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Convergence of Lyapounov Functions Along Trajectories of Nonexpansive Semigroups: Generic Convergence and Stability

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the degree of Doctor of Philosophy in Mathematics,
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Abstract

The main aim of this thesis is to study the convergence of Lyapounov functions along the trajectories of nonexpansive semigroups in a Hilbert space.

The outline of the thesis is as follows. In Chapter 3, it is shown that a regularly Lyapounov function for a semigroup of contractions on a Hilbert space converges to its minimum along the trajectories of the semigroup. In Chapter 4, we show that while a convex Lyapounov function for a semigroup of contractions on a Hilbert space may not converge to its minimum along the trajectories of the semigroup, it converges generically along the trajectories of the semigroups generated by a class of bounded perturbations of the semigroup generator. In Chapter 5, we show that the regularly Lyapounov function nearly converges to its minimum along the trajectories of the semigroups generated by small bounded perturbations of the semigroup generator. Besides that we study a problem of interest in its own right, about the direction of movement of the element of minimal norm in a moving convex set, in Section 4.9. We show that if $C$ is a nonempty closed convex subset of a real Hilbert space $H$, $e$ is a non-zero arbitrary vector in $H$, and for each $t \in \mathbb{R}$, $z(t)$ is the closest point in $C + te$ to the origin, then the angle $z(t)$ makes with $e$ is a decreasing function of $t$ while $z(t) \neq 0$. 
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