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The University of Auckland

Galiev, Sh. U., Galiyev, T. Sh.

**Nonlinear quantum waves in the light of recent slit
experiments**

**Auckland
2017**

ABSTRACT

The history of the double-slit experiment (DSE), is more than two centuries old. Its results played a significant role in the discovery of the wave nature of light, and then formed the basis for important provisions of quantum mechanics. However, the results are still difficult to interpret unequivocally. In particular, they have been treated recently even outside the framework of the universally accepted, experimentally verifiable science. For example, they were associated with the influence on the experiment of the experimenter's consciousness or with the influence of worlds which exist parallel to our world. At the same time, indeed, the interpretation of the results is extremely difficult and the situation with the understanding of these results, especially in the light of recent researches, is becoming more and more complicated. The purpose of our study is to analyze the results of several recent experiments and to introduce a new understanding of them based on taking into account the possibility of synchronous interaction of all the components of the process of the experiment. We believe that it is necessary to take into account the nonlinear nature of the interacting fields.

First, we consider the most significant experiments and their analysis in the framework of traditional approaches. Then non-traditional approaches are considered. Some of them seem implausible, but nevertheless they lie in line with the most high-profile ideas of modern physics. We draw reader's attention to the fact that practically no researcher pays attention to the influence of quantum fluctuations in slits and the interaction of the physical fields on the results of the experiment.

A systematic study of these issues has apparently not been carried out. Therefore, we propose a number of thought experiments. At the end of the study, we use the mathematical modelling of the physical processes responsible for the results of the experiments. Plasma waves and waves in vacuum are considered. The interaction of these waves is studied in the attempts to model some processes occurring in the DSE.

Thus, the main goal of our research is to introduce the reader to the new understanding of certain positions of quantum mechanics on the basis of the new understanding of the results of the DSE. We believe that in order to explain the results of the DSE, it is first necessary to introduce greater clarity in quantum mechanics.

Wave quantum effects in the light of recent slit experiments

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Part one. Experiments, its results and discussions of them

1. Introduction

Niels Bohr said to Werner Hisenberg “Your theory is crazy, but it is not crazy enough to be true”. At the same time Albert Einstein wrote “Most of the fundamental ideas of science are essentially simple, and may, as a rule, be expressed in a language comprehensible to everyone.” “The great tragedy of science”, Thomas Huxley observed, “the slaying of a beautiful hypothesis by an ugly fact”. Science is the clash of opinions.

Short history of the puzzling experiment. Quantum mechanics is the best theory we have for describing the world at the nuts-and-bolts level of atoms and subatomic particles. According to this theory particles behave both as a particle and as a wave. This point of view was supported by the results of the double-slit experiment (DSE), which is one of the most beautiful experiments in physics. It is frequently used in classic textbooks on quantum mechanics. The experiment demonstrates the principle of [wave–particle duality](#). The double –slit experiment is schematically illustrated in Fig. 1. As we see from Fig. 1 particles illuminate a thin solid (metal) barrier in which two slits are cut. A photographic plate records the light that gets through the slits – brighter areas of the photograph indicate more incident light.

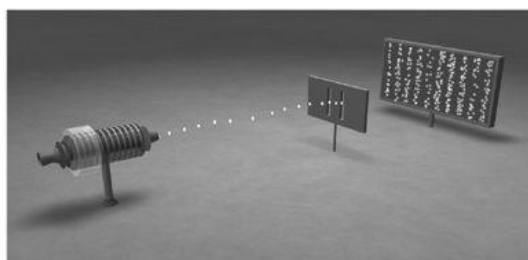


Fig. 1. The data obtained when electrons are fired and both slits are open [1].

Particles pass through two narrow slits in a screen and then impinge on a second screen making small dots. After many particles have hit the second screen, an interference patterns develops in the form of light and dark stripes similar to light interference fringes. It looks like we are dealing with waves passing through the two slits. On the other hand the particles make small dots on the second screen! It looks like

the particles behave as moving point in space. Most amazingly the pattern appears even if the particles arrive at the screen one at a time. It appears that a particle somehow interferes with itself, but how can this be possible?

Following Aatish Bhatia [2] we put a few questions and give several answers to highlight the situation with this experiment.

Let us consider an electron that arrives at the screen. Which slit did it go through?

1. Did the electron go through the left slit?

No! Because when you cover up the right slit, the stripey pattern disappears and you get a boring single band instead.

2. Did the electron go through the right slit?

No! For the same reason as above. When you cover up the left slit, instead of the stripey pattern you get a single band.

3. Does the electron go through both slits?

No! Because if that were true, we'd expect to see the electron split into two, and one electron (or maybe half) would go through each slit. But if you place detectors at the slits you find that this never happens. You always see only one electron at a time. It never, ever splits into two.

4. Did the electron go through neither slit?

No! Of course not, that's just silly. If you cover both slits, nothing happens.

However, to this day, physicists do not agree on the best way to interpret the results of these quantum experiments. It is hard to avoid the implication that we can describe quantum effects but we do not have a clear understanding of them. Richard Feynman famously said that "the double-slit experiment has in it the heart of quantum mechanics. It reality, it contains the only mystery" [3, 4]. Below there is an extract from "QED –The strange theory of light and matter" of R.P.Feynman [4].

«Before I go into the main part of this lecture, I'd like to show you another example of how light behaves. What I would like to talk about is very weak light of one color – one photon at a time- going from a source, at S, to a detector, at D (see Fig.2 (a)).

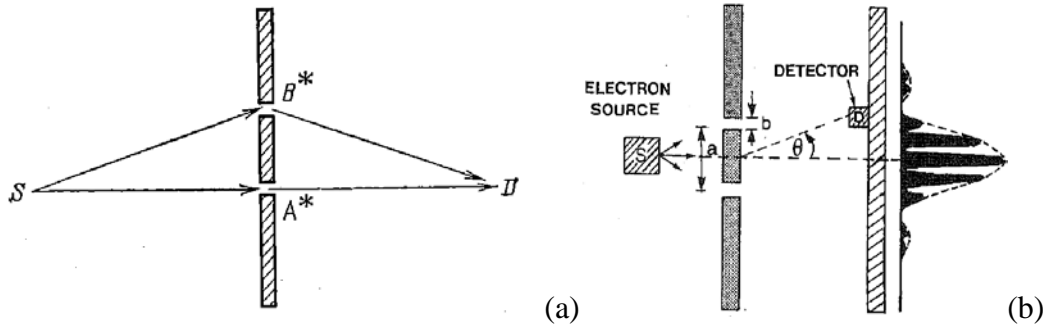


Fig. 2. A rough scheme of the double-slit experiment presented by R.P. Feynman (a) [4]. An electron propagates like a wave from source S through the two slits to the detector D, but always registers as a discrete particle. The particle distribution shows the oscillatory behaviour of wave interference. The broken line which envelops the interference fringes is the one-slit electron diffraction pattern (b) [3].

Two tiny holes (at A and B) in a screen that is between a source S and a detector D let nearly the same amount of light through (in this case 1%) when one or the other hole is open. When both holes are open, “interference” occurs: the detector clicks from zero to 4% of the time, depending on the separation of A and B – shown in Fig. 3(a).

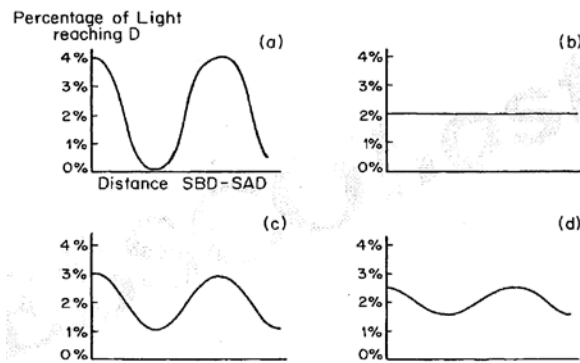


Fig. 3. Effects of the distance SBD-SAD and of the quality of detectors on the interference. (a) no detectors. In this case the detector clicks from zero to 4% of the time, depending on the separation of A and B; (b) perfect detectors. The amount of light is constant - 2%; (c) slightly reliable detectors. The amount of light varies from 3% to 1%; (d) detectors close to perfect. The amount of light varies near 2% .

When we put in detectors (*) at A and B (Fig. 2), we changed the problem. ...The complete story on this situation is very interesting: if the detectors at A and B are not perfect, and detect photons only *some* of the time, then there are three distinguishable final conditions” (see (a), (c) and (d) of Fig. 3). It is important for us that experimental results depend strongly on the distance SBD-SAD. In another words the results is determined by a location of an experimental equipment. In considering case the dependence on

the distance is described by the harmonic law. According to Fig. 3 there is the periodical amplification of the light up to maximal value when the distance SBD-SAD increases. Is it possible that this amplification is connected with own modes of oscillations of the system and certain resonances?

We consider the experimental equipment as a system of physical fields. Their strong interaction might be awoken by first particles. A part of them hits the barrier (the interslit material) and excites its oscillations. The following particles move in the space of strongly interacting wave fields. The trajectories and wave properties of these particles are determined by the "nodes" and "antinodes" of the interacting fields, and these "nodes" and "antinodes" are fixed on the detector (the photographic plate)

In contrast considering his thought experiment and its results Feynman declared “ The theory of quantum electrodynamics describes Nature are absurd from the point of view of common sense. And it agrees fully with experiments. So I hope you can accept Nature as She is – absurd».

At this point you are probably thinking that this is getting a bit ridiculous. *Why can't we just look at the damn electron and see which path it took?* The problem with this idea is that looking at something means shining light on it, and shining light on it means bumping it with a photon. If you're a tiny electron, this bump disturbs your original path. Perhaps the most renowned of its mysteries is the fact that the outcome of quantum experiment can change depending on whether or not we choose to measure some property of the particles involved.

The double-slit experiment is often used to highlight the differences and similarities between the various [interpretations of quantum mechanics](#). It must be remembered that the initial explanations of the double-slit experiments was obtained when concepts of quantum fields and vacuum energy were absent.

However, these explanations were very interesting. Furthermore, these explanations formed the basis of the quantum theory (quantum mechanics).

However, even from point of view of quantum mechanics the results of the double-slot experiments are difficult to explain. To clear these difficulties, theorists resort to ever-increasing levels of mathematical sophistication and abstraction. As a result of many attempts to explain, researchers again and again encounter difficulties that can be explained by a lack of understanding at the fundamental level.

Basic concept – briefly. Taking this into account we suggest new understanding of the fundamental experiment of quantum mechanics. As we see from Fig. 1 the vertical length of the slits is much greater than their width. We can consider the strip of the metal between the slits as a one-dimensional resonator. The slits can also be considered resonators for vacuum fluctuations. So we have three “pianos” which can play “resonantly”. We think that the results of the double-slot experiments are determined by resonant interaction of vacuum and plasmonic oscillations (waves) with particles.

Could such an almost classical model explain some results of the DSE, and might this possibility have been overlooked by the founders of quantum theory who were not aware of the existence of a fluctuating

vacuum field? This means that it might be possible to model many aspects of quantum theory on the basis of self-consistent, physical fields, where particles are only concentrations of density of the fields. We remind that the words “quantum mechanics” mean “wave quantum mechanics”.

However, until now, obtaining direct images of wavefunctions of particles had been notoriously difficult. Scientists can now identify roughly the location of only a single atom [5].

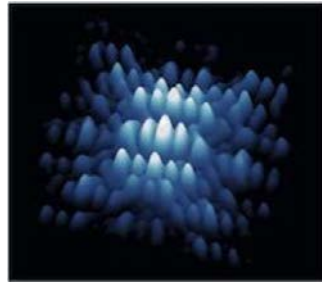


Fig. 4. It is possible now identify the location of a single atom in a silicon crystal [6].

The image in Fig. 5 (left) is not of an atom, but shows an alternative electron corral pattern, predicted by the Schrödinger wave equation and created by electrons in certain experiments [7-10]. We think that the image gives a certain understanding of a structure of the wavefunctions of the particles.

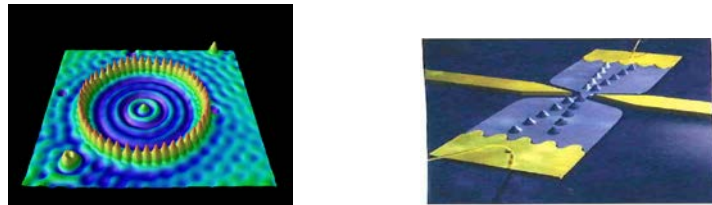


Fig. 5. The electron corral (left) [7-10]. Tiny waves that excite a single electron (right) [11, 12].

Electrons are traditionally thought of as spherical. Now a group called the ACME collaboration, led by David DeMille of Yale University and John Doyle and Gerald Gabrielse of Harvard University found no signs of an electric dipole moment in the electron. The electron appears to be spherical to within 0.00000000000000000000000001 centimeter, according to ACME’s results [13]. Tiny waves (pictured in Fig. 5 (right) as bumps) that excite a single electron at a time as they propagate in the “electron sea”.

Only very recently direct images of wavefunctions were obtained (for graphene) (Fig. 6). The images have been taken by physicists in the US and Japan [14, 15]. Similar images could provide a better understanding of a structure of so-called quantum particles.

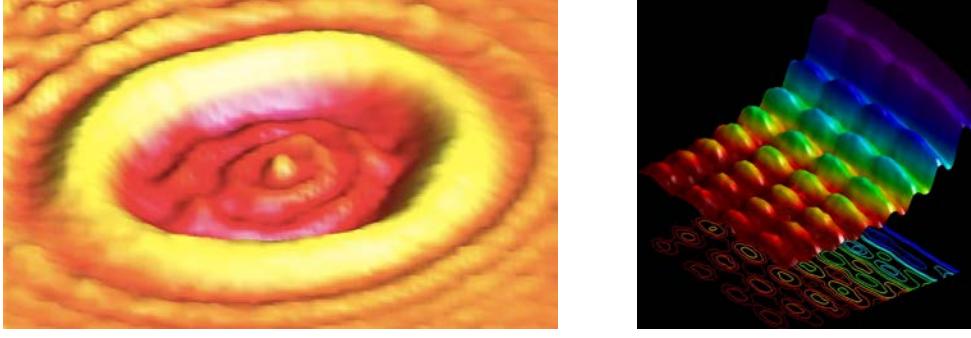


Fig. 6. Scanning-tunnelling-microscope image showing circular quantum interference patterns resulting from the confined Dirac fermions within the junction boundary, as well as scattering states exterior to the boundary. The diameter of this quantum dot is approximately 180 nm [15] (left). Light as a certain complex wave. The bottom “slice” and the top picture show different images of waves (right): <https://phys.org/news/2015-03-particle.html> (see, also, <https://www.zmescience.com/science/what-is-photon-definition-04322/>).

In our mathematical analysis of the problem we will use approximate solutions of the nonlinear Klein-Gordon equation (NKGE). It is assumed that these solutions describe certain quantum waves which correspond to so-called quantum particles. Examples of these wave solutions constructed for eight instants are shown in Fig. 7.

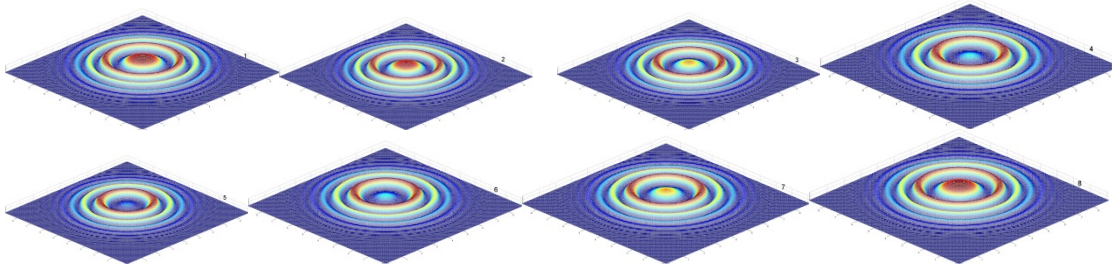


Fig. 7. Two-dimensional presentation of the wave packet calculated for eight moments of the time.

We emphasise that the wave function (Fig. 7) does not fully correspond to the wavefunction well-known in quantum mechanics. It is not also some kind of “pilot wave” (de Broglie-Bohm model) that is often identified with the wavefunction. It is a nonlinear wave with a periodically radially oscillating radius R which is strongly localized near centre but infinite in space. Generally speaking the amplitude l of the radial oscillations is very small relatively to R . However, so that to illustrate the phenomenon we used that $l = \frac{1}{2}R$ in Fig. 7. In this case during the half-cycle, the wave packet can be approximately considered as a particle, during the next half-period it is a typical wave packet. As a result, in the two-dimensional presentation the function resembles strongly nonlinear ripples which are generated after a

collision of a liquid drop with the surface of deep enough water. A jet erupts from the centre in this case. However, in reality $l \ll R$. Therefore the jet amplitude is very small and our function more resembles the wave shown in Fig. 6 (see more figures in the sections 14 -16).

In a three-dimensional case the solution describes a sphere with an infinite radius. Field oscillations take place very near the centre. We assumed that the wave packet corresponds to some “particle” of quantum mechanics. Indeed, the centre of the wave can demonstrate itself in experiments as a particle. Generally speaking, we can find the “mass” m of the wave packet according to de Broglie’s theory ($m = \hbar \lambda^{-1} v^{-1}$, \hbar is the Planck’s constant, λ is the wave length and v is the wave speed). In particular, roughly speaking the dark-solitons can correspond to our wave packet. And similar to solitons the superposition principle is approximately applicable for them.

According to this approach any wave packet (“particle”) (Fig.7) influences the whole Universe and all “particles” of the Universe influence the wave packet.

One can see that we do not agree with the Copenhagen interpretation of quantum theory, developed in the 1920s mainly by physicists Niels Bohr and Werner Heisenberg. They treat the wavefunction as nothing more than a tool for predicting the results of observations, and cautions physicists not to concern themselves with what the reality looks like underneath.

According to modern quantum field theory, absolutely everything is made of a field or a combination of fields. What we call “particles” are tiny local vibrations in these fields (Fig. 7). The results of the DSE are merely the results of interaction of quantum fields. We put these ideas in a basis our analysis of “the heart of quantum mechanics”.

In the beginning, we consider the most significant, in our opinion, experiments and their analysis in the framework of traditional approaches. Then non-traditional approaches are considered. Some of them seem implausible, but nevertheless they lie in line with the most high-profile ideas of modern physics. We draw reader's attention to the fact that practically no researcher pays attention to the influence of quantum fluctuations, although experiments with single particles are conducted at a sufficiently deep vacuum. What is the effect of vacuum and the interaction of physical fields arising in the experiment on its results?

A systematic study of this issue has apparently not been carried out. Therefore, we propose a number of thought experiments that can help a reader formulate the answers we need most. At the end of the study, we use the mathematical modelling of the physical processes responsible for the results of the experiments. Plasma waves and waves in vacuum are considered. In particular, approximate solutions of the NKGE that describe highly localized wave packets (“particles”) are constructed. The interaction of these waves is studied in the attempts to model some processes occurring in the DSE.

