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A Genius' Story: Two Books on Gödel

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Undoubtly, Gödel was the greatest logician of the twentieth century.[1] There is no trace of exaggeration in saying, following Wang, that Gödel's contribution to mathematics has the same status as Freudian psychology, Einstein's theory of relativity, Bohr's principle of complementarity, Heisenberg's uncertainty principle, keynesian economics, and Watson and Crick double helix model of DNA. Yet, with a few notable exceptions, most of the personal details of Gödel's life remained a mystery.[2]

Gödel[3] was born on 28 April 1906 in Brünn, Austria-Hungary (now Brno, Czech Republic). He was aware, from early childhood, of his great capacities for concentration, accuracy, and thoroughness, for separating the essential from the inessential, for getting fast to the core. As a consequence, a central feature of his work and life was his choice to concentrate on what he considered to be *fundamental*, almost completely disregarding other issues. Sadly enough, he has got also very early signs of mental and physical problems.

Gödel attended school in Brünn, completing his high school studies in 1923. According to his brother Rudolf Gödel:

> ... to the astonishment of his teachers and fellow pupils [Gödel] had mastered university mathematics by his final Gymnasium years. ... Mathematics and languages ranked well above literature and history. At the time it was rumoured that in the whole of his time at High School not only was his work in Latin always given the top marks but that he had made not a single grammatical error.

Gödel entered the University of Vienna in 1923. He was taught by Furtwängler, Hahn, Wirtinger, Menger, Helly and others. As an undergraduate, he took part in a seminar run by Schlick which studied Russell's book *Introduction to Mathematical Philosophy.* Olga Tausky-Todd, a fellow student, recalled:

> It became slowly obvious that he would stick with logic, that he was to be Hahn's student and not Schlick's, that he was incredibly talented. His help was much in demand.

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*Hao Wang. *A Logical Journey—From Gödel to Philosophy*, MIT Press, Cambridge, MA, 1996. and John W. Dawson, Jr. *Logical Dilemmas—The Life and Work of Kurt Gödel*, A. K. Peters, Wellesley, MA, 1997.

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[1] R. Oppenheimer referred to Gödel as the "greatest logician since Aristotle". A. Weil observed that in 2500 years Gödel was the only person who could speak of "Aristotle and me", while J. Wheeler had even a stronger appreciation: "if you called him the greatest logician since Aristotle you'd be downgrading him".

[2] For an incomplete list see the bibliography.

[3] Gödel = "Pate" (godparent).

Gödel completed his doctoral dissertation under Hahn's supervision in 1929 and became a member of the faculty of the University of Vienna in 1930, where he belonged to the "school of logical positivism" until 1938.

In 1933 Hitler came to power. At first this had no effect on Gödel's life in Vienna as he had little interest in politics. However after Schlick, whose seminar had aroused Gödel's interest in logic, was murdered by a National Socialist student, Gödel was much affected and had a breakdown. His brother wrote:

> This event was surely the reason why my brother went through a severe nervous crisis for some time, which was of course of great concern, above all for my mother. Soon after his recovery he received the first call to a Guest Professorship in the USA.

In 1934 Gödel gave a series of famous lectures at the Institute for Advanced Study in Princeton entitled "On undecidable propositions of formal mathematical systems". At Veblen's suggestion S. C. Kleene, who had just completed his Ph.D. Thesis at Princeton, took notes of these lectures which have been subsequently published.

In 1938 Gödel visited again the Institute for one term and lectured on set theory,[4] returned to Vienna to marry Adele Porkert, but when the war started he was fortunate[5] to be able to return in 1940 to the USA (via trans-Siberian railway and ship from Yokohama to San Francisco).

In the USA he held immediately a temporary position at the Institute, but his position became a chair only in 1953.[6] Gödel remained with the Institute until his death, on 14 January 1978.[7] He received (among other honors) the Einstein Award[8] and the National Medal of Science.

Gödel's work falls into two almost distinct parts, the European (Vienna) and the American (Princeton) part, with 1940 as the dividing time-line. The first period contains Gödel's most celebrated results: the completeness theorem (1930), the incompleteness theorems (1931), the consistency of the axiom of choice and the generalized continuum hypothesis (1938-1940). This period contains other important, but less well-known results on the decision problem, intuitionistic logic and arithmetic, speed-up theorems, and geometry. After 1940, Gödel worked on a new quantifier-free functional interpretation of intuitionistic logic, on the independence of the axiom of choice and the continuum hypothesis[9], on relativistic cosmology and on the ontological argument[9]; in this period Gödel devoted much time to general philosophy and metaphysics.

