ. She never pushes but is good at guiding. She is very supportive, for example, always responding to emails promptly even when it is not convenient, and striving to provide efficient help as soon as possible when I meet problems. She has also supported me financially during the period of extension. Also, she is very encouraging. I still remember how she taught me not to feel stupid in the academic life and comforted me when my paper was rejected by a conference. Of course, she is brilliant, always able to find a way for me when I get stuck in the research. In a word, she is the most wonderful woman I have ever met, and I am deeply indebted to her for being my supervisor.

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### CO-AUTHORS

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<td>Work with and supervise the PhD candidate in the process of formulation and solving the problem, and revising the paper</td>
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### Certification by Co-Authors

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate’s contribution to this work, and the nature of the contribution of each of the co-authors; and
- that the candidate wrote all or the majority of the text.

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<td>Tava Olsen</td>
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Last updated: 19 October 2015
Chapter 1

INTRODUCTION

A cooperative (co-op) is a business organisation that is distinct from the more commonly recognised investor-owned firm (IOF). An IOF is owned and controlled by capital investors and pursues profit maximisation; whereas a co-op is owned and controlled by those who transact with the organisation, striving to maximise the benefits for its users/members (Lund 2013). Co-ops that are formed by groups of farmers are referred to as agricultural co-ops, and their creation is usually motivated by situations where farmers, individually, have less ability to pool production and/or resources. For example, it may be too expensive for farmers to manufacture products individually. Agricultural co-ops enable farmers to obtain better outcomes, typically economic benefits, through a coalition. This research is concerned with agricultural marketing co-ops wherein farmers pool their resources in certain areas of activity such as processing, transportation, packaging, distribution, and the marketing of farm products. This thesis will use “co-op” to denote a general cooperative organisation, or most of the time, an agricultural marketing cooperative.

Co-ops operate primarily to provide benefits to their farmers; in return, farmers have a responsibility to supply both products and equity capital. The success of a co-op requires a highly coordinated supply chain and a well-managed capital structure. Many studies, from both the operational and the financial side, have identified several problems inherent in co-ops that create disadvantages for cooperative members (Cook 1995). The following paragraphs describe two important challenges.

A major challenge for co-ops is to combine farmers’ collective action with vertical coordination in the supply chain (Cechin et al. 2013). This challenge becomes more severe when quality requirements from downstream customers become stricter than before, so that farmers must exert extra effort to improve production at the farm level. Traditional co-ops
normally do not have supply agreements with farmers, and they are expected to accept a broad range of quality (Coltrain 2000). However, with increasing concerns surrounding the quality and safety of agro-food products, there has been a fundamental change, in that co-ops are beginning to propose quality provisions in the contracts so as to control the quality more closely (Li et al. 2010). Under this circumstance, supply chain coordination requires that members of an agricultural co-op have a common interest in delivering products that comply with quality requirements and build up a collective reputation through the co-op. However, the challenge is that, if there are no proper incentives, they might, individually, not reach such a consensus.

Another challenge for co-ops is to raise enough capital to support their business. Like other business organisations, co-ops require capital to finance their assets, marketing programmes, and operational expenses before revenue can be collected (Moore and Hardesty 1991). Co-ops that are constrained by insufficient access to capital may be limited in their growth and, ultimately, their competitiveness. However, investment constraints are often the “Achilles’ heel” of co-ops, especially within an increasingly concentrated, coordinated, and capital-intensive food system (Vitaliano 1980). According to the co-op financial constraint hypothesis (Chaddad et al. 2005), it is difficult for co-ops to acquire sufficient capital because their equity is limited to their members and is also tied to members’ economic transactions with the co-op (e.g., in farm products).

Finding solutions to these problems was the original motivation for the research that follows. Therefore, this thesis studies contracts and incentive mechanisms in agricultural co-ops with an emphasis on quality coordination and financial decisions associated with operations. This thesis consists of seven chapters. Chapter 2 provides background information for co-ops and Chapter 3 provides a literature review. Chapter 4 studies incentive mechanisms for cooperative quality coordination and Chapter 5 pays attention to cooperative financial issues and constructs theoretical models to integrate financial and operational decisions. Chapter 6 carries out a case study of cooperative capital structure optimisation for the New Zealand Dairy Industry operating within Fonterra Co-operative Group Ltd.
The thesis is concluded in Chapter 7. The following paragraphs give a brief synopsis of each of Chapters 4, 5, and 6.

Chapter 4 investigates the contractual coordination of an agricultural marketing co-op when a quality standard is specified in its contract with farmers who can exert quality-related effort at the farm level. Earlier studies on traditional cooperative coordination have turned out to be inappropriate because various business practices have been changed in response to customers’ increasing concerns surrounding food quality and safety. Although traditional co-ops do not have quality agreements with farmers, current co-ops often specify quality provisions in the contract. Also, in contrast to uniform payment schemes and paying farmers the spot market price when the commodity is marketed, current co-ops often offer quality premiums and pay farmers in multiple periods. That is, they pay different contracted prices when the product is submitted, and they distribute further progressive payments based upon market returns, which is defined as a “multi-period payment scheme.” These changes have a fundamental influence on the contractual coordination; however, little research has ever been done to answer the following questions:

1. When a quality standard is specified in the contract, can the supply chain be coordinated?
2. What are the optimal multi-period payment schemes?

This chapter proposes a two-stage stochastic programme to determine the quality standard and the payments offered in two periods. It also investigates quality coordination under different multi-period payment schemes. It first examines the market-price-guarantee payment scheme, under which high-quality products are guaranteed to be rewarded at the premium market price. The results show that farmers are over-motivated unless they have a high time preference. This chapter also proposes an improved multi-period payment scheme, under which a quality premium is offered in the first stage. The results show that the supply chain can be better coordinated regardless of farmers’ time preferences.

Chapter 5 studies both operational and financial decisions for a so-called proportional investment co-op, where farmers’ equity is required to be in proportion to their products. That is, farmers who supply a greater quantity of the given product are required to supply
a proportionately higher amount of equity for the co-op. Therefore, the co-op’s decisions about processing quantity interact with the financial decisions of retained earnings and short-term loans. This work is devoted to answering the following questions:

1. How should an agricultural co-op coordinate operational and financial decisions when equity is in proportion to patronage?
2. What does the capital position look like when the optimal decisions are applied?
3. With uncertain yield and market revenue, what financial crises are possible, and how do they influence the optimal decisions?

In the presence of yield and market uncertainty, this chapter models this situation using a Markov decision process. The objective is to maximise a weighted combination of the expected present value of the co-op’s retained profits and the farmers’ rewards over a finite horizon. The results include (1) the characterisation of the properties of the value function and the optimal policy; (2) explicit expressions for the deterministic-yield dynamic programme, wherein the myopic policy is optimal; and (3) identification of financial risks associated with uncertain market and yield.

Chapter 6 implements a case study on cooperative capital structure optimisation for the New Zealand Dairy Industry within Fonterra Co-operative Group Ltd. In response to increasing pressure from highly competitive global markets, accompanied with a capital shortage, Fonterra has since 2009 restructured its capital structure with two significant changes. The first gives farmers greater flexibility to hold more or fewer shares than are required by their milk production. Another far-reaching change is the launch of Trading Among Farmers (TAF) which allows farmers to trade shares among themselves and also provides outside investors an opportunity to invest in a special equity type called Units. However, the new capital structure has sparked concern about demutualisation, namely that the farmers may lose the control of the company. It is therefore of significant value for Fonterra to maximise equity holders’ returns while maintaining the optimal capital structure.

This chapter proposes a Markov decision process (MDP) to make decisions on equity, borrowing, dividend, and reinvestment, with exogenous information about milk price,
share price, farmers’ behaviour in share-trading, and uncertain milk production. It carries out numerical experiments and derives results via an Approximate Dynamic Programming (ADP) algorithm. It also investigates the efficiency of several policies by comparing numerical results and conducts a thought experiment on milk production to test the resilience of the system under adversity. The results reflect the trade-offs involved in the capital structure optimisation, show the importance of designing effective policies, and yield some suggestions to mitigate financial risks.

This thesis has contributions in three key aspects. First, it provides operational and financial insights for current agricultural co-ops. Thus far, there are very limited guidelines on how to implement quality provisions and how to make financial decisions for co-ops. Second, it enriches the theoretical and empirical literature on co-ops. Most studies of co-ops are qualitative or empirical, while this thesis constructs models and adopts quantitative methodologies. Last, but not least, it contributes to the interface of operations and financial management. Although there are an increasing number of papers paying attention to financial constraints when operational decisions are made, those papers apply only to IOFs, not to co-ops. However, a non-cooperative operational or financial model is usually an inappropriate tool to use for the study of cooperative operations and finance. Therefore, this study is an effort to incorporate cooperative features into a practical model for analysing cooperative operations and finance; as such, it contributes to the interface of operations and finance in operations management.
Chapter 2

Background Information

This chapter gives some background information on co-ops related to operational practices and finance. Section 2.1 provides an overview of co-ops including definitions, principles, and features. Then, Section 2.2 introduces cooperative operational practices and quality management issues, and Section 2.3 presents cooperative finance and capital structure innovations. As this thesis is driven by real-life case studies, Section 2.4 introduces two representative agricultural co-ops in New Zealand.

2.1 Definition, Principles, and Features of Co-ops

As early as 1844, the first co-op, known as “the Rochdale Society of Equitable Pioneers”, was founded in Britain on behalf of a group of workers. Now, many co-ops are distributed worldwide in almost every country and every kind of industry like consumer, producer, work and service, etc. All co-ops, regardless of sector or size, operate following a common set of principles that distinguish co-ops from other business organisations.

<table>
<thead>
<tr>
<th>Rochdale Principles</th>
<th>ICA (International Co-op Alliance) Principles</th>
<th>US Principles</th>
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<tr>
<td>1) Open membership;</td>
<td>1) Voluntary and open membership;</td>
<td>1) User-ownership;</td>
</tr>
<tr>
<td>2) One-member-one-vote;</td>
<td>2) Democratic member control;</td>
<td>2) User-benefit;</td>
</tr>
<tr>
<td>3) Cash trading;</td>
<td>3) Member economic participation;</td>
<td>3) User-control.</td>
</tr>
<tr>
<td>4) Membership education;</td>
<td>4) Autonomy and independence;</td>
<td></td>
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<tr>
<td>5) Political and religious neutrality;</td>
<td>5) Education, training and information;</td>
<td></td>
</tr>
<tr>
<td>6) No unusual risk assumption;</td>
<td>6) Co-operation among cooperatives, and</td>
<td></td>
</tr>
<tr>
<td>7) Limitation on the number of shares owned;</td>
<td>7) Concern for community.</td>
<td></td>
</tr>
<tr>
<td>8) Limited interest on investment;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9) Goods sold at regular retail prices;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10) Net margins distributed based on patronage;</td>
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Figure 2.1: Principles of Co-ops; sourced from Evans and Meade (2005)
Some general principles are shown in Figure 2.1. The earliest principles were proposed by the Rochdale Equitable Pioneers and were known as the “Rochdale Principles,” which determine co-operatives’ unique operating principles and also provided the foundation for the principles on which current co-ops operate. In 1966, the International Cooperative Alliance (ICA) updated the principles, which are now internationally recognised as the “Seven Principles.” The ICA also gave a definition that underscores that a co-op is a coalition of a group of people who have the same goals:

“A co-op is an autonomous association of persons united voluntarily to meet their common economic, social, and cultural needs and aspirations through a jointly-owned and democratically controlled enterprise.”

In 1987, the United States Department of Agriculture (USDA) adopted just three main principles of user-ownership, user-benefit, and user-control, with the argument that co-ops, particularly agricultural marketing co-ops, cannot follow all principles but must focus on fewer, more representative principles (Ortmann and King 2007). It gave a definition that directly provides the answer to the question: “who owns, who controls, and who benefits from, the business?”

“A co-op is a user-owned and user-controlled business that operates for the benefit of members.”

<table>
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<tr>
<td>Members/Patrons/Users</td>
</tr>
<tr>
<td>Patronage</td>
</tr>
<tr>
<td>Patronage Refund</td>
</tr>
<tr>
<td>IOFs</td>
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<td>Equity</td>
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Evans and Meade (2005), focusing on a study of agricultural co-ops, gave the following definition.
“A co-op is an organisation in which those who transact with (patronise) the organisation also own and formally control the organisation, and derive significant benefits from those transactions over and above any financial returns they derive from their investment in the organisation.”

This definition highlights an important aspect that distinguishes co-ops from other business organisations: a co-op’s equity is invested by members and also tied to their economic transactions with the co-op. In addition to different financing methods, co-ops are also distinctive in many other aspects. Table 2.1 provides some terms that are commonly referred to in the literature on co-ops.

1. Cooperative Ownership

Ownership yields the right of control of that enterprise and is also linked to the rights to share a business’s net income. In most companies or IOFs, the ownership is linked to the money investment, and both control and financial rewards are driven by the amount of invested money. However, co-ops are distinguished from IOFs in that the ownership of the enterprise is linked to members’ economic transactions with the organisation or what is called patronage (e.g., pallets of fruit or gallons of milk supplied in a season).

2. Cooperative Control

IOFs are controlled by shareholders on the basis of one-share, one-vote. However, the control of a co-op is allocated evenly among the users of the co-op following the principle of “one member, one vote”, regardless of how much money is invested. In other words, a co-op shareholder cannot enhance his/her voting or power through the acquisition of cooperative shares, suggesting a democracy (Duft 1914).

3. Cooperative Management

Co-ops are businesses owned and controlled by the people who use them; however, it is difficult for member-owners to directly make all the decisions. Cooperative management usually has several important elements consisting of member-owners, a board of elected directors, a hired manager, and other paid employees (USDA 1994). The control is preserved by members electing directors in many of the operations.
4. Cooperative Objective

Co-ops neither aim to maximise profits as IOFs do, nor pursue zero-profit like non-profit organisations. As specified in the definition, they are business enterprises that operate for the benefit of members (Lund 2013). However, since benefits may be defined in a myriad of ways: for example, economic returns, like product payments, or non-economic benefits, like stability, or growth opportunities. It becomes quite controversial when trying to measure a co-op’s performance in the sense of objectives. The theory that focuses on the distinction between co-ops and other businesses reflects the dual nature of the co-op; that is, member profit and firm profitability (Soboh et al. 2009). Co-ops may show much variation between maximising member profit and maximising firm profitability, which depends on the size of business, industry, state of operations, etc. Therefore, this thesis has made different objective assumptions in Chapters 4, 5, and 6, due to different research questions.

5. Cooperative Equity

In co-ops, the “user-owner” principle not only entitles members to the right of sharing net profits, but it also implies the responsibility of members to make sure the co-op has the capital it needs to operate effectively. One way of investment is through the subscription of cooperative shares. However, a co-op’s equity is basically limited to its members and tied to members’ economic participation. A further explanation of this is given in Section 2.3.

6. Cooperative Profit Distribution

When the co-op earns a net profit at the end of a financial year, the board decides what portion of the net profit is distributed to each member, and what portion remains in the co-op (see Figure 2.2). Note that the net profit is calculated by deducting product payments to members as a cost. The portion that is allocated to members is known as a “patronage refund” or “patronage dividend,” which is distributed based upon the members’ transactions with the co-op (or patronage), instead of capital investment. For example, if a farmer submits 2% of the co-op’s total product in a selling season, then he/she will receive 2% of the annual net profit allocated to
member-owners. Using the mechanism of distributing financial return in proportion to patronage is a defining characteristic of the cooperative model. The unallocated portion becomes equity that is owned by all members as a whole.

Figure 2.2: Cooperative Profit Distribution

7. Cooperative Return on Equity

In any business enterprise, equity funds are fundamentally at risk because equity holders are the last to be paid when a business suffers bankruptcy or dissolution. IOFs compensate for the risk of equity by distributing dividend payments or changes of share prices; however, co-ops fundamentally differ in that they do not promise particularly high rates of return in exchange for the risk of ownership (Lund 2013). Also, cooperative shares are neither transferable nor appreciable. Instead, the compensation of a co-op is to provide some advantages beyond a simple economic return on invested capital; for example, a democratically governed business.

Table 2.2: Co-ops vs. IOFs

<table>
<thead>
<tr>
<th></th>
<th>Co-ops</th>
<th>IOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ownership</td>
<td>Members</td>
<td>Capital investors</td>
</tr>
<tr>
<td>Control</td>
<td>One-member, one-vote</td>
<td>One-share, one-vote</td>
</tr>
<tr>
<td>Objective</td>
<td>To meet member’s needs</td>
<td>To earn a return on owners’ investment</td>
</tr>
<tr>
<td>Financial Structure</td>
<td>Sales of shares to members</td>
<td>Sales of stock to public shareholders</td>
</tr>
<tr>
<td>Equity Investment</td>
<td>Patronage-associated</td>
<td>None</td>
</tr>
<tr>
<td>Distribution of Net Profit</td>
<td>Based on patronage</td>
<td>Based on capital investment</td>
</tr>
<tr>
<td>Return on Capital Investment</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>
Table 2.2 summarises some features of co-ops and their differences from IOFs. It is important to note that, although the cooperative organisation has been around for a long time, the operating practices and capital structures they utilise have been evaluated and updated in light of new conditions. The next section describes how traditional co-ops have changed their practices in doing business with farmers, and how they have innovated their organisational and capital structures in response to significant changes in the agricultural economy.

2.2 Quality Management and Practice Changes

The purpose of this section is to introduce the challenge of quality coordination in agricultural co-ops. In the economic literature on co-ops, one classic problem of traditional co-ops has been identified as the quantity coordination problem (Albæk and Schultz 1998). This is because the co-op has no direct control over its input and farmers individually decide how much to deliver to the co-op. The fact of the decentralised decision making of members, as a result, leads to the inefficiency of over-production and free-rider problems on quantity. This fact also causes the problem of free-riding on quality when the co-op has no quality control over the product supplied by farmers. Compared to the inefficiency of quantity coordination, the problem of quality coordination is considered even more detrimental to the co-op (Pennerstorfer and Weiss 2012a).

The quality of a co-op’s product is significantly dependent on the behaviour of members (Baiman et al. 2000). Although it is in the common interest of all its members to deliver products that comply with quality and safety requirements and build up a collective reputation, individually, being focused on their short-term benefit, they might not take these actions (Ostrom 2014). Some farmers could enjoy the benefit obtained by others who make efforts to improve their quality. Conversely, those who improve the quality would often not be adequately rewarded, causing them to either exit from the co-op or be unwilling to exert effort in the future (Cechin et al. 2013). If members just act according to their interests, their behaviour may run contrary to the best interests of the integrated entity, and thus set back the sustainable development of the co-op (Harris et al. 1996a). The situation could
be more complicated when members are very heterogeneous due to differences in farm size, farm technologies and practices, and even cultural backgrounds. More individual members’ goals make collective action more difficult, thus challenging traditional cooperative principles (Hovelaque et al. 2009).

The changing world provides both opportunities and challenges for cooperative quality coordination. Customers are becoming more demanding about food safety and quality, and they are willing to pay a higher price for safety guarantees. For example, parents in China prefer infant milk formula from NZ over and above domestic products, due to NZ’s high-quality standards. In reaction to enhanced competition, co-ops must increase their emphasis on improving the quality of the product. Thus, in the 1990s, many new types of co-ops spread quickly all around the world (Coltrain 2000). They not only followed some tried and true principles such as those developed and used by traditional co-ops, but also certain new principles that have been developed much more recently. Both traditional co-ops and new co-ops are member-owned and member-controlled with similar objectives; however, they are quite different in the way they operate and manage the supply and marketing functions.

### 2.2.1 Quality Provision

The most far-reaching change in co-op management is to specify quality provisions in the contract with farmers. Traditional co-ops usually do not have quality agreements with farmers, and they are expected to find markets for all qualities of the product that are delivered. However, current co-ops usually have specific quality provisions which indicate the characteristics and traits of product that can be delivered (Coltrain 2000).

Quality and safety guarantees imply less freedom and more control over suppliers’ behaviour at the farm level (Bijman et al. 2010). To secure the highest quality supply to the market, the effort taken by suppliers is of high importance, since they are the patrons of co-ops (Li et al. 2010). Also, quality control by co-ops is crucial to ensure high-quality products being delivered. Various methods have been implemented in practice, including testing technologies or sampling inspections (Resende-Filho and Hurley 2012a).
2.2.2 Payment Mechanism

The method for paying or payment mechanism rests at the heart of any contract, and it is also the main means for allocating profits/costs and risks and providing incentives. However, a payment mechanism means more than pricing, since it also refers to payment time, payment methods, and so on. Most studies only focus on pricing; however, Chapter 4 will consider two dimensions including both pricing and payment time.

- Non-Uniform Payments

The payment mechanism should be a sufficient incentive to guarantee the participation of suppliers as well as motivation for quality improvement. The traditional practice of uniform payment schemes, under which all farmers receive the same price for their products, is outdated. Instead, many co-ops have resorted to non-uniform payment mechanisms that resemble those used by other regular companies. This is to offer different prices to quality-differentiated products (Fulton et al. 1995). A higher quality product corresponds to a higher price.

- Multi-Period Payments

Another significant difference is reflected in how a co-op distributes payments to farmers. Traditional co-ops pay the spot market price when the commodity is marketed. In contrast, current co-ops adopt multi-period payment schemes: that is an initial or advanced payment—a price stipulated in the contract, is made upon delivery and then one or more progress payments are made based on market returns from the further marketing of products (Coltrain 2000). Total income is based on the initial payment and progress payments.

2.2.3 Market Strategies

Pooling is the way each co-op member markets through the co-op. However, traditional co-ops pool all products to a homogeneous market, which has been proven to be of low-profitability and disadvantageous to success in meeting markets’ demands for quality
(Saitone and Sexton 2009). Instead, many current co-ops deliver their products to quality-differentiated markets (Fultion and Sanderson 2002). Basically, high-quality products are targeted to high-end markets where customers are willing to pay a higher market price.

Table 2.3 summarises the notable changes considered in Chapter 4. Although the problem of supply chain coordination has been well studied in the literature on co-ops, those papers fail to consider the changes introduced above. Therefore, it is quite unclear whether the supply chain can be coordinated under these practices. Further, although payment schemes under quality provisions have been addressed extensively in the literature on IOFs, the role of these mechanisms in aligning interests and actions of farmers within co-ops has never been fully-explored. Therefore, the purpose of Chapter 4 is to examine the quality coordination problem existing in agricultural co-ops under these new strategies.

<table>
<thead>
<tr>
<th>Table 2.3: Traditional Co-ops vs. Current Co-ops</th>
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<tbody>
<tr>
<td>Traditional Co-ops</td>
</tr>
<tr>
<td>Without quality provisions</td>
</tr>
<tr>
<td>Uniform payment schemes</td>
</tr>
<tr>
<td>One-period payment</td>
</tr>
<tr>
<td>Homogeneous market pooling</td>
</tr>
</tbody>
</table>

2.3 Financial Management and Capital Structure Innovations

Like other businesses, co-ops require capital to buy equipment and plant, pay staff, and cover other operating expenses. The money they need will vary depending on the size of the business, stage of operations, and industry. However, they are capital-constrained because of their limited sources of funding. This section provides an introduction to cooperative financial sources, financial constraints, and capital structure innovations.

2.3.1 Financial Sources

There are three major sources of funding that traditional co-ops can access as follows:

1. From Members
The “user-owner” principle implies the requirement for a co-op to be capitalised by its members (Li et al. 2014). In practice, members help finance the operations and growth of the co-op in two main ways as follows.

- Cooperative Shares

A co-op may issue a specific class of stock that is required to be purchased by members as a certificate to join the organisation. This type of stock is commonly known as “voting stock” because it is tied to members’ voting rights and represents individual member commitment to the co-op. The par value of a cooperative share varies greatly from co-op to co-op.

- Retained Patronage Refund (Allocated Equity)

In addition to purchasing cooperative shares, another major source of equity is the retained patronage refund. As introduced earlier, the co-op distributes a portion of net income to members as patronage refunds. However, the board also decides what portion of each member’s patronage refund will be retained by the co-op for several periods as a source of member equity to finance ongoing operations. The retained patronage refund is recorded in each individual’s member account, while the other portion is distributed to members (usually in cash). The general principle of retaining members’ designated patronage refunds is based upon their use of the co-op in a proportional method.

2. From Cooperative Business Surpluses (Unallocated Equity)

Another important source is through the retention of business surpluses that are not allocated to members, which is similar to IOFs. As introduced earlier, the co-op retains a portion of net profit as the property of the co-op as a whole. Different from the retained patronage refund, which is a part of allocated equity, this fund of retained earnings becomes an unallocated equity that would typically only be divided among members in the case of dissolution. However, the unallocated equity represents a reduced payment to the individual members who willingly accept this reduction only when the benefits it creates for them are clear and worthwhile. The amount
of internal co-op capital is largely a result of payments to members in the form of payments for product and patronage refunds (Chaddad and Cook 2004).

3. From Outsiders

In addition to member-provided capital and retained earnings, co-ops often make use of external funds to run a business. External funding may be provided in different ways, and debt instruments are the most common ones, like long-term or short-term loans, etc. Of course, the higher the amount of borrowing, the higher the risk the co-op suffers in the event of inability to repay a loan. Besides, borrowing bears financial costs that need to be paid eventually.

Determining the type and source of capital is crucial because different sources of capital imply different return and risk. Member-provided equity (co-op shares) is cost-efficient with low return on equity but is resolvable because those shares can be redeemed. Retained earnings are safe and permanent, but may be in conflict with members’ expectation on payments. Borrowing is a good strategy only when the returns from borrowing are larger than the cost of borrowing.

2.3.2 Financial Constraints and Problems

The literature argues that one of the weaknesses of the cooperative organisation lies in its ability to raise capital (Karantininis and Nilsson 2007). Agricultural co-ops, especially those focused on processing and value-added food systems, are usually subject to financial constraints, which are assumed to be the major constraints for them to grow and sustain themselves. Chaddad (2006) argues that financial constraints largely result from the incentive mechanism inherent in the organisational structure of co-ops. Several reasons are given below.

1. Cooperative residual claims are restricted

Since co-ops are farmer-owned and controlled organisations, co-ops have restricted residual claims (Condon and Vitaliano 1983). In other words, only active members may provide the co-op with voting equity capital. Therefore, the amount of capital is limited by the number, the wealth, and the risk-bearing capacity of its current
members (Chaddad 2006). Also, cooperative residual claim rights, or voting stock, are usually non-transferable. Restrictions on transferability prevents the functioning of a secondary market for cooperative shares, and hence leads to portfolio and horizon problems (see below) because members cannot adjust their investment portfolio to match their risk preferences (Jensen and Meckling 1979).

2. Cooperative members do not have appropriate incentives to invest

In co-ops, there is a very limited return on capital investment because co-ops distribute their earnings by patronage, not invested capital. In addition, cooperative shares are non-appreciable. The co-op provides returns to members mainly through farm-gate prices for product and patronage refunds. Therefore, members have an incentive to increase their patronage compared to capital investment (Knoeber and Baumer 1983).

3. Cooperative equity capital is tied to members’ patronage

Cooperative equity capital is tied to member patronage in two senses. First, in an IOF, it is not necessary to transact any business with the enterprise to be a shareholder. However, in co-ops, the right of purchasing cooperative shares is tied to members’ economic participation. Second, traditional co-ops rely primarily on internally generated capital, in particular, retaining patronage refunds. This is because there are few incentives for members to invest in the organisation. Therefore, a farmer’s decision on transacting with a co-op is tied to his/her decision on investing in the co-op (Peterson et al. 1992).

However, a co-op’s ability to generate internal earnings is constrained by two factors. First, the co-op does not strive to maximise profits as other IOFs do; instead, it strives to maximise members’ returns. Second, co-ops are usually focussing more on low margin than on high margin businesses. Therefore, some economists are concerned that the user-owner principle may hinder a co-op’s ability to generate profits (Staatz 1987). For example, the co-op may have to forego some profitable investment opportunities, whereas the profit-maximising firms can explore all opportunities for profit (Helmberger 1966).
4. Cooperative equity capital is not permanent

In traditional co-ops, members provide equity capital through share acquisition or retained patronage refund. However, neither is permanent. Since cooperative shares are tied to members’ voting rights, they are redeemed at par value (original purchase price) by the co-op when a member leaves (Lund 2013). Furthermore, the retained patronage refunds in the co-op can be viewed as a pool of deferred cash that the co-op temporarily employs and should finally return to individual members (Parliament et al. 1990). In other words, the retained patronage refund recorded in a co-op’s balance sheet represents a claim that can be redeemed sooner or later depending on the co-op’s plan.

5. Co-ops have limited access to external sources of funds

Unlike IOFs, co-ops cannot raise money from public shares because their ownerships are restricted to members (Vitaliano 1980). Furthermore, co-ops also lack access to adequate sources of debt capital, because cooperative equity is not sufficiently permanent in the eyes of lenders. Usually, the more equity an organisation owns, the more others are willing to lend.

The literature lists some problems resulting from unique cooperative organisational structures, which includes the free-rider problem, horizon problem, portfolio problem, control problem, and influence costs problem (Cook 1995). This thesis, associated with financial issues, is mainly relevant to the free-rider, horizon, and portfolio problems.

1. Free Rider Problem on Investment

The free rider problem occurs when those who benefit from resources, goods, or services do not pay for them in totality (Olson 1971). The free-rider problem on investment is likely to occur when new members obtain the same residual rights as existing members and are paid the same price for their patronage. This set of equally distributed rights, combined with non-appreciable shares due to the lack of a secondary market, creates an inter-generational conflict. For example, a farmer may find it optimal to patronise, and invest in, the co-op until the cooperative investment has been undertaken by other individuals. This problem is of particular significance.
during a co-op’s start-up phase, when a co-op relies heavily on direct contribution from members (Giannakas et al. 2016).

2. Horizon Problem

Cook (1995) summarises the horizon problem in this way: “The horizon problem occurs when a member’s residual claim on the net income generated by the asset is shorter than the productive life of that asset. In a co-op, this problem is due to the non-transferability of residual claimant rights and the lack of a secondary market.” The general view in the literature indicates that since some members will exit the co-op before they harvest the full benefits from their investment, they are reluctant to invest in co-ops.

3. Portfolio Problem

Another problem associated with members’ investment is the portfolio problem. Since cooperative equity is non-transferable, non-appreciable, and tied to the patronage, it prevents members from adjusting their portfolios to match their individual risk preferences. Therefore, members hold suboptimal portfolios.

4. Control Problem

The control problem is caused by a divergence of interests between the membership and their representative board of directors and management in a co-op. This problem is also known as the principal-agent problem.

5. Influence Costs Problem

The influence costs problem is a result of the diverse objectives among its members. When organisational decisions affect the distribution of wealth or benefits among different groups, the affected individual or groups attempt to influence the decisions to their benefit.

2.3.3 Capital Structure Innovations

In response to environmental and structural changes in the food system, agricultural co-ops adopt several competitive strategies like value-added processing, branding, global market participation, etc. These strategies require substantial capital investment in contrast to
traditional business. To acquire the necessary risk capital to survive in the competitive market, agricultural co-ops are adapting to new conditions and employing organisational innovations.

Figure 2.3: Alternative Cooperative Models: An Ownership Rights Perspective; sourced from Chaddad and Cook (2004)

Chaddad and Cook (2004) summarise several innovative organisational models that have evolved from traditional cooperative structure regarding ownership and control rights. They characterise these emerging models by describing various organisational attributes in terms of ownership structure, membership policy, voting rights, governance structures, residual claim rights, profit distribution, etc. In the Chaddad and Cook (2004)’s typology, five non-traditional cooperative models are identified (see Figure 2.3) including “Proportional Investment Co-ops”, “Member-Investor Co-ops”, “New Generation Co-ops”, “Co-ops with Capital Seeking Companies”, and “Investor-Share Co-ops.”

In Chapter 5, a model for a Proportional Investment Co-op is constructed. In Chapter 6, a case study, Fonterra, whose capital structure fits more like a New Generation Co-op, is carried out. These two types are introduced below:

1. Proportional Investment Co-ops
• Cooperative ownership is restricted to members;
• Cooperative shares are non-transferable, non-appreciable, and redeemable;
• Members are required to invest in the co-op in proportion to patronage;
• Profit is distributed in proportion to patronage.

Proportional investment co-ops should adopt equity management policies to ensure proportionality of equity, and a commonly adopted management tool is known as the base capital plan which is implemented as follows:

(a) Choosing a base period, which may vary from 1 to 10 years;
(b) Determining a measurement unit. For example, a ton of fruit or a hundred-weight of milk;
(c) Determining the investment level according to each member’s patronage. For example, if a member’s average patronage over the base period accounts for 15% of total patronage, then his investment level shall be 15% of total investment accordingly;
(d) Identifying over-invested members and under-invested members, and making equity adjustments to achieve proportionality.

2. New Generation Co-ops

• Cooperative ownerships are restricted to members;
• Cooperative shares are non-redeemable but transferable;
• Profit is distributed in proportion to patronage in addition to shareholdings.

In this model, ownership rights are in the form of tradable and appreciable delivery rights restricted to current member-patrons. The transferability provides liquidity and capital appreciation through secondary market valuation. Also, member-patrons are required to acquire delivery rights on the basis of expected patronage such that usage and capital investment are perfectly aligned.
The structures described above suggest that agricultural co-ops are increasingly relaxing some of the structural constraints imposed by the traditional model, hence financial constraints are ameliorated. However, under those transformations, new organisational costs may surface. Members may have to share profits and control rights with outside investors who are not necessarily patrons of the co-op and thus may have diverging interests. Conflicting goals between maximising returns to investors and maximising returns to member-patrons may occur as a result.

2.4 New Zealand Agricultural Co-ops

New Zealand is famous for its advanced agricultural industry and prominent products all over the world. Co-ops and other forms of farmer-controlled business are major players in some agricultural sectors, which together account for a significant share of the economic activity of the country. According to a report from PwC, the agricultural sector in 2012 comprised approximately 3% of GDP, which represents a very large percentage of the economy of the country (www.pwc.com). This section introduces the New Zealand Kiwifruit Industry under Zespri International Ltd, and the New Zealand Dairy Industry under Fonterra Cooperative Group Ltd.

2.4.1 Zespri

Zespri is a consumer-driven, 100 per cent grower-owned company dedicated to the global marketing of kiwifruit (https://www.Zespri.com). Not only a leading kiwifruit pioneer, it is also the sole exporter of New Zealand kiwifruit outside of New Zealand and Australia. To counteract the power of overseas buyers, the government established producer boards with “single desk selling” provisions that enabled it to be the sole marketer and export distributor of their product. Currently, Zespri is comprised of almost 3000 domestic and international growers and is the biggest marketer of kiwifruit in the world with over 30% of market share. As a growers’ co-op, Zespri takes responsibility for a large range of activities from collecting kiwifruit from growers, storing them in inventory, transport and shipping, marketing promotion, research, and so forth.
Kiwifruit thrives in New Zealand’s temperate climate and thus enjoyed significant cost advantages over other farmers for most of the 1980s. However, when the kiwifruit industry was first developed in NZ, it confronted great challenges from its domestic competition as well as competition from Italy and Chile. Oversupply and undercutting of price led to the collapse of this industry in international markets. Also New Zealand failed to differentiate their product from Chilean and Italian counterparts with the common product name “kiwifruit.” Under the concern of high risks, heavy fixed and variable costs, long-term investment, being at the mercy of consumer demand, and climatic conditions, the New Zealand kiwifruit industry was a price taker rather than a price maker in export markets (Crocombe et al. 1991). In response to these problems, kiwifruit growers in New Zealand raised their hands and signed an agreement to form a coalition, developing a new structure with an exclusive brand name: Zespri. The strategy of being “customer driven” marks it out as a premium product, moving it from the category of a perishable commodity into the premium-priced consumer goods bracket.

- Quality Provision

Zespri has a specialised requirement regarding quality attributes as they find that increasingly consumers are prepared to pay more for better quality fruit. Considering the quality attributes of kiwifruit, this refers mainly to the size and taste of the fruit, which is tested by the Taste Zespri programme through a sampling methodology that calculates the Taste Zespri Grade (TZG) by examining the dry matter content. Using this methodology, only fruit that meets the basic quality requirements are delivered to inventory and then separated into several categories according to quality and variety.

- Quality Premium

In general, growers are paid on the number of trays they supply to Zespri. However, as well as a base fruit payment, there are some premiums that act as commercial incentives to encourage the supply of fruit demonstrating a range of product specifications in demand by customers. Like most co-ops, Zespri does not pay growers by
individual slot, and payments are often pooled at an entity level to disperse the risk and maintain a greater orientation to the end market.

- Multi-Period Payments

The payments of Zespri can be split into two types: an “Advance Payment” and “Progress Payments.”

1. The Advance Payment is paid at the time the pack-house submits the growers fruit into Zespri’s inventory. This payment could vary due to the volume submitted as well as incentive premiums for quality, service, and time, which encourage growers to supply and deliver kiwifruit with characteristics that benefit customers as well as growers.

2. Progress Payments are made monthly from the start to the end of the season and are discretionary based on market returns. Although fruit is pooled in the market, it is categorised by the degree of quality and variety. Generally, high-quality fruit bring more market returns than low-quality fruit.

- Market Strategy

Zespri differentiates markets by exporting high-quality fruit to global markets like Japan while pooling low-quality fruit to the local markets. Further, Zespri has either separate sales pools or a combined sales pool. For example, Zespri has separate sales pools for GREEN and GREEN ORGANIC Kiwifruit; while it has a combined sales pool for GOLD and GOLD ORGANIC Kiwifruit.

2.4.2 Fonterra

Fonterra is owned by around 10,500 New Zealand farmers and collects approximately 85% of New Zealand’s milk production. It is also New Zealand largest company and is responsible for 30% of the world’s dairy exports. Its core business consists of exporting dairy products, which accounts for 95% of its New Zealand production, and a fast-moving consumer goods business for dairy products; it produces over 1000 dairy-based ingredient products for the international food industry under the NZMP brand (www.fonterra.com). In New Zealand,
dairy marketing has gone from some competing co-ops to one dominated by a single co-op in a short period. The primary motivations for the creation of Fonterra were to achieve cost savings and provide a more effective platform to compete in international markets.

Fonterra has restructured its capital structure since 2009, although it was never a traditional co-op on both operations and capital structure since it launched in 2001. Table 2.4 summarises Fonterra’s capital structure features before 2009, and is sourced from Trechter et al. (2003).

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<table>
<thead>
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<tbody>
<tr>
<td>1</td>
<td>Fonterra is a user-owned, user-controlled, and user-benefit business.</td>
</tr>
<tr>
<td>2</td>
<td>The voting rights are based on the amount of milk delivered rather than one person-one vote.</td>
</tr>
<tr>
<td>3</td>
<td>There is a contractual relationship between Fonterra and its milk producers. Fonterra requires its farmers to match their shareholding with their milk production by owning one cooperative share for each kilogramme of milk solids (KgMS) produced annually. However, there is no real penalty if the farmer provides less milk than his shares due to unexpected conditions.</td>
</tr>
<tr>
<td>4</td>
<td>The capital investment is limited to farmer suppliers, restricted from public shareholders.</td>
</tr>
<tr>
<td>5</td>
<td>The dominance of Fonterra in the New Zealand dairy industry requires it to, generally, accept any new suppliers.</td>
</tr>
<tr>
<td>6</td>
<td>The operating income is distributed based upon milk production.</td>
</tr>
<tr>
<td>7</td>
<td>Fonterra is obligated to redeem shares when production drops or issue shares when production increases.</td>
</tr>
<tr>
<td>8</td>
<td>Fonterra’s stock is transferable but only to the co-op itself.</td>
</tr>
<tr>
<td>9</td>
<td>The amount of a farmer’s initial investment depends on the amount of milk estimated and the fair market value of Fonterra’s shares. The share price is determined by an independent valuer. The valuation reflects Fonterra’s profit expectations and hence varies year by year.</td>
</tr>
</tbody>
</table>

The old capital structure mitigated many classic problems that existed in traditional co-ops, like free-rider issues on investment because of the proportional requirement between capital investment and milk production (Trechter et al. 2003). However, like most co-ops who are striving to compete in global markets, the most pressing issue for Fonterra was to source sufficient equity capital. The previous capital structure stipulated that cooperative shares were limited to farmers only, and also required that farmers matched their shareholding with their milk production in proportion. Therefore, if farmers’ milk production dropped in any season, Fonterra had to redeem shares to ensure the proportionality.
Consequently, Fonterra faced the risk of losing large amounts of share capital through redemption during times of declining milk production. For instance, after milk production fell during the 2007/08 drought, Fonterra had to pay out $742 million of share capital to farmers via redemptions.

To deal with those problems, the board of Fonterra announced a two-year consultation programme regarding their capital re-restructuring option in 2007, namely putting the business operations in a separate listed company, but maintaining the control within farmers. However, the proposals encountered significant opposition from both Farmer Shareholders and the government, which had to pass enabling legislation, because farmers were very worried about the risk of losing control, in what was described as demutualisation. After continuing consultation with Farmer Shareholders, in 2009, the board announced a three-step process to revamp Fonterra’s capital structure, which abandoned thoughts of a public listing of Fonterra shares but retained 100% farmer control and ownership. Now the new capital structure is characterised by the following features, summarised in Table 2.5.

Table 2.5: The Comparison between Fonterra Old and New Capital Structure

<table>
<thead>
<tr>
<th>Old Capital Structure</th>
<th>New Capital Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>In proportion to last year milk production</td>
<td>In proportion to three-year average</td>
</tr>
<tr>
<td>No flexibility on the Share Standard</td>
<td>With flexibility on the Share Standard</td>
</tr>
<tr>
<td>Shares are non-transferable among farmers</td>
<td>Shares are transferable among farmers</td>
</tr>
<tr>
<td>Restricted from outsider investors</td>
<td>Issuing Units to outsider investors</td>
</tr>
<tr>
<td>Milk-based profit distribution</td>
<td>Equity-based profit distribution</td>
</tr>
</tbody>
</table>

1. Three-year Rolling Average Standard

The Share Standard that the number of shares a Farmer Shareholder is required to hold in accordance with the patronage is calculated based on a three-year rolling average of a farm’s milk production. A new member is required to invest based on an estimation of average milk production. This is designed to smooth out seasonal production fluctuations and reduce the need to buy or sell shares in any season.

2. Flexible Share Requirement

Fonterra gives farmers greater flexibility in the number of shares they can own. Farmers are allowed to hold more or fewer shares, than required by the Share Standard,
to an extent which Fonterra permits. However, voting rights remain based on share-
backed milksolids, just like before.

3. Fonterra Shareholders’ Market (FSM)

The Fonterra Shareholders’ Market is a private market on which only Farmer Share-
holders (or specially appointed markets) are allowed to trade shares. In other words,
Fonterra enables the transferability of its stock, not only to itself but among farm-
ers. Since there is no need to directly trade with farmers in shares, this provides
Fonterra with a permanent share capital regardless of any individual changes in milk
production in any season.

4. Fonterra Shareholders’ Fund (FSF)

This fund is set up to financially help farmers purchase or sell shares. Outsider in-
vestors who are not allowed to hold shares in Fonterra can invest in Units in the Fund,
which are listed on the NZX Main Board (https://www.nzx.com/markets/NZZX).
For example, a Farmer Shareholder can sell his shares to the Fund based upon his
milk production or personal risk preferences. The Fund pays Farmer Shareholders for
the economic rights of shares (receiving dividends and the gain/loss from any changes
in share values), not voting rights.

5. Fluctuating Share Price

With the launch of “Trading Among Farmers (TAF)” consisting of Fonterra Share-
holders’ Market and Fonterra Shareholders’ Fund, the share price is no longer valued
by an independent valuer who had determined the share price of Fonterra for the next
season based on Fonterra’s estimated market value in the future. The share price, like
other freely traded public shares, is now driven by what Farmer Shareholders trade
at.

6. Dividend Distribution

As a financial incentive for farmers to hold more shares than production, Fonterra
distributes any profits (after farm-gate milk payments) as a dividend based on shares
held, rather than milksolids produced.
2.5 Brief Summary

This chapter provides some background information on co-ops, that is relevant to research questions in the following chapters. It first clarifies cooperative unique features, which will be incorporated into the theoretical models of this thesis. Since Chapter 4 investigates the quality coordination problem for a co-op, an overview of cooperative operational practices and quality management is provided in Section 2.2. In addition, both Chapter 5 and Chapter 6 make financial decisions for co-ops; therefore, Section 2.3 introduces cooperative finance and various capital structures. Finally, both of case studies in this thesis, namely Zespri and Fonterra, are introduced.
Chapter 3

Literature Review

This chapter provides a literature review of the studies on co-ops from different aspects. Section 3.1 first reviews the work comparing the differences and similarities between co-ops and IOFs. Section 3.2 gives a review of contracts and game theoretical models on co-ops, mainly focusing on quality provisions and payment schemes. Section 3.3 reviews some papers considering different uncertainties in an agricultural supply chain. Section 3.4 reviews studies on co-ops’ financial issues and also refers to some papers making both operational and financial decisions for IOFs. Finally, Section 3.5 gives a brief review of studies on New Zealand agricultural co-ops.

3.1 Co-ops vs. IOFs

The most prominent differences between co-ops and IOFs are their ownership and their objectives. The objective of the IOF is usually assumed to be maximisation its own profit; however, this objective does not quite fit the co-op because it is defined as a user-owned, user-controlled organisation that aims to benefit its members (Sexton 1984). The identification of a cooperative objective is essential because not only does the performance of co-ops depend on their objectives, the propositions in theoretical studies are also followed by the assumed objectives. However, the definition of co-ops, the heterogeneity of their organisational forms, and their various capital structures are stumbling blocks to identifying a uniform and consistent objective. Therefore, in the literature on co-ops, the assumed objectives are defined in different ways.

Soboh et al. (2009) identify three distinct views on co-ops in the theoretical economic literature: (a) a vertically integrated firm, (b) an independent business enterprise, and
(c) a coalition of firms. They also summarise the objectives into two classes: assuming a single objective and assuming multiple objectives. Both the first two views assume a single-objective, while the third assumes multiple objectives. Figure 3.1 summarises those objectives.

<table>
<thead>
<tr>
<th>View of the Co-op</th>
<th>Assumed Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Vertically Integrated Firm</td>
<td>Maximising members’ returns after paying the highest possible price</td>
</tr>
<tr>
<td>An Independent Business Enterprise</td>
<td>Maximising profit, considering the price paid to farmers as a variable cost</td>
</tr>
<tr>
<td>As an IOF</td>
<td>(1) Maximising the joint profit of the co-op’s and members’ profit;</td>
</tr>
<tr>
<td>As a variant of IOF</td>
<td>(2) Maximising patronage refund;</td>
</tr>
<tr>
<td></td>
<td>(3) Maximising the size of the total output.</td>
</tr>
<tr>
<td>A Coalition of Firms</td>
<td>The optimal allocation of the optimal profit across the different parties (i.e., different members’ categories, managers’ board, etc.)</td>
</tr>
</tbody>
</table>

Figure 3.1: Different Theoretical Views and Objectives of Co-ops; sourced from Soboh et al. (2009)

Maximising economic benefits to members (including both the price paid and the value of patronage refund) is assumed by many researchers like Sexton (1984), Li et al. (2010), and Bontems and Fulton (2009). They usually realise this function by composing a technical condition on the optimisation model: all profit has to be shared by the members. This condition is referred to as the budget balance condition or the break-even condition with the justification that farmers own rights to the residual earnings due to their ownership of the co-op (Bogetoft and Olesen 2007). Considering the problem of quality coordination, Chapter 4 shares this objective, that is to maximise the profit in the whole supply chain as a centralised decision maker, while being subject to the budget balance condition.

However, from a financial perspective, the objective of maximising members’ returns is not appropriate because it predicts zero (or substantially low) profit value and suggests no retained equity capital. Some researchers who compare the performance of co-ops with the performance of IOFs, especially from a financial perspective, view the co-op as similar to an IOF with the objective of profit-maximisation (Sexton et al. 1993). The model with this objective considers the price paid for their products as an additional variable cost, irrespective of members’ objectives in the decision-making process. This objective
implies the principle-agent problem, where the co-op is actually managed and controlled by entrepreneurs (as the absolute decision makers) who have overwhelming power over its members. However, this objective is challenged by many theorists from the perspective of organisational features.