Thus, the main goal of our research is to introduce the reader to the new understanding of certain positions of quantum mechanics on the basis of the new understanding of the results of the DSE.

Remarks. The use of solutions of the NKGE for the analysis of certain fundamental physical problems is a long time interest of the authors [8-10]. We tried to show that nonlinearity and different resonances play important role in different physical processes from the origin of the Universe to the formation of the tsunami [8-10, 16, 17].

2. Experiments using different kind of "slits" and the beginning of the discussion

Quantum mechanics is built on experiments and solutions of mathematical equations. It is important that the solutions describe the experimental data very well. But these solutions, in themselves do not bring the understanding of the experiments. As Richard Feynman wrote some 50 years ago, as soon as we try to build the understanding, we come to conclusions that contradict common sense. Therefore, as he writes, "nobody understands quantum mechanics." Apparently the situation in quantum mechanics has not improved over the past 50 years. Perhaps it even worsened. This is well demonstrated by the data of the DSE that were obtained in the last few decades. There is a significant variety of experiments in both the particles used and schemes and experimental equipment. Below we describe three types of experiments that follow the DSE and a briefly discuss of their results.

Experiments. 1. Diffraction of electrons. Until 1961 the double-slit experiments were performed with light beams. In 1961 this experiment was performed with electron beams (Claus Jönsson of the University of Tübingen) [18].

There is a source that emits a stream of electrons onto photosensitive screen (Fig. 8). And there is an obstruction in the way of these electrons, a copper plate with two slits. What kind of picture can be expected on the screen if the electrons are imagined as small charged balls? Two bands illuminated opposite to the slits.

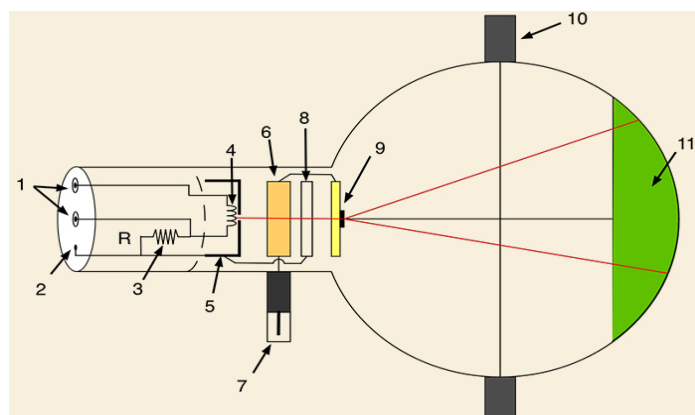


Fig. 8. There is a source that emits a stream of electrons onto photosensitive screen. And there is an obstruction in the way of these electrons, a copper plate with two slits.

In fact, the screen displays a much more complex pattern of alternating black and white bands. This is due to the fact that, when passing through the slit, electrons begin to behave not as particles, but as waves. These waves interact in space and as a result, a complex pattern of alternating light and dark bands appears on the screen. In 1974, the Italian physicists [19] repeated the experiment using single electrons and biprism (instead of slits, see Fig. 9), showing that each electron interferes with itself as predicted by quantum theory. In 2002, the single-electron version of the experiment was voted "the most beautiful experiment" by readers of Physics World [20]. What was its nature?

2. In 1989 the wave-particle duality of electrons was demonstrated in a kind of double-slit interference experiment using an electron microscope equipped with an electron biprism and a position-sensitive electron-counting system [21] (see Fig. 9).

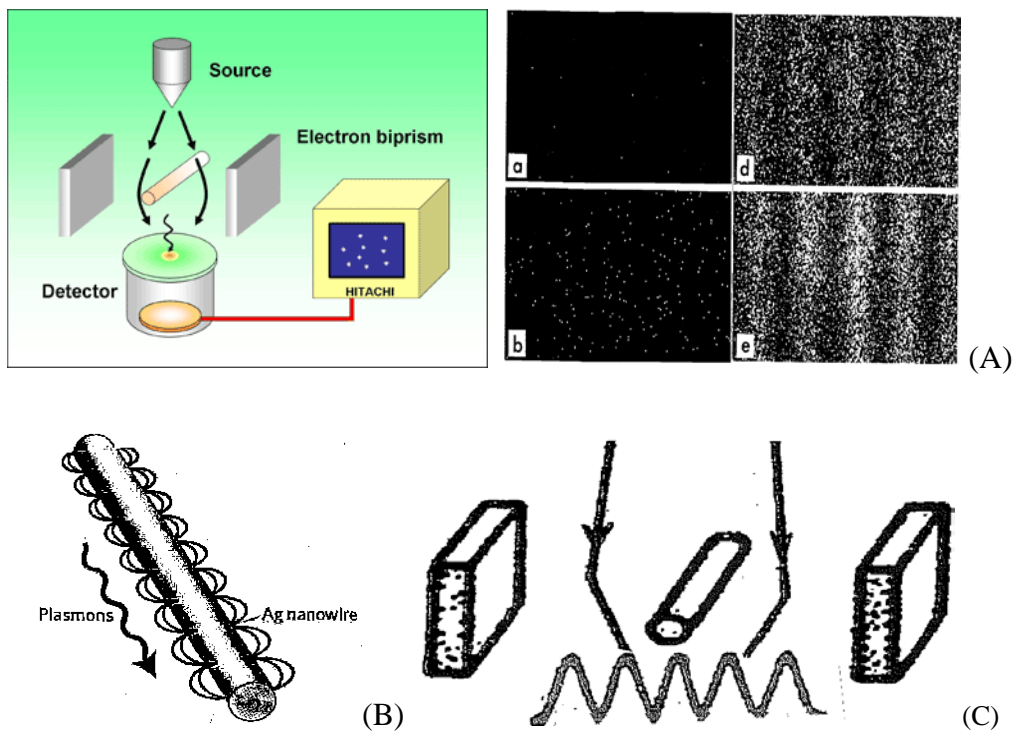


Fig. 9. One electron double slit experiment by Akira Tonomura (see <http://www.hitachi.com/rd/portal/highlight/quantum/doubleslit/index.html>) (left (A)). Single electron events build up to form an interference pattern in the DSE (right (A)). Number electrons 10(a), number electrons 100(b), number electrons 20000(c), number electrons 70000(d)]. Each time a bright spot is seen, as a result, an electron detected as a “particle”. However, over time an unmistakable interference pattern which is undoubtedly a signature of waves [21] (A). Plasmons excited by electrons in a fine filament (nanowire) (see, also, Figs.

17 and 21) (B). Vacuum fluctuations (standing waves of the Casimir effect) between of two parallel plates which interact with the electrons and the plasmons (C).

Electrons are emitted one by one from the source in the electron microscope. They pass through a device called the “electron biprism”, which consists of two parallel plates and a fine filament (nanowire) at the centre. The filament is thinner than 1 micron (1/1000 mm) in diameter. Electrons having passed through on both sides of the filament are detected one by one as particles at the detector. This detector was specially modified for electrons from the photon detector produced by Hamamatsu Photonics (PIAS). It could detect even a single electron with almost 100 % detection efficiency.

Interference fringes are produced only when two electrons pass through both sides of the electron biprism simultaneously. If there were two electrons in the microscope at the same time, such interference might happen. But this cannot occur, because there is no more than one electron in the microscope at one time, since only 10 electrons are emitted per second. When a large number of electrons is accumulated, something resembling regular fringes begin to appear in the perpendicular direction as Fig. 9 ((A), c) shows. Clear interference fringes can be seen in the last scene of the experiment (Fig. 9 ((A), d)). It should also be noted that the fringes are made up of bright spots, each of which records the detection of an electron.

We have reached a mysterious conclusion. Although electrons were sent one by one, interference fringes could be observed. These interference fringes can form only when electron waves pass through on both sides of the electron biprism at the same time but we do not have any of that. Whenever electrons are observed, they are always detected as individual particles. When accumulated, however, interference fringes are formed. Please recall that at any one instant there was at most one electron in the microscope. We have reached a conclusion which is far from what our common sense tells us.

Let us consider Fig. 9 (A) attentively. At the beginning of the experiment, the points on the screen are scattered randomly, but an increasingly more ordered picture appears and bands appear at the end of the experiment. Therefore, the idea arises that at the beginning (Fig. 9 ((A), a and b)) the wave properties of the electrons did not appear, and then they began to manifest themselves (Fig. 9 ((A), c and d)). The element of periodicity is barely discernible after the passing of a few thousand electrons through the biprism. We think, it is the result of the influence of the periodical plasmons excited in the nanowire by these electrons (Fig. 9). On the other hand, we emphasise that the cylindrical wire and plates form a pair where the Casimir force is large enough (see the section 6 and Fig. 17). Thus, we think, after the passing of a few thousand electrons from the source to the detector elements of system (nanowire, electrons and plates) begin to interact. In particular, the vacuum fluctuations can begin to demonstrate themselves. The interference fringes are a “photograph” of the results of this interaction.

3. Interference patterns similar to those presented in Figs. 1, 2 and 9 have been observed in experiments using beams of electrons, neutrons and even heavier particles like fullerenes moving through carefully designed slits. The challenge in these experiments is that the slits should be sufficiently close to each other — the distance between slits must be comparable to the wavelength associated with the particle beam. In this case it is possible to determine through which slit the particle passes. However, according to the quantum mechanics position and momentum of particle cannot be measured accurately at the same time. One of the most famous discussions of this hypothesis involved the two great physicists Albert Einstein and Niels Bohr. Einstein suggested a gedanken, or thought, experiment — the so-called Einstein–Bohr recoiling double-slit gedanken experiment — in which one of the two slits in the experiment is allowed to move.

Xiao-Jing Liu and colleagues report the first experimental realization of the Einstein–Bohr gedanken experiment using a molecular double slit [22]. Elegant experiments performed with X-rays and a double slit formed from molecular oxygen have finally made it possible to realize and test a long-standing and famous gedanken experiment in quantum mechanics (Fig. 10).

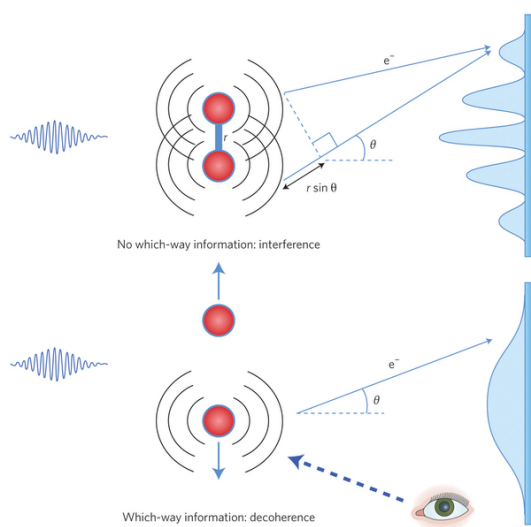


Fig. 10. Upper: A pulse of light (left) impinges on a diatomic molecule, leading to coherent electron ejection (e^- , blue arrows) from the two atomic centres (the “two slits”, red circles) in the form of waves (black lines). These waves reach a screen (right) where the intensity of the electron signal is recorded. Constructive interference between electron waves generated by the two atomic centres is obtained at observation angles θ satisfying $n\lambda = r \sin \theta$, where r is the internuclear distance, λ is the electron wavelength and n is an integer. No information about the path followed by the electron can be obtained from this experiment. Bottom: molecular version of the Einstein–Bohr gedanken experiment. Same as above except that the two atomic centres separate in opposite directions (thick blue arrows) as a result of dissociation. An observer measures the momentum of the atomic fragment from which the electron wave

is emitted. As a result of this measurement, which provides information about the path followed by the electron, interference patterns are no longer observed at the screen [23].

According to quantum mechanics, the determination of which slit the particle passes through inevitably destroys the wave aspects and implies the disappearance of the interference. In [22, 23] is noted that results of this experiment are in full agreement with the Bohr's complementarity principle.

Here we should emphasise that it is difficult to name the last two experiments fully as the double slits. Indeed, Figs. 9 and 10 more resemble the one-slit experiment when the interference was not observed. However, these figures show clearly the interference patterns. The noted discrepancy is very surprising. Might the interference pattern be some kind of “photograph” of the interaction of the electrons with the vacuum fluctuations which exist between the two atomic centres? We emphasise that the Casimir effect is largest between metal balls (see the section 6).

4. One of the earliest and strangest predictions of quantum physics is the idea of wave -particle duality, that everything in the universe has both particle and wave nature. Generally speaking, it is true only for elementary particles. However, the experiment can be done with entities much larger than electrons and photons, although it becomes more difficult as size increases. The largest entities for which the double-slit experiment has been performed were molecules (Fig. 11) that each comprised 810 atoms (whose total mass was over 10,000 atomic mass units) [24-26].

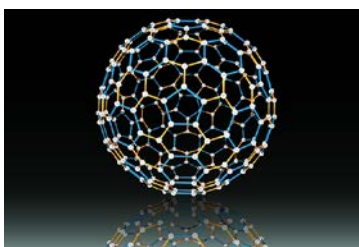


Fig. 11. Experiments were also held with large molecules in a form very similar to a soccer ball: a hollow sphere made of pentagons and hexagons.

These experiments demonstrated interference between matter waves passing through two or more slits cut in a barrier. Back in the day Richard Feynman famously said that interference of particles captures the essential mystery of quantum physics. Indeed, it is the mystery since the matter waves of large molecules are negligible small relative to the dimensions of the molecules!

The interference of waves is determined in part by the wavelength. According to quantum physics, the wavelength of a massive particle is inversely proportional to its momentum: the mass multiplied by the

particle's speed. In other words, the heavier the object, the shorter its wavelength at a given speed. A kicked football (for example) has a very tiny wavelength compared to the size of the ball because it has a relatively large mass and speed measured in meters per second (rather than nanometers or such). In contrast, an electron has a relatively large wavelength (though still microscopic) because it has a small mass. Longer wavelengths make it easier to generate interference so, while it is not going to be possible to make two footballs interfere with each other (in the quantum sense!), it is comparatively straightforward to produce electron interference.

The researchers observed the particle nature of the molecules in the form of individual light spots appearing singly in the fluorescent detector as they arrived. But, over time, these spots formed an interference pattern due to the molecules' wave-like character.

As the Juffmann et al. [26] point out, no other explanation but quantum interference can account for the pattern that appears in the fluorescent detector. Since the phthalocyanine and phthalocyanine-derived molecules are relatively large and massive, their behavior approaches the limits at which macroscopic properties begin to exhibit themselves. Future experiments with even larger molecules may be able to examine the transition between everyday physics, in which quantum interference does not play a role, and the underlying quantum world.

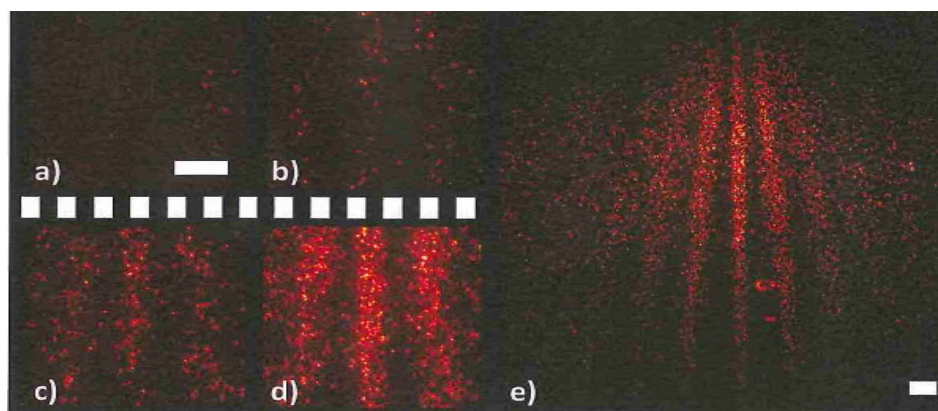


Fig. 12. Image of interference of phthalocyanine molecules; each dot is a single molecule detected by fluorescence, the pattern of bands is indicative of wave behavior. The images were recorded a) before deposition and after Pch2 deposition for b) 2 min, c) 20 min, d) 40 min and e) 90 min [26].

A beam of phthalocyanine molecules passes through a nanofabricated diffraction grating. The fluorescence light detects single molecules striking a glass plate on the far side. Each dot in Fig. 12 represents a single molecule, and their arrival times and locations are essentially random. The compilation of all the detections, though, reveals a pattern of bright and dark bands that is the result of interference. You can even spot the relationship between momentum and wavelength in the angle of the bands-- the

spacing is wider toward the bottom of the figure because those molecules are moving slower, and took more time to cover the distance between the grating and the detector. Slower speeds mean lower momentum, though, which means a longer wavelength, and thus a larger spacing between the bands. During the experiments moving molecules were irradiated by a laser ray. After being heated by an external source, molecules began glowing; thus, they became visible to the observer. After this change was added and used in multiple experiments, molecules altered their behaviour.

In this case, it was possible to heat the molecules by means of a laser beam, which changed their internal temperature. Now let us recall that any heated body, including a molecule, emits thermal photons. In this case, it is possible to determine the trajectory of the emitting molecule with accuracy of the wavelength of the emitted quantum. In the experiment, it was found that in the absence of laser heating, one observes an interference pattern, completely analogous to the picture from two slits in the experiment with electrons. The addition of laser heating leads first to a weakening of the interference contrast, and then, as the heating power increases, to the complete disappearance of interference effects. It was found that at temperatures $T < 1000\text{K}$, the molecules behave like quantum particles, and at $T > 3000\text{K}$ they behave like classical bodies.

The beginning of the discussion. The possibility of controlled heating of the molecules made it possible in this experiment to study the transition from the quantum to the classical regime. In other words, proceeding from this experiment, it can be concluded that the observed reality is based on a non-localized and “invisible” quantum reality that becomes localized and “visible” during the experiment, namely, when the effect of irradiation is reduced.

In other words, we connect the results with the fact that at low temperatures some “coherent invisible” quantum reality exhibits some resonant properties.

Another complex problem. If we place a detector inside or just behind one slit, we can find out whether any given particle goes through it or not. In that case, however, the interference vanishes. Simply by observing a particle's path – even if that observation should not disturb the particle's motion – we change the outcome. Until a particle is observed, an act that causes the wavefunction to “collapse”, we can say nothing about its location. Albert Einstein, among others, objected to this idea. He asked “does the moon exists only when I look at it?”

Can the observation disturb the results of the experiments? Taking into account similar results of the experiments certain researchers *declared that objectivity is an illusion*. This point of view was supported by a paper [27] where the authors consider closely some results of the DSE.

Then, in 2011 Aephraim Steinberg and his colleagues from the University of Toronto [28] succeeded in showing that the measurement does not necessarily lead to the collapse of the wave function and that the trajectory of photons before the collapse of the wave function exists! We emphasise that after much debate among the *Physics World* editorial team, year's honour (2011) went to Aephraim Steinberg and colleagues for their experimental work on the fundamentals of quantum mechanics. Using an emerging technique called "weak measurement", the team is the first to track the average paths of single photons passing through the Young's double-slit experiment – something that Steinberg says physicists had been "brainwashed" into thinking is impossible).