Gödel is best known for his incompleteness theorems. He proved that *in any recursively enumerable axiomatic consistent mathematical system there are propositions that cannot be*

[4] These lectures will be published as a monograph two years later.

[5] Chaitin (email to to C. Calude, 7 June, 1997) noted that "human beings are delicate plants, and sometimes transplanting them is no good. I think that Gödel & Einstein were unhappy in many ways in their new environment, but they were certainly better off than if they had stayed in Europe. Of course, it would have been better still if Hitler had never existed; then Einstein & Gödel would have stayed in Europe and would probably have been happier and more productive." See more in [9].

[6] It took 13 years and a lot of pain to be finally awarded.

[7] Towards the end of his life Gödel became convinced that he was being poisoned and, refusing to eat to avoid being poisoned, starved himself to death.

[8] The award was given by Einstein himself to his friend. J. von Neumann's tribute delivered with this occasion described Gödel's achievements as "a landmark which will remain visible far in space and time."

[9] He never published his results.

proved or disproved within the axioms of the system.[10] In particular, the consistency of the axioms cannot be proved. This ended a hundred years of attempts to establish axioms to put the whole of mathematics on an axiomatic basis.[11]

Anticipating resistance to his conclusions Gödel wrote very carefully his papers, avoiding any subjective reference, e.g., to the notion of mathematical truth.[12] He took pain to convince various people (P. Finsler, E. Post, E. Zermelo[13] C. Perelman, M. Barzin, J. Kuczyński[14]) about the validity of his assertions and results, but he avoided any public debate and considered his results to have been accepted by those whose opinion mattered to him.[15] The reactions of two great philosophers are also of interest. Wittgenstein's negative comments (dated 1938 and posthumously published in "Remarks on the foundations of mathematics" in [1]) are now considered an embarrassment to the work of a great philosopher. Russell realized the importance of Gödel's work, but expressed his continuous puzzlement in a rather ambiguous way.[16]

In the long run Gödel interpretations of incompleteness prevailed: the incompleteness theorems neither rejected the notion of formal system (quite the opposite) nor caused despair over the imposed limitations; they just re-affirmed the creative power of human reason.[17] It is intriguing and unfortunate that the new light shed by Chaitin's information-theoretic version of incompleteness (see Chaitin [6, 7, 8], Davis [12], Casti [4, 5], and various sources listed in Calude [3]) is completely ignored by both Wang and Dawson (as well as by the comments included in Gödel's three volumes of collected works).

Gödel's work on the consistency of the axiom of choice and of the generalized continuum-hypothesis with the axioms of set theory (1940) is another remarkable achievement. Tactically there is an interesting similarity between Gödel's arguments in 1931 and 1940: a fundamental property is highlighted ("to be primitive recursive" and "to be absolute for the constructible

[10] Wang cites the following alternative (informal) versions: GT: *Mathematics is inexhaustible.* GT1: *Any consistent formal theory of mathematics must contain undecidable propositions.* GT2: *No theorem-proving computer (program) can prove all and only the true propositions of mathematics.* GT3: *No formal system of mathematics can be both consistent and complete.* GT4: *Mathematics is mechanically (or algorithmically) inexhaustible (or incompletable).* See a detailed discussion in Casti's books [4, 5].

[11] One major attempt had been by Russell and Whitehead with *Principia Mathematica* (1910-13); another was Hilbert's formalism which was dealt a severe blow by Gödel's results. Gödel's theorem does not destroy the fundamental idea of formalism, but it did demonstrate that any system would have to be more comprehensive than that envisaged by Hilbert's.

[12] Feferman [11] speculating on his extreme caution states that Gödel "could have been more centrally involved in the development of the fundamental concepts of modern logic—*truth* and *computability*—than he was."

[13] Gödel met Zermelo in Bad Elster in 1931. Olga Taussky-Todd, who was at the same meeting, wrote: *The trouble with Zermelo was that he felt he had already achieved Gödel's most admired result himself. Scholz seemed to think that this was in fact the case, but he had not announced it and perhaps would never have done so. ... The peaceful meeting between Zermelo and Gödel at Bad Elster was not the start of a scientific friendship between two logicians.*

[14] Finsler, Post and Zermelo, were concerned with priority issues, while Perelman, Barzin, Kuczyński asserted that Gödel had in fact discovered another *antinomy.* Unlike the others, Post expressed "the greatest admiration" for Gödel's work, conceding that "after all it is not ideas but the execution of ideas that constitute[s]... greatness". Gödel's result provoked Hilbert's anger (Hilbert died in 1943), but, according to Bernays, he soon accepted its correctness. However, neither of Hilbert's papers nor his books cites Gödel's work.