Some researchers study the co-op as a variant of an IOF, assuming that the co-op’s objective is to maximise the joint profit of the members and the co-op as a whole (Enke 1945, LeVay 1983). The joint profit maximisation is represented by the total profits of both the co-op and members, and the allocation of the profit is assumed to be distributed equally or unequally depending on various factors like bargaining power, or the co-op’s capital needs, etc (Carson 1977). This objective comprises both objectives of maximising members’ returns and maximising its own profit, showing the trade-offs between the preferences of members’ returns and the needs of the co-op to have sufficient member capital for start-up and growth (Lund 2013). Since Chapter 5 makes both operational and financial decisions, it assumes such an objective — to maximise both the profit retained in the organisation and the payments distributed to farmers.

It is not simple to differentiate co-ops and IOFs solely through an analysis of their objectives. Zusman (1982) points out, the use of an objective function to capture the behaviour of a co-op is subject to difficulties that arise because of the way in which group decisions are typically made. Explicit modelling of group decision-making processes would greatly complicate the model. Thus, a more exact and accurate viewpoint is proposed by Srinivasan and Phansalkar (2003) who differentiated IOFs and co-ops via the residual claim specification (RCS). RCS specifies that in an IOF, equity capital suppliers are the residual claimants while in a co-op, the members in specific exchange relationships are the residual claimants. Similarly, Katz and Boland (2002) differentiated co-ops from IOFs by their basic purpose, property rights, and decision-making processes. Generally speaking, the objective of the co-op is to maximise the welfare for its members. The common theoretical strategies adopted by co-ops to achieve this objective are also categorised by Peterson and Anderson (1996) through conducting interviews with 21 US agricultural co-ops.
The question of cooperative efficiency and financial performance has been the subject of many studies. Such studies typically fall into two classes: one focuses on estimated production, and the other is about economic efficiency such as pricing, and scale efficiency. However, Hardesty et al. (2004) point out that considering cooperative performance in isolation is not exhaustive, one should also compare the total benefits derived by these owner-patrons about what they would achieve from alternative channels, like IOFs. Mosheim (2002) compares the performance of 16 IOFs and 28 co-ops by using data analysis techniques in the Costa Rican coffee processing sector and finds that even though co-ops are more price efficient than IOFs, the former are less scale efficient than the latter.

It seems that there are no definite answers; both have their own strengths and weaknesses. Karantininis and Zago (2001) establishes a model to research the choice of producers to join a co-op or sell their products to an IOF and find that the relative advantage of the cooperative vanishes with an open policy and its optimal size is lower compared to a closed membership. They also find that the cooperative produces more than the investor-owned firm in aggregate; however at the individual level, farmers delivering to the co-op produce less than those selling to the IOF.

Any advantages farmer-owned co-ops offer versus IOFs must also be considered in the light of the relative costs of ownership arising under each type in determining which one is more efficient. The survey by Hansmann (2009) regarding agricultural co-ops involved in bargaining, handling, processing, and marketing/distribution suggests that farm marketing co-ops enjoy significant advantages related to ownership costs compared to IOFs and suggests that this feature largely accounts for their popularity. He also points out that multiple farmers producing homogeneous highly seasonal and perishable outputs are often exposed to potential market power, abused by concentrated processors and other downstream agents. Hence, the formation of farmer-owned co-ops provides them with additional bargaining power. This conclusion coincides with the evidence of cooperative formation in New Zealand along with the fact that New Zealand’s agricultural producers are exposed to market power arising from processor scale of economy as well as coordination issues related
to international shipping. Other benefits Hansmann identified include alleviating the costs of asymmetric information between farmers and customers and diversifying risks, etc.

Co-ops have also been criticised for operational inefficiencies. Boyle (2004) investigated the economic efficiency of Irish dairy co-ops between 1961 and 1987 and noted that co-ops give rise to inefficiencies in two ways: (1) they suffer from technical inefficiency due to principal-agent problems and allocative inefficiency because of the horizon problem; (2) price inefficiency—the co-op will have a tendency to pay farmers over and above the internal value of the marginal profit, leading to less capital being available for development. Bogetoft and Olesen (2002) also criticised co-ops for little investment in product and market innovation, because members producing standard products have conflicting interests with those producing high-quality products. However, it is worth noting that certain inefficiencies that were predicted grew out of traditional co-ops rather than the non-traditional co-ops that have emerged in response to economic development.

In addition to comparing IOFs and co-ops, researchers are also interested in studying the situation whereby co-ops compete with IOFs in the same market. Albæk and Schultz (1998) use a model to describe the situation wherein a cooperative faces an IOF in a Cournot duopoly setting, arguing that in a Cournot market, it is beneficial to produce at a high level as products supplied by the co-op and the IOF can be substituted for one another. Thus, they conclude that the co-op would gain a larger market share than the IOF if the co-op did not control the supply quantity and thus farmers would gain more from a co-op than from an IOF. This conclusion is contrary to the argument that a co-op is not as efficient as an IOF due to overproduction resulting from the rule that it is its growers rather than the co-op who decide how much to produce. Nevertheless, even though this report supports the successful existence of co-ops in competition with IOFs, the assumption of this study that farmers have greater bargaining power over IOFs by themselves may not be justified. In reality, farmers, particularly those who grow perishable products, are more likely to be bullied by IOFs, which is one reason they are motivated to join a co-op, to enhance their bargaining power.
IOFs and co-ops are not opposite to each other. Based on their survey, Evans and Meade (2005) predicted that co-ops will become more prominent than IOFs as a competitive discipline and argue this coexistence could be beneficial, although not uniformly so. Furthermore, a co-op could be in a state of transition and, as previously mentioned, the five types of the newly-formed co-ops, more or less employ some characteristics of IOFs. Some co-ops even converted to IOFs; for example, NZ meat processor AFFCO became a listed company in 1995.

3.2 Contracts and Game Theoretical Analysis on Co-ops

The cooperative concept has been widely studied by economists and researchers. In the first 30 years of modelling for farmer co-ops, the concept was viewed by economists in one of three ways (1) as a form of vertical integration, (2) as an independent firm, and (3) as a coalition of firms. These three distinct theoretical approaches have been reviewed by Staatz (1987). Later, Cook et al. (2004) extended Staatz’s work and reviewed the theoretic contribution to farm co-ops since 1990, categorising co-ops into three major streams: a) extensions of the “co-op as a firm”, b) the co-op as a “coalition”, and c) the co-op as a “nexus of contracts.”

In a review of the various theories applied to the model of co-ops, game theory is perhaps the most common method advanced by theorists. Although Sexton (1984) did not apply the method of game theory, he draws attention to club theory and game-theoretic discussions about the incentives to form groups or coalitions and the requirements for stability within an organising cooperative. The review of Staatz (1983) points out the trend and importance of game-theoretical analysis on co-ops. A game theoretic model is a good method because it analyses situations wherein players make decisions beneficial to them while taking strategies of others into consideration. It is well known that game theory has two major branches, cooperative and non-cooperative, both of which have been adopted by researchers to solve coordination problems. For example, some decisions in a farmer co-op, such as how to allocate joint costs or profits by pooling, can be modelled as cooperative games, whilst some others, such as how to ensure members’ commitment
in a “competitive yardstick” cooperative, are more likely to be appropriate than non-cooperative games (Staatz 1983). “The competitive yardstick” concept maintains that the existence of a co-op in a market will force an IOF to behave more competitively to avoid losing patrons. Cooperative participants have their own objectives and incentives to act independently within a coalition.

Different from non-cooperative game theory, which is strategy-oriented, the cooperative theory looks at what the players can achieve, what coalitions will form, how to divide the outcome and whether the coalitions are stable and robust (Nagarajan and Sošić 2008). Thus, in the work of Staatz (1983), decision-making within the context of the agricultural cooperative with respect to the allocation of costs and profits is viewed as an “n-person cooperative game.” The bargaining power among members depends on two factors: the costs one member could impose on other members if he were to leave and the cost he would impose on himself. Thus, Staatz concluded that large suppliers would have more bargaining power than small ones.

Still, Staatz (1983) also includes non-cooperative game models when dealing with problems, ranging from profit sharing to governance. For example, when the output of a co-op is limited, demand sharing among its members can be regarded as non-cooperative as each member has an incentive to expand their own supply. Thus, Staatz establishes a game of Prisoner’s Dilemmas to analyse the cooperative behaviour and demonstrates that members should choose “defect” in one single game, albeit this conclusion would not hold in an infinite horizon. Thus, maintaining a farmer’s loyalty to the cooperative is a Prisoner’s Dilemma super-game (infinitely iterated). The author also criticises the assumptions within game-theoretic analyses of perfect knowledge. Information costs induced by imperfect information and transaction costs imply that participation in a co-op’s governance may not be beneficial.

Hendrikse and Veerman (2001) formulated an incomplete-contract theory regarding the choice of governance structure in an agricultural chain of production. The incompleteness of contracts refers to the situation wherein there is no specification for the division of the surplus ex-ante, instead of ex-post. This would cause problems on divisions that
may depend on the distribution of bargaining power and position. Incomplete-contracts are relatively common in co-ops, as, in essence, co-ops could not assume all risks for their members. This is different from that in IOFs where suppliers would not be exposed to market risk after submitting their products. This feature also holds in the study of Chapter 4 in that, part of the payment to farmers would not be paid until the realisation of gains from the market return, although the sharing rule as well as the paying mechanism has already been specified ex-ante. Later, Hendrikse and Veerman (2001) used another new institutional economics approach-transaction cost theory to study the relationship between investment constraints and control constraints. Both these papers address the hold-up problem wherein two parties may be able to work more efficiently by co-operating; however, they refrain from doing so due to concerns that they may benefit the other party by enabling them to have greater bargaining power. Sykuta and Cook (2001) also used the approach of new institutional economics to proffer a comparative conceptual framework to examine efficiency for co-ops, which suggests that by designing contracts, a co-op is more capable of ensuring economic efficiency than an IOF. Those papers are further expanded by Hendrikse and Bijman (2002) with the quest to determine what structure is beneficial for producers to integrate downstream through their investment.

Many researchers have described the relationship between the co-op and its members as a principal-agent relationship, concentrating on modelling the effects of the preference structure of contracting parties, the natural uncertainty, and the informational structure of the environment. Bontems and Fulton (2009) used principal-agent theory to explore the importance of alignment goals of its members on the relative efficiency of the cooperative. This work considers 1) the situation wherein the co-op is regarded as a principal whilst the farmers are the agents, and 2) there is asymmetric information where the processor does not know the productivity of its suppliers. The authors compared the situations where the processor is a cooperative or an IOF. In the case of an IOF, the goal is to extract the surplus from both farmers and consumers, whilst a co-op returns the rents from both to the farmers (breaks-even). They also state that goal alignment becomes less efficient when the cooperative uses its procurement policies to redistribute income. This conclusion
implies the influence of a payment mechanism on its members. Another contribution of this work is the consideration of heterogeneous farmers who are either high-type or low-type in production skill. The processor offers a series of contracts designed for particular farmers with pricing schedules that ensure self-selection.

Most papers related to the principal-agent problem consider incentives, actions, and performance for a single period; however, King et al. (2007) propose a dynamic principal-agent model with incentive systems, which allows for the consideration of the supplier’s performance history in the control of salmonella. They argue that the length of the agency relationship can provide strong incentives for increased effort. In this work, hog producers would receive an identical price per hog and a quality premium determined by the salmonella control programme, and the test results would become part of a supplier’s history. In this incentive system, quality premiums and charges for testing and penalties are determined when and if the salmonella’s prevalence level exceeds thresholds set by the plant. The manager of the plant is regarded as the principal who can influence the farmers’ behaviour through the design of the compensation/testing system whilst not being able to observe producers’ quality control efforts. Besides that, this work considers the two ownership structures — IOFs and co-ops — to show how the ownership structure affects overall performance. Under the structure of co-ops, the manager of the plant aims to maximise the certainty equivalent of the producers’ net gain subject to the participation constraint. This modelling framework provides a good insight into quality control in the food agribusiness and adaptive settings where homogeneous or heterogeneous suppliers interact repeatedly with a processing firm.

These papers assume that the moral-hazard problem existing between suppliers and buyers is due to asymmetric information in that buyers are often not able to observe the actions of suppliers. In reality, suppliers also face moral-hazard issues interacting with buyers. Suzuki et al. (2011) pointed out that contract farming has suffered from mutual or systematic moral-hazard problems: suppliers would defect by selling output to other marketers at higher prices (so-called side-selling), whilst marketers may defect by failing to pay a price that reflects the quality of the products, referred to as a hold-up problem.
Through examining the relationship between a risk-neutral principal (exporter) and a risk-averse agent (smallholder) via a contract-theory model with asymmetric information, this study demonstrates that the exporter would use the smallholder as a buffer to avoid market risks. Therefore, he cautions that under a rational contract, the exporters’ ability to reject product on quality grounds should be specified to avoid moral-hazard concerns that the product is rejected for reasons of demand uncertainty rather than for quality issues. This work provides us another perspective on the moral-hazard problem that exists between a co-op and the contracted farmers.

3.2.1 Quality Coordination in Co-ops

Chapter 4 studies the quality coordination problem in agricultural co-ops; an institutional fact that is specific to this problem is the specification of a quality provision and payment schemes. Although there has been extensive theoretical and empirical research on the behaviour and performance of co-ops, the issue of quality has received little attention. Neither a comprehensive survey of studies on producer co-ops two decades ago (Bonin et al. 1993) nor a more recent review on agricultural marketing co-ops (Soboh et al. 2009) has ever mentioned product quality. However, this gap has been unanimously noted in studies concerning quality dimension (Mérel et al. 2009, Pennerstorfer and Weiss 2012a,b).

Economides (1999) examined quality choice and vertical integration in a cooperative setting and argues that vertical disintegration has a significant adverse effect on quality provision. Mérel et al. (2009) analysed the challenges and opportunities for agricultural marketing co-ops in quality-differentiated markets and also proposed appropriate modelling frameworks in the quality dimension. They stress that various traditional cooperative business practices are not conducive to meeting consumers’ demands for quality. Saitone and Sexton (2009) studied the role of quality-differentiated markets and tried to figure out the optimal pooling practices from independent pools for the high and low product, to the traditional principle of complete pooling (a single pool). Complete pooling is regarded as disadvantageous in quality-differentiated markets due to the potential for adverse selection. This work, however, states that pooling can counteract the tendency of competitive farm-
ers to over-produce high-quality product. Pennerstorfer and Weiss (2012a,b) investigated the incentive to provide high-quality product in a farmer-owned co-op and an IOF. They characterise the co-op as having decentralised decision-making and suggest that members of the co-op have an incentive to overproduce and to free-ride on quality.

Eilers and Hanf (1999) addressed the issue of contract design between a potato processing cooperative and its members with conflicts of interest, information asymmetries, and opportune behaviour. This paper presents a new principal-agent approach that allows for an exchange in the position of principal and agent without changing the utility function and offers solutions for both, wherein principals offer a contract or contracts to agents. The payment for growers is the sum of the basic price based on quantity and a quality premium for each percentage point of starch and the remaining share of the surplus. The asymmetric information here refers to the fact that the processor is unable to control the farmers’ actions (effort) and likewise the farmers are unable to discern the effectiveness and efficiency of the co-op. It found that the total income of each actor is higher in the principal position whilst the agent ends up with the reservation level of utility. Risk attitude is also considered wherein a more risk-averse actor would receive a larger basic income and a lower share in surplus, thereby shifting risk to the other, whereas in another case with identical risk aversion, the agent and principal share the profit almost equally.

This paper relates to the study of this thesis, as it considers a quality premium based on the starch content of potatoes inspected by the processing cooperative and also analyses inconsistent expectations between the cooperative and farmer members. This inspection process has been implemented in many co-ops recently responding to the global concern for food safety. As noted in Section 2.4, Zespri kiwifruit company also has a quality inspection process.

Many studies on the problem of quality product in co-ops are carried out by comparing them with non-cooperative organisations (IOFs). Hoffmann (2005) examined how ownership structure affects endogenous quality choice by comparing a co-op and an IOF. He showed that, depending on different cost functions, either the co-op or the IOF can promote higher levels of quality. Drivas and Giannakas (2010) analysed the effect of co-ops
on quality-enhancing product innovation activity in the context of a mixed duopoly where a co-op competes with an IOF for differentiated products. Their results showed that the members’ welfare-maximising co-op has a greater incentive to invest in quality-enhancing innovation than its IOF counterparts. Schamel (2015) analysed how German cooperative wineries compete with non-cooperative wineries and implied that the reason why co-ops lag behind in terms of quality reputation is organisation principles and the difficulty to manage growers supplying grapes of different qualities.

These papers only deal with the competition or comparison between co-ops and IOFs without specifying the contracts to motivate the quality provision. Yu et al. (2009) analysed the quality provision of a farmer-owned co-op when it competes with a private-owned firm (IOF) in setting the quality standard and quoting raw product prices. The results showed that whether or not the co-op specifies a high-quality standard depends on a comparison of costs and benefits of raising the quality standard. Li et al. (2010) studied the incentives for quality provision in a farmer-owned co-op. The co-op maximises the total welfare of the farmers subjected to the constraint of budget balance. The contract offered by the co-op is a unit price linked with each farmer’s quality and others’ qualities as well. The optimal payment scheme derived in this work is that each farmer who produces a low-quality intermediate good gets a small amount of money, and the remaining revenue is shared equally by the high-quality producers. However, these papers do not develop more on payment schemes.

3.2.2 Payment Schemes of Co-ops

Since this thesis is directed to the study of supply chain coordination, emphasis is placed on the individual decision makers and their incentives to undertake joint action. In the review of papers on co-ops, the payment schemes have been mostly analysed in the study of co-op formation. Although not very close to the setting of Chapter 4, those payment schemes provide insights into individual motivation.

The economic literature on co-ops has focused most of its attention on uniform payment mechanisms where all members pay or receive the same price (Helmberger and Hoos 1962).
However, this mechanism has been proven to show economic inefficiency because it does not provide a method of allocating the profits or losses of a co-op among its members, and thus fails to provide individual incentives (Fulton et al. 1995). Marginal cost pricing turns out to be first-best optimal (in the single-product case); however, it often fails to generate revenues that satisfy the budget balance constraint (Ohm 1956). Holmstrom (1982) demonstrated that when the quality of goods is unobservable, free-rider problems cannot be overcome once the budget balance condition is imposed. Li et al. (2010), on the other hand, demonstrated that co-ops can achieve the first-best outcome when the quality is observable and can be contracted on.

Also, a non-uniform payment, which can further be classified into linear and non-linear payment types, has been carefully examined to mitigate the economic inefficiencies. The linear payment rule takes the form of a basic price plus a premium for farmers’ individual performance. Carter (1987) examines both uniform and linear payment schemes. He shows that the supply chain cannot be coordinated by taking uniform payments, while the linear payment rule is efficient to achieve the first best. Eilers and Hanf (1999) developed a model with a linear payment scheme when analysing the principal-agent problem within a farmers’ processing co-op for the potato starch industry. They argued that compared with non-linear payments, linear payments are easily implementable and comprehensible in practice. However, some theorists do not buy this argument and believe that non-linear payment schemes are potentially superior to the linear rule (Vercammen et al. 1996).

Research on non-linear payment schemes was performed by Sexton (1986), who proposed a multi-part pricing scheme that pursues the overall optimum by retaining marginal pricing and instituting an additional set of tariffs or rebates. Several feasible allocation methods based on the marginal cost-based pricing systems include (1) non-discriminatory two-part pricing where the tariffs are the same for all farmers, and (2) benefit/patronage-based financing where the tariffs are associated with (and specifically in proportion to) the benefit/patronage. Sexton’s payment schemes can lead to a stable equilibrium at the theoretical level; however, they have been criticised for their difficulty in implementation. The first one does not apply to heterogeneous membership, and the second one may fail
if farmers untruthfully disclose their patronage due to asymmetric information. Fulton et al. (1995) developed a model of a non-uniform pricing scheme in which different prices are offered to different volumes of supply. The authors demonstrated the efficiency of the non-uniform pricing scheme and also the distributional effects when members are heterogeneous. Vercammen et al. (1996) analysed the impact of non-linear pricing schemes by deriving a constrained efficient pricing rule for a constant-cost marketing co-op. Bourgeon and Chambers (1999) derived another pricing rule with heterogeneous members who differ by their cost efficiency and their bargaining power.

These articles advance the understanding of how pricing schemes affect the efficiency of the co-op in coordination with its members. However, most papers have not captured the quality provision, and none of these papers have captured the practice of multiple period payments. Eilers and Hanf (1999) noticed the structure of multi-period payments in terms of the dividend refund; however, they did not elaborate analytically its role in the individual motivation as Chapter 4 does. Furthermore, the multi-period payment arrangement in Chapter 4 is more broadly defined than dividend distribution at the end of the year. A similar practice observed in IOFs is termed as “the delayed payment method”, under which a delayed payment period or the trade credit period would be granted to fulfill the amount of purchasing cost (Jamal et al. 1997, 2000, Abad and Jaggi 2003, Chung and Liao 2004, Teng et al. 2007). Chan et al. (2010) proposed a coordination mechanism that incorporates a delayed payment method. However, the setting in Chapter 4 is different from these papers in that the decisions of payments in the later stages are made after the realisation of uncertainties. That decisions are postponed until uncertainties are resolved is referred to as operation hedging (Van Mieghem 2003). Tomlin and Wang (2008) show that the producer benefits a lot from delaying the pricing decision until after all uncertainties are realised.
3.3 Uncertain Yield, Quality, and Price

Uncertainty is an inherent consideration in an agricultural supply chain, and one of the challenges faced by both IOFs and co-ops. Therefore, this section reviews some papers that consider the uncertainties of random yield, random quality, and random market price.

Research on yield uncertainty has been reviewed by Yano and Lee (1995) who generalise three most prominent yield models adopted by researchers, known as stochastically proportional, binomial, and interrupted geometric yield. The model of stochastically proportional yield specifies the distribution of the fraction of good units (or yield rate), and forces the distribution of the fraction good to be independent of the batch size. For example, yield $Y(Q) = z \cdot Q$ is assumed to be a random variable, where $Q$ is the farm size and $z$ as the yield rate is a random number in $(0, 1)$ with mean $\mu_z$ and variance $\sigma_z^2$. It assumes that yield rate $z$ and farm size $Q$ are independent, thus the total yield mean is $E(Y(Q)) = Q\mu_z$ and variance $Var(Y(Q)) = Q^2\sigma_z^2$. This model has been widely adopted in agricultural operations management, like Tan and Çömden (2012), Noparumpa et al. (2011), Kazaz (2004), and Kazaz and Webster (2011). In Chapter 4, this model is briefly mentioned to discuss the robustness of the payment schemes when farmers are different in their farm size.

In the model described by Boyabatli and Wee (2013), the processor reserves farm space under the uncertainties of yield and open market price. At the time those uncertainties are realised, the firm can then make decisions about sourcing from the spot market. Also, the authors model the situation where yield uncertainty affects the total input volume as well as the unit processing rate. This assumption is reasonable for processing commodities, such as sugar, olive oil, starch, etc., as they need to be processed from raw material. In the content of an agricultural product, Tan and Çömden (2012) presented a planning methodology for a firm that deals with fruit and vegetables. The objective was to determine the farm areas to be contracted and the seeding times, hence maximising the total profit during the planning period. However, not limited to random yield, the supply uncertainty also results from random maturation time and harvest time. Thus, under those uncertainties,
Tan and Çömden (2012) proposed a methodology to match the random supply from some contracted farms with the random demand from the retailers during the planning period.

Kazaz (2004) studied production planning of olive oil with random yield and stochastic demand. A two-stage decision-making process was proposed. In the first stage, the oil producer decides on the size of the olive farm to be leased, while in the second stage, the producer has to decide how much to supply and how much to purchase from the spot market after the realisation of market demand. In other words, the processor needs to deal with yield uncertainty in the first stage while facing demand risk in the second stage. It is quite common to consider yield and demand uncertainties no matter whether they occur in the agribusiness industry or a cooperative organisation. Furthermore, this paper also assumes that the sale price of olive oil and purchase cost of olives from growers are affected by uncertain yield. In other words, the price of olive oil and the purchasing cost of olives are inversely related to the observed olive yield.

One research that takes dynamic pricing into consideration was conducted by Noparumpa et al. (2011) who examined the relationship between downward-substitution, fruit-trading, and price-setting flexibilities. By considering price-setting flexibilities, it models the situation of both exogenous and endogenous price. In the second case, they assume that the price of high-quality fruit is dependent on demand, while the price for low-quality fruit is the spot market price. Quality uncertainty is seldom considered by researchers, while in this paper, quality uncertainty is revealed by the proportion of high-quality and low-quality grades of fruit. The high-quality fruit and low-quality fruit would go to two selling markets with different demand and price.

Resende-Filho and Hurley (2012b) employed another method to estimate the price of agricultural products. They presented a stochastic tactical planning model for the production and distribution of fresh agricultural products, considering the uncertainties encountered in the fresh produce industry when developing growing and distribution plans due to the variability of weather and demand. The distribution for prices in this paper was determined by analysing the historical weekly price data, suggesting a lognormal distribution. Also, the seasonal patterns followed by price were estimated. Various scenarios for
the prices of the crops were generated by taking weekly price distributions and seasonality factors into account.

3.4 Operational and Financial Problems

Chapter 5 considers both operational and financial decisions and so it is related to the interface of operations and financial management. However, the literature review has revealed that there is a general lack of models on the integration of financial and operational decisions for co-ops. Although financial problems within co-ops have been noticed for a long time, most of the relevant studies are qualitative or empirical (Hart and Moore 1998, Cook 1995, Borgen 2004, Bekkum and Bijman 2006, Raab et al. 2013). On the other hand, quantitative papers for co-ops mainly focus on operational decisions, like production, payments, or supply chain coordination, and seldom consider the capital structure and financial decisions (Hovelaque et al. 2009, Li et al. 2010, Noparumpa et al. 2011, Kazaz 2004).

In a typical investor-owned firm (IOF), there may be some space for a dichotomy between finance and operations because, in perfect capital markets, the firm’s capital structure and its financial decisions should be independent of the firm’s operational decisions (Modigliani and Miller 1959). However, this segregation is less likely to hold in a co-op, whose ownership or equity structure is directly linked to its economic transactions with members (Chaddad 2006). Even within IOFs, there are an increasing number of papers paying attention to financial constraints when making operational decisions (Buzacott and Zhang 2004, Hu and Sobel 2005, Li et al. 2013, Zhang and Sobel 2010, Dada and Hu 2008, Raghavan and Mishra 2011). Since their capital structures are quite different, the results and conclusions for IOFs may not hold for co-ops. Therefore, the study in Chapter 5 provides a quantitative analysis of a co-op’s financial and operational strategies; as such, it contributes to the interface of operations and finance in operations management.

Financial management issues, and, in particular, acquiring and redeeming members’ equity capital, are identified in the literature as major constraints to co-op growth and sustainability (Hart and Moore 1998, Cook 1995, Chaddad and Cook 2003, Caves and Petersen 1986). As discussed in Section 2.3, Cook (1995) outlines five prominent financial
problems for co-ops: the Free Rider Problem, the Horizon Problem, the Portfolio Problem, the Control Problem, and the Influence Cost Problem.

Some researchers examine the financial performance of co-ops and IOFs to see whether their respective capital structures influence their performance. Hardesty et al. (2004) argue that the difference in financial performance between co-ops and IOFs is not significant despite their differences in capital structure. However, this is not agreed to by Gentzoglanis (1997) who examines empirically the financial and economic performance of dairy co-ops and IOFs in Canada. He found that performance differs in terms of liquidity and working capital management. Further, Cook and Iliopoulos (2000) collected data from 127 agricultural co-ops in the US, and suggest that the property rights structure significantly affects members’ incentives to invest in their organisations. In Chapter 5, the financial performance of co-ops is not compared with IOFs; however, the model agrees with Gentzoglanis (1997) in that the capital structure has an influence on financial decisions. The model also demonstrates that this influence results from IOFs’ and co-ops’ different business objectives.

In Chapter 5, three main sources of capital financing in co-ops are considered: member-provided equity, retained earnings, and borrowing. Chaddad et al. (2005) examine the presence of financial constraints in US agricultural co-ops, and find that an agricultural co-op’s capital is significantly affected by the availability of internal funds. The analysis of Chapter 5 supports this point and shows that funding from members’ direct investment and retained earnings is more profitable than borrowing.

Considering the lack of quantitative papers on the financial and operational interface in co-ops, much recent work on the interface in IOFs is valuable for reference. Buzacott and Zhang (2004) consider asset-based financing in production decisions. This paper is different from other papers, which assume the loan is unlimited. Whereas, this paper assumes that the amount of the loan that a firm can borrow is linked to the firm’s assets and inventory decisions. Xu and Birge (2008) consider the conflict of interest between managers and equity-owners or debt-owners and the effects of this conflict on the firm’s optimal decisions. They show that the manager prefers aggressive investment decisions and
conservative debt policies. Ding et al. (2007) study the integrated operational and financial hedging decisions faced by a global firm with both home and foreign markets. This paper derives the joint optimal capacity and financial decisions and also analyses the impact of the delayed allocation option and financial options on capacity commitment and the firm’s performance.

In the aforementioned papers, the objective is assumed to either be to maximise profit or to minimise cost. However, some papers, such as Li et al. (1997), assume a different objective from the financial perspective: maximising the equity value, that is often measured by dividends in the long term. Xu and Birge (2006) propose an integrated corporate planning model to make production and financial decisions for a firm with market uncertainty by applying a stochastic programming methodology. At each decision node, the managers make operational and financing decisions to: stop or continue operations; determine the amount of loans, dividend payouts or new equity issues; and determine the levels of production output. Zhang and Sobel (2010) elicit the structure of a policy for inventory and financial decisions that maximises the expected present value of the time-stream of dividends.

Babich and Sobel (2004) consider a start-up firm that issues an initial public offering (IPO). Their paper examines what is the right time to make an IPO and how to coordinate financial and operational decisions related to capacity expansions and production quantities. The IPO is treated as a stopping time in an infinite-horizon discounted MDP with multiple state variables, and an optimal threshold stopping rule is derived. The model also presents explicit expressions for the firm’s production schedule and loan size as functions of an optimal capacity increment. The only sources of capital in this paper are current assets and bank loans, where the rate depends on the financial condition of the firm. The model’s conclusions agree with the “pecking order” in finance, and support the idea that the least expensive source of capital for a firm is current assets. Last, but not least, it illustrates how to improve computational efficiency based on these conclusions.

Hu and Sobel (2005) examine the interdependence of a firm’s capital structure and its short-term operating decisions concerning inventories, dividends, and liquidity. This
paper employs the same framework as Li et al. (1997) and assumes that the objective of the firm is also to maximise the expected present value of the stream of dividends. The authors conclude that the optimal dividend and liquidity policy depends on the operating decisions, and the firm’s optimal capital structure is entirely debt or equity depending only on financing parameters rather than production parameters. However, Chapter 5 shows a different capital structure in co-ops with the capital coming from either debt or equity depending on both financing and production parameters, because the members’ investment includes the purchase of the membership stock and a retained allocation of the profits in the form of equity. Furthermore, the model in Chapter 5 shows that the optimal payments and liquidity policy not only depend on the operating decisions but also the co-op’s objective, which is different from that of IOFs.

Li et al. (2013) construct another dynamic model of coordination of inventory and financial decisions in an equity-financed firm, which is based on their unpublished work Li et al. (1997). In their model, the dividends are allowed to be negative under which the shareholders are obliged to subscribe additional capital. They extend the model by imposing a non-negativity constraint on dividends and argue that the firm has lower inventories and higher short-term loans than a counterpart that may obtain additional capital. This model yields an optimal myopic policy that stipulates that the firm produces just enough to raise the product stock to a target level and issues dividends to bring the cash stock to a target level as well. Chapter 5 shows a similar structure: when the capital is sufficient, the co-op retains just enough internal working capital to process product and distribute payments to farmers to bring the cash stock to a target level.

3.5 Research on New Zealand Co-ops

This section reviews previous work on New Zealand agricultural co-ops. Evans and Meade (2005) wrote a comprehensive report prepared for the New Zealand Ministry of Agriculture and Forestry. This report introduces cooperative organisations including basic concepts and definitions and includes information on cooperative generation and evolution, as well as related theoretical and empirical literature. This work also separately describes the role and
significance of NZ agricultural co-ops including the dairy industry, the meat industry, the wool industry, the fishing, and aquaculture industry, the horticulture industry, and forestry. In the section on horticulture, Zespri is mentioned as the dominant kiwifruit organisation. The author describes Zespri as a grower-controlled IOF, whose structure came out of responding to industry reforms when the statutory export monopoly of the former Kiwifruit Marketing Board was relinquished. This semi-cooperative, centralised marketing model indicates the homogeneity of growers’ interest that favour further cooperative development.

The development and survival of NZ co-ops have gone through a very challenging journey. For example, the NZ dairy industry, according to Ohlsson (2004), had over 400 dairy co-ops in 1935, which had decreased to 180 by 1960/61, and with transportation improvements and advancement in large-scale processing technologies, fell further to only 19 by the beginning of the 1990s. In 2001, two major co-ops, Cooperative Dairies Limited and New Zealand Dairy Group, merged to form Fonterra. Other industries such as the New Zealand apple industry, as shown in the report of Evans and Meade, represent an extreme example of industry fragmentation after the abolition of a single seller desk. A previously formed New Zealand cooperative meat processor, AFFCO, converted to a listed company in 1995, and another meat processing cooperative, PPCS, recently acquired listed IOF processor Richmond.

Although a lot of agricultural co-ops did arise in New Zealand, the failure of these co-ops increasingly arouses concerns about how efficient these organisations are. Chan and Robb (1998) present evidence on the relative profitability of a NZ cooperative (Foodstuffs) and IOF (Progressive Enterprises Limited), albeit in the wholesale grocery industry, rather than in agriculture. These authors compared the performance of both organisations between 1991 and 1997 and found that Foodstuffs outperformed Progressive, had higher profitability, less leverage, better solvency and liquidity, and greater asset efficiency. They also suggest that as co-ops are more accountable to their owners than IOFs they are required to more carefully monitor profitability and cash flow.

Nevertheless, NZ co-ops do not generally outperform IOFs. Sullivan and Scrimgeour (1995) considered the performance of the NZ Dairy Board (NZDB) compared with that of
Nestlé, a Swedish IOF from 1969 to 1992. They found the cost of the NZDB’s product to be 27% higher in cost than Nestlé and Nestlé added 12 percent more value to every dollar of sales compared to the NZDB. Hence, they concluded that Nestlé seemed to be more efficient than the NZDB. This conclusion contrasts with that of Lerman and Parliament (1990) who analysed the fruit and vegetable processing and dairy industries and concluded that co-ops in both industries performed as well as or better than IOFs by profitability, leverage, and interest coverage measures. Another comparison between Fonterra and the Australia Wheat Board (AWB) was conducted by Trechter et al. (2003) from the aspect of strategies responding to the cancellation of the single seller desk, which authorizes or certifies the monopoly rights to export. Fonterra was able to achieve cost savings through supply chain integration under a cooperative structure and to mitigate capital-raising issues by leveraging member capital through a joint venture, for example, with Nestlé. This, however, gave rise to farmer-partner conflicts of interest. By contrast, AWB established an IOF structure by issuing dual shares, A shares for farmers alone and B shares for any investors. The authors argue that even though this approach better resolves the horizon, portfolio, agency cost, and capital raising issues than Fonterra does, it makes farmer-investor conflicts worse. Based on these research studies, there seems to be no clear support, from a theoretical point of view, that co-ops will be more or less efficient than IOFs.

Other papers about NZ co-ops are mainly from the viewpoint of marketing strategies and production planning. Beverland (2001) conducted a survey of Zespri kiwifruit co-ops. This article was to identify Zespri’s brand strategy as well as its influence on Zespri market return and customer choice. Through this research, the author proposed some plausible suggestions on fruit promotion and marketing. For example, he suggested that to fulfill the aim of increasing returns to growers, Zespri should work with members of the supply chain, rather than focusing solely on raising consumer awareness. Thus, instead of selling at auction, Zespri should try to develop price and supply contracts with distributors and retailers. Another case study of co-ops in NZ, also studied by Beverland (2005), is The Game Industry Board. Similar to the study of Zespri, this work also identified its brand strategy. Based on previous research, Beverland (2007) compared the ability of five
NZ agricultural co-ops to develop and support market-based assets including brands and long-term relationships with buyers. These case studies include The Game Industry Board, Zespri, Merino NZ, Sealord, and Fonterra. Each has a high profile in NZ and has experienced a transition from farmer orientation to market orientation. By using interviews, the author suggests that traditional co-ops can develop a market-orientation but not support it in the long-term, whilst new generations of co-ops are more effective. The author argues that cooperative structures with well-defined property rights can break commodity price cycles, and become market leaders regarding both market share and rate of return. Hence, to enhance their long-term market success, cooperative leaders and members should push for new generation structures.

To mitigate conflict between suppliers and buyers, vertical integration/coordination related to supply chain management has long been a topic of study. In the agricultural co-operative, it seems that conflicts not only exist between farmers and their cooperative, they also occur among farmers. Wood (2010)’s research is based on case studies in NZ to analyse issues in horizontal coordination. One of the case studies relates to the NZBrand venture, a cluster formed by fruit exporters to increase their competitiveness in the international market. Even though they co-operated with each other for the one purpose of reducing shipping costs, they still maintained a competitive outlook about market share. This combined co-operation and competition is referred to as co-opetition. Co-opetition widely exists in the cooperative organisation. For example, with regarding to Zespri, kiwifruit farmers would compete with each other regarding assigned quota to certain high-value markets, such as Japan. Some strategies are proposed to overcome this problem, like information sharing, profit and cost sharing, and so forth. The author provides a good insight to consider the coordination problem horizontally from a game theory point of view in co-ops, especially with heterogeneous farmers.

Guan and Philpott (2011) studied the case of Fonterra, from the aspect of production planning by proposing a stochastic programming model. This work takes into account uncertain supply, a linear price-demand curve, and contracting. Uncertain milk supply is predicted through an autoregressive forecast model with historical data, and the pro-
gramming model is also proposed to mitigate the risk of uncertain supply. A Dynamic Outer-Approximation Sampling algorithm is adopted to solve this model and simulation experiments are used to test the results.

3.6 Brief Summary

This chapter provides a literature review of the studies on co-ops that are relevant to the research of this thesis. As Chapter 4 adopts a game theoretical approach, Section 3.2 gives a review of contracts and game theoretical models with an emphasis on quality provisions and payment schemes. Since, the agricultural supply chain is characterised by various uncertainties, Section 3.3 reviews the work that considers uncertain yield, quality, and market price. Furthermore, financial decisions are considered in both Chapter 5 and 6, therefore, relevant studies on operational and financial decisions within both co-ops and corporates are reviewed in Section 3.4. Since this thesis also contributes to the studies on New Zealand agricultural co-ops, Section 3.5 gives a brief review of relevant previous studies.
4.1 Introduction

This chapter investigates contractual coordination of a co-op that proposes quality provisions in its contracts with farmers. Improving product quality enables the agricultural supply chain to be more competitive, and also generates a higher value. However, providing quality products involves extra costs for farmers to exert effort at the farm level. The challenge is, individually, farmers may not have a common interest in delivering products complying with quality requirements if there are no proper incentives. Therefore, the success of quality provisions requires that the contracts that the co-op applies to farmers must be both value maximising for the supply chain and also incentive compatible for the individual farmers to undertake optimal collective effort. The first one depends on the co-op’s decision on the quality level driven by the whole value chain, and the second one lies mainly in the payment schemes that the co-op offers to the farmers.

Furthermore, in the pursuit of quality, the traditional practice of uniform payments has turned out to be ineffective, since all products are pooled without quality distinctions (Carter 1987). Recently, many co-ops have changed to non-uniform payment mechanisms that resemble those used by investor-owned firms (IOFs). That is, they offer quality-differentiated prices to high or low-quality products. However, there are two main differences between the two organisational structures. First, the IOF’s objective is to maximise its own profit, while the co-op aims to maximise the profit of the whole supply chain (Li et al. 2010). Second, maximising a co-op’s objective usually subjects it to a technical condition which maximising an IOF’s objective does not need to follow: all profit is assumed to be shared by the farmers. This condition is known as the budget balance condition.
and is usually justified by the fact that farmers own rights to the residual earnings due to their ownership of the co-op (Bogetoft and Olesen 2007). Compared with theoretical studies on IOFs, the imposed budget balance condition makes it complicated to design sensible payment schemes (Bogetoft and Olesen 2007).

This chapter also considers the strategy of distributing payments to farmers in multiple periods. A multi-period payment scheme allows the co-op to secure enough capital for marketing functions; however, it affects farmers’ welfare relating to their time preferences. In economics (Frederick et al. 2002), time preference indicates the relative valuation placed on a good at an earlier date compared with its valuation at a later date. Mathematically, the higher the time preference, the higher the discount placed on future returns. Staatz (1987) has mentioned farmers’ time preference for money to indicate the heterogeneity of cooperative members. However, as far as I am aware, none quantitative studies on co-ops have even taken this factor into account in their formulations, despite its importance in the analysis of supply chain coordination. This work has simplified the multiple periods into two stages: the initial stage when the product is submitted and the second stage after the commodity is marketed.

In addition to payment schemes, another item that has an impact on members’ utilities is the profitability of the co-op (Fulton et al. 1995). Pooling all products to a homogeneous market has been proven to be of low-profitability and disadvantageous in successfully meeting market demands for quality (Saitone and Sexton 2009); many co-ops now deliver products to quality-differentiated markets (Fultion and Sanderson 2002). Pooling high-quality products to high-end markets creates profitable opportunities for farmers and their co-ops to position themselves on the quality spectrum. However, it further complicates the process of determining each member’s marginal contribution.

In this chapter, the co-op makes decisions on the optimal level of quality standard, how many products to sell in either high- or low-end markets, and how to distribute payments in two periods. This chapter investigates different multi-period payment schemes, and seeks to answer the questions: whether a co-op can coordinate farmers’ efforts on quality, and what payment schemes are optimal.
The remainder of this chapter is organised as follows. Section 4.2 describes the model and assumptions. Section 4.3 explores the centralised decisions made by the co-op on the quality standard and optimal level of collective effort. Section 4.4 explores farmers’ behaviours under “the market-price-guarantee scheme”, and further proposes an improved payment scheme. Section 4.5 examines the impact of multi-period payment schemes on the initial payment and also its role in supply chain coordination, in comparison to the traditional payment scheme. Section 4.6 presents the main conclusions, and all notation of this chapter are summarised in Appendix A.

4.2 Model Description and Assumptions

This section describes the details of the problem and presents a two-stage stochastic model that features the quality provision and multi-period payment scheme. The first stage corresponds to the growing season, and the second stage is the selling season. The two-stage process is shown in Figure 4.1, and is also described below:

Figure 4.1: The Timing of Events

1. At the beginning of a growing season, the co-op determines a quality level \( l \). For example, in the kiwifruit industry, the quality level is measured by the percentage of dry weight of kiwifruit or the measure of mass when kiwifruit is completely dried. The product above this level is categorised as the high-quality product marked by \( H \); while the product below is the low-quality product represented as \( L \). Furthermore, the co-op determines the first-stage prices offered to both the \( H \) and the \( L \) product.
2. The individual farmers make an effort to increase the proportion of the $H$ product.

3. At the end of the growing season, the product is harvested and delivered to the co-op where the quality is measured and the first-stage payment is paid accordingly.

4. In stage 2 at the time of selling, the co-op delivers products to either the high-end or the low-end market with the realised $H$ and $L$ product.

5. At the end of the selling season, the co-op realises the market profits from both markets and determines the second-stage payments accordingly.

Assuming that there are a constant $n$ farmers in the co-op, each individual is expected to realise two grades of products depending on both the quality level $l$ and each individual’s effort level $e_i$ ($i = 1, 2, ..., n$). Let $h(e, l)$ denote the proportion of $H$ versus $L$ where $0 \leq h(e, l) \leq 1$. For the function $h(e, l)$, this chapter has the following assumption.

Assumption 4.1. Assume $h(e, l)$ possesses the following properties:

(1) $h(e, l)$ is increasing in $e$, while decreasing in $l$;

(2) $h(e, l)$ is jointly concave in $e$ and $l$;

(3) $h(e, l)$ is a submodular function, i.e., $\frac{\partial^2 h(e, l)}{\partial e \partial l} \leq 0$.

Both properties (1) and (2) can be easily justified by reality. The submodular function assumed in property (3) implies that the improvement of $h(e, l)$ by an increased effort decreases as the quality level increases. This assumption is quite sensible in this application because an increase of quality level makes it harder to improve the proportion of high-quality products with additional effort. A simple example for the generic function is $h(e, l) = c_1 e - c_2 l$ where $c_1$ and $c_2$ are certain parameters. Note that this model can be easily extended to include quality improvement uncertainty by introducing a random error term, e.g., $h(e, l) + \xi$. However, this would make the analysis of the model more difficult because of high-dimensional uncertainty.

At the farm level, assume that the total yield (both $H$ and $L$ products) of each farmer $Y_i$ is an independent and identically distributed (i.i.d) random variable, with a mean of $E[Y_i] = \mu_y$, a standard deviation of $\sigma_y$, and a cumulative distribution function of $F_i(y_i)$. Define $Y$ to be the random variable equal to the sum of all individual yields, and let $y$ be a realisation of $Y$. The total farm production is a function of the output of the given $n$
farmers in the co-op. This chapter uses $F(y)$ to denote the joint distribution of $n$ farmers, and $F_{-i}(y_{-i})$ for the other $n-1$ farmers. An upper limit to the realisation of total yield is set to be $\bar{y}$, which is an intuitively appealing assumption.

Then, the contribution of each individual on $H$ and $L$ production is specified as

$$Q_i^H = h(e_i, l)Y_i, \quad Q_i^L = (1 - h(e_i, l))Y_i,$$

and the total $H$ and $L$ farm production in the co-op as

$$Q^H = \sum_i h(e_i, l)Y_i, \quad Q^L = \sum_i (1 - h(e_i, l))Y_i.$$

At the end of the growing season, the co-op realises the total $H$ and $L$ production in the amount of $q^H = \sum_i h(e_i, l)y_i$ and $q^L = \sum_i (1 - h(e_i, l))y_i$, where $y_i$ is a realisation of $Y_i$.

The second-stage problem can be described as maximising profit from the sales of two differentiated markets for given realisations of $H$ and $L$ products, respectively. In this stage, the co-op makes decisions on the quantity of $H$ product delivered to the high-end markets, denoted as $q_s^H$. For the high-end market structure, the model has the following assumption.

**Assumption 4.2.** Assume that in the high-end market the co-op is able to determine the selling price for $H$ products, and the price is a function of the supply quantity of $H$ product and the quality level. The price is expressed as

$$p^H(q^H_s, l) = a - b(l)q^H_s,$$  \hspace{1cm} (4.1)

where $b(l) > 0$, and is decreasing and strictly convex in $l$ (i.e., $b'(l) < 0$ and $b''(l) > 0$), and satisfies the inequality $b(l)b''(l) - 2(b'(l))^2 > 0$.