Steinberg's work stood out because it challenges the widely held notion that quantum mechanics forbids us any knowledge of the paths taken by individual photons as they travel through two closely spaced slits to create an interference pattern

In these new experiments the technique of "weak measurements" was developed [29-31]. As a result, physicists managed to measure the average momentum of photons passing through a specific slit, i.e. determine their averaged trajectories and at the same time preserve the interference pattern on the screen. We emphasize that this result corresponds to the experiments with molecules.

This contradicts the Copenhagen interpretation of quantum mechanics and is more consistent with the so-called hidden-parameter theory, that Einstein defended in his time - "God does not play dice".

A group of scientists tried a variation of the DSE, called the delayed choice experiment [32, 33]. This experiment was first proposed as a thought experiment (gedanken experiment) by John Wheeler as a way of exploring the counterintuitive aspects of particle-wave duality. According to Wheeler "It is hard to avoid the implication that consciousness and quantum mechanics are somehow linked" [34, 35].

That possibility was admitted in the 1930s by the Hungarian physicist Eugene Wigner. "It follows that the quantum description of objects is influenced by impressions entering my consciousness," he wrote.

"Solipsism may be logically consistent with present quantum mechanics."

Wheeler even entertained the thought that the presence of living beings, which are capable of "noticing", has transformed what was previously a multitude of possible quantum pasts into one concrete history. In this sense, Wheeler said, we become participants in the evolution of the Universe since its very beginning. In his words, we live in a "participatory universe."

Wheeler's idea was to imagine a "cosmic interferometer" (see, also, [3, 61] and Fig. 29). Suppose light from a distant distant quasar were to be **gravitationally lensed** by closer galaxy. As a result, light from a single quasar would appear as coming from two slightly different locations. Wheeler then noted that this light could be observed in two different ways. The first would be to have a detector aimed at each lensed image, thus making a particle measurement. The second would be to combine light from these two images in an interferometer, thus making a wave measurement. According to quantum theory, the results of these

two types of experiments (particle or wave) would be exactly as we have observed in their standard form. But the light began its journey billions of years ago, long before we decided on which experiment to perform. Through this “delayed choice” it would seem as if the quasar light “knew” whether it would be seen as a particle or wave billions of years before the experiment was devised (see, also, Fig. 29 (Schematic diagram of a hybrid Hanbury Brown-Twiss and Aharonov-Bohm experiment)).

Although the quasar experiment Wheeler proposed is not practical, modern experimental equipment allows us to perform a similar experiment in the lab, where the decision to measure a particle or wave is done at random after the quantum system is “committed.” For example, in 2007 [32] a delayed-choice experiment was [made using laser light](#) to create a delayed-choice double slit experiment. In this new paper [33], the team used an ultracold helium atom to do a similar delayed-choice interference experiment. With both experiments the results were exactly as predicted by quantum theory. So both matter and light exhibit this strange quantum effect.

While that might seem strange, it is not magical or mystical. The Moon would not vanish from existence if everyone closed their eyes, and reality is not dependent upon us observing it. Although if the second particle is detected after the first particle hits the screen, it still ruins the interference pattern. This means that observing a particle can change events that have *already happened*.

Scientists are still unsure how exactly this whole thing works. It is one of the greatest mysteries of quantum mechanics. Perhaps someday someone will finally be able to solve it.

Conclusion. Quantum theory is strange, but very real. Through countless experiments we have found that quantum objects have both particle-like and wave-like properties. In some experiments the particle nature dominates, while in others the wave nature dominates. Some experiments can even show the effects of both properties. This duality between particles and waves in quantum theory is deeply counterintuitive, which means often the results of quantum experiments are interpreted incorrectly.

3. Classical explanations of the experimental results

We should remember that the greatest puzzles of the DSE must have explanations. Let us consider the most well-known explanations.

1. Particles can be thought of as a kind of wave, and when waves emerge from two slits like this they can interfere with each other. If their peaks coincide, they reinforce each other, whereas if a peak and a trough coincide, they cancel out. This diffraction produces a series of alternating bright and dark bands on the back screen, where the waves are either reinforced or cancel out (see, also, Fig. 33 (right)).

However, in this case we do not take into account the particle as an element of physical reality. This is just a wave in which there is nothing from the particle. However, for example, when experimenting with electrons, each electron forms on the screen a tiny dot, as if it was a particle! At the same time, the mathematical description of the experiment as the wave process is in good agreement with its results.

2. On the other hand, the basis of the quantum mechanics lies in the mathematical formalism, in which there is no absolute predictions, characteristic of classical physics. If you made identical experiments with two particles, then there is a chance you will get different results. In particular, during the passage through the slits, the particle can be fixed at different points on the screen. However, if through the slits consistently pass a lot of particles, they in the process form an increasingly clear interference pattern! And this picture does not depend on the type of particles or the features of the experiment. As in the case of a coin toss, you can not guarantee the result of tossing. However, when the number of tosses is increased, the total result will tend to the probability equal to 0.5. And this result, in an ideal experiment, does not depend on a size of the coin. Is it possible that during the experiments the particles also exhibit their corpuscular properties !?

The standard interpretation by which a wave function expresses probability helps to understand the results of the experiment, since it can be argued that we are dealing with probabilities rather than with actual events. It was this strange phenomenon and the inability to explain it that motivated Niels Bohr to develop his idea of complementary. After having struggled with this riddle and discussing it with leading scientists over a long time, he came to believe that it was impossible to explain the wave-particle duality and we just have to accept that nature is strange. Somehow matter is both particle and wave. It is well known that quantum objects have both particle-like and wave-like properties. However, Bohr suggested that particles can demonstrate corpuscle-like or wave-like properties during the same experiment. For example, near slits the wave nature dominates, while on the detector the particle-like nature is demonstrated. According to the classical quantum superposition the physical system simultaneously exist in all theoretically possible states; but when measured or observed gives a result corresponding to only one of the possible states. But in this case we can ask “what is really real?” [34, 35].

When physicists, during similar experiments, tried to determine with the help of instruments which slit the electron actually passes through, the image on the screen had changed dramatically and became a “classic” pattern with two illuminated sections opposite to the slits and no alternating bands displayed. Electrons did not seem to show their wave nature under the watchful eye of observers. At the same time

the particle demonstrates its corpuscle-like properties in the detector! Is this some kind of mystery? There is a simple explanation if we accept it. No observation of a system can be carried out without physically impacting it.

However, we can imagine a very weak impact which practically does not perturbs the particles and at the same time determines the path.

3. The latest experiments [28-31] perturb the role of the classical quantum superposition. There is the alternative view, known as pilot-wave theory. According to de Broglie–Bohm theory, every particle has an actual, definite location, even when it is not being observed. Changes in the positions of the particles are given by the “pilot wave” equation. The theory is fully deterministic; if you know the initial state of a system, and you have got the wave function, you can calculate where each particle will end up.

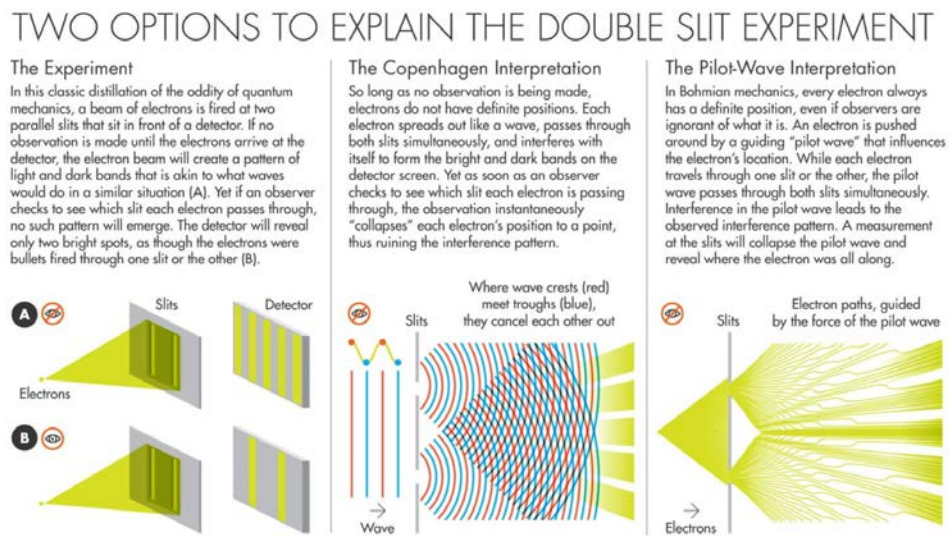


Fig. 13. Scheme of the DSE and basic explanations of its results [31].

According to the de Broglie–Bohm model, particles have definite locations and properties, but are guided by some kind of “pilot wave” that is often identified with the wavefunction. The de Broglie -Bohm interpretation imagined that the wavefunction of a particle is additional part of reality which exists in addition to the particle itself. In the DSE, for example, each particle goes through one slit or the other, while its wave function goes through both and suffers interference.

On a historical note, Einstein lived just long enough to hear about Bohm’s revival of de Broglie’s proposal — and he was not impressed, dismissing it as too simplistic to be correct. In a letter to physicist Max Born, in the spring of 1952, Einstein weighed in on Bohm’s work:

“Have you noticed that Bohm believes (as de Broglie did, by the way, 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me. But you, of course, can judge this better than I”.

4. Nonclassical explanations of experimental results

Thus, there are a number of approaches which tried to explain the DSE results. Further we describe three more interpretations.

1. Feynman proclaimed that each electron that makes it through to the phosphorescent screen actually goes through both slits. Feynman argued even more that in travelling from the source to a given point on the phosphorescent screen each individual electron actually traverses every possible trajectory simultaneously. As an example, some of the infinity of trajectories for a single electron travelling from the source to the phosphorescent screen are shown in Fig. 14. We stress that this one electron actually goes through both slits.

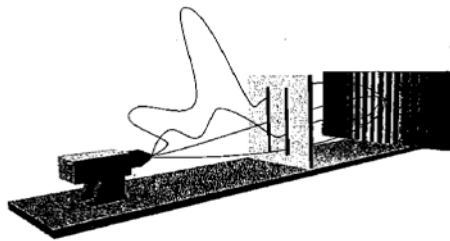


Fig. 14. According to Feynman's formulation of quantum mechanics, particles must be viewed as travelling from one location to another along every "possible" path.

The effect of one slit on another in the quantum language is easier to explain using the alternative descriptions of quantum physics developed by Feynman. According to his approach, known as "path integrals", when a particle moves from one point to another, it passes right through all trajectories connecting these points, but each trajectory has its own "weight". The greatest contribution is made by trajectories close to those predicted by classical physics. Therefore the quantum laws reduce to classical ones in the limit. But other trajectories are also important.

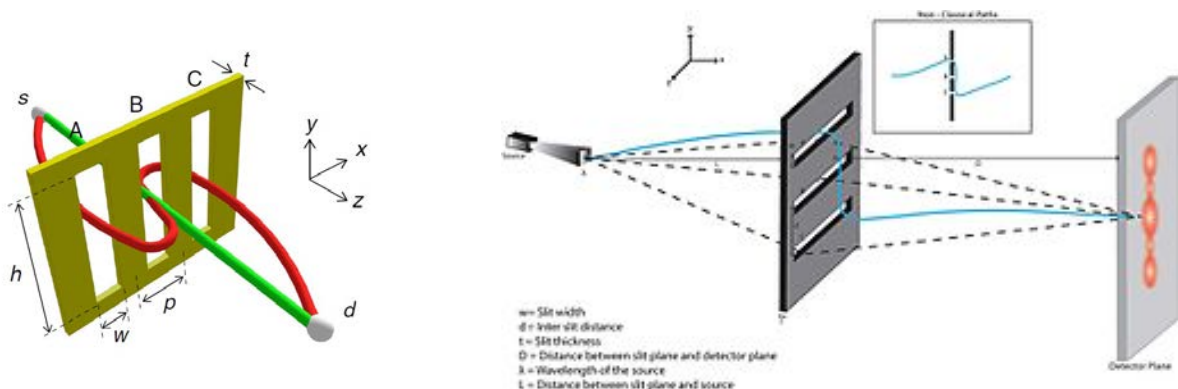


Fig. 15. Schemes of the three-slit interference [36]. Path integrals in a laboratory. The green line demonstrates a representative classical path. The purple line demonstrates a representative nonclassical path (left). The dashed lines demonstrate a representative classical paths, the blue line demonstrates a nonclassical path (right).

Among these trajectories there may be those that are completely impossible in the classical sense [36, 37]. They can contain areas on which the particle moves in the opposite directions (Fig.15). In the case of an experiment with slots, for example, these are trajectories that first enter one slit, then pass through the other, and then exit through a third one. These strange trajectories explain the effect of one slit on another, because only they are missing when one of the slits is closed.

2. The probabilistic interpretation of quantum laws within the framework of the Copenhagen interpretation was not very popular with many researchers in particular, Einstein. American scientist Hugh Everett proposed a different look at the process of collapse (reduction) of the wave function. It seems that he came under the influence of Richard Feynman. Everett generalized Feynman's idea and suggested that each trajectory corresponds to its own universe.

The many-worlds picture was used so that roughly describe some results of the DSE. Wiseman and colleagues [38] showed that with just 41 worlds, it is possible produce the same pattern as in some two-slit experiment. Their publication does not contain a wavefunction: particles obey classical rules such as Newton's laws of motion. The weird effects seen in quantum experiments arise because there is a repulsive force between particles and their clones in parallel universes.

However, the theory of the Multiverse is not the most incredible, which can be told with reference to quantum paradoxes and riddles. Is quantum teleportation or quantum coupling applicable for explanation of the results of the DSE?

3. Can an existence of the observer influence on the experimental results? Can quantum mechanics deal with the intervention of conscious thought in material reality. Or is it mystic?

We are only one step away from admitting that the world around us is just an illusory product of our mind [27, 34, 39]. Scary, isn't it? Let us then again try to appeal to physicists. Especially when in recent years, they favor less the Copenhagen interpretation of quantum mechanics, with its mysterious collapse of the wave function, giving place to another quite down to earth and reliable terms like the coherence (decoherence) and resonances.

We emphasise that in all these experiments with the observations, the experimenters inevitably impacted the system. This is a common and very important principle: you cannot observe the system or measure its properties without interacting with it. And where there is an interaction, there will be a modification of

properties. Especially when a tiny quantum system interact with colossal quantum objects. In this case, it is difficult to build a good theory describing experiments.

At the same time, quantum mechanics is good theory which often give very good predictions of the results of experiments. However, physics is not just about making predictions. There's a difference between *making* predictions and *understanding* them. The ability to predict behaviour is a big part of physics' power, but the heart of physics would be lost if it did not give us a deep understanding of the hidden reality underlying what we observe.

5. Some comments on the experiments and its results

We are not going to discuss the subtleties of the experiments and their interpretations. However, there are some features that are common to most of them.

First, coherent particles are used, in particular even in the case of experiments with single particle, the same particles are used (for example, only photons or electrons). The same is for particle beams.

Second, In a typical DSE the slits are located close enough to each other (in some one-slit experiments the slit is formed by plates, or plate and cylinder (Fig. 9) or balls (Fig. 10)). The ratio of the width of the slit to the distance between them varies in some cases over a wide range, but for the case of photons and electrons this ratio is often approximately 1/ 5. The slit width was often of the order of 100 nm. We remind that the de Broglie wavelength λ of an electron depends on its energy. For example, an electron with energy of 1.5 eV has $\lambda = 1$ nm, an electron with energy of 15 keV has $\lambda = 0.01$ nm and the energy of 40 keV has $\lambda = 0.006$ nm. The wavelength of photons also reduces when the energy increases. In particular, the photon wavelength was typically near 800 nm [28, 36]. Thus, typically, the electron wavelength was much smaller than the width of the slits and the photon wavelength could be larger than the width of the slits.

The wavelength of particles is usually much smaller than the width of the slits (for example, 50 pm relatively 62 nm in [40]). We have very counterintuitive results when the wavelength of photon was 800 nm while the slit width was 200 nm [36]. Generally speaking, the slits should be close enough to each other so that the distance between them must be comparable with particle wavelength. If the distance between the slits increases greatly, then the interference pattern disappears.

In the DSE, the part of the screen between the slits is either metallic or covered by a metal that conducts electricity well. The wavelength of the particles is significantly smaller than the distance between the slits.

Third. Conditions are created under which the particles move almost in a vacuum.

Fourth circumstance is due to the fact that, in interpreting the results, the Schrödinger equation is mainly used, and its linear version (classical version).

Fifth, it was found recently that if we use very weak measurements, then we manage to overcome the classical prohibitions of quantum mechanics

Sixth, the interference patterns are also formed by very heavy molecules (drops) that do not have wave properties that can affect the intensity picture. But the patterns are forming!

All the DSE show that when one slit is closed, the interference pattern disappears. In particular, this effect was thoroughly studied in [40]. However, there are experiments in which there is one “slit”, but the interference was strong (see Fig. 10). We think that the question of the existence of interference in the case of one slit requires additional careful study. Probably, in this case the interference is very weakly expressed and therefore practically not visible. We will discuss this issue below during the theoretical modelling of the problem.

Thus, the results of the latest experiments contradict with Feynman’s ideas, the Multiverse hypothesis and basis of quantum mechanics. This does not mean that the quantum mechanics does not describe the reality, but it means that certain results of it require of new understanding.

We think that the understanding of the results of the experiments depends on the idea underlying the method of its analysis. For example, the method used by Everett is a new understanding of the solutions offered by the Schrödinger equation. Another way is to reject the use of the Schrödinger linear equation. In general, we can use the nonlinear Schrödinger equation that takes into account the interaction of the probability peaks of the wave function. We will use nonlinear wave equations in the future to try to somehow represent the motion of particles and their interactions, but now we will focus on using a more radically new approach. We try to give a physically new understanding which is not completely a mathematical description of the results of the experiments.

6. Casimir’s effect

We emphasised that during the experiments the particles move almost in a vacuum (for example, 10^{-8} mbar in [26]). Thanks to the uncertainty principle, the vacuum buzzes with particle-antiparticle pairs popping in and out existence. They include, among many others, electron-positron pairs and pairs of photons, which are their own antiparticles. Ordinarily, those “virtual” particles cannot be directly captured. But like some spooky Greek chorus, they exert subtle influences on the “real” world. For example, the virtual photons fitting in and out of existence produce a randomly fluctuating electric field. In 1947, physicists found that the field shifts the energy level of an electron inside a hydrogen atom and hence the spectrum of radiation the atom emits.

Two plates. A year later, Dutch theorist Hendric Casimir predicted that a generalized version of van der Waals forces would arise between two metal plates (or conducting materials) due to quantum fluctuations of the electromagnetic field (Fig.16). When Casimir first calculated the effect, he used perfect “ideal” conductors. Later, more detailed calculations showed the effect for realistic conductors, and in 1997 the effect was confirmed experimentally. The most recent experiments get results to within 1% of the theoretical result. Strange as it is, the Casimir effect is very real. The Casimir effect is a great example the strangeness of quantum theory, and how even some of its strangest predictions turn out to be right [41-44].