[15] He must have found these attacks unjustified and stressful; Rudolf Gödel reported later that shortly after the publication of his famous work his brother exhibited signs of depression so serious that his family feared he might become suicidal.

[16] According to Dawson [20], Gödel remarked, in a letter addressed to A. Robinson, that "Russell evidently misinterprets my result; however he does so in a very interesting manner..."

[17] In Post celebrated words: "mathematical proof is [an] essentially creative [activity].

submodel") and the argument consists mainly in proving that a series of predicates possess this property. Finally, in each case a specific predicate ("being provable in the system" and "being a cardinal number within the model of set theory") **fails** to satisfy the property. The scenario worked because he looked at the same objects from two points of view: internally and externally (mathematical vs. metamathematical notions, in the first case, functions which exist in the given submodel and those that exist outside it, in the second one).

Gödel was also interested by the problem of mind and matter, a notoriously elusive issue. The responses to this problem are very diverse; however, there are two main trends, *monism* which claims that the distinction between mind and matter is only apparent, simply, the mind is identical with the brain and its function, and *dualism*[18] which maintains that they are fundamentally distinct. There are many types of dualism: a) "categorical dualism" (the mind and the body are different logical entities), b) "substance dualism" (the mind exists in a mental space outside space or time, and the brain is just a complex organ which "translates" thoughts into the corporeal movements of the body), c) "property dualism" (the mind and our experiences are "emergent" properties of the material brain), d) "epistemic dualism" (from a "theoretical reason" the states of the mind are reducible to the states of the brain, but from a "practical reason" such a reduction is not possible).

Gödel rejected monism by saying (in Wang's words) that parallelism—i.e., the belief that there is a one-to-one correlation between one's mental states and brain states—*is a prejudice of our time which will be disproved scientifically—perhaps by the fact that there aren't enough nerve cells to perform the observable operations of the mind.*[19] This claim remains as a challenging *scientific conjecture.* According to Wang, Gödel asserted in 1972 that *the brain functions basically like a digital computer.*[20] How to conciliate this position with Gödel's incompleteness theorem and his rejection of parallelism? First note Gödel's remark that

> *... it remains possible that there may exist (and even be empirically discoverable) a theorem-proving machine which* **is** *in fact equivalent to mathematical intuition, but cannot be* **proved** *to be so, nor even be proved to yield only correct theorems of finitary number theory.*

Secondly, from an operational point of view, the *mind* is the user of the brain functioning as a computer. This seems to put Gödel in the camp of substance dualism. We may further suppose that perhaps the brain, and the mind, are simply *unsimulatable* and the reason for this claim may be the fact, already noticed by von Neumann, that *the only explanation of the brain is a complete wiring diagram.* This property suggests the information-theoretic notion of *randomness.*

Was Gödel interested in randomness? To the best of our knowledge this notion appears twice in Gödel's writings. The first occurrence is in Gödel's lecture at Brown University on 15 November 1940, dealing with Cantor's continuum hypothesis (CH).[21] His exact words (quoted from [16] pp. 184-185) are:

[18]The dualism can be traced to Descartes.

[19]In a short paper "Les mânes de Gödel" published in the French *La Recherche* (January 1996) D. Berlinski says: "Gödel pense que le mécanisme en biologie est un préjugé de notre époque qui ne résistera pas à l'épreuve du temps. L'une des démonstrations à venir sera un théorème mathématique qui montrera que la formation dans les temps géologiques d'un corps humain, avec les lois de la physique - ou d'autres lois de nature similaire - à partir d'une distribution aléatoire de particules élémentaires et d'un champ quantique, est aussi improbable que la séparation par hasard de l'atmosphère en ses composants simples."

[20]This is the thesis of computabilism for brains according to Wang [33], p. 169.

[21]The text of which was only published posthumously in [16].

It is to be expected that also ˜A will be consistent with the axioms of mathematics, because an inconsistency of ˜A would imply an inconsistency of the notion of a random sequence, where by a random sequence I mean one which follows no mathematical law whatsoever, and it seems very unlikely that this notion should imply a contradiction.