The assumption of price-maker resembles the real-world scenario of some large companies who take up a high market share and can set the selling price for their premium products (Noparumpa et al. 2011). This is also the case for some large co-ops, like Zespri who is able to set prices on its high-end markets. Also, assume that the quality contributes to the market price by reducing price elasticity, indicated by $b(l)$, which measures the responsiveness of price to a change in $q^H_s$. The dependency of quality on price has long been noticed.
in the economic literature (Stiglitz 1987); Coibion et al. (2007) argue that price elasticities for high-quality products are likely to be lower. In other words, as quality levels increase, consumers become less sensitive to the price. This assumption appears intuitive since customers should desire higher quality products (Ding et al. 2010). However, the reduction of price elasticity is assumed to decrease as the quality level increases. This assumption makes sense because as a product improves, consumers may be less likely to detect an improvement in quality (Ding et al. 2010). The technical condition $b(l)b''(l) - 2(b'(l))^2 > 0$ is needed for the proof of the joint concavity of the objective function (see below). One specific functional form that satisfies these assumptions is

$$b(l) = \frac{1}{lk}, 0 < k < 1,$$

where the coefficient $k$ captures the sensitivity of the elasticity to quality. For the same value of quality, the elasticity with a larger $k$ derives less value than that with a low $k$. Therefore, the co-op can influence the high-end market price by appropriately choosing the level of quality specified in the contract and the quantity delivered to the high-end market, which shall be no more than the realised yield, i.e., $q^H_s \leq q^H$.

On the other hand, $L$ products are targeted to the low-end market, which is populated with many similar products at a lower price. Thus, the low-end market price is assumed to be a random variable, denoted by $P$, where $p$ is a realisation and $G(\cdot)$ is its cumulative distribution function which has support $[\underline{p}, \overline{p}]$. In addition, similar to the assumptions in the work of Noparumpa et al. (2011), the value of $p$ is available to the co-op before the selling season and there is no demand restriction in the low-end market. Since $L$ products are delivered to the low-end market which is populated with many similar products, the low-end market price can be well estimated before the decision of processing quantity. Assuming that the co-op makes decisions before the realisation of uncertainty does not appear analytically practical because it results in a three-stage, not a two-stage, stochastic model.

This chapter also considers the flexibility of downward substitution where the co-op can deliver some of its $H$ products to the low-end market. However, the possibility of downward substitution depends on the pooling arrangements of the co-op. When the co-op
has a combined sales pool for the $H$ and the $L$ product, it is able to substitute the $H$ product for the $L$ product. However, when the co-op has separate sales pools for the $H$ and the $L$ product, downward substitution is not allowed and the co-op delivers its entire $H$ production to the high-end market. For example, Zespri has a combined sales pool for GOLD and GOLD ORGANIC Kiwifruit, but separate sales pools for GREEN and GREEN ORGANIC. Therefore, Zespri obtains a combined market revenue for GOLD and GOLD ORGANIC Kiwifruit, but separate revenue pools for GREEN and GREEN ORGANIC. This chapter does not aim to identify the optimal practice of pooling as Saitone and Sexton (2009) did; instead, it investigates the optimal incentive mechanisms in different settings.

**Assumption 4.3.** Assume that when there is no downward substitution (e.g., in the case of separate sales pools), the selling price in the high-end market clears the production given any realisation of the $H$ product, i.e., $a - 2b(0)\bar{y} \geq 0$. In addition, to ensure that the $H$ product is sold at a higher price than the $L$ product, the random low-end market price is bounded by $p \leq a$.

This assumption captures the real-world scenarios that the profit margin of the $H$ product is high enough, so that it is never more profitable to dump or hold back excessive $H$ products in the high-end market. It does not rule out downward substitution being optimal in the case of a combined sales pool.

At the end of the selling season, the total profit from both high- and low-end markets are realised as $R(q^H, q^L)$. For simplicity, the fixed costs are assumed to be zero, while variable costs that are mainly associated with farmers’ payments are discussed below.

As described earlier, farmers’ payments are completed in two stages. In the first stage, the pre-determined prices are paid to the $H$ or $L$ product, denoted by $w_1^H$ and $w_1^L$, respectively. It is reasonable to assume that the price of the $H$ product is at least higher than that of the $L$ product, i.e., $w_1^H \geq w_1^L$. The difference $\beta = w_1^H - w_1^L \geq 0$ captures the quality premium for the $H$ product. At the end of the selling season, the co-op distributes the net surplus to farmers. Similarly, let $w_2^L$ and $w_2^H$ denote the second-stage prices. Therefore,
with the realisation of each individual’s production, each individual’s total income is:

\[
\begin{align*}
I_1^i &= w_1^L q_1^L + w_1^H q_1^H, \\
I_2^i &= w_2^L q_2^L + w_2^H q_2^H,
\end{align*}
\]  
(4.3)

where \(I_1^i\) and \(I_2^i\) denote each individual’s income in the first stage and second stage, respectively.

As described earlier, farmers have time preferences towards the payment time. Since the second-stage payment is lagging after the realisation of market revenue, a discount factor \(\alpha\) \((\alpha > 0)\) is assigned to indicate the degree of farmers’ time preferences. Let \(E[U(e_i)]\) denote the expected utility of farmer \(i\) expressed as below:

\[
E[U(e_i)] = E[I_1^i] + \alpha E[I_2^i] - c(e_i),
\]  
(4.4)

where \(c(\cdot)\) is the farmer’s cost of effort. A lower value of \(\alpha\) indicates a higher time preference. The time preference is slightly different from the common “discount rate” used in the future cash flow, in that the determination of time preference is more subjective, such as risk or personal preference (Becker and Mulligan 1997). To capture farmers’ attitudes towards the delayed payment, this chapter lets \(\delta\) \((0 < \delta < 1)\) denote the interest rate used in the discounted cash flow and assumes \(\alpha \leq \delta\). The equality \(\alpha = \delta\) indicates farmers are time-indifferent, while the inequality \(\alpha < \delta\) implies farmers are time-sensitive. Section 4.5 discusses how farmers’ time preferences influence the decision on the initial payment. As is usual, the effort cost \(c(\cdot)\) is assumed to be an increasing and convex function.

Finally, the co-op is subject to the budget balance condition in the second stage, shown as:

\[
\sum_{i=1}^{n} I_1^i + \delta \sum_{i=1}^{n} I_2^i = \delta R(q^H, q^L).
\]  
(4.5)

This chapter assumes a homogeneous membership where farmers have the same production and cost structure. In other words, the productivity of a farmer is mainly determined by his/her effort, instead of other forces beyond the individual’s control (e.g., quality of land, weather conditions), which specifically affect his/her productivity. This assumption does not reduce the contribution of this chapter too much because it focusses more on the sharing rule of a co-op when members differ in their effort level, instead of asymmetric information (e.g., farming technology investment) involved in the decision process. In
addition, the assumption of the same time preference for all farmers will not be perfectly accurate, due to personal preferences, for example; however, the analysis of the following sections implies that the payment schemes are also robust to heterogeneous farmers with different time preferences.

4.3 Centralised Decisions

The purpose of this section is to derive the optimal centralised decisions for the co-op. As a standard of comparison for decentralised optimal effort, the co-op’s centralised optimal effort is defined as the effort value $e^*$ that maximises the supply chain profit $E[\pi(e, l)]$. Since all individuals have identical utility functions defined over effort, therefore, the collectively assigned effort will be the same for each individual. This chapter first presents the two-stage stochastic problem as follows:

**Stage 1:** Maximising the expected profit of the whole supply chain.

$$\max_{e, l} E[\pi(e, l)] = \delta E[R(Q^H(e, l), Q^L(e, l))] - nc(e). \quad (4.6)$$

**Stage 2:** Given the realisation of $Q^H$, $Q^L$, and the random low-end market price $P$, maximising the market profit,

$$R(q^H, q^L) = \max_{0 \leq q_s^H \leq q^H} [(a - b(l)q^H_s)q^H_s + p(y - q_s^H)], \quad (4.7)$$

where $y = q^H + q^L$.

In the following subsections, the problem defined in Eq.(4.6) and (4.7) is solved using backward induction, starting from Stage 2, by incorporating the optimal second-stage decisions into the first-stage objective function.

4.3.1 Case 1: Separate Sales Pools.

With separate sales pools, the co-op delivers the entire $H$ production to the high-end market and the $L$ production to the low-end market. Using Assumption 4.3, this section has the following results in the second stage stated in Remark 1.

**Remark 1.** With separate sales pools, the selling price on the high-end market is $p^H = a - b(l)q^H$, and the realised total market profit is $R(q^H, q^L) = [a - b(l)q^H]q^H + pq^L$. 

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Since the collectively assigned effort is the same for each individual, that is $e^*_i = e^*_j$ where $i \neq j$, the joint high-quality production is:

$$q^H = \sum_{i=1}^{n} h(e, l) y_i = h(e, l) \sum_{i=1}^{n} y_i = h(e, l) y.$$  

Under Remark 1, the first-stage objective function is therefore

$$E[\pi(e, l)] = \delta E\left[(a - P - b(l)Q^H(e, l))Q^H(e, l) + Y P\right] - nc(e). \quad (4.8)$$

To show that the objective function $E[\pi(e, l)]$ is a jointly concave function of $e$ and $l$, this chapter first proposes a necessary condition that always motivates a positive effort.

**Lemma 4.1.** The optimal effort $e$ is strictly positive, i.e., $e^* > 0$ when

$$E[(a - P)Y] > 0. \quad (4.9)$$

**Proof of Lemma 4.1.**

According to Eq.(4.6), a positive collective effort requires that $\frac{\partial E[R(Q^H(e, l), Q^L(e, l))]}{\partial e} > 0$ for all $l$; otherwise, the optimal effort is zero. According to Remark 1, $R(q^H, q^L) = [[(a - b(l)q^H)q^H + p(y - q^H)]; therefore,

$$\frac{\partial E\left[(a - b(l)Q^H)Q^H + P(Y - Q^H)\right]}{\partial e} = E[(a - P - 2b(l)Q^H)Y] \frac{\partial h(e, l)}{\partial e}.$$  

A positive marginal profit requires $E[(a - P)Y - 2b(l)h(e, l)Y^2] > 0$ for all $l$. Since $0 \leq h(e, l) \leq 1$, and $b(l)$ is decreasing in $l$ where $l \geq 0$, a sufficient condition is $E[(a - P)Y - 2b(0)Y^2] > 0$.

This yields the following proposition.

**Proposition 4.1.** Under the condition of Lemma 4.1, the co-op’s first-stage objective function, Eq.(4.8), is jointly concave in $e$ and $l$, and there exists a unique optimal centralised effort $e^*$ and quality level $l^*$ satisfying:

$$\delta E[(a - P - 2b(l)Q^H)Y] \frac{\partial h(e, l)}{\partial e} - n \frac{\partial c(e)}{\partial e} = 0, \quad (4.10)$$

$$E[(a - P - 2b(l)Q^H)Y] \frac{\partial h(e, l)}{\partial l} - E[Y^2]h^2(e, l) \frac{\partial b(l)}{\partial l} = 0, \quad (4.11)$$

where $e^* > 0$, $l^* > 0.$

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Proof of Proposition 4.1.

According to Eq.(4.8), the first and second-derivatives are:

\[
\begin{aligned}
\frac{\partial E[\pi(e, l)]}{\partial e} &= \delta E[(a - P)Y - 2b(h(e, l))Y^2] \frac{\partial h(e, l)}{\partial e} - n \frac{\partial c(e)}{\partial e}, \\
\frac{\partial E[\pi(e, l)]}{\partial l} &= \delta \left( E[(a - P)Y - 2b(h(e, l))Y^2] \frac{\partial h(e, l)}{\partial l} - E[h^2(e, l)Y^2] \frac{\partial b(l)}{\partial l} \right).
\end{aligned}
\]

and

\[
\begin{aligned}
\frac{\partial^2 E[\pi(e, l)]}{\partial e^2} &= \delta E \left[ (a - P)Y - 2b(h(e, l))Y^2 \right] \frac{\partial^2 h(e, l)}{\partial e^2} - \delta \left( \frac{\partial h(e, l)}{\partial e} \right)^2 - n \frac{\partial^2 c(e)}{\partial e^2}, \\
\frac{\partial^2 E[\pi(e, l)]}{\partial l^2} &= \delta E \left[ (a - P)Y - 2b(h(e, l))Y^2 \right] \frac{\partial^2 h(e, l)}{\partial l^2} - \delta \left( \frac{\partial h(e, l)}{\partial l} \right)^2 - 4h(e, l)Y^2 b'(l) \frac{\partial h(e, l)}{\partial l} - h^2(e, l)Y^2 b''(l), \\
\frac{\partial^2 E[\pi(e, l)]}{\partial e \partial l} &= \delta E \left[ (a - P)Y - 2b(h(e, l))Y^2 \right] \frac{\partial h(e, l)}{\partial e} \frac{\partial h(e, l)}{\partial l} - \delta \left( \frac{\partial h(e, l)}{\partial e} \right)^2 - 2h(e, l)Y^2 \frac{\partial h(e, l)}{\partial e} \frac{\partial h(e, l)}{\partial l}.
\end{aligned}
\]

The determinant of the Hessian matrix is:

\[
\Delta = \frac{\partial^2 E[\pi(e, l)]}{\partial e^2} \frac{\partial^2 E[\pi(e, l)]}{\partial l^2} - \left( \frac{\partial^2 E[\pi(e, l)]}{\partial e \partial l} \right)^2
\]

\[
= \delta^2 \left\{ E[(a - P)Y - 2b(h(e, l))Y^2] \left[ \frac{\partial^2 h(e, l)}{\partial e^2} \frac{\partial h(e, l)}{\partial l} \right] - \left( \frac{\partial^2 h(e, l)}{\partial e \partial l} \right)^2 - 2 \frac{\partial^2 h(e, l)}{\partial e \partial l} \frac{\partial h(e, l)}{\partial e} \right\} - 4E[Y^2] E[(a - P)Y - 2b(h(e, l))Y^2] h(e, l) \frac{\partial b(l)}{\partial l} \\
* \left[ \frac{\partial^2 h(e, l)}{\partial e^2} \frac{\partial h(e, l)}{\partial l} - \frac{\partial^2 h(e, l)}{\partial e \partial l} \frac{\partial h(e, l)}{\partial e} \right] - h^2(e, l) E[Y^2] E[(a - P)Y - 2b(h(e, l))Y^2] \\
* \frac{\partial^2 h(e, l)}{\partial e^2} \frac{\partial^2 b(l)}{\partial l^2} + 2(E[Y^2])^2 h^2(e, l) \left( \frac{\partial h(e, l)}{\partial e} \right)^2 [b(l)b'(l) - 2(b'(l))^2] \\
- \Delta_n \frac{\partial^2 c(e)}{\partial e^2} \frac{\partial^2 E[\pi(e, l)]}{\partial l^2}.
\]

The conditions that $E[(a - P)Y - 2b(h(e, l))Y^2] > 0$ (shown in Lemma 4.1), $\frac{\partial^2 h(e, l)}{\partial e^2} < 0$, $\frac{\partial^2 b(l)}{\partial e^2} < 0$, $\frac{\partial h(e, l)}{\partial l} < 0$, $\frac{\partial h(e, l)}{\partial e} < 0$, and $\frac{\partial^2 c(e)}{\partial e^2} > 0$, $\frac{\partial^2 b(l)}{\partial e^2} > 0$, $\frac{\partial h(e, l)}{\partial e} \leq 0$, and $[b(l)b'(l) - 2(b'(l))^2] > 0$, imply that $\frac{\partial^2 E[\pi(e, l)]}{\partial e^2} < 0$, $\frac{\partial^2 E[\pi(e, l)]}{\partial l^2} < 0$, and $\Delta > 0$. Since the Hessian is negative definite, $E[\pi(e, l)]$ is jointly concave in $e$ and $l$ (Aubin 2007). Furthermore, the first-order conditions, $\partial E[\pi(e, l)]/\partial e = 0$ and $\partial E[\pi(e, l)]/\partial l = 0$, provide the optimal solutions.
The concavity of the objective function is important for sufficiency of the first-order optimality conditions. Although the optimal \( e^* \) and \( l^* \) cannot be expressed in a closed-form expression, they can be calculated by equating the first-order derivative of \( E[\pi(e, l)] \) to zero.

The above proposition also provides general results regarding the behaviour of the optimal effort and quality level. Eq.(4.10) shows that the optimal effort is to balance the marginal contribution of effort and the total marginal effort cost. Eq.(4.11) shows the trade-off introduced by the quality level. On the one hand, it contributes to the market profit by increasing market price and reducing the price-elasticity; on the other hand, it decreases the market profit with fewer \( H \) products.

### 4.3.2 Case 2: Combined Sales Pool

In this case, when the \( H \) production is low, pricing flexibility enables the co-op to set higher prices to compensate for the lack of production. When the high-quality production is high, the co-op can deliver excessive \( H \) products to the low-end market. As a result, the co-op sets a threshold for a maximum supply level in the high-end market, where the remaining product will be sold in the low-end market. This chapter denotes this threshold, which determines the co-op’s switching decision from the high-end market to the low-end market, by \( \hat{q} \). This threshold can be determined by ignoring the realisation of high-quality production.

**Proposition 4.2.** The threshold for the amount of product delivered to the high-end market is

\[
\hat{q} = \frac{a - p}{2b(l)} , \tag{4.12}
\]

and is increasing in \( l \).

**Proof of Proposition 4.2.**

Since the revenue function \((a - b(l)q_s^H)q^H + p(y - q_s^H)\) is concave in \( q_s^H \), the optimal decision can be derived from the first-order condition, i.e., \( a - 2b(l)q_s^H - p = 0 \) and the optimal \( \hat{q} = \frac{a - p}{2b(l)} \). Furthermore, since \( b(l) \) is decreasing in \( l \), then \( \hat{q} \) is increasing in \( l \).
Proposition 4.2 shows that the co-op’s desired level of production \( \hat{q} \) changes with the quality level. In traditional modelling approaches where the market price is independent of quality, the desired level of the \( H \) product would be constant for all realisations of \( q^H \). However, Proposition 4.2 shows that the desired level increases in the quality level, manifesting the contribution of quality provisions. As will be shown below, when the realised high-quality supply is below the desired level, the co-op delivers all \( H \) products to the high-end market and charges a market-clearing price. Otherwise, it delivers up to the threshold and sells the remaining products in the low-end market.

The following proposition proposes the optimal selling price and market revenue in the second stage. Subscripts 1 and 2 are used to denote the cases of \( q^H \leq \hat{q} \) and \( q^H > \hat{q} \), respectively.

**Proposition 4.3.** With a combined sales pool, the optimal selling prices and the revenues in the second stage are

\[
\begin{align*}
    p_H^1 &= a - b(l)q^H & \text{if } q^H \leq \hat{q}, \\
    p_H^2 &= \frac{a + p}{2} & \text{if } q^H > \hat{q},
\end{align*}
\]

and

\[
\begin{align*}
    R_1(q^H, q^L) &= (a - b(l)q^H - p)q^H + py & \text{if } q^H \leq \hat{q}, \\
    R_2(q^H, q^L) &= \frac{(a - p)^2}{4b(l)} + py & \text{if } q^H > \hat{q}.
\end{align*}
\]

**Proof of Proposition 4.3.**

As proven in Proposition 4.2, the revenue function \((a - b(l)q^H)q^H + p(y - q^H)\) is concave, and \( \hat{q} \) is the optimal value that maximises the revenue function without the constraint of \( 0 \leq q^H \leq \hat{q} \). Therefore, \( q^H = \min\{q^H, \hat{q}\} \) when \( 0 \leq q^H \leq \hat{q} \).

1. If \( q^H \leq \hat{q} \), then \( q^H = \hat{q} \). Put \( q^H = \hat{q} \) into Eq.(4.1) and Eq.(4.7), hence derive \( p_H^1 = a - b(l)q^H \) and \( R_1(q^H, q^L) = (a - b(l)\hat{q})\hat{q} + p(y - \hat{q}) \).

2. If \( q^H > \hat{q} \), then \( q^H = \hat{q} = \frac{a - p}{2b(l)} \). Put \( q^H = \frac{a - p}{2b(l)} \) into Eq.(4.1) and Eq.(4.7), thus \( p^H = a - b(l)\hat{q} = \frac{a + p}{2} \) and \( R_2(q^H, q^L) = \frac{(a - p)^2}{4b(l)} + py \).
This section next considers the first-stage objective function by incorporating the optimal second-stage decisions into Eq. (4.6) as follows:

$$E[\pi(e, l)] = \delta \left[ \int \int_{q^H \leq \hat{q}} R_1(q^H, q^L) dF(y) dG(p) + \int \int_{q^H > \hat{q}} R_2(q^H, q^L) dF(y) dG(p) \right] - nc(e).$$  \hspace{1cm} (4.15)

Note that the dependent of $R_1(\cdot)$ and $R_2(\cdot)$ on $p$ has been suppressed in the above notation and it should be understood that $q^H = h(e, l)y$ and $q^L = (1 - h(e, l))y$. The expected profit in Eq. (4.15) can be interpreted as a combination of the expected profits from two cases: switching to the low-end market or not. As shown below, the concavity of the objective function is assured without enforcing any limitations on the probability distribution of yield uncertainty and the low-end market price.

**Proposition 4.4.** The co-op’s first-stage objective function Eq. (4.15) is jointly concave in $e$ and $l$, and there exists a unique optimal centralised effort $e^*$ and quality standard $l^*$ satisfying

$$\delta \left[ \int \int_{q^H \leq \hat{q}} [(a - p)y - 2b(l)h(e, l)y^2] dF(y) dG(p) \right] \frac{\partial h(e, l)}{\partial e} - nc'(e) = 0, \hspace{1cm} (4.16)$$

$$\int \int_{q^H \leq \hat{q}} [(a - p)y - 2b(l)h(e, l)y^2] dF(y) dG(p) \right] \frac{\partial h(e, l)}{\partial l} - \int \int_{q^H \leq \hat{q}} h^2(e, l)y^2 dF(y) dG(p)$$

$$+ \int \int_{q^H > \hat{q}} \frac{(a - p)^2}{4b^2(l)} dF(y) dG(p) \right] \frac{\partial b(l)}{\partial l} = 0, \hspace{1cm} (4.17)$$

where $e^* > 0$, $l^* > 0$.

**Proof of Proposition 4.4.**

Let $\pi_1(e, l)$ and $\pi_2(e, l)$ denote the co-op’s objective given realisation $y$ and $p$ as follows:

$$\pi(e, l) = \begin{cases} 
\pi_1(e, l) = \delta [(a - b(l)h(e, l)y - p)h(e, l)y + py] - nc(e) & \text{if } q^H \leq \hat{q}, \\
\pi_2(e, l) = \delta \left[ \frac{(a - p)^2}{4b(l)} + py \right] - nc(e) & \text{if } q^H > \hat{q}.
\end{cases}$$

When $q^H = \hat{q} = \frac{a - p}{2b(l)}$, $\pi_1(e, l) = \pi_2(e, l)$, thus $\pi(e, l)$ is a continuous function.

$$\frac{\partial \pi(e, l)}{\partial e} = \begin{cases} 
\frac{\partial \pi_1(e, l)}{\partial e} = \delta [(a - p)y - 2b(l)h(e, l)y^2] \frac{\partial h(e, l)}{\partial e} - n \frac{\partial c(e)}{\partial e} & \text{if } q^H \leq \hat{q}, \\
\frac{\partial \pi_2(e, l)}{\partial e} = -n \frac{\partial c(e)}{\partial e} & \text{if } q^H > \hat{q}.
\end{cases}$$
\[
\partial \pi(e, l) \over \partial l = \begin{cases} 
\frac{\partial \pi_1(e, l)}{\partial l} = \delta \left[ (a - p)y - 2b(l)h(e, l)y^2 \right] \frac{\partial h(e, l)}{\partial l} - h^2(e, l)y^2 \frac{\partial b(l)}{\partial l} & \text{if } q^H \leq \hat{q}, \\
\frac{\partial \pi_2(e, l)}{\partial l} = -\delta \left[ \frac{(a - p)^2 \partial b(l)}{4b^2(l)} \right] & \text{if } q^H > \hat{q}.
\end{cases}
\]

When \( q^H = \hat{q} = \frac{a - p}{2b(l)} \), then \( (a - p)y - 2b(l)h(e, l)y^2) = 0 \), thus
\[
\frac{\partial \pi_1(e, l)}{\partial e} = \frac{\partial \pi_2(e, l)}{\partial e} = -n \frac{\partial c(e)}{\partial e},
\]
and
\[
\frac{\partial \pi_1(e, l)}{\partial l} = \frac{\partial \pi_2(e, l)}{\partial l} = -\delta \frac{(a - p)^2 \partial b(l)}{4b^2(l)} \frac{\partial l}{\partial l}.
\]

Therefore, \( \pi(e, l) \) is a continuously differentiable function in \( e \) and \( l \) (Gamboa 2000).

As is required in the proof of Proposition 4.1, the technical condition \([b(l)b''(l) - 2b^2(l)] > 0\) (in Assumption 4.2) is also needed to prove the jointly concavity of \( E[\pi(e, l)] \) as below:
\[
\frac{\partial^2 \pi(e, l)}{\partial e^2} = \begin{cases} 
\frac{\partial^2 \pi_1(e, l)}{\partial e^2} < 0 & \text{if } q^H \leq \hat{q}, \\
\frac{\partial^2 \pi_2(e, l)}{\partial e^2} = -n \frac{\partial c^2(e)}{\partial e^2} < 0 & \text{if } q^H > \hat{q},
\end{cases}
\]
and
\[
\frac{\partial^2 \pi(e, l)}{\partial l^2} = \begin{cases} 
\frac{\partial^2 \pi_1(e, l)}{\partial l^2} < 0 & \text{if } q^H \leq \hat{q}, \\
\frac{\partial^2 \pi_2(e, l)}{\partial l^2} = \delta \left[ 2(b'(l))^2 - b(l)b''(l) \right] \frac{(a - p)^2}{4b^3(l)} < 0 & \text{if } q^H > \hat{q}.
\end{cases}
\]

Also
\[
\frac{\partial^2 \pi(e, l)}{\partial e \partial l} = \begin{cases} 
\frac{\partial^2 \pi_1(e, l)}{\partial e \partial l} = \delta \left[ (a - P)Y - 2b(l)h(e, l)y^2 \right] \frac{\partial h(e, l)}{\partial e} \frac{\partial b(l)}{\partial l} - 2b(l)y^2 \frac{\partial h(e, l)}{\partial e} \frac{\partial h(e, l)}{\partial l} & \text{if } q^H \leq \hat{q}, \\
\frac{\partial^2 \pi_2(e, l)}{\partial e \partial l} = -n \frac{\partial c'(e)}{\partial l} = 0 & \text{if } q^H > \hat{q}.
\end{cases}
\]

Proposition 4.1 has shown that \( \pi_1(e, l) \) is jointly concave in \( e \) and \( l \). Therefore, \( \pi_2(e, l) \)
is jointly concave in \( e \) and \( l \) as well, because
\[
\frac{\partial^2 \pi_2(e, l)}{\partial e^2} \frac{\partial^2 \pi_2(e, l)}{\partial l^2} - \left( \frac{\partial^2 \pi_2(e, l)}{\partial e \partial l} \right)^2 = \frac{\partial^2 \pi_2(e, l)}{\partial e^2} \frac{\partial^2 \pi_2(e, l)}{\partial l^2} > 0.
\]

Therefore, \( \pi(e, l) \) is a continuously differentiable function that is jointly concave in \( e \) and \( l \) for any realisation \( y \) and \( p \). As a result, the expectation \( E[\pi(e, l)] \) is jointly concave in \( e \) and \( l \) (Kazaz and Webster 2011).
Since \( R_1(q^H, q^L) = R_2(q^H, q^L) \) when \( q^H = \hat{q} \), the first order conditions are:

\[
\begin{align*}
\frac{\partial E[\pi(e, l)]}{\partial e} &= \delta \left[ \int_{q^H} \frac{\partial R_1(q^H, q^L)}{\partial e} dF(y) dG(p) + \int_{q^H > \hat{q}} \frac{\partial R_2(q^H, q^L)}{\partial e} dF(y) dG(p) \right] \\
- nc'(e) &= 0, \\
\frac{\partial E[\pi(e, l)]}{\partial l} &= \delta \left[ \int_{q^H} \frac{\partial R_1(q^H, q^L)}{\partial l} dF(y) dG(p) + \int_{q^H > \hat{q}} \frac{\partial R_2(q^H, q^L)}{\partial l} dF(y) dG(p) \right] = 0.
\end{align*}
\]

Using the following results:

\[
\begin{align*}
\frac{\partial R_1(q^H, q^L)}{\partial e} &= [(a - p)y - 2b(l)h(e, l)y^2] \frac{\partial h(e, l)}{\partial e}, \\
\frac{\partial R_2(q^H, q^L)}{\partial e} &= 0, \\
\frac{\partial R_1(q^H, q^L)}{\partial l} &= [(a - p)y - 2b(l)h(e, l)y^2] \frac{\partial h(e, l)}{\partial l} - h^2(e, l)y^2 \frac{\partial b(l)}{\partial l}, \\
\frac{\partial R_2(q^H, q^L)}{\partial l} &= \frac{(a - p)^2 b(l)}{4b^2_l(l)} \frac{\partial b(l)}{\partial l}.
\end{align*}
\]

Eq.(4.16) and (4.17) are derived as a result.

The results are quite similar to Case 1 (i.e., Section 4.3.1) except that Eq.(4.16) further implies that if the high-end market is over-supplied, the marginal contribution of effort is zero.

Defining the optimal decisions in Case 1 as \((e^0, l^0)\) and in Case 2 as \((e^*, l^*)\), the model yields the following conclusion:

**Corollary 4.1.** The market profit with separate sales pools is no more than that with a combined sales pool, shown as \(E[\pi(e^0, l^0)] \leq E[\pi(e^*, l^*)]\).

**Proof of Corollary 4.1.**

When \(q^H(e^0, l^0) > \hat{q}(e^0, l^0)\), then \(\pi_1(e^0, l^0) \leq \pi_2(e^0, l^0)\), and the optimal solution is \(e^*\) and \(l^*\) in Case 2; therefore,

\[
E[\pi(e^0, l^0)] = \int_{q^H(e^0, l^0) \leq \hat{q}(e^0, l^0)} \pi_1(e^0, l^0) dF(y) dG(p) + \int_{q^H(e^0, l^0) > \hat{q}(e^0, l^0)} \pi_1(e^0, l^0) dF(y) dG(p) 
\leq \int_{q^H(e^0, l^0) \leq \hat{q}(e^0, l^0)} \pi_1(e^0, l^0) dF(y) dG(p) + \int_{q^H(e^0, l^0) > \hat{q}(e^0, l^0)} \pi_2(e^0, l^0) dF(y) dG(p) 
\leq \int_{q^H(e^*, l^*) \leq \hat{q}(e^*, l^*)} \pi_1(e^*, l^*) dF(y) dG(p) + \int_{q^H(e^*, l^*) > \hat{q}(e^*, l^*)} \pi_2(e^*, l^*) dF(y) dG(p) 
= E[\pi(e^*, l^*)],
\]

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that is, \( E[\pi(e^0, l^0)] \leq E[\pi(e^*, l^*)] \).

With a combined sales pool, the co-op can still implement the solution for the separate sales pools (in other words, the co-op acts as if there is no pooling). So the separate sales is a feasible but not necessarily optimal solution of the combined sales pool. This corollary shows the value of pooling, specifically, pooling attenuates the risks from a stochastic production of \( H \) and \( L \) products. Offsetting this benefit is perhaps a limitation that pooled revenue makes it more difficult for the co-op to pay farmers fairly for \( H \) and \( L \) products.

### 4.3.3 The Value of Quality Provisions

This section explores the value of quality provisions by comparing the models with and without a quality standard. To accomplish this, this model assumes \( \frac{\partial^2 h(e, l)}{\partial e \partial l} = 0 \) to eliminate the impact of a quality standard. Thus, the model without the quality standard corresponds to a specific case of the original model when assuming \( l = 0 \). It is helpful to mark a superscript of \( N \) to all notation in the case without the quality standard. Note that \( N \) does not indicate the number of farmers. Therefore, the objective function of the co-op can be written as follows:

\[
E[\pi^N(e, 0)] = \delta \int \int_{q^N \leq \hat{q}^N} [(a - b(0)h(e, 0)y - p)h(e, 0)y + py]dF(y)dG(p) \\
+ \delta \int \int_{q^N > \hat{q}^N} \left[ \frac{(a - p)^2}{4b(0)} + py \right]dF(y)dG(p) - nc(e). \tag{4.18}
\]

Following Proportion 4.4, it is easy to conclude that Eq.(4.18) is a concave function, and the optimal effort satisfies Eq. (4.16) as well. The following proposition shows the comparative results.

**Proposition 4.5.** By comparison, the model without a quality standard indicates that the market threshold is decreased, i.e., \( \hat{q}^N < \hat{q} \), the market profit is decreased, i.e., \( E[\pi(e^N, 0)] < E[\pi(e^*, l^*)] \), and the optimal effort is reduced as well, i.e., \( e^N < e^* \).

**Proof of Proposition 4.5.**

First, since \( b(l) \) is decreasing in \( l \) as assumed; therefore, \( b(0) > b(l^*) \) where \( l^* > 0 \). Then \( \frac{a - p}{2b(0)} < \frac{a - p}{2b(l^*)} \), thus \( \hat{q}^N < \hat{q} \).
Second, since $E[\pi(e^*, l^*)]$ is the maximum profit for all $e$ and $l$, it is obvious that $E[\pi(e^*, l^*)] > E[\pi(e^N, 0)]$.

Last, $E[\pi(e, l)]$ has been proven to be concave in $e$, thus, $E[\pi^N(e, 0)]$ is concave in $e$ as well. Then,

$$
\frac{\partial E[\pi(e^*, l^*)]}{\partial e} = \delta \left[ \int \int \left[ (a - p) y - 2b(l) h(e, l) y^2 \right] dF(y) dG(p) \right] \frac{\partial h(e^*, l^*)}{\partial e}
$$

$$
- n \frac{\partial c(e^*)}{\partial e} = 0,
$$

$$
\frac{\partial E[\pi^N(e^N, 0)]}{\partial e} = \delta \left[ \int \int \left[ (a - p) y - 2b(0) h(e^N, 0) y^2 \right] dF(y) dG(p) \right] \frac{\partial h(e^N, 0)}{\partial e}
$$

$$
- n \frac{\partial c(e^N)}{\partial e} = 0.
$$

Define

$$m(e, l) = \int \int_{y \leq \frac{(a-p)}{2b(l)h(e, l)}} \left[ (a - p) y - 2b(l) h(e, l) y^2 \right] dF(y) dG(p).$$

Since $b(l)$ and $h(e, l)$ are decreasing in $l$, then

$$
\left[ (a - p) y - 2b(0) h(e^N, 0) y^2 \right] < \left[ (a - p) y - 2b(l) h(e^N, l^*) y^2 \right]; \text{and}
$$

$$
\frac{(a-p)}{2b(0)h(e^N, 0)} < \frac{(a-p)}{2b(l^*)h(e^N, l^*)}.
$$

Therefore, $m(e^N, 0) < m(e^N, l^*)$.

Now, suppose $e^N \geq e^*$, since $h(e, l)$ is increasing in $e$, then

$$
\left[ (a - p) y - 2b(l^*) h(e^N, l^*) y^2 \right] \leq \left[ (a - p) y - 2b(l^*) h(e^*, l^*) y^2 \right], \text{and}
$$

$$
\frac{(a-p)}{2b(l^*)h(e^N, l^*)} \leq \frac{(a-p)}{2b(l^*)h(e^*, l^*)},
$$

therefore, $m(e^N, l^*) < m(e^*, l^*)$. Then, it shows

$$m(e^N, 0) < m(e^N, l^*) < m(e^*, l^*).$$

On the other hand, since $e^N \geq e^*$, the following inequality is satisfied because $c(e)$ is convex in $e$, $h(e, l)$ is concave in $e$ and $\frac{\partial^2 h(e, l)}{\partial e \partial l} = 0$.

$$
\frac{\partial c(e^N)}{\partial e} \frac{\partial h(e^N, 0)}{\partial e} > \frac{\partial c(e^*)}{\partial e} \frac{\partial h(e^*, l^*)}{\partial e},
$$

which, however, contradicts the FOC conditions. Therefore, $e^N < e^*$.
This proposition demonstrates the contribution of quality provisions in the contract. Without the quality requirement, each farmer exerts a lower effort, and the total profit has decreased as well.

4.4 Decentralised Decisions

This section analyses the decentralised decisions by farmers and supply chain coordination under different payment schemes. It first examines a widely adopted payment mechanism in reality, called the “market-price-guarantee payment.” Further, it proposes another payment scheme called the “upfront incentive payment scheme.” In both these schemes, the budget balance condition must be satisfied.

According to Eq.(4.4), each farmer’s problem can be expressed as:

$$\max_{e} E[U_i(e)] = E [w^L_1 Q^L_i(e, l) + w^H_1 Q^H_i(e, l)] + \alpha E [w^L_2 Q^L_i(e, l) + w^H_2 Q^H_i(e, l)] - c(e), \quad (4.19)$$

where $l, w^L_1, w^H_1, w^L_2, w^H_2$ are determined by the co-op.

4.4.1 Market-Price-Guarantee Payment Scheme

Under the market-price-guarantee payment scheme, the prices offered to farmers are basically in accordance with the market prices. In order to adequately reward high-quality producers, the co-op ensures that the price offered to the $H$ product is equal to the realised high-end market price, which is therefore described as:

$$\frac{1}{\delta} w^H_1 + w^H_2 = p^H(q^H, l). \quad (4.20)$$

Separate Sales Pools.

This section first analyses the behaviour of each individual when the sales pools for $H$ and $L$ products are separate. According to Eq.(4.5) and (4.20), the unit prices of $H$ and $L$ products are given in the following lemma:
Lemma 4.2. With separate sales pools, the total unit prices of $H$ and $L$ products are just equal to their market unit prices as:

$$\begin{align*}
\frac{1}{\delta}w_L^1 + w_L^2 &= p, \\
\frac{1}{\delta}w_H^1 + w_H^2 &= p^H(q^H, l) = a - b(l)q^H,
\end{align*}$$

(4.21)

and the first and second-stage incomes are:

$$\begin{align*}
I_1^i &= w_L^1 q_L^i + w_H^1 q_H^i, \\
I_2^i &= (p - \frac{1}{\delta}w_L^1)q_L^i + \left[p^H(q^H, l) - \frac{1}{\delta}w_H^1\right]q_H^i.
\end{align*}$$

(4.22)

Proof of Lemma 4.2.

According to the budget balance condition in Eq.(4.5), one can drive that

$$\frac{1}{\delta}(w_L^1 q^L + w_H^1 q^H) + (w_L^2 q^L + w_H^2 q^H) = \left(\frac{1}{\delta}w_L^1 + w_L^2\right)q^L + \left(\frac{1}{\delta}w_H^1 + w_H^2\right)q^H = pq^L + p^H q^H.$$

According to the condition that $\left(\frac{1}{\delta}w_H^1 + w_H^2\right) = p^H$, then $\left(\frac{1}{\delta}w_L^1 + w_L^2\right) = p$. Therefore, the unit prices of $H$ and $L$ products just equal their market unit prices.

Supply chain coordination requires that each individual farmer exerts an effort expected by the co-op. Therefore, this model analyses the behaviour of farmer $i$ who maximises his/her own utility function with the individual’s conjecture about the behaviour of the other $n - 1$ individuals. The assumption here is that the individual takes others’ behaviour as given. Let $Q^H_{-i}$ denote the individual’s conjecture about the high-quality supply of others, where $Q^H_{-i} = \sum_{j \neq i} h(e_j, l)Y_j$. The utility of the farmer $i$ is

$$E[U_i(e)] = \left(1 - \frac{\alpha}{\delta}\right)E[w^i Y_i + \beta Q^H_i(e, l)] + \alpha E\left[a - P - b(l)(Q^H_i(e, l) + Q^H_{-i})\right]Q^H_i(e, l) + Y_iP] - c(e).$$

(4.23)

Proposition 4.6. Under the condition of Lemma 4.1, the farmer’s utility function Eq.(4.23) is concave in $e$, and the optimal effort of individual $i$, $e_i^*$ satisfies

$$\left[(1 - \frac{\alpha}{\delta})\beta E(Y_i) + \alpha E[(a - P - 2b(l)Q^H_i - b(l)Q^H_{-i})Y_i]\right] \frac{\partial h(e, l)}{\partial e} - c'(e) = 0,$$

(4.24)

where $e_i^* > 0$. 

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Proof of Proposition 4.6.

From Eq.(4.23), the first and second-derivatives are:

\[
\begin{align*}
\frac{\partial E[U_i(e)]}{\partial e} &= E \left[ \left(1 - \frac{\alpha}{\delta}\right) \beta Y_i + \alpha [(a - P) Y_i - (Q^H_i(e, l) + Q^H) b(l) Y_i] \right] \frac{\partial h(e, l)}{\partial e} - \frac{\partial c(e)}{\partial e}, \\
\frac{\partial^2 E[U_i(e)]}{\partial e^2} &= E \left[ \left(1 - \frac{\alpha}{\delta}\right) \beta Y_i + \alpha [(a - P) Y_i - (Q^H_i(e, l) + Q^H) b(l) Y_i] \right] \frac{\partial^2 h(e, l)}{\partial e^2} \\
&\quad - 2b(l) Y_i^2 \left( \frac{\partial h(e, l)}{\partial e} \right)^2 - \frac{\partial^2 c(e)}{\partial e^2}.
\end{align*}
\]

With the condition of \( E[(a - P)Y - 2b(0)Y^2] > 0 \) (see Lemma 4.1), the following inequality is satisfied for all \( l > 0 \) and \( h(e, l) < 1 \):

\[
E[(a - P)Y_i - b(l) (Q^H_i(e, l) + Q^H) Y_i] > E[(a - P)Y_i - b(0)(Y_i + Y) Y_i] > \frac{1}{n} E[(a - P)Y - 2b(0)Y^2] > 0.
\]

Therefore, \( E[U_i(e)] \) is concave in \( e \). According to the FOC, the optimal individual effort satisfies \( \frac{\partial E[U_i(e)]}{\partial e} = 0 \).

The optimal individual effort lies at the point where the marginal reward of individual effort equals the marginal individual effort cost. The left-hand section of Eq.(4.24) indicates that the effort motivations are two-fold: one is the quality premium provided in the first stage, and the other is the individual’s market return when others’ effort is given. The effort by individual \( i \) will vary with the value of \( \beta \) as well as the individual’s conjecture \( Q^H_i \) about the behaviour of the other \( n - 1 \) individuals. In addition, Eq.(4.24) implies that a higher quality premium upfront is more encouraging; however, it is decreased by the realisation of \( Q^H_i \). This can be justified because the high-end market price is decreasing in the total supply of \( H \) product. In other words, the more \( H \) product is supplied by others, the lower the price each can receive from the high-end market.

**Theorem 4.1.** There is a threshold value of \( \hat{\alpha} \),

(1) when \( 0 < \alpha \leq \hat{\alpha} \), the supply chain is coordinated with a non-negative quality premium \( \beta^* \) which equals

\[
\beta^* = \frac{1}{(1 - \frac{\alpha}{\delta})\mu_y} \left\{ \delta E \left[ (a - P - 2b(l^*)Q^H) \frac{Y}{n} \right] - \alpha E \left[ (a - P - b(l^*) (Q^H_i + Q^H) Y_i) \right] \right\}, \quad (4.25)
\]
and is decreasing in $\alpha$.

(2) when $\hat{\alpha} < \alpha \leq \delta$, the supply chain cannot be coordinated, and a high effort strategy dominates the strategy of $e^*$, i.e., $e^*_i > e^*$.

**Proof of Theorem 4.1.**

(1) Compare Eq.(4.10) with (4.24), the supply chain is coordinated when $e^*_i = e^*_j = e^*$ and

$$
\frac{\partial E[\pi(e,l)]}{\partial e} = \delta E[(a - P - 2b(l)h(e^*, l^*)Y)\frac{\partial h(e^*, l^*)}{\partial e} - n\frac{\partial c(e^*)}{\partial e} = 0,
$$

$$
\frac{\partial E[U_i(e)]}{\partial e} = E[(1 - \frac{\alpha}{\delta})\beta Y_i] + \alpha E[(a - P - b(l)(h(e^*, l^*)Y_i + h(e^*, l^*)Y)Y_i)]\frac{\partial h(e^*, l^*)}{\partial e} - \frac{\partial c(e^*)}{\partial e} = 0.
$$

Define

$$
\begin{align*}
E1 &= E[(a - P - 2b(l)h(e^*, l^*)Y)\frac{Y_i}{n}], \\
E2 &= E[(a - P - b(l)(h(e^*, l^*)Y_i + h(e^*, l^*)Y)Y_i)].
\end{align*}
$$

When $n \geq 2$,

$$E2 > E1.$$

(a) If $\alpha E2 > \delta E1$, then the supply chain cannot be coordinated because the coordination condition results in $\beta < 0$ which contradicts the condition that $\beta \geq 0$.

(b) If $\alpha E2 \leq \delta E1$, then the supply chain can be coordinated with a non-negative $\beta$ which equals

$$
\beta^* = \frac{1}{(1 - \frac{\alpha}{\delta})\mu_y} \left[ \delta E[(a - P - 2b(l^*)Q^H)\frac{Y_i}{n}] - \alpha E[(a - P - b(l^*)Q^H)Q^H Y_i] \right].
$$

Together, they imply that a threshold value for $\alpha$ exists.

It is easy to deduce that $\beta^*$ is decreasing in $\alpha$ as follows:

$$
\frac{\partial \beta^*}{\partial \alpha} = - \frac{\mu_y}{((1 - \frac{\alpha}{\delta})\mu_y)^2\delta} \left[ \delta E[(a - P - 2b(l^*)Q^H)\frac{Y_i}{n}] - \alpha E[(a - P - b(l^*)Q^H)Q^H Y_i] \right] - \frac{1}{(1 - \frac{\alpha}{\delta})\mu_y} E[(a - P - b(l^*)Q^H)Q^H Y_i] < 0
$$

(2) Suppose farmer $i$ exerts a lower effort than required by the co-op, that is $e^*_i < e^*$. Compare Eq.(4.10) with (4.24), then, $$(1 - \frac{\alpha}{\delta})\beta E(Y_i) + \alpha E[(a - P - 2b(l^*)h(e^*_i, l^*)Y_i - b(l)Q^H e^*_i, l^*)Y_i)] \leq \delta E[(a - P - 2b(l^*)Q^H e^*, l^*)\frac{Y_i}{n}].$$
Let 
\[ E_3 = E \left[ (a - P - 2b(l^*)h(e_i^*, l^*)Y_i - b(l)Q^H_{-1}(e^*, l^*))Y_i \right]. \]
Since \( e_i \leq e^* \), and \( h(e, l) \) is increasing in \( e \), therefore,
\[ E_3 \geq E_2. \]

When the supply chain is not coordinated, which implies the condition of \( \alpha E_2 > \delta E_1 \), then the following inequality is satisfied:
\[ \alpha E_3 \geq \alpha E_2 > \delta E_1. \]
However, since \( \beta \geq 0 \), this condition contradicts the inequality: \( (1 - \frac{\alpha}{\delta})\beta E(Y_i) + \alpha E_3 \leq \delta E_1 \), therefore, \( e_i^* > e^* \).

Figure 4.2: Equilibrium Effort under the Market-Price-Guarantee Payment Scheme with Different Values of \( \alpha \)

Figure 4.2 illustrates the equilibrium of optimal effort with different values of \( \alpha \). The red line depicts the collective optimal effort, and the blue lines correspond to each farmer’s effort. The higher line represents a higher \( \alpha \). The solid one indicates the equilibrium under the market-price-guarantee payment scheme. It shows that when \( \alpha > \hat{\alpha} \), farmers always exert a higher individual optimal effort than the collective effort, i.e., \( e_i^* > e^* \). In other words,
if the individual assumes that everyone else will work $e^*$, they will maximise their own utility by defecting from the cooperative allocation to exert a higher effort. The problem of over-motivation actually reflects the free-ride problem on quantity in traditional co-ops where each farmer decides individually how much to deliver to the co-op (Helmberger and Hoos 1962). The intuition behind this behaviour is that although an individual farmer may realise that an increase in $H$ production will decrease the price in the high-end market, they only internalise their own part of the profit-loss stemming from the decreased price. In other words, the loss to the whole supply chain caused by his own over-production is shared by all farmers.