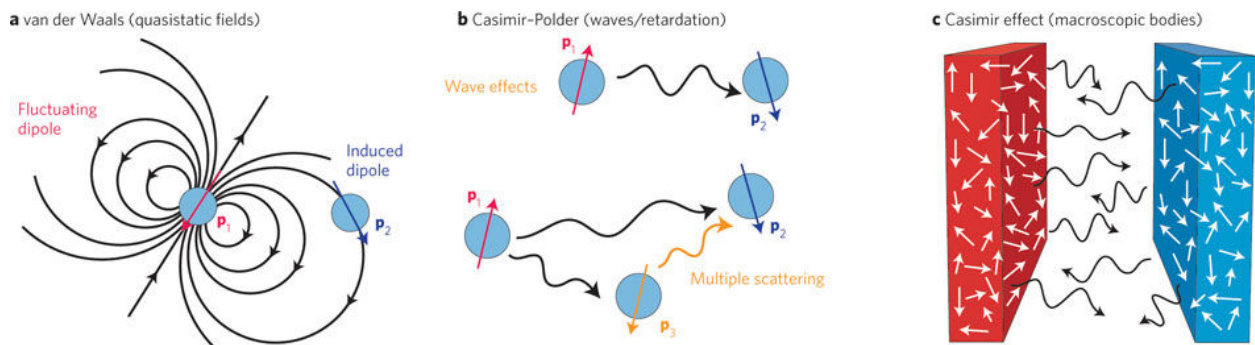


Fig. 16. Relationship between van der Waals, Casimir–Polder and Casimir forces, whose origins lie in the quantum fluctuations of dipoles. **a**, A fluctuating dipole p_1 induces a fluctuating electromagnetic dipole field, which in turn induces a fluctuating dipole p_2 on a nearby particle, leading to van der Waals forces between the particles. **b**, When the particle spacing is large, retardation/wave effects modify the interaction, leading to Casimir–Polder forces. When more than two particles interact, the non-additive field interactions lead to a breakdown of the pairwise force laws. **c**, In situations consisting of macroscopic bodies, the interaction between the many fluctuating dipoles present within the bodies leads to Casimir forces [42].

Two parallel plates can be considered as a resonator, in which exist only those waves for which the resonance condition is met: at an interval L between plates an integer n of half-waves is stacked. The force of attraction is inversely proportional to the fourth power of the distance between the plates. As the distance decreases, the force increases sharply. But even at submicron distances the Casimir force remains so small that it was observed only ten years after the prediction. It was measured for plates only in 1996.

Curvature effect on the Casimir force. Ensuring of the parallelism of the plates with a submicron slit is extremely difficult, so most experiments with the Casimir effect were carried out by replacing one of the plates with a sphere (ball). In this case the force of attraction is inversely proportional to the cube of the

distance between the sphere and the plate. The similar is true for the interaction of plate and cylinder (Fig.17) [43].

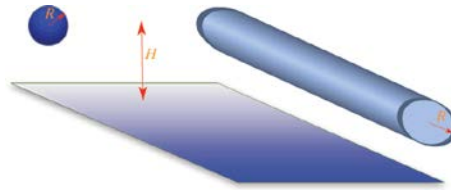


Fig. 17. Quasi-one-dimensional structures such as nano-wires or carbon nanotube and a plate [43].

This explains why in the DSE, cylinder and spheres are often used as certain element of the system (Figs. 9 and 10).

Another important circumstance related to the Casimir effect (or the van der Waals force) and the double-slit experiments is determined by the resonance. Apparently only when the slits, plate, cylinder or balls form a resonant system, only then the interference picture appears on the screen.

Photons (energy) from nothing. More recently, Chris Wilson et al. [45] have tried to prove another eccentric prediction: that it is possible to use the effect to release latent energy. Instead of allowing the fluctuations to tug on the plates, you rapidly force the plates together to squeeze their wavelengths – and force out photons (see Fig. 18).

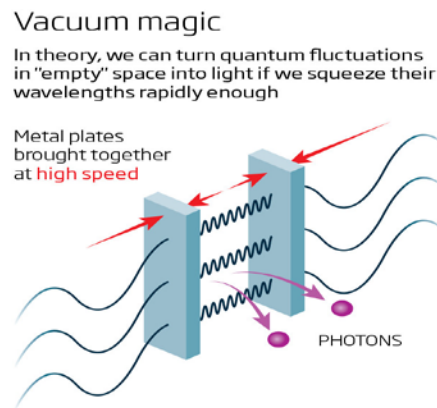


Fig. 18. This scheme illustrates a much old prediction on how to pull energy from empty space and produce light [46].

Generally speaking, in this experiment vacuum fluctuations manifests itself only indirectly. However, possibly, taking into account nonlinear and resonant coupling of purely virtual particles allow to detect effects origination of them [47].

The acoustic Casimir force. The term acoustic Casimir force (ACF) refers to the force between two parallel plates when they are placed in an acoustic random field [47]. This is a classical analogue of the quantum Casimir force that results from quantum vacuum fluctuations. Unlike the unbounded spectrum of

the quantum case, the ACF has very interesting physical consequences. The most significant being that the ACF changes from attractive to repulsive depending on the plate separation and the frequency bandwidth.

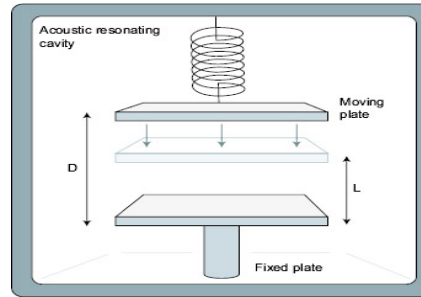


Fig. 19. Simple-lumped one-degree-of-freedom system considered in the calculation of the acoustic Casimir force [47].

It might be considered the schemes shown in Fig. 18 and 19 as some coarse analogues of the situation inside of the slits that exists in the DSE. On the other hand it might be considered the schemes as source of the real particles (waves). If we excite one plate with certain resonant frequency of the system, perhaps, the real particles will be radiated in this case. Might this situation be realised in the certain DSE?

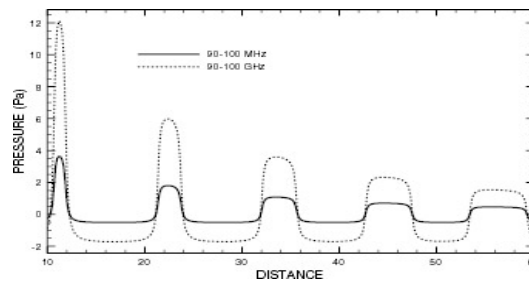


Fig. 20. Acoustic Casimir pressure as a function of separation for the frequency bandwidths MHz (solid line) and GHz (dotted line). There is the change in sign from attractive (negative) to repulsive (positive) The horizontal axis is in microns for the MHz bandwidth and in nanometers for the GHz bandwidth [47].

Thus, the vacuum is not “empty”, but filled with virtual particles that are very difficult to register, but under certain conditions become real - for example, when an external field of high energy is applied. In addition, they can have an effect on the actual particles and fields introduced into the vacuum. According to our theory (see the Part 2) the virtual particles (the standing waves of them) form in the slits. They change the particle trajectories. As a result, the particles fix the structure of these standing waves on the detector.

We think that the results of certain experiments considered above might be explained by the interaction of

these standing waves and the tiny local vibrations which move through the slits. However, for experiments with the molecules the influence of the van der Waals force might be the most important.

7. Thin metal layer and plasmons as the synchronizers

In order to prove the existence of “nonclassical” trajectories, Robert W. Boyd [36] and his colleagues proposed to excite the near-surface plasmons in the interslit material. The existence of plasmons increases the influence of one slit on another. In the experiment, the slits were in a layer of gold deposited on a transparent glass. Since gold is a good conductor, the plasmons are easily excited in it. We emphasise that trajectories of photons were considered in [36]. However, collective oscillations of electrons in the thin metal, that is, surface plasmons, can radiate the electromagnetic field [48]. Can this radiation disturb the electromagnetic near-fields in the vicinity of the slits? and change right understanding of the experimental results?

Preliminary information. Plasma of typical metals can be viewed as a kind of an electron Fermi liquid. Plasma is on the average electrically neutral. Due to fluctuations, plasma oscillations arise in a good conductor. Surface plasmons are quanta of vibrations of the density of free electrons of a metal, propagating only along its boundary with a dielectric or vacuum. Surface plasmons can be excited by means of a laser beam directed toward the metal surface. Scientists established that under certain conditions, plasmon waves can oscillate at the same frequency as external electromagnetic waves. So plasmons are a bound state of a photon and an electron in a metal which can form the standing plasmon wave between the edges. Similar to what happens with the standing waves on springs, only waves with appropriate characteristics can appear for a certain width. Surface plasmon polaritons (SPP) can confine electromagnetic fields in subwavelength spaces. In analogy to photons, they exhibit wave–particle duality [48] (Fig. 21).

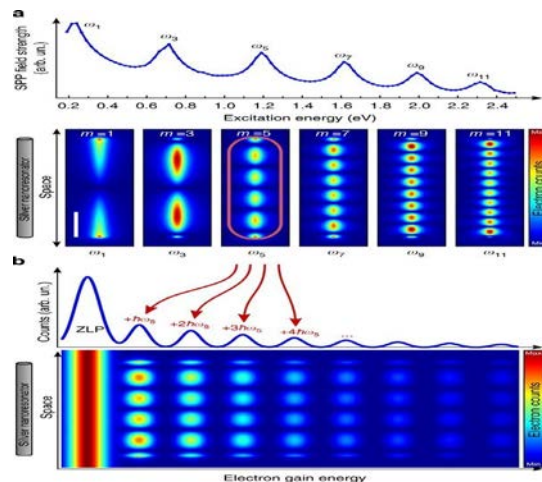


Fig. 21. Excitation energy-dependent imaging versus energy-space imaging. **a**, Finite-element simulation of the excitation energy dependence of the photoinduced SPP (surface plasmon polaritons) field strength. The vertical scale bar in the image of the the $m=1$ SPP mode corresponds to 500 nm and holds for all images. **b**, Selectively photoexciting only one of the SPP modes (here $m=5$) [48].

We believe that plasmons could be excited in interslit material of the typical of the DSE. Indeed, there is material coated with a thin layer of well conductive metal (silver or gold) [18, 36, 40, 44, 49]. When both slots are open, each of them influences the other, and the particle, generally speaking, now passes the slits differently than if the other slit was closed. The difference is not great. However, as it turns out, the influence of the slits on each other can be amplified.

Of course, the question of the possible excitation of plasmons by successive hits of particles remains unclear [50-54]. However, recently similar questions were discussed.

Can a sequence of electrons (photons) awake plasmons in the DSE? The answer is depended on the interslits dimension and its material.

Everyday experience, of course, indicates that big objects behave classically. In special labs and with a lot of effort, we can observe the quantum properties of photons or electrons. But even the best labs and greatest efforts are yet to find them in anything approaching the size of a cat.

One of the most important experimental questions in quantum physics is whether or not there is a point or boundary at which the quantum world ends and the classical world begins.

Riedinger et al. [55] report the quantum pairing of light and vibrations of microscopic mechanical oscillators comprising more than 10^{12} atoms — large for a quantum object (see, also, [56-60]). We should remind that the plasmon exhibits wave–particle duality and wavelength of photon may be of order 1000 nm. It is comparable to the length of plasmons.

Thus, there are testimonies that quantum particles (electrons and photons) can excite waves in sufficiently large atomic systems. Therefore, we do not exclude a very significant effect of plasmons on the results of the DSE. Thus, the results of the DSE are determined by the resonance interaction of fields. In the slits there are waves of virtual particles, and in the interslit space excited plasmons exist. Due to this resonance nature, the interference may be only for certain parameters of the experimental equipment. In particular, results of the DSE can change periodically with changing parameters as indicated by Feynman (see Fig. 3).

Synchronization and comments. In the second half of the 17th century, Christiaan Huygens described the first time the synchronization two pendulum clocks located on common shelf due to weak interaction through vibrations of a shelf. The same, pianos in neighboring rooms can interact resonantly. Similarly the

virtual oscillations within slits in the DSE can be synchronized due to plasmons in the metal layer. It is as if you had two pianos that were completely out of tune with respect to each other, and ordinarily would not resonate at all; but there is a third instrument in the room, like a violin, that has enough flexibility to resonate with both of them.

Plasmons synchronize the oscillations in the slits. We assume that in the case of the DSE the plasmons appear in the surface layer of the metal at the very initial moments of the experiment, as a result of the impact of particles on the metal, when it is impossible to speak of any occurrence of bands on the screen (see Fig. 9 (A) a and b). The bands arise when synchronization of resonance oscillations occurs in the metal and the slits. The latter is valid only for the DSE (Fig. 1). The results of other experiments are explained by the Casimir's effect (that is, by vacuum oscillations) or the van der Waals forces. Indeed, the bands (fingers) finely demonstrate themselves in the one-slit experiments (Figs. 9 and 10).

Thus, there is a strong interaction of physical fields of the equipment elements with moving particles. The foregoing determines that the study and description of these fields are of the utmost importance for understanding the experiment.

Proceeding from what has been said, we assume that in the series of the DSE the wave packet (wave-particle) passes through the regions of standing waves formed by vacuum fluctuations. Namely, the wave packet interacts with standing waves (vacuum fluctuations) during passing through the slit. For example, it can be assumed that the particles can not pass through the compression zones, but they can pass the rarefied zones of the standing waves. With this understanding, the interference on the fixing screen does not arise as a result of the interaction of the particles emitted in the experiment. The bands correspond to the zones of condensation and rarefaction existing on the standing wave in the slits like in the famous Rutherford's gold foil experiment the flying particles shown the condensation zone (Fig. 22).

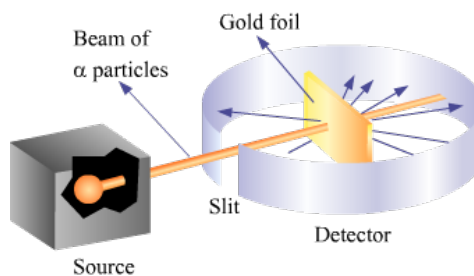


Fig. 22. Scheme of Rutherford's gold foil experiment.

In our case the particles show the zones of condensation of the vacuum fluctuations on the detector (screen) (see, also, Figs. 34-38).

When we talk about particle physics, we do not usually emphasize that we are actually talking about field physics. But we are. Our aim is to reorient our intuition, in order to appreciate how quantum fields are the ultimate building block of reality as we currently understand it.

Traditionally, during the analysis of the experimental results, it is not taken into account that we are dealing with describing the all-embracing interaction of field waves. This introduces confusion and misunderstanding of the results of experiments. We are not dealing with points and their trajectories, but with wave systems and effects of the resonant amplification.

In particular, the thin metal (gold) layers are resonators for the photon (electron) gas. On the other hand, the ripple of the field in the slits can change the trajectory of the motion of the moving waves (particle) so that they are fixed in certain zones of the screen!

Effects that manifest themselves in experiments are very weak and capricious. Any attempts at tracking the experiment disturb the synchronization of oscillations in the elements of the system and can completely destroy the interference. Together with this carefully conducted very weak control - does not fundamentally change the results as it would be expected. Apparently, in single-slit experiments with elementary particles, it is possible to obtain the bands (fingers) if we manage to strengthen the Casimir effect.

We can change (in this case mentally) the components of the experiment so as to explicitly test our conclusions. For example, one can remove the thin layer of metal to exclude the appearance of plasmons and the synchronization of vacuum oscillations in the slits. As a result the Casimir's effect disappears and the standing waves in the slit are not excited - and the fingers disappear!

We formulated several provisions which, on the whole, are not directly confirmed by experiment. However, following the great physicists, we can suggest the thought experiments so that to make our assumptions more real. This suggestion is no more than a suggestion. But there are several ways in which its usefulness can be tested.

8. Testing thought experiments

Following Feynman (Fig. 3), we have no doubt that changing the geometric and other characteristics of the experiments can affect the final interference pattern. In thought experiments, instead of slits (or in slits), one can place the Casimir plates. Or, it is possible to significantly change the interslit distance. We can introduce in the system disturbing electromagnetic field. As a result, the interference bands can change or disappear. This is due to the fact that as a result of these actions, the coherence of vacuum oscillations

in the slits can reduce or disappear. We emphasize that, in our opinion, this coherence (synchronism) of oscillations in the slits is ensured by waves (plasmons) in the interslit metal.

In the general case, we can mentally change certain components of the experiment and trace the possible results so that to explicitly check our conclusions.

Vacuum waves and corresponding bands (fingers). 1. For example, it would be interesting to use two plates of Casimir instead of bioprism in the experiment illustrated by Fig. 9. In this case we must bear in mind that for the bioprism (for the plate and cylinder) the Casimir effect is an order of magnitude greater than for the plates. But if we take this into account (that is, pick up the right material for the plates and the correct distance between them), then it will probably be possible to obtain an interference pattern, even for a single slit.

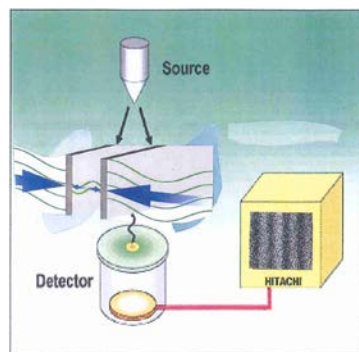


Fig. 23. The thought experiment where the biprism effect is replaced by the Casimir effect (the two plates).

Since the Casimir force is determined by the virtual harmonics arising between the plates, we assume that the experiment will reflect this (Fig. 23). The effect of these harmonics on the moving particles will be shown on the screen in the form of bands (fingers).

2. Another important circumstance of the double-slit experiment associated with the Casimir effect is determined by the resonance. Apparently only when the slits and the material between them (Fig. 2 and 3), or the plates and cylinder (Fig. 9) or closely located balls (see Fig. 10) form resonant systems, only then the interference takes place and the bands appear on screen.

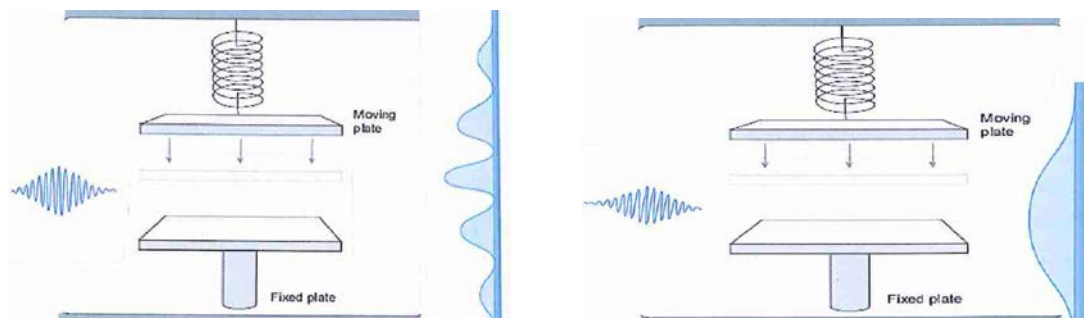


Fig. 24. The thought experiments with the plates (the Casimir effect) and the wave packets (showed as short harmonics). Resonant version of the experiment (left), nonresonant version of the experiment (right).