In his introductory note to this lecture, R. Solovay comments ([16] p. 118) :

> At first glance this seems a foreshadowing of my notion of a real being random over a transitive model of set theory. (Cf. *Solovay 1970.*) In this latter notion, a real x is random over a transitive model of set theory M iff x lies in no Borel set of Lebesgue measure zero coded by a real of M. The analogous notion (of an absolutely random real) would be a real that lies in no ordinal-definable set of measure zero. It is of course [now known that it is] consistent that such reals exist ...

Solovay goes on to say,

> Upon reflection, however, I doubt that this notion is what Gödel had in mind. More likely, it seems to me that by "random" he meant a real which is not ordinal definable. This seems to be what the phrase "no mathematical law whatsoever" was intended to express.

Gödel's reference to *random sequences* is indeed extraordinary for that time, because Cohen and Solovay had announced their results on the independence of the CH much later, and algorithmic information theory was developed only in mid sixties.[22] And of course, it was just six years later that Gödel first broached the notion of ordinal definability, in his lecture at the Princeton Bicentennial conference.

The second reference to random sequences is contained in Gödel's 1970 letter addressed, but not sent, to Tarski; see [16] pp. 424-425. The last paragraph on p. 424 reads:

My conviction that $2^{\aleph_0} = \aleph_2$ of course has been somewhat shaken. But it still seems to be plausible to me. One of my reasons is that I don't believe in any kind of irrationality such as, e.g., random sequences in an absolute sense.

Finally, in a post scriptum Gödel added:

A measure theory of zero sets would be very interesting, but I am doubtful the definition given in my paper is the one to be chosen.

There are a few points of interest here:

- the letter was written 30 years after his Brown University lecture; in 1970 we had the right information-theoretic definition of randomness, but the lack of a complexity-theoretical characterization of random reals was a barrier to a proper understanding of the notion of randomness; specifically, only after Chaitin's 1975 paper,[23] algorithmic information theory was able to explain why there is no absolute definition of random sequence;

[22] See Chaitin [7] and the historical notes in Calude [3].
[23] See [7].

- it is probably unknown to what extent Gödel was familiar with algorithmic information theory, specifically, with Martin-Löf and Solovay measure-theoretic characterizations of randomness; this theory is a possible candidate to the "measure theory of zero sets" referred to by Gödel.

The above references to randomness give us a glimpse of Gödel's unusual flair for suggesting prescience ideas. Dawson[24] noted: *There are also other stunning instances of Gödel's prescience. One such is his lecture to the Zilsel circle in 1938, in which he anticipated Kreisel's "no-counterexample interpretation". In studying Gödel's papers, I sometimes had the eerie feeling I was dealing with someone not quite human, someone with a direct pipeline to mathematical truth (a genius, that is, possessed of an extraordinary mathematical intuition).*

Finally, let me enumerate some of Gödel's main conceptions, as they emerge from the books by Wang and Dawson[25]: a) the universe is rationally organised and comprehensible to the human mind, b) the universe is causally deterministic,[26] c) there is a conceptual and mental realm apart from the physical world, and d) the conceptual understanding is to be sought through introspection.

Gödel succeeded where others failed because of a) his careful distinction between syntax and semantics, b) his self-imposed restriction to clearly specified formal systems, c) his concern for relative rather than absolute notions (e.g., undecidability), and d) his philosophical views.[27] Gödel's results, in spite of their great variety, share a number of common characteristics: they all a) challenged firmly held preconceptions, b) were motivated by philosophical issues outside the concerns of the scientific community, c) have a paradoxical air, d) appear to be more theoretical curiosities, of little relevance to the main stream mathematics or physics. His own life—conspicuously governed by his obsessive wish to see order and attain security—was a strange cocktail of triumph, tragedy, inner turmoil, paradox, and eccentricity. The books by Wang and Dawson represent, each and both, an invaluable contribution to the understanding of Gödel's work and life; in fact, it is most important to be read together, preferably first Dawson, than Wang.[28]

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[24] Email to C. Calude, 21 May, 1997.

[25] Gödel's results in relativistic cosmology have not (yet) received the definitive interpretation.

[26] Both Einstein and Gödel rejected the idea of indeterminacy or chaos in physics. "God may be subtle, but He isn't malicious" (Einstein); "nothing that happens" in our world "is due to accident or stupidity" (Gödel).

[27] In two letters addressed to Wang (7 December 1967 and 7 March 1968) he strongly stated that his philosophical views— an "objectivistic conception of mathematics and metamathematics in general, and of transfinite reasoning in particular"—played a most essential role in his work, from the very beginning, in 1929.

[28] It's intriguing that Wang had more direct human contact with Gödel, but Dawson is more sympathetic and warmer.

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