The problem of over-motivation is caused by the payment scheme where the $H$ product is adequately rewarded according to the market price. One can imagine that this opportunistic behaviour becomes more significant when the co-op has a larger membership base. In addition, while farmer $i$’s utility at this point exceeds that at the collective equilibrium, $e^*_i$ is not a sustainable position if the other $n - 1$ individuals duplicate $i$’s action. For example, suppose that in a game with complete information, all other individuals would duplicate this action to produce more high-quality products. Then, the motivation for each individual shall be compromised. Finally, the equilibrium (Nash Equilibrium) may be close to, but slightly higher than the collective effort.

However, when farmers have a relatively high time preference, i.e., $0 < \alpha \leq \hat{\alpha}$, the supply chain can be coordinated with a positive quality premium in the first stage. It can be explained that when farmers are time-sensitive, their lower expectation of the second-stage payment will suppress their motivation of supplying more $H$ products than expected, hence alleviating the problem of over-motivation. In this case, the quality premium offered in the first stage is able to compensate for this discouragement and enable quality coordination. In addition, since $\beta^*$ is decreasing in $\alpha$, it implies that a larger quality premium corresponds to a higher degree of time preference (a smaller value of $\alpha$). This conclusion is intuitive because when farmers have a lower expectation of the second-stage payment, they may have a higher expectation of a quality premium in the first stage.
According to the analysis of Case 1 with separate sales pools, it can be reasonably expected that most of the results extend to the formulation of Case 2 where the co-op has a combined sales pool for $H$ and $L$ products. Since the co-op can deliver excessive $H$ products to the low-end market to maintain a premium price of $H$ products, the problem of over-motivation shall be further exacerbated. For simplicity, this chapter does not explicitly analyse Case 2.

In conclusion, the market-price-guarantee payment scheme can coordinate the supply chain only when farmers have a high time preference; otherwise, they are over-motivated. Since people generally prefer to receive a reward sooner rather than later (Arrondel et al. 2007), this payment scheme still has a potential, especially in the field of agriculture where farmers are commonly risk averse (Binswanger 1980). Furthermore, this payment scheme is expected to be robust to heterogeneous farmers with different time preferences, i.e., $\alpha_i$ for farmer $i$, if $\alpha_i < \hat{\alpha}$, for all $i = 1, 2...n$. Note that, the threshold value $\hat{\alpha}$ is the same to all farmers under the i.i.d assumption. In this setting, the optimal quality premium is also determined by Eq.(4.25), except that the individual quality premium $\beta_i^*$ is dependent on the value of $\alpha_i$, hence varies among farmers. Specifically, a larger $\beta_i^*$ will correspond to a lower $\alpha_i$. The implicit assumption is that the information of farmers’ time preferences are known to the co-op; the consideration of asymmetric information is interesting, but this is left as a subject for future research.

The limitation of the market-price-guarantee payment scheme lies in the fact that it cannot achieve supply chain coordination when farmers have a low time preference. A desirable co-op contract must be incentive compatible for the individual farmers unconditionally. Therefore, in the following section, an improved payment scheme is proposed, which can enable quality coordination regardless of farmers’ time preferences.

4.4.2 Upfront Incentive Payment Scheme

Consider the following scheme: in the first stage, the co-op offers differentiated prices $w_i^L$ and $w_i^H$ to the $L$ and the $H$ product, respectively; in the second stage, the co-op equally distributes the net surplus to all farmers. This payment scheme is different from complete
income sharing where collective income is simply shared equally (Carter 1987). Instead, it is a combination of income sharing and an individual incentive mechanism. This payment scheme is expressed as:

\[
\begin{align*}
I^i_1 &= w^H_1 q^L_i + w^H_1 q^H_i = (w^H_1 - w^L_1) q^H_i + w^L_1 y_i, \\
I^i_2 &= \frac{1}{n} \left[ R(q^H, q^L) - \frac{1}{\delta} \sum_{i=1}^n I^i_1 \right].
\end{align*}
\] (4.26)

The first-stage quality premium \( \beta = w^H_1 - w^L_1 \) is an individual incentive parameter, and the second-stage payment is complete income sharing tied to the aggregate production of the collective. The total payment to farmers satisfies the budget balance condition defined in Eq. (4.5).

Under this scheme, the individual optimisation problem of each farmer is:

\[
E[U_i(e)] = E[I^i_1] + \frac{\alpha}{n} E \left[ R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^n I^i_1 \right] - c(e). \] (4.27)

**Separate Sales Pools**

With separate sales pools, the market revenue in the second stage is \( R(q^H, q^L) = R_1(q^H, q^L) \) defined in Eq. (4.14), and the objective function of farmer \( i \) is

\[
E[U_i(e)] = E[w^L_1 Y_i + \beta Q^H_i(e, l)] + \frac{\alpha}{n} E \left[ R_1(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^n (w^L_1 Y_i + \beta Q^H_i(e, l)) \right] - c(e). \] (4.28)

**Theorem 4.2.** Under the condition of Lemma 4.1:

(a) The farmers’ objective function defined in Eq. (4.28) is strictly concave in \( e \), and the unique optimal individual effort \( e^*_i \) satisfies:

\[
\left\{ (1 - \frac{\alpha}{n \delta}) E[Y_i] \beta + \frac{\alpha}{n} E[(a - P - 2b(l)Q^H)Y_i] \right\} \frac{\partial h(e, l)}{\partial e} - c'(e) = 0, \] (4.29)

where \( e^*_i > 0 \).

(b) The following value of the quality premium \( \beta^* \) results in each farmer choosing the collectively optimal effort.

\[
\beta^* = \frac{\delta}{\mu y n^*} E[(a - P - 2b(l^*)Q^H)Y^*], \] (4.30)

where \( \beta^* > 0 \).
Proof of Theorem 4.2.

According to Eq.(4.28), the first and second-derivatives are

\[
\begin{align*}
\frac{\partial E[U_i(e)]}{\partial e} &= \left(1 - \frac{\alpha}{n\delta}\right) E[Y_i] \beta + \frac{1}{n} \alpha E[(a - P - 2b(l)Q^H)Y_i] \frac{\partial h(e, l)}{\partial e} - c'(e), \\
\frac{\partial^2 E[U_i(e)]}{\partial e^2} &= \left(1 - \frac{\alpha}{n\delta}\right) E[Y_i] \beta + \frac{1}{n} \alpha E[(a - P - 2b(l)Q^H)Y_i] \frac{\partial^2 h(e, l)}{\partial e^2} \\
&\quad - 2b(l)E[Y_iY] \left(\frac{\partial h(e, l)}{\partial e}\right)^2 - c''(e).
\end{align*}
\]

Since \( E[(a - P)Y - 2b(0)Y^2] > 0 \), indicated in Lemma 4.1, and \( E[Y_iY] = \frac{1}{n} E[Y^2] = (n\mu^2 + \sigma^2_y) \), therefore,

\[
E[(a - P - 2b(l)Q^H)Y_i] > E[((a - P) - 2b(0)Y)Y_i] = \frac{1}{n} E[((a - P) - 2b(0)Y)Y] > 0.
\]

The function of \( E(U_i(e)) \) is concave in \( e \). Then, according to the FOC, the optimal effort satisfies \( \frac{\partial E(U_i(e))}{\partial e} = 0 \).

By comparing farmers’ optimal effort in Eq.(4.29) with the co-op’s optimal effort in Eq.(4.10), the following equation is satisfied:

\[
\left(1 - \frac{\alpha}{n\delta}\right) \beta E[Y_i] + \frac{1}{n} \alpha E[(a - P - 2b(l)Q^H)Y_i] = \frac{1}{n} \delta E[(a - P - 2b(l)Q^H)Y],
\]

Since \( E[Y_i] = \mu_y, E[Y_iY] = \frac{1}{n} E[Y^2] \), then, \( E[(a - P - 2b(l)Q^H)Y_i] = \frac{1}{n} E[(a - P - 2b(l)Q^H)Y] \),

and the optimal quality premium is derived as:

\[
\beta^* = \frac{\delta}{\mu_y n} E[(a - P - 2b(l^*)Q^H)Y] > 0.
\]

Figure 4.3 illustrates the equilibrium under the upfront incentive payment scheme. It is clear that without the individual incentive parameter \( \beta \), the farmer is under-motivated because all farmers are paid the same price regardless of quality. This further demonstrates the ineffectiveness of complete income sharing and its associated under-motivation problem. The quality premium offered in the first stage can overcome this disadvantage and motivate each farmer to exert the collective effort required by the co-op. Furthermore, Eq.(4.30) implies that the quality premium offered in the first stage is the average marginal contribution of collective effort.
Next, this section analyses the decisions in Case 2. The utility function of farmer $i$ is:

$$E\left[U_i(e)\right] = E[I_i^1] + \frac{1}{n} \left\{ \int \int \int_{q_H \leq \hat{q}} R_1(q^H, q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I_i^1 \right\} dF_i(y_i) dF_{-i}(y_{-i}) dG(p) + \int \int \int_{q_H > \hat{q}} R_2(q^H, q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I_i^1 \right\} dF_i(y_i) dF_{-i}(y_{-i}) dG(p) - c(e),$$  \hspace{1cm} (4.31)

where $R_1(q^H, q^L)$ and $R_2(q^H, q^L)$ are expressed in Eq(4.14). In the following analysis, the indicator function $1_{Q_H \leq \hat{q}}$ is used to represent the probability of event $q^H \leq \hat{q}$.

**Theorem 4.3.** Under the condition of Lemma 4.1:

(a) The farmers’ objective function defined in Eq.(4.31) is concave in $e$, and the optimal individual effort $e_i^*$ satisfies:

$$\left\{ \left(1 - \frac{\alpha}{n\delta} \right) \mu_y \beta + \frac{1}{n} \int \int \left[ (a - p - 2b(l)q^H) y_i \right] dF_i(y_i) dF_{-i}(y_{-i}) dG(p) \right\} \frac{\partial h(e, l)}{\partial e} - c'(e) = 0.$$  \hspace{1cm} (4.32)

(b) The following value of quality premium $\beta$ results in each farmer choosing the collectively optimal effort,

$$\beta^* = \frac{\delta}{n\mu_y} E \left[ (a - P - 2b(l^*)Q^H) Y \right] \left[ 1_{Q_H \leq \hat{q}} \right],$$  \hspace{1cm} (4.33)

and $\beta^* > 0$.  

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Proof of Theorem 4.3.

Let \( U_i^1(e) \) and \( U_i^2(e) \) denote farmer \( i \)'s objective given realisation of \( y_i \) and \( y_{-i} \) and \( p \) as follows:

\[
\begin{align*}
    U_i^1(e) &= \left[ w^T_i y_i + \beta h(e, l) y_i \right] - \frac{\alpha}{n\delta} \left[ \sum_{i=1}^n \left( w^T_i y_i + \beta q_i^H(e, l) \right) \right] + \frac{\alpha}{n} R_1(q^H, q^L) - c(e), \quad \text{if } q^H \leq \hat{q}, \\
    U_i^2(e) &= \left[ w^T_i y_i + \beta h(e, l) y_i \right] - \frac{\alpha}{n\delta} \left[ \sum_{i=1}^n \left( w^T_i y_i + \beta q_i^H(e, l) \right) \right] + \frac{\alpha}{n} R_2(q^H, q^L) - c(e), \quad \text{if } q^H > \hat{q}.
\end{align*}
\]

When \( q^H = \hat{q} = \frac{a-p}{2b(l)} \), \( R_1(q^H, q^L) = R_2(q^H, q^L) \), then \( U_i^1(e) = U_i^2(e) \), thus \( U_i(e) \) is a continuous function.

In addition, when \( q^H = \hat{q} = \frac{a-p}{2b(l)} \),

\[
\begin{align*}
    \frac{\partial U_i^1(e)}{\partial e} &= \left( 1 - \frac{\alpha}{n\delta} \right) \beta y_i \frac{\partial h(e, l)}{\partial e_i} + \frac{\alpha}{n} \left[ (a - b(l)q^H - p) \frac{\partial q_i^H}{\partial e_i} - b(l)q^H \frac{\partial q_i^H}{\partial e_i} \right] - c'(e), \quad \text{if } q^H \leq \hat{q}, \\
    \frac{\partial U_i^2(e)}{\partial e} &= \left( 1 - \frac{\alpha}{n\delta} \right) \beta y_i \frac{\partial h(e, l)}{\partial e_i} - c'(e), \quad \text{if } q^H > \hat{q}.
\end{align*}
\]

Since \( \frac{\partial U_i^1(e)}{\partial e} = \frac{\partial U_i^2(e)}{\partial e} \), then, \( U_i(e) \) is a continuously differentiable function in \( e \).

Since when \( q^H \leq \hat{q} \), \( (a - 2b(l)q^H - p) \geq 0 \), \( \frac{\partial^2 q^H}{\partial e_i^2} = \frac{\partial^2 h(e, l)}{\partial e_i^2} y_i < 0 \) and \( c''(e) > 0 \), then

\[
\begin{align*}
    \frac{\partial^2 U_i^1(e)}{\partial e^2} &= \left( 1 - \frac{\alpha}{n\delta} \right) \beta \frac{\partial^2 q_i^H}{\partial e_i^2} + \frac{\alpha}{n} \left[ (a - 2b(l)q^H - p) \frac{\partial^2 q_i^H}{\partial e_i^2} - 2b(l) \left( \frac{\partial q_i^H}{\partial e_i} \right)^2 \right] - c''(e) < 0, \quad \text{if } q^H \leq \hat{q}, \\
    \frac{\partial^2 U_i^2(e)}{\partial e^2} &= \left( 1 - \frac{\alpha}{n\delta} \right) \beta \frac{\partial^2 q_i^H}{\partial e_i^2} - c''(e) < 0, \quad \text{if } q^H > \hat{q}.
\end{align*}
\]

Therefore, \( U_i(e) \) is a continuously differentiable function that is concave in \( e \) for any realisation of \( y_i, y, \) and \( p \). As a result, \( E[U_i(e)] \) is concave in \( e \).

According to the FOC condition, the optimal individual effort is:

\[
\frac{\partial E[U_i(e)]}{\partial e} = \left\{ (1 - \frac{\alpha}{n\delta}) \mu_y \beta + \frac{\alpha}{n} E \left[ (a - P - 2b(l)Q^H)Y_i \right] \right\} \frac{\partial h(e, l)}{\partial e} - c'(e) = 0.
\]

By the symmetry assumption, the solution is symmetric for all farmers, then it can be written as:

\[
\frac{\partial E[U_i(e)]}{\partial e} = \left\{ (1 - \frac{\alpha}{n\delta}) \mu_y \beta + \frac{1}{n} E \left[ (a - P - 2b(l)Q^H)Y \right] \right\} \frac{\partial h(e, l)}{\partial e} - c'(e) = 0.
\]
Comparing the above equation and Eq.(4.16) where

$$\frac{\partial E[\pi(e,l)]}{\partial e} = \left\{ \frac{\delta}{n} E \left[ \left( (a - P - 2b(l)Q^H)Y \right)[1_{Q_H \leq \hat{q}}] \right] \right\} \frac{\partial h(e,l)}{\partial e} - c'(e) = 0.$$ 

The optimal quality premium can be derived:

$$\beta^* = \frac{\delta}{n\mu_y} E \left[ \left( (a - P - 2b(l^*)Q^H)Y \right)[1_{Q_H \leq \hat{q}}] \right] > 0.$$ 

The upfront incentive payment scheme can better coordinate the supply chain because it is not conditional on the degree of farmers’ time preferences. Since the quality premium is independent of farmers’ time preferences ($\alpha$), it implies that the upfront incentive payment scheme is robust to heterogeneous farmers with different time preferences.

Furthermore, due to symmetrical solutions, the quality premium offered in the first stage is the average marginal contribution of collective effort. It can be reasonably expected that this payment scheme is also robust to heterogeneous farmers differing in their farm size. For example, in a two-farmer example that one farmer is bigger than the other, the optimal scheme remains the same form. In this setting, the upfront payment scheme can be revised slightly: in the first stage, the co-op offers differentiated prices to the $L$ and the $H$ product respectively; in the second stage, the co-op distributes the net surplus to farmers in proportion to their farm size. For example, one can consider such an extension by assuming a stochastically proportional (SP) yield model: a farmer’s production size $Q_i$ is a percentage of the aggregate farm size $Q$, i.e., $Q_i = \theta_i Q$ where $\sum_i \theta_i = 1$, and the yield of each farmer $Y_i(Q_i)$ is a random variable on farm size $Q_i$, i.e., $Y_i(Q_i) = z * Q_i$, wherein the yield rate $z$ is a random variable, and independent of farm size. As introduced in Section 3.3, the SP yield model is widely assumed in the research on yield uncertainty. In addition, assume each farmer’s production cost is in proportion to their farm size as well, i.e., $Q_i c(e_i)$. It can be expected that the upfront incentive payment scheme is still able to coordinate the supply chain by offering a quality premium in the first stage.

Last but not least, the upfront incentive payment scheme is also easier to implement in practice because the individual’s incentive parameter is offered in the first stage. The co-op can distribute the net surplus in the second stage among the farmers regardless of their quality level.
However, a potential drawback is the implied financial burden that the co-op may suffer. Since farmers are fully motivated by the first-stage payment, the differentiated market prices realised in the second stage may be reflected in the earlier stage. If products are highly differentiated, it means the co-op has to pay out a large amount of capital before the realisation of market revenue. Financial constraints are not discussed in this chapter; however, they need to taken into consideration when applying the schemes in practice.

4.5 Discussion

This section discusses two relevant issues for the problem of quality coordination. The first is the decision on the initial payment, and the second is the role of multi-period payment schemes.

4.5.1 Discussion on the Initial Payment

With the optimal decisions $e$, $l$, and $\beta$ derived in the above sections, it seems one can get the utility of farmers under different payment schemes. However, the astute reader may note that the price of the $L$ products in the first stage has not yet been determined. The quality premium only captures the difference for the $H$ and $L$ products. Although the decision of $w_1^L$ does not influence the incentives on effort, it affects a farmer’s participation (or loyalty) in the co-op. There will be a range of $w_1^L$ that satisfy the budget balance condition but lead to different farmer utilities when he has time preferences. This section analyses how farmers’ time preferences influence the allocation of payments in two periods.

Define $\underline{w}_1^L$ and $\overline{w}_1^L$ as the minimum and maximum value of $w_1^L$. To analyse the influence of $\alpha$ on the decision of $w_1^L$, two constraints are considered, which determine $\underline{w}_1^L$ and $\overline{w}_1^L$, respectively. The first one is the participation constraint. Besides the assumption on homogeneous farmers, this section further assumes that the minimum utility is only determined by the farmer’s effort, which shall be the same for all when the supply chain is coordinated. Therefore, the participation constraint is defined as

\[ E[U_i(e)] \geq \overline{U} \geq 0. \]  

(4.34)
Besides that, this section constrains the second-stage payments (in expected value) to be non-negative. This constraint prohibits the possibility that the co-op may overpay farmers in the first stage, i.e.,
\[ E[I^*_2] \geq 0. \] (4.35)

The last section has shown that the optimal quality premium under the upfront incentive payment scheme is independent of farmers’ time preferences \( \alpha \). Therefore, the following corollary presents the influence of farmers’ time preferences on farmers’ utilities.

**Corollary 4.2.** Under the upfront incentive payment scheme, farmers’ utilities are increasing in \( \alpha \).

**Proof of Corollary 4.2.**

A farmer’s utility function is \( E[U_i(e)] = E[I^*_1] + \alpha E[I^*_2] - c(e) \), and \( E[I^*_1] = E[\beta q^H_i + w^L_i y_i] \), \( E[I^*_2] = \frac{1}{n} E[R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I^*_i] \). Since
\[
\frac{\partial E[I^*_1]}{\partial \alpha} = \frac{\partial \beta}{\partial \alpha} = 0 = \frac{\partial E[I^*_2]}{\partial \alpha},
\]
then,
\[
\frac{\partial E[U_i(e)]}{\partial \alpha} = \frac{\partial E[I^*_1]}{\partial \alpha} + E[I^*_2] + \alpha \frac{\partial E[I^*_2]}{\partial \alpha} = E[I^*_2] > 0.
\]
Therefore, farmers’ utilities are increasing in \( \alpha \).

**Proposition 4.7.** Under constraint Eq. (4.34) and (4.35), the lower bound \( w^L_i \) is decreasing in \( \alpha \) and the upper bound \( \pi^L_i \) is independent of \( \alpha \).

**Proof of Proposition 4.7.**

(1) The lower bound \( w^L_i \) is determined by the constraint \( E[U_i(e)] \geq U \) as follows:
\[
E[\beta q^H_i + w^L_i y_i] + \frac{\alpha}{n} E\left[R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I^*_i \right] - c(e) \geq U.
\]
Then,
\[
E[w^L_i y_i] \geq U - \left[ E[\beta q^H_i] + \frac{\alpha}{n} E[R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I^*_i] - c(e) \right],
\]
and the lower bound \( w^L_i \) is identified as:
\[
w^L_i E[y_i] = U - \left[ E[\beta q^H_i] + \frac{\alpha}{n} E[R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I^*_i] - c(e) \right].
\]
Since the right-side $[E[\beta q^H] + \frac{\alpha}{n}E[R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I_i] - c(e)]$ is increasing in $\alpha$ (see the proof of Corollary 4.2), then $w^L$ is decreasing in $\alpha$.

(2) The upper bound $\overline{w}^L$ is determined by the condition $E[I_2] \geq 0$ as follows:

$$E[I_2] = \frac{1}{n} E \left[ R(Q^H, Q^L) - \frac{1}{\delta} \sum_{i=1}^{n} I_i \right] \geq 0,$$

then,

$$E \left[ \frac{1}{\delta} \sum_{i=1}^{n} I_i \right] \leq E[R(Q^H, Q^L)],$$

that is:

$$\frac{1}{\delta} E \left[ \sum_{i=1}^{n} [\beta q^H_i + w^L \gamma_i] \right] \leq E[R(Q^H, Q^L)].$$

Therefore, the upper bound $\overline{w}^L$ is identified as:

$$\frac{1}{\delta} E \left[ \sum_{i=1}^{n} \overline{w}^L \gamma_i \right] = E[R(Q^H, Q^L)] - \frac{1}{\delta} E \left[ \sum_{i=1}^{n} \beta q^H_i \right].$$

Since the right-hand $E[R(Q^H, Q^L)] - \frac{1}{\delta} E \left[ \sum_{i=1}^{n} \beta q^H_i \right]$ is independent of $\alpha$ (recall that $\beta$ is independent of $\alpha$), therefore, $\overline{w}^L$ is independent of $\alpha$ as well.

This proposition implies that the initial payment is influenced by farmers’ time preferences. If farmers have a higher time preference with a smaller $\alpha$, the co-op is more likely to offer a higher initial price $w^L$, hence a larger portion of payment in the first stage; otherwise, the co-op can distribute less upfront. This proposition sheds light on the solution to the financial problem of the co-op. The co-op typically prefers to retain/delay the payments to the later stages because it requires a large amount of capital for its business, like for value-adding activities. In practice, the retained patronage refund (the profit allocated in proportion to patronage) is an important source of capital for co-ops. However, individually, farmers may require capital to cover their production or support the business coming in the next season. Therefore, there is a trade-off between the members’ individual interest and the co-op’s financial needs (Lund 2013). The financial decision is interesting, but not the focus in this chapter. However, it will be considered in the next chapter.
4.5.2 Discussion on the Role of Multi-Period Payment Schemes

This section examines the role of multi-period payment schemes on supply chain coordination by comparing it to the traditional payment practice where the co-op pays the current market prices when the commodity is marketed (Coltrain 2000). With the traditional payment scheme, the co-op does not pay in the first stage, that is \( I_1 = 0 \). Therefore, farmers’ utility function under the upfront incentive payment scheme is expressed as

\[
E[U_i(e)] = \alpha[w_2^iY_i + \beta Q_i^H] - c(e) \\
= \alpha E\left[R(Q^H, Q^L) - \sum_{j \neq i} [w_2^jY_j + \beta Q_j^H]\right] - c(e).
\]

**Corollary 4.3.** The results of supply chain coordination under the traditional payment scheme are summarised in Table 4.1.

<table>
<thead>
<tr>
<th>( \alpha &lt; \delta )</th>
<th>With Separate Sales Pools</th>
<th>With a Combined Sales Pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = \delta )</td>
<td>( e_i^* &lt; e^* )</td>
<td>( e_i^* &lt; e^* )</td>
</tr>
<tr>
<td></td>
<td>( e_i^* = e^* )</td>
<td>( e_i^* = e^* )</td>
</tr>
</tbody>
</table>

**Proof of Corollary 4.3.**

When \( I_1 = 0 \), then the budget balance condition in the second stage implies that

\[
\sum_{i=1}^n [w_2^iY_i + \beta q_i^H] = R(q^H, q^L).
\]

Then, the farmer \( i \)'s utility is:

\[
E[U_i(e)] = \alpha E[w_2^iY_i + \beta Q_i^H] - c(e) = \alpha E\left[R(Q^H, Q^L) - \sum_{j \neq i} [w_2^jY_j + \beta Q_j^H]\right] - c(e)
\]

The FOC condition is

\[
\frac{\partial E[U_i(e)]}{\partial e} = \alpha \frac{\partial E[R(Q^H, Q^L)]}{\partial e_i} - c'(e).
\]

In Case 1 where \( E[R(Q^H, Q^L)] = E[R_1(Q^H, Q^L)] = E[(a - b(l)Q^H - P)Q^H + PY] \), then

\[
\frac{\partial E[U_i(e)]}{\partial e} = \alpha E[(a - P - 2b(l)Q^H)Y_i] \frac{\partial h(e, l)}{\partial e} - c'(e) = 0
\]

Recall that the co-op’s optimal effort is

\[
\frac{\partial E[\pi(e, l)]}{\partial e} = \delta E[(a - P - 2b(l)Q^H)Y] \frac{\partial h(e, l)}{\partial e} - nc'(e) = 0.
\]
(1) When $\alpha = \delta$, $e^*_i = e^*$, because

$$E[(a - P - 2b(l)Q^H)Y] = \frac{1}{n}E[(a - P - 2b(l)Q^H)Y].$$

(2) When $\alpha < \delta$, $e^*_i < e^*$, because

$$\alpha E[(a - P - 2b(l)Q^H)Y] < \frac{\delta}{n}E[(a - P - 2b(l)Q^H)Y].$$

then,

$$c'(e^*_i) < c'(e^*)$$

Since $c(e)$ is convex, then $e^*_i < e^*$. The proof is similar in Case 2.

Under the traditional payment scheme, there are two situations: if farmers have time preferences, the supply chain cannot be coordinated and farmers are under-motivated; if farmers have no time preferences, the supply chain can be coordinated. This is explained as follows: if farmers have a time preference, the second-stage payment to farmers is always discounted greater than that applied to the co-op; hence, farmers are under-motivated. However, if farmers are time-indifferent, the supply chain can be fairly coordinated, because all market revenue is distributed to farmers under the budget balance condition.

### 4.6 Conclusions

This chapter investigates the problem of quality coordination for an agricultural co-op with multiple farmers. In response to the significant changes in the agricultural economy, various traditional cooperative business practices have been transformed. New co-ops have emerged to specify quality provisions in the contract, and also to adopt payment schemes characterised by non-uniform prices and multiple period arrangements. However, this problem has not been well studied heretofore. The analysis of this chapter aims to fill the gap by proposing a two-stage stochastic programme to investigate the centralised decisions of the co-op and the decentralised decisions of farmers under these changes. The model considers the case of separate sales pools and the case of a combined sales pool, and determines the optimal level of quality standards, the amount of product delivered to the high or low-end
market, and the prices offered to both \( H \) and \( L \) products in two stages. This chapter shows that the quality requirement is conducive to market profits and motivates farmers’ efforts as well.

Note that, the optimal quality level \( l^* \) and optimal collective effort \( e^* \) are determined in the centralised problem, and the co-op can simply coordinate the supply chain with this optimal quality through a payment scheme in the decentralised problem, under which each individual farmer is motivated to exert the optimal collective effort. Therefore, this chapter investigates the behaviour of farmers under a widely adopted payment scheme—the market-price-guarantee payment scheme, under which the \( H \) products are guaranteed to be rewarded at the premium market price. The results show that whether the supply chain can be coordinated depends on the degree of farmers’ time preference. If farmers have a relatively high time preference (higher than a threshold), the co-op can coordinate farmers’ behaviour by providing a quality premium in the first stage. However, if farmers have a low time preference (lower than the threshold), they will be over-motivated and produce too much \( H \) products. In this sense, farmers’ preference on payment time may not be a bad thing, because it alleviates the problem of over-motivation, hence contributes to supply chain coordination under the market-price-guarantee payment scheme.

This chapter also proposes an improved payment scheme—the upfront incentive payment scheme, under which a quality premium is provided as an individual incentive in the first stage, and shares net surplus among farmers irrespective of quality in the second stage. The results show that this payment scheme can coordinate the supply chain regardless of farmers’ time preferences. In addition, this payment scheme is superior to the market-price-guarantee payment scheme in that it is robust to the model with heterogeneous farmers with different time preferences or with different farm size.

Finally, this chapter briefly discusses the influence of farmers’ time preferences on the decision of the initial payment. The results show that when farmers have a higher time preference, the co-op should distribute no less than that distributed when farmers have a lower time preference. This conclusion sheds light on the solution to co-op financial problems. The role of the multi-period payment scheme is also investigated by comparing
it with the traditional payment scheme where the co-op pays the current market prices when
the commodity is marketed. The results imply that the multi-period payment arrangement
plays an important role in supply chain coordination when farmers have a time preference.

While a homogeneous membership is assumed in the first analytical step for conve-
nience, it is still important to consider a heterogeneous membership as noticed by Staatz
(1987). An immediate extension would be to extend the model for heterogeneous farm-
ers who are different in their time preferences or farm size. In these cases, asymmetric
information needs to be carefully considered and is left as the subject for future research.

In addition, this chapter shows the problem of over-motivation under the market-price-
guarantee payment scheme: farmers put in too much effort relative to the season’s payoff.
However, over motivation may not actually be a bad thing when the building of market
share and brand is considered (i.e, Zespri may be wise to over motivate farmers), but this
is left as the subject of possible future research.
5.1 Introduction

This chapter explores both operational and financial decisions for a so-called Proportional Investment Co-op (PIC), where farmers’ equity is required to be in proportion to their economic transaction with the organisation (or patronage). The “proportionality principle” requires that both the equity investment and financial return by and to members be proportional to their use of the co-op (Lund 2013). To ensure proportionality, PICs usually adopt a policy called the base capital plan to manage equity, as introduced in Section 2.3. With this plan, each member’s equity contribution is readjusted periodically according to the co-op’s capital needs and the member’s proportion of the co-op’s total patronage during a moving base period (Royer 2003). An under-invested member is required to purchase more shares to reach his required investment level. An over-invested member would be due an equity redemption. Therefore, the co-op receives additional capital from under-invested members, while it refunds excesses to over-invested members. Fonterra, which was discussed in Chapter 6, is a specific example of a co-op using such a policy. Farmer suppliers are required to hold a minimum of one share for every kilogramme of milksolids supplied, known as the share standard, and this standard is based on a three season rolling average of a farm’s production.

In addition to member-provided equity, this chapter also considers another two capital resources for processing: internal retained earnings and external debt instruments. The former represents a cost to the individual members who otherwise would have had that portion of the surplus allocated to them; while the later introduces a financial cost and risk.
This chapter is in the style of Li et al. (1997) and Li et al. (2013) who propose a Markov decision process (MDP) model that makes production, borrowing, and dividend decisions with uncertain demand; the study described in this chapter also uses a MDP model. In their work, the IOF aims to maximise the expected present value of the dividends in the infinite horizon with constraints on loan size, production size, and liquidity. While maximising the expected present value of the dividend stream is highly appropriate for an IOF, it may not be appropriate for a co-op. The co-op will need to balance the preferences of members to receive a return on their patronage with the needs of the co-op to have sufficient member capital for start-up and growth (Lund 2013). Therefore, this chapter assumes that the objective function is a weighted sum of the expected present value of both the aggregated profit retained within the co-op and the aggregated payments distributed to members in multiple periods. A co-op may place different weights on either term, and the results in this chapter show that different objectives result in different financing behaviours and performance.

Another key feature of agricultural businesses is the variability of patronage. In agriculture, the quantity of product submitted by farmers is usually not in response to orders, but instead is whatever their land yields. Therefore, the discrete-time MDP in this chapter assumes that the yield in the future is an exogenous random variable, largely dependent on weather conditions. However, as described above, uncertain patronage is associated with uncertain equity investment. Therefore, this chapter identifies a financial risk, defined as redemption default, when there is insufficient liquidity to redeem members’ equity as required by the co-op’s policy. This type of financial risk is unique to co-ops, but commonly exists within co-ops rather than IOFs. Furthermore, considering the reality that agricultural markets are volatile, especially at a global level, this chapter also assumes that market revenues are randomly realised. This chapter, therefore, like Li et al. (2013), considers the probability of loan-default risk when the co-op cannot fully repay its borrowing at the end of each period. Further, because agricultural products are typically perishable, this chapter does not allow the carrying of inventory between periods (which stands in contrast to Li et al. (2013)). These assumptions will be discussed further in Section 5.2.
This chapter is devoted to answering the following questions: (1) How should an agricultural co-op coordinate operational and financial decisions when equity is in proportion to patronage? (2) What does the capital position look like when applying the optimal decisions? (3) What financial crises are possible with uncertain yield and market revenue, and how do they influence the optimal decisions?

This chapter appears to be the first work to quantitatively model the integration of financial and operational decisions for co-ops, and that is the key contribution of this work.

The remainder of this chapter is organised as follows. Section 5.2 presents the model. Section 5.3 answers the first question above. Then, in order to answer the second question, Section 5.4 assumes a deterministic yield, with the justification that many new types of co-ops can now actively control farmers’ patronage (Harris et al. 1996b). This model is more tractable because the myopic policy turns out to be optimal. Section 5.5 examines the third question by modelling the financial crises of loan default and redemption default. Section 5.6 implements numerical experiments to validate the model and visualise the results under uncertain yield. Finally, Section 5.7 concludes the work and describes opportunities for future research.

5.2 Problem Description

This chapter considers an agricultural co-op that makes both financial and operational decisions in multiple periods, \( n = 1, 2, \ldots, N \). The co-op is responsible for processing and marketing agro-product submitted by farmers (or “members”). To support the business, the co-op requires farmers to hold a quantity of shares in proportion to their patronage, and the member-provided equity is managed by a base capital plan, as described in Section 2.3.

Due to the perishable nature of most agricultural products, it is assumed that there is no carried inventory between periods; this assumption is further justified by the statement of Kazaz and Webster (2011) that “agricultural businesses cannot use inventories strategically to battle supply uncertainty.” At the end of each financial year, the co-op distributes market profits to farmers, but is allowed to retain partial profits. Besides that, the co-op
can borrow a short-term loan from the bank. The specifics of the model are laid out in the following subsections on the state variables, decision variables, the payoffs and costs, the model timing and constraints, transition functions, and the full problem formulation. Throughout the chapter \(x^+ = \max(x, 0)\) and \(x^- = \max(-x, 0)\), for \(x \in \mathbb{R}\).

### 5.2.1 State Variables

There are two types of state variables considered. Namely, \(s_n \equiv\) the cash position at the beginning of period \(n\); and variables corresponding to yield, which will be discussed below. Similar to the paper of Li et al. (2013), this study allows the cash position in a co-op to be either positive or negative (i.e., its range is the real line), although a penalty cost is imposed if it becomes negative (see Section 5.2.3).

Define \(Y_n\) to be the random variable equal to the sum of all individual member yields in period \(n\), and let \(y_n\) be a realisation of \(Y_n\). This chapter assumes that \(\{Y_i\}\) forms an independent and identically distributed (i.i.d.) sequence of random variables from some common distribution function \(F(\cdot)\) with compact support \([0, \overline{y}]\). While the assumption of a stationary distribution is primarily for ease of exposition, it corresponds to a model with a stable member base. The assumption of independent yields across periods will not be perfectly accurate, due to long range dependency on weather, for example, but is needed for model tractability. This chapter assumes an exogenous distribution for yields because the explicit modelling of farmers’ efforts is not relevant to the key questions of this work.

As discussed earlier, this chapter assumes a base capital plan where the co-op will use the length of the base period to calculate each member’s average patronage. The base period is denoted by some fixed \(m \in \{1, 2, \ldots\}\), and the measurement units for the product yield are assumed to have been scaled to be equivalent to dollars. Recall that \(y_n\) is the total yield across all farmers in period \(n\). Then, letting \(e_n\) denote the total capital requirement in period \(n\), the calculation of the equity requirement is

\[
e_n = \frac{\sum_{i=n-m}^{n-1} y_i}{m}.
\]
This implies that the adjustment of capital required by the co-op (across all members) in period $n$ is

$$\Delta e_n = e_n - e_{n-1} = \frac{y_{n-1} - y_{n-m-1}}{m}. \quad (5.1)$$

If $\Delta e_n > 0$, the co-op requires members to purchase more shares, whereas if $\Delta e_n < 0$, the co-op will refund extra shares. Note that this is the aggregate change. Some individual members may receive a refund while others need to purchase extra shares. Correlation between different farmers’ yields can easily be accommodated by the aggregate yield distribution $F(\cdot)$. Share purchases and redemption are assumed to be all completed in cash. Although, in practice, share purchase can be realised through retaining a farmer’s payment at the end of period, this does not influence the internal working capital in each period (see below).

In order to track equity requirements, the model must keep track of $\{y_n, y_{n-1}, \ldots, y_{n-m-1}\}$, which is defined as the multi-dimensional state vector $\tilde{y}_n$. If $\tilde{y}_n^i$ is the $i$th element of $\tilde{y}_n$, $1 \leq i \leq m + 2$, then $\tilde{y}_n^1$ represents current yield $y_n$, and $\tilde{y}_n^2$ is immediate past yield $y_{n-1}$. Also define

$$\tilde{y}_{n}^{-2} \equiv \{y_{n-2}, \ldots, y_{n-m-1}\}$$

as the $m$ dimensional vector that excludes $y_n$ and $y_{n-1}$.

In conclusion, this model has $m + 3$ state variables (for each period $n$): $s_n$ and $\tilde{y}_n = \{y_n, y_{n-1}, \ldots, y_{n-m-1}\}$.

### 5.2.2 Decision Variables

In each period, the co-op makes three decisions:

1. How much to pay out to farmers, $w_n$.

   In practice, the payments to farmers by the co-op are never fulfilled in one stage, especially when the farmers are both suppliers and investors, as assumed here. The co-op pays a farm-gate price as the reward for product submission and also a patronage refund as the residual return. In this model, they can be combined because both payments are proportional to the farmers’ patronage. It can reasonably be assumed that farmers are indifferent to the categories of payments and only care about their total reward. This dual
role assumption also indicates that the payments to farmers not only depend on their contribution to the market profit, but also on the financial conditions in each period. This chapter assumes that this decision is made after random yield is realised but before revenue is received (see below).

(2) How much money to borrow, $b_n$.

Borrowing can be provided as short-term or long-term loans. However, this chapter assumes the loan is short-term because short-term loans are usually used for seasonal businesses while long-term loans are mainly used for long-term decisions like building a new factory, etc. A similar assumption is used in Li et al. (1997).

(3) How much product to process, $q_n$.

The co-op is a centralised decision maker who tries to maximise the profit of the whole supply chain. In practice, given some yield $y_n$, the co-op will solve a product mix problem to find out how to maximise profit. As a member-owned and controlled organisation, this chapter assumes the co-op should accept all products submitted by member farmers. Furthermore, this chapter does not consider the possibility of sourcing from non-members, because the amount of sourcing from outsiders is generally insignificant compared to members’ patronage (Cracogna 2013). This chapter will abstract this problem by using a single decision variable $q_n \leq y_n$ to represent the quantity of premium product provided to the market. For example, Fonterra will sometimes dump excess raw milk due to the high volume of milk collected. Such dumping occurred in 2013, and the Fonterra spokesman said that they also adjusted their product mix to process the maximum amount of raw milk (Ewing 2013). The following subsection revisits this assumption when discussing the revenues and costs associated with $q_n$.

5.2.3 Payoffs and Costs

Let $\beta$ denote the single-period discount factor ($0 < \beta < 1$). Assume that the co-op earns a capital return of $\theta_1 s_n$ from the cash position $s_n \geq 0$, for some $\theta_1 \geq 0$. In a co-op, profits retained or distributed to members are equivalent if the co-op and its members are regarded as a community of interest; however, it seems reasonable to assume that
capital accumulated within the co-op produces extra benefits for members such as product research, etc. This chapter also takes the assumption assumed by Zhang and Sobel (2010, p. 480) that the interest rate \( \theta_1 \) earned by cash is less than the discount rate \( (1 - \beta)/\beta \) expressed as \( \theta_1 < (1 - \beta)/\beta \). However, if \( s_n < 0 \), there is a debt default which introduces a penalty cost of \((\theta_1 + \theta_2)s_n\) for some \( \theta_2 \geq 0 \). Although in co-ops, members are always required to keep the business solvent, the expense charged to them will not be automatically cleared from the account book of the co-op.

The interest rate for borrowing is denoted by \( \rho \), and is assumed to be constant in each period and subject to \( \rho > \frac{1}{\beta} + \theta_1 - 1 \). This assumption makes sure that borrowing will not occur for the sole purpose of accumulating money. If the interest rate is less than the opportunity cost of capital (i.e., \( \rho \leq \frac{1}{\beta} + \theta_1 - 1 \)), then the co-op would like to borrow as much as possible, which does not make much sense in practice. This argument is similar to one in Li et al. (2013, p. 1110), except with the addition of \( \theta_1 \). Since \( \theta_1 \) is the capital gain associated with the cash position, it makes sense that this value increases the opportunity cost of borrowing.

As discussed above, the co-op’s key operational decision is represented by \( q_n \leq y_n \) to reflect that yield will bound profits. One can think of \( q_n \) as the quantity of premium product produced, where the rest of the product is sold locally, frozen, or processed into pulp (in the case of fruit). The cost associated with \( q_n \) is \( c(q_n) \), which is assumed to be strictly convex and increasing in \( q_n \) (and thus has a well defined inverse function \( c^{-1}(\cdot) \)). It may consist of a fixed as well as variable cost so long as that fixed cost is paid even when \( q_n = 0 \). The cost is assumed to be paid in advance of the realisation of market revenue, since it is a processing cost. Note that processing cost is the largest expense for a value-added co-op after payments to farmers.

Define \( R(q_n, \omega) \) to be the random market revenue under some random realisation \( \omega \) and quantity \( q_n \). Then the random variable \( R(q_n) \) is realised by the end of the period with expected payout \( r(q_n) \equiv E[R(q_n)] = E[R(q_n, \omega)] \), for a given processing quantity \( q_n \). For any random realisation \( \omega \), \( R(q, \omega) \) is assumed to be concave increasing in \( q \). This general form allows the processor to be either a price maker or a price taker in the market.
Further, given $q$, $R(q)$ is also assumed to be independent of yield, which corresponds to an assumption that the world market for this product is significantly broader than the local conditions for the co-op.

The horizon length is assumed to be the natural planning horizon for the co-op and the terminal value is its ongoing value as a firm after this time. This is assumed to depend on its cash position (and all parameters, such as $\alpha$ (see Section 5.2.6)) but not (in a significant way) on the previous yield. Therefore, this chapter will assume in the finite horizon model that there is a terminal value in period $N + 1$, $V_{N+1}(s_{N+1})$, which is continuous, bounded, concave, and non-decreasing in $s_{N+1}$, and independent of $\tilde{y}_{N+1}$. This chapter further assumes that the terminal value has been normalized by $\beta^N s_{N+1}(1 + \theta_1 - \alpha \theta_1 - 2\alpha)$. In other words, if $\tilde{V}_{N+1}(s_{N+1})$ is the actual terminal value function, then this chapter will use a terminal value of $V_{N+1}(s_{N+1}) = \tilde{V}_{N+1}(s_{N+1}) - \beta^N s_{N+1}(1 + \theta_1 - \alpha \theta_1 - 2\alpha)$. Thus, all assumptions on $V_{N+1}(s_{N+1})$ should be translated into assumptions on the actual terminal value function $\tilde{V}_{N+1}(s_{N+1})$ by adding $\beta^N s_{N+1}(1 + \theta_1 - \alpha \theta_1 - 2\alpha)$ to $V_{N+1}(s_{N+1})$. The reason for this technical assumption may be seen in the proof of Lemma 5.3. Note that the effect of any assumption on terminal value becomes increasingly diminished as $N$ increases due to the discount factor $\beta^N$.

### 5.2.4 Model Timing and Constraints

This chapter assumes that the co-op is able to make all decisions after the realisation of current yield. This can be justified by the fact that the co-op, in essence, works on behalf of farmers. Further, even though processing and harvest can occur concurrently, yield is often well estimated by the time harvest begins. Therefore, an assumption of decisions occurring after yield is realised seems more appropriate than an assumption of decisions occurring before yield is realised; mixing the two does not appear analytically practical. The sequence of events in period $n$, shown in Figure 5.1, are as follows:

1. The co-op posts its current capital position, $s_n$, and gains (or loses) an additional capital value $\theta_1 s_n - \theta_2 s_n^-$;
2. The co-op calculates the equity requirement based on the patronage in the past \( m \) years (not including the current one) \((y_{n-1}, y_{n-2}, \ldots, y_{n-m}, y_{n-m-1})\), and updates the equity by \( \Delta e_n \) as per equation (5.1);

3. After observing the realisation of current yield, \( y_n \), the co-op makes decisions on payments to farmers, the level of borrowing, as well as the quantity of processing \((w_n, b_n, q_n)\); and

4. The market profit is realised and the co-op repays the loan (if any), fulfills the payments to farmers, and retains earnings for the next period.

Notice that the co-op must decide \( w_n \) before actual market profit is realised. If profit is unusually high then this will be reflected in a high cash position leading to a high payout to farmers in the following period.

The reward to farmers is considered to be non-negative, i.e., \( w_n \geq 0 \). In practice, given farmers’ production costs, there should be a minimum per unit payment, \( \bar{w} \), in each period, which, for simplicity, is normalised to zero. Note that, adding a term equal to \( \bar{w} y_n \) to the objective function in each period would make no difference to the optimisation (since yield is assumed to be independent of all co-op decisions). Further, as introduced earlier, farmers’ equity is redeemable at a par value. Therefore, assuming away the opportunity cost, this chapter can ignore the initial cost of a long-term equity investment. Although this assumption is primarily for ease of exposition, the opportunity cost can be counteracted by other benefits of co-ops beyond a simple economic return (Lund 2013). In addition, as
observed in practice, this chapter implicitly assumes that farmers are allowed to fulfill their equity requirement over time rather than forcing bankruptcy when the cost of membership is significant (Lund 2013).

The decision variables in each period are $w_n$, $b_n$, and $q_n$, but it is more convenient in the analysis to replace $w_n$ with the following decision variable:

$$v_n = (1 + \theta_1)s_n - \theta_2s_n^- + \Delta e_n - w_n - \rho b_n.$$  

Variable $v_n$ may be interpreted as the amount of internally generated working capital. Specifically, $v_n$ is the working capital after the equity adjustment, the payment to farmers, and loan interest are paid, and before the loan is received and revenue and processing cost are realised. Rewriting the constraint that $w_n \geq 0$, the incentive constraint is

$$(1 + \theta_1)s_n - \theta_2s_n^- + \Delta e_n - v_n - \rho b_n \geq 0. \quad (5.2)$$

With the short-term loan, the total working capital available in period $n$ is $b_n + v_n$. Therefore, the liquidity constraint is:

$$c(q_n) \leq b_n + v_n. \quad (5.3)$$

Inequality (5.3) prevents the expenditures in period $n$ from exceeding the total working capital from both internal and external funds. In addition, this chapter has previously assumed that the processing quantity and the loan are bounded by

$$0 \leq q_n \leq y_n \quad (5.4)$$

and

$$b_n \geq 0, \quad (5.5)$$

respectively. From the above analysis, this chapter has transformed decisions of $(w_n, b_n, q_n)$ into decisions of $(v_n, b_n, q_n)$ subject to constraints (5.2), (5.3), (5.4), and (5.5).