On the left (Fig. 24), the case is presented when the distance between the plates is optimal for the Casimir effect. At the same time, one plate vibrates with the resonant frequency of the vacuum oscillations in the slit. As a result, the bands appear on the screen. If the oscillations do not occur with the resonance frequency, then the bands do not appear.

3. Generally speaking, we do not need the passing particles for the formation of the bands. This possibility is illustrated by Fig. 25.

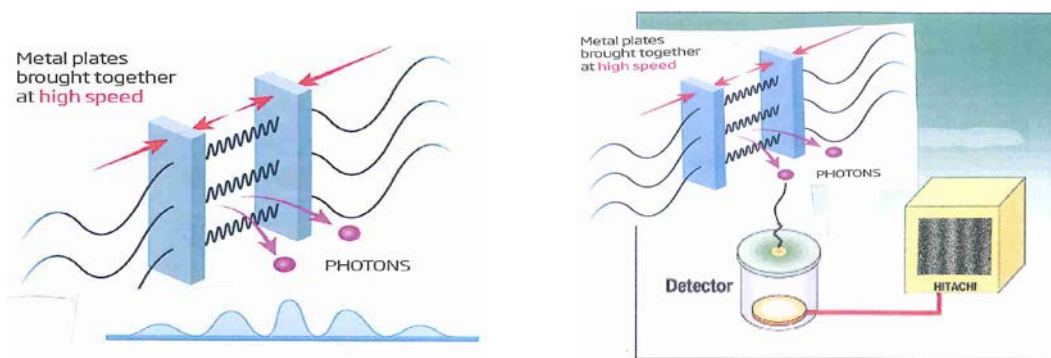


Fig. 25. Two versions of the thought one-slit experiment with radiating vacuum and the Casimir plates.

In Fig. 25 the plates oscillate with the resonant frequency and an amplitude such that the vacuum begins to radiate particles. Naturally, these particles arise at the tops of harmonics excited between the slits. The particles are fixed in the detectors in the form of interference fringes corresponding to the vacuum harmonics excited in the slit.

We have shown that the presence of the Casimir force in experiments ensures the appearance of the bands even if there is only one slit. Consequently, the appearance of the bands in no way, in the general case, is connected with the passage of particles through two slits. Reducing the power of the Casimir effect can achieve the disappearance of the bands. For example, this can be achieved by placing sensors near the slits or irradiating them with a laser beam.

Interaction and synchronization of the fields. A certain model of the thought double-slit experiment of Feynman (Fig. 2 (a)) is schematically shown in Fig. 26. We present the slits as springs (the vacuum oscillations are the springs). The particles light a thin solid (metal) barrier and the springs. The detector D records the particles that get through the slits. The vertical length of the springs is much greater than the thickness of them. We can consider the strip of the metal and the springs as certain one-dimensional resonators (indeed, for example, let the length of the strips is L and its ends are fixed as in a typical one-dimensional resonator).

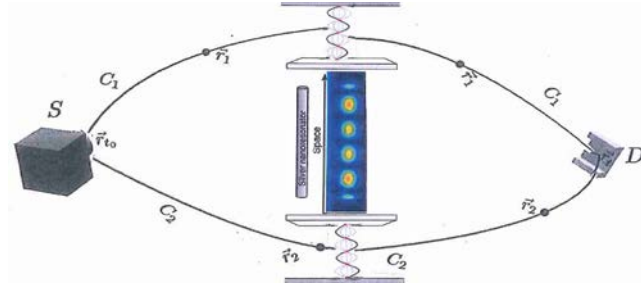


Fig. 26. Certain model of the thought experiments of Feynman. The springs model the influence of van de Waals or Casimir forces.

However, the frequencies of the longitudinal vibrations of the metal layer and the springs can be different. In this case the waves radiated by them can be incoherent. Therefore, no interference bands can form in the detector.

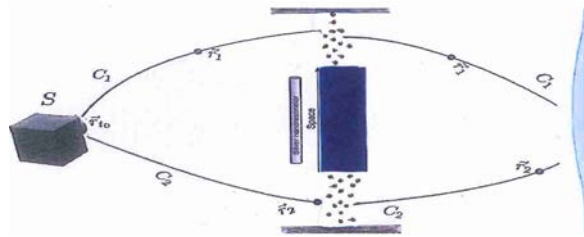


Fig. 27. Certain model of the thought experiments of Feynman. The thin layer of metal is excluded from the experiment. As a results, the plasmons, the synchronization and the fingers disappear from the experiment. The vacuum particles are shown as the points in the slits.

If there is no the synchronization of the vacuum oscillations in the slots, then the virtual particles can appear independently (even chaotically). Therefore, one can not speak of the possibility of the appearance of interference fringes. Even if Casimir forces (or van de Waals forces) appear there, the vacillations will still occur with different frequencies. As a result, in this case the appearance of interference fringes is unlikely (Fig. 27). The synchronizing role of the metal coating (plasmons) is illustrated by the mental experiments shown in Fig. 28.

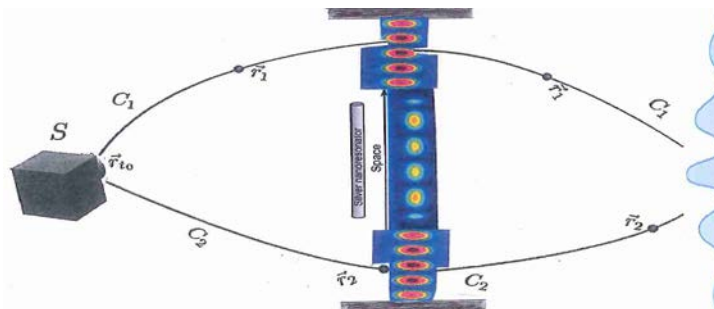


Fig. 28. Certain model of the thought experiments of Feynman.

Of course, we can imagine that the bands disappear if the particle energy is so small that the plasmons do not appear or if the passing particles are separated by very long time intervals (for example, several minutes) sufficient to completely attenuate of plasmon oscillations. In these cases, the experiment with slits will not show the appearance of the bands.

Above experiments (Figs. 23-28) complement the results of the Feynman's thought experiment. We believe that the synchronization was lost and restored (Fig. 3) when the geometric dimensions of the elements of the experiment were changed.

Plasmons and corresponding bands (fingers). We have considered several thought experiments designed primarily to show the possibility of the influence of quantum fluctuations on the interference. Another important element of the proposed understanding is plasmons. According this understanding the interference fingers will be changed if the plasmons are disturbed.

In [61] we described thought experiments designed to demonstrate the effect of an external electric field on final results of experiments.

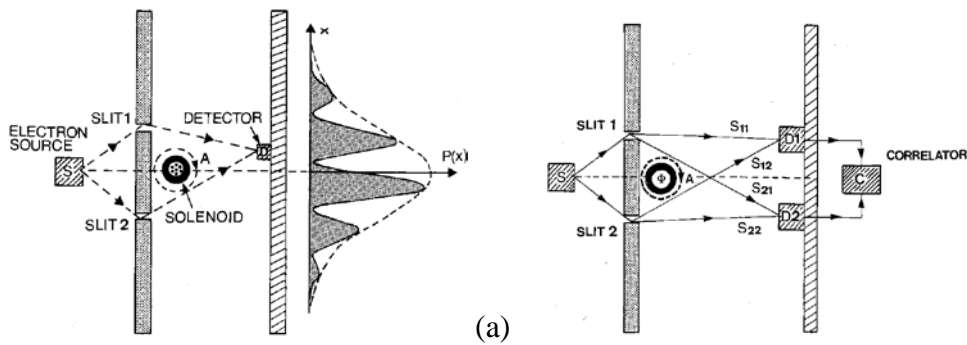


Fig. 29. Schematic diagram of two-slit electron interference in the presence of a force-free vector potential field (a). Schematic diagram of a hybrid Hanbury Brown-Twiss and Aharonov-Bohm experiment. Electrons emitted from source S pass through slits 1 and 2, around the solenoid (with magnetic flux F directed into the page and vector potential field A circulating clockwise) and are received at detectors D1 and D2 (b) [61].

In Fig. 29 (a) the magnetic field inside the solenoid is directed perpendicularly into the page; the external vector potential field has a clockwise sense about the solenoid axis. Although the diffraction envelope remains undeviated from the forward direction, the interference fringes are displaced by a relative phase shift between the components of the electron wave issuing from slits 1 and 2. This phase shift is proportional to the magnetic field within the solenoid, the region from which the electrons are excluded. The schematic diagram (Fig. 29 (b)) illustrates the effect of strong amplification of the solenoid field. We think that the confined magnetic flux changes the picture of plasmon oscillations in the interslit metal as a result the interference pattern is changed.

Part two. Equations, solutions and understanding of certain experimental results

Einstein suggested that light is particles, later called photons. Louis de Broglie then made a bold speculation: if light really is made up of particles that act like waves, then why should not particles in general also have their own waves? Later Erwin Schrödinger, one of the founders of quantum theory, picked up de Broglie's idea and carried it further to arrive at the famous Schrödinger wave equation (SWE). However, this equation is just a wave equation and it can only describe waves! How can it describe quantum particles, for example, photons?

Therefore, Bohr was faced with the enigmatic problem that matter seems to behave both like particles and waves (the particle-wave duality). Bohr insisted that matter has two different faces which show itself differently in different situations.

In the beginning of the 1950's, David Bohm found that, if particles were assumed to move under the influence of a certain guiding function derived from the SWE all the results of quantum mechanics (for example connected with the results of the DSE) could be explained. After this it was found that the Bohm's idea was suggested earlier by de Broglie in 1927.

Today, more than anything, the SWE is associated with quantum mechanics. However, although the SWE is a very powerful tool, it is certain particular case of more fundamental equations.

We showed earlier that the above approaches and the SWE do not completely describe the results of the DSE. A more powerful method of theoretical investigation is required to prepare the base for more purposeful experimentation. As such an equation, we have chosen the NKGE for describing of the waves in a vacuum. We will use the equations for plasma waves to describe plasmons arising, in our opinion, in the interslit material.

Vacuum waves. Thanks to the uncertainty principle, the vacuum buzzes with particle-antiparticle pairs popping in and out of existence.

In particular, a chaotic appearance and disappearance of virtual particles take place in the slit space. However, because of this randomness, virtual particles do not manifest themselves. Indeed, their total effect is zero. In particular, they do not in any way affect the particles passing through a sole slit. The situation fundamentally changes if there are two slits separated by a certain resonator (for example, a finite layer of a good conducting metal). If in this layer there are ordered vibrations of charged particles (for example, photons or electrons - a photon or electron gas), then these oscillations order the vibrations of virtual particles in the slits. There standing waves of virtual particles are formed (see Figs. 26-28).

Each set of waves has its own characteristic set of nodes and crests. These waves begin to affect the passing particles in such a way that their trajectories vary depending on whether the nodes or the crests

these particles meet during the passing through the slit. As a result, the particles in the detector “draw” the interference pattern which is certain kind of “photograph” of the standing wave in the slit (in the slits). Thus, in our opinion, along with the Casimir effect and the Lamb shift, the results of the DSE are an evidence of the existence of virtual particles in a vacuum. (The LAMB SHIFT, named after the American physicist, Willis Lamb. This work was carried out in the late 1940's, using techniques developed for wartime radar, showed that the effect of zero-point fluctuations of the electromagnetic field their atomic orbits, leading to a shift in frequency of transitions of about 1000 MEGAHERTZ).

Might this possibility have been overlooked by the founders of quantum theory who were not aware of the existence of the fluctuating background (vacuum) field?

Surface plasmons. Plasma of solids consists of ions, which are usually inactive and electrons are moving. Plasma is on the average electrically neutral. However, due to fluctuations, plasma oscillations arise in it. For their description, a quantum of plasma oscillations called the plasmon is introduced. Surface plasmons, which, in particular, take place in the DSE are quanta of vibrations of free electrons of the metal, propagating only along its boundary with a dielectric or vacuum. Free electrons in the metal can be excited by hits of photons or electrons. As a results collective oscillations (surface plasmons) may be excited in the surface thin metal layer.

9. Linear Schrödinger and Klein-Gordon equations

The kinetic energy written for a particle of mass m is

$$E = \frac{1}{2} \bar{p}^2 m^{-1}. \quad (1)$$

Here E is energy, \bar{p} is a vector of the particle momentum. Considering the expression

$\psi = \psi_0 \exp i(\omega t - \bar{p} \times \bar{r})$ of a particle wavefunction, it is natural to associate the momentum \bar{p} with the operator $i\hbar \nabla$, and energy E with the operator $i\hbar \partial / \partial t$. As a result the Schrödinger equation follows from (1):

$$i\psi_t = -\frac{1}{2} \hbar m^{-1} \nabla^2 \psi, \quad (2)$$

where \hbar is Planck's constant and $\psi_t = \partial \psi / \partial t$. This equation is valid for nonrelativistic free particles.

For relativistic particles we have that

$$E^2 = \bar{p}^2 c^2 + m^2 c^4. \quad (3)$$

Here c may be considered as a particle speed. The same transformations lead to the Klein-Gordon equation

$$\varphi_{tt} - c^2 \nabla^2 \varphi + m^2 \hbar^{-2} c^4 \varphi = 0. \quad (4)$$

Since all reference to imaginary number has been eliminated from this equation, it can be applied to fields that are real valued as well as those that have complex values. We will consider cases when

$$\nabla^2 \varphi = \sum_{n=1}^N \varphi_{x_n x_n}, \quad n = 1, 2, 3, \dots, N. \text{ Here } x_n \text{ are axes of a rectangular coordinate system.}$$

So that to simplify the equation (4) we will use new coordinates

$$T = tc^2 / \hbar \quad \text{и} \quad X_n = x_n c^2 / \hbar. \quad (5)$$

In this case the equation is written in a form

$$\varphi_{TT} - c^2 \sum_{n=1}^N \varphi_{X_n X_n} + m^2 \varphi = 0. \quad (6)$$

At first glance, the equations (2) and (6) are very different. However, it is well known that they can have similar solutions. The nonlinear Schrödinger equation (NSE) may be considered as certain particular case of the NKGE.

10. The nonlinear Schrödinger equation as a particle case of the nonlinear Klein-Gordon equation

Let us write the NKGE in so-called φ^4 field form:

$$\varphi_{tt} - c^2 \varphi_{xx} + m^2 \varphi - \lambda \varphi^3 = 0, \quad (7)$$

Here λ is a constant. Following to [62-64] we will seek the solution in the form of the quasi-harmonic wave with slowly varying complex amplitude $A(x, t)$ (an envelope function) and high-wavenumber modulated function $e^{i\theta}$ (so called “carrier wave”):

$$\varphi(x, t) = A(x, t)e^{i\theta} + A^*(x, t)e^{-i\theta}. \quad (8)$$

Here

$$\theta = \omega t - kx, \quad (9)$$

ω and k are constants. Thus, we assumed that the wave φ may spontaneously self-modulate.

Generally speaking, the modulation, which arises due to the overtones induced by nonlinearity, can split of the wave into “wave packets” which behave like solitons. These solitons are made of a carrier wave modulated by an envelope signal and this is why they are called envelope solitons. In (8) the function

$A(x,t)$ corresponds to the envelope of the wave. We think that similar soliton-like waves can describe the forms of “quantum particles” and its motion.

A modulated wave includes space and time scales: 1) A fast time and space variation of the carrier wave; 2) A much slower variation of the envelope.

We found that

$$\begin{aligned}\varphi_{tt} &= A_{tt}e^{i\theta} + 2A_t i\omega e^{i\theta} - A\omega^2 e^{i\theta}, \quad \varphi_{xx} = A_{xx}e^{i\theta} - 2A_x i k e^{i\theta} - A k^2 e^{i\theta}, \\ \varphi^3(x,t) &= A^3 e^{3i\theta} + 3A^2 A^* e^{i\theta} + 3A A^{*2} e^{-i\theta} + A^{*3} e^{-3i\theta}.\end{aligned}\quad (10)$$

We assume, approximately, that $\varphi^3(x,t) \approx 3A|A|^2 e^{i\theta}$. These expressions are substituted into (7) and equated the coefficients of the exponentials $e^{i\theta}$. This yields an equation for the complex amplitude $A(x,t)$:

$$A_{tt} - c^2 A_{xx} + 2i\omega(A_t + c^2 k \omega^{-1} A_x) + (c^2 k^2 + m^2 - \omega^2)A - 3\lambda A|A|^2 = 0. \quad (11)$$

Let

$$-\omega^2 + m^2 + c^2 k^2 = 0. \quad (12)$$

Then

$$A_{tt} - c^2 A_{xx} + 2i\omega(A_t + c^2 k \omega^{-1} A_x) - 3\lambda A|A|^2 = 0. \quad (13)$$

Let $X = x - v_{gr}t$ and $T = t$, then

$$A_x = A_X, \quad A_{xx} = A_{XX}, \quad A_t = A_T - v_{gr} A_X \quad \text{and} \quad A_{tt} = A_{TT} - 2v_{gr} A_{XT} + v_{gr}^2 A_{XX}. \quad (14)$$

In this case, if $v_{gr} = c^2 k_0 \omega_0^{-1}$, the equation (11) yields

$$A_{TT} - 2v_{gr} A_{XT} + v_{gr}^2 A_{XX} - c^2 A_{XX} + 2i\omega_0 A_T - 3\lambda A|A|^2 = 0. \quad (15)$$

We emphasise that the wave packet is just transported at the group velocity v_{gr} and vary very slowly within this frame. In particular, there

$$(A_T - 2v_{gr} A_X)_T \ll 2i\omega_0 A_T. \quad (16)$$

In this case we obtain well known equation usually referred to as the NSE:

$$iA_T + 0.5\omega_0^{-1}(v_{gr}^2 - c^2)A_{XX} - 1.5\omega_0^{-1}\lambda|A|^2 A = 0. \quad (17)$$

Thus, we showed that the nonlinear Klein-Gordon equation is the generalization of the NSE. With all this in mind further, we can look for solutions to the NKGE in the form of envelope solitons. These solutions will allow us to consider the motion of the quantum wave packets to the slits. To describe other elements of the experiment, we also consider equations describing the plasma oscillations in the interslit material.

11. Linear wave packet for the electron

Consider now an electron motion in a uniform electric field E oriented along the coordinate x [63]. In quantum mechanics, the electron is described by certain version of (2):

$$i\psi_t + \frac{1}{2}\hbar m^{-1}\psi_{xx} - U(x)\psi = 0, \quad (18)$$

where $U(x) = Ex$ is the potential energy of the electron. The solution is sought in the form

$$\psi(x, t) = \Psi(x)e^{-i\omega t}, \quad (19)$$

where $\omega = E\hbar^{-1}$ is the frequency of the wave and the function $\Psi(x)$ satisfies the equation

$$\Psi_{xx} + 2m\hbar^{-2}E(1-x)\Psi = 0. \quad (20)$$

The change of variable $z = -(2m\hbar^{-2}E)^{1.5}(1-x)$ transform (20) into the Airy equation

$$\Psi_{zz} - z\Psi = 0. \quad (21)$$

The solution that meets the condition of boundedness at $|z| \rightarrow \infty$ is expressed through the Airy function $CAi(z)$, where C is a constant. The wave function describing the behaviour of the electron in the uniform electric field has the form

$$\psi = CAi(z)e^{-i\omega t}. \quad (22)$$

Fig. 30 presented the distribution of the function $|\psi|^2$ that describes the probability for this electron to be found at the point with the coordinate z .