Let $h(s, \tilde{y}) = (1 + \theta_1)s - \theta_2s^- + \Delta e$ be the adjusted cash position after the equity update (where $\Delta e$ is defined in terms of $\tilde{y}$ by (5.1)), and let $A(s, \tilde{y})$ be the set of $(v, b, q)$ defined
by the following constraints (subscript \( n \) is omitted):

\[
\begin{align*}
    v + \rho b & \leq h(s, \tilde{y}); \\
    v + b - c(q) & \geq 0; \\
    0 & \leq q \leq y; \text{ and} \\
    b & \geq 0.
\end{align*}
\]  

(5.6)

Then \( A(s_n, \tilde{y}_n) \) is the feasible region for \( (v_n, b_n, q_n) \), representing constraints (5.2), (5.3), (5.4), and (5.5).

Define a set to be “expanding” in some variable if for any two values of the variable the set for the smaller value is a subset of the set for the larger value; define it to be “shrinking” if the reverse is true. The following lemma examines set \( A(s, \tilde{y}) \).

**Lemma 5.1.** Region \( A(s, \tilde{y}) \) is non-empty and convex; expanding in \( s \), \( \tilde{y}^1 \), and \( \tilde{y}^2 \); and shrinking in \( \tilde{y}^2 \).

**Proof of Lemma 5.1.**

For a fixed \( q \), Figure 5.2 shows the two-dimensional feasible region of \( A(s, \tilde{y}) \). From the action set, one can see that as long as \( b \) is large enough, the action set \( A(s, \tilde{y}) \) is non-empty for any \( s \). Furthermore, for any \( (s, \tilde{y}) \), the set is defined by three polyhedral constraints on

![Figure 5.2: The Two-Dimensional Feasible Region of \( A(s, \tilde{y}) \)](image)

\((v, b, q)\) plus the constraint \( v + b \geq c(q) \). Since \( c(q) - v - b \) is convex in \((v, b, q)\), \( \{ (v, b, q) : c(q) - v - b \leq 0 \} \) is a convex set. Therefore, \( A(s, \tilde{y}) \) is a convex set. One can immediately see that if the value of \( s \) increases, the inequality \( v + \rho b \leq (1 + \theta_1)s - \theta_2 s^- + \Delta e \) is weakened.
while all other constraints remain the same, and thus $A(\cdot)$ is expanding (non-shrinking) in $s$. It is similarly easy to show that $A(s, \tilde{y})$ is expanding in $\tilde{y}^1$ and $\tilde{y}^2$, and shrinking (non-expanding) in $\tilde{y}^{-2}$.

Lemma 5.1 shows that as the cash position grows, the feasible region also grows, which is quite intuitive. It also shows that the feasible region is increasing in current yield and last yield, but decreasing in previous yields, denoted by $\tilde{y}^{-2}$.

5.2.5 Transition Functions

After the loan is repaid, the cash position is specified by:

$$s_n + 1 = (1 + \theta_1)s_n - \theta_2 s_n^- + \Delta e_n - w_n - \rho b_n + R(q_n) - c(q_n)$$

$$= v_n + R(q_n) - c(q_n). \quad (5.7)$$

This chapter assumes that at time 0, $\tilde{y}_0$ is still well defined with $m+1$ elements (i.e., the co-op has data on yields from before the beginning of the decision horizon). The transition function requires that $\tilde{y}_n + 1 = \tilde{y}_n^{i-1}$ for $i \geq 2$ and $\tilde{y}_n + 1$ is the realisation of $Y_n$. Notice here that decisions are made after the realisation of $y_n$, but before the realisation of $Y_n + 1$ in the next period; thus, the future state depends on a random variable which is independent of actions and past state variables. To include this exogenous uncertainty, the future yield state variables is given as a random vector

$$\tilde{Y}_n + 1 \equiv \{Y_n + 1, y_n, y_n - 1, \ldots, y_n - m\}. \quad (5.8)$$

Note that, for ease of notation, the explicit dependence of $\tilde{Y}_n + 1$ on $\tilde{y}_n$ has been suppressed, but it should be taken as given in what follows. The following lemma regards the transition function for cash position.

**Lemma 5.2.** Suppose that, for each random revenue outcome $\omega$, $R(\cdot, \omega)$ and $c(\cdot)$ are continuous and differentiable, then the transition function of $s_n + 1$ is continuous, differentiable, jointly concave in $q_n$ and $v_n$, and non-decreasing in $v_n$.

**Proof of Lemma 5.2.**
Since \( s_{n+1} = v_n + R(q_n) - c(q_n) \), the transition function is a concave function of \( q_n \) because \( R(\cdot) \) is concave and \( c(\cdot) \) is convex. Furthermore, it is separable in \( v_n \) and \( q_n \) and linear in \( v_n \); thus, \( s_{n+1} \) is jointly concave in \( q_n \) and \( v_n \). It is also easy to see \( s_{n+1} \) is continuous, and differentiable (for a given randomness). Therefore, the desired properties of \( s_{n+1} \) are obtained.

Lemma 5.2 will be useful in proving structural results for the value function.

### 5.2.6 Full Problem Formulation

In broad terms, capital retained within co-ops is used to maintain existing operations or to increase sales and profits by growing the business, and internal savings is one of the most important financing sources. However, members may also need additional capital for their farming operations. In order to balance the benefits of both the co-op and the farmers, the one-period objective of the co-op is assumed to maximise the net profit retained within the co-op and the payment distributed to farmers in each period. Note that the net profit of the co-op is the market revenue after processing cost, payments to farmers, and other expenses, i.e., \( R(q_n) - c(q_n) - w_n - \rho b_n + \theta_1 s_n - \theta_2 s_n^- \). The one-period objective is therefore:

\[
\alpha \left( R(q_n) - c(q_n) - w_n - \rho b_n + \theta_1 s_n - \theta_2 s_n^- \right) + (1 - \alpha) w_n,
\]

for some \( 0 \leq \alpha \leq 1 \). The first part is the co-op’s retained net profit, and the second part is the payment distributed to farmers. Note that the farmers’ payment includes both a product payment and a patronage refund. Adding a weight factor \( \alpha \) allows the model to reflect the fact that a co-op may place more or less importance on the co-op’s savings rather than on members’ profits. The value of \( \alpha \), in practice, is largely dependent on a co-op’s size, business, and the stage of operations (Lund 2013), and also the bargaining power of the members versus the co-op (Soboh et al. 2009). For example, a large processing co-op that is capital-intensive is more likely to have a larger \( \alpha \).
Then, the objective of the finite-horizon decision problem is to maximise the expected value of:

\[ U \equiv \sum_{n=1}^{N} \beta^{n-1} \left[ \alpha (R(q_n) - c(q_n) - \rho b_n + \theta_1 s_n - \theta_2 s_n^-) + (1 - 2\alpha) w_n \right] + \tilde{V}_{N+1}(s_{N+1}). \]

Recall from Section 5.2.3 that \( V_{N+1}(s_{N+1}) \equiv \tilde{V}_{N+1}(s_{N+1}) - \beta^N s_{N+1} (1 + \theta_1 - \alpha \theta_1 - 2\alpha). \)
This normalisation yields the following lemma.

**Lemma 5.3.** Define the new single-period objective as

\[ L(v, b, q) \equiv [(\alpha+\beta-2\alpha\beta + (\theta_1 \beta - 1)(1-\alpha)]v + [(\alpha+\beta-2\alpha\beta + \beta \theta_1 (1-\alpha)]r(q) - c(q)) + (1-\alpha) \rho b. \]

Then maximising \( E[U] \) is equivalent to maximising

\[ E\left[ \sum_{n=1}^{N} \beta^{n-1} [-(1 - \alpha) \theta_2 s_n^- + L(v_n, b_n, q_n)] + V_{N+1}(s_{N+1}) \right]. \]

(5.9)

**Proof of Lemma 5.3.**

Substituting \( w_n = (1 + \theta_1) s_n - \theta_2 s_n^- + \Delta e_n - v_n - \rho b_n \) into the objective yields:

\[
\begin{align*}
U &= \alpha \sum_{n=1}^{N} \beta^{n-1} (R(q_n) - c(q_n) - w_n - \rho b_n + \theta_1 s_n - \theta_2 s_n^-) + (1 - \alpha) \sum_{n=1}^{N} \beta^{n-1} w_n + \tilde{V}_{N+1}(s_{N+1}) \\
&= \alpha \sum_{n=1}^{N} \beta^{n-1} (R(q_n) - c(q_n) - \rho b_n + \theta_1 s_n - \theta_2 s_n^-) + (1 - 2\alpha) \sum_{n=1}^{N} \beta^{n-1} \left( (1 + \theta_1) s_n - \theta_2 s_n^- + \Delta e_n - v_n - \rho b_n \right) + \tilde{V}_{N+1}(s_{N+1}) \\
&= \alpha \sum_{n=1}^{N} \beta^{n-1} (R(q_n) - c(q) - \rho b_n) + \sum_{n=1}^{N} \beta^{n-1} [(1 - 2\alpha)(1 + \theta_1) s_n + \alpha \theta_1 s_n] + \tilde{V}_{N+1}(s_{N+1}) \\
&\quad + \sum_{n=1}^{N} \beta^{n-1} \left[ (1 - 2\alpha)(-\theta_2 s_n^- + \Delta e_n - v_n - \rho b_n) - \alpha \theta_2 s_n^- \right] \\
&= \alpha \sum_{n=1}^{N} \beta^{n-1} (R(q_n) - c(q) - \rho b_n) + \left[ [(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1] (s_1 - \beta^N s_{N+1}) \right] + \tilde{V}_{N+1}(s_{N+1}) \\
&\quad + \sum_{n=1}^{N} \beta^{n-1} \left[ (1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1 \right] s_{n+1} + (1 - 2\alpha)(\Delta e_n - v_n - \rho b_n) \\
&= \sum_{n=1}^{N} \beta^{n-1} \left[ \alpha (R(q_n) - c(q_n) - \rho b_n) + \beta [(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1] s_{n+1} + (1 - 2\alpha)(-v_n - \rho b_n) \right] \\
&\quad + \sum_{n=1}^{N} \beta^{n-1} \left[ -(1 - \alpha) \theta_2 s_n^- + (1 - 2\alpha)(\Delta e_n) \right] + \left[ [(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1] (s_1 - \beta^N s_{N+1}) \right] \\
&\quad + \tilde{V}_{N+1}(s_{N+1})
\end{align*}
\]
\[
\sum_{n=1}^{N} \beta^{n-1} \left[ \alpha(R(q_n) - c(q_n) - \rho b_n) + \beta[(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1]s_{n+1} + (1 - 2\alpha)(-v_n - \rho b_n) \right] \\
+ \sum_{n=1}^{N} \beta^{n-1}[-(1 - \alpha)\theta_2 s_n^+] + \left[ \tilde{V}_{N+1}(s_{N+1}) + [(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1](s_1 - \beta^N s_{N+1}) \right] \\
+ (1 - 2\alpha) \sum_{n=1}^{N} \beta^{n-1}(\Delta e_n) \\
= \sum_{n=1}^{N} \beta^{n-1} \left[ \alpha(R(q_n) - c(q_n) - \rho b_n) + \beta[(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1]s_{n+1} + (1 - 2\alpha)(-v_n - \rho b_n) \right] \\
+ \sum_{n=1}^{N} \beta^{n-1}[-(1 - \alpha)\theta_2 s_n^-] + (1 - 2\alpha) \sum_{n=1}^{N} \beta^{n-1}(\Delta e_n) + \left[ V_{N+1}(s_{N+1}) + [(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1]s_1 \right].
\]

Then, using that \( s_{n+1} = v_n + R(q_n) - c(q_n) \) and

\[
L(v_n, b_n, q_n) \equiv \mathbb{E} \left[ \alpha(R(q_n) - c(q_n) - \rho b_n) + \beta[(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1]s_{n+1} + (1 - 2\alpha)(-v_n - \rho b_n) \right] \\
= \mathbb{E} \left[ \alpha(R(q_n) - c(q_n) - \rho b_n) + \beta[(1 - 2\alpha)(1 + \theta_1) + \alpha \theta_1][v_n + R(q_n) - c(q_n)] \right] \\
+ (1 - 2\alpha)[-v_n - \rho b_n] \\
= \left[ (\alpha + \beta - 2\alpha \beta) + (\theta_1 \beta - 1)(1 - \alpha) \right] v_n + \left[ (\alpha + \beta - 2\alpha \beta) + \beta \theta_1 (1 - \alpha) \right] (r(q_n) - c(q_n)) \\
- (1 - \alpha)\rho b_n.
\]

Then,

\[
\mathbb{E}[U] = \mathbb{E} \left[ \sum_{n=1}^{N} \beta^{n-1}[-(1 - \alpha)\theta_2 s_n^- + L(v_n, b_n, q_n)] + (1 - 2\alpha) \sum_{n=1}^{N} \beta^{n-1}\Delta e_n \right] \\
+ V_{N+1}(s_{N+1}) + (1 + \theta_1 - 2\alpha - \alpha \theta_1)s_1.
\]

Since \( \mathbb{E} \left[ \sum_{n=1}^{N} \beta^{n-1}\Delta e_n \right] \) is independent of decision variables and the initial state \( s_1 \) cannot be controlled, the problem is equivalent to choosing \( \{(v_n, b_n, q_n)\}_{n=1}^{N} \) to maximise

\[
\mathbb{E} \left[ \sum_{n=1}^{N} \beta^{n-1}[-(1 - \alpha)\theta_2 s_n^- + L(v_n, b_n, q_n)] + V_{N+1}(s_{N+1}) \right].
\]

\( \square \)

In order to make the equivalence in Lemma 5.3, \( [(1 + \theta_1 - \alpha \theta_1 - 2\alpha)s_1] \) has been dropped from \( \mathbb{E}[U] \), because the initial state is not controllable. This has resulted in the objective function in Lemma 5.3 not having an explicit dependence on any state variables (beyond
the terminal value) when \( s_n \geq 0 \) for all \( n \). Note that \( s_n \) can be inferred from \((v_n, b_n, q_n)\), and \( \tilde{y}_n \) is only relevant in the constraints on the decision variables and the transition functions. Then, the full stochastic optimisation is as follows.

**Definition 1.** Stochastic optimisation: Find the supremum over \( \{v_n, b_n, q_n\}_{n=1}^{N} \) of (5.9) subject to \((v_n, b_n, q_n) \in A(s_n, \tilde{y}_n) \) (for all \( n \)) and state transitions governed by (5.7) and (5.8).

### 5.3 Properties of the Value Function and the Optimal Policy

Let \( V_n(s, \tilde{y}) \) be the value function of the finite-horizon recursion from period \( n \) to \( N \) with terminal value \( V_{N+1}(\cdot) \). For \( n = 1, 2, \ldots, N \), the Bellman equation that corresponds to the stochastic optimisation in Definition 1 is:

\[
V_n(s, \tilde{y}_n) = -(1 - \alpha)\theta_2 s^2 + \sup_{(v, b, q) \in A(s, \tilde{y}_n)} \left\{ J_n(v, b, q, \tilde{y}_n) \right\}
\]

where

\[
J_n(v, b, q, \tilde{y}_n) = L(v, b, q) + \beta E \left[ V_{n+1}(v + R(q) - c(q), \tilde{Y}_{n+1}) \right].
\]

The following proposition states that the value function is non-decreasing in the cash position, which is intuitive. It also shows that the value function is non-increasing in past (but not present yield). Because a higher current yield means more revenue now but a lower future value, monotonicity results on current yield do not appear to exist.

**Proposition 5.1.** The value function \( V_n(s, \tilde{y}) \) is non-decreasing in \( s \) and non-increasing in \( \tilde{y}^{-2} \) for \( n = 1, 2 \ldots, N \).

**Proof of Proposition 5.1.**

Notice that both \( L(v, b, q) \) and the transition function \( s_{n+1} \) (without constraints) are independent of \( s_n \), and \(-\theta_2 s^2\) is non-decreasing in \( s_n \). Furthermore, the feasible region \( A(s, \tilde{y}) \) is non-shrinking in \( s \) and the terminal value is (by assumption) non-decreasing in \( s \); thus, an inductive argument yields that \( V_n(\cdot) \) is non-decreasing in \( s_n \) because the maximum of a non-decreasing function over a non-shrinking set is non-decreasing.
Suppose $V_{n+1}(\cdot)$ is non-increasing in $\tilde{y}_{n+1}^{-1}$. It is therefore non-increasing in $\tilde{y}_{n+1}^{-1}$ (from (5.8)) and hence particularly it is non-increasing in $\tilde{y}_{n+1}^{-2}$. Therefore, since $L(v, b, q)$ is independent of yield, $J_n(\cdot)$ is non-increasing in $\tilde{y}_{n+1}^{-2}$. Further, since the constraint set is non-expanding in $\tilde{y}_{n+1}^{-2}$, then $V_n(s, \tilde{y}_n)$ is non-increasing in $\tilde{y}_{n+1}^{-2}$. Noting that $V_{N+1}(\cdot)$ is independent of yield (by assumption), completes the inductive argument.

In addition, this chapter has the following properties for $J_n(\cdot)$ and $V_n(\cdot)$.

**Proposition 5.2.** For each $n$ and any $\tilde{y}_n$, the function $J_n(\cdot, \tilde{y}_n)$ is a jointly concave function on $(v, b, q)$ and the value function $V_n(s, \tilde{y}_n)$ is a concave function of $s$.

**Proof of Proposition 5.2.**

Recall that

$$V_n(s, \tilde{y}) = -(1 - \alpha)\theta_2 s^- + \sup_{(v, b, q) \in A(s, \tilde{y})} \left\{ J_n(v, b, q, \tilde{y}) \right\},$$

where

$$J_n(v, b, q, \tilde{y}_n) = L(v, b, q) + \beta E \left[ V_{n+1}(v + R(q) - c(q), \tilde{Y}_{n+1}) \right].$$

Assume that for some $n$ and all $\tilde{y}_n$, $V_{n+1}(\cdot, \tilde{y}_n)$ is a concave, and non-decreasing function of $s$. This is true for $n = N$ by assumption. Also, note that $L(v, b, q) = [(\alpha + \beta - 2\alpha\beta) + (\theta_1\beta - 1)(1 - \alpha)]v + [(\alpha + \beta - 2\alpha\beta) + \beta\theta_1(1 - \alpha)](r(q) - c(q)) - (1 - \alpha)\rho b$ is clearly concave in $(v, b, q)$ since it is separable in these variables and each separate term is concave. From Lemma 5.2, for any given revenue realisation $R(q)$, $v + R(q) - c(q)$ is jointly concave in $v$ and $q$ and independent of $b$. Therefore, $V_{n+1}(v + R(q) - c(q), \tilde{Y}_{n+1})$ is also jointly concave in $v$ and $q$ as is its expected value (Heyman and Sobel, 2003, p. 529). Which implies, since $R(\cdot)$ and $\tilde{Y}_{n+1}$ are independent and $L(\cdot)$ has the desired properties, that $J_n(v, b, q, \tilde{y}_n)$ is jointly concave in $(v, b, q)$ for any given $\tilde{y}_n$. It is also independent of $s_n$.

Since this model is taking the maximum of a concave function over a convex set, it has that $\sup_{(v, b, q) \in A(s, \tilde{y}_n)} \left\{ J_n(v, b, q, \tilde{y}_n) \right\}$ is concave in $s$ (Heyman and Sobel, 2003, p. 525). In addition, $-\theta_2 s^-_n$ is a piece-wise linear and concave function. Since the sum of two concave functions is concave, therefore, the value function $V_n(s, \tilde{y})$ is concave in $s$ for all $\tilde{y}$. It is non-decreasing in $s$ by Proposition 5.1. This model has thus shown the inductive hypothesis to be true for period $n$, which completes the proof.
Propositions 5.1 and 5.2 have a vital role in the analysis of the problem. Monotonicity and concavity facilitate the proof of the existence of a threshold cash position. In addition, they lead to efficient analysis of the optimal policy.

Next, this chapter analyses the properties of the optimal policy. Assume for each \( n \) and \((s, \tilde{y})\) that the maximum of \( J_n(\cdot) \) is achieved at \((v_n, b_n, q_n) = (v_n(s, \tilde{y}), b_n(s, \tilde{y}), q_n(s, \tilde{y}))\).

From \( v_n = (1 + \theta_1)s_n - \theta_2 s_n^- + \Delta e_n - w_n - \rho b_n, \) if the cash position is \( s \), then the payments to farmers in period \( n \) are

\[
w_n(s, \tilde{y}) = (1 + \theta_1)s - \theta_2 s^- + \Delta e_n - v_n(s, \tilde{y}) - \rho b_n(s, \tilde{y}). \tag{5.10}
\]

**Proposition 5.3.** If \([\alpha + \beta - 2\alpha\beta + (\theta_1 \beta - 1)(1 - \alpha)] \geq 0\), then the optimal policy has \( v_n(s, \tilde{y}) = h(s, \tilde{y}) - \rho b_n(s, \tilde{y})\) for all \( n = 1, 2, ..., N \) and \((s, \tilde{y}) \in \mathbb{R}^{n+2}\). Further, the optimal payment to farmers \( w_n(s, \tilde{y}) = 0\) for all \( n = 1, 2, ..., N \).

**Proof of Proposition 5.3.**

Under the assumption of this proposition, this chapter has that \( L(v, b, q) \) is increasing in \( v \). This implies that \( J_n(\cdot) \) is increasing in \( v \) since the value function \( V_{n+1}(\cdot) \) is increasing in \( s \) and since \( s_{n+1} \) is increasing in \( v_n \). Therefore, it is optimal to choose the value for the decision variable \( v_n \) as large as possible. The constraints that restrict the choice of \( v_n \) are \( v + \rho b \leq h(s_n, \tilde{y}_n) \) and \( v + b - c(q) \geq 0 \), that is \( c(q_n) - b_n \leq v_n \leq h(s_n, \tilde{y}_n) - \rho b_n \). Therefore, \( v_n(s, \tilde{y}) = h(s, \tilde{y}) - \rho b_n(s, \tilde{y})\) is an optimal policy in each period. Putting this into (5.10) and using the definition of \( h(\cdot) \) completes the proof.

This proposition states that the optimal retained earnings are always equal to the capital in hand minus the borrowing cost. This, in turn, implies that the farmers do not receive any payments under this condition. This conclusion seems to be counter-intuitive and does not seem practical. However, it can be explained by the assumed inequality \([\alpha + \beta - 2\alpha\beta + (\theta_1 \beta - 1)(1 - \alpha)] \geq 0\), which represents the marginal return on \( v_n \) for the current period’s utility. If the marginal return on \( v_n \) for the current period is positive and it also contributes to the future utility (as is likely), it is optimal to increase \( v_n \) as much as possible in all periods.
This behaviour is further justified from the objective of the co-op because the inequality is equivalent to \( \alpha \in \left[ \frac{1}{2} - \frac{\theta_1 \beta}{2 - 2 \beta - \theta_1 \beta}, 1 \right] \). Recall that \( \alpha \) denotes the weight of importance placed by the co-op on its own profit. Thus, this assumption implies that when the co-op values its own benefits more than \( \frac{1}{2} - \frac{\theta_1 \beta}{2 - 2 \beta - \theta_1 \beta} \), it would try to increase the internal working capital as much as possible in each period, and as a result, pay nothing to farmers. The extreme case is when \( \alpha = 1 \) under which the objective of the co-op is just to maximise its own profit, which resembles the special case of IOFs. Although there will be some minimum payment to suppliers under a participation constraint in IOFs, their behaviour of extracting the maximum possible from suppliers is consistent with this result.

This chapter does not consider the principal-agent problem between the co-op and its farmers; however, it implies a potential challenge of keeping a balance between the co-op’s capital requirement and members’ capital requirements. Readers interested in this kind of issue should refer to Porter and Scully (1987), Richards et al. (1998), etc. To capture farmers’ role as owners, this chapter makes the following assumption.

**Assumption 5.1.** In the following analysis, this chapter assumes that the co-op puts more weight on the benefits of farmers, shown by \( [\alpha + \beta - 2\alpha\beta + (\theta_1\beta - 1)(1 - \alpha)] < 0 \).

**Lemma 5.4.** Assumption 5.1 plus \( \rho > \frac{1}{\beta} + \theta_1 - 1 \) implies that \( -(1 - \alpha)\rho < [\alpha + \beta - 2\alpha\beta + (\theta_1\beta - 1)(1 - \alpha)] \), which means that the reward function \( L(\cdot) \) is decreasing in \( v \) and the marginal cost of borrowing is higher than the marginal return from internal working capital.

**Proof of Lemma 5.4.**

Let \( g(\alpha) = [\alpha + \beta - 2\alpha\beta + (\beta\theta_1 - 1)(1 - \alpha)] + (1 - \alpha)\rho \) and \( 0 \leq \alpha \leq 1 \). This model gets \( g(1) = 1 - \beta > 0 \) and \( g(0) = \beta + \beta\theta_1 - 1 + \rho \). As \( \rho > \frac{1}{\beta} + \theta_1 - 1 \), then \( g(0) > \beta + \beta\theta_1 - 1 + \frac{1}{\beta} + \theta_1 - 1 = (1 + \beta)\theta_1 + \beta + \frac{1}{\beta} - 2 \geq (1 + \beta)\theta_1 \geq 0 \). Since \( g(0) > 0 \), \( g(1) > 0 \), and \( g(\cdot) \) is a linear function of \( \alpha \), I have \( g(\alpha) > 0 \) for all \( \alpha \in [0, 1] \), which proves that \( [\alpha + \beta - 2\alpha\beta + (\beta\theta_1 - 1)(1 - \alpha)] > -(1 - \alpha)\rho \). From \( L(v, b, q) \), the marginal cost of borrowing \( b \) is \( (1 - \alpha)\rho \). Since the negative of the left hand side equals the marginal return from internal working capital and the negative of the right hand side is the marginal cost of borrowing, the proof is complete.
The cost of capital is an important economic and financial concept because it plays a role in identifying profitable uses of cash (Pederson 1998). Lemma 5.4 argues that, compared with borrowing, a cheaper method of financing is to generate internal working capital. Note that this does not imply borrowing is always unacceptable, but instead implies that priority should be given to the use of internal working capital.

The following theorem characterises the structure of the value function and the optimal policy for \((v, b, q)\).

**Theorem 5.1.** Under the condition of Lemma 5.4, for each \(n = 1, 2, \ldots, N\):

1. The optimal policy for the processing quantity is 
   \[ q_n(s, \tilde{y}) = \min \{ \arg \max_{q \geq 0} \{ \max_{(v, b) \in A_q^N(s, \tilde{y})} J_n(v, b, q, \tilde{y}_n) \} \}, \ y_n, \ c^{-1}(v_n(s, \tilde{y}) + b_n(s, \tilde{y})) \} \]

2. The loan amount \(b_n(s, \tilde{y}) = [c(q_n(s, \tilde{y})) - v_n(s, \tilde{y})]^+\) is optimal for all \((s, \tilde{y})\); and

3. There exists a threshold point \(\hat{s}_n(\tilde{y})\) (possibly \(+\infty\)) such that for all \(s \geq \hat{s}_n(\tilde{y})\), the optimal decisions \((v_n, b_n, q_n)\) do not change with \(s\).

**Proof of Theorem 5.1.**

(1) Proposition 5.2 proved that \(J_n(\cdot)\) is concave in \(q_n\). Furthermore, the constraints in the feasible region \(A(\cdot)\) relevant to \(q_n\) are \(0 \leq q_n \leq y_n\) and \(v_n + b_n \geq c(q_n)\), which implies that \(0 \leq q_n(s, \tilde{y}) \leq \min \{y_n, \ c^{-1}(v_n(s, \tilde{y}) + b_n(s, \tilde{y})) \} \). Further, we have shown that the action set \(A(\cdot)\) is non-empty; therefore, \(A^q(\cdot)\) is not empty for all \(q \geq 0\). Since we assume that the maximization over the empty set of \(A^q(\cdot)\) is \(-\infty\), which implies (together with concavity) that in the set of \(A^q_n(s, \tilde{y})\), \(\arg \max_{q \geq 0} \{ \max_{(v, b) \in A^q_n(s, \tilde{y})} J_n(v, b, q, \tilde{y}_n) \}\) exists. Therefore, an optimal point exists in the constraint set where the optimal processing quantity is the given minimum value.

(2) Since \(L(\cdot)\) is decreasing in \(b\) and \(E[V_{n+1}(v + R(q) - c(q), \tilde{Y}_{n+1})]\) is independent of \(b\), it is optimal to choose the value of \(b\) as small as possible (given values of the other decision variables). The constraints that restrict the choice of \(b_n\) are \(b_n \geq 0\) and \(b_n \geq c(q_n) - v_n\), thus \(b_n = \max\{0, c(q_n) - v_n\}\).

(3) Since the actions \((v, b, q)\) are located within the feasible region, which is expanding in \(s\), the optimal unconstrained actions of \((v, b, q)\) are more likely to be included within
$A(s, \tilde{y})$ with an increase of $s$. When $s$ is larger than a certain point $\tilde{s}_n(\tilde{y})$, the constraint $v + pb \leq h(s, \tilde{y})$ becomes relaxed. Past that point an increase in $s$ would no longer affect the optimal decisions, and the maximum value remains constant beyond this point.

Theorem 5.1 shows that the optimal solution for the processing quantity is to maximise the market profit. This result is quite intuitive: given the financial decisions determine how the profits are allocated but not the overall profit, so it is always optimal to choose the processing quantity that maximises the market profit. The constraint $v_n + b_n \leq c(q_n)$ can be viewed as the liquidity constraint and $0 \leq q_n \leq y_n$ as the supply constraint, while the optimal point that depends on a function of $R(q)$ and $c(q)$ is the marketing constraint. All of these conditions constrain the optimal processing quantity as well as other decisions. However, a co-op in different times may be constrained by different conditions. For example, when the cash position is low, the main constraint may be the financial condition, and when the cash is high, the supply constraint or marketing constraint may become the bottleneck. Therefore, identifying the main constraints in different states will improve the value of the business.

The amount the co-op should borrow from the bank is exactly the shortage of capital from product processing. If the co-op possesses sufficient capital, then it should not borrow anything. This conclusion is consistent with the “pecking order” in finance that internal sources of funds are used prior to using external funds. Li et al. (2013) reach a similar conclusion in their work on IOFs. This chapter has therefore shown that the “pecking order” theory applies in both co-ops and IOFs, although they have different capital structures. However, they differ in the payments to farmers, as discussed below.

Theorem 5.1 presents the existence of a threshold for the capital position within the co-op. This threshold results from the relaxation of the constraint $v + pb \leq h(s, \tilde{y})$. Since the cost of borrowing is higher than using internal working capital, as shown in Lemma 5.4, without this constraint, the decisions of $v_n$, $b_n$, and $q_n$ remain constant in $s$. Note that this result follows from the assumption that there is no carry-over inventory.
The following proposition presents the behaviour of the co-op in terms of borrowing and the payment to farmers in different financial conditions.

**Proposition 5.4.** For each $n = 1, 2, \ldots, N$,

1. When $s_n \geq \tilde{s}_n(\tilde{y}_n)$, the optimal borrowing $b_n = 0$, so that the optimal retained capital is $v_n \geq c(q_n)$ and the payment to farmers is $w_n \leq (1 + \theta_1)s_n - c(q_n)$;
2. When $s_n < \tilde{s}_n(\tilde{y}_n)$, if the optimal policy has $b_n > 0$, then the optimal retained capital is $v_n = h(s_n, \tilde{y}_n) - \rho b_n$, and hence the payment to farmers is zero.

**Proof of Proposition 5.4.**

The problem is to

$$\max \{ L(v, b, q) + \beta E(V_{n+1}(v_n + R(q_n) - c(q_n))) : v_n + \rho b_n \leq h(s_n, \tilde{y}_n), v_n + b_n \geq c(q_n), \quad b_n \geq 0, 0 \leq q_n \leq y_n \}.$$  

Therefore, a Lagrangian function is as follows:

$$L(v, b, q) + \beta E(V_{n+1}(v_n + R(q_n) - c(q_n))) + \mu_1(h(s_n, \tilde{y}_n) - v_n - \rho b_n) + \mu_2(v_n + b_n - c(q_n)) + \mu_3b_n + \mu_4(y_n - q_n) + \mu'_4q_n.$$  

Assuming differentiability for ease of exposition, the Karush-Kuhn-Tucker conditions are as follows:

$$\frac{\partial L(v, b, q)}{\partial v} + \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v} - \mu_1 + \mu_2 = 0$$  

(5.11)

$$\frac{\partial L(v, b, q)}{\partial b} - \mu_1\rho + \mu_2 + \mu_3 = 0$$  

(5.12)

$$\frac{\partial L(v, b, q)}{\partial q} + \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial q}(R'(q) - c'(q)) - \mu_2c'(q) - \mu_4 + \mu'_4 = 0$$  

(5.13)

1) If the capital is enough, then $\mu_1 = 0$ which implies the constraint of $v_n + \rho b_n \leq h(s_n, \tilde{y}_n)$ is relaxed, as indicated in Theorem 5.1. Thus Eq.(5.11) implies that $\mu_2 = -\frac{\partial L(v, b, q)}{\partial v} - \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v} (\text{which is clearly non-negative}).$

Put $\mu_2 = -\frac{\partial L(v, b, q)}{\partial v} - \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v}$ into Eq.(5.12), then $\mu_3 = \frac{\partial L(v, b, q)}{\partial v} - \frac{\partial L(v, b, q)}{\partial b} + \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v}$, and $\mu_3 > 0$ because $\frac{\partial L(v, b, q)}{\partial v} - \frac{\partial L(v, b, q)}{\partial b} > 0$ (proved in Lemma 5.4) and $\frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v} \geq 0$. Since $\mu_3 > 0$, $b_n = 0$ and hence $v_n \geq c(q_n)$.  

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2) If the capital is not large, from Eq.(5.11) and Eq.(5.12), then,
\[
(1 - \rho)\mu_1 = \frac{\partial L(v, b, q)}{\partial v} - \frac{\partial L(v, b, q)}{\partial b} + \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v} - \mu_3.
\] (5.14)

Note that \( \mu_1 \) represents the shadow price of the constraint \( v_n + \rho b_n \leq h(s_n, \tilde{y}_n) \), if \( \mu_1 > 0 \), then \( v_n + \rho b_n = h(s_n, \tilde{y}_n) \), and hence \( w_n = 0 \). Otherwise, it is not. However, \( \mu_1 \) depends on \( \mu_3 \) and \( \beta \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v} \). It is impossible to identify the Lagrange multiplier of each constraint; however, there are some identified situations:

(1) when \( \mu_3 = 0 \) then \( \mu_1 > 0 \) because \( \frac{\partial L(v, b, q)}{\partial v} - \frac{\partial L(v, b, q)}{\partial b} > 0 \) (proved in Lemma 5.4) and \( \frac{\partial EV_{n+1}(v + R(q) - c(q))}{\partial v} \geq 0 \). In other words, when \( b_n > 0 \), then \( v_n + \rho b_n = h(s_n, \tilde{y}_n) \), and thus \( w_n = 0 \).

(2) when \( \mu_1 = 0 \), then \( \mu_3 > 0 \). This follows from Eq.(5.14) because \( \frac{\partial L(v, b, q)}{\partial v} - \frac{\partial L(v, b, q)}{\partial b} > 0 \).

In other words, if \( v + \rho b < h(s_n, \tilde{y}_n) \) which implies \( w > 0 \), then \( b = 0 \).

Together from (1) and (2), It can be concluded that in case of capital shortage, stopping payments to farmers is preferable to borrowing from the bank.

The first part of Proposition 5.4 states that when the capital is sufficient, the optimal strategy is to borrow nothing, and retain at least enough capital for processing. This is quite intuitive since borrowing always introduces a cost. The second part of the proposition analyses the situation when capital is not in excess. Note that it does not mean that when the capital is less than the threshold value, the co-op would stop paying farmers. It underscores that in the case when the co-op has to start borrowing from the bank because of capital shortage, the co-op should stop paying farmers, and instead, retain as much money as possible for business operations. On the contrary, if the co-op is still able to afford to pay farmers \((w > 0)\) (for example, when it is better to compromise the processing quantity instead of borrowing), then it should not borrow from the bank. In other words, retaining the payments to farmers is superior to borrowing when there is a shortage of capital. This makes sense because retaining the farmers’ payments is part of the accumulation of internal working capital, which is cheaper and also safer than borrowing. It also implies that when the financial condition is not good enough, it is possible for the co-op to stop paying farmers.
who, in principle, are obligated to sacrifice returns, at least temporarily, to make sure the co-op has enough working capital in the near future.

Notice that $\hat{s}_n(\tilde{y})$ and $V_n(s, \tilde{y})$ are functions of $\tilde{y}$. This implies that the value function and threshold value are influenced by the yield vector, although in an ambiguous way. Given that $V_n(s, \tilde{y})$ is not monotone in $\tilde{y}$, further results regarding $\tilde{y}$'s influence appear unlikely. The astute reader may also note that this chapter has not shown monotonicity results for $v_n(s, \tilde{y})$ and $q_n(s, \tilde{y})$ in $s$. This is because the traditional method for showing such results requires supermodularity of $J_n(\cdot)$, which requires the transition function $s_{n+1} = v_n + R(q_n) - c(q_n)$ to be monotone, which it is not true for general $R(\cdot) - c(\cdot)$. While it is likely the case that both $v_n$ and $q_n$ increase in $s$, a proof of this result eludes the author, thus far. One possibility is to assume linear revenues and costs, which would introduce the monotonicity required for the transition functions, however it would be a relatively restrictive assumption.

In conclusion, this section characterises the optimal policy of both operational and financial decisions. The results confirm the influence of co-op’s objective on financing behaviours in the way of retaining earnings. If the co-op positions itself as a profit-maximising business, it will accumulate as much money as possible, and cut down payments to farmers like an IOF. This conclusion implies a potential principal-agent problem within co-ops. Conversely, if the co-op acts more on behalf of farmers, the optimal policy is such that: (1) The optimal processing quantity in each period is to maximise profit; (2) the optimal amount of borrowing is to meet the shortage of capital; and (3) the decisions on retained earnings and farmers’ payments depend on financial conditions and market profit.

Compared with papers for IOFs, it can be seen that in both IOFs and co-ops, the cheapest way to raise capital is internally, instead of loans. However, there is a difference between co-ops and IOFs in terms of farmers’ payments. In IOFs, farmers are treated as profit competitors, the firm will pay farmers as little as possible to maximise its own profit (usually subject to participation constraints). However, in co-ops, payments to farmers depend on the co-op’s financial condition. When the co-op is not in a good financial condition, the co-op may not pay farmers, instead, use all capital available to process product. When
the financial condition is healthy enough, the co-op retains only sufficient internal working
capital as is needed for processing, and distributes the surplus to farmers.

5.4 Optimal Myopic Policies under Deterministic Yield

This section considers the case of deterministic yield. It turns out that, with this assump-
tion, the myopic policy becomes optimal. Myopic policies are important for several reasons.
First of all, they are simple to implement and easy to compute because they are based on
maximising $L(\cdot)$ rather than $J(\cdot)$. Second, they form a reference point for the stochastic
problem. Finally, for the problem of this chapter, they are in fact optimal for deterministic
yield, hence likely close to optimal for low yield variability.

This section assumes that yield is fixed at some value $y \in [0, y]$. Further, the random
market profit at the optimal $q^*$ (defined below) is assumed to be always positive, that is
$R(q^*) - c(q^*) \geq 0$. Without this assumption, the myopic policy may not be optimal because
it needs to hedge against the future risk of a negative cash position.

With deterministic yield, the state variables reduce to a single scalar $s$ and this section
has $\Delta e_n = 0$ for all $n$. The problem is now:

$$V_n(s) = -(1 - \alpha)\theta_2 s^- + \sup_{(v, b, q) \in A(s)} \left\{ L(v, b, q) + \beta E[V_{n+1}(v + R(q) - c(q))] \right\}, \quad \text{(5.15)}$$

where $A(s)$ is defined by (5.6) with $h(s, \bar{y})$ replaced by $(1 + \theta_1)s - \theta_2 s^-$. According to the analysis in the last section, when the capital is large enough, the
constraint $v + \rho b \leq (1 + \theta_1)s - \theta_2 s^-$ is relaxed. Therefore, this section defines the relaxed
myopic problem as

$$\max_{v, b, q} \{ L(v, b, q) : v + b - c(q) \geq 0; b \geq 0; 0 \leq q \leq y \}$$

and $(v^*, b^*, q^*) \equiv \arg \max_{v, b, q} \{ L(v, b, q) : v + b - c(q) \geq 0; b \geq 0; 0 \leq q \leq y \}$. Recall that $r(q_n)$ denotes
expected revenue.

Proposition 5.5. Let

$$\hat{q} \equiv \arg \max_{0 \leq q \leq \bar{y}} \left( [\alpha + \beta - 2\alpha\beta + \theta_1 \beta(1 - \alpha)]r(q) - (1 - \alpha)c(q) \right). \quad \text{(5.16)}$$

1) If $y > \hat{q}$, the optimal policy of the relaxed myopic problem is characterised by $(v^*, b^*, q^*) = (c(\hat{q}), 0, \hat{q})$; otherwise, the optimal myopic policy is $(v^*, b^*, q^*) = (c(y), 0, y)$.

2) The optimal payment to farmers $w = (1 + \theta_1)s - c(q^*)$. 

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Proof of Proposition 5.5.

Since both \( r(q) \) and \( c(q) \) are continuous on a closed interval \([0, y]\) (yield \( y \) is bounded by \([0, \hat{q}]\) as assumed), then there exists a point of global maximum \( \hat{q} \). Considering the constraints that \( b \geq 0 \), \( v + b \geq c(q) \) and \( 0 \leq q \leq y \), the Lagrangian function with Lagrange multipliers \( \mu_1, \mu_2, \mu_3, \mu'_3 \geq 0 \) is constructed as follows:

\[
L(v, b, q) + \mu_1 b + \mu_2 (v + b - c(q)) + \mu_3 (y - q) + \mu'_3 q.
\] (B1)

The Karush-Kuhn-Tucker (KKT) conditions are as follows:

\[
\frac{\partial L(v, b, q)}{\partial v} + \mu_2 = 0; \quad (B2)
\]

\[
\frac{\partial L(v, b, q)}{\partial q} - \mu_2 c'(q) - \mu_3 + \mu'_3 = 0; \quad \text{and} \quad (B3)
\]

\[
\frac{\partial L(v, b, q)}{\partial b} + \mu_1 + \mu_2 = 0. \quad (B4)
\]

Set

\[
\mu_1 = \alpha + \beta - 2\alpha \beta + (\beta \theta_1 + \rho - 1)(1 - \alpha); \quad (B5)
\]

\[
\mu_2 = -[(\alpha + \beta - 2\alpha \beta + (\beta \theta_1 - 1)(1 - \alpha)]; \quad (B6)
\]

\[
\mu_3 = [(\alpha + \beta - 2\alpha \beta + \beta \theta_1(1 - \alpha)) r'(q) - (1 - \alpha) c'(q)]; \quad (B7)
\]

and \( \mu'_3 = 0 \). It is easy to show that these satisfy the KKT conditions above. Further, \( \mu_2 > 0 \) because \( \frac{\partial L(v, b, q)}{\partial v} < 0 \), therefore, \( v + b = c(q) \) is binding. Put \( \mu_2 \) into (B4), then \( \mu_1 > 0 \) because

\[
\mu_1 = \frac{\partial L(v, b, q)}{\partial v} - \frac{\partial L(v, b, q)}{\partial b} > 0,
\]

thus \( b = 0 \) is binding. Furthermore, if \( \mu_3 > 0 \) when \( q = y \), then the optimal \( q^* = y \), otherwise, \( q^* = \hat{q} \) when \( [(\alpha + \beta - 2\alpha \beta + \beta \theta_1(1 - \alpha)) r'(\hat{q}) - (1 - \alpha) c'(\hat{q})] = 0 \).

In conclusion, the optimal \( v^*, b^*, q^* \) satisfies \( b^* = 0 \), \( v^* = c(q^*) \), and \( q^* \) equals \( y \) or \( \hat{q} \). Thus, \( w^* = (1 + \theta_1) s - v^* - \rho b^* = (1 + \theta_1) s - c(q^*) \).

The myopic policy in Proposition 5.5 stipulates that, without the liquidity constraint \( v + \rho b \leq (1 + \theta_1) s - \theta_2 s^- \), the optimal processing quantity depends only on the supply constraint and marketing parameters. In other words, with enough financial support and market support, the co-op will process as much product as is optimal. Furthermore, with enough capital, it retains just enough internal working capital to cover the processing cost.
without borrowing. This is easy to explain because the cost of borrowing is higher than the cost of internal working capital. The payment to farmers is the difference between the capital available and the processing cost.

Define \( \hat{s} \) as the cash threshold, which equals \( \hat{s} = v^* + \rho b^* \). Then only one condition is sufficient for the relaxed myopic policy to be feasible, namely that \((1 + \theta_1)s_n - \theta_2 s_n^r \geq \hat{s}\) for all \( n \). This implies the following proposition.

**Proposition 5.6.** Let \( V_{N+1}(s_{N+1}) = 0 \), then if \( s_k \geq \hat{s} \) for some \( k \), \((v^*, b^*, q^*)\) is optimal for all \( k \leq n \leq N \).

**Proof of Proposition 5.6.**

If \( s_n \geq \hat{s} = c(q^*) \) for some \( n \), then \((v_n, b_n, q_n) = (v^*, b^*, q^*)\) is feasible because \( v^* = c(q^*), b^* = 0 \) satisfies all constraints in (5.6). Therefore, \( s_{n+1} = v_n + R(q_n) - c(q_n) = R(q^*) \geq c(q^*) \) (by assumption), so \((v_{n+1}, b_{n+1}, q_{n+1}) = (v^*, b^*, q^*)\) is feasible as well. Furthermore, it is optimal for the following reason. Define the inductive hypothesis so that for some \( n \), \( s_n \geq \hat{s}, \)

\[
V_n(s_n) = L(v^*, b^*, q^*)(1 + \beta + \ldots + \beta^{N-n-1})
\]

and for \( s_n < \hat{s} \),

\[
V_n(s_n) \leq L(v^*, b^*, q^*)(1 + \beta + \ldots + \beta^{N-n-1}).
\]

This holds for \( n = N \) because for \( s_N \geq \hat{s} \), it has \( V_N(s_N) = L(v^*, b^*, q^*) \) because \( V_{N+1}(s_{N+1}) = 0 \) by assumption and for \( s_N < \hat{s} \) the additional constraint can only decrease profit. Now suppose the hypothesis holds for period \( n + 1 \). Then, for \( s \geq \hat{s} \),

\[
V_n(s_n) = L(v^*, b^*, q^*) + \beta E[V_{n+1}(s_{n+1})] = L(v^*, b^*, q^*)(1 + \beta + \ldots + \beta^{N-n-1}).
\]

For \( s < \hat{s} \) profit is bounded above by this term.

\[ \Box \]

This section takes \( V_{N+1}(s_{N+1}) = 0 \) for expositional convenience in Proposition 5.6; a necessary condition is that there is no incentive to build cash up towards the end of the horizon.

The myopic policy stipulates that the co-op retains just enough \( v^* \) to process quantity \( q^* \) and distributes payments to farmers to bring the capital position to a random final
level $R(q^*)$. It therefore resembles the base-stock policy in inventory management. Thus, if $s_1 \geq \hat{s}$ then $v_n = v^*$, $b_n = b^*$ and $q_n = q^*$ for all $n$. That is, if the initial capital is large enough, then an optimal decision rule is determined by three scalars $(v^*, b^*, q^*)$. Figure 5.3 illustrates the capital position when $s_1 \geq \hat{s}$.

Corollary 5.1. Under the assumption in Proposition 5.6, $\hat{s}$ is non-decreasing in $y$.

Proof of Corollary 5.1.

From Proposition 5.5, one can know that when $y < \hat{q}$, then $(v^*, b^*, q^*) = (c(y), 0, y)$, thus the threshold value $\hat{s} = v^* + \rho b^* = c(y)$, which is non-decreasing in $y$ when $y < \hat{q}$. For $y \geq \hat{q}$ $\hat{s} = c(\hat{q})$, which is constant in $y$.

This corollary implies that a yield shortage can be a bottleneck in the long run. The co-op can accumulate capital by internal savings to release the financial constraint, but it cannot release the supply constraint, especially when it cannot control the supply. This study does not allow the possibility that the co-op purchases product from outside farmers. However, if it can, assuming that the per unit cost sourced from outside farmers is $c_o > 0$, yields the following corollary.

Corollary 5.2. Let $\mu$ be the shadow price of the supply constraint, where $\mu = [\alpha + \beta - 2\alpha\beta + \beta\theta_1(1 - \alpha)]r'(y) - (1 - \alpha)c'(y)$,

1. if $\mu > c_o$, it is profitable to purchase from outside farmers, otherwise, it is not; and
2. Suppose $\mu > c_o$, the optimal total processing quantity satisfies: 
\[
(\alpha + \beta - 2\alpha\beta + \beta\theta_1(1-\alpha))r'(q) - (1-\alpha)c'(q) = c_o \text{ and the outside amount purchased equals } q - y.
\]

Proof of Corollary 5.2.
Based on the proof of Proposition 5.5, 
\[
\mu = (\alpha + \beta - 2\alpha\beta + \beta\theta_1(1-\alpha))r'(q) - (1-\alpha)c'(q).
\]
If $q = y$ is binding, then $\mu > 0$, and the shadow price is $(\alpha + \beta - 2\alpha\beta + \beta\theta_1(1-\alpha))r'(y) - (1-\alpha)c'(y)$.

If $((\alpha + \beta - 2\alpha\beta + \beta\theta_1(1-\alpha))r'(y) - (1-\alpha)c'(y) > c_o$, then the optimal quantity satisfies 
\[
((\alpha + \beta - 2\alpha\beta + \beta\theta_1(1-\alpha))r'(q) - (1-\alpha)c'(q) = c_o, \text{ and the outside amount equals } q - y.
\]

This corollary evaluates the possibility of outside sourcing. If the marginal cost of purchasing from outside is less than the marginal profit, it is profitable to purchase from outsiders, otherwise, it is not. However, since the marginal cost is larger than 0 ($c_o > 0$), one can see that the optimal processing quantity is still less than the optimal quantity without the supply constraint. This implies that purchasing from outsiders is more expensive than supply by cooperative members, which is quite sensible in reality. Note that, it still might be worth considering such a strategy when the co-op suffers from a supply shortage.

The following proposition states that successive payments and the target cash position comprise a sequence of independent and identically distributed random vectors.

**Proposition 5.7.** Assume that revenues in each period are independent and let initial cash $s_1 = R(q^*)$ for some random realisation of revenue $R(\cdot)$. Then, if $v_n = v^*$, $b_n = b^*$, and $q_n = q^*$ for all $n = 1, \ldots, N$, then $(w_i, s_i)$ for $i = 2, \ldots N$ form a sequence of independent and identically distributed random vectors that are perfectly correlated with the previous period’s random market revenue, so that $w_i = (1 + \theta_1)R(q^*) - c(q^*)$ and $s_i = R(q^*)$, where the same realisation of $R(q^*)$ is used for both variables.

Proof of Proposition 5.7.
Utilising $w_n = (1 + \theta_1)s_n - \theta_2s_n - v_n - \rho b_n$ with $v_n = c(q^*)$, $b_n = 0$, and $q_n = q^*$ for $n = 1, 2, \ldots N$, if $n > 1$ then $w_n = (1 + \theta_1)s_n - \theta_2s_n - c(q^*)$. Since $s_n = v^* + R(q^*) - c(q^*) = R(q^*)$, then $w_n = (1 + \theta_1)R(q^*) - c(q^*)$. The fact that the revenue realisations are independent in each period completes the proof.

\[
\square
\]
A consequence of Proposition 5.7 is that each period’s payment to farmers is a reflection of the previous period’s market performance and not based on expected values. The payments are dependent on operational decisions \( q^* \), market structure \( R(\cdot) \), \( c(\cdot) \), and financial characteristics \( \theta_1 \). This correlation between farmers’ payments and the business’s market performance is one that exists in co-ops more than in IOFs.

In conclusion, this section analyses the optimal policy under deterministic yield, and also characterises the capital position when cash is, or is not, in shortage. Deterministic yield provides sufficient conditions for the application of the optimal myopic policy, where the cash position simply needs to be larger than a threshold value.

Although the assumption of deterministic yield sounds less reasonable, it can be justified in certain settings. The first justification is the pooling of supply. Each individual suffers from variability in yield, while the total supply summing from all individuals is less inherently variable (as measured by the coefficient of variation). This is one of many operational situations where there are benefits of pooling. For example, investors diversify portfolios in finance, and manufacturers find multiple suppliers to reduce supply risk, etc.

Another more applicable setting for deterministic yield is when the co-op can actively control its product submissions. In practice, there are several strategies adopted to achieve this purpose, especially in the value-added processing of agricultural commodities. One is to change from open membership to closed membership, under which the number of members depends upon the proposed capacity of the co-op’s operations. Another strategy is to issue delivery rights where members purchase the right to deliver a specific quantity of the commodity each year. Take Fonterra for example, it sells delivery rights to members to raise upfront capital. Delivery rights give a member the right and obligation to deliver a given quantity of goods. Its rights and obligations are two sides of the same coin: members receive a guarantee that they will always have a buyer for their product, which is especially crucial for dairy producers who have daily deliveries. On the other hand, the co-op has a stable supply of raw material, also important for a food processing co-op.
This section identifies the potential risks associated with random yield and random market revenue. In reality, co-ops, like IOFs, also suffer from debt-default risk especially when the market is volatile.

In this model, the capital in hand at the end of the period by applying the optimal decision is $v_n(s, \tilde{y}) + R(q_n(s, \tilde{y})) - c(q_n(s, \tilde{y}))$. If $v_n(s, \tilde{y}) + R(q_n(s, \tilde{y})) - c(q_n(s, \tilde{y})) \geq 0$, then the short-term loan (if any) is fully repaid. Define

$$p_n^b(s, \tilde{y}) = \mathbb{P} (v_n(s, \tilde{y}) + R(q_n(s, \tilde{y})) - c(q_n(s, \tilde{y})) \geq 0),$$

which is the probability that debt-default does not occur in period $n + 1$ when the initial state in period $n$ is $(s, \tilde{y})$. Therefore, the debt-default is more likely to happen when the capital position is low, which is dependent on $(s, \tilde{y})$. Theorem 5.1 has shown that $b_n(s, \tilde{y}) = [c(q_n(s, \tilde{y}) - v_n(s, \tilde{y})]^+$. Thus, if $b_n = 0$, then $p_n^b(s, \tilde{y}) = 1$.

Note that, the probability of debt-default will be decreasing in $s$ if $v_n(s, \tilde{y}) + R(q_n(s, \tilde{y})) - c(q_n(s, \tilde{y}))$ is stochastically increasing in $s$. However, as noted in Section 5.3, this chapter has not been able to show monotonicity of actions $v_n$ and $q_n$ in $s$. It is well-known that debt-default is likely to happen when the market price falls dramatically. However, one can also see that the probability of debt-default is also influenced by the financial condition and the yield supply. In this sense, both the co-op and farmers share the responsibility of debt-default risk.

Furthermore, from the viewpoint of capital lenders, it is better to have symmetric information on financial conditions as well as patronage before they make decisions on loan amount or loan rate to a cooperative. In the paper of Babich and Sobel (2004), the loan rate is assumed to be dependent on the firm’s capacity and current assets. However, the above implies that the loan rate for a co-op could also be dependent on patronage, which may be related to members’ involvement as well as weather conditions. Exploring this further is left as a subject for future research.

Another financial risk identified is defined as redemption risk, when the co-op is unable to redeem members’ shares as required by the co-op’s policy. In an agricultural supply
chain, the most notable risk is weather-related such as drought, excess rainfall or temperature, or natural disasters (Jaffee et al. 2010). The consequences of these weather-related risks are mostly associated with patronage reduction, hence capital reduction due to the proportionality principle. Therefore, this chapter considers the possibility of redemption risk. This risk commonly but uniquely exists in co-ops, where they are required to redeem members’ equity when the patronage greatly drops or when a large number of members leave. For example, in 2008, when drought, coupled with the effects of the global financial crisis, meant Fonterra had to write cheques of around a total of $742 million to over-invested farmers (Gray 2012).

The condition for redemption default is:

\[
(1 + \theta_1)(v_n + R(q_n) - c(q_n)) - \theta_2(v_n + R(q_n) - c(q_n)) - \Delta v_{n+1} < 0. \tag{5.17}
\]

In this situation, there is insufficient liquidity to redeem members’ equity immediately. Define \( p_r^n(s, \tilde{y}) \) be the probability that redemption-default does not occur in period \( n+1 \) when the state in period \( n \) is \((s, \tilde{y})\), so that

\[
p_r^n(s, \tilde{y}) = \mathcal{P} \left( (1 + \theta_1)[v_n + R(q_n) - c(q_n)] - \theta_2[v_n + R(q_n) - c(q_n)] - (\tilde{y}^1 - \tilde{y}^m)/m \geq 0 \right).
\]

Note the explicit dependence of \( v_n \) and \( q_n \) on \((s, \tilde{y})\) has been dropped in the above but they should be read as given.

The monotonicity of \( p_r^n(\cdot) \) in \( s \) depends on the monotonicity of \( v_n(s, \tilde{y}) + R(q_n(s, \tilde{y}) - c(q_n(s, \tilde{y})) \). The following proposition presents the relationship between the probability of loan-default risk and redemption-default risk.

**Proposition 5.8.**

1. If there is no loan default in period \( n+1 \), that is \( v_n + R(q_n) - c(q_n) \geq 0 \), when \( y_n \) is low enough compared with its history such that \( y_n < y_{n-m} - m(1 + \theta_1)(v_n + R(q_n) - c(q_n)) \), then there is a redemption risk, otherwise not; and

2. If there is a loan default in period \( n+1 \), that is \( v_n + R(q_n) - c(q_n) < 0 \), when \( y_n \) is high enough compared with its history such that \( y_n \geq y_{n-m} - m(1 + \theta_1 + \theta_2)(v_n + R(q_n) - c(q_n)) \), then there is no redemption risk, otherwise not.
Proof of Proposition 5.8.

If \( v_n + R(q_n) - c(q_n) \geq 0 \), then Eq.(5.17) becomes \((1 + \theta_1)[v_n + R(q_n) - c(q_n)] + \Delta e_{n+1} < 0\). As long as \( \Delta e_{n+1} < -(1 + \theta_1)[v_n + R(q_n) - c(q_n)] \), there is redemption risk. Recall that \( \Delta e_{n+1} = \frac{y_n - y_{n-m}}{m} \), which implies \( y_n < y_{n-m} - m(1 + \theta_1)[v_n + R(q_n) - c(q_n)] \). Conversely, if \( v_n + R(q_n) - c(q_n) < 0 \), then Eq.(5.17) becomes \((1 + \theta_1 + \theta_2)[v_n + R(q_n) - c(q_n)] + \Delta e_{n+1} < 0\). As long as \( \Delta e_{n+1} \geq -(1 + \theta_1 + \theta_2)[v_n + R(q_n) - c(q_n)] \), there is no redemption risk, under which \( y_n \geq y_{n-m} - m(1 + \theta_1 + \theta_2)[v_n + R(q_n) - c(q_n)] \).

This proposition states that even if there is no risk of loan default, there is still a risk of redemption default if the patronage is low, and the co-op has to redeem a large number of member shares. However, if there is a risk of loan default, the co-op may not suffer from redemption risk if the patronage is high enough.

One implication this chapter can derive is the influence of \( m \), the base period selected in the base capital plan. In the first case, when there is no loan default, increasing \( m \) can reduce the redemption risk because \( y_{n-m} - m(1 + \theta_1)(v_n + R(q) - c(q_n)) \) is decreased. However, in the second case, increasing \( m \) will increase the probability of redemption risk because \( y_{n-m} - m(1 + \theta_1 + \theta_2)(v_n + R(q) - c(q_n)) \) is increased. It is commonly known that the smoothing effect of a longer base period could benefit situations with a commodity where yield may change significantly year to year, or where the influence of weather-related factors is significant (Royer 2003). This chapter, from the market side, suggests that when the market condition is stable, a longer base period can help reduce redemption risk. However, when the market condition is fluctuating, a shorter base period is preferred because it makes the programme and the adjustment of capital levels more responsive to changes in market levels.

5.6 Numerical Experiments

This section describes numerical experiments to study the proposed theoretical model and the analytical results. Section 5.4 has derived optimal myopic policies under deterministic
yield and has also visualised the cash position; therefore, Section 5.6.1 assumes deterministic yield and derives numerical results to validate the model. Furthermore, in the theoretical model, this chapter didn’t show any explicit influence of random yield on the value function and optimal decisions because it is analytically intractable due to multidimensional randomness in the yield vector. Therefore, Section 5.6.2 undertakes numerical experiments with an assumption on the distribution of random yield, aiming to provide a visual impression of how uncertain yield influences the process.

There are a wide-range of algorithms for finding the value function and an optimal policy in a dynamic programme, and this chapter uses the Value Iteration algorithm, programmed in MATLAB R2016. Furthermore, this chapter uses the data sourced from the case study in Chapter 6 — Fonterra. For computational simplicity, this section assumes deterministic market revenue throughout the experiments. This assumption would not compromise the results significantly because this chapter didn’t focus too much on the random market revenue in the analytical model. The basic version of the Value Iteration algorithm is given in Table 5.1.

<table>
<thead>
<tr>
<th>Table 5.1: Value Iteration Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 0:</strong> Initialisation:</td>
</tr>
<tr>
<td><strong>Step 0a:</strong> Set $V_0(s) = 0$ for $\forall s \in S$.</td>
</tr>
<tr>
<td><strong>Step 0b:</strong> Fix a tolerance parameter $toler &gt; 0$, a maximum iteration number $maxite &gt; 1$, and an initial different $dif$</td>
</tr>
<tr>
<td><strong>Step 0c:</strong> Set $n=1$.</td>
</tr>
<tr>
<td><strong>Step 1:</strong> For each $s \in S$ compute:</td>
</tr>
<tr>
<td>$V_1(s) = \max_{(v,b,q) \in A_n(s)} \left( L_n(v, b, q) + \beta E(V_0(s')) \right)$</td>
</tr>
<tr>
<td><strong>Step 2:</strong> If there is no significant improvement in the value function $|V_1 - V_0| &lt; toler$, STOP; otherwise, let $V_0 = V_1$ and $n = n+1$, if $n \leq maxite$, return to <strong>Step 1</strong>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.2: Computational Parameter Initialisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue Function $R(q) = -0.057q^2 + 167.65q - 103,080$</td>
</tr>
<tr>
<td>Cost Function $c(q) = 7.6q - 1841$</td>
</tr>
<tr>
<td>Weight Factor $\alpha = 0.3$</td>
</tr>
<tr>
<td>Capital Growth Rate $\theta_1 = 2.68%$</td>
</tr>
<tr>
<td>Penalty Rate $\theta_2 = 4%$</td>
</tr>
<tr>
<td>Discount Factor $\beta = 93.5%$</td>
</tr>
<tr>
<td>Interest Rate $\rho = 7.69%$</td>
</tr>
</tbody>
</table>
The discount factor is estimated to be $\beta = 93.5\%$ (see Chapter 6), and the interest rate is estimated as the average rate in the past several years, that is $\rho = 7.69\%$. The penalty cost for delayed repayment is calculated on the amount not paid and the additional interest rate which is estimated as $\theta_2 = 4\%$ (sourced from Inland Revenue). The value of $\alpha$ is an assumed parameter. This section chooses $\alpha = 0.3$ to indicate the case where the co-op values more on the farmers’ benefit. The revenue and cost function is estimated from Fonterra’s financial statements. Note that the revenue function is assumed to be deterministic (see Chapter 6) and the cost function is estimated without farmers’ payments. From the cost function, one may notice that it is illogical to have a negative cost when $q = 0$, i.e., $c(0) = -1841$. However, in reality, the milk production is normally larger than a value (e.g., 242.2), above which the overall cost is positive. All parameters and functions are presented in Table 5.2.

### 5.6.1 Deterministic Yield

Theorem 5.1 proves that the optimal policy for the processing quantity is constrained by the supply constraint and the financial constraint. Without those constraints, the optimal processing quantity is around 1400 with the given revenue and cost functions assumed above. In order to assess the influence of yield, this section set $y$ to be either 1250 or 1500, where $y = 1250$ represents the case where yield is in shortage, while $y = 1500$ represents the case of excessive yield.
Figure 5.4: Deterministic Yield with $y=1250$ and $y=1500$

Figure 5.4 depicts the value functions and optimal policies for two cases. One can see that the value function is non-decreasing in the cash position as stated in Proposition 5.1. Furthermore, it also verifies Proposition 5.2 and Theorem 5.1 that the value function is a concave function, and it becomes stable when the cash position is higher than a threshold. The threshold value $\hat{s}$ lies around the point of 5759 when $y = 1250$ and 8693 when $y = 1500$.

Other charts in Figure 5.4 illustrate the optimal policies of $(v, b, q)$, and they can be explained from two aspects: when $s \geq \hat{s}$ and when $s < \hat{s}$.

(1) When $s \geq \hat{s}$, the optimal decisions $(v, b, q)$ do not change with the cash position any more, which demonstrates the third part of Theorem 5.1. Furthermore, when the co-op has enough capital, the optimal borrowing is reduced to zero, and the processing quantity is released from financial constraint, only limited to yield realisation and market constraint, hence stabilises at either $y$ or $\hat{q}$. 
When $s < \hat{s}$, one can see that in general, the value of $v$ is increasing, $b$ is decreasing, and $q$ is increasing in the cash position. Note that, this chapter has not shown monotonicity results for $v, b, q$ because it does not want to strengthen the assumption on the concavity of $R(\cdot) - c(\cdot)$ into linearity. One can observe two (perhaps more) distinct spikes in $q$, and also in $v$ and $b$ (although not that clearly) when $y = 1500$. The reason has been given in Proportion 5.4, that when the capital is not in excess, there are trade-offs among the decisions on how much product to process, how much money to retain, and how much to borrow. If the marginal profit of the product is higher than both the costs of borrowing and internal working capital, then the co-op would increase the processing quantity by borrowing or by reducing payments to farmers. Otherwise, the co-op may reduce borrowing or increase farmers’ payments by limiting processing quantity.

Having solved the model by the Value Iteration, there are a collection of points for the state variables and a decision rule that relates the control variables to this collection of points. In other words, there is Lagrange data that can be used to get a continuous decision rule, and this decision rule can be used to simulate the model. Figure 5.5 shows a simulation of the cash position under deterministic yield. Section 5.4 proposed the optimal myopic policy in Proposition 5.6, which stipulates that if the cash position becomes high enough, the co-op retains just enough working capital to process optimal quantity and distributes payments to farmers to bring the cash position to a random final level $R(q^*)$. Figure 5.5 clearly validates this conclusion since the cash position achieves a stabilised target level after several periods. Note that; the target level is randomised by the uncertain market price which is assumed away in the numerical experiments. Furthermore, since the cash position is increasing in several initial periods, one can deduce that if the cash position is not high enough, the co-op shall accumulate a cash position up to the target level where the myopic policy becomes optimal.
Table 5.3: Numerical Results vs. Analytical Results

<table>
<thead>
<tr>
<th>y=1,250</th>
<th>Comparison</th>
<th>v*</th>
<th>b*</th>
<th>q*</th>
<th>s*</th>
<th>V*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical Results</td>
<td>7,659</td>
<td>0</td>
<td>1,250</td>
<td>17,420</td>
<td>64,300</td>
<td></td>
</tr>
<tr>
<td>Numerical Results</td>
<td>7,714</td>
<td>0</td>
<td>1,250</td>
<td>17,475</td>
<td>64,287</td>
<td></td>
</tr>
<tr>
<td>y=1,500</td>
<td>Analytical Results</td>
<td>8,693</td>
<td>0</td>
<td>1,386</td>
<td>19,786</td>
<td>73,357</td>
</tr>
<tr>
<td>Numerical Results</td>
<td>9,143</td>
<td>0</td>
<td>1,408</td>
<td>20,254</td>
<td>73,094</td>
<td></td>
</tr>
</tbody>
</table>

To verify the value-iteration results, this section compares the numerical results to the analytical results in Table 5.3. From \( [(\alpha + \beta - 2\alpha\beta + \beta \theta (1 - \alpha))] R'(q) - (1 - \alpha)c'(q) = 0 \), one can get \( \hat{q} = 1386 \). When \( s \geq \hat{s} \), the processing quantity equals \( q^* = \min\{\hat{q}, y\} \), \( v^* = c(q^*) \), and \( b^* = 0 \). Further, one can estimate the value of \( s^* = R(\hat{q}) \) and \( V^* = \frac{L(v^*, b^*, q^*)}{1 - \beta} \). As indicated in Table 5.3, the numerical results are very close to the analytical results. The minor discrepancy stems from the discretisation of State Space and Action Spaces.

In the Value Iteration algorithm, one notorious challenge is the “curse of dimensionality” (Rust 1997), under which an optimal policy cannot be computed because the State Space is too large. In this model, there are three control variables \( (v, b, \text{and } q) \), \( m + 3 \) state variables \( (s_n, y_n, y_{n-1}, \ldots y_{n-m-1}) \) and one stochastic random variable \( (Y_{n+1}) \). Each variable has a specific State Space which needs to be discretised. Creating a high resolution grid will provide more precise solutions, but may take longer to converge, because multiple dimensions increase the burden of computation at an exponential rate of growth. Therefore, a very important problem that arises whenever using Value Iteration is the identification
of the State Spaces and Action Spaces. As introduced by Babich and Sobel (2004), one can use the analytical results found for the model to improve computational efficiency; although, that is beyond the scope of this work. All computation parameters are summarised in Table 5.4, and the solution time takes almost 7.5 hours as shown in Figure 5.6.

<table>
<thead>
<tr>
<th>Space</th>
<th>discretisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Space of (s)</td>
<td>(S = [0, 25000])</td>
</tr>
<tr>
<td>Action Space of (v)</td>
<td>(v = [-2000, 12000])</td>
</tr>
<tr>
<td>Action Space of (b)</td>
<td>(b = [0, 8000])</td>
</tr>
<tr>
<td>Action Space of (q)</td>
<td>(q = [0, y])</td>
</tr>
<tr>
<td>Yield</td>
<td>(y = 1, 250) or (y = 1, 500)</td>
</tr>
<tr>
<td>Maximum Number of Iteration</td>
<td>(maxite = 1, 000)</td>
</tr>
<tr>
<td>Tolerance Condition</td>
<td>(toler = 1e - 6)</td>
</tr>
<tr>
<td>Initial Difference</td>
<td>(dif = 100)</td>
</tr>
</tbody>
</table>

Table 5.4: Computation Parameters in Deterministic Yield

5.6.2 Stochastic Yield

This section considers uncertain yield. In the application of the Value Iteration algorithm on the Markov decision process, there are many challenges, and one is to identify the one-step transition matrix. Finding the one-step transition matrix means that all one has to do is to sum over all possible outcomes of random information and add up the probabilities that take us from a particular state-action pair to a particular state (Powell 2007). In many cases, this calculation is impossible especially when the distribution is continuous. Further, it would be an extremely heavy task regarding computation time. However, one condition, that dramatically facilitates the computation is the assumption that the distribution of yield is independent of both the state variable of cash position, \(s\), and the actions of \((v, b, q)\). This can be justified because agricultural products are mainly weather-dependent.

Figure 5.6: Computation Time

5.6.2 Stochastic Yield

This section considers uncertain yield. In the application of the Value Iteration algorithm on the Markov decision process, there are many challenges, and one is to identify the one-step transition matrix. Finding the one-step transition matrix means that all one has to do is to sum over all possible outcomes of random information and add up the probabilities that take us from a particular state-action pair to a particular state (Powell 2007). In many cases, this calculation is impossible especially when the distribution is continuous. Further, it would be an extremely heavy task regarding computation time. However, one condition, that dramatically facilitates the computation is the assumption that the distribution of yield is independent of both the state variable of cash position, \(s\), and the actions of \((v, b, q)\). This can be justified because agricultural products are mainly weather-dependent.
In other words, one can explicitly compute the one-step transition probability matrix if the process of yields across periods is known. Therefore, this section makes two assumptions as follows to make it possible and simple to calculate the one-step transition matrix.

First, this section assumes the uncertain yield follows a two-point distribution. Since the yield is mainly weather-dependent, then the variation of weather is more significant than individual differences. Without considering the regional disparity of farmers, this section can reasonably assume that there are two possible results for the yield: high or low. Assume that $Y$ follows a two-point distribution as follows:

$$Y = \begin{cases} H, & p \\ L, & 1 - p \end{cases}$$

(5.18)

where $H$ represents high yield, while $L$ represents low yield. This assumption implies that, in each year, the probability of a high yield is $p$, while that of a low yield is $1 - p$.

Second, the base period determined in the base capital plan is assumed to be two years. In reality, the base period can range from 1-10 years. For example, Fonterra adopts a base period of 3 years. However, the more years it takes, the higher the dimensions of the State Space shall be in the calculation. For example, if the base period is 3 years where $\tilde{Y} = \{y_n, y_{n-1}, y_{n-2}, y_{n-3}, y_{n-4}\}$, the dimension of the yield vector is $2^5 = 32$; while when the base period is 2 years where $\tilde{Y} = \{y_n, y_{n-1}, y_{n-2}, y_{n-3}\}$, the dimension is reduced by half as $2^4 = 16$. Therefore, to reduce the dimensionality of state, this section chooses a base period as small as possible. However, this reduction would not change the structure of the results.

According to the assumptions stated above, the state variables of $\tilde{Y}$ are now chosen from 16 combinations as shown in Table 5.5. Considering the transition function of $\tilde{Y}$, the transition probability matrix is shown in Table 5.6. For example, if the current yield vector is $\tilde{Y}1 = [L, L, L, L]$, then the possible outcome of yield vector in the next period is $\tilde{Y}1 = [L, L, L, L]$ with the probability of $1 - p$ and $\tilde{Y}9 = [H, L, L, L]$ with the probability of $p$.

Further, the variation of yields influences the capital position by $\Delta e$. The theoretical model assumes that the measurement units for the product yield have been scaled to be
Table 5.5: The State Space of $\tilde{Y}$ (H-high, L-low)

<table>
<thead>
<tr>
<th>$y_n$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
<th>$Y_7$</th>
<th>$Y_8$</th>
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<th>$Y_{11}$</th>
<th>$Y_{12}$</th>
<th>$Y_{13}$</th>
<th>$Y_{14}$</th>
<th>$Y_{15}$</th>
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<tbody>
<tr>
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<td>L</td>
<td>L</td>
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<td>L</td>
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<tr>
<td>$y_{n-1}$</td>
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<td>L</td>
<td>L</td>
<td>L</td>
<td>H</td>
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Table 5.6: The Transition Probability Matrix of Yield Vector

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<thead>
<tr>
<th></th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
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<th>$Y_{15}$</th>
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<tr>
<td>$Y_5$</td>
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<td>$Y_{16}$</td>
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</tbody>
</table>

equivalent to dollars. In reality, they are measured in the number of shares. For example, Fonterra requires its farmers to purchase one share for one milksolid supplied. Therefore, the numerical experiments scale the yield to dollars by multiplying the share price, which is assumed to be the fixed par value. This assumption makes sense because cooperative shares are redeemable and non-appreciable. All computation parameters are presented in Table 5.7.
Table 5.7: Computation Parameters in Stochastic Yield

<table>
<thead>
<tr>
<th>Space</th>
<th>discretisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Space of $s$</td>
<td>$S = [0, 25,000]$</td>
</tr>
<tr>
<td>State Space of $\tilde{Y}$</td>
<td>$\tilde{Y}1 - \tilde{Y}16$</td>
</tr>
<tr>
<td>Action Space of $v$</td>
<td>$v = [-2,000, 12,000]$</td>
</tr>
<tr>
<td>Action Space of $b$</td>
<td>$b = [0, 8,000]$</td>
</tr>
<tr>
<td>Action Space of $q$</td>
<td>$q = [0, Y]$</td>
</tr>
</tbody>
</table>

- High Yield: $H = 2,000$
- Low Yield: $L = 1,250$
- Probability: $p = 0.5$
- Share Price: $\$5$
- Maximum Number of Iteration: $\text{maxite} = 1,000$
- Tolerance Condition: $\text{toler} = 1e-6$
- Initial Difference: $\text{dif} = 100$

Figure 5.7: $V(s)$ by Projected Cash Position for All Yield Vectors

Since there are two categories of state variables ($s$ and $\tilde{Y}$), theoretically, one can expect that the value function of $V(s, \tilde{Y})$ is three-dimensional. However, as the continuous distribution of yield is discretised, the problem becomes a discrete-state Markov decision process (MDP). Therefore, this section plots the value functions $V(s)$ with regard to the
cash position when the initial yield vector is specific to $\tilde{Y}_i$, $i = 1, \ldots, 16$ in Figure 5.7. For convenience, this section uses $V(s|\tilde{Y})$ to indicate the value with regard to $s$. Due to the level of discretisation, the curve is not that smooth. However, the value functions still follow the structure proposed earlier in this chapter. In addition, when the cash position is high enough, the different values of the objective result from a different current yield $y_n$, of $H$ or $L$, not depending on yields in the past years ($y_{n-1}, y_{n-2}, y_{n-3}$). It can be understood that: when the cash position is high enough, the yield variation is unable to decrease the cash position below the threshold ($\tilde{s}(|\tilde{y}|)$); hence, the optimal value becomes stable. However, when the cash position is lower, the influence of uncertain yield is significant.

Figure 5.8: Comparison among Models with $\Delta e = 0$, $\Delta e > 0$ and $\Delta e < 0$

To investigate the influence of uncertain yield, this section needs to clarify how the cash position is affected. According to the base capital plan, the capital position is adjusted in each period by $\Delta e_n = \frac{y_{n-1} - y_{n-3}}{2}$. The numerical experiments characterise three scenarios
that are $\Delta e = 0$, $\Delta e < 0$, and $\Delta e > 0$. The inequality $\Delta e < 0$ represents the practice of share redemption; while, $\Delta e > 0$ indicates share subscription. For example, the yield vector of $\tilde{Y}_1 = [L, L, L, L]$ implies $\Delta e = 0$, $\tilde{Y}_2 = [L, L, L, H]$ implies $\Delta e = \frac{1250 - 2000}{2} \times 5 = -1875$; while in $\tilde{Y}_5 = [L, H, L, L]$, $\Delta e = \frac{2000 - 1250}{2} \times 5 = 1875$.

**Influence on the Threshold Value**

Figure 5.8 illustrates the value of $V(s|\tilde{Y}_1)$, $V(s|\tilde{Y}_2)$ and $V(s|\tilde{Y}_5)$ and their corresponding decision policies. As one can see, these figures show the same structures as those proven in the analytical study, so this section does not repeat those here. What this section wants to emphasise is the influence of uncertain yield on the threshold cash position $\hat{s}$. Theorem 5.1 has proposed the existence of a threshold point $\hat{s}(\tilde{y})$, and this threshold is a function of $\tilde{y}$, instead of a scalar value. The experiments do not explicitly show the function of $s(\tilde{y})$; however, they clearly visualise the influence of yield on the threshold value, that $\Delta e < 0$ corresponds to a higher threshold value while $\Delta e > 0$ corresponds to a lower threshold. This trend can be understood: when the cash position is low, the additional capital investment from $\Delta e > 0$ would greatly contribute to the processing quantity, and also to its accumulated capital position.

It needs to be noted that in the value-iteration algorithm, the Action Spaces predetermined would influence the optimal decisions. This experiment sets the Action Space of $v$ to be $[-2000, 10000]$. However, the lower bound of $v$ constrains the choice of borrowing when $s$ is very low (say, $< 1000$). One needs to pay attention to this problem when applying the algorithm.

**Influence on the Cash Position**

Similar to the case of deterministic yield, one can simulate the cash position in the future with the collection of state variables and the decision rule. The difference is one needs to simulate the random yield in the forward progress. With the transition probability matrix determined earlier, this section used the Markov Chain Monte Carlo (MCMC) methods to realise this function. Therefore, with a given initial state of $\tilde{Y}$, one can generate a
sequence of random yields. In the numerical experiments, the Markov Chain is stationary and time-discrete.

Figure 5.9: The Simulation of Cash Position with Stochastic Yield

Figure 5.9 reports the results of the simulation on the cash position in the future with different initial yield vectors. In this experiment, the initial cash position is set to be 100, and the progress lasts for 40 periods. The left chart depicts the progress of the cash position when the initial yield vector is $\tilde{Y}_1$. In the first few periods, the cash position is dramatically increasing, and then, after a few periods, it varies within quite a stable range. The right one compares different processes with the initial yield vectors of $\tilde{Y}_1$, $\tilde{Y}_2$, and $\tilde{Y}_5$, and it illustrates that there is no difference in the range within which the cash position varies.

It is not hard to understand that a low cash position will stimulate the organisation to increase cash by retaining profits. However, it is very interesting to notice that the target level is varied by the random yield realisation in the current period. The upper limit is determined by the market revenue when $y_n = H$, while the lower limit is determined by the market revenue when $y_n = L$. Section 5.4 only proposes the optimal myopic policy when assuming deterministic yield. However, the numerical experiments imply that when the yield is uncertain, the myopic policy is also optimal when the cash position becomes high enough. This can be further explained by the theoretical analysis: as long as the variation of $\Delta e$ (specifically when $\Delta e < 0$) does not deactivate the relaxation of the constraint.
\( v + \rho b \leq (1 + \theta_1)s - \theta_2 s^- + \Delta e \) when the process moves through, then myopic policy is optimal through all the periods.

However, the conclusion of myopic policy with stochastic yields is followed by the assumption of an independent and identical distribution. This section does not experiment on other assumptions due to their complexity.

5.7 Conclusions and Future Research

This chapter formulates a dynamic stochastic model of operational and financial decisions within a proportional investment co-op whose equity investment by members is tied to patronage and whose objective is to maximise the expected discounted benefits for both the member-owners as well as the co-op. Yield in the future is an exogenous random variable that is largely dependent on weather conditions, and the market revenue is random as well and realised after the decisions. Each period the co-op makes decisions on payments to farmers, processing quantity, as well as any short-term loan. However, this chapter transforms the decision of farmers’ payments to a decision on the internal working capital.

This chapter shows the concavity and monotonicity of the value function in the cash position. These properties imply the existence of a threshold for the cash position where the optimal decisions become stable, and the business achieves the maximum value. This chapter also characterises the optimal policy to be that the optimal processing quantity is to maximise the market profit and that the optimal amount of borrowing is exactly the shortfall between the capital available and the capital required, which agrees with the paper of Babich and Sobel (2004). However, the decision on farmers’ payments or retained earnings is affected by the co-op’s objective. If the co-op weights itself as most important, then it accumulates as much profit as possible and pays little to farmers, as in an IOF, whereas if the co-op puts more weight on farmers, then the farmers’ payment or the retained earnings depend on the financial conditions and market profit. When the capital is sufficient, the co-op just retains what is required for optimal processing cost, and distributes the surplus to farmers. However, when there is a shortage of capital, the co-op
may borrow from the bank and also retain all payments to farmers. In any case, borrowing is more expensive than retaining internal working capital.

With the assumption of deterministic yield, Section 5.4 characterised the capital position under sufficient initial capital as an optimal base-stock policy. That is, if the capital is larger than a threshold, the payment to farmers in each period is so determined that the capital position before random revenue is realised is maintained at a fixed target level. A possible strategy when there is patronage shortage is to source from outside farmers as an alternative when the marginal cost of outside sourcing is lower than its marginal profit. Formalising this is left as the subject of future research.

Section 5.5 identifies the financial crises associated with uncertain market revenue and patronage. The first is the debt-default risk and the second is the redemption-default risk when the co-op is unable to redeem members’ equity as required. Both risks can be mitigated by an increased capital position. From the market perspective, this chapter suggests that when the market is stable, it is better to choose a long base period in the base capital plan; while if the market is unstable, a short one is preferred.

Numerical experiments are undertaken in Section 5.6 where the Value Iteration algorithm in MATLAB is applied to consider both deterministic and stochastic yields. The results validate the properties and conclusions this chapter has proposed through the analysis of the theoretical model. It also provides a visualised understanding of the structure of the value function, the optimal decisions, and the cash position. Furthermore, by simplifying the assumption on random yield, it allows us to have a simple understanding on the impact of random yield.

An immediate extension would be to constrain the liquidity condition on both loan repayment and equity redemption. One may interpret the illiquidity by the fact that farmers are the owners of the organisation that shall pay out as needed. However, defaulting on a loan is never best for the organisation because co-ops have limited access to other outside investors compared to IOFs. Furthermore, legislative and judicial officials have suggested that actions that do not provide timely redemption of older equity are unfair and inequitable (Centner 1982). Therefore, one could try to extend results to a default-free
model as discussed in Section 5.5. However, it can be reasonably expected that most of the results extend to this formulation if the added constraints are independent of \( s_n \).

This chapter assumes the sequence of random yields to be i.i.d.; modelling the entry and exit of members, which may be interesting, is outside the scope of this work and left as a possible topic for future research. Further, it may be worthwhile to consider more general processes in the future, and examine if there is any explicit influence of yield on the value function and optimal policies, which this chapter has not yet shown. Ideally, one would allow dependence of yields across periods because weather, and hence yields, often show long-range dependencies. This is also left as a topic of future research.

Investment constraints are supposed to be the “Achilles’ heels” of co-ops, especially within an increasingly concentrated, coordinated, and capital-intensive food system (Vitaliano, 1983). According to the co-op financial constraint hypothesis, agricultural co-ops are unable to acquire sufficient risk capital for several reasons, one of them being that equity capital acquisition is tied to member patronage (Chaddad 2006). Therefore, another attractive direction for future research is to allow a more flexible equity requirement, which may not require members’ investment strictly in proportion to patronage. For example, Fonterra allows farmers to hold 20% (maximum) shares over their share standard according to its new equity policy introduced in 2012. In practice, there are some other strategies adopted by co-ops to realise this objective, for example, via issuing preferred shares in addition to members’ shares. In this case, the coordination of patronage payment, as well as dividends, needs to be carefully considered.

In addition, this chapter simply introduces the three most common types of funding: member-provided equity, retained earnings (or institutional capital), and bank loans. However, there are other methods available, such as, financing from outside investors. Fonterra permits outside investors to invest in a security that gives them access to the Economic Rights which are the rights to receive dividends, but not to vote. In this situation, the challenge is how co-ops can find capital in this way that doesn’t at the same time compromise the principle of member control and ownership. Such extensions will be considered in Chap-
ter 6, where Fonterra’s capital structure is characterised by a flexible equity requirement and outsider investors.
Chapter 6

A Case Study of Cooperative Capital Structure

6.1 Introduction

A Markov decision process (MDP) model was proposed in Chapter 5 for a Proportional Investment Co-op where farmers’ equity is required to be in proportion to their patronage. However, there are many variations of cooperative organisational models summarised by Chaddad and Cook (2004) in terms of capital structure. Those models may change and reform under different circumstances from time to time. For example, Fonterra, as introduced in Chapter 2, fit in the definition of Proportional Investment Co-ops more with its previous capital structure; however, it is now more like a New Generation Co-op after the capital structure reformation. Since a cooperative financial structure is different from a non-cooperative’s financial structure, research dealing with capital structure optimisation in corporations is hence inappropriate for study of cooperative finance. Therefore, this chapter is an effort to incorporate unique cooperative features into a practical case study for analysing cooperative finance.

This section introduces more of Fonterra’s dynamic equity system and its relevant policies, then outlines the typical challenges encountered in the co-op capital structure optimisation as well as solutions. The information in this section mainly comes from Fonterra (https://www3.fonterra.com/nz/en/our-financials/fonterra-shareholders-fund.html). All data employed in the numerical experiments are mainly sourced from Fonterra Financial Statements (https://www3.fonterra.com/nz/en/our-financials/financial-results.html). Several specific terms used in this chapter are listed in Table 6.1, and capital letters are used to denote them throughout this chapter. The terms will be explained in detail in later sections.
<table>
<thead>
<tr>
<th><strong>Table 6.1: Fonterra Glossary</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry Shares</strong></td>
</tr>
<tr>
<td><strong>EBIT</strong></td>
</tr>
<tr>
<td><strong>Economic Rights</strong></td>
</tr>
<tr>
<td><strong>Farmer Shareholder</strong></td>
</tr>
<tr>
<td><strong>Farmgate Milk Price</strong></td>
</tr>
<tr>
<td><strong>Farmgate Milk Price Manual</strong></td>
</tr>
<tr>
<td><strong>Fonterra</strong></td>
</tr>
<tr>
<td><strong>Fonterra Farmer Custodian</strong></td>
</tr>
<tr>
<td><strong>Fonterra Shareholders’ Market (FSM)</strong></td>
</tr>
<tr>
<td><strong>Fonterra Shareholder’s Fund (FSF)</strong></td>
</tr>
<tr>
<td><strong>Fonterra Unit</strong></td>
</tr>
<tr>
<td><strong>Fund Size</strong></td>
</tr>
<tr>
<td><strong>Fund Size Risk Management Policy</strong></td>
</tr>
</tbody>
</table>
Fund Transfer Limit

The limit set by Fonterra Board from time to time in relation to the maximum number of Wet Shares in respect of which shareholders may sell Economic Rights to the Fund.

GDT

GlobalDairyTrade, the auction platform for internationally-traded commodity dairy product.

KgMS

A kilogram of milksolids.

Reference Commodity

The commodity dairy products used in the calculation of the Farmgate Milk Prices, which are currently whole milk powder, skim milk powder, and their by-products, buttermilk powder, butter, and anhydrous milk fat.

Season

A period of 12 months (Fonterra may specify from time to time in each year).

Share

A fully paid cooperative share in Fonterra.

Share Standard

The number of Shares a Farmer Shareholder is required to hold from time to time as determined in accordance with the Constitution.

Trading Among Farmers (TAF)

The share trading system of Fonterra including FSM and FSF.

Unit Holder

The holder of a Unit, e.g., outsider investors.

Voucher

A certificate that will apply to a Farmer Shareholder upon transfer of the Economic Rights of a Wet Share to the Fund.

Wet Shares

Any shares held by a Farmer Shareholder which are required to be held in accordance with the Share Standard for a Season.
6.1.1 Fonterra’s Dynamic Equity System

Chapter 2 has introduced Fonterra’s capital structure transformation. There are two significant changes: the first gives farmers greater flexibility to hold more or fewer Shares than is prescribed by the Share Standard. Another far-reaching change to the capital structure is the launch of Trading Among Farmers (TAF) — the share-trading system of Fonterra. Under the TAF, Farmer Shareholders trade Shares among themselves on a new market — the Fonterra Shareholders’ Market (FSM), where they are able to adjust their shareholdings as their milk production increases or decreases, instead of trading with Fonterra as before. In addition, as part of the TAF, Fonterra set up a special Fund—Fonterra Shareholders’ Fund (FSF), which provides outside investors, who are restricted from holding Shares, an opportunity to invest in the business by subscribing to Units in the FSF. In other words, Fonterra’s subscribed equity instrument comprises Shares which are issued and traded at market prices in the farmer-only market of FSM, and Units issued in the public market of FSF. Although the two markets are separate, they have been designed to work together by allowing a Share and a Unit to be effectively changed on a one-for-one basis. However, only Farmer Shareholders are able to exchange Units/Shares for Shares/Units; while other investors are only allowed to buy/sell Units.

Farmer Shareholders who want to exchange Shares for Units are able to place Shares on the FSF, which would then pay farmers for the right to receive dividends and gain/losses from any changes in value of those Shares (called Economic Rights). The FSF, in turn, raises the money it needs to pay Farmer Shareholders by selling Units to investors who will receive benefits from Economic Rights of Shares. To allow each Unit to essentially pass the Economic Rights of one Share through to the Unit Holder, the number of Units in the FSF must correspond to the number of Shares in which Economic Rights are held for the FSF by the Fonterra Farmer Custodian. Note that, farmers are still the owners of the Shares, and they can convert any Units they hold back into Shares whenever they wish. When farmers sell Economic Rights to the FSF, the Custodian holds the legal title to Shares and it ensures 100% farmer control and ownership.
On launching of the FSF in the Season of 2012/2013, Fonterra, on behalf of the FSF, has made an offer to Farmer Shareholders to acquire Economic Rights of Shares from those who want to transfer Shares to the Custodian. In addition, Fonterra provided the shortfall by issuing additional Shares to the Custodian to target the number of Units it planned to issue in the FSF. Finally, Fonterra issued 89,808,526 Units in the FSF (accounting for almost 7% of the total number of Shares on issue). Following the launch of the FSF, farmers are free to sell the Economic Rights of their Shares to Units in response to the changes in milk production in each dairy Season, and Fonterra manages the size of the FSF within parameters which are outlined in the FSF Size Risk Management Policy.

Figure 6.1: Dynamic Equity System of FSM and FSF

Figure 6.1 illustrates the dynamic equity systems of TAF, which operates as follows:

1. Being the first day of a Season (currently, 1 June), the Share Standard is determined and advised to farmers according to the average milk production in the last three Seasons.
2. According to the policy of flexibility, Farmer Shareholders are permitted to hold more or fewer Shares than required by the Share Standard. To match production needs and business cash flows, farmers adjust their shareholdings through buying or selling Shares, leading to a change in farmers’ shareholdings (represented by $\Delta e^f$).
3. In response to the change of farmers’ Shares, Fonterra will either issue or redeem Shares, resulting in the change of the number of Shares on issue by $\Delta e$, or transfer Shares to Units, or Units to Shares, resulting in the change in the number of Units by $\Delta e^u$. Fonterra can make a combination action to optimise the capital structure.

Therefore, under the dynamic equity system, it maintains the equity balance condition throughout each Season, that is, the total Shares on issue equal the number of Shares held by farmers (farmers’ shareholdings), plus the number of Units in the FSF. The dynamic relationship is expressed as:

$$\Delta e = \Delta e^f + \Delta e^u.$$  (6.1)

It is important to note that a change in the number of Units on issue does not, of itself, result in a dilution in per unit measures of financial performance (such as earnings per Unit). That is because changes in the number of Units on issue need not (and typically will not) reflect any change in the total number of Shares on issue. No matter how many Shares are placed with the FSF by farmers, the total number of Shares on issue will stay the same unless Fonterra adjusts its Shares by issuing or redeeming. Therefore, by controlling the total number of Shares, Fonterra is able to maintain the stability of Fund Size in response to the variability of milk production and farmers’ shareholdings.

### 6.1.2 Fonterra’s Policies

To manage the financial risks faced by the organisation, Fonterra has established a series of relevant policies. This chapter examines three main policies: the Fund Size Risk Management Policy, the Flexibility Policy, and the Liquidity Policy. The first one is to manage the Fund Size measured by the ratio between the number of Units and Shares, and to ensure that it remains within the target limits. The Flexibility Policy allows Fonterra to control the extent to which Farmer Shareholders can hold more or fewer Shares than are required by the Share Standard. Furthermore, Fonterra has a Liquidity Policy by retaining cash to ensure it has sufficient cash to meet its financial obligations as they fall due. These policies are introduced in detail below:

1. **Fund Size Risk Management Policy**
Fonterra has an interest in ensuring the stability of the Fund Size measured by the ratio of Units and Shares, i.e., \( e^u/e \); therefore, it has established a Fund Size Risk Management Policy which is designed to address the following requirements:

- the key goals of the FSF are to supplement liquidity in the FSM and to provide flexibility for Farmer Shareholders;
- the size of the FSF should remain within specified limits; and
- the Fund Size should be managed based on graduated responses.