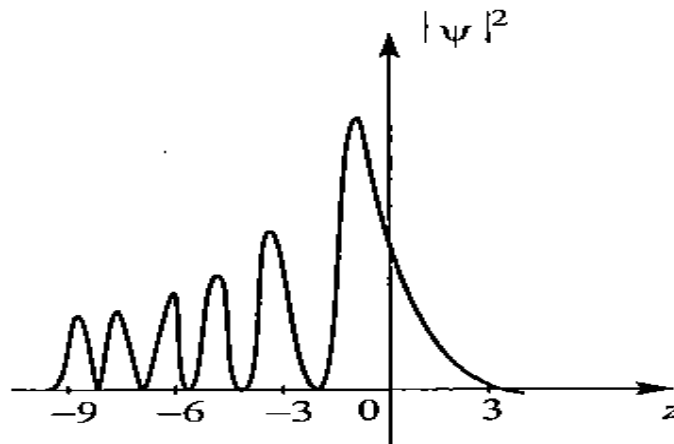


Fig. 30. The spatial distribution of the probability density for the electron.

One can see from last figure that for $z > 0$ this probability drops to zero abruptly as if there were an obstacle “reflecting” the electron at the point $z = 0$. Therefore, the point $z = 0$ is usually referred to as the turning or return point. The expression (22) for $z < 0$ is a nonuniform standing wave.

12. Linear model of plasmon oscillations in the interslit metal

Considering the electrons in the resonator we can assume that particles propagate as free electrons at speed much less than light.

The ion-electron and electron–electron interactions are completely ignored. In this case we have the model of the ideal electron gas. This model might be described with the help of the Schrödinger equation or as some kind of a plasma.

Plasma model. The simplest model of a plasma assumes that positively charged ions stay at rest while electrons move in the field of the electromagnetic wave under the action of the Lorentz force [63]. Since the electron speed is small respectively the light one can neglect the magnetic component of this force. In this case, the electrons move primarily along the y -axis and is described by a Newtonian equation

$$mu_{tt} = -eE(x, t), \quad (23)$$

where m is the electron mass and u is its displacement. Local separation of electrons and ions gives rise to plasma polarization, which is equal to $P = -eNy(x, t)$. Here N is the number density of electrons.

Consequently, a transverse electromagnetic wave propagating in such a medium is described by the equations:

$$E_x = -\mu_0 H_t, \quad (24)$$

$$H_x = -(\varepsilon_0 E + P)_t, \quad (25)$$

$$P_{tt} = m^{-1} e^2 N E. \quad (26)$$

The system (23) – (26) readily reduces to the linear Klein-Gordon equation:

$$E_{tt} - c^2 E_{xx} + \omega_p^2 E = 0, \quad (27)$$

where $\omega_p = e(N / \varepsilon_0 m)^{0.5}$. The parameter ω_p is called the plasma frequency of the electron gas.

We will use equation (27) to describe the plasma oscillations in the interslit metal. The boundary conditions are that

$$E(x, t) = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L. \quad (28)$$

In this case the solutions of (27) may be presented as

$$E(x, t) = A \sin k_n x \cos \omega t . \quad (29)$$

Here

$$k_n^2 = n^2 \pi^2 L^{-2} \quad \text{and} \quad \omega^2 = \omega_p^2 + c^2 k_n^2 . \quad (30)$$

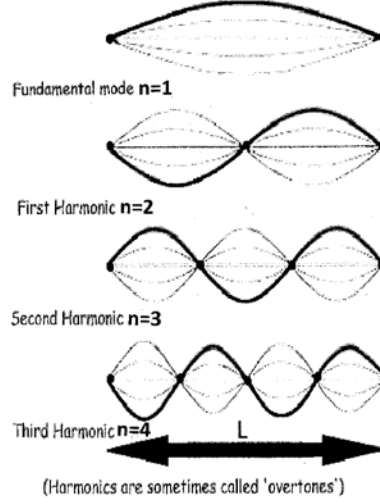


Fig. 31. Examples of standing waves of plasmons that can be excited by hits of flying particles to the interslit metal.

Examples of standing waves of plasmons excited in the interslit metal and described by the solution (29) are shown in Fig. 31.

13. Strongly localised nonlinear sphere-like wave packets

The behavior of many quantum particles traditionally thought both as a particle and as a wave. At the same time we can image the particles as some wave packets having spherical shapes (see Figs. 6 and 7). We will describe quantum particles by the NKGE (the φ^4 - theory). First we rewrite (7) as

$$\varphi_{tt} - c^2 \sum_{n=1}^N \varphi_{x_n x_n} + m^2 \varphi - \lambda \varphi^3 = 0 . \quad (31)$$

We introduce new variable

$$r = R^2 - \sum_n^N (x_n - c_n t)^2 . \quad (32)$$

Here

$$R = R_0 + l \cos \Omega t \quad \text{and} \quad R_t = -l \Omega \sin \Omega t . \quad (33)$$

and $R_0 \gg l$. The variable (32) allows us to describe wave packets having spherical shapes. Since

$$\varphi_{tt} = [(R^2)_t + 2 \sum_n^N (x_n - c_n t) c_n]^2 \varphi_{rr} + [(R^2)_{tt} - 2 \sum_n^N c_n^2] \varphi_r \quad \text{and} \quad \Phi_{x_n x_n} = 4 \Phi_{rr} [(x_n - c_n t)]^2 + 2 \Phi_r \quad (34)$$

the equation (31) yields

$$\Phi_{rr} \{ [(R^2)_t + 2 \sum_n^N (x_n - c_n t) c_n]^2 - 4c_*^2 \sum_n^N (x_n - c_n t)^2 \} - [2c_*^2 N - (R^2)_{tt} + 2 \sum_n^N c_n^2] \Phi_r = -m^2 \Phi + \lambda \Phi^3. \quad (35)$$

We shall consider a very strongly localized near points $R_0^2 = \sum_n^N (x_n - c_n t)^2$ solutions of (35). For such solutions, the equation (35) takes the form

$$R_0^2 (2l^2 \Omega^2 - 4c_*^2) \varphi_{rr} - [2c_*^2 N - (R^2)_{tt} + 2 \sum_n^N c_n^2] \varphi_r = -m^2 \varphi + \lambda \varphi^3. \quad (36)$$

Below we consider three cases.

First version of solution in the form of the wave packet. We can look for the solution as

$$\varphi = A \operatorname{sech} Br \sin \bar{B}r. \quad (37)$$

Here B is an arbitrary constant, A and \bar{B} will be found later. The solution (37) is obvious written in the form of the envelope solitons. In (37) $\sin \bar{B}r$ is the carrier part of the wave packet, and $\operatorname{sech} Br$ is the envelope. Let $\bar{B} \gg B$. Thus, we assume that $\operatorname{sech} Br$ varies very slowly respectively the variation of $\sin \bar{B}r$. At the same time both parts move in space with the same speed.

Now using (37) we can rewrite the equation (36) in the form

$$\begin{aligned} & R_0^2 (2l^2 \Omega^2 - 4c_*^2) (-2AB^2 \sec^3 Br \sin \bar{B}r + AB^2 \operatorname{sech} Br \sin \bar{B}r - 2AB\bar{B} \operatorname{sech}^2 Br \sinh Br \cos \bar{B}r \\ & - A\bar{B}^2 \operatorname{sech} Br \sin \bar{B}r) - [2c_*^2 N - (R^2)_{tt} + 2 \sum_n^N c_n^2] (-AB \operatorname{sech}^2 Br \sinh Br \sin \bar{B}r + A\bar{B} \operatorname{sech} Br \cos \bar{B}r) \\ & = -m^2 A \operatorname{sech} Br \sin \bar{B}r + \frac{1}{4} \lambda A^3 \operatorname{sech}^3 Br (3 \sin \bar{B}r - \sin 3\bar{B}r). \end{aligned} \quad (38)$$

Then we equate to zero the coefficients of $\operatorname{sech} Br \sin \bar{B}r$ and $\sec^3 Br \sin \bar{B}r$. As a result we have two algebraic equations. We found from them

$$B^2 - \bar{B}^2 = -m^2 R_0^{-2} (2l^2 \Omega^2 - 4c_*^2)^{-1}, \quad (39)$$

$$A^2 = -\frac{8}{3} \lambda^{-1} R_0^2 (2l^2 \Omega^2 - 4c_*^2) B^2. \quad (40)$$

Let $B^2 \ll \bar{B}^2$ and $l^2 \Omega^2 \gg 2c_*^2$. In this case the frequency Ω and the amplitude l of radial oscillations of the wave packet and \bar{B} of the carrier part of the wave packet are connected by the relation

$$\bar{B}_{\pm} \approx \pm 0.71 m R_0^{-1} l^{-1} \Omega^{-1} \quad (41)$$

and

$$A \approx \pm 2.3 (-\lambda)^{-0.5} R_0 l \Omega B. \quad (42)$$

The last expression is real only if $\lambda < 0$.

Second version of solution in the form of the wave packet. In this case approximate solution of (36) may be written in the form

$$\varphi = A \tanh Br \sin \bar{B}r, \quad (43)$$

where

$$\bar{B}_{\pm} \approx \pm 0.71 m R_0^{-1} l^{-1} \Omega^{-1} \quad \text{and} \quad A \approx \pm 2.3 \lambda^{-0.5} R_0 l \Omega B. \quad (44)$$

The amplitude is real only if $\lambda > 0$.

Third version of solution in the form of the wave packet. We can look for the solution as

$$\varphi = A \operatorname{sech}(Br) \sin^2 \bar{B}r. \quad (45)$$

In this case the equation (36) yields

$$\begin{aligned} & R_0^2 (2l^2 \Omega^2 - 4c_*^2) (-2AB^2 \operatorname{sech}^3 Br \sin^2 \bar{B}r + AB^2 \operatorname{sech} Br \sin^2 \bar{B}r \\ & - 2A\bar{B}\bar{B} \operatorname{sech}^2 Br \sinh Br \sin 2\bar{B}r + 2A\bar{B}^2 \operatorname{sech} Br \cos 2\bar{B}r) \\ & + [2c_*^2 N - (R^2)_t + 2 \sum_n^N c_n^2] (-AB \operatorname{sech}^2 Br \sinh Br \sin^2 \bar{B}r + A\bar{B} \operatorname{sech} Br \sin 2\bar{B}r) \\ & = -m^2 A \operatorname{sech} Br \sin^2 \bar{B}r + \frac{1}{4} \lambda A^3 \operatorname{sech}^3 Br \sin^6 \bar{B}r. \end{aligned} \quad (46)$$

Then we equate to zero the coefficients of $\operatorname{sech} Br$ and $\operatorname{sech}^3 Br$. As a result we have two algebraic equations. We found from them

$$B = \pm m R_0^{-1} (4c_*^2 - 2l^2 \Omega^2)^{-0.5}, \quad (47)$$

$$A = \pm 3.8 \lambda^{-0.5} m. \quad (48)$$

We have obtained solutions of the equation (31), which is strongly localized near points

$R_0^2 = \sum_n^N (x_n - c_n t)^2$. The solutions describe give a purely qualitative description of the shape and law of

motion of wave packets in space. They contain several indeterminate constants and is limited, of course, to the range of applicability of the initial equation and the coefficients involved in it.

If the scalar field and the scalar potential is known, one can calculate the energy pressure p and energy density ρ of this field according to expressions [65, 66]

$$p = \frac{1}{2} \varphi_t^2 - \frac{1}{6} (\nabla \varphi)^2 - V(\varphi), \quad (49)$$

$$\rho = \frac{1}{2} \varphi_t^2 + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi). \quad (50)$$

We used the scalar potential $V(\varphi)$ earlier in (31) as

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 + C . \quad (51)$$

Here C is a constant.

According to (49)-(51) we have real expressions for p and ρ both for $\lambda < 0$ and $\lambda > 0$.

14. "Nonclassical" trajectories, wave packets interaction and discussions

Despite the noted shortcomings, the solutions found allow us to proceed to the modeling of certain aspects of wave processes, which, according to the assumptions made above, determine the results of the DSE.

At the beginning of the 20th century, it was clarified that light still consists of particles called photons, but these particles also possess the mysterious wave property. The concept of the wave-particle dualism arose, which was also extended to particles of matter. In particular, the presence of wave properties was observed in electrons, and later in atoms and molecules (see Part 1).

The influence of one slit on another in the quantum language is easier to explain through one of the alternative descriptions of quantum physics developed by Richard Feynman. According to his approach, known as path integrals, when a particle moves from one point to another, it passes right through all possible trajectories connecting these points, but each trajectory has its own "weight". The greatest contribution is made by trajectories close to those predicted by classical physics, which is why quantum laws in the limit reduce to classical ones. But other trajectories are also important. Among these trajectories there may be those that are completely impossible in the classical sense [36, 37] (Fig. 15).

To prove the existence of "nonclassical" trajectories, Robert Boyd and his colleagues [36] suggested strengthening their influence by excitation of the surface plasmons. According to [36] the plasmons increase the influence of one slit on the other, and, correspondingly, the "weight" of "nonclassical" trajectories going from one slit to the other.

To clear up the effect of the slits on each other, the experimenters proposed to use a light source whose beam width is smaller than the distance between the slits. Only one slit was illuminated by them. As a result, when the light was used with such a polarization that the plasmons could not be excited, a small illuminated strip opposite the illuminated slit was observed on the screen. But when the polarization was changed, and the plasmons started to be excited effectively, a characteristic interference pattern appeared on the screen. The experimenters decided that it proves the existence of "nonclassical" trajectories.

However, it only proves the strong influence of plasmons on the experiments results how we have shown this above.

However, the outstanding publication [37] in which very broad generalizations are made deserves further consideration. The interaction of the particles is a very complicated process. If we try to describe the interaction of many thousands of the particles based on the Schrödinger equation, then we get an unsolvable problem. Feynman radically simplified it and achieved an exceptionally good description of many experiments by the theory. Let us consider two extracts from [4].

Is there.....

a limited number of bits and pieces that can be compounded to form *all* the phenomena that involve light and electrons? Is there a limited number of “letters” in this language of quantum electrodynamics that can be combined to form “words” and “phrases” that describe nearly every phenomenon of Nature?

The answer is yes; the number is three. There are only three basic actions needed to produce all of the phenomena associated with light and electrons. . . .

. . . In 1924 Louis De Broglie found that there was a wavelike character associated with electrons, and soon afterwards, C. J. Davisson and L. H. Germer of the Bell Laboratories bombarded a nickel crystal with electrons and showed that they, too, bounced off at crazy angles (just like X-rays do), and that these angles could be calculated from De Broglie’s formula for the wavelength of an electron.

When we look at photons on a large scale—much larger than the distance required for one stopwatch turn—the phenomena that we see are very well approximated by rules such as “light travels in straight lines,” because there are enough paths around the path of minimum time to reinforce each other, and enough other paths to cancel each other out. But when the space through which a photon moves becomes too small (such as the tiny holes in the screen), these rules fail—we discover that light doesn’t have to go in straight lines, there are interferences created by two holes, and so on. The same situation exists with electrons: when seen on a large scale, they travel like particles, on definite paths. But on a small scale, such as inside an atom, the space is so small that there is no main path, no “orbit”; there are all sorts of ways the electron could go, each with an amplitude. The phenomenon of interference becomes very important, and we have to sum the arrows to predict where an electron is likely to be. . . .

. It appears that *all* the “particles” in Nature—quarks, gluons, neutrinos, and so forth (which will be discussed in the next lecture)—behave in this quantum mechanical way.

So now, I present to you the three basic actions, from which all the phenomena of light and electrons arise.

- ACTION #1: A photon goes from place to place.
- ACTION #2: An electron goes from place to place.
- ACTION #3: An electron emits or absorbs a photon.

Our consideration is related to the fundamental results of Feynman. In the general case (in the philosophical sense) any interaction can not be considered in isolation from the influence of entire Universe. Apparently, Feynman first realized this with reference to the problems of quantum mechanics. Taking into account this kind interaction he introduced the so-called path integrals (integration over trajectories), instead of the wave function of Copenhagen interpretation of quantum mechanics. Feynman's ideas are very original. We presented some of them above.

Thus, the problem of the DSE in some respects turns out to be connected with the fundamental problems of quantum mechanics, - what is a "particle" and how they interact. On the one hand we have the Schrödinger equation (the Copenhagen interpretation of quantum mechanics), and on the other hand the integrals above trajectories. And how does the model of the section 13, based on the solutions of the Klein-Gordon nonlinear equation, fit into this confrontation?

In Fig. 32 the results obtained according to the solution (45) are presented.

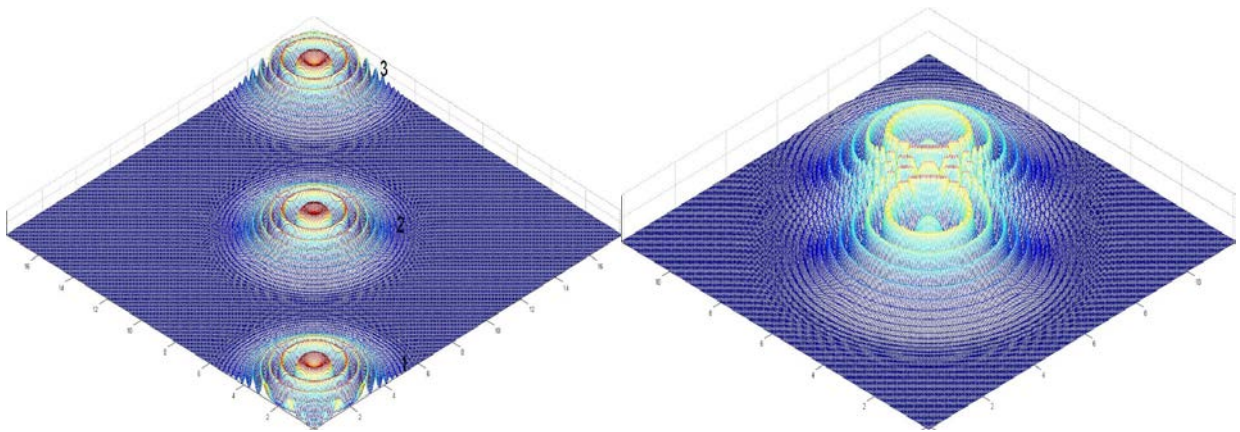


Fig. 32. 2D picture of wave packet motion calculated for 3 instants of the time (left). 2D picture of the interaction of wave packets (left).

It can be seen from Fig. 32 and the calculations that the interaction of the particles can be approximately described as the interaction of wave packets in an infinite space. Of course, a significant influence on the final result is exerted by waves located in the center of the packet, however, the influence of waves adjacent to the center is also not excluded. Thus, the influence of the wave trajectories of the strongly removed elements of the wave packets on each other and on the final result of the interaction can be according to the theory of the section 13 and Fig. 32. Therefore, the interaction of the "particles" described as the wave packets is a certain development of the Feynmann's ideas.

Perhaps this is not surprising, since some authors associate the simplest Feynman diagrams with solutions of the nonlinear Klein-Gordon equation (31). To illustrate this provision, we give below a few pages from the book of Helling K.C. Solving Classical Field Equations. http://homepages.physik.uni-muenchen.de/~helling/classical_fields.pdf [67].

4. Solving the interacting theory perturbatively

Armed with this ability to solve arbitrary inhomogeneous equations we now come back to the ϕ^4 -equation

$$(\square + m^2)\phi = g\phi^3.$$

We want to view this as a family of equations parametrised by the coupling constant g . Similarly, the solutions to all these equations will depend on g . Underlying the idea of perturbation theory is the idea that these solutions can be written as a power-series in g , i.e. that they are analytic in g around $g = 0$.

Unfortunately, this is not really the case as can be seen as follows: Power series (in the complex plane) have a radius of convergence (which can be zero or infinite): Everywhere inside a circle of this radius the power series converges and outside it diverges. Thus, if the power-series would converge for any $g > 0$ it would as well have to converge for some $g < 0$. But for $g < 0$, again, the potential is unbounded from below and the system is unstable: Solutions will be radically different from solutions of the free equation and not be small perturbations. In fact, as is shown in the appendix, the kink solutions have energy and action scaling like $1/g$ which has a singularity at $g = 0$. In a path integral (which in a stationary phase approximation reproduces the classical behavior), these solutions appear as saddle points contributing $e^{-S} \approx e^{-1/g}$. These contributions are exponentially small for small g . In fact, this function has an essential singularity at $g = 0$ and is invisible in a Taylor expansion around this point. Indeed, "solitonic" solutions like the kink are believed to be what is missed by the perturbative treatment. Their contributions are exponentially small for small g and can thus be safely ignored if one is interested in solutions to a finite precision.

Nevertheless, we will just proceed and pretend that solutions to the ϕ^4 -equation can be written as a power series

$$\phi = \sum_{n=0}^{\infty} \phi_n g^n$$

for some coefficient functions $\phi_n(x)$.

Now plug this Ansatz into the equation and collect powers of g :

$$\sum_{n=0}^{\infty} (\square + m^2) \phi_n g^n = g \left(\sum_{n=0}^{\infty} \phi_n g^n \right)^3 = \sum_{n=0}^{\infty} \left(\sum_{k+l+m=n} \phi_k \phi_l \phi_m \right) g^n$$

Comparing coefficients we find

$$(\square + m^2) \phi_n = \sum_{\substack{k,l,m \\ k+l+m+1=n}} \phi_k \phi_l \phi_m.$$

This simple manipulation has helped us a lot: We can now work our ways up starting from $n = 0$ to larger n . The important observation here is that this is a differential equation for ϕ_n in terms of a right-hand side given in terms of ϕ_k , ϕ_l , and ϕ_m where all $k, l, m < n$. That is, when computing ϕ_n we already know these ϕ_k , ϕ_l , and ϕ_m !

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Let's see how this works out for the first couple of n :

$$(\square + m^2) \phi_0 = 0$$

Nothing to be done. We know the solution is given in terms of plane waves obeying the dispersion relation. Next is

$$(\square + m^2) \phi_1 = \phi_0^3$$

That was simple. Using the Green's function, we can write down the solution:

$$\phi_1(x) = \int dy \phi_0(x-y) \phi_0(y)^3.$$

Now comes

$$(\square + m^2) \phi_2 = 3\phi_0^2 \phi_1.$$

The 3 arises as there are three possible assignments of two 0's and one 1 to (k, l, m) . The solution is

$$\begin{aligned} \phi_2(x) &= 3 \int dy \phi_0(x-y) \phi_0(y)^2 \phi_1(y) \\ &= 3 \int dy \int dy' \phi_0(x-y) \phi_0(y)^2 \phi_0(y-y') \phi_0(y')^3 \end{aligned}$$

Now for $n = 3$:

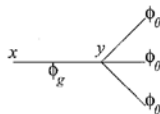
$$(\square + m^2) \phi_3 = 3\phi_0^2 \phi_2 + 3\phi_0 \phi_1^2.$$

The iterated solution gets longer and longer:

$$\begin{aligned} \phi_3(x) &= \int dy \phi_0(x-y) (3\phi_0(y)^2 \phi_2(y) + 3\phi_0(y) \phi_1(y)^2) \\ &= 9 \int dy \int dy' \int dy'' \phi_0(x-y) \phi_0(y)^2 \phi_0(y-y') \phi_0(y')^2 \phi_0(y-y'') \phi_0(y'')^3 \\ &\quad + 3 \int dy \int dy' \int dy'' \phi_0(x-y) \phi_0(y) \phi_0(y-y') \phi_0(y')^3 \phi_0(y-y'') \phi_0(y'')^3 \end{aligned}$$

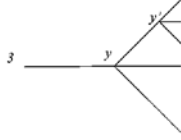
5. Feynman graphs in position space

Obviously, continuing like this will be more and more cumbersome. However, we see a simple pattern of these terms emerging: We can represent the solution for ϕ_1 like this:

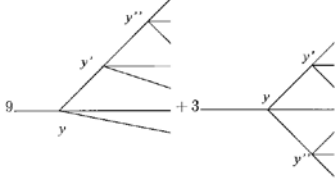


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We obtain the solution by bringing together three ϕ_0 's at one point y and then transport this to x using the Green's function ϕ_g . At higher orders, this pattern is iterated. For $n = 2$, we have



where the factor 3 arises because the graph for ϕ_1 can be substituted at any of the three legs. At level $n = 3$, there are two different graphs



again with "symmetry factors" indicating the number of possibilities of obtaining these graphs.

In this graphical notation, it should be clear what we have to do to obtain the expression for ϕ_n : We have to draw all possible graphs according to these rules:

- Draw n vertices for the expression for ϕ_n at order g^n .
- Each vertex gets one in-going line at the left and three outgoing lines to the right.
- A line can either connect to the in-going port of another vertex or to the right-hand side of the diagram.
- Write down an integral for the point of each vertex.
- For a line connecting two vertices at points y_1 and y_2 , write down a Green's function $\phi_g(y_1 - y_2)$.
- For a line ending on the right, write down a factor of ϕ_0 evaluated at the point of the vertex at the left of the line.
- Multiply by the number of permutations of outgoing lines at the vertices which yield different diagrams ("symmetry factor").

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6. More general field equations

Looking back at how these rules came up, we can immediately guess the generalisation to other field equations: The fourth order potential $V(\phi) = \frac{g}{4}\phi^4$ resulted in a field equation with a cubic right-hand side. The cube in the field equation became $\phi_k \phi_l \phi_m$ with the constrained sum over k, l , and m in the equation for ϕ_n and eventually resulted in the rule that each vertex has to have three outgoing lines. This suggests that for a potential $V(\phi) = \frac{g}{p}\phi^p$ we would derive similar rules but now with $(p-1)$ outgoing lines at each vertex.

If we have a potential which consists of a sum of more than one monomial there will be a separate coupling constant for each monomial.

$$V(\phi) = \sum_{i=1}^I \frac{g_i}{p_i} \phi^{p_i}$$

As a result, we would express ϕ in a multi-dimensional power series over all coupling constants:

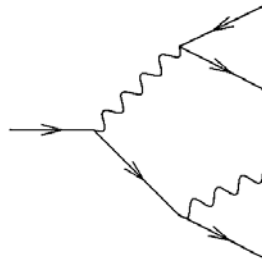
$$\phi = \sum_{n_1 \dots n_I} \phi_{n_1 \dots n_I} g_1^{n_1} \dots g_I^{n_I}.$$

Consequently, we now have I different types of vertices. To compute the solution for $\phi_{n_1 \dots n_I}$ we draw all diagrams with n_1 vertices of type 1 (which have $(p_1 - 1)$ outgoing legs), n_2 vertices of type 2 (with $(p_2 - 1)$ outgoing legs) and so on to n_I vertices of type I .

Of course, we can have also more than one type of field. In that case each field comes with its own equation of motion which we solve perturbatively: Each field has its Green's function and thus, in the graphical notation, we have different types of lines denoting the different fields and the vertices have "ports" connecting to the different types of lines. For example in Quantum electrodynamics, there is an electron field denoted by a straight line and a photon field denoted by a wavy line. There is a cubic term in the action which reads $e\bar{\psi}\gamma^\mu A_\mu \psi$. Here ψ is the electron field, A_μ is the photon field (which is a fancy name for the vector potential of electromagnetism), e is the charge of the electron playing the role of the coupling constant and γ^μ is some matrix needed to write down the Dirac equation which is the analogue of the Klein-Gordon equation for spin 1/2 fermions like the electron. The bar indicates a conjugate (which is finally responsible for the difference between electrons and positrons) which results in the electron lines having a direction which is indicated by an arrow. This cubic term in the action ends up in quadratic right hand sides of the field equations: The field equation for the photon has a term quadratic in the electron on its right-hand side (namely the expression for the electromagnetic current) whereas the electron has a right-hand side which is bilinear in the electron and the photon. A typical

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diagram then looks like this:



7. Relation to the quantum theory

Looking back at the “Feynman rules” above for the ϕ^4 -theory which describe the allowed graphs we notice that these rules simply describe all graphs with 4-valent vertices (there are in total four lines at each vertex) that do not contain any loops. In quantum field theory, there is a similar perturbative expansion which similarly is most easily written down in a graphical notation in which each Feynman diagram corresponds to an integral expression. It turns out that the rules which translate graphs to integrals are identical to our rules above and the only difference between our classical theory here and the quantum theory is that we drop the distinction between in-going and outgoing lines at a vertex and only require that there are four lines at each vertex (for ϕ^4 -theory), no matter if in- or out-going. Weakening this rule has the consequence that also graphs containing loops are allowed. This fact that the perturbative solution in classical field theory and in the quantum theory are so similar is then explained by the observation that each loop in a Feynman graph contributes a factor \hbar and thus the contribution of the loop graphs vanish in the classical limit of $\hbar \rightarrow 0$.

15. The virtual particles and the fundamental puzzles of the wave quantum mechanics (from the DSE to the Universe)

Physicists have established that in addition to electric and magnetic fields, there exists a whole panoply of others with names like strong and weak nuclear fields and electron, quark and neutrino fields.

All these fields are limited by the Heisenberg Uncertainty Principle. It states that it is not possible to know position and momentum of a particle with absolute accuracy, and the more precisely you measure one quantity, the less you know about the other.

The mathematical basis of quantum mechanics consists of very complex and abstract equations. They are so complex that sometimes there is doubts, that they are correct. Such complex theory can not be correct, because it is not beautiful for many researchers! Perhaps R. Feynman basing on the same feeling came up the beautiful way to depict the processes described by these equations with the help of the graphs (pictures). These graphs can be simplified to a few elements (see the extracts from Feynman's book placed in the section 14).

According to the Feynman's view, all phenomena in Nature can be reduced to collisions of particles. These collisions are described by the Feynman's graphs. In particular, new fields and particles are produced as a result of these collisions. However, these omnipotent graphs do not fully explain the results of the slit experiments. These graphs like all quantum mechanics fail with a puzzle of the another supertask of the modern physics.

The point is that all models give a very large energy of own quantum oscillations of the vacuum. At the same time, there is no evidence that the real vacuum has something similar. The situation resembles the situation at the end of the 19th century, namely the situation with the ultraviolet catastrophe.

We recall it briefly using the results of the section 12. The results are applicable to electromagnetic waves propagating between two reflecting mirrors. Since the results remain valid for all wavelengths, we have an infinite value of the energy enclosed between the boundaries. In order to reconcile the theoretical treatment with the experimental results, M. Planck introduced the conception of quantization of the radiation energy. After unsuccessful attempts to deduce this conception from the principles of classical physics, the quantization began to be considered as the fundamental principle of Nature. It was put in the basis of the new physical theory - quantum mechanics.

At the present time "classical theories of quantum field theory" estimate the vacuum energy as almost infinite. Of course, we do not fully know the real energy of the vacuum, so there is some uncertainty. However, this energy can be estimated indirectly if we assume that the vacuum energy determines the cosmological constant introduced in science hundred years ago by Albert Einstein. According to astrophysical studies (space geodetic survey), this constant is extremely small, but it is not zero, namely it is somewhere in the value $4.33 \cdot 10^{-66} eV^2$ in natural units.

If we accept the correspondence of the vacuum energy to the cosmological constant [68-70], then we come to the new scientific catastrophe determined by the extreme discrepancy of them.

Perhaps, we should accept a revolutionary hypothesis similar to that which Planck declared in 1900 so that to destroy this discrepancy.

Complex wave packet. Thus, we have come to the question of vacuum fluctuations (virtual particles). Apparently they should be distinguished from real particles, which, for example, passing through the slits in the DSE. Let us assume that the virtual particles consist from positive and negative halves. Due to the quantum uncertainty these halves can slightly shift relative to each other in time and space. This shift is of order of the Planck's constant, so the observer cannot see it. In the sea of virtual particles called vacuum, there are all kinds of elementary particles. The vacuum energy is the sum of the energies of these particles. Each type of particle contributes in the energy.

Although the virtual particles exist, they can not manifest themselves in any way outside of the interaction with normal (real) matter. Thus, we arrive to a model of particles that have properties attributed to particles of dark energy. In order to get at least some idea of virtual particles, we give results of calculations based on the solution (45), (47) and (48). We will assign the sign + to the positive part of the wave packet, and assign the sign - to the negative part of the wave packet. The results of the calculations are shown in Fig. 33.

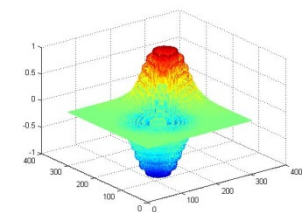
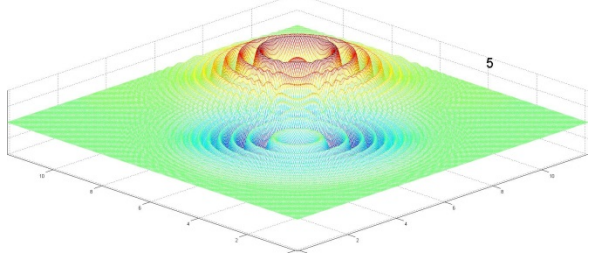
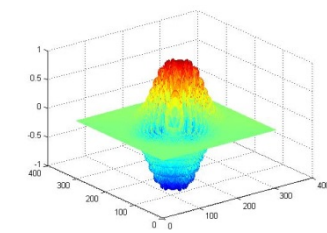
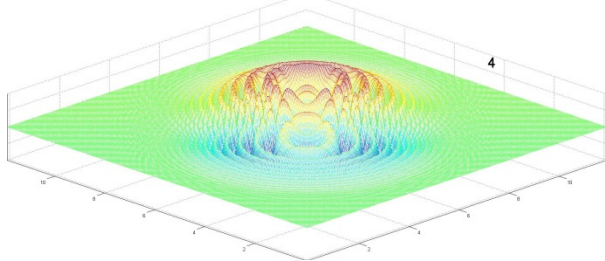
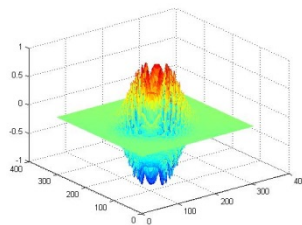
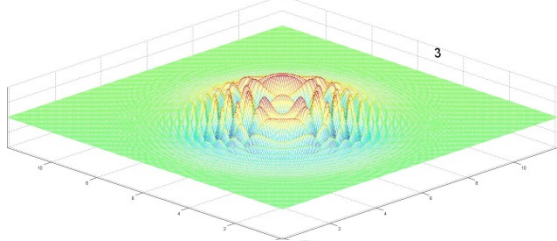
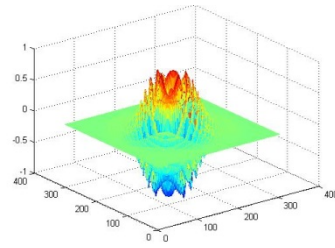
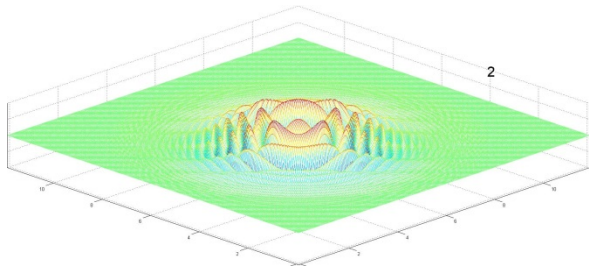
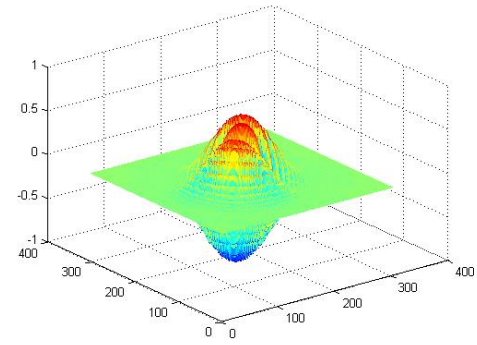
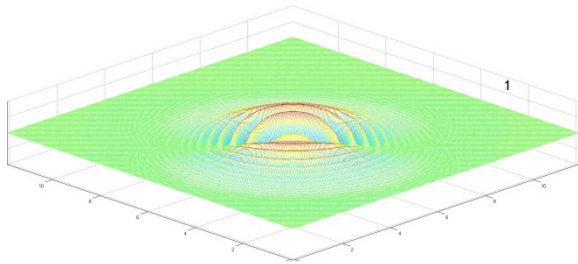
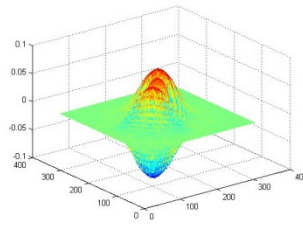
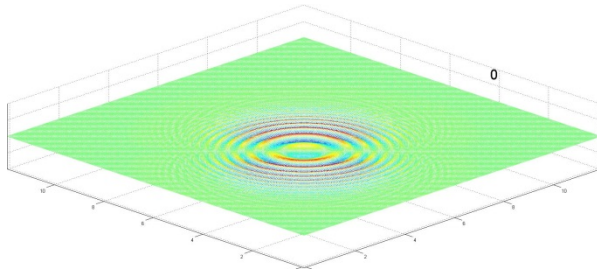


Fig. 33. 6 versions of virtual particles considered at different angles and differing by the shift of the corresponding halves in the space (see, also, Fig. 6). The particle has zero energy if the shift is zero. The particle can begin to exhibit properties of energy if there is a shift of the halves.