The current policy is intended to achieve a Fund Size between 7% to 12% of the total number of Shares on issue. If the Fund Size falls outside this range, certain actions will be taken, and the extent and nature of actions required will vary according to the extent to which the target range is exceeded. Table 6.2 summarises all boundaries and indicates their corresponding responses as Low, Medium, and High levels. For example, if the Fund Size breaches 18%, Fonterra will suspend the ability for further Economic Rights to be sold to the FSF.

<table>
<thead>
<tr>
<th>The extent of threshold</th>
<th>Response level</th>
</tr>
</thead>
<tbody>
<tr>
<td>If the Fund Size breaches 12%</td>
<td>Low</td>
</tr>
<tr>
<td>If the Fund Size breaches 15%</td>
<td>Medium</td>
</tr>
<tr>
<td>If the Fund Size breaches 18%</td>
<td>High</td>
</tr>
</tbody>
</table>

2. **Flexibility Policy**

Farmer Shareholders are required to hold the number of Shares needed to meet the Share Standard. Any Shares held by farmers that are required to be held in accordance with the Share Standard in a Season are called Wet Shares; while any Shares in excess of the Share Standard are called Dry Shares. Under its current Flexibility Policy, Farmer Shareholders are allowed to:

- hold Dry Shares above the Share Standard with an overall cap at 20%;
- sell, at most, 25% of Wet Shares into Vouchers whose Economic Rights will be placed on the FSF for Units.
Both of the policies on holding Dry Shares and Vouchers are able to influence the potential size of FSF. This is because Farmer Shareholders can sell them freely to the Fund. In other words, the Dry Shares and Vouchers represent a readily available pool that can be exchanged for Units.

3. Liquidity Policy

Fonterra’s approach to manage liquidity is to ensure that it will always have sufficient funds to meet its liabilities when due, under both normal and stressed conditions, without incurring unacceptable losses or risking damage to its reputation. Fonterra has a policy in place to ensure that it has sufficient cash to meet expected operational expenses for a period of at least 80 days. This excludes the potential impact of extreme circumstances that cannot reasonably be predicted.

It needs to be emphasised that, for all those policies, Fonterra has discretion under the Constitution to change them from time to time for a variety of reasons, for example, meeting new circumstances. Therefore, this chapter examines the value of these policies and their influences on the financial performance under different hypothesised scenarios.

6.1.3 Fonterra Farmgate Price Manual

Fonterra currently collects around 85% of New Zealand’s milk production; hence, there is no valid market-based price that is independent of the price paid by Fonterra for liquid milk within New Zealand. Neither is it appropriate to use prices paid in foreign markets, due to the significant variances in cost structures (Trechter et al. 2003). It has therefore become necessary for Fonterra to establish an alternative mechanism to determine the price for milk supplied to Fonterra, that is the Fonterra Farmgate Milk Price Manual, which is overseen by the competition law regulator in New Zealand (www.fonterra.com.).

The calculation is summarised as follows:

1. Determine the revenue that the co-op would earn if the equivalent of all the milk Fonterra collects was converted into “Reference Commodity Products” including whole milk powder, skim milk powder, and their by-products, which are buttermilk
powder, butter, and anhydrous milk fat. Prices primarily reflect US dollar prices achieved on the GlobalDairyTrade (GDT) trading events every two weeks.

2. Deduct costs, including the cost of transport, manufacturing, and other selling and administration expenses.

Those “Reference Commodity Products” comprise approximately 75% of total milk production. Fonterra is committed to paying the highest sustainable Farmgate Milk Price that is linked to actual GDT prices. In other words, the payment to farmers for milk production is based upon the GDT prices. However, the Farmgate Milk Price approach does not include any returns earned from value-added products, such as infant formula. These premium prices are recognised in Fonterra’s earnings rather than in the Farmgate Milk Price. This enables total returns to be allocated between payments for milk and returns on the capital invested by Farmer Shareholders in the co-op.

6.1.4 Challenges in Fonterra’s Capital Structure Optimisation and Solutions

In an attempt to study the cooperative capital structure optimisation, the underlying structure of the model in this chapter represents a simplified version of Fonterra’s current capital structure, which is mainly characterised by the Flexibility Policy and the equity system of TAF. The aim of these strategies is to alleviate financial constraints and to remove Fonterra’s obligation of issuing Shares to, and redeeming Shares from, Farmer Shareholders. However, this new capital structure has sparked concern about demutualisation, that the farmers may lose the control of the company. Furthermore, as a financial incentive for equity holders (both farmers and investors) to invest in the business, Fonterra is expected to pay competitive dividends to them, which, however, allows less money available for paying off debt, reinvesting in growth, or increasing cash reserves. This also raises concern about profitability and liquidity. It is therefore of significant value for Fonterra to maximise equity holders’ returns while maintaining the optimal capital structure.

In addition, Fonterra is expected to have proper policies to mitigate financial risks that are inherent in the new capital structure. This chapter only considers two significant risks: the Fund Size risk and the liquidity risk. The Fund Size risk reflects the concern about
demutualisation, and is controlled by the Fund Size Risk Management Policy. The key goal of this policy is that the number of Units compared to the number of Shares is managed appropriately within several limits. On the other hand, the liquidity risk is managed by the Liquidity Policy which ensures Fonterra has sufficient cash reserve to meet its liabilities under both normal and stressed conditions.

For all those policies, Fonterra has discretion to change them from time to time for a variety of reasons. For example, to manage the Fund Size risk, Fonterra is able to adjust the Flexibility Policy: it can constrain the extent to which Farmer Shareholders can sell Economic Rights of Shares they are required to hold to meet the Share Standard (Vouchers); or it can manage the aggregate number of Shares on issue, thereby influencing the total number of Shares that are in excess of those required under the Share Standard (Dry Shares). Also, Fonterra may require a high cash reserve to reduce the liquidity risk, which results in less capital available for reinvestment, or paying off debt. Therefore, it would be valuable to examine how these policies impact performance.

This study first proposes an MDP model which considers four sources of capital: Shares by farmers, Units by investors, retained earnings, and debt. In each Season, Fonterra makes decisions on how to adjust Shares and Units on issue, how much to borrow, how much to reinvest in growth, and how much it pays in dividends to equity holders. The random information includes milk price, share price, farmers’ behaviour in share-trading, and milk production. However, such a model is analytically intractable with so many state variables and uncertainties. A numerical experiment is a simple, yet useful, way to investigate theoretical models in identifying properties and patterns, and there is no need to make strict assumptions to make the model analytically tractable; hence, it mostly captures real world scenarios. In addition, it enables examination the process from many aspects that involve various sources of randomness, to investigate the impact of different policies, and derives results under various hypothesised scenarios.

This chapter uses an Approximate Dynamic Programming (ADP) algorithm to derive results for the proposed model. ADP is a powerful tool to overcome the well-known curse of dimensionality in Markov decision processes. One challenge in ADP algorithms is the
simulation of random information. This work uses three approaches. First, random realisations coming from real physical processes are used. Through data analysis on historical milk and share prices, a forecast model for uncertain milk and share prices is constructed based on the autoregressive model. Second, computer simulations are used to generate random observations from a distribution using a process referred to as Monte Carlo sampling. Those distributions are estimated from historical data and theoretical assumptions. Lastly, thought experiments are used to derive results under different assumptions.

This chapter contributes mainly in three aspects. First, it develops a stochastic dynamic programming model, which can be used to enable Fonterra to make better decisions in capital structure optimisation. Second, via numerical analysis, it explores the impact of different policies on performance, hence offers the potential for policy-making in different settings. Furthermore, it also experiments on the resilience of systems with milk production in adversity and makes suggestions to improve the situation.

Next, Section 6.2 describes the mathematical model of the optimal capital structure problem. Section 6.3 proposes a financial model for Fonterra’s net profit and Section 6.4 briefly reviews the implementation of the ADP algorithm. Numerical experiments are carried out in Section 6.5, and a thought experiment on milk production is implemented in Section 6.6. The chapter concludes in Section 6.7. The identification of several functions are presented in Appendix C.

6.2 The Mathematical Model

This section formulates a stochastic dynamic model to study the capital structure optimisation for Fonterra in each dairy Season. The goal is to balance the trade-off between maximising equity holders’ returns over time and minimising financial risks. The timeline is illustrated in Figure 6.2.

1. At the beginning of a Season, Fonterra calculates the Share Standard of each farmer according to his three-year rolling average milk production.

2. Based upon the Share Standard, Farmer Shareholders adjust their shareholdings under the guidance of the Flexibility Policy.
3. In response to the change of farmers’ shareholdings, Fonterra makes decisions to control Shares and Units on issue.

4. Throughout a Season, Fonterra processes and sells the majority of dairy products in the GlobalDairyTrade (GDT) platform wherein the global milk price is randomly realised. Based upon the milk price, the farmgate milk price is calculated using the Farmgate Milk Price Manual.

5. Based upon the cash flow, Fonerra makes decisions on the borrowing, dividend paid to equity holders, and the cash needed for reinvestment.

6. At the end of each financial year, Fonterra publishes its annual financial report which updates the information on Shares $e$, Units $e^u$, equity $eq$, borrowing $b$, and cash position $s$.

![Figure 6.2: Chronology of Events in a Season $t$](image)

The specifics of the model are laid out in the following subsections on the state variables, decision variables, exogenous information process, transition functions, the objective function, and the full problem formulation.

### 6.2.1 State Variables

Table 6.3 presents all state variables considered in this study. At the end of each financial year, Fonterra publishes its annual financial report including the Balance Sheet, Income
Statement, and Cash Flow Statement, from which the information of Shares $e$, Units $u$, equity $eq$, borrowing $b$, and cash position $s$ are tracked.

Table 6.3: State Variables

<table>
<thead>
<tr>
<th>The set of state variables $S_t$</th>
<th>$=(e_{t-1}, e_{t-1}^f, e_{t-1}^u, eq_{t-1}, s_{t-1}, b_{t-1}, \tilde{y}_t, p_t^s), t = 1, \ldots, T.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{t-1}$</td>
<td>Opening number of Shares on issue in Season $t$</td>
</tr>
<tr>
<td>$e_{t-1}^f$</td>
<td>The number of farmers’ Shares at the end of the last Season</td>
</tr>
<tr>
<td>$e_t^f$</td>
<td>The number of farmers’ Shares realised in the current Season</td>
</tr>
<tr>
<td>$e_{t-1}^u$</td>
<td>Opening number of Units on issue in Season $t$</td>
</tr>
<tr>
<td>$eq_{t-1}$</td>
<td>Opening Equity in Season $t$</td>
</tr>
<tr>
<td>$s_{t-1}$</td>
<td>Opening cash position in Season $t$</td>
</tr>
<tr>
<td>$b_{t-1}$</td>
<td>Opening borrowing in Season $t$</td>
</tr>
<tr>
<td>$\tilde{y}<em>t = (y</em>{t-2}, y_{t-1}, y_t)$</td>
<td>Vector of milk production in Seasons $t-2, t-1, t$</td>
</tr>
<tr>
<td>$p_t^s$</td>
<td>Realised average share price in Season $t$</td>
</tr>
</tbody>
</table>

In each Season, there are two main drivers in the dynamic equity system. One is the variability of milk production, by which the Share Standard is calculated based on a rolling average of three seasons’ historical milk production, tracked in the vector of $\tilde{y}$. It is assumed that the information of current annual milk production becomes known when the co-op makes decisions. Although the milk is produced and collected daily throughout the production Season, which lasts from June to May, it often well estimated by the time harvest begins. A similar assumption was made in Chapter 5.

Another driver is the flexibility of farmers’ behaviour in share-trading. Upon the launch of TAF, Farmer Shareholders can effectively exchange Shares/Units for Units/Shares based upon the Share Standard. While they are allowed to trade at any time; it is impractical to monitor and track the information every minute. Therefore, this work only tracks the annual information of farmers’ shareholdings $e^f$, and assume that it becomes known as well in the current period. This assumption makes sense because Fonterra is able to adjust and control the equity in response to the changes of Shares and Units. Therefore, the change of farmers’ shareholdings is expressed as:

$$\Delta e^f_t = e^f_t - e^f_{t-1}.$$
In addition, the price at which Shares and Units trade varies from minute to minute, therefore, this study uses the average annual price to represent the price when equity trading is implemented. Further, the Shares and Units are traded at very similar prices, which is a key design objective for TAF. Similarly, this study assumes that the average Share/Unit price can be well estimated when Fonterra makes decisions. This can be justified by the fact that the FSM is a restricted market wherein the share price is not that variable as those traded in the free market.

Note that, the random information is available when Fonterra makes decisions in the current Season, but unknown in the next Season. The main justification is, Fonterra, in essence, works on behalf of farmers; therefore, it is able to control the system before uncertain information is realised. The model can be easily extended to include more dimensions in the state definition.

6.2.2 Decision Variables

In each Season, Fonterra makes four decisions to control the capital structure in Table 6.4.

<table>
<thead>
<tr>
<th>Table 6.4: Decision Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_t$: The decision of equity, given the information available in period $t$, $t = 1,\ldots,T$.</td>
</tr>
<tr>
<td>$D_t$: The decision of dividend, given the information available in period $t$, $t = 1,\ldots,T$.</td>
</tr>
<tr>
<td>$B_t$: The decision of borrowing, given the information available in period $t$, $t = 1,\ldots,T$.</td>
</tr>
<tr>
<td>$I_t$: The decision of reinvestment, given the information available in period $t$, $t = 1,\ldots,T$.</td>
</tr>
</tbody>
</table>

1. Managing the Number of Shares and Units.

Fonterra needs to manage the number of Shares on issue to enable farmers to be able to adjust their shareholdings as their milk production increases or decreases. In response to the change of farmers’ shareholdings, Fonterra can issue/redeem Shares. The alternative way is to transfer the Economic Rights of farmers’ Shares into Units (or vice versa). This model uses $Z_t$ and $0 \leq Z_t \leq 1$ to denote the portion of Shares that Fonterra decides to issue/redeem in response to the changes of farmers’ shareholdings.
The changes of Shares on issue and Units on issue are:

\[
\Delta e_t^f = Z_t \ast (e_t^f - e_{t-1}^f),
\]

\[
\Delta e_t^u = -(1 - Z_t) \ast (e_t^f - e_{t-1}^f).
\]

When \(Z_t = 1\), it indicates Fonterra manages the aggregate number of Shares by issuing or redeeming Shares; while \(Z_t = 0\) indicates Fonterra manages the aggregate number of Units by transferring Shares into Units (or vice versa).

2. Dividend Payout Ratio

After the realisation of net profit, Fonterra needs to decide how much to pay out to Shareholders as dividends. This study uses the decision variable — the Dividend Payout Ratio \(D_t\) — rather than dividend per Share \(d_t\). The payout ratio shows whether the dividend payments made by a company make sense given their earnings. The non-distributed profits are normally retained or reinvested for future operations. If the payout ratio is too high, it means there is too small a percentage of profits being reinvested for future operations, hence casting doubt on a company’s ability to maintain future returns. The dividend per Share \(d_t\) can be calculated as follows:

\[
d_t \ast e_t = D_t \ast NP_t(\cdot),
\]

where \(NP_t(\cdot)\) denotes the net profit. Fonterra’s dividend policy is to pay out 65-75% of net profit after tax.

3. The Gearing Ratio

In each Season, Fonterra borrows in the form of bonds, bank facilities, and other financial instruments, and it also needs to repay borrowings that may fall due. To make those decisions, Fonterra closely monitors its gearing ratio, which is calculated as debt divided by total capital (equity plus debt). This model uses \(B_t\) to denote this ratio, thus,

\[
\frac{b_t}{b_t + eq_t} = B_t,
\]

where \(b_t\) is the amount of borrowing in the end of each period. Instead of making decisions on the net borrowing (“borrowing - repayment”), it is more convenient to
use the decision variable of the gearing ratio \((B_t)\) to monitor the size of borrowing. Fonterra’s target level of gearing ratio falls in the range of 45%-50%.

4. Reinvestment Rate.

The reinvestment rate, indicated by the proportion of after-tax operating income, measures how much a firm is plowing back to generate future growth. To move up the value chain by producing products with higher margins, Fonterra has to reinvest some or a large portion of earnings back into the business. Since 2012, Fonterra has invested $1.5 billion in increasing manufacturing capacity in NZ. New plants have contributed to gains in efficiency, better yields, and improved quality performance. A firm’s reinvestment rate can be ebb and flow, and the amount of reinvestment will depend upon the return on capital on the new investments and capital available. This model uses \(I_t\) to denote the reinvestment rate, and the capital used for reinvestment in each period \(Invest_t\) is hence calculated as:

\[
Invest_t = I_t \times \text{EBIT}_t \times (1 - \text{Tax}),
\]

where \(\text{EBIT}\) represents the Earnings before Interest and Taxes, and \(\text{EBIT}_t \times (1 - \text{Tax})\) is hence the after-tax operating income. There is no specific policy on reinvestment and this decision is dependent on financial parameters like return on capital and cost of capital (see Section 6.3).

6.2.3 Exogenous Information Process

In the process of capital structure optimisation, the exogenous information becomes known when the decisions are made in the current period, however, it is unknown in the next period. Given the description above, the exogenous information is listed in Table 6.5.

| \(P_{t+1}\) | Exogenous random milk price |
| \(P_{s_{t+1}}\) | Random share price |
| \(Flex_{t+1}\) | The flexibility of farmers’ Shares based upon the Share Standard |
| \(Y_{t+1}\) | Random milk production |
1. Random Share Price and Milk Price

Before 2012, Fonterra made decisions on the share price for each next Season, so the share price was changed year by year. After the introduction of FSM in 2012, the share price is set by what Farmer Shareholders trade at, and it is changed from minute to minute. The share price is influenced by many factors. Since Fonterra determines the farm-gate milk price based upon the global milk price, one can expect that the random share price is highly correlated to the milk price. Therefore, by analysing the historical data of milk prices and share prices, this study uses an autoregressive distributed lag model (ARDL) to predict the dynamic process of share price, in which the effect of both the lagged share price and the lagged milk price occurs over time (Hassler and Wolters 2006). In this process, the milk price is the independent variable (or regressor) and is estimated based on an autoregressive integrated moving average (ARIMA) model. ARIMA is a forecasting technique which is usually applied in time series analysis.

**Assumption 6.1.** Assume that the share price $P_{s,t}$ is mainly influenced by the global milk price $P_{t}$ and the model for uncertain share price follows an autoregressive distributed lag model $ARDL(p,q)$ with the dependency on random milk price, which is estimated from an $ARIMA(p,d,q)$ model.

The estimation of parameters is carried out in Appendix C, and the models are identified in the form of $P_{s,t} = \alpha_1 P_{s,t-1} + \beta_0 P_t + \mu_t$ and $P_t = \alpha_2 P_{t-1} + \gamma e_{t-1}$. Therefore, with the estimated models, the sample path can be simulated in the computation to generate the growth trajectory of milk prices and share prices. Note that the process of random milk price is not directly included into the model and this will be further explained in the following section.

2. The Flexibility of Farmers’ Shares

Under the new capital structure, farmers are allowed to hold more or fewer Shares than the Share Standard. Therefore, this study defines the variable of $Flext_{t+1}$ as the ratio between farmers’ shareholdings and the Share Standard, and assume it to be a
random variable within a limited range because the extent to which farmers can buy or sell is stipulated by Fonterra. Therefore, this work has the following assumption:

**Assumption 6.2.** Assume that the ratio of farmers’ shareholdings and the Share Standard follows a Triangular Distribution \((L, M, H)\) where the lower limit \(L\) and the upper limit \(H\) are stipulated by Fonterra’s policy and the Mode \(M\) is identified at 1.04 based upon historical data.

This study creates a Triangular Distribution object with specified parameter values using `makedist` in Matlab (see Appendix C for details of this estimation). Therefore, with the calculation of the Share Standard, one can produce the number of farmers’ Shares by simulating a random ratio.

Like all Fonterra Policies, the lower limit and upper limit can be changed by Fonterra from time to time to meet new circumstances. In the numerical experiments, the impact of the Flexibility Policy will be analysed by changing the value of the lower limit and the upper limit, respectively.

3. **Uncertain Milk Production**

Fonterra faces uncertain milk production, due to a number of factors, one of which is weather. From the historical data displayed in Figure 6.3, it can be observed that there is an increasing trend in milk production and the annual growth of milk supply is close to linear. This trend may be introduced by developed farming technology or service. Therefore, as assumed in Guan and Philpott (2011) who study the production planning problem for Fonterra, the average milk production is assumed to be a linear function in time. In their model, the variance of milk production is estimated based on an autoregressive model because they consider the milk supply in a typical region. In contrast to their model, this work considers the total production across all farmers in Season \(t\), hence assumes that the uncertain milk production follows a normal distribution according to the central limit theorem (CLT).
Assumption 6.3. Assume that the random milk production follows a normal distribution each year $Y_t \sim N(\mu_t, \sigma^2)$, and the average milk production has an increasing trend in time while the variance remains stable throughout the planning horizon, shown as:

$$\mu_t = 38 \times t + 1550,$$

(6.4)

where $t$ is the period of time, $t = 1, \ldots, T$.

The parameters are estimated from historical data using maximum likelihood estimation. The regression model sets 2002/2003 to be 1, 2003/2004 to be 2, and so on. Therefore, Eq.(6.4) is estimated as $\mu_t = 38t + 1550$ where the time of $t$ starts from the Season of 2015/2016. The variance of milk production is estimated by fitting the distribution specified to the historical data (the function of fitdist in MATLAB). While the assumption of constant variance is primarily for ease of computation, it corresponds to a model with a stable farm base. In this model, $y_t$ is a realisation of milk production in Season $t$.

According to the historical data, an increasing milk production with time is considered. However, the pattern of random milk production is quite unpredictable, especially in the distant future. Therefore, Section 6.5 carries out a thought experiment assuming that the milk supply is decreasing in time, so as to test resilience of the system under adversity.
6.2.4 Transition Functions

In the dynamic programming process, the transition functions of state variables are functions of current states and decisions, as well as the exogenous information. Since the random information becomes known in the current period, the state transition functions can be divided into two groups. The stochastic group of the transition function models the random information of milk prices, share prices, flexibility, and milk production. The deterministic group describes the transitions of Shares on issue \( e \), Units on issue \( e^u \), equity \( eq \), borrowing \( b \), and cash position \( s \). The process of exogenous information has been described above, therefore, this section describes the deterministic transition functions which are function of current state variables and decisions.

1. Shares on Issue

After the decision of \( Z_t \) is made, the total number of Shares on issue is updated as follows:

\[
e_t = e_{t-1} + Z_t \ast (e^f_t - e^f_{t-1}).
\] (6.5)

If \( Z_t = 0 \), it indicates Fonterra does not issue or redeem any Shares; hence there is no change in the number of Shares.

2. Units on Issue

Similar to Shares, the total number of Units is updated as well after the decision of \( Z_t \):

\[
e^u_t = e^u_{t-1} - (1 - Z_t) \ast (e^f_t - e^f_{t-1}).
\] (6.6)

If \( Z_t = 1 \), it means Fonterra does not transfer Farmers’ Shares into Units (or vice versa); hence these is no change in the number of Units.

3. Equity

In the financial report, Equity mainly consists of subscribed equity and retained earnings. Therefore, it is updated by additional subscribed equity \( (e_t - e_{t-1}) \ast p^s_t \) and the retained net profit \( (1 - D_t) \ast NP(\cdot) \):

\[
eq_t = e_{q_{t-1}} + (e_t - e_{t-1}) \ast p^s_t + (1 - D_t) \ast NP(\cdot)
\]

\[
= e_{q_{t-1}} + Z_t \ast (e^f_t - e^f_{t-1}) \ast p^s_t + (1 - D_t) \ast NP(\cdot),
\] (6.7)

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where $NP(\cdot)$ denotes the net profit of Fonterra and it is estimated in Section 6.3. It is important to note that the subscribed equity is only increased or decreased by issuing or redeeming Shares.

4. Cash Position

In finance and accounting, the Cash Flow Statement is divided into three categories: net cash flows from operating activities, net cash flows from investing activities, and net cash flows from financing activities. Net cash flow from operating activities refers to money generated by its core business activities. Net cash flow from investing activities relates to its investment in businesses through acquisition of business operations, property, plant, equipment etc. Net cash flow from financing activities mainly includes new borrowing/repayment of borrowing, equity subscription/equity redemption, interest paid, and dividends paid to equity holders.

Therefore, the cash position is updated as:

$$s_t = s_{t-1} + NP_t(\cdot) + \left((e_t^{f} - e_{t-1}^{f}) \ast Z_t \ast p_s^t + (b_t - b_{t-1}) - D_t \ast NP(\cdot) \right) - Invest_t, \quad (6.8)$$

where the first term is the initial cash position, the second is the net cash flow from operating, the third indicates the cash flow from financing, and the last one $Invest_t$ denotes the capital used for reinvestment, which is estimated in Section 6.3. Note that in this transition function, the interest paid to borrowing, taxation, and operating costs are included in the net profit $NP(\cdot)$ (see Section 6.3). In addition, the discount factor in the cash flow is considered in the objective function.

6.2.5 Objective Function

As Fonterra’s financial report states “The objective is to maximise equity holders’ returns over time by maintaining an optimal capital structure.” Therefore, this study uses multiple objectives, which are to maximise equity holders’ returns over time and to minimise financial risks by maintaining an optimal capital structure.

1. Maximising Dividends
Fonterra provides returns to Farmer Shareholders through milk prices, and to equity holders through dividends and changes in the share price. However, the milk price is mainly determined by the exogenous global milk price. Besides that, and as assumed earlier, the share price is mainly dependent on the milk price as well, because of the restricted Share market. Therefore, it is assumed that the main drivers of equity holders’ returns are dividends. According to the accounting literature on investor-owned firms (IOFs), the most popular model for equity valuation is the dividend discount model, which forecasts dividends for equity holders and measures the equity holders’ returns as the present value of expected dividends. The objective of maximising dividends is assumed by a lot of researchers like Xu and Birge (2008), Hu and Sobel (2005), and Li et al. (2013). While assuming a similar objective to that of an IOF seems to be inappropriate for a co-op, it can still hold in this model because the majority equity holders are Farmer Shareholders. Therefore, the objective of maximising dividends in each period is expressed as

$$\max d_t \times e_t.$$  

2. Minimising Financial Risks

- The Fund Size Risk

As introduced earlier, the Fund Size Risk Management Policy requires the Fund Size to remain within a specified limit; otherwise, actions will be taken accordingly. Therefore, this chapter uses $k$ to indicate the threshold of Fund Size and defines the Fund Size risk as being in a state of not meeting a certain target level. Under this policy, the ratio of $e_{t}^n/e_t$ in each period should satisfy

$$e_{t}^n/e_t \leq k. \quad (6.9)$$

Fonterra’s target range is between 7% and 12%. Both the cases of under-target ($< 7\%$) or over-target ($> 12\%$) are undesirable; however, the over-target case is more prohibitive because the co-op may suffer from the risk of demutualisation. Therefore, in this study, a lower limit is not specified (except larger than 0); instead, an upper limit is specified to reflect the risk of demutualisation.
Furthermore, as specified in the Fund Size Risk Management Policy, there are three indicated boundaries that the Fund Size may break, that are 12%, 15%, and 18%. Those boundaries can be regarded as Fonterra’s degree of tolerance on the Fund Size risk. A larger boundary implies a higher tolerance, and vice versa. This study explores how the tolerance on the Fund Size influences the optimisation process and financial risks.

- Liquidity Risk

Similar to the Fund Size risk, Fonterra also has a policy in place to ensure that it has sufficient cash or facilities to meet its financial obligations by requiring a minimum cash position. Therefore, this chapter uses $s_{low}$ to indicate the cash reserve and define the liquidity risk as being in a state of not meeting a certain required cash level. Under this policy, the cash position of $s_t$ in each period should satisfy

$$s_t \geq s_{low}. \quad (6.10)$$

Similarly, the level of cash reserve can be regarded as Fonterra’s degree of tolerance on the liquidity risk. A larger value of $s_{low}$ implies a low tolerance, and vice versa. According to the policy, the cash position should meet the expected operational expenses for a period of at least 80 days. However, it is quite difficult to identify this value because Fonterra’s business is characterised by seasonality. In addition, this requirement excludes the potential impact of extreme circumstances. Therefore, the numerical experiments will try different values of $s_{low}$ and examine how the tolerance on liquidity influences the results.

### 6.2.6 Full Problem Formulation

In finance, it is quite common to have two or more conflicting objectives — a desire to have the expected value of returns to be as high as possible, and a desire to have risks be as low as possible. This study formulates the multi-objective optimisation problem by the $\varepsilon$-constraint method defined below:

$$\max d_t e_t, \quad (6.11)$$
\[\begin{align*}
\begin{aligned}
\text{s.t.} & & \\
& & \\
& & \begin{cases}
  k - e^u_t/e_t \geq \varepsilon_1, \\
  s_t - s_{\text{low}} \geq \varepsilon_2,
\end{cases}
\end{aligned}
\end{align*}\]

where lower bounds \(\varepsilon_1\) and \(\varepsilon_2\) are the degree of tolerance on the Fund Size and on liquidity.

The experiments on changing the parameters of \(\varepsilon_1\) and \(\varepsilon_2\) (with fixed \(k\) and \(s_{\text{low}}\)) are actually equal to the problem with constraints of \(e^u_t/e_t \leq k\) and \(s_t \geq s_{\text{low}}\) by applying different values on \(k\) and \(s_{\text{low}}\). For example, with fixed \(k = 12\%\), the model with \(\varepsilon_1 = 0\) equals the model with \(e^u_t/e_t \leq 12\%\). Therefore, for ease of exposition, this model constrains the Fund Size and liquidity, i.e., \(e^u_t/e_t \leq k\) and \(s_t \geq s_{\text{low}}\), and applies different values of \(k\) and \(s_{\text{low}}\) to examine the influence of objective parameters.

The objective of the finite-horizon decision problem is to seek a policy for choosing \(\{Z_t, D_t, B_t, I_t\}\), which achieves

\[\max E \left[ \sum_{t=1}^{T} \beta^{t-1} d_t e_t \right], \quad (6.13)\]

s.t.

\[\begin{align*}
\begin{aligned}
& e^u_t/e_t \leq k; \\
& s_t \geq s_{\text{low}}; \\
& d_t \cdot e_t = D_t \cdot NP_t(\cdot); \\
& \frac{b_t}{b_t + eq_t} = B_t; \\
& b_t \geq 0; e^u_t \geq 0; e_t \geq 0; eq_t \geq 0; \text{and} \\
& 0 \leq Z_t \leq 1; D_t \geq 0; B_t \geq 0; I_t \geq 0;
\end{aligned}
\end{align*}\]

where \(\beta\) (\(0 \leq \beta \leq 1\)) is the discount factor.

In dynamic programming, the problem is divided into time stages. The value associated with each state can be computed using Bellman’s optimality equations, which are typically written as:

\[V_t(S_t) = \max_{x_t \in \Omega(S_t)} \left( L(S_t, x_t) + \beta E(V_{t+1}(S_{t+1})|S_t, x_t) \right), \quad (6.15)\]

where \(V_t(S_t)\) is the value of being in state \(S_t\), \(\Omega(S_t)\) is the set of allowable decisions given the information available in period \(t\), defined by Eq.(6.14), and \(L(S_t, x_t)\) is the current period utility function with state \(S_t\) and action \(x_t = [Z_t, D_t, B_t, I_t]\).
In this model, there are several trade-offs, such as paying dividends, reinvesting in the business, or paying off borrowing or retaining cash. Those decisions are dependent on financial parameters and are also influenced by the proposed policies.

6.3 A Financial Model for Fonterra’s Net Profit

This section identifies the financial model of Fonterra’s net profit. Before that, Table 6.6 provides several terms that are generally used and defined in accounting and finance. According to the Income Statement, this section introduces the following categories and assumptions used throughout this work.

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td></td>
<td>The amount of money that a company actually receives from its business activities.</td>
</tr>
<tr>
<td>Cost of Goods Sold</td>
<td></td>
<td>The direct costs attributable to the production of the goods sold by a company</td>
</tr>
<tr>
<td>Operating Costs</td>
<td>$c_o$</td>
<td>The expenses associated with the maintenance and administration of a business</td>
</tr>
<tr>
<td>Interest</td>
<td>$\rho_b$</td>
<td>The charge for the privilege of borrowing money, usually expressed as annual percentage rate</td>
</tr>
<tr>
<td>Taxes (tax rate)</td>
<td>$Tax$</td>
<td>The percentage of a corporation’s earnings that is owed to the state.</td>
</tr>
<tr>
<td>Gross Profit</td>
<td>$GP$</td>
<td>“Revenue - Cost of Goods Sold”</td>
</tr>
<tr>
<td>Earning Before Interest and Taxes</td>
<td>$EBIT$</td>
<td>“Gross Profit - Operating Costs”</td>
</tr>
<tr>
<td>Net Profit</td>
<td>$NP$</td>
<td>“EBIT - Interest - Taxes”</td>
</tr>
<tr>
<td>Return on Invested Capital</td>
<td>$ROC$</td>
<td>“EBIT/Capital Invested”</td>
</tr>
<tr>
<td>Weighted Average Cost of Capital</td>
<td>$WACC$</td>
<td>A calculation of a firm’s cost of capital in which equity and debt are proportionately weighted.</td>
</tr>
</tbody>
</table>

1. Revenue

Fonterra is an integrated dairy business organised in three major business segments including ingredients, the consumer and food-service business, and international farming. Ingredients is the core of the business, where milk supplied by farmers is manufactured into export products such as Whole Milk Powder ($WMP$), Skim Milk
Powder (\textit{SMP}), etc. Revenue from sales of goods is mainly driven by sales volume and dairy commodity prices. Since the 2012 Season, GlobalDairyTrading (\textit{GDT}) has been the sole source of the dairy price indicator.

2. Cost of Goods Sold

Cost of goods sold is primarily made up of NZ sourced cost of milk which comprises the volume of milksolids supplied at the Farmgate Milk Price as determined by the Board for the relevant Season. According to the farm-gate milk price manual, the GDT milk price is the most significant driver of the Farmgate Milk Price.

3. Operating Costs, $c_o$

Operating costs mainly consist of distribution expenses, administrative expenses, and other operating expenses. For simplicity, the operating cost is assumed to be constant and fixed, and this value is calculated by the average operating costs over the last few years.

4. Interest/Financial Costs, $\rho_b$ 

Finance costs mainly comprise interest expenses on borrowings. Therefore, finance cost is a function of borrowing $\rho_b$ where $\rho_b$ is the interest rate. Fonterra borrows a mixture of fixed and variable rate debts; therefore, this work uses the weighted average interest rate estimated by Fonterra and assumes it to be constant in the near future.

5. Taxation, $Tax$

Taxes are generally an involuntary fee levied on corporations that is enforced by a government entity, and income tax is a percentage of corporate earnings filed to the federal government.

The Estimation of Gross Profit $GP_t$

Since the Gross Profit is calculated by “Revenue - Cost of Goods Sold,” and both Revenue and Cost of Goods Sold are greatly dependent on the market prices (represented by $GDT$ prices), the influence of random market prices on the Gross Profit has been significantly
cancelled out, just as Fonterra states “the increase or decrease in revenue would be offset by the increase or decrease in cost of goods sold” (PDF-Fonterra Shareholder’s Fund Prospectus and Investment Statement). Therefore, the following assumption is made in Fonterra’s model.

**Assumption 6.4.** Assume that Fonterra’s Gross Profit is mainly dependent on the milk volume, not on the global milk price, it is estimated as:

\[ GP(y_t) = 3.3y_t - 1680, \]  \hspace{1cm} (6.16)

where \( y_t \) is a realisation of \( Y_t \) in Season \( t \).

The estimation of Gross Profit implies a linear relationship with \( y_t \) (see Appendix C). It should be noted that this assumption follows from the reality of the dominance of Fonterra in the New Zealand Dairy Industry, which results in a government-regulated farm-gate price; it may not apply to other co-ops.

**The Estimation of EBIT with Growth**

After Gross Profit, it is easy to calculate EBIT by “Gross Profit - Operating Costs.” However, this dynamic model needs to consider the growth rate that the co-op may experience in the near future or in the long run. This consideration is essential because Fonterra, as introduced earlier, is now taking the strategy of moving up in the value chain by investment activities. The investments in plant capacity in NZ in recent years have improved the ability to respond to price volatility and to channel milk to the highest-returning products over time, and to better match the product mix to customer demands and global price signals, delivering higher price premiums above GDT prices.

Therefore, the growth in operating income can be related to total reinvestment made into the business, and the return earned on capital invested. However, the challenge in valuation is to forecast future returns on investments. In firm valuation models, the expected growth rate on operating income is a product of the proportion of after-tax operating income that is reinvested back into the business (reinvestment rate, \( I \)) and the quality of these reinvestments (measured as the return on capital, \( ROC \)), that is \( ROC * I \) (Damodaran
2007). In each period, \( ROC \) is calculated as “EBIT/Capital Invested;” therefore, the value of \( ROC \) varies from period to period. However, for ease of computation, this study has the following assumption:

**Assumption 6.5.** Assume that the return on capital \( ROC \) invested in new assets equals the return on capital invested in current assets, which is estimated at an the average of the last three years, and the \( ROC \) remains unchanged in the following periods.

Since the return on capital invested in new assets is assumed to be equal to the current financial parameter of \( ROC \), then the increase of the operating income can be estimated at a product of the return on capital reinvested and the capital used for reinvestment, that is \( ROC \times Invest \). In addition, this study assumes that it takes one period for an investment to start paying off. Therefore, the operating income (\( EBIT \)) in each period can be estimated as:

\[
EBIT_t(y_t) = GP(y_t) - c_o + ROC \times Invest_{t-1}
\]

\[
= GP(y_t) - c_o + ROC \times I_{t-1} \times EBIT_{t-1}(y_{t-1}) \times (1 - Tax).
\]

The first item, \( GP(y_t) - c_o \), can be regarded as the business income of the current period, and the second term, \( ROC \times Invest_{t-1} \), is the economic return on capital reinvested in the last period.

Finally, the net profit is calculated as: “Net Profit \( \equiv EBIT \)-Financial Costs -Taxation,” that is:

\[
NP_t(y_t) = (1 - Tax) \left[ EBIT_t(y_t) - \rho_b b_t \right].
\]

Note that, the financial model of net profit is a dynamic model and is not stationary.

### 6.4 The ADP Algorithm

Solving Eq.(6.15) encounters the three curses of dimensionality: the state vector, the action space, and the outcome space (the expectation is over a vector of random variables). Further, it is difficult to solve the problem of Eq. (6.15) using classical backward dynamic programming because the transition function is intractable. Therefore, rather than solving for the value of each state exactly, this work approximates the solution of Eq.(6.15) using
an Approximate Dynamic Programming (ADP) algorithm. ADP steps forward through
time via simulation and proceeds by iteratively estimating and updating the approximate
value of being in a state, and has emerged as a powerful technique for solving dynamic
programmes that are computationally intractable.

This work largely follows the notation of Powell (2007), and a brief note on notation is in
order here. Let $V_t(S_t)$ be an approximation of the value function in a state at time $t$, while
$\overline{V}_t^n(s)$ is a statistical estimate after $n$ sample observations. Let $\omega^n$ represent the specific
value of $\omega$ that it is sampled for iteration $n$. The value $\hat{v}_t^n$ is computed using information
from sample path $\omega^n$, and is used to update the value function approximation to produce
$\overline{V}_t^n(s)$.

Several technical issues are introduced in the following subsections.

6.4.1 Finding the Post-Decision State Variables

In Approximate Dynamic Programming, the post-decision state is defined as the state of
the system after a decision is made, but before any new information has arrived. Therefore,
the transition functions can be broken into the two steps:

$$S^x_t = S^{M,x}(S_t, x_t),$$
$$S_{t+1} = S^{M,W}(S^x_t, W_{t+1}),$$

where $S_t$ is the state of the system immediately before a decision is made, known as
the pre-decision state variable, while $S^x_t$ is the post-decision state variable. The function
$S^{M,x}(S_t, x_t)$ is the “pre- to post-” transition function, and $S^{M,W}(S^x_t, W_{t+1})$ is the “post- to
pre-” transition function.

Table 6.7 summarises $S_t$, $S^x_t$ and $S_{t+1}$ as below.

<table>
<thead>
<tr>
<th>Table 6.7: Pre-decision and Post-decision State Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_t = (e_{t-1}^l, e_{t-1}^f, e_{t-1}^n, eq_{t-1}, s_t-1, b_t-1, \bar{y}_t, p_t^e)$, $t=1,...,T$</td>
</tr>
<tr>
<td>$S^x_t = (e_t, e_{t-1}^l, e_t^f, eq_t, s_t, b_t, \bar{y}_t, p_t^e)$, $t=1,...,T$</td>
</tr>
<tr>
<td>$S_{t+1} = (e_t, e_{t+1}^l, e_t^f, eq_t, s_t, b_t, \bar{y}<em>{t+1}, p</em>{t+1}^e)$, $t=1,...,T$</td>
</tr>
</tbody>
</table>

When making decisions, the following function is used:

$$\hat{v}_t^n(S_t^n) = \max_{x_t \in \Omega(S_t^n)} (L(S_t^n, x_t) + \overline{V}_t^{n-1}(S^{M,x}(S_t^n, x_t))). \quad (6.19)$$
Notation $S^n_t$ is used to indicate that it is at a particular state, rather than a set of all possible states. Let $x^n_t$ be the value of $x_t$ that solves Eq.(6.19) and $\tilde{v}^n_t$ is a sample realisation of the value of being in state $S^n_t$. Note that $\tilde{v}^n_t$ is a sample of the value of being in state $S^n_t$, and it is also a sample of the value of being in state $S^{x,n}_{t-1}$, which is the post-decision of the last period. Thus, one can update the post-decision value function approximation using

$$V^{n}_{t-1}(S^{x,n}_{t-1}) = (1 - \alpha_{n-1})V^{n-1}_{t-1}(S^{x,n}_{t-1}) + \alpha_{n-1}\tilde{v}^n_{t}(S^n_{t}), \quad (6.20)$$

where $\alpha_{n-1}$ is a smoothing factor (stepsize) governed by a particular stepsize rule.

The choice of a stepsize rule is essential to the convergence behaviour of an ADP algorithm and the quality of the solution (He et al. 2012). The stepsize $0 < \alpha_{n-1} \leq 1$ plays two roles in approximate value iteration. First, it smooths out the effects of noise in our observations (the lower the stepsize, the smoother the approximation). Second, it determines how quickly the one-period rewards are added up (the higher the stepsize, the more it is added). This work uses the polynomial stepsize rule $\alpha = 1/n^n$ with $\tau \in (0.5, 1]$. This rule satisfies the conditions for convergence but produces larger stepsizes for $\alpha > 1$ than the $1/n$ rule (George and Powell 2006).

6.4.2 Double Pass

In the classical approximate value iteration with a pure forward pass, it may take many iterations before the costs incurred in later time periods are correctly transferred to the earlier time periods. To overcome this, the ADP algorithm can also be used with a double pass approach consisting of a forward pass and a backward pass. The forward pass simulates decisions moving forward in time, remembering the trajectory of states, decisions and outcomes. Then the backward pass updates the value functions moving backwards in time using the trajectory information.

The difference between Single-Pass vs. Double-Pass is the timing of the value function updates. In the Single-Pass, update takes place immediately after the new estimation of the value of a state is available; while in the Double Pass, the update takes place after each iteration (Powell 2007). The single-pass approach is simple to implement, but not
necessarily very efficient. The alternative double-pass method, is generally more efficient.

The double-pass algorithm, as implemented here, is presented in Table 6.8.

**Table 6.8: The Double-Pass ADP Algorithm**

<table>
<thead>
<tr>
<th>Step 0: initialisation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 0a:</strong> initialise $V^0_t(S^x_t)$, $\forall S^x_t$, $\forall t$.</td>
</tr>
<tr>
<td><strong>Step 0b:</strong> Set $n=1$.</td>
</tr>
<tr>
<td><strong>Step 0c:</strong> initialise the state $S^n_0$ for all $n$.</td>
</tr>
<tr>
<td><strong>Step 1:</strong> Choose a sample path $\omega^n$.</td>
</tr>
<tr>
<td><strong>Step 2:</strong> The forward pass. Set $t=1$. Do for $t=0, ..., T$.</td>
</tr>
<tr>
<td><strong>Step 2a:</strong> For $S^n_t$, solve the optimisation problem:</td>
</tr>
<tr>
<td>$x^n_t = \arg\max (L_t(S^n_t, x_t) + V^{n-1}_t(S^{M,x}(S^n_t, x_t)))$.</td>
</tr>
<tr>
<td><strong>Step 2b:</strong> Observe the state transition and record the visited post-decision states $S^{x,n}_t$, and update the state:</td>
</tr>
<tr>
<td>$S^{x,n}_t = S^{M,x}(S^n_t, x^n_t)$,</td>
</tr>
<tr>
<td>$S^{x,n}<em>{t+1} = S^{M,W}(S^n_t, x^n_t, W^n</em>{t+1})$.</td>
</tr>
<tr>
<td><strong>Step 3:</strong> The backward pass. Do for $t = T, ..., 1$, and update its value function approximation:</td>
</tr>
<tr>
<td>$\widehat{v}^n_t = L(S^n_t, x^n_t) + \beta \widehat{v}^n_{t+1}$,</td>
</tr>
<tr>
<td>$\widehat{V}^n_t(S^{x,n}_t) = (1 - \alpha_n)\widehat{V}^{n-1}_t(S^{x,n}_t) + \alpha_n \widehat{v}^n_t$.</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Let $n = n + 1$. If $n \leq N$, go to <strong>Step 2</strong>.</td>
</tr>
<tr>
<td><strong>Step 5:</strong> Return the value functions $(\widehat{V}^N_t)_{t=0}^T$.</td>
</tr>
</tbody>
</table>

### 6.4.3 Aggregation

The ADP has the advantage of being independent of problem structures; however, it struggles with large state space. Recall that the problem has continuous state and outcome spaces. The ADP requires discretisation of the state space. A popular way to overcome large state spaces is to aggregate the state space. For example, a five-digit zip code can be aggregated up to a three-digit zip; a state variable can be discretised into fewer ranges; or a state variable can be completely ignored (George et al. 2008).

Note that aggregation reduces statistical errors, but introduces structural errors; therefore, the challenge is to find the optimal aggregation level. Powell (2007) introduces several strategies by estimating a set of weights for different levels of aggregation. This work chooses a fixed level of aggregation because of the extremely large number of values that need to be
estimated. With respect to the state variables, one can either reduce their levels or ignore them. Since this study pays more attention to financial conditions, the state variables of $e_t, e_t^n, s_t,$ and $b_t$ are retained, while others are ignored. An overview of the aggregation structure is given in the following table, where “−” corresponds to a state variable that is ignored.

<table>
<thead>
<tr>
<th>Table 6.9: Static Aggregation Structure of $S_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_t$</td>
</tr>
<tr>
<td>Level</td>
</tr>
</tbody>
</table>

6.5 Numerical Experiments

This section undertakes numerical experiments to study the proposed model by the ADP algorithm. In either theoretical studies or numerical experiments, the specified policies can greatly alter the results and conclusions. Considering the policies described earlier, this study examines the impact of these policies on performance. Section 6.5.1 first estimates the relevant parameters and functions. Section 6.5.2 provides results of a benchmark case. Then Sections 6.5.3, 6.5.4, and 6.5.5 analyse the impact of the Fund Size Risk Management Policy, the Liquidity Policy, and the Flexibility Policy, respectively.

6.5.1 Parameter Estimation

An important part of numerical experiments is to configure the model input data. Next, the model parameters and technical parameters are discussed and determined.

Model Parameters

Table 6.10 summarises all parameter values estimated from Fonterra’s financial reports. In finance, the weighted average cost of capital $WACC$ is often used as a discount rate (Bruner et al. 1998), whose average value is around 6.95% (Fonterra Farmgate Milk Price Statement 2015); therefore, the discount factor is identified as $\beta = \frac{1}{1 + WACC} = 93.5%$. Furthermore, $ROC$ is estimated upon the average $ROC$ from the last three years. Notice that, $ROC > WACC$, indicates that the return on capital invested is larger than the cost
of capital. This condition has a significant implication because when a company makes
decisions on investing, they often compare the rates of return on investment and the cost
of raising money. If the company can get a high return on investment in a new project,
it is a costly mistake to keep the cash in the bank. However, if the project’s return is less
than the company’s cost of capital, the cash should be returned to shareholders.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WACC</strong></td>
<td>6.95%</td>
<td>The weighted average cost of capital (used as the discount rate)</td>
</tr>
<tr>
<td><strong>β</strong></td>
<td>93.5%</td>
<td>The discount factor</td>
</tr>
<tr>
<td><strong>ROC</strong></td>
<td>9.5%</td>
<td>Return on invested capital</td>
</tr>
<tr>
<td><strong>ρ₀</strong></td>
<td>6%</td>
<td>A weighted average interest rate</td>
</tr>
<tr>
<td><strong>c₀</strong></td>
<td>2058</td>
<td>Operating costs ($ million)</td>
</tr>
<tr>
<td><strong>Tax</strong></td>
<td>28%</td>
<td>Tax rate</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.75</td>
<td>The lower bound of flexibility</td>
</tr>
<tr>
<td><strong>H</strong></td>
<td>1.2</td>
<td>The upper bound of flexibility</td>
</tr>
<tr>
<td><strong>k</strong></td>
<td>[12%, 15%, 18%]</td>
<td>The possible upper limit of Fund Size</td>
</tr>
<tr>
<td><strong>s_{low}</strong></td>
<td>[0, 200, 500]</td>
<td>The minimum required cash position ($ million)</td>
</tr>
</tbody>
</table>

Furthermore, Fonterra borrows a mixture of fixed and variable rate debt in a range of
currencies. Weighted average interest rate in 2016 was 6% (6.3% in 2015, 5.97% in 2014),
therefore, an estimated interest rate is ρ₀ = 6%, and it is assumed to be constant in the
following years. In the financial report, the taxation charge would rise at the standard rate
of corporation tax in NZ, that is 28%. Lastly, the operating costs, which have been quite
stable throughout those years, are estimated at an average value between 2010-2016.

**Technical Parameters**

The algorithm also needs to determine several technical parameters including state spaces,
action spaces, and initial values, etc.

Fonterra’s dividend policy is to pay out 65-75% of net profit after tax and Table 6.12
summarises the ratio of dividends in past years. In extreme cases, dividend payout ratios
exceed 100%, which means more dividends were paid out than profits. This study allows
D_t = 0 when the cash position is pretty low, and D_t = 120% when the cash position is very
high. Therefore, the action space of dividends is \( D_t \in [0, 50\%, 65\%, 75\%, 100\%, 120\%] \). In addition, Fonterra’s target level of gearing ratio falls in the range of 45-50% and Table 6.12 shows the gearing ratio in past years. The action space of borrowing is determined as \( B_t \in \{35\%, 40\%, 45\%, 50\%\} \). Furthermore, unlike the retention ratio, reinvestment rate \( I_t \) can be well in excess of 100% because firms can raise new equity or borrow from the bank; therefore, the action space of investment rate is \( I_t \in \{0, 30\%, 60\%, 100\%, 150\%, 200\%\} \).

### Table 6.11: Technical Parameters in ADP

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z )</td>
<td>( {0, 0.2, 0.5, 0.75, 1} )</td>
<td>The Action Space of choice</td>
</tr>
<tr>
<td>( D )</td>
<td>( {0, 0.5, 0.65, 0.75, 1, 1.2} )</td>
<td>The Action Space of dividend payout ratio</td>
</tr>
<tr>
<td>( B )</td>
<td>( {0.35, 0.4, 0.45, 0.5} )</td>
<td>The Action Space of gearing ratio</td>
</tr>
<tr>
<td>( I )</td>
<td>( {0, 0.3, 0.6, 1, 1.5, 2} )</td>
<td>The Action Space of reinvestment rate</td>
</tr>
<tr>
<td>([e_{\text{low}}, e_{\text{upp}}])</td>
<td>([1200, 2000])</td>
<td>The Action Space of Shares</td>
</tr>
<tr>
<td>([e_{\text{low}}, e_{\text{upp}}])</td>
<td>([0, 300])</td>
<td>The State Space of Units</td>
</tr>
<tr>
<td>([\text{cash}<em>{\text{low}}, \text{cash}</em>{\text{upp}}])</td>
<td>([0, 3000])</td>
<td>The State Space of cash position</td>
</tr>
<tr>
<td>([\text{b}<em>{\text{low}}, \text{b}</em>{\text{upp}}])</td>
<td>([4000, 10000])</td>
<td>The State Space of borrowing</td>
</tr>
<tr>
<td>( T )</td>
<td>5</td>
<td>The length of planning horizon</td>
</tr>
<tr>
<td>( \text{numPaths} )</td>
<td>10000</td>
<td>The number of iterations</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.6</td>
<td>The step-size parameter</td>
</tr>
</tbody>
</table>

### Table 6.12: Gearing Ratio and Dividend Payout Ratio from 2011-2016

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_t )</td>
<td>78.4%</td>
<td>86.2%</td>
<td>100%</td>
<td>72.7%</td>
<td>78%</td>
<td>56.6%</td>
</tr>
<tr>
<td>( B_t )</td>
<td>44.3%</td>
<td>49.7%</td>
<td>42.3%</td>
<td>39.6%</td>
<td>39.1%</td>
<td>41.8%</td>
</tr>
</tbody>
</table>

Since this work employed the technique of aggregation, it only specified the state space of \( e_t, e^n_t, s_t, \) and \( b_t \). These parameters were derived based upon Fonterra’s historical data. Note that, in the discretisation of both state space and action space, one should take care of the trade-off between accuracy and dimensionality. In addition, it was decided that the length of the planning horizon be as short-term as five years. This is because some relevant parameters and policies may vary across time; however, may not change in the near future. Furthermore, this study applied the data in 2015 as the initial state values shown in Table 6.13.
Table 6.13: Initial Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_0$</td>
<td>1,599</td>
<td>The opening number of issued Shares ($ million)</td>
</tr>
<tr>
<td>$e_0$</td>
<td>109.7</td>
<td>The opening number of issued Units ($ million)</td>
</tr>
<tr>
<td>$s_0$</td>
<td>303</td>
<td>The opening cash position ($ million)</td>
</tr>
<tr>
<td>$b_0$</td>
<td>7,560</td>
<td>Borrowings ($ million)</td>
</tr>
<tr>
<td>$eq_0$</td>
<td>6,959</td>
<td>Opening equity ($ million)</td>
</tr>
<tr>
<td>$e_f^0$</td>
<td>1,490</td>
<td>Farmers’ Shares realised in the current period ($ million)</td>
</tr>
<tr>
<td>$e_f^{−1}$</td>
<td>1,493</td>
<td>Farmers’ Shares in the last period ($ million)</td>
</tr>
<tr>
<td>$p_s^0$</td>
<td>5.95</td>
<td>Initial share price</td>
</tr>
<tr>
<td>$y_{−2}$</td>
<td>1,584</td>
<td>Milk collected two years ago (million kgMS)</td>
</tr>
<tr>
<td>$y_{−1}$</td>
<td>1,614</td>
<td>Milk collected in the last year (million kgMS)</td>
</tr>
<tr>
<td>$y_0$</td>
<td>1,566</td>
<td>Milk collected in the current year (million kgMS)</td>
</tr>
</tbody>
</table>

6.5.2 Benchmark Case

This section first experiments on the benchmark case when the limit of Fund Size is $k = 12\%$ and the limit of cash position is $s_{low} = 200$. Then the objective defined in Eq.(6.13) is subject to the constraints of $e_t/e_t \leq 12\%$ and $s_t \geq 200$. In the ADP algorithm, the optimal decision is derived from $\arg \max L(S_t^*, x_t) + V_{t+1}^m(S_t^*)$. It was set that if there was more than one optimal decision, one of these would be chosen at random from the optimal decisions. There is also a possibility that decisions in the action spaces are unable to satisfy all constraints (specifically the constraint of Fund Size and liquidity). If there are no optimal decisions, then one is chosen at random. After running the model, the percentage of the cash position less than $s_{low}$ or the Fund Size larger than $k$ would be counted to reflect the probability of financial risks when the cash position is unable to achieve the minimum requirement or the ratio of Units and Shares breaks the limit. This chapter uses $P(\cdot)$ to indicate the probability of an event, e.g., $P(s < 200)$ indicates the probability of the liquidity risk.
Figure 6.4: The Objective Value and Financial Risk of the Benchmark Case When $k = 12\%$ and $s_{low} = 200$

Figure 6.5: The Results of the Benchmark Case When $k = 12\%$ and $s_{low} = 200$

The model was run using parameters in Tables 6.10, 6.11, and 6.13. Figure 6.4 plots the approximated objective values up to 10,000 iterations. The objective value stabilised at around 4,000 iterations and the optimal value is around 3,450 ($\text{million}$). The right-hand picture illustrates the financial risks in each period which are reflected by the probability of
\(e^n/e > 12\%\) or \(s < 200\), i.e., \(P(e^n/e > 12\% \cup s < 200)\). Note that, the states are recorded as the post-decision states, and the initial period is represented by 1. Therefore, in these figures, one should understand the states as post-decision states in accordance with the periods starting from 1 to 6.

Figure 6.5 shows the average share levels, gearing ratio, dividend payout ratios, and reinvestment rates. The top-left chart illustrates that the average share level increases with time. This makes sense, because milk production is increasing as assumed earlier. It also indicates that, in response to the increased milk production, Fonterra is likely to issue new Shares. This is consistent with Fonterra’s statement that, “Fonterra intends to respond to growing Share-backed milk production by increasing the number of Shares on issue to the extent necessary to promote liquidity.” Other figures depict the optimal decisions of borrowing, dividends, and reinvestment. In the first stage, when the initial cash position is low, the organisation tends to borrow as much as possible and not to reinvest. With an increase in cash flow, the organisation is able to increase dividend, decrease gearing ratio, and increase the cash used for reinvestment. Since the average dividend payout ratios are quite high and stable in the following periods, it can be deduced that return to Shareholders is preferable to reinvesting in growth and paying off debt.

### 6.5.3 Impact of Fund Size Risk Management Policy

This section analyses the impact of the Fund Size Risk Management Policy by relaxing the constraint of Fund Size and comparing the results with \(k = 12\%, 15\%,\) and \(18\%\). This experiment used \(k = 15\%\) to indicate the case of a low-tolerance on Fund Size; while \(k = 18\%\) indicates a high-tolerance. In all cases, the minimum cash position was specified as \(s_{low} = 200\).

Figure 6.6 compares the objective values and financial risks using the same measurement of \(e^n/e < 12\%\) and \(s > 200\) when \(k = 12\%, k = 15\%,\) and \(k = 18\%.\) As would be expected, the model with a higher tolerance on Fund Size generates a larger objective value. However, a high-yield is accompanied with a high-risk. The results reflect the trade-off between conflicting objectives of maximising dividends and minimising financial risks. Essentially, a
higher tolerance on Fund Size generates a larger objective value, but also a higher financial risk.

![Objective Value Comparison](image1)

![Financial Risks Comparison](image2)

Figure 6.6: The Objective Value and Financial Risks Comparison When $k = 12\%$, $k = 15\%$, and $k = 18\%$

The probabilities of the liquidity risk and the Fund Size risk are shown in Figure 6.7 to analyse the relationship between the liquidity risk and Fund Size risk. The left-hand side figures illustrate the projected liquidity risk and one can observe that with the increase of tolerance, the probabilities of the liquidity risk decrease. The right-hand side figures depict the Fund Size risk under different tolerances. It can be seen that in period 1, the Fund Size is quite stable within $[7\%, 12\%]$. This stability results from the same initial values in all iterations. However, with the exogenous information coming in the next period, variabilities begin to show. Furthermore, Fonterra’s target Fund Size is between 7% and 12%; however, one can observe that the probabilities of the Fund Size less than 7%, i.e., $P(e_u/e < 7\%)$, are quite significant in the following periods. There are two reasons for this: first, milk production is increasing with time, which is likely to increase the number of Farmers’ Shares as required by the Share Standard. Under the TAF, Fonterra can either issue new Shares or transfer Units to Shares; both options are likely to decrease the ratio of $e_u/e$. In addition, a lower limit for Fund Size is not specified (except larger than 0). Therefore, the Fund Size is likely to be less than 7%. Finally, one can observe that with the increase of $k$, the probabilities of the Fund Size larger than 12%, i.e., $P(e_u/e > 12%)$, are increasing as well. This indicates that a higher tolerance on Fund Size leads to a higher Fund Size risk, although it alleviates the liquidity risk.
Figure 6.7: The Results of the Model with $k = 12\%$, $k = 15\%$, and $k = 18\%$
Overall, it can be implied that a higher Fund Size boundary is able to increase equity holders’ returns and also able to reduce the liquidity risk by increasing the cash flow, however, it is at a greater risk of demutualisation. The results show the trade-off between the objectives of maximising equity holders’ returns and minimising financial risks, also reflecting the trade-off between minimising Fund Size risk and minimising liquidity risk.

6.5.4 Impact of Liquidity Policy

This section relaxes the constraint on the cash position to examine the impact of the Liquidity Policy. The experiment chose the minimum cash position to be $s_{\text{low}} = 0$, and $s_{\text{low}} = 500$, in comparison with the benchmark case where $s_{\text{low}} = 200$. In all cases, the limit of Fund Size is $k = 12\%$.

![Figure 6.8: The Objective Value and Financial Risks Comparison When $s_{\text{low}} = 0$, $s_{\text{low}} = 200$, and $s_{\text{low}} = 500$](image)

Figure 6.8 compares the objective values and financial risks in three cases when $s_{\text{low}} = 0$, $s_{\text{low}} = 200$, and $s_{\text{low}} = 500$. The relaxation of liquidity constraints also increases the objective value; for example, the objective value when $s_{\text{low}} = 0$ is significantly larger than the value when $s_{\text{low}} = 500$. This is understandable as a higher cash reserve implies less cash for distributing to shareholders, for reinvesting, or for paying off debt. Furthermore, the results also show the same pattern of high profit, and high risk. In the right-hand figure, it can be seen that when $s_{\text{low}} = 0$, the financial risk is quite significant in comparison with the other two cases.
To figure out the reasons concerning financial risks, the distribution of the cash position is illustrated in Figure 6.9. When $s_{low} = 0$, the probabilities of the cash position in the range of $[0, 200)$, i.e., $P(0 \leq s < 200)$, are quite significant under the constraint of $s_t \geq 0$. Particularly in the first period, the cash position is actually stabilised at around 30 (recall that the same initial values were set in all iterations). The probability of the Fund Size risk in Figure 6.9 were also compared, and it is shown that the probability of Fund Size risk is slightly reduced when a higher cash reserve is required.

Overall, it can be concluded that a higher cash reserve is able to reduce both the liquidity risk and the Fund Size risk; however, at the cost of greatly reduced profits and returns for equity holders.

Figure 6.9: The Results of the Model with $s_{low} = 0$ and $s_{low} = 500$
6.5.5 Impact of the Flexibility Policy

Before the transformation of capital structure, there was no flexibility and farmers were required to hold the number of Shares in proportion to the Share Standard. However, after the transformation, farmers (as a whole) are allowed to hold more or less than the Share Standard. The lower limit indicates the extent to which farmers are allowed to hold less than required; while the upper limit represents the extent to which farmers are allowed to hold more. In order to examine the impact of the Flexibility Policy, two parts will be examined: one is to examine the influence of the lower limit and the other is the influence of the upper limit.

The Impact of the Lower Limit in the Flexibility Policy

This section examines the impact of the lower limit within the Flexibility Policy by comparing models with the flexibility of \([0.75, 1.2]\), \([0.9, 1.2]\), and \([1, 1.2]\) as presented in Table 6.14. A greater lower limit indicates a lower tolerance in allowing fewer Shares. In the model with the flexibility of \([1, 1.2]\), there is no tolerance on holding fewer Shares.

Table 6.14: The Probability of Financial Risks and Objective Values with Different Lower Limits

<table>
<thead>
<tr>
<th>With (s_{low} = 200, k = 12%)</th>
<th>(t = 1)</th>
<th>(t = 2)</th>
<th>(t = 3)</th>
<th>(t = 4)</th>
<th>(t = 5)</th>
<th>(t = 6)</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>([L, H] = [0.75, 1.2])</td>
<td>0</td>
<td>0.057</td>
<td>0.0766</td>
<td>0.099</td>
<td>0.0963</td>
<td>0.0839</td>
<td>3,450</td>
</tr>
<tr>
<td>([L, H] = [0.9, 1.2])</td>
<td>0</td>
<td>0</td>
<td>0.0012</td>
<td>0.0094</td>
<td>0.0111</td>
<td>0.0077</td>
<td>3,800</td>
</tr>
<tr>
<td>([L, H] = [1, 1.2])</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3,900</td>
</tr>
</tbody>
</table>

Figure 6.10 illustrates the objective values and financial risks in three cases. It turns out that with a greater lower limit, the objective value is increased and the financial risks are reduced. Especially when there is no tolerance on holding fewer Shares (i.e., \([L, H] = [1, 1.2]\)) , the financial risks are reduced to zero in all periods. Figure 6.10 also compares the average cash position and Fund Size in each period. As shown in both charts, the average cash position increases and the average Fund Size decreases as the lower limit increases. The results can be easily explained where under a greater lower limit, farmers are less likely to hold fewer Shares. In other words, they are more likely to hold more Shares. In response to this, Fonterra issues more Share than before, which contributes to
the cash flow, hence reducing the liquidity risk. The decreased average Fund Size shown in the lower-right chart reflects the relatively increased number of Shares on issue, which results in a lower ratio of Shares and Units.

Figure 6.10: The Comparison among the Models with the Flexibility of [0.75, 1.2], [0.9, 1.2], and [1, 1.2]

The Impact of the Upper Limit in the Flexibility Policy

This section examines the impact of the upper limit within the Flexibility Policy by comparing models of no-flexibility, and the flexibility of [1, 1.2], and of [1, 1.5], presented in Table 6.14. The model of no-flexibility manifests the feature of the previous capital structure, under which farmers are required to hold the exact number of Shares by the Share Standard. A larger upper limit indicates a higher tolerance in allowing more Shares.
Figure 6.11: The Comparison of Objective Value and Average Dividend per Share with No-Flexibility, and with the Flexibility of $[1, 1.2]$ and $[1, 1.5]$

Table 6.15: The Probability of Financial Risks and Objective Values with Different Upper Limits

<table>
<thead>
<tr>
<th>With $s_{low} = 200, k = 12%$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
<th>$t = 4$</th>
<th>$t = 5$</th>
<th>$t = 6$</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Flexibility</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0006</td>
<td>0.0012</td>
<td>0.0004</td>
<td>3700</td>
</tr>
<tr>
<td>$[L, H] = [1, 1.2]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3900</td>
</tr>
<tr>
<td>$[L, H] = [1, 1.5]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4000</td>
</tr>
</tbody>
</table>

Table 6.15 presents the results for the objective values and the probability of financial risks with no-flexibility, and with the flexibility of $[1, 1.2]$ and $[1, 1.5]$. Unsurprisingly, with an increase of the upper limit, the objective value increases, and the financial risks become quite insignificant in all cases. This makes sense, because when farmers are allowed to hold more Shares, the capital invested is likely to increase accordingly, hence contributes to the liquidity and decreases the financial risks.

Figure 6.11 illustrates the dividend per Share under the three cases. It can be seen that the dividend per Share is increasing throughout the periods. This makes sense because milk production is increasing and it is also a sign that the co-op’s value-add strategy of reinvestment might be starting to pay off. However, it is quite surprising that the Flexibility Policy does not contribute to the average dividend per share, i.e., the average dividend per share in the model of $[1, 1.5]$ is less than the other two cases. In finance, this indicates the phenomenon of equity dilution, which results from the increased number of Shares on issue. The equity dilution is likely to occur when the growth of profitability does not keep pace with the increase in capital invested. Dilution can shift fundamental positions of the stock such as earnings per share, voting control, and the value of individual shares.
Furthermore, it will reduce the share price as well, further damaging an equity holder’s returns. Therefore, from the perspective of return on equity, allowing greater flexibility on more Shares is not always beneficial to the organisation.

Figure 6.12: The Comparison among the Models with No-Flexibility, and with the Flexibility of [1, 1.2] and [1, 1.5]

Figure 6.12 compares the results of the three cases in terms of cash position, the dividend payout ratio, gearing ratio, and reinvestment ratio. As the upper limit increases, the average cash position increases due to a larger equity investment. With abundant cash flow, the organisation needs to decide how to use the money it generates. The dividend payout ratio provides an indication of how much money a company is returning to its shareholders, versus how much money it is keeping on hand to reinvest in growth, pay off debts, or retain as cash reserve. An observation is that the organisation gives priority to paying out dividends since the average dividend payout ratio achieves the maximum allowable payout ratio throughout the periods (specified at 120%). After that, the organisation is likely to pay off debt or to reinvest, reflected by the decreased gearing ratio or the increased reinvestment ratio. Since the average cash position is decreasing gradually, it implies that
Fonterra is less likely to keep the cash in the organisation. This does make sense since retaining the cash does not generate profit, as it is only used for risk resistance.

![Objective Value Comparison](image1)

![Financial Risks Comparison](image2)

Figure 6.13: The Comparison between the Benchmark Case and the model of No-Flexibility

From the above analysis, it can be found that the role of allowing farmers to hold fewer Shares on the objective value and financial risks is opposite to that to allowing farmers to hold more Shares. Since the current Flexibility Policy allows both, the efficiency of Fonterra’s current Flexibility Policy is examined by comparing the benchmark case where 

\[ L, H \] = [0.75, 1.2]

with the model of no-flexibility in Figure 6.13. It is quite interesting to notice that both the objective value and the financial risks are damaged by the Flexibility Policy. This is because the advantage of allowing farmers to hold more Shares is counteracted and even surpassed by the disadvantage of allowing farmers to hold fewer Shares.

### 6.6 A Thought Experiment on Milk Production

Thought experiments consider a hypothesis, theory, or principle for the purpose of thinking through its consequences. Keith Woodford, a Professor of Agri-Food Systems at Lincoln University in New Zealand, has been doing a “thought experiment” as to how Fonterra might fare if its milk collected decreases. The specific scenario he has chosen to consider is where Fonterra’s processing volumes decline from 1600 million of milksolids in 2014/2015, down to 1200 million at some time in the future. Since Fonterra’s equity is associated with milk production, then what would happen to the Shares associated with the lost milksolids? Under TAF there are two options: the first would be for Fonterra to buy back the Shares
and the second option is that investors come in and buy the economic rights of these Shares (transferring into Units). However, he is concerned that the problem introduced by the first option is that Fonterra may suffer from liquidity risk; while the second option may cause the risk of demutualisation (Fund Size risk). This thought experiment concludes that Fonterra would be caught between a rock and a hard place, and the introduction of TAF does not eliminate redemption risk as it is intended to.

Therefore, this section examines the situation when milk production significantly decreases in the near future. In Section 6.2.3, the assumption (Assumption 6.3) was made where the average processing quantity is increasing in time. In this thought experiment, the assumption has changed as follows:

**Assumption 6.6.** Assume that the random milk production follows a normal distribution each year $Y_t \sim N(\mu_t, \sigma^2)$, but the average is decreasing in time as below:

$$\mu_t = -100t + 1600.$$ \hfill (6.21)

Note that, in this experiment, there is no intention to verify this assumption because the idea of a thought experiment, such as this, is not to precisely predict the future, but to test resilience of the system under adversity. Thus, this experiment is to run the model with predetermined parameters, setting the objective parameters with $k = 12\%$ and $s_{low} = 200$, and then compare the results with the benchmark case wherein milk production is increasing.

Figure 6.14 shows the objective value stabilising at approximately 1350, which is greatly decreased compared with the benchmark case. This result is not surprising, because a decreased milk production results in a lower market profit, hence a lower dividend. Furthermore, as shown in the top-right picture, the probability of financial risks dramatically increases with time. It is worth noting that the risk becomes quite significant in the near future.
Figure 6.14: The Results of the Thought Experiment with Decreasing Milk Production

High-risk is introduced by both the liquidity risk and the Fund Size risk, which can be seen in the charts below, that depict the probabilities of the liquidity risk and the probabilities of the Fund Size risk. With a decreased milk production, the liquidity suffers from two aspects: on the one hand, a decreased milk production leads to a decreased market profit and also a decreased cash flow accordingly, and on the other hand, the number of Farmers’ Shares associated with milk production are reduced as well, which requires the co-op to pay out capital to redeem Shares. In addition to the increased liquidity risk, there is also an obviously increased Fund Size risk. This is because, in alleviating the liquidity risk, the co-op has to transfer a large amount of Shares required to be redeemed into Units, which results in an increase of Fund Size and Fund Size risk.
Figure 6.15: The Comparison between the Benchmark Model and the Model with Decreasing Milk Production

The average Share and Unit level in the benchmark case and in the thought experiment are compared in Figure 6.15. When milk production is increasing, the average Share level is also increasing. However, when milk production is decreasing, the average Share level begins to decrease after the initial two periods. Note that, the trend of decreasing is lagged by the calculation method of the Share Standard, which is based on the three-year rolling average starting from the last period. Therefore, with initial values given in Table 6.13, there is still an increase of Shares in the first two periods. Furthermore, the Unit level is relatively stable in the benchmark case; however, it shows a remarkable increasing trend when milk production is decreasing. The trend of Share and Unit levels can be explained: on the one hand, the co-op transfers a portion of redeemed Shares into Units to maintain the liquidity; on the other hand, the co-op redeems a portion of Shares by itself to alleviate the pressure of Fund Size risk. However, since the influence of milk reduction is so significant, it can hardly maintain the Fund Size and liquidity at the same time.

In the last section, the impacts of the Fund Size Risk Management Policy, Liquidity Policy, and Flexibility Policy have been analysed. Since relaxing the constraint of Fund Size does not contribute to the financial risks, one possible alternative is to require a higher minimum cash position to cope with the adverse situation, and another possible choice is to constrain the flexibility on holding fewer Shares, but relaxing the flexibility on holding more Shares. Therefore, this section tries to mitigate this situation by running a model with $s_{low} = 500$ and the Flexibility Policy of $[1, 1.5]$ shown in Figure 6.16.
Figure 6.16: The Thought Experiment with $s_{\text{low}} = 500$ and the Flexibility of [1, 1.5]

According to the earlier analysis, one can expect that both of these actions can reduce financial risks. This improvement is illustrated in the right chart where the probability of financial risks is greatly reduced compared to the former model. Furthermore, since a higher minimum cash position results in a lower objective value as stated previously, the objective value is still compromised slightly although a higher flexibility on holding more Shares contributes to the objective value.

However, the challenge is to motivate farmers to purchase more Shares in practice, especially under adversity. Therefore, Fonterra should try to motivate farmers to invest more by providing other incentives beyond economic benefits.

6.7 Conclusions

This chapter formulates a dynamic stochastic model of capital structure optimisation for the Fonterra milk cooperative. Fonterra’s capital structure is characterised with a flexible equity requirement tied to farmers’ milk production, and with an introduction of Trading Among Farmers wherein farmers are free to trade Shares among themselves and outside investors are able to invest in Units. The capital structure is governed and controlled by the Fund Size Risk Management Policy, Liquidity Policy, and Flexibility Policy, aiming to maximise equity holders’ returns and minimise the Fund Size risk and liquidity risk. The
algorithm for solving this stochastic programme is based on the Approximate Dynamic Programming algorithm, with all parameters estimated from Fonterra’s financial reports and other relevant documents. The numerical experiments are implemented to derive the results of the benchmark case, investigate the impact of three policies on performance, and test resilience of the system under adversity, thus producing the following findings:

- Under Fonterra’s current capital structure, policies, and macro-environment, the profitability and equity holders’ returns are likely to increase in the following periods; however, there is also a possibility of financial risks. In comparison to the model of no-flexibility, there is an increased risk caused by the Flexibility Policy of allowing farmers to hold fewer Shares than the Share Standard. Therefore, Fonterra should constrain the extent to which farmers can sell Economic Rights of Wet Shares to Units.

- The optimal decision policy implies that Fonterra gives priority to distributing profit to equity holders, rather than to reinvestment or to repayment of borrowing, and then to increasing the cash reserve. Since equity holders comprise both Farmer Shareholders and Unit holders, this result casts a doubt on Fonterra’s ability to maintain future returns to Farmer Shareholders. Furthermore, it may encourage farmers to purchase more Shares, instead of investing in their farm production. Therefore, in the short term, Fonterra should focus more on maximising profitability, instead of maximising dividends.

- The analysis of the Fund Size Risk Management Policy and Liquidity Policy indicates that a higher tolerance on the Fund Size is able to increase the equity holders’ returns and reduce the liquidity risk; however, it is at a greater risk of demutualisation. In addition, a higher cash reserve contributes to both the liquidity risk and the Fund Size risk; however, it greatly compromises the equity holders’ returns (dividends). The results reflect the trade-offs among the objectives of maximising equity holders’ returns, minimising Fund Size risk, and minimising liquidity risk. Fonterra can take advantage of these policies in different circumstances. For example, in the case of
capital shortage, Fonterra may relax the Fund Size limit for a few periods and then constrain the Fund Size when the financial condition improves.

- The Flexibility Policy of allowing farmers to hold more Shares is able to increase the equity holders’ returns and reduce financial risks as well; while the policy of allowing farmers to hold fewer Shares plays the opposite role in performance. Therefore, the current Flexibility Policy proposed by Fonterra does not contribute to the organisation because the advantage of allowing farmers to hold more Shares is surpassed by the disadvantage of allowing farmers to hold fewer Shares. Fonterra should design a proper Flexibility Policy for different situations.

- It is worth noting that allowing farmers to hold more Shares to increase capital may lead to the problem of equity dilution indicated by the decreased dividend per share, and the severity of this problem depends on the extent to which farmers can purchase, and the growth of profitability. Therefore, Fonterra should strive to produce more value-added product and increase the return on invested capital to alleviate this problem.

- Last but not least, the thought experiment where assuming decreased milk production over time implies that Fonterra is very likely to suffer from redemption risk if milk production continues to decrease in the future. This result manifests the importance of farmers’ economic participation as well, and suggests that the poor condition can be mitigated by specifying higher cash reserves, or specifying larger flexibility to hold more Shares.

Note that, making decisions on dividends, borrowing, or reinvestment is largely dependent on financial parameters, such as interest rates, capital costs, and returns on capital investment, etc. For simplicity, this chapter assumes constant financial parameters, which indeed vary from period to period. The consideration of dynamic financial parameters is left for future research. In addition, this chapter assumes a random response of farmers behaviour in share-trading, hence it does not capture the reality that farmers may not want to invest in the business, especially when the economic condition is poor. For example, when
the global dairy price keeps going down, farmers may want to exit the organisation, which results in the reduction of milk production. Therefore, the challenge is to motivate farmers under adversity, which is worth considering in the future. Last but not least, it would be worthwhile to build analytical models for a specific type of co-op whose capital structure is similar to that of Fonterra. The analytical results coupled with empirical validation using real data would greatly contribute to the research on co-ops.
Chapter 7

Conclusions

This thesis studied contracts and incentive mechanisms in agricultural co-ops with an emphasis on quality coordination and financial decisions associated with operations. This work models the interaction between the co-ops and their farmers who pool their products under a single cooperative organisational structure and also invest in the business relative to the amount of product they supply. Though these models are simplified versions of reality, they capture the key trade-offs correctly and contribute to the management of the cooperative organisation in both operations and finance.

Chapter 4 investigated contractual coordination of a co-op when a quality standard is proposed in its contract with farmers who exert effort at the farm level. As current co-ops are significantly different from traditional co-ops in many aspects, this research explicitly considers the quality provisions and strategies of multi-period payment schemes, and shows that the quality provision is conducive to market profits and motivates farmers’ efforts. By examining the optimal centralised and decentralised decisions under the market-price-guarantee payment scheme, the results show that farmers are likely to be over-motivated unless they have a high time preference. This research also proposes an improved payment scheme—the upfront incentive payment scheme, and shows that it can efficiently achieve supply chain coordination regardless of farmers’ time preferences. This study also suggests that the strategy of the multi-period payment schemes contributes to supply chain coordination when subject to the budget balance condition. The analytical results from this chapter provide implications on the implementation of quality provisions and incentive mechanisms.

Chapter 5 studied both operational and financial decisions for a so-called proportional investment co-op, where farmers’ equity is required to be in proportion to their patronage.
This chapter models this situation, in the presence of yield and market uncertainty, using a Markov decision process where the decisions of processing quantity interact with the financial decisions of retained earnings and short-term loans. The results show that the optimal processing quantity is to maximise market profit and that the optimal amount of borrowing is exactly the shortfall between the capital available and the capital required. However, the decision on farmers’ payments or retained earnings is affected by the co-op’s objective.

Chapter 6 implemented a case study of an agricultural co-op and formulated a Markov decision process to study the capital structure optimisation for Fonterra. By applying numerical experiments via an Approximate Dynamic Programming algorithm, this study investigates the financial performance under different policies. The results show that placing a higher tolerance on Fund Size and liquidity contributes to a higher objective value, however, this is at the cost of increased Fund Size risk and liquidity risk. Furthermore, allowing farmers to hold more shares could counteract the negative effect of allowing farmers to hold fewer shares, however, it may lead to the problem of equity dilution. The thought experiment of assuming a decreased milk production shows that the organisation is very likely to suffer from redemption risk under adversity. Possible mitigating strategies include requiring a higher cash reserve or allowing farmers to purchase more shares. This research provides guidelines for a policy-maker to respond to different circumstances.

This thesis has analysed two problems faced by agricultural co-ops: quality coordination and financial challenges. For quality coordination, a game theoretical approach is used to examine the incentive mechanisms in the agricultural supply chain with multiple farmers. For financial decisions, two approaches have been employed: dynamic programming and a case study with numerical experiments. In the incentive mechanism for quality coordination, efficient multi-period payment schemes exist to achieve the first-best results. The model constructed for the Proportional Investment Co-op strives to deal with co-ops’ financial problems using quantitative methods. The numerical analysis in the case study reflects the trade-offs involved in capital structure optimisation and suggests the importance of designing effective policies. Overall, this thesis has contributions from both practical and...
theoretical perspectives: it provides both operational and financial insights on how to im-
plement quality provisions and how to manage cooperative finance, and it also contributes
to the theoretical and empirical literature on co-ops related to quality and financial issues.

Besides the future work mentioned in the conclusion of each chapter, there are other
directions that are worthy of future study. First, one may want to examine the co-op as a
colalition of firms which can be formed by heterogeneous member groups, managers, new
(active) members, and old (inactive) members. The objective of the different groups can be
conflicting, in which bargaining processes may be required to reach a compromise decision.
Second, it would be interesting to study the investors’ response to cooperative financial
decisions. Motivating outsiders to invest in the cooperative business is not as easy as in
other regular companies because they are not entitled to voting and ownership rights. It
awaits further investigation to see how to motivate outside investors without compromising
the benefits of members.
Appendix A

Appendix for Chapter 4

Notation and Parameters

$Y_i, y_i$ The uncertain yield of farmer $i$, and its realisation

$Y_\sim i, y_\sim i$ The uncertain yield of other $n - 1$ farmers, and its realisation

$Y, y$ The total yield, and its realisation

$\mu_y, \sigma^2_y$ The mean and variance of farmer $i$’s yield

$Q^H_i, q^H_i$ The amount of high-quality product of farmer $i$, and its realisation

$Q^L_i, q^L_i$ The amount of low-quality product of farmer $i$, and its realisation

$Q^H_\sim i, q^H_\sim i$ The amount of high-quality product of $n - 1$ farmers, and its realisation

$Q^L_\sim i, q^L_\sim i$ The amount of low-quality product of $n - 1$ farmers, and its realisation

$Q^H, q^H$ The amount of high-quality product of all farmers, and its realisation

$Q^L, q^L$ The amount of low-quality product of all farmers, and its realisation

$P, p$ The random low-end market price, and its realisation

$a$ The fixed price for the high-quality product

$\delta$ The discount factor for the time value of money

$\alpha$ The discount factor for time preference

$I_1^i$ The first-stage income of farmer $i$

$I_2^i$ The second-stage income of farmer $i$

$n$ The number of farmers.
Functions

$F_i(y_i)$ The cumulative distribution function of farmer $i$’s yield

$F_{-i}(y_{-i})$ The cumulative distribution function of the other $n - 1$ farmers’ yield

$F(y)$ The cumulative distribution function of the total yield

$G(p)$ The cumulative distribution function of the low-end market price

$h(e, l)$ The proportion of high-quality production

$b(l)$ The impact factor of quantity on price

$c(e)$ The effort cost

$p^H(q^H, l)$ The price of high-quality product, i.e., $p^H(q^H, l) = a - b(l)q^H$

$R(q^H, q^L)$ The total market revenue realised in the second-stage

$\pi(e, l)$ The market profit obtained by the co-op

$U_i(\cdot)$ The utility function of farmer $i$

Decision Variables

$e$ The collective effort level required by the co-op

$e_i$ The individual effort level of farmer $i$

$l$ The quality level

$w^L_1$ The first-stage unit price offered to the low-quality product

$w^H_1$ The first-stage unit price offered to the high-quality product

$w^L_2$ The second-stage unit price of the low-quality product

$w^H_2$ The second-stage unit price of the high-quality product

$\beta$ The quality premium, i.e., $\beta = w^H_1 - w^L_1$

$q^H_s$ The quantity delivered to the high-end market
Parameters and Functions

- $s_n$: Cash position at the beginning of period $n$
- $\tilde{y}_n$: The yield vector in period $n$
- $e_n$: The total equity in period $n$
- $\rho$: The interest of loan in period $n$, it is assumed to be constant through the time flow
- $\beta$: Single-period discount factor ($0 < \beta < 1$)
- $\alpha$: The weight factor put on the co-op’s benefit
- $\theta_1$: Capital gain
- $\theta_2$: Factor of penalty cost
- $Y_n$: The uncertain total yield in period $n$ ($random$)
- $R(q_n, \omega)$: The random market revenue in period $n$ ($random$)
- $c(q_n)$: The expenses of the co-op in period $n$
- $V_n(s, \tilde{y})$: Value function of the dynamic programming model in period $n$

Decision Variables

- $v_n$: The internal working capital
- $w_n$: The farm-gate price offered to farmers in period $n$
- $q_n$: The quantity processed and delivered to the market
- $b_n$: The amount of borrowing in period $n$
The Estimation of the Share Price

1. Model Identification

Figure C.1 displays the historical monthly data of milk prices (of whole milk powder) sourced from GlobalDairyTrade (https://www.globaldairytrade.info/), and share prices sourced from NZX Main Board (https://www.nzx.com) from 2012-2016. Through data analysis in Stata14.0, one can tell that these two series are positively correlated, which can be seen in Figure C.2.

![Figure C.1: Share Prices and Milk Prices from 2012-2016](image)

Therefore, it is assumed that there is a long-term relationship between share prices and milk prices, and this study uses the autoregressive distributed lag model ARDL$(p,q)$ to describe the dependency of share prices on milk prices, which is defined as

\[ y_t = m + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + \beta_0 x_t + \beta_1 x_{t-1} + \ldots + \beta_q x_{t-q} + \mu_t. \]
In this model, $y_t$ is a dependent variable representing share prices, while $x_t$ is an independent variable (or explanatory variable) indicating milk prices. The last term $\mu_t$ is a white noise which follows the definition that “A sequence \( \{\mu_t\} \) is a white-noise process if each value in the sequence has a mean of zero, a constant variance, and is serially uncorrelated.” The order $p$ is the lagged value of the dependent variable $y_t$, and $q$ is the lagged (or current) value of the explanatory variable $x_t$.

Figure C.2: Correlation between the Milk Price and the Share Price

2. Regression Parameters Estimation.

The next step is to estimate regression parameters. Note that, in the ARDL model, both $y_t$ and $x_t$ are stationary variables. Therefore, it is necessary to check and process data before running regression models. A stationary time series is one whose properties do not depend on the time at which the series are observed. So time series with trends, or with seasonality, are not stationary. One way to identify non-stationary time series is through the Dickey Fuller Test ($ADF$ test) and another useful tool is by looking at the Autocorrelation Function (ACF) plot. In this model, both milk prices and share prices are identified as non-stationary variables; or in statistics, as having one unit root, as shown in Figure C.3.

One can make a time series stationary by transformations such as logarithms and differencing which is to compute the differences between consecutive observations. Logarithms can help stabilise the variance while differencing can help stabilise the
mean of a time series by removing changes in the level of a time series, hence eliminating trend and seasonality. Figure C.4 shows the ACFs of share prices and milk prices after logarithms and first-differencing, from which the stationarity can be observed. This model indicates the transformation of share price and milk price as $\text{difflnshare}$ and $\text{difflnmilk}$ respectively.

![Figure C.3: The ACF of the Share Price (left) and of the Milk Price (right)](image)

![Figure C.4: The ACF of One-differencing Share Price (left) and of One-differencing Milk Price (right)](image)

After a time series has been stabilised by differencing, the next step in fitting an ARDL model is to identify the optimal lags of $p$ and $q$. One can follow the selection-order criteria of $\text{FPE}$, $\text{AIC}$, $\text{HQIC}$ and $\text{SBIC}$ (omitted here). According to the data analysis, the optimal model is identified as $\text{ARDL}(1, 0)$ which specifies the general form as $y_t = m + \alpha_1 y_{t-1} + \beta_0 x_t$. The results of the $\text{ARDL}(1, 0)$ regression model (with a suppressed constant) are shown in Figure C.5.

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After the identification of the ARDL model, the next step is to generate simulations of the share price based on the model. Since the share price is dependent on exogenous information related to the milk price; hence, it first needs to fit and simulate the model of the milk price. According to data analysis, the change in the milk price \textit{difflnmilk} fits an \textit{ARIMA}(p, d, q) model, which stands for Auto-Regressive Integrated Moving Average. It is expressed as:

\[ X_t = \alpha_1 X_{t-1} + \ldots \alpha_p X_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \ldots \beta_q \varepsilon_{t-q}, \]

where \( X_t \) represents a stationary variable (transforming original data if required) and \( p \) is the number of autoregressive terms, \( d \) is the number of differences needed for stationarity, and \( q \) is the number of lagged forecast errors. One challenge in an ARIMA model is to identify the numbers of \textit{AR}(p) and \textit{MA}(q) terms, and one can try different models following general principles and use \textit{AIC/BIC} tests to choose which one fits best (omitted here). By applying monthly data from 2012-2016, it is identified that the change of milk price fits the model of \textit{ARIMA}(0, 0, 1). The result is presented in Figure C.6 (note that the data of \textit{difflnmilk} has been differenced).

![ARDL regression output]

\textit{Figure C.5: The ARDL Regression Model of the Share Price}
Now, with the identified models of the share price and the milk price, one can simulate multiple sample paths in the future. To achieve this, the function of `vgxsim` in MATLAB 2016 is used. First, simulate 2000 sample paths from December – 2015 to August – 2016 with the known exogenous milk price. The left-hand panel in Figure C.7 plots the response data, the simulated responses, the identifications of 5%, 25%, 50%, 75%, and 95% percentiles, and the mean of the simulated series at each period. Next, simulate the paths of out-of-sample from August – 2016 to August – 2022 using the random milk price simulated from the ARIMA(0,0,1) model. All results are illustrated in the right-hand panel of Figure C.7.

```
.arima difflnmilk, noconstant arima(0,0,1)

(setting optimization to BHHH)
Iteration 0: log likelihood = 38.984651
Iteration 1: log likelihood = 39.826527
Iteration 2: log likelihood = 39.996475
Iteration 3: log likelihood = 40.001781
Iteration 4: log likelihood = 40.001895
(switching optimization to BFGS)
Iteration 5: log likelihood = 40.001896

ARIMA regression
Sample: 2012m12 - 2016m9
Number of obs = 46
Wald chi2(1) = 32.94
Log likelihood = 40.0019
Prob > chi2 = 0.0000

|         | COF  | Std. Err. | z     | P>|z|  | 95% Conf. Interval |
|---------|------|-----------|-------|------|-------------------|
| difflnmilk |      |           |       |      |                   |
| ARMA     |      |           |       |      |                   |
| ma       |      |           |       |      |                   |
| L1       | 0.5393726 | 0.0113977 | 4.74  | 0.000 | 0.4210372          | 0.6577081 |
| /sigma   | 0.1003866 | 0.0101102 | 9.97  | 0.000 | 0.0810211          | 0.1206522 |

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.
```

Figure C.6: The ARIMA Regression Model of the Milk Price
The Estimation of the Flexibility of Farmers’ Shareholdings

According to the policy, Farmer Shareholders are allowed to hold up to 120% of the Share Standard in an aggregate level; and also able to sell the Economic Rights of up to 25% of their minimum required Shares to the Fonterra Shareholders’ Fund. Therefore, the lower limit and upper limit are set at $L = 75\%$ and $H = 120\%$. The Mode $M$ is identified as $M = 1.0378$ based upon historical data presented in Table C.1 and Figure C.8. This experiment has only sourced the data from 2009 when is the launch time of the new capital structure.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_i^t/e_i^a$</td>
<td>0.959</td>
<td>0.987</td>
<td>1.038</td>
<td>0.998</td>
<td>1.065</td>
<td>1.071</td>
<td>1.048</td>
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Table C.1: The Flexibility of Farmers’ Shareholdings
The Estimation of the Gross Profit

Table C.2 lists the Gross Profit and Milk Volume from Fonterra’s Income Statements starting from 2002/2003 (Note that Fonterra was founded in 2001), and Figure C.9 plots a scatter diagram. The data was fit by *Polynomial Fitting*, thus obtaining the following equation:

\[ GP(y_t) = 3.3y_t - 1680. \]

<table>
<thead>
<tr>
<th>Season</th>
<th>Revenue ($ million)</th>
<th>Cost of Goods Sold ($ million)</th>
<th>Gross Profit ($ million)</th>
<th>Milk Collected (million KgMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002/2003</td>
<td>12474</td>
<td>9873</td>
<td>2601</td>
<td>1150</td>
</tr>
<tr>
<td>2003/2004</td>
<td>11830</td>
<td>9731</td>
<td>2099</td>
<td>1195</td>
</tr>
<tr>
<td>2004/2005</td>
<td>12323</td>
<td>10200</td>
<td>2023</td>
<td>1160</td>
</tr>
<tr>
<td>2005/2006</td>
<td>13001</td>
<td>10811</td>
<td>2090</td>
<td>1210</td>
</tr>
<tr>
<td>2006/2007</td>
<td>13687</td>
<td>10853</td>
<td>2834</td>
<td>1246</td>
</tr>
<tr>
<td>2007/2008</td>
<td>19512</td>
<td>16820</td>
<td>2692</td>
<td>1192</td>
</tr>
<tr>
<td>2008/2009</td>
<td>16035</td>
<td>13217</td>
<td>2818</td>
<td>1281</td>
</tr>
<tr>
<td>2009/2010</td>
<td>16726</td>
<td>13975</td>
<td>2751</td>
<td>1286</td>
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<tr>
<td>2010/2011</td>
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<td>16861</td>
<td>3010</td>
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<tr>
<td>2011/2012</td>
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<td>16721</td>
<td>3048</td>
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<td>15611</td>
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<td>2013/2014</td>
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<td>15567</td>
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<td>2015/2016</td>
<td>17199</td>
<td>13567</td>
<td>3632</td>
<td>1566</td>
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Figure C.9: The Linear Relationship between the Gross Profit and Milksolids
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