The magnitude of the shift is determined by the influence of the surrounding normal matter. For example, the Casimir force is not manifested if there is a very large distance between the plates. However, this shift (the effect of normal matter) occurs when the plates approach each other. At the same time the Casimir force manifests. Thus, according to the accepted hypothesis, virtual particles and the normal matter are interrelated environments. In depths of the cosmos, where there is almost no normal matter, virtual particles barely manifest themselves. Of course, if we do not take into account the presence there all possible finite resonators in which standing waves of virtual particles arise. However, only near stars and planets their influence becomes noticeable. For example, for plates having a size about a playing card and located at a distance of 0.0001 cm from each other, the force (the Casimir force) between them turned out to be approximately equal to the weight of one drop of dew. Another example, resonant forces arising inside the atoms. But apparently considering the colossal volume of vacuum in the Universe, the effect of these resonant oscillations on the energy of the entire Universe is not great. Perhaps, this resonance effect is determined the value of the cosmological constant.

The number of virtual particles grows and accordingly their influence on the space can grow during the expansion of the space. However, the influence of a normal matter decreases in parallel. As a result, we can have a complex picture of the interaction of virtual particles, the normal matter and the space. In particular, the interaction can lead to an oscillating law of the expansion of our universe (see Fig. 34).

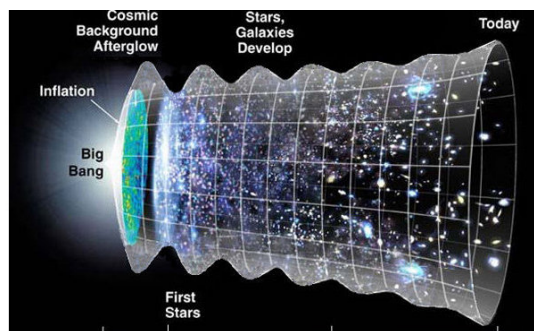


Fig. 34. The Universe itself may be oscillating through billions of years of cosmic time [71].

According to the presented hypothesis, these halves constitute an integer particle, although halves can be separated in time and space. How great these distances can be, we do not know, but they should be of the order of the Planck's constant.

At the same time, we are wondering - maybe the existence of these halves is somehow connected with the quantum entanglement and quantum coupling? It also is important that the sign of particle energy, apparently, depends on the position of the particle in space.

The question remains as a real field, for example, its maximum values are related to matter and energy. Of course there are formulas linking the field with its pressure and density (49) - (51). But our research is connected with limit cases of quantum mechanics. For example, according to the uncertainty principle, a violation of the law of conservation of energy is possible! Of course, it can be short-term, but it does not limit the magnitude of this violation. Indeed, according to the uncertainty principle, we have the following connection between the energy ΔE , the time interval Δt and the Planck constant

$$\Delta E \approx \hbar(2\pi * \Delta t)^{-1}.$$

If $\Delta t \rightarrow 0$, we have $\Delta E \rightarrow \infty$. Of course this is the limiting case near which, apparently, the principle itself does not work. Another aspect of this principle states that we can not predict, or at least accurately follow the trajectory of an individual particle (for example, an electron) in space and time. However, perhaps we are close to adjusting this principle [72]. In fact, it is possible to trace the trajectory of elementary particles with help of very weak measurements [24-26, 28-31].

16. Discussion and illustration of the main conclusions

Thus, we come to the conclusion that in the slit experiments with sole particles, the picture of processes is completely different from the Young's interference experiment (Fig. 35). In the last case the second slit provides the two sources of coherent waves. The presence of these waves and their superposition, as in the case of water surface waves, creates the interference pattern.

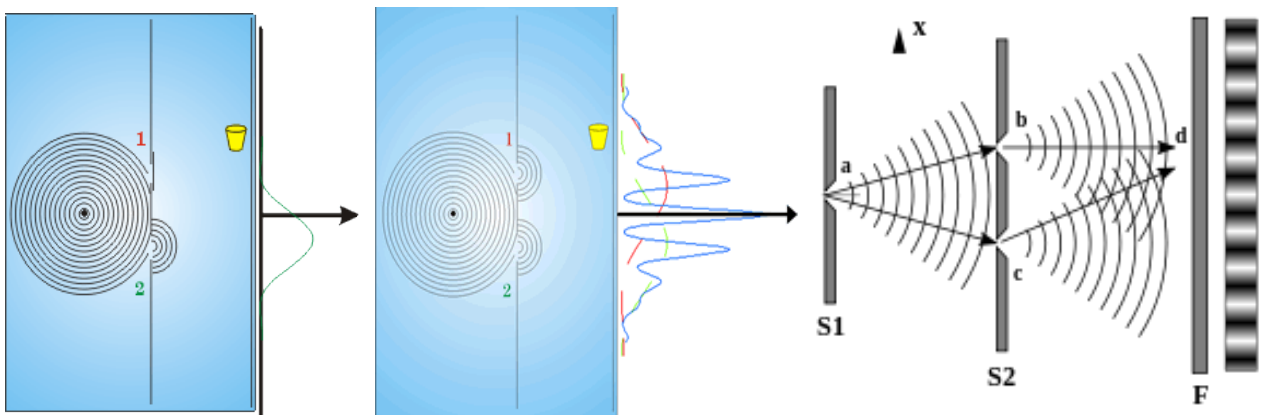


Fig. 35. Passing of the wave packet through one slit (on the left). Passing of the wave packet through two slits (center). The scheme and results of the Young's experiment (right).

In the case of experiments with sole particles, there are nothing like shown in Fig. 34 (right)! The particles passing through the slits meet there vacuum oscillations forming standing waves like those existing in the experiments of Casimir! These waves, like, for example, longitudinal waves in a spring, have compression and rarefied zones, and also nodes where the wave parameters practically do not change (see Fig. 36). We showed two instants of the wave packet motion, but did not show how the chain changed during this time. In accordance with the structure of these standing waves, the particles change their trajectories. Namely, a part of them easily passes through the rarefied zones, while the others pass through the zones of compression. In particular, some of these particles may not pass through compression zones, similar to the way it was in Rutherford's experiments (see Fig. 22).

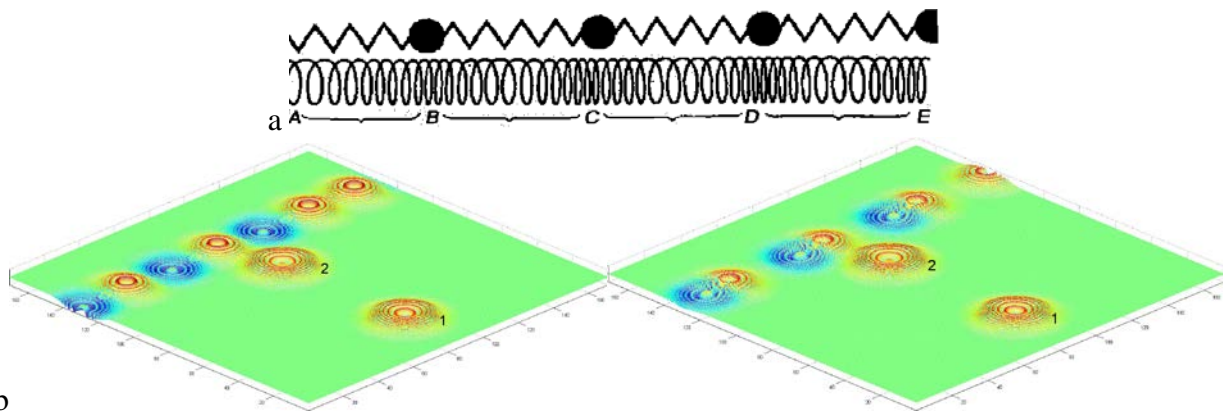


Fig. 36. a. Models of vacuum standing waves in slits. In the up chain the compression zones are modelling as atomic nuclei in the Rutherford's experiments. In the down chain the compression zones are modelling as the condensations of the spring. b. Two rough schemes of the beginning of the penetration of a wave packet (instants 1 and 2) into a chain of the wave packets simulating the vacuum standing wave in the slit (the red is the positive amplitude, the blue is the negative amplitude).

In particular, in the Rutherford's experiments, the atomic nucleus zone was fixed on the screen only as a shadow. Similarly, in slit experiments, the compression zones of vacuum particles are fixed by detectors as shadows (Fig. 37). However, the detectors fix also the light bands. But these light bands and shadows are not interference fringes! (Figs. 28 and 37)

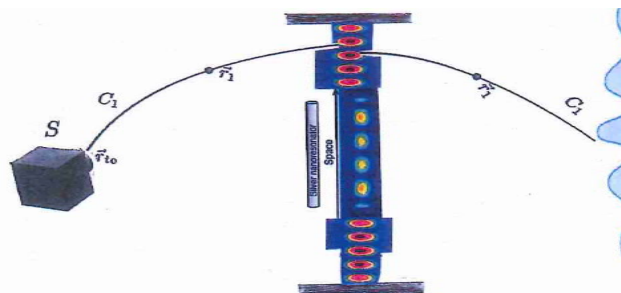


Fig. 37. The appearance of bright (light) bands and shadows in thought DSE experiment where only one slit is illuminated.

Thus, the appearance of these bands and shadows, in principle, is not related to the presence of the second slit! Of course, the presence of the second slit can enhance the effect, since it leads to the appearance of the interslit material. This material, under some additional conditions, forms a resonator. In particular, there resonant plasma oscillations (plasmons) arise. Under their influence, as experiments show [36], vacuum standing waves in the slits are amplified and synchronized.

As a result of the appearance of resonant oscillations of the plasmons, the vacuum fluctuations in the slits become coherent (this remind the appearance of coherent light waves due to the slits in the Young's experiment (Fig. 35)), and their amplitude is intensified strongly. As a result, the interaction of the passing particles with the vacuum standing waves increases and the resulting difference of light bands and shadows become so bright that they are fixed by the detectors.

We repeat, in general, the fringes arise in the case of the single slit, if there are conditions for the appearance of sufficiently strong quantum vacuum oscillations. The latter can be excited artificially as in the case of the resonantly excited Casimir force (see Fig. 19).

Thought experiments were presented to emphasize and illustrate these possibilities (see Figs. 23-29 and 37). Now let us illustrate the above by additional calculations.

17. Modeling the passage of wave packets through a slit (the interaction of vacuum waves with wave packets)

Despite its long legacy, the double-slit experiment remains the subject of researches. The analogy with epy Young's experiment turned out to be erroneous, in our opinion. On the other side from the very beginning, overly simplified mathematical models are used. In particular, the approach based on the Schrodinger equation and the superposition principle is not comprehensive. In our opinion, it is necessary to take into account the boundary conditions, the nonlinearity of the wave processes, and the possibility of resonances.

Thus, the essence of the physical processes taking place in the experiment excludes the use of the superposition principle. To some extent, we discussed these additional aspects in this publication. Of course, we raised more questions than gave answers. However, the developed theory makes it possible to approach some understanding of certain aspects of the wave processes taking place in the slits during the penetration a passing particle (see Figs. 38 and 39). We showed two instants of the wave packet motion, but did not show how the virtual particles changed during this time.

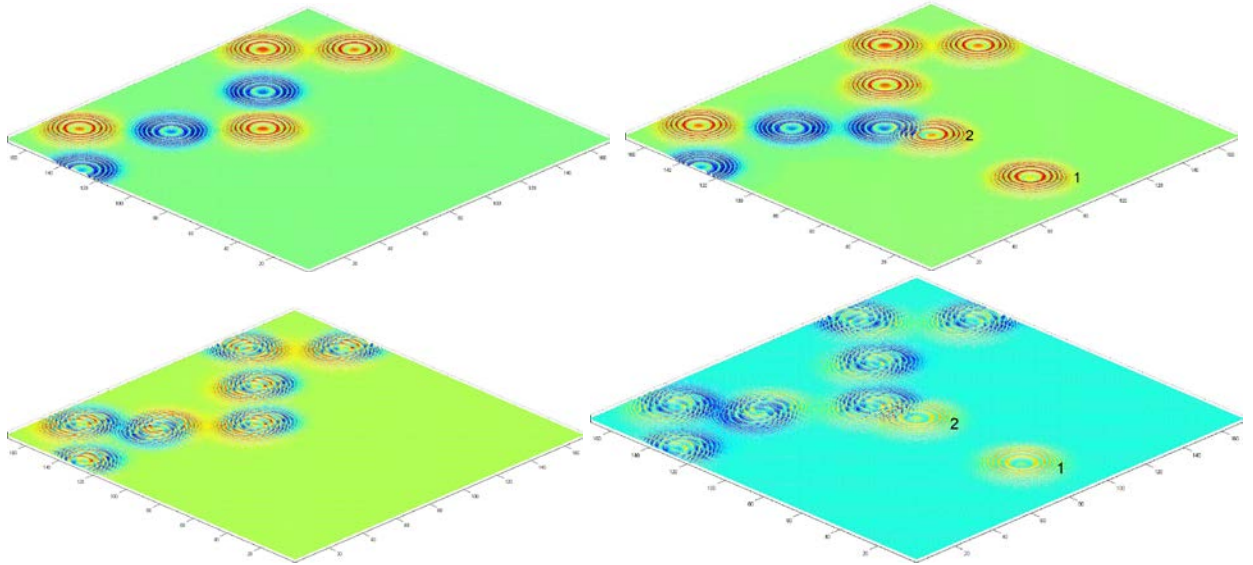


Fig. 38. Schemes illustrating the penetration of a passing particle into the slit. A standing wave from virtual particles (initial state, left). Two instants of the approach of a wave packet to the standing wave (right). Upper. The scheme using both positive and negative virtual particles. Bottom. The scheme using only neutral virtual particles.

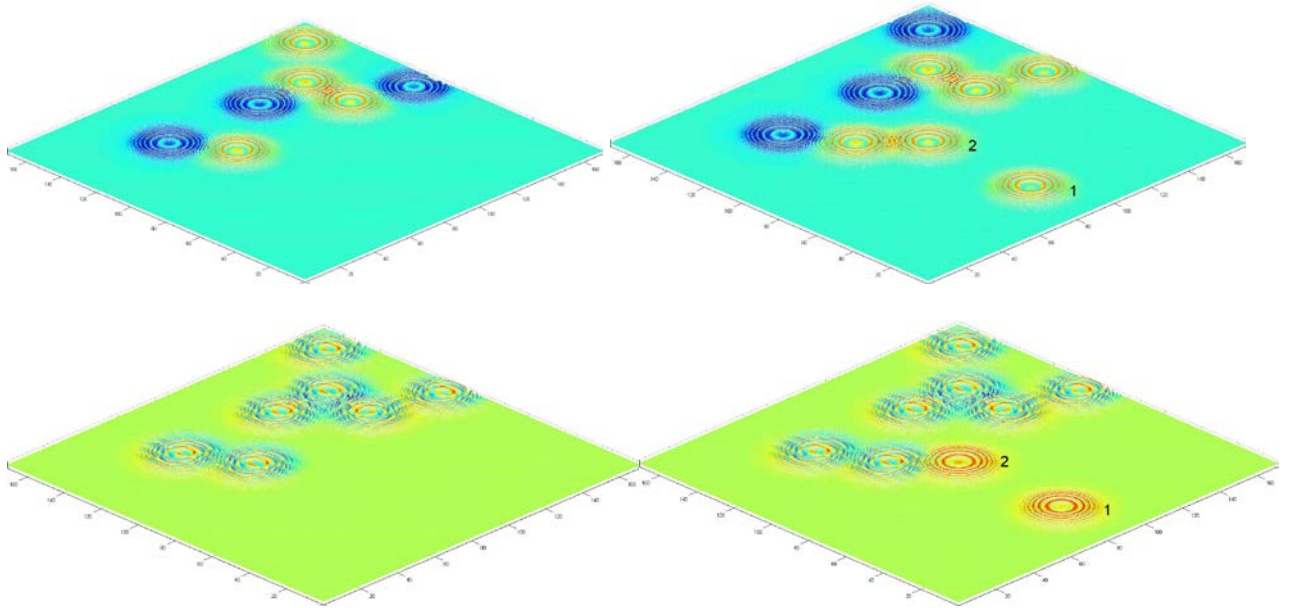


Fig. 39. Schemes illustrating the penetration of a passing particle into the slit. A chaotic located virtual particles (initial state, left, positive and negative particles). Two instants of the approach of a wave packet to the chaotic located virtual particles (right). Upper. The scheme using both positive and negative virtual particles. Bottom. The scheme using only neutral virtual particles.

We have considered different cases in Figs. 36, 38 and 39. Figs. 36 and 38 do not exclude the passage of the particles through the slit and the formation of bands resembling the interference like it was shown in

Figs. 37. These are the cases when there is a synchronizing and resonant effect of the interslit material. Fig. 39 shows the case when there is not the noted above influence. This is the case when gold-like metal between the slits absents or the geometric dimensions of the experimental setup exclude the required resonant phenomena.

We studied in the sections 14-17 the solution (45) which determines real values of the wave amplitude for all values λ . The solutions (37) and (43) describe also wave packets, but amplitudes of them may be imaginary. The study of these wave packets may interest the DSE too.

18. Final comments

Of course, many researchers get used to the oddities of the quantum world, especially researchers working there for many years! But still it is very, very strange world [73]! And over the years this situation is not simplified. This is well illustrated by slit experiments.

So maybe the oddities of the quantum world will clear up if we understand the results of these experiments?

With the beginning our study was intended to be limited to reviewing of the recent publications devoted to the slit experiments and analysing of results of them. But the very logic of a scientific research led us beyond this framework, in particular, required the analysis of quantum fluctuations in vacuum. That is, the logic forced us, in order to explain the phenomena studied, to draw the information from the most rapidly developing field of theoretical physics. What we found out was, in our opinion, enough to confirm our initial understanding of the results of the experiments. This understanding is based on resonant and synchronous oscillations of the elements of the experimental equipment and the presence of vacuum virtual fluctuations in the slits.

To illustrate this understanding, the approximate nonlinear theory of quantum waves is developed based on the solution of the scalar field equation. We use the simplest nonlinear field model (the φ^4 - theory). However, we emphasize that the approach used lies in the mainstream of the fundamental approach to understanding our Universe based on field equations, in particular on the nonlinear scalar field [62, 64, 66, 74]. Generally speaking, this approach might allow us to investigate both the interaction of many fields, and the origin and evolution of elementary particles. We decided to use the Klein-Gordon equation instead of the Schrödinger equation when we tried to analysis mathematically the results of the experiments. According to the analysis, the nonlinearity and resonances are important elements that open a new understanding of the results of the experiments. We abandoned the notion of probability in this understanding.

Of course, our goal was not to describe the huge amount of physical experimental data in a field of quantum phenomena, but to illustrate our understanding of the experiments examined.

The traditional mathematics of wave quantum mechanics is very successful. It is so successful that there is no understanding why it is so successful. We are trying to introduce the understanding of why quantum mechanics is so successful based on experiments and proposed models that take into account the nonlinearity of quantum processes and resonant nature many of them.

In doing so, we entered in a very complex area of analysis of the virtual particles of the vacuum. How true are the equations used by us? Even if they are applicable, how much do their coefficients randomly depend on the Heisenberg Uncertainty Principle?

So we came to the typical situation in dialectics of understanding - trying to understand the puzzle of the slit experiments, we came to the puzzles of the world of quantum vacuum fluctuations!

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